Giant K factors

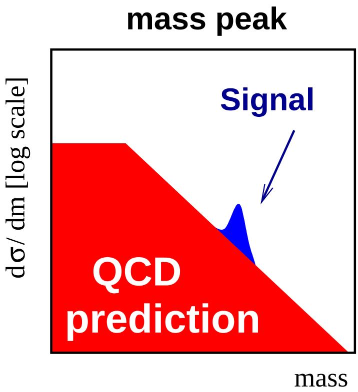
Gavin Salam

CERN, Princeton & LPTHE/CNRS (Paris)

Work performed with Mathieu Rubin and Sebastian Sapeta, arXiv:1006.2144

UMD/Hopkins joint seminar
Maryland, 1 December 2010

What kinds of searches at colliders?



New resonance (e.g. Z') where you see all decay products and reconstruct an invariant mass

QCD may:

- swamp signal
- smear signal

leptonic case easy; hadronic case harder

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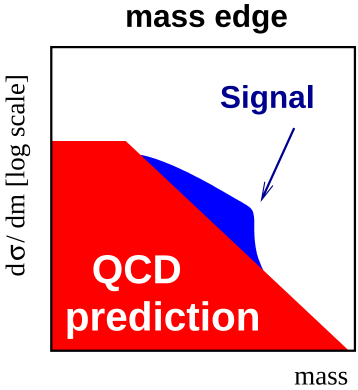

New resonance (e.g. R-parity conserving SUSY), where undetected new stable particle escapes detection.

Reconstruct only *part* of an invariant mass $\rightarrow$ kinematic edge.

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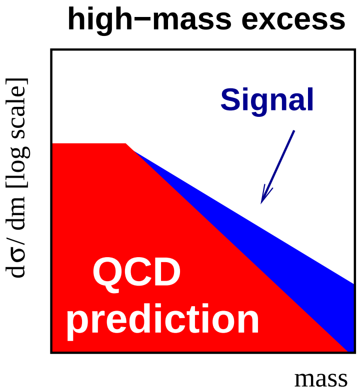
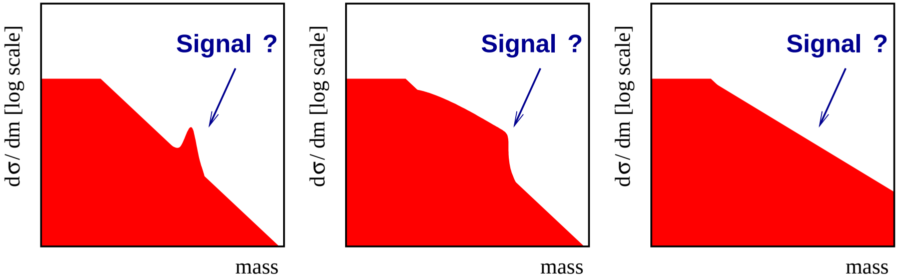


Unreconstructed SUSY cascade. Study *effective* mass (sum of all transverse momenta).

Broad excess at high mass scales.

Knowledge of backgrounds is crucial is declaring discovery.

QCD is *one way* of getting handle on background.


dσ/ dm [log scale]
Signal ?
mass
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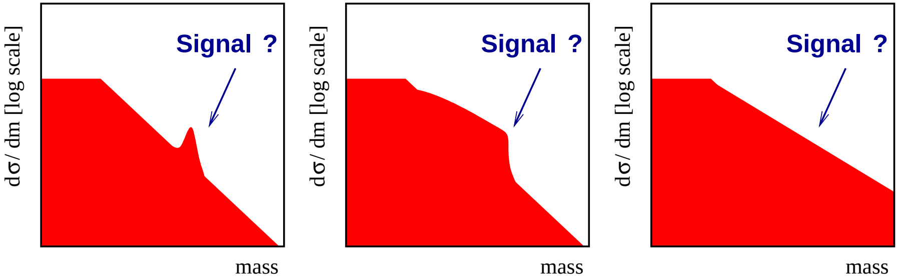
THIS
TALK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SUSY example: gluino pair production

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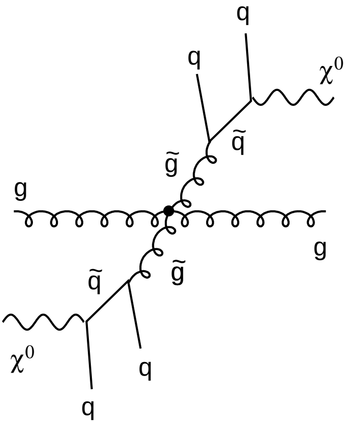

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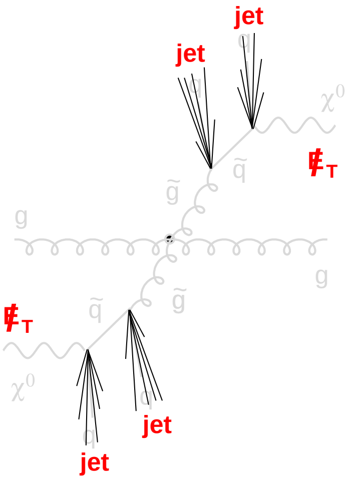

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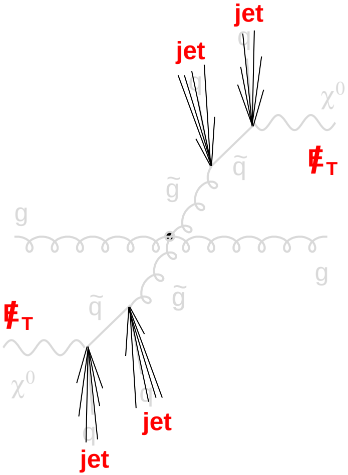

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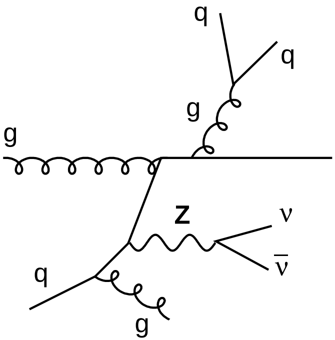

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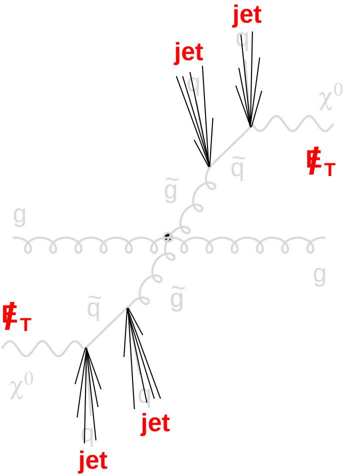

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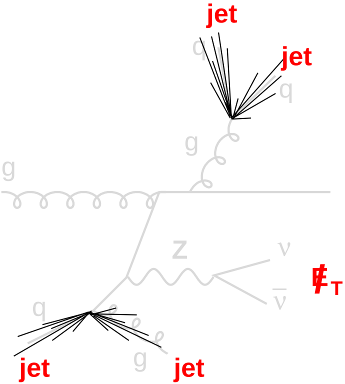

SUSY searches: what excesses?

Atlas selection [all hadronic]

- no lepton
- MET > 100 GeV
- 1^{st},2^{nd} jet > 100 GeV
- 3^{rd},4^{th} jet > 50 GeV
- MET / m_{eff} > 20%

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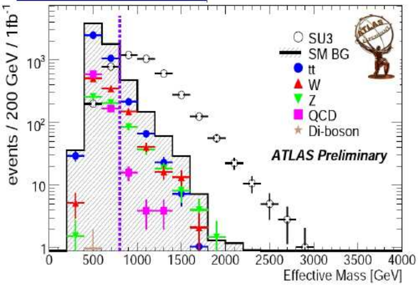
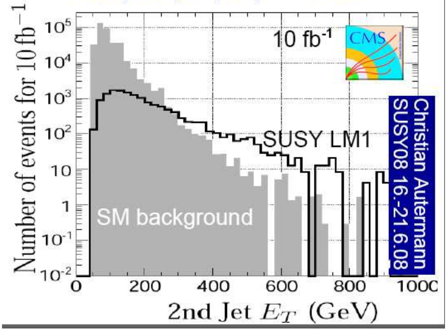


CMS selection [leptonic incl.]

(optimized for 10fb^{-1}, using genetic algorithm)

- 1 muon pT > 30 GeV
- MET > 130 GeV
- 1^{st}, 2^{nd} jet > 440 GeV
- 3^{rd} jet > 50 GeV
- −0.95 < cos(MET,1^{st}jet)<0.3
- cos(MET,2^{nd}jet) < 0.85



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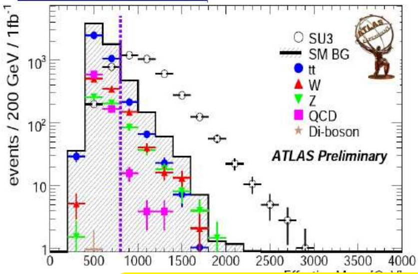

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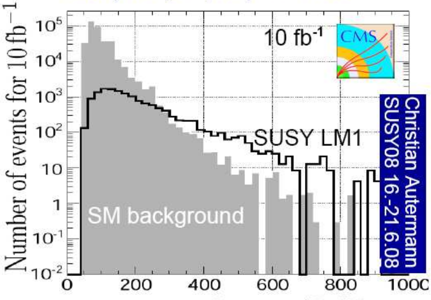

SUSY ≃ factor 5−10 excess

How accurate is perturbative QCD?

$$\sigma = c_0 + c_1\alpha_s + c_2\alpha_s^2 + \dots$$

$$\alpha_s \simeq 0.1$$

That implies LO QCD (just c_0) should be accurate to within 10%

It isn't

Rules of thumb:

LO good to within factor of 2

NLO good to within scale uncertainty

This talk is about an example where these rules fail spectacularly, the lessons we learn, and the solutions we can apply.

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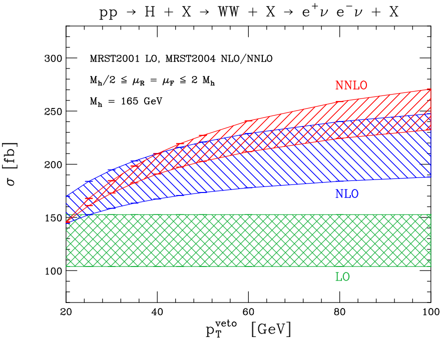

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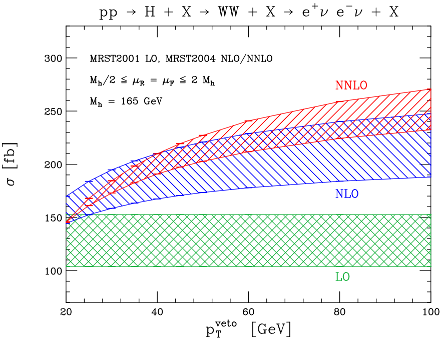

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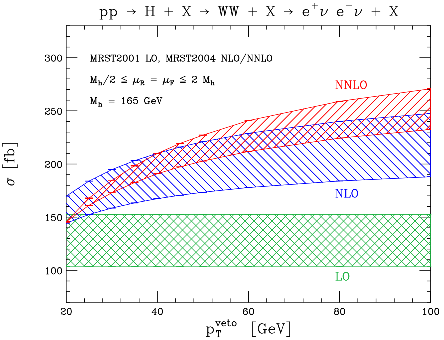

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Control samples

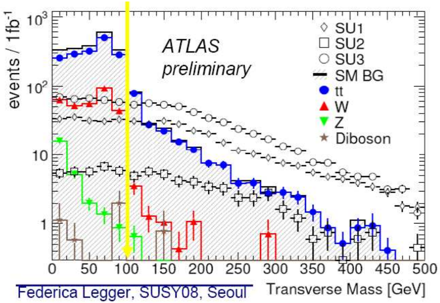
We don't always have NLO for the background (e.g. Z+4 jets, a $2 \to 5$ process).

Though amazing recent progress
$2 \to 4$: Blackhat, Rocket, Helac-NLO, BDDP
$2 \to 5$ (W+4j): Blackhat

Must then rely on LO (matched with parton showers). How does one verify it?

Common procedure (roughly):

- Get control sample at low p_t
- SUSY should be small(er) contamination there
- Once validated, trust LO prediction at high-p_t



Is this safe?

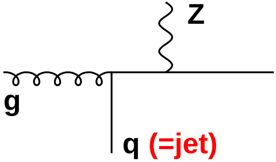
A conservative QCD theory point of view:

It's hard to be sure: since we can't (yet) calculate Z+4 jets beyond LO.

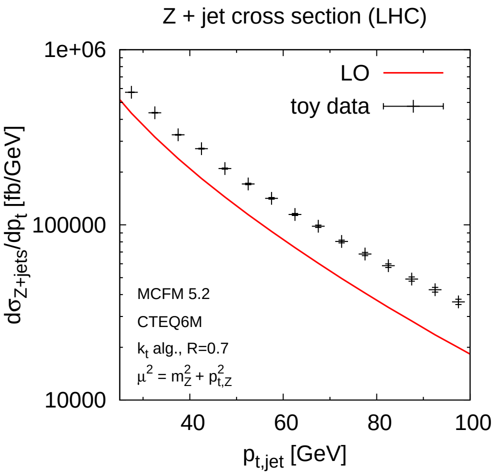
But we would tend to think it is safe, as long as control data are within usual factor of two of LO prediction

Illustrate issues with toy example: Z+jet production

- It's known to NLO — and a candidate for "first" $2 \to 2$ NNLO
  $\sim e^+e^- \to \gamma^*/Z \to 3$ jets, NNLO: Gehrman et al '08, Weinzierl '08
- But let's pretend we only know it to LO, and look at the p_t distribution of the hardest jet (no other cuts — keep it simple)



Toy data, control sample



stage 1: get control sample

Check LO v. data at low p_t

- normalisation off by factor 1.5 (consistent with expectations)

  So renormalise LO by K-fact

- shape OKish

  Don't be too fussy: SUSY could bias higher p_t

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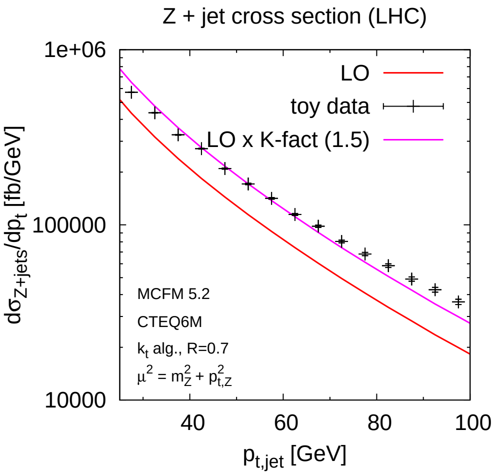

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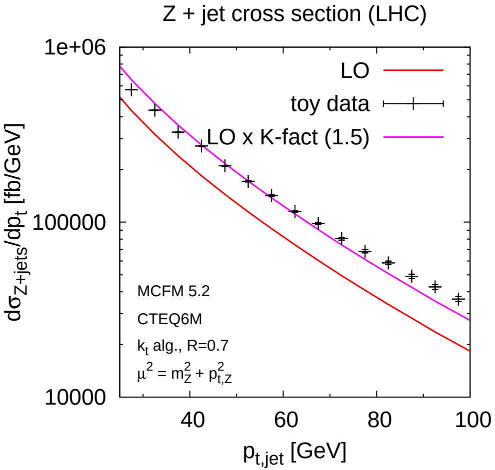

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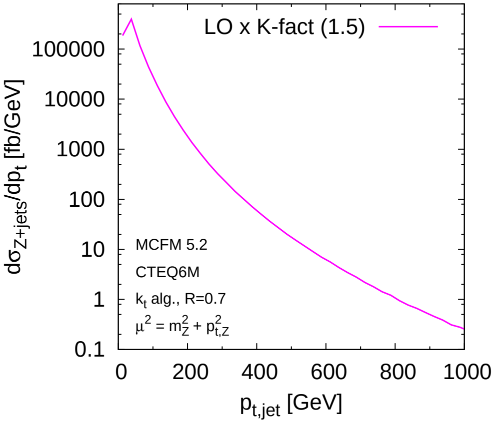
Toy data, high p_t

Z + jet cross section (LHC)



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- good agreement at low p_t, by construction
- excess of factor ~ 10 at high p_t
- check scale dependence of LO

[NB: not always done except e.g. Alwall et al. 0706.2569]

still big excess

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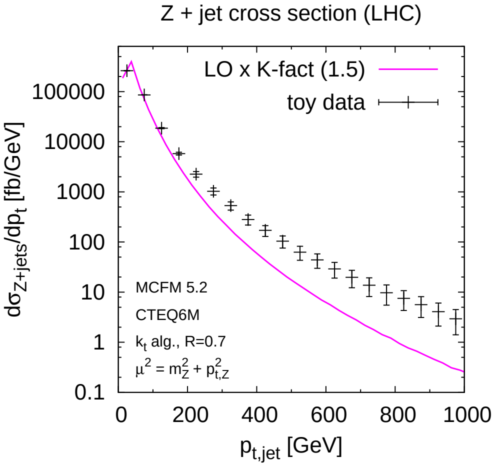

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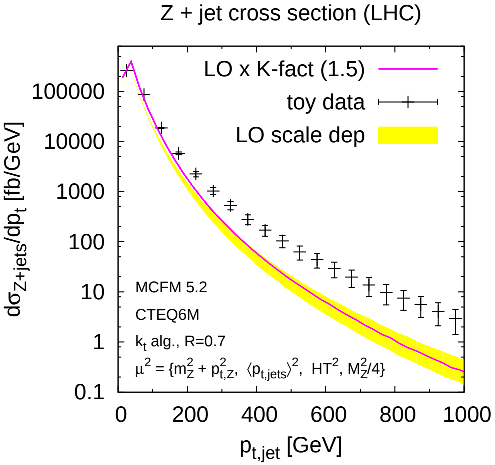

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What's in the toy data?

Is it:

- QCD + extra signal?
- just QCD? But then where does a K-factor of 10 come from?

Here it's just a toy illustration. Next year it may be for real:

- Do Nature / Science / PRL accept the paper?

  Discovery of New Physics at the TeV scale

  We report a 5.7σ excess in MET + jets production that is consistent with a signal of new physics . . .

- Do we proceed immediately with a linear collider?

  It'll take 10–15 years to build; the sooner we start the better

- At what energy? It would be a shame to be locked in to the wrong energy. . .

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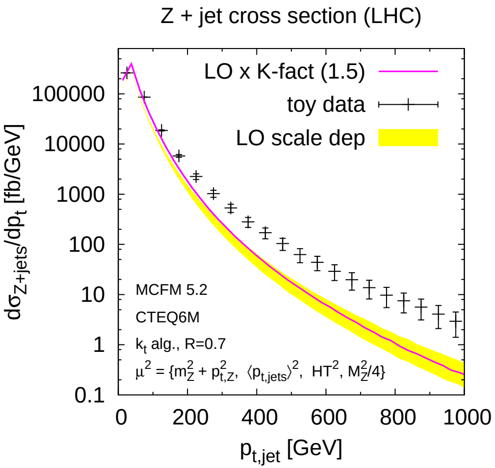

Unlike for SUSY multi-jet searches, in the Z+jet case we do have NLO.

Once NLO is included the excess disappears

The "toy data" were just the upper edge of the NLO band

Example based on background work for Butterworth, Davison, Rubin & GPS '08

Related observations also by Bauer & Lange '09; Denner, Dittmaier, Kasprzik & Muck '09

Hold on a second: how does QCD give a K-factor $\mathcal{O}(5-10)$?

NB: DYRAD, MCFM consistent

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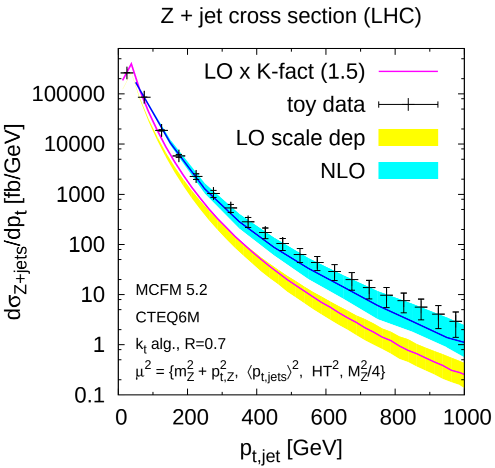

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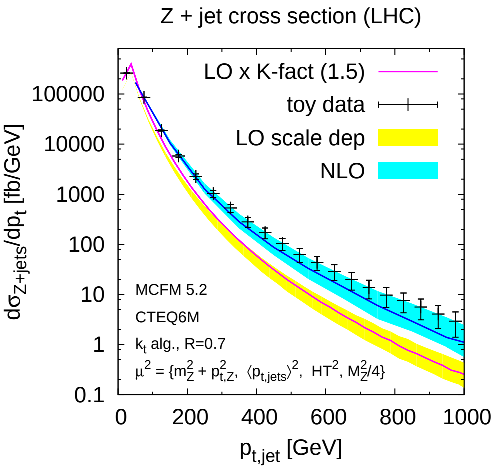

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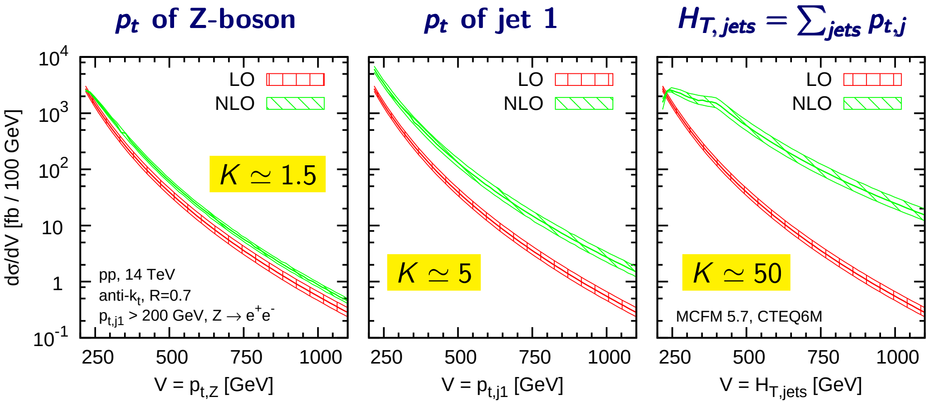
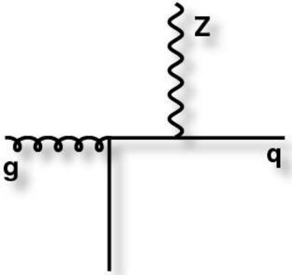
What about other observables?



"Giant *K*-factors"

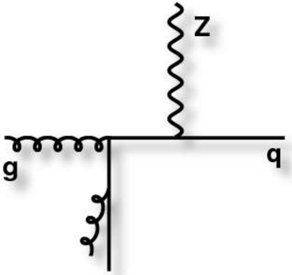
Why the large K-factor?

Leading Order



$\alpha_s \alpha_{EW}$

Next-to-Leading Order



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LHC will probe scales well above EW scale, $\sqrt{s} \gg M_Z$.
EW bosons are **light**.
New logarithmically enhanced topologies appear.

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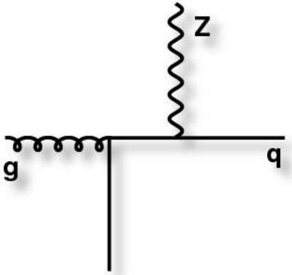

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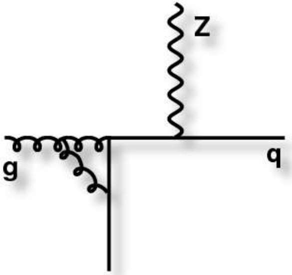

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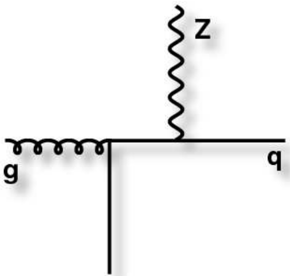

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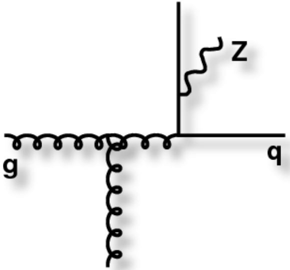

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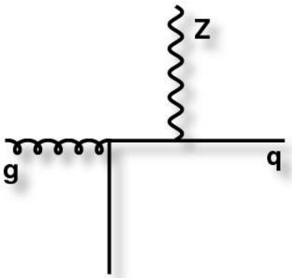

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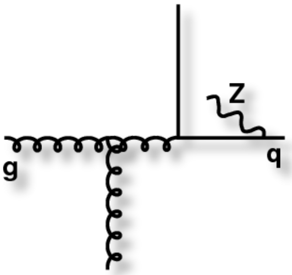

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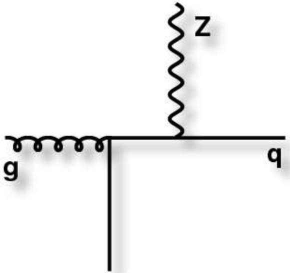

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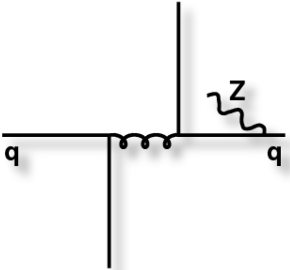

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Is this example not a little contrived?
After all, experiments would surely notice unexpected event topology such as that here.

We actually first saw the problem in a more complex process: $Wb\bar{b}$ as a background to boosted Higgs searches (with "wrong" cuts). *The more complicated the process, the trickier the diagnosis of the problem.*

It's enough to get this wrong once, leading to "unwarranted" press-releases and major subsequent embarassment.

We'll look at two questions:

1) In day-to-day experimental work, can standard techniques help reduce the likelihood of getting caught out by this type of problem? (Even without a NLO calculation)

2) What good is perturbative QCD if the "perturbative" convergence is so poor? What happens at the next order?

Standard technique: Matrix elements + parton showers

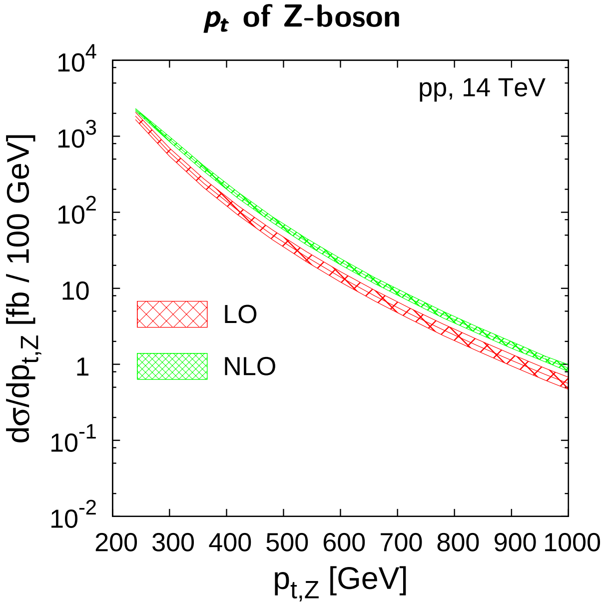
A standard predictive approach in Z+(multi)jet processes, widely used by experiments:

- take Alpgen/Madgraph/Sherpa to generate samples of Z, Z+1-jet, Z+2-jet, etc. tree-level events
  with some cuts to separate samples
- shower them with Herwig/Pythia/Sherpa, including some prescription to combine different topologies sensibly
  MLM matching, CKKW matching
  avoid double counting, approximate "Sudakovs"

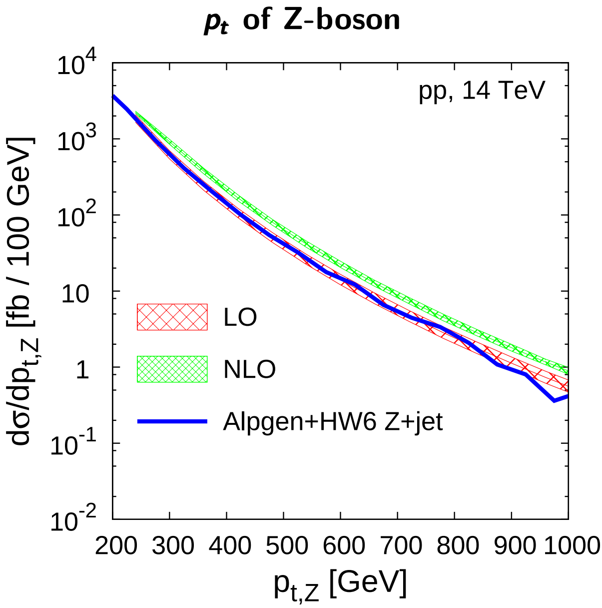
Does it work here?

Try showering Z + jet and Z + 2-jets samples

Testing Alpgen + Herwig + MLM Matching



Testing Alpgen + Herwig + MLM Matching



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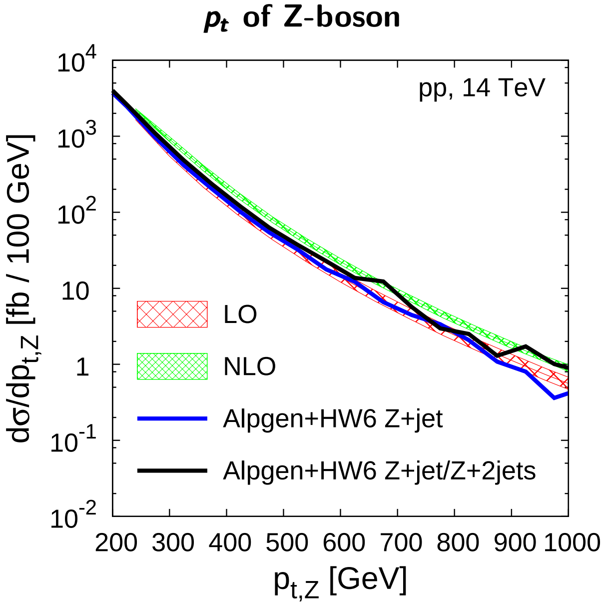
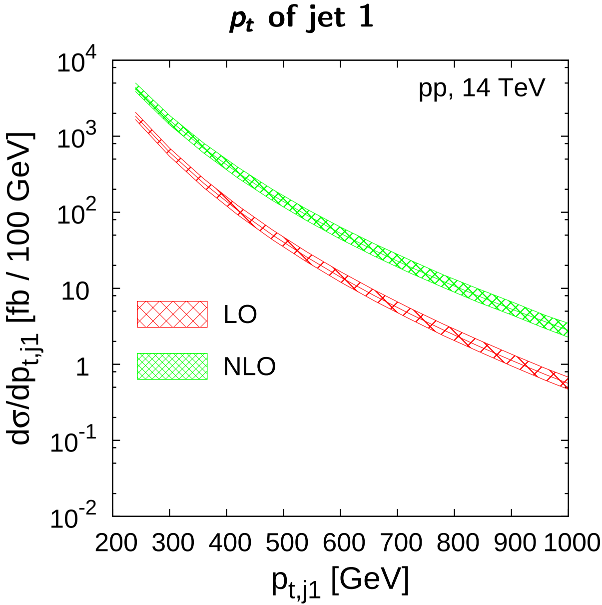


All predictions similar and stable

Testing Alpgen + Herwig + MLM Matching



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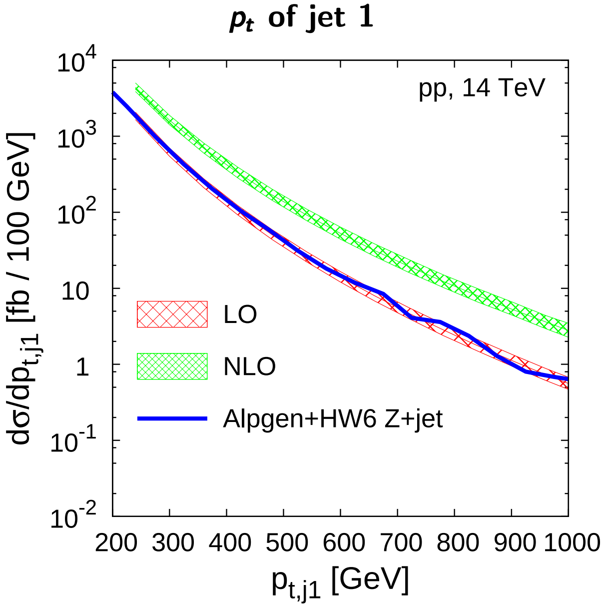

Showered Z+j $\simeq$ LO

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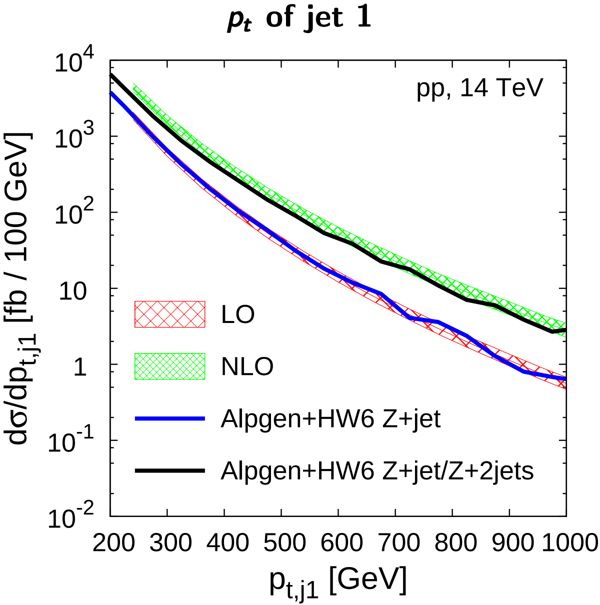
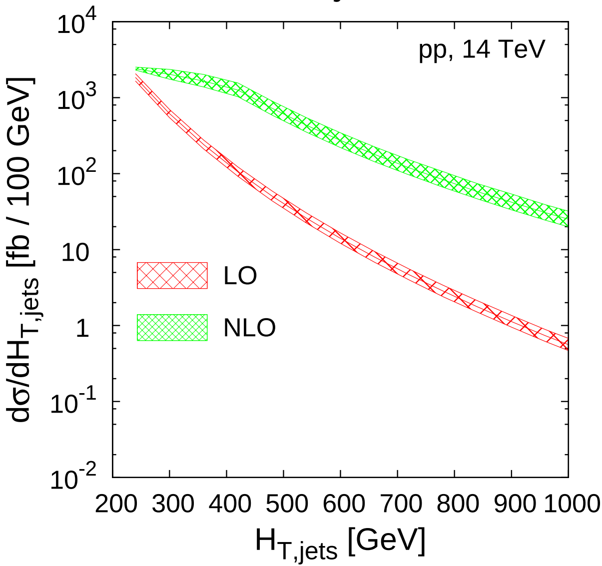


Showered Z+j $\simeq$ LO

Showered Z+j/Z+2j $\simeq$ NLO

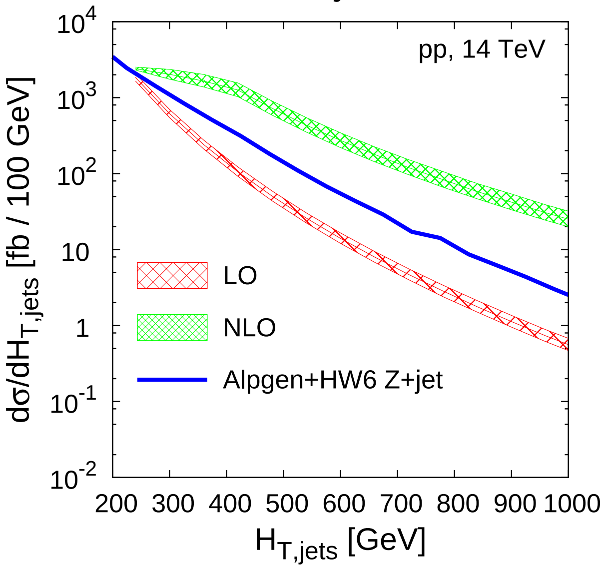
Testing Alpgen + Herwig + MLM Matching

$$H_{T,jets} = \sum_{jets} p_{t,j}$$



Testing Alpgen + Herwig + MLM Matching

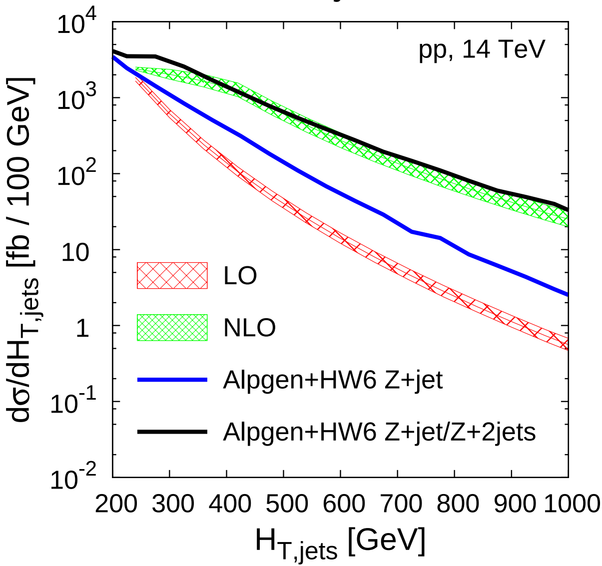
$H_{T,jets} = \sum_{jets} p_{t,j}$



Showered Z+j $\neq$ LO

Testing Alpgen + Herwig + MLM Matching

$H_{T,jets} = \sum_{jets} p_{t,j}$



Showered Z+j $\neq$ LO

Showered Z+j/Z+2j $\simeq$ NLO

1st lesson:

If you figure out the “leading” process
[Z + jet @ LO]

and add in process with one extra jet through
MLM/CKKW matching.
[i.e. include Z + 2 jets @ LO]

impact of new large topologies will often show up
This might be called “Pauper’s NLO”

It’s reassuring that **suitable use** of Alpgen catches this problem.

[Is it always being used “suitably” (with extra jet)? That’s far from clear.
What happens with heavy flavour? Also far from clear]

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Now suppose we want to be more ambitious, and get **accurate** predictions for such processes, i.e. "NLO quality"

When NLO Z+j is dominated by a new subprocess (Z+2j), it's effectively no better than LO for the new subprocess (Z+2j).

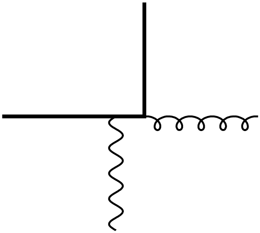
We really want somehow to include NLO for Z+2j.

Can we just merge Z+jet@NLO with Z+2jet@NLO?

To understand how, we're going to

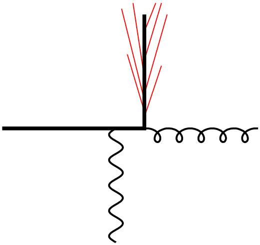
1. look at how MLM matching combines LO Z+jet and LO Z+2jets
2. simplify it (strip off the parton shower) $\rightarrow$ **LoopSim**
3. extend LoopSim to deal with NLO cases

Add Z+1jet, Z+2jet + shower



Z+parton

Add Z+1jet, Z+2jet + shower



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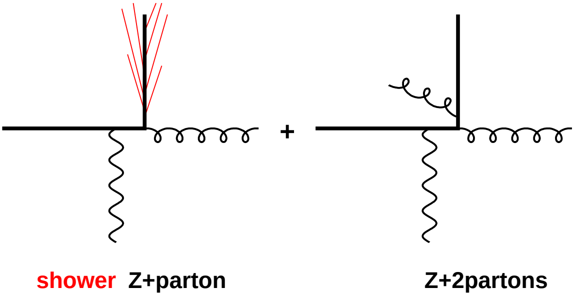

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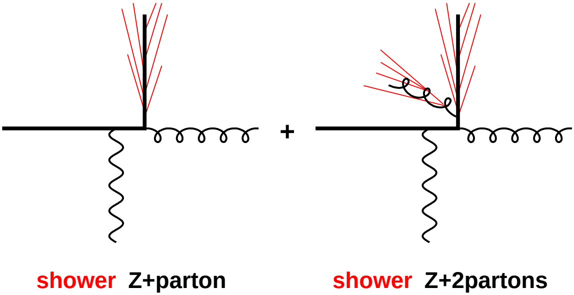
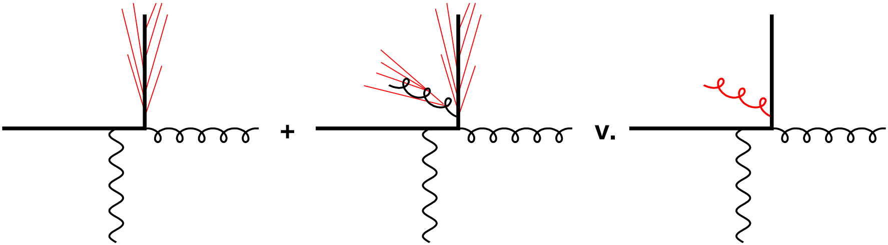
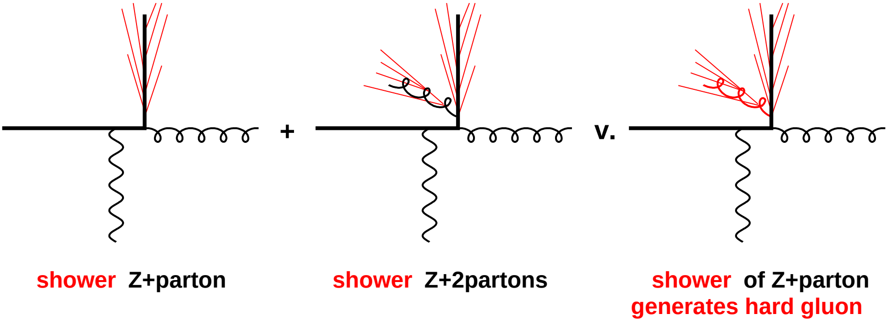


Add Z+1jet, Z+2jet + shower

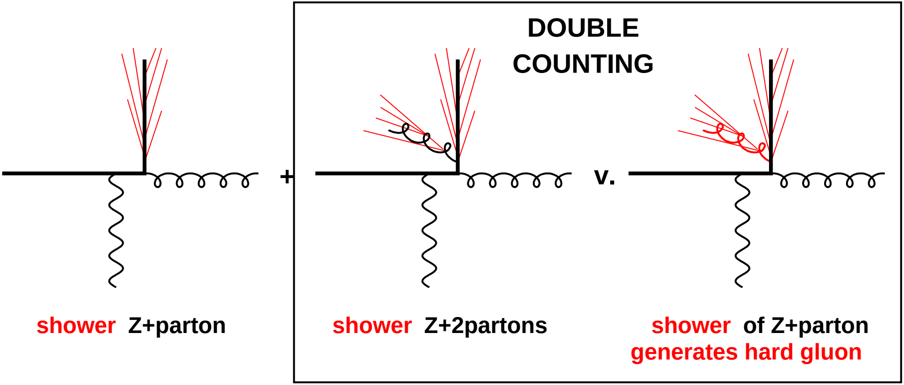


shower Z+parton **shower Z+2partons** **shower of Z+parton generates hard gluon**

Add Z+1jet, Z+2jet + shower



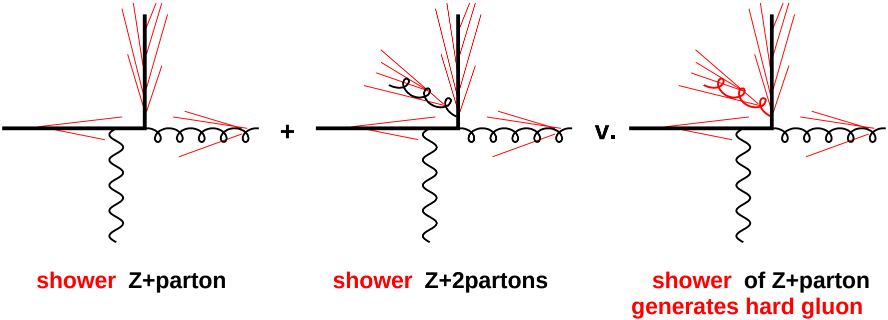
Add Z+1jet, Z+2jet + shower



Z + parton implicitly includes part of Z + 2 partons

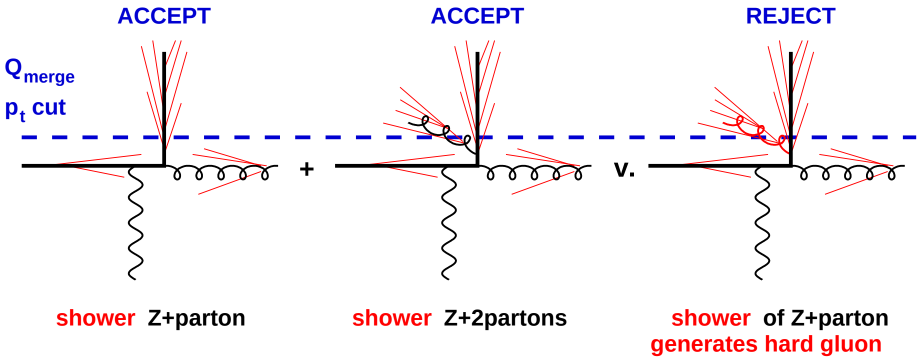
It's just that the 2nd parton isn't always explicitly "visible"

cartoon of MLM merging of Z+j and Z+2j



- MLM merging relies on parton shower to help figure out what fraction of Z + parton is really Z + 2 partons.
- Our aim is to do that without the parton shower

cartoon of MLM merging of Z+j and Z+2j



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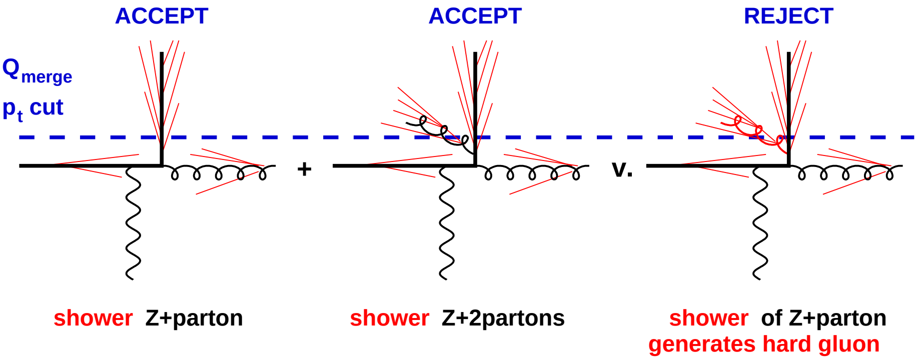

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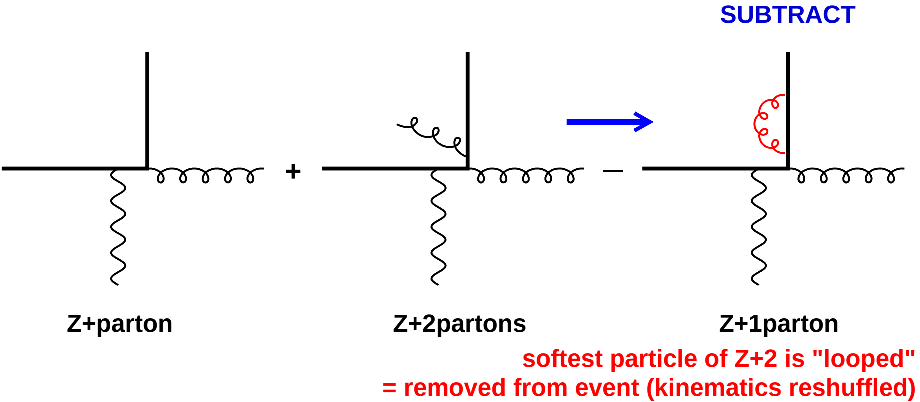
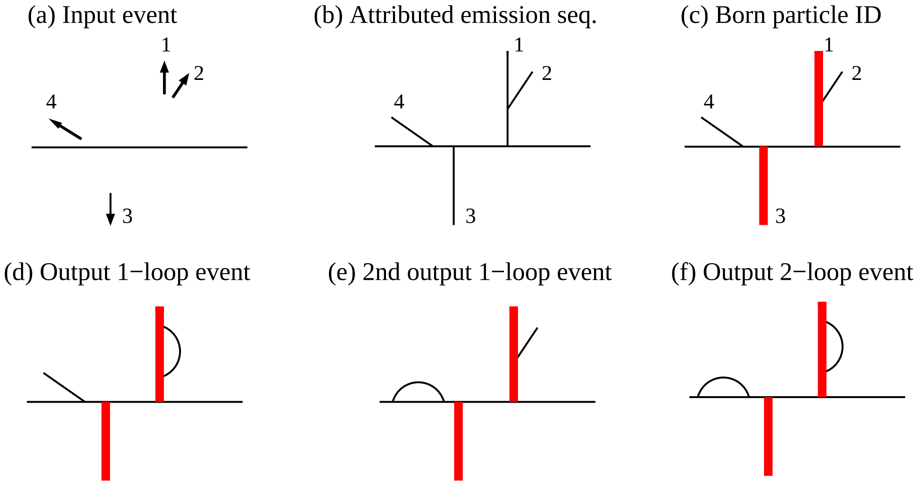
cartoon of the LoopSim idea



- For every $Z + 2$ parton $(2 \to 3)$ event, figure out what what $2 \to 2$ event it would really have come from
  "Loop" the softest parton
  [Don't actually explicitly calculate any loop diagrams: simulate the loops]
- Subtract that $2 \to 2$ event
  Unlike MLM, no cutoffs on $2 \to 3$ events
  If done properly, divergences will cancel

cartoon of LoopSim "looping" procedure



- Use jet algorithm to assign a branching structure to event à la CKKW
- The particles that are softest are the ones that will be "looped"

Define operators:

$$U_\ell(\text{event E}) \equiv \text{all simulated } \ell\text{-loop events from E}$$

$$U_\forall(\text{event}) \equiv \sum_{\ell=0} U_\ell(\text{event})$$

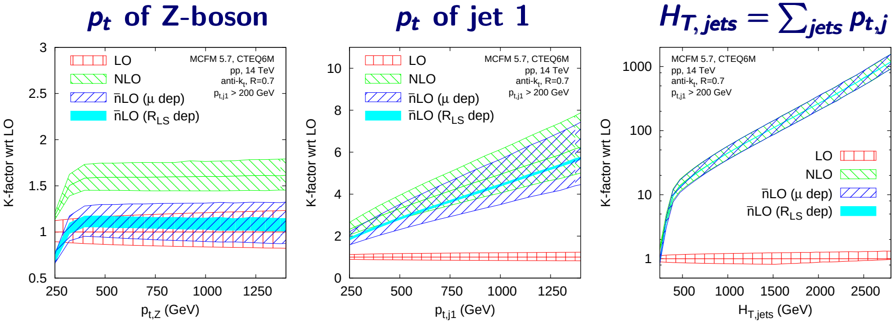
"U" stands for unitarisation (cancellation of all divergences)
sum of all diagrams (essentially) adds up to zero

Analogue of MLM Z+j combined with Z+2j is then

$$\text{Z+j@}\bar{n}\text{LO} \equiv \text{Z+j@LO} + U_\forall(\text{Z+2j@LO})$$

we use "$\bar{n}$LO" to emphasize that this is a crude approximation
to an actual NLO calculation — the exact loops are missing

$\bar{n}$LO results (K-factors, normalised to LO)



When the K-factors are large, $\bar{n}$LO agrees well with NLO

Just like MLM matching

Differences between and LoopSim and MLM/CKKW matching:

1. Does not rely on shower (✓: simple; ✗: not easily integrated with shower MCs)
2. Does not need arbitrary separation of Z+1/Z+2/etc. samples with (hard-to-choose) momentum cutoff
3. Can easily be extended beyond LO matching

$\bar{n}$NLO: merging Z+j and Z+2j, both @NLO

Just replace simulated loops with exact loops
Apply LoopSim to exact 1-loop to get (e.g.) simulated 2-loop terms

---

$E_{n,\ell} \equiv$ event with n partons and ℓ exact loops
$U_{\forall,\ell} \equiv$ operator to apply when ℓ exact loops known

$$U_{\forall,1}(E_{n,0}) = U_{\forall}(E_{n,0}) - U_{\forall}\left(U_1(E_{n,0})\right)$$
$$U_{\forall,1}(E_{n,1}) = U_{\forall}(E_{n,1})$$

$$\text{Z+j@}\bar{n}\text{NLO} = \text{Z+j@NLO} + U_{\forall,1}(\text{Z+2j@NLO}_{\text{only}})$$

Extension to NLO, NNLO, multi-leg, etc. is almost trivial in LoopSim

Not the case in methods that merge with parton showers too

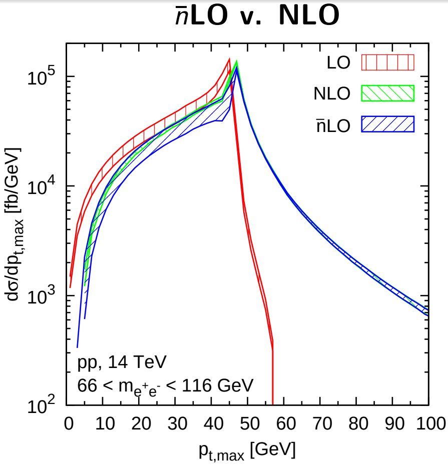
Testing NLO Merging, in 3 processes

1. Z@NLO with Z+j@NLO

2. Z+j@NLO with Z+2j@NLO

3. 2j@NLO with 3j@NLO

Validation: Drell-Yan lepton p_t, $\bar{n}$NLO v. NNLO

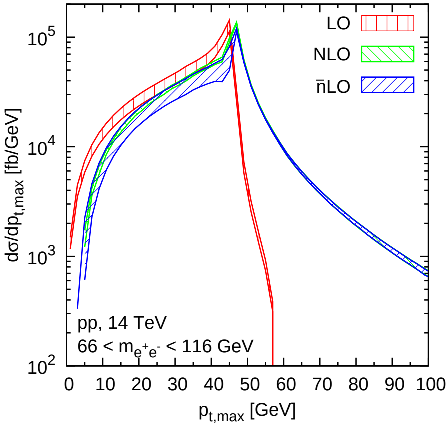


Z (i.e. DY) with Z+j from MCFM & LoopSim

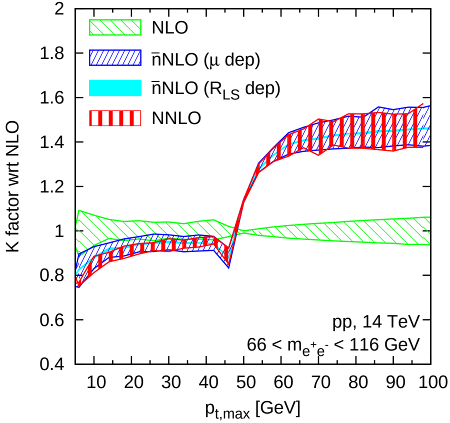
For $p_{t\,ell} \gtrsim \frac{1}{2}M_Z + \Gamma_Z$ (giant K-factor!) it had to work
For $p_{t,\ell} \lesssim \frac{1}{2}M_Z + \Gamma_Z$ it's remarkable that it still works

Validation: Drell-Yan lepton p_t, $\bar{n}$NLO v. NNLO

$\bar{n}$LO v. NLO



$\bar{n}$NLO v. NNLO



NNLO from DYNNLO, Z (i.e. DY) with Z+j from MCFM & LoopSim

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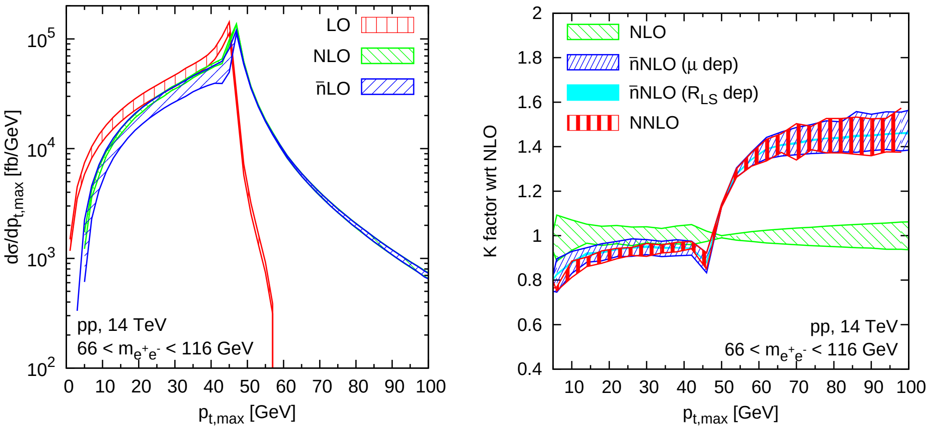
$\bar{n}$LO v. NLO | **$\bar{n}$NLO v. NNLO**



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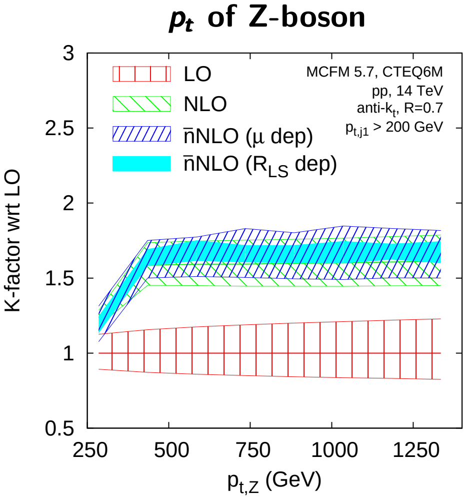
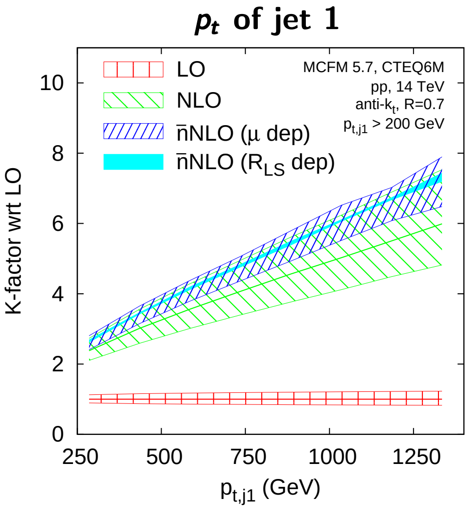
$\bar{n}$NLO for Z+j observables



- p_{tZ} distribution didn't have giant K-factors.
- $\bar{n}$NLO brings no benefit

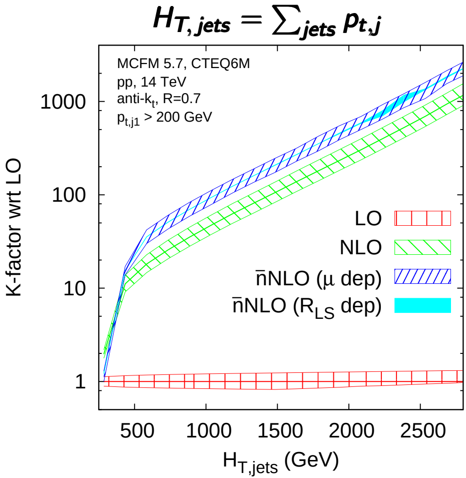
  To get improvement you would need exact 2-loop terms

n̄NLO for Z+j observables



- p_{tj} distribution seems to converge at n̄NLO
- scale uncertainties reduced by $\sim$ factor 2

$\bar{n}$NLO for Z+j observables



- Signficiant further enhancement for $H_{T,jets}$
- $\bar{n}$NLO brings clear message:

  $H_{T,jets}$ is not a good observable!

What's the problem with H_T?

H_T (effective mass) type observables are widely used in searches

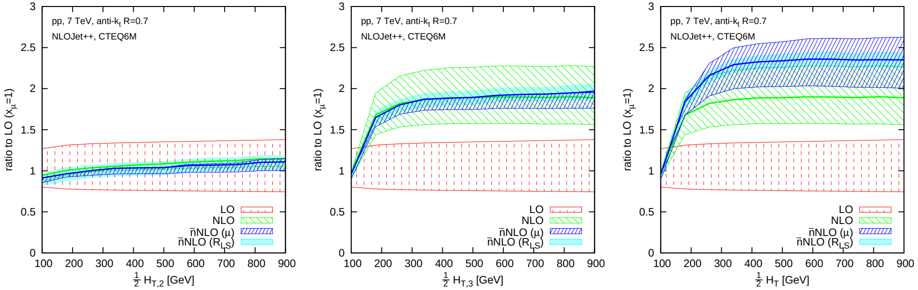
- H_T has a steeply falling distribution (like p_{tj}, p_{tZ})
- At each order (NLO, NNLO), an extra (soft) jet contributes to the H_T sum — e.g. from ISR
- That shifts H_T up, which translates to a substantial increase in the cross section

We can test this hypothesis for plain jet events, using a truncated sum,

$$H_{T,n} = \sum_{i=1}^{n} p_{t,\text{jet}\,i}$$

$H_{T,n}$ in (di)jet events

$H_{T,2}$ $H_{T,3}$ $H_{T,\infty}$



A clear message:
for a process with n objects at lowest order, use $H_{T,n}$

Do you know what gets used in your experiment's searches?
Many writers of ATLAS SUSY proceedings didn't...

Some take-home messages

Be aware that giant K-factors exist

Always look one order beyond the leading order, for example with MLM/CKKW matching

New tool to get good predictions in such cases: **LoopSim**

Basically an "operator" to generate approximations to unknown loops

Combine Z+j@NLO, Z+2j@NLO to get "$\bar{n}$NLO" Z+jet

It sometimes works even beyond "giant-K-factor" regions

Watch out for H_T

Even for simple processes, it converges very poorly unless you define it carefully (limit number of objects in sum)