

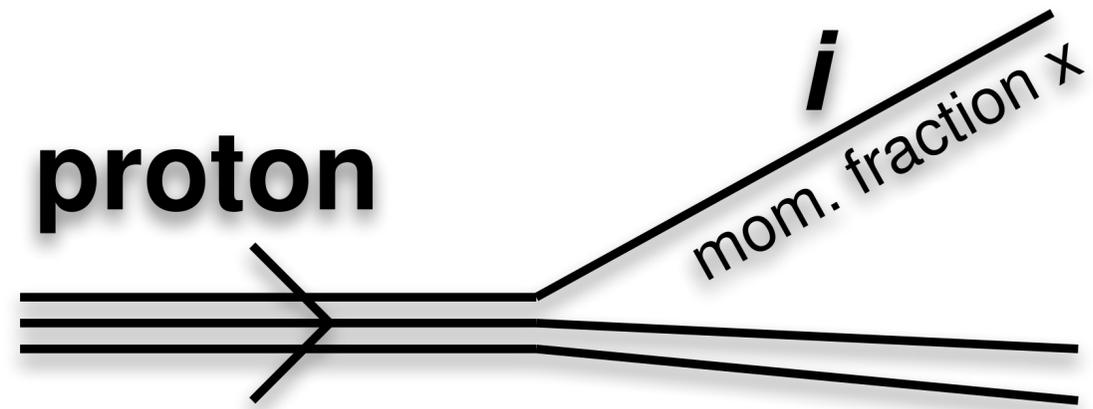
# How bright is the proton?

*Determining the photon content of the proton*

Gavin Salam (CERN)  
with Manohar, Nason & Zanderighi  
1607.04266 and work in progress

Moriond EW, March 2017, La Thuile

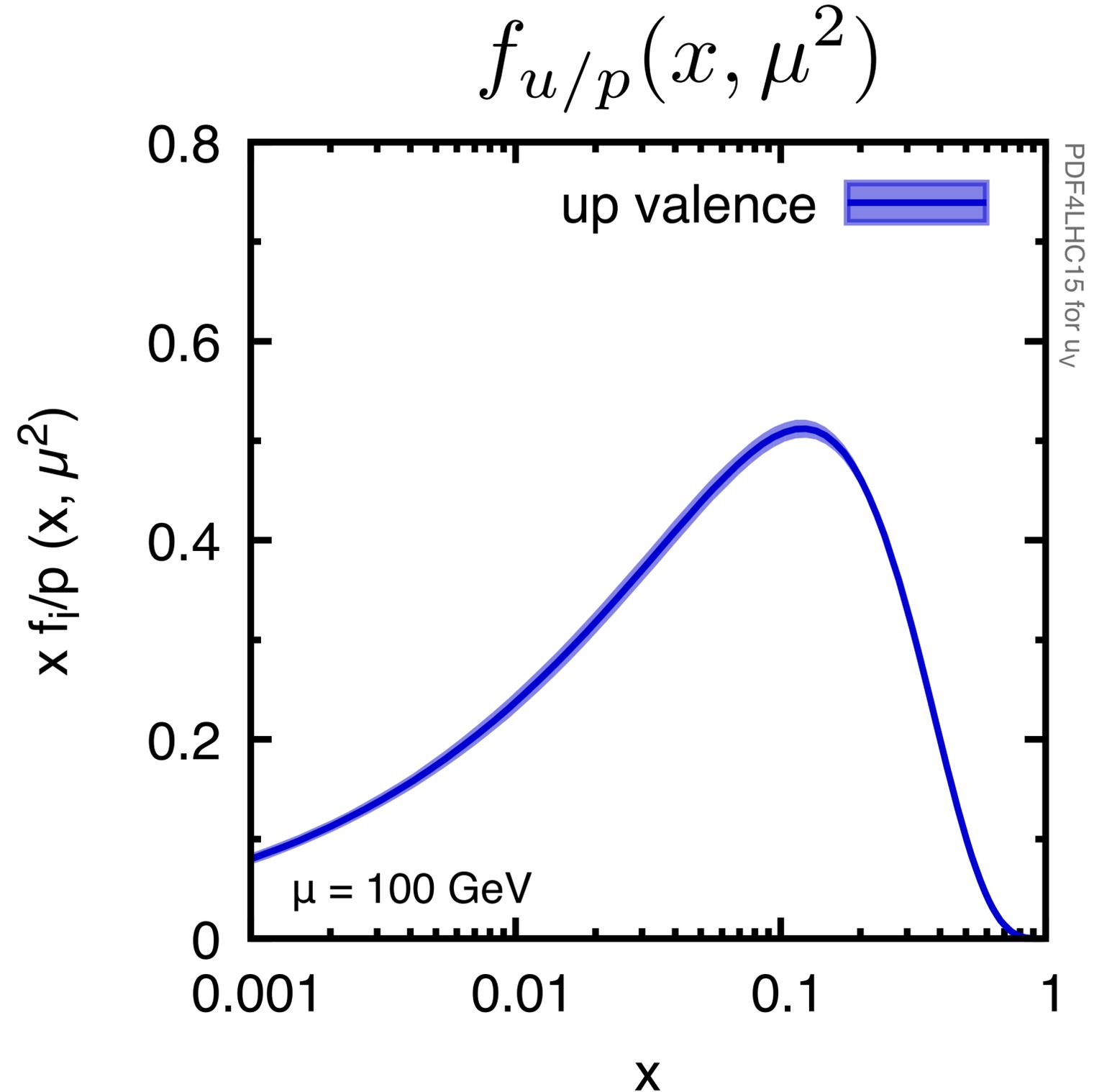
# parton distribution functions (PDFs)



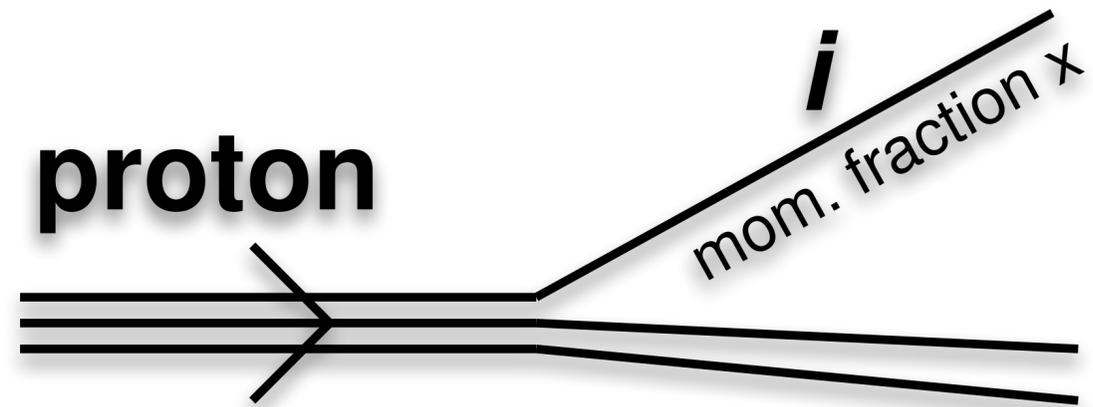
LHC physics  
needs PDFs in region  
 $\sim 10^{-3} - 0.5$

Typically known with good  
**precision  $\sim 1-3\%$**

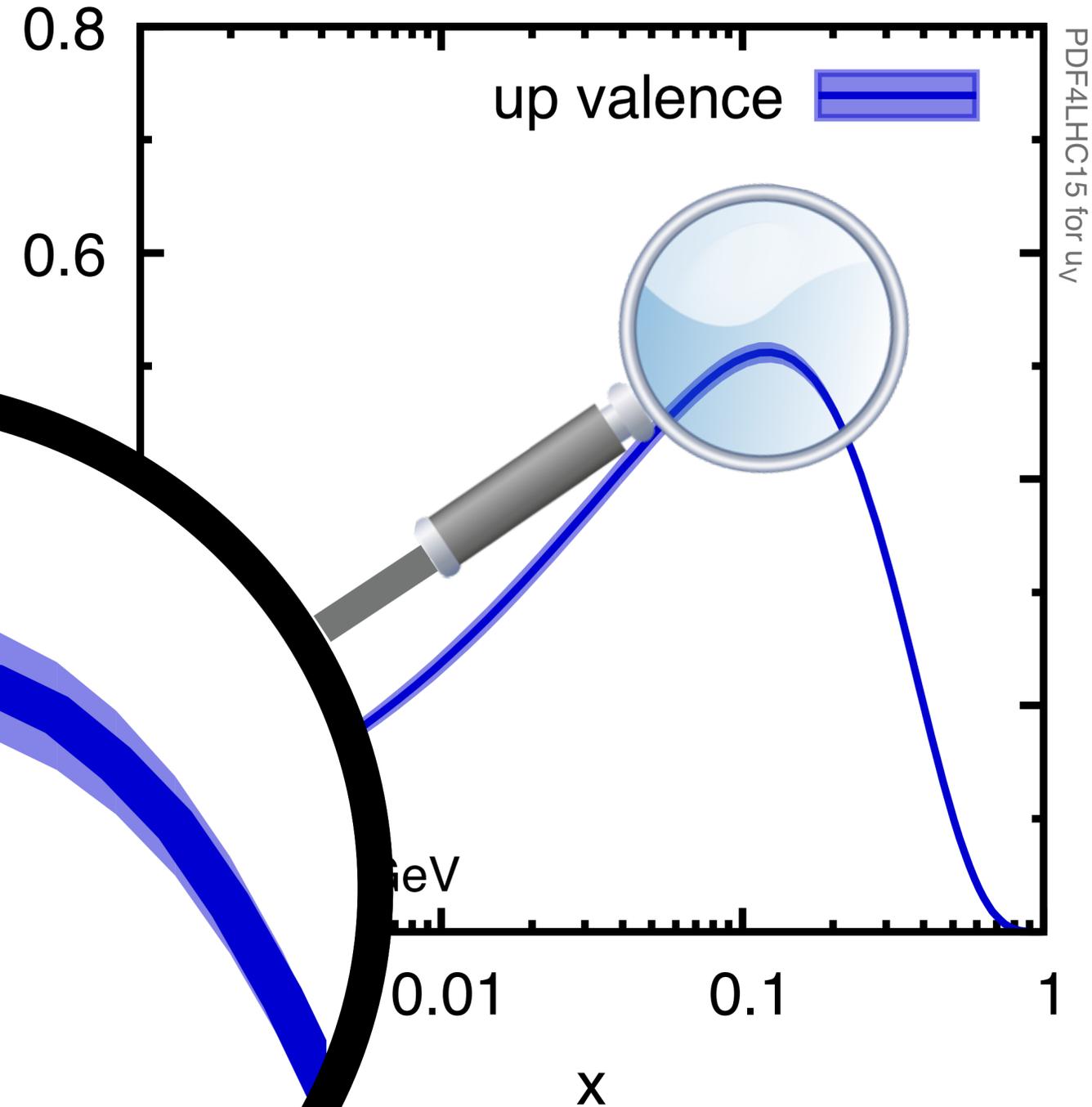
*E.g. NNPDF, MMHT, CT & PDF4LHC working group (+ also HERAPDF, ABM, ...)*



# parton distribution functions (PDFs)



$$f_{u/p}(x, \mu^2)$$



LHC physics  
needs PDFs in re  
 $\sim 10^{-3} - 0.$

Typically known w

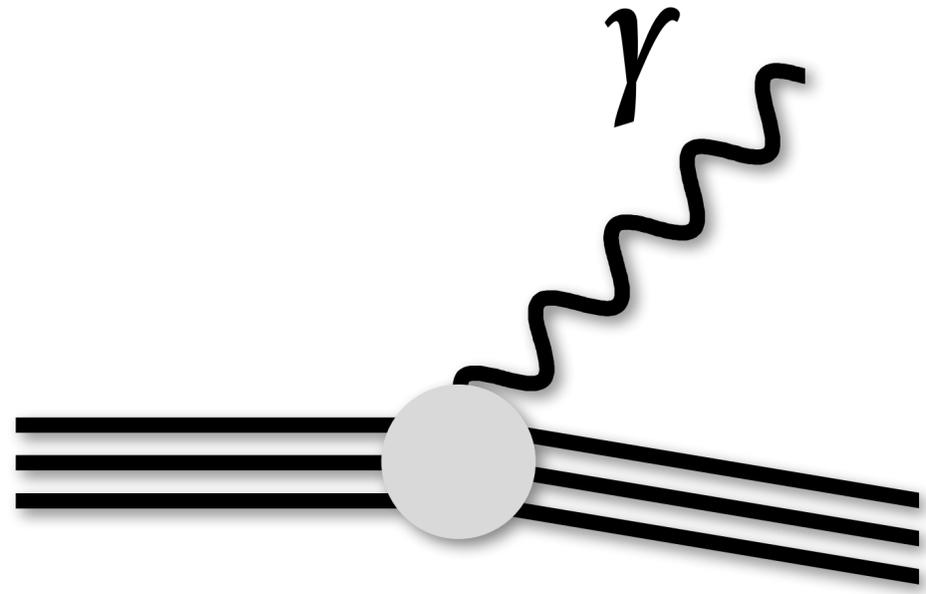
**precision  $\sim 1-$**

$\pm 2\%$

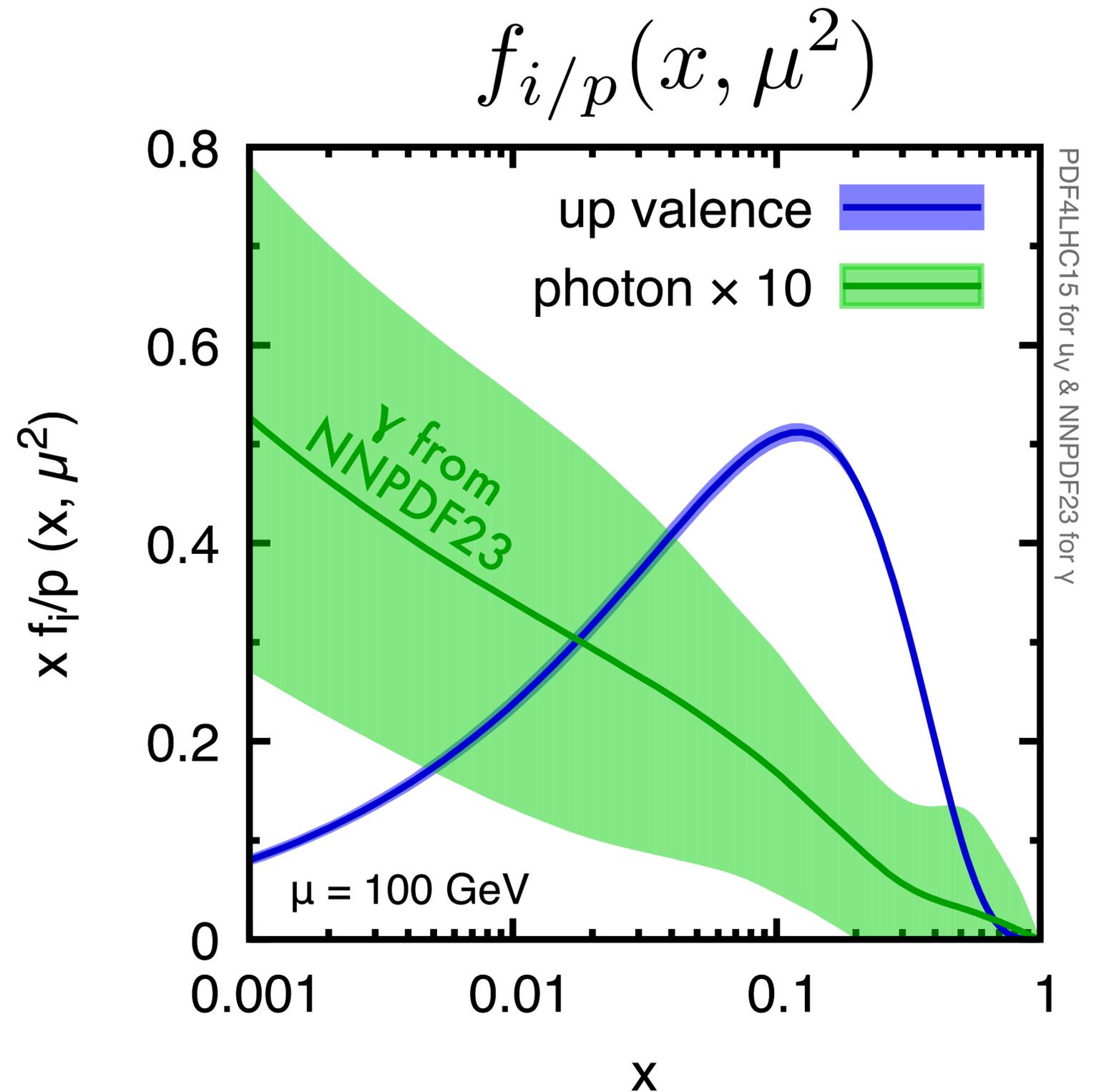
E.g. NNPDF, MMHT, CT & PDF4LHC

(ABM, ...)

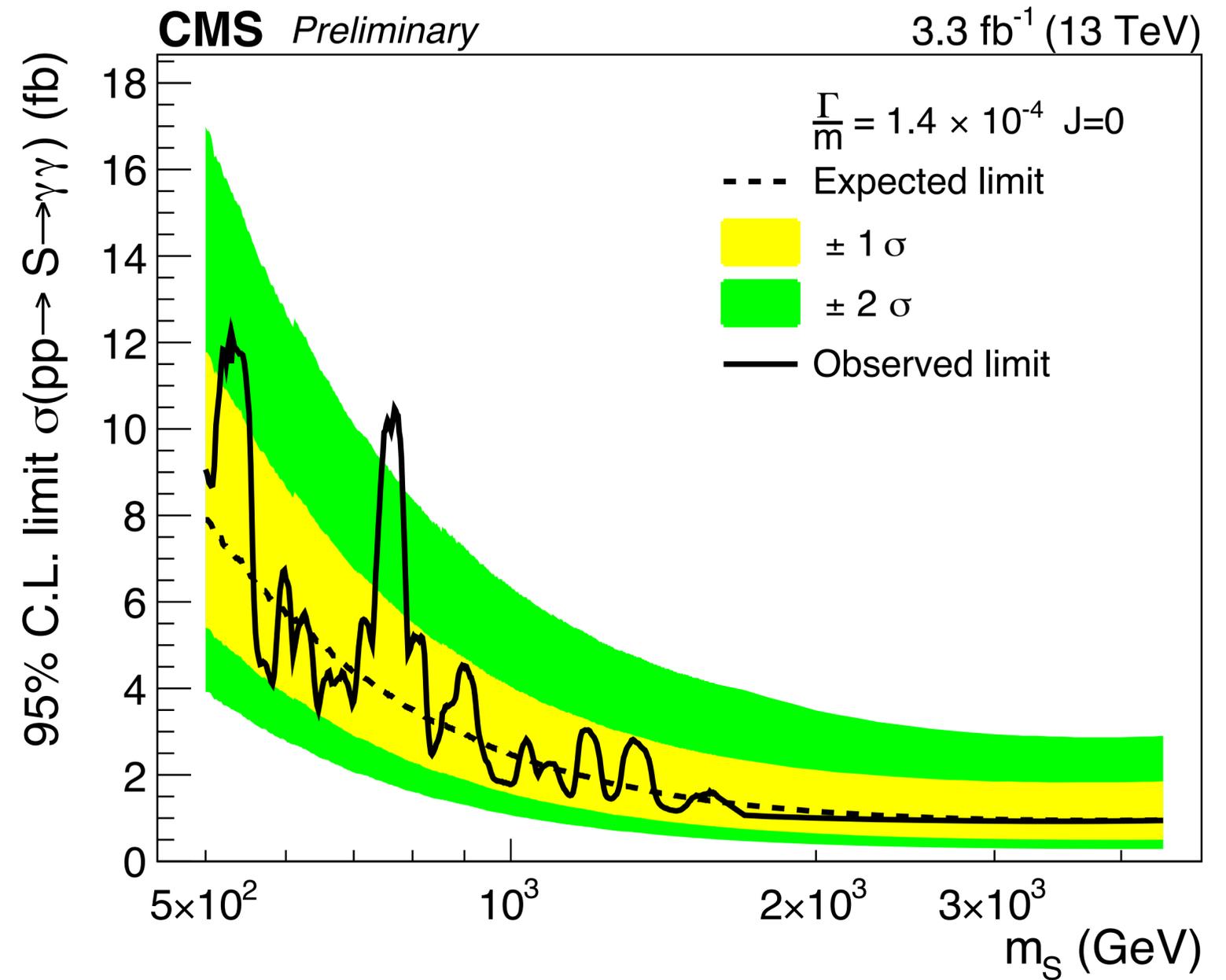
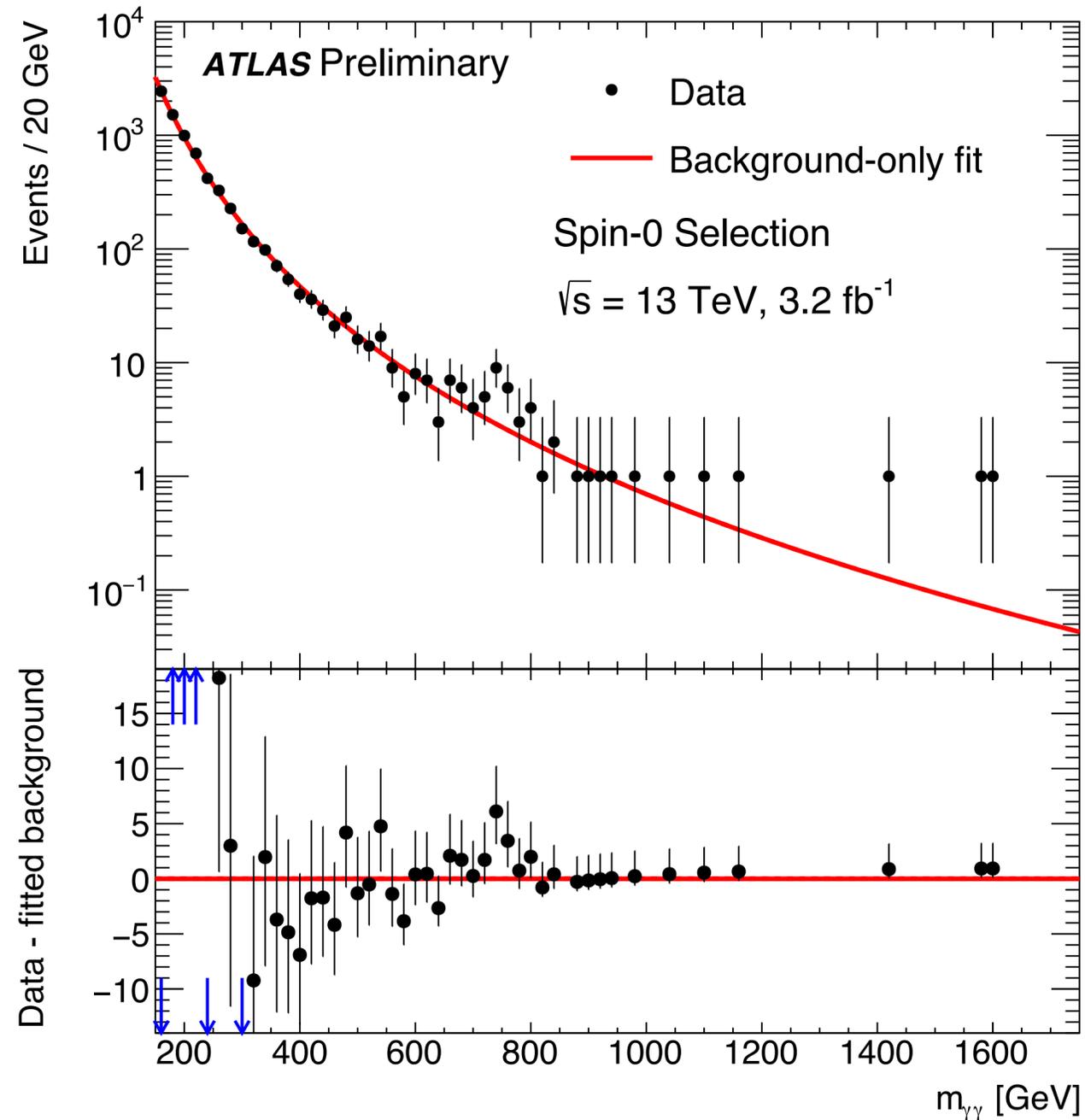
# parton distribution functions (PDFs)



One exception:  
the photon distribution  
inside the proton  
(had up to 100% uncertainty)



# one year ago: A $\Upsilon\Upsilon$ resonance? From $\Upsilon\Upsilon \rightarrow \Upsilon\Upsilon$ ?



# photon induced contribution to HW production

---

$pp \rightarrow H W^+ (\rightarrow l^+ \nu) + X$  at 13 TeV

non-photon induced contributions

$91.2 \pm 1.8$  fb

photon-induced contribs (NNPDF23)

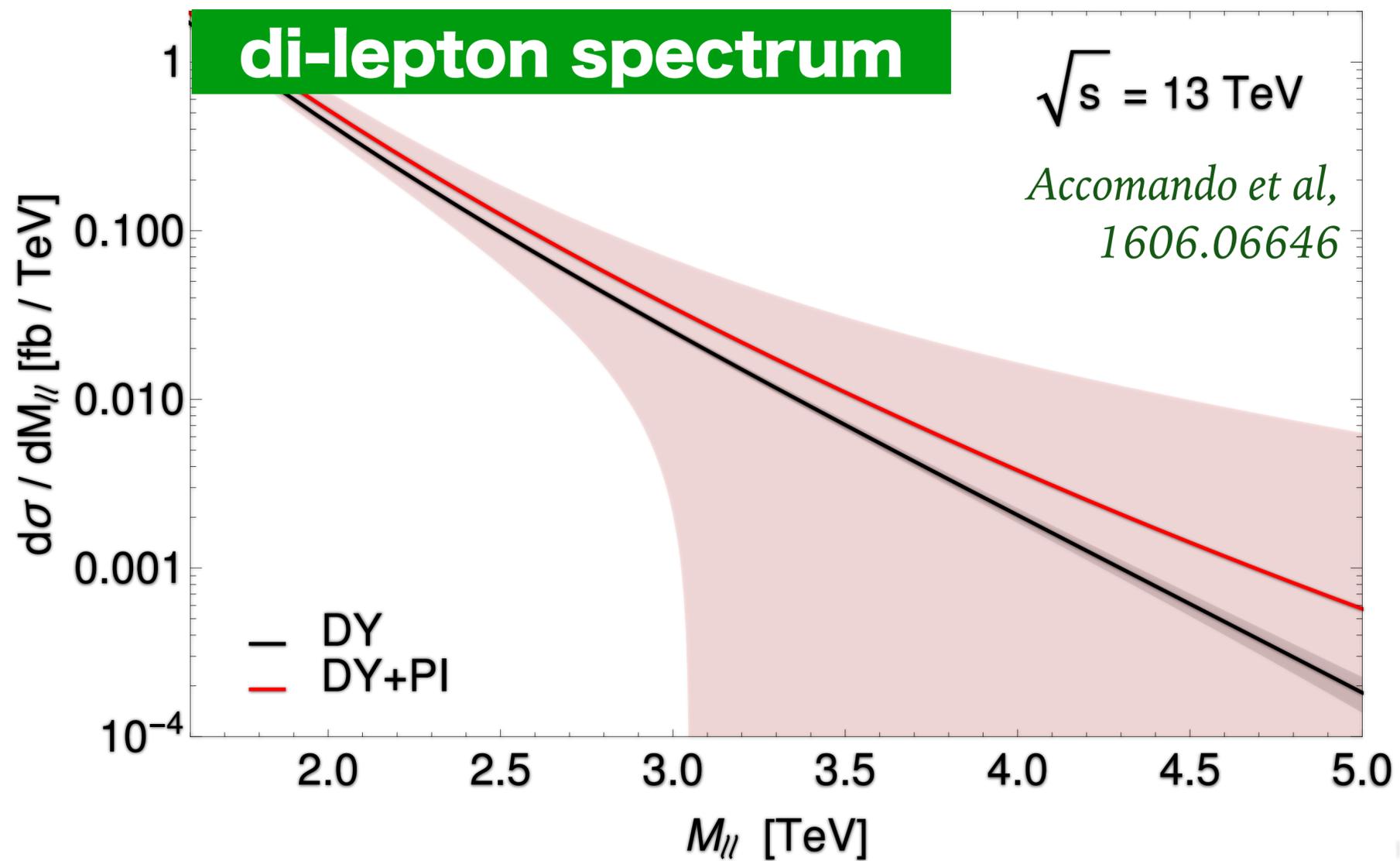
$6.0^{+4.4}_{-2.9}$  fb

**photon  
contribution  
brings the  
largest overall  
uncertainty**

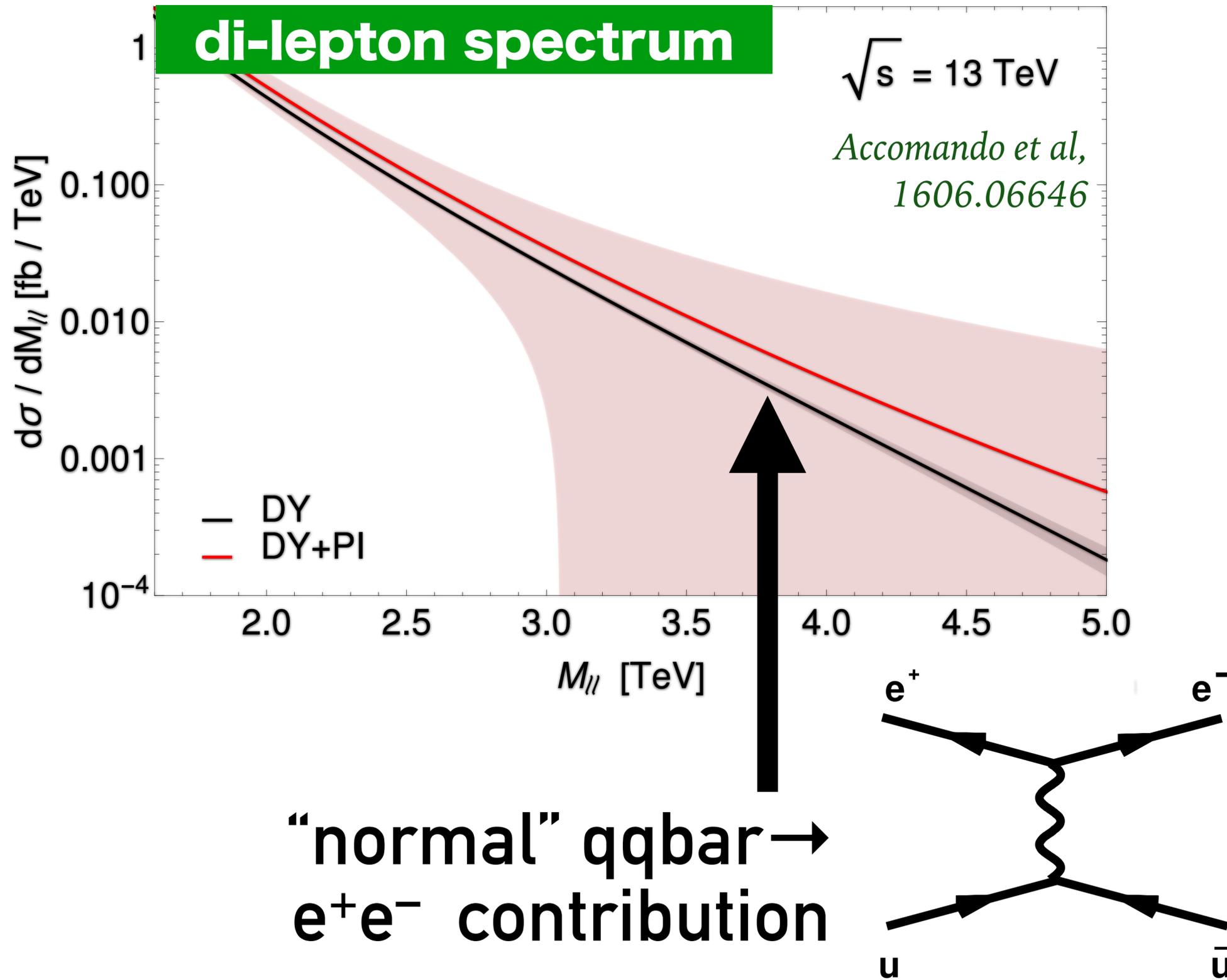


*non-photon numbers from LHCHSWG (YR4)  
including PDF uncertainties*

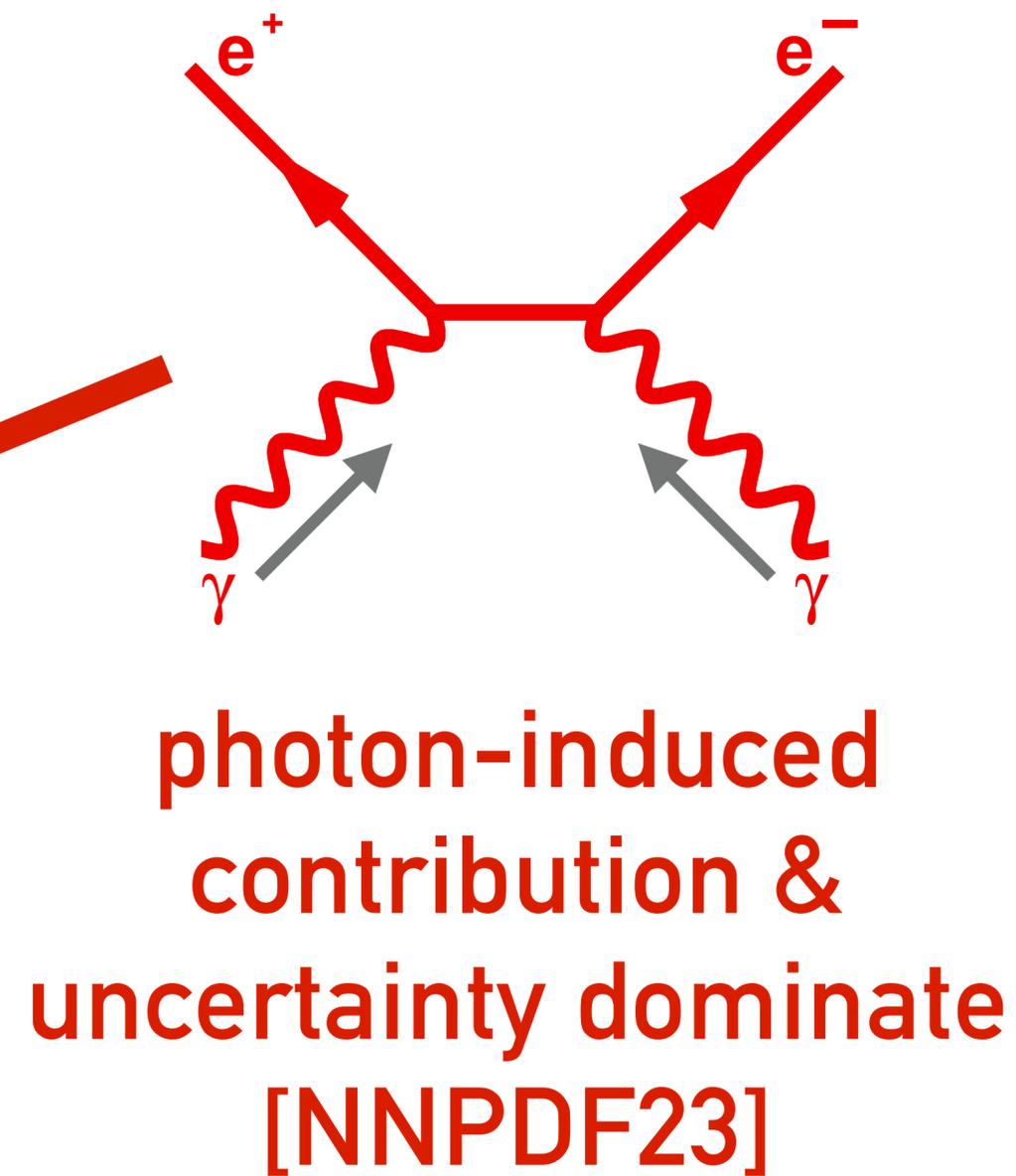
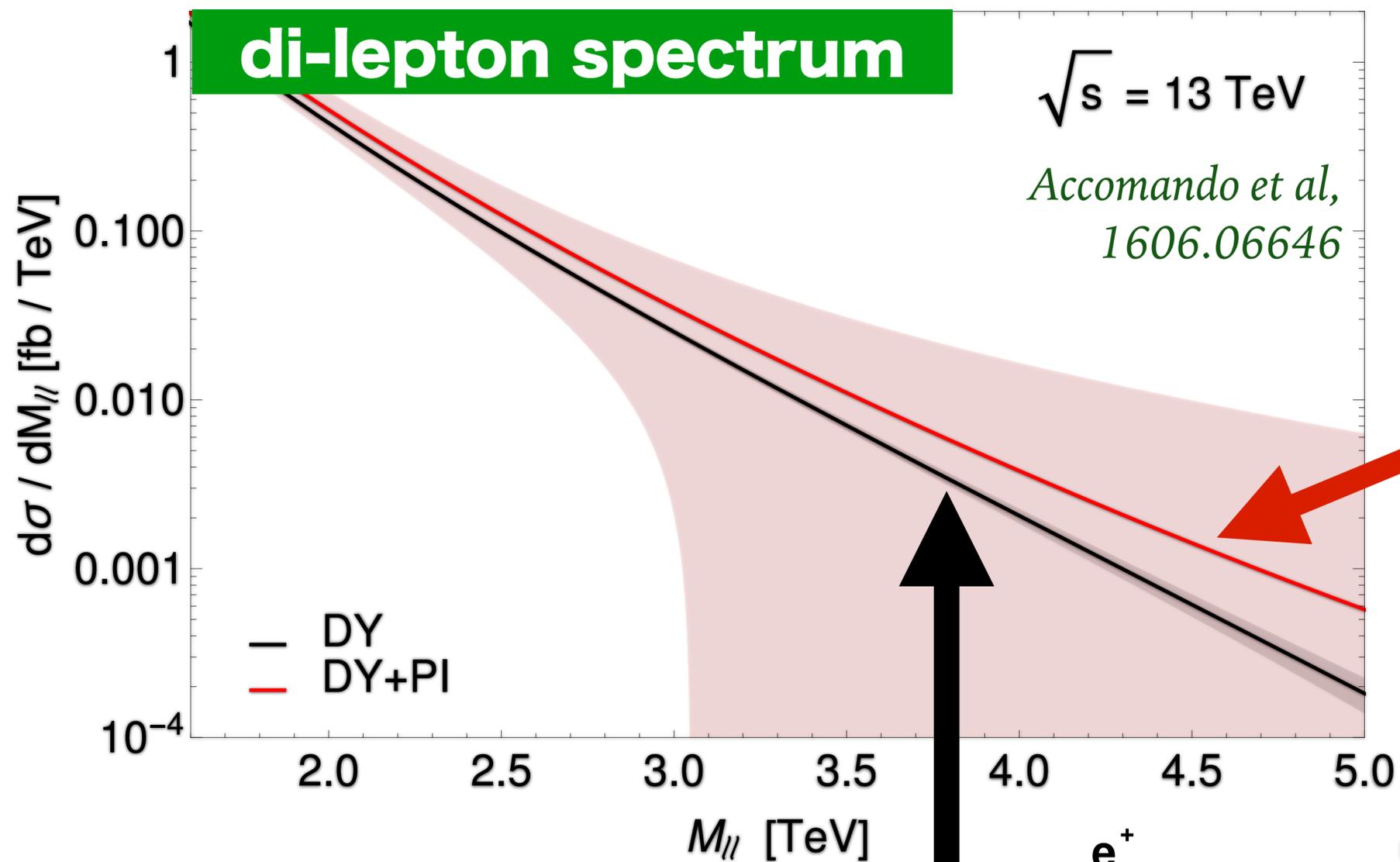
# it matters in new-physics searches



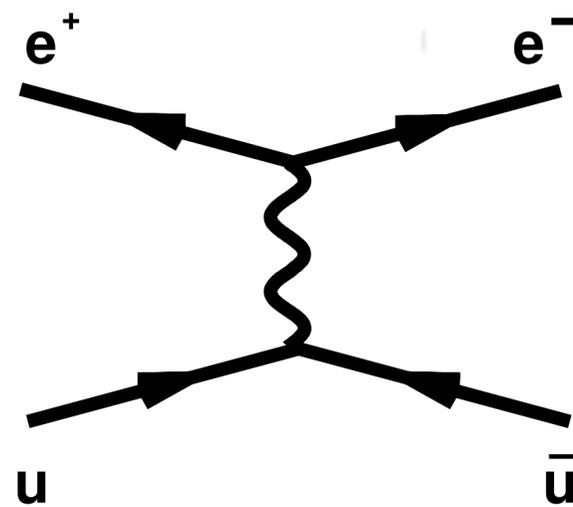
# it matters in new-physics searches



# it matters in new-physics searches



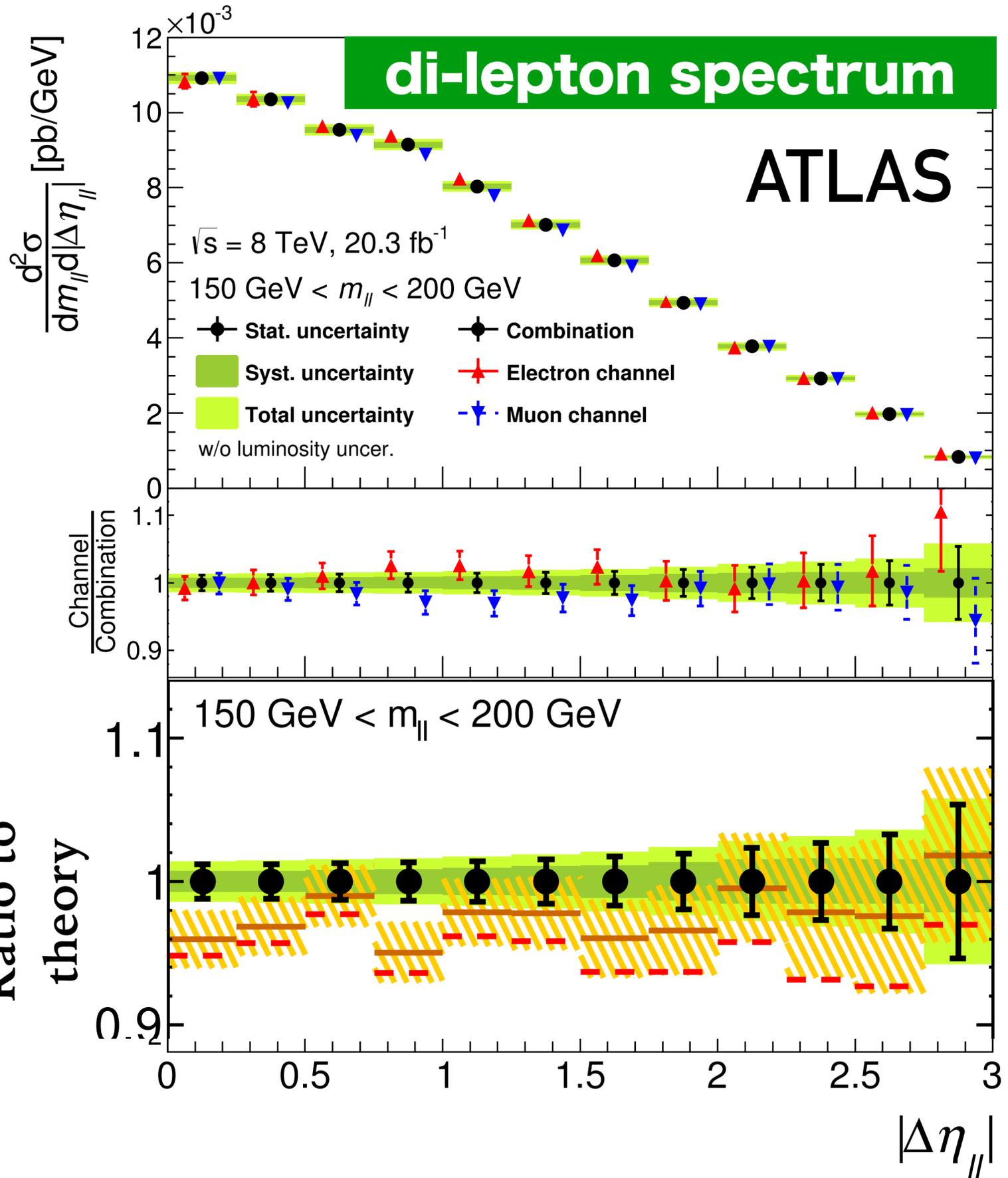
“normal”  $q\bar{q} \rightarrow e^+e^-$  contribution



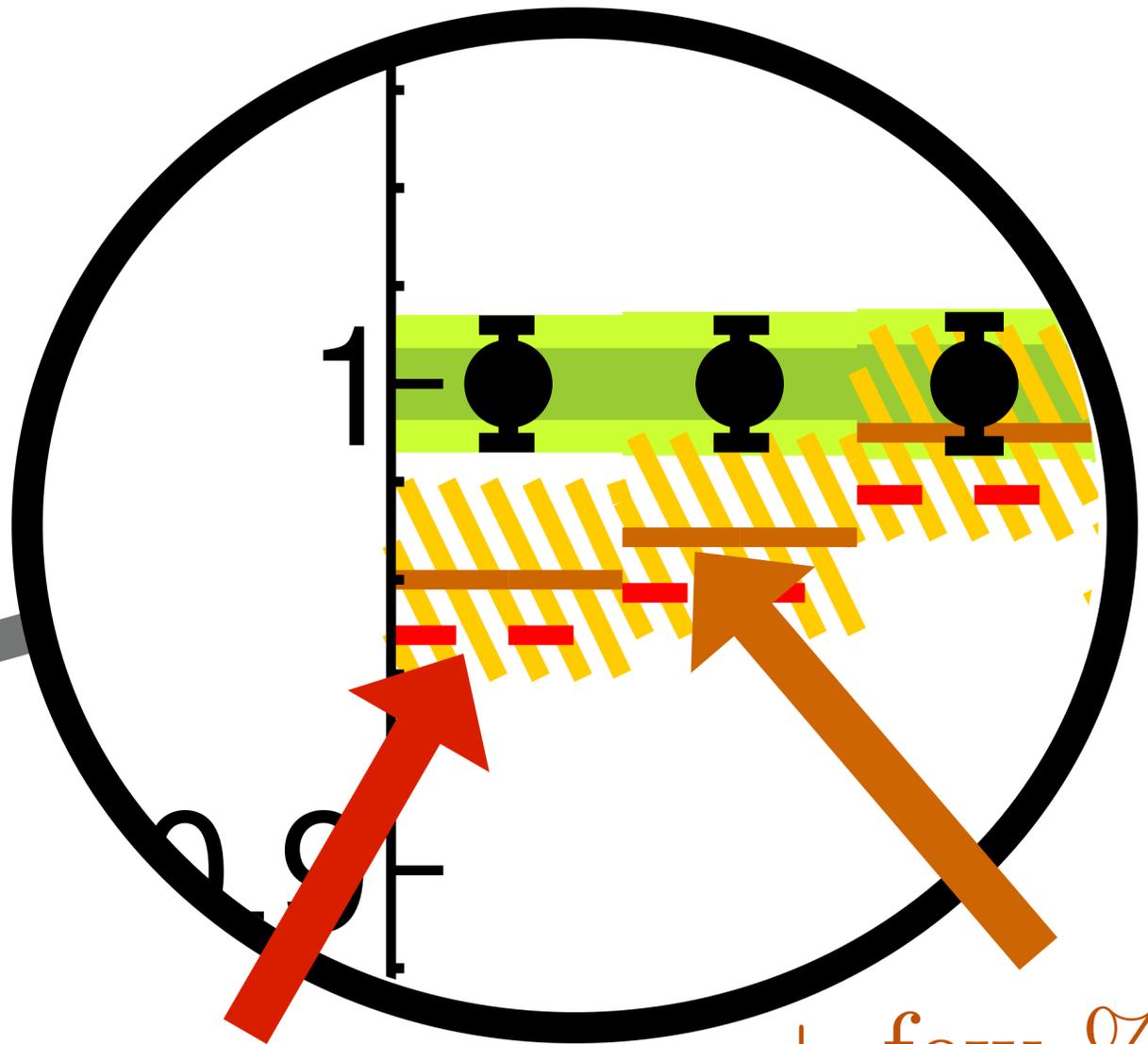
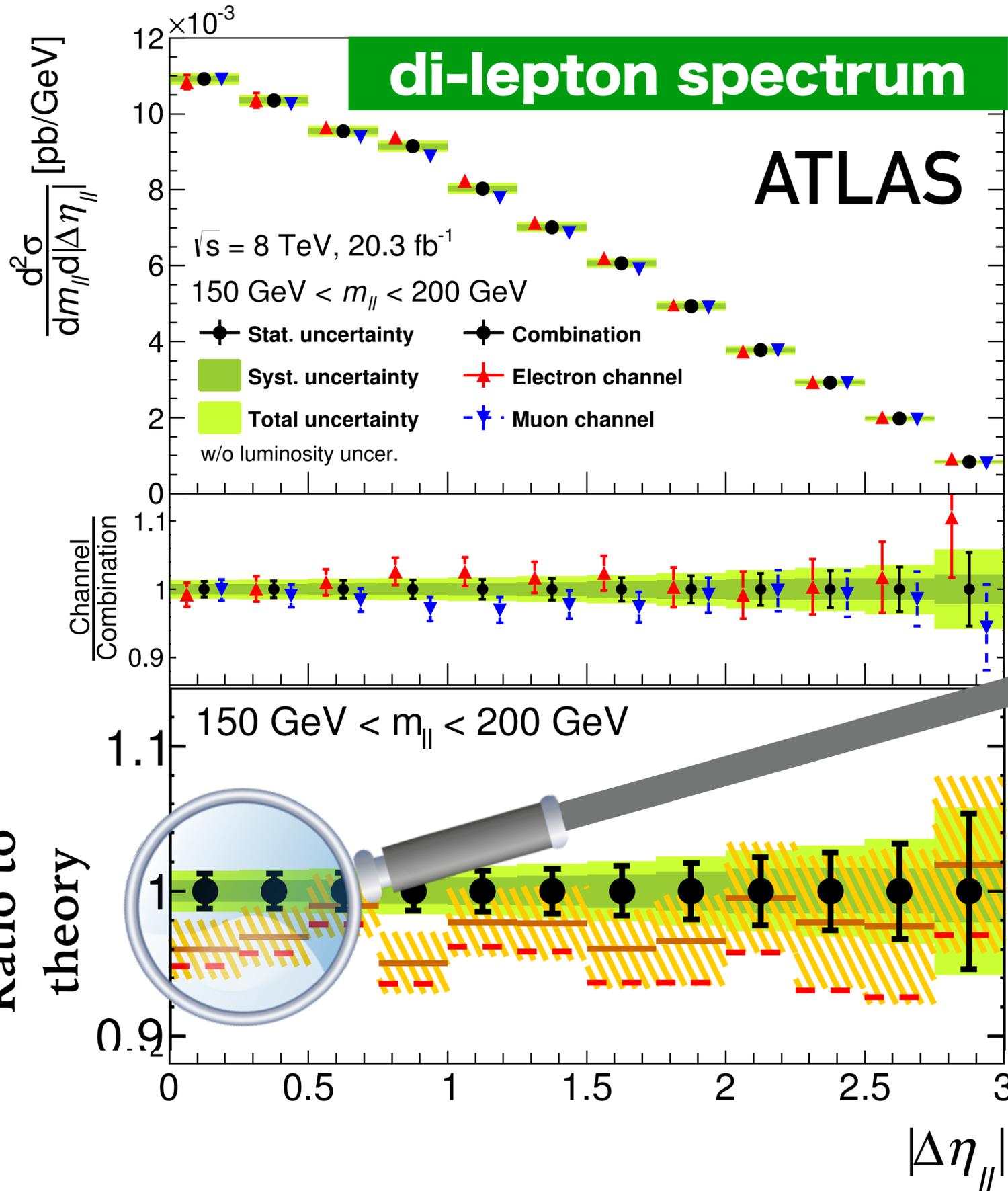
# model-independent $\gamma$ PDF fit (c. 2013)

## di-lepton spectrum

ATLAS



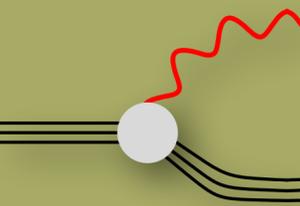
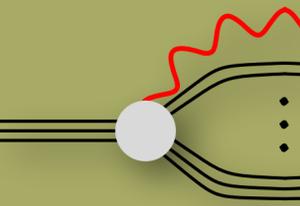
model-independent  $\gamma$  PDF fit (c. 2013)



95-99% from  $q\bar{q} \rightarrow e^+e^-$

+ few % from  $\gamma\gamma \rightarrow e^+e^-$

# Widely discussed photon-PDF estimates

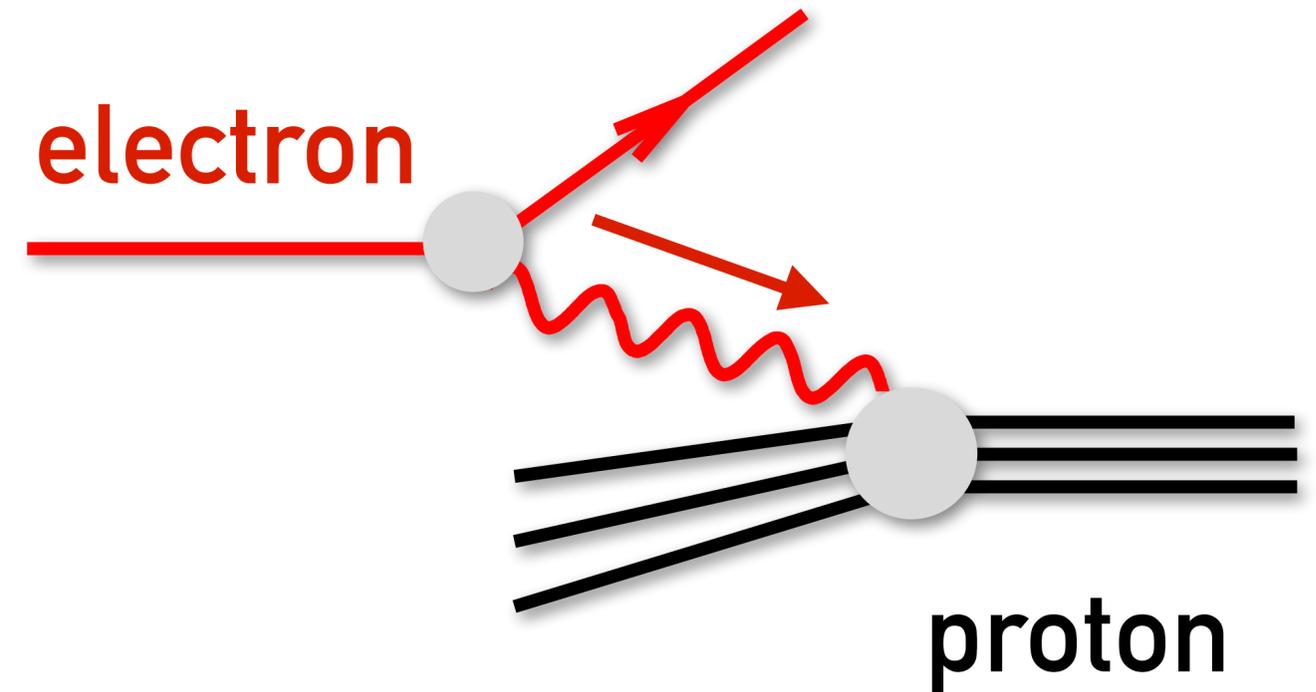
	 elastic	 inelastic	LHAPDF public computer-readable form?
Gluck Pisano Reya 2002	dipole	model	✗
MRST2004qed	✗	model	✓
CT14qed_inc	dipole	model (data-constrained)	✓
Martin Ryskin 2014	dipole (only electric part)	model	✗
Harland-Lang, Khoze Ryskin 2016	dipole	model	✗
NNPDF23qed (& NNPDF30qed)	no separation; fit to data		

*elastic part long known: Budnev, Ginzburg, Meledin & Serbo, Phys.Rept. 1974*

# How do you do better? → Use electron–proton scattering

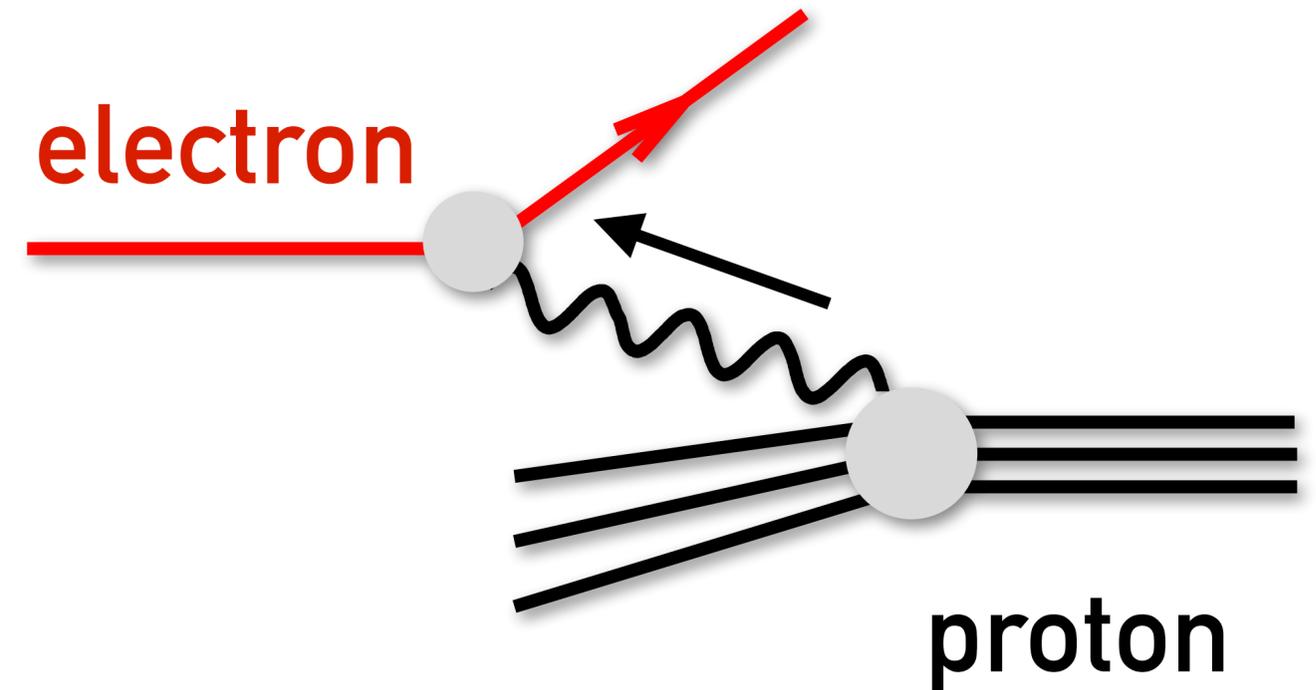
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- ▶ Experiments have been going on for decades
- ▶ Usually seen as photons from electron probing proton structure



# How do you do better? → Use electron–proton scattering

- ▶ Experiments have been going on for decades
- ▶ Usually seen as photons from electron probing proton structure
- ▶ But **can be viewed as electron probing proton's photonic field**
- ▶ Everything about unpolarized EM electron–proton interaction encoded in two “structure functions”  $F_2(x, Q^2)$  &  $F_L(x, Q^2)$



$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{xQ^4} \left( \left( 1 - y + \frac{y^2}{2} \left( 1 + 2x^2 \frac{m_p^2}{Q^2} \right) \right) F_2(x, Q^2) - \frac{y^2}{2} F_L(x, Q^2) \right)$$

# Photon PDF in terms of $F_2$ and $F_L$ — the **LUXqed** approach

$$x f_{\gamma/p}(x, \mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \right. \\ \left[ \left( z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right] \\ \left. - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right) \right\}$$

This includes terms

$$\alpha L (\alpha_s L)^n$$

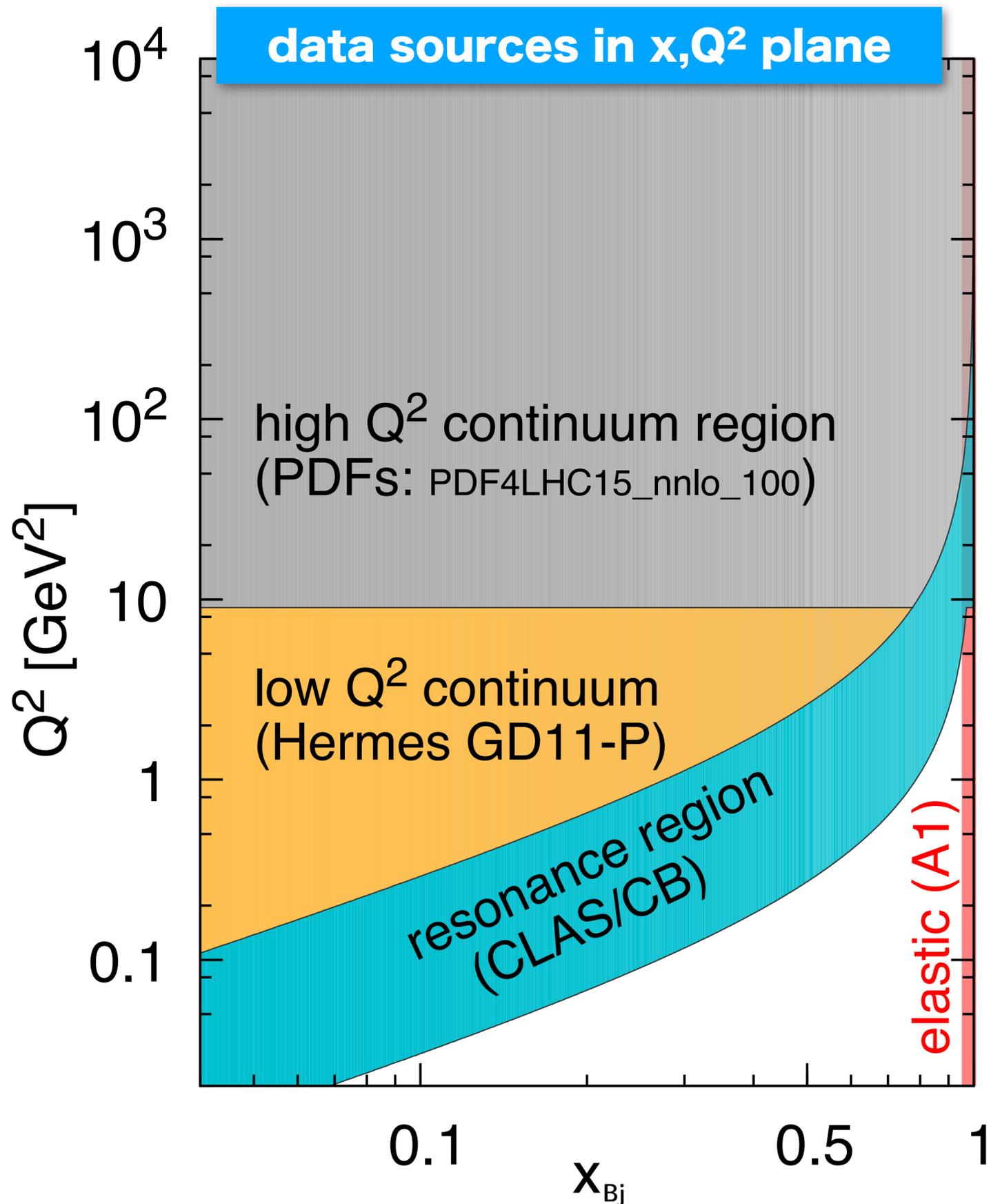
$$\alpha (\alpha_s L)^n$$

$$\alpha^2 L^2 (\alpha_s L)^n$$

$$(L = \ln \mu^2/\Lambda^2)$$

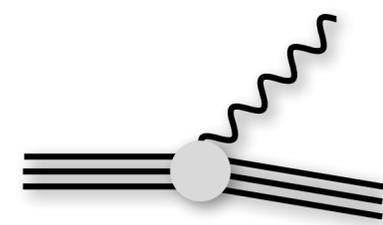
Work in progress goes one order higher (e.g. extra power of  $\alpha_s$ )

It subsequently emerged that two “forgotten” papers, Anlauf et. al, CPC70(1992)97 Mukherjee & Pisano, hep-ph/0306275, had the correct integrand (but not the limits)



## DATA

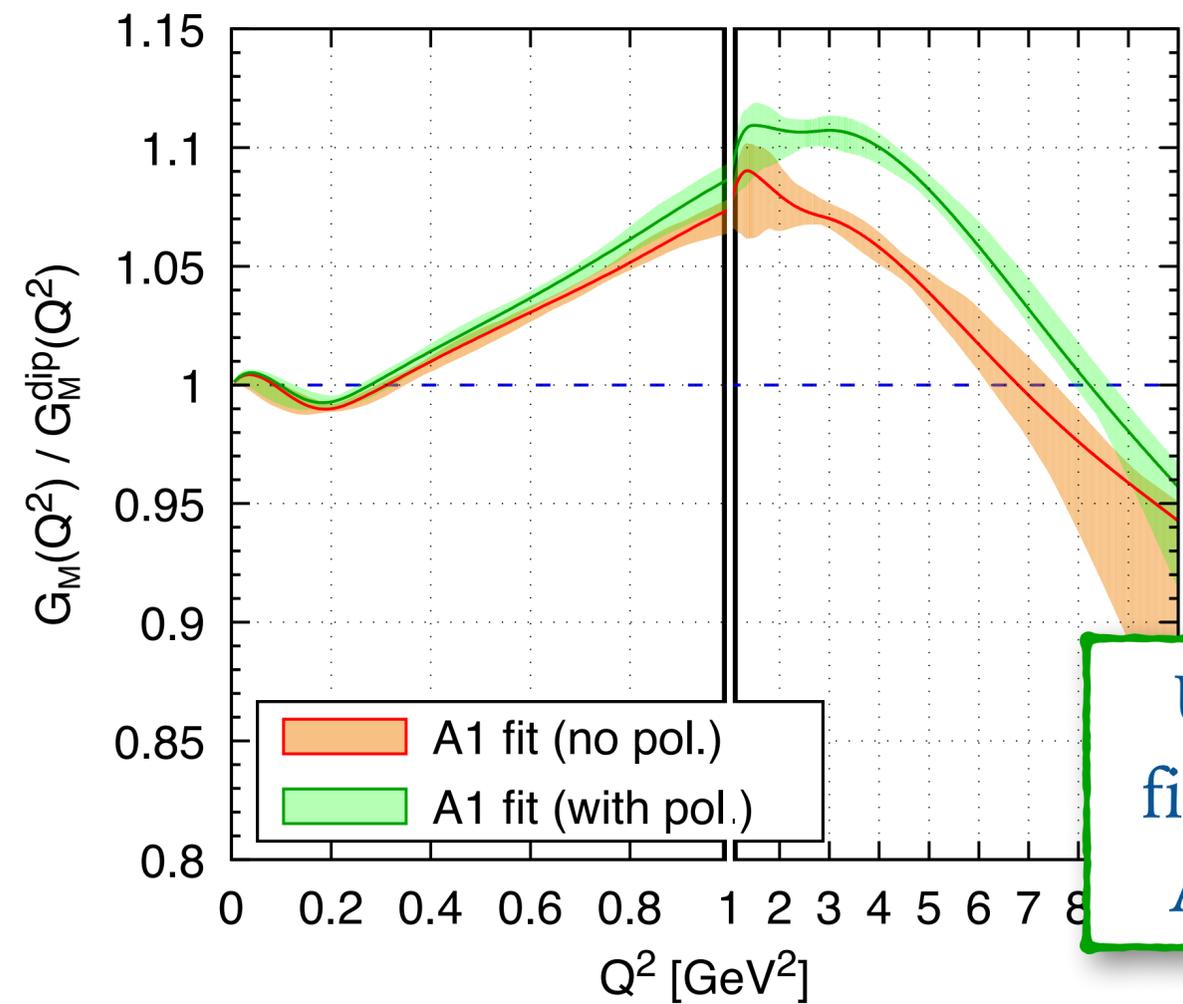
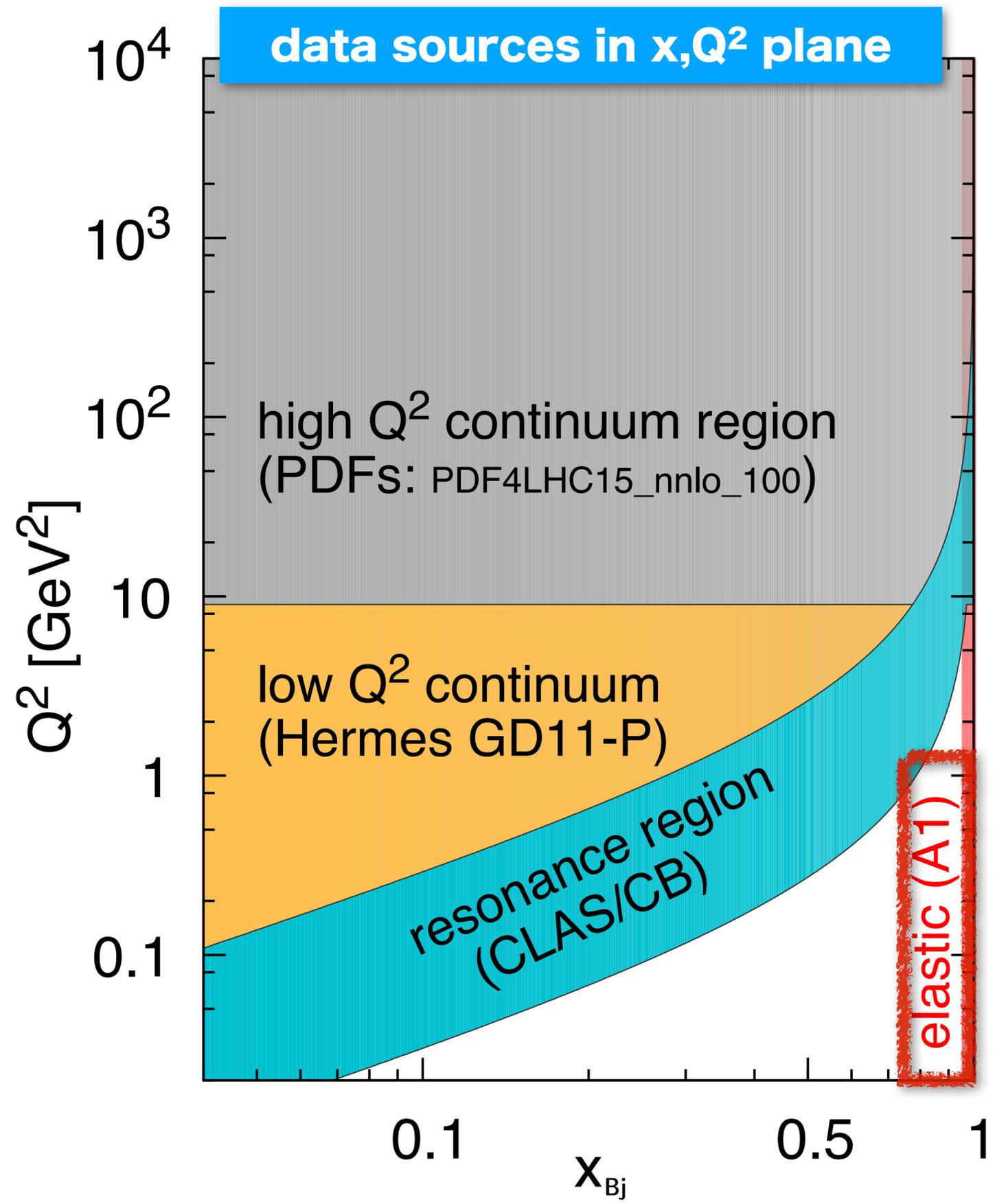
- $x, Q^2$  plane naturally breaks up into regions with different physical behaviours and data sources
- We don't use  $F_2$  and  $F_L$  data directly, but rather various fits to data



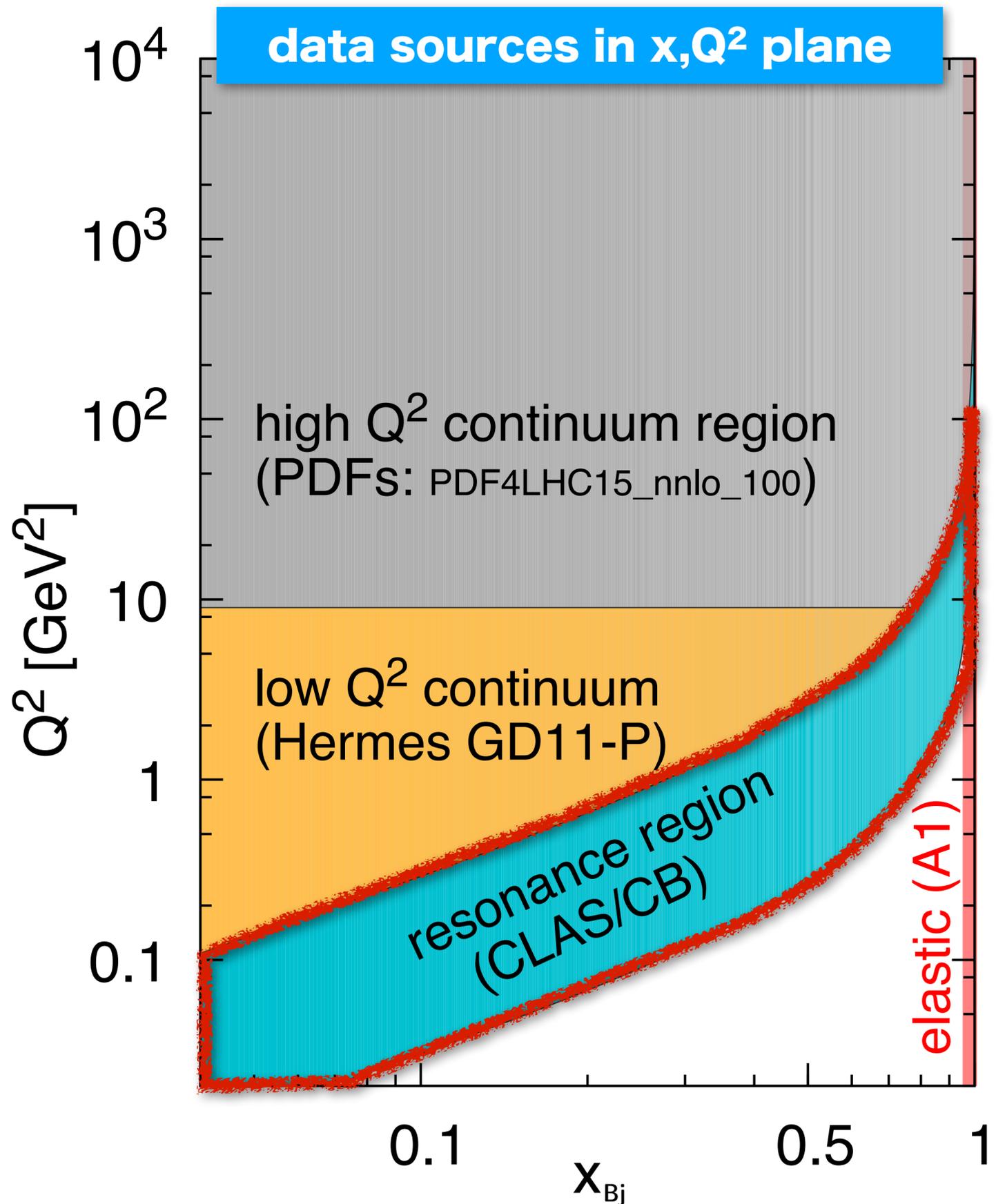
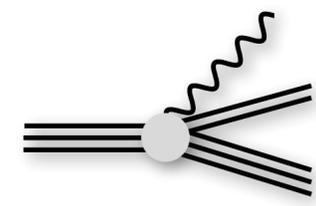
# ELASTIC COMPONENT

- Elastic component of  $F_{2/L}$  lives at  $x=1$
- Get from Sachs Form factors,  $G_E$ ,  $G_M$

$$F_2^{\text{el}}(x_{\text{bj}}, Q^2) = \frac{[G_E(Q^2)]^2 + [G_M(Q^2)]^2 \tau}{1 + \tau} \delta(1 - x_{\text{bj}}), \quad \tau = Q^2 / (4m_p^2)$$

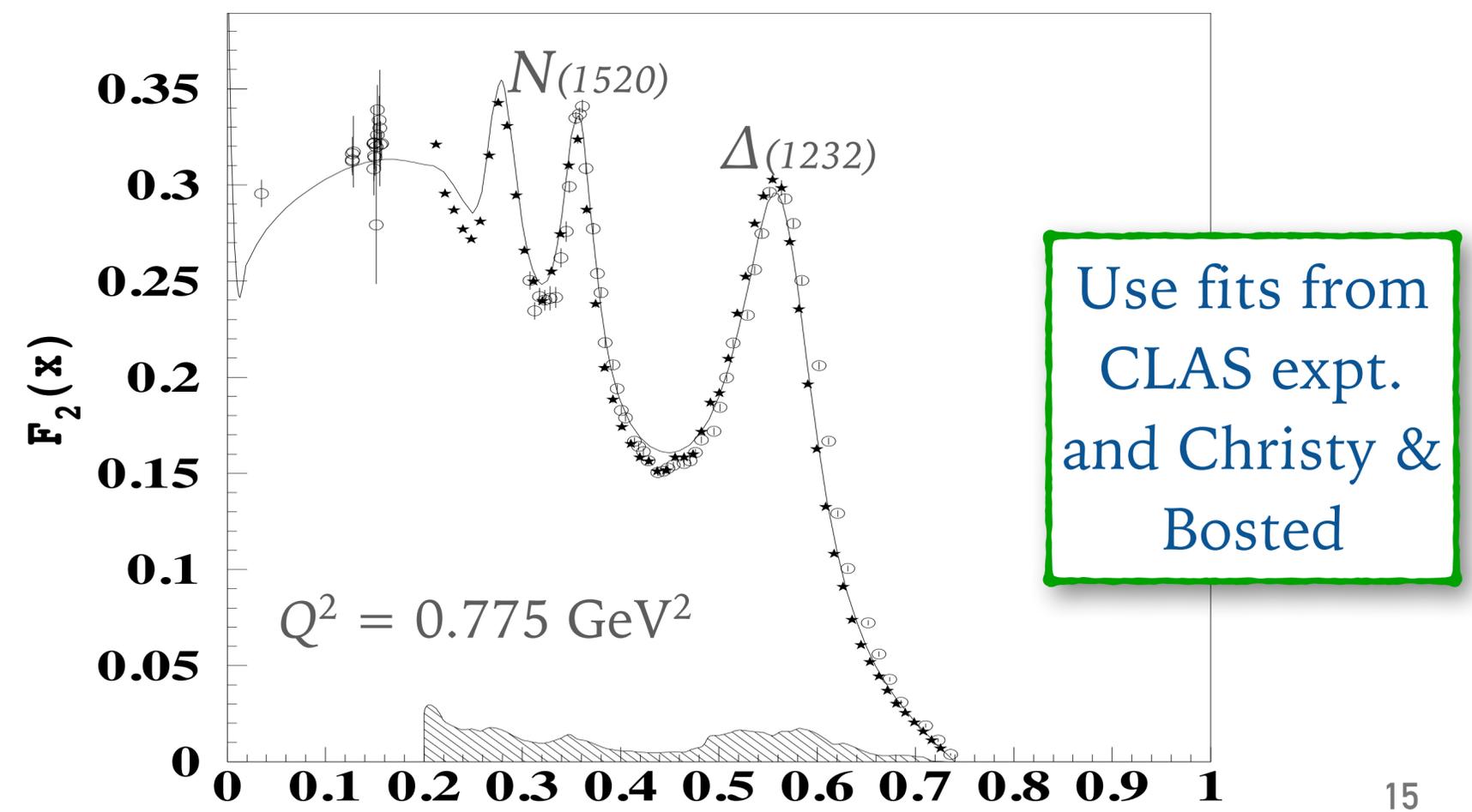


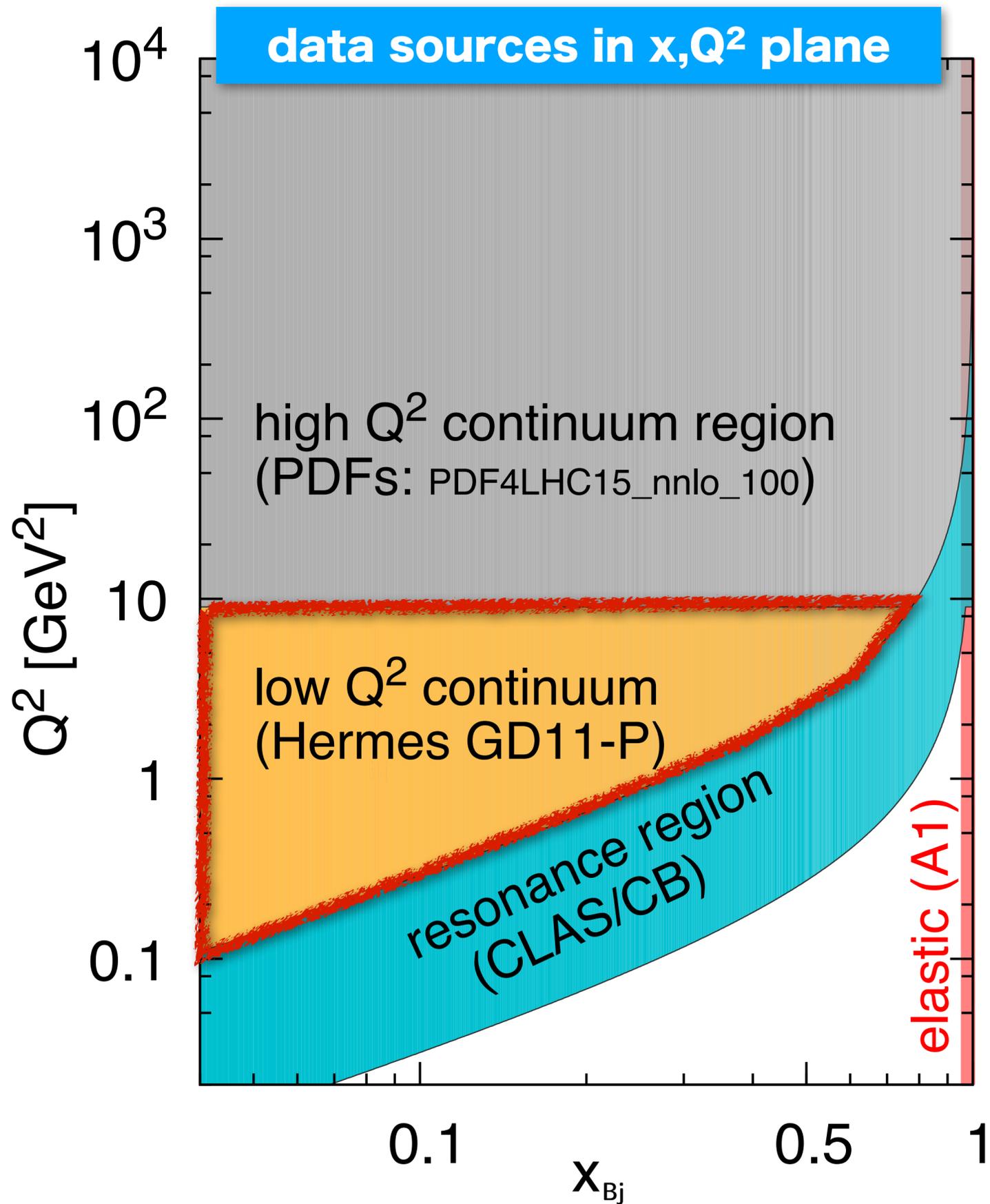
Use global fits from the A1 Collab.



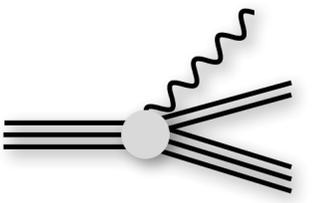
# RESONANCE COMPONENT

- proton gets excited, e.g. to  $\Delta \rightarrow p\pi$  and higher resonances
- relevant for  $(m_p + m_\pi)^2 < W^2 \lesssim 3.5 \text{ GeV}^2$

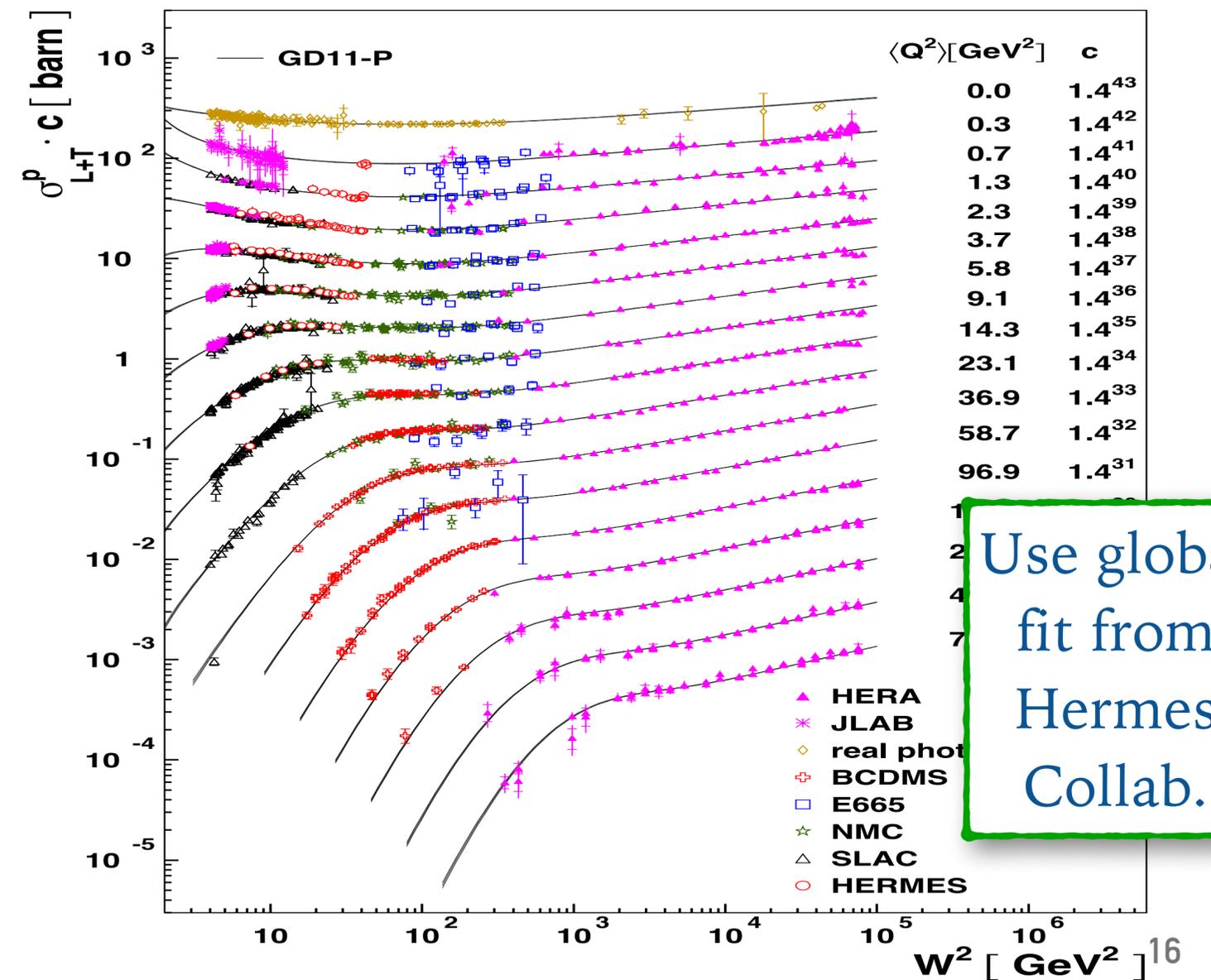




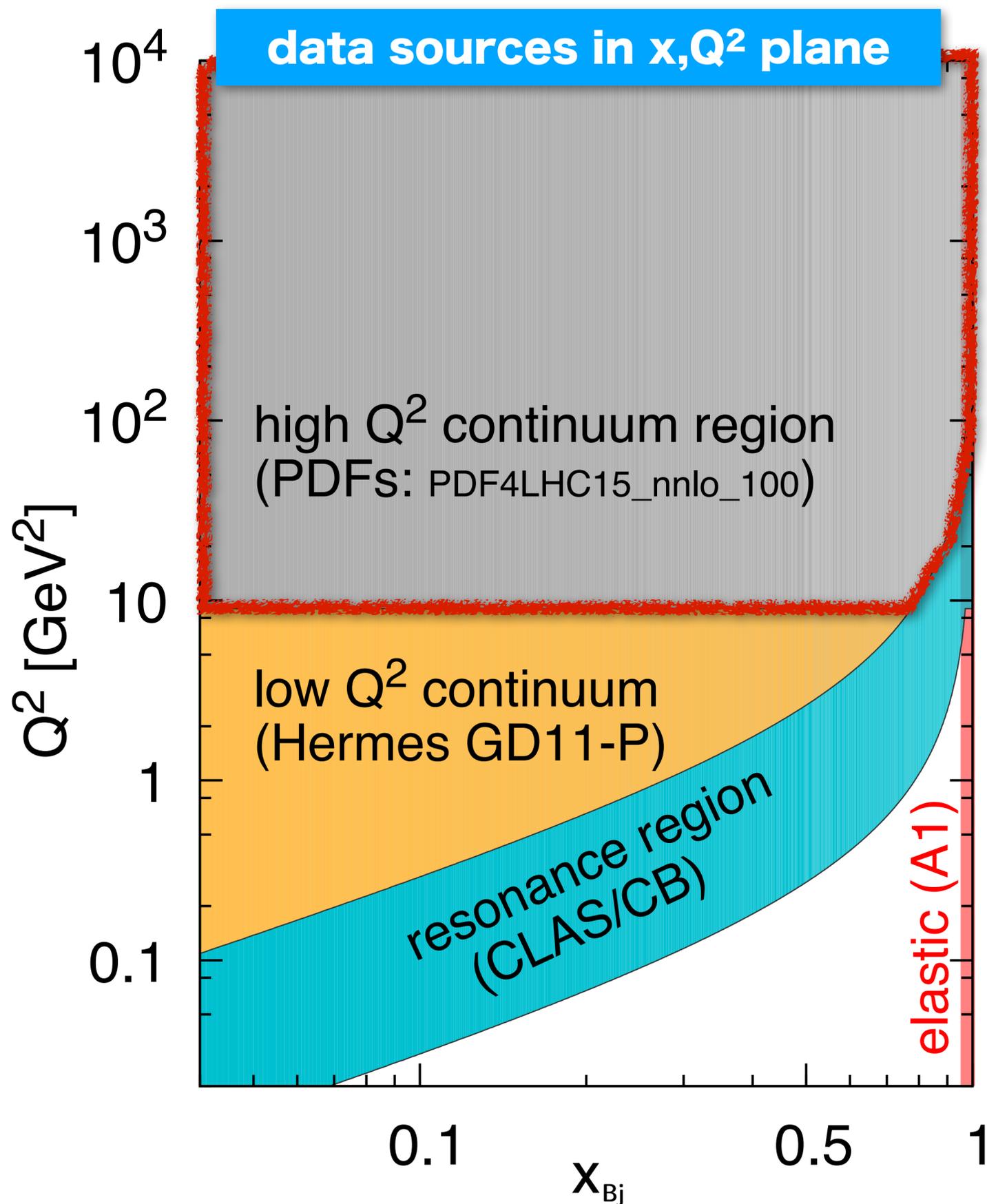
# CONTINUUM COMPONENT



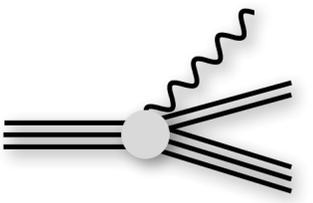
- Much data
- For  $Q^2 \rightarrow 0$ ,  $\sigma_{yp}$  indep. of  $Q^2$  at fixed  $W^2$



Use global fit from  
Hermes  
Collab.



## CONTINUUM COMPONENT

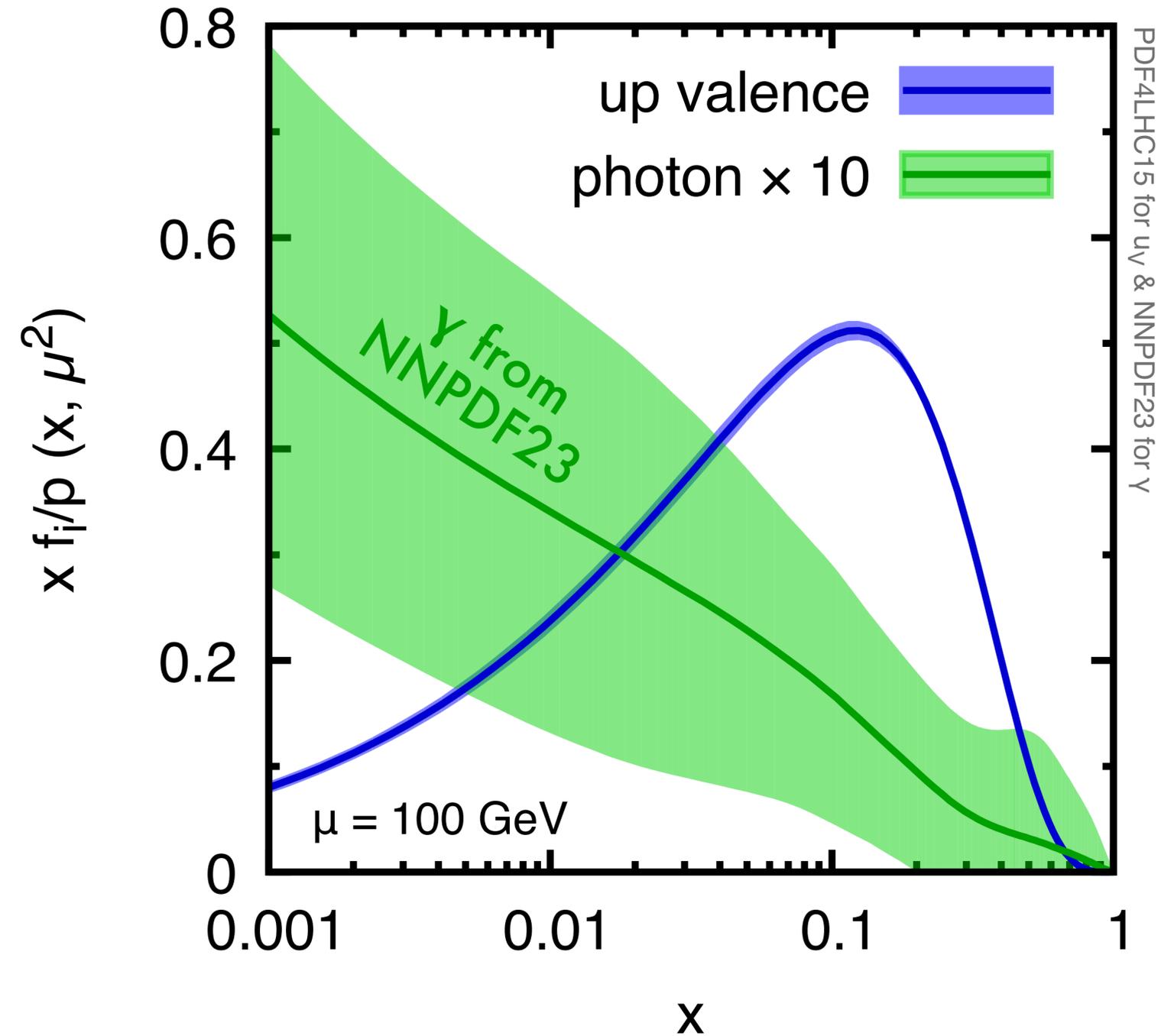


- ▶ Less direct data for  $F_2$  and  $F_L$  at high  $Q^2$
- ▶ But we can reliably use PDFs and coefficient functions to calculate them
- ▶ We use NNLO coefficient functions in a zero-mass variable flavour-number scheme

As a PDF we use  
**PDF4LHC15\_nnlo\_100**  
 from LHAPDF

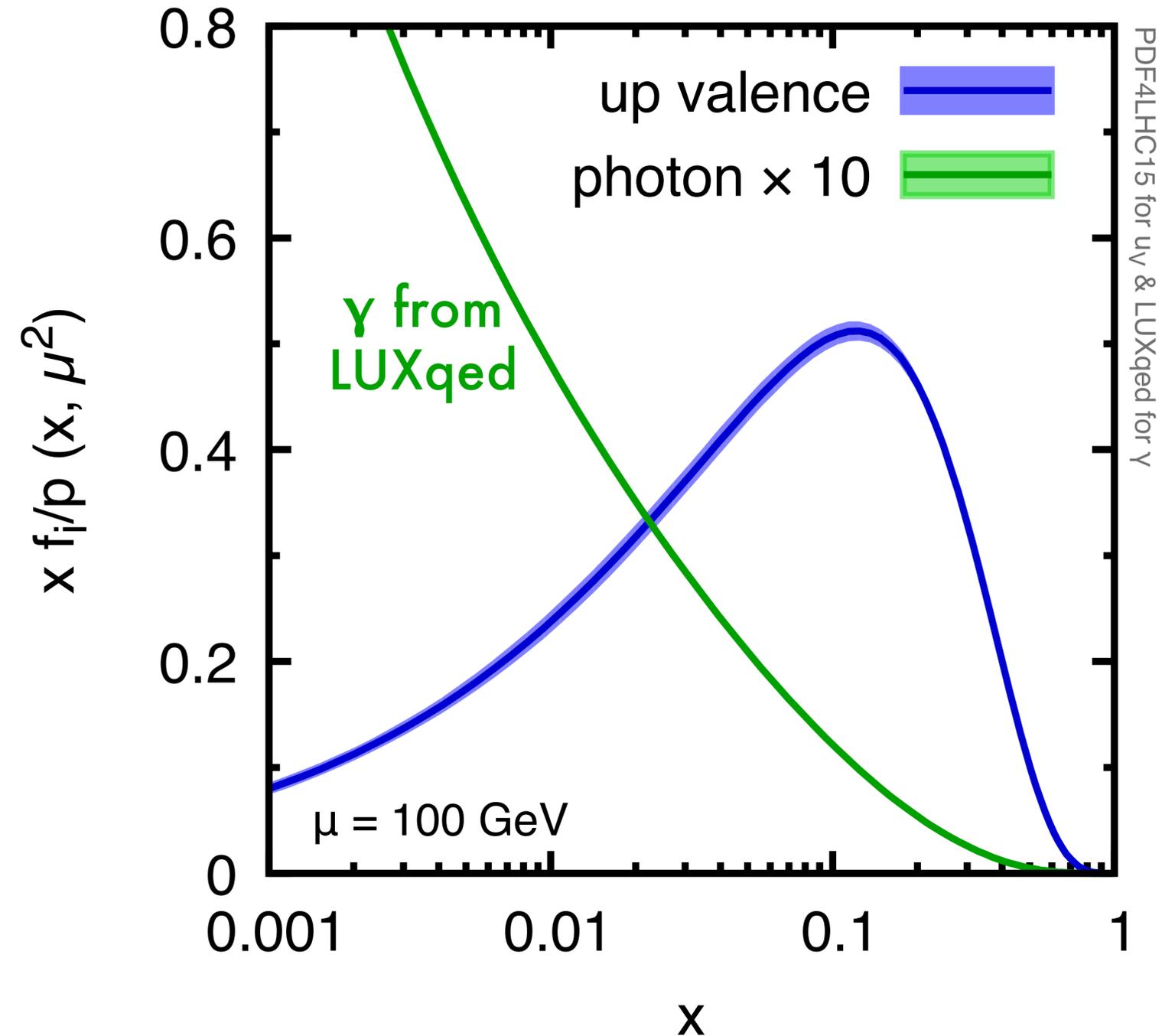
# photon PDF results

- Model-independent uncertainty (NNPDF) was 50–100%



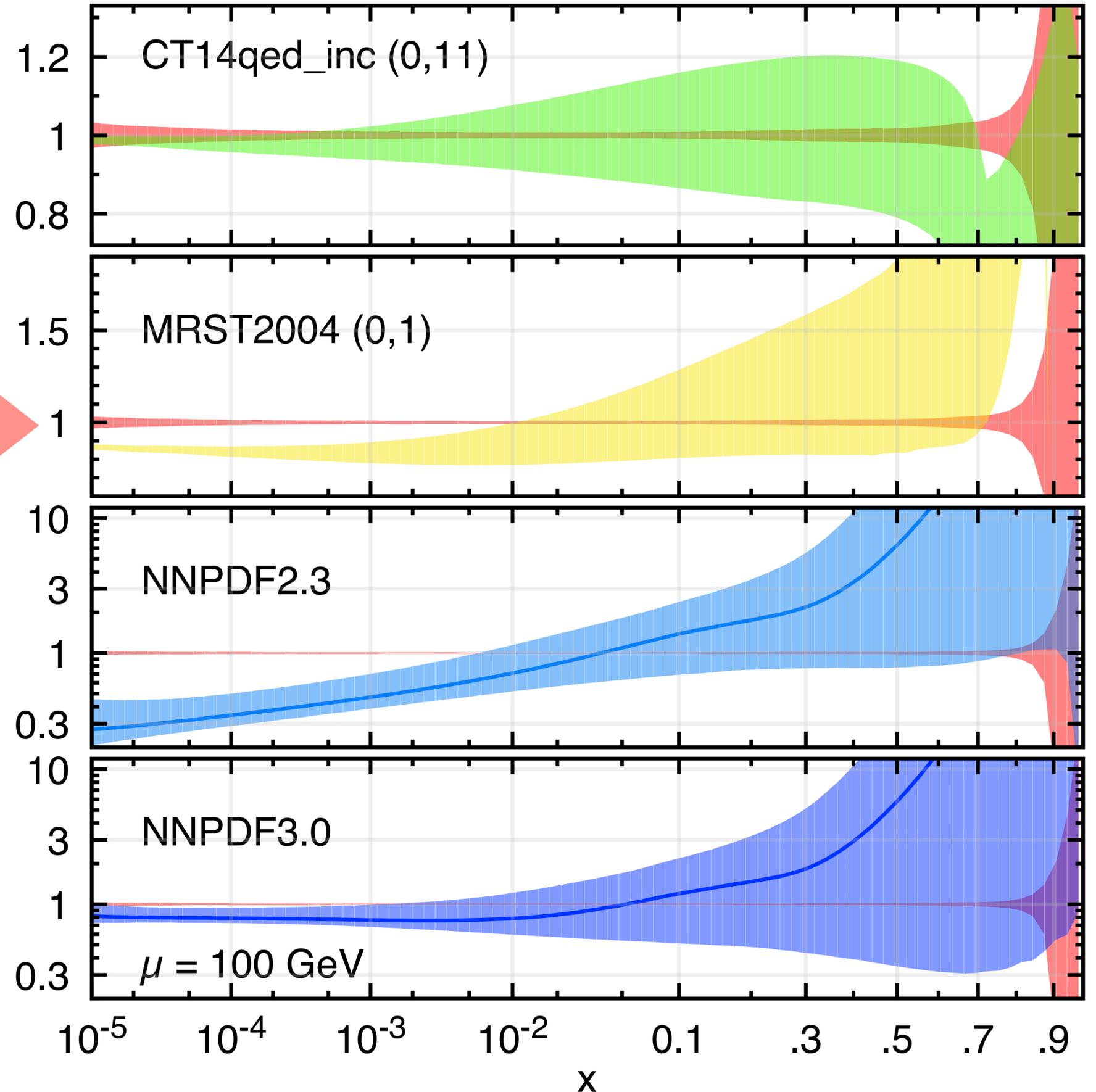
# photon PDF results

- ▶ Model-independent uncertainty (NNPDF) was 50–100%
- ▶ Goes down to  $O(1\%)$  with LUXqed determination



# LUXqed v. other photon PDFs

*LUXqed is  
the red band*



# Impact for Higgs + W production

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---

$pp \rightarrow H W^+ (\rightarrow l^+ \nu) + X$  at 13 TeV

non-photon induced contributions

$91.2 \pm 1.8$  fb

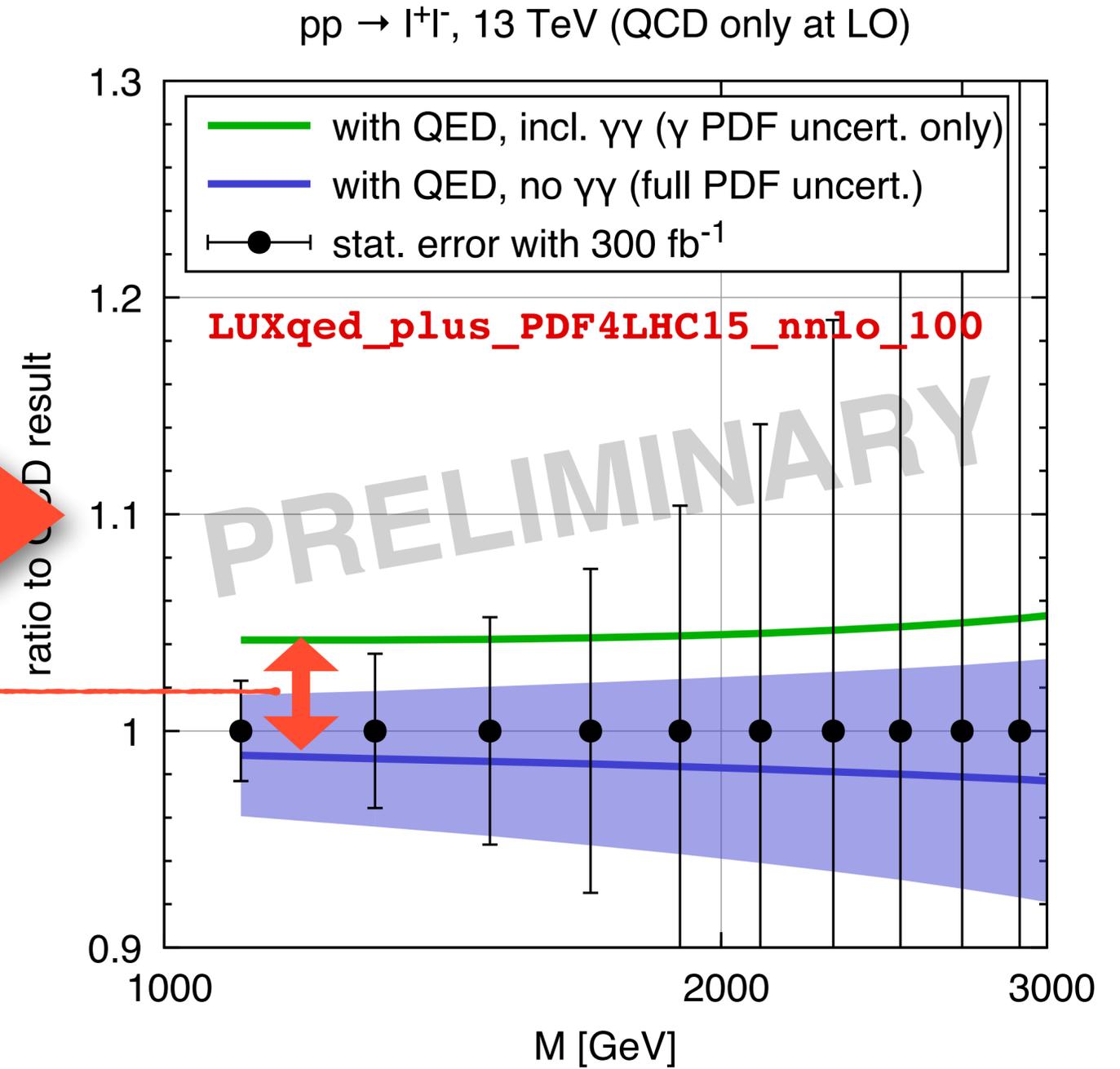
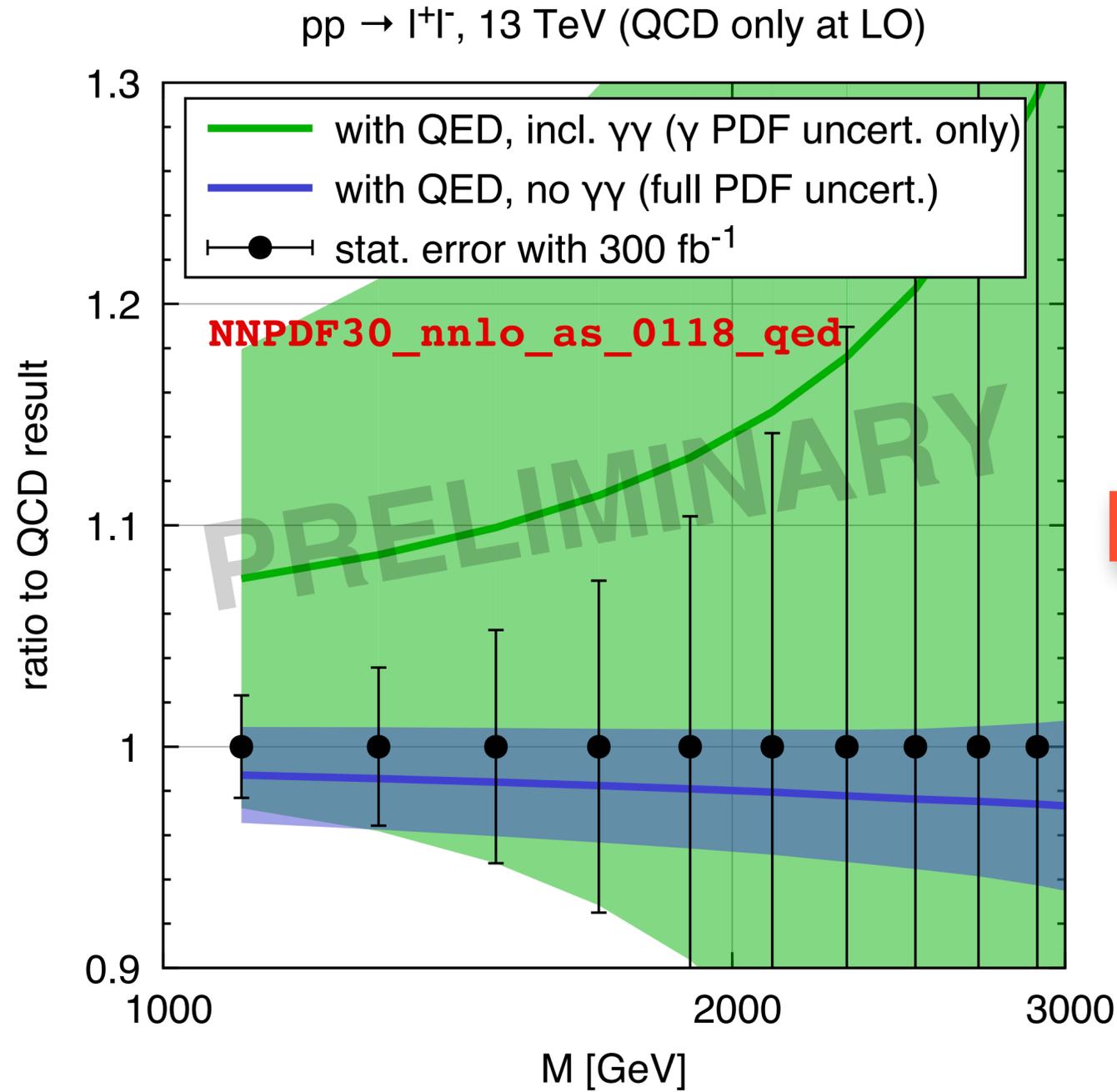
photon-induced contribs (NNPDF23)

$6.0^{+4.4}_{-2.9}$  fb

photon-induced contribs (LUXqed)

$4.4 \pm 0.1$  fb

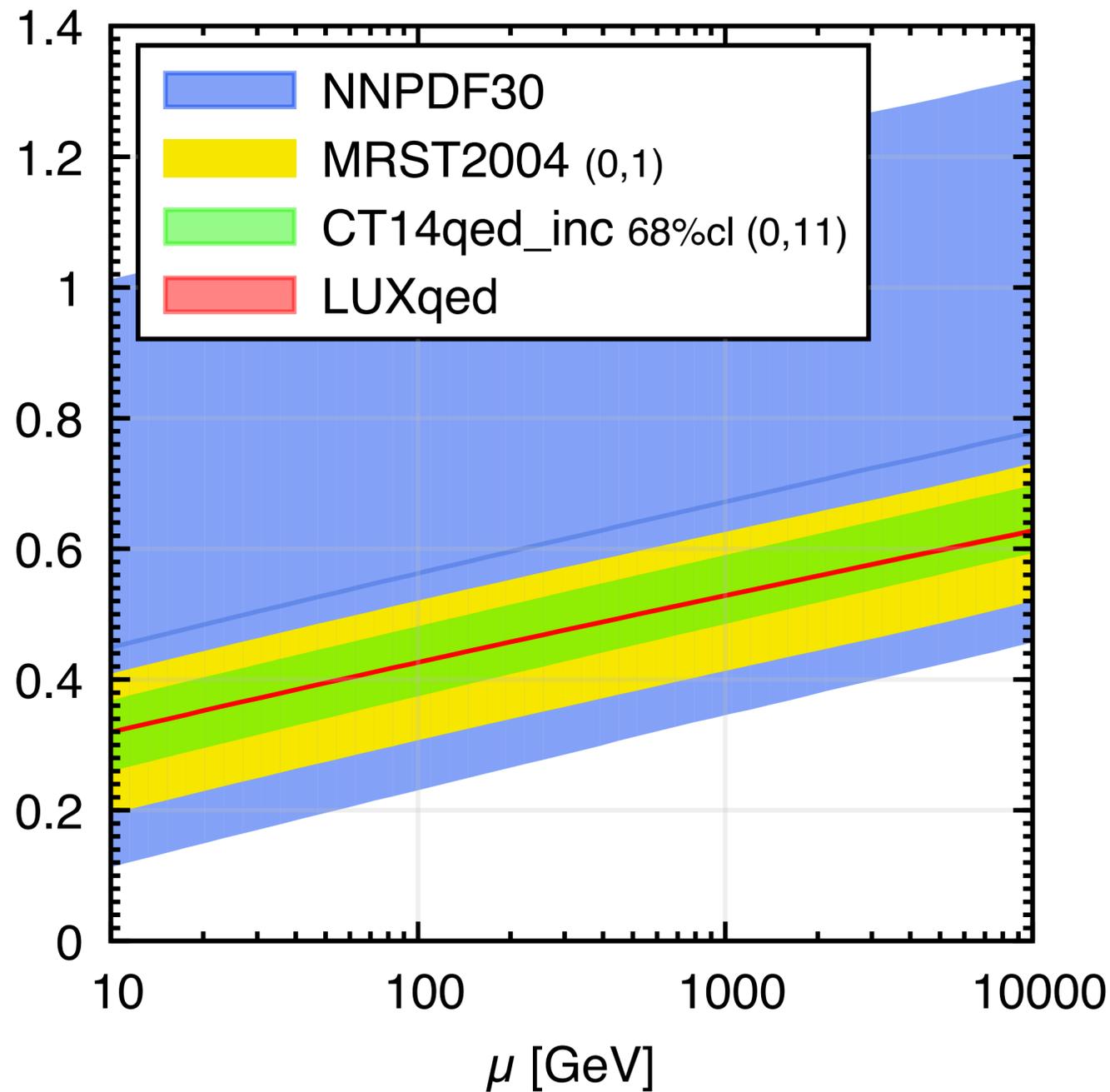
# di-lepton spectrum



$\gamma\gamma$  component has few-% effect on Drell-Yan spectrum; negligible uncertainty

# How much bright is the proton? [ $\gamma$ momentum fraction]

% of proton's momentum carried by photon



momentum ( $\mu = 100$ GeV)	
gluon	$46.8 \pm 0.4\%$
up valence	$18.2 \pm 0.3\%$
down valence	$7.5 \pm 0.2\%$
light sea quarks	$20.7 \pm 0.4\%$
charm	$4.0 \pm 0.1\%$
bottom	$2.5 \pm 0.1\%$
photon	$0.426 \pm 0.003\%$

LUXqed\_plus\_PDF4LHC15\_nnlo\_100

(1+107 members, symmhessian, errors handled by LHAPDF out of the box,

valid for  $\mu > 10$  GeV)

# CONCLUSIONS

# Summary

---

- The photon content of the proton matters starts to matter in many places at LHC
- Electron-proton scattering expts. ( $\rightarrow$  structure functions  $F_2$  and  $F_L$ ) have effectively been measuring proton's  $\gamma$  content for 50 years...
- Photon distribution can be determined from that data to within 1–2% — i.e. as precise as any “QCD” parton
- Available through LHAPDF as LUXqed\_plus\_PDF4LHC15\_nnlo\_100

**EXTRAS**

**physical picture**

# photon distribution from fast-moving charged particle

---

Point-like particle, e.g. electrons

► Fermi, Z. Phys. 1924 ; von Weizsäcker, Z. Phys 1924; Williams, Phys.Rev. 1934

$$f_{\gamma/e}(x, \mu^2) = \frac{\alpha}{2\pi} \left[ \frac{1 + (1-x)^2}{x} \log \left( \frac{1-x}{x^2} \frac{\mu^2}{m_e^2} \right) - 2 \frac{1-x - x^2 \frac{m_e^2}{\mu^2}}{x} \right]$$

# photon distribution from fast-moving charged particle

---

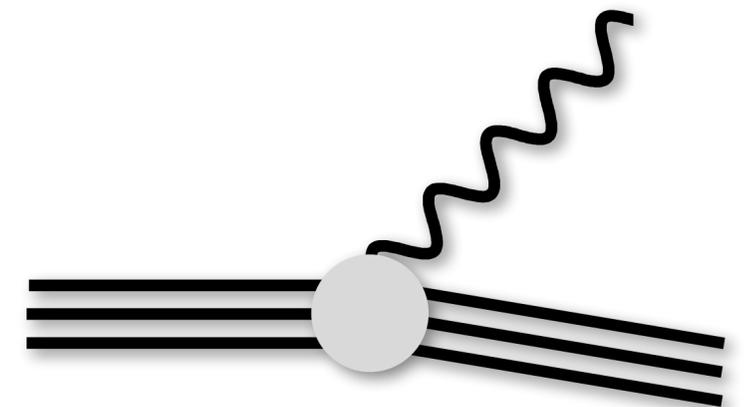
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**But protons are not point-like...**

- Budnev, Ginzburg, Meledin & Serbo, Phys.Rept. 1974  
→ an answer for the case where the proton remains intact after photon emission



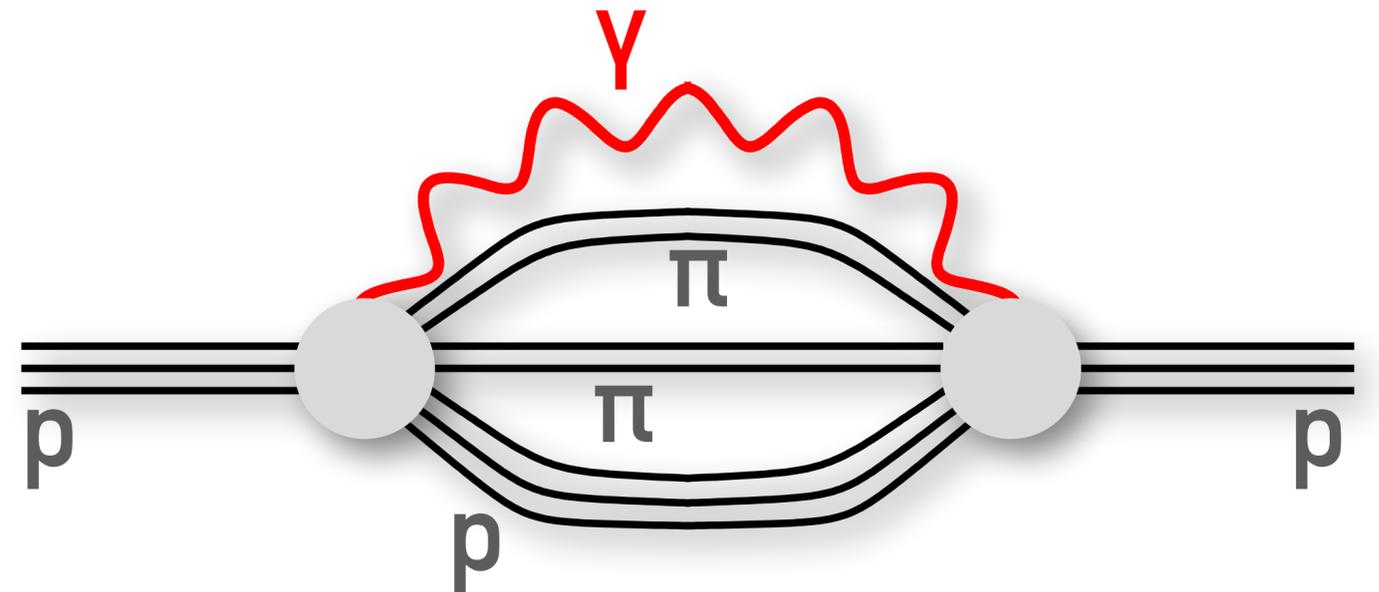
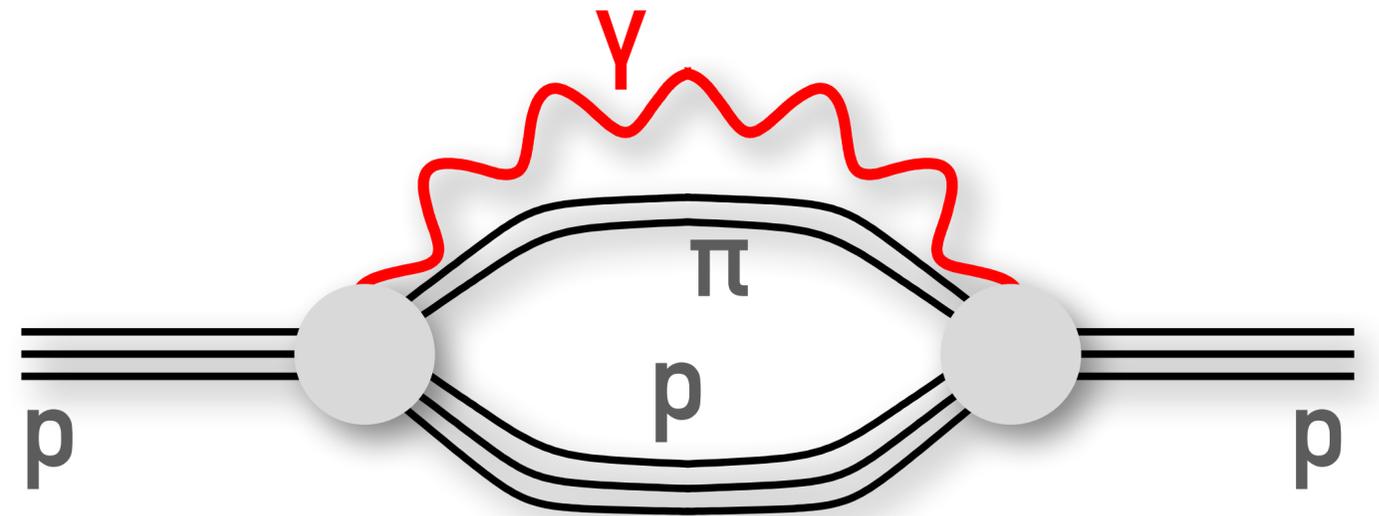
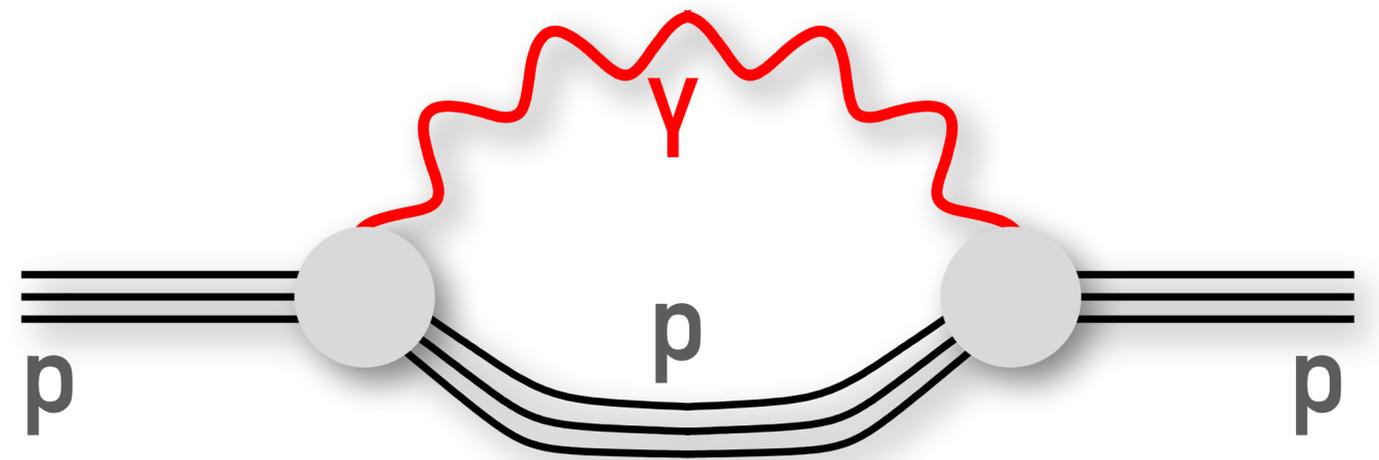
*given in terms of “proton form factors” (measurable from elastic ep scattering)*

# “number of photons” inside a proton?

Proton constantly fluctuates in & out of different Fock states, some of which have a photon.

The states with extra pions (etc.) are called the **inelastic** component

Intrinsically non-perturbative.

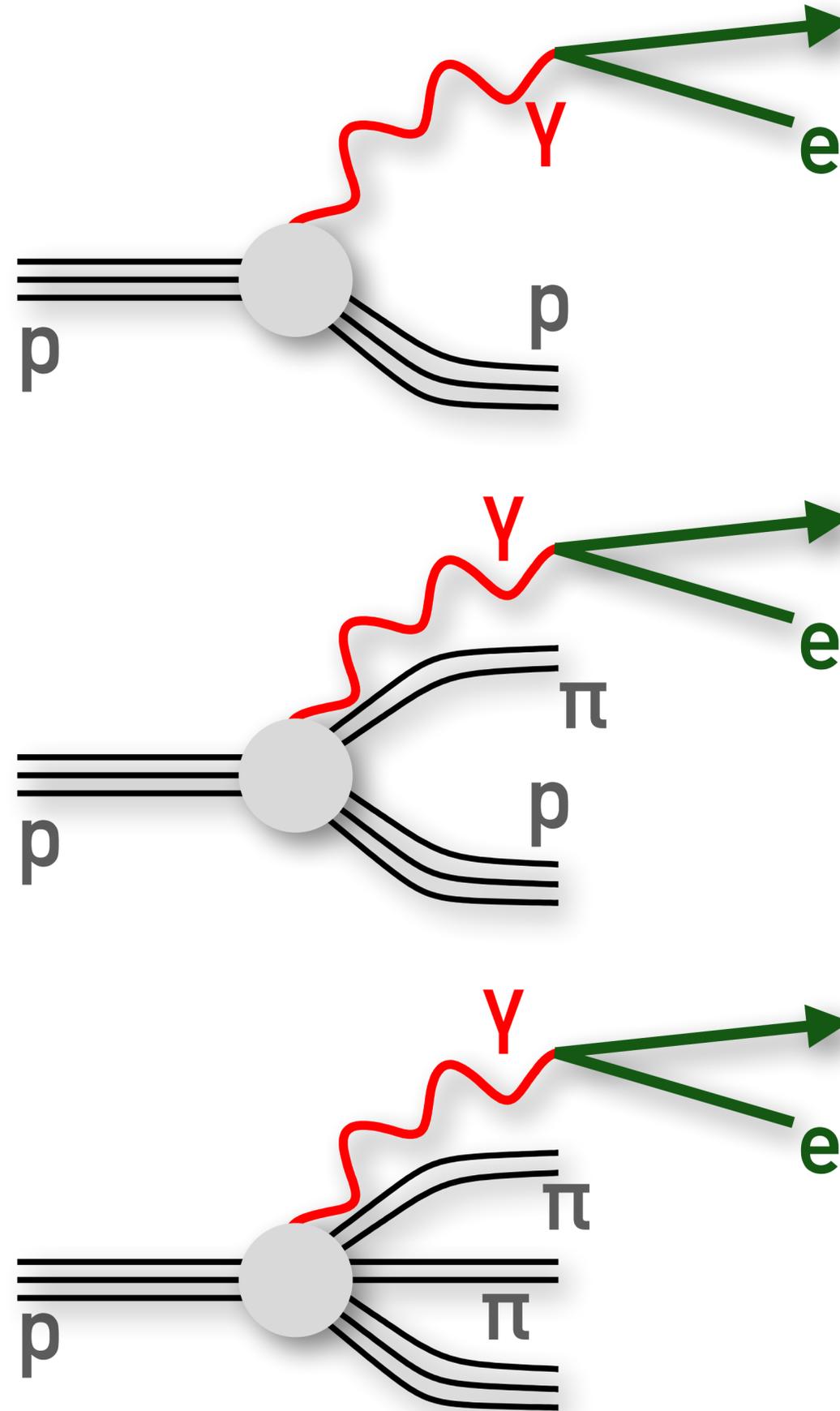


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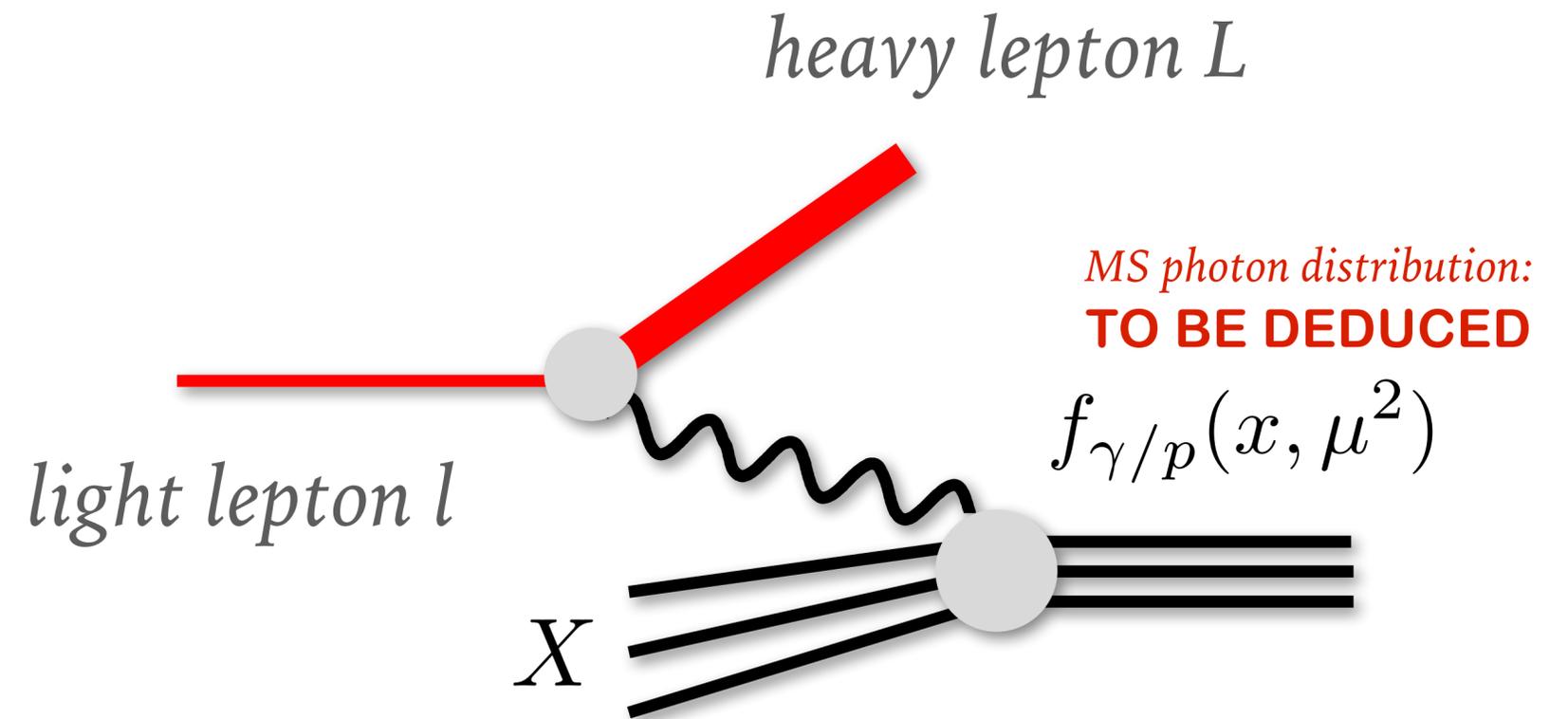
# derivation of LUX master formula

# How do you deduce photon distribution? Two approaches

---

## Approach 1

- Imagine production of BSM heavy lepton ( $L$ ), from a light neutral lepton  $l$  and a photon  $\gamma$ , i.e.  $l\gamma \rightarrow L$
- Calculate  $lp \rightarrow L + X$  in terms of (known) structure functions
- Calculate  $lp \rightarrow L + X$  in terms of (unknown) photon distribution
- **Equate them** to get the photon distribution



$$\mathcal{L}_{\text{int}} = (e/\Lambda) \bar{L} \sigma^{\mu\nu} F_{\mu\nu} l$$

# How do you deduce photon distribution? Two approaches

---

## Approach 2

- exploits operator definition of the photon distribution
- is especially powerful for going to higher order

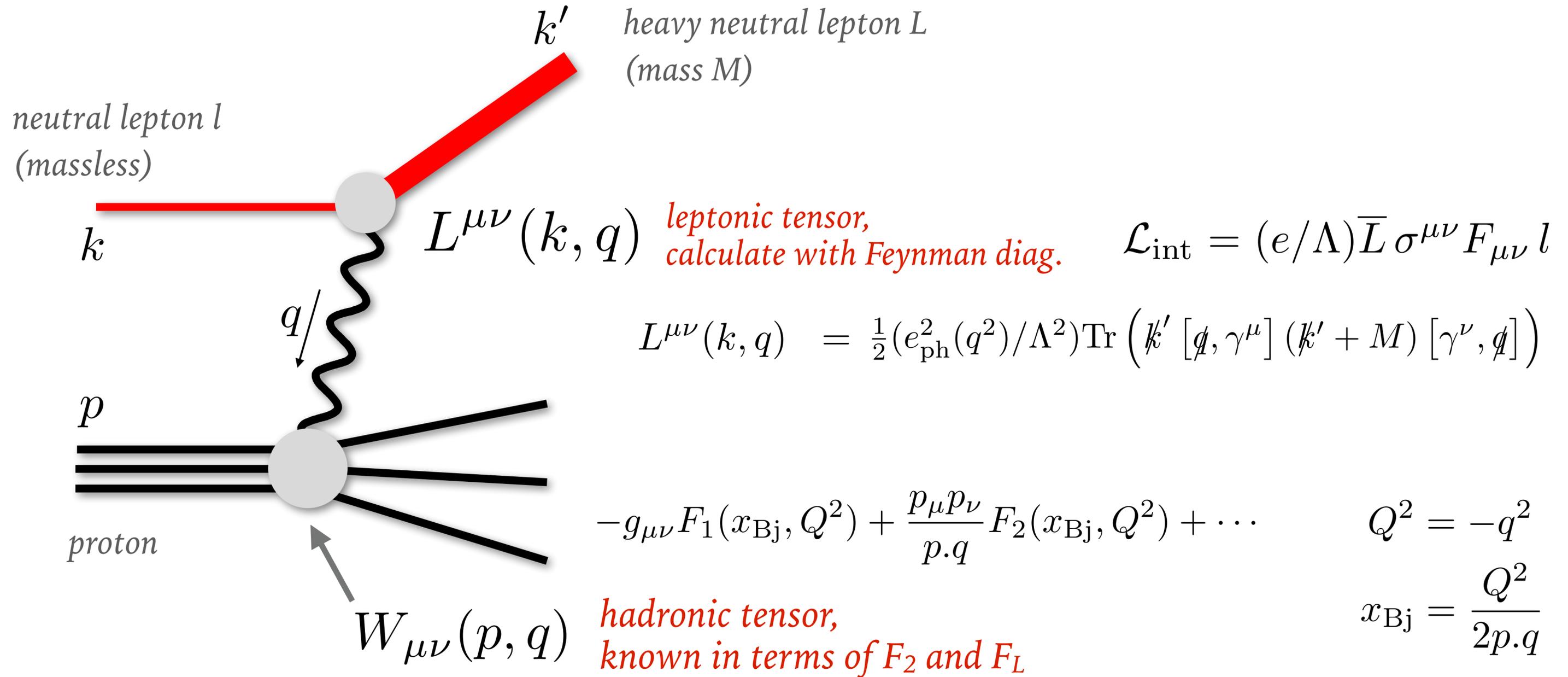
$$f_{\gamma}(x, \mu) = -\frac{1}{4\pi x p^+} \int_{-\infty}^{\infty} dw e^{-ixwp^+} \times \\ \langle p | F^{n\lambda}(wn) F^n_{\lambda}(0) + F^{n\lambda}(0) F^n_{\lambda}(wn) | p \rangle_c$$

Results with this approach  
are in progress

(and consistent with approach 1!)

# STEP 1

work out a cross section (exact) in terms of  $F_2$  and  $F_L$  struct. fns.



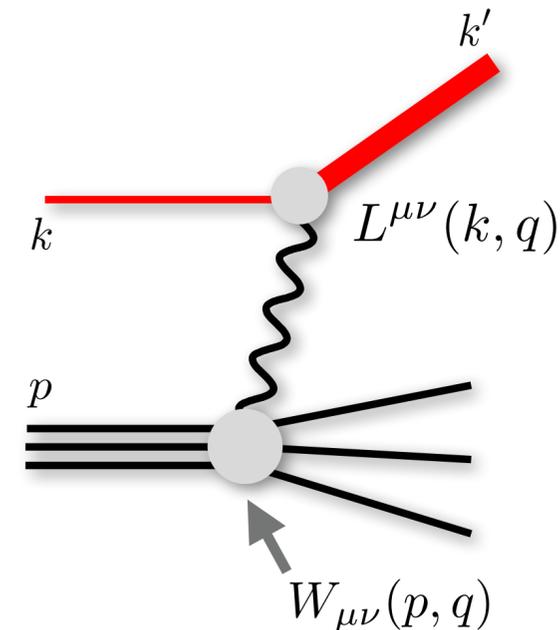
$$\sigma = \frac{1}{4p \cdot k} \int \frac{d^4q}{(2\pi)^4 q^4} e_{\text{ph}}^2(q^2) [4\pi W_{\mu\nu} L^{\mu\nu}(k, q)] \times 2\pi\delta((k - q)^2 - M^2)$$

# Cross section in terms of structure functions

- ▶ Lagrangian of interaction:  $\mathcal{L}_{\text{int}} = (e/\Lambda)\bar{L}\sigma^{\mu\nu}F_{\mu\nu}l$   
(magnetic moment coupling)
- ▶ Using leptons neutral and taking  $\Lambda$  large, ensure that only single-photon exchange is relevant
- ▶ **Answer is exact up to  $1/\Lambda$  corrections**

$$\sigma = \frac{c_0}{2\pi} \int_x^{1-\frac{2xm_p}{M}} \frac{dz}{z} \int_{Q_{\text{min}}^2}^{Q_{\text{max}}^2} \frac{dQ^2}{Q^2} \alpha_{\text{ph}}^2(-Q^2) \left[ \left( 2 - 2z + z^2 + \frac{2x^2m_p^2}{Q^2} + \frac{z^2Q^2}{M^2} - \frac{2zQ^2}{M^2} - \frac{2x^2Q^2m_p^2}{M^4} \right) F_2(x/z, Q^2) + \left( -z^2 - \frac{z^2Q^2}{2M^2} + \frac{z^2Q^4}{2M^4} \right) F_L(x/z, Q^2) \right]$$

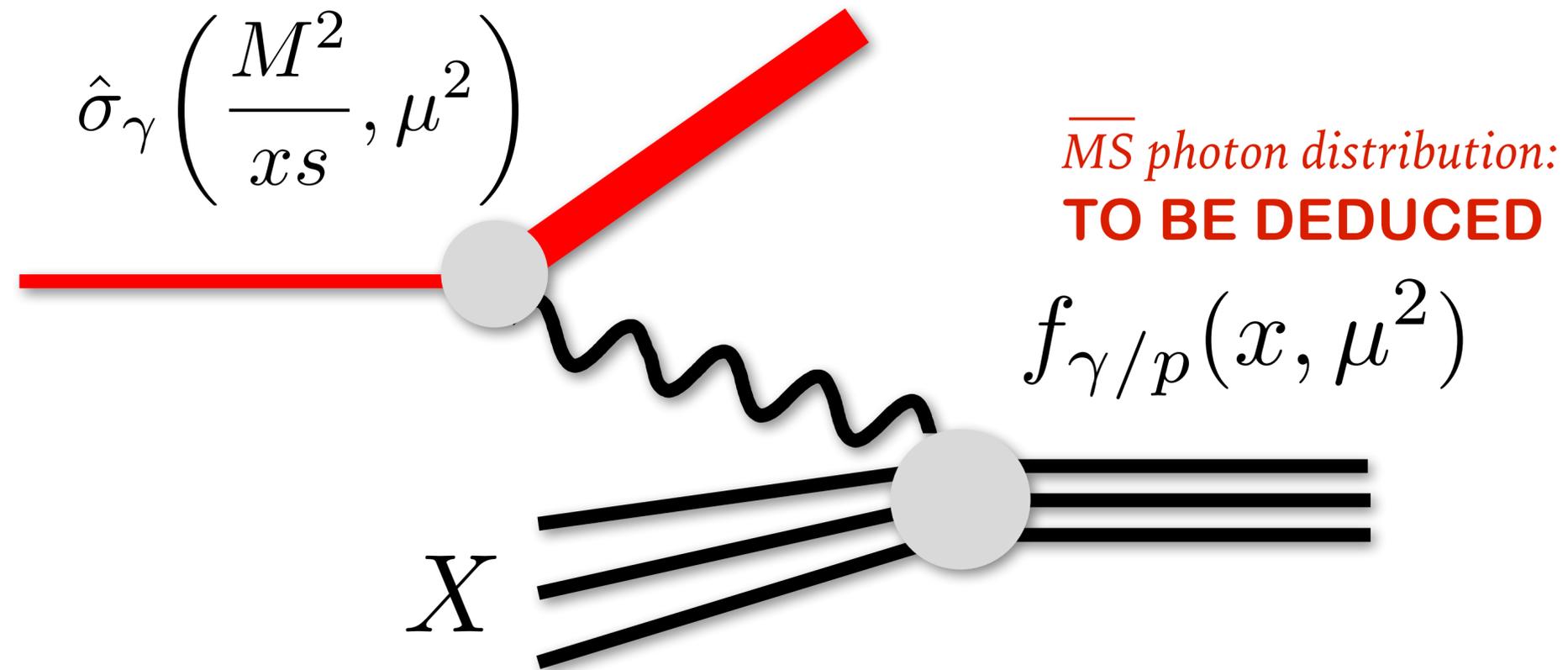
$$c_0 = 16\pi^2/\Lambda^2$$



## STEP 2

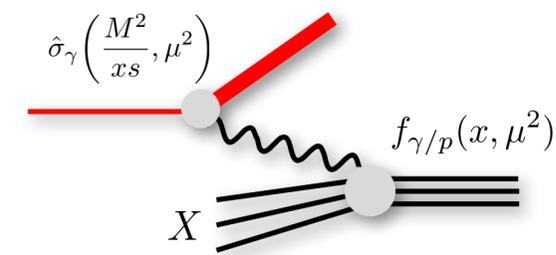
work out same cross section in terms of a photon distribution

*hard-scattering cross section  
calculate in collinear factorisation*



$$\sigma = c_0 \sum_a \int \frac{dx}{x} \hat{\sigma}_a \left( \frac{M^2}{xS}, \mu^2 \right) x f_{a/p}(x, \mu^2)$$

# Cross section in terms of structure functions



- Hard cross section driven by the photon distribution at LO

$$\hat{\sigma}_a(z, \mu^2) = \alpha(\mu^2)\delta(1-z)\delta_{a\gamma} + \frac{\alpha^2(\mu^2)}{2\pi} \left[ -2 + 3z + \right. \\ \left. + zp_{\gamma q}(z) \ln \frac{M^2(1-z)^2}{z\mu^2} \right] \sum_{i \in \{q, \bar{q}\}} e_i^2 \delta_{ai} + \dots, \quad (5)$$

- Quarks and gluons come in at higher orders

# Accuracy aim

---

- Take quark and gluon distributions  $\sim O(1)$
- $\alpha$  is QED coupling,  $\alpha_s$  is QCD coupling,  $L = \ln \mu^2/m_p^2$ 
  - Take  $L \sim 1/\alpha_s$ , so all  $(\alpha_s L)^n \sim 1$
  - Think of  $\alpha \sim (\alpha_s)^2$
- To first order, photon distribution  $\sim (\alpha L)$
- we aim to control all terms:
  - $\alpha L (\alpha_s L)^n$  [LO]
  - $\alpha_s \alpha L (\alpha_s L)^n \equiv \alpha (\alpha_s L)^n$  [NLO — extra  $\alpha_s$  or  $1/L$ ]
  - $\alpha^2 L^2 (\alpha_s L)^n$  [NLO — extra  $\alpha L$ ]
- Matching done at large  $M^2$  and  $\mu^2$  to eliminate higher twists

## Cross checks & literature comparisons

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- Repeat calculation for a different process ( $\gamma p \rightarrow H + X$ , via  $\gamma\gamma \rightarrow H$ ). Intermediate results differ, **final photon distribution is identical**.
- Substitute elastic-scattering component of  $F_2$  and  $F_L$ :

$$F_2^{\text{el}} = \frac{[G_E(Q^2)]^2 + [G_M(Q^2)]^2 \tau}{1 + \tau} \delta(1 - x),$$

$$F_L^{\text{el}} = \frac{[G_E(Q^2)]^2}{\tau} \delta(1 - x),$$

$$\tau = Q^2 / (4m_p^2)$$

and reproduce widely-used **Equivalent Photon Approximation** with electric ( $G_E$ ) and magnetic ( $G_M$ ) Sachs proton form factors

*Budnev et al., Phys.Rept.15(1975)181*

## Cross checks & literature comparisons

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- $\mu^2$  derivative of our answer should reproduce known DGLAP QCD-QED splitting functions
- At LO, this is trivial.
- At NLO we get relations between QED-QCD splitting functions ( $P$ ) and DIS coefficient functions ( $C$ )

$$P_{\gamma q}^{(1,1)} = e_q^2 \left[ p_{\gamma q} \otimes C_{2q} - h \otimes C_{Lq} + (\bar{p}_{\gamma q} - h) \otimes P_{qq}^{(1,0)} \right] ,$$

$$P_{\gamma g}^{(1,1)} = \sum_{q, \bar{q}} e_q^2 \left[ p_{\gamma q} \otimes C_{2g} - h \otimes C_{Lg} + (\bar{p}_{\gamma q} - h) \otimes P_{qg}^{(1,0)} \right] ,$$

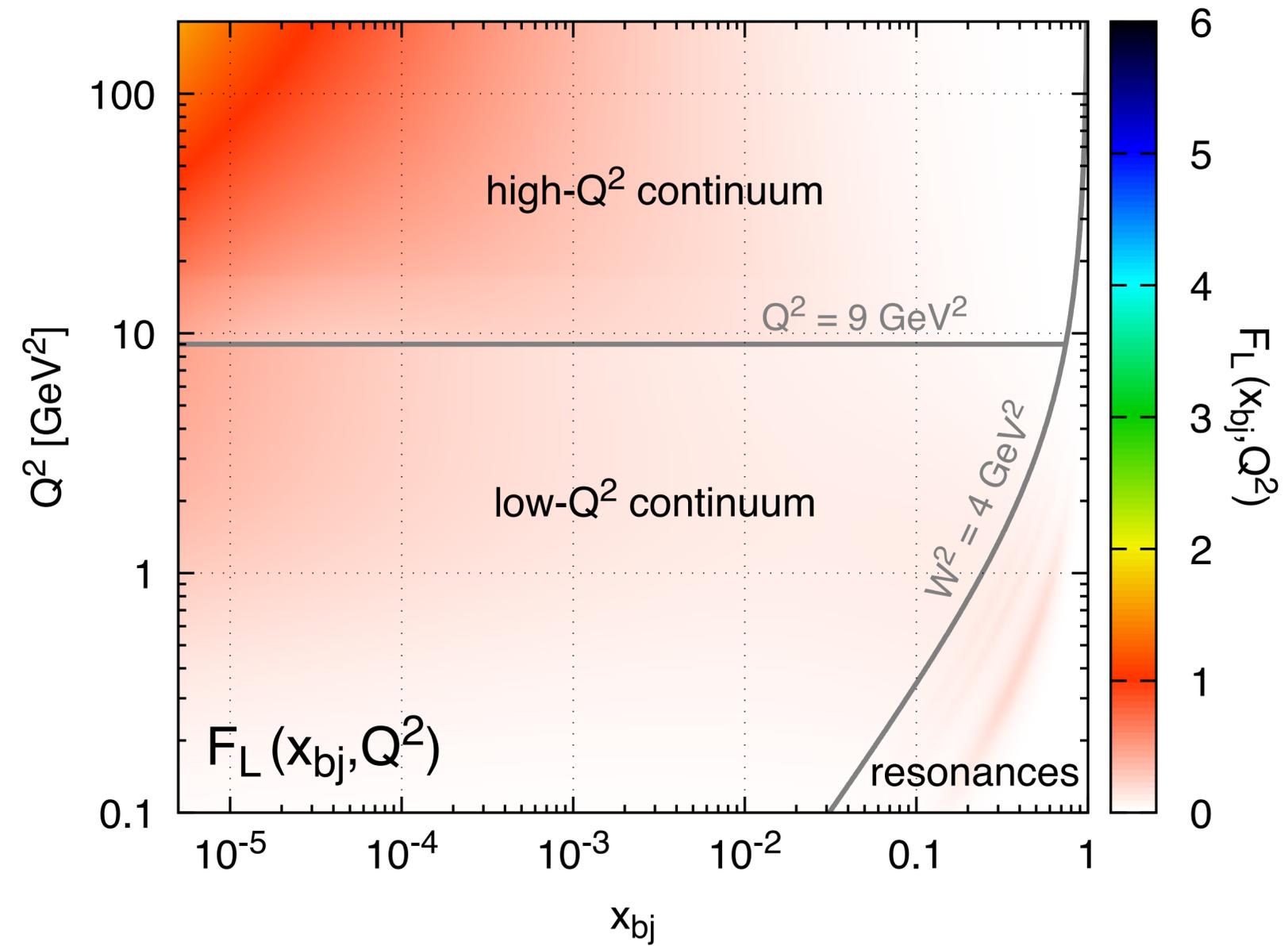
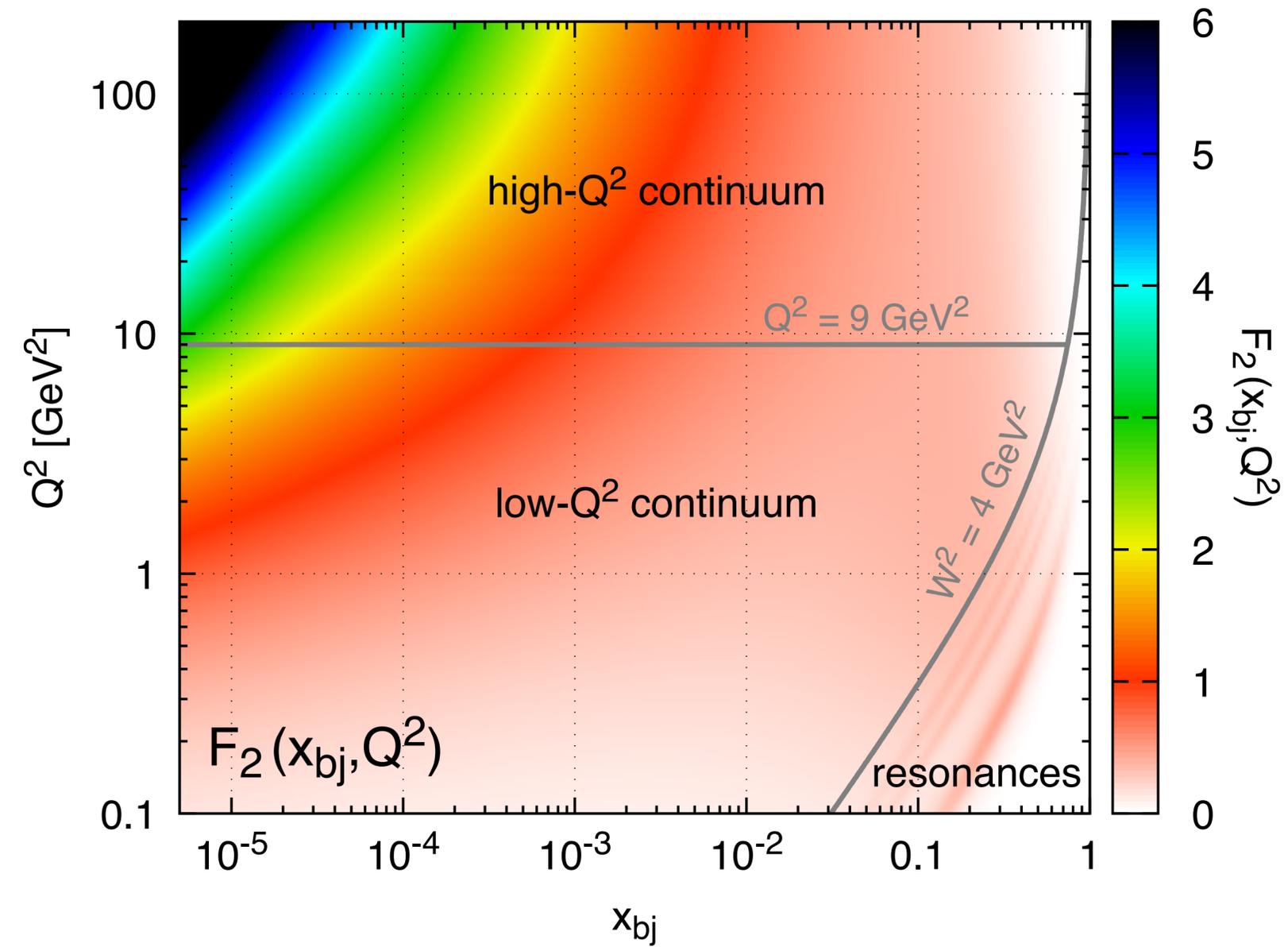
$$P_{\gamma\gamma}^{(1,1)} = (2\pi)^2 b_\alpha^{(1,2)} \delta(1-x) = -C_F N_C \sum_q e_q^2 \delta(1-x)$$

$$h(z) \equiv z \text{ and } \bar{p}_{\gamma q}(z) \equiv p_{\gamma q}(z) \ln \frac{1}{1-z}$$

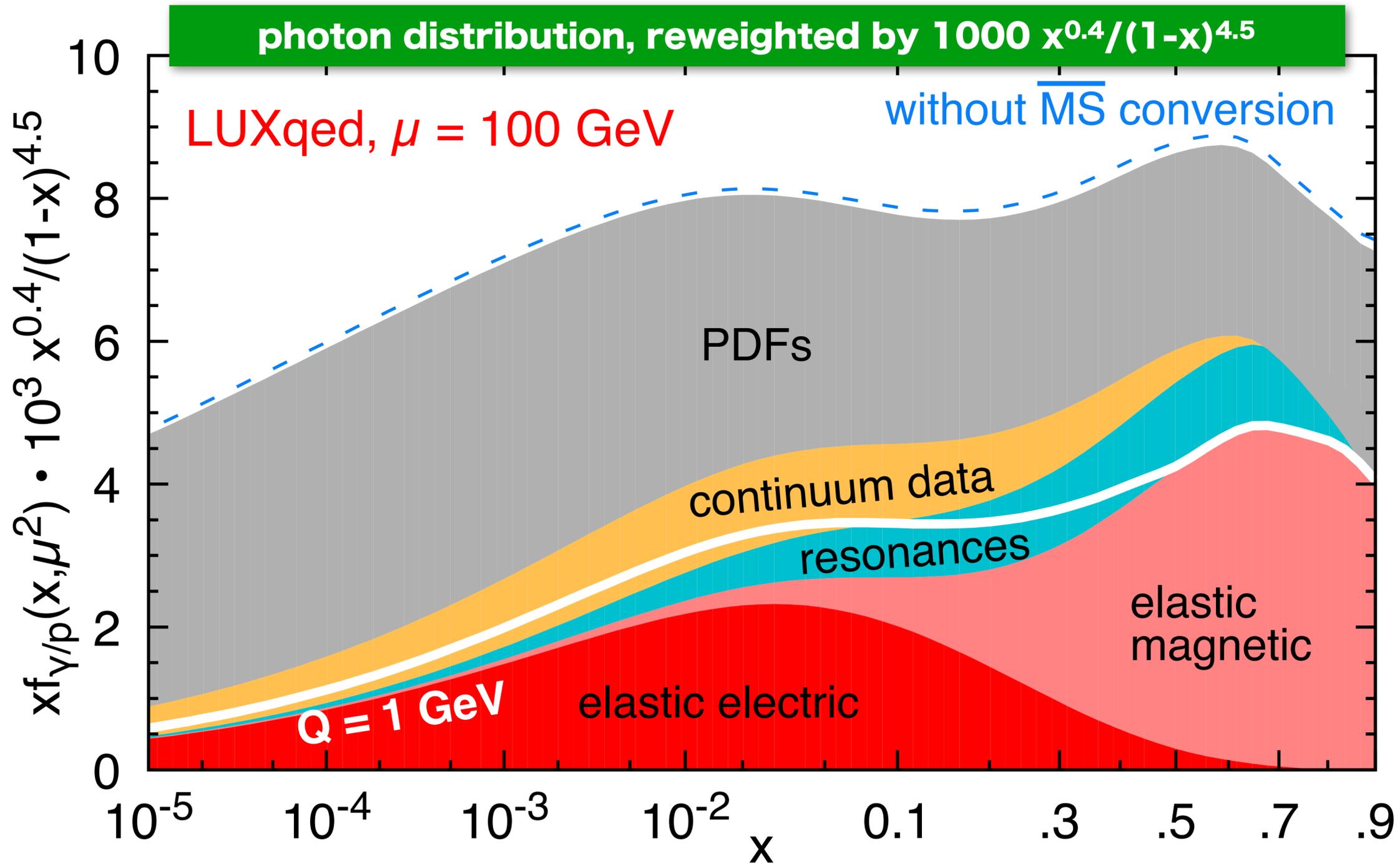
- These **agree with de Florian, Sborlini & Rodrigo results**

# contributions & uncertainties

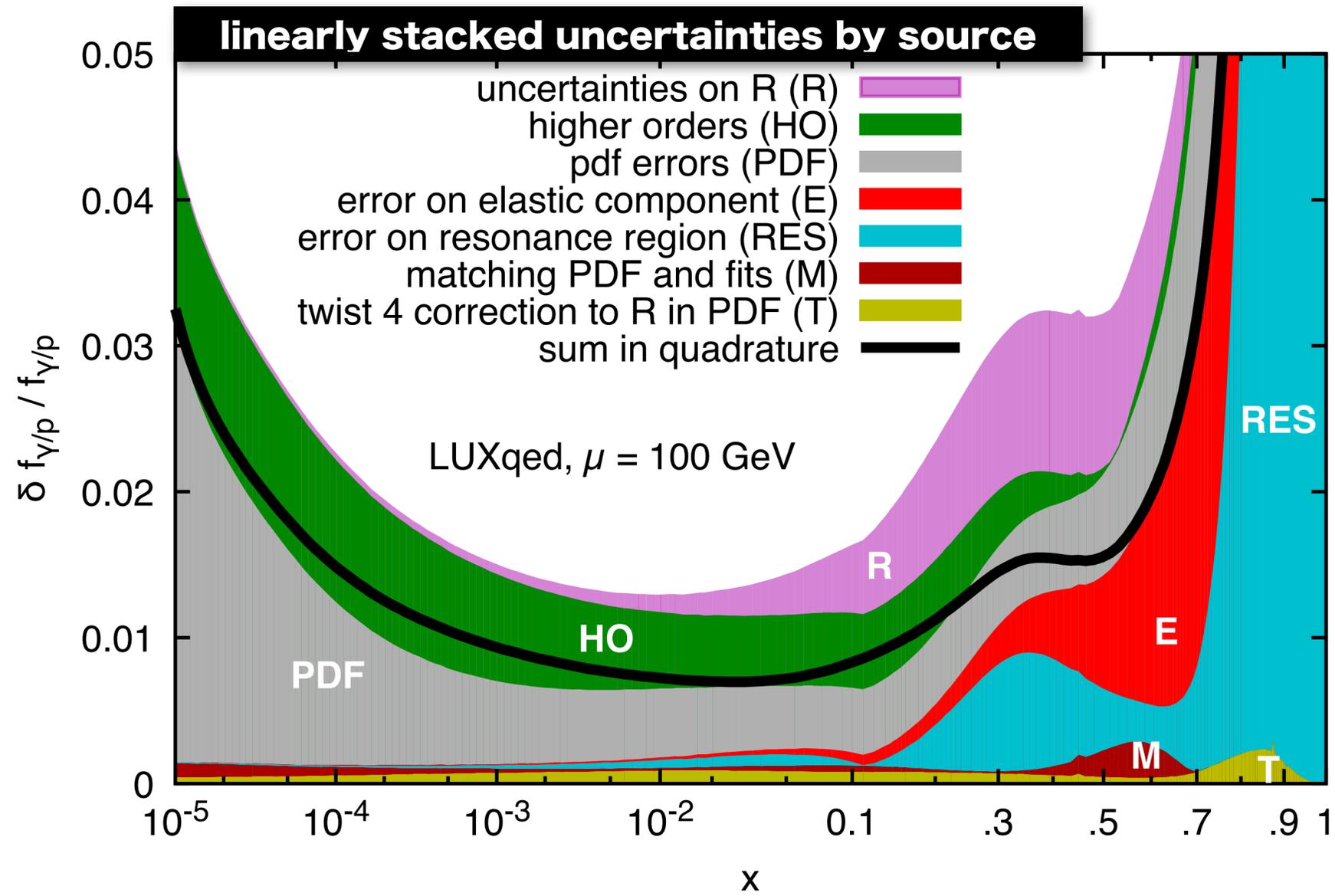
# $F_2$ and $F_L$ in our parametrisation



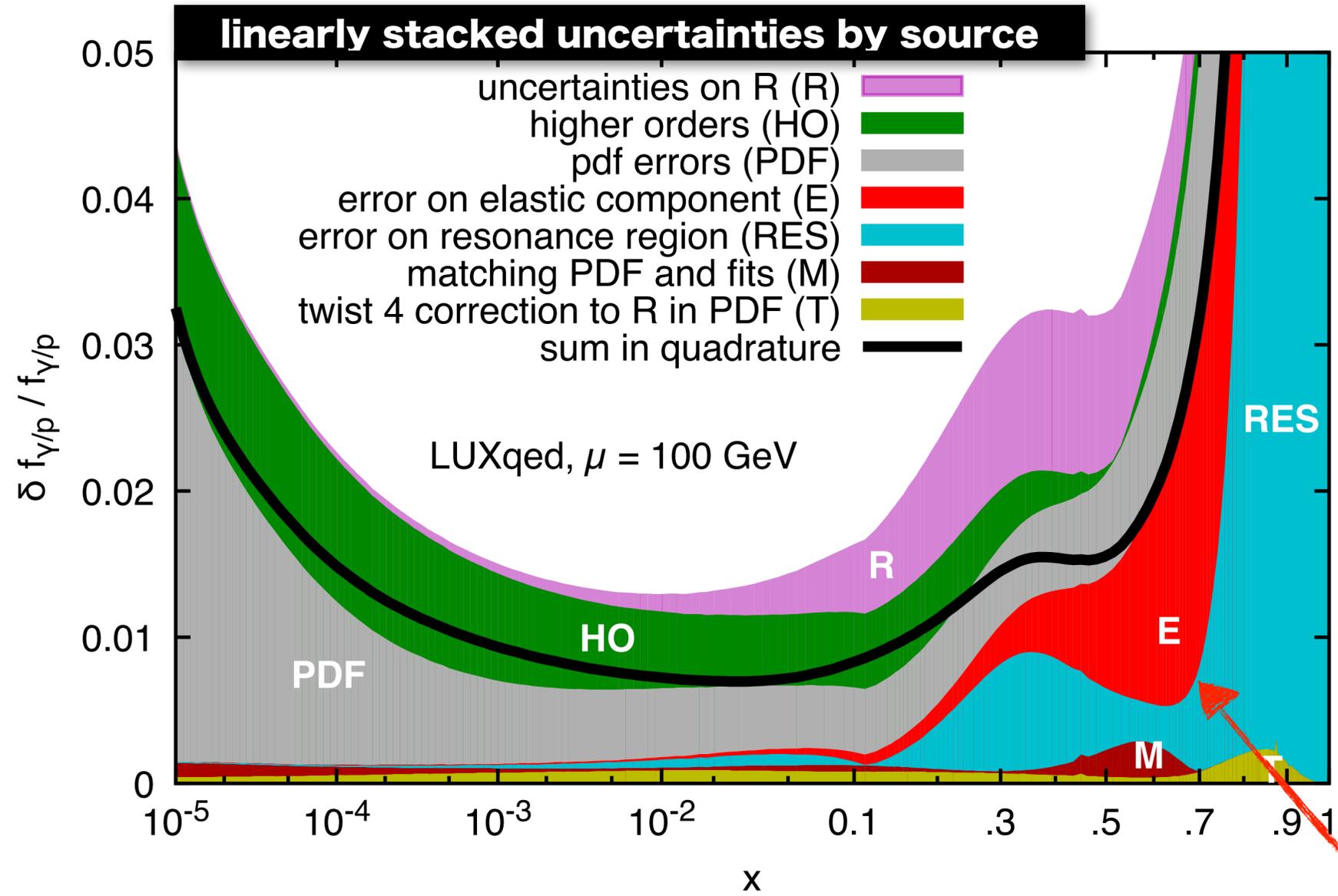
# SEPARATE CONTRIBUTIONS TO PHOTON PDF



# photon uncertainties (aim to be conservative & pragmatic)

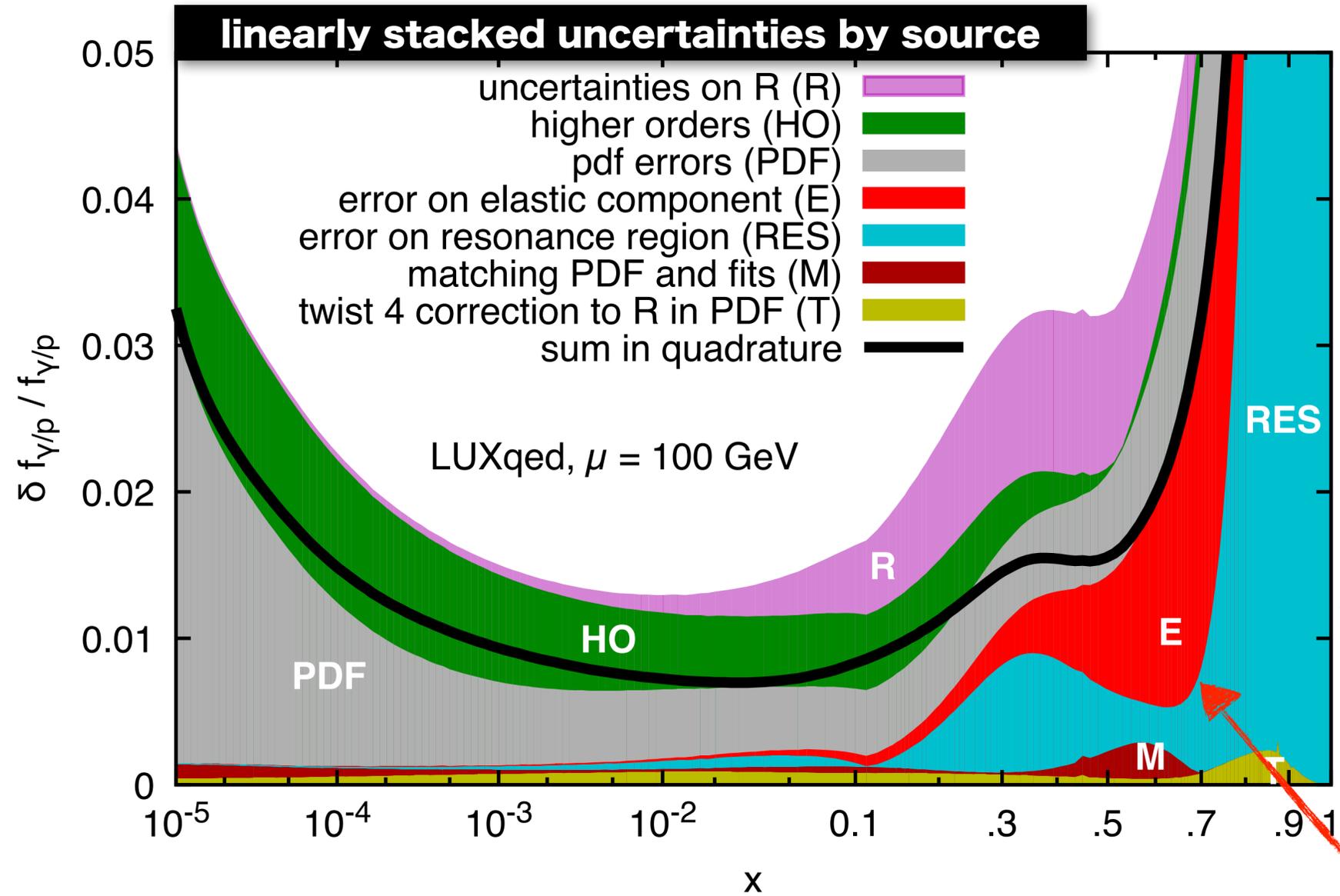


# photon uncertainties (aim to be conservative & pragmatic)



uncertainty on elastic component  
(quoted  $\oplus$   
unpol./pol.)

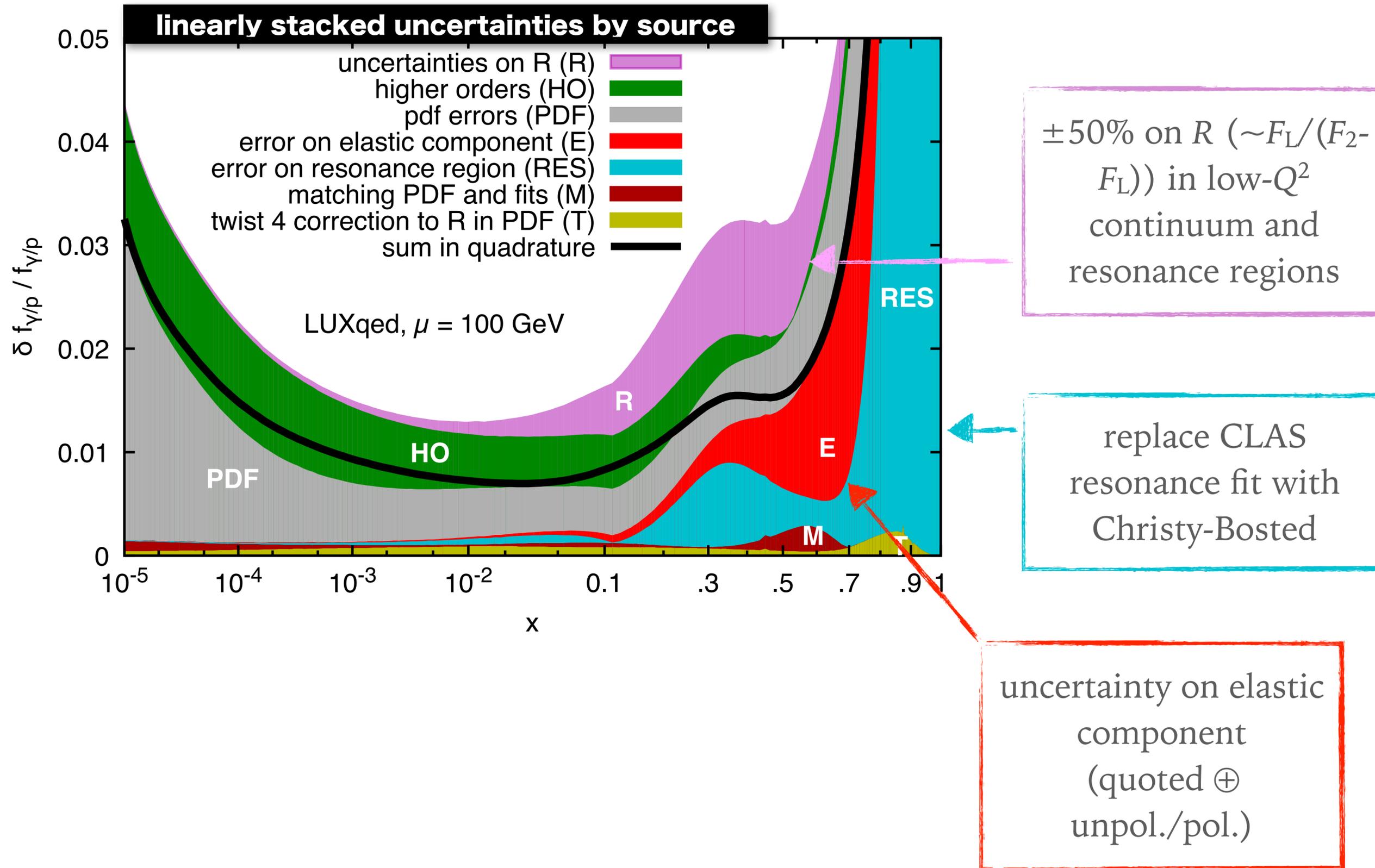
# photon uncertainties (aim to be conservative & pragmatic)



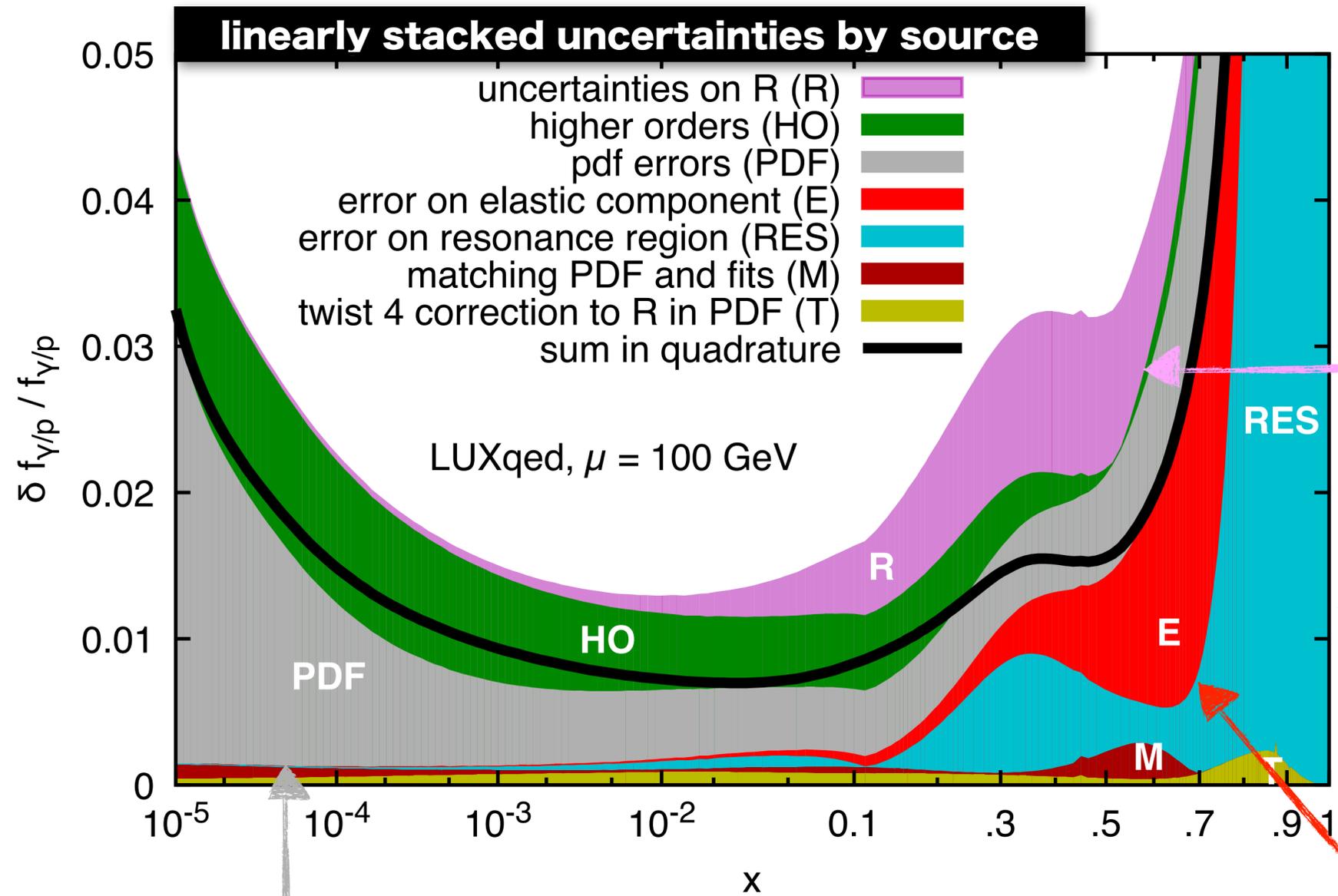
replace CLAS resonance fit with Christy-Bosted

uncertainty on elastic component (quoted  $\oplus$  unpol./pol.)

# photon uncertainties (aim to be conservative & pragmatic)



# photon uncertainties (aim to be conservative & pragmatic)



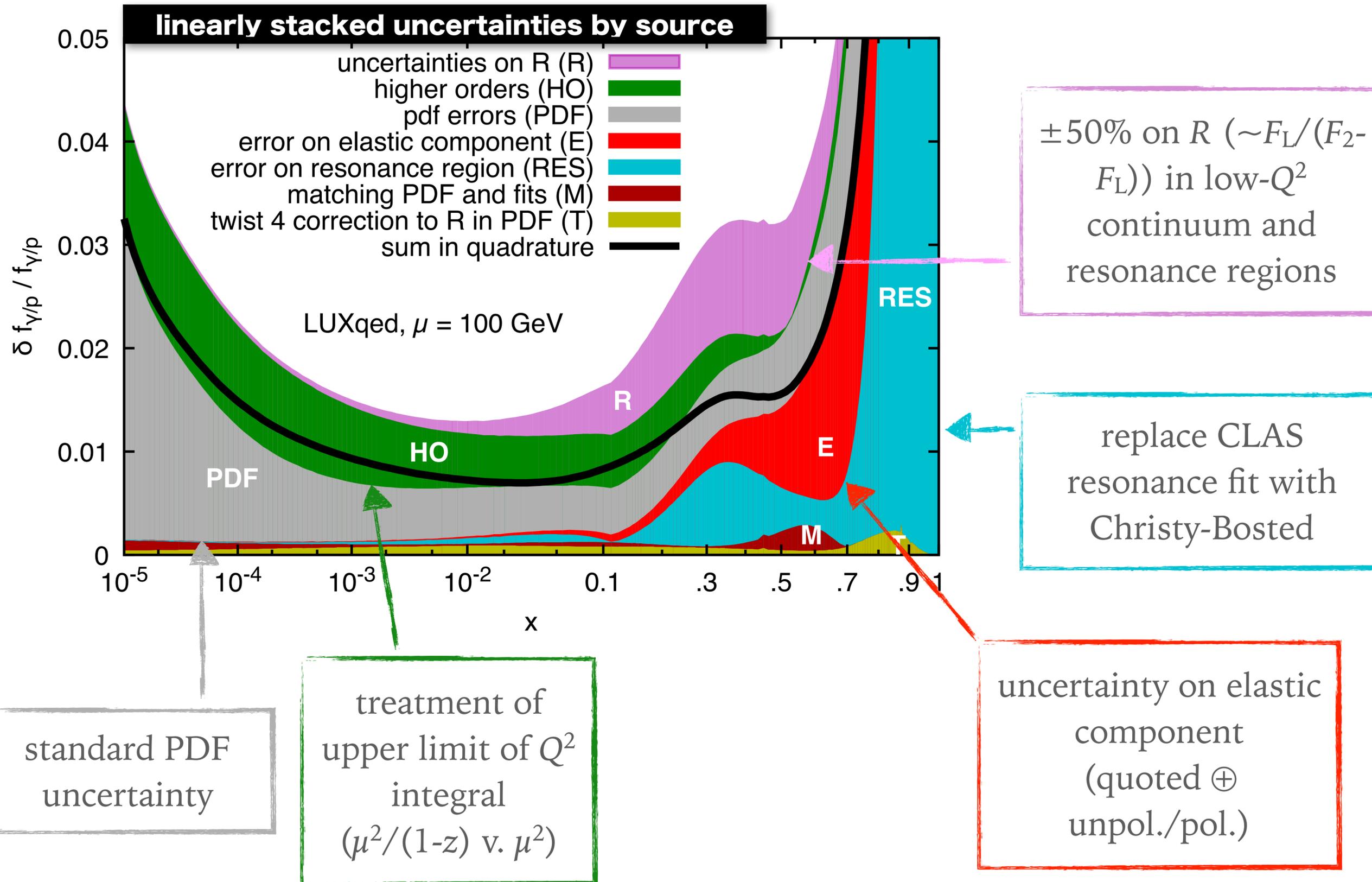
$\pm 50\%$  on R ( $\sim F_L / (F_2 - F_L)$ ) in low- $Q^2$  continuum and resonance regions

replace CLAS resonance fit with Christy-Bosted

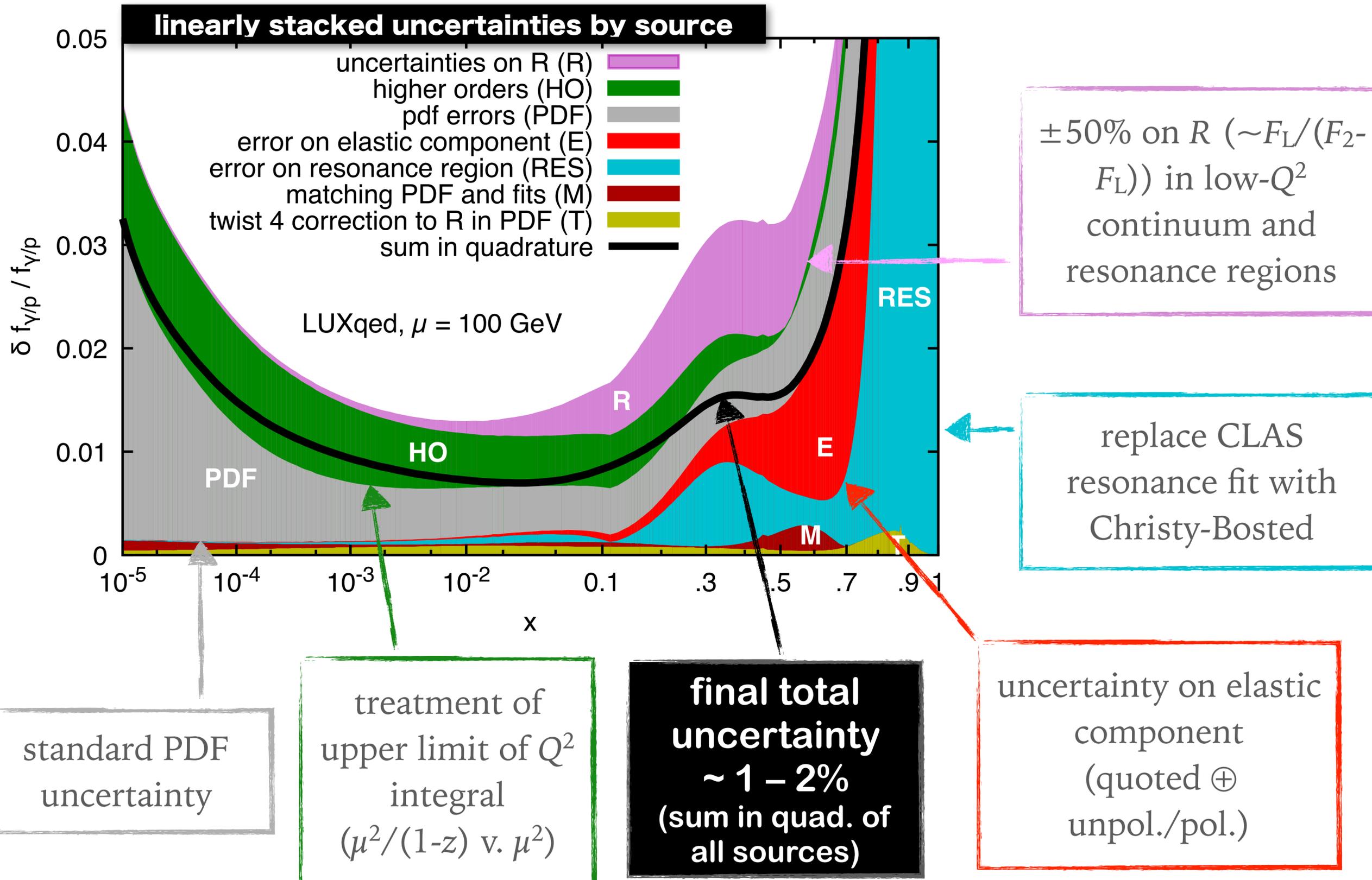
uncertainty on elastic component (quoted  $\oplus$  unpol./pol.)

standard PDF uncertainty

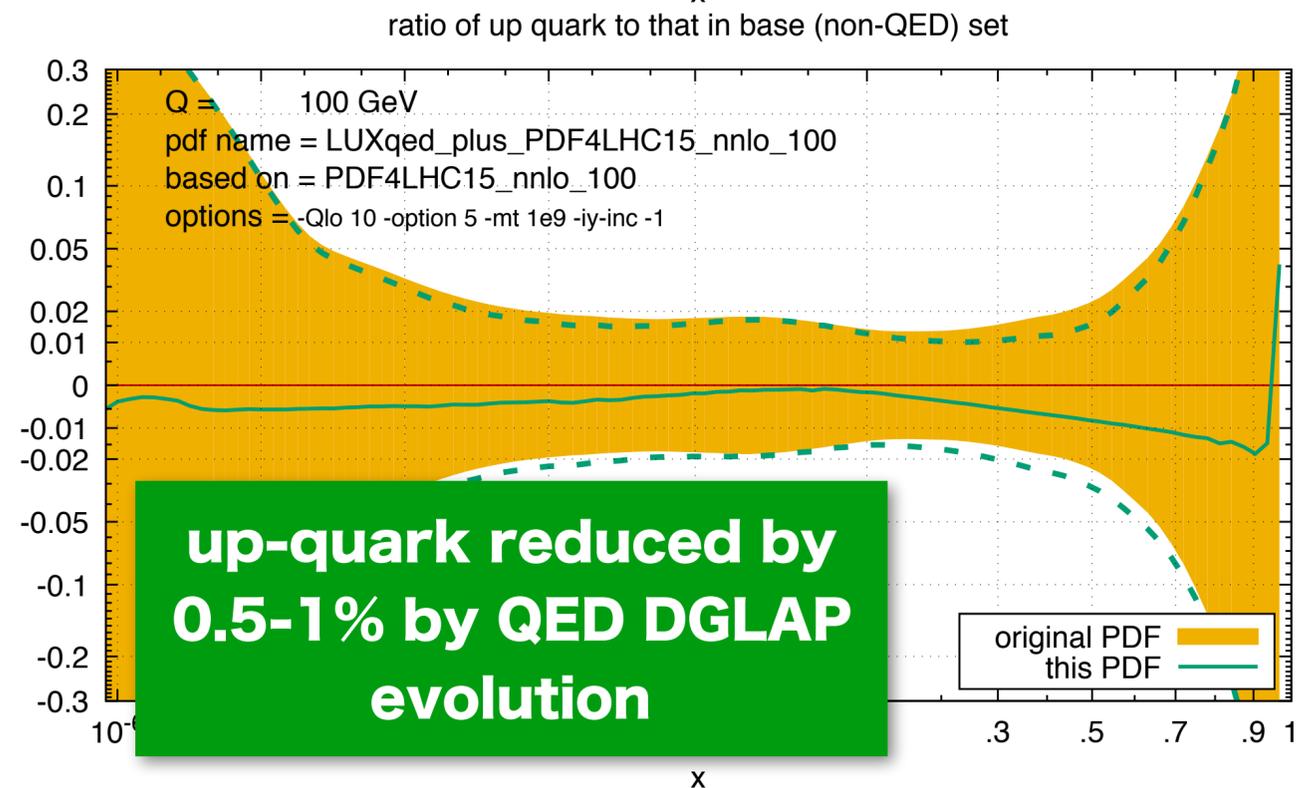
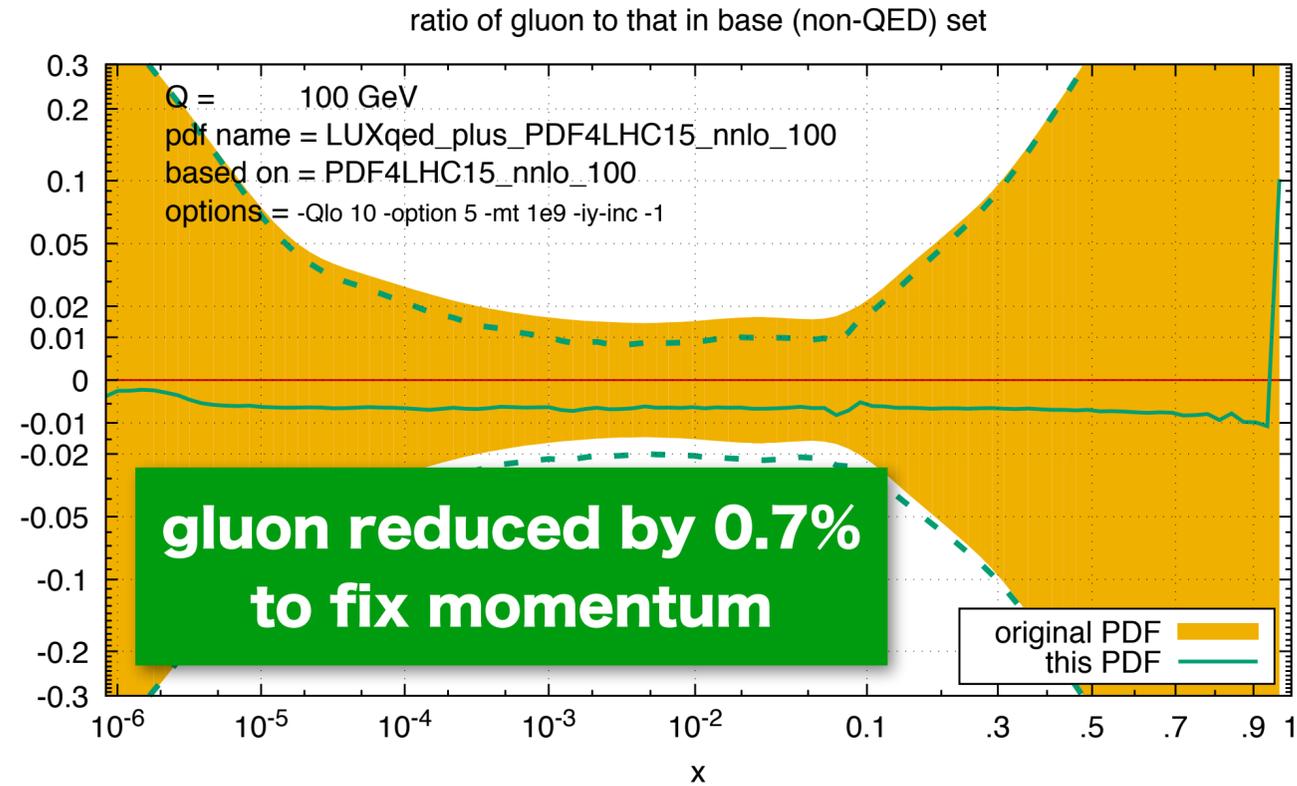
# photon uncertainties (aim to be conservative & pragmatic)



# photon uncertainties (aim to be conservative & pragmatic)



# matching procedure for full set of partons

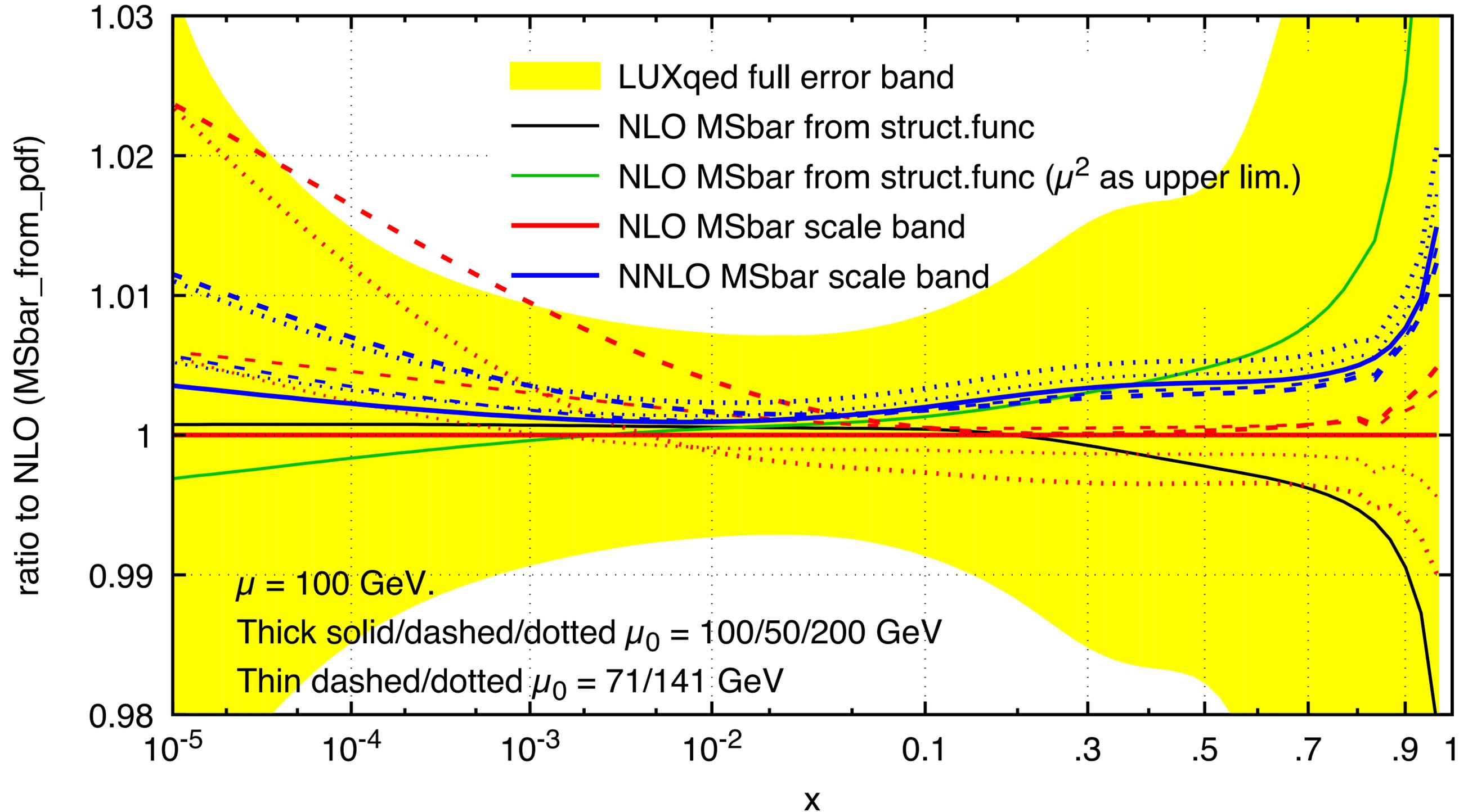


- ▶ evaluate master eqn. for  $\mu = 100$  GeV (with default PDF4LHC15\_nnlo partons)
- ▶ Do  $O(\alpha_s)$  photon evolution down to  $\mu = 10$  GeV (other partons: pure QCD evln.)
- ▶ Adjust momentum sum-rule by rescaling gluon  $g(x) \rightarrow 0.993g(x)$
- ▶ Evolve back up with NNLO-QCD &  $O(\alpha_s)$  QED for all partons

**better approach would be full PDF re-fit for QCD partons incl. EW/QED corrections & LUXqed photon**

# higher-order contributions

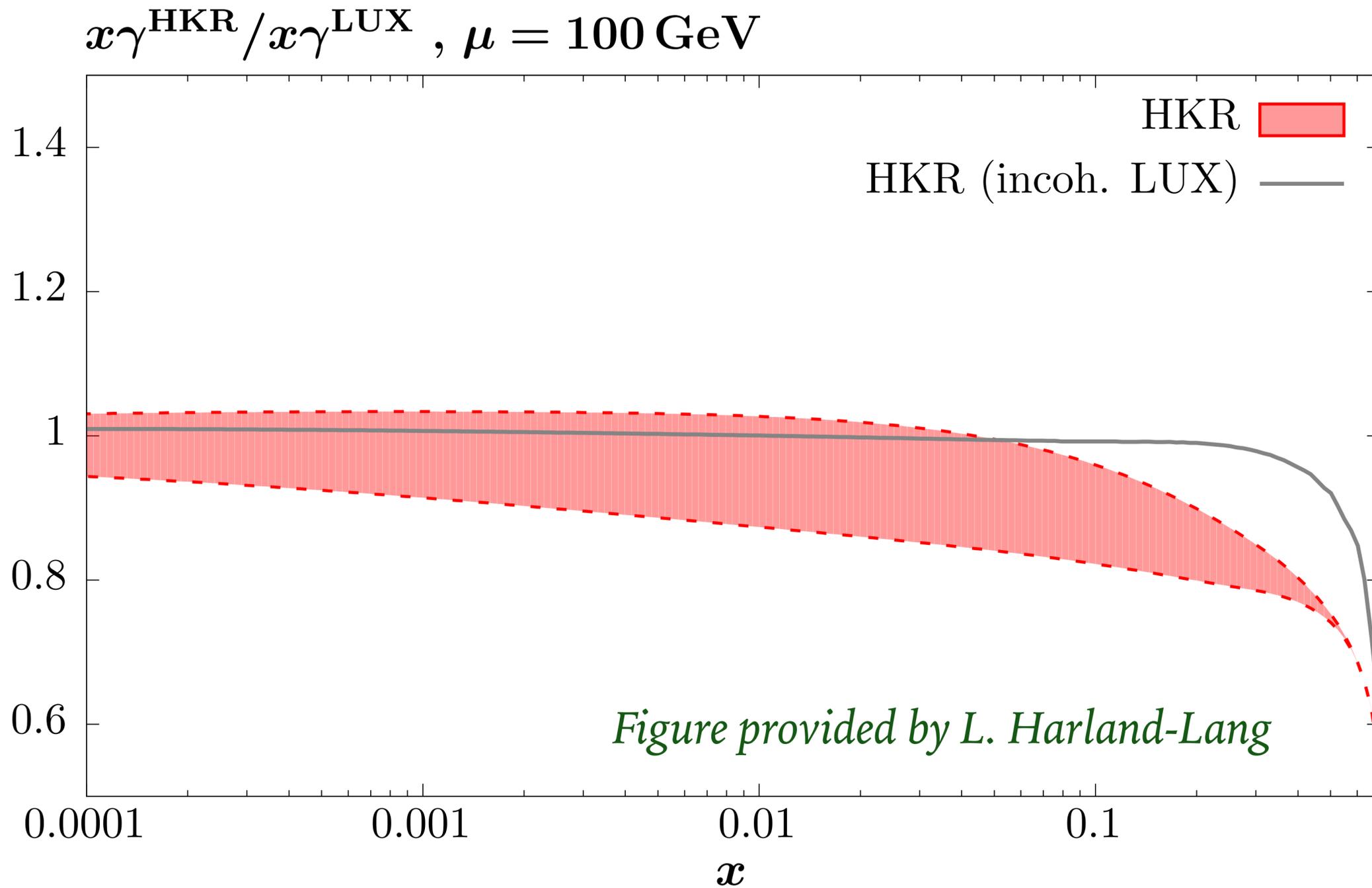
NLO and NNLO (in  $\alpha_s$ ) photon extraction and scale dep.



**LUXqed v. others**

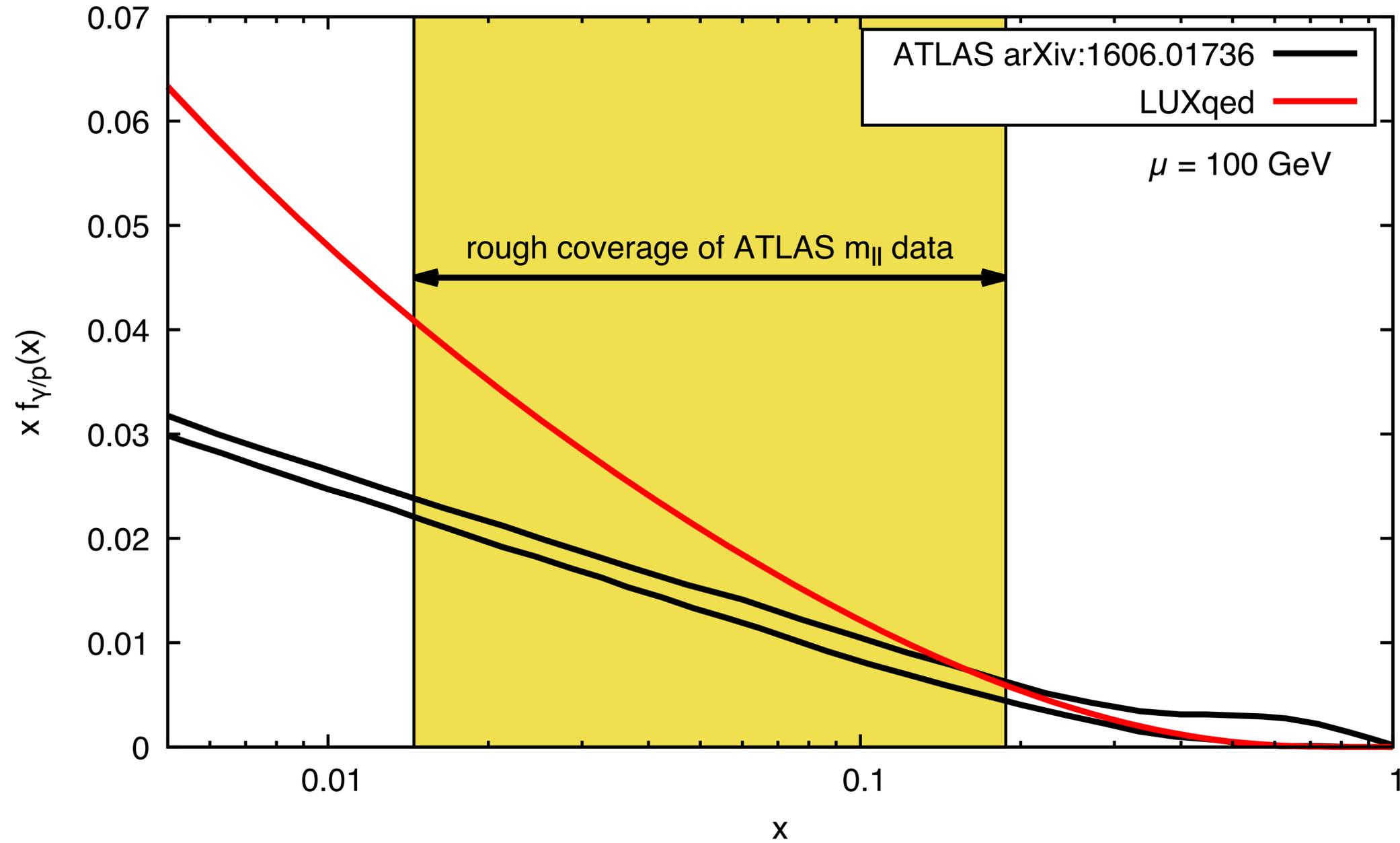
# ratio of HKR (1607.04635) to LUXqed

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HKR based on elastic contribution (dipole approx) + model for inelastic part + evolution

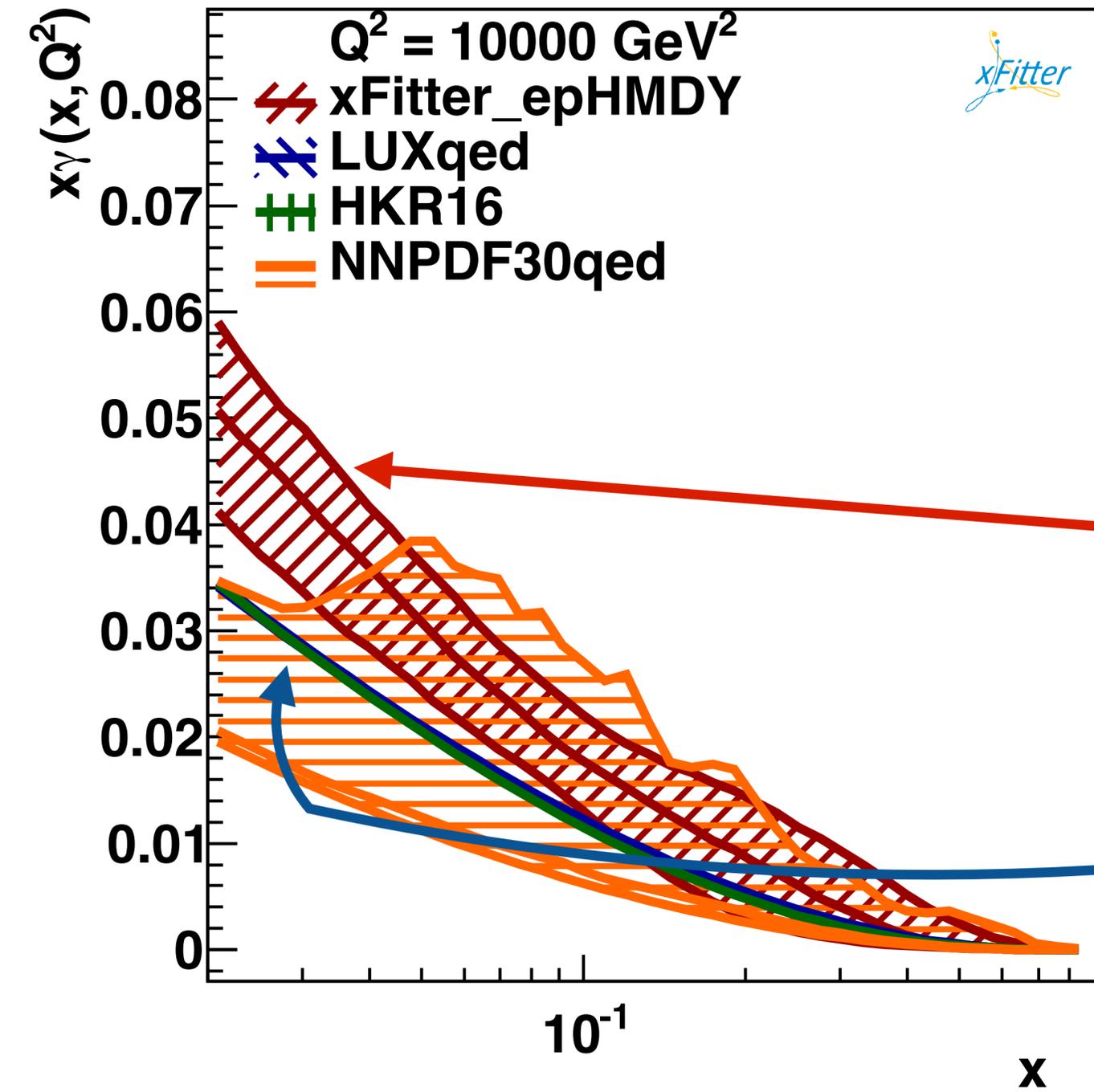
# ratio of ATLAS photon (1606.01736) to LUXqed



ATLAS result based on reweighting of NNPDF23 with high-mass ( $M_{ll} > 116 \text{ GeV}$ ) data.

**Fit is below LUXqed**

later fit (1701.08553) to same data



fit to

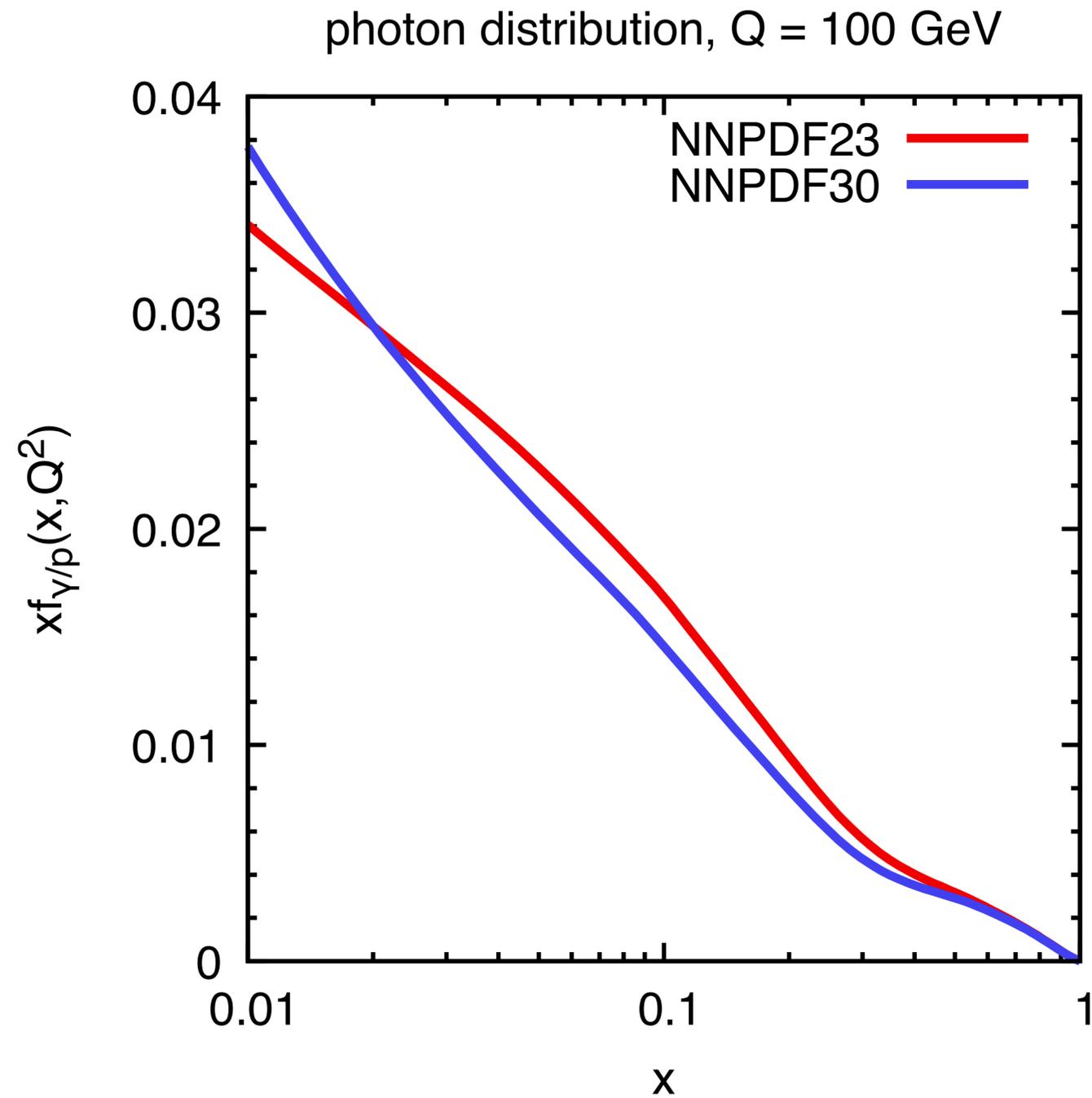
► HERA combined data

► ATLAS DY

**Fit is above LUXqed**

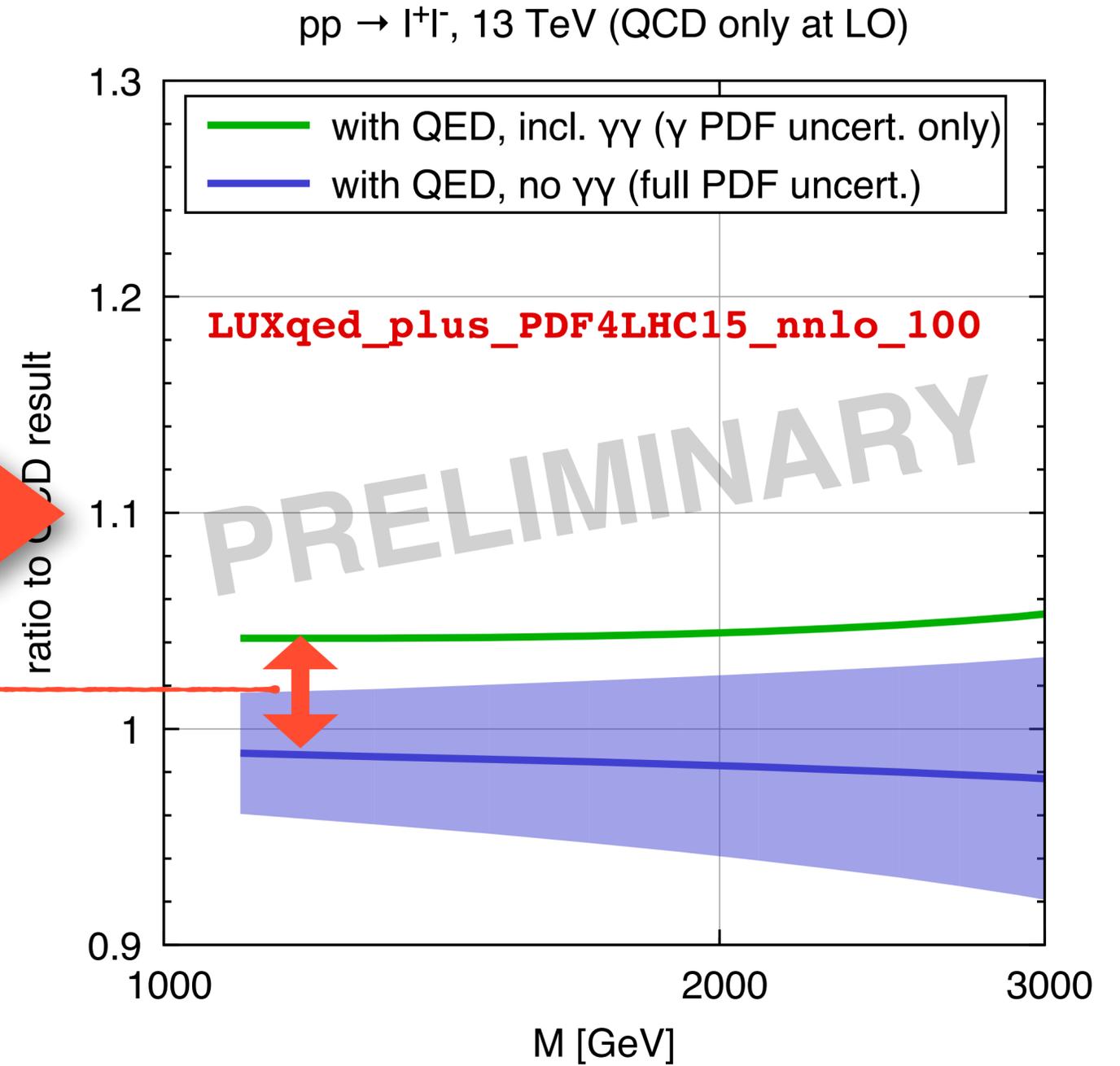
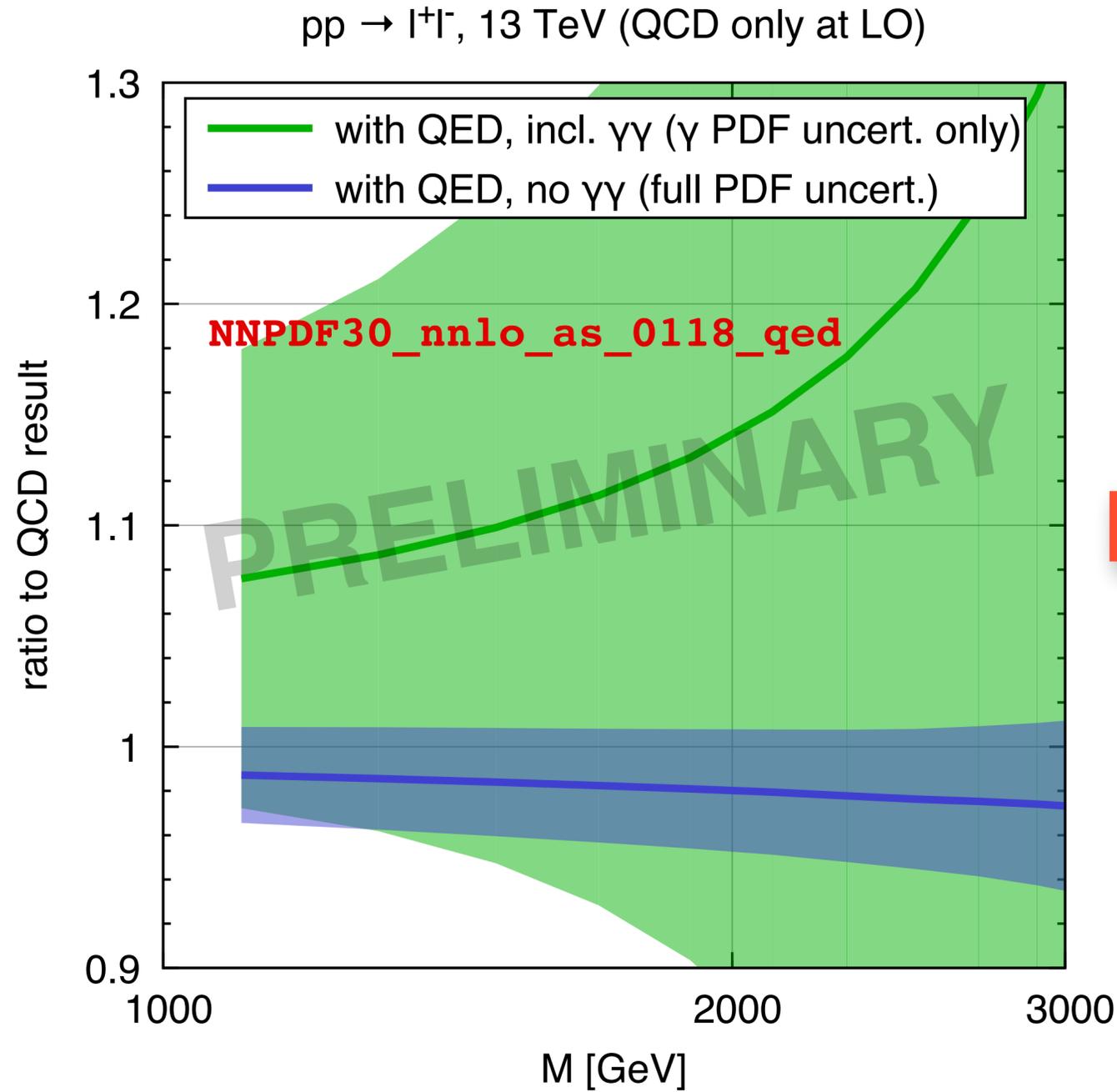
# explanation does not lie with NNPDF23 v. 30 evolution differences

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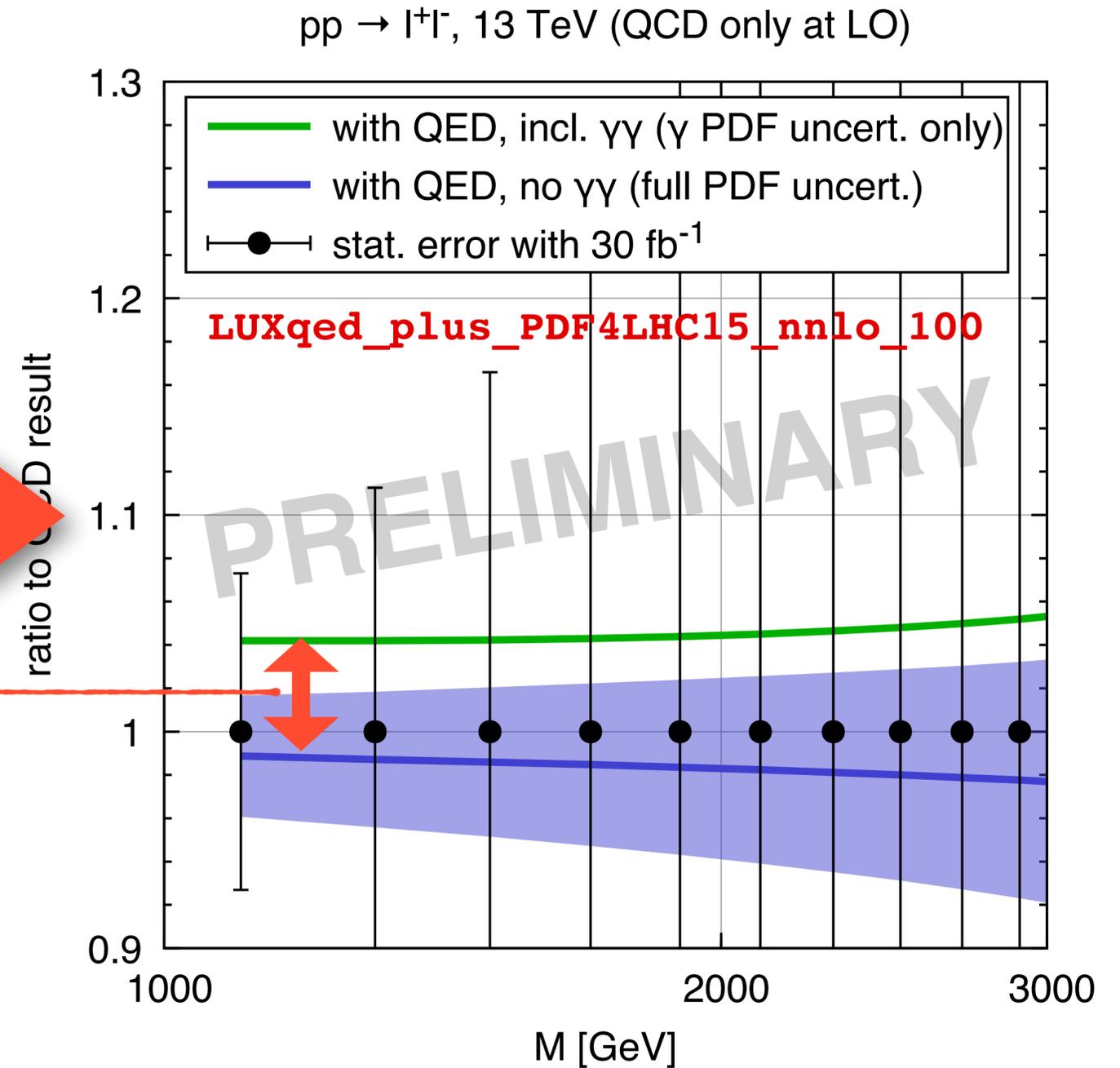
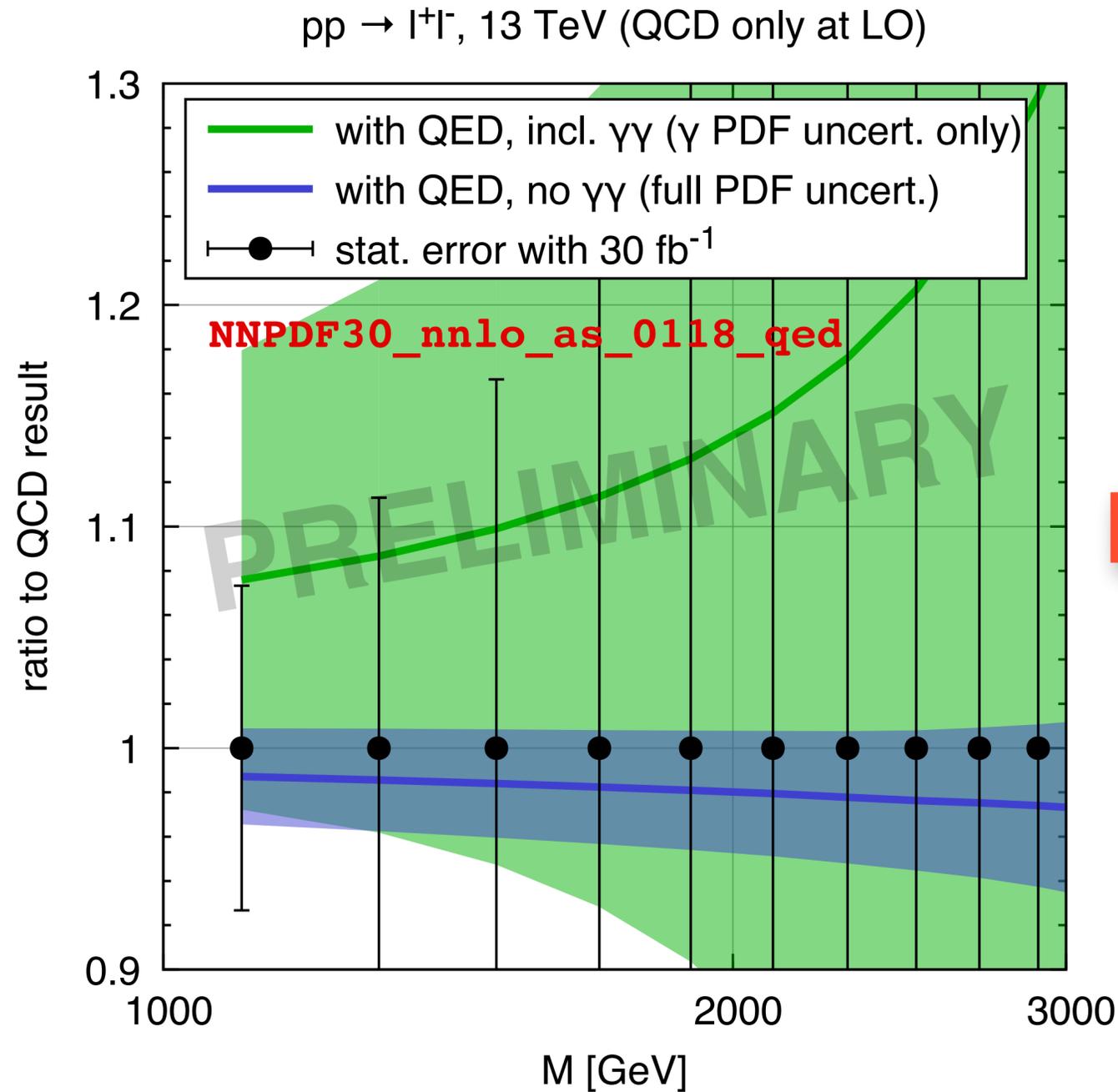
# extra dilepton plots

# di-lepton spectrum



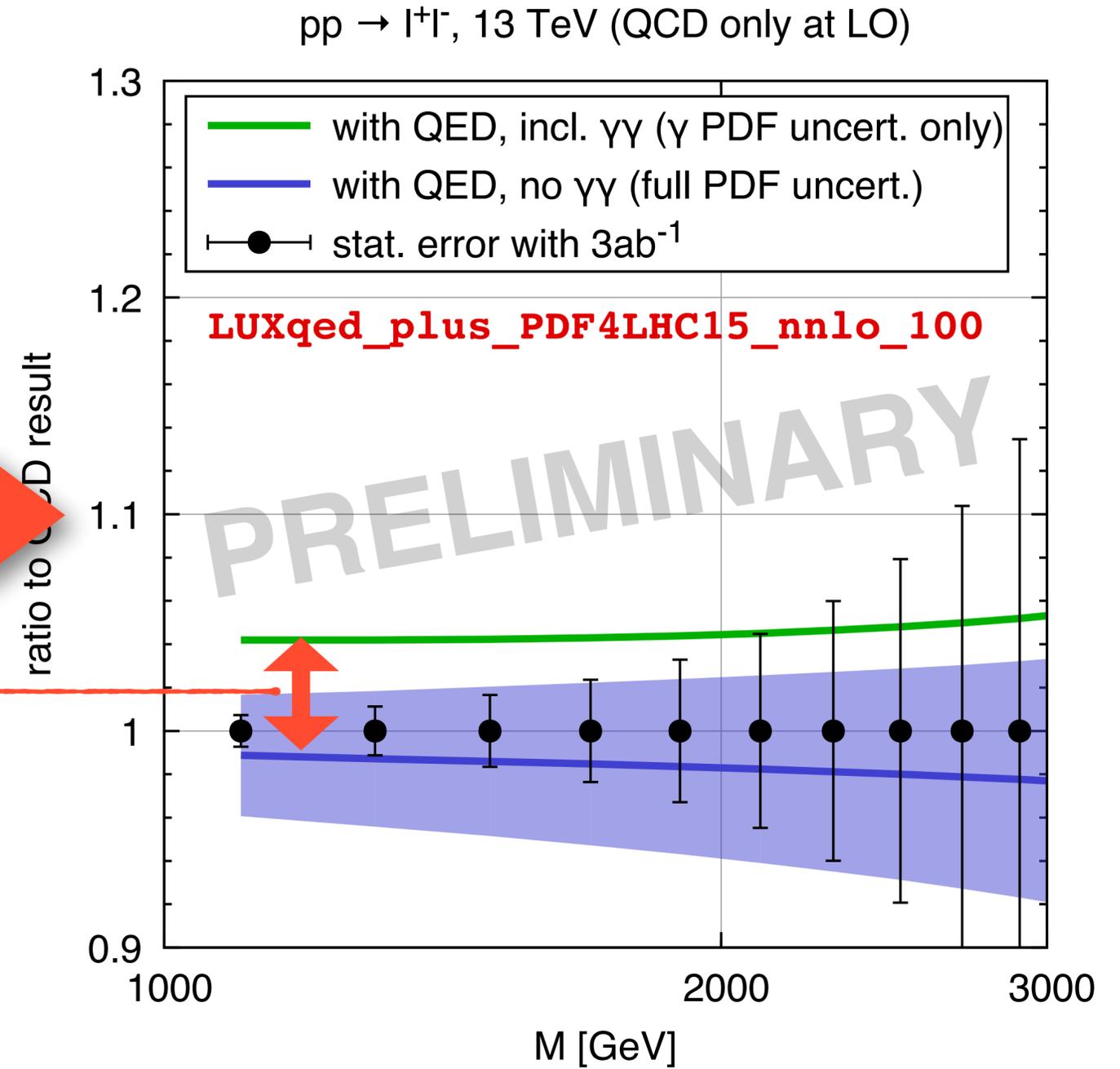
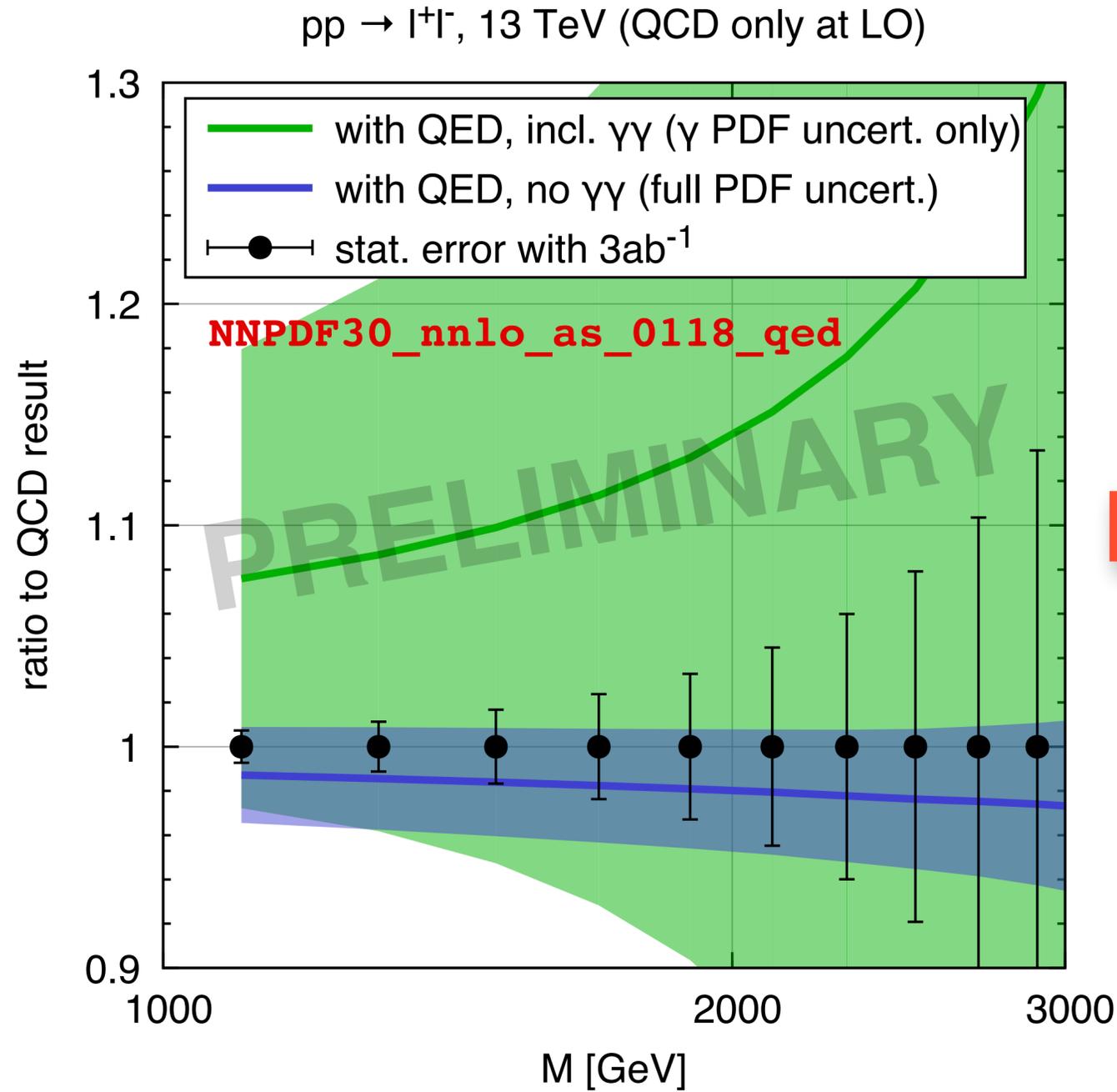
$\gamma\gamma$  component has few-% effect on Drell-Yan spectrum; negligible uncertainty

# di-lepton spectrum



$\gamma\gamma$  component has few-% effect on Drell-Yan spectrum; negligible uncertainty

# di-lepton spectrum



$\gamma\gamma$  component has few-% effect on Drell-Yan spectrum; negligible uncertainty