INTRODUCTION TO LUX QED

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how well do (did) we know the parton distributions?

PDF uncertainties (Q = 100 GeV)



core partons (up, down, gluon) are quite well known PDF uncertainties (Q = 100 GeV)



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strangeness ~10%

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 one other parton, the photon, had been debated. The only model-independent determination (NNPDF23qed) had O(100%) uncertainty

IT MATTERED FOR DI-LEPTON, DI-BOSON, TTBAR, EW HIGGS, ETC.



where else does the photon come in? Any time you produce charged particles

- Electroweak corrections to almost any process
- ► E.g. ~5% WH

LHC-HXSWG YR4

► top production

Pagani, Tsinikos, Zaro, arXiv:1606.01915

► VV production

1409.1803, 1510.08742, 1603.04874, 1601.07787, 1605.03419, 1604.04080,1607.04635, ...



EARLIER PHOTON PDF ESTIMATES (not exhaustive)

	elastic	inelastic	in LHAPDF?
Gluck Pisano Reya 2002	dipole	model	×
MRST2004qed	×	model	\checkmark
NNPDF23qed	no separation; fit to data		\checkmark
CT14qed	×	model (data-constrained)	\checkmark
CT14qed_inc	dipole	model (data-constrained)	\checkmark
Martin Ryskin 2014	dipole (only electric part)	model	×
Harland-Lang, Khoze Ryskin 2016	dipole	model	×
elastic: Budnev, Ginzburg, Meledin, Serbo, 1975			

YOU DON'T NEED A MODEL ep scattering (i.e. structure functions) contains all info about proton's EM field

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study hypothetical ("BSM") heavy-neutral lepton production process Calculate it in two ways

(1) in terms of structure functions (known)(2) in terms of photon distribution (unknown)

Equivalence gives us photon distirbution

Manohar, Nason, GPS & Zanderighi, arXiv:1607.04266 (use of BSM inspired by Drees & Zeppenfeld, PRD39(1989)2536) can also use PDF operator formalism, arXiv:1708.01256

calculation

work out a cross section (exact) in terms of F2 and FL struct. fns.



 $\sigma = \frac{1}{4p \cdot k} \int \frac{d^4q}{(2\pi)^4 q^4} e_{\rm ph}^2(q^2) \left[4\pi W_{\mu\nu} L^{\mu\nu}(k,q)\right] \times 2\pi \delta((k-q)^2 - M^2)$

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- ► Lagrangian of interaction: $\mathcal{L}_{int} = (e/\Lambda)\overline{L}\sigma^{\mu\nu}F_{\mu\nu}l$ (magnetic moment coupling)
- Using a neutral lepton and taking Λ large, ensure that only single-photon exchange is relevant
- > Answer is exact up to $1/\Lambda$ corrections

 $c_0 = 16\pi^2/\Lambda^2$

work out same cross section in terms of a photon distribution

hard-scattering cross section calculate in collinear factorisation

$$\hat{\sigma}_{\gamma}\left(\frac{M^{2}}{xs},\mu^{2}\right)$$

$$\xrightarrow{\text{MS photon distribution:}} \text{TO BE DEDUCED} f_{\gamma/p}(x,\mu^{2})$$

$$X$$

$$\sigma = c_0 \sum_{a} \int \frac{dx}{x} \,\hat{\sigma}_a \left(\frac{M^2}{xs}, \mu^2\right) \, x f_{a/p} \left(x, \mu^2\right)$$

Cross section in terms of structure functions



► Hard cross section driven by the photon distribution at LO

$$\hat{\sigma}_a(z,\mu^2) = \alpha(\mu^2)\delta(1-z)\delta_{a\gamma}$$



► Hard cross section driven by the photon distribution at LO

$$\hat{\sigma}_{a}(z,\mu^{2}) = \alpha(\mu^{2})\delta(1-z)\delta_{a\gamma} + \frac{\alpha^{2}(\mu^{2})}{2\pi} \left[-2+3z+zp_{\gamma q}(z)\ln\frac{M^{2}(1-z)^{2}}{z\mu^{2}}\right] \sum_{i\in\{q,\bar{q}\}} e_{i}^{2}\delta_{ai} + \dots$$
Quarks and gluons come in at higher orders

- > Take quark and gluon distributions $\sim O(1)$
- ▶ α is QED coupling, α_s is QCD coupling, $L = \ln \mu^2 / m_p^2$
 - ► Take $L \sim 1/\alpha_s$, so all $(\alpha_s L)^n \sim 1$
 - ► Think of $\alpha \sim (\alpha_s)^2$
- ► To first order, photon distribution $\sim (\alpha L)$
- ► we aim to control all terms:
 - $\succ \alpha L (\alpha_{\rm s} L)^{\rm n} \qquad [LO]$
 - $\succ \alpha_{\rm s} \alpha L \ (\alpha_{\rm s} L)^{\rm n} \equiv \alpha \ (\alpha_{\rm s} L)^{\rm n}$

[NLO — extra α_s or 1/L]

 $\succ \alpha^2 L^2 (\alpha_s L)^n \qquad [NLO - extra \alpha L]$

► Matching done at large M^2 and μ^2 to eliminate higher twists

equate them to deduce the photon distribution (LUXqed)

$$xf_{\gamma/p}(x,\mu^{2}) = \frac{1}{2\pi\alpha(\mu^{2})} \int_{x}^{1} \frac{dz}{z} \left\{ \int_{\frac{x^{2}m_{p}^{2}}{1-z}}^{\frac{\mu^{2}}{1-z}} \frac{dQ^{2}}{Q^{2}} \alpha^{2}(Q^{2}) \left[\left(zp_{\gamma q}(z) + \frac{2x^{2}m_{p}^{2}}{Q^{2}} \right) F_{2}(x/z,Q^{2}) - z^{2}F_{L}\left(\frac{x}{z},Q^{2}\right) \right] - \alpha^{2}(\mu^{2})z^{2}F_{2}\left(\frac{x}{z},\mu^{2}\right) \right\}$$

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with $F_2 \sim \sum_q e_q^2 x q(x)$ this is just (LO) DGLAP-like piece

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At low Q^2 , F_2 and F_L come directly from data (non.pert.) At high Q^2 , get them from PDFs, including $O(\alpha_s)$ (NLO) terms

equate them to deduce the photon distribution (LUXqed)

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Terms at boundaries are suppressed by 1/L (NLO)

equate them to deduce the photon distribution (LUXqed)

$$xf_{\gamma/p}(x,\mu^{2}) = \frac{1}{2\pi\alpha(\mu^{2})} \int_{x}^{1} \frac{dz}{z} \left\{ \int_{\frac{x^{2}m_{p}^{2}}{1-z}}^{\frac{\mu^{2}}{1-z}} \frac{dQ^{2}}{Q^{2}} \alpha^{2}(Q^{2}) \left[\left(zp_{\gamma q}(z) + \frac{2x^{2}m_{p}^{2}}{Q^{2}} \right) F_{2}(x/z,Q^{2}) - z^{2}F_{L}\left(\frac{x}{z},Q^{2}\right) \right] - \alpha^{2}(\mu^{2})z^{2}F_{2}\left(\frac{x}{z},\mu^{2}\right) \right\}$$

terms at boundary $\sim \mu^2$ ensure $\overline{\text{MS}}$ fact. scheme

equate them to deduce the photon distribution (LUXqed)

$$xf_{\gamma/p}(x,\mu^{2}) = \frac{1}{2\pi\alpha(\mu^{2})} \int_{x}^{1} \frac{dz}{z} \left\{ \int_{\frac{x^{2}m_{p}^{2}}{1-z}}^{\frac{\mu^{2}}{1-z}} \frac{dQ^{2}}{Q^{2}} \alpha^{2}(Q^{2}) \right.$$
$$\left[\left(zp_{\gamma q}(z) + \frac{2x^{2}m_{p}^{2}}{Q^{2}} \right) F_{2}(x/z,Q^{2}) - z^{2}F_{L}\left(\frac{x}{z},Q^{2}\right) \right] - \alpha^{2}(\mu^{2})z^{2}F_{2}\left(\frac{x}{z},\mu^{2}\right) \right\}$$

QED running of α accounts for most $(\alpha L)^2$ effects (NLO) (others come in the way we match to normal PDFs)

cross-checks

- ➤ Repeat calculation for a different process (γp→H+X, via γγ→H). Intermediate results differ, final photon distribution is identical.
- ► Substitute elastic-scattering component of F_2 and F_L :

$$F_2^{\text{el}} = \frac{[G_E(Q^2)]^2 + [G_M(Q^2)]^2 \tau}{1 + \tau} \delta(1 - x),$$

$$F_L^{\text{el}} = \frac{[G_E(Q^2)]^2}{\tau} \delta(1 - x), \qquad \tau = \frac{Q^2}{4m_p^2}$$

and reproduce widely-used **Equivalent Photon Approximation** with electric (G_E) and magnetic (G_M) Sachs proton form factors

Budnev et al., Phys.Rept.15(1975)181

► A core part of our answer

$$\left[\left(zp_{\gamma q}(z) + \frac{2x^2m_p^2}{Q^2}\right)F_2(x/z,Q^2) - z^2F_L\left(\frac{x}{z},Q^2\right)\right]$$

appears in literature for QED compton process $ep \rightarrow e\gamma X$ (but with inexact treatment of the upper and lower limits for Q^2 integration)

Anlauf et. al, CPC70(1992)97 Mukherjee & Pisano, hep-ph/0306275

 [NB other literature has expression for photon distribution in terms of F₂ and F₁ that doesn't reproduce DGLAP limit] Luszczak, Schäfer & Szczurek, arXiv:1510.00294

- µ² derivative of our answer should reproduce known DGLAP QCD-QED splitting functions
- ► At LO, this is trivial.
- At NLO we get relations between QED-QCD splitting functions (P) and DIS coefficient functions (C)

$$P_{\gamma q}^{(1,1)} = e_q^2 \left[p_{\gamma q} \otimes C_{2q} - h \otimes C_{Lq} + (\bar{p}_{\gamma q} - h) \otimes P_{qq}^{(1,0)} \right] ,$$

$$P_{\gamma g}^{(1,1)} = \sum_{q,\bar{q}} e_q^2 \left[p_{\gamma q} \otimes C_{2g} - h \otimes C_{Lg} + (\bar{p}_{\gamma q} - h) \otimes P_{qg}^{(1,0)} \right] ,$$

$$P_{\gamma \gamma}^{(1,1)} = (2\pi)^2 b_{\alpha}^{(1,2)} \delta(1-x) = -C_F N_C \sum_q e_q^2 \delta(1-x)$$

$$h(z) \equiv z \text{ and } \bar{p}_{\gamma q}(z) \equiv p_{\gamma q}(z) \ln \frac{1}{1-z}$$

These agree with de Florian, Sborlini & Rodrigo results

for $O(\alpha \alpha_s)$ terms, arXiv:1512.00612 both they and we have gone to higher order — but pheno accuracy not there yet

data inputs



DATA

- x, Q² plane naturally breaks up into regions with different physical behaviours and data sources
- We don't use F₂ and F_L data directly, but rather various fits to data



Q² [GeV²]





CONTINUUM COMPONENT



- ► Much data
- ► For $Q^2 \rightarrow 0$, $\sigma_{\gamma p}$ indep. of Q^2 at fixed W^2





CONTINUUM COMPONENT



- Less direct data for F₂ and F_L at high Q²
- But we can reliably use PDFs and coefficient functions (up to NNLO) to calculate them
- Our default choice is PDF4LHC15_nnlo_100 (and zero-mass variable flavournumber scheme)



INTEGRATION REGION

• depends on momentum fraction of the photon (x_{γ}) and factorisation scale (μ^2)

$$xf_{\gamma/p}(x,\mu^{2}) = \frac{1}{2\pi\alpha(\mu^{2})} \int_{x}^{1} \frac{dz}{z} \left\{ \int_{\frac{x^{2}m_{p}^{2}}{1-z}}^{\frac{\mu^{2}}{1-z}} \frac{dQ^{2}}{Q^{2}} \alpha^{2}(Q^{2}) \left[\left(zp_{\gamma q}(z) + \frac{2x^{2}m_{p}^{2}}{Q^{2}} \right) F_{2}(x/z,Q^{2}) - z^{2}F_{L}\left(\frac{x}{z},Q^{2}\right) \right] - \alpha^{2}(\mu^{2})z^{2}F_{2}\left(\frac{x}{z},\mu^{2}\right) \right\}, \quad (6)$$










SEPARATE CONTRIBUTIONS TO PHOTON PDF

















PHOTON PDF ESTIMATES (not exhaustive)

	elastic	inelastic	in LHAPDF?
Gluck Pisano Reya 2002	dipole	model	×
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NNPDF23qed	no separation; fit to data		\checkmark
CT14qed	×	model (data-constrained)	\checkmark
CT14qed_inc	dipole	model (data-constrained)	\checkmark
Martin Ryskin 2014	dipole (only electric part)	model	×
Harland-Lang, Khoze Ryskin 2016	dipole	model	×
LUXqed 2016/17	data	data	\checkmark

examine result

PHOTON UNCERTAINTY (1-2%) COMPARED TO OTHER FLAVOURS

PDF uncertainties (Q = 100 GeV)



MOMENTUM CARRIED BY PHOTON



momentum ($\mu = 100 \text{ GeV}$)			
gluon	46.8 ± 0.4%		
up valence	18.1 ± 0.3%		
down valence	7.5 ± 0.2%		
light sea quarks	20.5 ± 0.4%		
charm	4.0 ± 0.1%		
bottom	2.5 ± 0.1%		
photon	0.425 ± 0.003%		

(1+107 members, symmhessian, errors handled by LHAPDF out of the box)

NNPDF approach — LUXqed photon approach, refits other flavours



Figure 2.2. Flow diagram representing the NNPDF3.1luxQED fitting strategy. In the last iteration $n_{\rm ite}$, once the procedure has converged, the additional LUXqed17 are added to $\gamma(x, Q)$, see Sect. 2.5.

NNPDF LUXqed (1712.07053) v. our LUXqed



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MMHT-framework "ad lucem" (1907.02750) v. our LUXqed



MMHT-framework "ad lucem" (1907.02750) v. our LUXqed

$$\begin{aligned} x\gamma(x,Q_0^2) &= \frac{1}{2\pi\alpha(Q_0^2)} \int_x^1 \frac{dz}{z} \Big\{ \int_{\frac{x^2m_p^2}{1-z}}^{Q_0^2} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \Big[\Big(zP_{\gamma,q}(z) + \frac{2x^2m_p^2}{Q^2} \Big) F_2(x/z,Q^2) \\ &- z^2 F_L(x/z,Q^2) \Big] - \alpha^2(Q_0^2) \Big(z^2 + \ln(1-z)zP_{\gamma,q}(z) - \frac{2x^2m_p^2z}{Q_0^2} \Big) F_2(x/z,Q_0^2) \Big\}. \\ P_{\gamma,q}^{(0,1)}(z) \to P_{\gamma,q}^{(0,1)}(z) + \frac{2x^2m_p^2}{zQ^2}. \end{aligned}$$

$$q(x,Q^2) \to q(x,Q^2) \Big(1 + \frac{A_2'}{Q^2} \int_x^1 \frac{dz}{z} C_2(z)q(\frac{x}{z},Q^2) \Big), \end{aligned}$$

$$\delta x \gamma^{(el)}(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \frac{1}{x} \left[\left(x P_{\gamma,q}(x) + \frac{2x^2 m_p^2}{Q^2} \right) \frac{[G_E(Q^2)]^2 + \tau [G_M(Q^2)]^2}{1 + \tau} \right] \frac{d\gamma^{(el)}}{dt} = P_{\gamma\gamma} \otimes \gamma^{(el)} + \delta x \gamma^{(el)}. \qquad \qquad -x^2 \frac{[G_E(Q^2)]^2}{\tau} \right].$$

formulated so as to allow evolution with photon from low scale (whereas in LUXqed we take point of view that γ PDF makes sense only at high Q^2)

applications

PARTONIC LUMINOSITIES



γγ luminosity is about 1000 times smaller than $q\bar{q}$ luminosity

$pp \rightarrow H W^+ (\rightarrow l^+v) + X \text{ at } 13 \text{ TeV}$		
non-photon induced contributions	91.2 ± 1.8 fb	
photon-induced contribs (NNPDF23)	6.0 +4.4 _{-2.9} fb	
photon-induced contribs (LUXqed)	4.4 ± 0.1 fb	

non-photon numbers from LHCHXSWG (YR4) including PDF uncertainties

YY luminosity







Harland-Lang (1910.10178): in some cases photon distribution is too indirect



exact $\gamma \gamma \rightarrow \ell^+ \ell^-$ contribution, using structure function approach

conclusions & resources

LHAPDF PDF sets with modern photon distributions

LUXqed17_plus_PDF4LHC15_nnlo_100

NNPDF31_nlo_as_0118_luxqed

NNPDF31_nnlo_as_0118_luxqed

MMHT2015qed_nnlo_total

MMHT2015qed_nnlo_inelastic

MMHT2015qed_nnlo_elastic

- Istribution of photons in the proton depends on the nonperturbative QCD physics of the proton
- But perturbative QED enables you to deduce the photon density from measured (non-pert.) proton structure functions
- ► photon distributions needed for some EW higher-order calculations, but not necessarily for $\gamma\gamma \rightarrow \ell^+\ell^-$

"If you think about it, it's awesome: we are made of protons, and protons are, in some part, made of light... And now we know how much of it."

<u>blog post</u> by Tommaso Dorigo

extra slides

input data & procedures

ELASTIC COMPONENT & COMPARISON TO "DIPOLE" MODEL



CLAS DATA



MATCHING PROCEDURE FOR FULL SET OF PARTONS



- ➤ evaluate master eqn. for µ=100 GeV (with default PDF4LHC15_nnlo partons)
- ➤ Do O(aa_s) photon evolution down to µ=10 GeV (other partons: pure QCD evln.)
- ➤ Adjust momentum sum-rule by rescaling gluon $g(x) \rightarrow 0.993g(x)$
- Evolve back up with NNLO-QCD & O(aa_s) QED for all partons

better approach would be full PDF re-fit for QCD partons incl. EW/QED corrections & LUXqed photon

other PDFs v. LUXqed



central NNPDF result much higher at large x (but consistent within errors)

at small x, with corrected evolution (NNPDF30), about 20% smaller
other PDFs v. LUXqed

Others are numerically closer

Error bands don't always overlap with LUXqed, but within ~10-20%

