CUTS FOR 2-BODY DECAYS AT COLLIDERS

Particle theory seminar, Würzburg July 2022

Gavin Salam, with Emma Slade, arXiv:2106.08329
Rudolf Peierls Centre for Theoretical Physics &
All Souls College, University of Oxford

European Research Council
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Precision is crucial part of LHC programme: e.g. establishing the Higgs sector

Over the next 15 years

Today's ~8% on H→γγ

(5% on all-channel combination)

→ ~2% at HL-LHC

We wouldn't consider QED established if it had only been tested at O(10%) accuracy

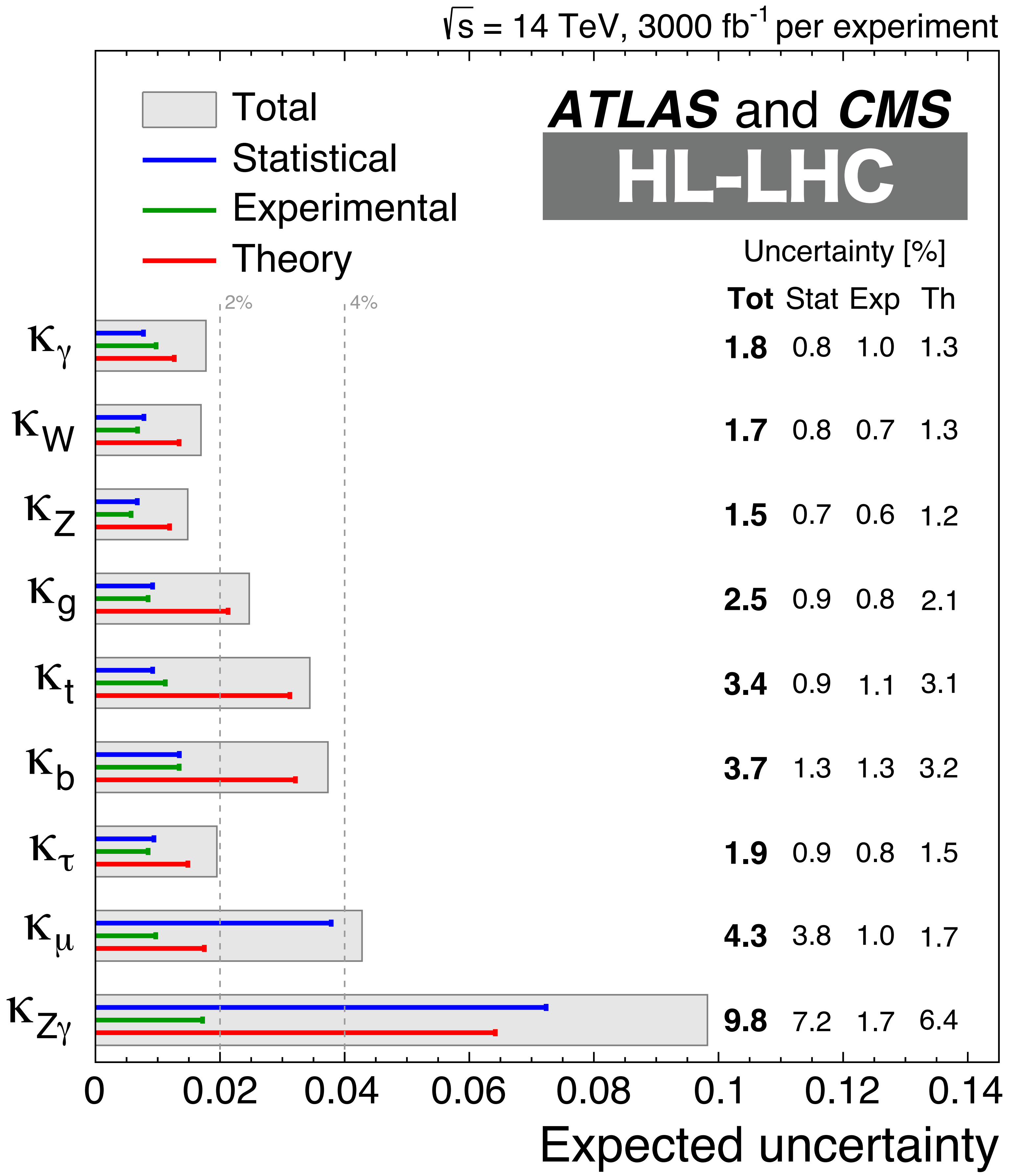


Figure 1. Projected uncertainties on κ_i, combining ATLAS and CMS: total (grey box), statistical (blue), experimental (green) and theory (red). From Ref. [2].

Starting point for any hadron-collider analysis: acceptance (fiducial) cuts

E.g. ATLAS/CMS $H \to \gamma\gamma$ cuts

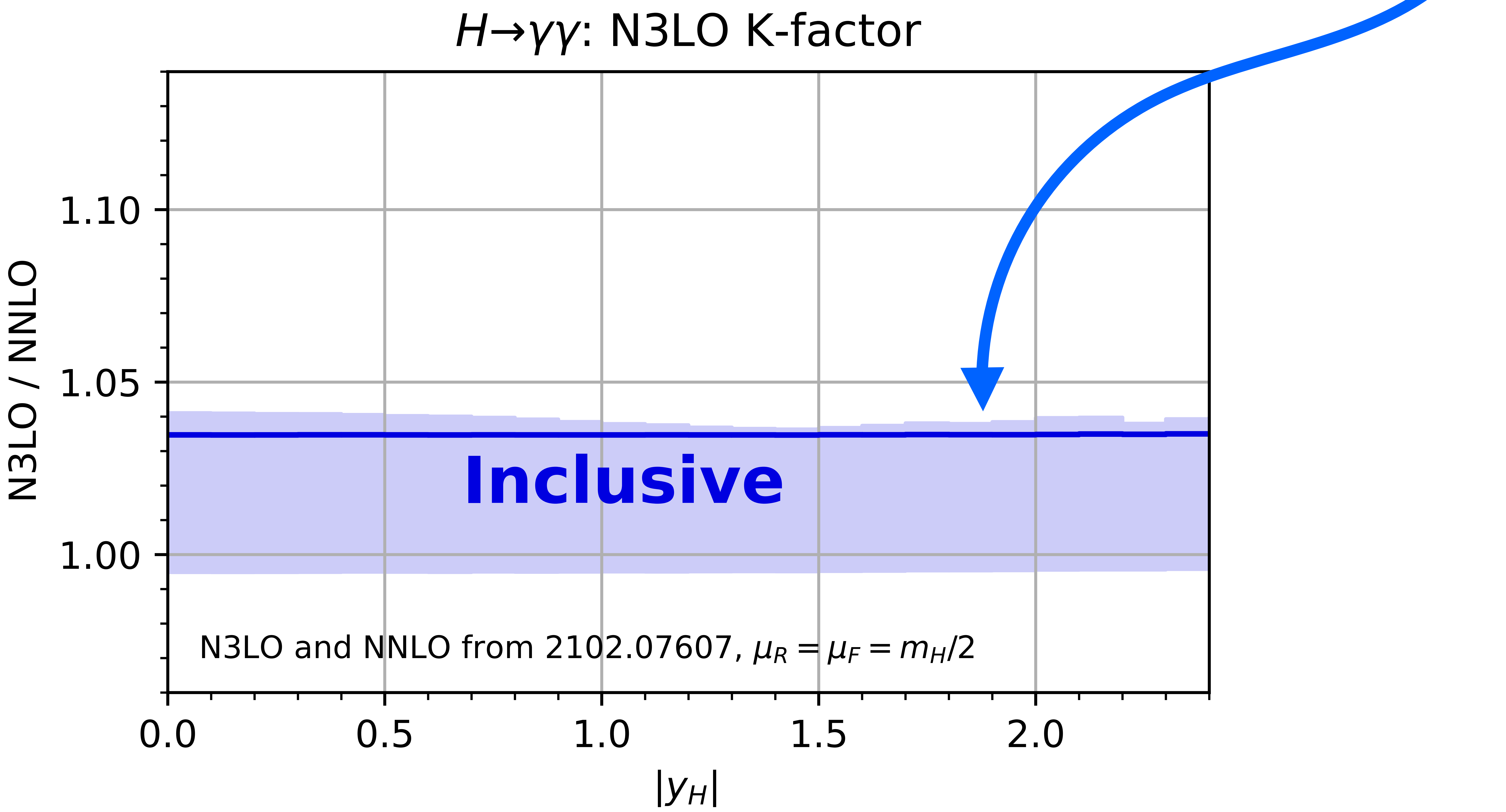
- Higher-p_t photon: $p_{t,\gamma} > 0.35 m_{\gamma\gamma}$ (ATLAS) or $m_{\gamma\gamma}/3$ (CMS)
- Lower-p_t photon: $p_{t,\gamma} > 0.25 m_{\gamma\gamma}$
- Both photons: additional rapidity and isolation cuts

Essential for good reconstruction of the photons and for rejecting large low-p_t backgrounds.

Theory-experiment comparisons with identical "fiducial" cuts often considered the Gold Standard of collider physics

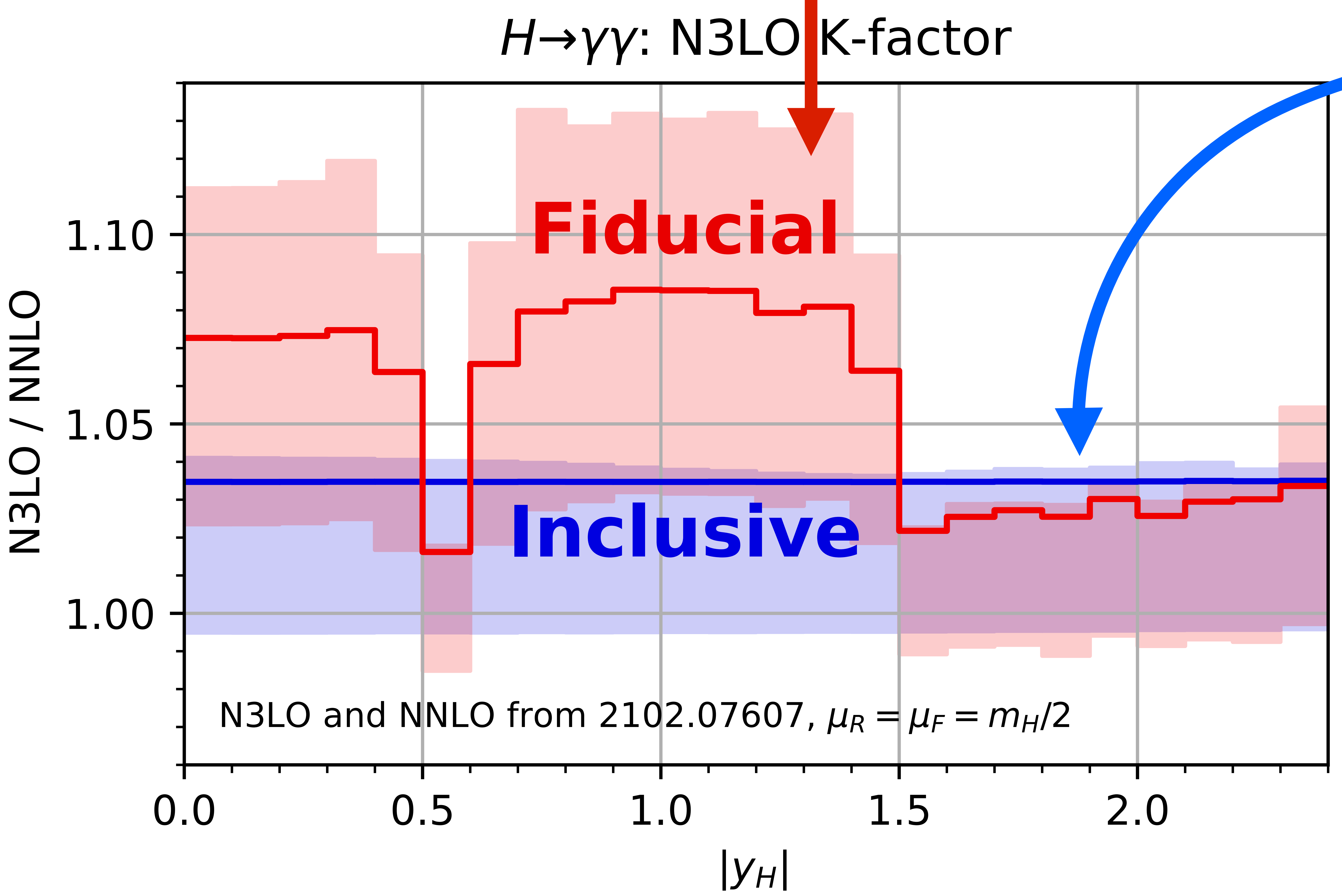
Recent surprise: H→γγ

inclusive N3LO σ uncertaities



Chen, Gehrmann, Glover, Huss, Mistlberger & Pelloni, 2102.07607

Recent surprise: H→γγ fiducial N3LO σ uncertainties ~2× greater than inclusive N3LO σ uncertaities



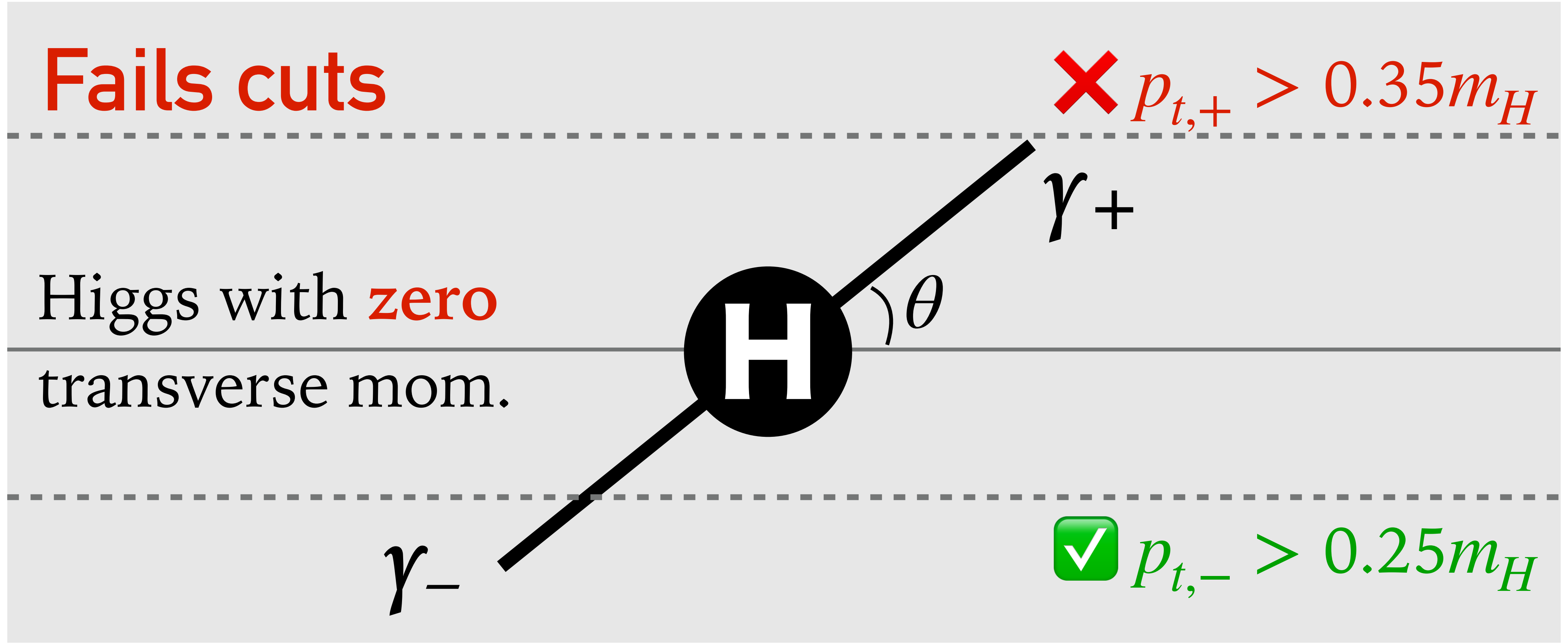
"Gold standard" fiducial cross section gives much worse prediction

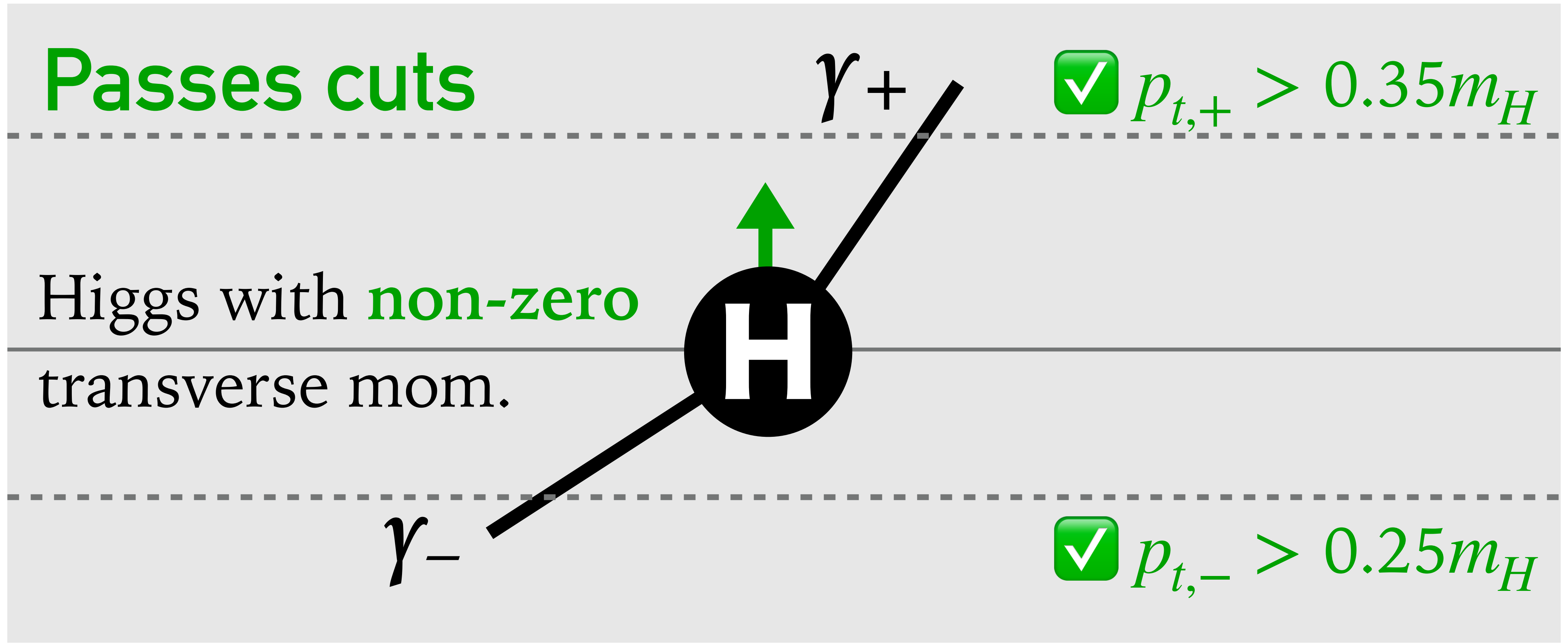
Why?
And can this be solved?

Chen, Gehrmann, Glover, Huss, Mistlberger & Pelloni, 2102.07607

the origin of the problem

Standard $p_{t,\gamma}$ cuts → Higgs p_t dependence of acceptance



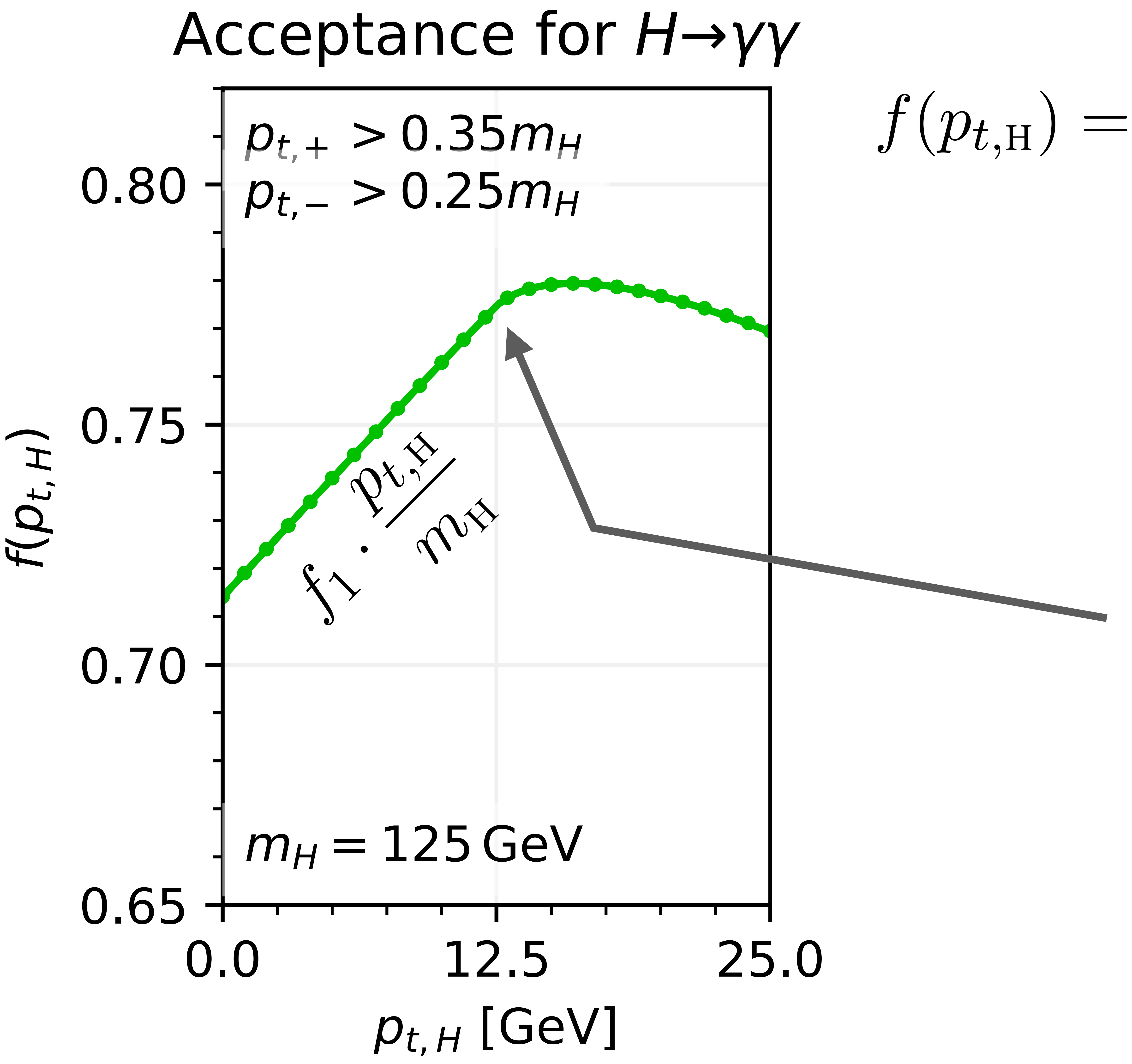


Numbers are for ATLAS H→ γγ p_t cuts, CMS cuts are similar

Expect acceptance to **increase with increasing $p_{t,\mathrm{H}}$**

$$p_{t,\pm}(p_{t,\mathrm{H}}, \theta, \phi) = \frac{m_\mathrm{H}}{2} \sin\theta \pm \frac{1}{2} p_{t,\mathrm{H}} |\cos\phi| + \frac{p_{t,\mathrm{H}}^2}{4 m_\mathrm{H}} \left(\sin\theta \cos^2\phi + \csc\theta \sin^2\phi\right) + \mathcal{O}_3\,,$$

Linear p_{tH} dependence of H acceptance ≡ $f(p_{tH})$



$$f(p_{t,\mathrm{H}}) = f_0 + f_1 \cdot \frac{p_{t,\mathrm{H}}}{m_\mathrm{H}} + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

See e.g. Frixione & Ridolfi '97
Ebert & Tackmann '19
idem + Michel & Stewart '20
Alekhin et al '20

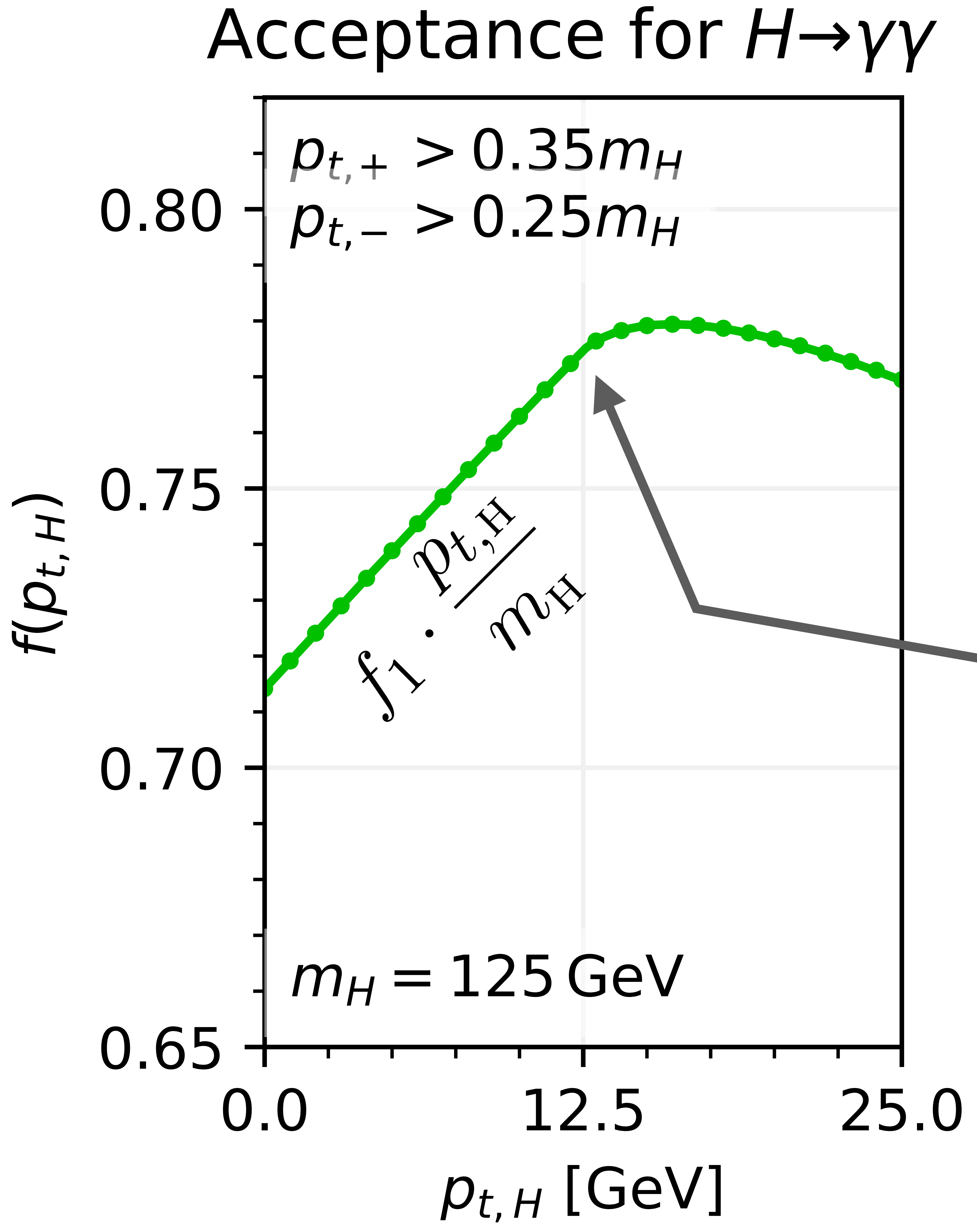
f_0 and f_1 are coefficients whose values depend on the cuts

effect of $p_{t,-}$ cut sets in at $0.1 m_\mathrm{H}$

define $s_0 = \frac{2p_{t+,\mathrm{cut}}}{m_H}$: $f_0 = \sqrt{1-s_0^2} \simeq 0.71$, $f_1 = \frac{2s_0}{\pi f_0} \simeq 0.62$

transition is at $p_{t+,cut} - p_{t-,cut}$

Linear p_{tH} dependence of H acceptance ≡ f(p_{tH})



$$f(p_{t,\text{H}}) = f_0 + f_1 \cdot \frac{p_{t,\text{H}}}{m_\text{H}} + \mathcal{O}\left(\frac{p_{t,\text{H}}^2}{m_\text{H}^2}\right)$$

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f_0 and f_1 are coefficients whose values depend on the cuts

effect of $p_{t,-}$ cut sets in at $0.1 m_\text{H}$

$p_{t,\text{H}}$ dependence of acceptance (at 10% level) → relating measured cross section and total cross section requires info about the $p_{t,\text{H}}$ distribution.

perturbative series for fiducial cross sections

$$f(p_{t,\mathrm{H}}) = f_0 + f_1 \cdot \frac{p_{t,\mathrm{H}}}{m_\mathrm{H}} + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

Fiducial cross section depends on acceptance and Higgs p_t distribution

$$\sigma_\mathrm{fid} = \int \frac{d\sigma}{dp_{t,\mathrm{H}}} f(p_{t,\mathrm{H}}) dp_{t,\mathrm{H}}$$

To understand qualitative perturbative behaviour consider simple (double-log) approx for p_t distribution

$$\frac{d\sigma^\mathrm{DL}}{dp_{t,\mathrm{H}}} = \frac{\sigma_\mathrm{tot}}{p_{t,\mathrm{H}}} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2 \log^{2n-1} \frac{m_\mathrm{H}}{2p_{t,\mathrm{H}}}}{(n-1)!} \left(\frac{2C_A \alpha_s}{\pi}\right)^n$$

$$\int_0^{m_H} \frac{dp_{t,H}}{p_{t,H}} \frac{\alpha_s^n}{(n-1)!} \left(\log \frac{m_H}{p_{t,H}}\right)^{2n-1} \cdot \left(\frac{p_{t,H}}{m_H}\right) \sim \alpha_s^n \frac{(2n-1)!}{(n-1)!} \sim \alpha_s^n \, 2^{2n} n!$$

perturbative series: results in DL approximation

$$\frac{f_1^{\text{asym}}}{f_0} \simeq 0.87$$

$$\frac{\sigma_{\text{asym}}^{\text{DL}}}{f_0 \sigma_{\text{tot}}} - 1 \simeq \frac{f_1^{\text{asym}}}{f_0} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2n)!}{2(n!)} \left(\frac{2C_A \alpha_s}{\pi} \right)^n + \cdots$$

$$\simeq \frac{f_1^{\text{asym}}}{f_0} \left(\underbrace{0.16}_{\alpha_s} - \underbrace{0.33}_{\alpha_s^2} + \underbrace{0.82}_{\alpha_s^3} - \underbrace{2.73}_{\alpha_s^4} + \underbrace{11.72}_{\alpha_s^5} + \ldots \right) \simeq \frac{f_1^{\text{asym}}}{f_0} \times \underbrace{0.05}_{\text{resummed}} .$$

- Alternating signs imply the asymptotic series can be Borel-resummed (reflecting the fact that the resummed p_{tH} distribution is well-behaved)
- but fixed-order truncation is dangerous

Behaviour of perturbative series in various log approximations

Resummed results

$$\frac{\sigma_{\text{asym}} - f_0\sigma_{\text{inc}}}{\sigma_0 f_0} \simeq 0.15_{\alpha_s} - 0.29_{\alpha_s^2} + 0.71_{\alpha_s^3} - 2.39_{\alpha_s^4} + 10.31_{\alpha_s^5} + \ldots \simeq 0.06 \quad \text{@DL},$$

$$\simeq 0.15_{\alpha_s} - 0.23_{\alpha_s^2} + 0.44_{\alpha_s^3} - 1.15_{\alpha_s^4} + 3.86_{\alpha_s^5} + \ldots \simeq 0.06 \quad \text{@LL},$$

$$\simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.29_{\alpha_s^3} + \ldots \simeq 0.10 \quad \text{@NNLL},$$

$$\simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.31_{\alpha_s^3} + \ldots \simeq 0.12 \quad \text{@N3LL}.$$

Thanks to Pier Monni & RadISH for supplying NN(N)LL distributions & expansions, $\mu = m_H/2$

(relative to previous slide, this now has full expression for acceptance)

- At DL & LL (DL+running coupling) **factorial divergence sets in from first orders**
- Poor behaviour of N3LL is qualitatively similar to that seen by Billis et al ‘21
- Theoretically similar to a power-suppressed ambiguity $\sim (\Lambda_{\text{QCD}}/m_{\text{H}})^{0.205}$ [inclusive cross sections expected to have Λ^2/m^2]

where in phase space does the bad behaviour come from?

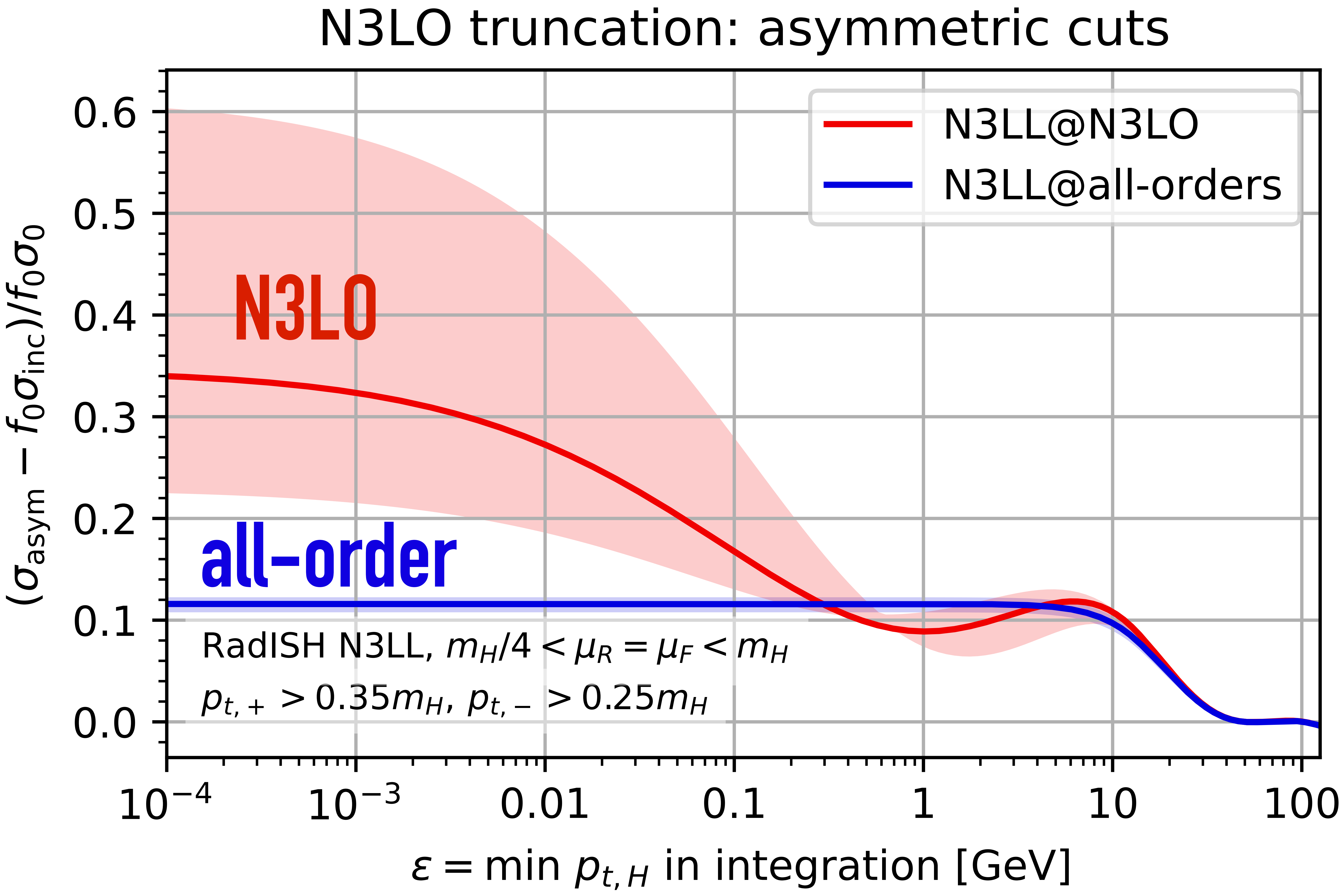
$$\int_0^{m_H} \frac{dp_{t,H}}{p_{t,H}} \frac{\alpha_s^n}{(n-1)!} \left(\log \frac{m_H}{p_{t,H}}\right)^{2n-1} \cdot \left(\frac{p_{t,H}}{m_H}\right)$$

log-enhancement pushes support to small p_{tH}

50% of integral comes from $p_{tH} \lesssim m_H\, e^{-(2n-1)-2/3} \sim 0.4$ GeV for $n = 3$

This is **pathological**: all-order p_{tH} distribution is almost zero for such small p_{tH} values

Sensitivity to cut on minimal Higgs p_t (in real & virt.): N3LO v. all-orders

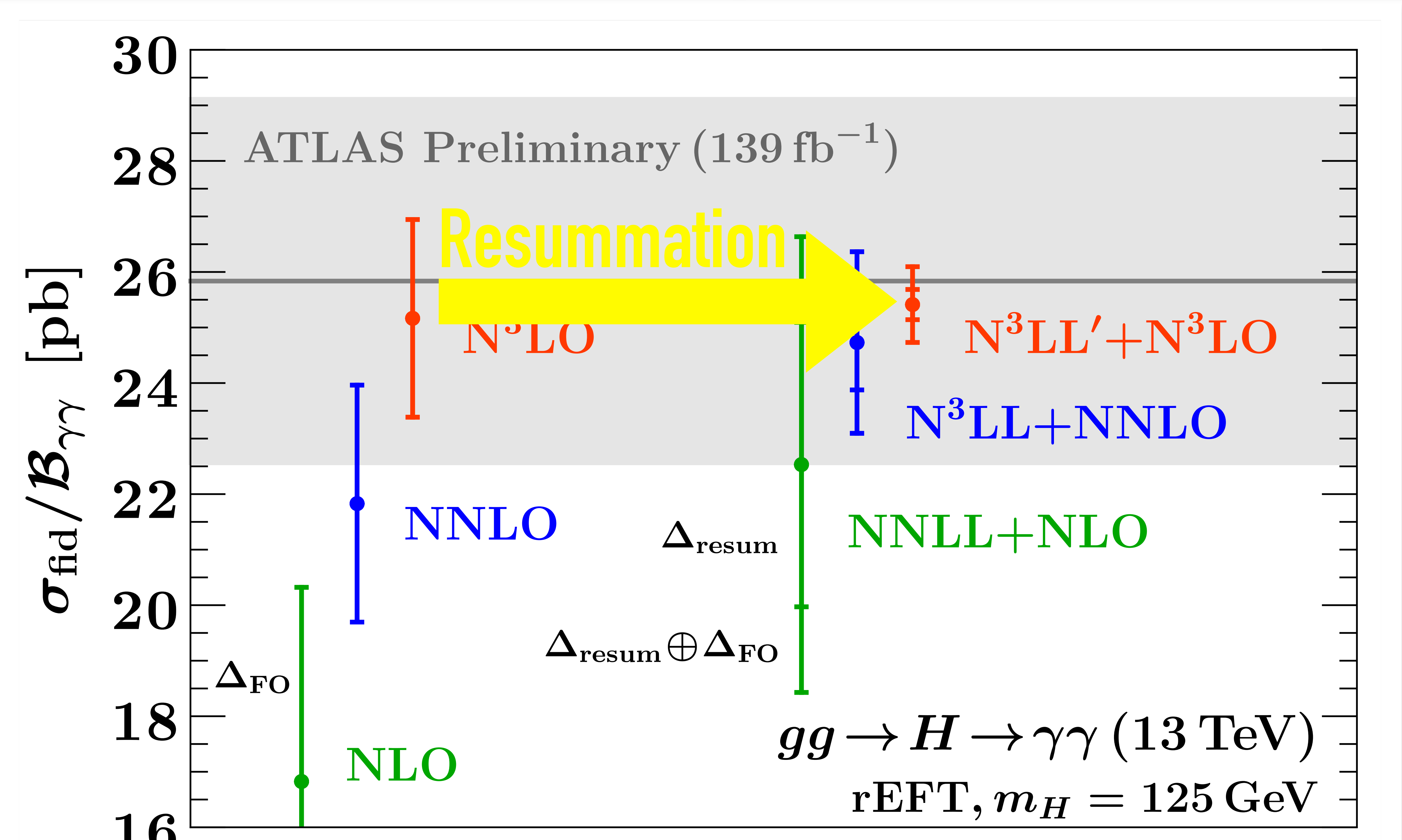


- fixed-order result very sensitive to minimum $p_{t,\mathrm{H}}$ value explored in phase-space integration
- only converges once you explore down to $p_{t,\mathrm{H}} \sim 1\,\mathrm{MeV}$
- i.e. extremely difficult to get reliable fixed-order result and once you have it, it is of dubious physical meaning

solutions

Solution #1: only ever calculate σ_{fid} with help of p_{tH} resummation

- Billis, Dehnadi, Ebert, Michel & Tackmann, 2102.08039, argue you should evaluate the fiducial cross section only after resummation of the p_{tH} distribution.
- For legacy measurements, resummation is only viable solution
- **Our view: not an ideal solution**
  - Fiducial σ is a hard cross section and shouldn't need resummation
  - losing the ability to use fixed order on its own would be a big blow to the field (e.g. flexibility; robustness of seeing fixed-order & resummation agree)
  - sensitivity to variation of acceptance at low $p_{t,H}$ → complications (e.g. sensitivity to heavy-quark effects in resummation and PDFs — not consistently treated in any N3LL resummation today)



Billis, Dehnadi, Ebert, Michel & Tackmann, 2102.08039

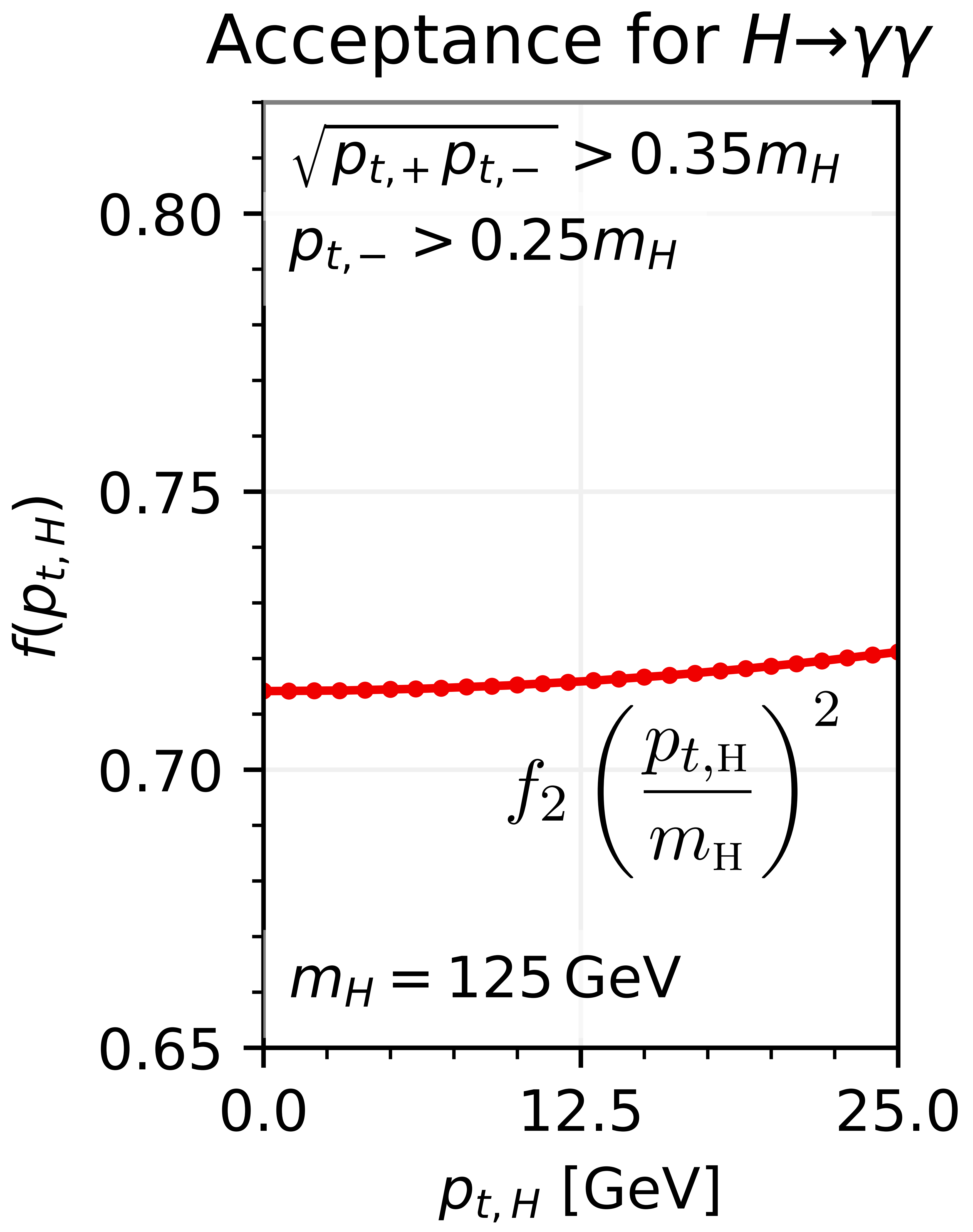
Solution #2a: for future measurements, make simple changes to the cuts

- Simplest option is to replace the cut on the leading photon with a **cut on the product of the two photon p_t's**
- E.g. $p_{t,\gamma+} \times p_{t,\gamma-} > (0.35 m_H)^2$ (and still keep softer photon cut $p_{t,\gamma-} > 0.25 m_H$)
- The product has **no linear dependence on $p_{t,H}$**

$$p_{t,\text{prod}}(p_{t,\text{H}}, \theta, \phi) = \sqrt{p_{t,+} p_{t,-}} = \frac{m_\text{H}}{2} \sin\theta + \frac{p_{t,\text{H}}^2}{4 m_\text{H}} \frac{\sin^2\phi - \cos^2\theta \cos^2\phi}{\sin\theta} + \mathcal{O}_4$$

[Several other options are possible, but this combines simplicity and good performance]

Replace cut on leading photon → cut on product of photon p_t's



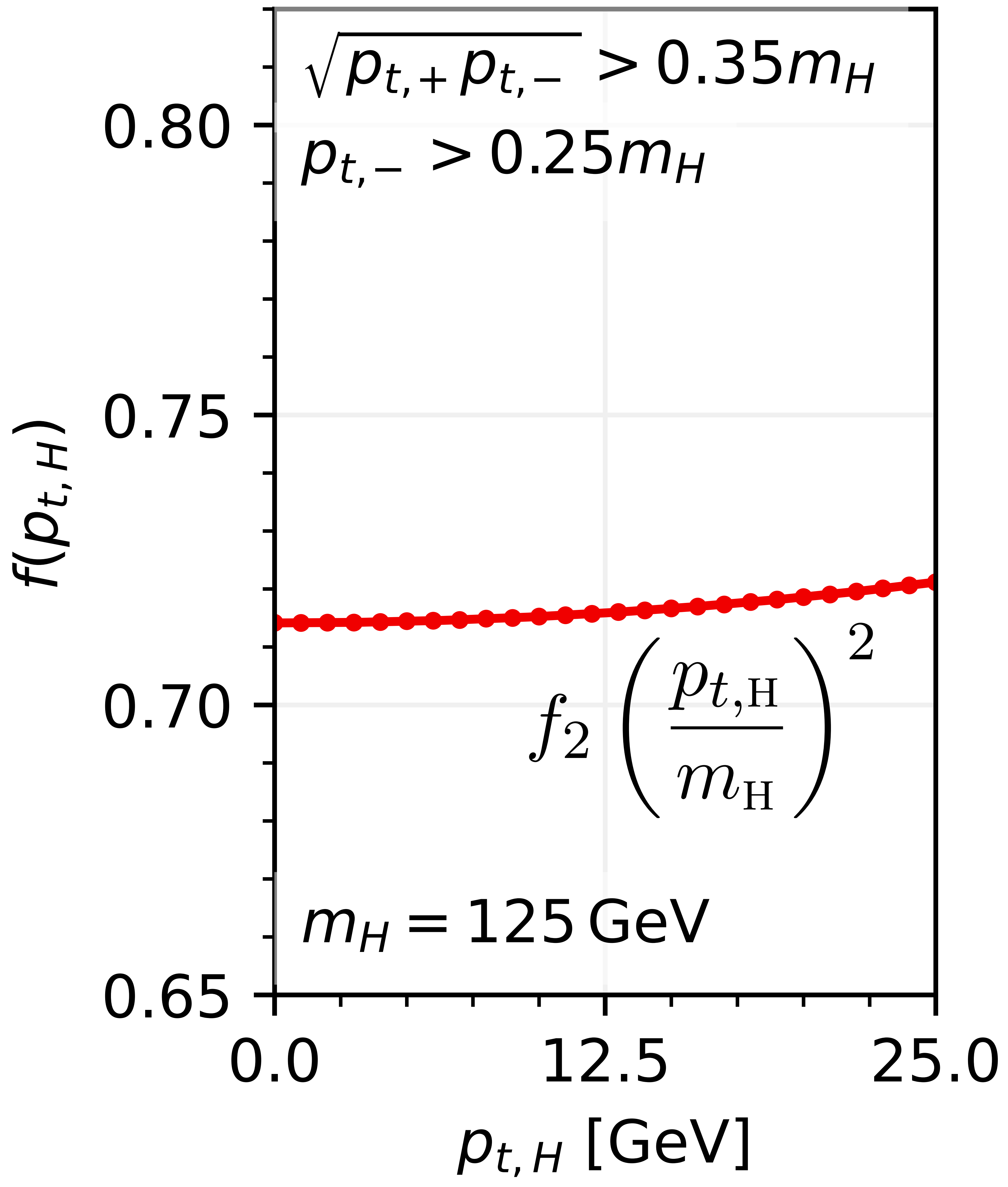
$$f(p_{t,\mathrm{H}}) = f_0 + f_2\left(\frac{p_{t,\mathrm{H}}}{m_\mathrm{H}}\right)^2 + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

linear → quadratic

NB: the cut on the softer photon is still maintained

Replace cut on leading photon → cut on product of photon p_t's

Acceptance for $H\to\gamma\gamma$



$$f(p_{t,\mathrm{H}}) = f_0 + f_2\left(\frac{p_{t,\mathrm{H}}}{m_\mathrm{H}}\right)^2 + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

linear → quadratic

$$\frac{(2n)!}{2(n!)}\left(\frac{2C_A\alpha_s}{\pi}\right)^n \rightarrow \frac{1}{4^n}\frac{(2n)!}{4(n!)}\left(\frac{2C_A\alpha_s}{\pi}\right)^n$$

Using product cuts dampens the factorial divergence

NB: the cut on the softer photon is still maintained

Behaviour of perturbative series with product cuts

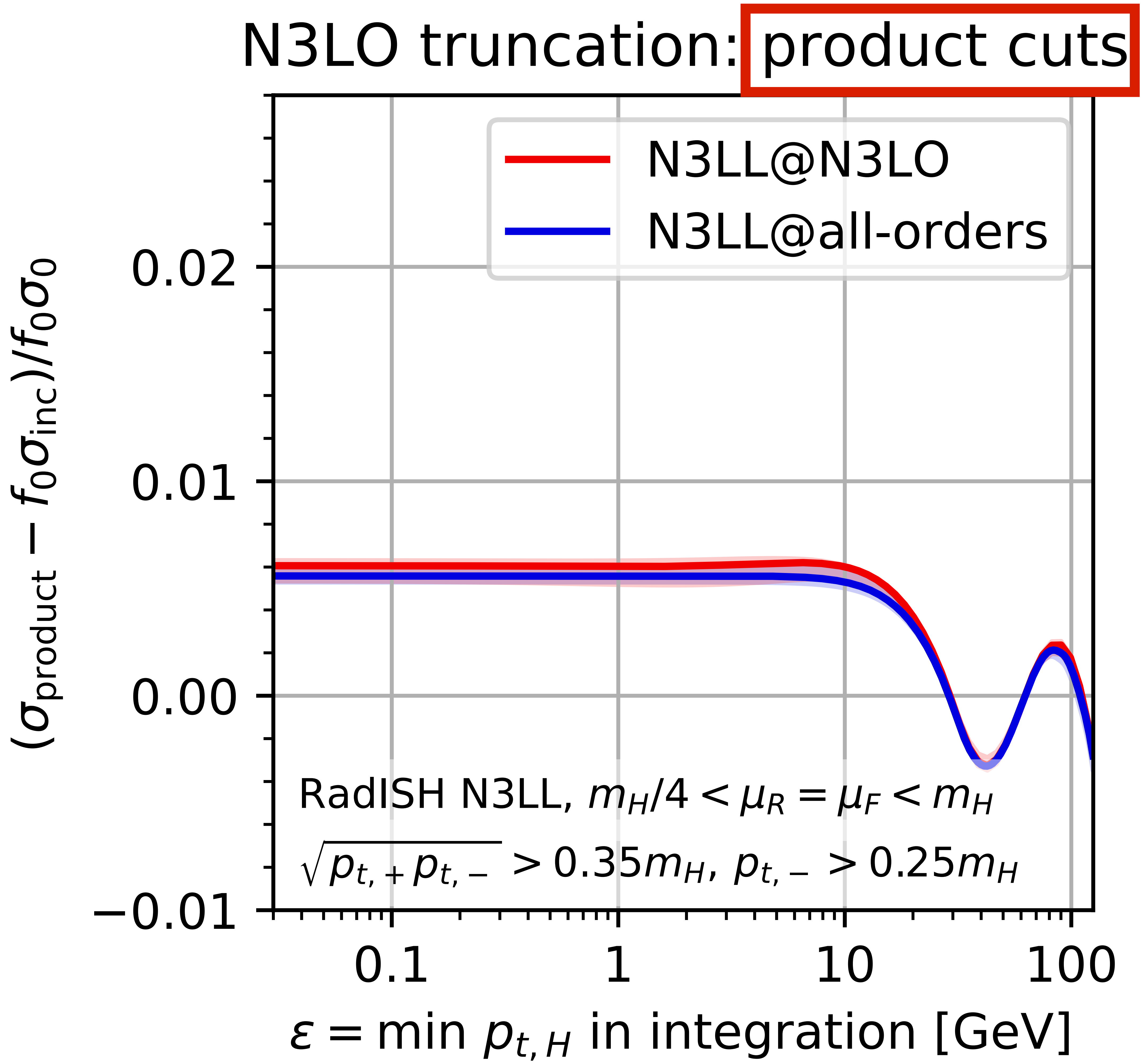
$$\frac{\sigma_{\text{prod}} - f_0\sigma_{\text{inc}}}{\sigma_0 f_0} \simeq 0.005_{\alpha_s} - 0.002_{\alpha_s^2} + 0.002_{\alpha_s^3} - 0.001_{\alpha_s^4} + 0.001_{\alpha_s^5} + \ldots \simeq 0.003 \quad @\text{DL},$$
$$\simeq 0.005_{\alpha_s} - 0.002_{\alpha_s^2} + 0.000_{\alpha_s^3} - 0.000_{\alpha_s^4} + 0.000_{\alpha_s^5} + \ldots \simeq 0.003 \quad @\text{LL},$$
$$\simeq 0.005_{\alpha_s} + 0.002_{\alpha_s^2} - 0.001_{\alpha_s^3} + \ldots \simeq 0.005 \quad @\text{NNLL},$$
$$\simeq 0.005_{\alpha_s} + 0.002_{\alpha_s^2} - 0.001_{\alpha_s^3} + \ldots \simeq 0.006 \quad @\text{N3LL}.$$

Resummed results (right-hand column)

Thanks to Pier Monni & RadISH for supplying NN(N)LL distributions & expansions, $\mu = m_H/2$

- Factorial growth of series strongly suppressed
- **N3LO truncation agrees well with all-order result**
- Per mil agreement between fixed-order and resummation **gives confidence that all is under control**

fixed-order sensitivity to low p_{tH} is gone



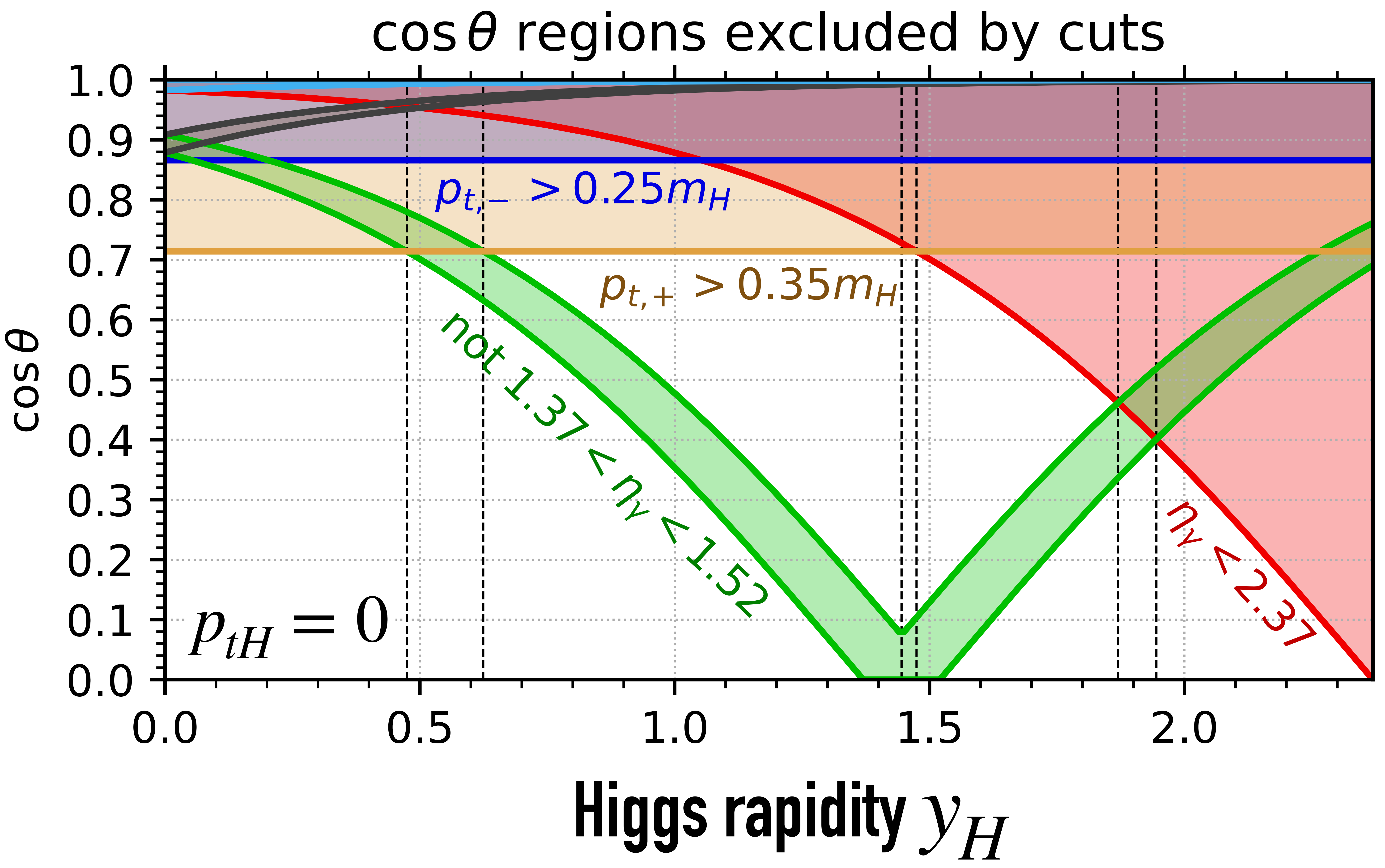
- fixed-order becomes insensitive to $p_{t,\mathrm{H}}$ values below a few GeV
- overall size of (non-Born part of) fiducial acceptance corrections much smaller
- resummation and fixed order agree at per-mil level

rapidity cuts

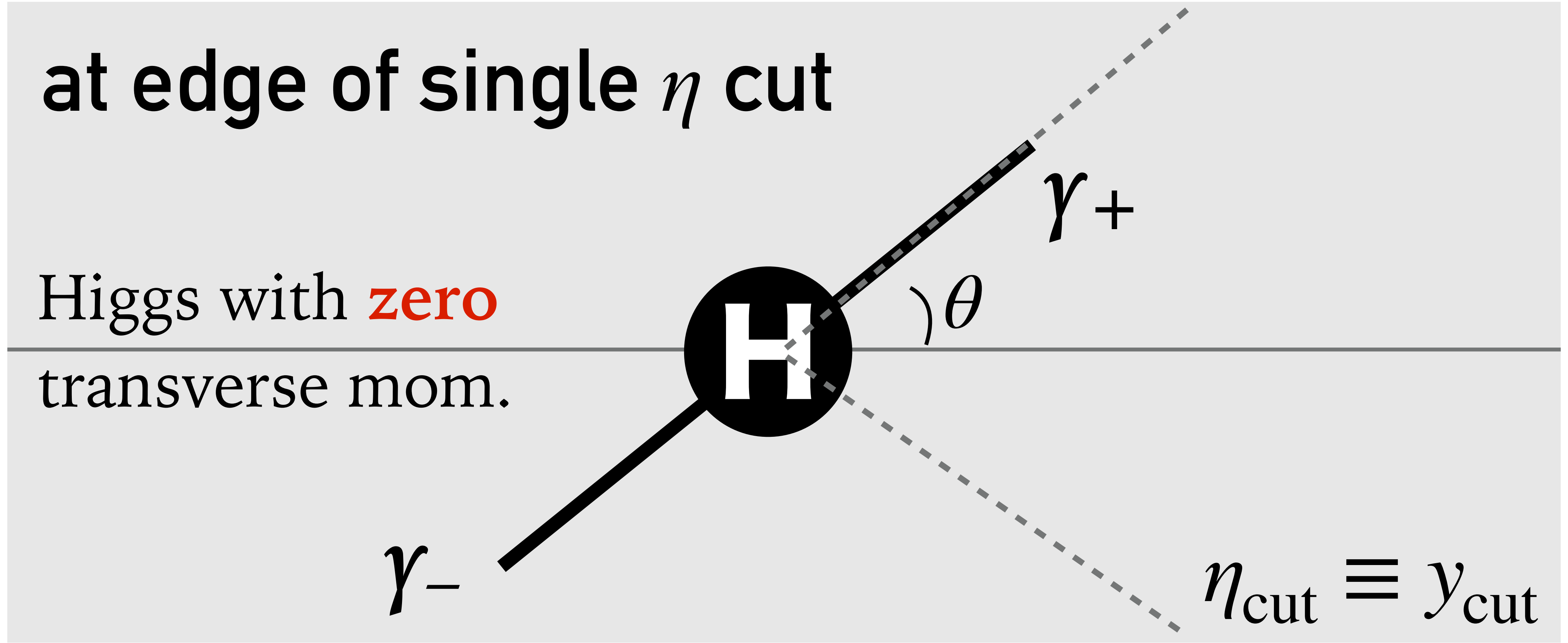
Real life measurements have rapidity cuts

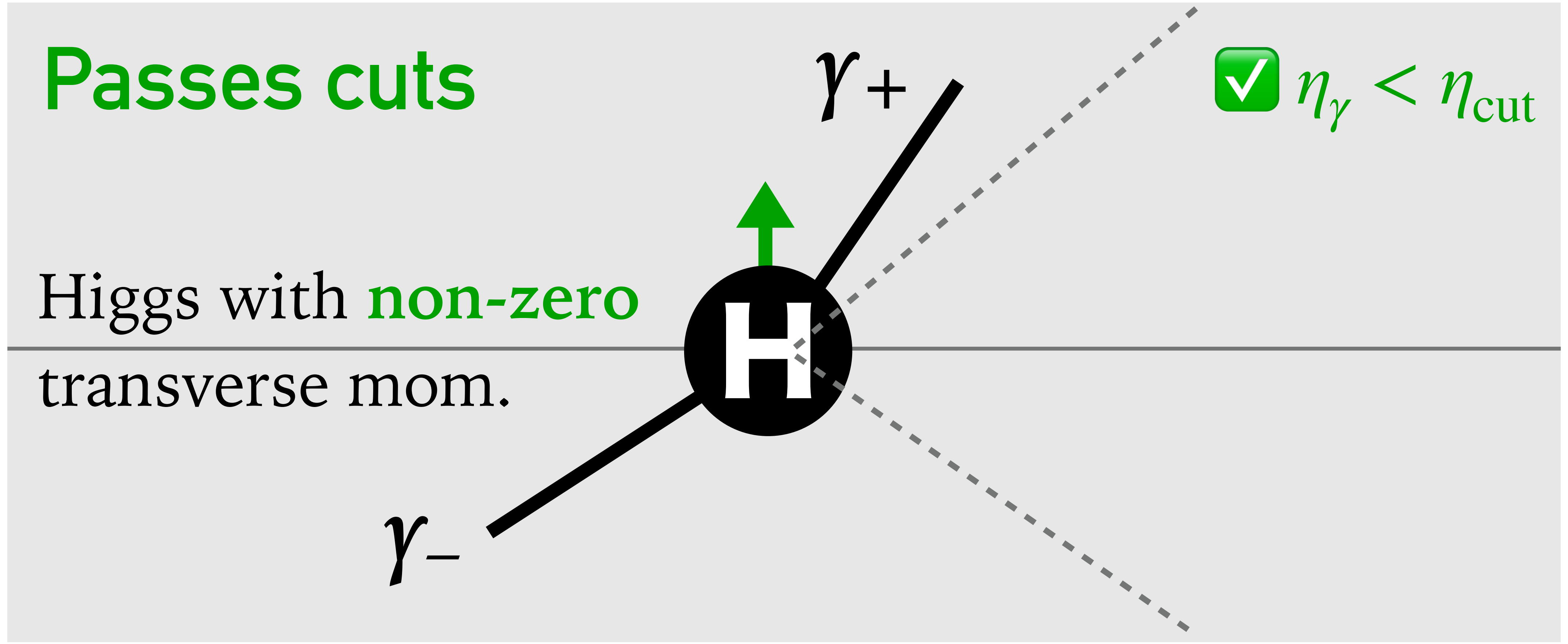
For example in the ATLAS detector:

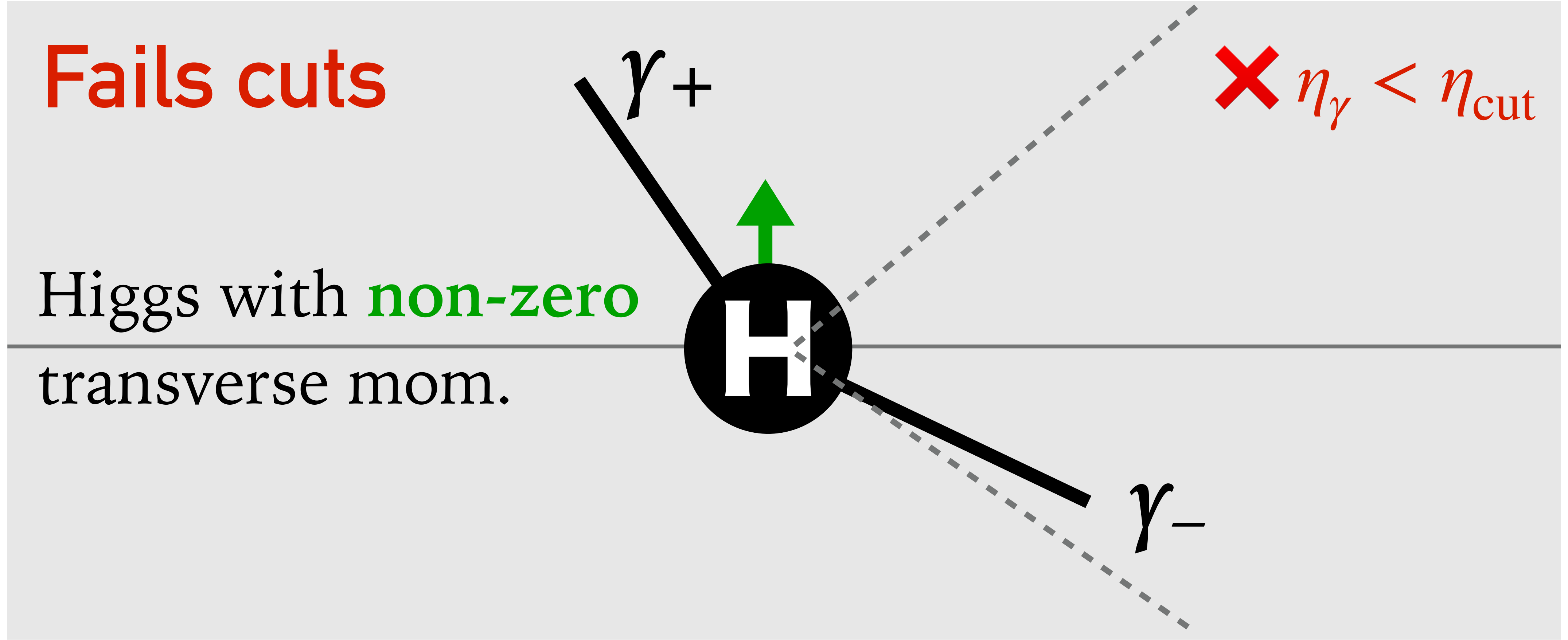
- $|\eta_\gamma| < 2.37$
  (region where EM calorimeter has sufficiently fine segmentation to distinguish γ from $\pi^0 \to \gamma\gamma$)
- not $1.37 < |\eta_\gamma| < 1.52$
  transition region between barrel and end-cap calorimeters



Visualising rapidity cuts



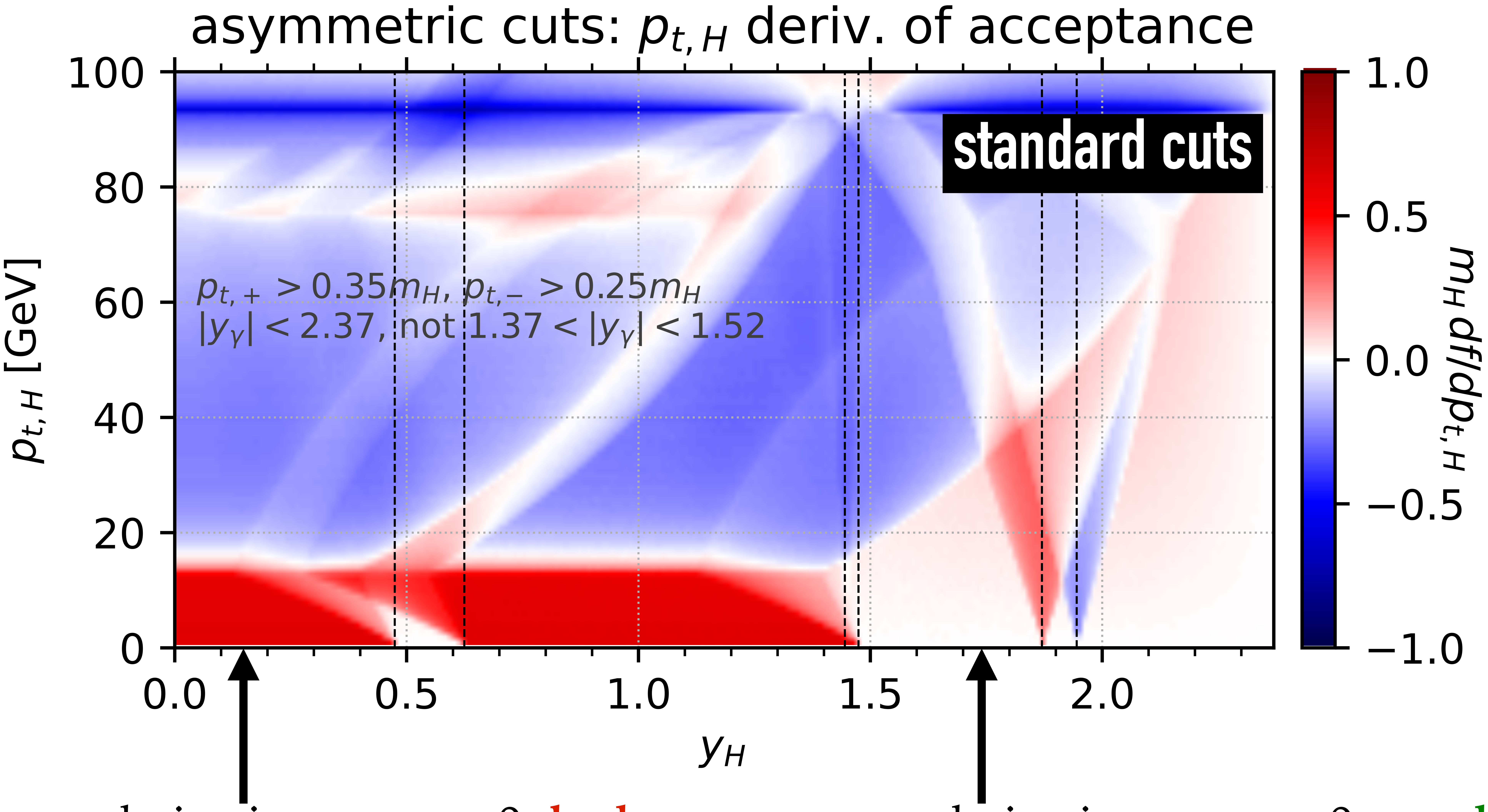




$$\cos\theta < \tanh y_{\rm cut}\left[1 + \frac{\cos\phi}{\cosh y_{\rm cut}}\cdot\frac{p_{t,\rm H}}{m_{\rm H}} + \frac{1}{2}\left(\operatorname{csch}^2 y_{\rm cut} - \cos 2\phi\right)\tanh^2 y_{\rm cut}\cdot\frac{p_{t,\rm H}^2}{m_{\rm H}^2} + \mathcal{O}_3\right]$$

Acceptance has linear dependence on Higgs p_t, but sign depends on decay orientation so **linear-p_{tH} term vanishes after azimuthal averaging**

visualising acceptance versus Higgs rapidity and p_t: look at derivative wrt p_{tH}



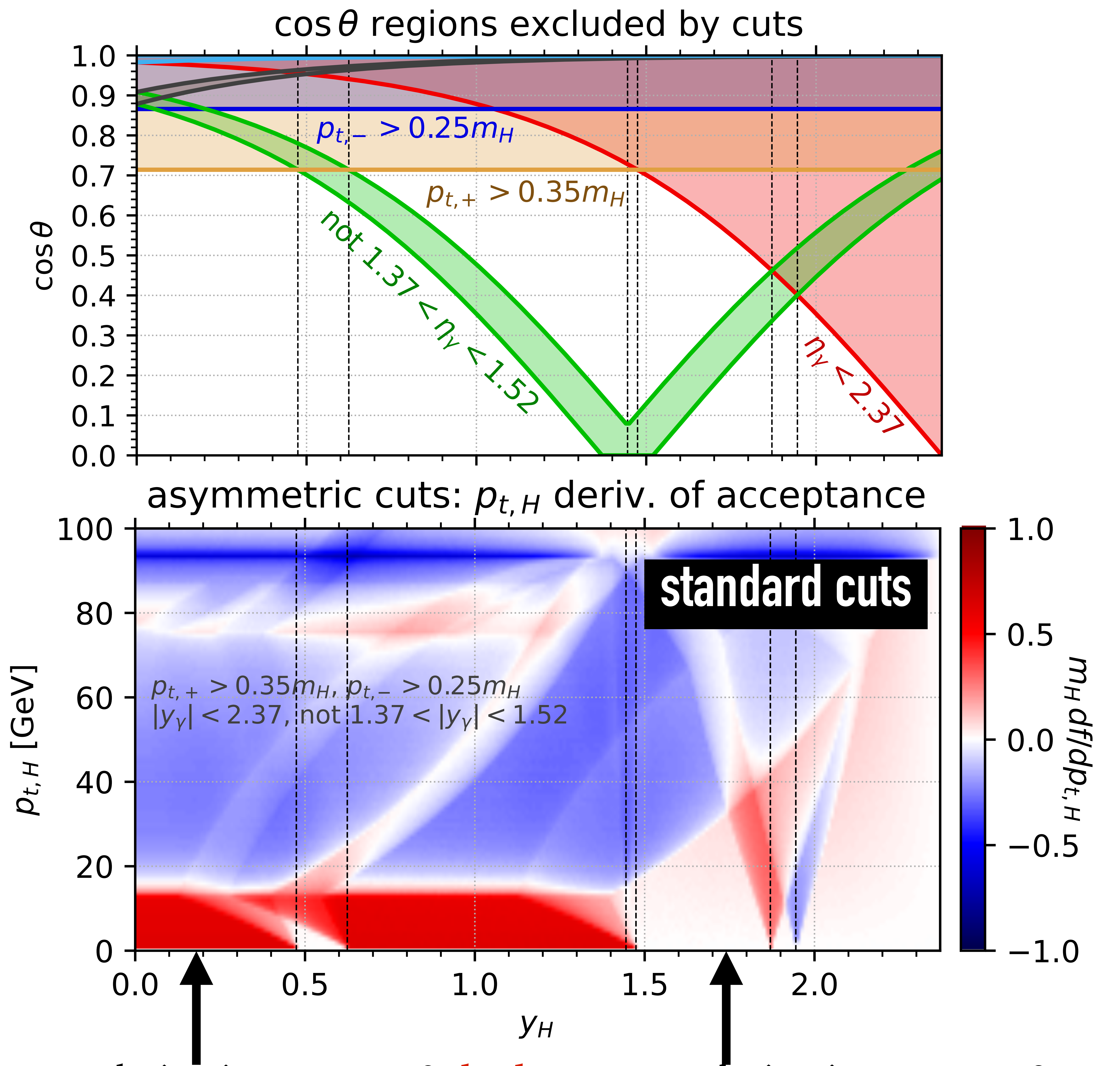
non-zero derivative at $p_{tH} = 0$: bad

zero derivative at $p_{tH} = 0$: good

interplay with η_γ cuts

Regions with bad behaviour (linear p_{tH} derivative) are those where the photon p_t cuts are active at Born level

Regions with good behaviour are those where the rapidity cuts make the photon p_t cuts irrelevant

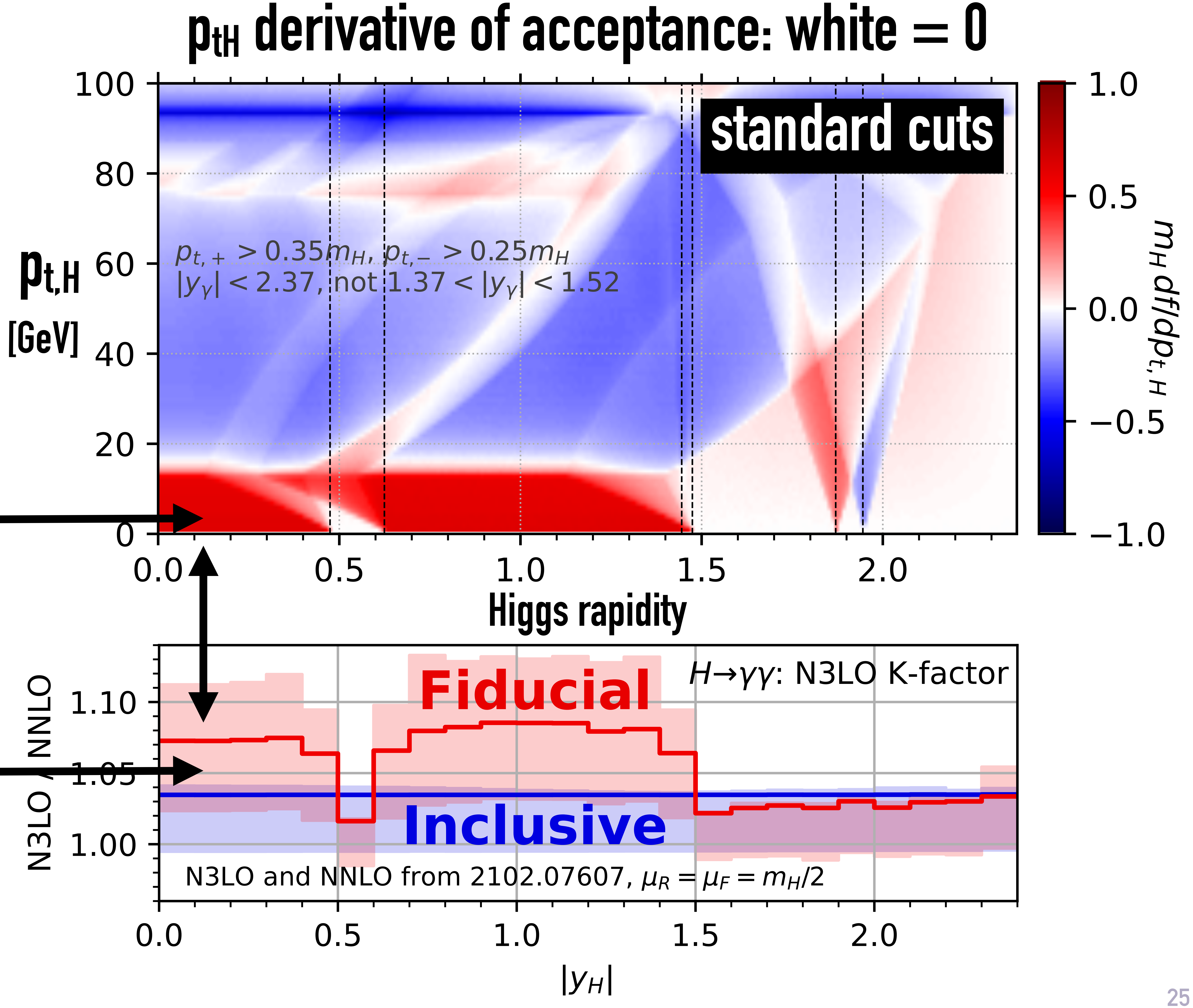


non-zero derivative at $p_{tH} = 0$: **bad**   **zero** derivative at $p_{tH} = 0$: **good**

interplay with η_γ cuts

$f(p_{t,\mathrm{H}}, y_\mathrm{H})$ has **non-zero** linear $p_{t,\mathrm{H}}$ derivative at $p_{t,\mathrm{H}} = 0$

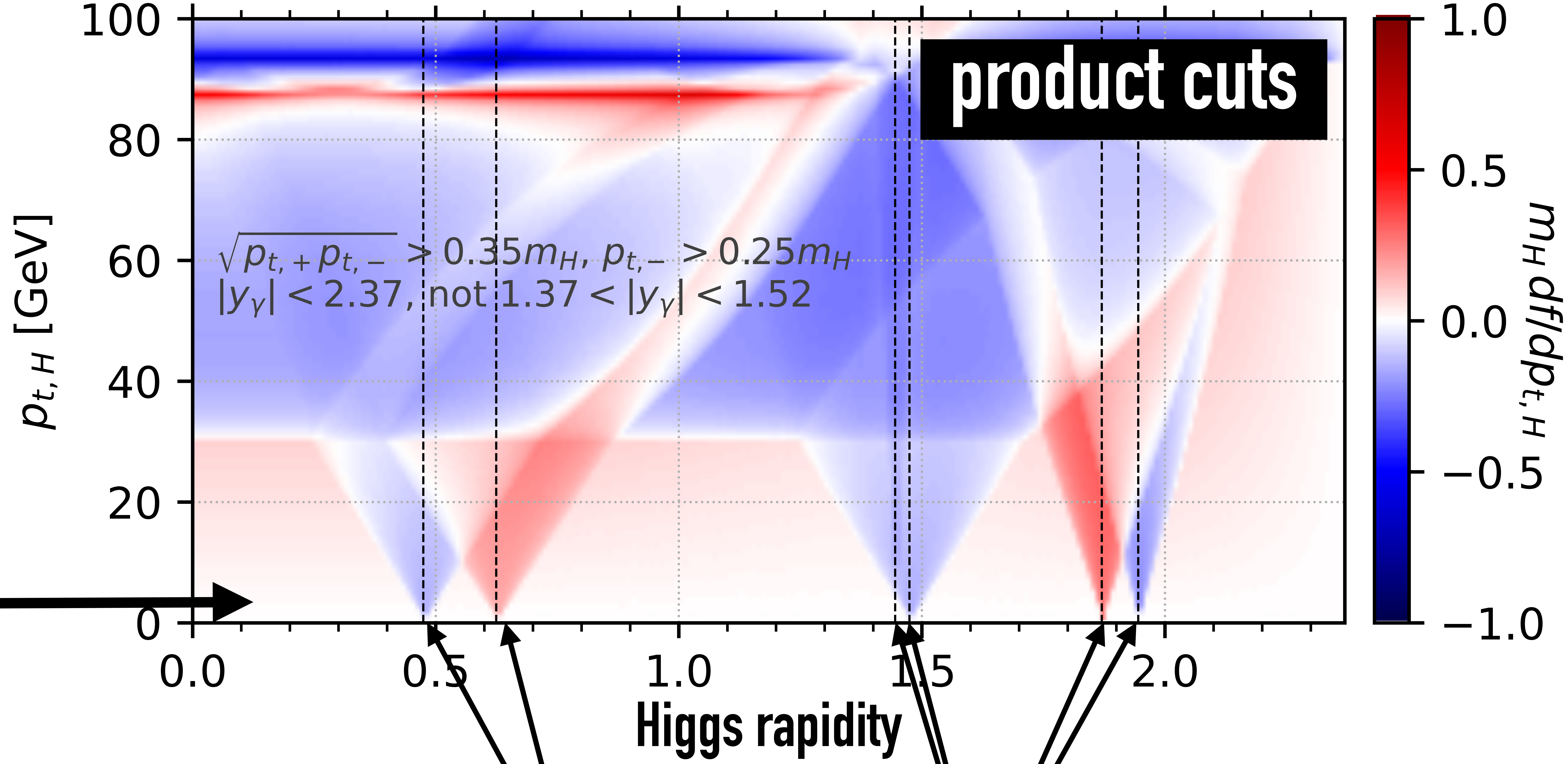
fixed-order perturbation theory has trouble



interplay with η_γ cuts

$f(p_{t,\mathrm{H}}, y_\mathrm{H})$ has **zero** linear $p_{t,\mathrm{H}}$ derivative at $p_{t,\mathrm{H}} = 0$

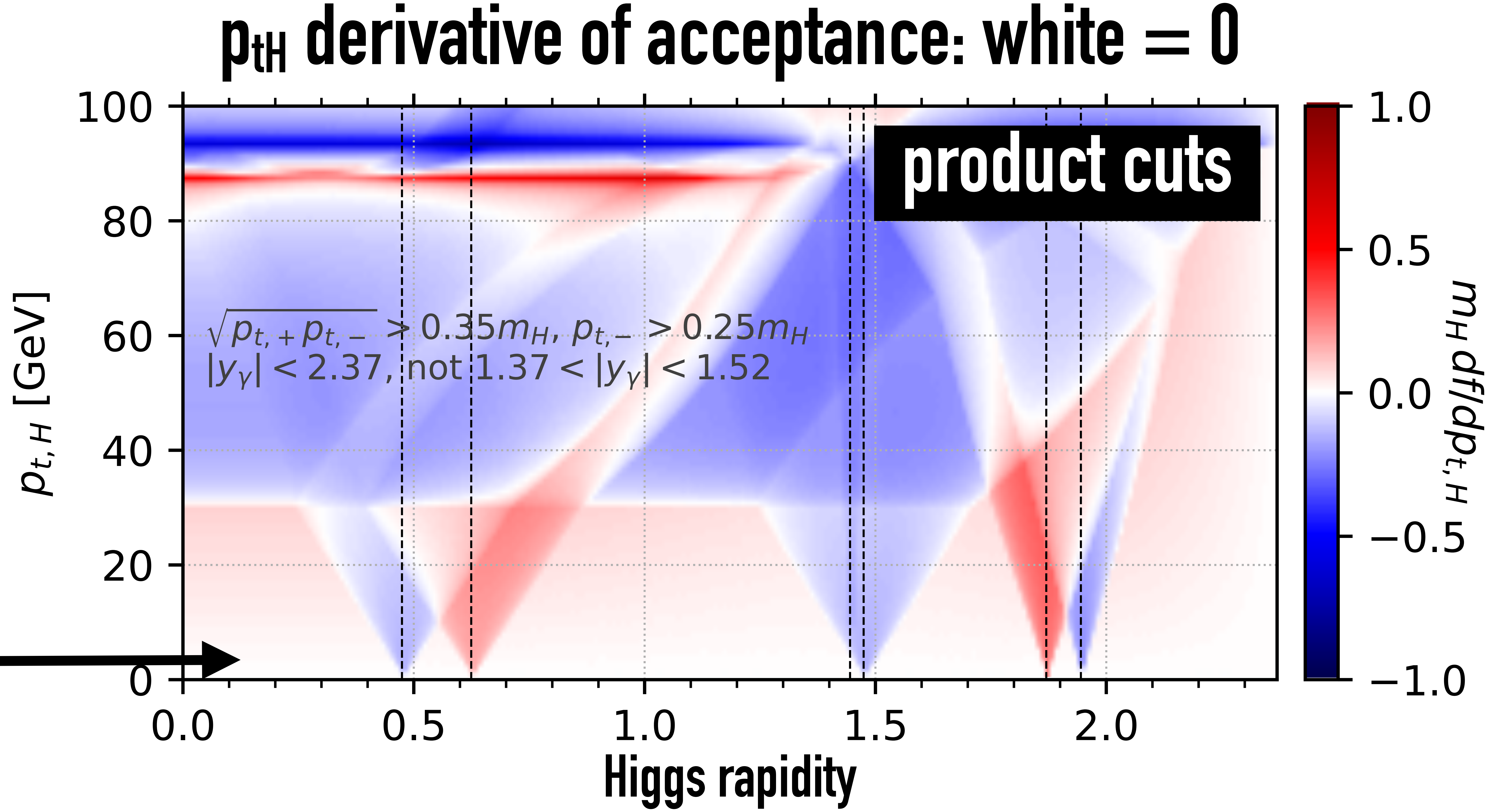
fixed-order perturbation theory will be fine



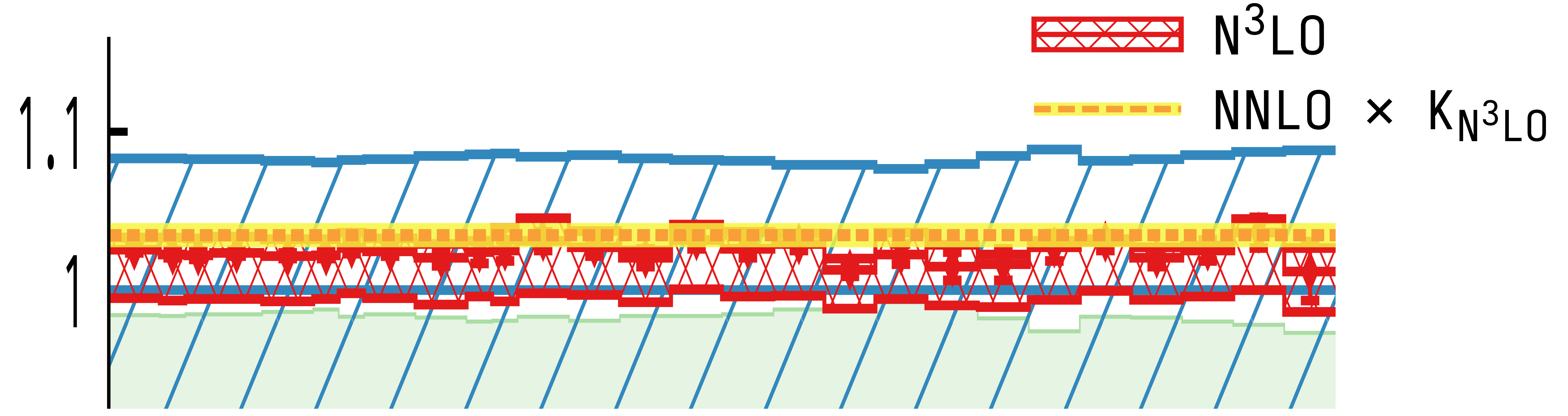
NB: at these points Born η_γ and $p_{t,\gamma}$ cuts are degenerate. If doing rapidity binning, choose bins that are not too narrow (e.g. ±0.1 around them)

interplay with η_γ cuts

$f(p_{t,\mathrm{H}}, y_\mathrm{H})$ has **zero** linear $p_{t,\mathrm{H}}$ derivative at $p_{t,\mathrm{H}} = 0$



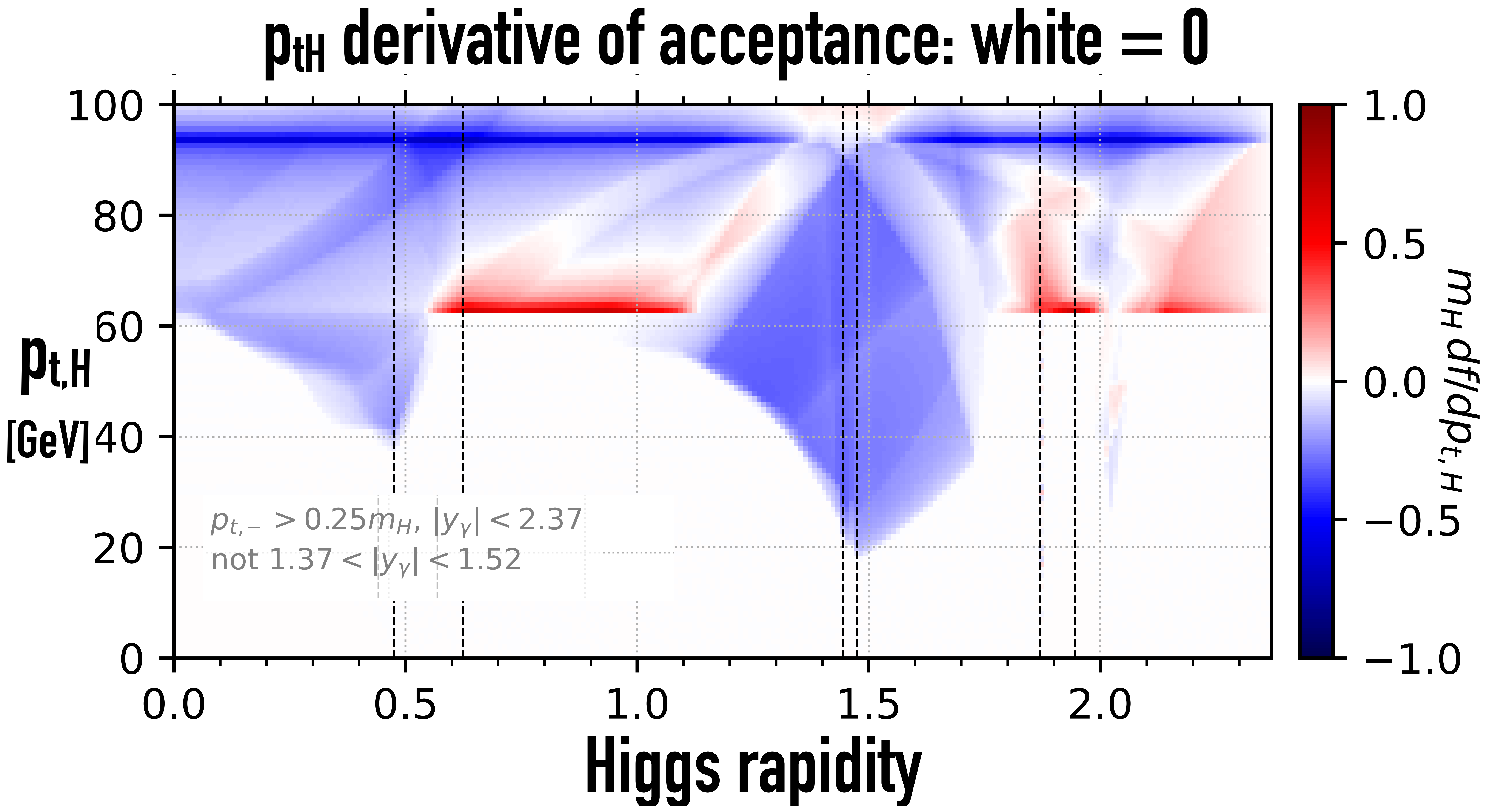
fixed-order perturbation theory will be fine



Huss et al preliminary @ Higgs 2021

Solution #2b: design cuts whose acceptance is independent of p_{tH} (at small p_{tH})

- keep standard cuts on softer photon p_t and on photon rapidities
- replace harder-photon p_t cut with Collins-Soper angle cut (transverse boost-invariant)
- selectively loosen CS angle cut to keep p_{tH}-independent acceptance as far as possible



hardness and rapidity compensating (CBI_{HR}) cuts

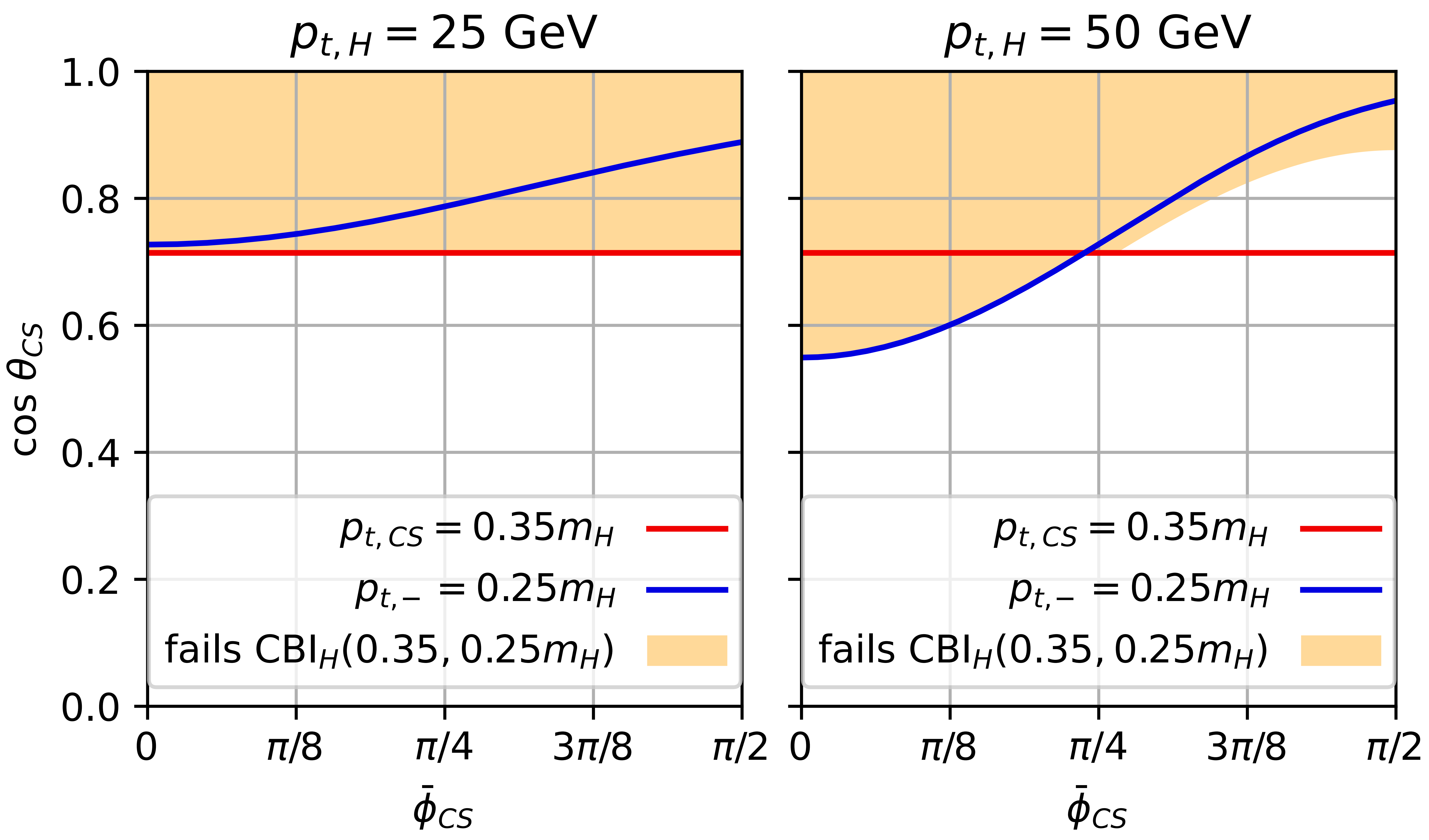
details in arXiv:2106.08329 + code at https://github.com/gavinsalam/two-body-cuts

Hardness [and rapidity] compensating boost invariant cuts (CBI_H and CBI_{HR})

Core idea 1: cut on decay p_t in Collins-Soper frame

$$\vec{p}_{t,\mathrm{CS}} = \frac{1}{2}\left[\vec{\delta}_t + \frac{\vec{p}_{t,12}.\vec{\delta}_t}{p_{t,12}^2}\left(\frac{m_{12}}{\sqrt{m_{12}^2+p_{t,12}^2}} - 1\right)\vec{p}_{t,12}\right], \qquad \vec{\delta}_t = \vec{p}_{t,1} - \vec{p}_{t,2}$$

Core idea 2: relax $p_{t,CS}$ cut at higher $p_{t,H}$ values to maintain constant / maximal acceptance



Solution #3: defiducialise (cf. Glazov 2001.02933 for DY)

- **Option 3a**: divide out both $p_{t,\mathrm{H}}$ and y_H dependence of acceptance from fiducial differential cross section

$$\sigma_\mathrm{defid} = \int_{-y_\mathrm{H}^\mathrm{max}}^{+y_\mathrm{H}^\mathrm{max}} dy_\mathrm{H} \int_0^{p_{t,\mathrm{H}}^\mathrm{max}} dp_{t,\mathrm{H}} \frac{d\sigma^\mathrm{fid}}{dy_\mathrm{H} dp_{t,\mathrm{H}}} \frac{1}{f(y_\mathrm{H}, p_{t,\mathrm{H}})}\,, \equiv \int_{-y_\mathrm{H}^\mathrm{max}}^{+y_\mathrm{H}^\mathrm{max}} dy_\mathrm{H} \int_0^{p_{t,\mathrm{H}}^\mathrm{max}} dp_{t,\mathrm{H}} \frac{d\sigma}{dy_\mathrm{H} dp_{t,\mathrm{H}}}\,,$$

- **Option 3b**: divide out just $p_{t,\mathrm{H}}$ dependence of acceptance from fiducial differential cross section (adapted from suggestion by referee of paper)

$$\sigma_{\mathrm{defid},p_{t,\mathrm{H}}} = \int_{-y_\mathrm{H}^\mathrm{max}}^{+y_\mathrm{H}^\mathrm{max}} dy_\mathrm{H} \int_0^{p_{t,\mathrm{H}}^\mathrm{max}} dp_{t,\mathrm{H}} \frac{d\sigma^\mathrm{fid}}{dy_\mathrm{H} dp_{t,\mathrm{H}}} \frac{f(y_\mathrm{H},0)}{f(y_\mathrm{H}, p_{t,\mathrm{H}})}\,, \equiv \int_{-y_\mathrm{H}^\mathrm{max}}^{+y_\mathrm{H}^\mathrm{max}} dy_\mathrm{H} \int_0^{p_{t,\mathrm{H}}^\mathrm{max}} dp_{t,\mathrm{H}} \frac{d\sigma}{dy_\mathrm{H} dp_{t,\mathrm{H}}} f(y_\mathrm{H},0)\,,$$

NB1: some care needed in choice of integration limits, to avoid division by zero (or, for 3a, by small numbers for $y_\mathrm{H} \gtrsim 2$)

NB2: defiducialisation is theoretically robust for a scalar particle (in a way that it is not for DY)

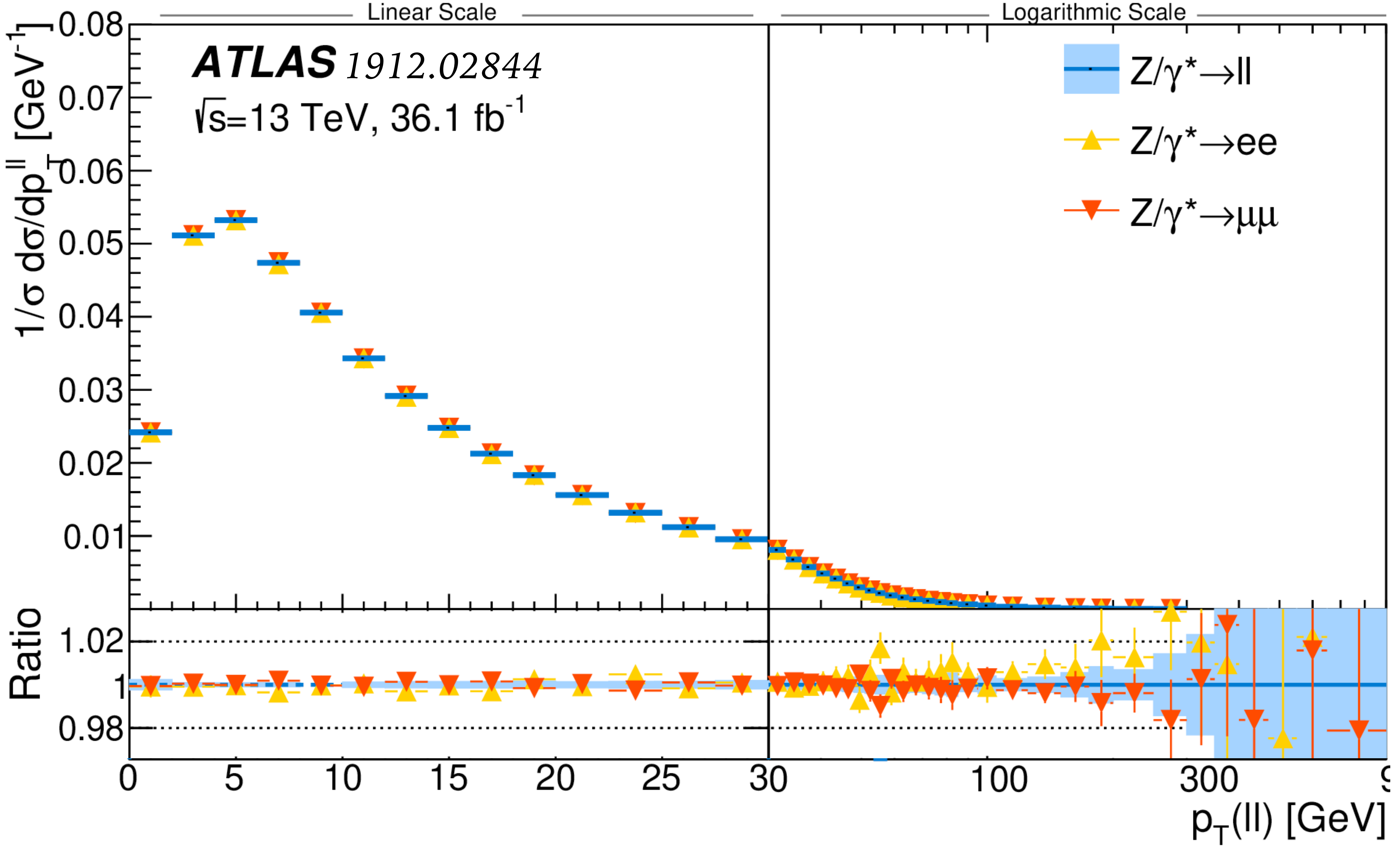
NB3: code at https://github.com/gavinsalam/two-body-cuts can also help with defiducialisation for Higgs

other processes, e.g. Drell-Yan

the problem is not just for Higgs

- any time you have a 2-body final state that is symmetric at LO, one should ask if the analysis involves the asymmetry induced when there is a non-zero system p_t
- Drell-Yan measurements often use asymmetric (or symmetric) lepton cuts
- continuum γγ production
  (but very large NLO/NNLO corrections from new topologies are probably more important)
- $t\bar{t}$ studies, e.g. plot p_t of leading and subleading top-quark
  (NB: those observables can be relevant for separating out different production mechanisms, but there are better ways of doing that cf. Caola, Dreyer, McDonals & GPS, 2101.06068)

Z p_T distribution — a showcase for LHC precision



Normalised distribution's statistical and systematic errors well below 1% all the way to $p_T \sim 200$ GeV

Largest normalisation err is luminosity then lepton ID

$$\sigma_{\text{fid}} = 736.2 \pm 0.2\ \text{(stat)} \pm 6.4\ \text{(syst)} \pm 15.5\ \text{(lumi)}\ \text{pb}$$

Precision luminosity measurement in proton-proton collisions at $\sqrt{s} = 13$ TeV in 2015 and 2016 at CMS

Table 4: Summary of contributions to the relative systematic uncertainty in σ_{vis} (in %) at $\sqrt{s} = 13$ TeV in 2015 and 2016. The systematic uncertainty is divided into groups affecting the description of the vdM profile and the bunch population product measurement (normalization), and the measurement of the rate in physics running conditions (integration). The fourth column indicates whether the sources of uncertainty are correlated between the two calibrations at $\sqrt{s} = 13$ TeV.

Source	2015 [%]	2016 [%]	Corr
Normalization uncertainty			
Bunch population			
Ghost and satellite charge	0.1	0.1	Yes
Beam current normalization	0.2	0.2	Yes
Beam position monitoring			
Orbit drift	0.2	0.1	No
Residual differences	0.8	0.5	Yes
Beam overlap description			
Beam-beam effects	0.5	0.5	Yes
Length scale calibration	0.2	0.3	Yes
Transverse factorizability	0.5	0.5	Yes
Result consistency			
Other variations in σ_{vis}	0.6	0.3	No
Integration uncertainty			
Out-of-time pileup corrections			
Type 1 corrections	0.3	0.3	Yes
Type 2 corrections	0.1	0.3	Yes
Detector performance			
Cross-detector stability	0.6	0.5	No
Linearity	0.5	0.3	Yes
Data acquisition			
CMS deadtime	0.5	<0.1	No
Total normalization uncertainty	1.3	1.0	—
Total integration uncertainty	1.0	0.7	—
Total uncertainty	1.6	1.2	—

Luminosity: the systematic common to all measurements

- has hovered around 2% for many years (except LHC*b*)
- CMS has recently shown that they can get it down to 1.2%
- a major achievement, because it matters across the spectrum of precision LHC results

Drell-Yan harmonic decomposition (with Collins-Soper angles)

$$\frac{d\sigma}{d^4q\,d\cos\theta\,d\phi} = \frac{3}{16\pi}\frac{d\sigma^{\text{unpol.}}}{d^4q}\left(h_u(\theta,\phi) + \sum_{i=0}^{7} A_i(q)\,h_i(\theta,\phi)\right)$$

modulo certain classes of electroweak correction

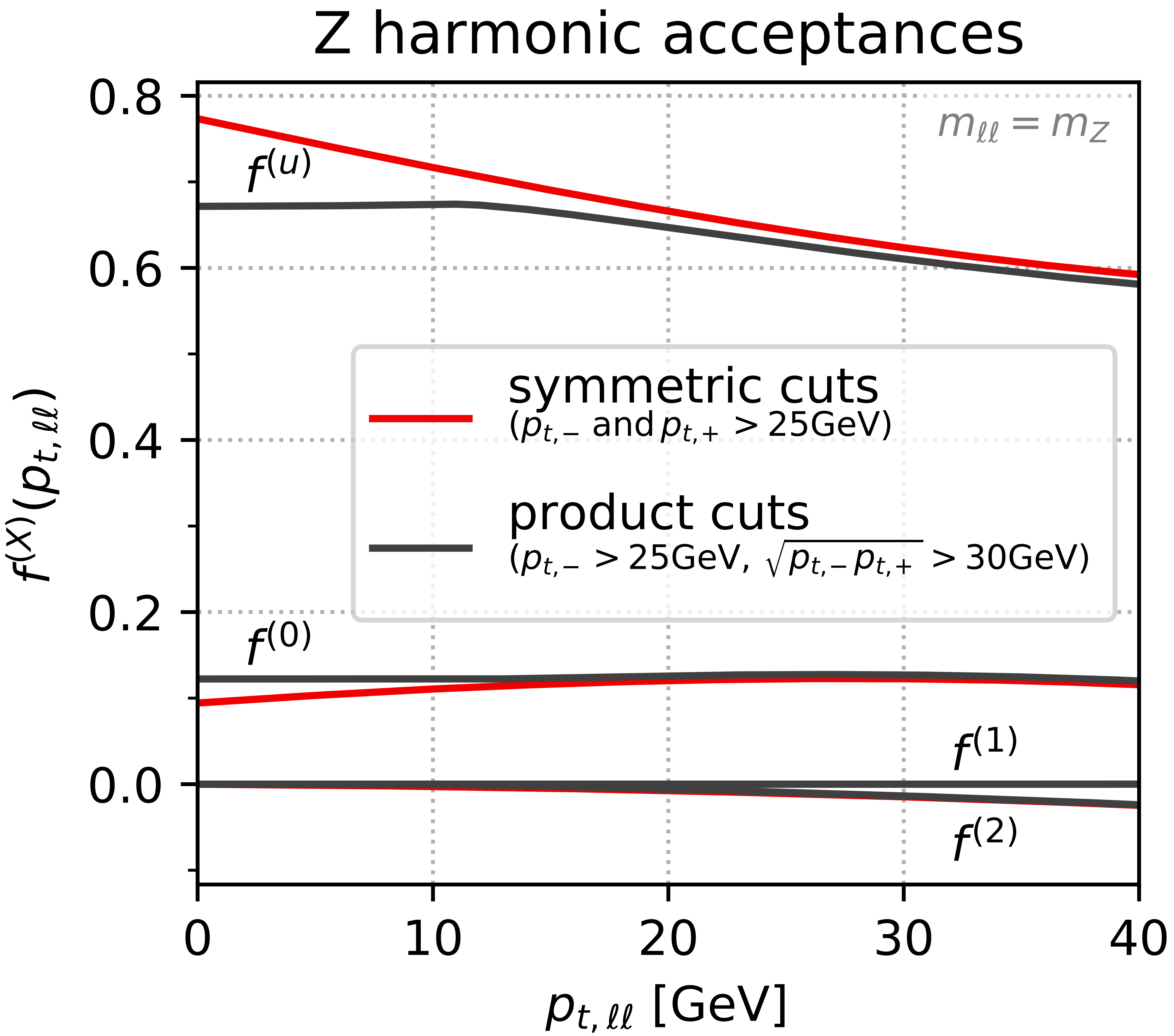
$$\begin{aligned}
h_u &= 1+\cos^2\theta\,, & h_0 &= \tfrac{1}{2}(1-3\cos^2\theta)\,, & h_1 &= \sin 2\theta\cos\phi\,,\\
h_2 &= \tfrac{1}{2}\sin^2\theta\cos 2\phi\,, & h_3 &= \sin\theta\cos\phi\,, & h_4 &= \cos\theta,\\
h_5 &= \sin^2\theta\sin 2\phi\,, & h_6 &= \sin 2\theta\sin\phi\,, & h_7 &= \sin\theta\sin\phi\,.
\end{aligned}$$

$$\frac{d\sigma_{\text{fid}}}{d^4q} = \frac{d\sigma^{\text{unpol.}}}{d^4q}\left[f^{(u)}(q) + \sum_{i=0\ldots7} A_i(q) f^{(i)}(q)\right]$$

the $f^{(i)}(q)$ are the acceptances for each harmonic i

$$f^{(\mathrm{X})}(q) = \frac{3}{16\pi}\int_{-1}^{1} d\cos\theta \int_{-\pi}^{\pi} d\phi\; h_{\mathrm{X}}(\theta,\phi)\;\Theta_{\text{cuts}}(\theta,\phi,q)$$

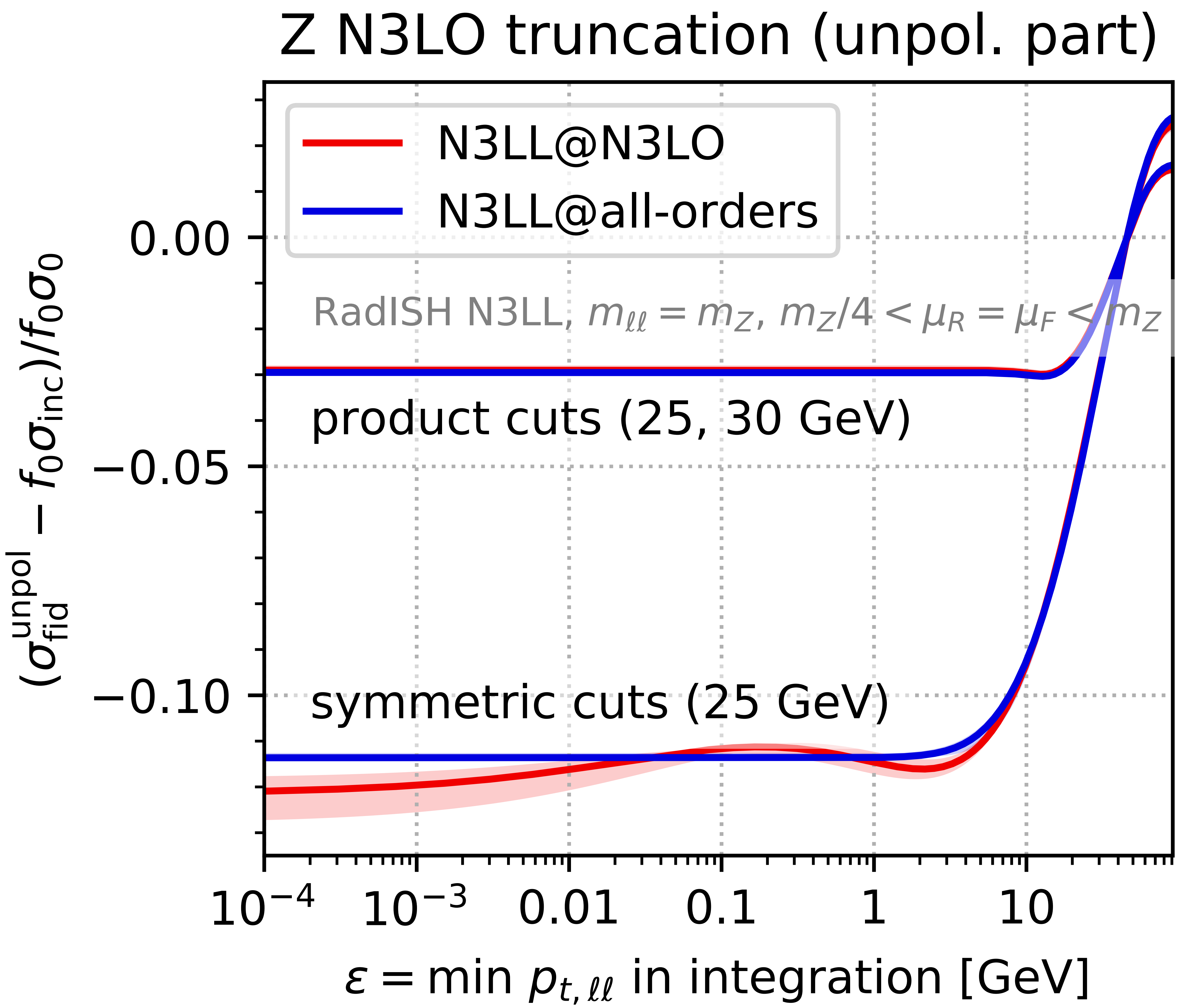
Example in Drell-Yan case



- harmonic acceptances $f^{(i)}(q)$ are zero for $i = 3...7$ (if we treat $\ell^{\pm}$ equivalently)
- cross section weights multiplying them
  - $A_0, A_2 \sim p_t^2$, $A_1 \sim p_t$
- if $f^{(u)}$ has at most quadratic dependence on p_t and $f^{(1)}$ is zero at $p_t = 0$, effective cross section acceptance will have quadratic dependence and we should be safe

Example in Drell-Yan case

- problems are %-level, i.e. much smaller than in Higgs case, because $C_A \to C_F$
- but experimental precisions are higher too

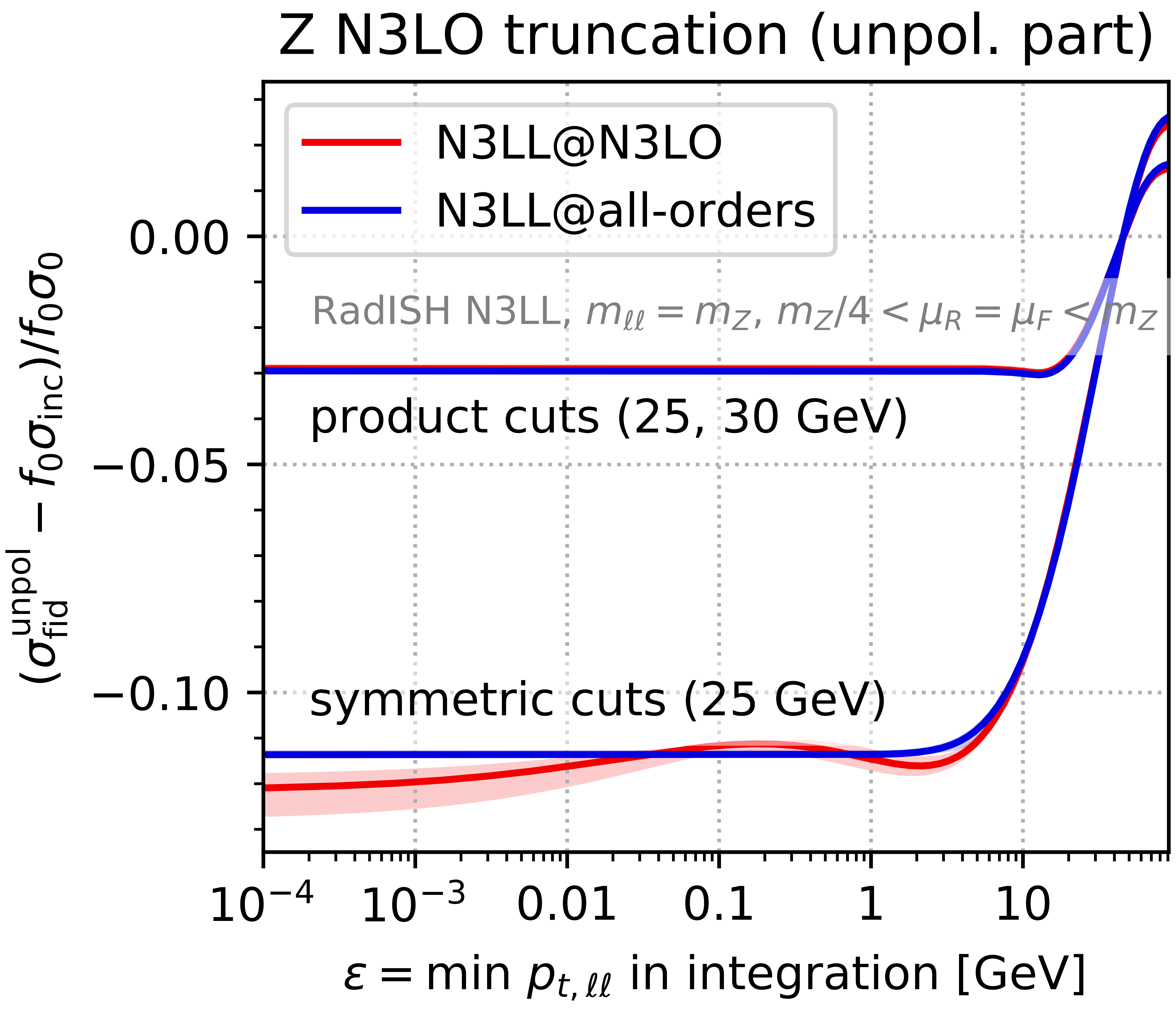


$$
\begin{aligned}
\frac{\sigma^{(\rm u)}_{\rm sym} - f_0\sigma_{\rm inc}}{\sigma_0 f_0} &\simeq -0.074_{\alpha_s} + 0.051_{\alpha_s^2} - 0.057_{\alpha_s^3} + 0.090_{\alpha_s^4} - 0.181_{\alpha_s^5} + \ldots \simeq -0.047 \quad @\text{DL},\\
&\simeq -0.074_{\alpha_s} + 0.027_{\alpha_s^2} - 0.014_{\alpha_s^3} + 0.010_{\alpha_s^4} - 0.010_{\alpha_s^5} + \ldots \simeq -0.055 \quad @\text{LL},\\
&\simeq -0.118_{\alpha_s} + 0.012_{\alpha_s^2} - 0.016_{\alpha_s^3} + \ldots \simeq -0.114 \quad @\text{NNLL},\\
&\simeq -0.118_{\alpha_s} + 0.012_{\alpha_s^2} - 0.016_{\alpha_s^3} + \ldots \simeq -0.114 \quad @\text{N3LL}.
\end{aligned}
$$

symmetric cuts

Example in Drell-Yan case (unpol.)

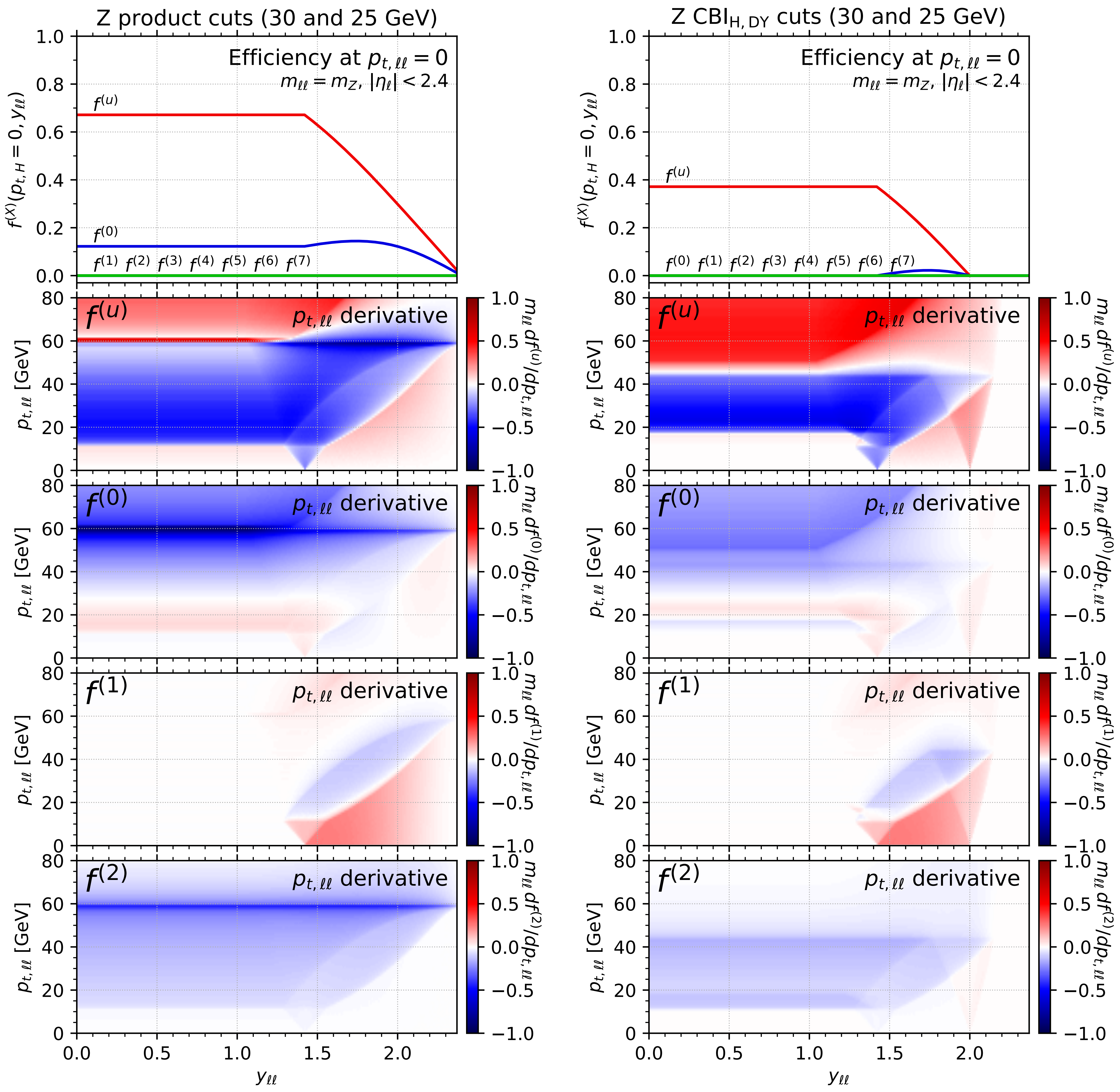
- problems are %-level, i.e. much smaller than in Higgs case, because $C_A \to C_F$
- but experimental precisions are higher too
- **product cuts are much more convergent and stable**



$$
\begin{aligned}
\frac{\sigma^{(\mathrm{u})}_{\mathrm{prod}} - f_0\sigma_{\mathrm{inc}}}{\sigma_0 f_0} &\simeq -0.006_{\alpha_s} - 0.000_{\alpha_s^2} + 0.000_{\alpha_s^3} - 0.000_{\alpha_s^4} - 0.000_{\alpha_s^5} + \ldots \quad \simeq -0.006 \quad \text{@DL},\\
&\simeq -0.006_{\alpha_s} - 0.000_{\alpha_s^2} - 0.000_{\alpha_s^3} + 0.000_{\alpha_s^4} - 0.000_{\alpha_s^5} + \ldots \quad \simeq -0.007 \quad \text{@LL},\\
&\simeq -0.018_{\alpha_s} - 0.009_{\alpha_s^2} - 0.003_{\alpha_s^3} + \ldots \quad \simeq -0.030 \quad \text{@NNLL},\\
&\simeq -0.018_{\alpha_s} - 0.009_{\alpha_s^2} - 0.002_{\alpha_s^3} + \ldots \quad \simeq -0.029 \quad \text{@N3LL}.
\end{aligned}
$$

product cuts

DY p_t dependence of harmonic acceptances with product and boost invariant cuts



It is possible to get identically zero p_t dependence for all harmonic acceptances (at central rapidity) with an extra cut

$$\cos\theta > \bar{c} = \frac{-c_0 + \sqrt{4 - 3c^2}}{2}$$

Full N3LO calculation (all harmonics)

Chen, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli, 2203.01565

Order	σ [pb] Symmetric cuts		σ [pb] Product cuts	
k	$\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$	$\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

+3pb ~ 0.5%

–0.2 pb ~ 0.02%

fixed-order & resummed for fiducial σ agree better with product cuts than symmetric cuts
(scale uncertainty also lower with product cuts, but only moderately)

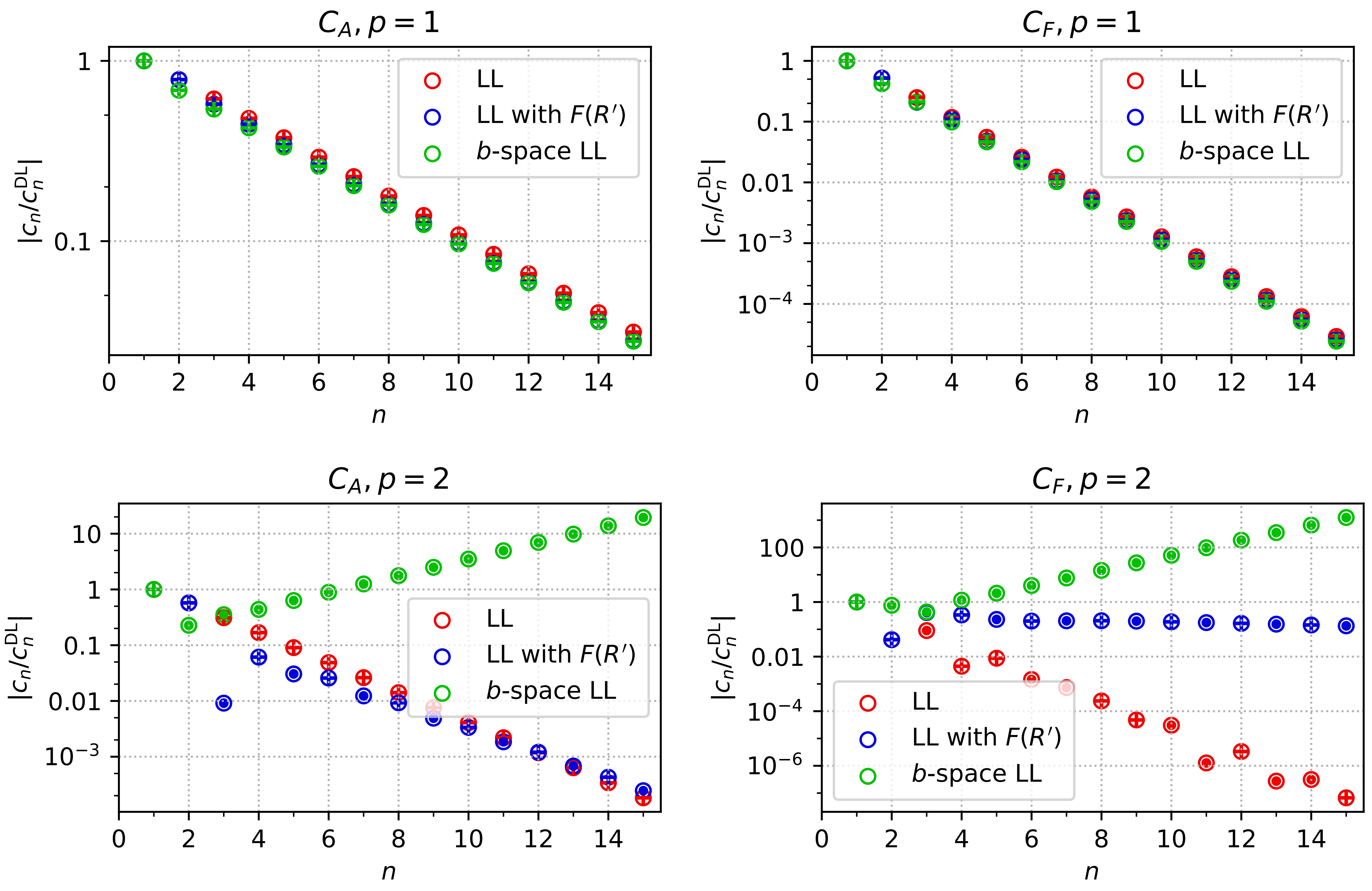
Conclusions

- Fixed-order perturbation theory can be badly compromised by existing (2-body) cuts (→ intriguing questions about asymptotics of QCD perturbative series)
- In simple cases (e.g. $H \to \gamma\gamma$), can be solved by resummation. But physics will be more robust if we can reliably use both fixed-order and resummed+FO results (and both yield similar central values & uncertainties)
- A better long-term solution may be to **revisit experimental cuts**:
  - product and boost-invariant cuts give much better perturbative series
  - Potentially relevant also for other processes (for DY: effects at the 0.5–1%-level)
- Alternatively: in Higgs case, you can **defiducialise**
- Cuts with little p_{tH} dependence (or defiducialisation) may be useful also, e.g., for extrapolating measurements to STXS or more inclusive cross sections, with limited dependence on BSM or non-perturbative effects.
- **Needs addressing in future LHC measurements for robust accuracy in Run 3 & HL-LHC**

Backup

Effective power-ambiguity in truncation of perturbative series (for p_t^p dependence)

$$\left(\frac{\Lambda}{Q}\right)^{|r|\frac{(11C_A - 2n_f)p^2}{48C}}$$



C is C_A (for $gg \to X$) or C_F (for $q\bar{q} \to X$)

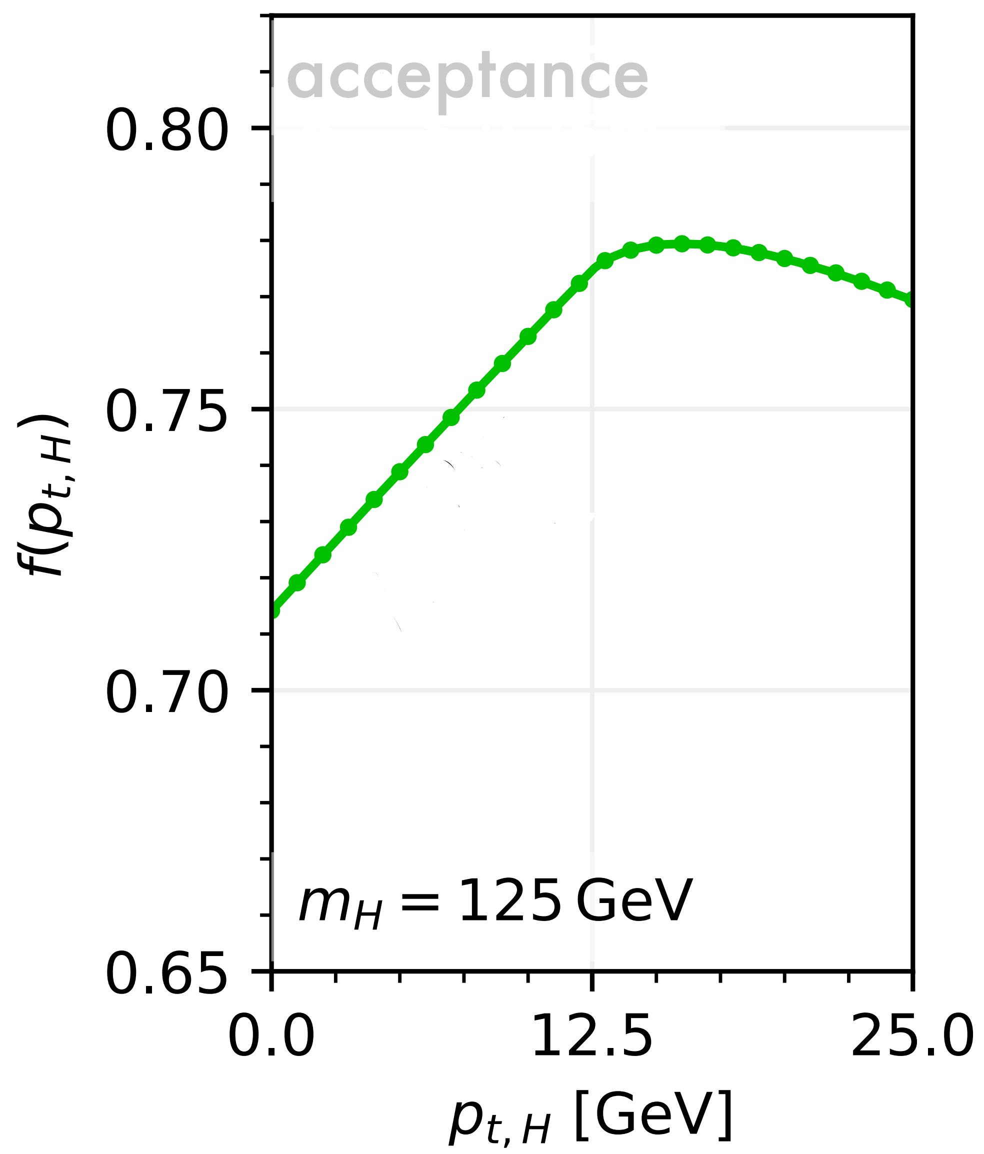
r captures difference true all-order scaling and DL approx

Numerical studies give stable result for $p = 1$ (linear dep.) giving $(\Lambda/Q)^{0.205}$ for $gg \to X$ and $(\Lambda/Q)^{0.76}$ for $q\bar{q} \to X$. Scaling with quadratic cuts ($p = 2$) remains tbd

Cut Type	cuts on	small-$p_{t,\mathrm{H}}$ dependence	f_n coefficient	$p_{t,\mathrm{H}}$ transition
symmetric	$p_{t,-}$	linear	$+2s_0/(\pi f_0)$	none
asymmetric	$p_{t,+}$	linear	$-2s_0/(\pi f_0)$	Δ
sum	$\frac{1}{2}(p_{t,-} + p_{t,+})$	quadratic	$(1+s_0^2)/(4f_0)$	2Δ
product	$\sqrt{p_{t,-} + p_{t,+}}$	quadratic	$s_0^2/(4f_0)$	2Δ
staggered	$p_{t,1}$	quadratic	$s_0^4/(4f_0^3)$	Δ
Collins-Soper	$p_{t,\mathrm{CS}}$	none	—	2Δ
CBI_H	$p_{t,\mathrm{CS}}$	none	—	$2\sqrt{2}\Delta$
rapidity	y_γ	quadratic	$f_0 s_0^2/2$	

Table 1: Summary of the main hardness cuts, the variable they cut on at small $p_{t,\mathrm{H}}$, and the small-$p_{t,\mathrm{H}}$ dependence of the acceptance. For linear cuts $f_n \equiv f_1$ multiplies $p_{t,\mathrm{H}}/m_\mathrm{H}$, while for quadratic cuts $f_n \equiv f_2$ multiplies $(p_{t,\mathrm{H}}/m_\mathrm{H})^2$ (in all cases there are additional higher order terms that are not shown). For a leading threshold of $p_{t,\mathrm{cut}}$, $s_0 = 2p_{t,\mathrm{cut}}/m_\mathrm{H}$ and $f_0 = \sqrt{1-s_0^2}$, while for the rapidity cut $s_0 = 1/\cosh(y_\mathrm{H} - y_\mathrm{cut})$. For a cut on the softer lepton's transverse momentum of $p_{t,-} > p_{t,\mathrm{cut}} - \Delta$, the right-most column indicates the $p_{t,\mathrm{H}}$ value at which the $p_{t,-}$ cut starts to modify the behaviour of the acceptance (additional $\mathcal{O}\left(\Delta^2/m_\mathrm{H}\right)$ corrections not shown). For the interplay between hardness and rapidity cuts, see sections 4.2, 4.3 and 5.2.

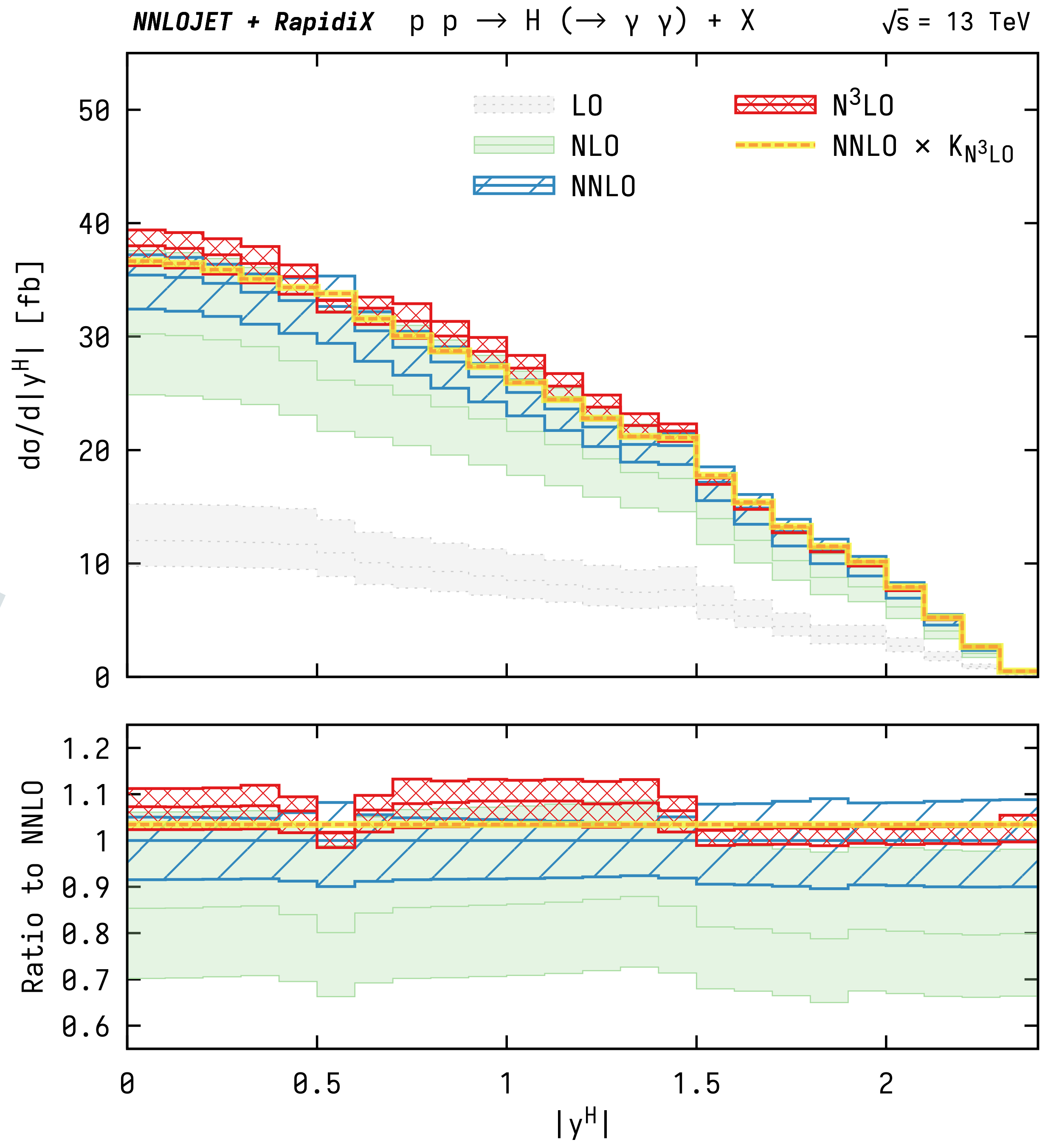
CUTS TO REMOVE THE IR SENSITIVITY



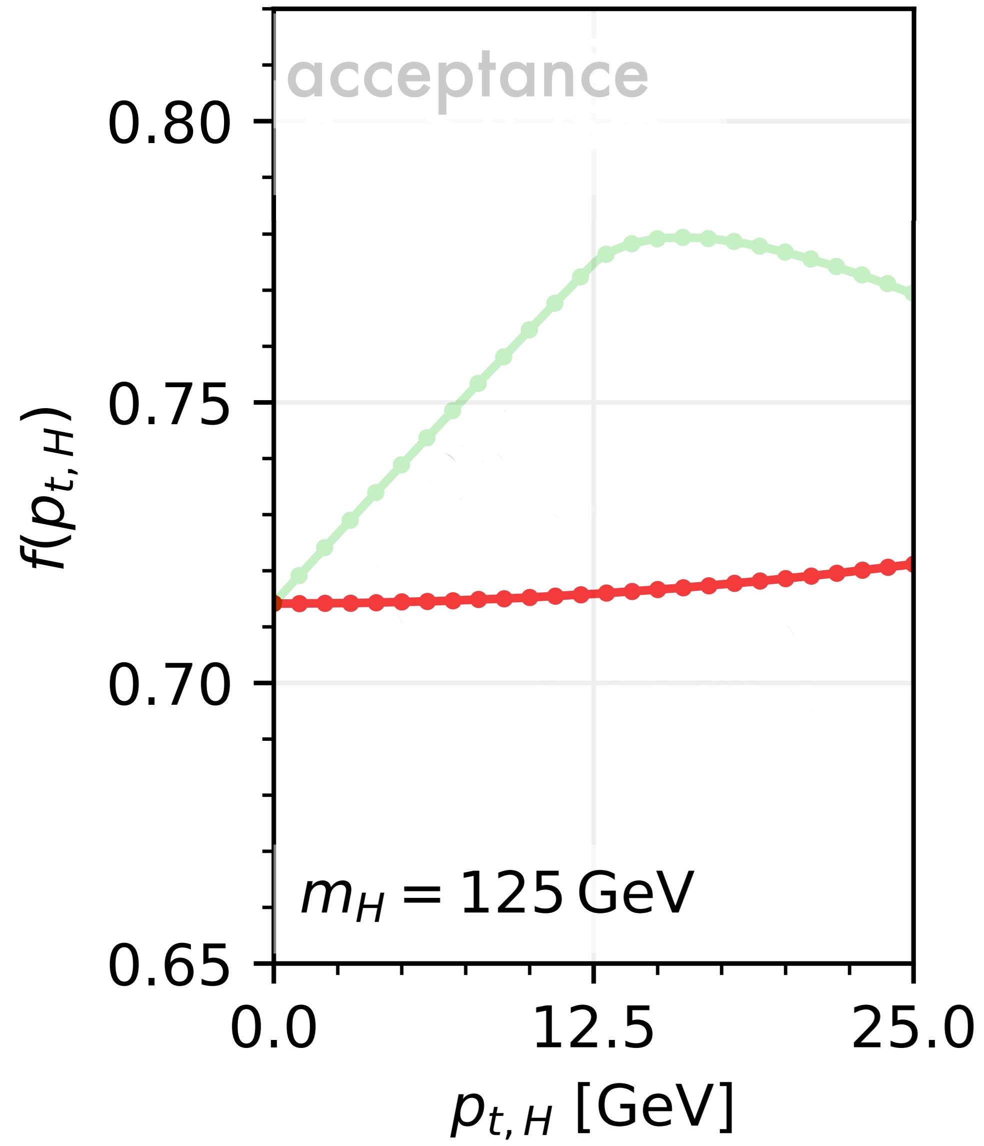
ATLAS

$$p_T^{\gamma_1} \geq 0.35 \cdot M_H$$
$$p_T^{\gamma_2} \geq 0.25 \cdot M_H$$

$$f(p_T^H) = f_0 + f_1 \cdot p_T^H + \mathcal{O}((p_T^H)^2)$$



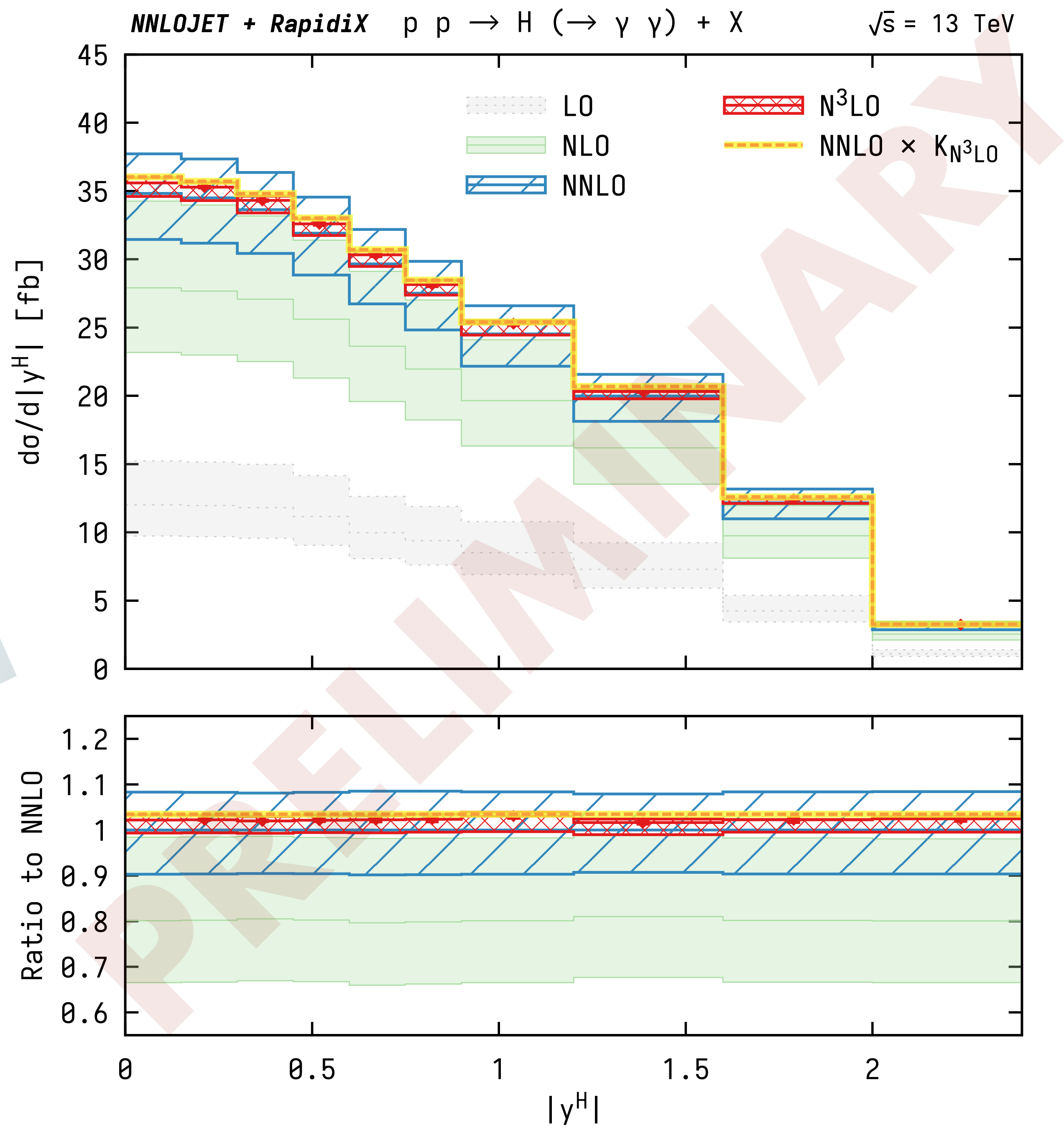
Alex Huss @ Higgs 2021



Product cuts [Salam, Slade '21]

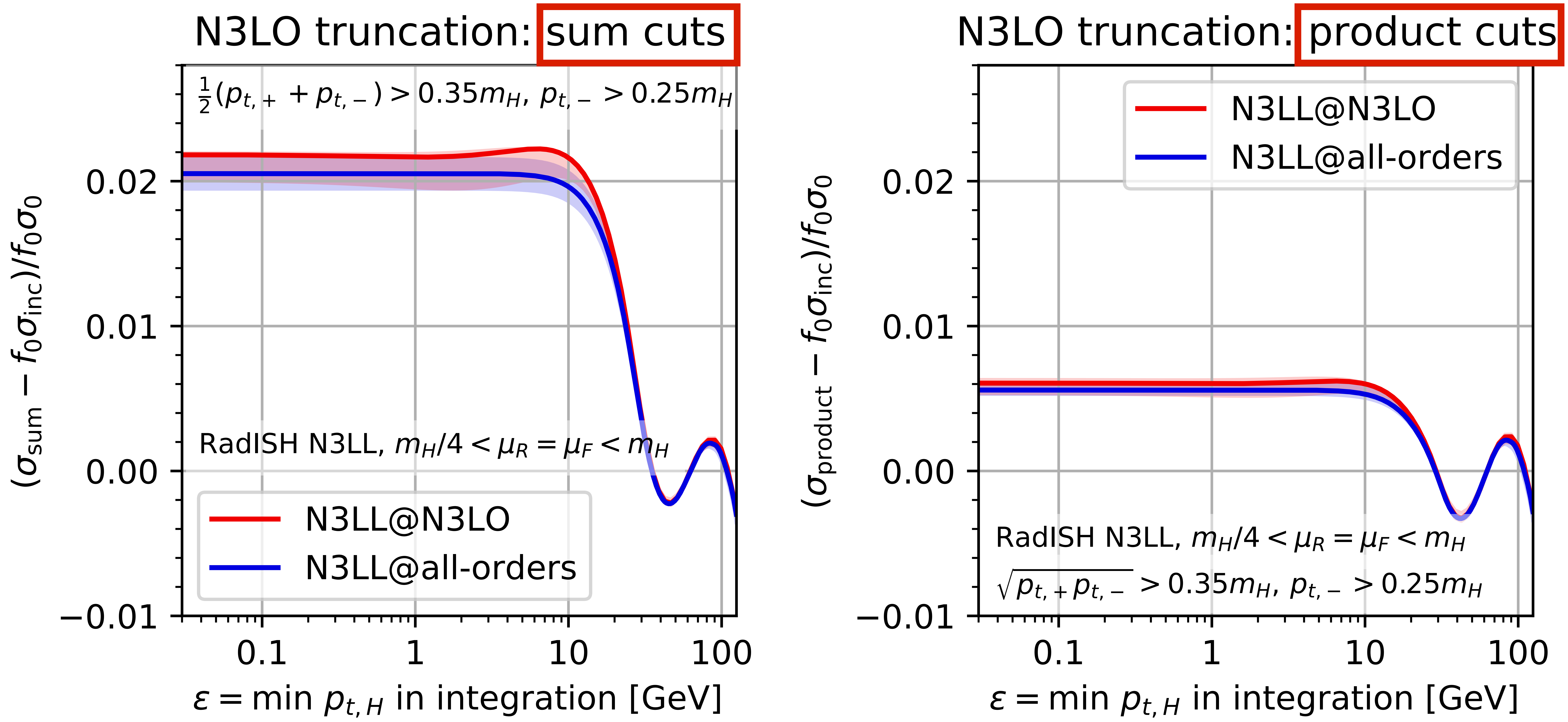
$$\sqrt{p_T^{\gamma_1} p_T^{\gamma_2}} \geq 0.35 \cdot M_H$$
$$p_T^{\gamma_2} \geq 0.25 \cdot M_H$$

$$f(p_T^H) = f_0 + f_1 \cdot p_T^H + f_2 \cdot (p_T^H)^2 + \mathcal{O}((p_T^H)^3)$$

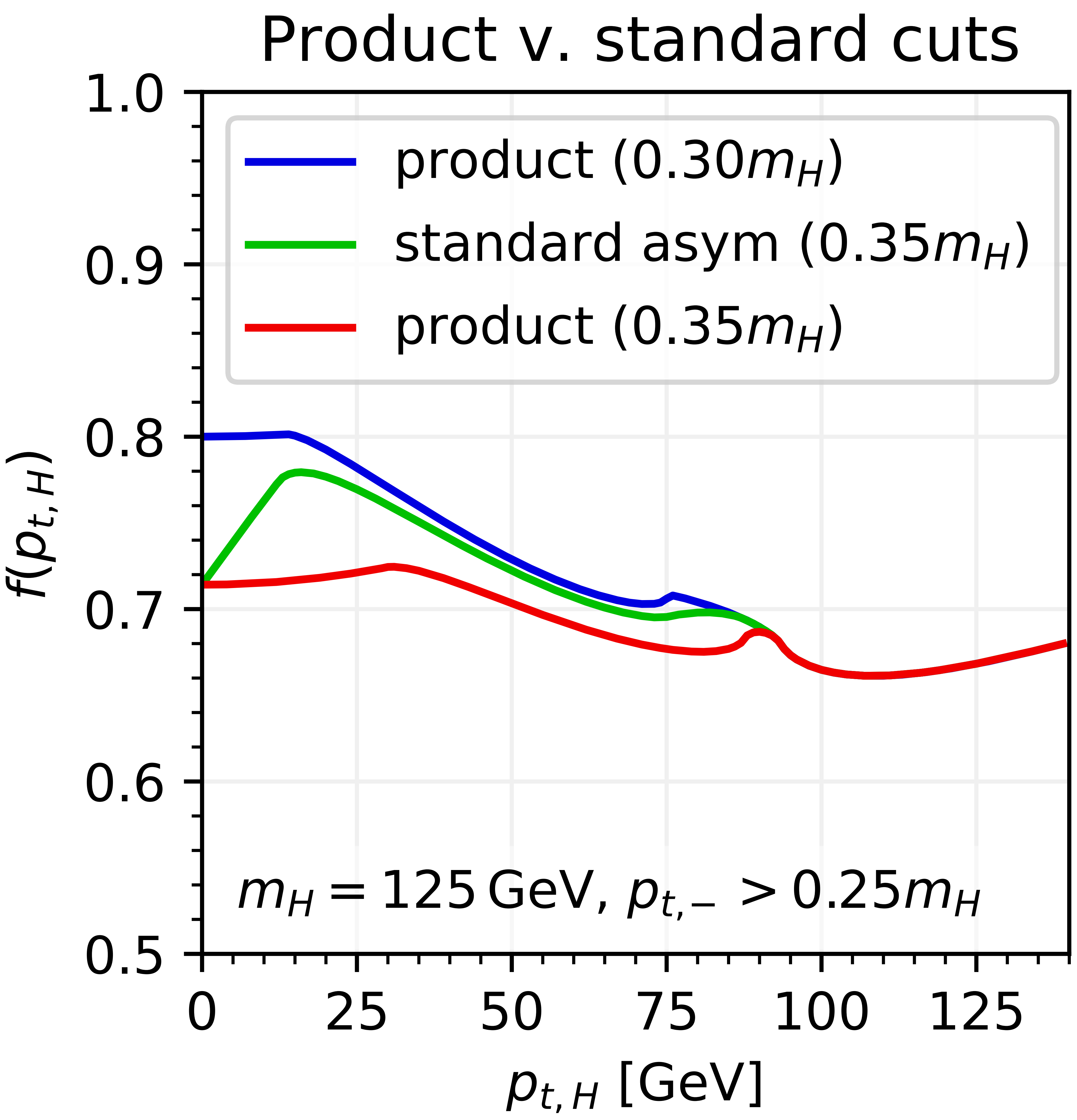


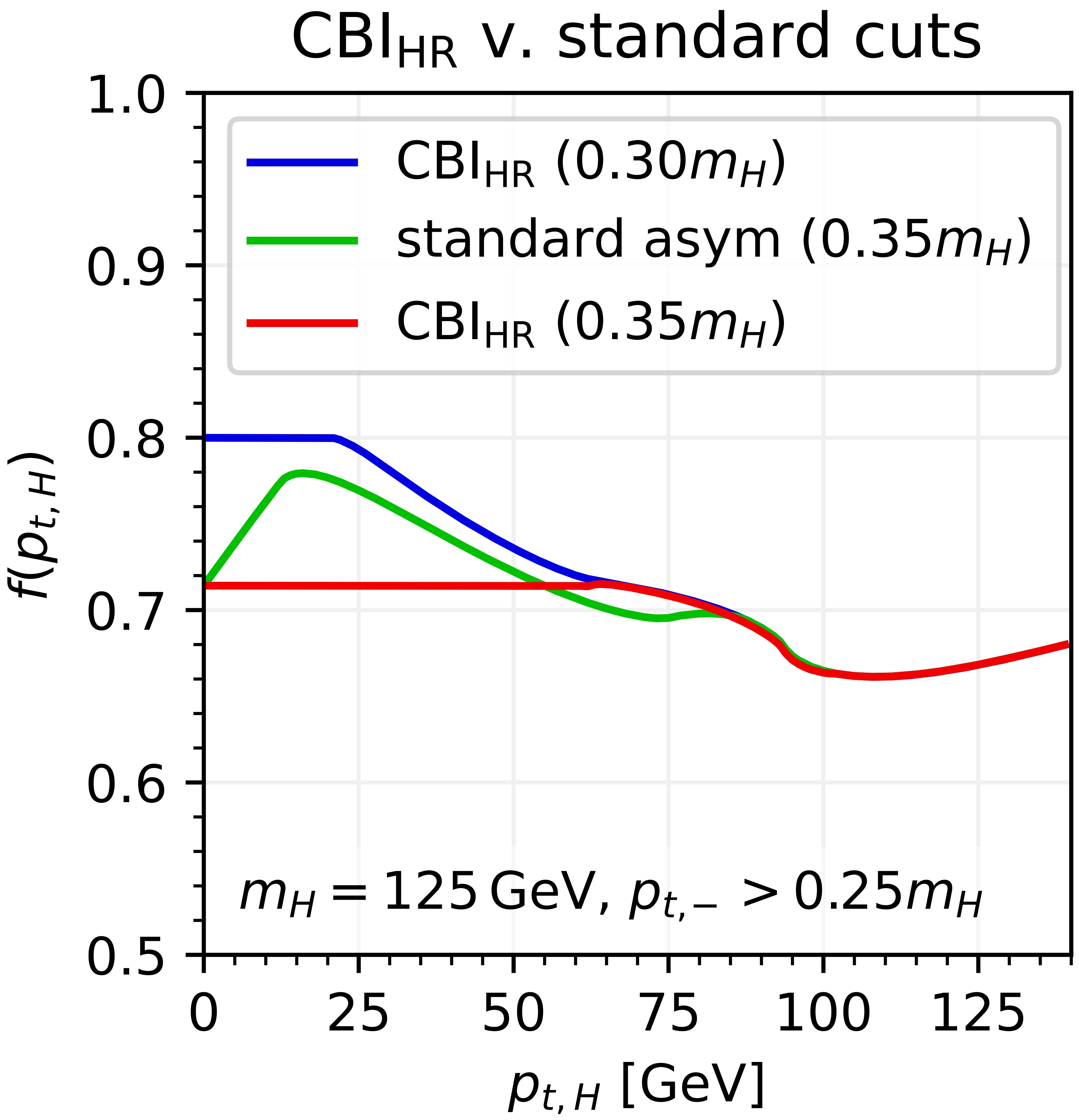
- NNLO × K_{N^3LO} ≈ N³LO
- very flat
- no "features"
- robust (v.s. resummation)

Sensitivity to low Higgs p_t (and also scale bands): sum & product cuts

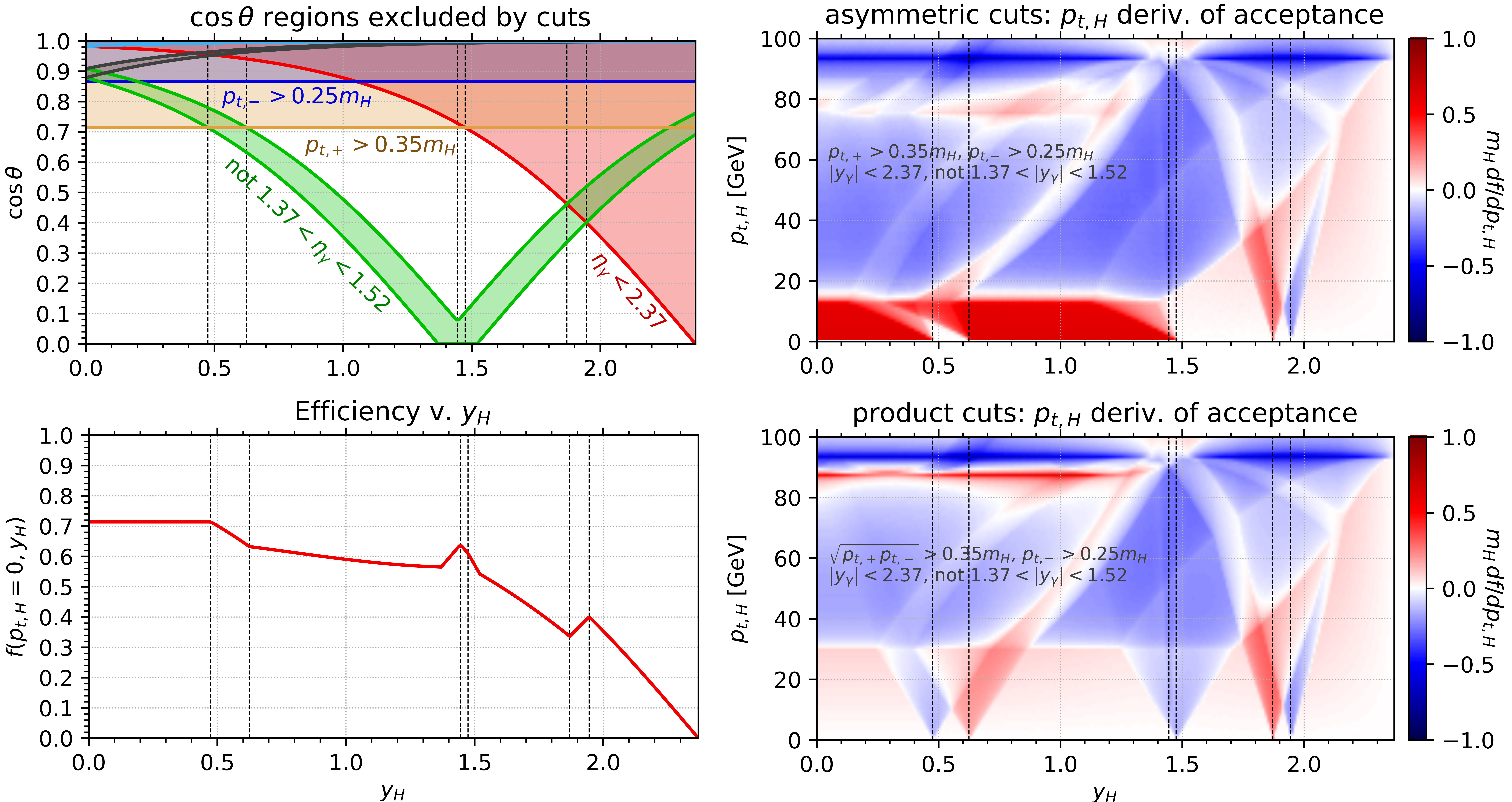


Option of changing thresholds

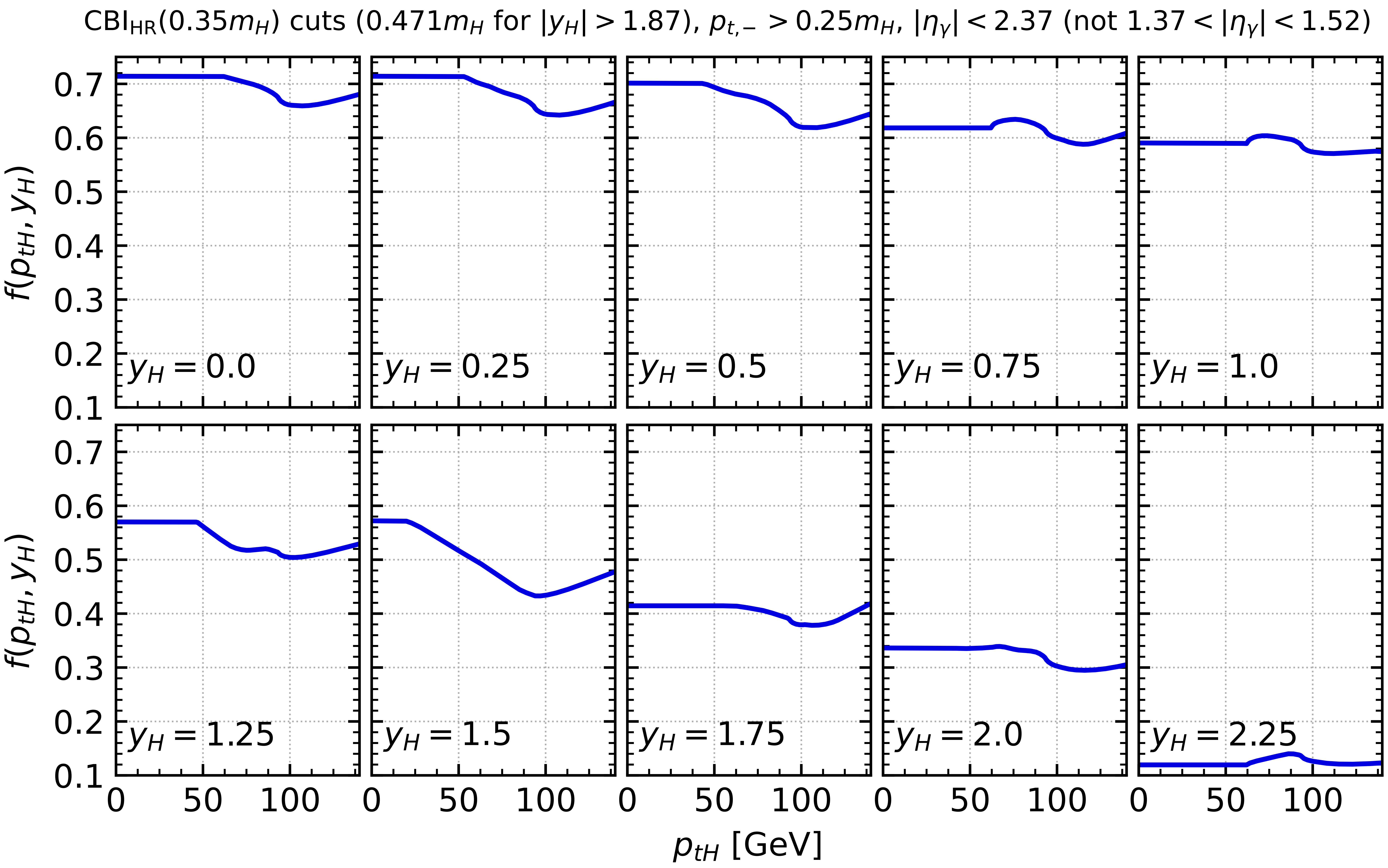




Interplay with rapidity cuts



CBI_{HR} cuts: acceptance v. p_{tH} at several y_H values



CBI_{HR} w. CMS rapidity cuts