Phenomenology

Gavin P. Salam

LPTHE, Universities of Paris VI and VII and CNRS

BUSSTEPP
Ambleside, August 2005

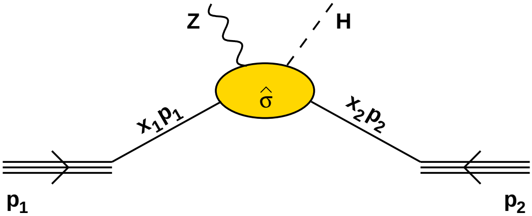
Phenomenology

Lecture 4

(Processes with incoming protons)

Factorization & parton distributions

Recall Higgs production in hadron-hadron collisions:

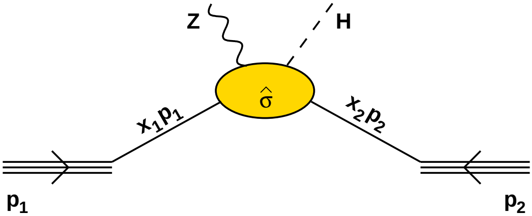


$$\sigma = \int dx_1 f_{q/p}(x_1,\mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2,\mu^2)\,\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)\,, \quad \hat{s} = x_1 x_2 s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section ↔ *need to know PDFs* to be able to test hard part (*e.g.* Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

Factorization & parton distributions

Recall Higgs production in hadron-hadron collisions:

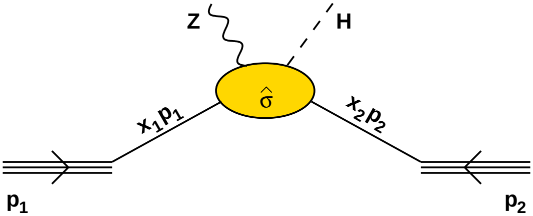


$$\sigma = \int dx_1 f_{q/p}(x_1,\mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2,\mu^2)\, \hat{\sigma}(x_1p_1, x_2p_2, \mu^2)\,, \qquad \hat{s} = x_1x_2s$$

- Total X-section is *factorized* into a ‘hard part’ $\hat{\sigma}(x_1p_1, x_2p_2, \mu^2)$ and ‘normalization’ from parton distribution functions (PDF).
- Measure total cross section $\leftrightarrow$ *need to know PDFs* to be able to test hard part (*e.g.* Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

Factorization & parton distributions

Recall Higgs production in hadron-hadron collisions:

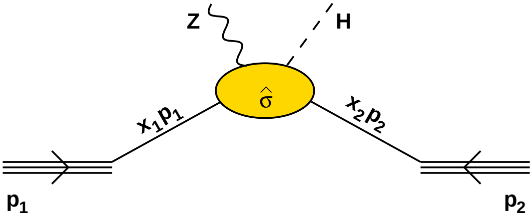


$$\sigma = \int dx_1 f_{q/p}(x_1,\mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2,\mu^2)\,\hat{\sigma}(x_1p_1, x_2p_2, \mu^2)\,, \quad \hat{s} = x_1x_2s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1p_1, x_2p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section $\leftrightarrow$ *need to know PDFs* to be able to test hard part (*e.g.* Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

Factorization & parton distributions

Recall Higgs production in hadron-hadron collisions:

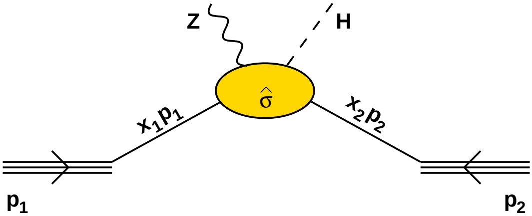


$$\sigma = \int dx_1 f_{q/p}(x_1, \mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2, \mu^2)\, \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)\,, \quad \hat{s} = x_1 x_2 s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section $\leftrightarrow$ *need to know PDFs* to be able to test hard part (*e.g.* Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

Factorization & parton distributions

Recall Higgs production in hadron-hadron collisions:

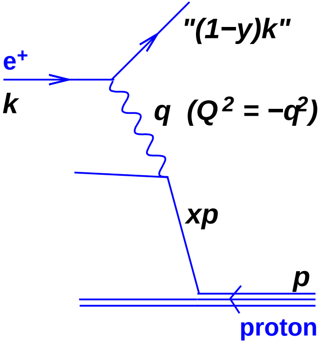


$$\sigma = \int dx_1 f_{q/p}(x_1, \mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2, \mu^2)\, \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)\,, \quad \hat{s} = x_1 x_2 s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section $\leftrightarrow$ *need to know PDFs* to be able to test hard part (*e.g.* Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

Deep Inelastic Scattering: kinematics

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



Kinematic relations:

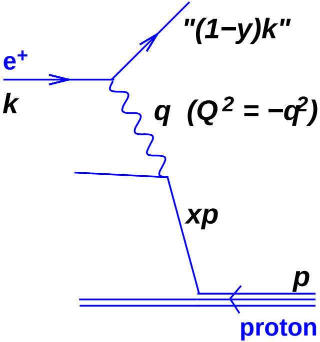
$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

- Q^2 = photon virtuality $\leftrightarrow$ *transverse resolution* at which it probes proton structure
- x = *longitudinal momentum fraction* of struck parton in proton
- y = momentum fraction lost by electron (in proton rest frame)

Deep Inelastic Scattering: kinematics

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



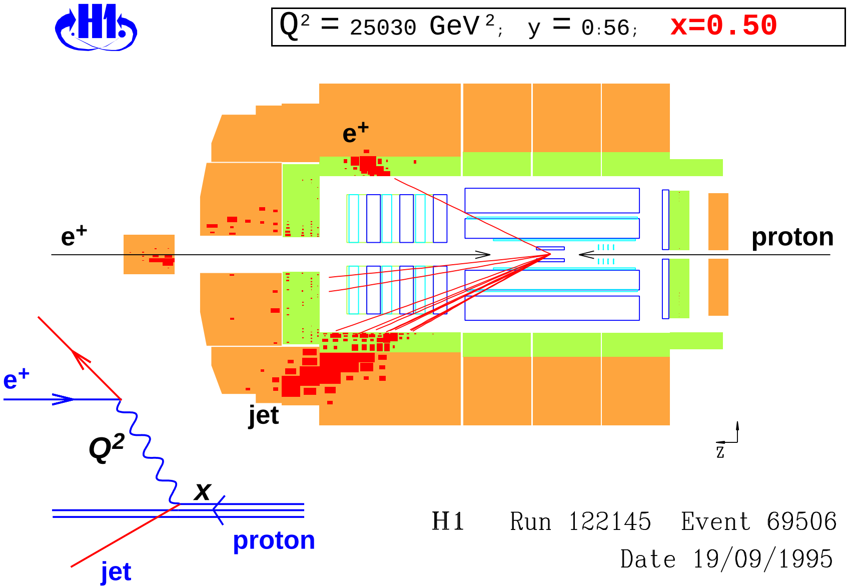
Kinematic relations:

$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

- Q^2 = photon virtuality $\leftrightarrow$ *transverse resolution* at which it probes proton structure
- x = *longitudinal momentum fraction* of struck parton in proton
- y = momentum fraction lost by electron (in proton rest frame)

Deep Inelastic scattering (DIS): example



E.g.: extracting u & d distributions

Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

NB:

- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

E.g.: extracting u & d distributions

Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

NB:

- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

F_2 gives us *combination* of u and d.
How can we extract them separately?

Using neutrons

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:
proton = uud; neutron = ddu

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$
$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

Using neutrons

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:
proton = uud; neutron = ddu

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$
$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

Using neutrons

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:
proton = uud; neutron = ddu

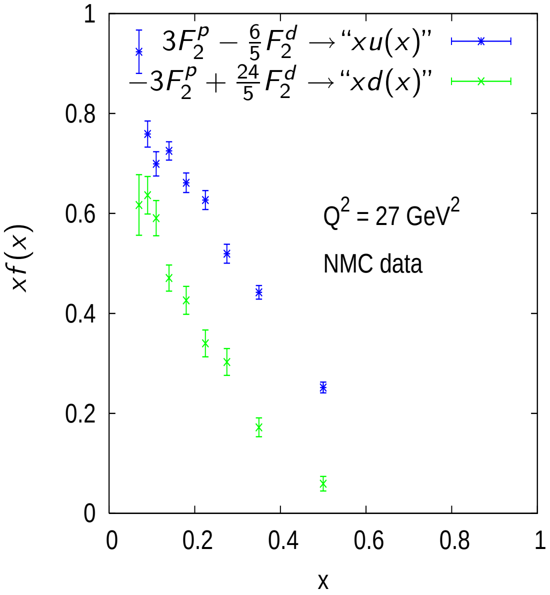
Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$
$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

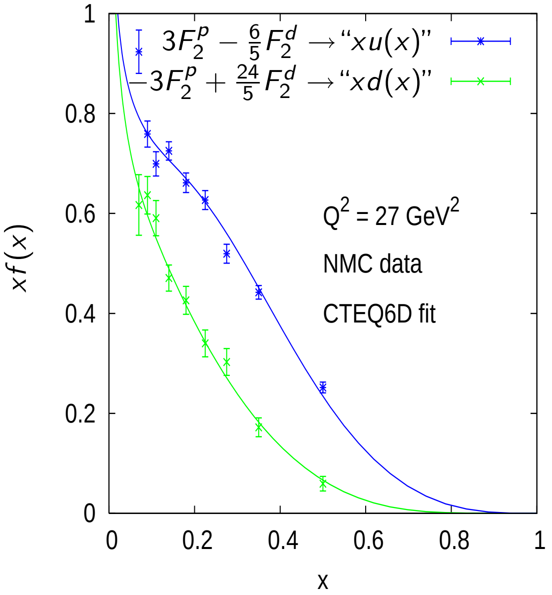
How much u and d?

- Total $U = \int dx\ u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

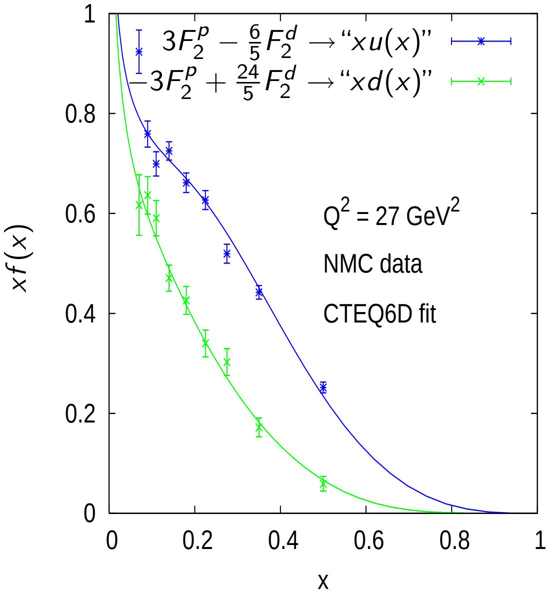
How much u and d?

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

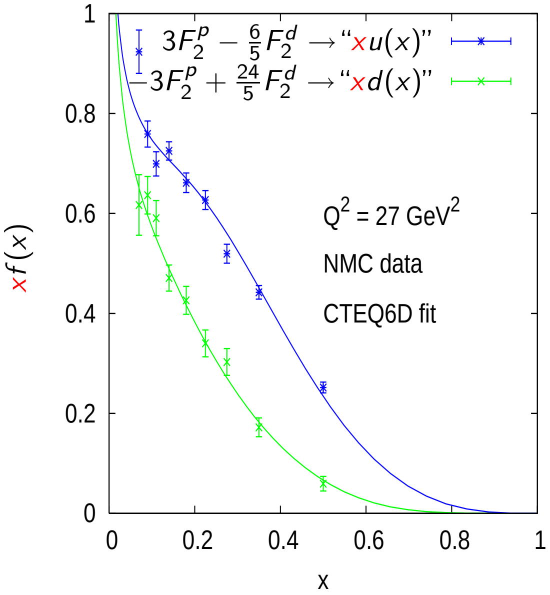
<u>How much u and d?</u>

- Total $U = \int dx\ u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

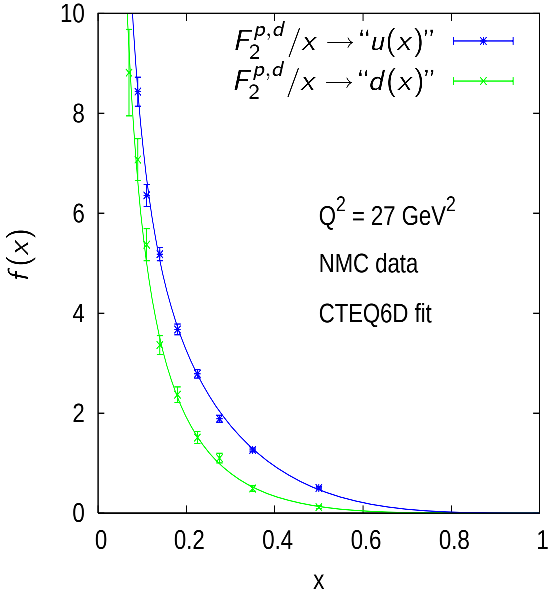
How much u and d?

- Total $U = \int dx\; u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

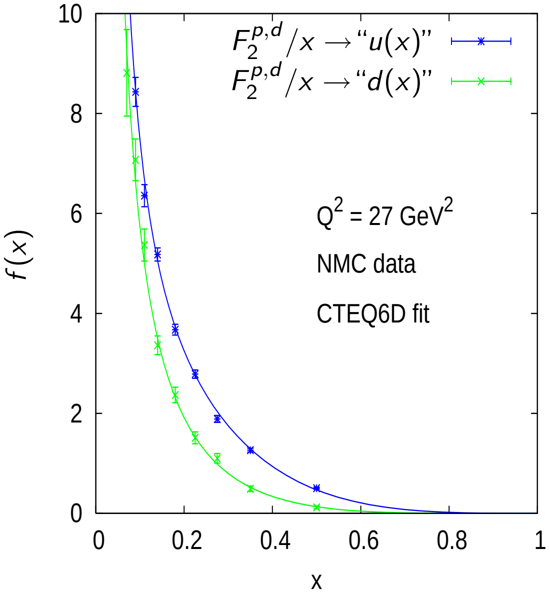
<u>How much u and d?</u>

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

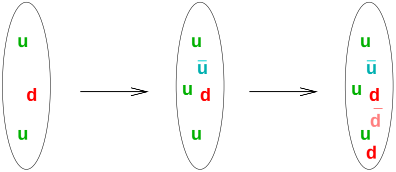
<u>How much u and d?</u>

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

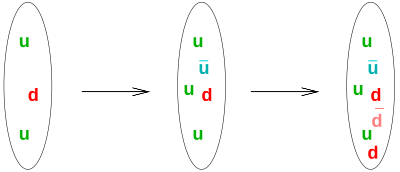
NB: photon interaction $\sim$ square of charge $\to$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite*, so

$$\int dx\,(u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

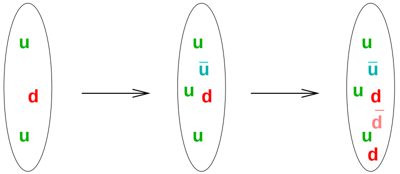
NB: photon interaction $\sim$ square of charge $\rightarrow$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

$$\int dx\,(u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

NB: photon interaction $\sim$ square of charge $\rightarrow$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

$$\int dx\,(u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

"Valence" quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both $\rightarrow$ no good. . .

Question: what interacts differently with particle & antiparticle?

Answer: W^+ *or* W^-

See question sheet for more details. . .

"Valence" quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both → no good...

Question: what interacts differently with particle & antiparticle?

Answer: W^+ or W^-

See question sheet for more details...

"Valence" quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

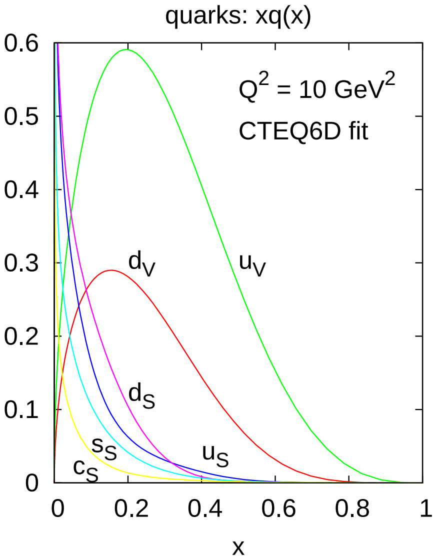
How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both → no good...

Question: what interacts differently with particle & antiparticle?

Answer: W^+ *or* W^-

See question sheet for more details...

All quarks

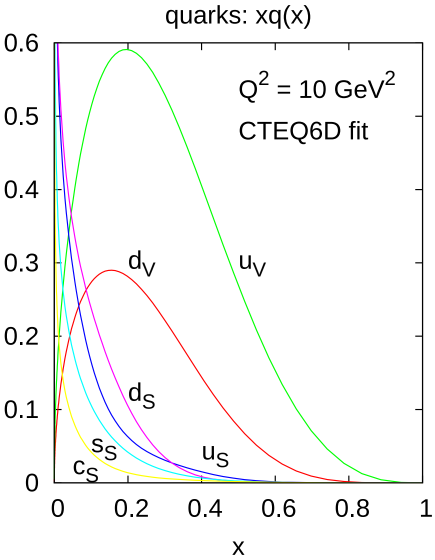


These & other methods → whole set of quarks & antiquarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*
  $x \to 1 : xq_V(x) \sim (1-x)^3$
  quark counting rules
  $x \to 0 : xq_V(x) \sim x^{0.5}$
  Regge theory
- sea quarks ($u_S = 2\bar{u}, \dots$) fairly *soft* (low-momentum)
  $x \to 1 : xq_S(x) \sim (1-x)^7$
  $x \to 0 : xq_S(x) \sim x^{-0.2}$

All quarks



These & other methods $\rightarrow$ whole set of quarks & antiquarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*

  $x \rightarrow 1 : xq_V(x) \sim (1 - x)^3$

  quark counting rules

  $x \rightarrow 0 : xq_V(x) \sim x^{0.5}$

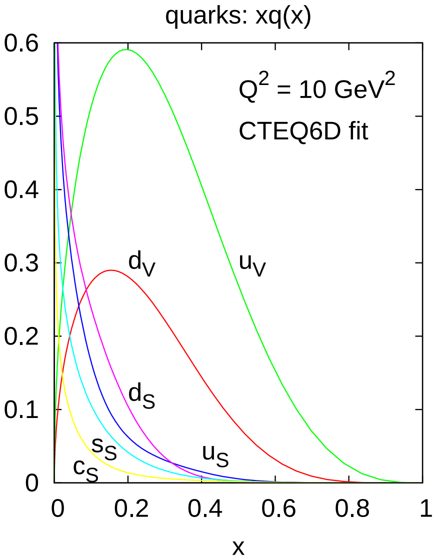
  Regge theory

- sea quarks ($u_S = 2\bar{u}, \dots$) fairly *soft* (low-momentum)

  $x \rightarrow 1 : xq_S(x) \sim (1 - x)^7$

  $x \rightarrow 0 : xq_S(x) \sim x^{-0.2}$

All quarks



These & other methods $\rightarrow$ whole set of quarks & antiquarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*

  $x \longrightarrow 1 : xq_V(x) \sim (1-x)^3$

  quark counting rules

  $x \longrightarrow 0 : xq_V(x) \sim x^{0.5}$

  Regge theory

- sea quarks ($u_S = 2\bar{u}, \ldots$) fairly *soft* (low-momentum)

  $x \longrightarrow 1 : xq_S(x) \sim (1-x)^7$

  $x \longrightarrow 0 : xq_S(x) \sim x^{-0.2}$

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^\pm$.

NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting. . .

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx \, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^{\pm}$.

NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting...

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^{\pm}$.

NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting...

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^{\pm}$.

NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting. . .

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, xq_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?
Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^{\pm}$.

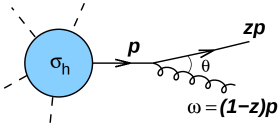
NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting...

Recall final-state splitting

Previous lecture: calculated $q \to qg$ $(\theta \ll 1,\ \omega \ll p)$ for final state of arbitrary hard process (σ_h):

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta^2}{\theta^2}$$



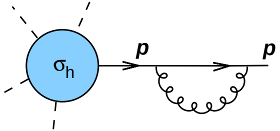
Rewrite with different kinematic variables

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

$$\omega = (1-z)p$$
$$k_t = \omega \sin\theta \simeq \omega\theta$$

If we avoid distinguishing $q+g$ final state from q (infrared-collinear safety), then divergent real and virtual corrections *cancel*

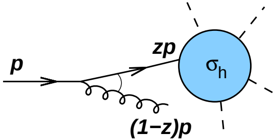
$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



Initial-state splitting

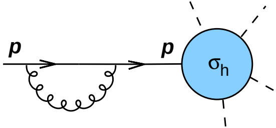
For initial state splitting, hard process occurs *after splitting*, and momentum entering hard process is modified: $p \to zp$.

$$\sigma_{g+h}(p) \simeq \sigma_h(zp) \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



For virtual terms, momentum entering hard process is unchanged

$$\sigma_{V+h}(p) \simeq -\sigma_h(p) \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



Total cross section gets contribution with *two different hard X-sections*

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int \frac{dk_t^2}{k_t^2} \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]$$

NB: We assume σ_h involves momentum transfers $\sim Q \gg k_t$, so ignore extra transverse momentum in σ_h

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \int \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles
So how do we do QCD calculations in such cases?

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \underbrace{\int \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles
So how do we do QCD calculations in such cases?

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int \frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles
So how do we do QCD calculations in such cases?

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int \frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

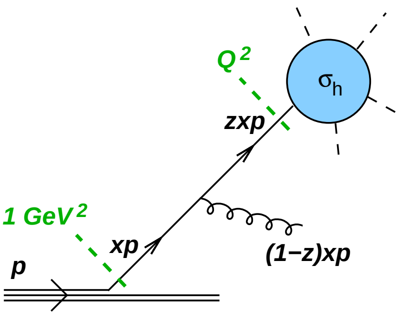
This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles

So how do we do QCD calculations in such cases?

Collinear cutoff



By what right did we go to $k_t = 0$?

We assumed pert. QCD to be valid for all scales, but *below* 1 GeV *it becomes non-perturbative*.

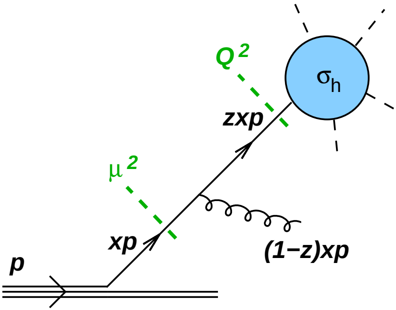
Cut out this divergent region, & instead put non-perturbative quark distribution in proton.

$$\sigma_0 = \int dx\, \sigma_h(xp)\, q(x, 1\text{ GeV}^2)$$

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_{1\text{ GeV}^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large)}} \underbrace{\int \frac{dx\,dz}{1-z} \left[\sigma_h(zxp) - \sigma_h(xp)\right] q(x, 1\text{ GeV}^2)}_{\text{finite}}$$

In general: replace 1 GeV^2 cutoff with arbitrary *factorization scale* μ^2.

Collinear cutoff



By what right did we go to $k_t = 0$?

We assumed pert. QCD to be valid for all scales, but *below* 1 GeV *it becomes non-perturbative*.

Cut out this divergent region, & instead put non-perturbative quark distribution in proton.

$$\sigma_0 = \int dx\, \sigma_h(xp)\, q(x, \mu^2)$$

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_{\mu^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large)}} \underbrace{\int \frac{dx\, dz}{1-z} \left[\sigma_h(zxp) - \sigma_h(xp)\right] q(x, \mu^2)}_{\text{finite}}$$

In general: replace 1 GeV^2 cutoff with arbitrary *factorization scale* μ^2.

Summary so far

- Collinear divergence for incoming partons *not cancelled* by virtuals.
  Real and virtual have different longitudinal momenta
- Situation analogous to renormalization: need to *regularize* (but in IR instead of UV).
  Technically, often done with dimensional regularization
- Physical sense of regularization is to separate *(factorize)* proton non-perturbative dynamics from perturbative hard cross section.
  Choice of factorization scale, μ^2, is arbitrary between 1 GeV^2 and Q^2
- In analogy with running coupling, we can *vary factorization scale* and get a *renormalization group equation* for parton distribution functions.
  Dokshizer Gribov Lipatov Altarelli Parisi equations (DGLAP)

Summary so far

- Collinear divergence for incoming partons *not cancelled* by virtuals.

  Real and virtual have different longitudinal momenta

- Situation analogous to renormalization: need to *regularize* (but in IR instead of UV).

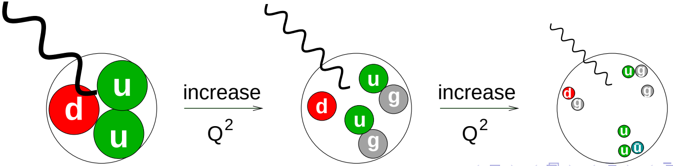
  Technically, often done with dimensional regularization

- Physical sense of regularization is to separate *(factorize)* proton non-perturbative dynamics from perturbative hard cross section.

  Choice of factorization scale, μ^2, is arbitrary between 1 GeV^2 and Q^2

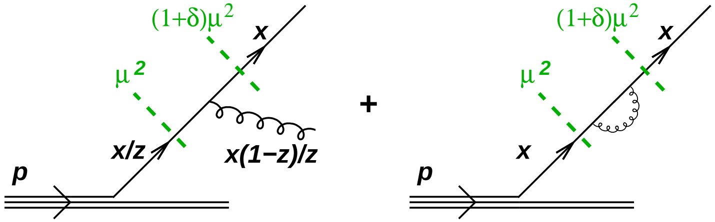
- In analogy with running coupling, we can *vary factorization scale* and get a *renormalization group equation* for parton distribution functions.

  Dokshizer Gribov Lipatov Altarelli Parisi equations (DGLAP)



DGLAP equation ($q \leftarrow q$)

Change convention: (a) now *fix outgoing* longitudinal momentum x; (b) *take derivative* wrt factorization scale μ^2



$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz\, p_{qq}(z)\,\frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu^2)$$

p_{qq} is real $q \leftarrow q$ **splitting kernel:** $p_{qq}(z) = C_F\dfrac{1+z^2}{1-z}$

Until now we approximated it in soft ($z \to 1$) limit, $p_{qq} \simeq \frac{2C_F}{1-z}$

DGLAP rewritten

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\underbrace{\int_x^1 dz\, P_{qq}(z)\, \frac{q(x/z,\mu^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

This involves the *plus prescription:*

$$\int_0^1 dz\, [g(z)]_+\, f(z) = \int_0^1 dz\, g(z)\, f(z) - \int_0^1 dz\, g(z)\, f(1)$$

$z = 1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z = 1$

DGLAP flavour structure

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

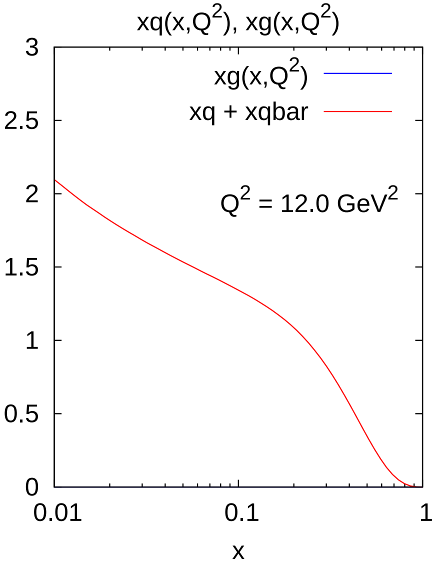
$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, *e.g.*

- P_{qg}, P_{gg}: *symmetric $z \leftrightarrow 1-z$* (except virtuals)
- P_{qq}, P_{gg}: *diverge for $z \to 1$* soft gluon emission
- P_{gg}, P_{gq}: *diverge for $z \to 0$* Implies PDFs grow for $x \to 0$

Effect of DGLAP (initial quarks)

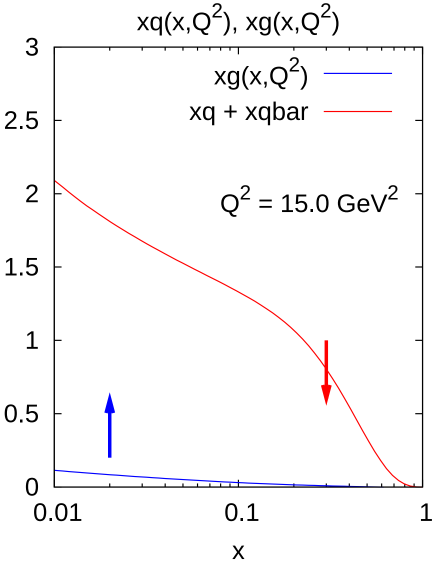


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q \leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

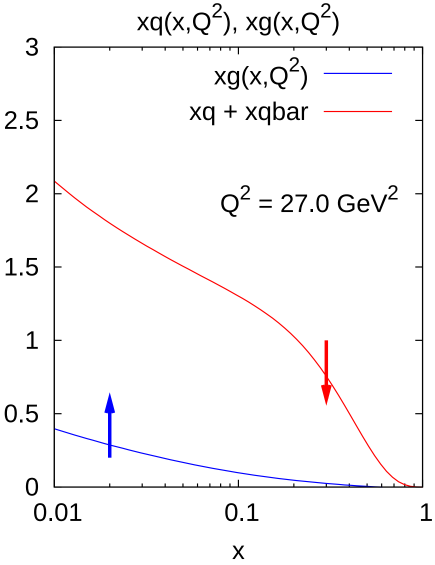


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

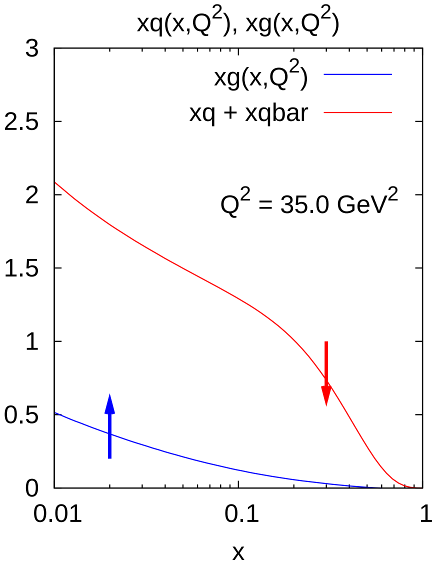


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

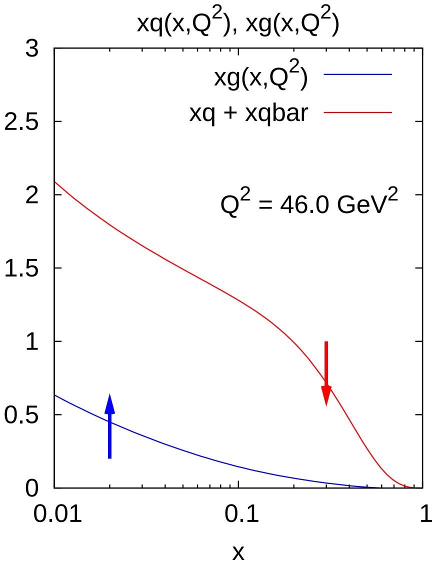


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q \leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

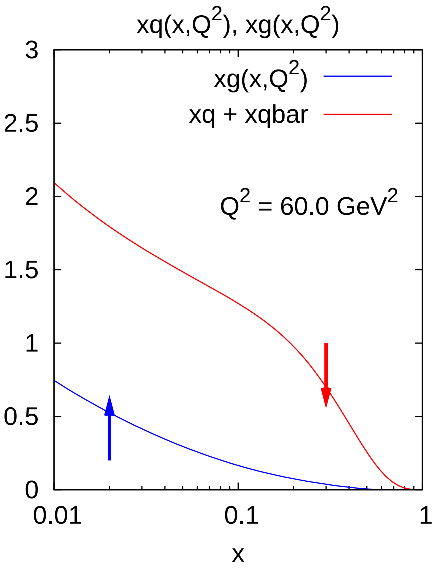


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q \leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

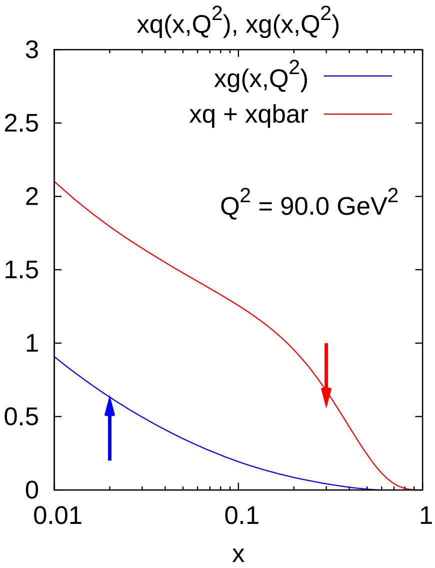


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

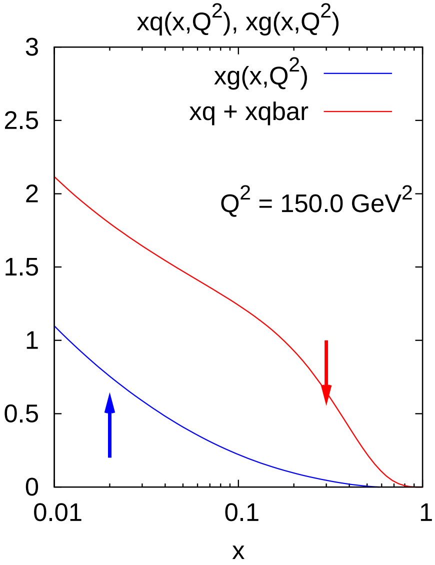


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q \leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

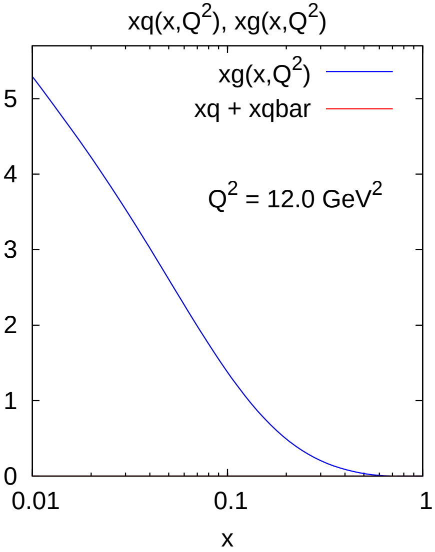


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q \leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial gluons)

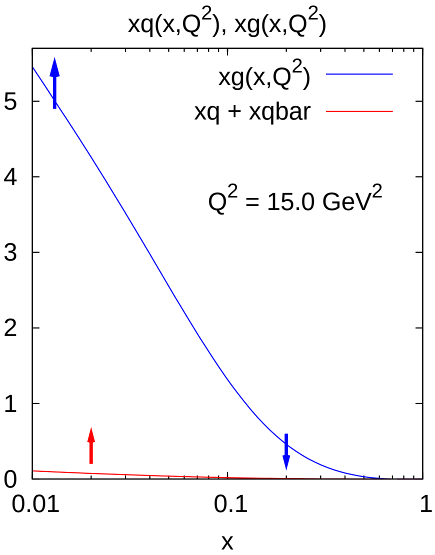


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

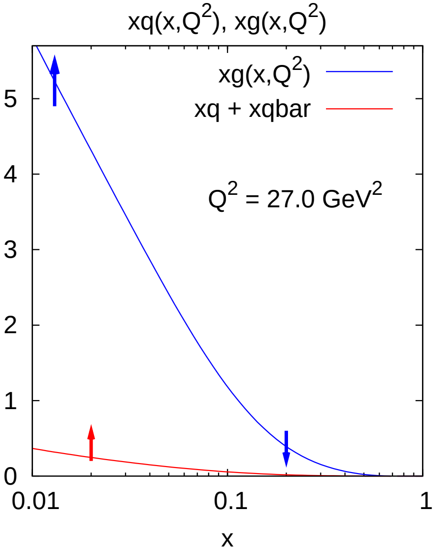


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

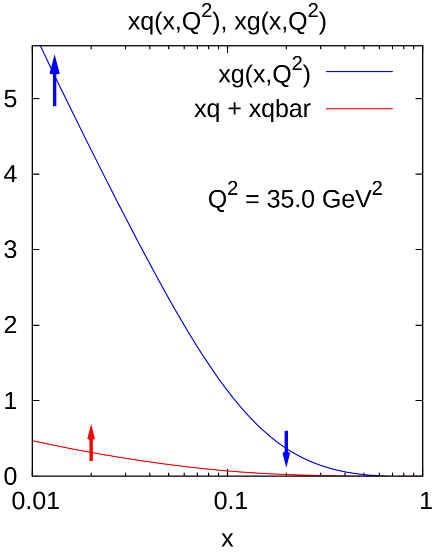


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

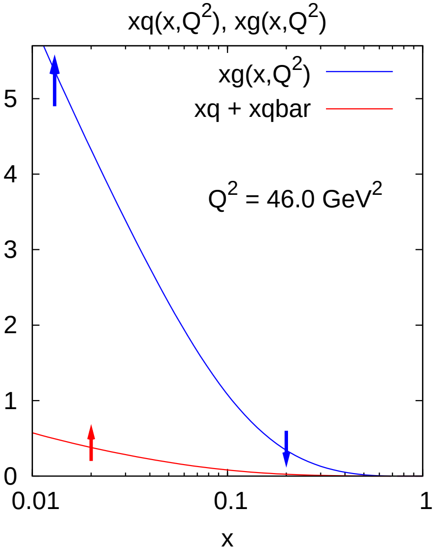


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

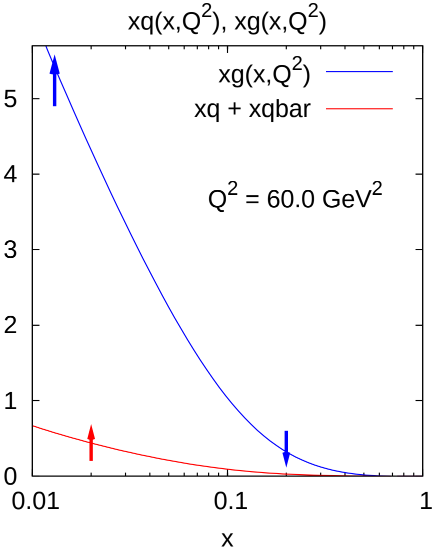


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

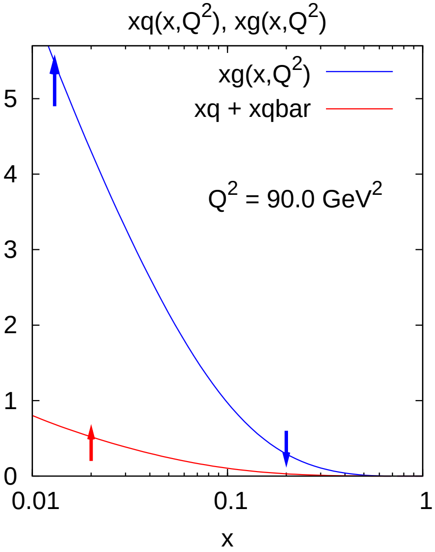


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

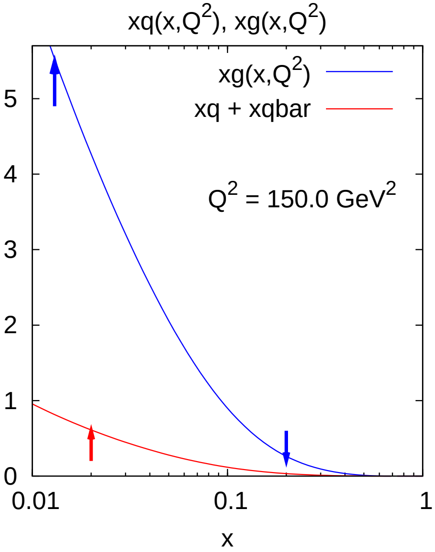


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)



2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$

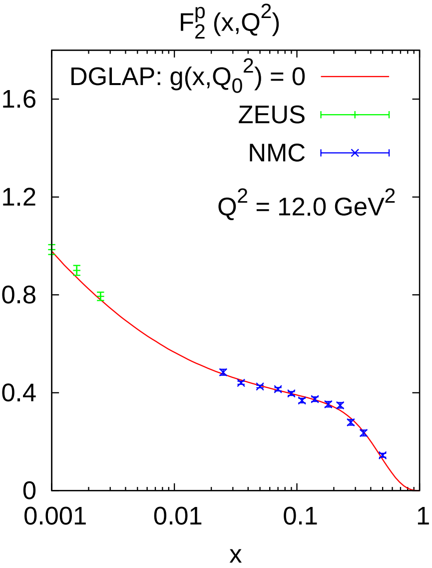
- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

DGLAP evolution

- As Q^2 increases, partons lose longitudinal momentum; distributions all shift to lower x.
- gluons can be seen because they help drive the quark evolution.

Now consider data

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\ \mathrm{GeV}^2$.

NB: Q_0 often chosen lower

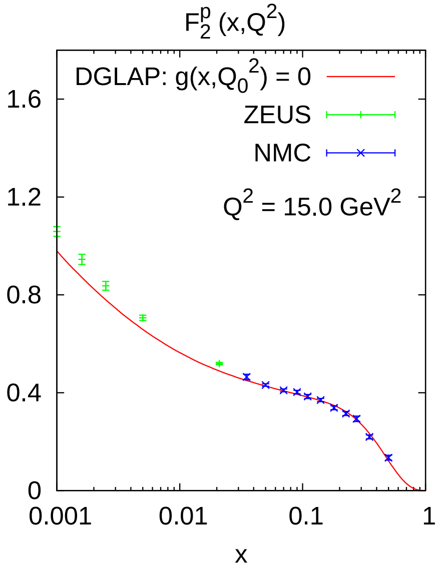
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon $= 0$



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

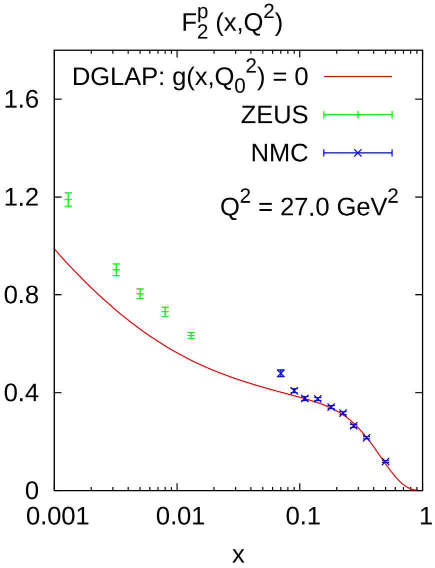
NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\ \mathrm{GeV}^2$.

NB: Q_0 often chosen lower

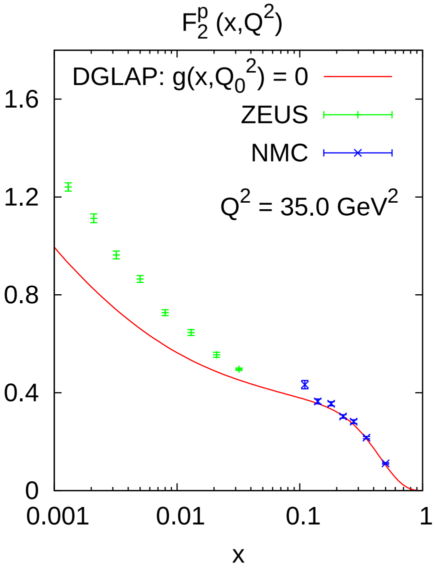
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\ \mathrm{GeV}^2$.

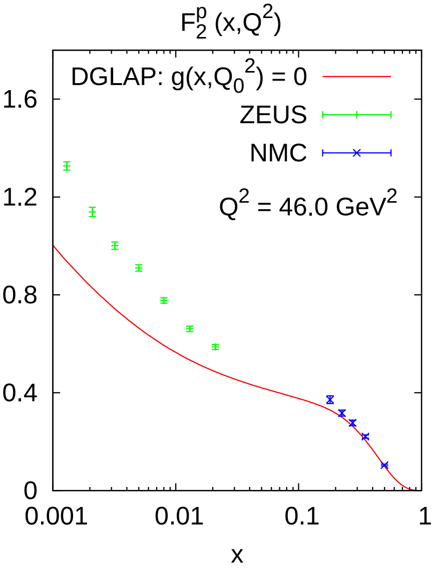
NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

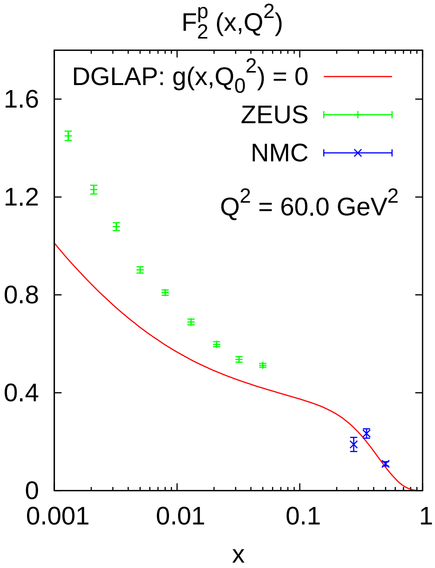
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

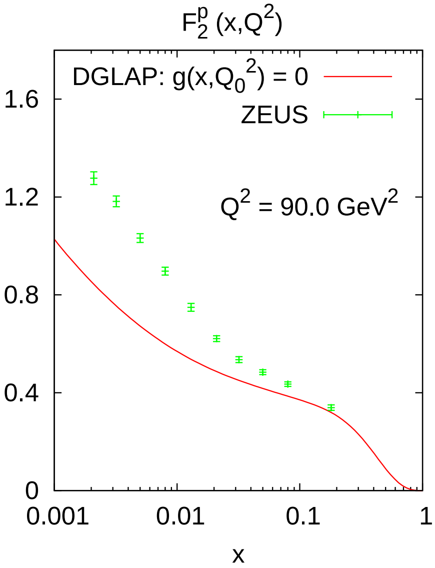
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

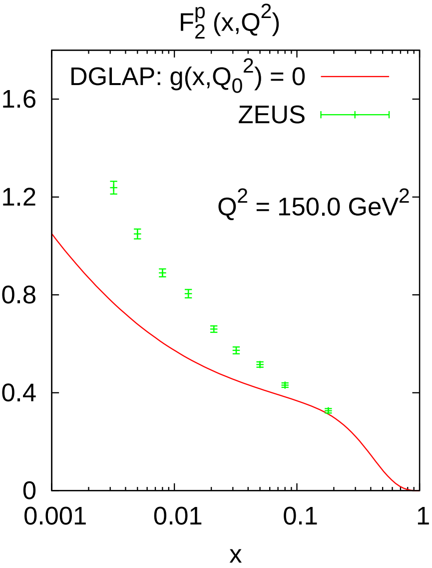
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\ \mathrm{GeV}^2$.

NB: Q_0 often chosen lower

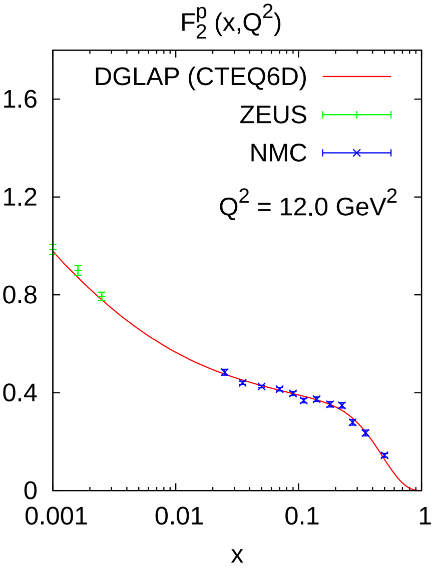
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon $\neq 0$



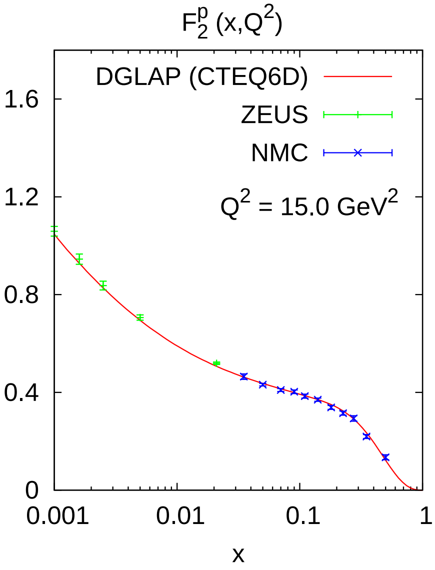
If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, . . .
PDF fitting collaborations.

DGLAP with initial gluon $\neq 0$



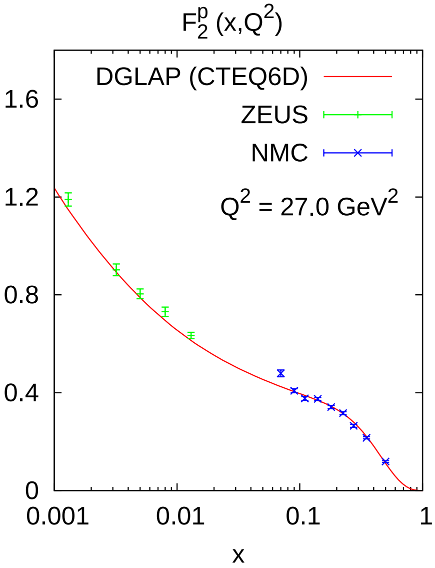
If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

DGLAP with initial gluon $\neq 0$



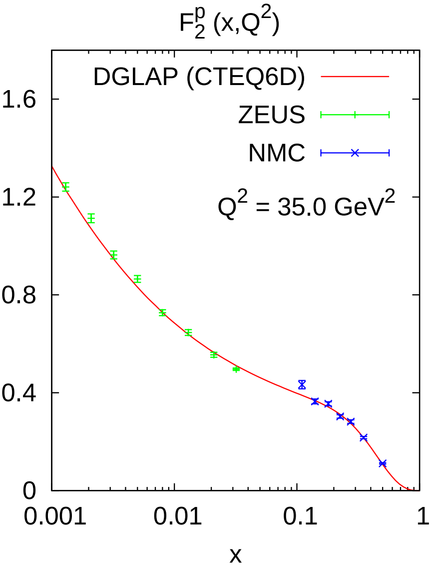
If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, . . .
PDF fitting collaborations.

DGLAP with initial gluon $\neq 0$



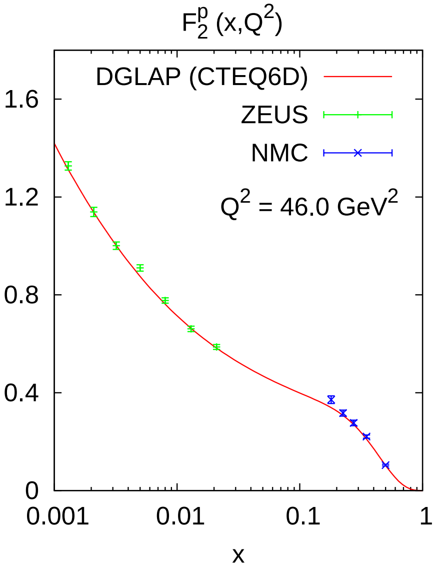
If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

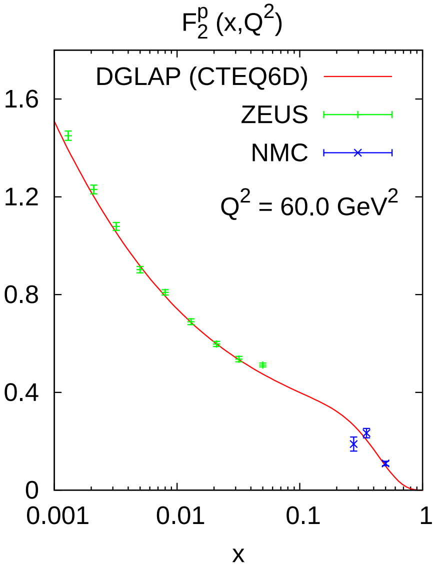
Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...

PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

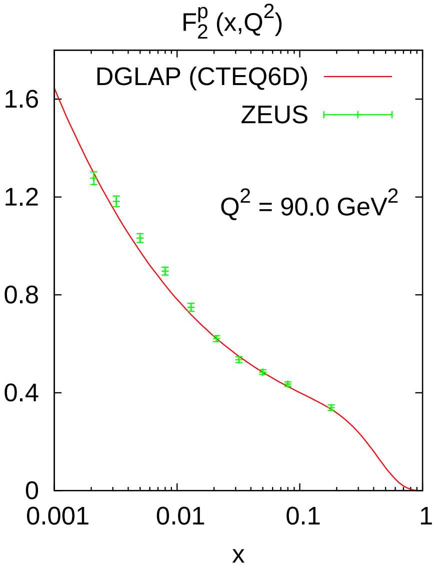
Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...

PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

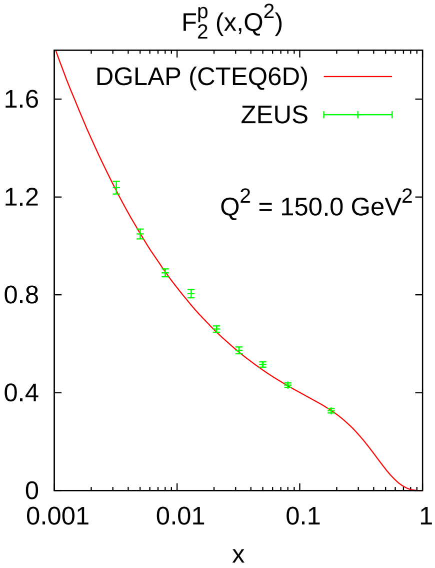
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, . . .
PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

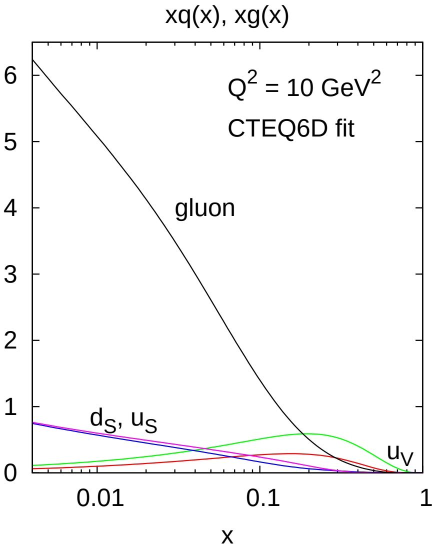
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, . . .
PDF fitting collaborations.

Success!

Gluon distribution



Gluon distribution is **HUGE!**

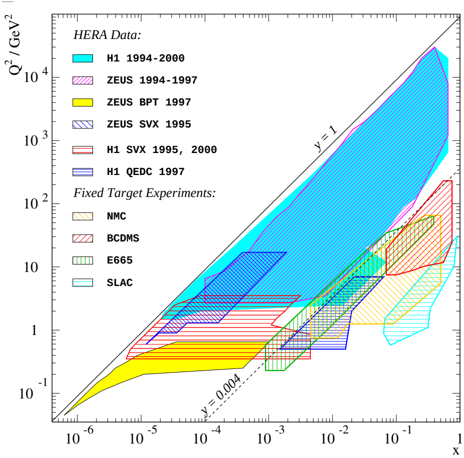
Can we really trust it?

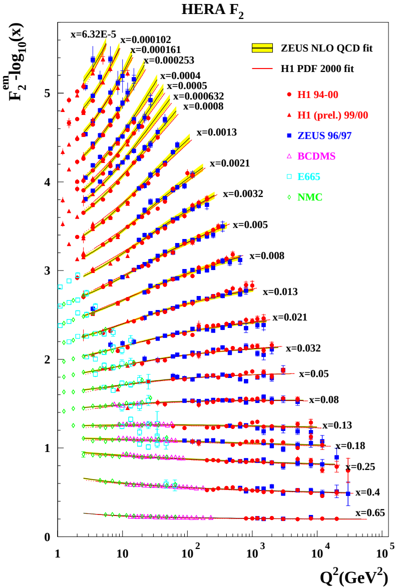
- Consistency: momentum sum-rule is now *satisfied*.

  NB: gluon mostly at small x

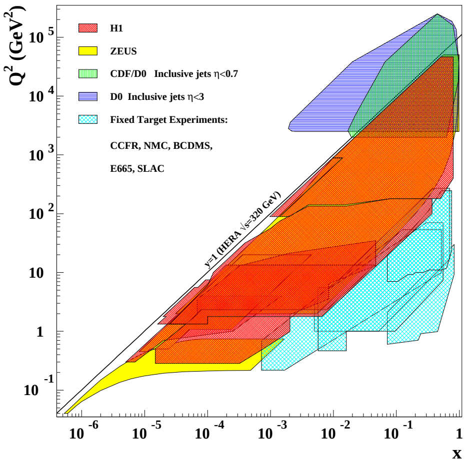
- Agrees with vast range of data

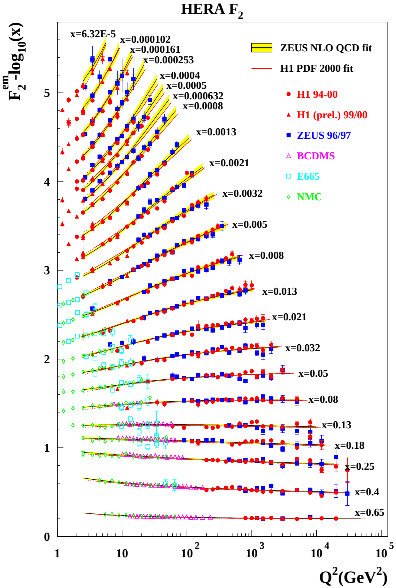
DIS data


HERA Data:
H1 1994-2000
ZEUS 1994-1997
ZEUS BPT 1997
ZEUS SVX 1995
H1 SVX 1995, 2000
H1 QEDC 1997
Fixed Target Experiments:
NMC
BCDMS
E665
SLAC
y = 1
y = 0.004
Q^2 / GeV^2
x


HERA F_2
F_2^{em}-$log_{10}(x)$
$Q^2(GeV^2)$
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65

DIS data

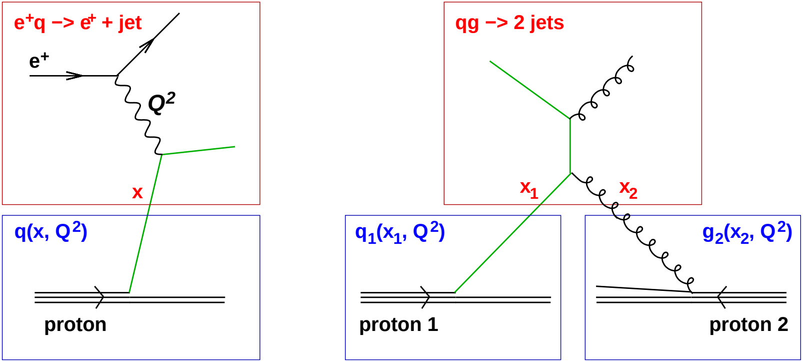

H1
ZEUS
CDF/D0 Inclusive jets η<0.7
D0 Inclusive jets η<3
Fixed Target Experiments:
CCFR, NMC, BCDMS,
E665, SLAC
y=1 (HERA √s=320 GeV)
Q^2 (GeV^2)
x


HERA F_2
F_2^{em}-log$_{10}$(x)
Q^2(GeV^2)
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65

Crucial check: other processes

Factorization of QCD cross-sections into convolution of:

- hard (perturbative) process-dependent **partonic subprocess**
- non-perturbative, process-independent **parton distribution functions**

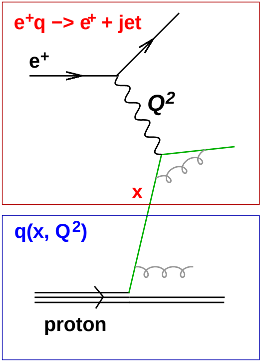


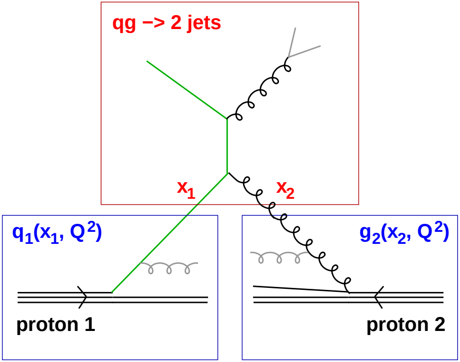
$$\sigma_{ep} = \sigma_{eq} \otimes q \qquad \sigma_{pp\to 2\,jets} = \sigma_{qg\to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$$

Crucial check: other processes

Factorization of QCD cross-sections into convolution of:

- hard (perturbative) process-dependent **partonic subprocess**
- non-perturbative, process-independent **parton distribution functions**

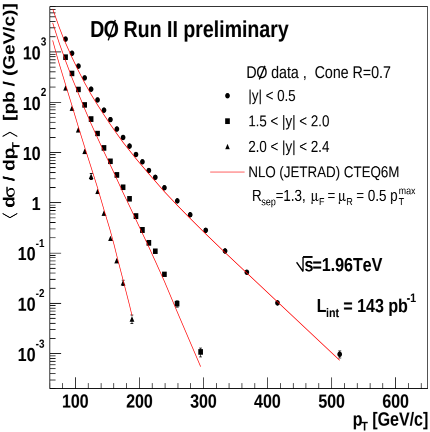




$\sigma_{ep} = \sigma_{eq} \otimes q$

$\sigma_{pp\to 2\,jets} = \sigma_{qg\to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$

Jet production in $p\bar{p}$



Jet production in proton-antiproton collisions is *good test of large gluon distribution,* since there are large direct contributions from

$$gg \to gg\,, \quad qg \to qg$$

NB: more complicated to interpret than DIS, since many channels, and x_1, x_2 dependence.

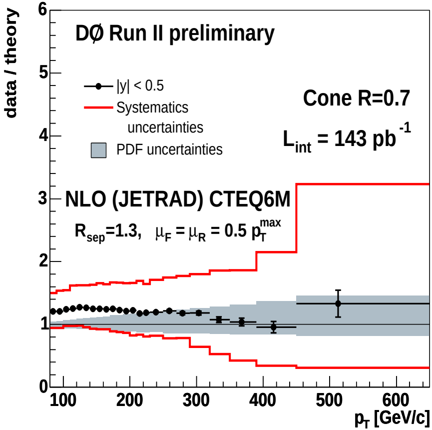
$p_T \sim \sqrt{x_1 x_2 s}$ jet transverse mom. $\sim Q$

$y \sim \frac{1}{2} \log \frac{x_1}{x_2}$ $\quad y = \log \tan \frac{\theta}{2}$

jet angle wrt $p\bar{p}$ beams

Good agreement confirms factorization

Jet production in $p\bar{p}$



Jet production in proton-antiproton collisions is *good test of large gluon distribution,* since there are large direct contributions from

$$gg \to gg\,, \quad qg \to qg$$

NB: more complicated to interpret than DIS, since many channels, and x_1, x_2 dependence.

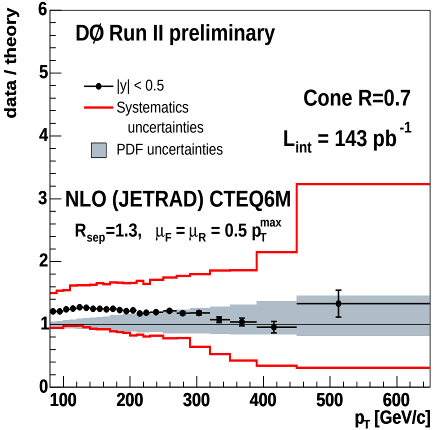
$p_T \sim \sqrt{x_1 x_2 s}$ jet transverse mom. $\sim Q$

$$y \sim \frac{1}{2} \log \frac{x_1}{x_2} \qquad y = \log \tan \frac{\theta}{2}$$

jet angle wrt $p\bar{p}$ beams

Good agreement confirms factorization

Jet production in $p\bar{p}$



Jet production in proton-antiproton collisions is *good test of large gluon distribution,* since there are large direct contributions from

$$gg \to gg\,, \quad qg \to qg$$

NB: more complicated to interpret than DIS, since many channels, and x_1, x_2 dependence.

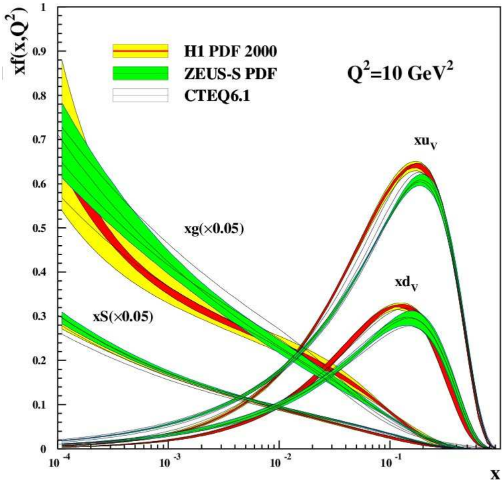
$p_T \sim \sqrt{x_1 x_2 s}$ jet transverse mom. $\sim Q$

$$y \sim \frac{1}{2}\log\frac{x_1}{x_2} \qquad y = \log\tan\frac{\theta}{2}$$

jet angle wrt $p\bar{p}$ beams

Good agreement confirms factorization

PDF uncertainties



Major recent activity is translation of experimental errors (and theory uncertainties) into *uncertainty bands on extracted PDFs.*

PDFs with uncertainties allow one to estimate *degree of reliability* of future predictions

Higher-order calculations

Earlier, we saw leading order (LO) DGLAP splitting functions, $P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)}$:

$$P_{qq}^{(0)}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right] ,$$

$$P_{qg}^{(0)}(x) = T_R \left[x^2 + (1-x)^2 \right] ,$$

$$P_{gq}^{(0)}(x) = C_F \left[\frac{1+(1-x)^2}{x} \right] ,$$

$$P_{gg}^{(0)}(x) = 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x)\frac{(11C_A - 4n_f T_R)}{6} .$$

Higher-order calculations

NLO:

$$P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)} + \frac{\alpha_s^2}{16\pi^2} P^{(1)}$$

Curci, Furmanski & Petronzio '80

$$P^{(1)}_{\rm ps}(x) = 4\, C_F n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2 \left[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9} \right] + (1+x) \left[5\mathrm{H}_0 - 2\mathrm{H}_{0,0} \right] \right)$$

$$P^{(1)}_{\rm qg}(x) = 4\, C_A n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x)\mathrm{H}_{-1,0} - 2p_{\rm qg}(x)\mathrm{H}_{1,1} + x^2 \left[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9} \right] \right.$$
$$+ 4(1-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1 \right] - 4\zeta_2 x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0 \Big) + 4\, C_F n_f \Big(2p_{\rm qg}(x) \Big[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2$$
$$\left. - \zeta_2 \right] + 4x^2 \left[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2} \right] + 2(1-x)\left[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4} \right] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0 \Big)$$

$$P^{(1)}_{\rm gq}(x) = 4\, C_A C_F \left(\frac{1}{x} + 2p_{\rm gq}(x) \left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1 \right] - x^2 \left[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9} \right] + 4\zeta_2 - 2 \right.$$
$$- 7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1 x + (1+x)\left[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9} \right] - 2p_{\rm gq}(-x)\mathrm{H}_{-1,0} \Big) - 4\, C_F n_f \Big(\frac{2}{3}x$$
$$- p_{\rm gq}(x)\left[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9} \right] \Big) + 4\, C_F^2 \Big(p_{\rm gq}(x)\left[3\mathrm{H}_1 - 2\mathrm{H}_{1,1} \right] + (1+x)\left[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0 \right] - 3\mathrm{H}_{0,0}$$
$$\left. + 1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1 x \right)$$

$$P^{(1)}_{\rm gg}(x) = 4\, C_A n_f \left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2 \right) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x) \right) + 4\, C_A^2 \Big(27$$
$$+ (1+x)\left[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2} \right] + 2p_{\rm gg}(-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2 \right] - \frac{67}{9}\left(\frac{1}{x} - x^2 \right) - 12\mathrm{H}_0$$
$$- \frac{44}{3}x^2\mathrm{H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2 \right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3 \right] \Big) + 4\, C_F n_f \Big(2\mathrm{H}_0$$
$$+ \frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0} \right] - \frac{1}{2}\delta(1-x) \Big) \; .$$

Higher-order calculations

NNLO, $P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

Conclusions on PDFs

- Experiments tell us that proton really is what we expected (*uud*)
- Plus lots more: large number of 'sea quarks' ($q\bar{q}$), gluons (50% of momentum)
- *Factorization* is key to usefulness of PDFs
  - Non-trivial beyond lowest order
  - PDFs depend on factorization scale, evolve with *DGLAP equation*
  - Pattern of *evolution gives us info on gluon* (otherwise hard to measure)
  - PDFs really are universal!
- *Precision* of data & QCD calculations steadily increasing.
- Crucial for understanding future signals of *new particles, e.g.* Higgs Boson production at LHC.