

# QCD at hadron colliders

## Lecture 2: Showers, Jets and fixed-order predictions

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An extended (differently ordered, 2009) version of these lectures is available from:

<http://www.lpthe.jussieu.fr/~salam/teaching/PhD-courses.html>

or equivalently

<http://bit.ly/dqoIpj>

Take squared matrix element and rewrite in terms of  $E$ ,  $\theta$ ,

$$\frac{2p_1 \cdot p_2}{(2p_1 \cdot k)(2p_2 \cdot k)} = \frac{1}{E^2(1 - \cos^2 \theta)}$$

So final expression for soft gluon emission is

$$d\mathcal{S} = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin \theta} \frac{d\phi}{2\pi}$$

NB:

- ▶ It *diverges* for  $E \rightarrow 0$  — *infrared (or soft) divergence*
- ▶ It *diverges* for  $\theta \rightarrow 0$  and  $\theta \rightarrow \pi$  — *collinear divergence*

Soft, collinear divergences derived here in specific context of  $e^+e^- \rightarrow q\bar{q}$   
 But they are a very general property of QCD

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**But they are a very general property of QCD**

Yesterday we discussed the total cross section &  
real-virtual cancellation

Today let's look at more “exclusive” quantities —  
structure of final state

Let's try and integrate emission probability to get the mean number of gluons emitted off a quark with energy  $\sim Q$ :

$$\langle N_g \rangle \simeq \frac{2\alpha_s C_F}{\pi} \int^Q \frac{dE}{E} \int^{\pi/2} \frac{d\theta}{\theta}$$

This diverges unless we cut the integral off for transverse momenta ( $k_t \simeq E\theta$ ) below some non-perturbative threshold,  $Q_0 \sim \Lambda_{QCD}$ .

On the grounds that perturbation no longer applies for  $k_t \sim \Lambda_{QCD}$   
Language of quarks and gluons becomes meaningless

With this cutoff, result is:

$$\langle N_g \rangle \simeq \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{Q_0} + \mathcal{O}(\alpha_s \ln Q)$$



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## Naive gluon multiplicity (cont.)

Suppose we take  $Q_0 = \Lambda_{QCD}$ , how big is the result?

Let's use  $\alpha_s = \alpha_s(Q) = 1/(2b \ln Q/\Lambda)$

[Actually, over most of integration range this is optimistically small]

$$\langle N_g \rangle \simeq \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{\Lambda_{QCD}} \rightarrow \frac{C_F}{2b\pi} \ln \frac{Q}{\Lambda_{QCD}}$$

NB: given form for  $\alpha_s$ , this is actually  $\sim 1/\alpha_s$

Put in some numbers:  $Q = 100 \text{ GeV}$ ,  $\Lambda_{QCD} \simeq 0.2 \text{ GeV}$ ,  $C_F = 4/3$ ,  $b \simeq 0.6$ ,

$$\longrightarrow \langle N_g \rangle \simeq 2.2$$

Perturbation theory assumes that first-order term,  $\sim \alpha_s$  should be  $\ll 1$ .

But the final result is  $\sim 1/\alpha_s > 1 \dots$

**Is perturbation theory completely useless?**

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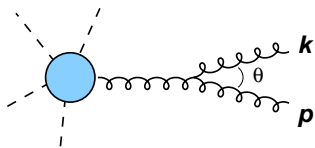
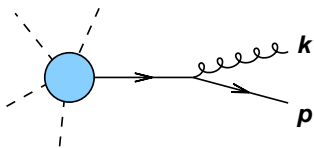
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But the final result is  $\sim 1/\alpha_s > 1 \dots$

**Is perturbation theory completely useless?**

Given this failure of first-order perturbation theory, two possible avenues.

1. Continue calculating the next order(s) and see what happens
2. Try to see if there exist other observables for which perturbation theory is better behaved

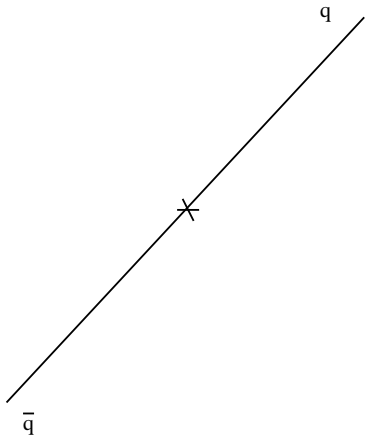
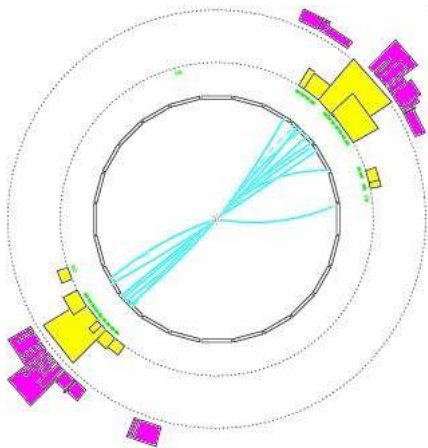


Gluon emission from quark:  $\frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$

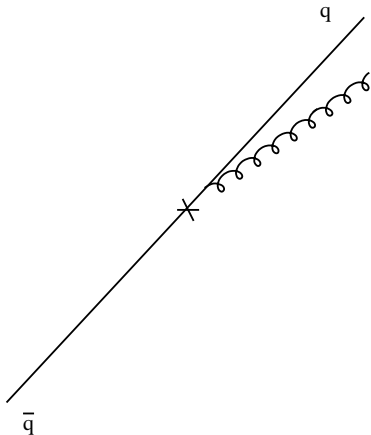
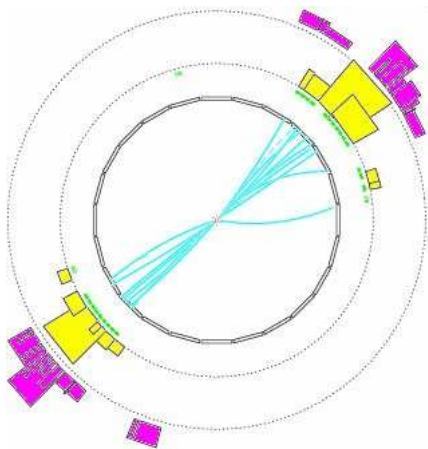
Gluon emission from gluon:  $\frac{2\alpha_s C_A}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$

Both expressions valid only if  $\theta \ll 1$  and energy soft relative to parent

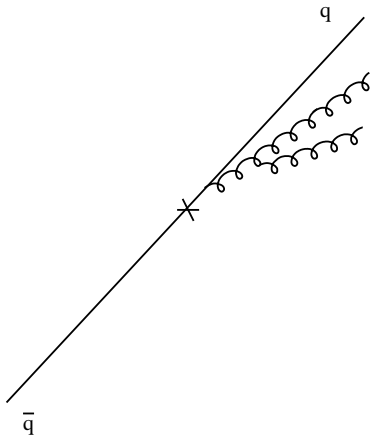
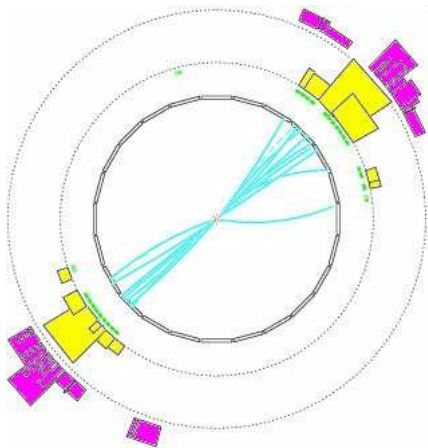
- ▶ Same divergence structures, regardless of where gluon is emitted from
- ▶ All that changes is the colour factor ( $C_F = 4/3$  v.  $C_A = 3$ )
- ▶ Expect low-order structure ( $\alpha_s \ln^2 Q$ ) to be replicated at each new order



Start of with  $q\bar{q}$

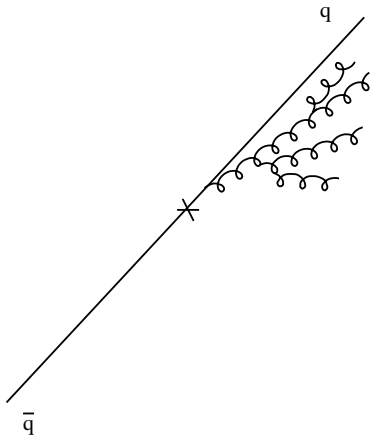
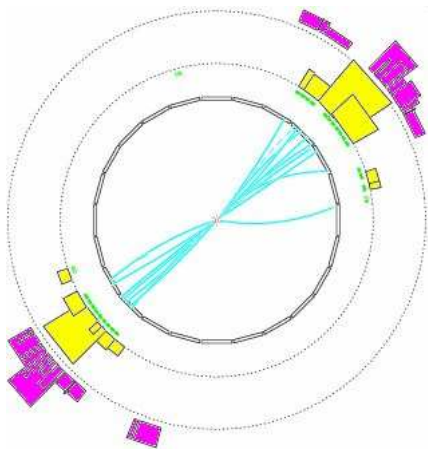


**A gluon gets emitted at small angles**

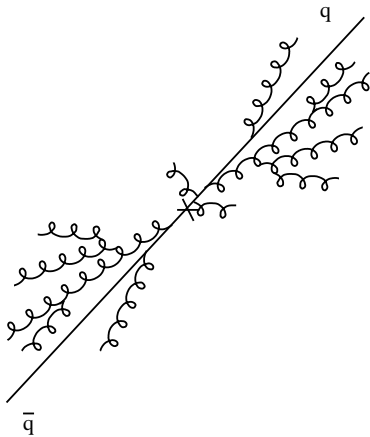
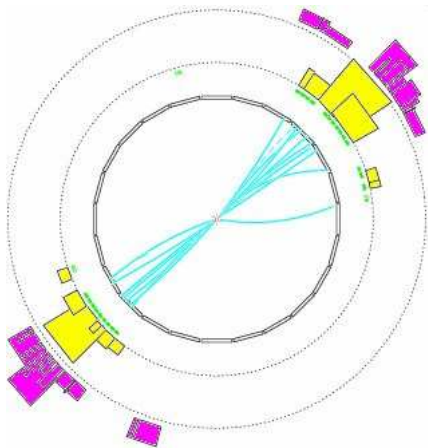


**It radiates a further gluon**

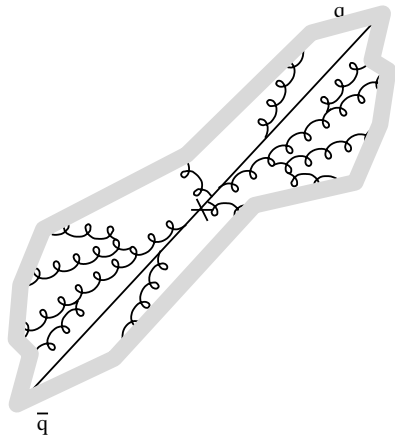
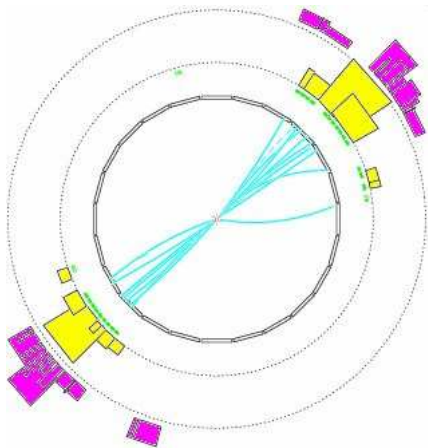




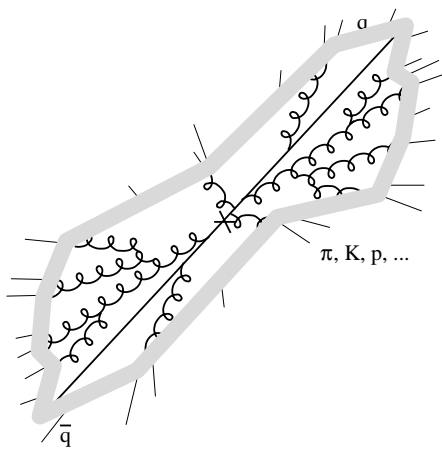
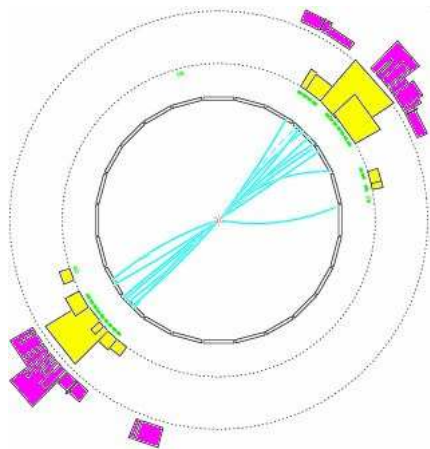
**And so forth**



**Meanwhile the same happened on other side of event**



**And then a non-perturbative transition occurs**



**Giving a pattern of hadrons that “remembers” the gluon branching**

Hadrons mostly produced at small angle wrt  $q\bar{q}$  directions or with low energy

It turns out you can calculate the gluon multiplicity analytically, by summing all orders ( $n$ ) of perturbation theory:

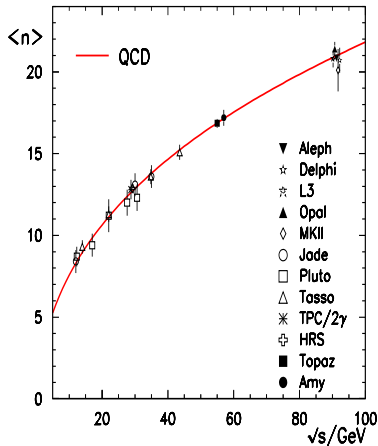
$$\langle N_g \rangle \sim \sum_n \frac{1}{(n!)^2} \left( \frac{C_A}{\pi b} \ln \frac{Q}{\Lambda} \right)^n$$

$$\sim \exp \sqrt{\frac{4C_A}{\pi b} \ln \frac{Q}{\Lambda}}$$

Compare to data for **hadron** multiplicity ( $Q \equiv \sqrt{s}$ )

Including some other higher-order terms and fitting overall normalisation

**Agreement is amazing!**



charged hadron multiplicity  
in  $e^+e^-$  events  
adapted from ESW

We don't want to have to do analytical calculations for every observable an experimenter measures.

[too many experimenters, observables too complex, too few theorists]

Resort to **parton showers**

Using the soft-collinear approximation to make predictions about events' detailed structure

How can we get a computer program to generate all the nested ensemble of soft/collinear emissions?

The way to frame the question is: *what is the probability of **not** radiating a **gluon** above a scale  $k_t$ ?*

$$P(\text{no emission above } k_t) = 1 - \frac{2\alpha_s C_F}{\pi} \int^Q \frac{dE}{E} \int^{\pi/2} \frac{d\theta}{\theta} \Theta(E\theta - k_t)$$

In the soft-collinear limit, it's quite easy to calculate the full probability of nothing happening: it's just the exponential of the first order:

$$P(\text{nothing} > k_t) \equiv \Delta(k_t, Q) \simeq \exp \left[ -\frac{2\alpha_s C_F}{\pi} \int^Q \frac{dE}{E} \int^{\pi/2} \frac{d\theta}{\theta} \Theta(E\theta - k_t) \right]$$

NB1:  $\Delta$  is bounded —  $0 < \Delta(k_t, Q) < 1$

NB2: to do this properly, running coupling should be inside integral  
 + replace  $dE/E$  with full collinear splitting function

$\Delta(k_t, Q)$  is known as a **Sudakov Form Factor**

Probability distribution for first emission (e.g.  $q\bar{q} \rightarrow q\bar{q}g$ ) is simple

$$\frac{dP}{dk_{t1}} = \frac{d}{dk_{t1}} \Delta(k_{t1}, Q)$$

Easy to generate this distribution by Monte Carlo

Take flat random number  $0 < r < 1$  and solve  $\Delta(k_t, Q) = r$

Now we have a  $q\bar{q}g$  system.

We next work out a Sudakov for there being no emission from the  $q\bar{q}g$  system above scale  $k_{t2}$  ( $< k_{t1}$ ):  $\Delta^{q\bar{q}g}(k_{t2}, k_{t1})$ , and use this to generate  $k_{t2}$ .

Then generate  $k_{t3}$  emission from the  $q\bar{q}gg$  system ( $k_{t3} < k_{t2}$ ). Etc.

Repeat until you reach a non-perturbative cutoff scale  $Q_0$ , and then stop.

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That was a description that roughly encompasses:

- ▶ The New Pythia shower    Pythia 8.1, and the  $p_t$  ordered option of Pythia 6.4
- ▶ The Ariadne shower

Other showers:

- ▶ Old Pythia (& Sherpa): order in virtuality instead of  $k_t$  and each parton branches independently (+ angular veto)    works fine on most data  
but misses some theoretically relevant contributions  
by far the most widely used shower
- ▶ Herwig (6.5 & ++): order in angle, and each parton branches independently    Herwig++ fills more of phase space than 6.5

That was all for a “final-state” shower

- ▶ Initial-state showers also need to deal carefully with PDF evolution

## 1. You select the beams and their energy

---INITIAL STATE---

IHEP	ID	IDPDG	IST	M01	M02	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
1	P	2212	101	0	0	0	0	0.00	0.00	7000.0	7000.0	0.94
2	P	2212	102	0	0	0	0	0.00	0.00	-7000.0	7000.0	0.94
3	CMF	0	103	1	2	0	0	0.00	0.00	0.0	14000.0	14000.0

2. You select the hard process (here  $Z + jet$  production)  
Herwig generates kinematics for the hard process

---HARD SUBPROCESS---

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
4	UQRK	2	121	6	8	9	5	0.00	0.00	590.8	590.8	0.32
5	GLUON	21	122	6	4	17	8	0.00	0.00	-232.1	232.1	0.75
6	HARD	0	120	4	5	7	8	0.40	-9.40	358.7	823.0	740.63
7	Z0/GAMA*	23	123	6	7	22	7	-261.59	-217.31	329.3	481.6	88.56
8	UQRK	2	124	6	5	23	4	261.59	217.31	29.4	341.3	0.32

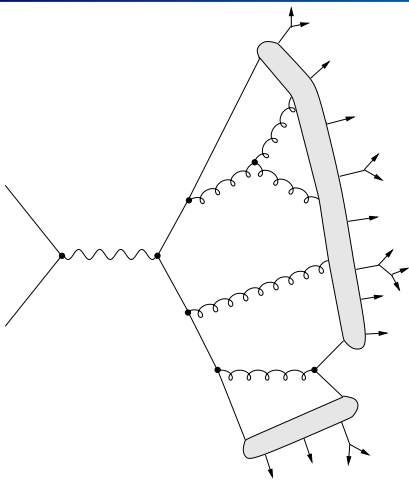
## 3. Herwig “dresses” it with initial and final-state showers

---PARTON SHOWERS---

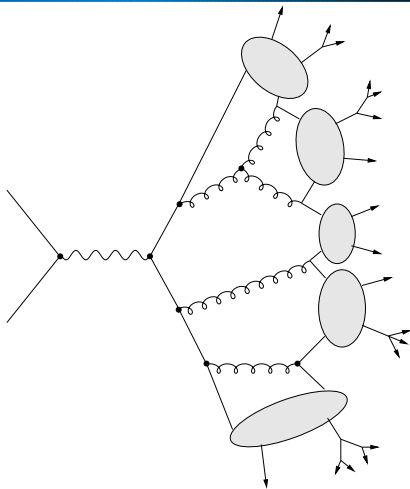
IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
9	UQRK	94	141	4	6	11	16	2,64	-9,83	592,2	590,2	-49,07
10	CONE	0	100	4	5	0	0	-0,27	0,96	0,1	1,0	0,00
11	GLUON	21	2	9	12	32	33	-1,02	3,59	5,6	6,7	0,75-
12	GLUON	21	2	9	13	34	35	0,25	1,46	3,6	4,0	0,75-
13	GLUON	21	2	9	14	36	37	-0,87	1,62	4,7	5,1	0,75-
14	GLUON	21	2	9	15	38	39	-0,81	4,17	3611,7	3611,7	0,75-
15	GLUON	21	2	9	16	40	41	-0,19	-1,01	1727,7	1727,7	0,75-
16	UD	2101	2	9	25	42	41	0,00	0,00	1054,6	1054,6	0,32-
17	GLUON	94	142	5	6	19	21	-2,23	0,44	-233,5	232,8	-18,36
18	CONE	0	100	5	8	0	0	0,77	0,64	0,2	1,0	0,00
19	GLUON	21	2	17	20	43	44	1,60	0,58	-2,1	2,8	0,75
20	UD	2101	2	17	21	45	44	0,00	0,00	-2687,6	2687,6	0,32
21	UQRK	2	2	17	32	46	45	0,63	-1,02	-4076,9	4076,9	0,32
22	Z0/GAMA*	23	195	7	22	251	252	-257,66	-219,68	324,8	477,5	88,56
23	UQRK	94	144	8	6	25	31	258,06	210,29	33,9	345,5	86,10
24	CONE	0	100	8	5	0	0	0,21	0,17	-1,0	1,0	0,00
25	UQRK	2	2	23	26	47	42	26,82	24,33	23,7	43,3	0,32
26	GLUON	21	2	23	27	48	49	8,50	8,18	6,0	13,3	0,75
27	GLUON	21	2	23	28	50	51	73,27	61,24	12,0	96,2	0,75
28	GLUON	21	2	23	29	52	53	73,66	58,54	-6,3	94,3	0,75
29	GLUON	21	2	23	30	54	55	67,58	52,13	-7,3	85,7	0,75
30	GLUON	21	2	23	31	56	57	6,98	4,60	2,3	8,7	0,75
31	GLUON	21	2	23	43	58	59	1,24	1,26	3,6	4,1	0,75

**INITIAL  
STATE  
SHOWER**

**FINAL  
STATE  
SHOWER**

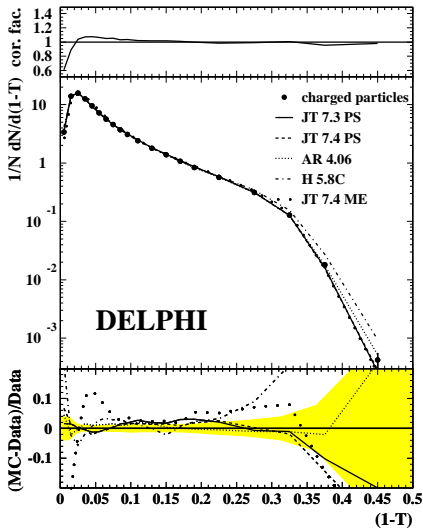


String Fragmentation  
(Pythia and friends)

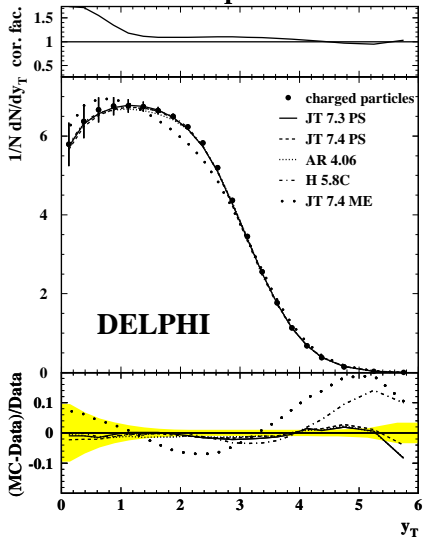


Cluster Fragmentation  
(Herwig)

### 1-Thrust



### $y_T$



It's amazing that just soft/collinear “showering” + a hadronisation-model gives such a good description of the physics.

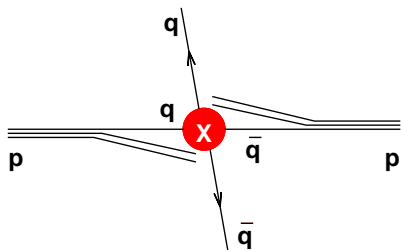
**BUT:**

1. haven't we left out all the information that comes from exact Feynman diagrams?
2. what if we want to get back at the information about the “original” quarks?

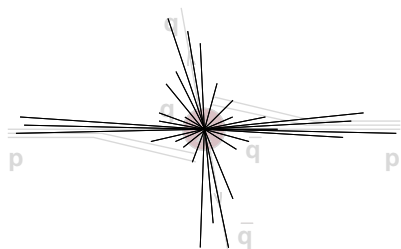


# Reconstructing $Z' \rightarrow q\bar{q}$ ?

Take the leading hadrons: how much “unlike” the original quarks are they?  
To find out, check their pair invariant mass.

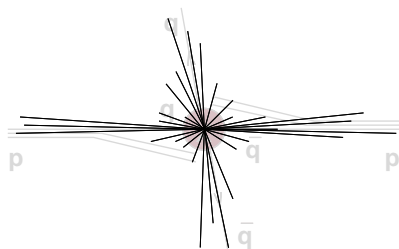
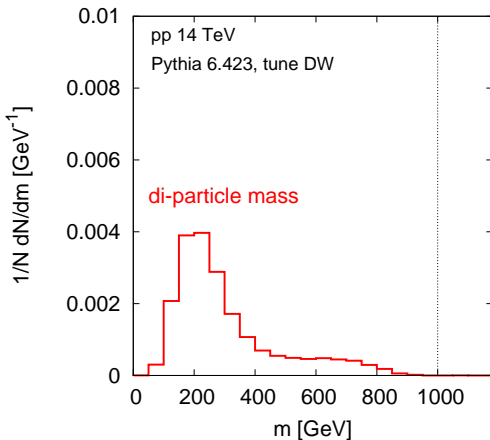


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Reconstructing  $Z'$  with  $m = 1000$  GeV



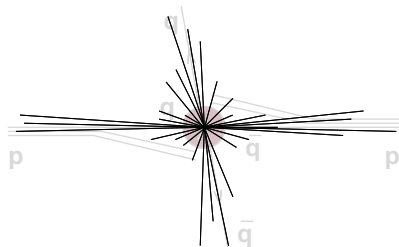
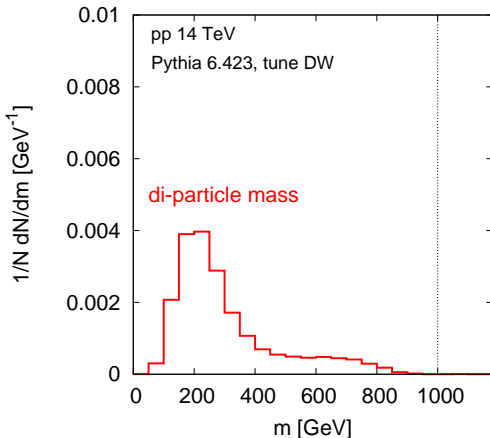
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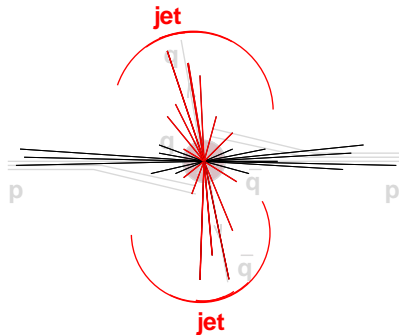
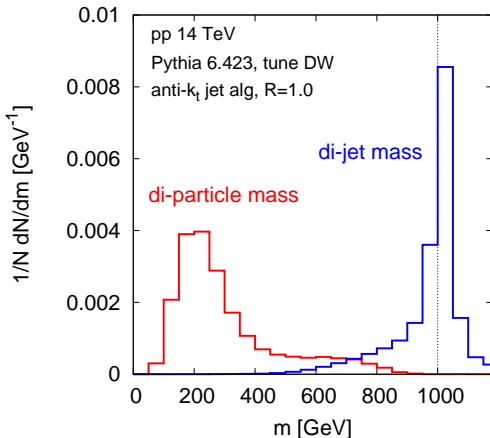
Reconstructing  $Z'$  with  $m = 1000$  GeV



# Reconstructing $Z' \rightarrow q\bar{q}$ (via jets)?

We really want to capture the bulk of the energy flow from the quark  
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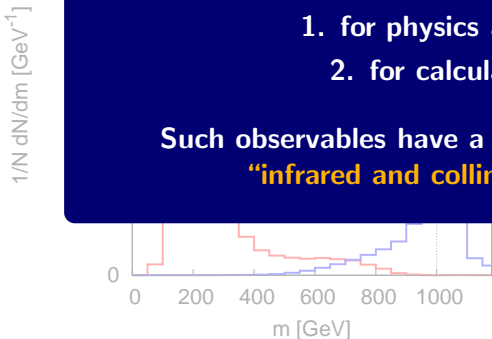
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Reconstructing  $Z'$  with  $m = 1000$  GeV

An observable that captures the essence of a parton's kinematics is good:

1. for physics analyses
2. for calculations

Such observables have a property known as  
"infrared and collinear safety"



## Infrared and Collinear Safety (definition)

*For an observable's distribution to be calculable in [fixed-order] perturbation theory, the observable should be infra-red safe, i.e. insensitive to the emission of soft or collinear gluons. In particular if  $\vec{p}_i$  is any momentum occurring in its definition, it must be invariant under the branching*

$$\vec{p}_i \rightarrow \vec{p}_j + \vec{p}_k$$

*whenever  $\vec{p}_j$  and  $\vec{p}_k$  are parallel [collinear] or one of them is small [infrared].*

[QCD and Collider Physics (Ellis, Stirling & Webber)]

### Examples

- ▶ Multiplicity of gluons is *not* IRC safe [modified by soft/collinear splitting]
- ▶ Energy of hardest particle is *not* IRC safe [modified by collinear splitting]
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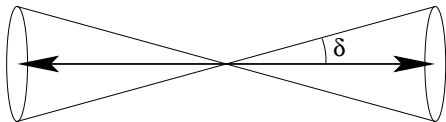
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## The *original* (finite) jet definition

An event has 2 jets if at least a fraction  $(1 - \epsilon)$  of event energy is contained in two cones of half-angle  $\delta$ .

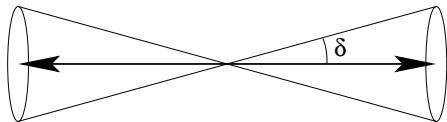


$$\sigma_{2\text{-jet}} = \sigma_{q\bar{q}} \left( 1 + \frac{2\alpha_s C_F}{\pi} \int \frac{dE}{E} \frac{d\theta}{\sin\theta} \left( R \left( \frac{E}{Q}, \theta \right) \times \right. \right. \\ \left. \left. \times \left( 1 - \Theta \left( \frac{E}{Q} - \epsilon \right) \Theta(\theta - \delta) \right) - V \left( \frac{E}{Q}, \theta \right) \right) \right)$$

- ▶ For small  $E$  or small  $\theta$  this is just like total cross section — full cancellation of divergences between real and virtual terms.
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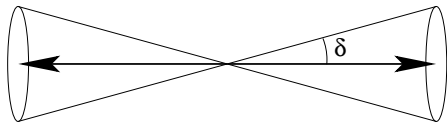


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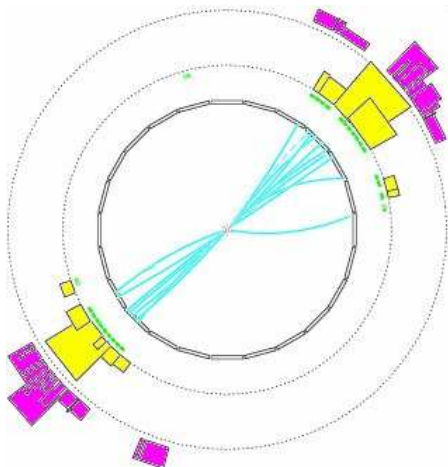
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## Near 'perfect' 2-jet event

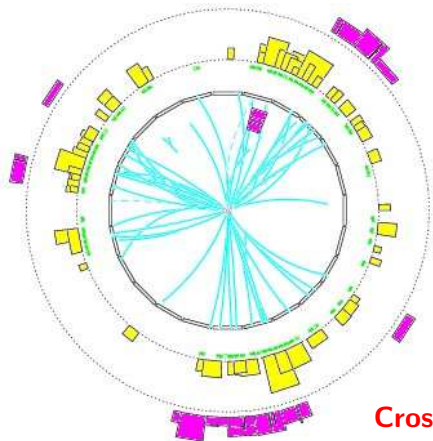
2 well-collimated jets of particles.

Nearly all energy contained in two cones.

Cross section for this to occur is

$$\sigma_{2\text{-jet}} = \sigma_{q\bar{q}}(1 - c_1\alpha_s + c_2\alpha_s^2 + \dots)$$

where  $c_1, c_2$  all  $\sim 1$ .



## How many jets?

- ▶ Most of energy contained in 3 (fairly) collimated cones
- ▶ Cross section for this to happen is

$$\sigma_{3\text{-jet}} = \sigma_{q\bar{q}}(c'_1\alpha_s + c'_2\alpha_s^2 + \dots)$$

where the coefficients are all  $\mathcal{O}(1)$

**Cross section for extra gluon diverges**  
**Cross section for extra jet is small,  $\mathcal{O}(\alpha_s)$**

NB: Stermen-Weinberg procedure gets complex for multi-jet events. 4th lecture will discuss modern approaches for defining jets.

- ▶ Soft-collinear divergences are universal property of QCD
- ▶ Lead to divergent predictions for many observables
- ▶ Regularising the divergences near  $\Lambda_{QCD}$  and summing over all orders in a soft/collinear approximation works remarkably well.
- ▶ A parton shower is an easily-used computer-implementation of that idea.  
[parton showers are ubiquitous in any collider context]
- ▶ **But:** not all observables are affected by these soft/collinear splittings. Those that are unaffected are called “IR/Collinear safe”:
  - ▶ Tend to be good for practical physics studies (e.g. new physics searches)
  - ▶ Can also be calculated within plain fixed-order QCD