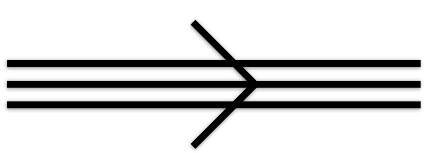
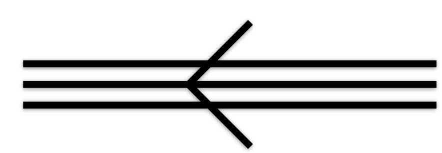
Introduction to QCD at high-energy colliders
Lectures 6 & 7: Parton Distribution Functions

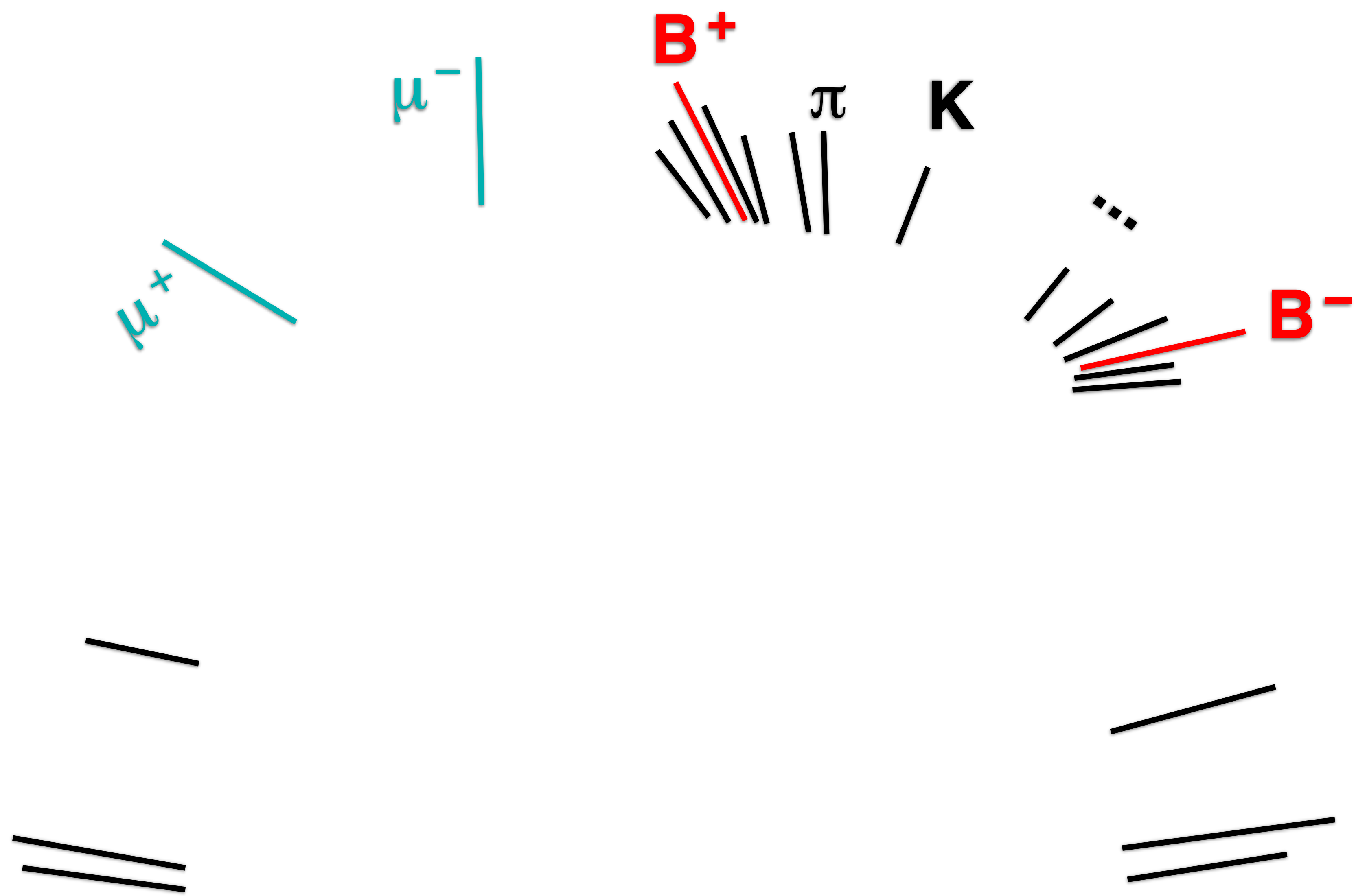
Gavin Salam, Oxford, February 2024
as part of the QCD PhD course with
Fabrizio Caola, Jack Helliwell, Peter Skands

A proton-proton collision: INITIAL STATE



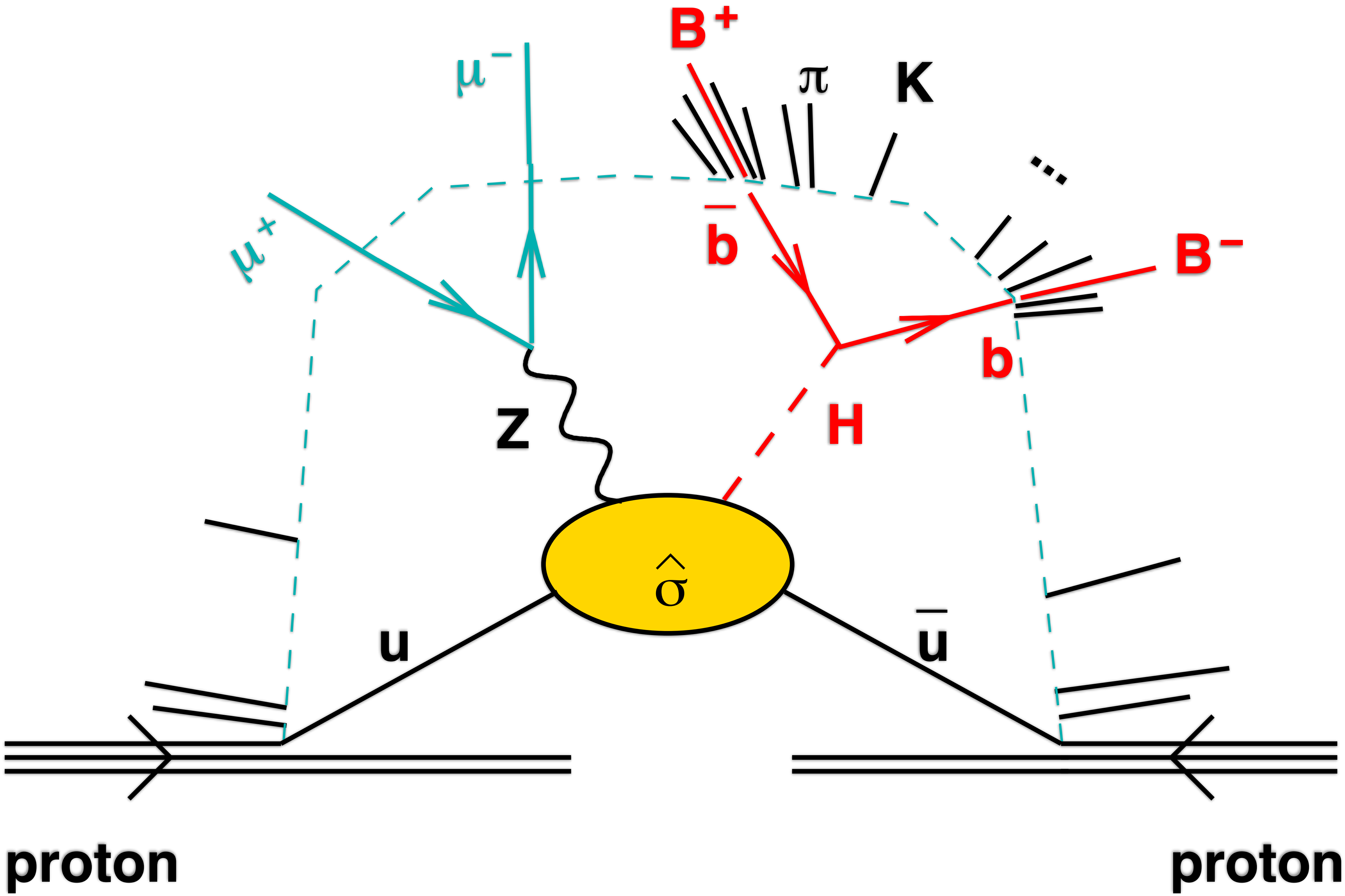


A proton-proton collision: FINAL STATE

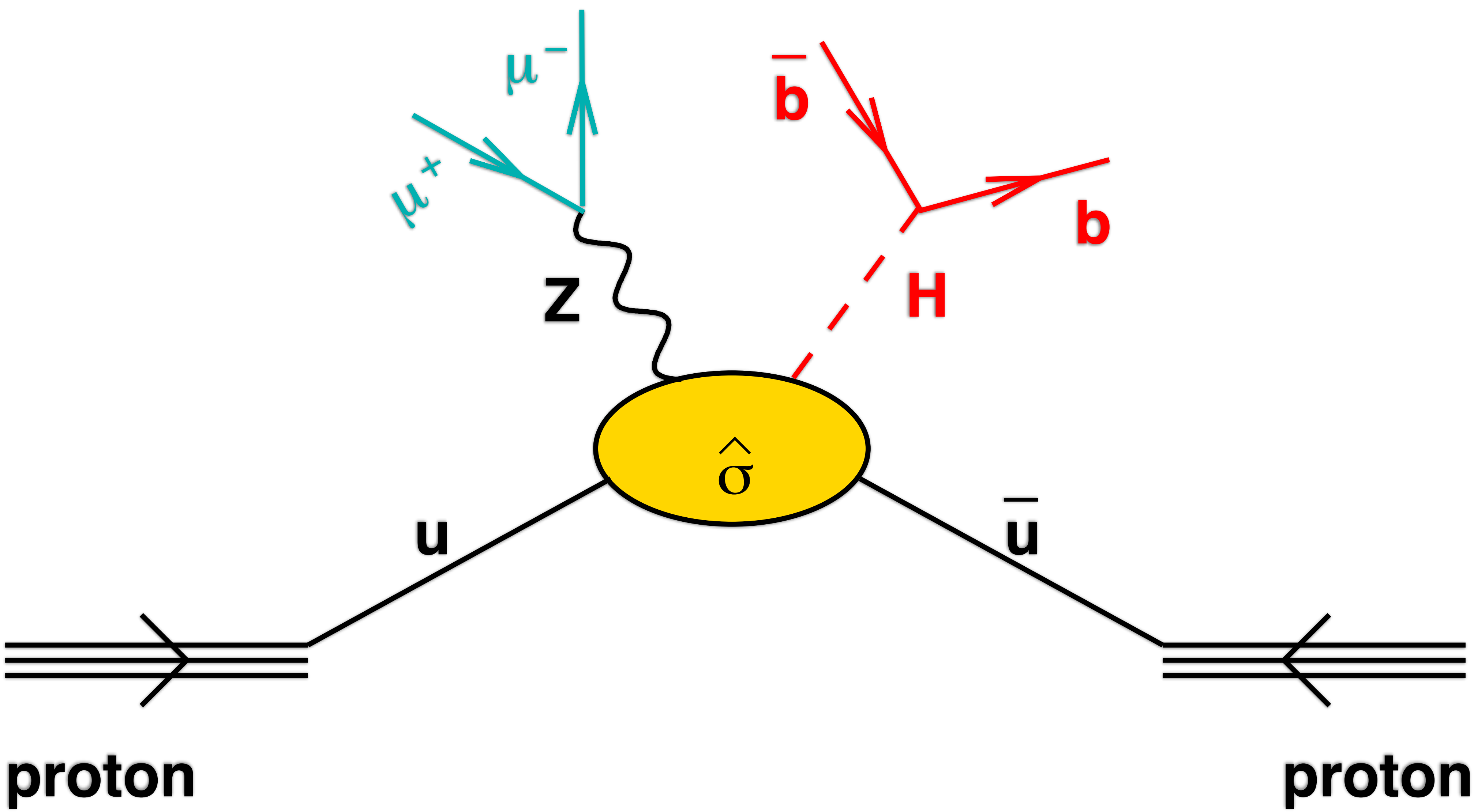


(actual final-state multiplicity ~ several hundred hadrons)

A proton-proton collision: FILLING IN THE PICTURE



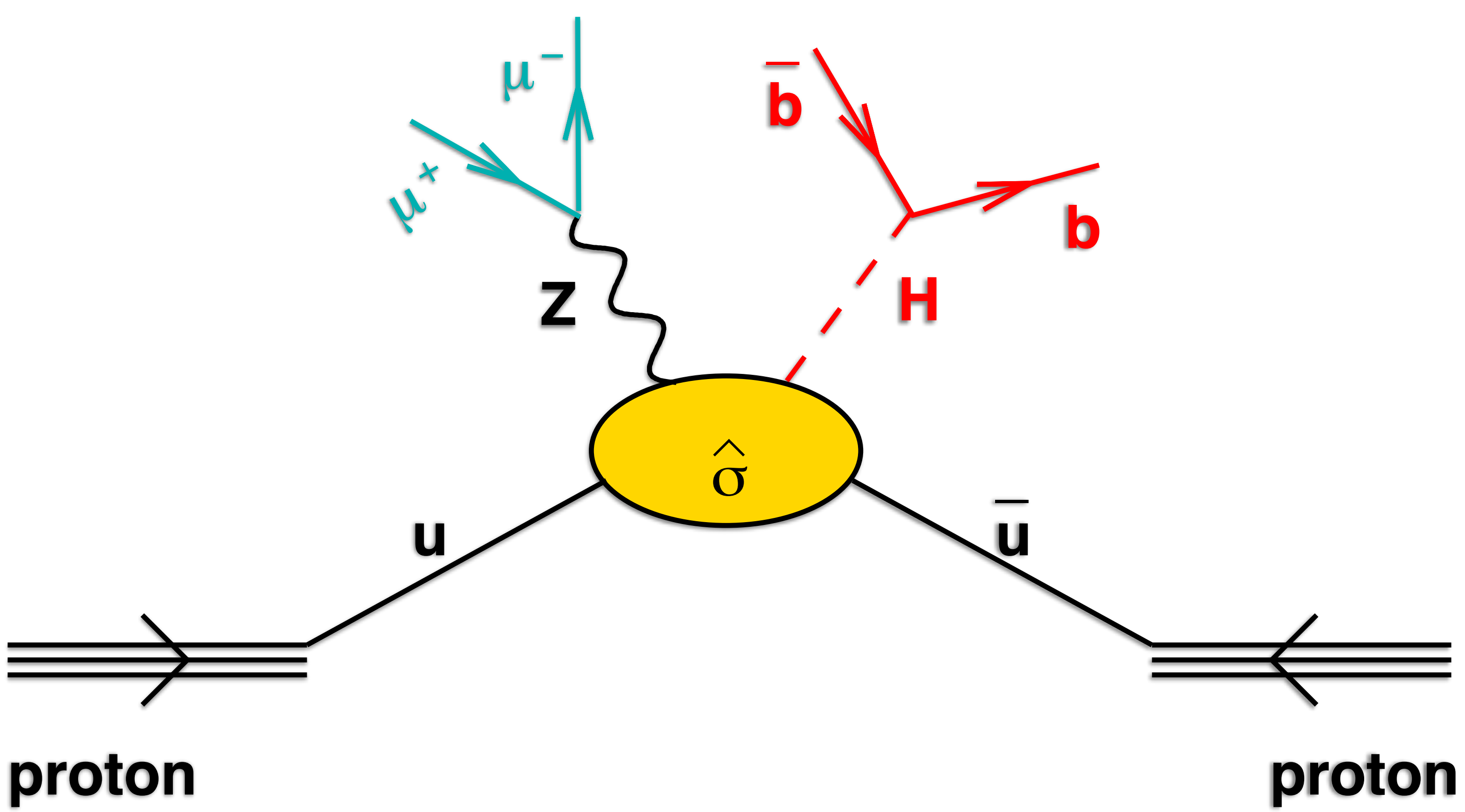
A proton-proton collision: SIMPLIFYING IN THE PICTURE



THE MASTER EQUATION — FACTORISATION

$$\sigma\left(h_1 h_2 \to ZH + X\right) = \sum_{n=0}^{\infty} \alpha_s^n\left(\mu_R^2\right) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right)$$

$$\times\, \hat{\sigma}^{(n)}_{ij\to ZH+X}\left(x_1 x_2 s, \mu_R^2, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right),$$



Perturbative sum over powers of the strong coupling: typically we know first 2-4 orders

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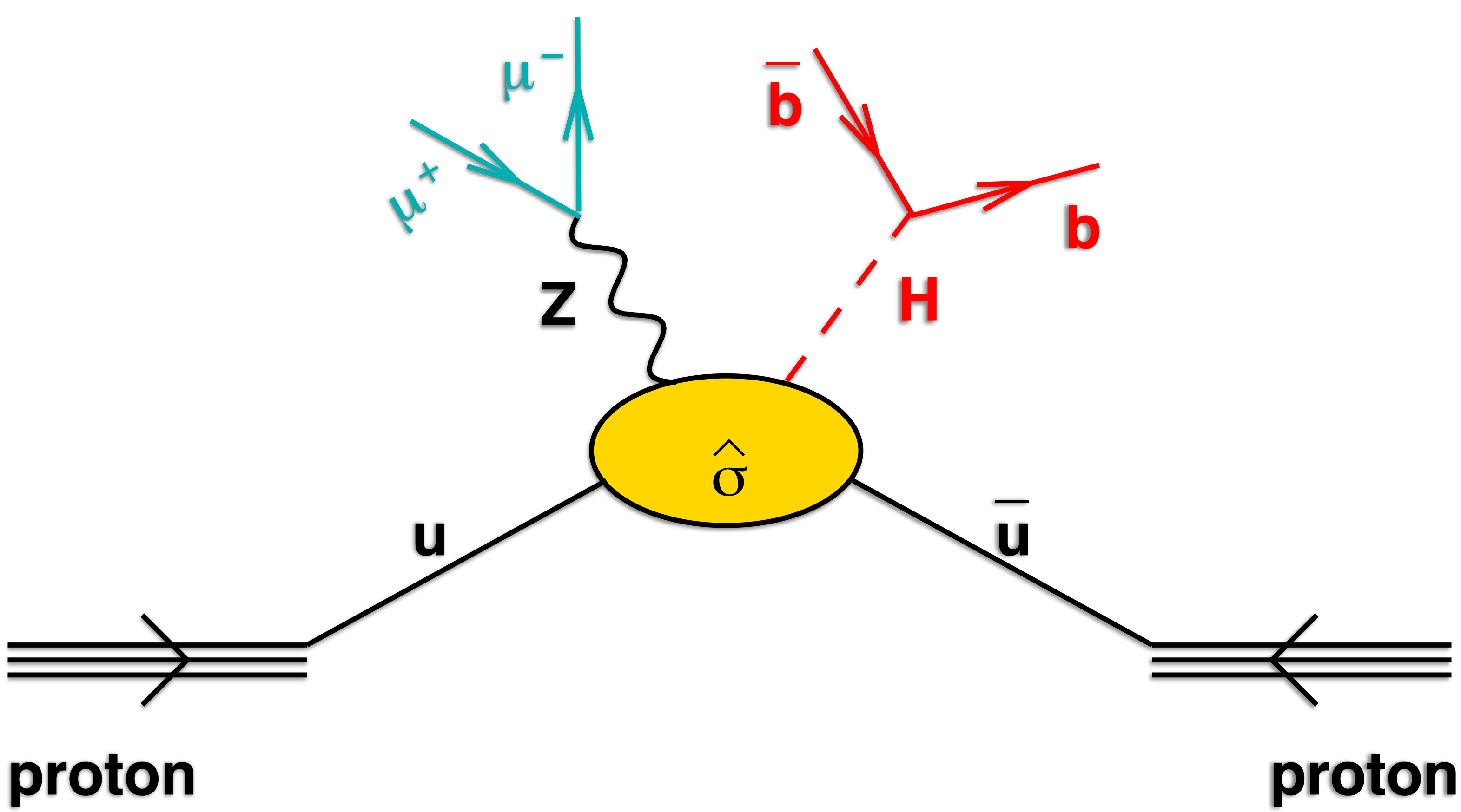

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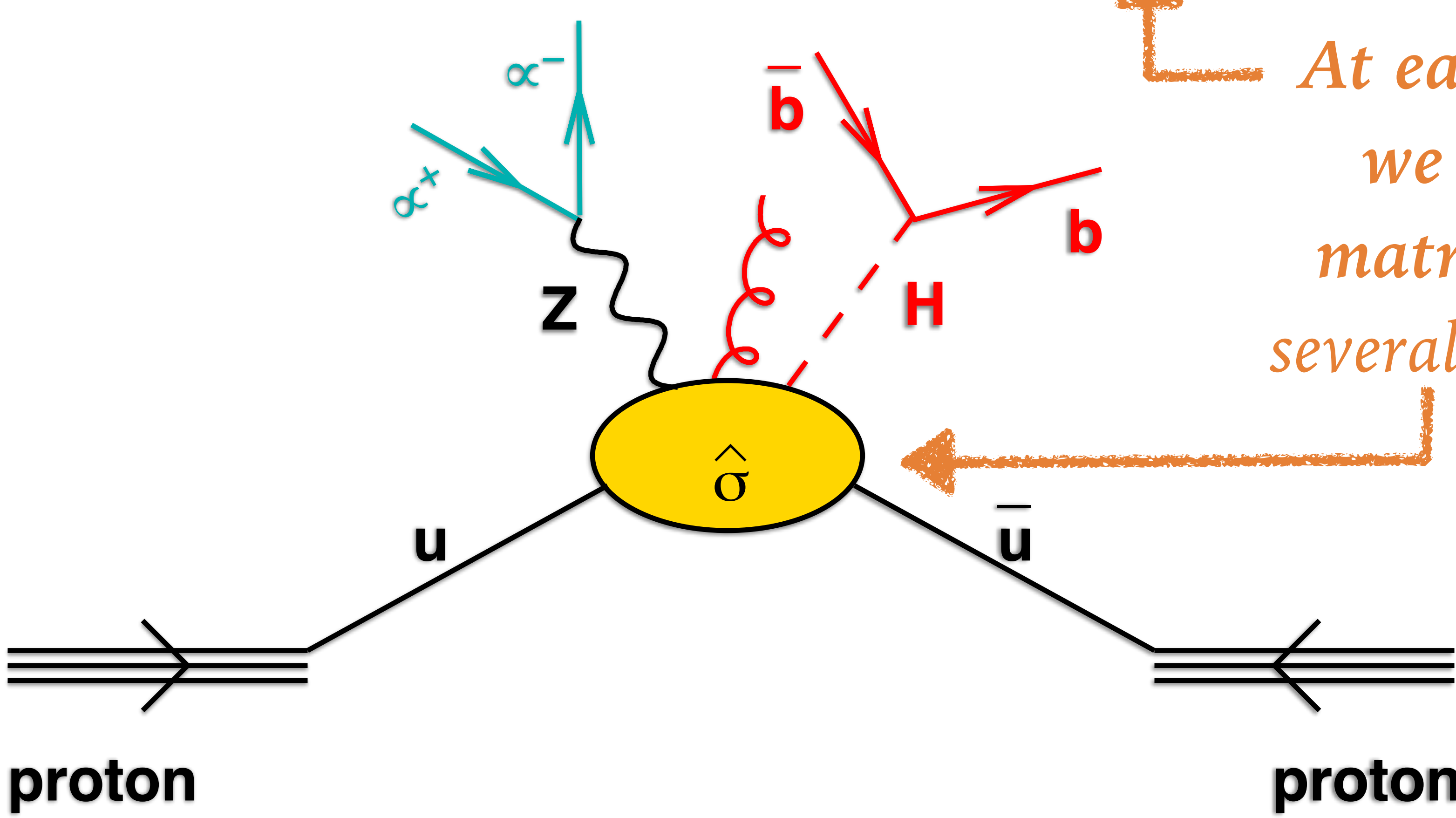
At each perturbative order n we have a specific "hard matrix element" (sometimes several for different subprocesses)



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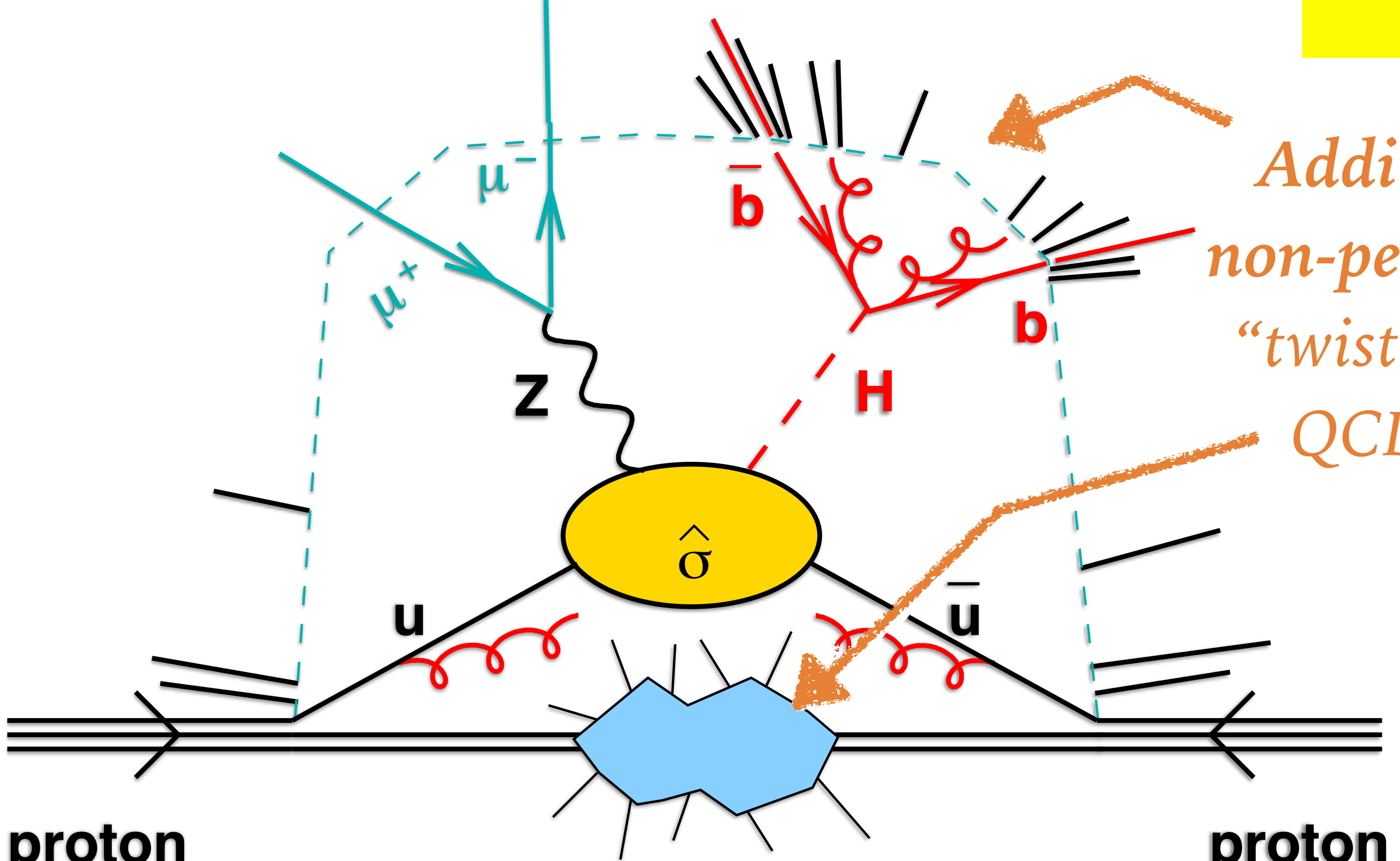

Additional corrections from non-perturbative effects (higher "twist", suppressed by powers of QCD scale (Λ) / hard scale)

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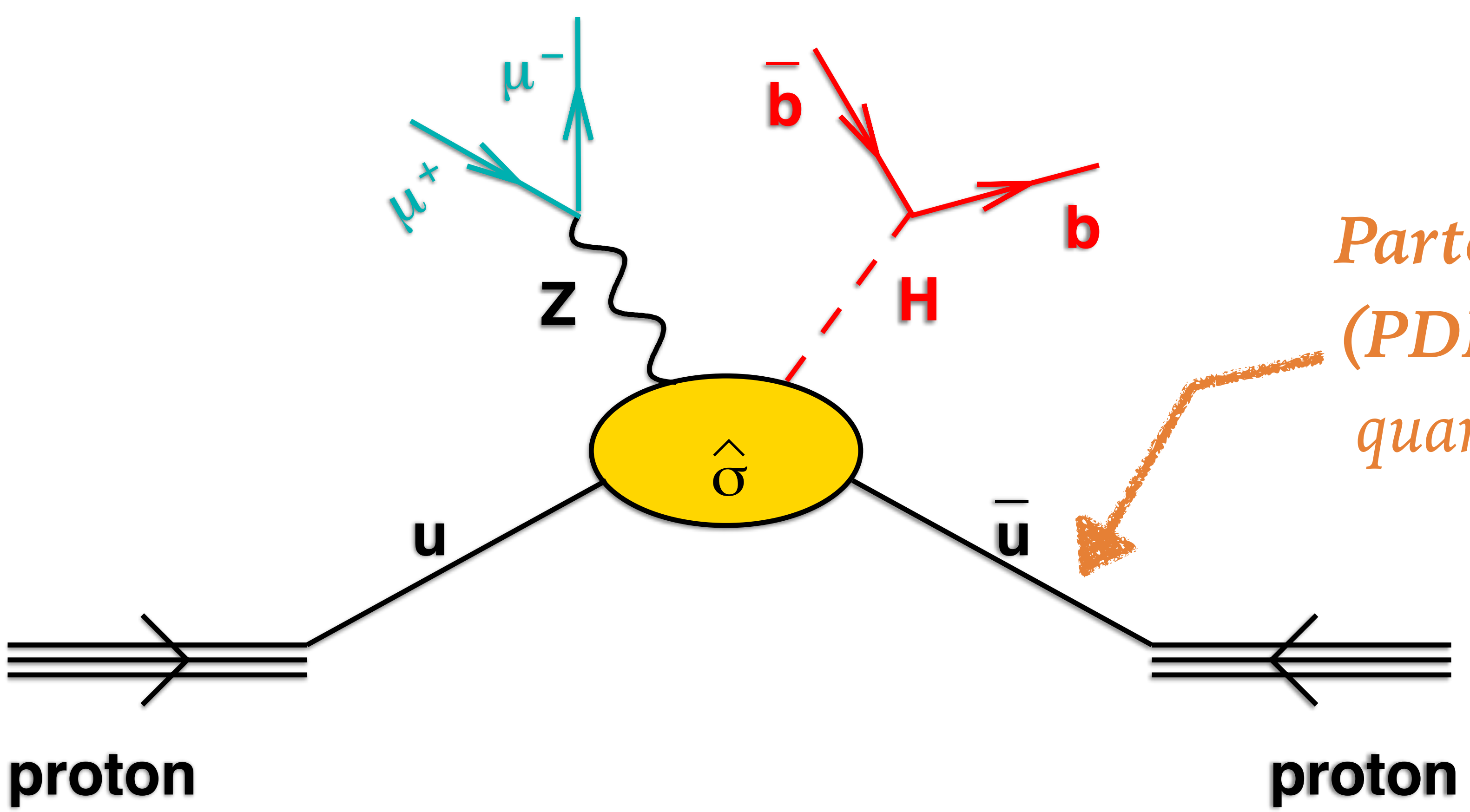

Parton distribution function (PDF): e.g. number of up anti-quarks carrying fraction x_2 of proton's momentum

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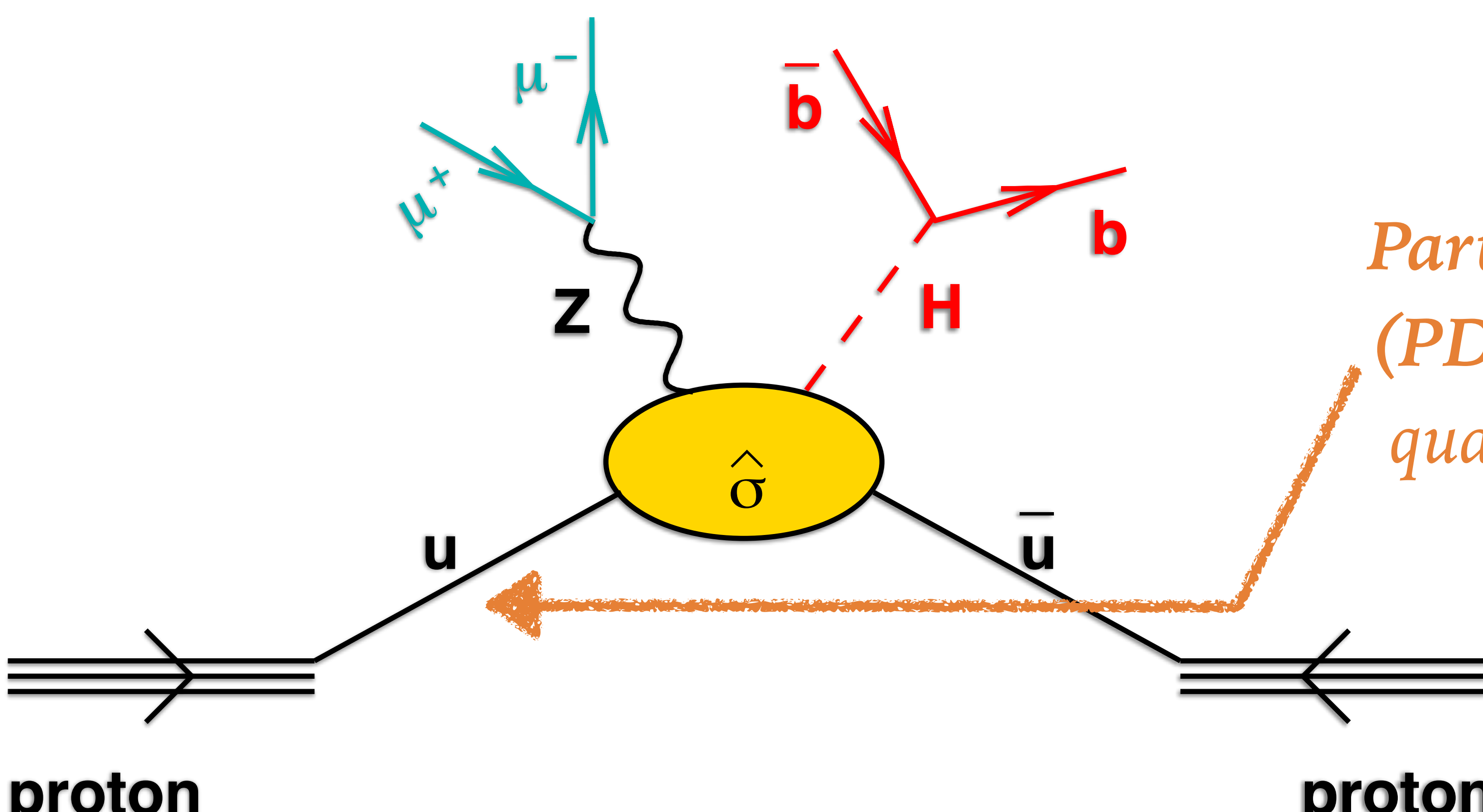
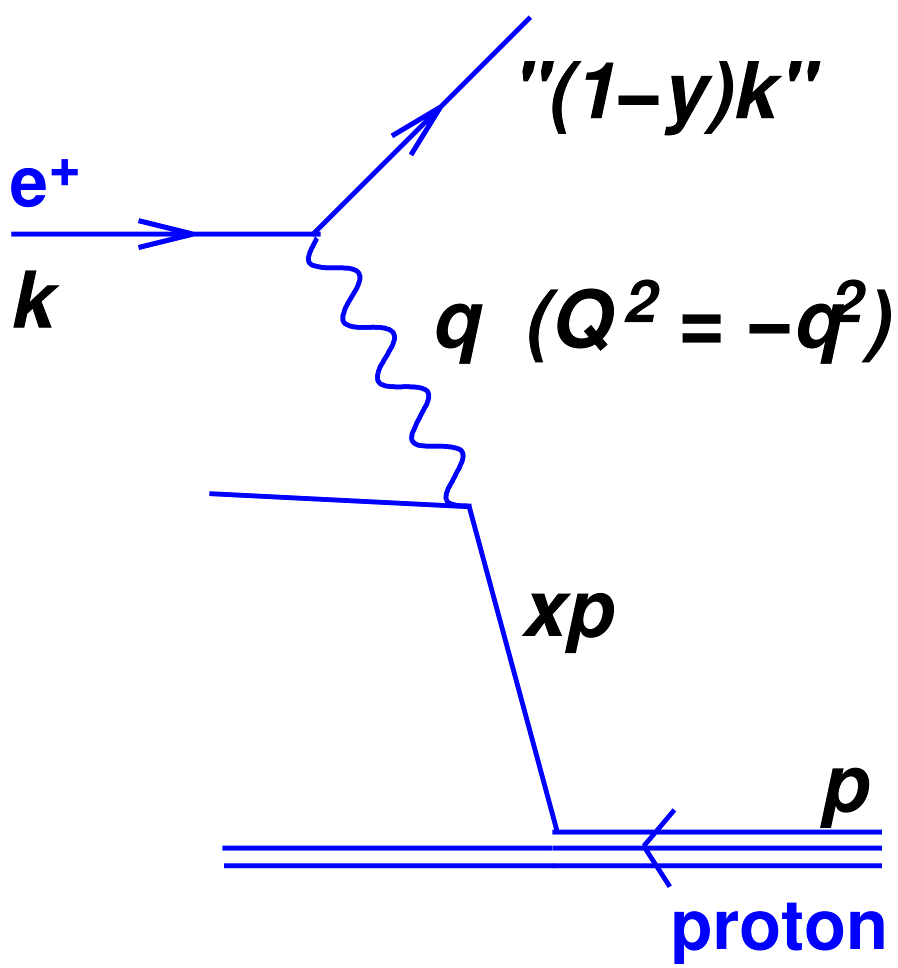


Parton distribution function (PDF): e.g. number of up quarks carrying fraction x_1 of proton's momentum

PARTON DISTRIBUTION FUNCTIONS (PDFs)

DEEP INELASTIC SCATTERING

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



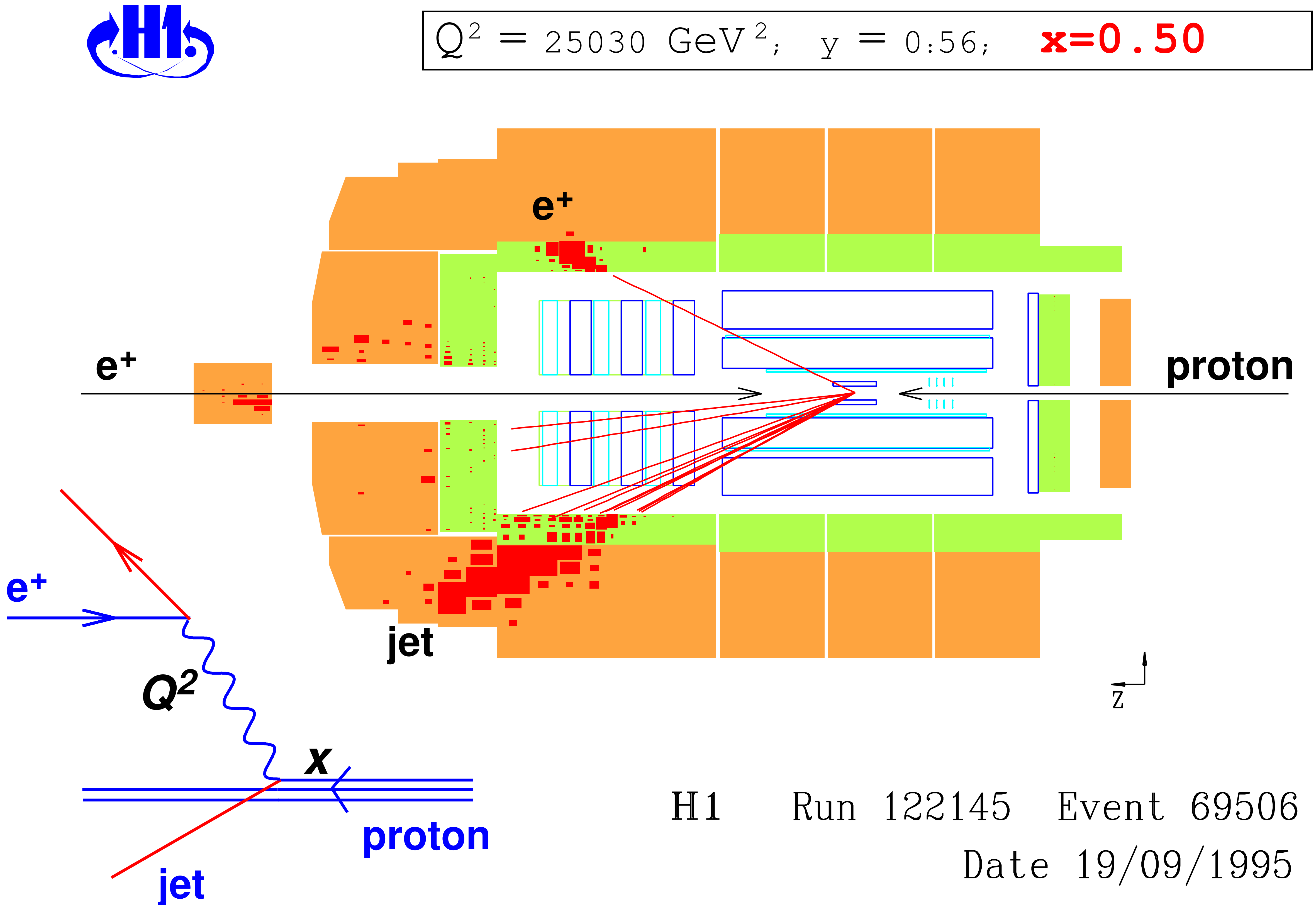
Kinematic relations:

$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

- Q^2 = photon virtuality ↔ *transverse resolution* at which it probes proton structure
- x = *longitudinal momentum fraction* of struck parton in proton
- y = momentum fraction lost by electron (in proton rest frame)

DEEP INELASTIC SCATTERING



Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dx dQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

NB:

- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

F_2 combines up and down-quark distributions: how do we separate them?

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:
proton = uud; neutron = ddu $[-2 \times \frac{1}{3} + 2 \times \frac{1}{3} = 0]$

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

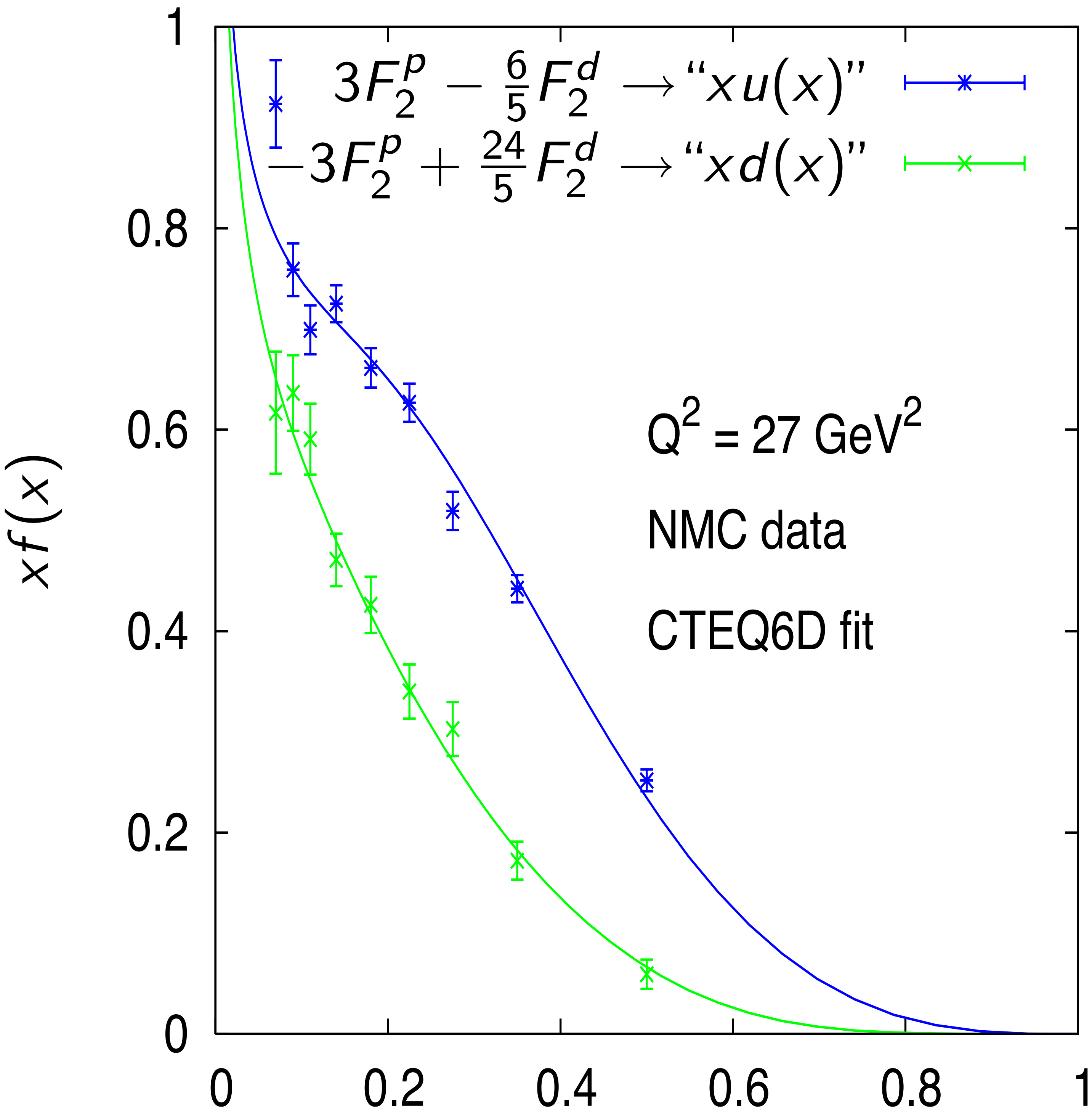
$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$
$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

Beware: isospin symmetry is a good approximation, but it is not exact

An example: NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

How much u and d?

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

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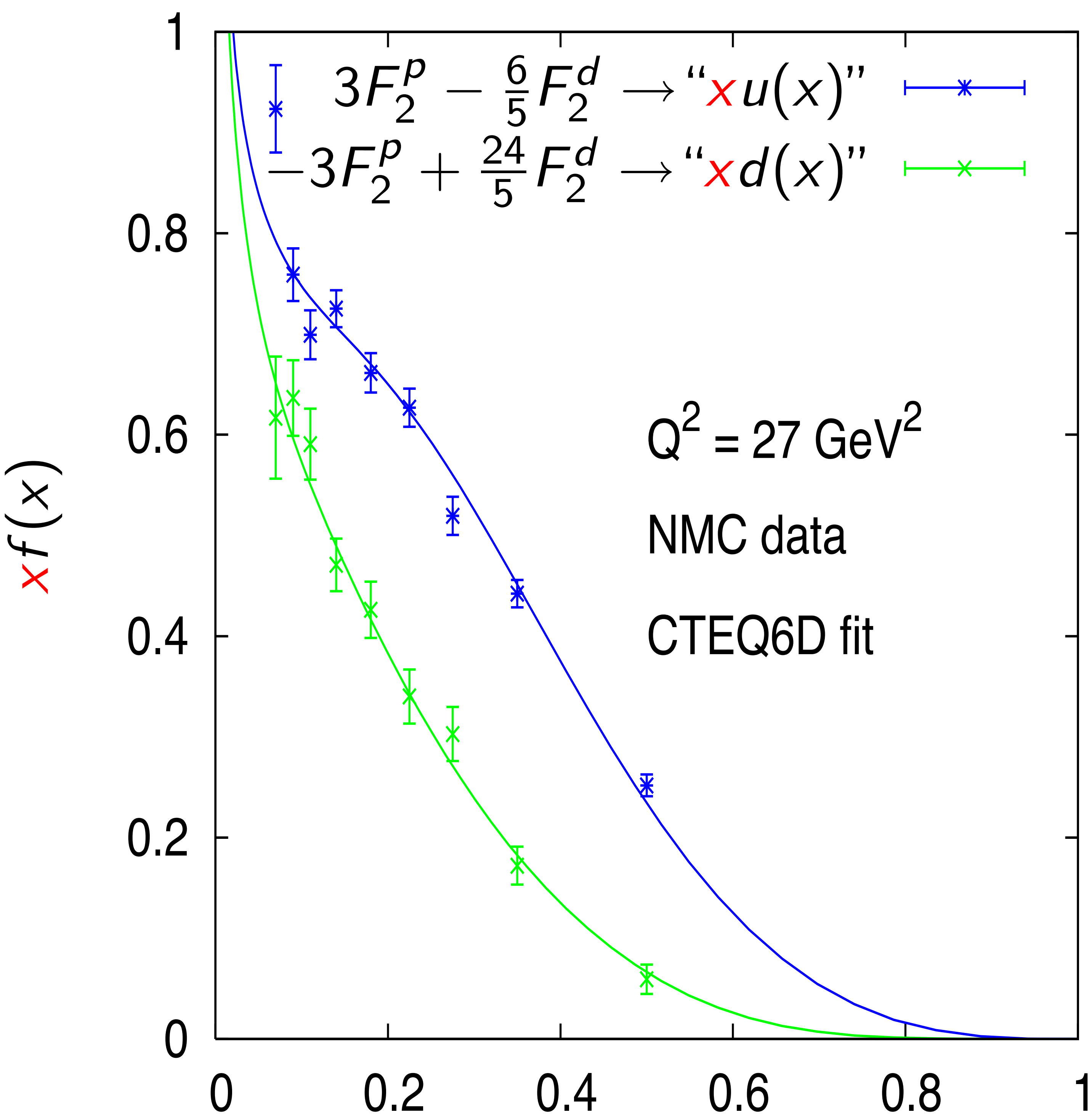

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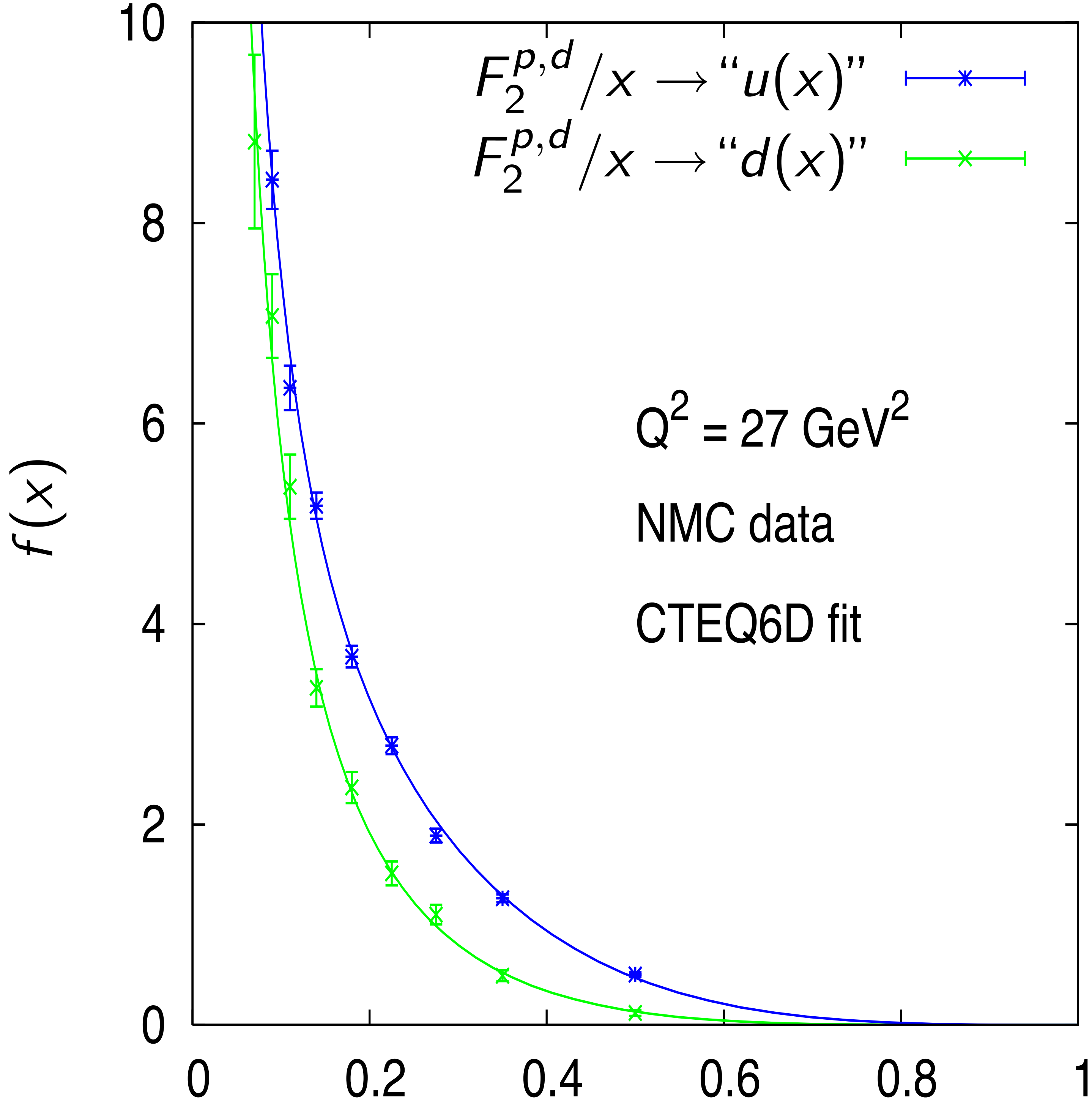

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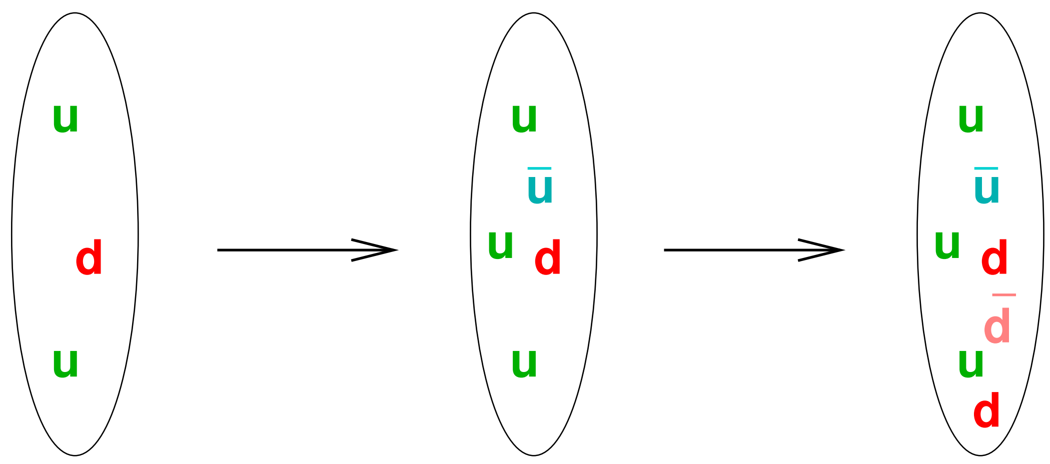
Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

NB: photon interaction $\sim$ square of charge $\to$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

$$\int dx\,(u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

“Valence” quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

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How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both $\rightarrow$ no good...

Question: what interacts differently with particle & antiparticle?

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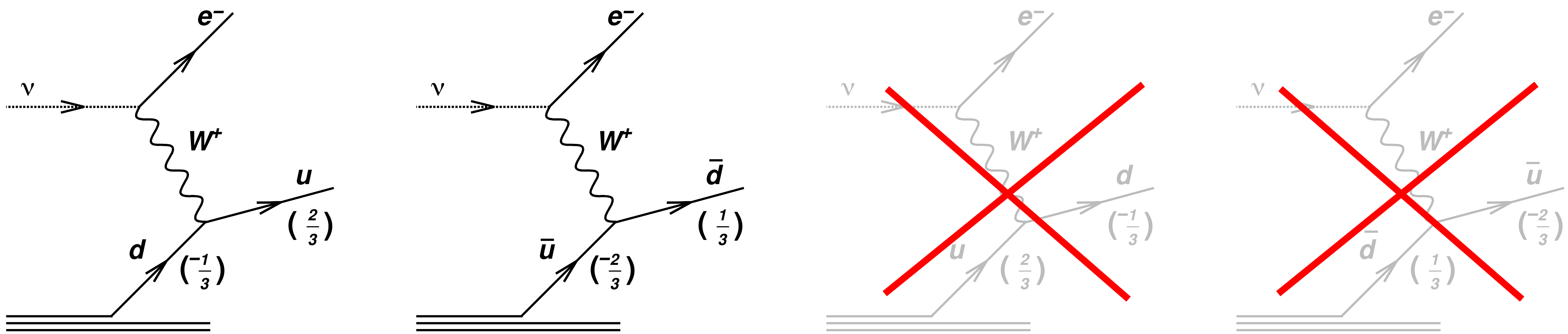
Answer: W^+ *or* W^-

"Valence" quarks



$$\frac{d^2\sigma^{W^\pm p}}{dxdQ^2} \propto \left(\frac{1+(1-y)^2}{2} F_2^{W^\pm p} \pm \frac{2y-y^2}{2} x F_3^{W^\pm p} + \mathcal{O}(\alpha_s) \right)$$

$$F_2^{W^+p} = 2x(d(x) + \bar{u}(x)), \qquad F_3^{W^+p} = 2(d(x) - \bar{u}(x))$$

$$F_2^{W^-p} = 2x(u(x) + \bar{d}(x)), \qquad F_3^{W^-p} = 2(u(x) - \bar{d}(x))$$

Combination of νp and $\bar{\nu} p$ scattering in principle provides all necessary information for getting separately u, d, $\bar{u}$ and $\bar{d}$.

“Valence” quarks

NB: Instead of neutrino beams, you can use charged current DIS with incoming electron/positron, and outgoing neutrino (as done at HERA)

Problem: experiments with neutrinos are *difficult* (small cross sections).

Look at collisions on *nuclei* (e.g. Fe) to increase cross section, and use *isospin symmetry* ($d_n = u_p$) to relate $F_3^{W^+p}$, $F_3^{W^+n}$

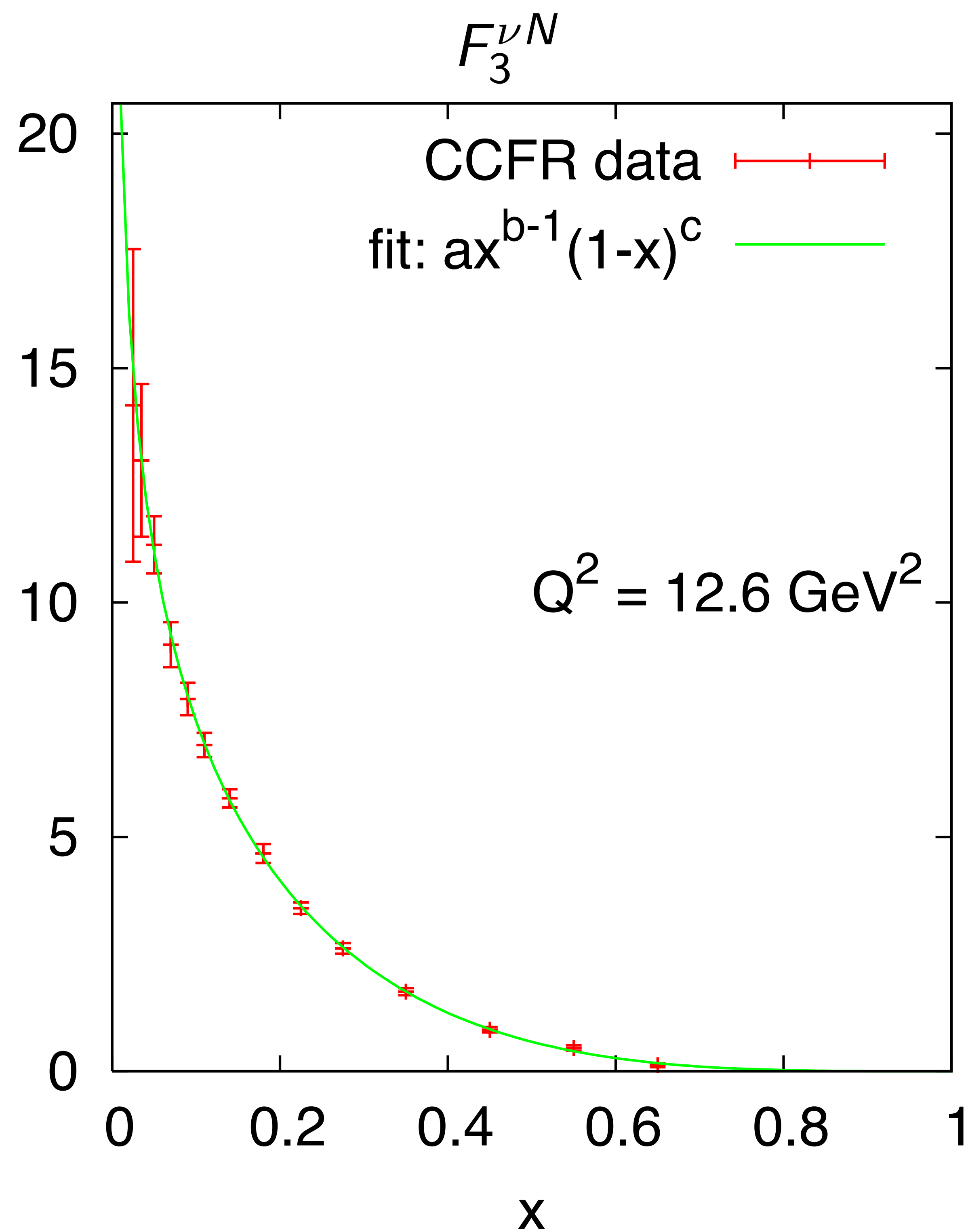
$$F_3^{W^+N} = \frac{1}{2}(F_3^{W^+p} + F_3^{W^+n}) = d_p(x) - \bar{u}_p(x) + d_n(x) - \bar{u}_n(x)$$
$$= d_p(x) - \bar{u}_p(x) + u_p(x) - \bar{d}_p(x)$$
$$= d_V(x) + u_V(x)$$

E.g.: use this to check total number of *valence quarks is 3:*

$$\int dx\, F_3^{W^+N}(x) = \int dx\, (\, d_V(x) + u_V(x)\,) = 3$$

Gross LLewellyn Smith sum rule

[Beware: nucleus is only ≈ sum of protons & neutrons]



- $xF_3^{\nu N} \simeq x(u_V + d_V)$ vanishes for $x \to 0$

  Regge theory: $xu_V, xd_V \sim x^{0.5}$

- $F_3^{\nu N} \simeq u_V + d_V$ should be *integrable*

  ➥ $\int dx\, F_3^{\nu N} = 2.50 \pm 0.08$

  CCFR, $Q^2 = 3$ GeV^2

  We expected 3 (uud)...

QCD corrections

We believe proton really does have 3 valence quarks!

But interaction with W^+ receives *higher order QCD corrections:*

$$\int dx\, F_3^{\nu N} = 3\left(1 - \frac{\alpha_s}{\pi} - 3.25\frac{\alpha_s^2}{\pi^2} - 12.2\frac{\alpha_s^3}{\pi^3} + \cdots\right)$$

$$\simeq 2.52 \qquad [\alpha_s(3\ \text{GeV}^2) \simeq 0.34]$$

Bardeen et al. '78

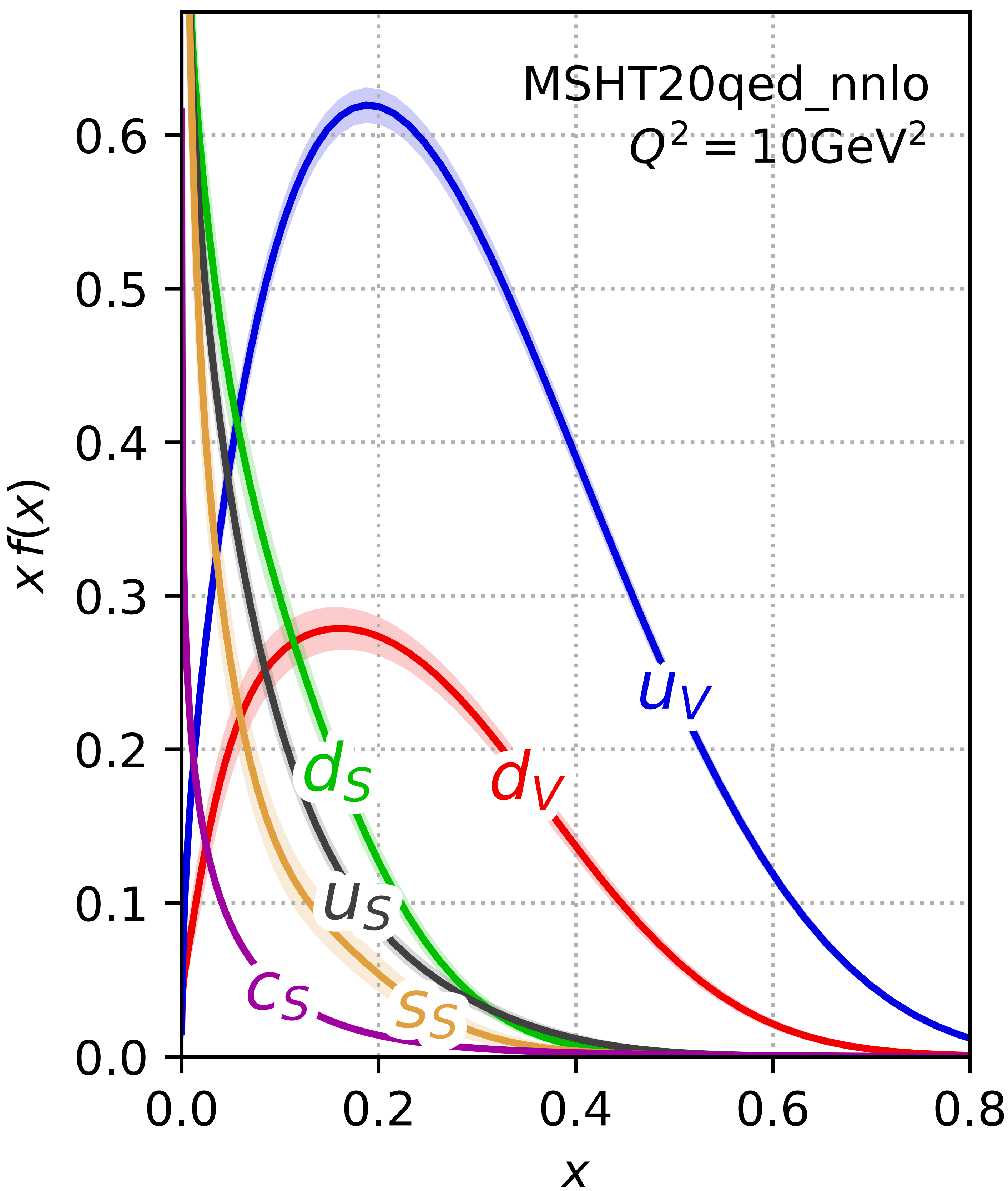
Gorishny & Larin '86

Larin & Vermaseren '91

Reconstruct number of valence quarks from data:

Data $(2.50 \pm 0.08) \Rightarrow 2.98 \pm 0.10$ valence quarks

An overview of quark PDFs



Modern PDF fits use a range of data (see later)

- **valence quarks** ($u_V = u - \bar{u}$) are ***hard***
  $x \to 1 : xq_V(x) \sim (1-x)^n$
  $x \to 0 : xq_V(x) \sim x^{\lambda}$
  with $n \simeq 3$, $\lambda \simeq 0.5$
- **sea quarks**
  ($u_S = 2\bar{u}$, $s_S = s + \bar{s}, \ldots$)
  are ***soft***, i.e. mostly at low x,
  with $n \simeq 7$, $\lambda \simeq -0.2$

(values of n, λ semi-predicted from quark counting rules and Regge theory)

Momentum sum rule

Sum of momentum carried in all constituents should be 1:

$$\sum_i \int_0^1 dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.104
u_V	0.262
d_S	0.078
u_S	0.064
s_S	0.046
c_S	0.015
total	0.570

Where is missing momentum?

Only parton we've neglected so far is the

gluon

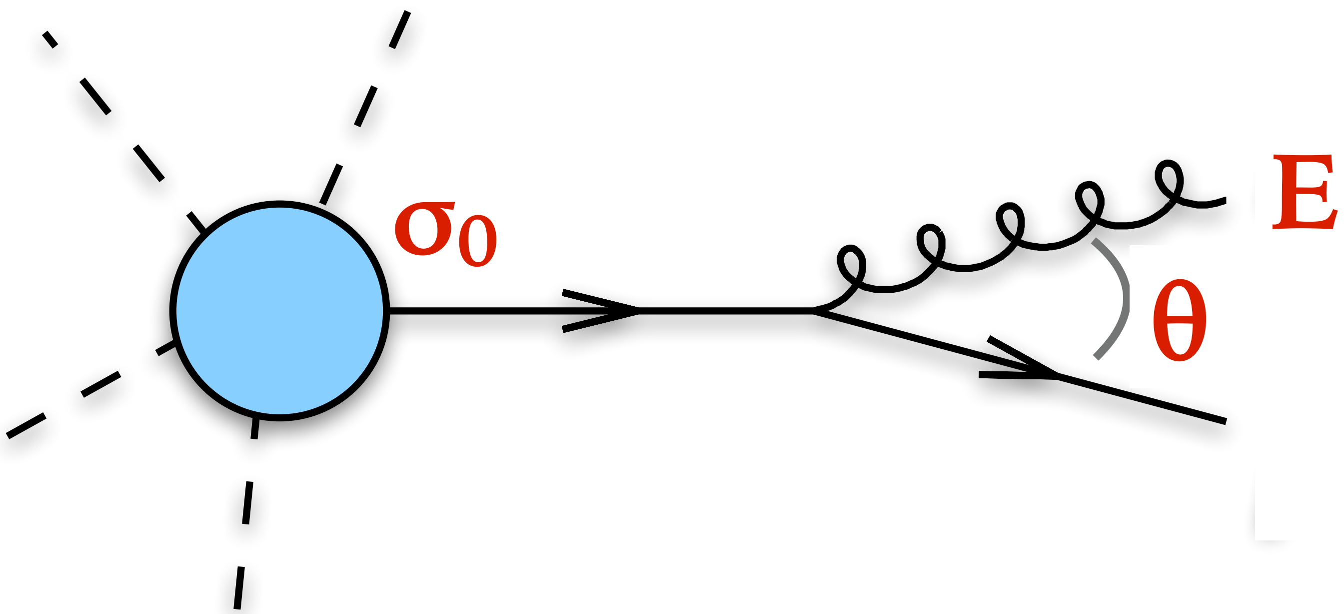
Not directly probed by photon or $W^{\pm}$
Approaches to measuring it include

1) at hadron colliders (e.g. jets)
2) through DGLAP evolution equations

DGLAP evolution

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation

GLUON EMISSION FROM A QUARK



Consider an emission with

- energy $E \ll \sqrt{s}$ ("soft")
- angle $\theta \ll 1$ ("collinear" wrt quark)

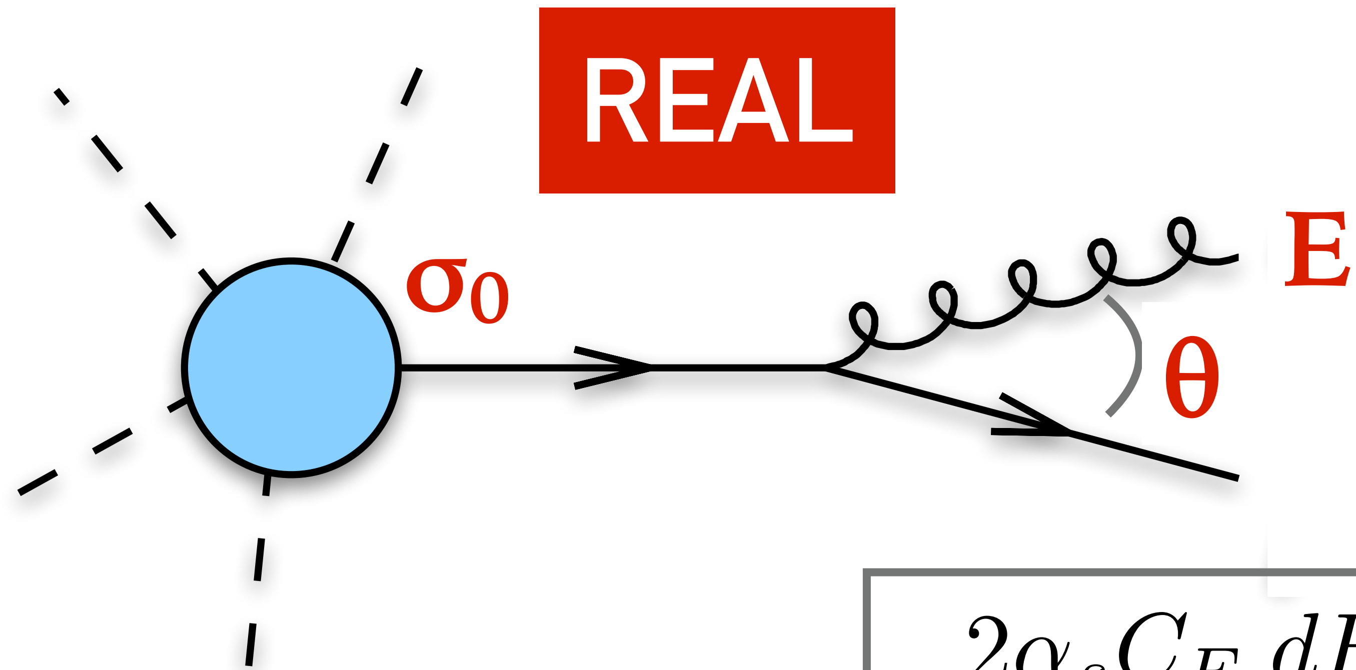
Examine correction to some hard process with cross section σ_0

$$d\sigma \simeq \sigma_0 \times \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$$

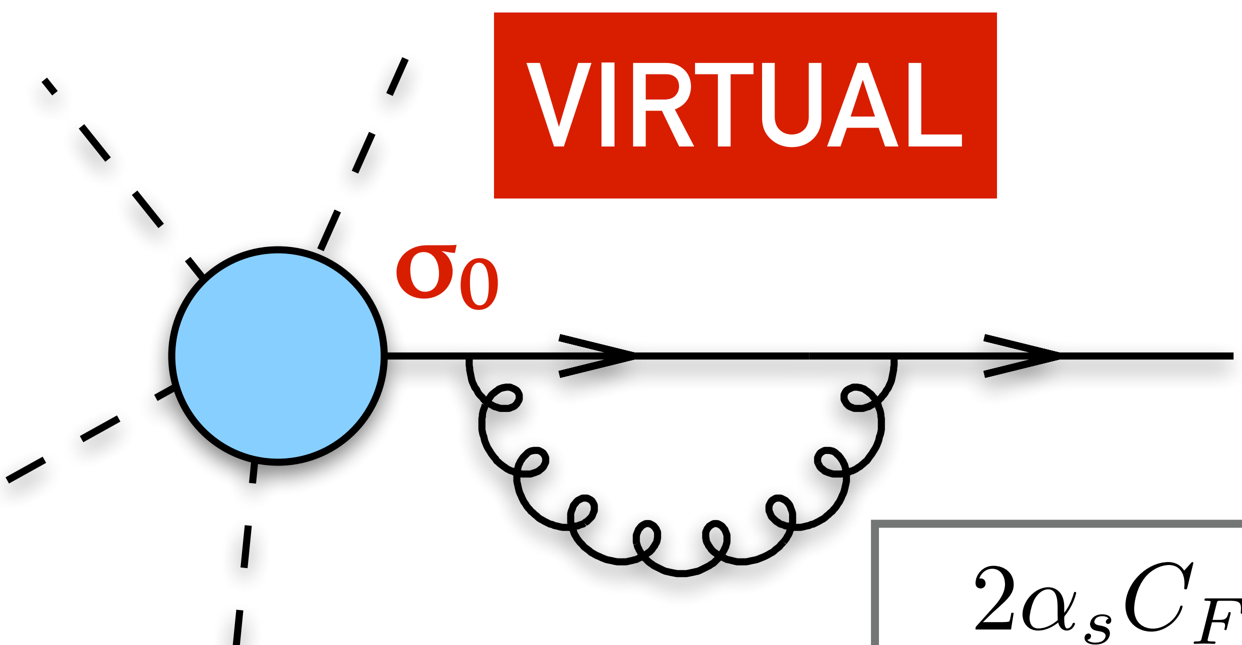
This has a divergence when E→0 or θ→0

[in some sense because of quark propagator going on-shell]

How come we get finite cross sections?



$$+\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$



$$-\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$

Divergences are present in both real and virtual diagrams.

If you are “**inclusive**”, i.e. your measurement doesn’t care whether a soft/collinear gluon has been emitted then the **real and virtual divergences cancel.**

Beyond inclusive cross sections: infrared and collinear (IRC) safety

For an observable's distribution to be calculable in [fixed-order] perturbation theory, the observable should be infra-red safe, i.e. insensitive to the emission of soft or collinear gluons. In particular if $\vec{p}_i$ is any momentum occurring in its definition, it must be invariant under the branching

$$\vec{p}_i \to \vec{p}_j + \vec{p}_k$$

whenever $\vec{p}_j$ and $\vec{p}_k$ are parallel [collinear] or one of them is small [infrared].

[QCD and Collider Physics (Ellis, Stirling & Webber)]

<u>Examples</u>

Multiplicity of gluons is not IRC safe

[modified by soft/collinear splitting]

Energy of hardest particle is not IRC safe

[modified by collinear splitting]

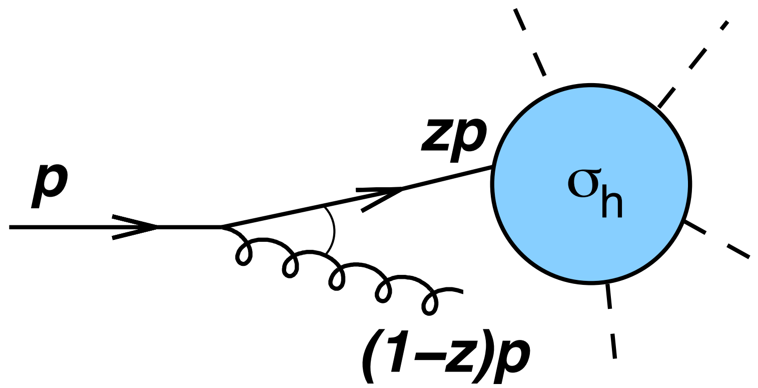
Energy flow into a cone is IRC safe

[soft emissions don't change energy flow,
collinear emissions don't change its direction]

Higher order corrections from initial state splittings?

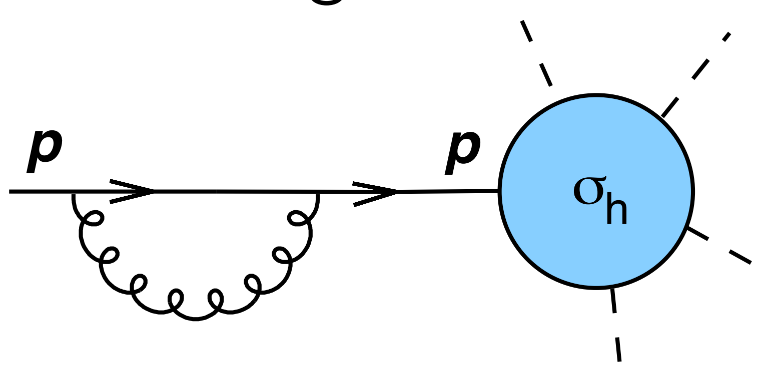
For initial state splitting, hard process occurs *after splitting*, and momentum entering hard process is modified: $p \to zp$.

$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



For virtual terms, momentum entering hard process is unchanged

$$\sigma_{V+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



Total cross section gets contribution with *two different hard X-sections*

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi}\int\frac{dk_t^2}{k_t^2}\frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]$$

NB: We assume σ_h involves momentum transfers $\sim Q \gg k_t$, so ignore extra transverse momentum in σ_h

Higher order corrections from initial state splittings?

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int \frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.
Cross section with incoming parton is not collinear safe!

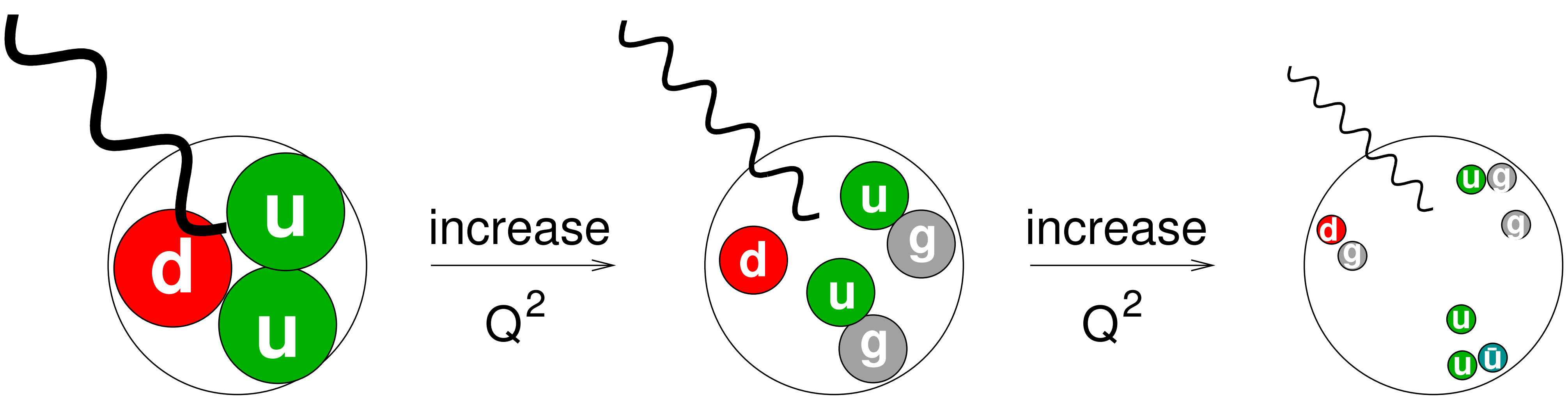
This always happens with coloured initial-state particles
So how do we do QCD calculations in such cases?

Parton distributions and DGLAP

- Write up-quark distribution in proton as

$$u(x, \mu_F^2)$$

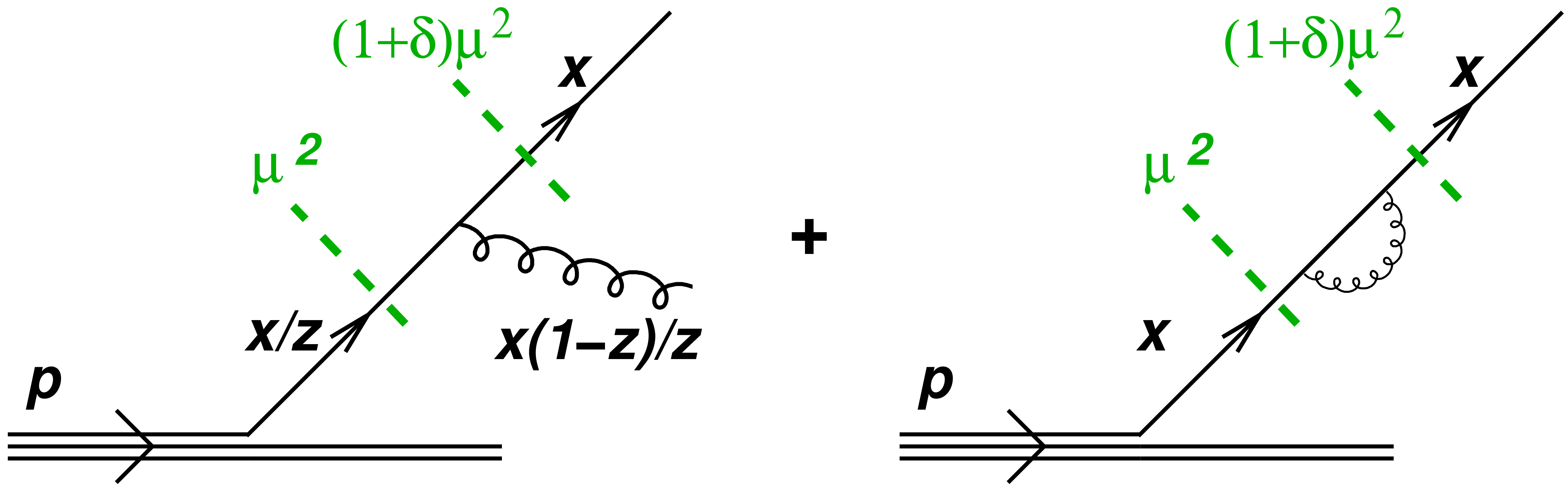
- Perturbative collinear (IR) divergence absorbed into the parton distribution (NB divergence not physical: non-perturbative physics provides a physical cutoff)
- μ_F is the **factorisation scale** — a bit like the renormalisation scale (μ_R) for the running coupling.
- As you vary the factorisation scale, the parton distributions evolve with a renormalisation-group type equation



Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations

DGLAP EQUATION

take derivative wrt factorization scale μ^2



$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz\, p_{qq}(z)\, \frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu^2)$$

p_{qq} is real $q \leftarrow q$ **splitting kernel:** $p_{qq}(z) = C_F \frac{1+z^2}{1-z}$

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\underbrace{\int_x^1 dz\, P_{qq}(z)\,\frac{q(x/z,\mu^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

This involves the *plus prescription:*

$$\int_x^1 dz\,[g(z)]_+ f(z) = \int_x^1 dz\, g(z) f(z) - \int_0^1 dz\, g(z) f(1)$$

$z=1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z=1$

DGLAP EQUATION

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

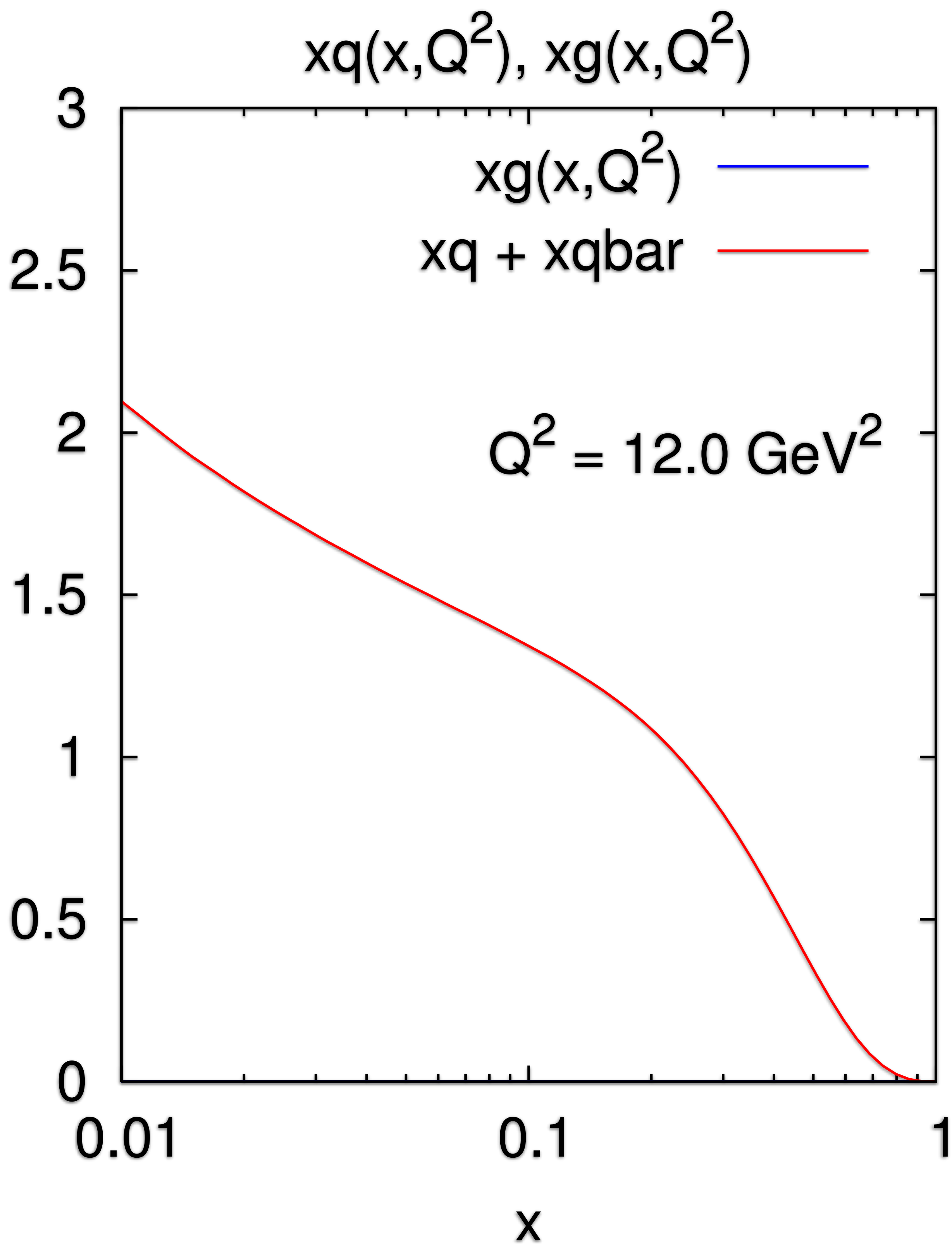
$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- P_{qg}, P_{gg}: *symmetric $z \leftrightarrow 1-z$* (except virtuals)
- P_{qq}, P_{gg}: *diverge for $z \to 1$* soft gluon emission
- P_{gg}, P_{gq}: *diverge for $z \to 0$* Implies PDFs grow for $x \to 0$

2015 EPS HEP prize to Bjorken, Altarelli, Dokshitzer, Lipatov & Parisi

DGLAP evolution (initial quarks only)



Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

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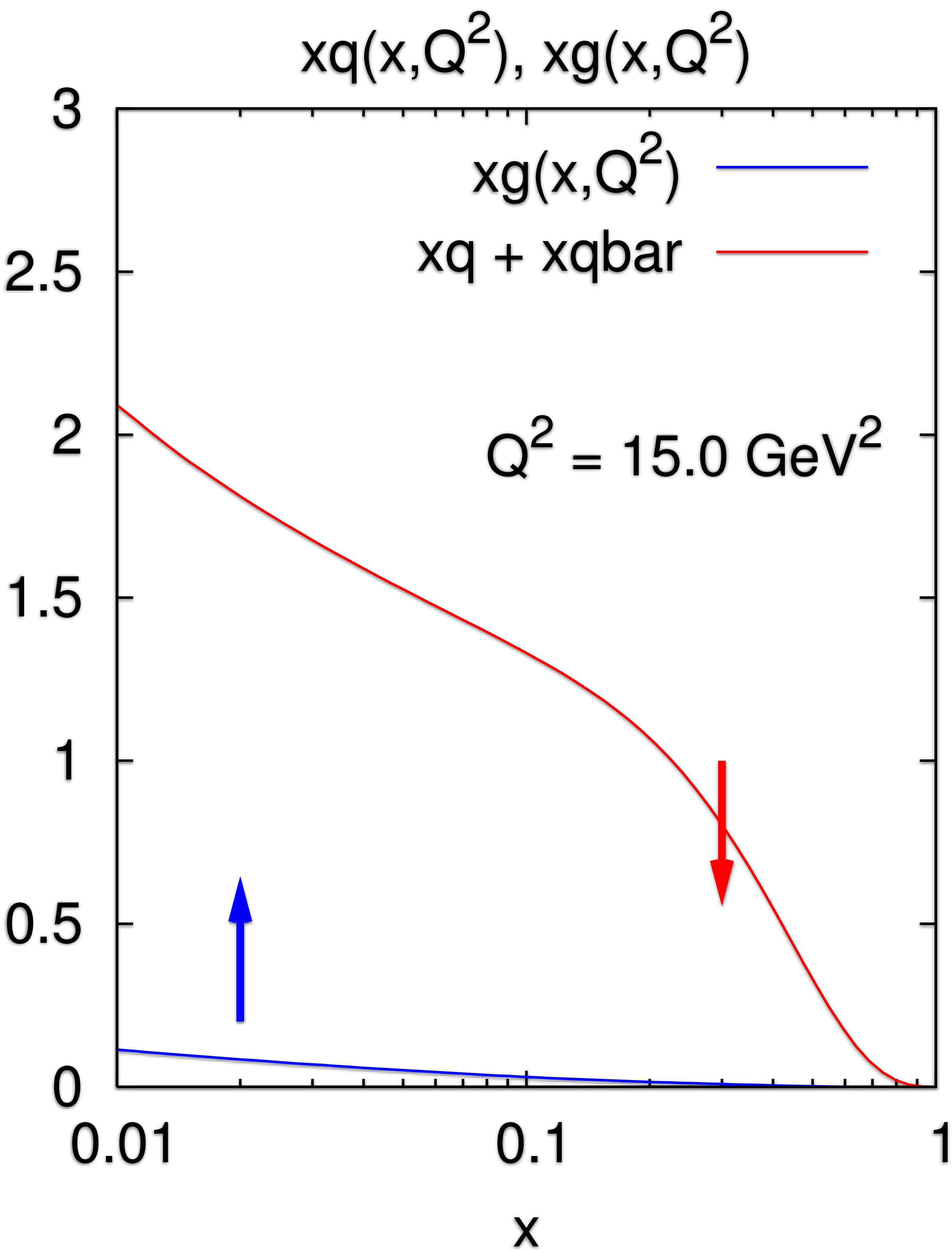

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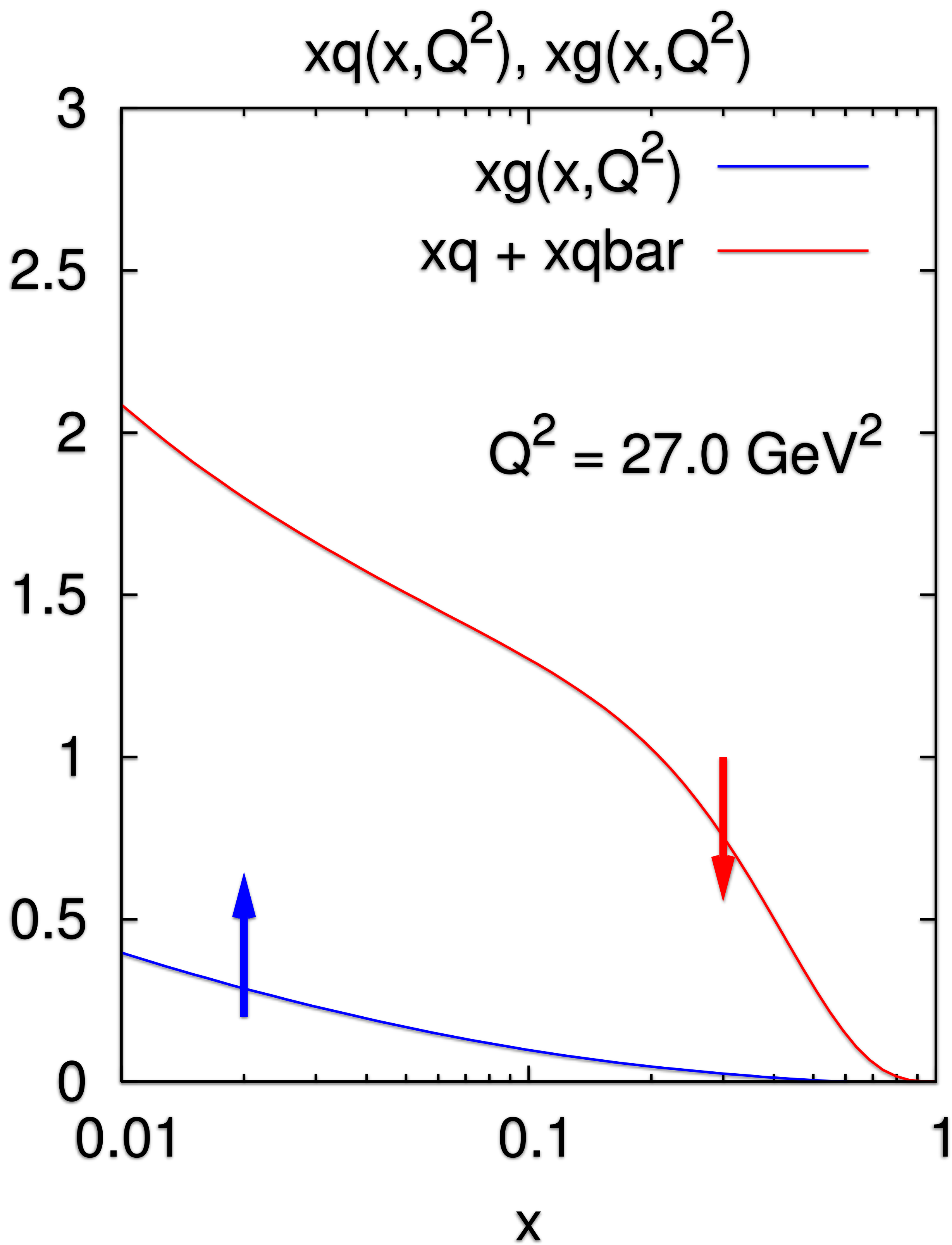

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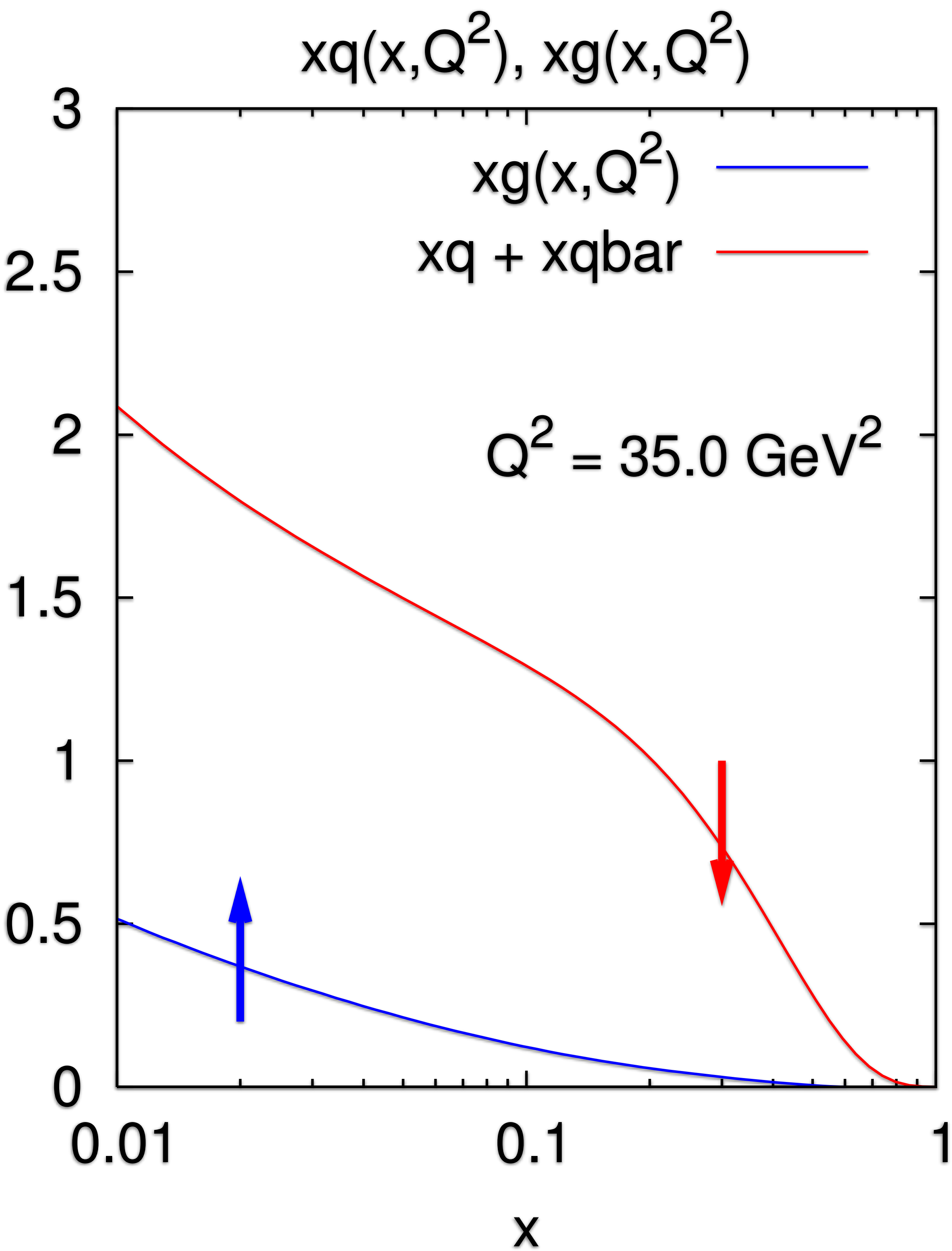

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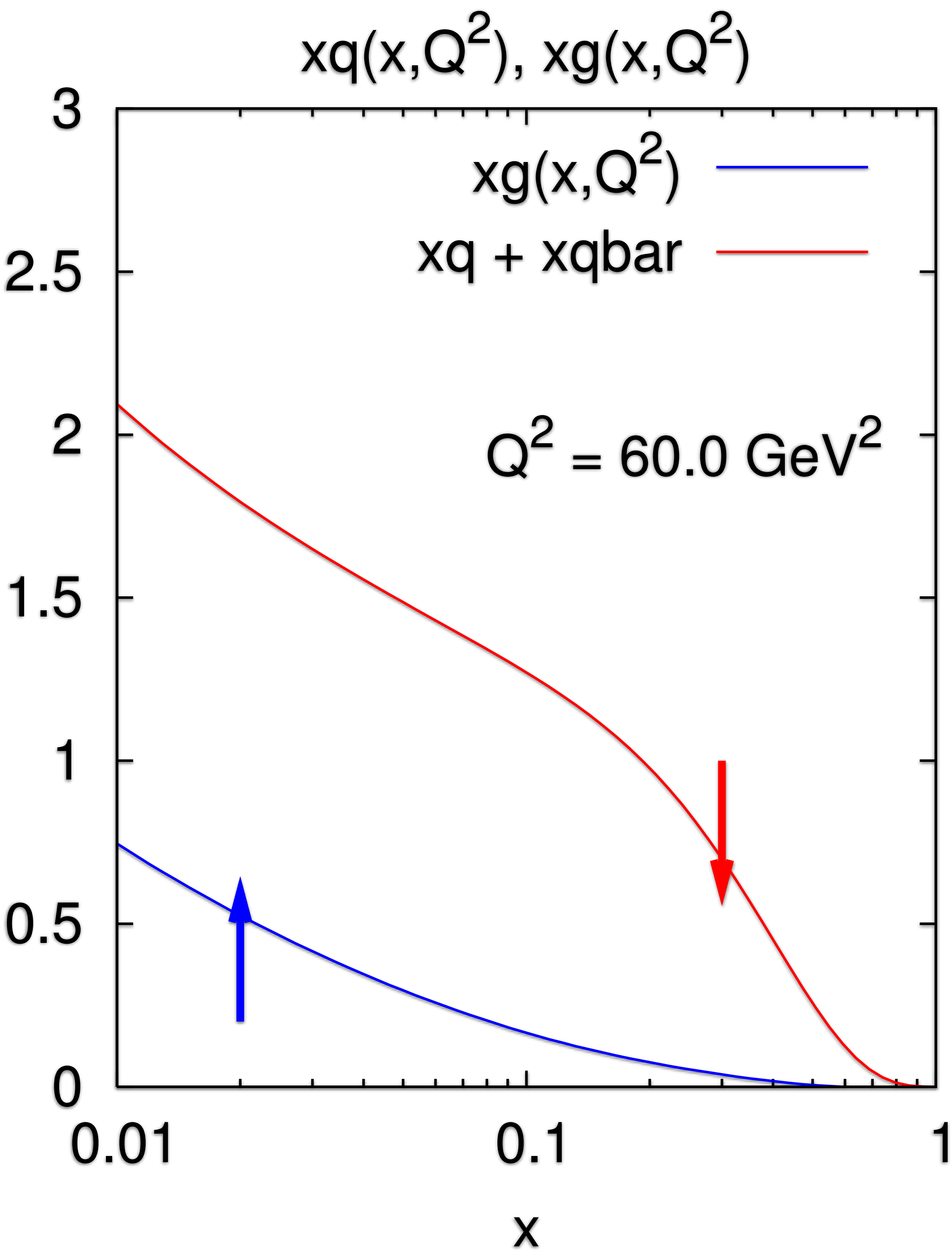

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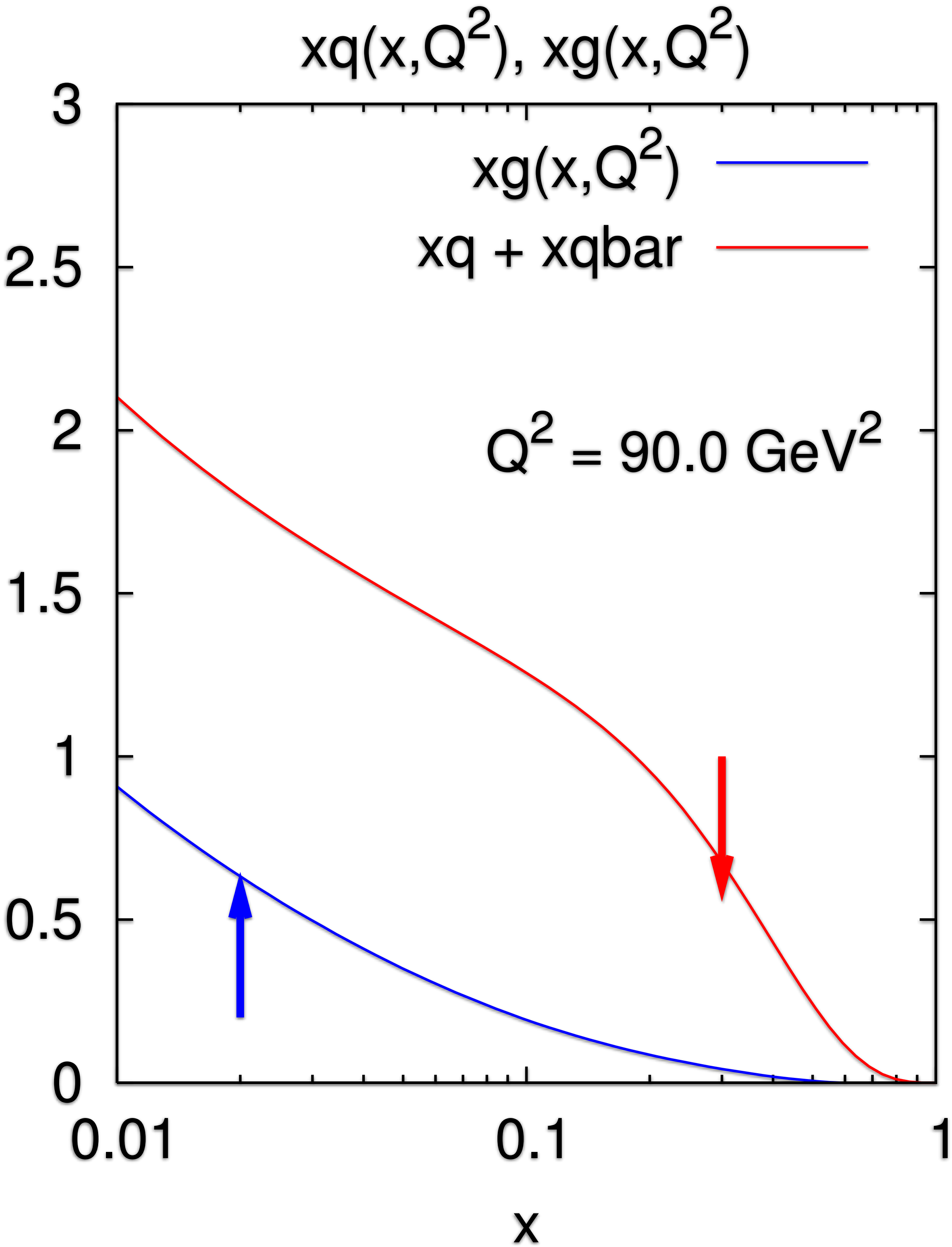

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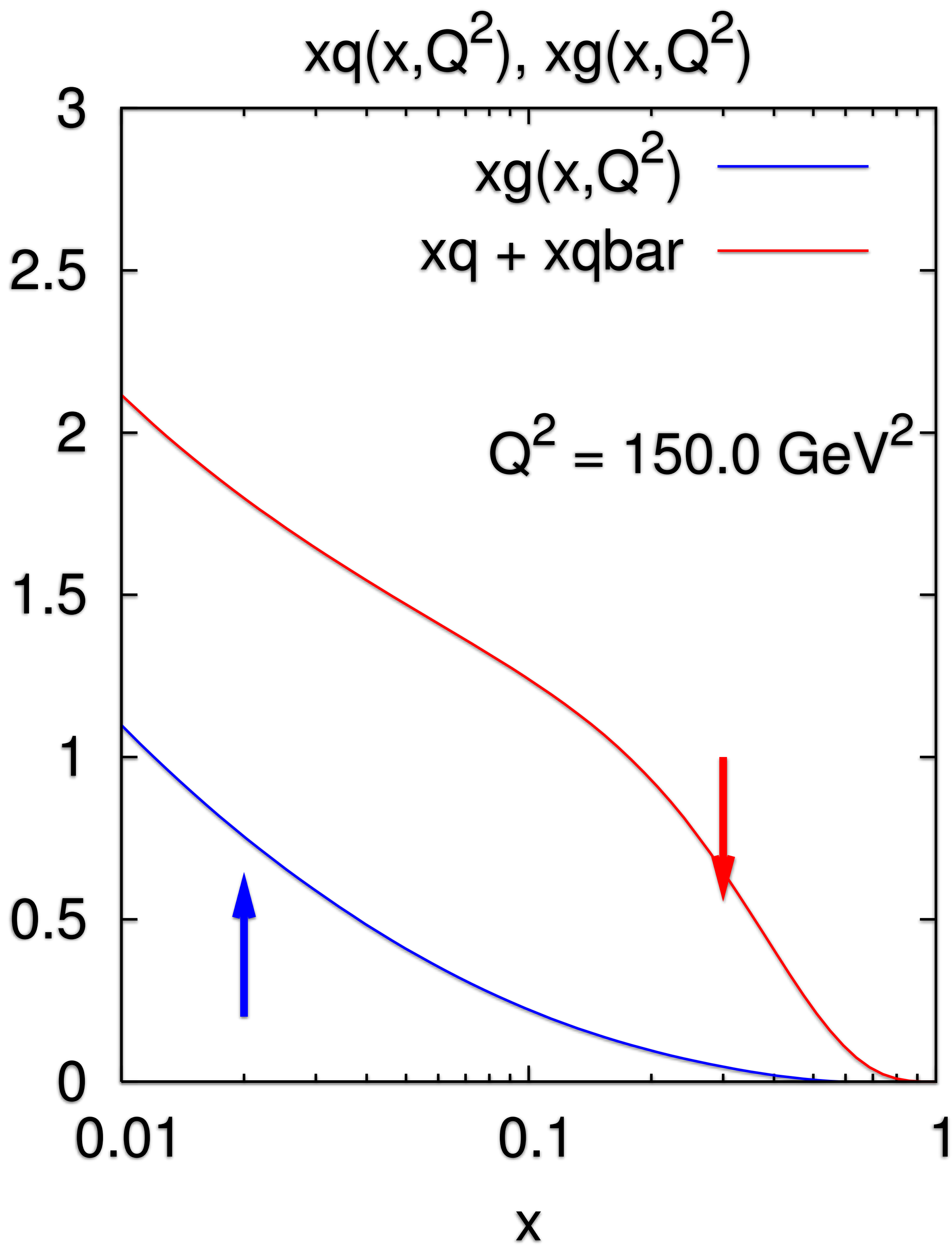

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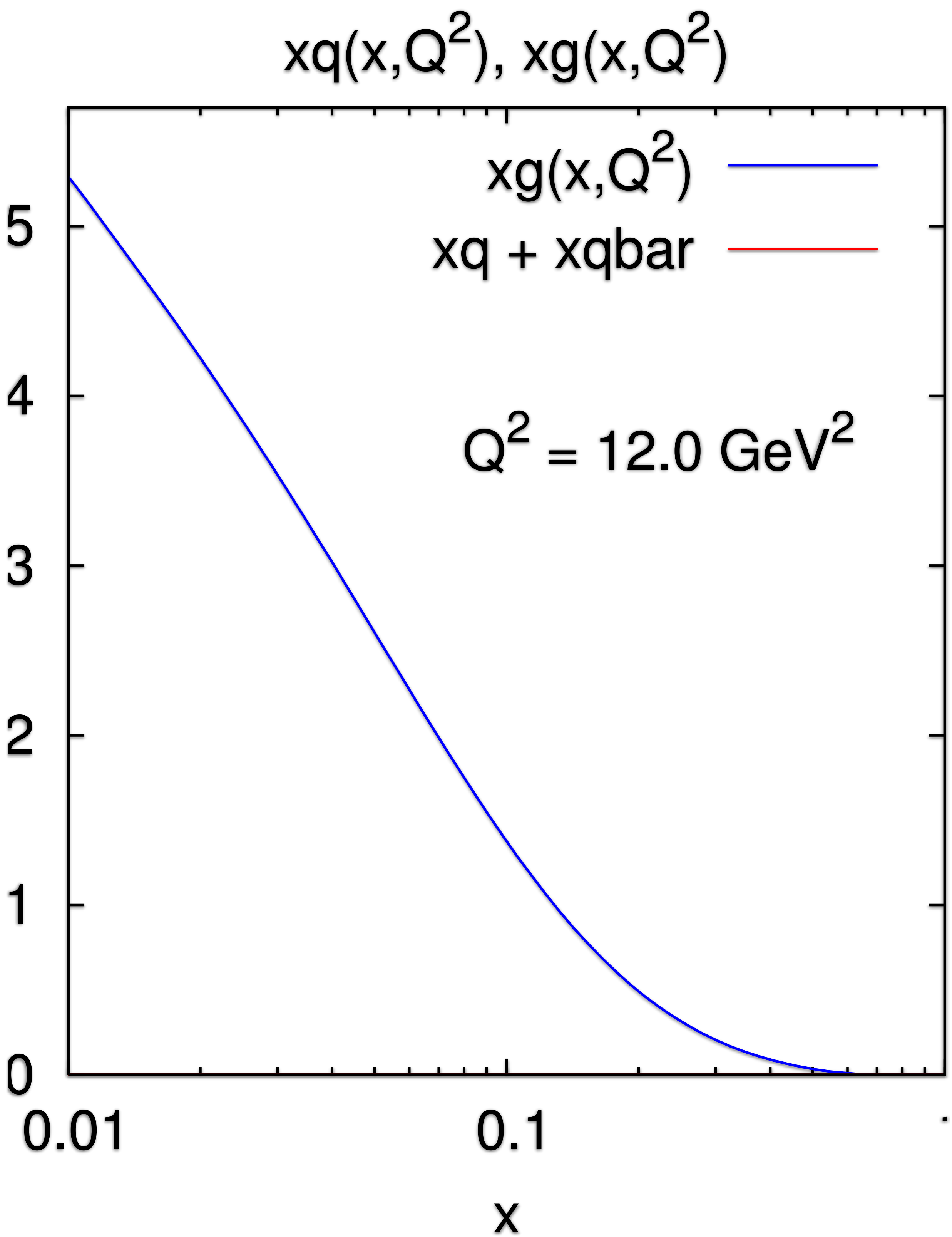
DGLAP evolution (initial gluons only)

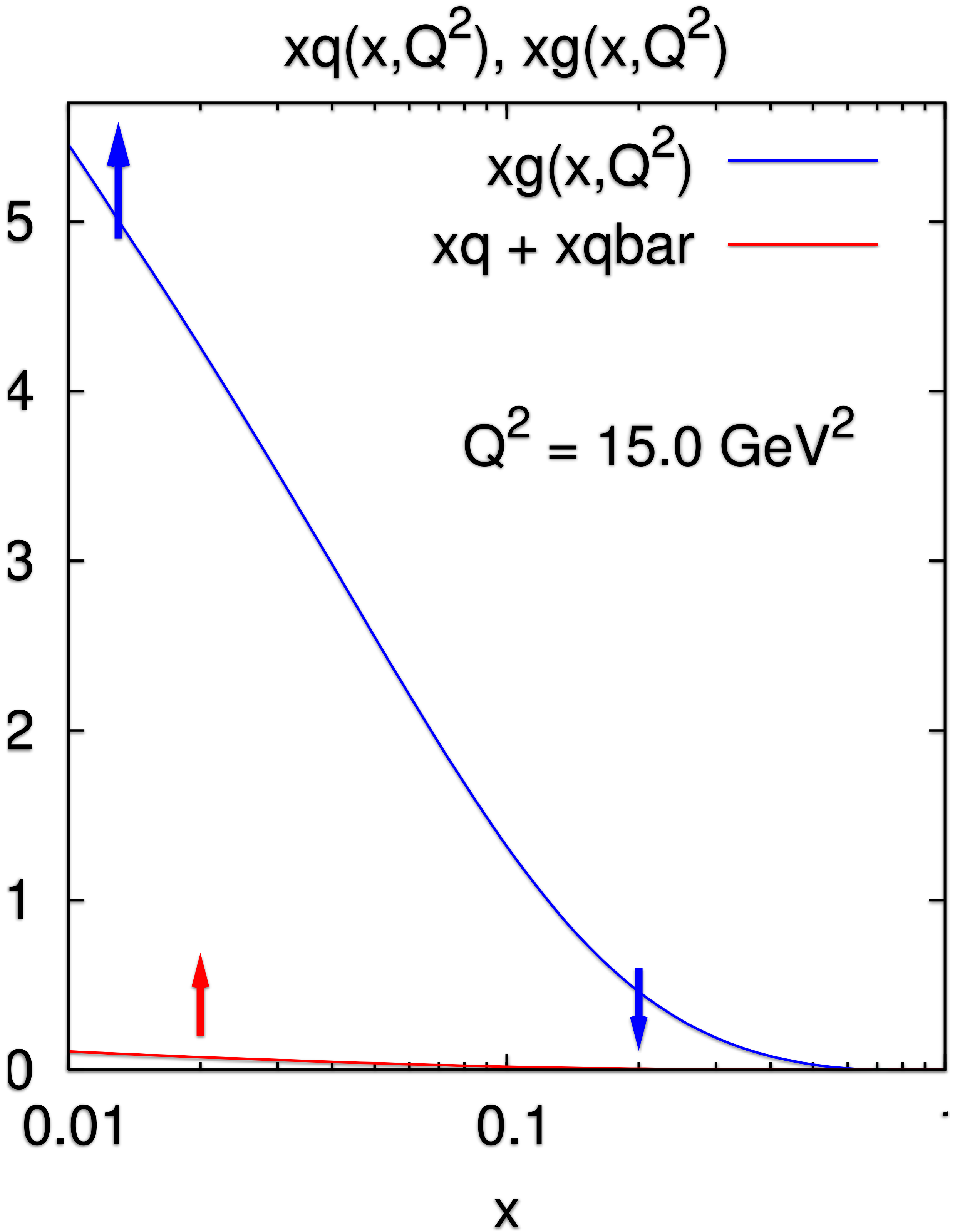


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

DGLAP evolution (initial gluons only)



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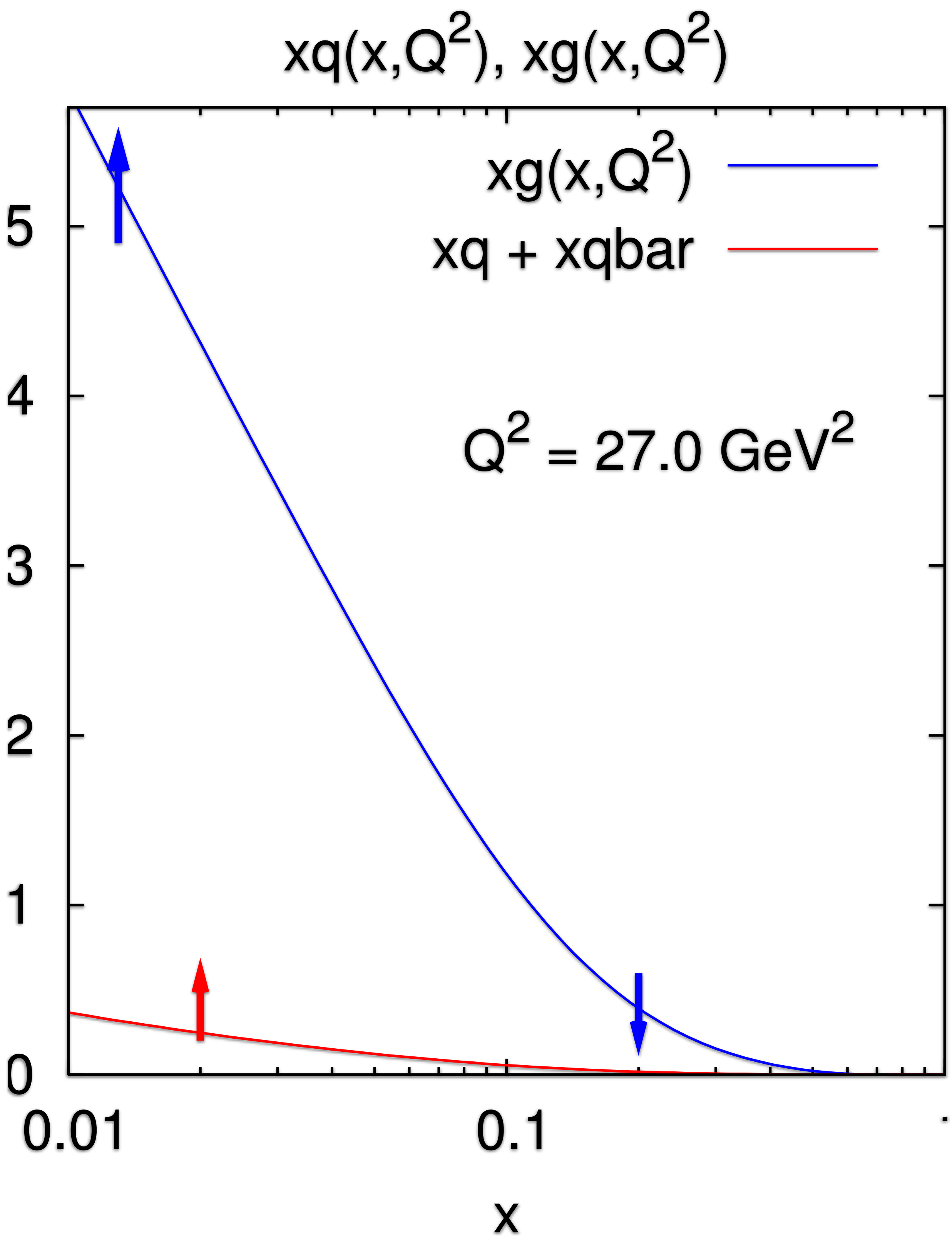

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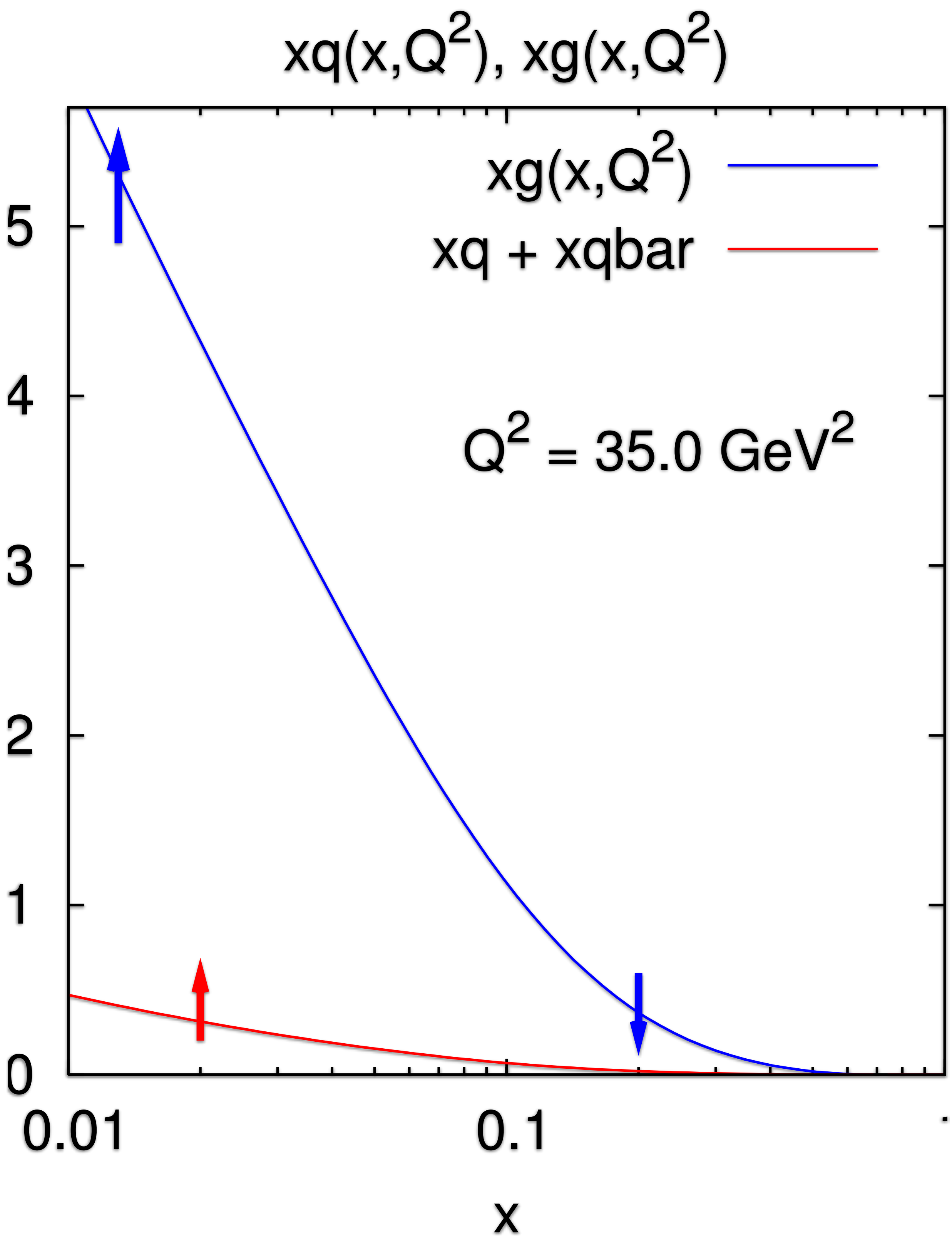

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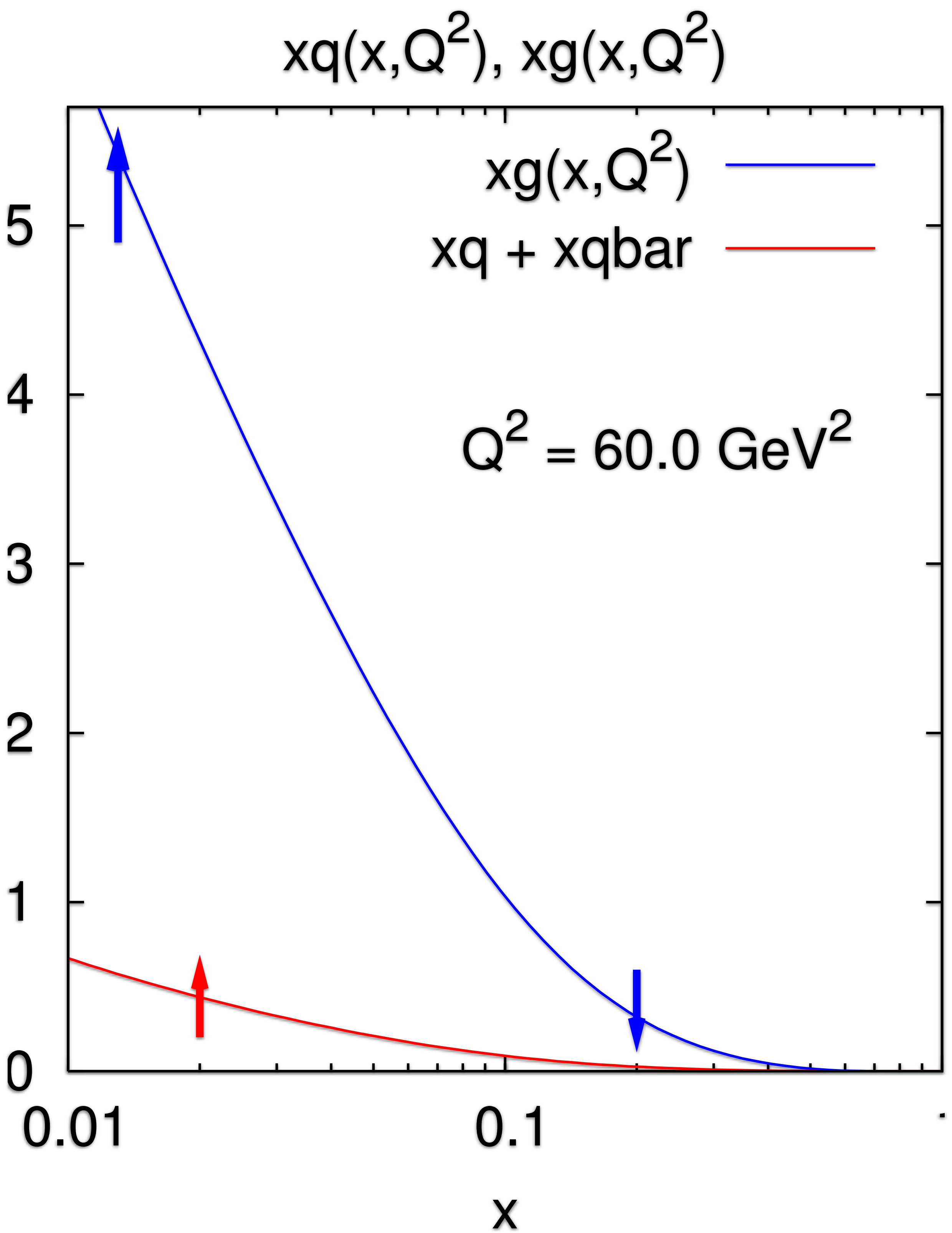

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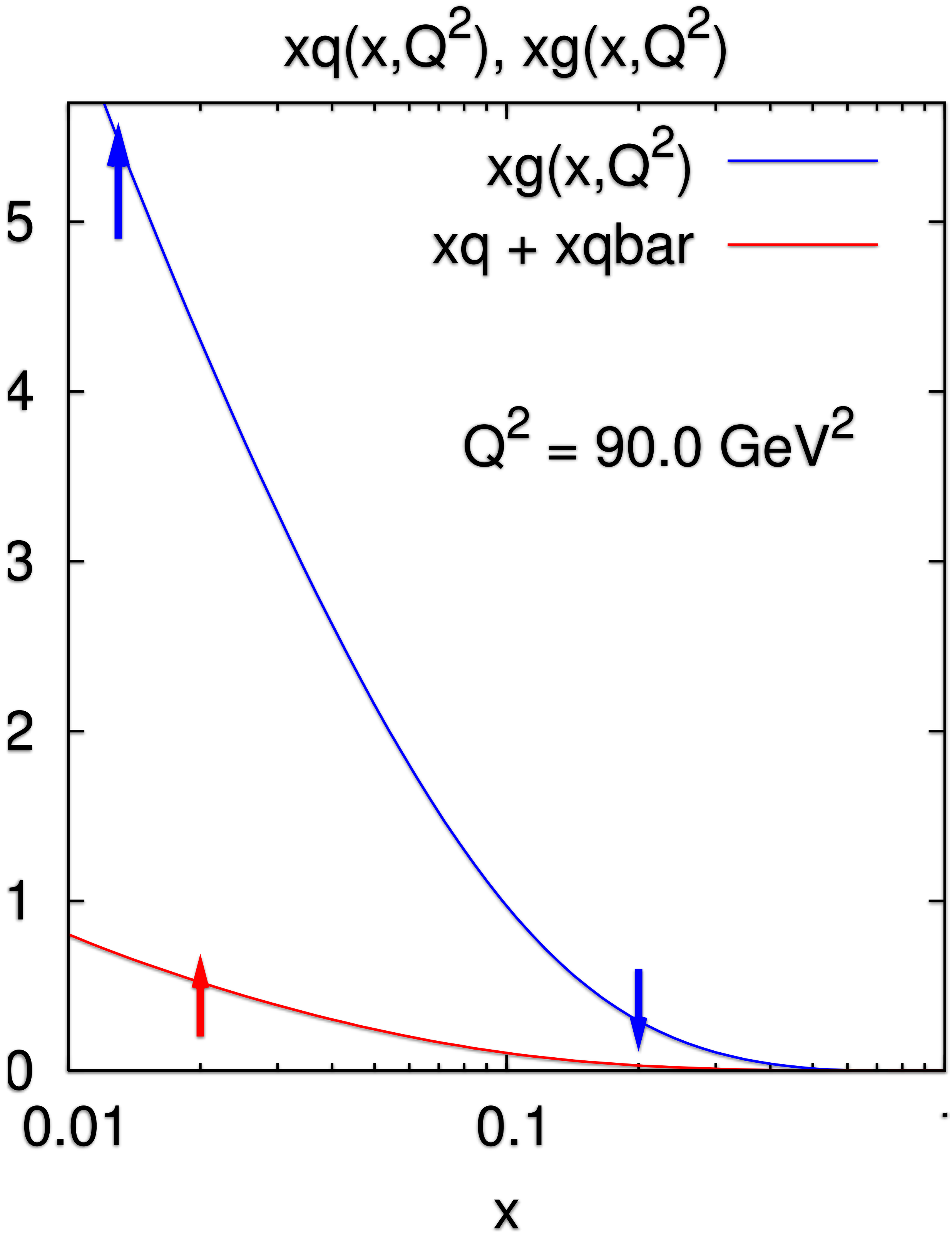

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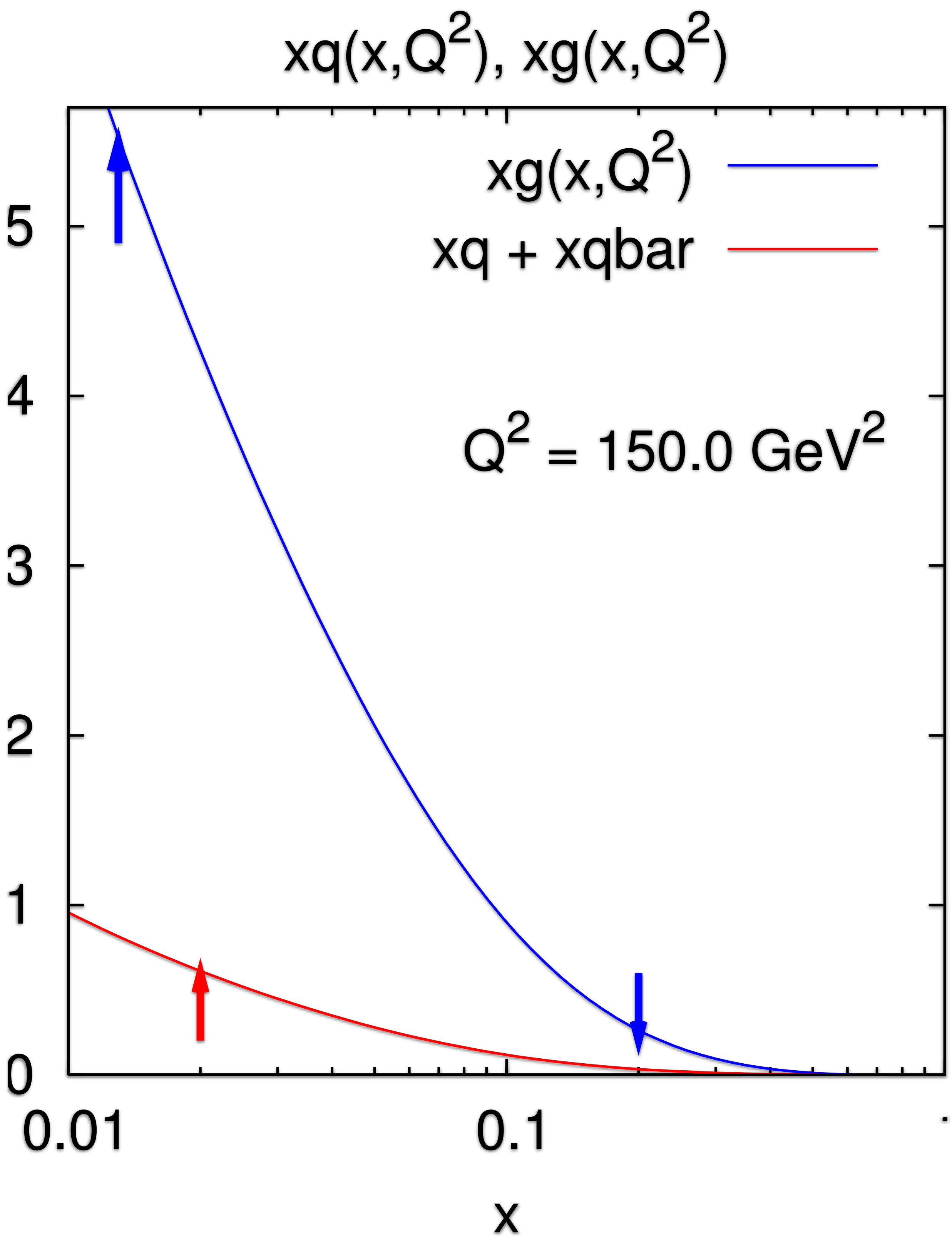

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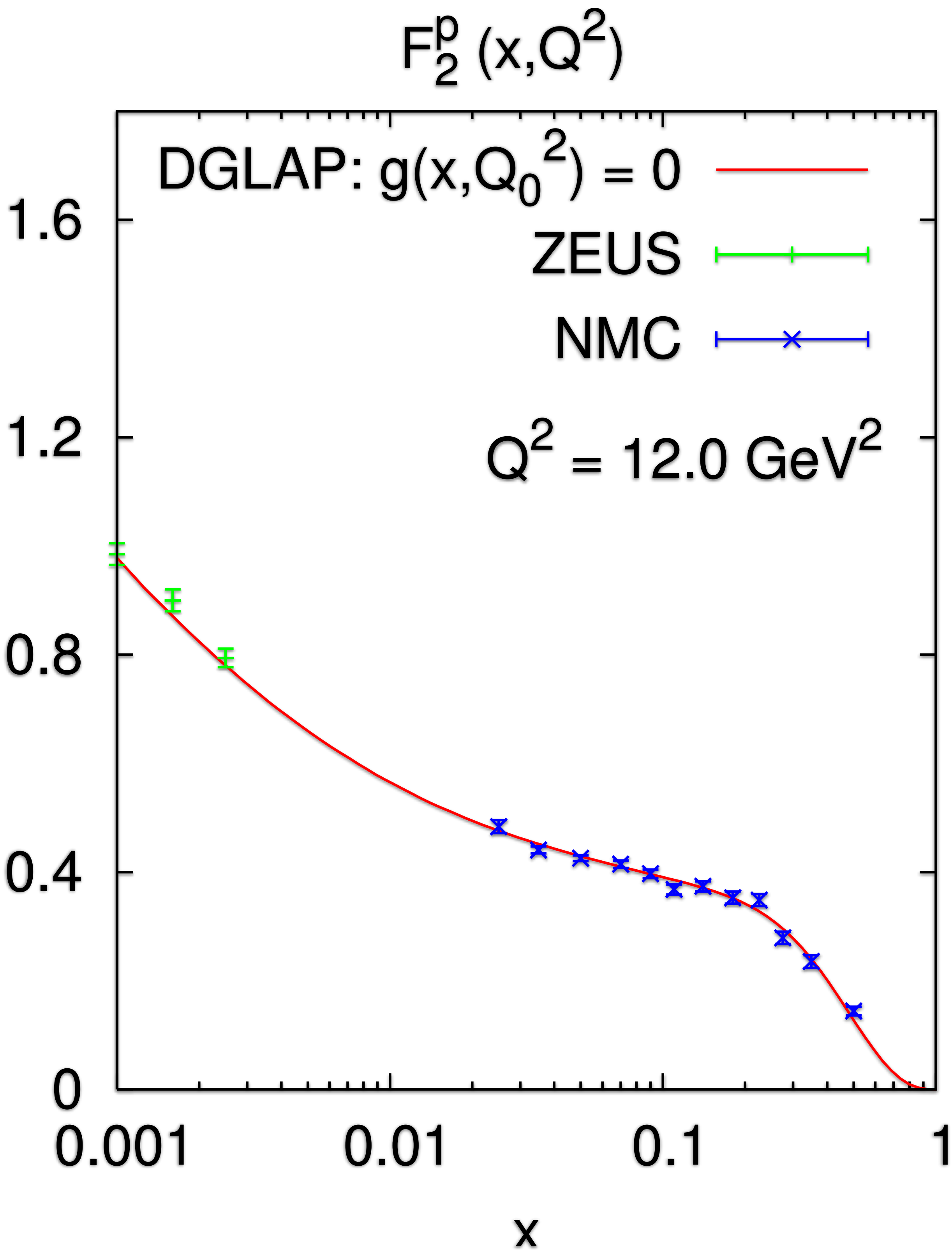
DGLAP evolution:

- ➤ partons lose momentum and shift towards smaller x
- ➤ high-x partons drive growth of low-x gluon

determining the gluon

which is critical at hadron colliders (e.g. ttbar, Higgs dominantly produced by gluon-gluon fusion), but not directly probed in Deep-Inelastic-Scattering

Consider DIS data — $F_2(x,Q^2)$ — in a world where the proton just had quarks



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\ \text{GeV}^2$.

NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

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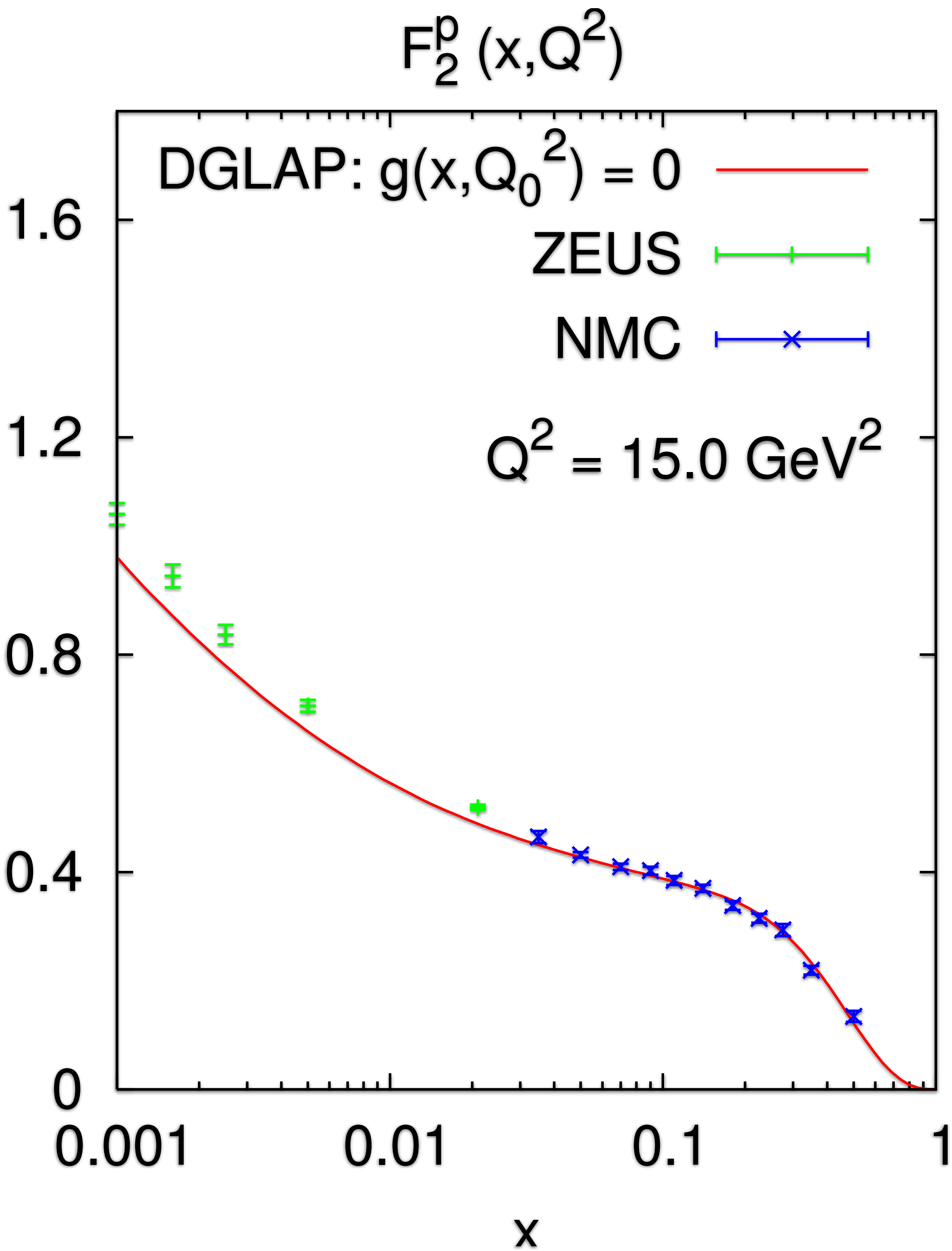

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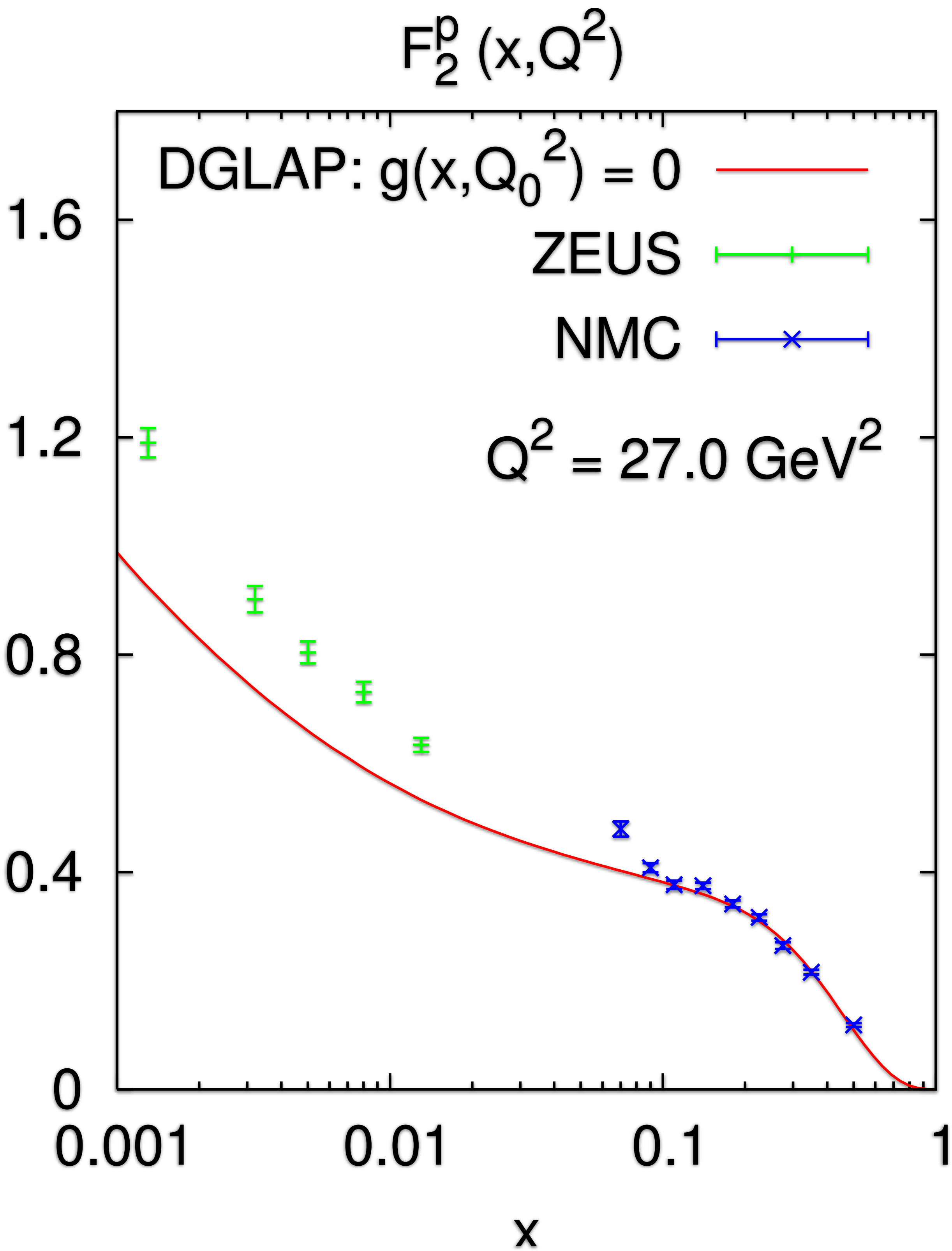

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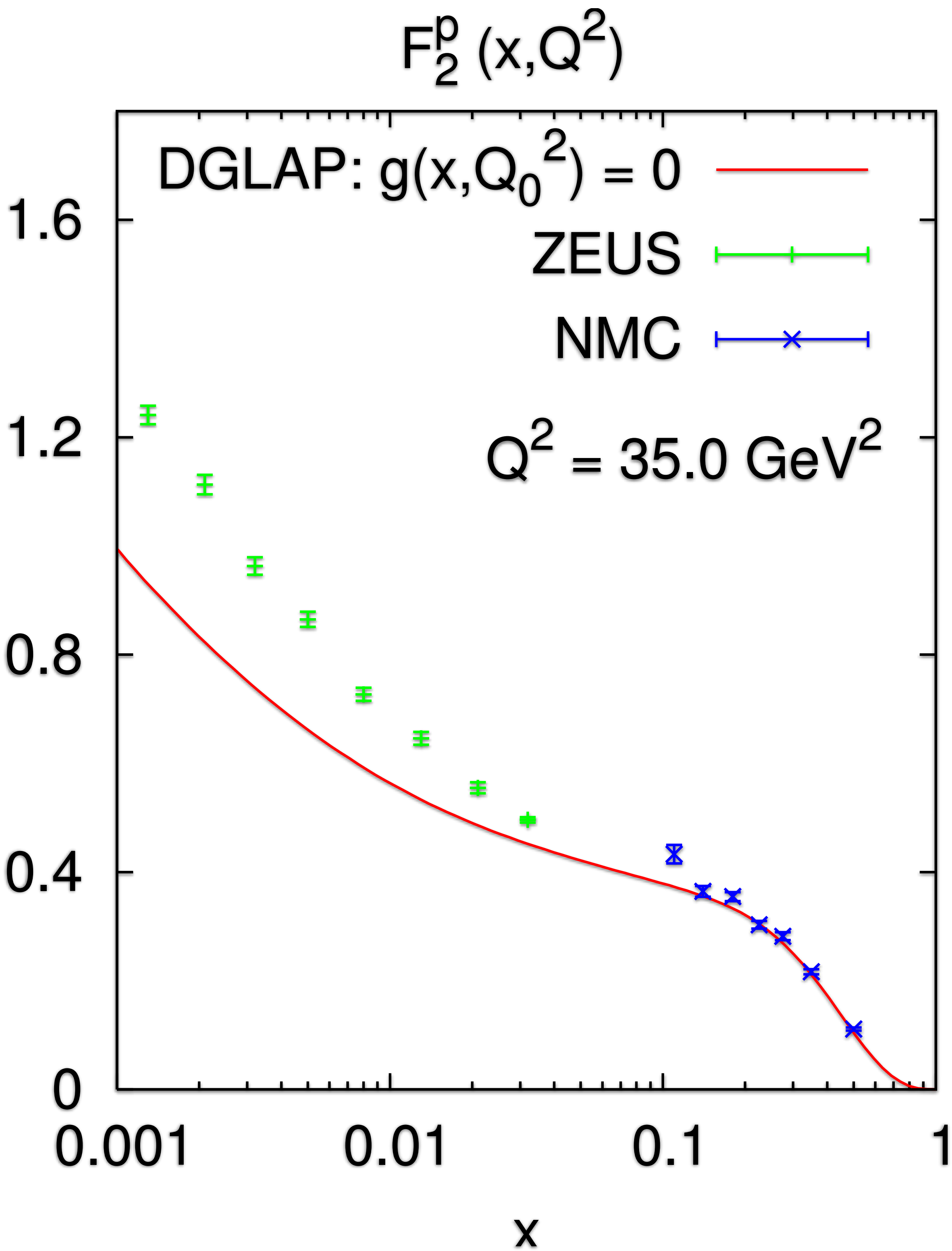

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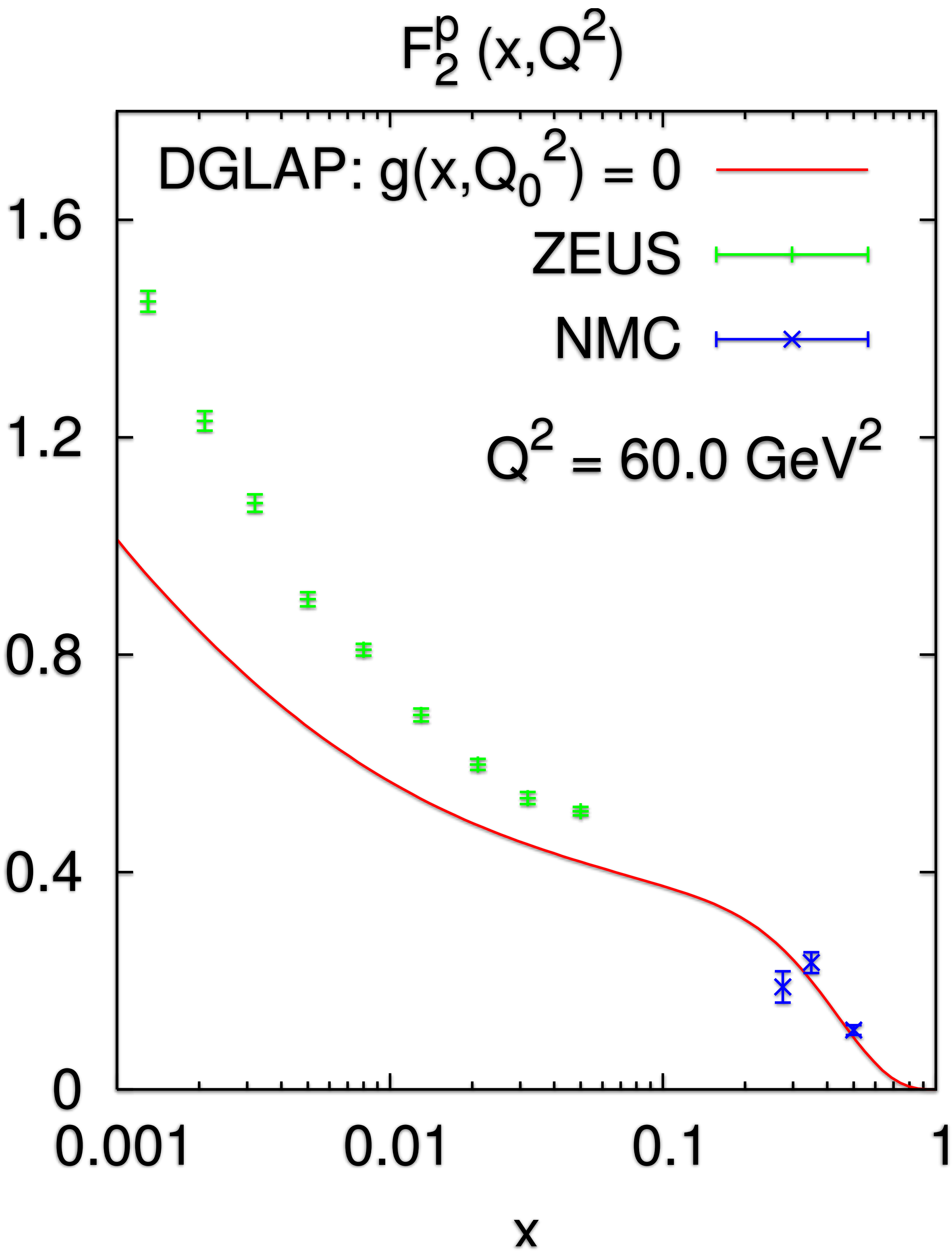

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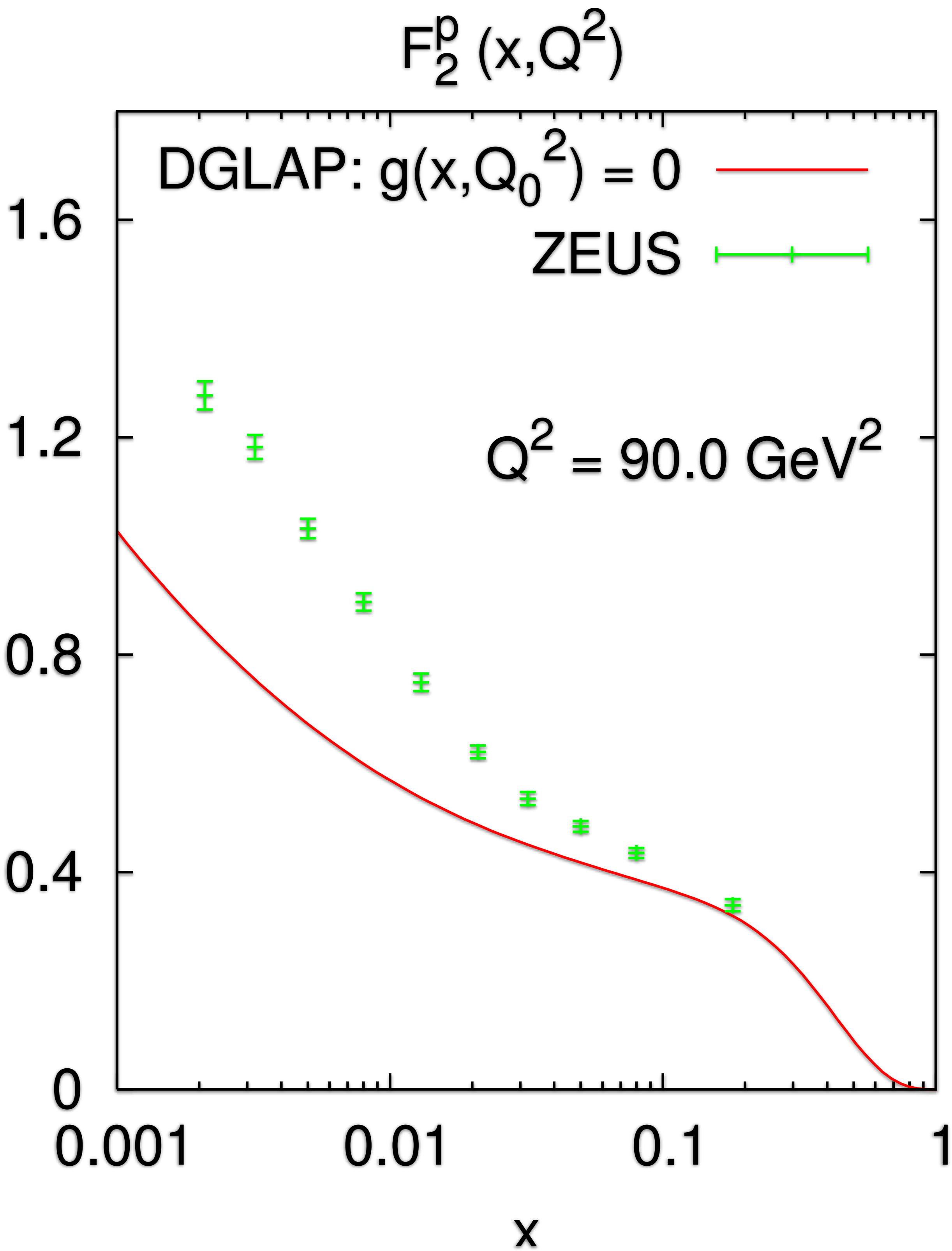

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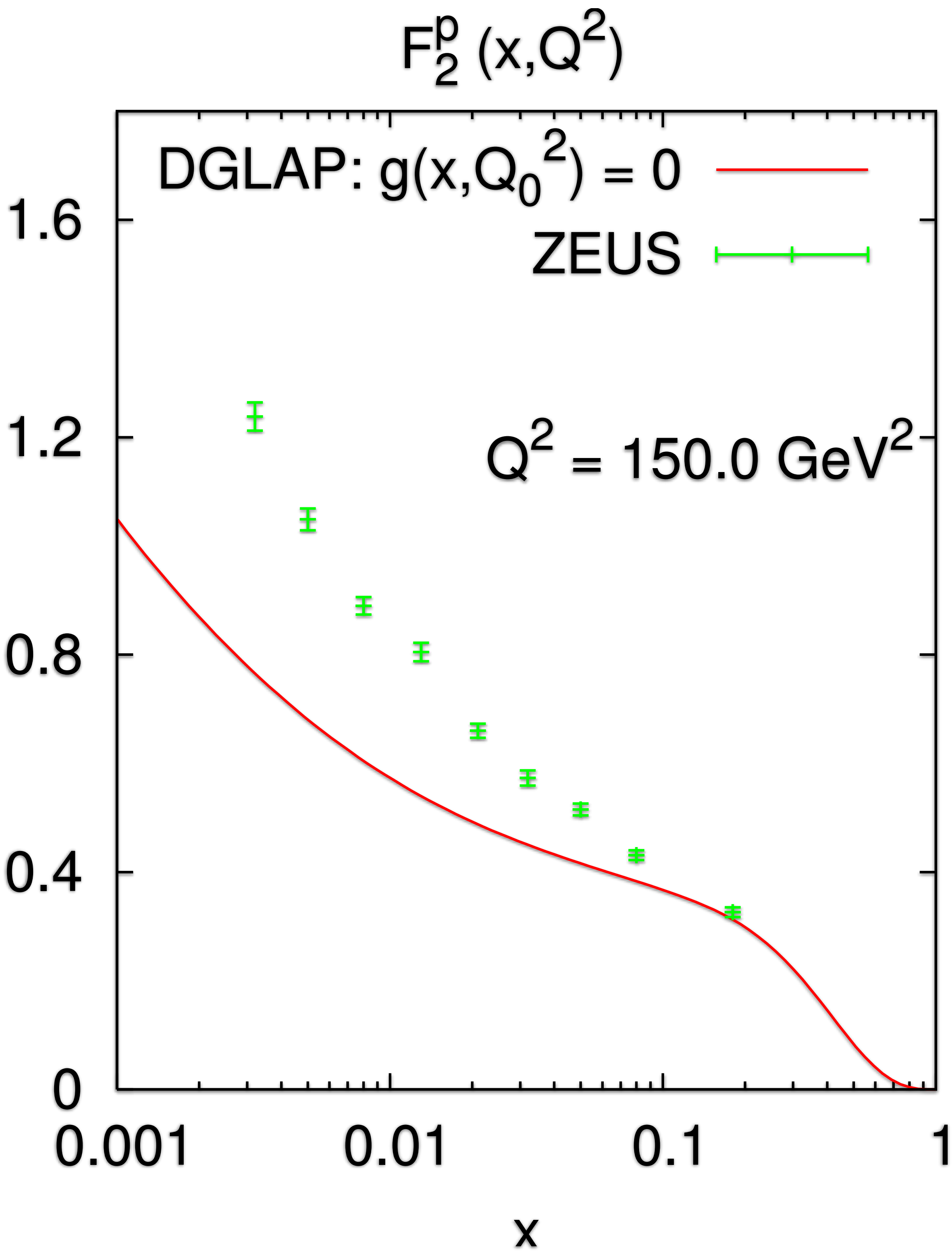

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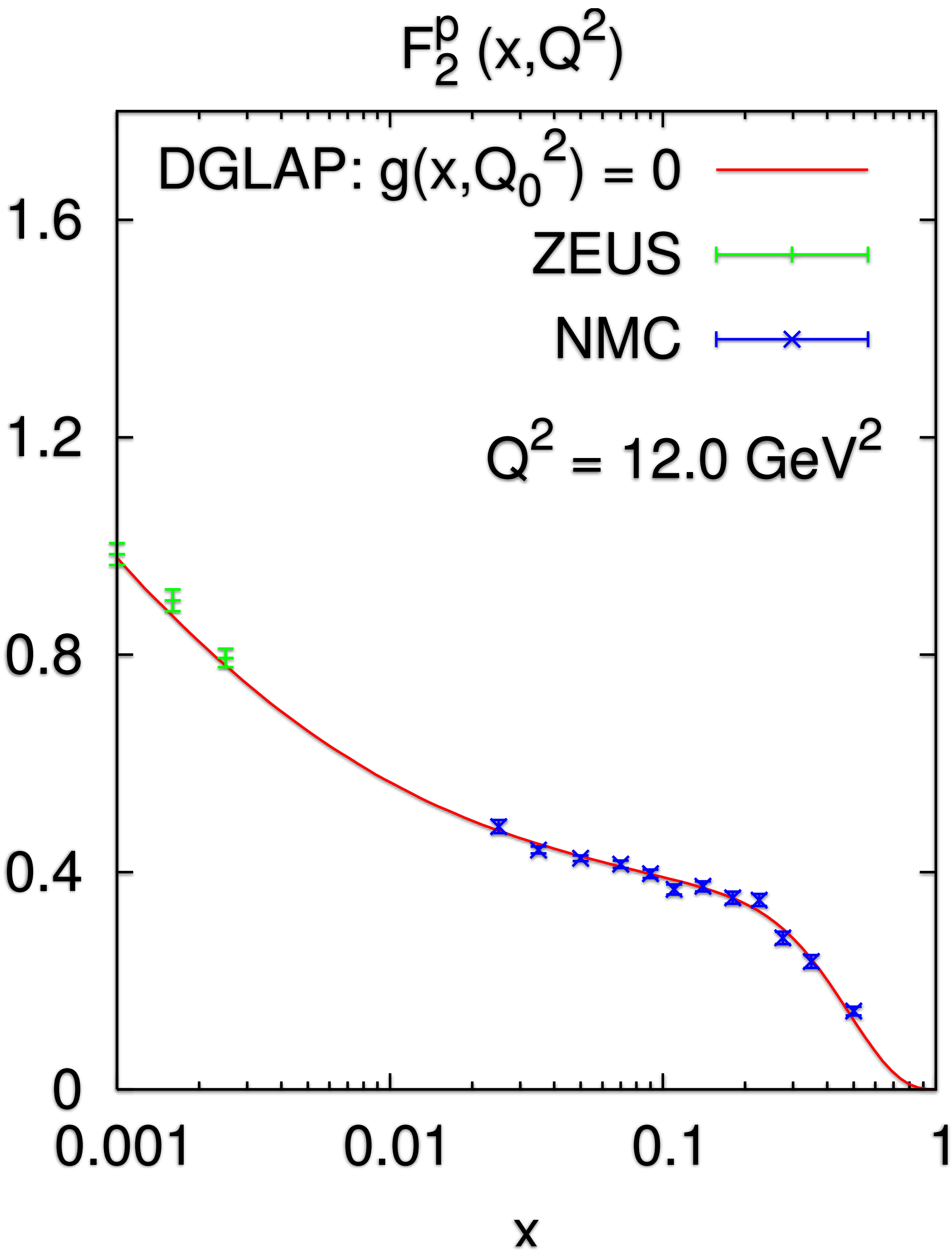
COMPLETE FAILURE
to reproduce data evolution

Consider DIS data — $F_2(x,Q^2)$ — with specially tuned gluon



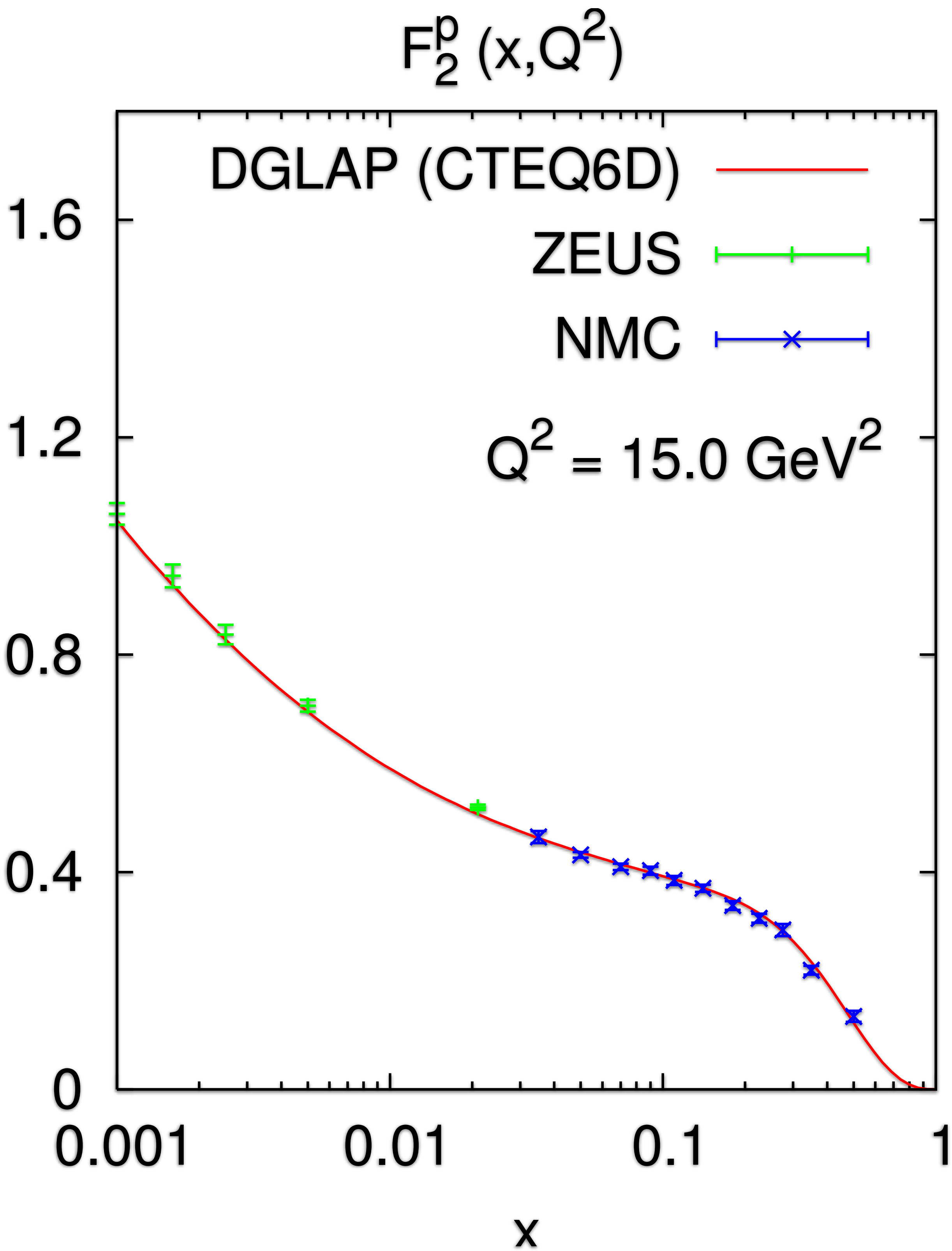
If gluon ≠ 0, splitting

$$g \to q\bar{q}$$

generates extra quarks at large Q2 ➠ faster rise of F2

Global PDF fits (CT, MSHT, NNPDF, etc.) choose gluon distribution that leads to the correct Q2 evolution.

Consider DIS data — $F_2(x,Q^2)$ — with specially tuned gluon



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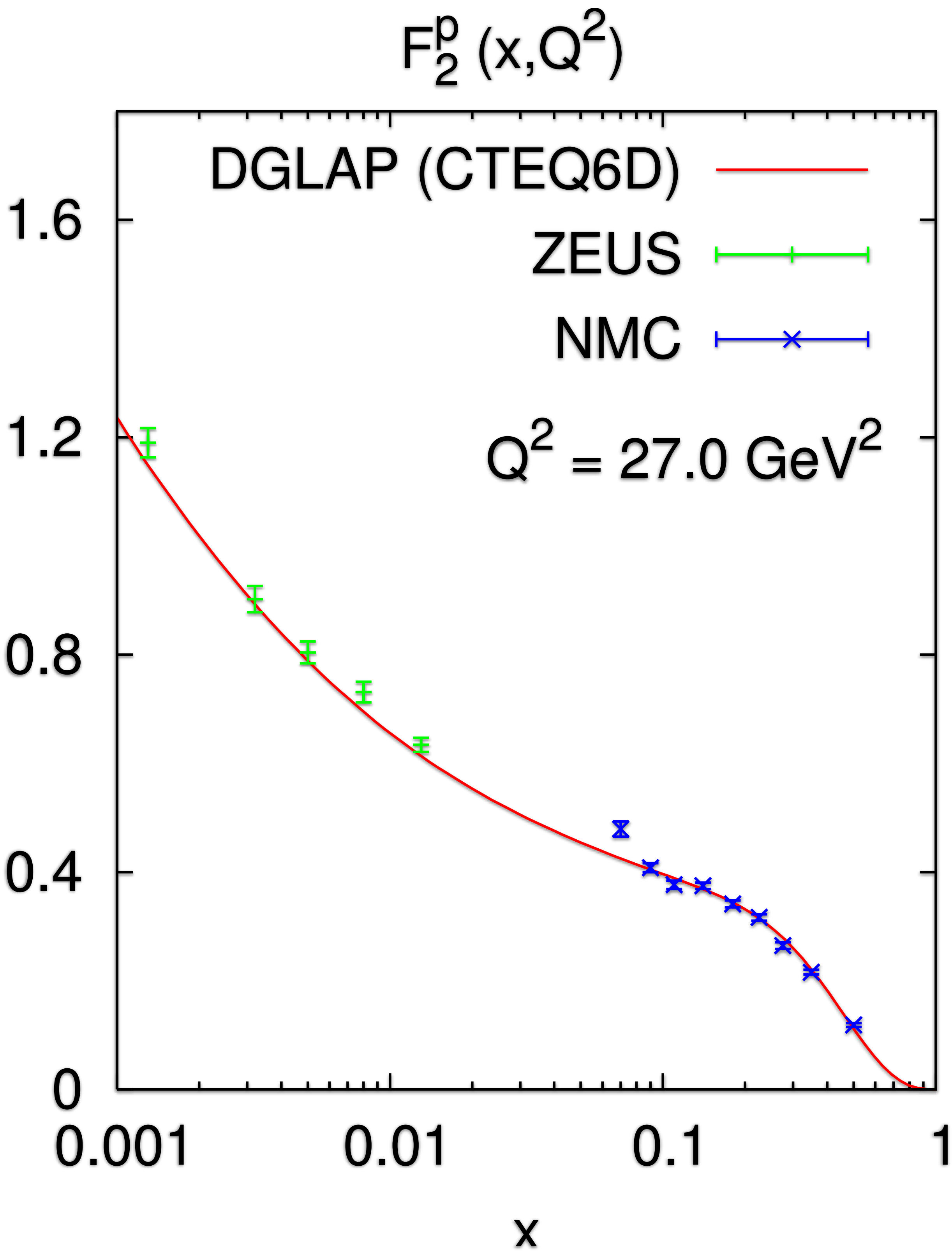

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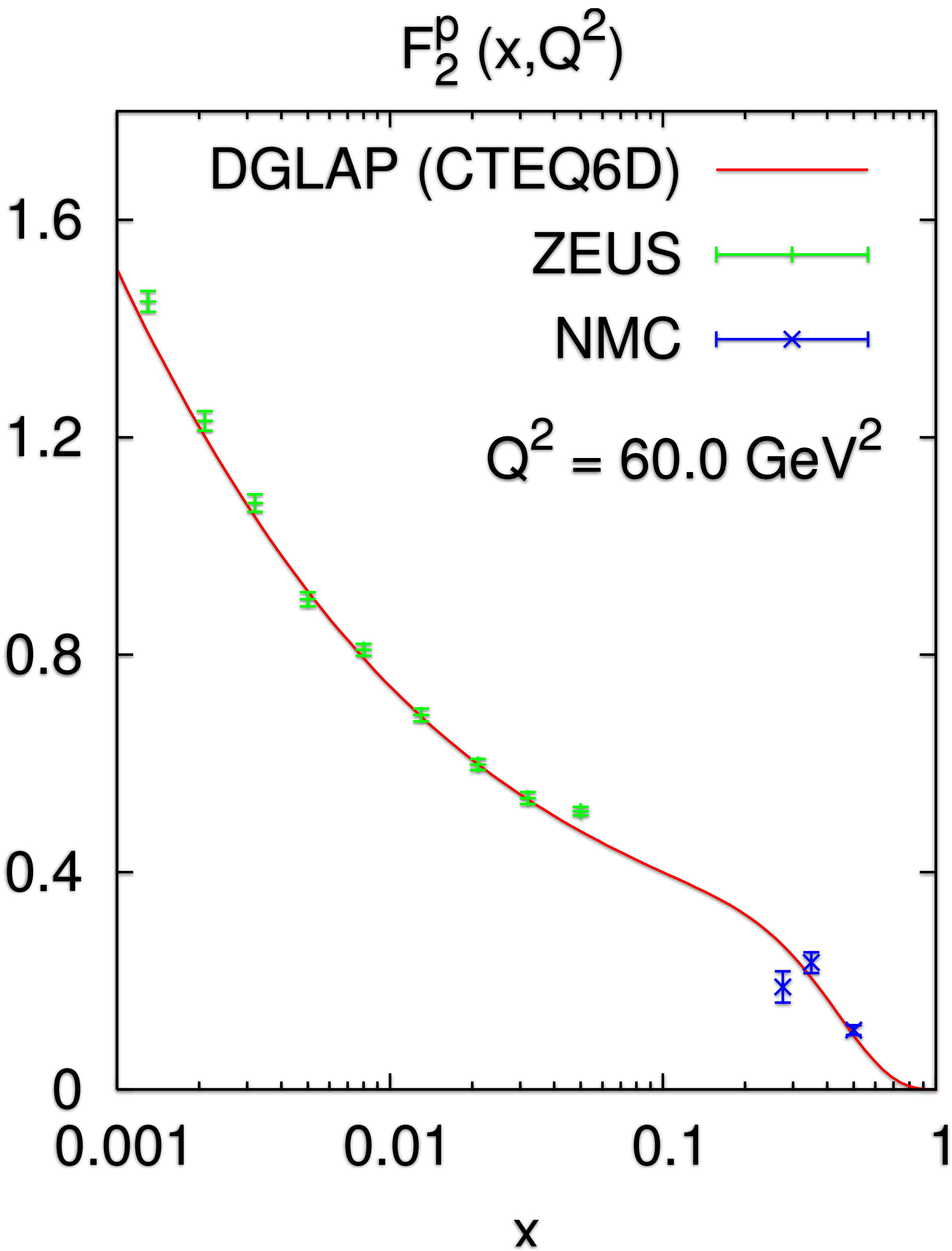

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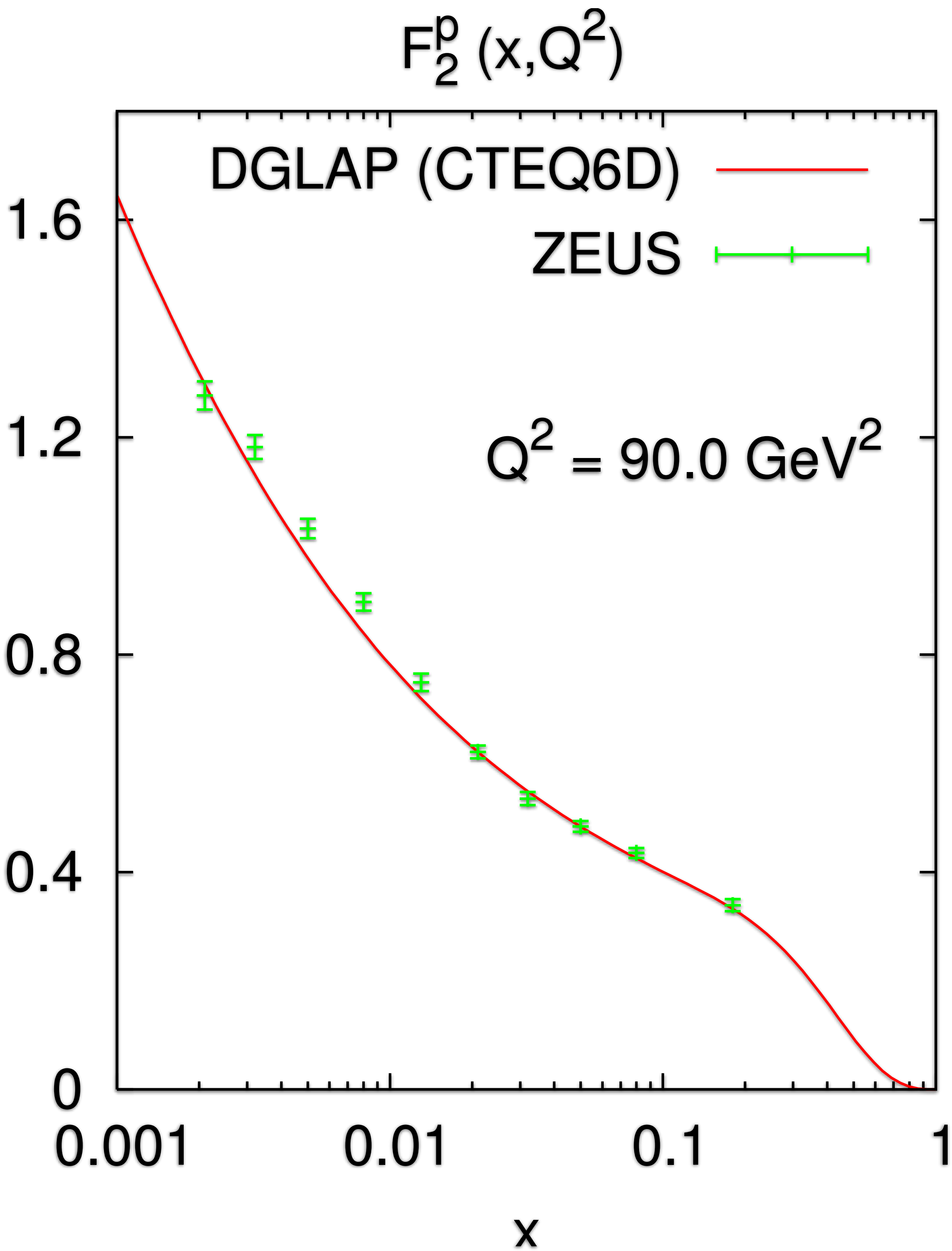

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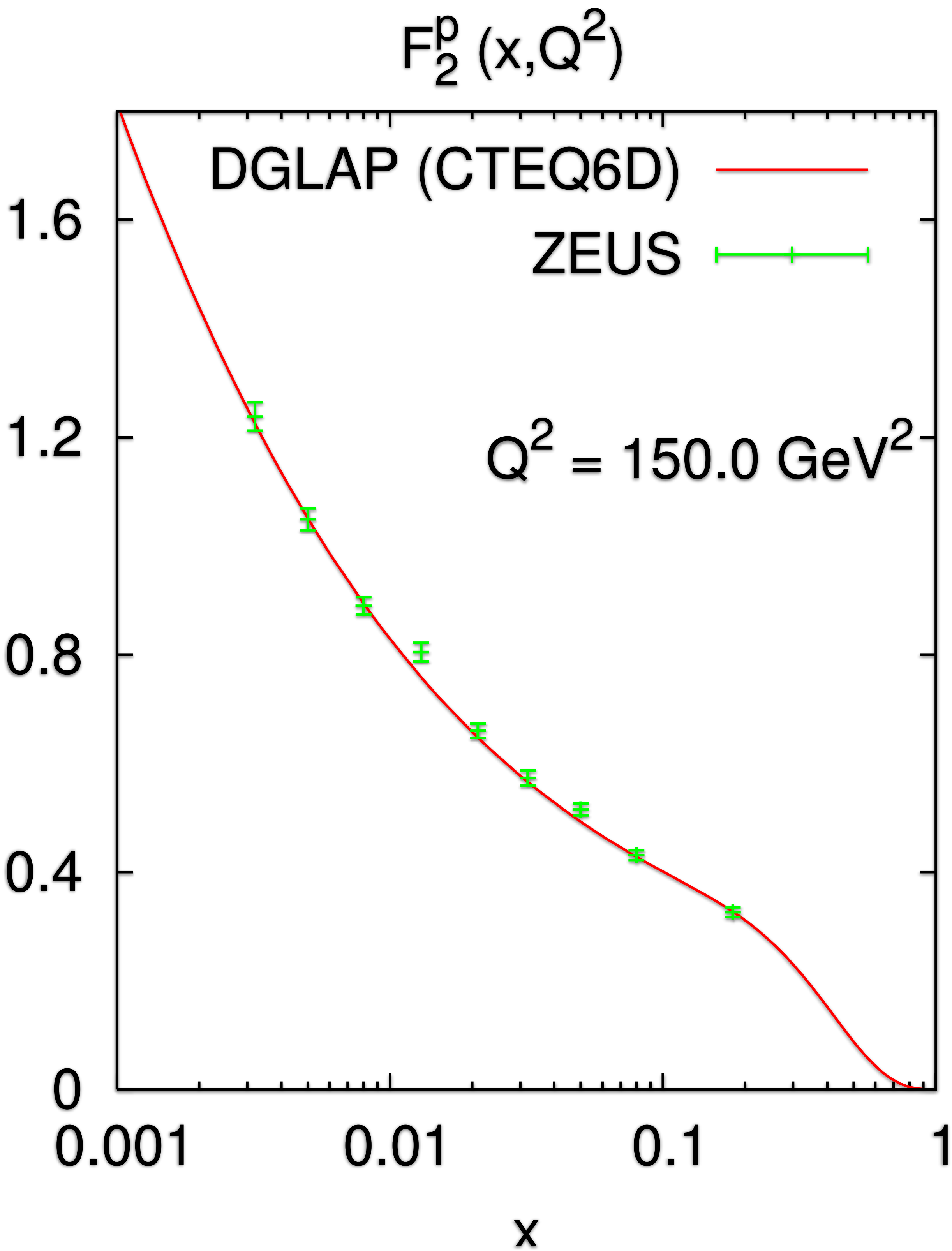

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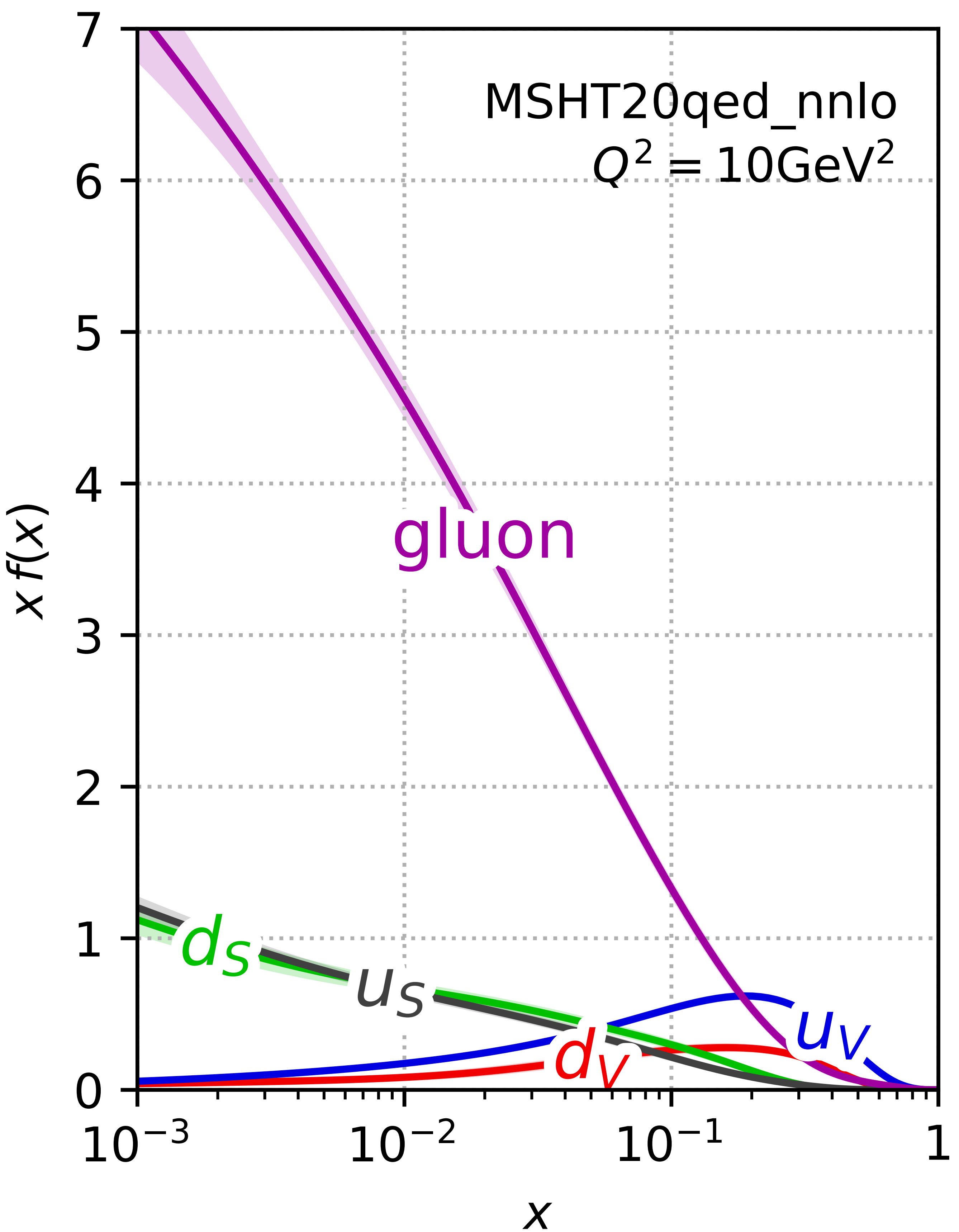
SUCCESS

Resulting gluon distribution, compared to quarks



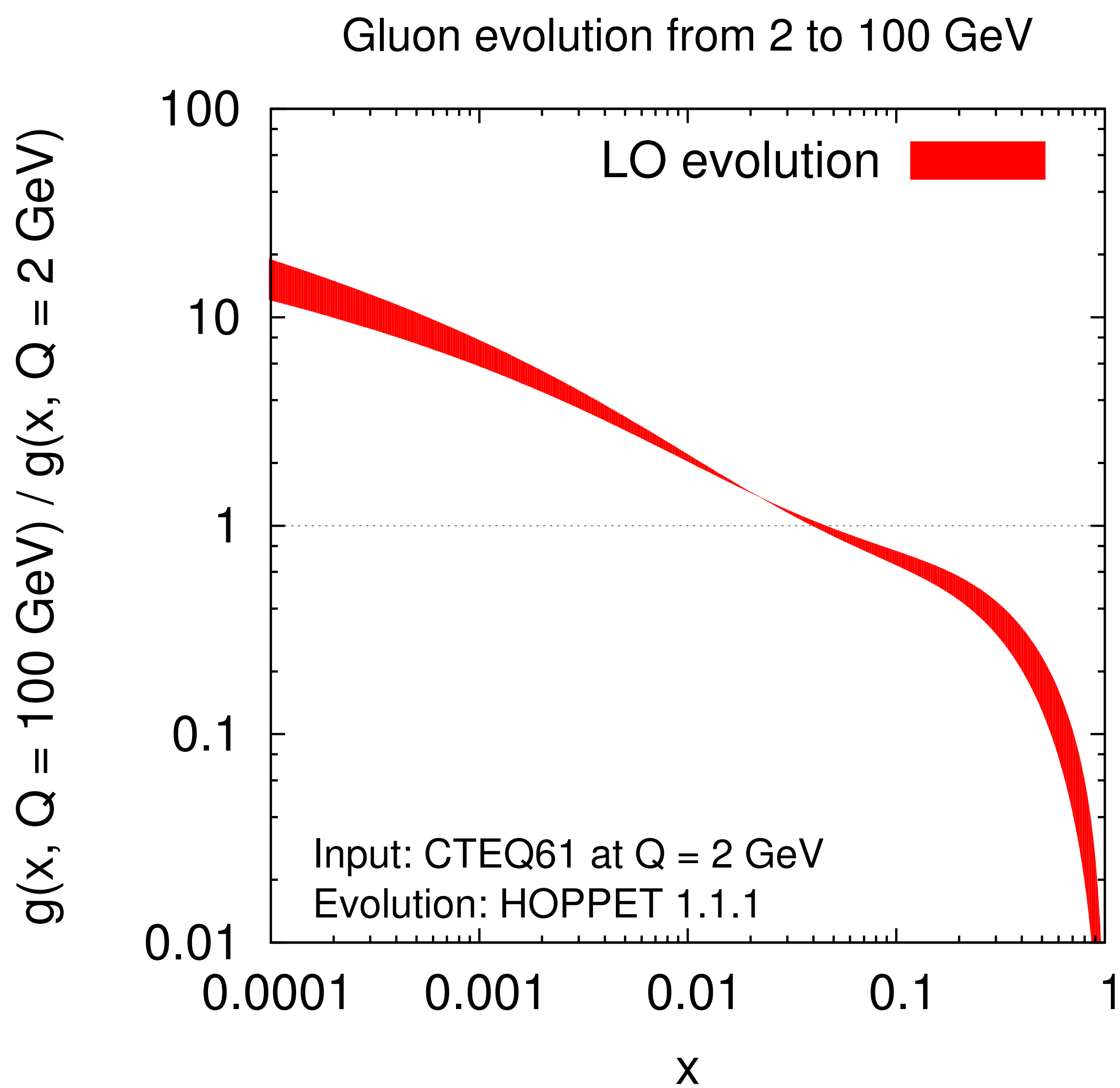
Resulting gluon distribution is **HUGE!**

Carries 43% of proton's momentum
(at scale of 10 GeV^2)

Crucial in order to satisfy momentum sum rule.

Large value of gluon has big impact on phenomenology, Higgs production is mostly (~90%) through gg→H

by how much does the gluon evolve?



Illustrate for the gluon distribution

Here using fixed Q scales
But for HERA $\to$ LHC
relevant Q range is x-dependent

- See factors $\sim 0.1 - 10$
- Remember: LHC involves product of two parton densities

It's crucial to get this right!

Without DGLAP evolution, you couldn't predict anything at LHC

NLO DGLAP

$$P^{(1)}_{\rm ps}(x) = 4\, C_F n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9}\right] + (1+x)\left[5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right]\right)$$

$$\begin{aligned}
P^{(1)}_{\rm qg}(x) &= 4\, C_A n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x)\mathrm{H}_{-1,0} - 2p_{\rm qg}(x)\mathrm{H}_{1,1} + x^2\left[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9}\right]\right.\\
&\left. +4(1-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1\right] - 4\zeta_2 x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0\right) + 4\, C_F n_f \left(2p_{\rm qg}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2\right.\right.\\
&\left.\left. -\zeta_2\right] + 4x^2\left[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2}\right] + 2(1-x)\left[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4}\right] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0\right)
\end{aligned}$$

$$\begin{aligned}
P^{(1)}_{\rm gq}(x) &= 4\, C_A C_F \left(\frac{1}{x} + 2p_{\rm gq}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1\right] - x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9}\right] + 4\zeta_2 - 2\right.\\
&\left. -7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1 x + (1+x)\left[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9}\right] - 2p_{\rm gq}(-x)\mathrm{H}_{-1,0}\right) - 4\, C_F n_f \left(\frac{2}{3}x\right.\\
&\left. -p_{\rm gq}(x)\left[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9}\right]\right) + 4\, {C_F}^2 \left(p_{\rm gq}(x)\left[3\mathrm{H}_1 - 2\mathrm{H}_{1,1}\right] + (1+x)\left[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0\right] - 3\mathrm{H}_{0,0}\right.\\
&\left. +1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1 x\right)
\end{aligned}$$

$$\begin{aligned}
P^{(1)}_{\rm gg}(x) &= 4\, C_A n_f \left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2\right) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x)\right) + 4\, {C_A}^2 \left(27\right.\\
& +(1+x)\left[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2}\right] + 2p_{\rm gg}(-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2\right] - \frac{67}{9}\left(\frac{1}{x} - x^2\right) - 12\mathrm{H}_0\\
&\left. -\frac{44}{3}x^2\mathrm{H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2\right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3\right]\right) + 4\, C_F n_f \left(2\mathrm{H}_0\right.\\
&\left. +\frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right] - \frac{1}{2}\delta(1-x)\right) .
\end{aligned}$$

<u>NLO:</u>

$$P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)} + \frac{\alpha_s^2}{16\pi^2} P^{(1)}$$

Curci, Furmanski & Petronzio '80

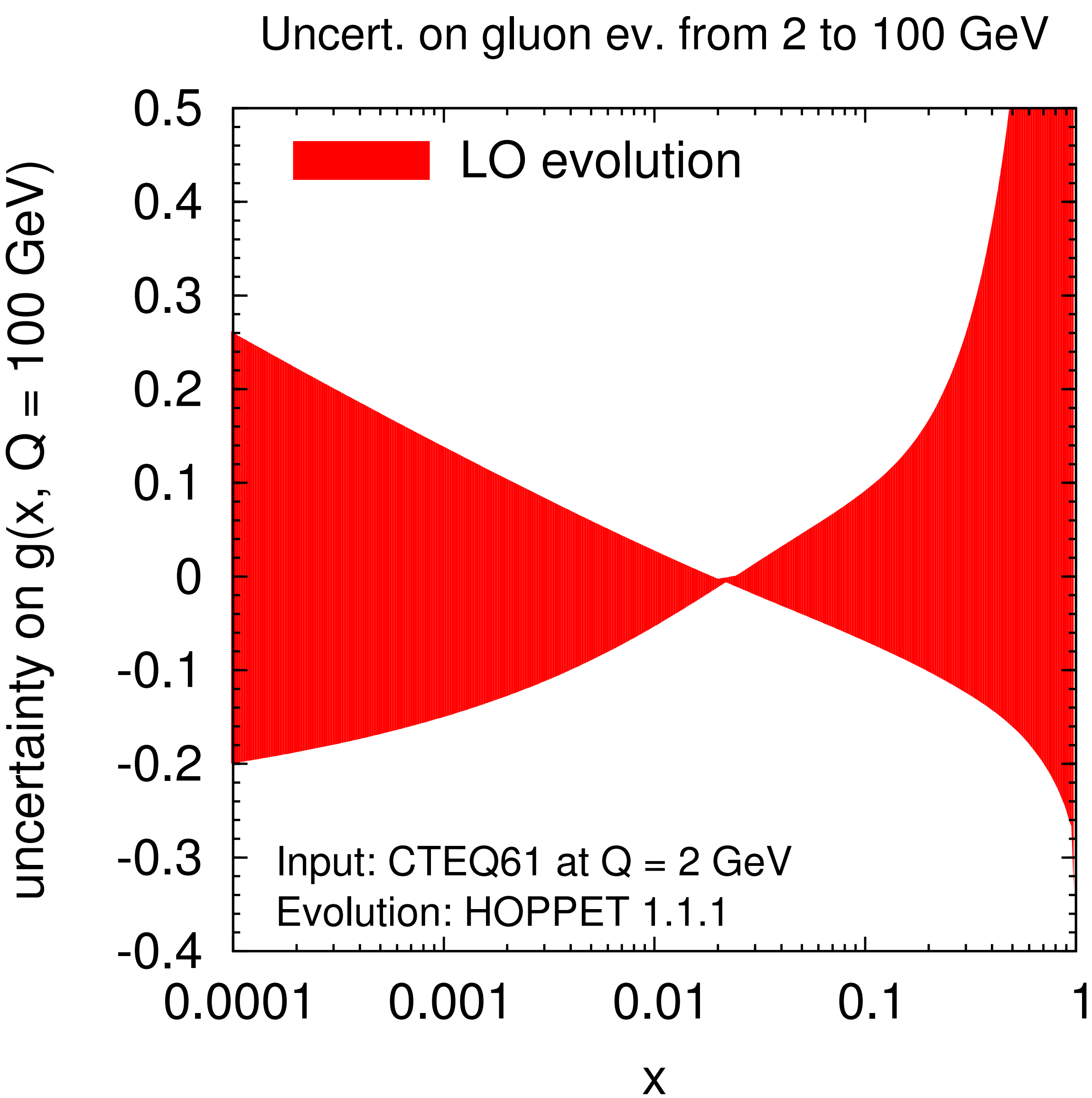
NNLO DGLAP

and

$$P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{ab}^{(2)}$$

NNLO, $P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

by how much does the gluon evolve?



Estimate uncertainties on evolution by changing the scale used for α_s inside the splitting functions

- with LO evolution, uncertainty is $\sim 30\%$
- NLO brings it down to $\sim 5\%$
- NNLO $\rightarrow 2\%$ Commensurate with data uncertainties

$$P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{ab}^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^4 P_{ab}^{(3)}$$

Four-Loop Non-Singlet Splitting Functions in the Planar Limit and Beyond

S. Moch[a], B. Ruijl[b,c], T. Ueda[b], J.A.M. Vermaseren[b] and A. Vogt[d]

arXiv:1707.08315v2 [hep-ph] 5 Oct 2017

Four-loop splitting functions in QCD – The quark-quark case –

G. Falcioni[a], F. Herzog[a], S. Moch[b] and A. Vogt[c]

arXiv:2302.07593v1 [hep-ph] 15 Feb 2023

heavy-quarks PDFs

if we have time, discuss at blackboard

modern PDF fits

Today's PDF fits

PDFs are obtained from "global" fits to large & diverse data sets

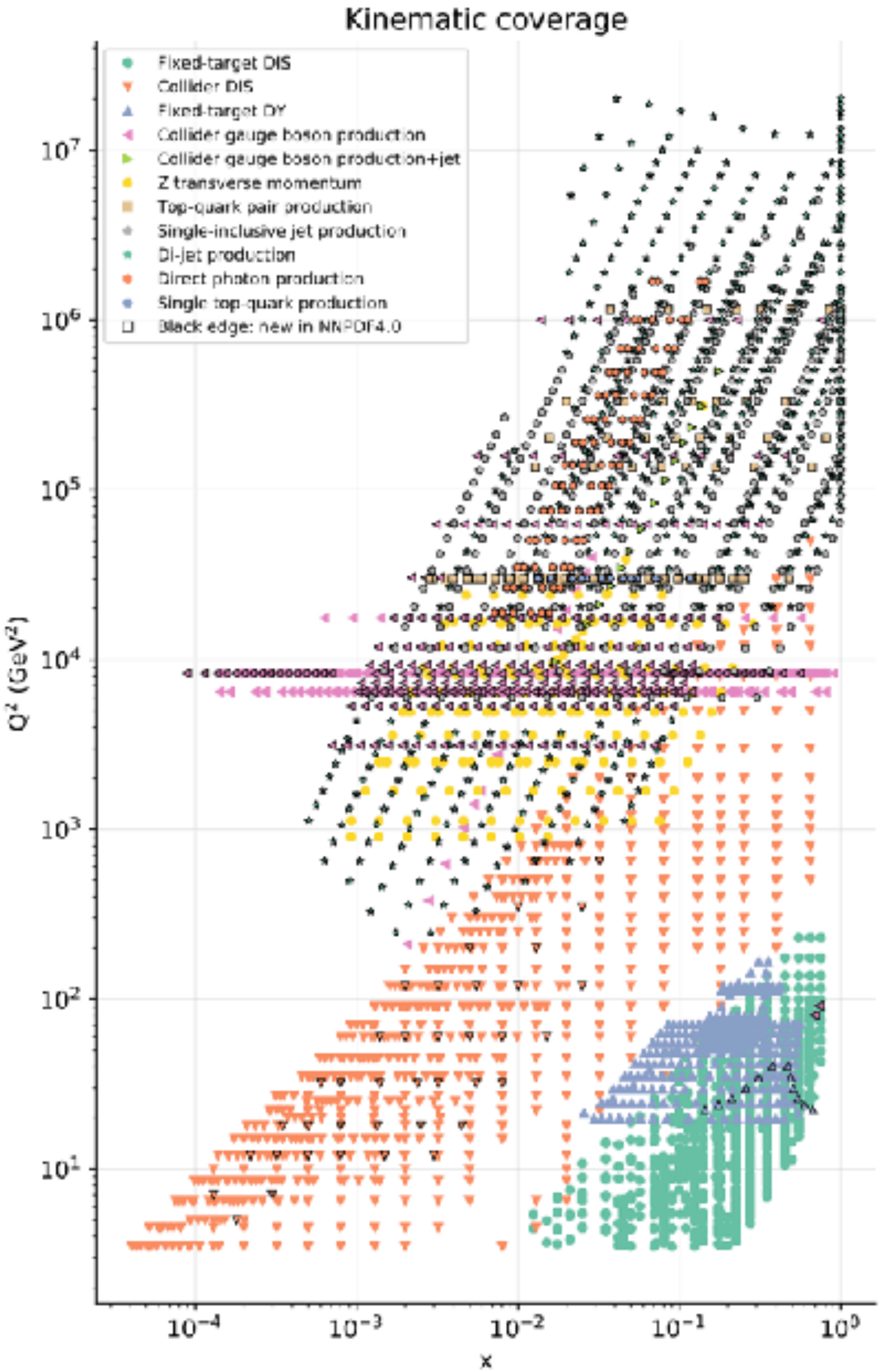
Several groups active, three stand out for core LHC applications

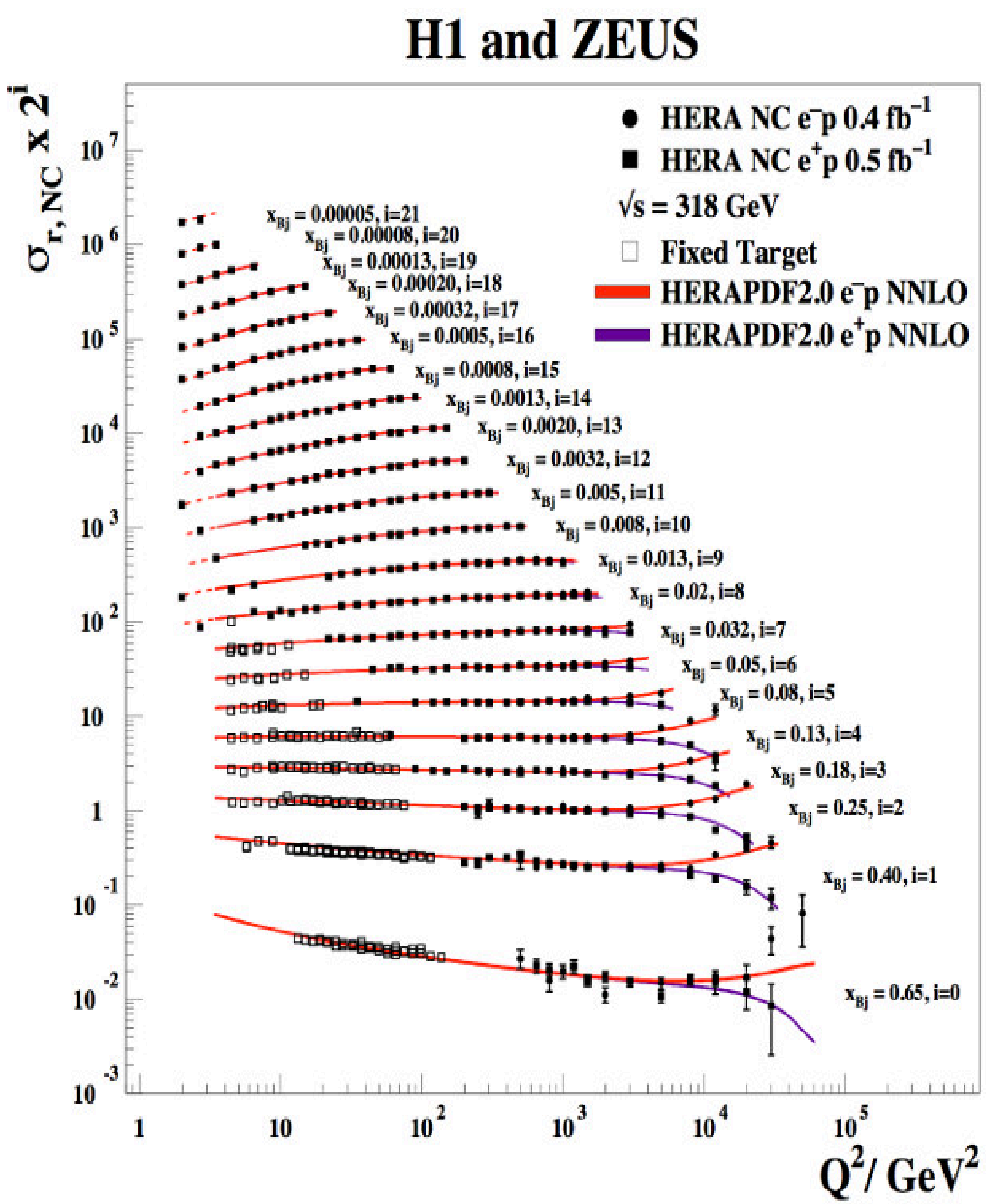
- CT (aka CTEQ, US+China based): 1912.10053 (CT18)
- MSHT (mostly UK based): 2012.04684 (MSHT20)
- NNPDF (pan-European): 2109.02653 (NNPDF40)

Next few slides illustrate some of the issues that they have to deal with and differences of approach.

Today's PDF fits: huge array of data (and choices about which data to use)

NNPDF4.0 dataset





Today's PDF fits: huge array of data (and choices about which data to use)

MSHT20 data sets & χ^2

Data set	NLO	NNLO
BCDMS μp F_2 [49]	169.4/163	180.2/163
BCDMS μd F_2 [49]	135.0/151	146.0/151
NMC μp F_2 [50]	142.9/123	124.1/123
NMC μd F_2 [50]	128.2/123	112.4/123
NMC $\mu n/\mu p$ [51]	127.8/148	130.8/148
E665 μp F_2 [52]	59.5/53	64.7/53
E665 μd F_2 [52]	50.3/53	59.7/53
SLAC ep F_2 [53, 54]	29.4/37	32.0/37
SLAC ed F_2 [53, 54]	37.4/38	23.0/38
NMC/BCDMS/SLAC/HERA F_L [49, 50, 54, 146–148]	79.4/57	68.4/57
E866/NuSea pp DY [149]	216.2/184	225.1/184
E866/NuSea pd/pp DY [150]	10.6/15	10.4/15
NuTeV νN F_2 [55]	43.7/53	38.3/53
CHORUS νN F_2 [56]	27.8/42	30.2/42
NuTeV νN xF_3 [55]	37.8/42	30.7/42
CHORUS νN xF_3 [56]	22.0/28	18.4/28
CCFR $\nu N \to \mu\mu X$ [57]	73.2/86	67.7/86
NuTeV $\nu N \to \mu\mu X$ [57]	41.0/84	58.4/84
HERA e^+p CC [84]	54.3/39	52.0/39
HERA e^-p CC [84]	80.4/42	70.2/42
HERA e^+p NC 820 GeV [84]	91.6/75	89.8/75
HERA e^+p NC 920 GeV [84]	553.9/402	512.7/402
HERA e^-p NC 460 GeV [84]	253.3/209	248.3/209
HERA e^-p NC 575 GeV [84]	268.1/259	263.0/259
HERA e^-p NC 920 GeV [84]	252.3/159	244.4/159
HERA ep F_2^{charm} [26]	125.6/79	132.3/79
DØ II $p\bar{p}$ incl. jets [125]	117.2/110	120.2/110
CDF II $p\bar{p}$ incl. jets [124]	70.4/76	60.4/76
CDF II W asym. [90]	19.1/13	19.0/13
DØ II $W \to \nu e$ asym. [151]	44.4/12	33.9/12
DØ II $W \to \nu\mu$ asym. [152]	13.9/10	17.3/10
DØ II Z rap. [153]	15.9/28	16.4/28
CDF II Z rap. [154]	36.9/28	37.1/28
DØ W asym. [21]	13.1/14	12.0/14

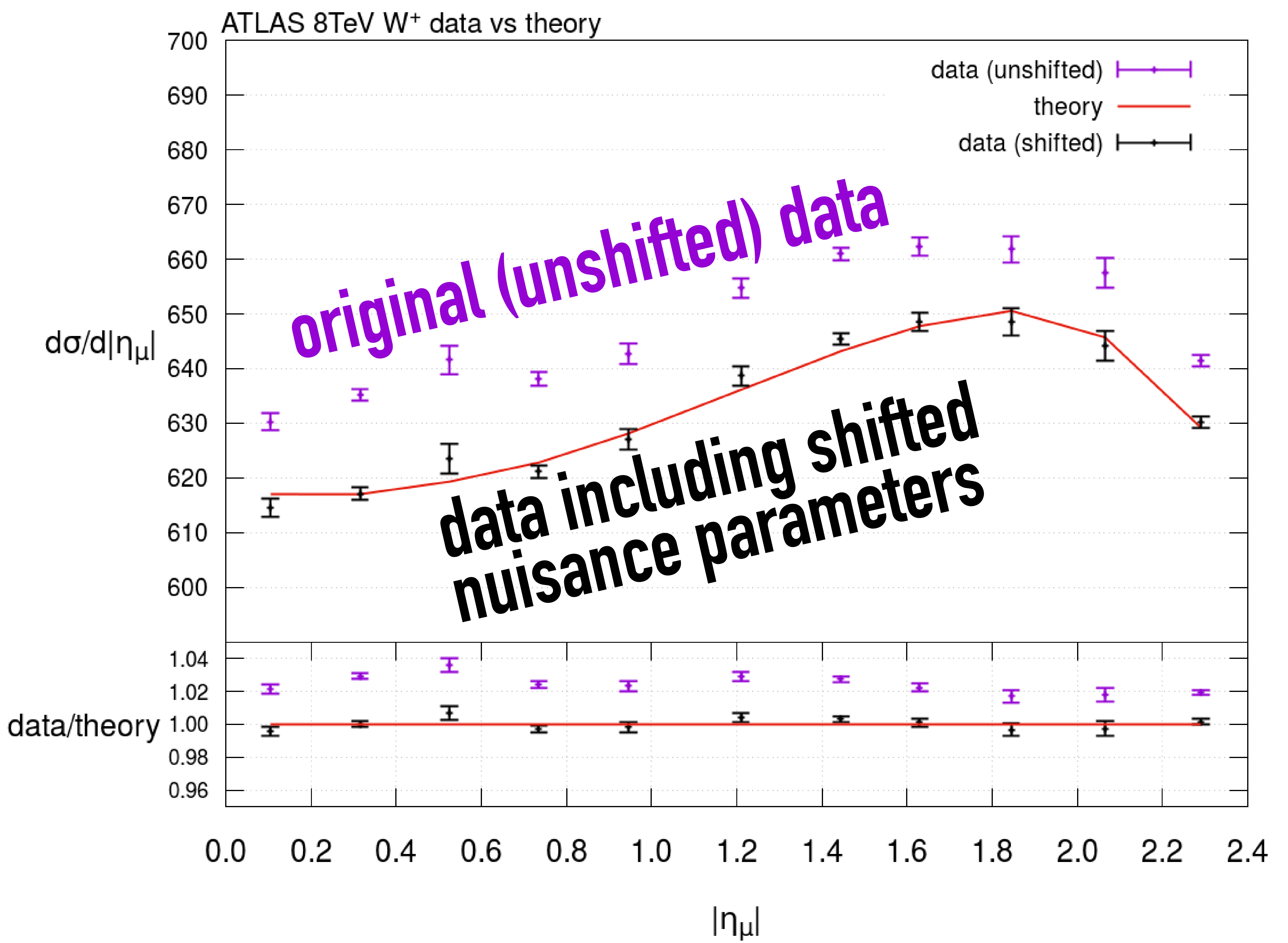
Table 6: The values of χ^2/Npts. for the non-LHC data sets included in the global fit at NLO and NNLO.

Data set	NLO	NNLO
ATLAS W^+, W^-, Z [119]	34.7/30	29.9/30
CMS W asym. $p_T > 35$ GeV [155]	11.8/11	7.8/11
CMS asym. $p_T > 25, 30$ GeV [156]	11.8/24	7.4/24
LHCb $Z \to e^+e^-$ [157]	14.1/9	22.7/9
LHCb W asym. $p_T > 20$ GeV [158]	10.5/10	12.5/10
CMS $Z \to e^+e^-$ [159]	18.9/35	17.9/35
ATLAS High-mass Drell-Yan [160]	20.7/13	18.9/13
CMS double diff. Drell-Yan [72]	222.2/132	144.5/132
Tevatron, ATLAS, CMS $\sigma_{t\bar{t}}$ [93]- [94]	22.8/17	14.5/17
LHCb 2015 W, Z [95, 96]	114.4/67	99.4/67
LHCb 8 TeV $Z \to ee$ [97]	39.0/17	26.2/17
CMS 8 TeV W [98]	23.2/22	12.7/22
ATLAS 7 TeV jets [18]	226.2/140	221.6/140
CMS 7 TeV $W + c$ [99]	8.2/10	8.6/10
ATLAS 7 TeV high precision W, Z [20]	304.7/61	116.6/61
CMS 7 TeV jets [100]	200.6/158	175.8/158
CMS 8 TeV jets [101]	285.7/174	261.3/174
CMS 2.76 TeV jet [107]	124.2/81	102.9/81
ATLAS 8 TeV Z p_T [75]	235.0/104	188.5/104
ATLAS 8 TeV single diff $t\bar{t}$ [102]	39.1/25	25.6/25
ATLAS 8 TeV single diff $t\bar{t}$ dilepton [103]	4.7/5	3.4/5
CMS 8 TeV double differential $t\bar{t}$ [105]	32.8/15	22.5/15
CMS 8 TeV single differential $t\bar{t}$ [108]	12.9/9	13.2/9
ATLAS 8 TeV High-mass Drell-Yan [73]	85.8/48	56.7/48
ATLAS 8 TeV W [106]	84.6/22	57.4/22
ATLAS 8 TeV W + jets [104]	33.9/30	18.1/30
ATLAS 8 TeV double differential Z [74]	157.4/59	85.6/59
Total	5822.0/4363	5121.9/4363

Table 7: The values of χ^2/Npts. for the LHC data sets included in the global fit and the overall global fit χ^2/N at NLO and NNLO. The corresponding values for the non-LHC data sets are shown in Table 6, and the total value corresponds to the sum over both tables.

data is precise, correlations between systematics are crucial

e.g. from MSHT20 (2012.04684)



A generic function $f(x)$ involves an infinite number of degrees of freedom. How can you fit this with a finite number of data points?

CT / MSHT use parameterisations with hand-picked number of terms, e.g. up to $n = 6$ in Chebyshev series:

$$xf(x, Q_0^2) = A(1-x)^\eta x^\delta \left(1 + \sum_{i=1}^{n} a_i T_i^{\mathrm{Ch}}(y(x))\right)$$

NNPDF use a ***neural network*** as a generic fit function, and separate data into training / validation. Fit is done using just the training subset, and stops when χ^2 on training + validation starts to increase. (Supplemented with closure tests)

today's PDF fits: uncertainty estimation

With fits to O(60) data sets, chances are they won't all be consistent (plainly inconsistent data sets may simply be excluded, but that can be biased)

CT / MSHT do a Hessian fit, with error eigenfunctions, scaled by a tolerance T that is like replacing $\Delta\chi^2 = 1$ with $\Delta\chi^2 = T$.

Squared error on a cross section is obtained by summing squared variations from each of the eigenfuntions.

NNPDF fits ***Monte Carlo replica data sets***
i.e. fluctuate the data according to errors, and fit the fluctuated data; repeat over and over, to get O(100) replica fits; prediction for any cross section is then average and std.dev. across the replicas

today's PDF fits: treatment of charm

Charm-quark mass is around 1.5 GeV. Is this perturbative enough to treat it as purely perturbative generated? Or should one fit the charm as a light flavour?

CT / MSHT treat charm perturbatively, turning on its evolution from (almost zero) at the charm mass.

NB: CT also explores "fitted" charm

NNPDF fits by default treat the charm as light, but also provide PDF sets with perturbative charm

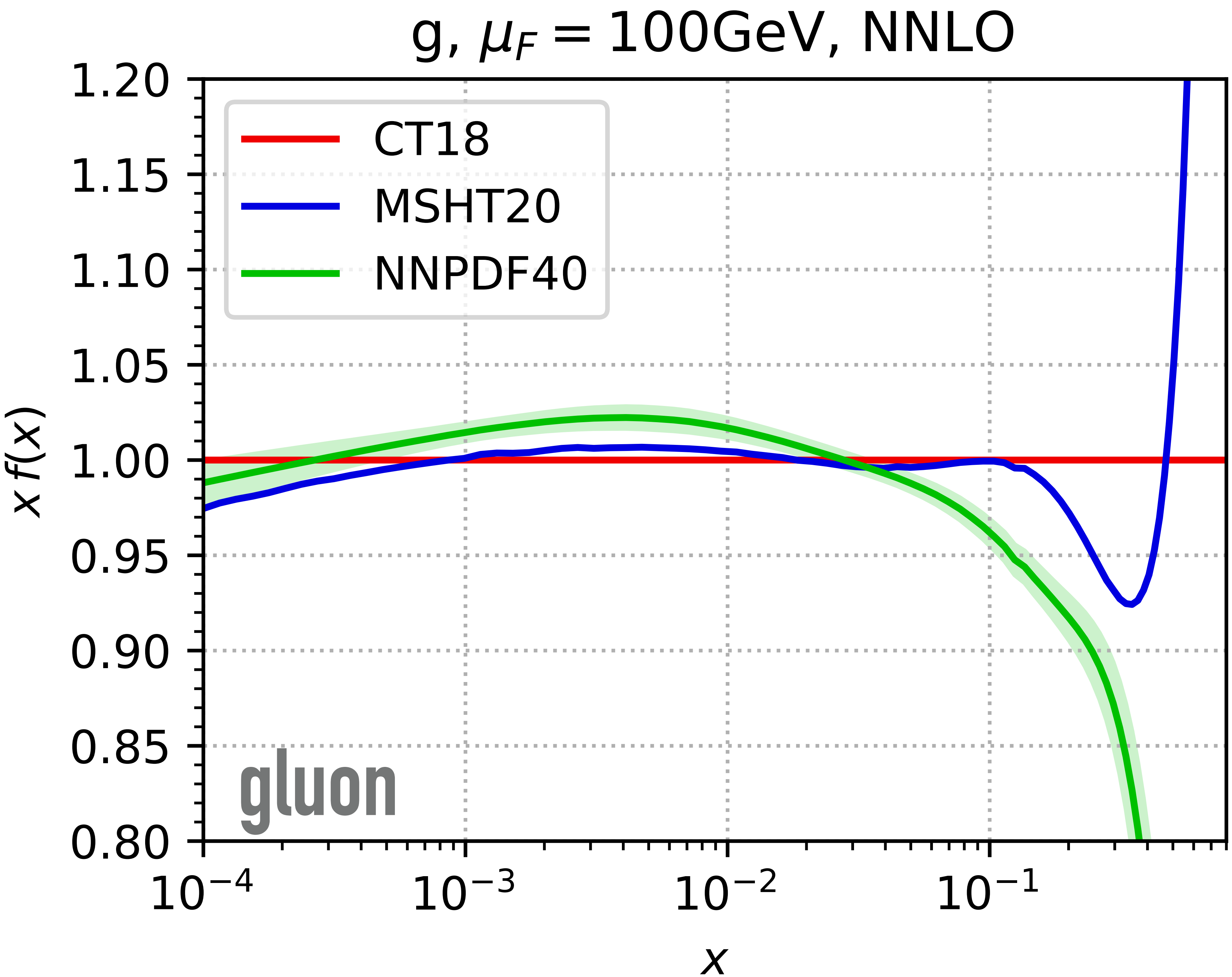
Using PDFs: LHAPDF

Standard software for accessing PDFs, in C++ and Python

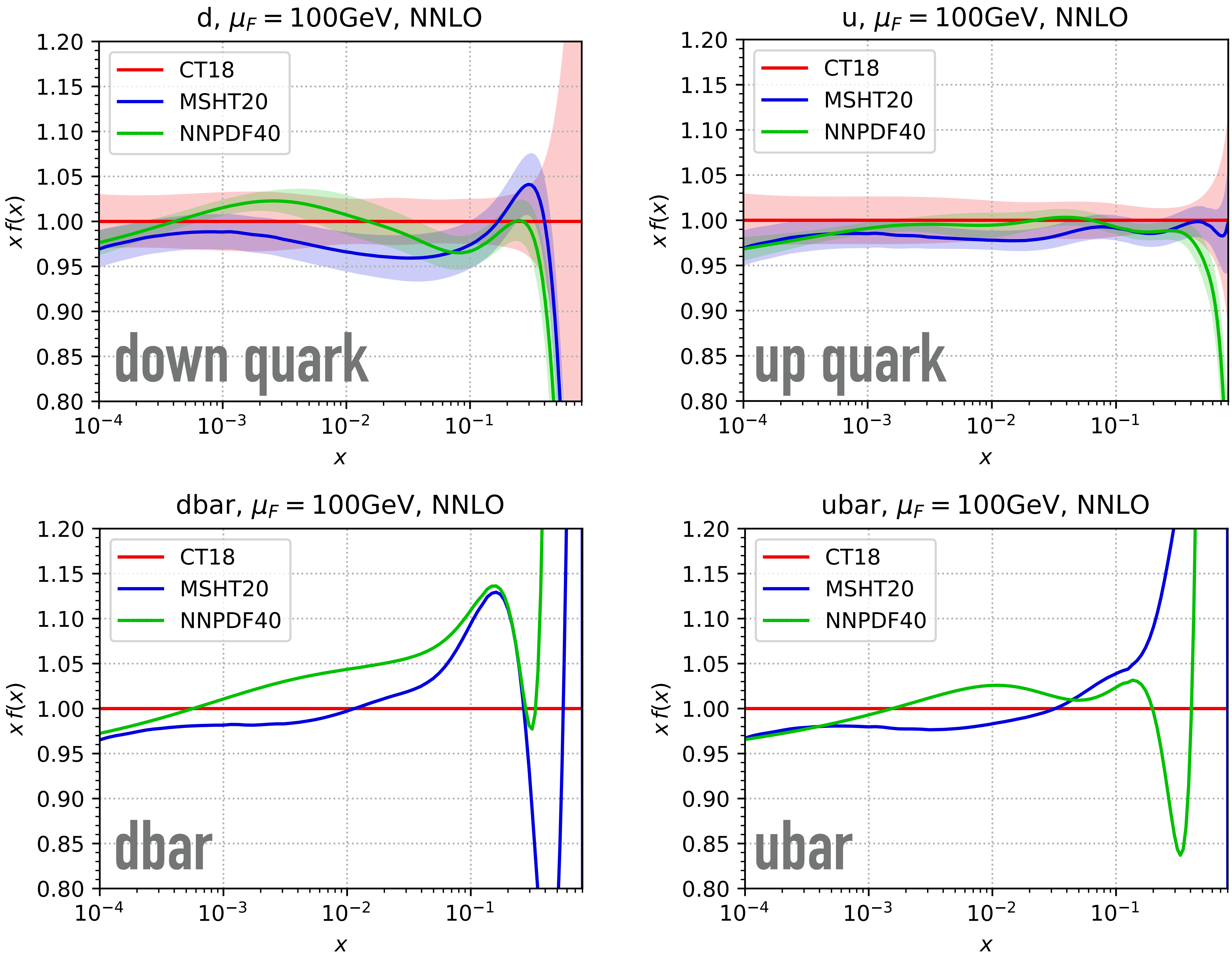
```
import numpy as np
import lhapdf

# get the pdf set
pdfname = "MSHT20nnlo_as118"
pdfset = lhapdf.getPDFSet(pdfname)
pdfs = pdfset.mkPDFs()

# decide the x and factorisation scale
x = 0.1
muF = 100.0

# evaluate the central (pdfs[0]) value of the gluon
pdgid = 21
xgluon = pdfs[0].xfxQ(pdgid, x, muF)
print(f"x * gluon(x={x}, muF={muF}) = {xgluon}")

# evaluate also the uncertainty on the gluon
# First step is to access the gluon for each of the
# members of the pdf set
print(f"Looping over the {len(pdfs)} members")
xgluon_members = np.empty(len(pdfs))
for i in range(len(pdfs)):
    xgluon_members[i] = pdfs[i].xfxQ(pdgid, x, muF)

# LHAPDF can then calculate the uncertainty
uncertainty = pdfset.uncertainty(xgluon_members)
print(f"uncertainty = {uncertainty.errsymm}")
```

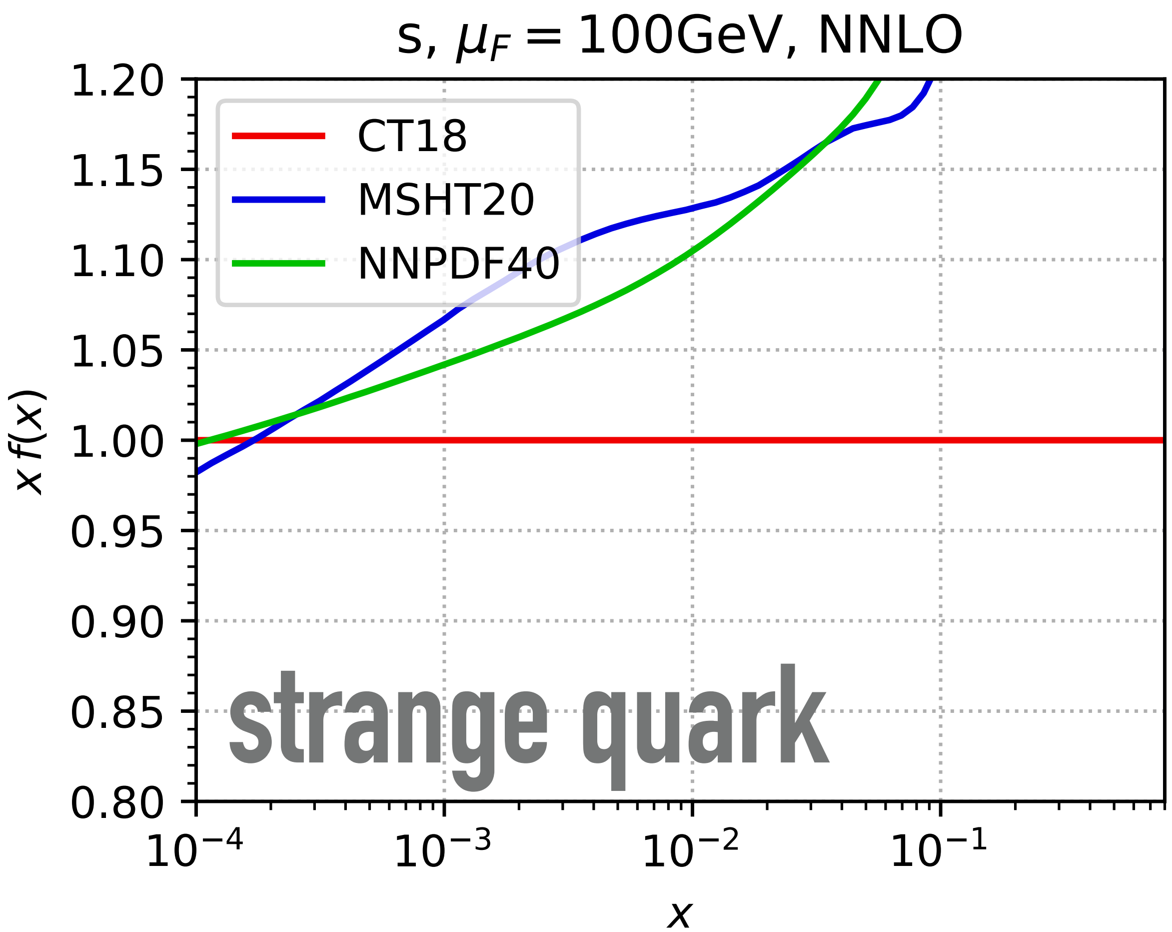
```
LHAPDF 6.5.3
MSHT20nnlo_as118, version 3; 65 PDF members

x * gluon(x=0.1, muF=100.0) = 0.88816059

Looping over the 65 members
uncertainty = 0.01519004859884556

Thanks for using LHAPDF 6.5.3. Please make sure to cite
the paper:
  Eur.Phys.J. C75 (2015) 3, 132
  http://arxiv.org/abs/1412.7420)
```

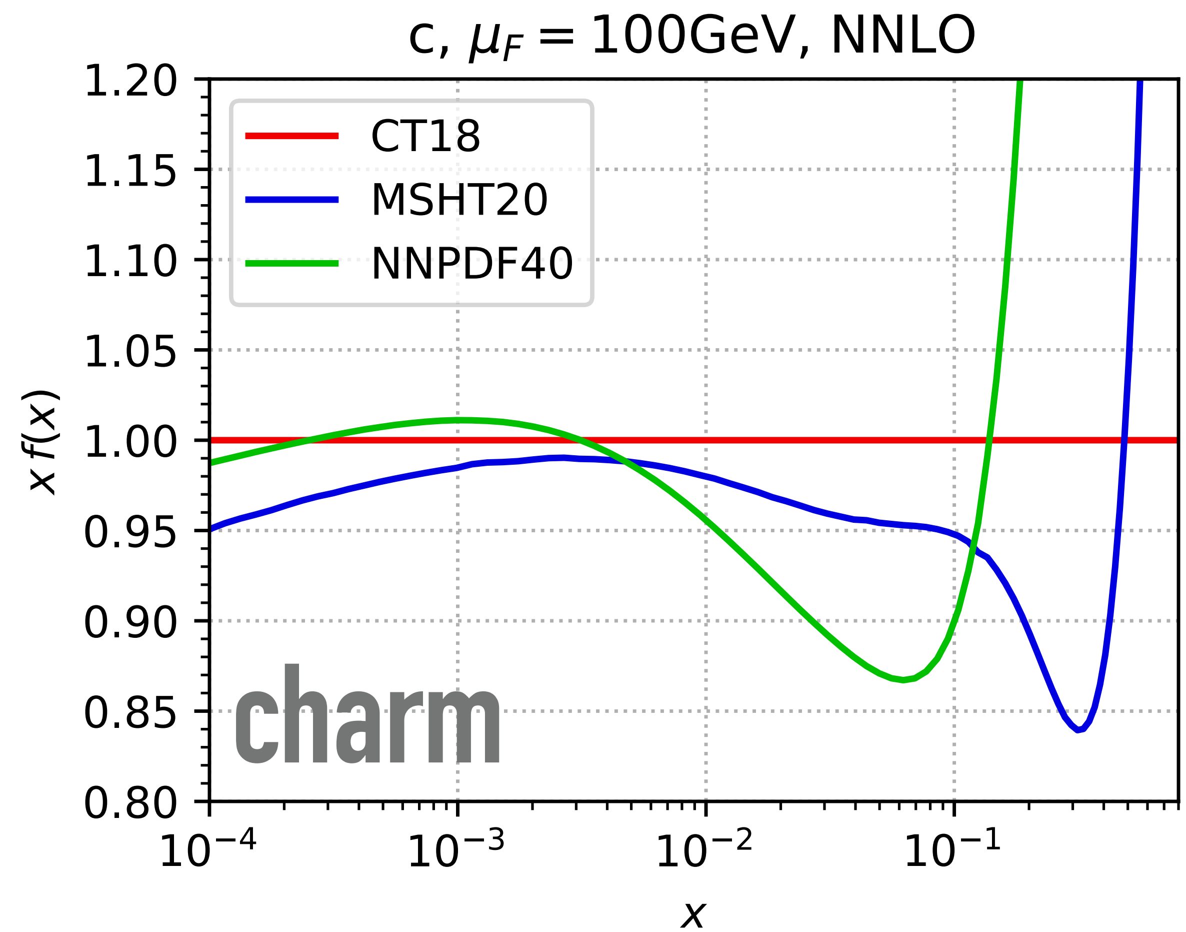
Comparing different sets: the gluon

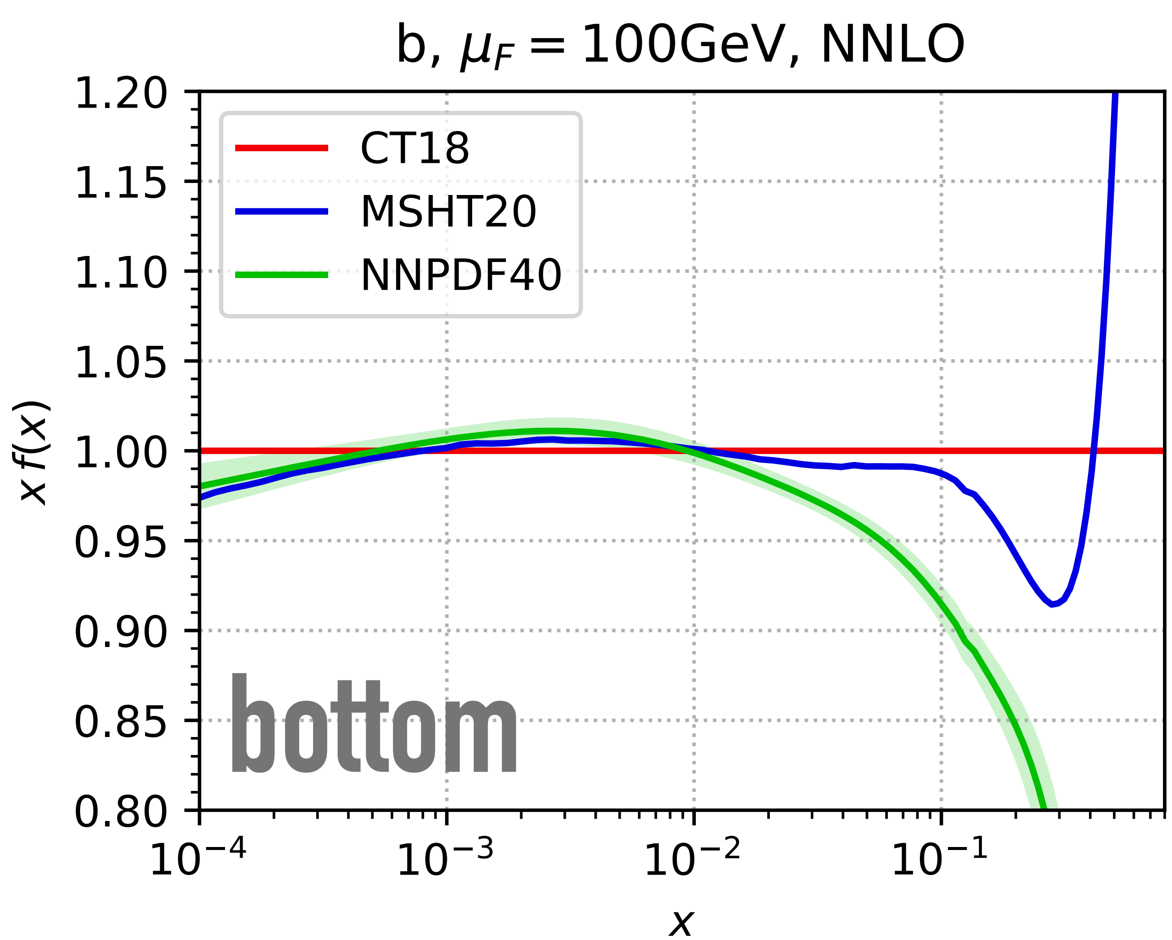


In much of region relevant to LHC, uncertainty is in the 1-2% range


d, $\mu_F = 100$GeV, NNLO
u, $\mu_F = 100$GeV, NNLO
dbar, $\mu_F = 100$GeV, NNLO
ubar, $\mu_F = 100$GeV, NNLO
CT18
MSHT20
NNPDF40
down quark
up quark
dbar
ubar
$x\,f(x)$
x
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
10^{-4}
10^{-3}
10^{-2}
10^{-1}







- strange (anti-)quark is least well known PDF (small charge, few good experimental handles)
- charm: current debate about intrinsic charm
- bottom: mostly driven by gluon

the concept of a PDF luminosity

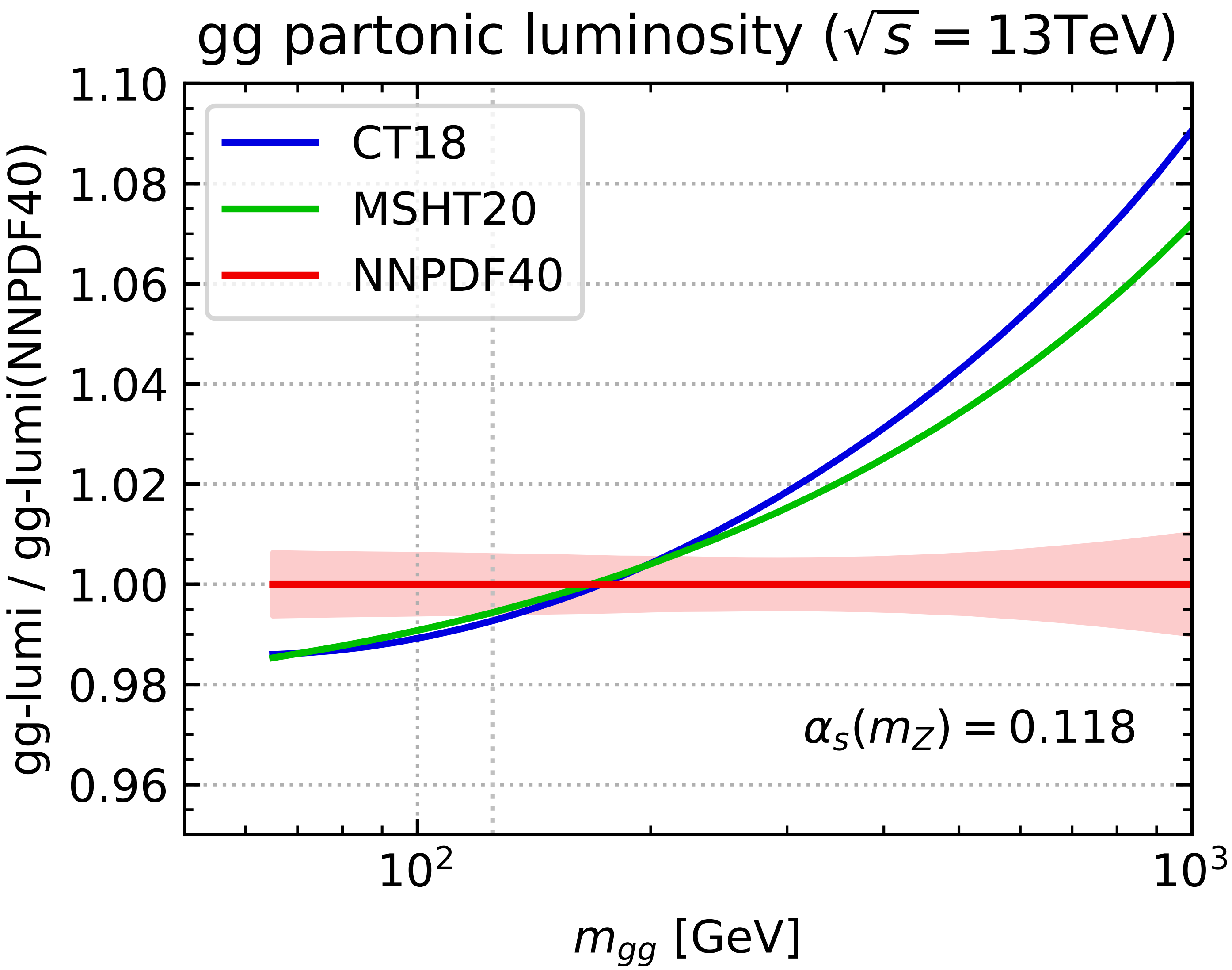
"Think" at Leading Order (LO) in QCD:

- collide protons at CoM energy $\sqrt{s}$,
- take momentum fractions x_1 and x_2 from the two protons
- producing a system of mass m requires $x_1 x_2 s = m^2$

Number of parton-parton collisions with flavours i and j is proportional to **partonic luminosity** $\mathscr{L}_{ij}(m^2)$

$$\mathscr{L}_{ij}(m^2) = \int dx_1 dx_2\, f_{i,p}(x_1, \mu_F^2)\, f_{j,p}(x_2, \mu_F^2)\, \delta(x_1 x_2 s - m^2)$$

comparing PDF "luminosities"



gg-lumi, ratio to PDF4LHC15

PDF4LHC15	1.0000	±	0.0184
PDF4LHC21	0.9930	±	0.0155
CT18	0.9914	±	0.0180
MSHT20	0.9930	±	0.0108
NNPDF40	0.9986	±	0.0058

$\times 3$

NB: PDF4LHC21 uses CT18/MSHT20/NNPDF31

Amazing that MSHT20 & NNPDF40 are reaching %-level precision

At this level, QED effects probably no longer optional (MSHT20QED: 0.9870)

FINAL REMARKS ON PDFS

- In range $10^{-3} < x < 0.1$, core PDFs (up, down, gluon) known to ~ few % accuracy
- For many LHC applications, you can use PDF4LHC21 set, which merges CT18, MSHT20, NNPDF31
- PDFs are not a settled issue: e.g. uncertainties are quite different between fitting groups; central values don't always agree; disagreements about intrinsic charm; N3LO splitting functions are being calculated

For visualisations of PDFs and related quantities, a good place to start is

https://apfel.mi.infn.it/ (ApfelWeb, but a little out of date)