La QCD à hautes énergies

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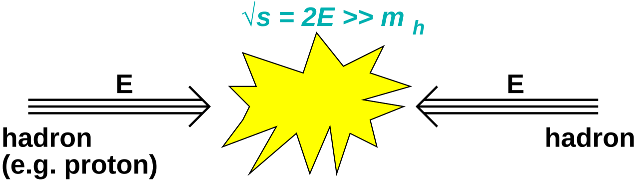
High-energy limit

One of the major unsolved problems of QCD (and Yang-Mills theory in general) is the understanding of its *high-energy limit.*

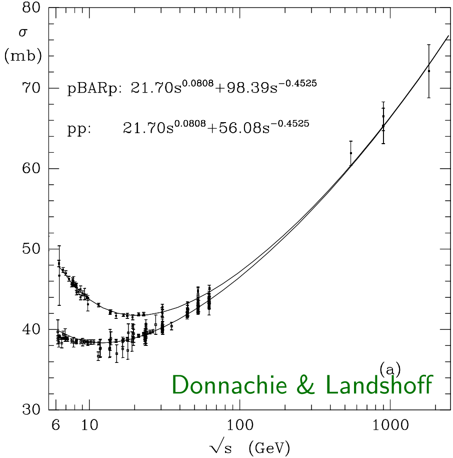
I.e. the limit in which C.O.M. energy ($\sqrt{s}$) is much larger than *all other scales* in the problem.



Want to examine perturbative QCD predictions for

- asymptotic behaviour of cross section, $\sigma_{hh}(s) \sim ??$
- properties of final states for large s.

Experimental knowledge



- Some knowledge exists about behaviour of cross section experimentally
- Slow rise as energy increases
- Data insufficient to make reliable statements about functional form
  - $\sigma \sim s^{0.08}$?
  - $\sigma \sim \ln^2 s$?
- Understanding of final-states is $\sim$ inexistent
- Would like theoretical predictions. . .

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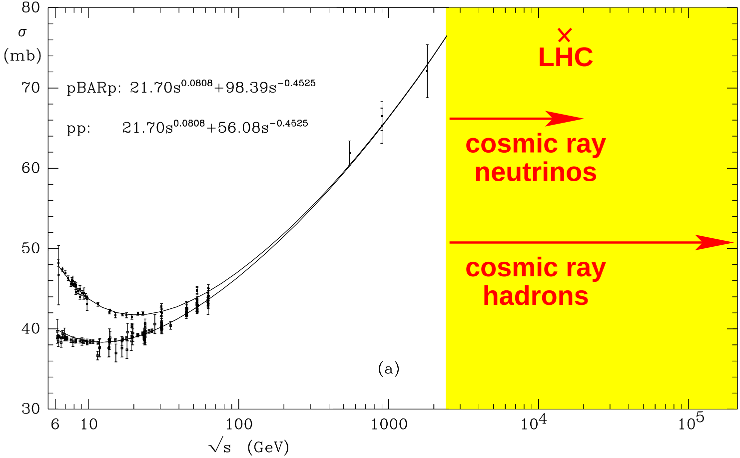
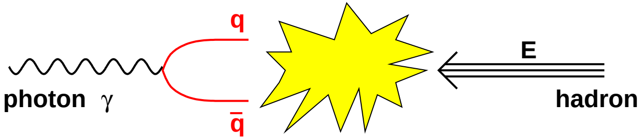


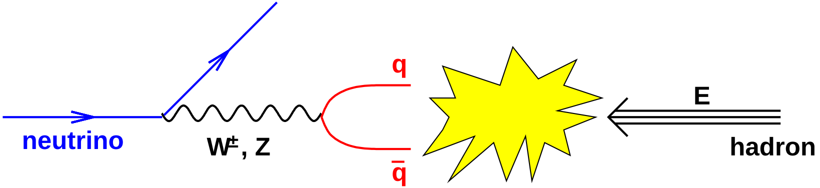
Future experiments go to much higher energies.

Not just for hadrons

Problem is must more general than just for hadrons. E.g. photon can *fluctuate* into a quark-antiquark (hadronic!) state:

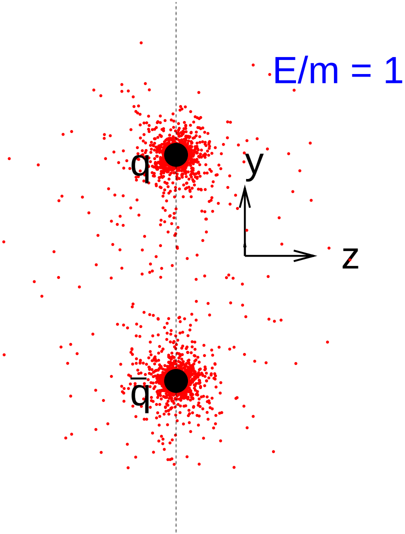


Even a neutrino can behave like a hadron



Hadronic component dominates high-energy cross section

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)



Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy* $\equiv$ *large boost* (along z axis), by factor
- Fields flatten into *pancake*.
  - simple longitudinal structure

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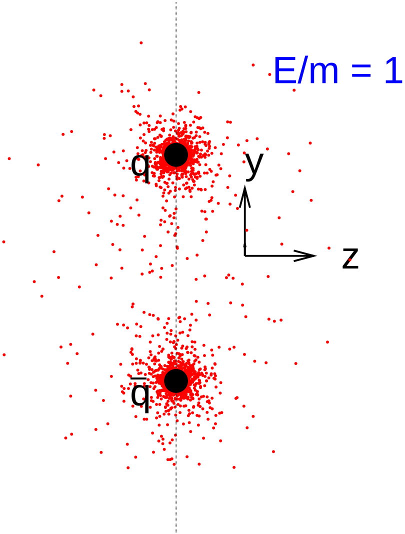

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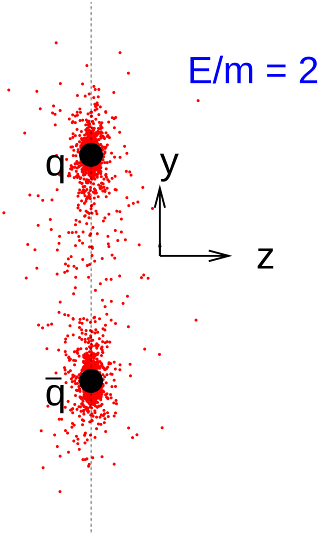

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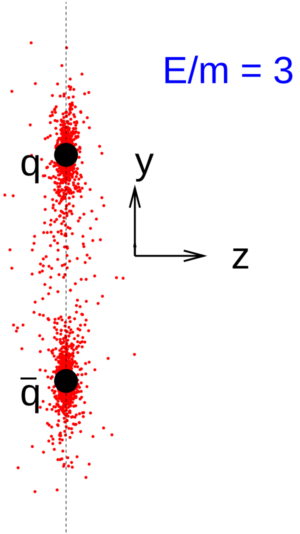

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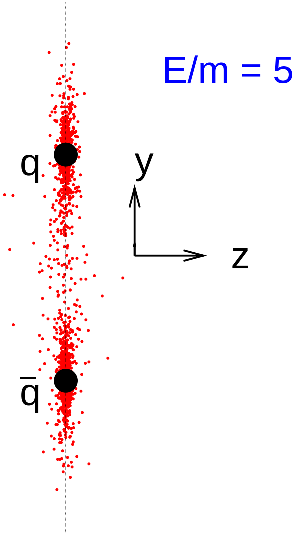

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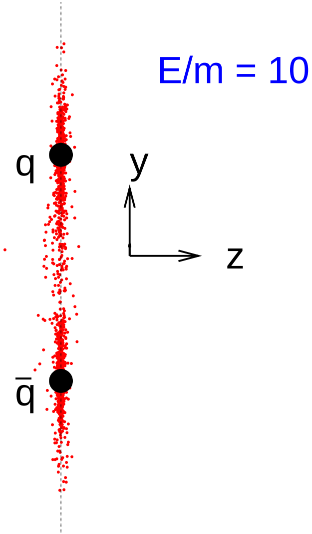

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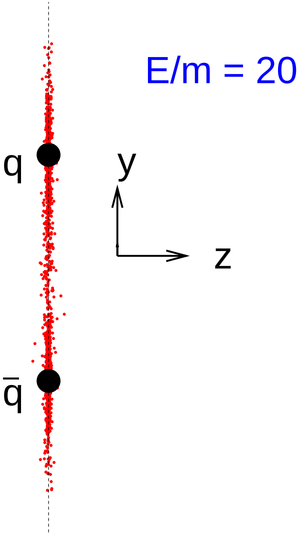

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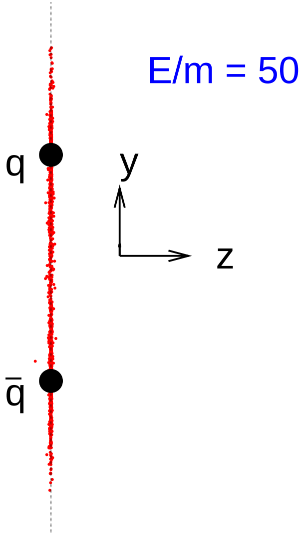

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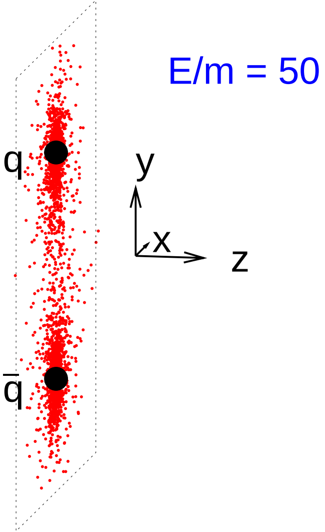

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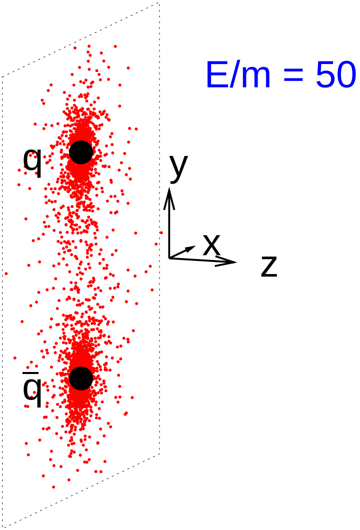

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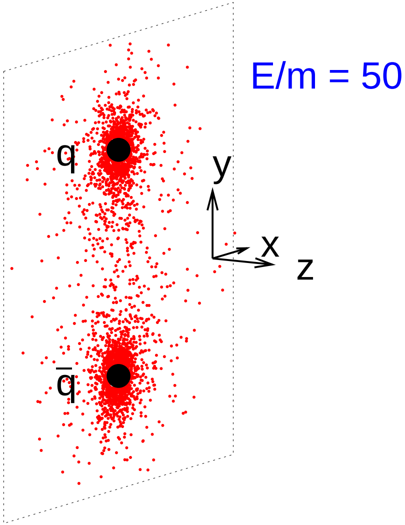

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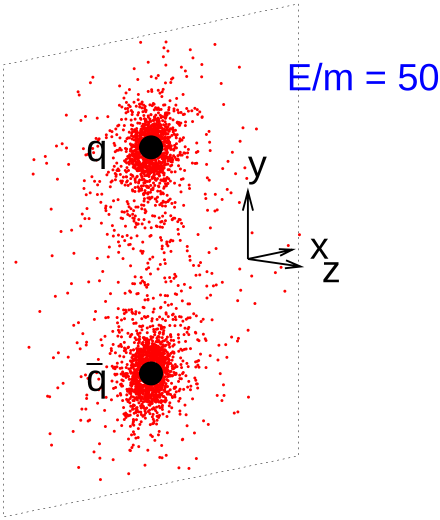

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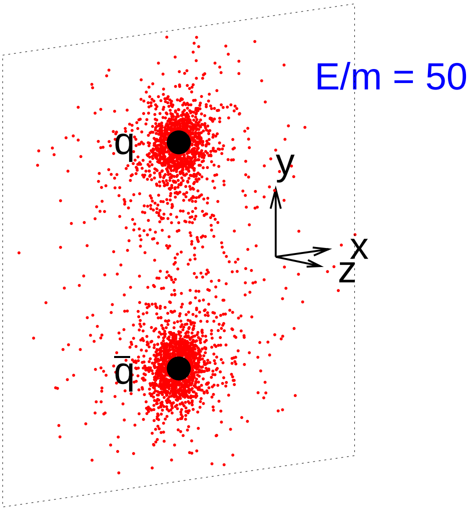

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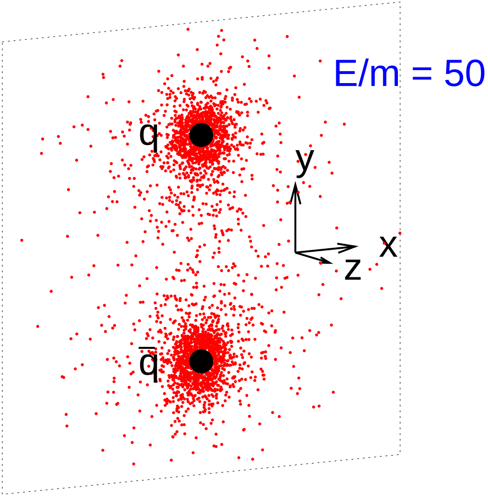

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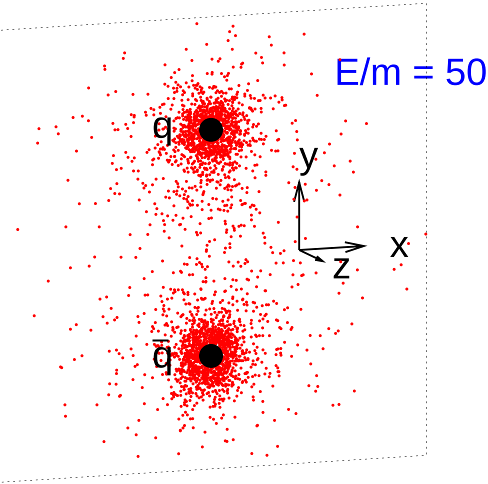

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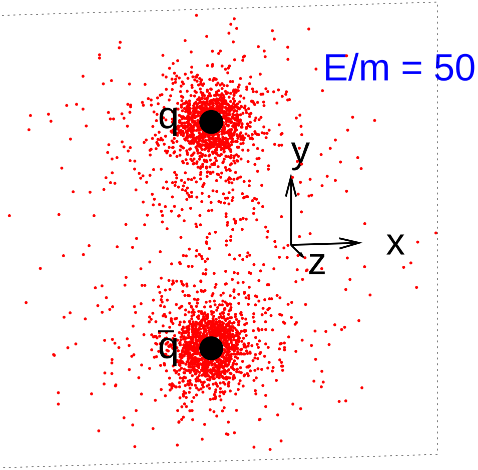

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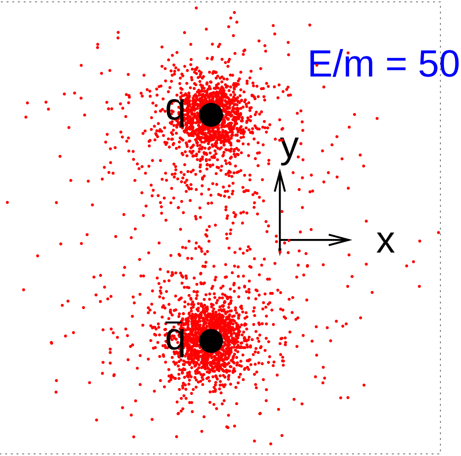

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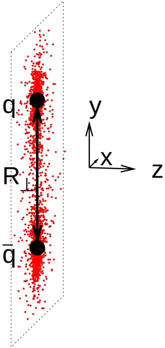
Total number of gluons

Longitudinal structure of energy density (N_c = # of colours):

$$\frac{d\epsilon}{dz} \sim \frac{\alpha_s N_c}{\pi} \times E\delta(z) \times \text{transverse}$$



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$$\frac{d\epsilon}{dp_z} \sim \frac{\alpha_s N_c}{\pi} \times \text{transverse}, \qquad m \ll p_z \ll E$$

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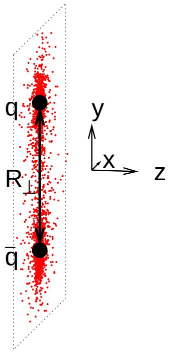

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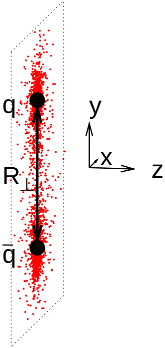

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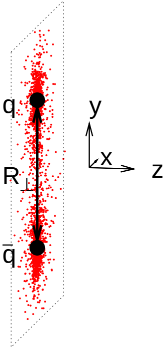

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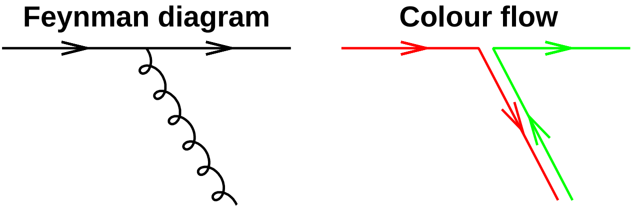
High-energy limit $\sqrt{s}, E \to \infty$

- Calculation so far is first-order perturbation theory.
- Fixed order perturbation theory is reliable if series converges quickly.
- At high energies, $n \sim \alpha_s \ln E \sim 1$.
- What happens with higher orders?

$$(\alpha_s \ln E)^n?$$

Leading Logarithms (LL). Any fixed order potentially non-convergent...

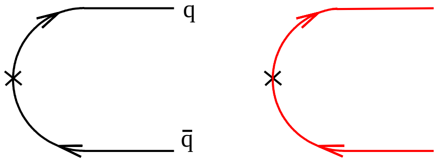
Large-N_c approx.



- Quarks come in 3 'colours' ($N_c = 3$). Gluons emission 'repaints' the colour of the quark.
- i.e. gluon carries away one colour and brings in a different one [this simple picture $\equiv$ approx of many colours].
- gluon itself is charged with both colour and anti-colour [*c.f.* two lines with different directions].

Multiple gluon emission

Start with bare $q\bar{q}$ dipole:



Emission of 1 gluon is like QED case — modulo additional colour factor (number of different ways to repaint quark):

$$\alpha \to \alpha_s N_c/2$$ (approx)

- In QED subsequent photons are emitted by *original dipole*
- In QCD original dipole is converted into two new dipoles, which *emit independently*.

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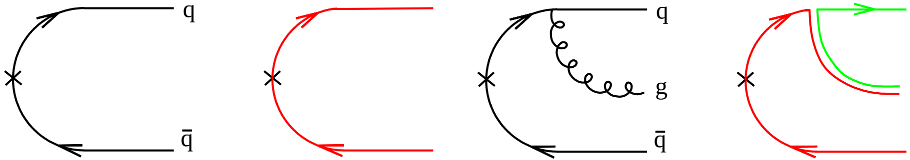

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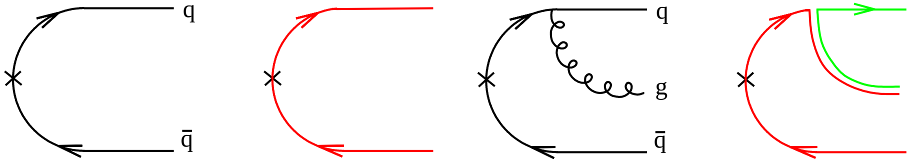

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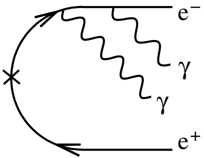

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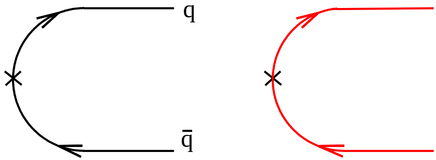

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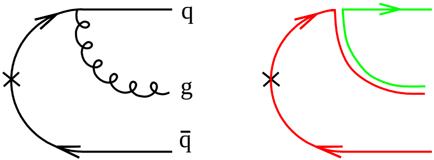

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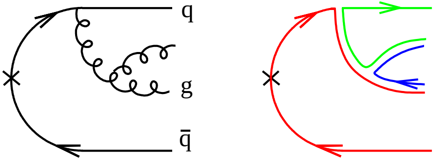

Towards evolution equation

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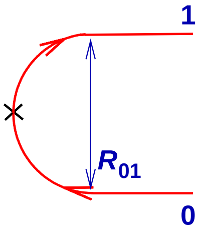
- Instead examine *total* number of dipoles as a function of energy:



Start with dipole of size R_{01}.

Define *number of dipoles of size r* obtained after evolution in energy to a *rapidity* $Y = \ln s$:

$$n(Y; R_{01}, r)$$

- Write an equation for the *evolution* of $n(Y; R_{01}, r)$ with energy.

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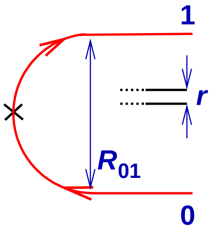

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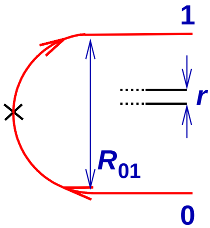

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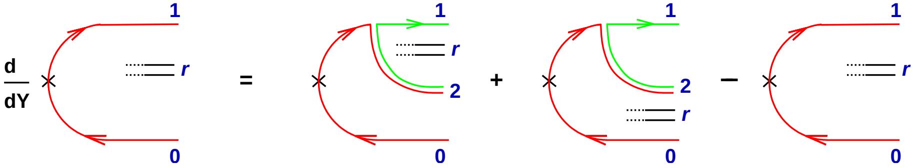
Dipole evolution equation



$$\frac{\partial n(Y; R_{01}, r)}{\partial Y} = \frac{\alpha_s N_c}{2\pi^2} \int \frac{d^2 R_2 \, R_{01}^2}{R_{02}^2 R_{12}^2} \left[n(Y; R_{12}, r) + n(Y; R_{02}, r) - n(Y; R_{01}, r) \right]$$

Transverse struct:
2-dim dipole-field (squared)

Balitsky-Fadin-Kuraev-Lipatov (BFKL)

Formulation of Mueller + Nikolaev & Zakharov '93

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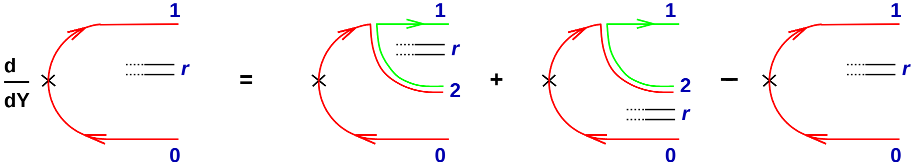
Dipole evolution equation



$$\frac{\partial n(Y; R_{01}, r)}{\partial Y} = \frac{\alpha_s N_c}{2\pi^2} \int \frac{d^2 R_2\, R_{01}^2}{R_{02}^2 R_{12}^2} \left[n(Y; R_{12}, r) + n(Y; R_{02}, r) - n(Y; R_{01}, r) \right]$$

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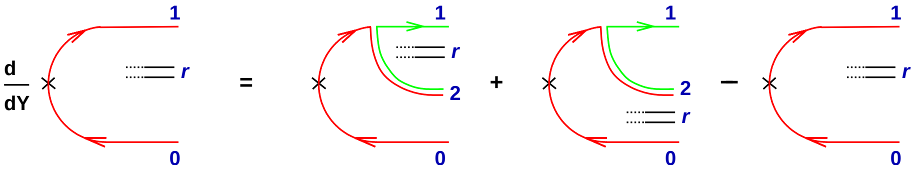

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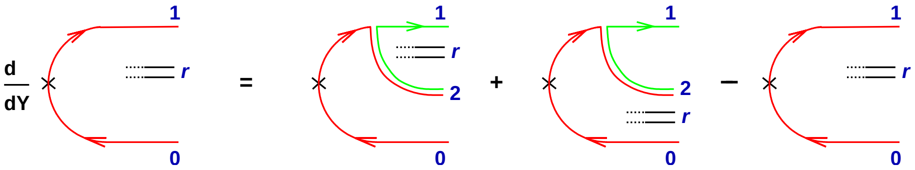

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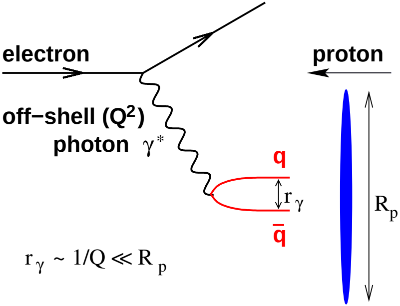
Simplest case is *double asymptotic limit:* $\ln s \sim e^Y \ll 1$ & $r \ll R$.



This is just *Deep Inelastic Scattering* at small longitudinal momentum fraction x:

$$\frac{1}{x} \sim \frac{s}{Q^2} \gg 1$$

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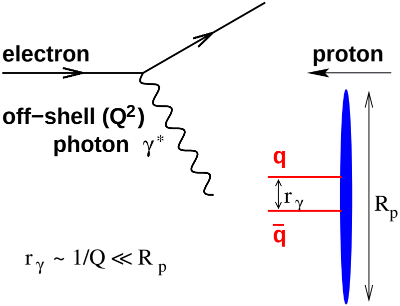

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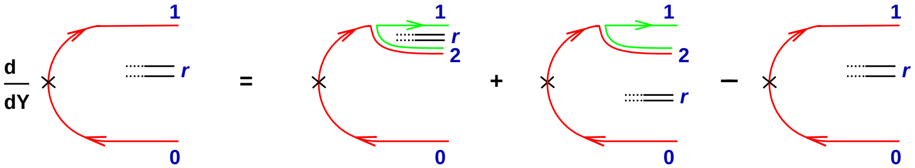

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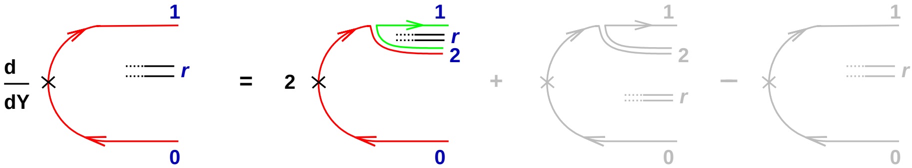

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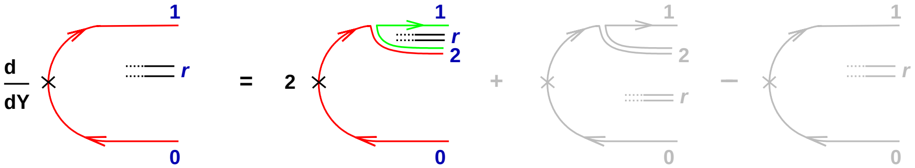

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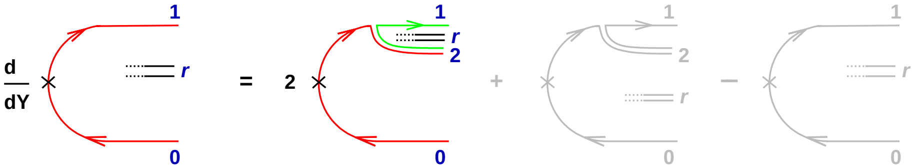

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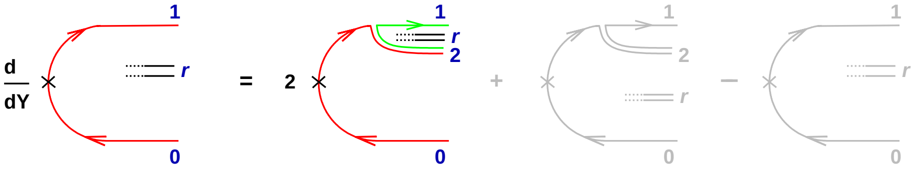

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Double Log (DL) Solution

Make *zeroth order approx:* $n^{(0)}(Y; R, r) = \Theta(R - r)$

count number of dipoles larger than r

Solve *iteratively* to get j^{th} order contribution:

$$n^{(j)}(Y; R, r) = \bar{\alpha}_s \int_0^Y dy \int_r^R \frac{dR'^2}{R'^2} n^{(j-1)}(y; R', r)$$

Result:

$$n^{(j)}(Y; R, r) = \bar{\alpha}_s^j \frac{Y^j}{j!} \frac{(\ln R^2/r^2)^j}{j!}$$

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Do sum:

$$n(Y; R, r) = \sum_{j=0}^{\infty} \frac{(\bar{\alpha}_s Y \ln R^2/r^2)^j}{(j!)^2} \sim \exp\left[2\sqrt{\bar{\alpha}_s Y \ln R^2/r^2}\right]$$

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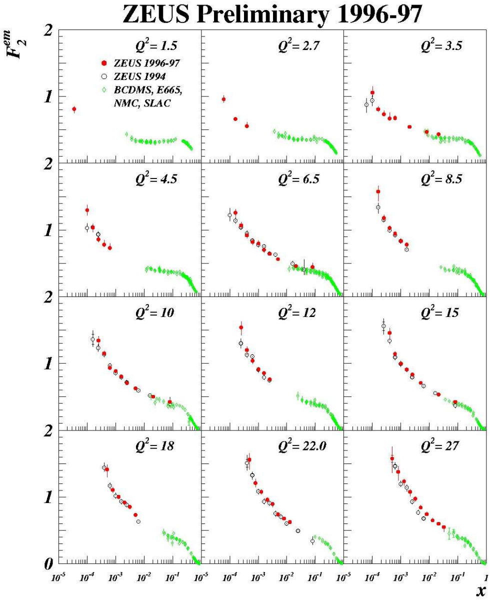
Test in Deep Inelastic Scattering



DIS X-sctn $\sim$ n dipoles:

$$F_2(x, Q^2) \sim n(\ln \frac{1}{x}; \frac{1}{\Lambda^2}, \frac{1}{Q^2})$$

$$\sim \exp \left[\frac{2}{\beta_0^2} \sqrt{\ln \frac{1}{x} \ln \ln \frac{Q^2}{\Lambda^2}} \right]$$

- Growth of cross section at small x
- Faster growth for high Q^2

NB: truly predict **features** of x-dependence, even for non-perturbative (NP) proton, since NP uncertainty $\equiv$ rescaling of Λ

+ can be made quantitative

(Ball & Forte '94–96)

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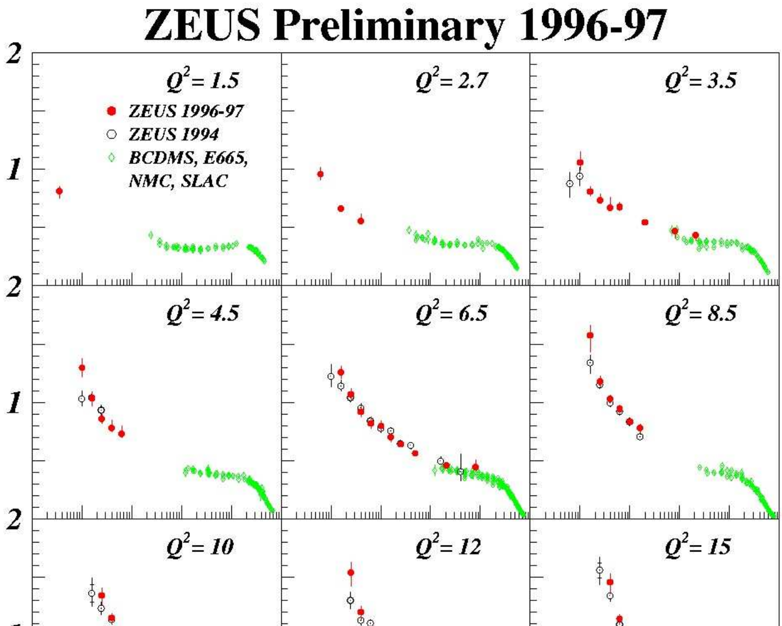

DIS X

$F_2($

$\sim$ e

- Gr
sm
- Fas

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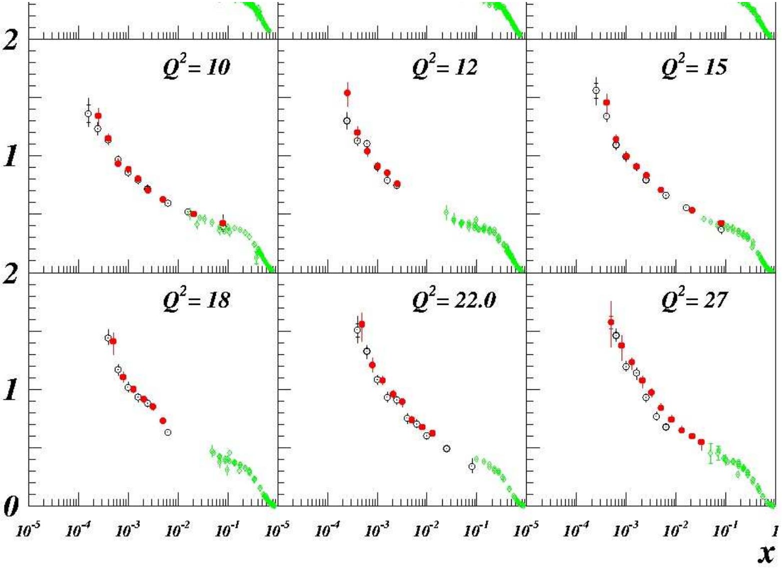

sm

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NB:

x-dep

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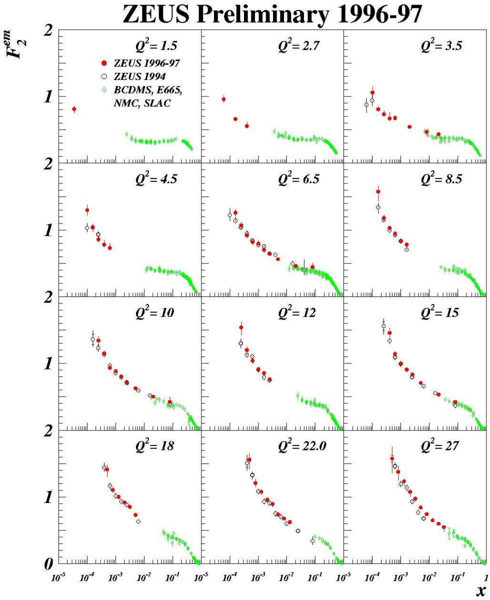

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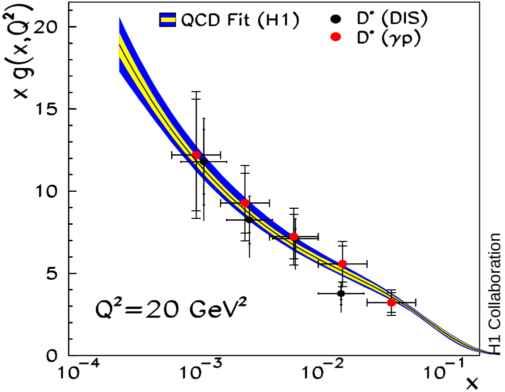
Large number of gluons



- Convert cross sections into estimate of number of gluons
- Various independent extractions
- *Up to 20 gluons per unit* $\ln x$ *(or unit* $\ln p_z$*)!*

NB: at resolution Q^2, area occupied by gluon $\sim 1/Q^2$ (area of proton $\sim 1/\Lambda^2$) $\Rightarrow$ the many gluons are *spread out thinly,*

density $\sim xg(x) \times \Lambda^2/Q^2 \lesssim 1$

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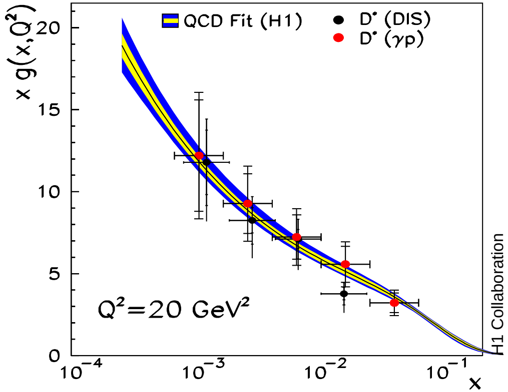

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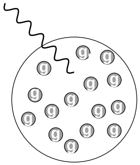

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True high-energy limit

Double-Log limit had $\ln s$ and $\ln Q^2$ growing *simultaneously.*

True high-energy limit is when c.o.m. energy $\sqrt{s} \gg$ *all other scales:*

$$\perp \text{ scale} = \text{fixed} \quad \textit{and} \quad \ln s \to \infty$$

Since all $\perp$ scales similar, problem is *self-similar:*

$$\text{dipole} \to 2 \text{ dipoles} \to 4 \text{ dipoles} \to \dots$$

Expect exponential growth:

$$n \sim \exp\left[\bar{\alpha}_s \ln s \times \text{transverse}\right] \sim s^{\bar{\alpha}_s \times \text{transverse}}$$

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BFKL eigenfunctions

BFKL equation is linear & homogeneous, kernel is *conformally invariant*

$$\frac{\partial n(Y;R_{01},r)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi}\int \frac{d^2R_2\, R_{01}^2}{R_{02}^2 R_{12}^2}\left[n(Y;R_{12},r)+n(Y;R_{02},r)-n(Y;R_{01},r)\right]$$

It has power-like *eigenfunctions:*

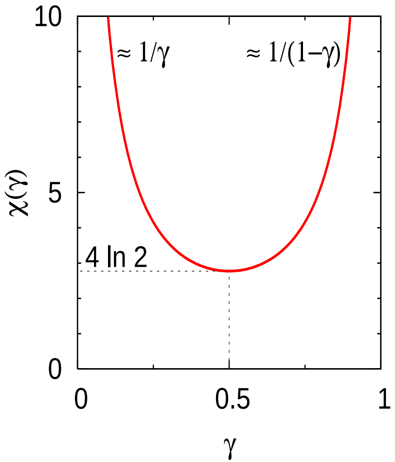
$$n(Y;R,r) = n_\gamma(Y)\left(\frac{R^2}{r^2}\right)^\gamma ,$$

which evolve exponentially (as expected):

$$\frac{\partial n_\gamma(Y)}{\partial Y} = \bar{\alpha}_s \chi(\gamma) n_\gamma(Y) \qquad \Rightarrow \qquad n_\gamma(Y) \propto \exp\left[\bar{\alpha}_s \chi(\gamma) Y\right]$$

$$\left[\underbrace{\chi(\gamma) = 2\psi(1)-\psi(\gamma)-\psi(1-\gamma)}_{\text{characteristic function}}, \qquad \psi(\gamma) = \frac{1}{\Gamma(\gamma)}\frac{d\Gamma(\gamma)}{d\gamma}\right]$$

Characteristic function



Eigenvalues for $(R^2/r^2)^\gamma$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

$\rightarrow$ high energy evolution, $n \sim e^{\bar{\alpha}_s \chi(\gamma) Y}$.

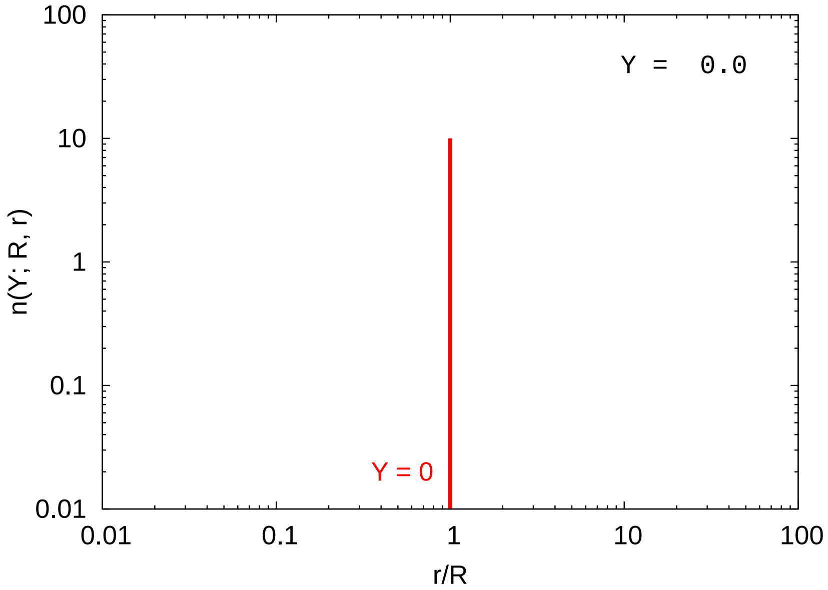
- pole $(1/\gamma)$ corresponds to $\perp$ logarithms $\rightarrow$ DL terms $\alpha_s Y \ln Q^2$
- dominant part at high energies is *minimum* (only stable solution)

$$n(Y; R, r) \sim \frac{R}{r} e^{4 \ln 2 \bar{\alpha}_s Y} \sim \frac{R}{r} e^{0.5 Y}$$

$\alpha_s \simeq 0.2$

Rapid power growth with energy of number of dipoles (and cross sections).

BFKL eqn solved numerically



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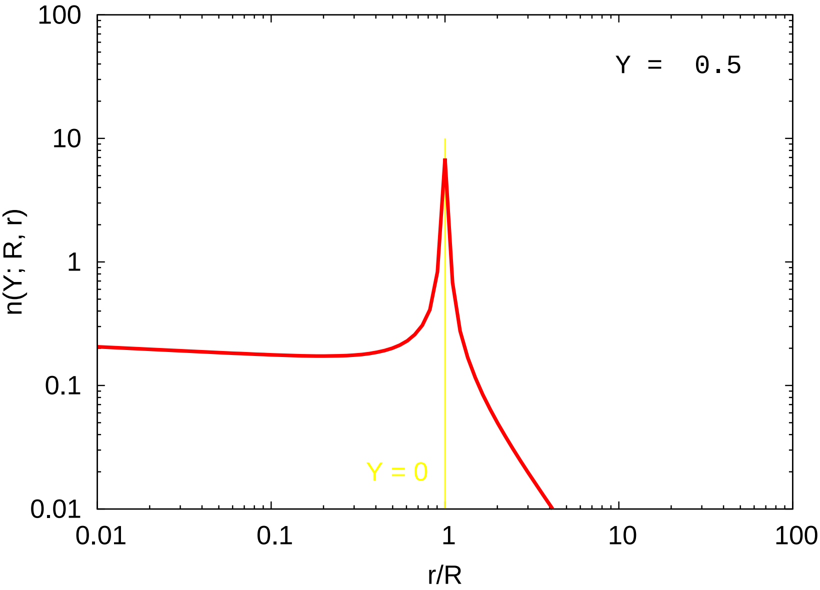

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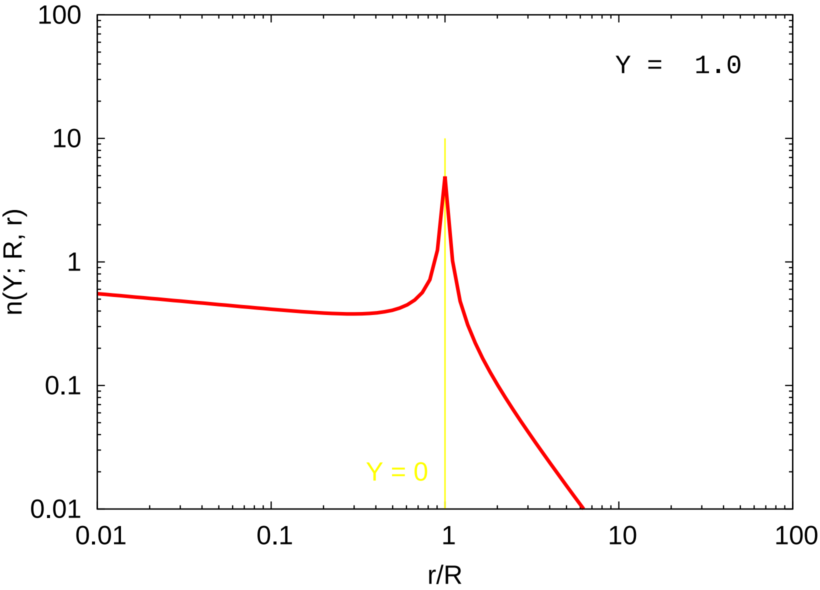

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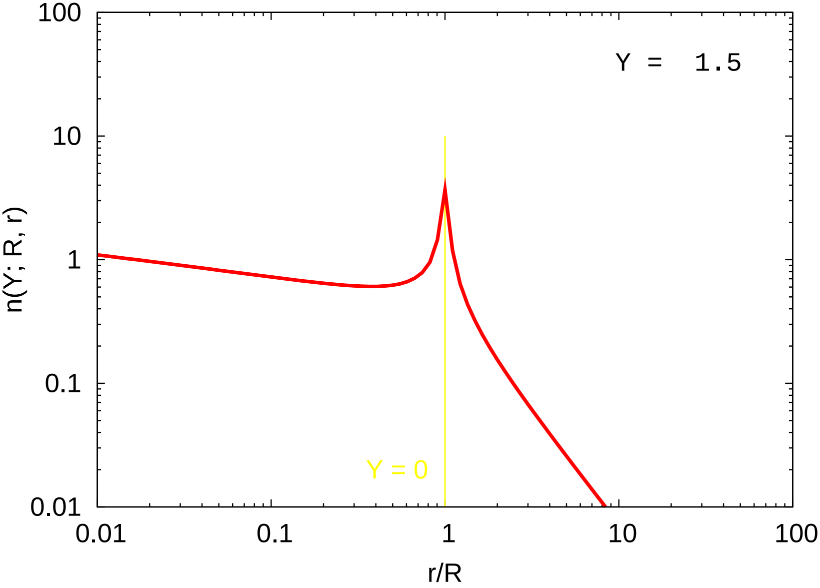

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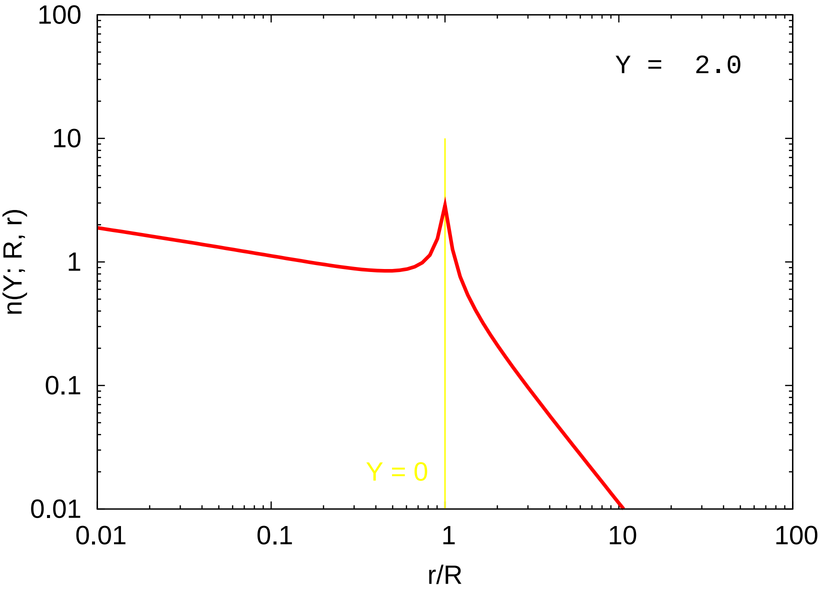

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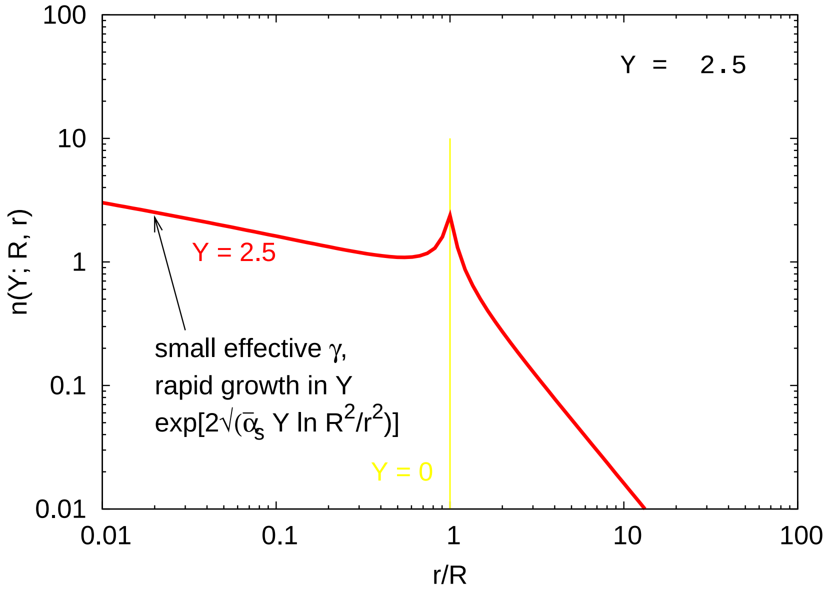

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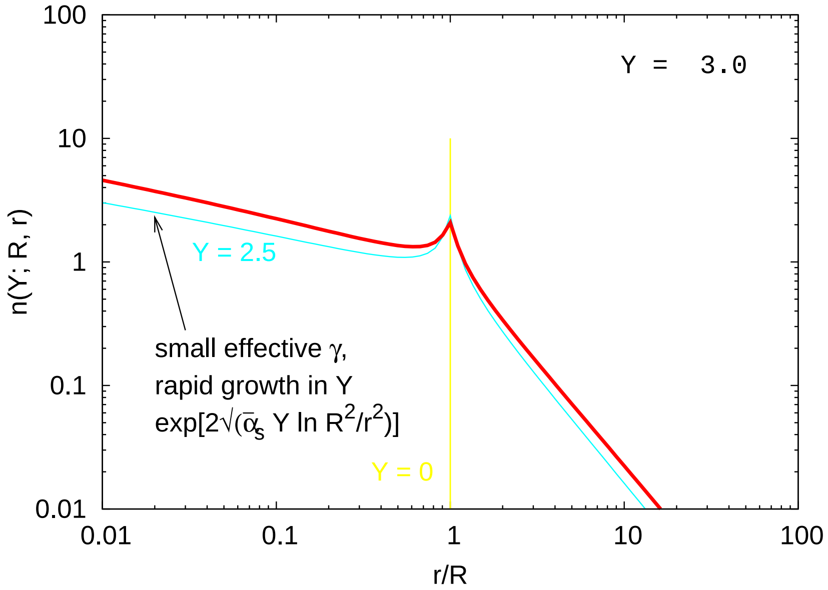

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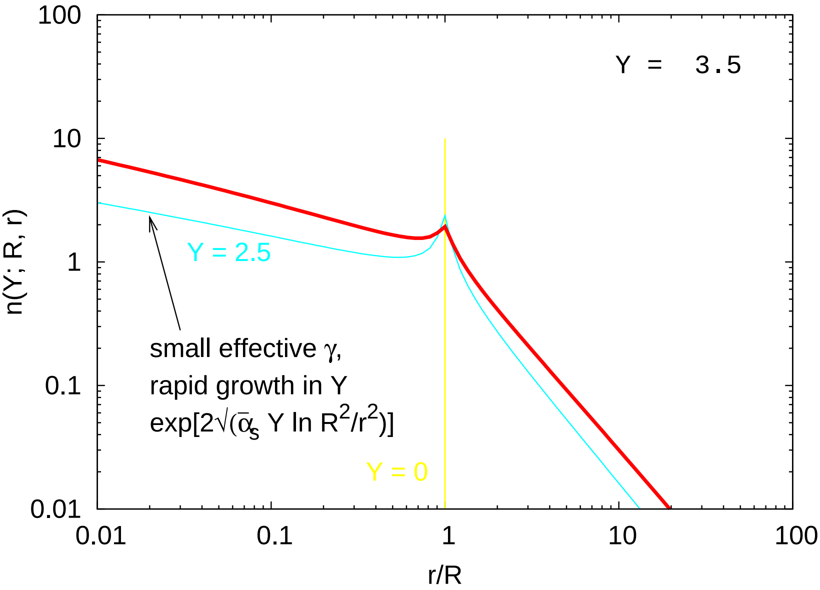

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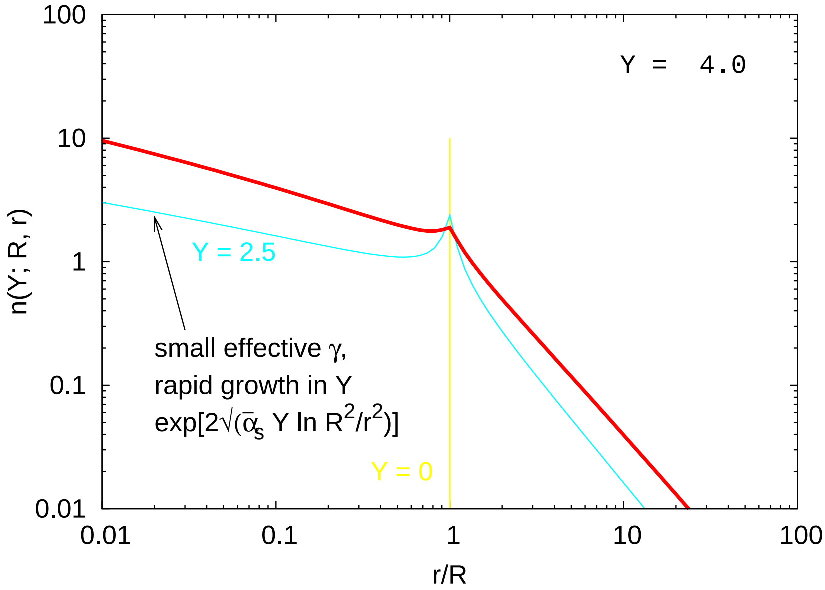

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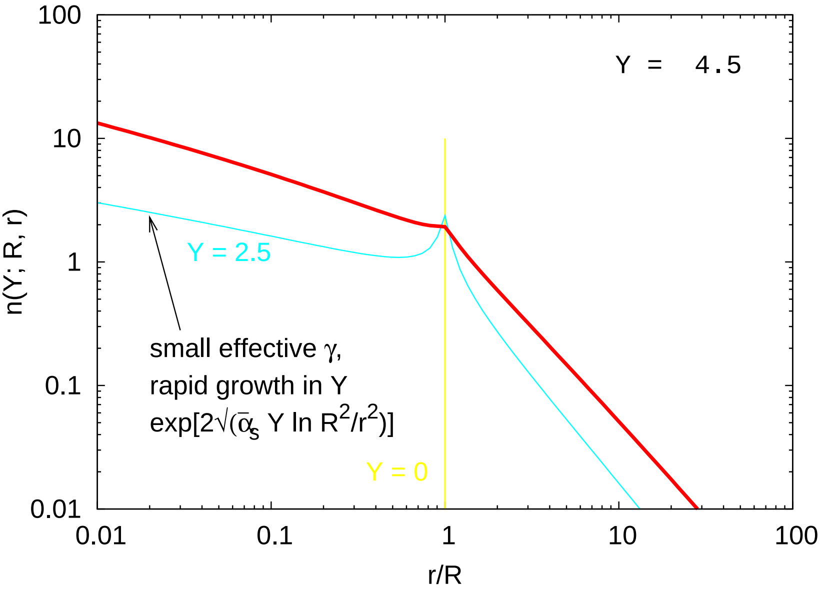

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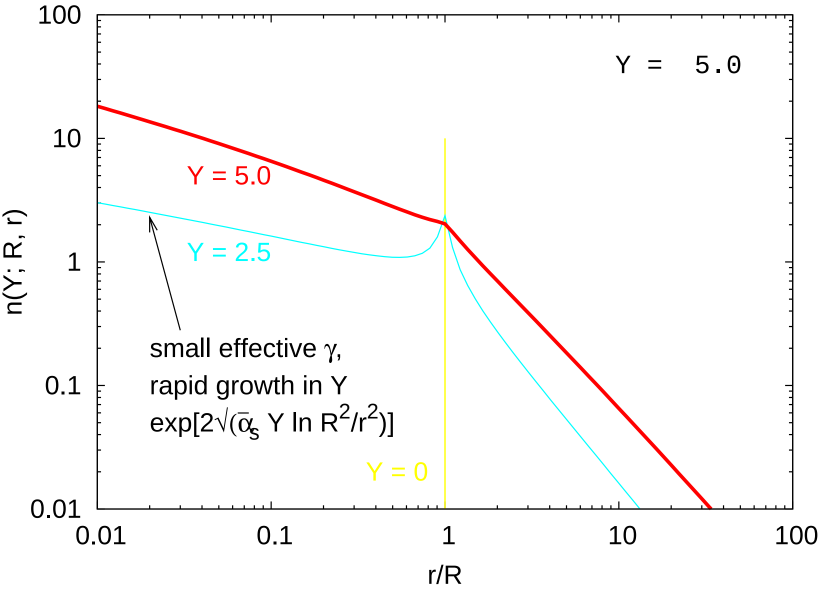

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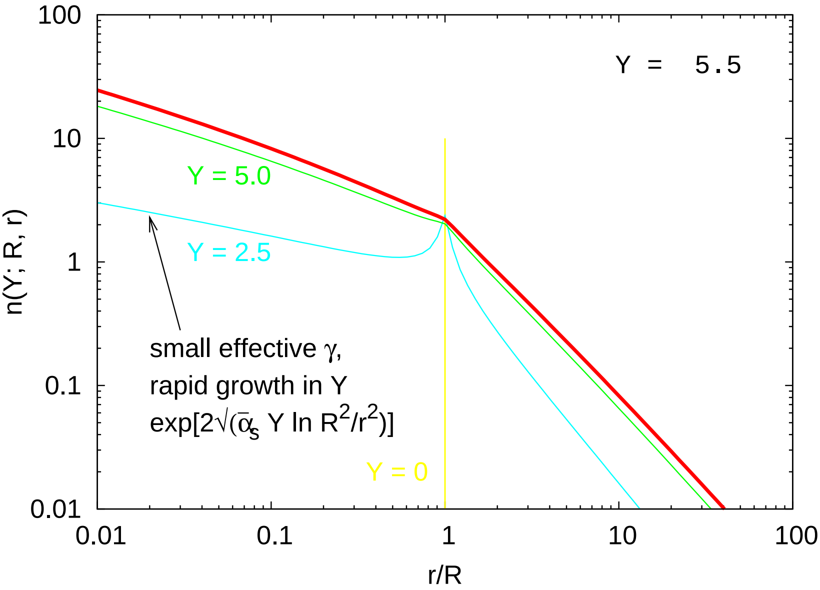

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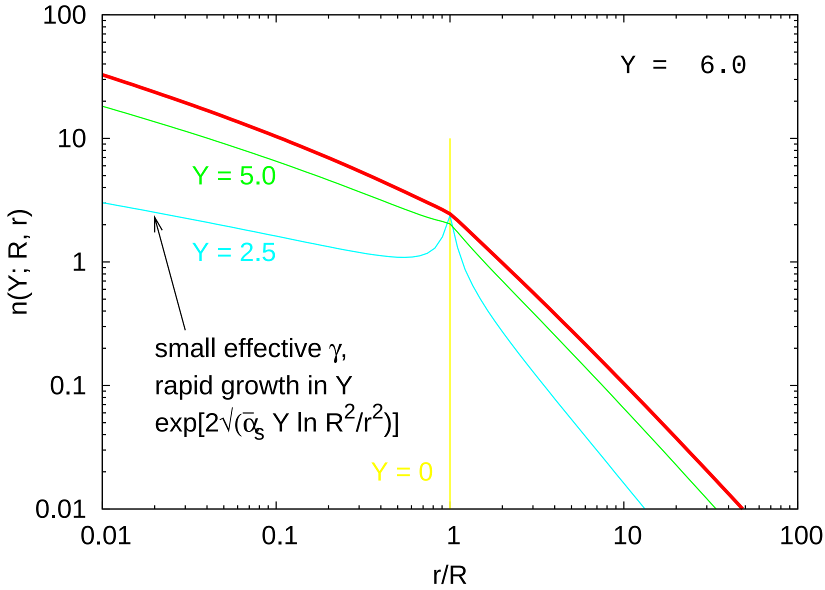

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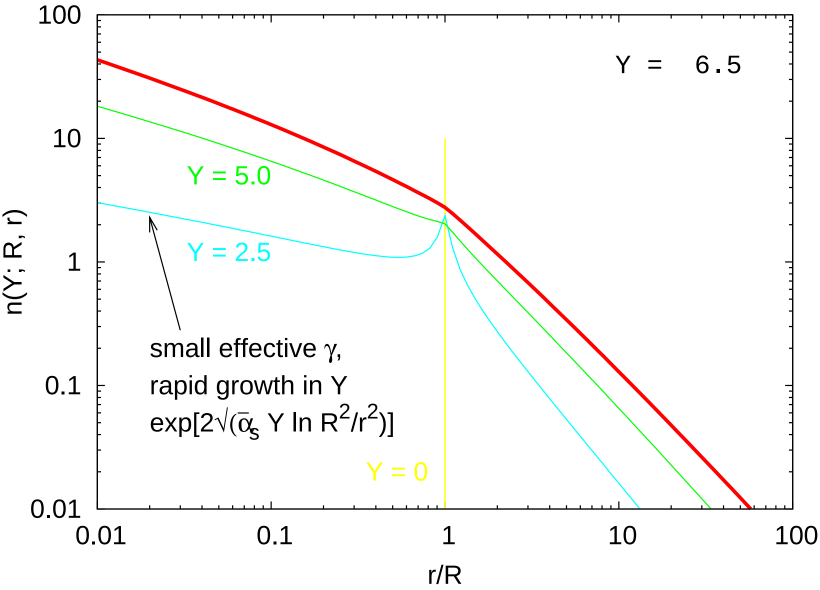

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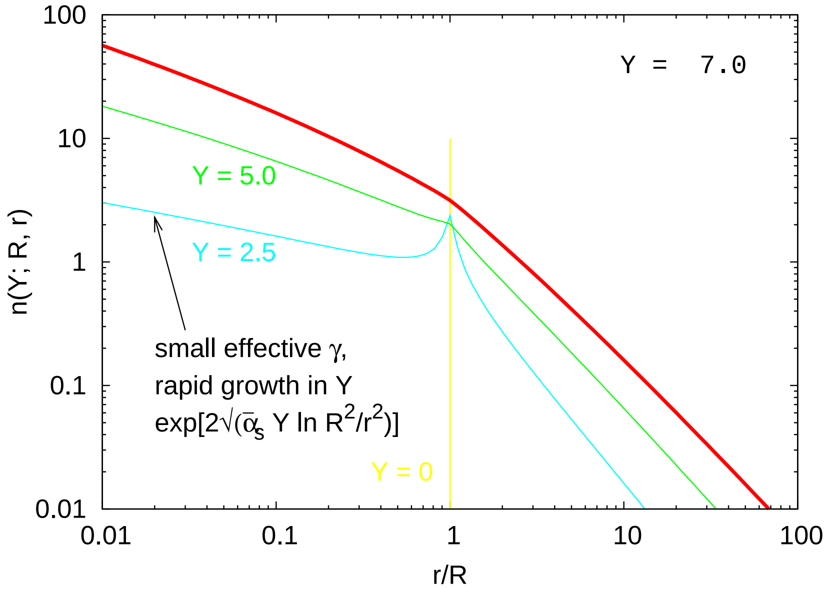

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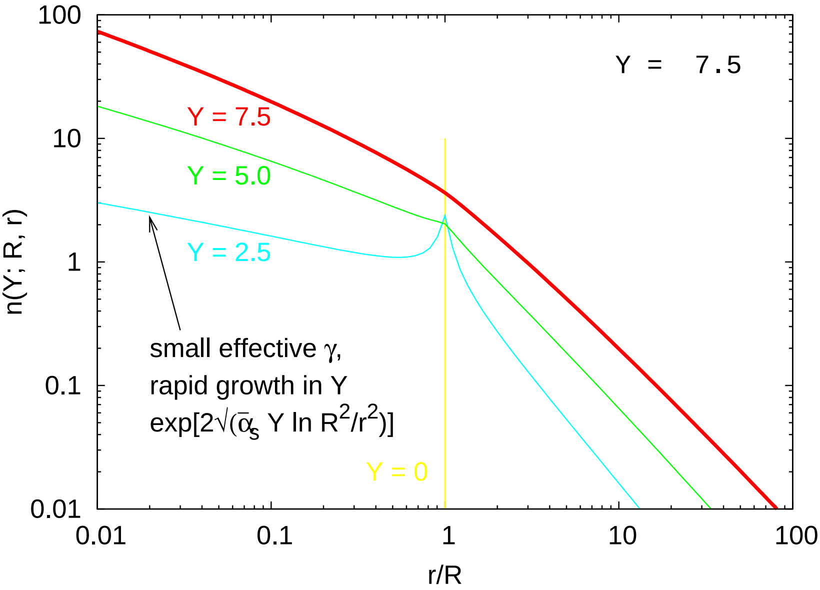

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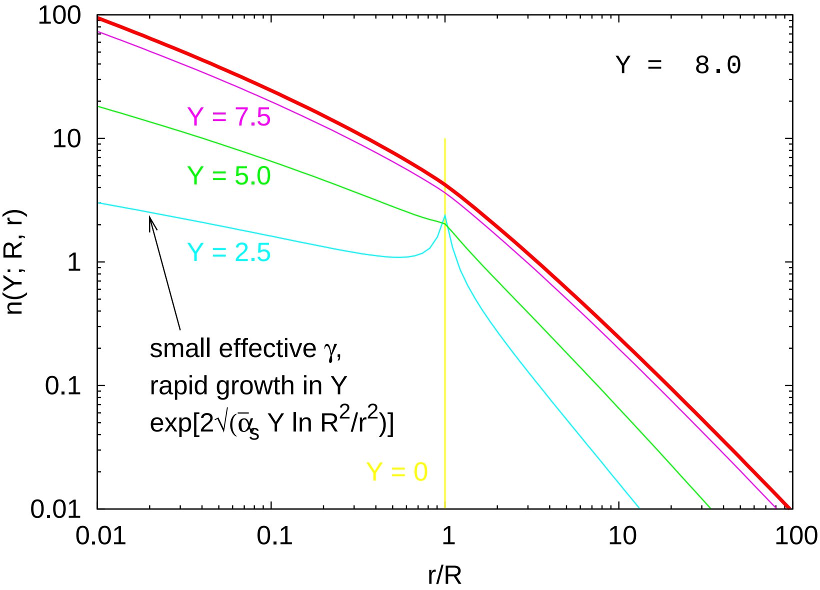

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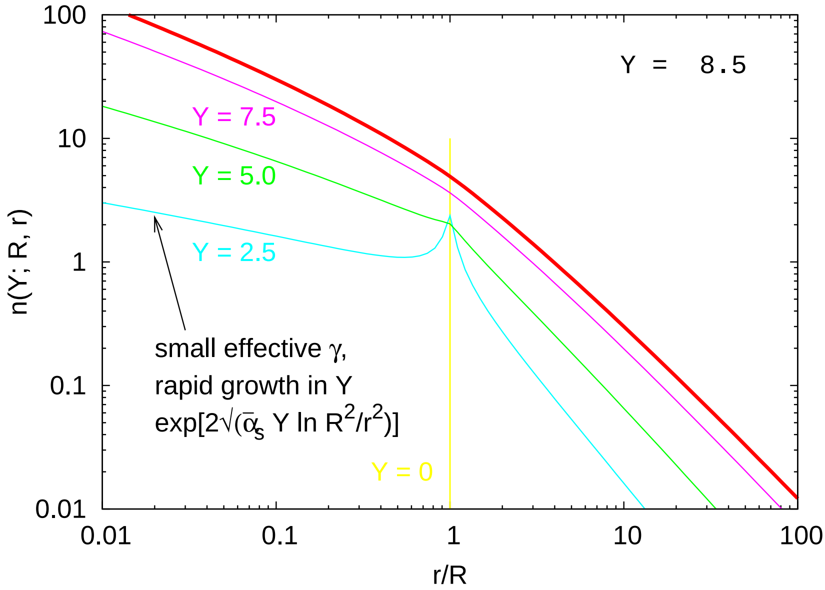

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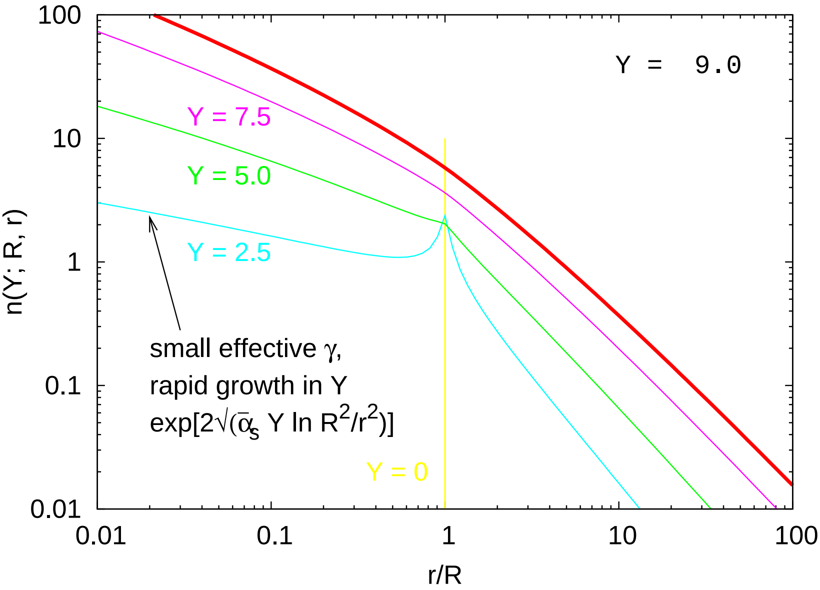

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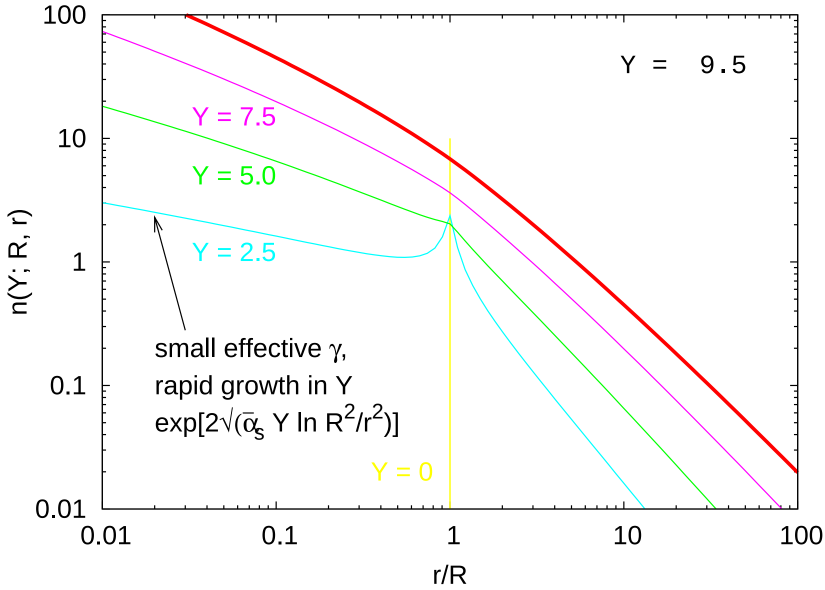

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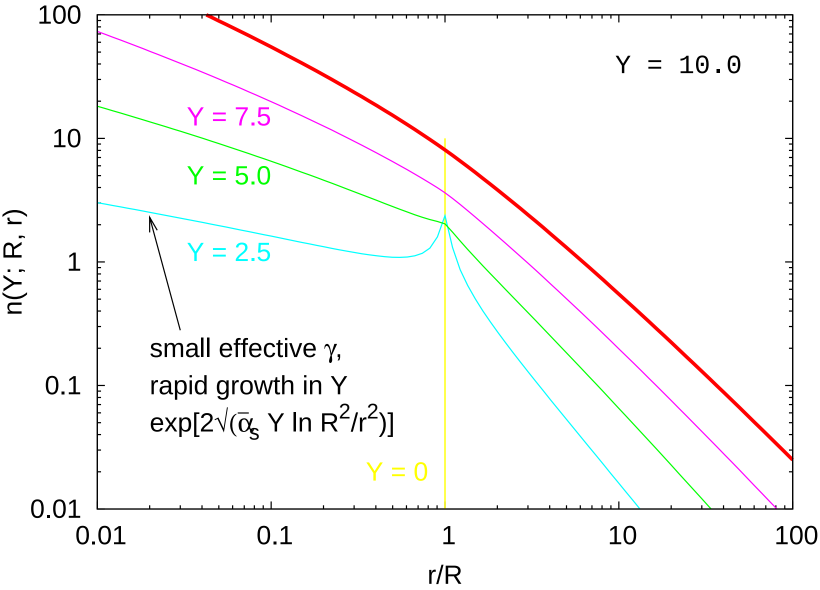

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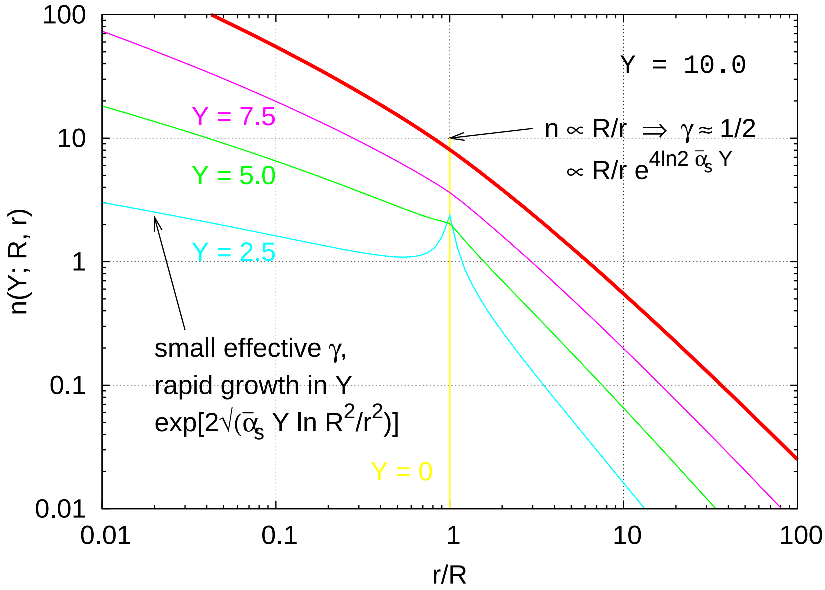

Look for BFKL in F_2 [$\gamma^* p$ X-sct]

BFKL 'predicts' (for low Q^2)

$$F_2(x, Q^2) \sim e^{4 \ln 2 \alpha_s Y} \sim x^{-0.5}$$

Fit λ in $F_2(x, Q^2) \sim x^{-\lambda(Q^2)}$.

Expect to find $\lambda \simeq 0.5$

may be larger at high Q^2 (DL)

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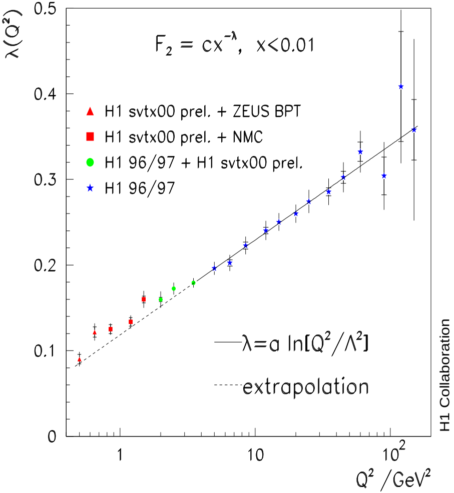

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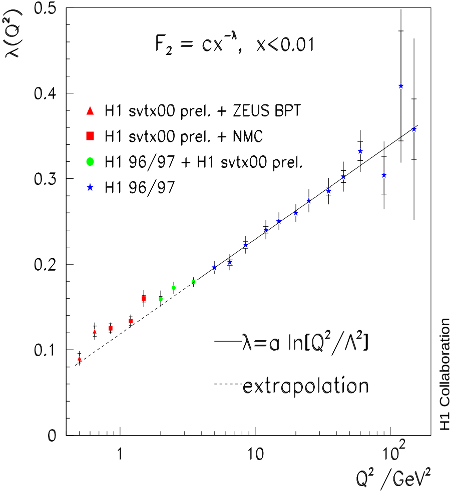
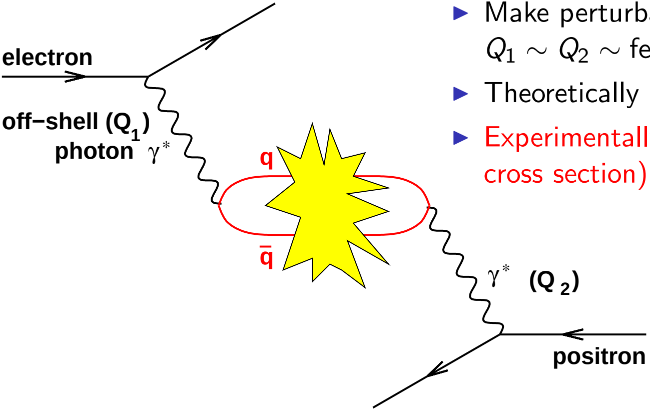


What's wrong?

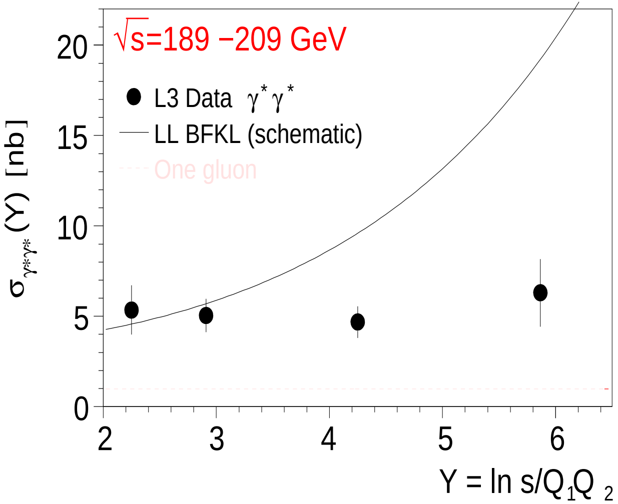
- proton is non-perturbative (NP)
- BFKL dynamics naturally concentrated at (NP) scales
- NB: DLs spread over range of scales $\Rightarrow$ less sensitive to NP region

$\gamma^*\gamma^*$ collisions



- Eliminate ratios of transverse scales by colliding two virtual photons $Q_1 \sim Q_2$
- Make perturbative by choosing $Q_1 \sim Q_2 \sim$ few GeV
- Theoretically clean
- Experimentally difficult (small cross section)

Results from LEP



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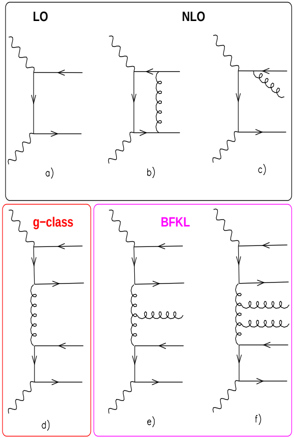

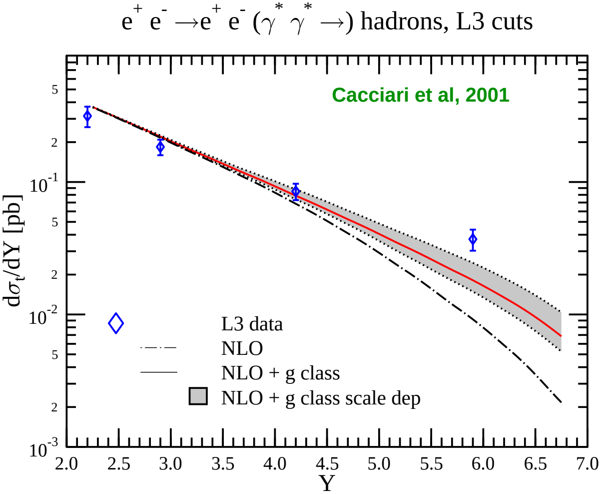

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Where is BFKL?

- BFKL is rigorous prediction of field theory, yet not seen in data
- Should we be worried?
- Calculations shown so far are in Leading Logarithmic (LL) approximation, $(\alpha_s \ln s)^n$: accurate only for

$$\alpha_s \to 0, \ln s \to \infty \text{ and } \alpha_s \ln s \sim 1.$$

- Need higher order corrections

Next-to-Leading-Logarithmic (NLL) terms: $\alpha_s(\alpha_s \ln s)^n$

Fadin, Lipatov, Fiore, Kotsky, Quartarolo; Catani, Ciafaloni, Hautmann, Camici; '89–'98

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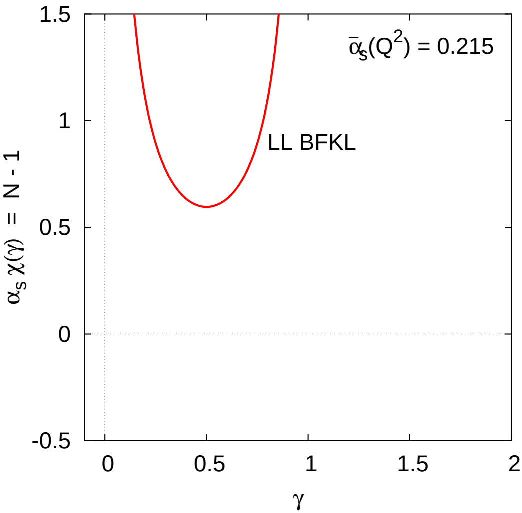
Higher orders in kernel. . .



Examine $\bar{\alpha}_s\chi(\gamma)$

minimum = BFKL power

$$\chi(\gamma) = \underbrace{\chi_0(\gamma)}_{LL} + \underbrace{\bar{\alpha}_s\chi_1(\gamma)}_{NLL} + \dots$$

- NLL terms are *pathologically large*
  oscillating X-sctns, . . .
- $\exists$ other constraints
  - DGLAP for $\gamma \sim 0$
  - symetries for $\gamma \sim 1$
- Assemble all constraints $\rightarrow$ stable, sensible kernel
  Ciafaloni, Colferai, GPS & Stasto; Altarelli, Ball & Forte; '98–'05

NB: DGLAP = 'rotated' plot of $\gamma(N)$

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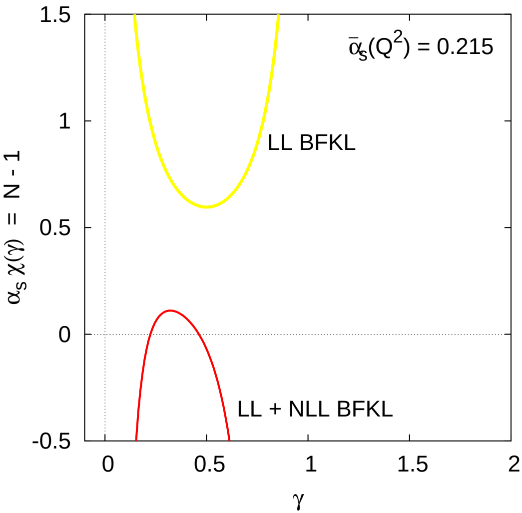

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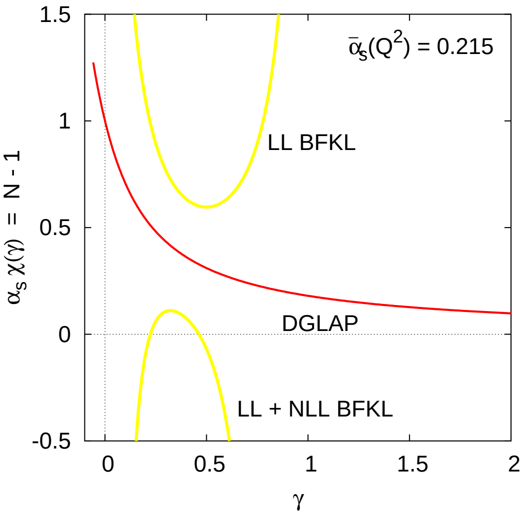

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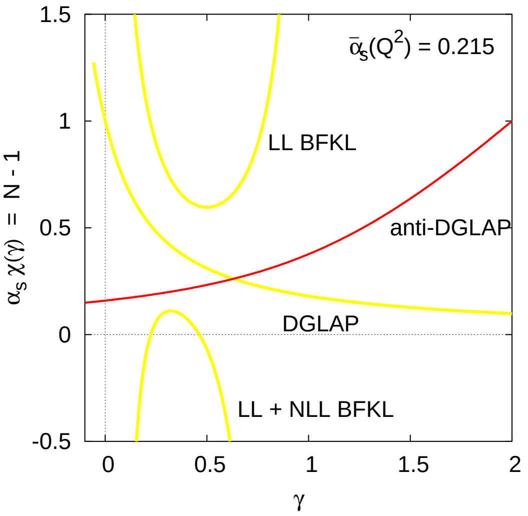

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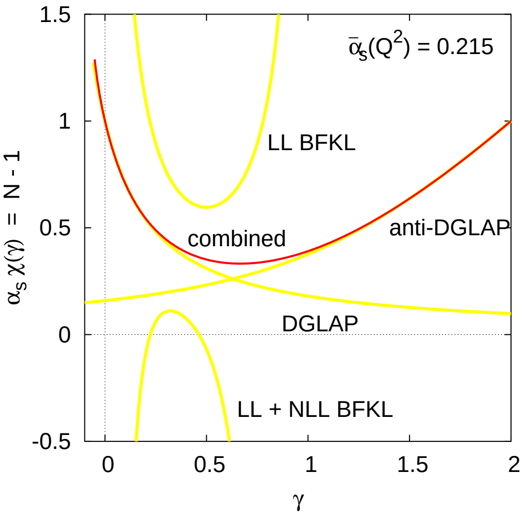

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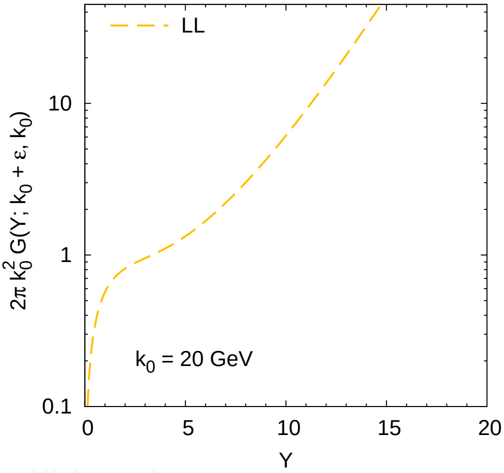

Examine solutions at LL, NLL, etc.

$G(Y; k, k_0)$ = Fourier transform of $n(Y; R, r)$

- LL grows rapidly with Y
- NLL unstable wrt subleading changes
- DGLAP-symmetry constrained higher-orders (schemes A, B) give stable predictions

- Higher orders
  - slow onset of growth ($Y \gtrsim 5$)
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- Detailed comparison with data not yet done — parts of NLL ('impact factors') missing
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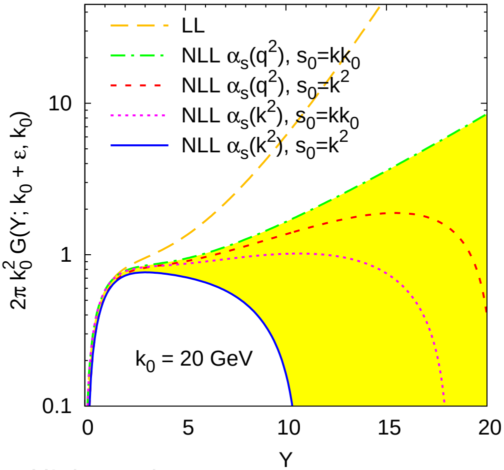

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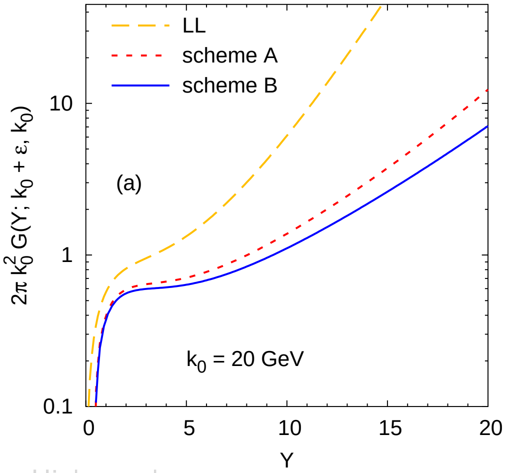

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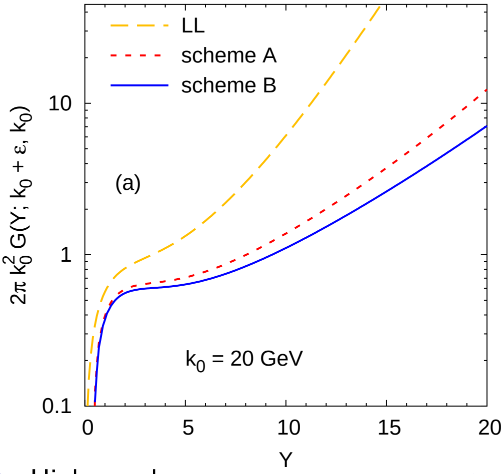

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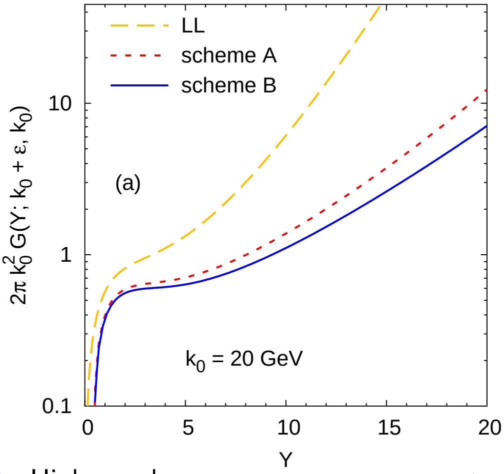

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What about protons?

- Higher-order corrections are sufficient to explain lack of growth in $\gamma^*\gamma^*$ data ($Y \lesssim 6$).
  NB: LHC and International Linear Collider can test perturbative BFKL up to $Y \simeq 10$
- But pp and low-Q^2 DIS go to higher energies, $Y \simeq 10 - 14$.
  NLL BFKL (+ DGLAP constraints) predicts $\sigma \gtrsim s^{0.3}$ by such energies.
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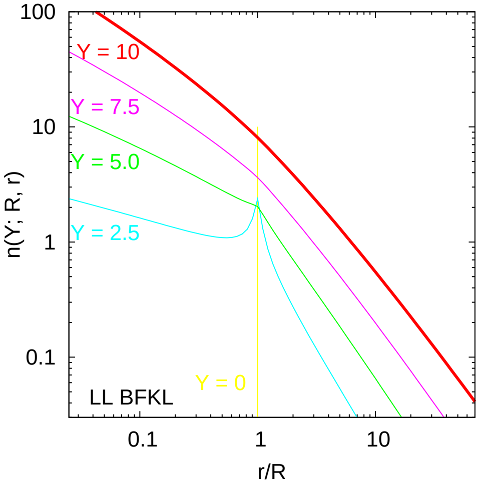
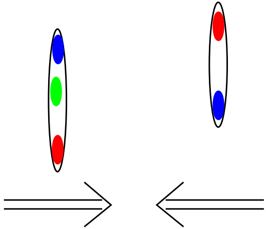
Two mechanisms for growth of σ



Cross sections grow:

- Increase in number of dipoles $r \sim R$
- Increase in size of biggest dipoles $r_{\max}$.



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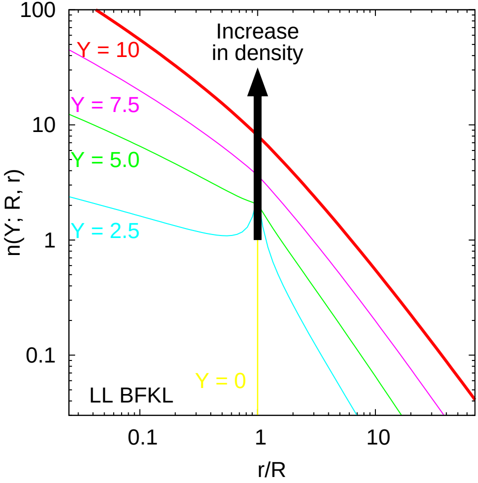

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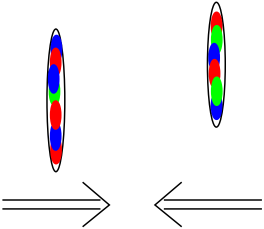

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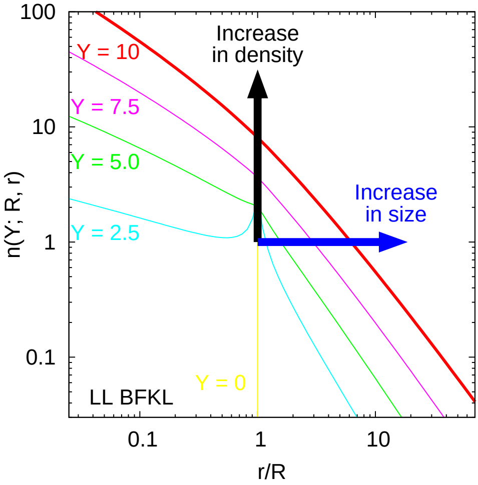

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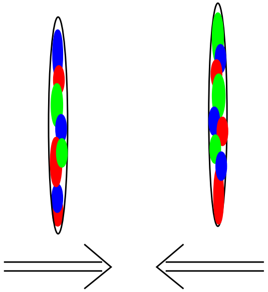

Density of gluons cannot increase indefinitely

- When dipole density is high ($\sim N_c/\alpha_s$) dipole branching compensated by *dipole merging* $\rightarrow$ *saturation of density*
- Reach maximxal 'occupation number' Colour Glass Condensate
- Closely connected issue: *unitarity* (interaction prob. bounded, ≤ 1)

Expressed (approx. . .) in BFKL equation via non-linear term

$$\frac{\partial n(Y;R_{01})}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{d^2R_2\, R_{01}^2}{R_{02}^2 R_{12}^2} \left[n(Y;R_{12}) + n(Y;R_{02}) - n(Y;R_{01}) - c\alpha_s^2 n(Y;R_{12}) n(Y;R_{02}) \right]$$

Gribov Levin Ryskin '83; Balitsky '96; Kovchegov '98; JIMWLK '97–98.

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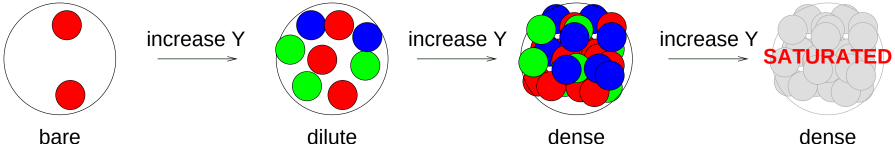

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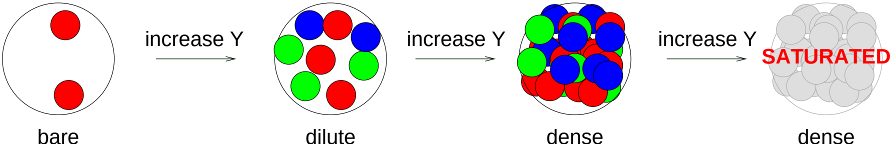

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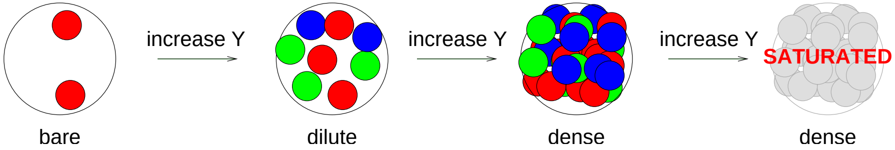


- Closely connected issue: *unitarity* (interaction prob. bounded, ≤ 1)

Expressed (approx. . .) in BFKL equation via non-linear term

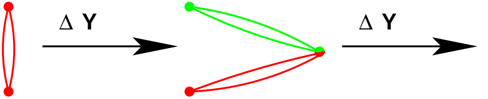
$$\frac{\partial n(Y; R_{01})}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{d^2 R_2 \, R_{01}^2}{R_{02}^2 R_{12}^2} \left[n(Y; R_{12}) + n(Y; R_{02}) - n(Y; R_{01}) - c\alpha_s^2 n(Y; R_{12}) n(Y; R_{02}) \right]$$

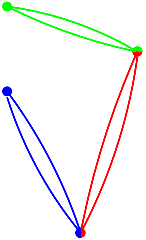
Gribov Levin Ryskin '83; Balitsky '96; Kovchegov '98; JIMWLK '97–98.

Cross-section with saturation

Kernel $\frac{R_{01}^2 d^2 \vec{R}_2}{R_{12}^2 R_{02}^2}$ is *conformally invariant* (even with non-linear term)

e.g. : Growth in area





BFKL growth is not just increase in number of gluons/dipoles.

Gluons can be produced *far* from original dipole — because of conformal (scale) invariance *each step* in Y translates to a constant *factor of increase in area.*

No other scales in problem.

Perturbative (fixed-coupling) *geometric* cross section for two dipoles in Balitsky-Kovchegov (= BFKL with saturation) grows as

$$\sigma \sim \exp\left[2.44 \times \bar{\alpha}_s Y\right] \qquad 2.44 \simeq \chi'(\bar{\gamma}) \quad \text{where} \quad \bar{\gamma}\chi'(\bar{\gamma}) = \chi(\bar{\gamma})$$

Only *marginally weaker* than $e^{4 \ln 2 \bar{\alpha}_s Y} = e^{2.77 \bar{\alpha}_s Y}$ of unsaturated BFKL.

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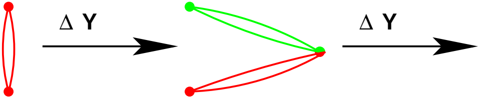

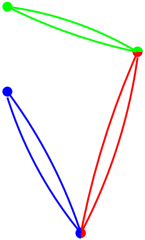

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Confinement

- Conformal invariance not an exact symmetry of high-energy QCD.
- Broken by *running of coupling.*
- For distances $\gtrsim 1/\Lambda_{QCD}$ perturbative treatment makes no sense
  - confinement sets in
  - cannot produce dipoles larger than $1/\Lambda_{QCD}$
  - exponential BFKL growth in size *stops*
  - (other than by additive amount $\sim 1/\Lambda_{QCD}$ per unit increase in Y)
- This is the semi-perturbative picture consistent with

  Froissart bound: $\sigma \sim Y^2/m_\pi^2$

  But no direct connection with $p\bar{p}$ X-section possible so far
- NB: *combination* of saturation & confinement are needed to get Froissart.

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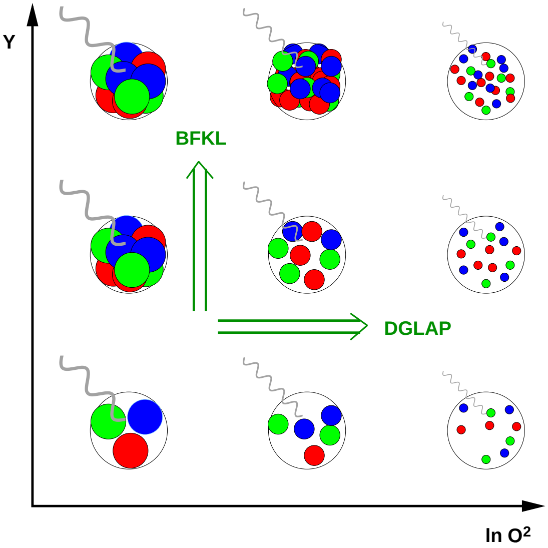
Saturation scale for proton



Plot Y-ln Q^2 plane

(as Prof. Veneziano)

Recall:

- Density $\Uparrow$ with Y
- Density $\Downarrow$ with ln Q^2

Classify:

- Dilute: $\frac{r^2}{R^2} n \lesssim \alpha_s^{-1}$
- Dense: $\frac{r^2}{R^2} n \gtrsim \alpha_s^{-1}$

Introduce boundary between them (in Q^2):

Saturation Scale
$Q_s^2(Y)$

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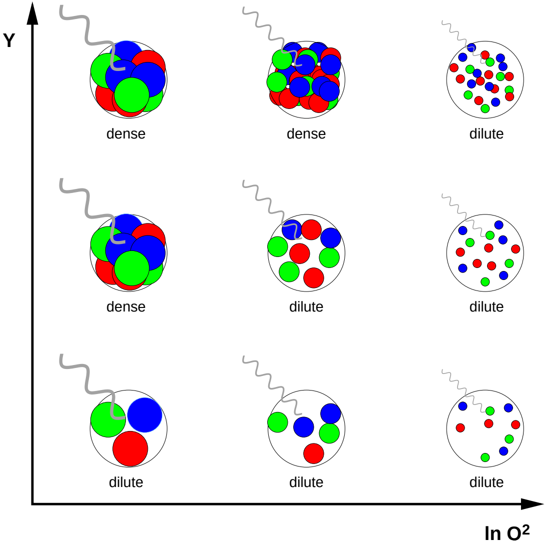

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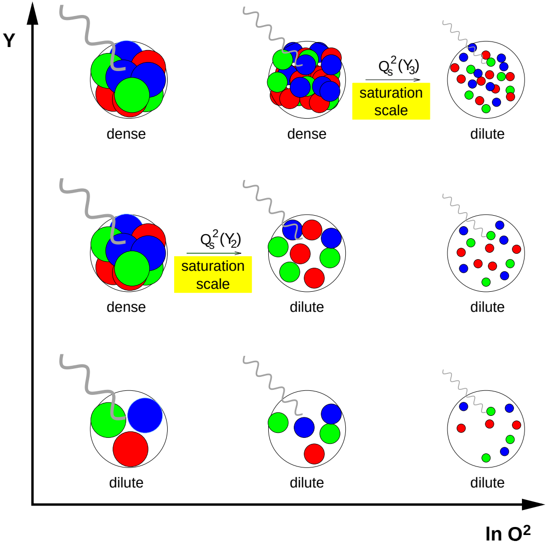

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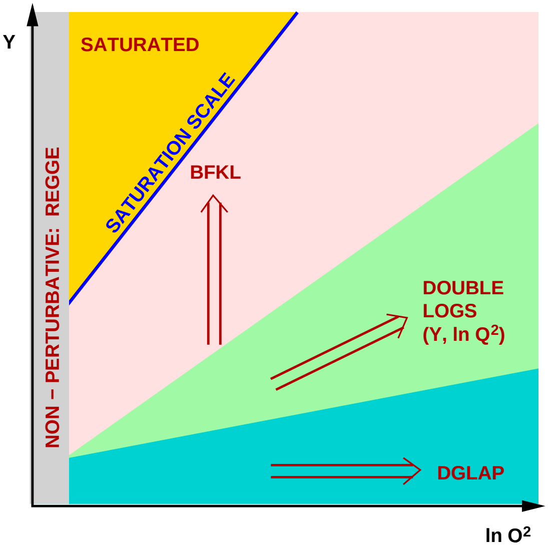

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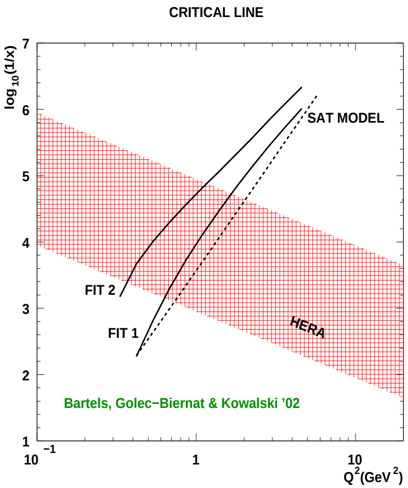
Saturation scale from data?

Big business at HERA collider

- Saturation ⇒ *strong non-Abelian fields (but $\alpha_s \ll 1$)* if $Q_s^2 \gtrsim 1$ GeV
- Use *diffraction* to measure degree of saturation
- Saturation sets in (perhaps?) just at limit of perturbative region
- NB: much interest also for *nuclei* (thickness increases density) (RHIC)



Dynamics at $Q_s^2(Y)$

- All gluon modes occupied up to $Q_s^2(Y)$.
- *pp* collisions always radiate gluons up to $Q_s^2(Y)$.
- $Q_s \gtrsim 1$ GeV ⇒ *pp* collisions partially perturbative.

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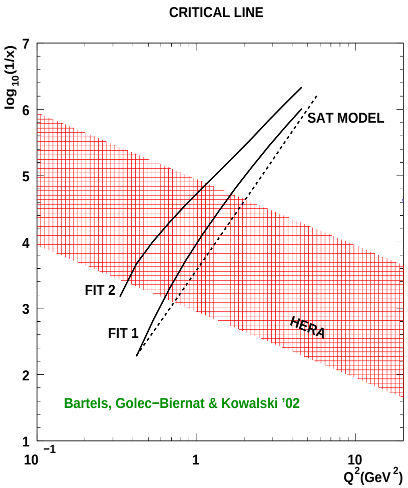

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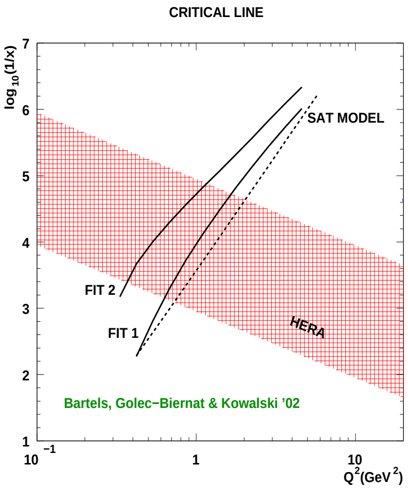

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A (biased) selection of recent work

Towards NLL comparisons with data

- NLL *couplings* to external particles (photons, jets) — 'impact factors'
  Bartels, Gieseke, Qiao, Colferai, Vacca, Kyrieleis '01–...
  Fadin, Ivanov, Kotsky '01–...
- Understanding *solutions* of NLL evolution equations
  Altarelli, Ball Forte '02–...; Andersen & Sabio Vera '03–...
  Ciafaloni, Colferai, GPS & Staśto '02–...

Evolution equations with saturation:

- Solutions of *multipole* evolution (BKP) Derkachov, Korchemsky, Kotanski & Manashov '02
  de Vega & Lipatov '02
- Connections between Balitsky-Kovchegov and *statistical physics (FKPP)*
  Munier & Peschanski '03
- Evolution eqns *beyond 'mean-field'*
  Iancu & Triantafyllopoulos '04-05
  Mueller, Shoshi & Wong '05
  Levin & Lublinsky '05
- Understanding of *solutions* beyond mean-field Mueller & Shoshi '04
  Iancu, Mueller & Munier '04
  Brunet, Derrida, Mueller & Munier (in progress)

Conclusions & outlook

- Basic field-theoretical framework for high-energy limit of perturbative QCD: *BFKL*
- Has many sources of corrections
  - Higher-orders in linear equation
  - Non-linearities
- These effects all combine together to provide a *picture* that looks *sensible* wrt data
- Progress still needed in order to be *quantitative*

Active in the Paris Area

- CPhT (X): Stéphane Munier, Bernard Pire
- LPT (Orsay): Gregory Korchemsky, Dominique Schiff, Samuel Wallon
- LPTHE (Paris 6 & 7): Hector de Vega, GPS
- SPhT (CEA): Jean-Paul Blaizot, François Gelis, Edmond Iancu, Robi Peschanski, Kazunori Itakura, Grégory Soyez, Dionysis Triantafyllopoulos, Cyrille Marquet.

Permanent
Postdoc
Ph.D.

- Senior visitors over the past few years: Ian Balitsky, Marcello Ciafaloni, Stefano Forte, Lev Lipatov, Larry McLerran, Alfred H. Mueller, Raju Venugopalan, . . .