Impact of higher orders in the high-energy limit of QCD

[OR: Is BFKL predictive?]

Gavin Salam
(work with M. Ciafaloni, D. Colferai & A.M. Staśto)

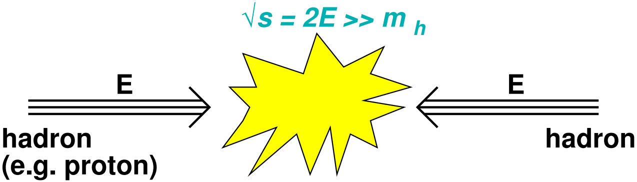
LPTHE, Universities of Paris VI and VII and CNRS

Cavendish Laboratory
Cambridge, March 2005

High-energy limit

One of the major unsolved problems of QCD (and Yang-Mills theory in general) is the understanding of its *high-energy limit.*

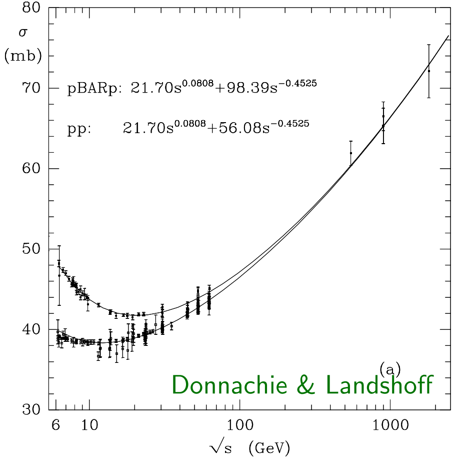
I.e. the limit in which C.O.M. energy ($\sqrt{s}$) is much larger than *all other scales* in the problem.



Want to understand:

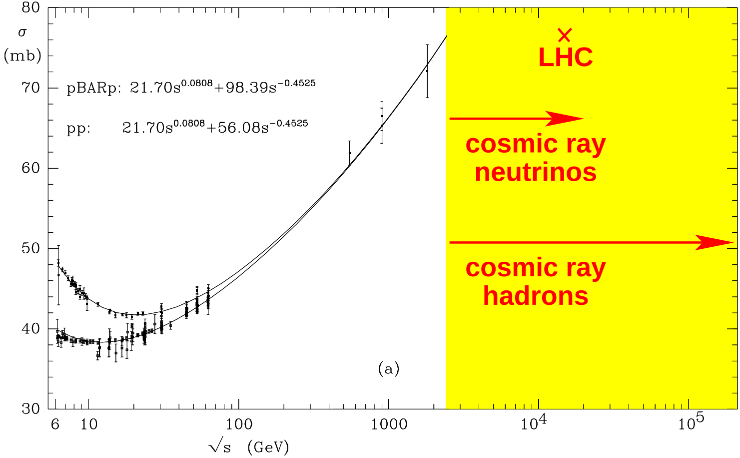
- asymptotic behaviour of cross section, $\sigma_{hh}(s) \sim ??$
- properties of final states for large s.

Experimental knowledge



- Some knowledge exists about behaviour of cross section experimentally
- Slow rise as energy increases
- Data insufficient to make reliable statements about functional form
  - $\sigma \sim s^{0.08}$?
  - $\sigma \sim \ln^2 s$?
- Understanding of final-states is $\sim$ inexistent
- Would like theoretical predictions...

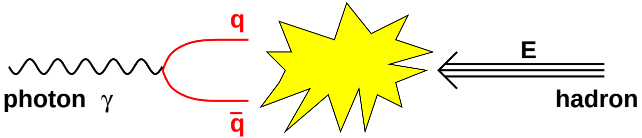
Experimental knowledge



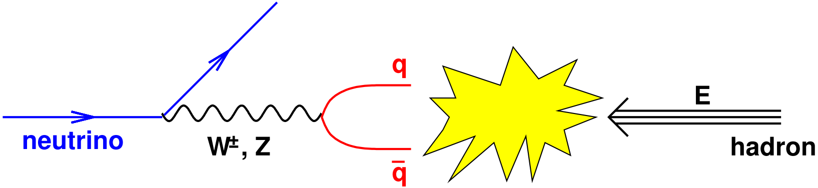
Future experiments go to much higher energies.

Not just for hadrons

Problem is must more general than just for hadrons. E.g. photon can *fluctuate* into a quark-antiquark (hadronic!) state:



Even a neutrino can behave like a hadron



Hadronic component dominates high-energy cross section

Outline

- Perturbative, leading-logarithmic (LL), calculation of cross-section growth — Using just classical field theory
- Failure of comparison to data
- Higher-order corrections
  - NLL corrections
  - Problems & solutions
- Splitting functions

Outline

- Perturbative, leading-logarithmic (LL), calculation of cross-section growth
  Using just classical field theory
- Failure of comparison to data
- Higher-order corrections
  - NLL corrections
  - Problems & solutions
- Splitting functions

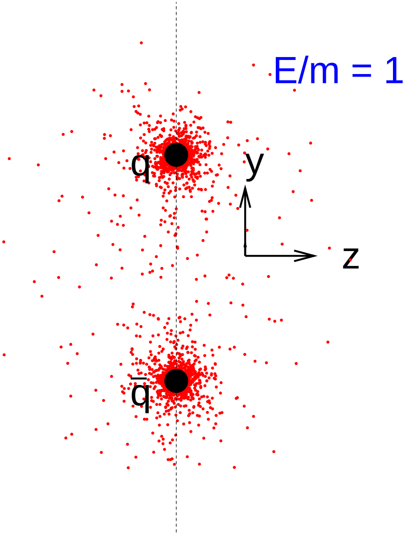
Outline

- Perturbative, leading-logarithmic (LL), calculation of cross-section growth Using just classical field theory
- Failure of comparison to data
- Higher-order corrections
  - NLL corrections
  - Problems & solutions
- Splitting functions

Outline

- Perturbative, leading-logarithmic (LL), calculation of cross-section growth — Using just classical field theory
- Failure of comparison to data
- Higher-order corrections
  - NLL corrections
  - Problems & solutions
- Splitting functions

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

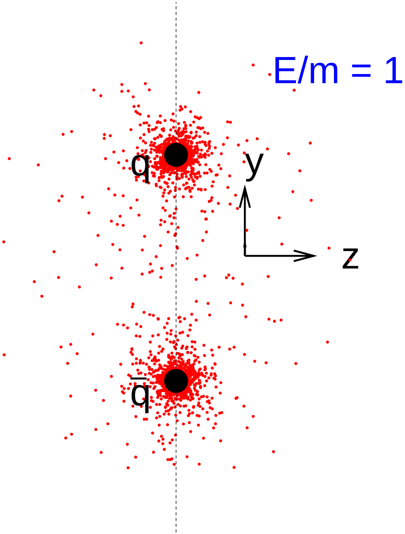


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy* $\equiv$ *large boost* (along z axis), by factor
- Fields flatten into *pancake*.
  - simple longitudinal structure

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

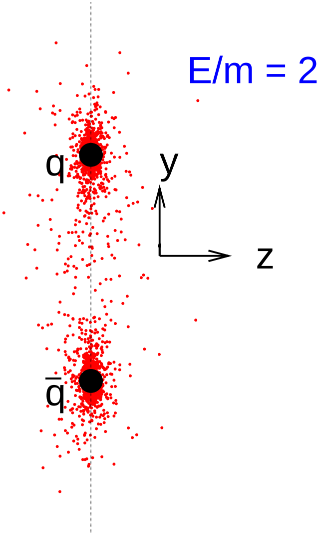


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake*.
  - simple longitudinal structure

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

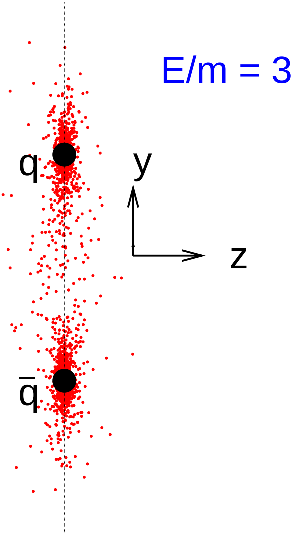


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

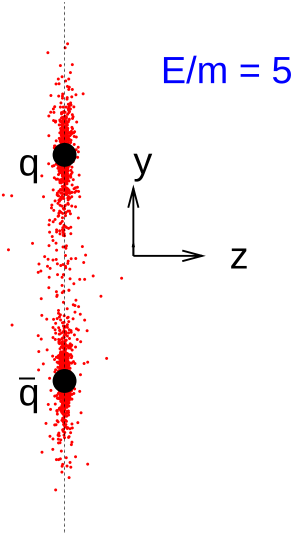


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

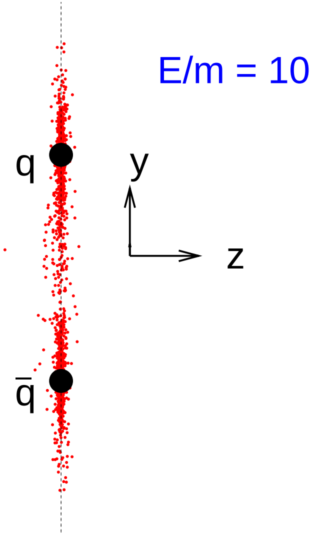


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy* $\equiv$ *large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

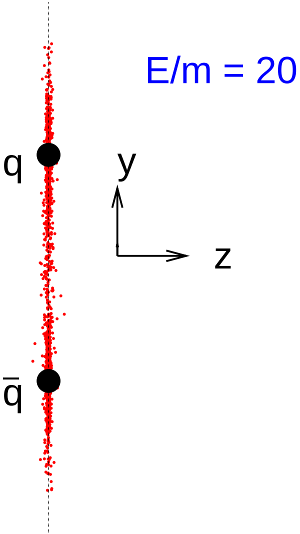


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy* $\equiv$ *large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

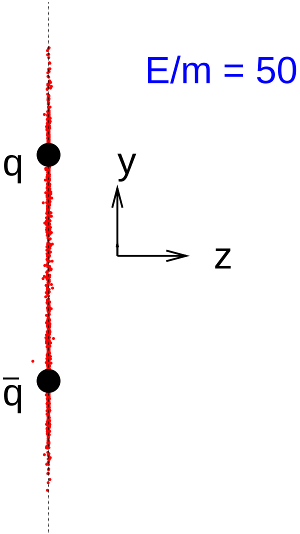


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

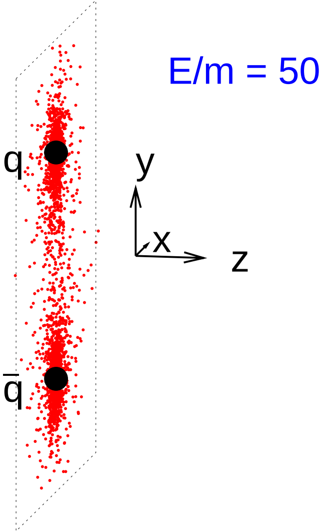


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

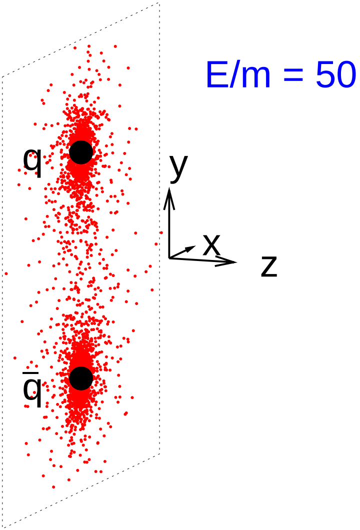


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

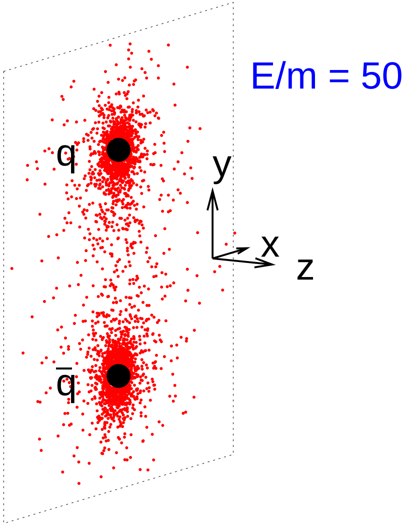


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

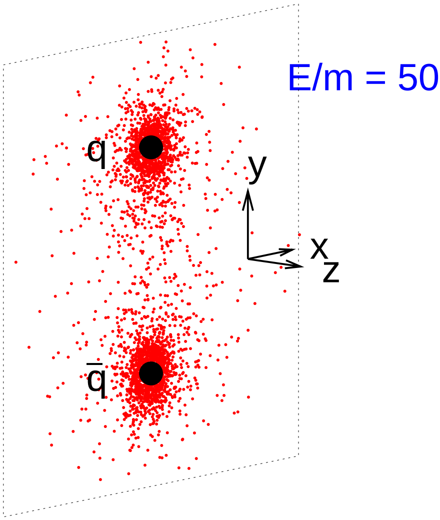


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

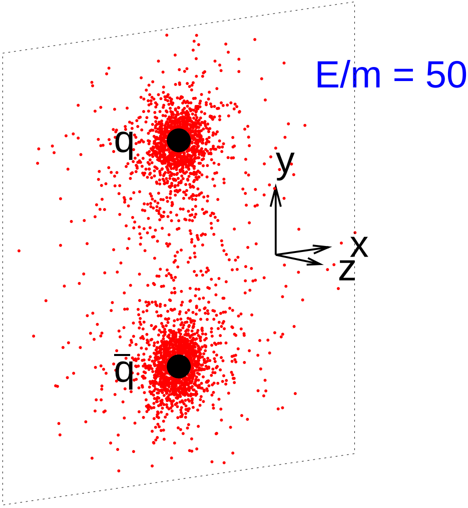


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

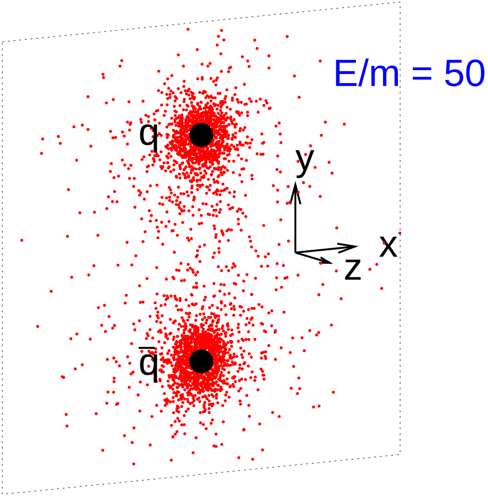


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

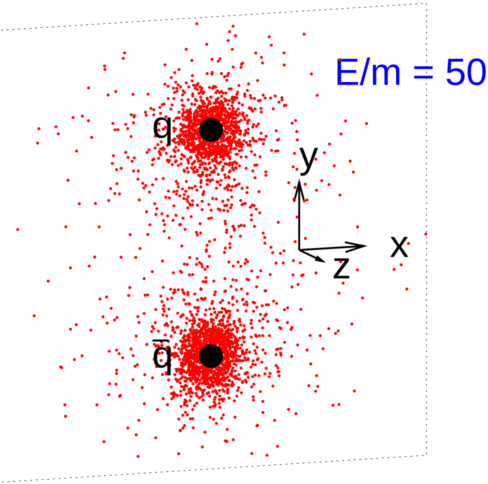


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

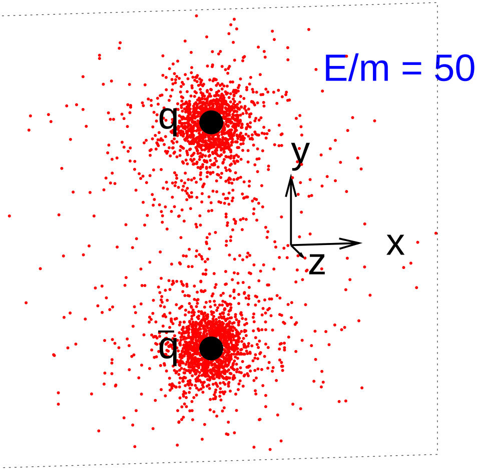


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)

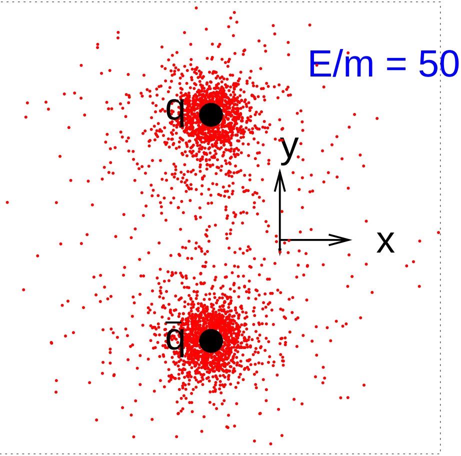


Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

Study field of $q\bar{q}$ dipole ($\simeq$ hadron)



Look at density of *gluons* from dipole field ($\sim$ energy density).

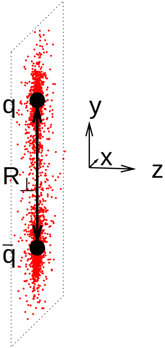
QCD $\simeq$ QED

- *Large energy $\equiv$ large boost* (along z axis), by factor
- Fields flatten into *pancake.*
  - simple longitudinal structure
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

Total number of gluons

Longitudinal structure of energy density (N_c = # of colours):

$$\frac{d\epsilon}{dz} \sim \frac{\alpha_s N_c}{\pi} \times E\delta(z) \times \text{transverse}$$



Fourier transform → energy density in field per unit of long. momentum (p_z)

$$\frac{d\epsilon}{dp_z} \sim \frac{\alpha_s N_c}{\pi} \times \text{transverse}\,, \qquad m \ll p_z \ll E$$

→ number (n) of gluons (each gluon has energy p_z):

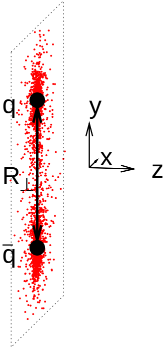
$$\frac{dn}{dp_z} \sim \frac{\alpha_s N_c}{\pi}\frac{1}{p_z} \times \text{transverse}\,, \qquad m \ll p_z \ll E$$

Total number of gluons:

Total number of gluons

Longitudinal structure of energy density (N_c = # of colours):

$$\frac{d\epsilon}{dz} \sim \frac{\alpha_s N_c}{\pi} \times E\delta(z) \times \mathrm{transverse}$$



Fourier transform → energy density in field per unit of long. momentum (p_z)

$$\frac{d\epsilon}{dp_z} \sim \frac{\alpha_s N_c}{\pi} \times \mathrm{transverse}\,, \qquad m \ll p_z \ll E$$

→ number (n) of gluons (each gluon has energy p_z):

$$\frac{dn}{dp_z} \sim \frac{\alpha_s N_c}{\pi} \frac{1}{p_z} \times \mathrm{transverse}\,, \qquad m \ll p_z \ll E$$

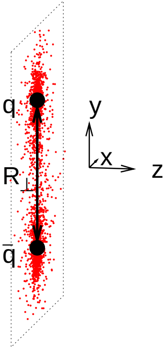
Total number of gluons:

$$n \sim \frac{\alpha_s N_c}{\pi} \ln \frac{E}{m} \times \mathrm{transverse}$$

Total number of gluons

Longitudinal structure of energy density (N_c = # of colours):

$$\frac{d\epsilon}{dz} \sim \frac{\alpha_s N_c}{\pi} \times E\delta(z) \times \text{transverse}$$



Fourier transform → energy density in field per unit of long. momentum (p_z)

$$\frac{d\epsilon}{dp_z} \sim \frac{\alpha_s N_c}{\pi} \times \text{transverse}\,, \qquad m \ll p_z \ll E$$

→ number (n) of gluons (each gluon has energy p_z):

$$\frac{dn}{dp_z} \sim \frac{\alpha_s N_c}{\pi} \frac{1}{p_z} \times \text{transverse}\,, \qquad m \ll p_z \ll E$$

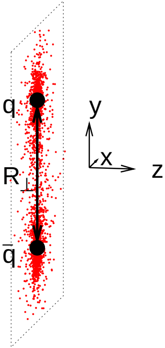
Total number of gluons:

$$n \sim \frac{\alpha_s N_c}{\pi} \ln \frac{E}{m} \times \text{transverse}$$

Total number of gluons

Longitudinal structure of energy density (N_c = # of colours):

$$\frac{d\epsilon}{dz} \sim \frac{\alpha_s N_c}{\pi} \times E\delta(z) \times \text{transverse}$$



Fourier transform → energy density in field per unit of long. momentum (p_z)

$$\frac{d\epsilon}{dp_z} \sim \frac{\alpha_s N_c}{\pi} \times \text{transverse}\,, \qquad m \ll p_z \ll E$$

→ number (n) of gluons (each gluon has energy p_z):

$$\frac{dn}{dp_z} \sim \frac{\alpha_s N_c}{\pi} \frac{1}{p_z} \times \text{transverse}\,, \qquad m \ll p_z \ll E$$

Total number of gluons:

$$\boxed{n \sim \frac{\alpha_s N_c}{\pi} \ln \frac{E}{m} \times \text{transverse}}$$

High-energy limit $\sqrt{s}, E \to \infty$

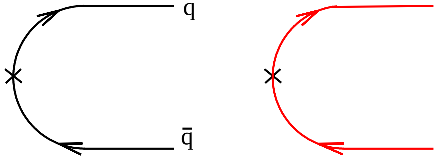
- Calculation so far is first-order perturbation theory.
- Fixed order perturbation theory is reliable if series converges quickly.
- At high energies, $n \sim \alpha_s \ln E \sim 1$.
- What happens with higher orders?

$$(\alpha_s \ln E)^n?$$

Leading Logarithms (LL). Any fixed order potentially non-convergent. . .

Multiple gluon emission

Start with bare $q\bar{q}$ dipole:



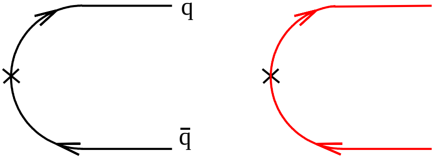
Emission of 1 gluon is like QED case — modulo additional colour factor (number of different ways to repaint quark):

$$\boxed{\alpha \to \alpha_s N_c/2} \quad \text{(approx)}$$

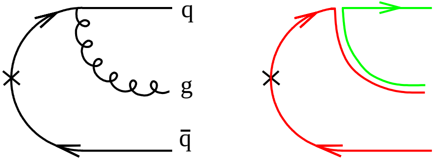
- In QED subsequent photons are emitted by *original dipole*
- In QCD original dipole is converted into two new dipoles, which *emit independently*.

Multiple gluon emission

Start with bare $q\bar{q}$ dipole:



Emit a gluon:



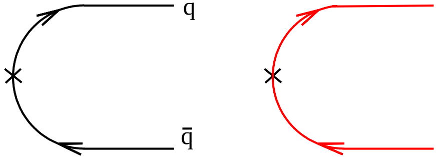
Emission of 1 gluon is like QED case — modulo additional colour factor
(number of different ways to repaint quark):

$$\boxed{\alpha \to \alpha_s N_c/2} \quad \text{(approx)}$$

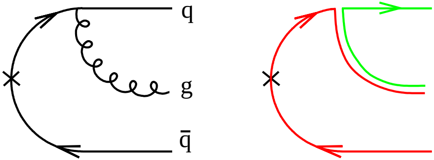
- In QED subsequent photons are emitted by *original dipole*
- In QCD original dipole is converted into two new dipoles, which *emit independently*.

Multiple gluon emission

Start with bare $q\bar{q}$ dipole:



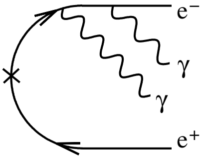
Emit a gluon:



Emission of 1 gluon is like QED case — modulo additional colour factor (number of different ways to repaint quark):

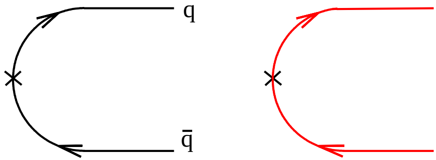
$$\boxed{\alpha \to \alpha_s N_c/2} \quad \text{(approx)}$$

- In QED subsequent photons are emitted by *original dipole*
- In QCD original dipole is converted into two new dipoles, which *emit independently*.

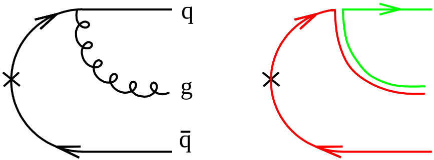


Multiple gluon emission

Start with bare $q\bar{q}$ dipole:



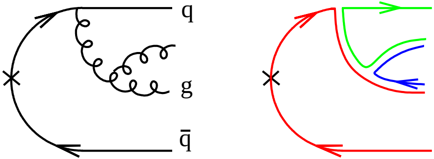
Emit a gluon:



Emission of 1 gluon is like QED case — modulo additional colour factor (number of different ways to repaint quark):

$$\boxed{\alpha \to \alpha_s N_c/2}$$ (approx)

- In QED subsequent photons are emitted by *original dipole*
- In QCD original dipole is converted into two new dipoles, which *emit independently*.



Iterating gluon emission

Problem is self-similar: dipole $\to$ 2 dipoles $\to$ 4 dipoles $\to \ldots$

Number of dipoles (or gluons) grows *exponentially:*

$$n \sim \exp\left[\frac{\alpha_s N_c}{\pi} \ln E \times \mathrm{transverse}\right] \sim E^{\frac{\alpha_s N_c}{\pi} \times \mathrm{transverse}}$$

Tranverse part $\to$ many complications/interest

- transverse part is *conformally invariant* ➥Extensive mathematical studies
- In high-energy limit it reduces to a pure number: $4 \ln 2$

$$n \sim E^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} \sim E^{0.5}$$

Balitsky Fadin Kuraev Lipatov Pomeron (1976)

Iterating gluon emission

Problem is self-similar: dipole $\rightarrow$ 2 dipoles $\rightarrow$ 4 dipoles $\rightarrow \dots$

Number of dipoles (or gluons) grows *exponentially:*

$$n \sim \exp\left[\frac{\alpha_s N_c}{\pi} \ln E \times \mathrm{transverse}\right] \sim E^{\frac{\alpha_s N_c}{\pi} \times \mathrm{transverse}}$$

Tranverse part $\rightarrow$ many complications/interest

- transverse part is *conformally invariant* ➥Extensive mathematical studies
- In high-energy limit it reduces to a pure number: $4 \ln 2$

$$n \sim E^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} \sim E^{0.5}$$

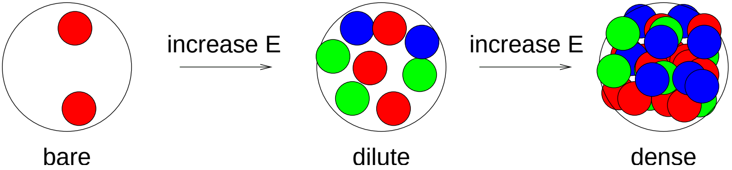
Balitsky Fadin Kuraev Lipatov Pomeron (1976)

BFKL: rising cross sections

$$n_{\text{dipoles}}(E) \sim E^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} \quad \Leftrightarrow \quad \sigma_{hh} \sim s^{\frac{\alpha_s N_c}{\pi} 4 \ln 2}, \qquad \frac{\alpha_s N_c}{\pi} 4 \ln 2 \simeq 0.5$$

- Completely incompatible with rise of $p\bar{p}$ cross section ($\sim s^{0.08}$)
  - $p\bar{p}$ is simply beyond perturbation theory
- experimentally spectactular — if observable in some process...
- Raises many theoretical issues — high gluon densities should lead to non-linear effects: *high fields, but still perturbative*

Colour Glass Condensate



BFKL: rising cross sections

$$n_{\text{dipoles}}(E) \sim E^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} \quad \Leftrightarrow \quad \sigma_{hh} \sim s^{\frac{\alpha_s N_c}{\pi} 4 \ln 2}, \qquad \frac{\alpha_s N_c}{\pi} 4 \ln 2 \simeq 0.5$$

- Completely incompatible with rise of $p\bar{p}$ cross section ($\sim s^{0.08}$)
  - $p\bar{p}$ is simply beyond perturbation theory
- experimentally spectactular — if observable in some process...
- Raises many theoretical issues — high gluon densities should lead to non-linear effects: *high fields, but still perturbative*

Colour Glass Condensate

How can we search for BFKL experimentally?

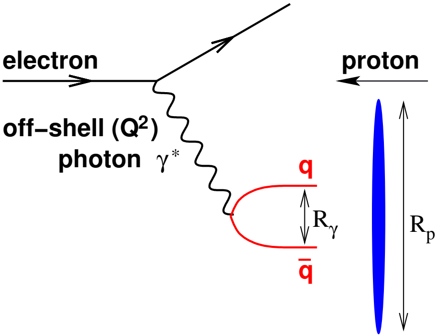
- Need to ensure we are in regime where perturbation theory can be applied
  - Choose appropriate hadronic scales (small R)

Deep Inelastic Scattering (DIS)

Getting small transverse sizes (needed for $\alpha_s \ll 1$) *and* asymptotically large collision energies is experimentally difficult.

In general collide two hadronic probes — try a compromise: *make one of them small*

$$R_\gamma \sim \frac{1}{Q} \ll R_p \sim \frac{1}{m_p}$$



- $q\bar{q}$ probe measures (roughly) number of gluons in proton up to scale Q
- NB: DIS more usually viewed as photon hitting quarks in proton

- Some of physics perturbative ($Q \gtrsim p_t \gg m_p$)
- But if $\ln Q^2 \gtrsim \ln s$ we have *competition* between

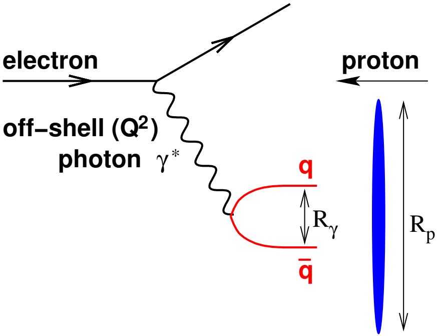
$$(\alpha_s \ln s)^n \quad \text{v.} \quad (\alpha_s \ln s \ln Q^2)^n$$

Deep Inelastic Scattering (DIS)

Getting small transverse sizes (needed for $\alpha_s \ll 1$) *and* asymptotically large collision energies is experimentally difficult.

In general collide two hadronic probes — try a compromise: *make one of them small*

$$R_\gamma \sim \frac{1}{Q} \ll R_p \sim \frac{1}{m_p}$$



- $q\bar{q}$ probe measures (roughly) number of gluons in proton up to scale Q
- NB: DIS more usually viewed as photon hitting quarks in proton

- Some of physics perturbative ($Q \gtrsim p_t \gg m_p$)
- But if $\ln Q^2 \gtrsim \ln s$ we have *competition* between

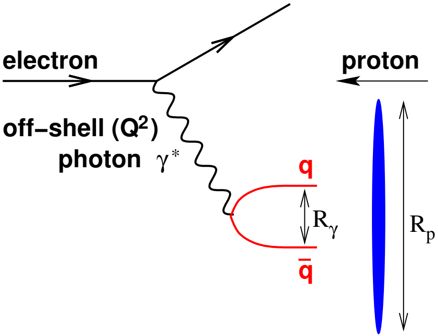
$$(\alpha_s \ln s)^n \quad \text{v.} \quad (\alpha_s \ln s \ln Q^2)^n$$

Deep Inelastic Scattering (DIS)

Getting small transverse sizes (needed for $\alpha_s \ll 1$) *and* asymptotically large collision energies is experimentally difficult.

In general collide two hadronic probes — try a compromise: *make one of them small*

$$R_\gamma \sim \frac{1}{Q} \ll R_p \sim \frac{1}{m_p}$$

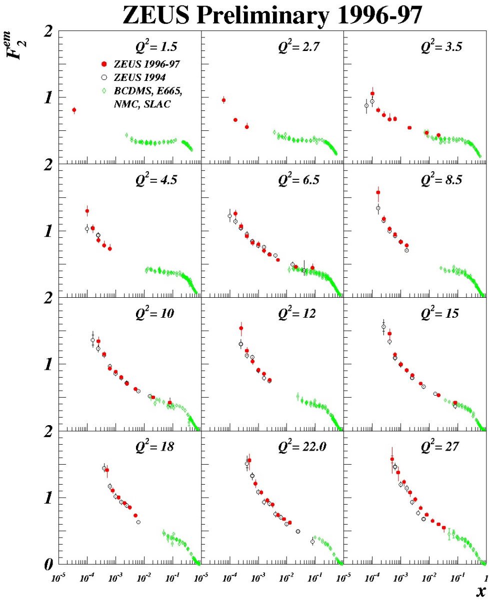


- $q\bar{q}$ probe measures (roughly) number of gluons in proton up to scale Q
- NB: DIS more usually viewed as photon hitting quarks in proton

- Some of physics perturbative ($Q \gtrsim p_t \gg m_p$)
- But if $\ln Q^2 \gtrsim \ln s$ we have *competition* between

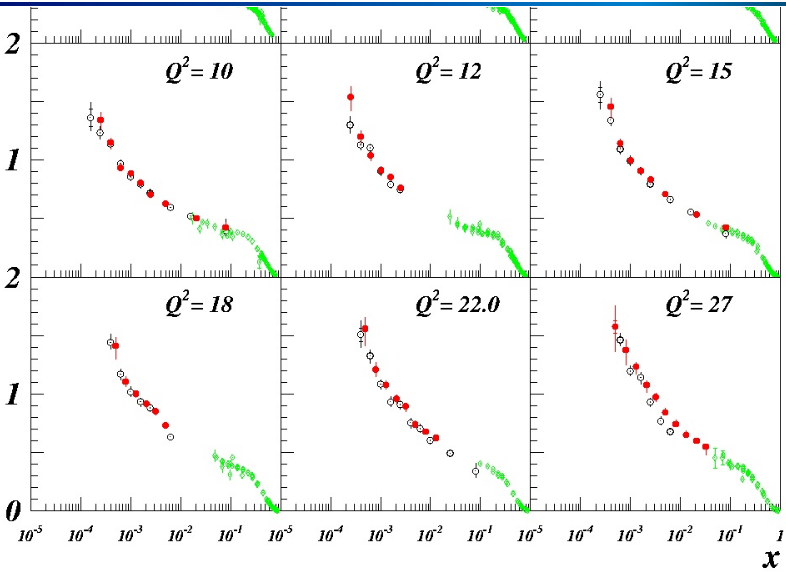
$$(\alpha_s \ln s)^n \quad \text{v.} \quad (\alpha_s \ln s \ln Q^2)^n$$

HERA F_2 data



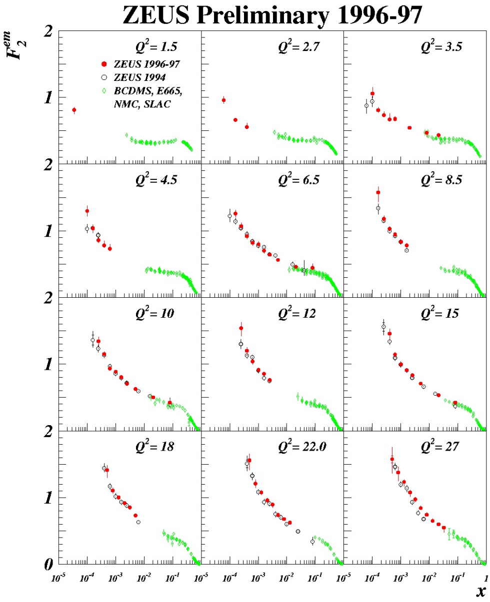
- F_2 is rescaled cross section
- $x = \dfrac{p_z}{p_{z,\text{proton}}} \sim \dfrac{1}{s}$
- Clear rise of cross section at high energies (low x).

HERA F_2 data



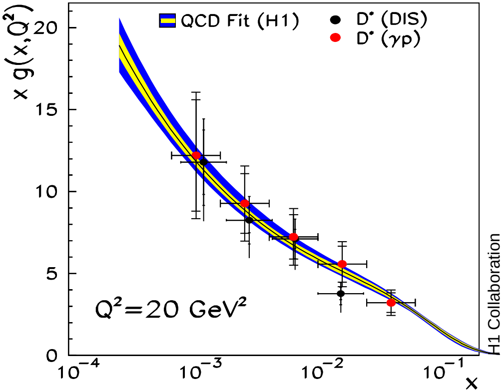
- x =
- Cle
hig

HERA F_2 data



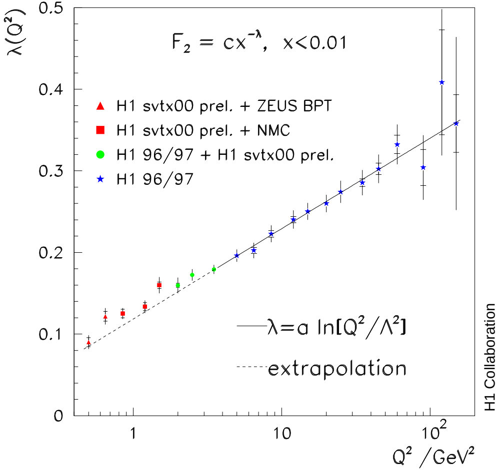
- F_2 is rescaled cross section
- $x = \dfrac{p_z}{p_{z,\text{proton}}} \sim \dfrac{1}{s}$
- Clear rise of cross section at high energies (low x).

Large number of gluons



- Convert cross sections into estimate of number of gluons
- Various independent extractions
- *Up to 20 gluons per unit* $\ln x$ *(or unit* $\ln p_z$*)!*

Is it BFKL?



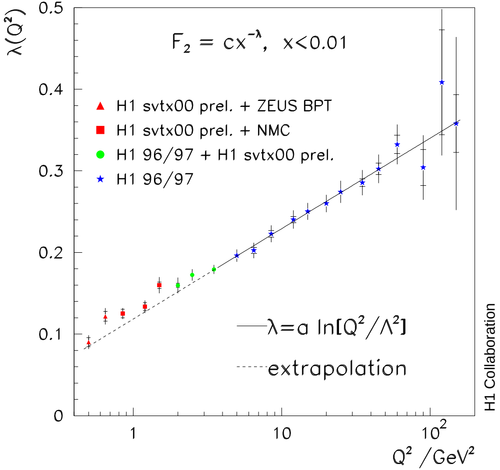
- Check if BFKL by looking at power (λ) of x
- For BFKL, expect $\lambda \simeq 0.5$
- *Definitely not LL BFKL!*

There is some growth — where does it come from?

It is due to combination of $x \ll 1$ and $Q^2 \gg m_p^2$ — resummation of terms $(\alpha_s \ln \frac{1}{x} \ln Q^2)^n$:

$$\sigma \sim \exp\left[c\sqrt{\alpha_s \ln Q^2 \ln x}\right]$$

Is it BFKL?



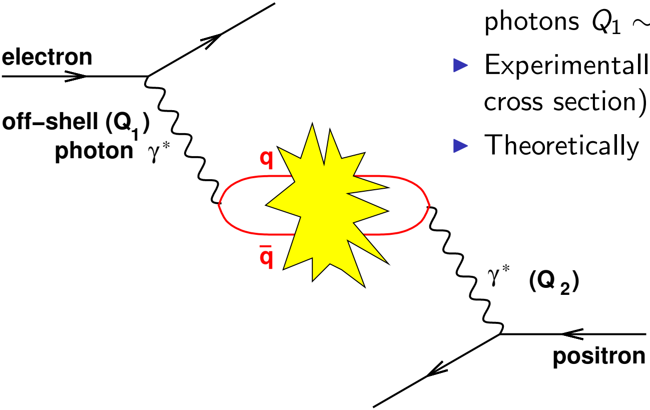
- Check if BFKL by looking at power (λ) of x
- For BFKL, expect $\lambda \simeq 0.5$
- *Definitely not LL BFKL!*

There is some growth — where does it come from?

It is due to combination of $x \ll 1$ and $Q^2 \gg m_p^2$ — resummation of terms $(\alpha_s \ln \frac{1}{x} \ln Q^2)^n$:

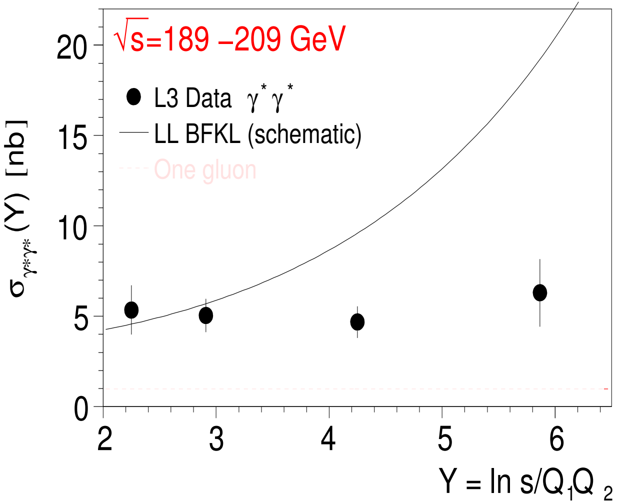
$$\sigma \sim \exp\left[c\sqrt{\alpha_s \ln Q^2 \ln x}\right]$$

$\gamma^*\gamma^*$ collisions



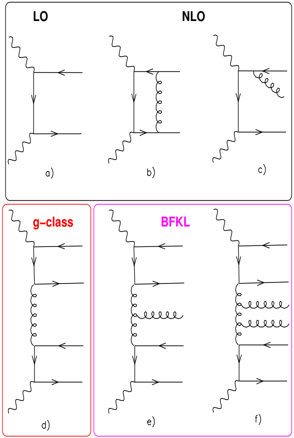
- Eliminate ratios of transverse scales by colliding two virtual photons $Q_1 \sim Q_2$
- Experimentally difficult (small cross section)
- Theoretically clean

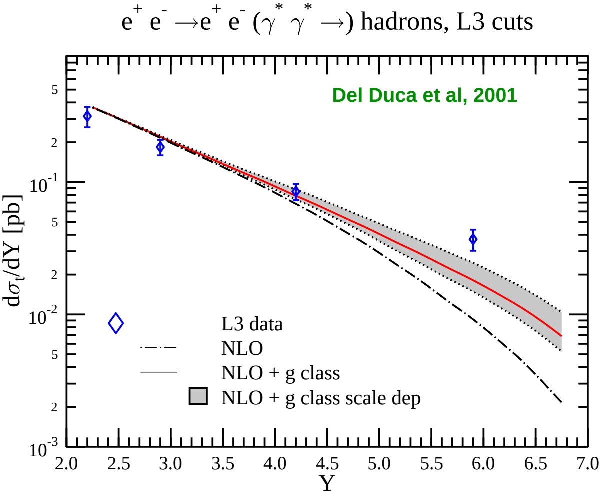
Results from LEP



- Here too, data clearly incompatible with LL BFKL
- But perhaps some evidence for weak growth

Results from LEP





- Here too, data clearly incompatible with LL BFKL
- But perhaps some evidence for weak growth

Where is BFKL?

- BFKL is rigorous prediction of field theory, yet not seen in data
- Should we be worried?
- Calculations shown so far are in Leading Logarithmic (LL) approximation, $(\alpha_s \ln s)^n$: accurate only for

$$\alpha_s \to 0, \ln s \to \infty \text{ and } \alpha_s \ln s \sim 1.$$

- Need higher order corrections

Next-to-Leading-Logarithmic (NLL) terms: $\alpha_s(\alpha_s \ln s)^n$

Where is BFKL?

- BFKL is rigorous prediction of field theory, yet not seen in data
- Should we be worried?
- Calculations shown so far are in Leading Logarithmic (LL) approximation, $(\alpha_s \ln s)^n$: accurate only for

$$\alpha_s \to 0, \ln s \to \infty \text{ and } \alpha_s \ln s \sim 1.$$

- Need higher order corrections

Next-to-Leading-Logarithmic (NLL) terms: $\alpha_s(\alpha_s \ln s)^n$

Where is BFKL?

- BFKL is rigorous prediction of field theory, yet not seen in data
- Should we be worried? No!
- Calculations shown so far are in Leading Logarithmic (LL) approximation, $(\alpha_s \ln s)^n$: accurate only for

$$\alpha_s \to 0, \ln s \to \infty \text{ and } \alpha_s \ln s \sim 1.$$

- Need higher order corrections

Next-to-Leading-Logarithmic (NLL) terms: $\alpha_s(\alpha_s \ln s)^n$

Where is BFKL?

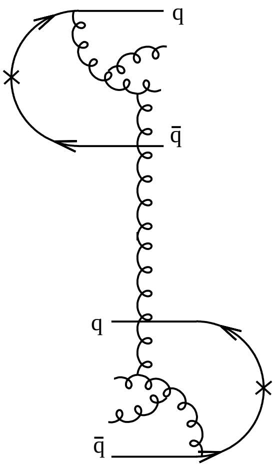
- BFKL is rigorous prediction of field theory, yet not seen in data
- Should we be worried? No!
- Calculations shown so far are in Leading Logarithmic (LL) approximation, $(\alpha_s \ln s)^n$: accurate only for

$$\alpha_s \to 0, \ln s \to \infty \text{ and } \alpha_s \ln s \sim 1.$$

- Need higher order corrections

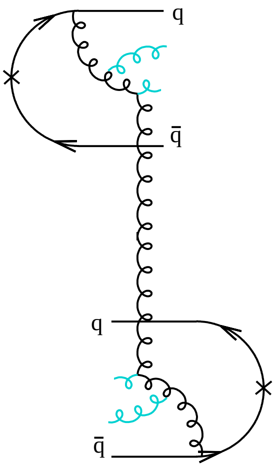
Next-to-Leading-Logarithmic (NLL) terms: $\alpha_s(\alpha_s \ln s)^n$

Wavefunction v. ladder graphs



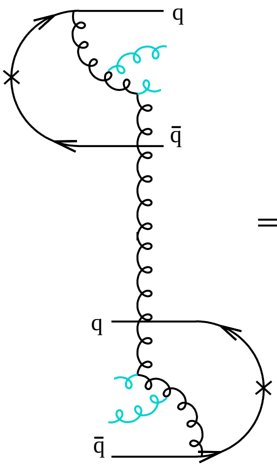
evolution in wavefunctions

Wavefunction v. ladder graphs

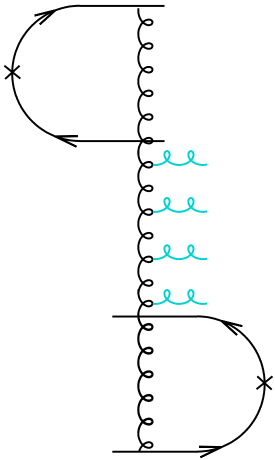


evolution in wavefunctions

Wavefunction v. ladder graphs



evolution in wavefunctions

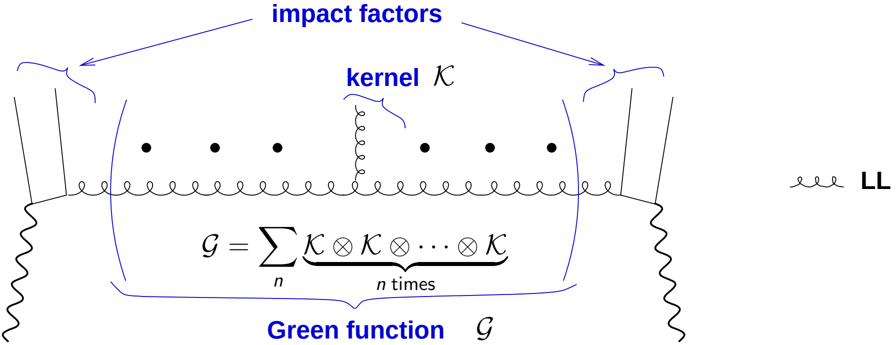


evolution as ladder diagram

NLL ingredients

Label various parts of cross-section calculation

NLL: include relative $\mathcal{O}(\alpha_s)$ corrections to each



Kernel (universal):

Associated with power growth

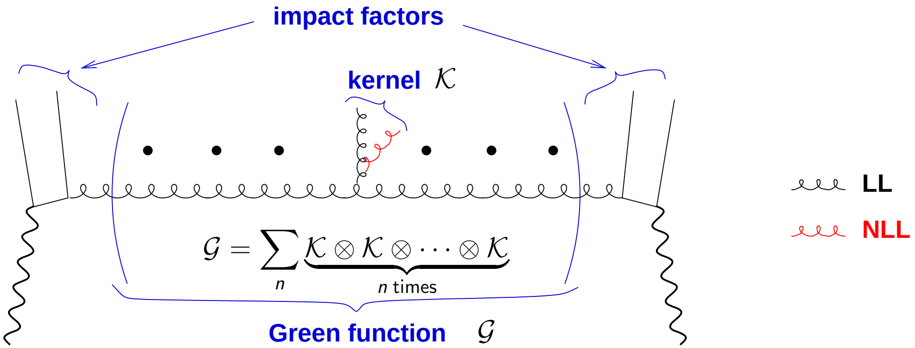
Impact factors (proc.-dependent):

Associated with normalisation

NLL ingredients

Label various parts of cross-section calculation

NLL: include relative $\mathcal{O}(\alpha_s)$ corrections to each



Kernel (universal):

Fadin, Lipatov, Fiore, Kotsky, Quartarolo; Catani, Ciafaloni, Hautmann, Camici '89–'98

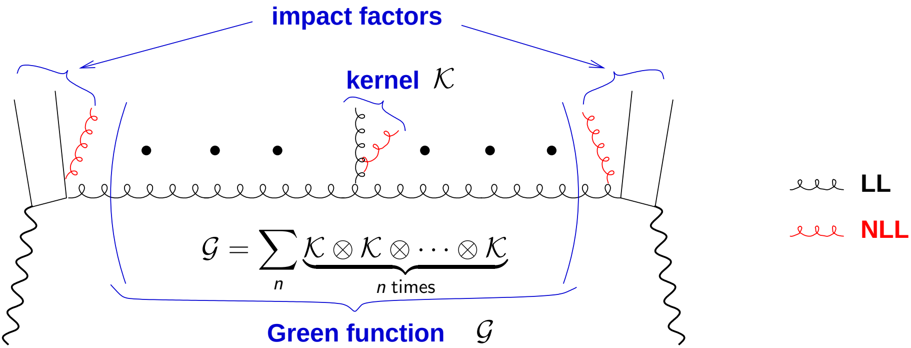
Impact factors (proc.-dependent):

Associated with normalisation

NLL ingredients

Label various parts of cross-section calculation

NLL: include relative $\mathcal{O}(\alpha_s)$ corrections to each



Kernel (universal):

Fadin, Lipatov, Fiore, Kotsky, Quartarolo; Catani, Ciafaloni, Hautmann, Camici '89–'98

Impact factors (proc.-dependent):

Bartels, Gieseke, Qiao, Colferai, Vacca, Kyrieleis; Fadin, Ivanov, Kotsky '01–...

NLL power

Cast NLL corrections to kernel as modification of power:

$$\sigma \sim G(Y,k,k) \sim \exp\left[4\ln 2\bar{\alpha}_s(1 - 6.5\bar{\alpha}_s)\,Y\right]$$

NB: k = transv. mom. scale

- Very *poorly convergent* ($\bar{\alpha}_s = \alpha_s N_c/\pi \simeq 0.15 \cdots 0.2$)
- Unstable perturbative hierarchy: *expansion of power has limited sense*

➥ Instead, try solving BFKL equation with full NLL kernel (including running coupling)

$$G(Y,k,k_0) = \frac{\delta(k-k_0)}{2\pi k_0} + \int_0^Y dy \int dk'^2\, \mathcal{K}(k,k')\, G(Y-y,k,k')$$

Andersen & Sabio-Vera '03
Ciafaloni, Colferai, GPS & Staśto '03

NLL power

Cast NLL corrections to kernel as modification of power:

$$\sigma \sim G(Y, k, k) \sim \exp\left[\, 4 \ln 2 \bar{\alpha}_s (1 - 6.5 \bar{\alpha}_s)\, Y \,\right]$$

NB: k = transv. mom. scale

- Very *poorly convergent* ($\bar{\alpha}_s = \alpha_s N_c/\pi \simeq 0.15 \cdots 0.2$)
- Unstable perturbative hierarchy: *expansion of power has limited sense*

➥ Instead, try solving BFKL equation with full NLL kernel (including running coupling)

$$G(Y, k, k_0) = \frac{\delta(k - k_0)}{2\pi k_0} + \int_0^Y dy \int d k'^2 \, \mathcal{K}(k, k') \, G(Y - y, k, k')$$

Andersen & Sabio-Vera '03
Ciafaloni, Colferai, GPS & Staśto '03

NLL power

Cast NLL corrections to kernel as modification of power:

$$\sigma \sim G(Y,k,k) \sim \exp\left[\, 4\ln 2\bar{\alpha}_s(1 - 6.5\bar{\alpha}_s)\, Y \,\right]$$

NB: k = transv. mom. scale

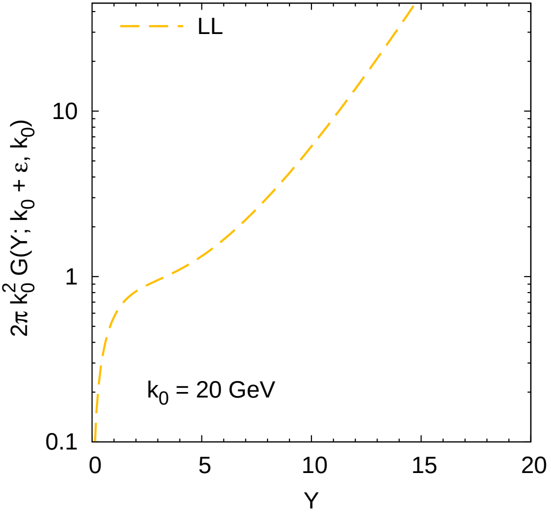
- Very *poorly convergent* ($\bar{\alpha}_s = \alpha_s N_c/\pi \simeq 0.15 \cdots 0.2$)
- Unstable perturbative hierarchy: *expansion of power has limited sense*

➥ Instead, try solving BFKL equation with full NLL kernel (including running coupling)

$$G(Y,k,k_0) = \frac{\delta(k-k_0)}{2\pi k_0} + \int_0^Y dy \int dk'^2\, \mathcal{K}(k,k')\, G(Y-y,k,k')$$

Andersen & Sabio-Vera '03
Ciafaloni, Colferai, GPS & Staśto '03

NLL Green function solution



Various convention choices affect higher orders (NNLLx):

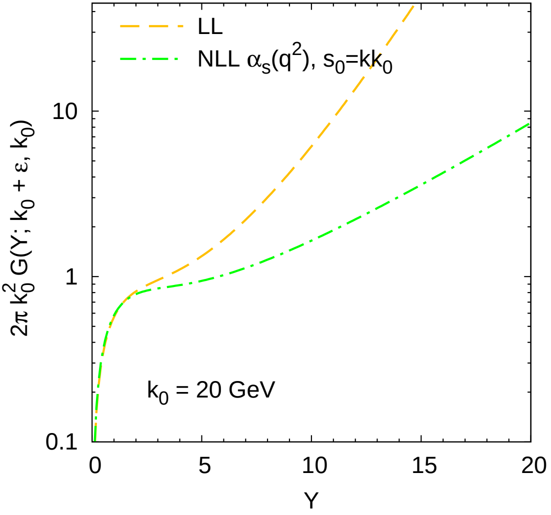
- scale of α_s
- 'energy-scale' s_0 ($Y = \ln s/s_0$).

Extreme sensitivity to choice of convention $\Leftrightarrow$ poor perturbative convergence.

NB: Andersen & Sabio Vera solutions $\sim$ green curve

Need to understand origin of instability

NLL Green function solution



Various convention choices affect higher orders (NNLLx):

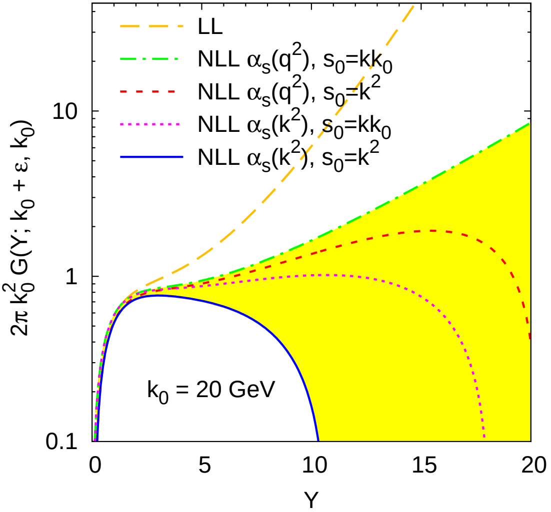
- scale of α_s
- 'energy-scale' s_0 ($Y = \ln s/s_0$).

Extreme sensitivity to choice of convention $\Leftrightarrow$ poor perturbative convergence.

NB: Andersen & Sabio Vera solutions $\sim$ green curve

Need to understand origin of instability

NLL Green function solution



Various convention choices affect higher orders (NNLLx):

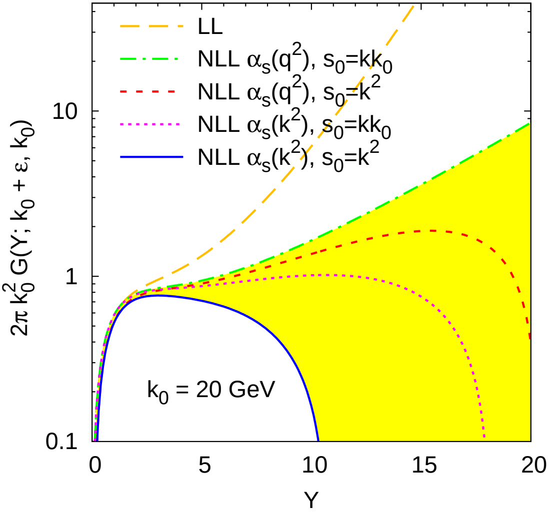
- scale of α_s
- 'energy-scale' s_0 ($Y = \ln s/s_0$).

Extreme sensitivity to choice of convention $\Leftrightarrow$ poor perturbative convergence.

NB: Andersen & Sabio Vera solutions $\sim$ green curve

Need to understand origin of instability

NLL Green function solution



Various convention choices affect higher orders (NNLLx):

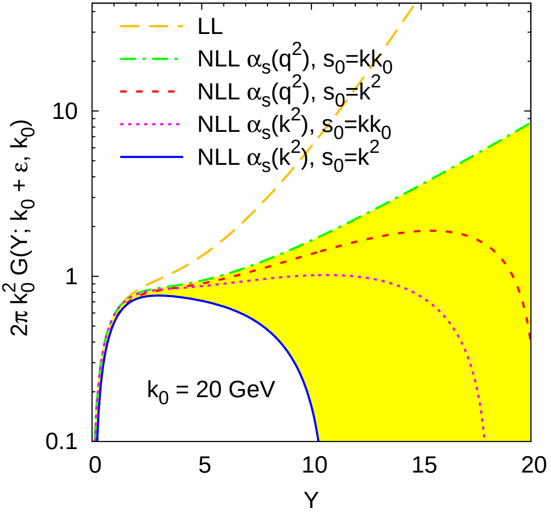
- scale of α_s
- 'energy-scale' s_0 ($Y = \ln s/s_0$).

Extreme sensitivity to choice of convention $\Leftrightarrow$ poor perturbative convergence.

NB: Andersen & Sabio Vera solutions $\sim$ green curve

Need to understand origin of instability

NLL Green function solution



Various convention choices affect higher orders (NNLLx):

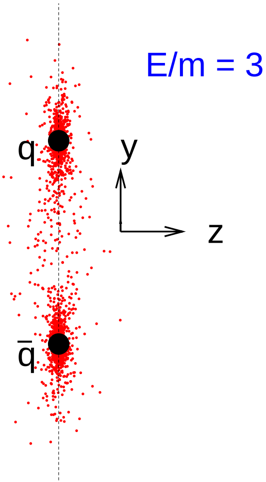
- scale of α_s
- 'energy-scale' s_0 ($Y = \ln s/s_0$).

Extreme sensitivity to choice of convention $\Leftrightarrow$ poor perturbative convergence.

NB: Andersen & Sabio Vera solutions $\sim$ green curve

Need to understand origin of instability

NLL: why so bad?



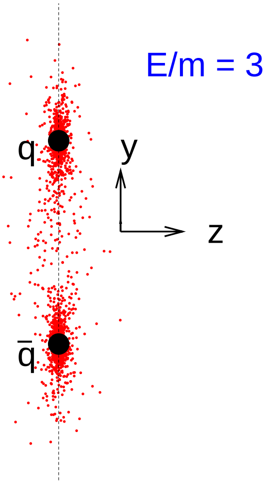
- First branching occurs for $Y \sim \frac{c}{\alpha_s}$
- In practice c is small: $e^Y \sim 2-5$
- Energy-distribution $\neq$ perfect $\delta(z)$
- 'degree of imperfection' depends on transverse position

  Ciafaloni '88

  Andersson et al; Kwiecinski et al '96
- Dominant part $\equiv$ double & single $\perp$ logs
  - Responsible for $\sim 90\%$ of NLL corrections
  - Can be used to supplement NLL at all orders

  GPS; Ciafaloni & Colferai, '98–99

NLL: why so bad?



- First branching occurs for $Y \sim \frac{c}{\alpha_s}$
- In practice c is small: $e^Y \sim 2 - 5$
- Energy-distribution $\neq$ perfect $\delta(z)$
- 'degree of imperfection' depends on transverse position

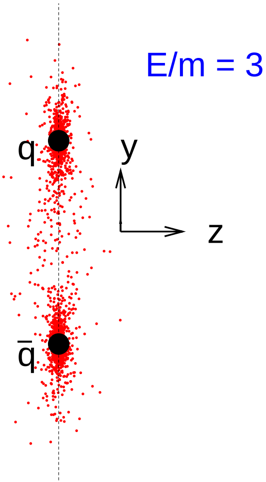
Ciafaloni '88

Andersson et al; Kwiecinski et al '96

- Dominant part $\equiv$ double & single $\perp$ logs
  - Responsible for $\sim 90\%$ of NLL corrections
  - Can be used to supplement NLL at all orders

GPS; Ciafaloni & Colferai, '98–99

NLL: why so bad?



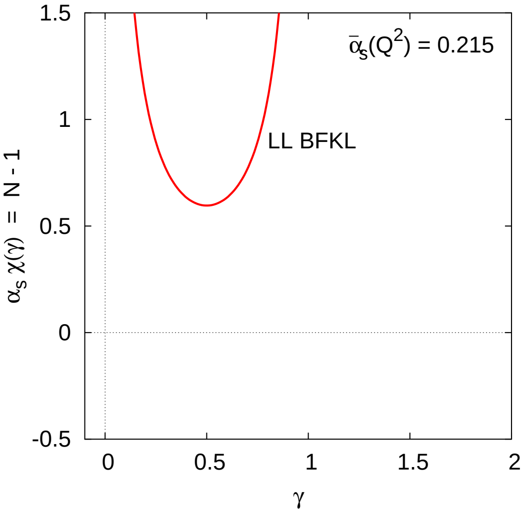
- First branching occurs for $Y \sim \frac{c}{\alpha_s}$
- In practice c is small: $e^Y \sim 2-5$
- Energy-distribution $\neq$ perfect $\delta(z)$
- 'degree of imperfection' depends on transverse position

  Ciafaloni '88

  Andersson et al; Kwiecinski et al '96
- Dominant part $\equiv$ double & single $\perp$ logs
  - Responsible for $\sim 90\%$ of NLL corrections
  - Can be used to supplement NLL at all orders

  GPS; Ciafaloni & Colferai, '98–99

Building up the kernel...



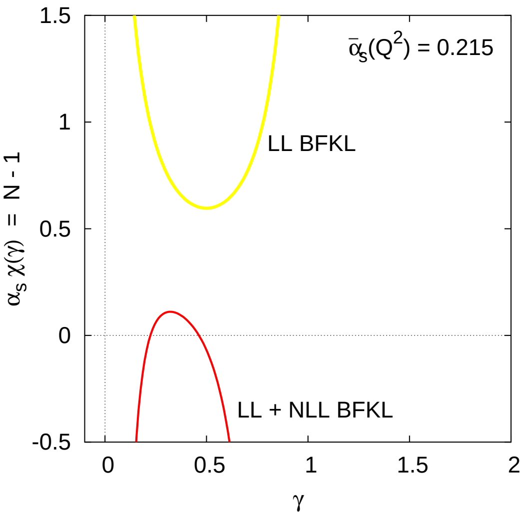
Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)

$$\bar{\alpha}_s \chi(\gamma) = \\ = \int \frac{dk^2}{k^2} \left(\frac{k^2}{k_0^2}\right)^{\gamma} \mathcal{K}(k, k_0)$$

Height of minimum is 'BFKL power'

$\gamma \to 0$ is small transverse distance region (normally described by DGLAP equations)

Building up the kernel...



Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)

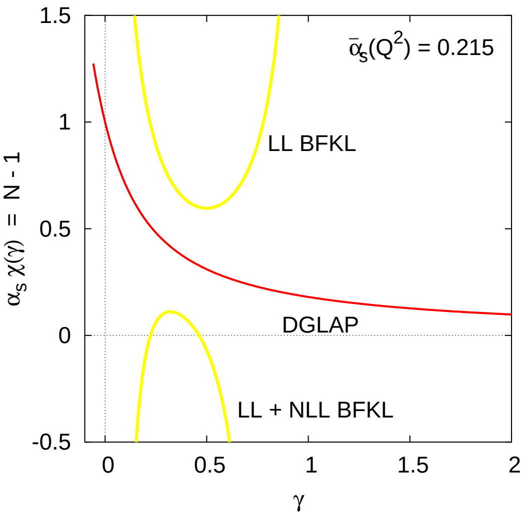
$$\bar{\alpha}_s \chi(\gamma) = \\ = \int \frac{dk^2}{k^2}\left(\frac{k^2}{k_0^2}\right)^{\gamma} \mathcal{K}(k, k_0)$$

Height of minimum is 'BFKL power'

$\gamma \to 0$ is small transverse distance region (normally described by DGLAP equations)

NB: DGLAP = 'rotated' plot of $\gamma(N)$

Building up the kernel...



Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)

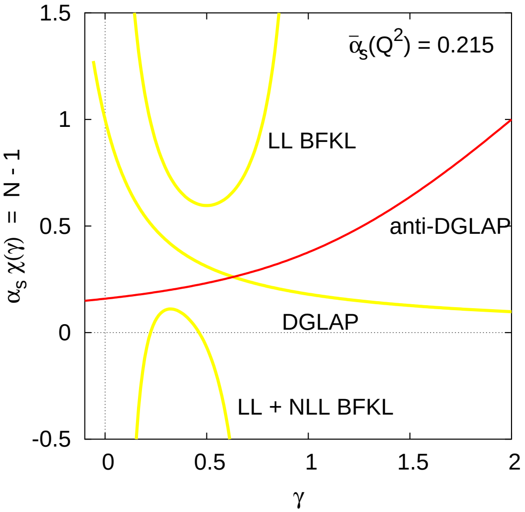
$$\bar{\alpha}_s \chi(\gamma) = \\ = \int \frac{dk^2}{k^2} \left(\frac{k^2}{k_0^2}\right)^{\gamma} \mathcal{K}(k, k_0)$$

Height of minimum is 'BFKL power'

$\gamma \to 0$ is small transverse distance region (normally described by DGLAP equations)

NB: DGLAP = 'rotated' plot of $\gamma(N)$

Building up the kernel. . .



Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)

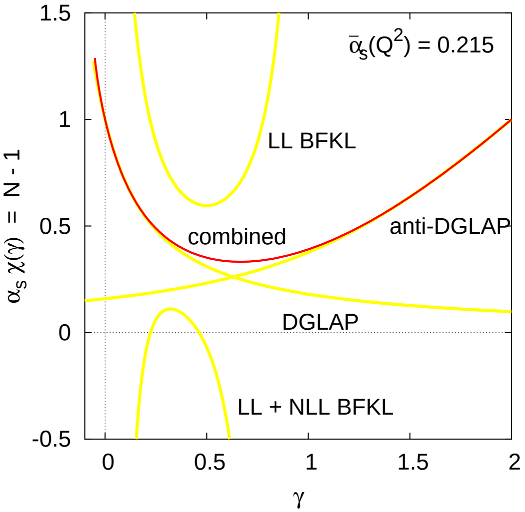
$$\bar{\alpha}_s \chi(\gamma) =$$
$$= \int \frac{dk^2}{k^2} \left(\frac{k^2}{k_0^2}\right)^{\gamma} \mathcal{K}(k, k_0)$$

Height of minimum is 'BFKL power'

$\gamma \to 0$ is small transverse distance region (normally described by DGLAP equations)

NB: DGLAP = 'rotated' plot of $\gamma(N)$

Building up the kernel...



Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)

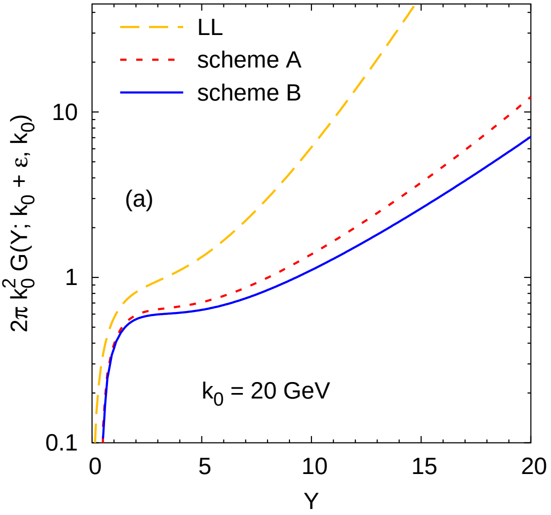
$$\bar{\alpha}_s\chi(\gamma) = \\ = \int \frac{dk^2}{k^2}\left(\frac{k^2}{k_0^2}\right)^\gamma \mathcal{K}(k, k_0)$$

Height of minimum is 'BFKL power'

$\gamma \to 0$ is small transverse distance region (normally described by DGLAP equations)

NB: DGLAP = 'rotated' plot of $\gamma(N)$

Green fn. from improved kernel



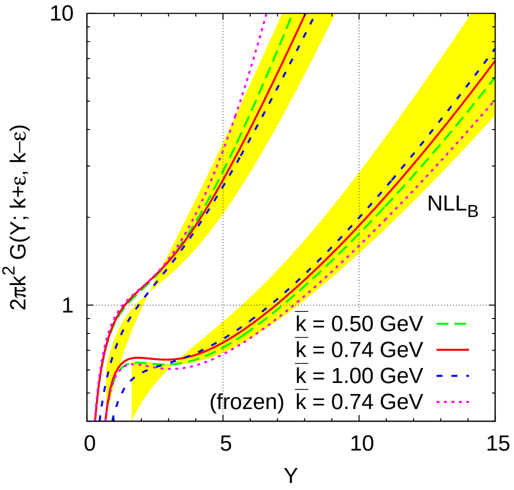
Various schemes for combining NLLx BFKL with DGLAP:

- scheme A ($\mathrm{NLL_A}$) violates mom. sum-rule at $\mathcal{O}\left(\alpha_s^2\right)$
- scheme B ($\mathrm{NLL_B}$) satisfies it at all orders

Different schemes $\longrightarrow$ similar results

more

Check stability of results



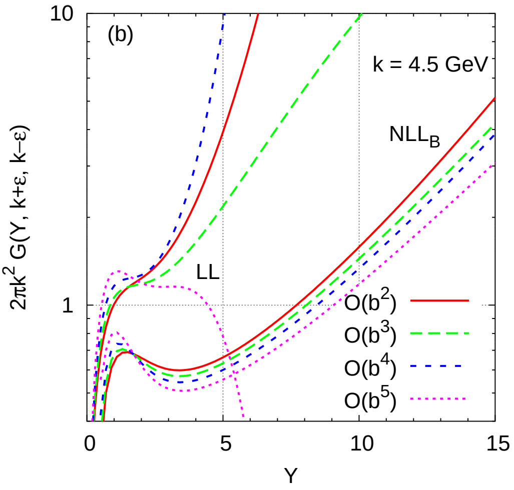
Check stability wrt:

- renorm. scale variation $1/2 < x_\mu < 2$
- change of infrared cutoff
- Instead of varying IR cutoff, write $\log G$ as *expansion in powers of b* (coeff. of β-fn) & truncate series

not quite renormalons

Uncertainties seem under control

Check stability of results



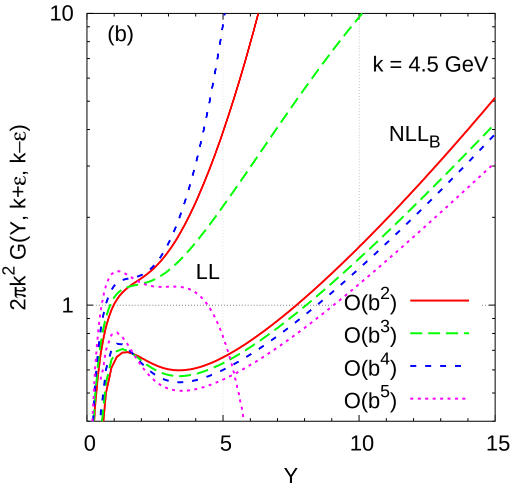
Check stability wrt:

- renorm. scale variation $1/2 < x_\mu < 2$
- change of infrared cutoff
- Instead of varying IR cutoff, write $\log G$ as *expansion in powers of b* (coeff. of β-fn) & truncate series

not quite renormalons

Uncertainties seem under control

Check stability of results



Check stability wrt:

- renorm. scale variation $1/2 < x_\mu < 2$
- change of infrared cutoff
- Instead of varying IR cutoff, write $\log G$ as *expansion in powers of b* (coeff. of β-fn) & truncate series

not quite renormalons

Uncertainties seem under control

Limitations of Green function

Green function

$G(Y, k, k_0)$ perturbatively calculable for $k, k_0 \gg \Lambda_{QCD}$.

- Fine for $\gamma^*\gamma^*$, Mueller-Navelet jets (hadron-hadron), Forward jets (DIS).
  But: rare processes – of interest mainly for testing BFKL

Recall:

We were interested in proton (e.g. $F_2(x, Q^2)$ structure fn in DIS).

- In best of cases, $k \sim Q \gg k_0 \sim \Lambda_{QCD}$
- Such structure functions *not perturbatively calculable*

Evolution in Q^2 *is* calculable

- via DGLAP *splitting functions*
- these also get small-x enhancements

➥ *Calculate them!*

Limitations of Green function

Green function

$G(Y, k, k_0)$ perturbatively calculable for $k, k_0 \gg \Lambda_{QCD}$.

- Fine for $\gamma^*\gamma^*$, Mueller-Navelet jets (hadron-hadron), Forward jets (DIS).

But: rare processes – of interest mainly for testing BFKL

Recall:

We were interested in proton (e.g. $F_2(x, Q^2)$ structure fn in DIS).

- In best of cases, $k \sim Q \gg k_0 \sim \Lambda_{QCD}$
- Such structure functions *not perturbatively calculable*

Evolution in Q^2 *is* calculable

- via DGLAP *splitting functions*
- these also get small-x enhancements

➥ *Calculate them!*

Limitations of Green function

Green function

$G(Y, k, k_0)$ perturbatively calculable for $k, k_0 \gg \Lambda_{QCD}$.

- Fine for $\gamma^*\gamma^*$, Mueller-Navelet jets (hadron-hadron), Forward jets (DIS).

But: rare processes – of interest mainly for testing BFKL

Recall:

We were interested in proton (e.g. $F_2(x, Q^2)$ structure fn in DIS).

- In best of cases, $k \sim Q \gg k_0 \sim \Lambda_{QCD}$
- Such structure functions *not perturbatively calculable*

Evolution in Q^2 *is* calculable

- via DGLAP *splitting functions*
- these also get small-x enhancements

➥ *Calculate them!*

Green function ⇒ splitting function

Construct a gluon density from Green function (take $k \gg k_0$):

$$xg(x, Q^2) \equiv \int^{Q} d^2k \; G^{(\nu_0 = k^2)}(\ln 1/x, k, k_0)$$

There should exist a *perturbative* splitting function, $P_{gg,\mathrm{eff}}(z, Q^2)$, such that

$$\frac{dg(x, Q^2)}{d \ln Q^2} = \int \frac{dz}{z} P_{gg,\mathrm{eff}}(z, Q^2)\, g\left(\frac{x}{z}, Q^2\right)$$

Factorisation

- Splitting function:

  red paths

- Green function:

  all paths

Splitting function ≡ evolution with cutoff

Green function $\Rightarrow$ splitting function

Construct a gluon density from Green function (take $k \gg k_0$):

$$xg(x,Q^2) \equiv \int^{Q} d^2k \; G^{(\nu_0 = k^2)}(\ln 1/x, k, k_0)$$

There should exist a *perturbative* splitting function, $P_{gg,\mathrm{eff}}(z,Q^2)$, such that

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \int \frac{dz}{z} P_{gg,\mathrm{eff}}(z,Q^2)\, g\left(\frac{x}{z}, Q^2\right)$$

Factorisation

- Splitting function:

  red paths

- Green function:

  all paths

Splitting function = evolution with cutoff

Green function $\Rightarrow$ splitting function

Construct a gluon density from Green function (take $k \gg k_0$):

$$xg(x, Q^2) \equiv \int^Q d^2k \; G^{(\nu_0 = k^2)}(\ln 1/x, k, k_0)$$

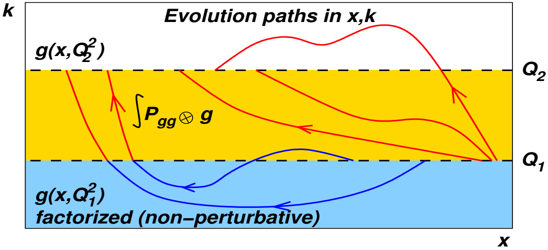
There should exist a *perturbative* splitting function, $P_{gg,\mathrm{eff}}(z, Q^2)$, such that

$$\frac{dg(x, Q^2)}{d \ln Q^2} = \int \frac{dz}{z} P_{gg,\mathrm{eff}}(z, Q^2) \, g\left(\frac{x}{z}, Q^2\right)$$

Factorisation

- Splitting function:
  red paths
- Green function:
  all paths

Splitting function $\equiv$ evolution with cutoff



Green function ⇒ splitting function

Construct a gluon density from Green function (take $k \gg k_0$):

$$xg(x, Q^2) \equiv \int^Q d^2k \; G^{(\nu_0 = k^2)}(\ln 1/x, k, k_0)$$

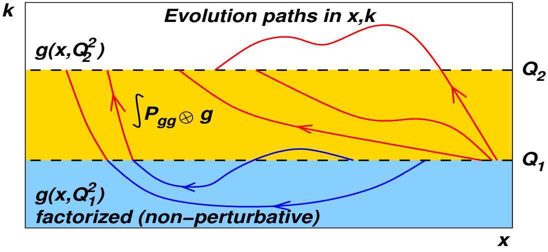
There should exist a *perturbative* splitting function, $P_{gg,\mathrm{eff}}(z, Q^2)$, such that

$$\frac{dg(x, Q^2)}{d \ln Q^2} = \int \frac{dz}{z} P_{gg,\mathrm{eff}}(z, Q^2)\, g\left(\frac{x}{z}, Q^2\right)$$

Factorisation

- Splitting function:
  red paths
- Green function:
  all paths

Splitting function ≡ evolution with cutoff



Perturbative structure of P_{gg}

- Small-x gluon splitting function has logarithmic enhancements:

$$xP_{gg}(x) = \sum_{n=1} \alpha_s^n \ln^{n-1}\frac{1}{x} + \sum_{n=2} \alpha_s^n \ln^{n-2}\frac{1}{x} + \dots$$

- NNLO (α_s^3): first small-x enhancement in gluon splitting function.

Leading Logs (LLx)

$$\bar{\alpha}_s + \frac{\zeta(3)}{3}\bar{\alpha}_s^4 \ln^3\frac{1}{x} + \frac{\zeta(5)}{60}\bar{\alpha}_s^6 \ln^5\frac{1}{x} + \cdots$$

Next-to-Leading Logs (NLLx)

$$A_{20}\bar{\alpha}_s^2 + A_{31}\bar{\alpha}_s^3 \ln\frac{1}{x} + A_{42}\bar{\alpha}_s^4 \ln^3\frac{1}{x} + \dots$$

Perturbative structure of P_{gg}

- Small-x gluon splitting function has logarithmic enhancements:

$$xP_{gg}(x) = \boxed{\sum_{n=1} \alpha_s^n \ln^{n-1} \frac{1}{x}}$$

$$+ \sum_{n=2} \alpha_s^n \ln^{n-2} \frac{1}{x} + \ldots$$

<u>Leading Logs (LLx)</u>

$$\bar{\alpha}_s + \frac{\zeta(3)}{3} \bar{\alpha}_s^4 \ln^3 \frac{1}{x} + \frac{\zeta(5)}{60} \bar{\alpha}_s^6 \ln^5 \frac{1}{x} + \cdots$$

<u>Next-to-Leading Logs (NLLx)</u>

$$A_{20}\bar{\alpha}_s^2 + A_{31}\bar{\alpha}_s^3 \ln \frac{1}{x} + A_{42}\bar{\alpha}_s^4 \ln^3 \frac{1}{x} + \ldots$$

- NNLO (α_s^3): first small-x enhancement in gluon splitting function.

Perturbative structure of P_{gg}

- Small-x gluon splitting function has logarithmic enhancements:

$$xP_{gg}(x) = \sum_{n=1} \alpha_s^n \ln^{n-1}\frac{1}{x}$$

$$+ \boxed{\sum_{n=2} \alpha_s^n \ln^{n-2}\frac{1}{x}} + \ldots$$

<u>Leading Logs (LLx)</u>

$$\bar{\alpha}_s + \frac{\zeta(3)}{3}\bar{\alpha}_s^4 \ln^3\frac{1}{x} + \frac{\zeta(5)}{60}\bar{\alpha}_s^6 \ln^5\frac{1}{x} + \cdots$$

<u>Next-to-Leading Logs (NLLx)</u>

$$A_{20}\bar{\alpha}_s^2 + A_{31}\bar{\alpha}_s^3 \ln\frac{1}{x} + A_{42}\bar{\alpha}_s^4 \ln^3\frac{1}{x} + \ldots$$

- NNLO (α_s^3): first small-x enhancement in gluon splitting function.

Understanding small-x becomes unavoidable

Perturbative structure of P_{gg}

- Small-x gluon splitting function has logarithmic enhancements:

$$xP_{gg}(x) = \sum_{n=1} \alpha_s^n \ln^{n-1} \frac{1}{x}$$

$$+ \sum_{n=2} \alpha_s^n \ln^{n-2} \frac{1}{x} + \ldots$$

Leading Logs (LLx)

$$\bar{\alpha}_s + \frac{\zeta(3)}{3} \bar{\alpha}_s^4 \ln^3 \frac{1}{x} + \frac{\zeta(5)}{60} \bar{\alpha}_s^6 \ln^5 \frac{1}{x} + \cdots$$

Next-to-Leading Logs (NLLx)

$$A_{20}\bar{\alpha}_s^2 + \boxed{A_{31}\bar{\alpha}_s^3 \ln \frac{1}{x}} + A_{42}\bar{\alpha}_s^4 \ln^3 \frac{1}{x} + \ldots$$

- NNLO (α_s^3): first small-x enhancement in gluon splitting function.

Understanding small-x becomes unavoidable

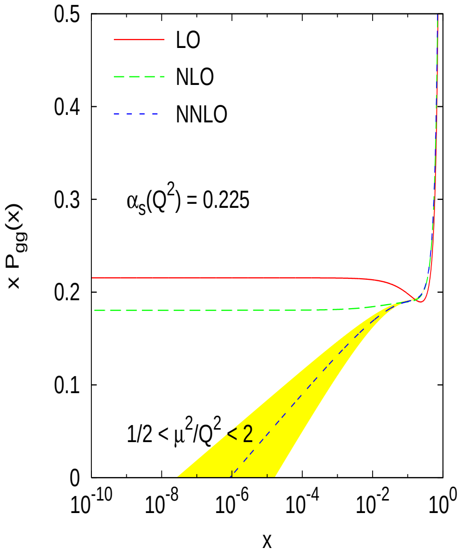
Perturbative structure of P_{gg}

- Small-x gluon splitting function has logarithmic enhancements:

$$xP_{gg}(x) = \sum_{n=1} \alpha_s^n \ln^{n-1}\frac{1}{x} + \sum_{n=2} \alpha_s^n \ln^{n-2}\frac{1}{x} + \ldots$$

- NNLO (α_s^3): first small-x enhancement in gluon splitting function.

Understanding small-x becomes unavoidable



Moch, Vermaseren & Vogt

LLx, NLLx

Problem:

- LLx terms rise very fast, $xP_{gg}(x) \sim x^{-0.5}$.

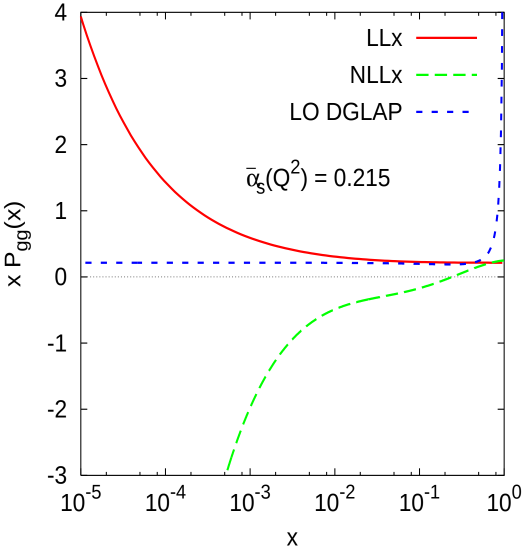
  Incompatible with data.

  Ball & Forte '95

- NLLx terms go negative very fast.

  No one's even tried fitting the data!

[NB: Taking NLLx terms of P_{gg} is almost the worst possible expansion]



BFKL splitting function 'power'

Two classes of correction, to power growth ω:

$$\omega = 4\ln 2\,\bar{\alpha}_s(Q^2)\left(1 - \underbrace{6.5\,\bar{\alpha}_s}_{NLL} - \underbrace{4.0\,\bar{\alpha}_s^{2/3}}_{running} + \cdots\right)$$

- NLL piece is *universal*

As before, add approximate higher orders via $\mathrm{NLL_B}$ kernel

- running piece appears only in problems with *cutoffs*
  - a consequence of *asymmetry* due to cutoff (only scales higher than cutoff contribute)

$$\alpha_s(Q^2) \to \alpha_s(Q^2 e^{-X/(b\alpha_s)^{1/3}})$$

Hancock & Ross '92

- Beyond first terms, not possible to separate effects of 'pure' higher orders & running coupling

Obtain $G(Y, k, k_0) \Rightarrow g(x, Q^2)$ with arbitrary non-pert. condition, deconvolute $\partial_{\ln Q^2} g = P_{gg} \otimes g \Longrightarrow P_{gg}$

BFKL splitting function 'power'

Two classes of correction, to power growth ω:

$$\omega = 4\ln 2\,\bar{\alpha}_s(Q^2)\left(1 - \underbrace{6.5\,\bar{\alpha}_s}_{NLL} - \underbrace{4.0\,\bar{\alpha}_s^{2/3}}_{running} + \cdots\right)$$

- NLL piece is *universal*

As before, add approximate higher orders via $\mathrm{NLL_B}$ kernel

- running piece appears only in problems with *cutoffs*
  - a consequence of *asymmetry* due to cutoff (only scales higher than cutoff contribute)

$$\alpha_s(Q^2) \to \alpha_s(Q^2 e^{-X/(b\alpha_s)^{1/3}})$$

Hancock & Ross '92

- Beyond first terms, not possible to separate effects of 'pure' higher orders & running coupling

Obtain $G(Y, k, k_0) \Rightarrow g(x, Q^2)$ with arbitrary non-pert. condition,
deconvolute $\partial_{\ln Q^2} g = P_{gg} \otimes g \Longrightarrow P_{gg}$

BFKL splitting function 'power'

Two classes of correction, to power growth ω:

$$\omega = 4\ln 2\,\bar{\alpha}_s(Q^2)\left(1 - \underbrace{6.5\,\bar{\alpha}_s}_{NLL} - \underbrace{4.0\,\bar{\alpha}_s^{2/3}}_{running} + \cdots\right)$$

- NLL piece is *universal*

As before, add approximate higher orders via $\mathrm{NLL_B}$ kernel

- running piece appears only in problems with *cutoffs*
  - a consequence of *asymmetry* due to cutoff (only scales higher than cutoff contribute)

$$\alpha_s(Q^2) \to \alpha_s(Q^2 e^{-X/(b\alpha_s)^{1/3}})$$

Hancock & Ross '92

- Beyond first terms, not possible to separate effects of 'pure' higher orders & running coupling

Obtain $G(Y,k,k_0) \Rightarrow g(x,Q^2)$ with arbitrary non-pert. condition,
deconvolute $\partial_{\ln Q^2} g = P_{gg} \otimes g \Longrightarrow P_{gg}$

BFKL splitting function 'power'

Two classes of correction, to power growth ω:

$$\omega = 4 \ln 2\, \bar{\alpha}_s(Q^2) \left(1 - \underbrace{6.5\, \bar{\alpha}_s}_{NLL} - \underbrace{4.0\, \bar{\alpha}_s^{2/3}}_{running} + \cdots \right)$$

- NLL piece is *universal*

As before, add approximate higher orders via $\mathrm{NLL_B}$ kernel

- running piece appears only in problems with *cutoffs*
  - a consequence of *asymmetry* due to cutoff (only scales higher than cutoff contribute)

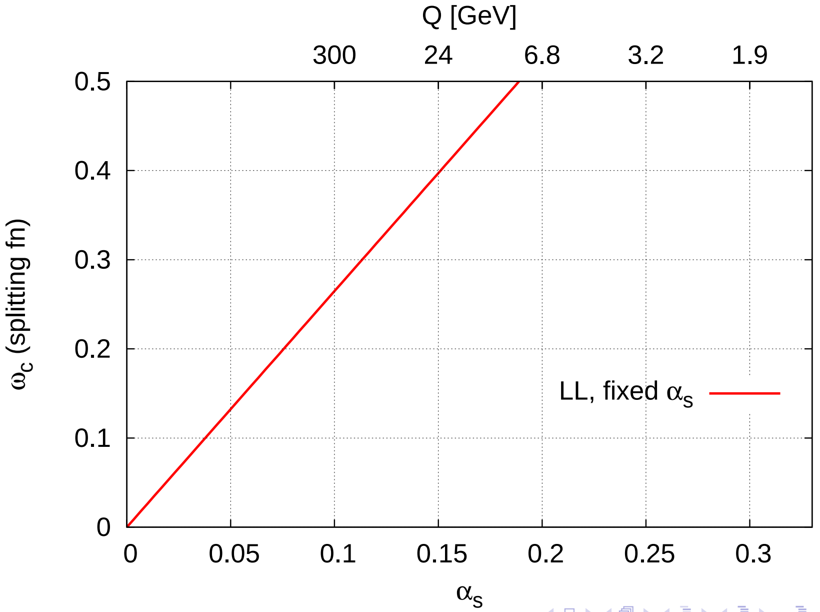
$$\alpha_s(Q^2) \to \alpha_s(Q^2 e^{-X/(b\alpha_s)^{1/3}})$$

Hancock & Ross '92

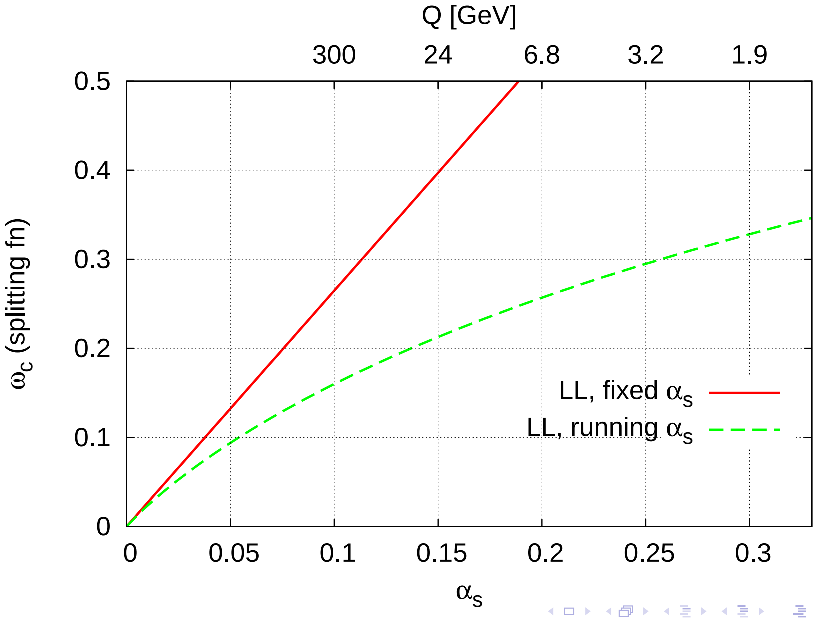
- Beyond first terms, not possible to separate effects of 'pure' higher orders & running coupling

Obtain $G(Y, k, k_0) \Rightarrow g(x, Q^2)$ with arbitrary non-pert. condition,
deconvolute $\partial_{\ln Q^2} g = P_{gg} \otimes g \Longrightarrow P_{gg}$

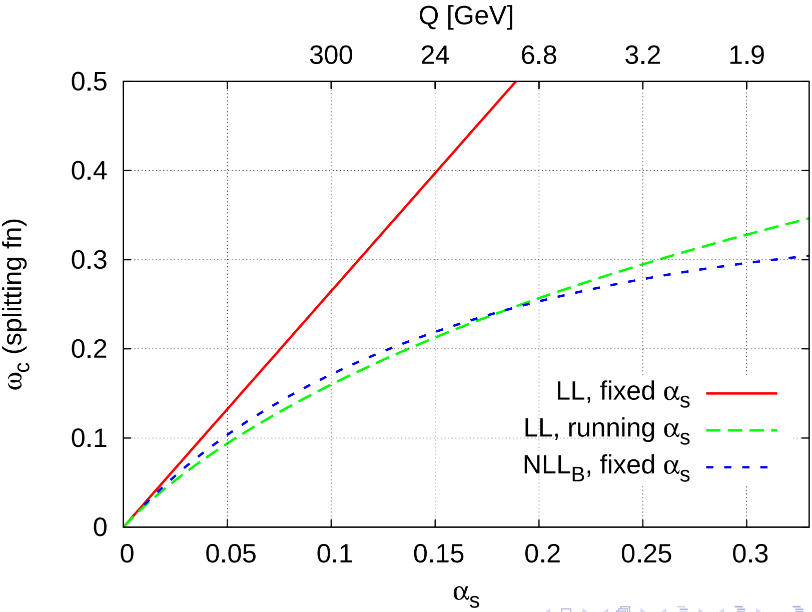
BFKL P_{gg} power growth — results



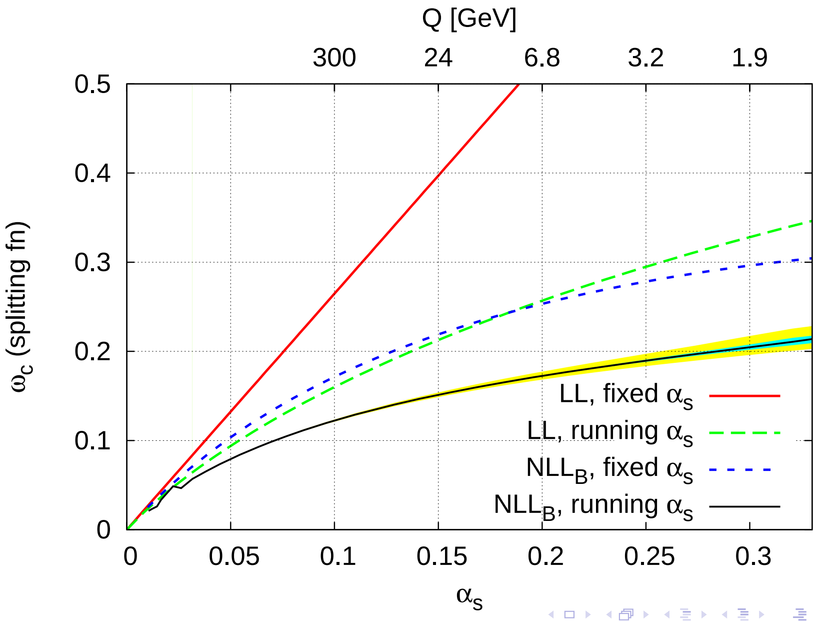
BFKL P_{gg} power growth — results



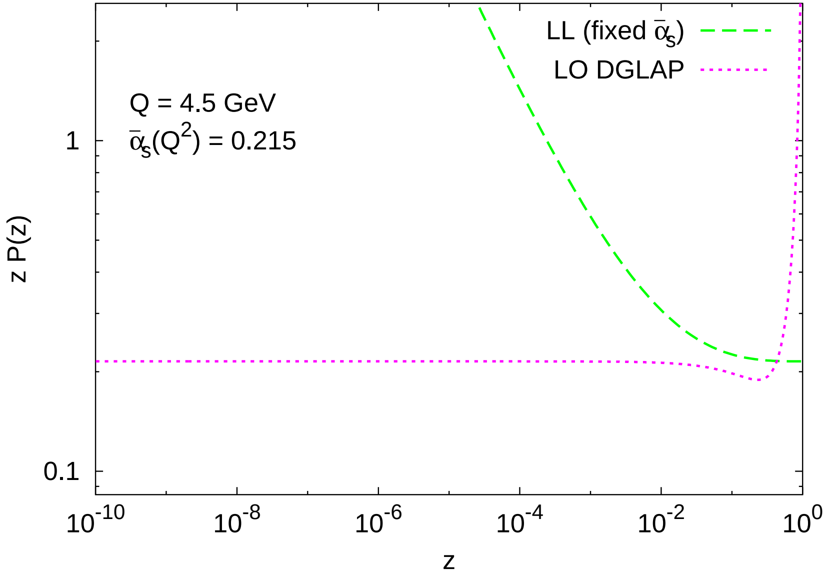
BFKL P_{gg} power growth — results



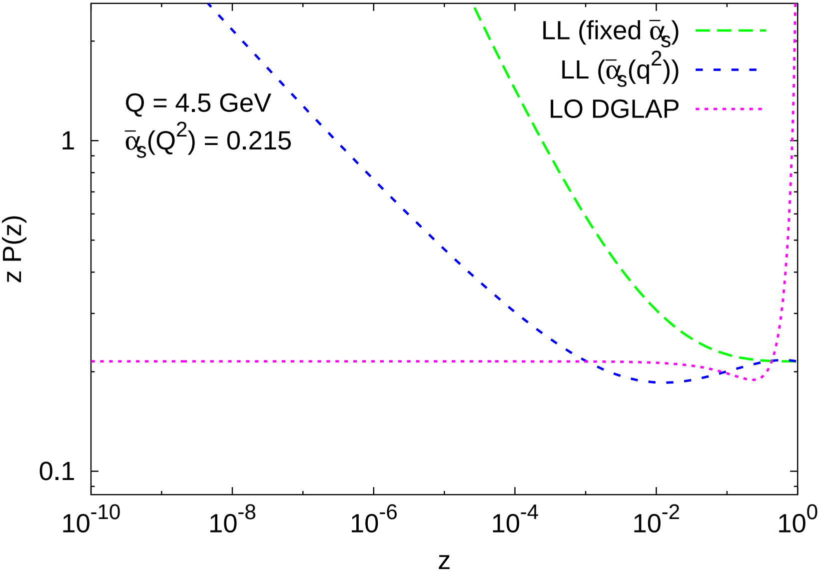
BFKL P_{gg} power growth — results



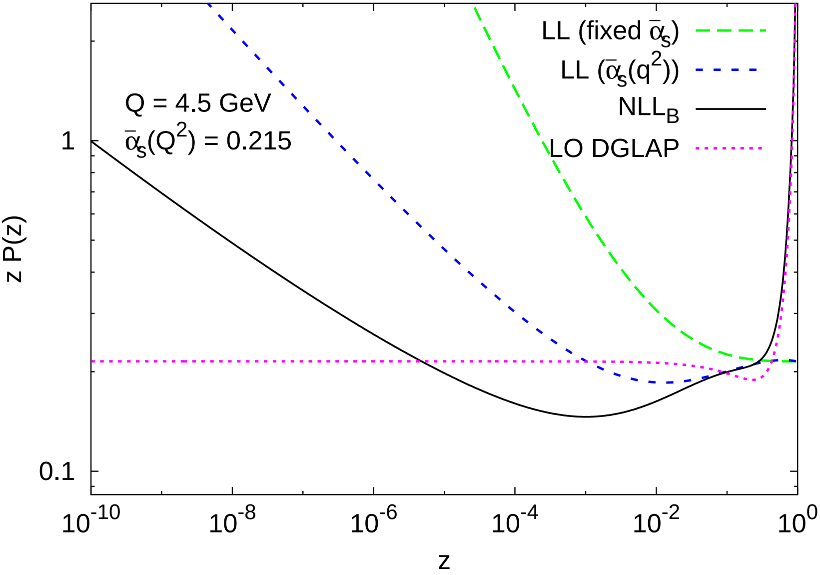
Full $P_{gg}(z)$ splitting fn



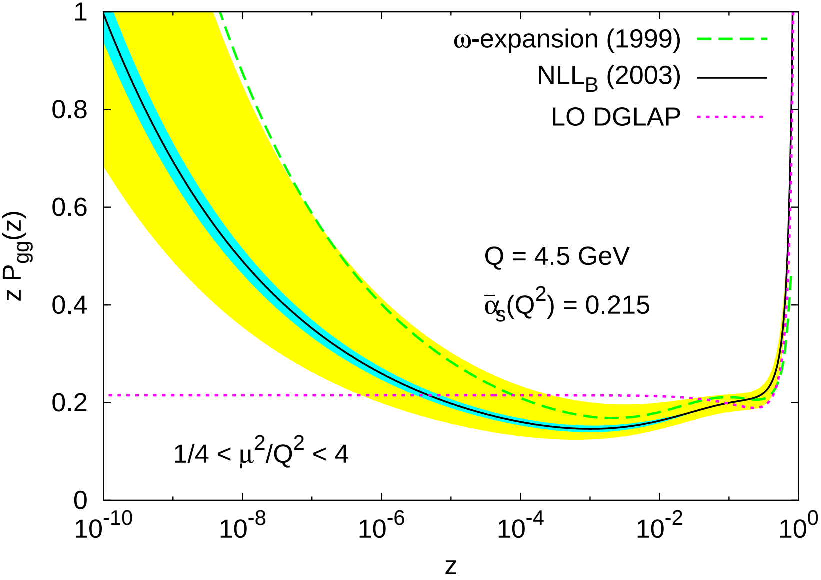
Full $P_{gg}(z)$ splitting fn



Full $P_{gg}(z)$ splitting fn



Full $P_{gg}(z)$ splitting fn



Phenomenology: dip dominates P_{gg}

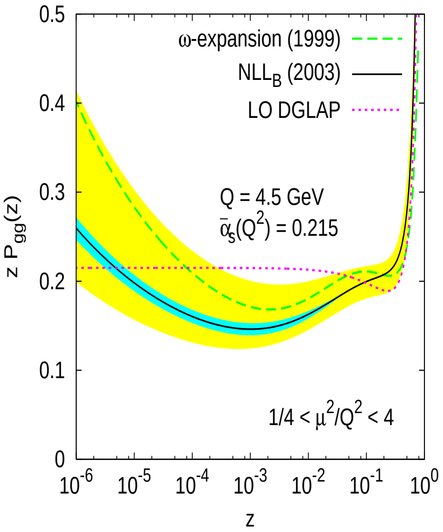
- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at $x \sim 10^{-3}$*

Questions:

- Various 'dips' have been seen
  Thorne '99, '01 (running α_s, NLLx)
  ABF '99–'03 (fits, running α_s)
  CCSS '01,'03 (running α_s, NLL$_B$)
  Is it always the same dip?
- Is the dip a rigorous prediction?
- What is its origin?
  Running α_s, mom. sum rule...?

NNLO DGLAP gives a clue...

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$



Phenomenology: dip dominates P_{gg}

- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at* $x \sim 10^{-3}$

Questions:

- Various 'dips' have been seen

  Thorne '99, '01 (running α_s, NLLx)

  ABF '99–'03 (fits, running α_s)

  CCSS '01,'03 (running α_s, NLL$_B$)

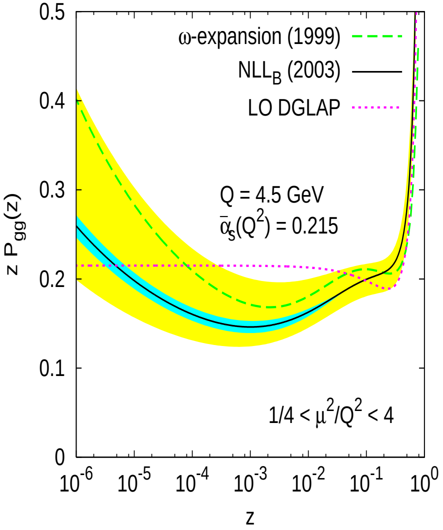
  Is it always the same dip?

- Is the dip a rigorous prediction?
- What is its origin?

  Running α_s, mom. sum rule. . . ?

NNLO DGLAP gives a clue. . .

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$



Phenomenology: dip dominates P_{gg}

- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at $x \sim 10^{-3}$*

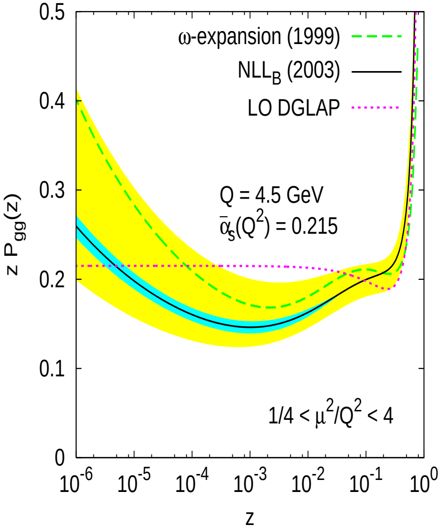
Questions:

- Various 'dips' have been seen
  Thorne '99, '01 (running α_s, NLLx)
  ABF '99–'03 (fits, running α_s)
  CCSS '01,'03 (running α_s, NLL$_B$)

  Is it always the same dip?
- Is the dip a rigorous prediction?
- What is its origin?
  Running α_s, mom. sum rule. . . ?

NNLO DGLAP gives a clue. . .

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$



Phenomenology: dip dominates P_{gg}

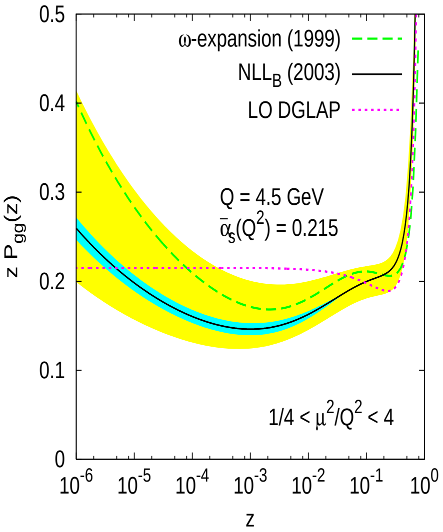
- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at $x \sim 10^{-3}$*

Questions:

- Various 'dips' have been seen
  Thorne '99, '01 (running α_s, NLLx)
  ABF '99–'03 (fits, running α_s)
  CCSS '01,'03 (running α_s, NLL$_B$)
  Is it always the same dip?
- Is the dip a rigorous prediction?
- What is its origin?
  Running α_s, mom. sum rule. . . ?

NNLO DGLAP gives a clue. . .

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$



Phenomenology: dip dominates P_{gg}

- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at $x \sim 10^{-3}$*

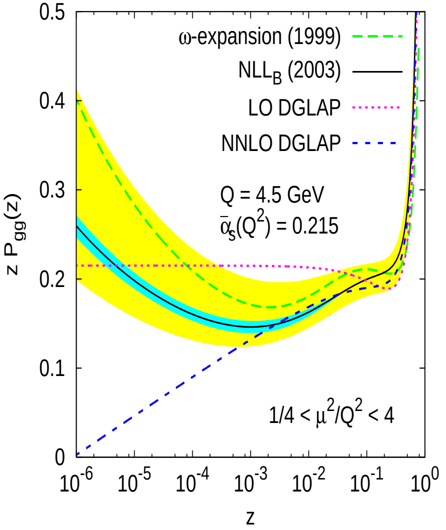
Questions:

- Various 'dips' have been seen
  Thorne '99, '01 (running α_s, NLLx)
  ABF '99–'03 (fits, running α_s)
  CCSS '01,'03 (running α_s, NLL$_B$)

  Is it always the same dip?
- Is the dip a rigorous prediction?
- What is its origin?
  Running α_s, mom. sum rule. . . ?

NNLO DGLAP gives a clue. . .

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$



Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
$\vdots$				

const.

ln 1/x

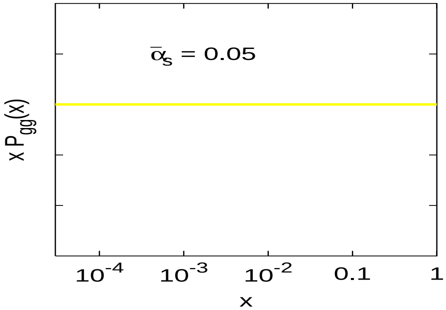
ln^2 1/x

ln^3 1/x

Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
$\vdots$				

const.
$\ln 1/x$
$\ln^2 1/x$
$\ln^3 1/x$



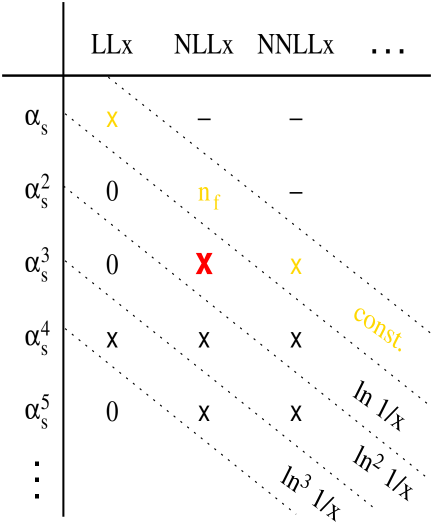
At moderately small x, first terms with x-dependence are

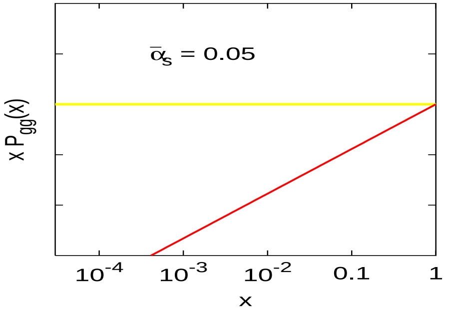
$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x} + 0.401\,\bar{\alpha}_s^4 \ln^3\frac{1}{x}$$

Minimum when

$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln\frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$

Reorganise perturbative series





At moderately small x, first terms with x-dependence are

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x} + 0.401\,\bar{\alpha}_s^4 \ln^3 \frac{1}{x}$$

Minimum when

$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln \frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$

Reorganise perturbative series

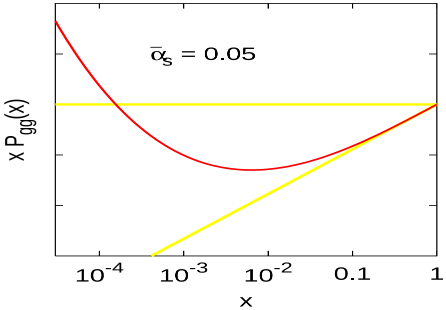
	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	**x**	x	x	
α_s^5	0	x	x	
⋮				

const.

$\ln 1/x$

$\ln^2 1/x$

$\ln^3 1/x$



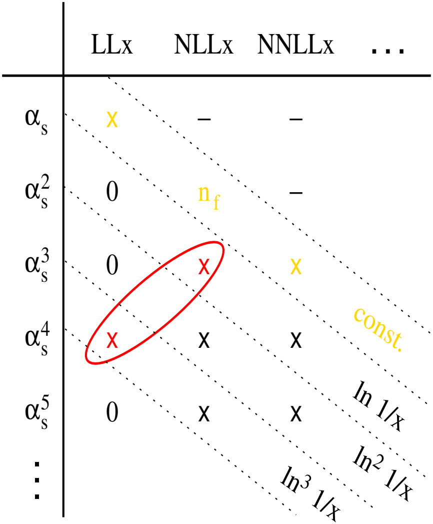
At moderately small x, first terms with x-dependence are

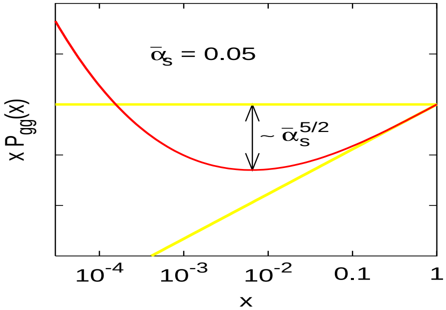
$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x} + \mathbf{0.401}\,\bar{\alpha}_s^4 \ln^3\frac{1}{x}$$

Minimum when

$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln\frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$

Reorganise perturbative series





At moderately small x, first terms with x-dependence are

$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x} + 0.401\,\bar{\alpha}_s^4 \ln^3\frac{1}{x}$$

Minimum when

$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln\frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$

Systematic expansion in $\sqrt{\alpha_s}$

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
$\vdots$				

const.
ln 1/x
ln^2 1/x
ln^3 1/x

Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}} + 6.947 + \cdots$$

Depth of dip

$$-d \simeq -1.237\bar{\alpha}_s^{5/2} - 11.15\bar{\alpha}_s^3 + \cdots$$

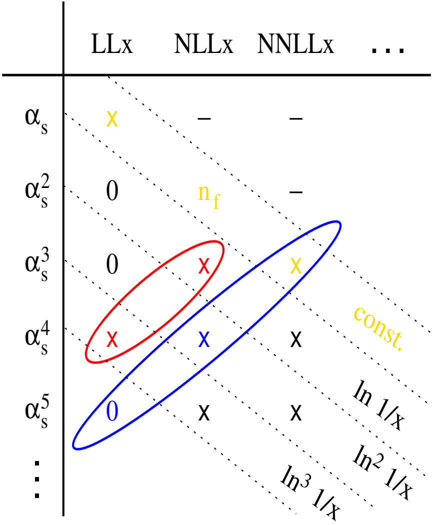
NB:

- convergence is very poor

  As ever at small x!

- higher-order terms in expansion need NNLLx info

Systematic expansion in $\sqrt{\alpha_s}$



Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}} + 6.947 + \cdots$$

Depth of dip

$$-d \simeq -1.237\bar{\alpha}_s^{5/2} - 11.15\bar{\alpha}_s^3 + \cdots$$

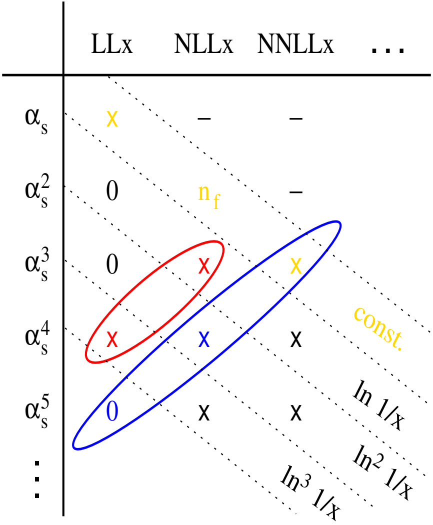
NB:

- convergence is very poor

  As ever at small x!

- higher-order terms in expansion need NNLLx info

Systematic expansion in $\sqrt{\alpha_s}$



Position of dip

$$\ln\frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}} + 6.947 + \cdots$$

Depth of dip

$$-d \simeq -1.237\bar{\alpha}_s^{5/2} - 11.15\bar{\alpha}_s^3 + \cdots$$

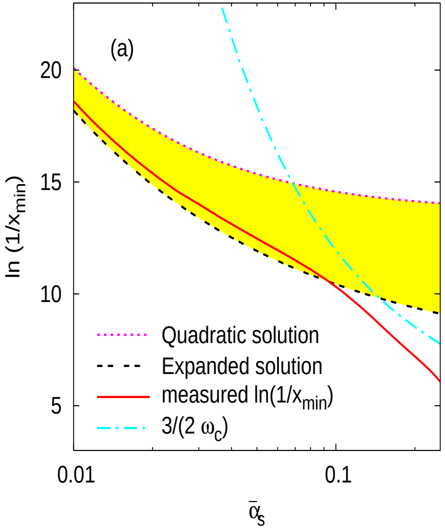
NB:

- convergence is very poor

  As ever at small x!

- higher-order terms in expansion need NNLLx info

Test $\sqrt{\alpha_s}$ dip expansion



Test *position* of dip v. α_s

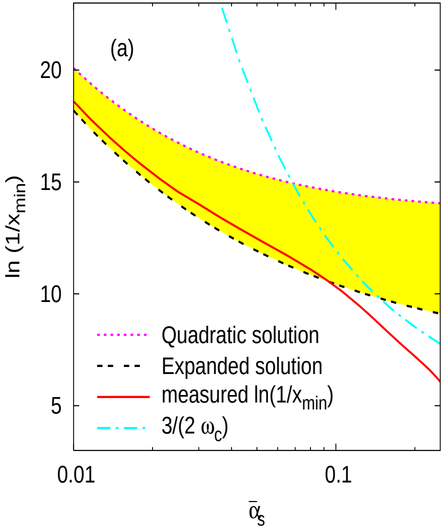
- Band is uncertainty due to higher orders in $\sqrt{\alpha_s}$
- At small α_s, good agreement $\rightarrow$ confirmation of 'dip mechanism'
- At moderate α_s, normal small-x resummation effects 'collide' with dip

$$\ln \frac{1}{x_{\min}} \lesssim \frac{3}{2\omega_c}$$

Dip then comes from interplay between $\alpha_s^3 \ln x$ (NNLO) term and full resummation.

[Actually, story more complex]

Test $\sqrt{\alpha_s}$ dip expansion



Test *position* of dip v. α_s

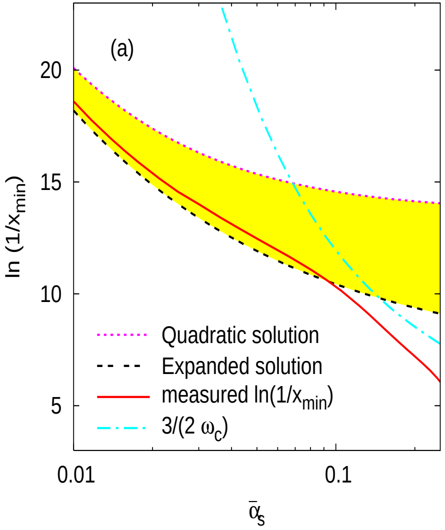
- Band is uncertainty due to higher orders in $\sqrt{\alpha_s}$
- At small α_s, good agreement $\rightarrow$ confirmation of 'dip mechanism'
- At moderate α_s, normal small-x resummation effects 'collide' with dip

$$\ln \frac{1}{x_{\min}} \lesssim \frac{3}{2\omega_c}$$

Dip then comes from interplay between $\alpha_s^3 \ln x$ (NNLO) term and full resummation.

[Actually, story more complex]

Test $\sqrt{\alpha_s}$ dip expansion



Test *position* of dip v. α_s

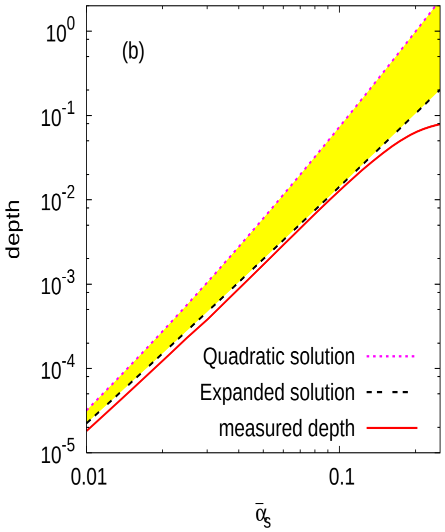
- Band is uncertainty due to higher orders in $\sqrt{\alpha_s}$
- At small α_s, good agreement $\to$ confirmation of 'dip mechanism'
- At moderate α_s, normal small-x resummation effects 'collide' with dip

$$\ln \frac{1}{x_{\min}} \lesssim \frac{3}{2\omega_c}$$

Dip then comes from interplay between $\alpha_s^3 \ln x$ (NNLO) term and full resummation.

[Actually, story more complex]

Test $\sqrt{\alpha_s}$ dip expansion



Test *depth* of dip v. α_s

- similar conclusions!

Phenomenological impact?

Phenomenological relevance comes through impact on growth of small-x gluon with Q^2.

$$\frac{\partial g(x, Q^2)}{d \ln Q^2} = P_{gg} \otimes g + P_{gq} \otimes q$$

At small x, $P_{gg} \otimes g$ dominates.

Take CTEQ6M gluon as 'test' case for convolution.

Because it's nicely behaved at small-x

Phenomenological impact?

Phenomenological relevance comes through impact on growth of small-x gluon with Q^2.

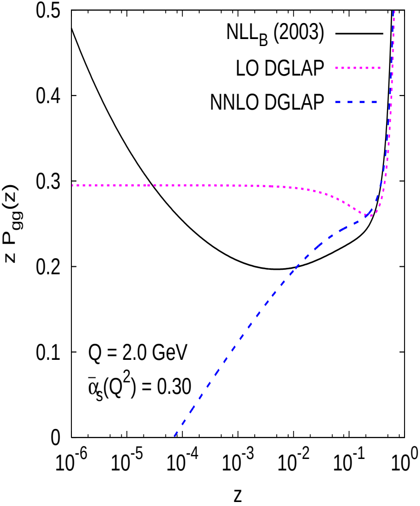
$$\frac{\partial g(x,Q^2)}{d\ln Q^2} = P_{gg}\otimes g + P_{gq}\otimes q$$

At small x, $P_{gg}\otimes g$ dominates.

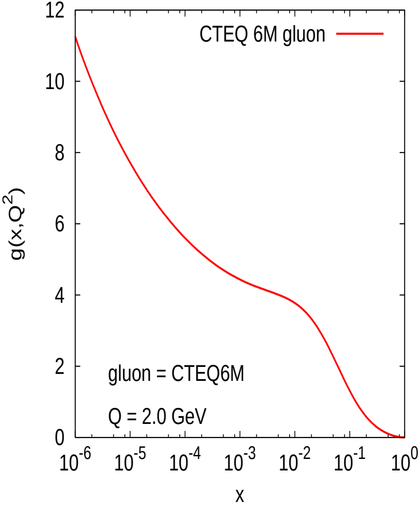
Take CTEQ6M gluon as ‘test’ case for convolution.

Because it’s nicely behaved at small-x

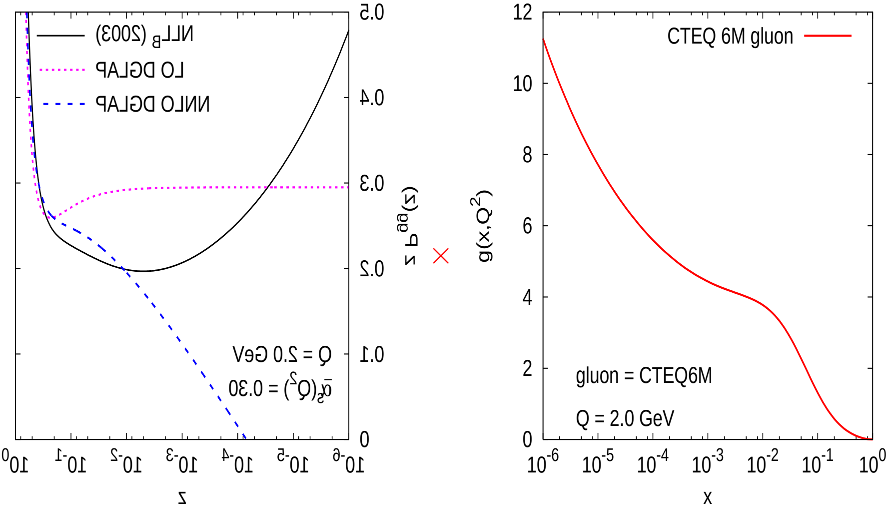
Phenomenological impact? $P_{gg} \otimes g(x)$



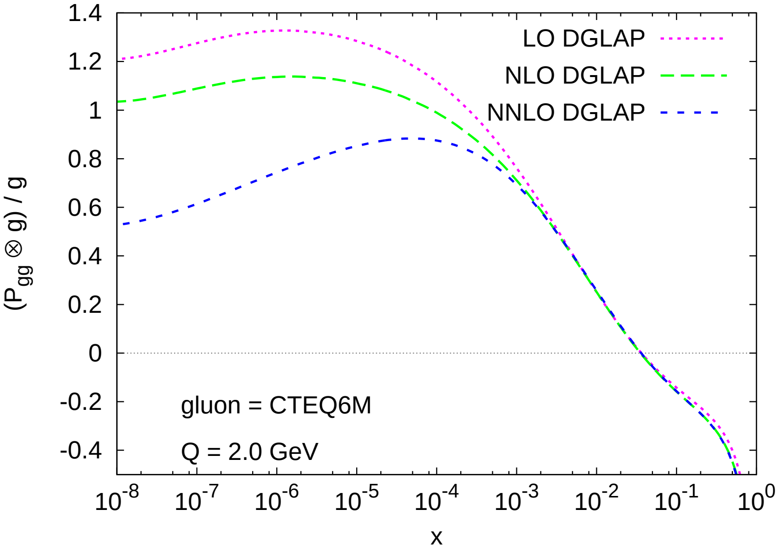
$\otimes$



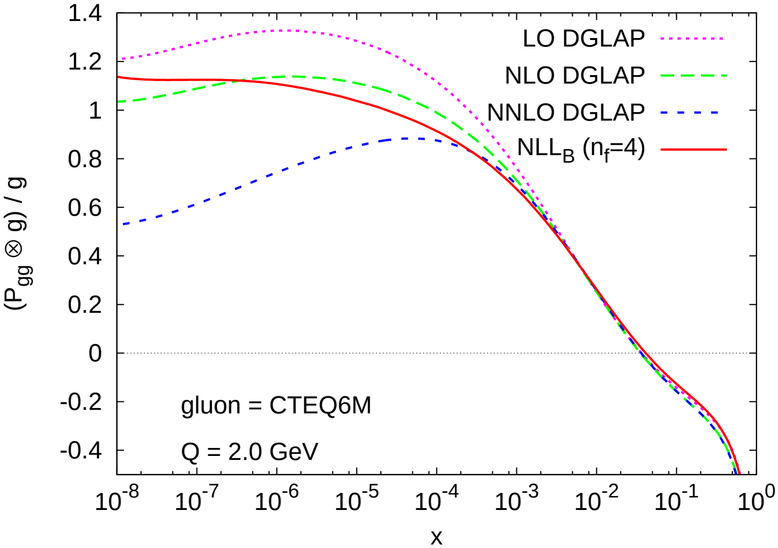
Phenomenological impact? $P_{gg} \otimes g(x)$



$P_{gg} \otimes g(x)$



$P_{gg} \otimes g(x)$



Conclusions

- High-energy limit is one of most challenging problems of QCD.
- Much is now understood about some central elements of small-x resummations:
  - gluon *Green* function
  - gluon *splitting* function

  It seems both can be predicted with confidence
- Phenomenological tests are *essential*
  - Mueller-Navelet jets at LHC, $\gamma^*\gamma^*$ at ILC
  - Structure functions from HERA
- Some ingredients still missing
  - NLL Impact factors
  - Full singlet matrix for splitting functions (not just P_{gg}).
- Big, active, question not touched on:

Saturation & limit of high gluon density

Conclusions

- High-energy limit is one of most challenging problems of QCD.
- Much is now understood about some central elements of small-x resummations:
  - gluon *Green* function
  - gluon *splitting* function

  It seems both can be predicted with confidence
- Phenomenological tests are *essential*
  - Mueller-Navelet jets at LHC, $\gamma^*\gamma^*$ at ILC
  - Structure functions from HERA
- Some ingredients still missing
  - NLL Impact factors
  - Full singlet matrix for splitting functions (not just P_{gg}).
- Big, active, question not touched on:

  Saturation & limit of high gluon density

Conclusions

- High-energy limit is one of most challenging problems of QCD.
- Much is now understood about some central elements of small-x resummations:
  - gluon *Green* function
  - gluon *splitting* function

  It seems both can be predicted with confidence
- Phenomenological tests are *essential*
  - Mueller-Navelet jets at LHC, $\gamma^*\gamma^*$ at ILC
  - Structure functions from HERA
- Some ingredients still missing
  - NLL Impact factors
  - Full singlet matrix for splitting functions (not just P_{gg}).
- Big, active, question not touched on:

Saturation & limit of high gluon density

Conclusions

- High-energy limit is one of most challenging problems of QCD.
- Much is now understood about some central elements of small-x resummations:
  - gluon *Green* function
  - gluon *splitting* function

  It seems both can be predicted with confidence
- Phenomenological tests are *essential*
  - Mueller-Navelet jets at LHC, $\gamma^*\gamma^*$ at ILC
  - Structure functions from HERA
- Some ingredients still missing
  - NLL Impact factors
  - Full singlet matrix for splitting functions (not just P_{gg}).
- Big, active, question not touched on:

  Saturation & limit of high gluon density

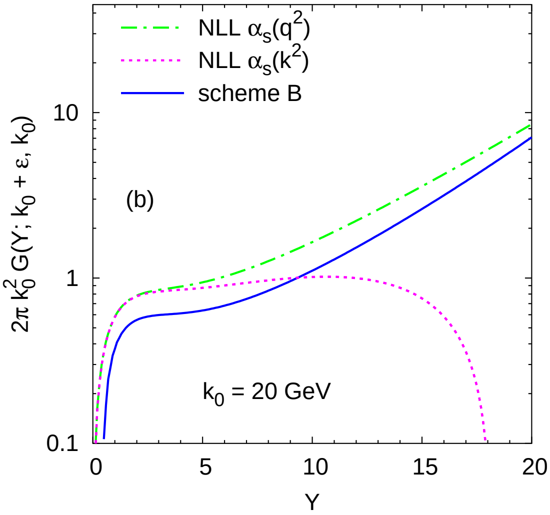
Conclusions

- High-energy limit is one of most challenging problems of QCD.
- Much is now understood about some central elements of small-x resummations:
  - gluon *Green* function
  - gluon *splitting* function

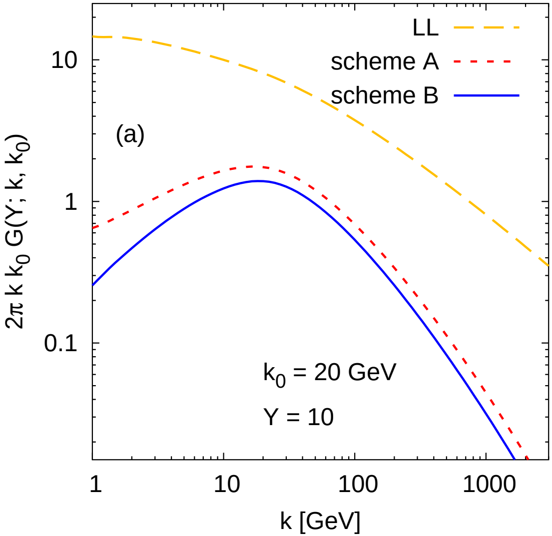
  It seems both can be predicted with confidence
- Phenomenological tests are *essential*
  - Mueller-Navelet jets at LHC, $\gamma^*\gamma^*$ at ILC
  - Structure functions from HERA
- Some ingredients still missing
  - NLL Impact factors
  - Full singlet matrix for splitting functions (not just P_{gg}).
- Big, active, question not touched on:

Saturation & limit of high gluon density

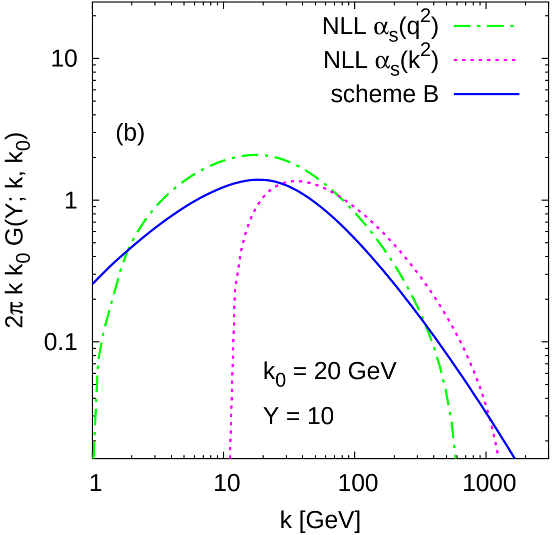
Green function (extra slides)



Green function (extra slides)



Green function (extra slides)



Towards phenomenology?

Steps missing for 'full' phenomenology:

- Resummation of all entries of singlet matrix & coefficient functions.
- Put results in $\overline{\text{MS}}$ factorisation scheme

↪illustrate nature of surprises that arise...

Factorisation scheme

Results shown so far in Q_0 scheme. [Catani, Ciafaloni & Hautmann '93]

$$xg(x, Q^2) \equiv \int d^2k \; G(\ln 1/x, k, k_0)\Theta(Q-k) \qquad G^{(0)} = f(x)\delta^2(k-k_0)$$

To translate to $\overline{\text{MS}}$ scheme

$$xg(x, Q^2) \equiv \int d^2k \; G(\ln 1/x, k, k_0) r\left(\frac{k^2}{Q^2}\right), \qquad r\left(\frac{k^2}{Q^2}\right) = \int \frac{d\gamma \, e^{\gamma \ln \frac{Q^2}{k^2}}}{2\pi i \, \gamma R(\gamma)}$$

Should be easy?!

$$R(\gamma) = \left\{\frac{\Gamma(1-\gamma)\chi(\gamma)}{\Gamma(1+\gamma)[-\gamma\chi'(\gamma)]}\right\}^{\frac{1}{2}} \exp\left\{\int_0^\gamma d\gamma' \frac{\psi'(1)-\psi'(1-\gamma')}{\chi(\gamma')}\right\}$$

Catani & Hautmann '94

[NB: involves $\chi(\gamma)$ — does this need to be collinearly improved? Ignore problem for now. . .]

Factorisation scheme

Results shown so far in Q_0 scheme. [Catani, Ciafaloni & Hautmann '93]

$$xg(x, Q^2) \equiv \int d^2k \; G(\ln 1/x, k, k_0)\Theta(Q - k) \qquad G^{(0)} = f(x)\delta^2(k - k_0)$$

To translate to $\overline{\text{MS}}$ scheme

$$xg(x, Q^2) \equiv \int d^2k \; G(\ln 1/x, k, k_0) r\left(\frac{k^2}{Q^2}\right), \qquad r\left(\frac{k^2}{Q^2}\right) = \int \frac{d\gamma \, e^{\gamma \ln \frac{Q^2}{k^2}}}{2\pi i \, \gamma R(\gamma)}$$

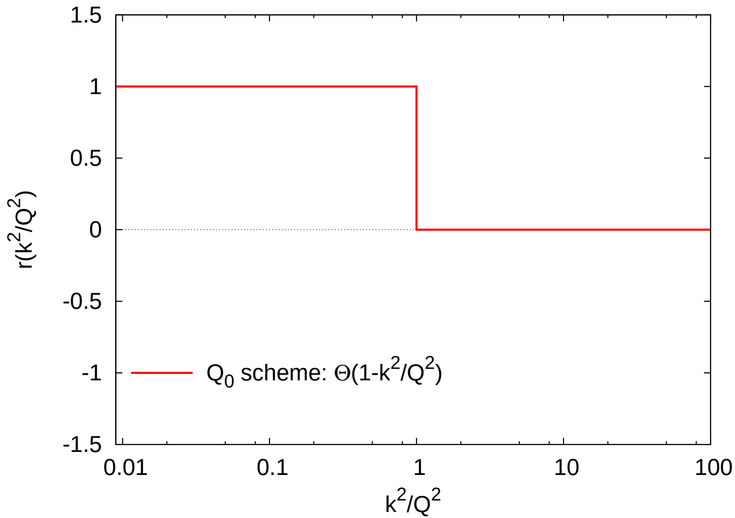
Should be easy?!

$$R(\gamma) = \left\{ \frac{\Gamma(1-\gamma)\chi(\gamma)}{\Gamma(1+\gamma)[-\gamma\chi'(\gamma)]} \right\}^{\frac{1}{2}} \exp\left\{ \int_0^\gamma d\gamma' \frac{\psi'(1) - \psi'(1-\gamma')}{\chi(\gamma')} \right\}$$

Catani & Hautmann '94

[NB: involves $\chi(\gamma)$ — does this need to be collinearly improved? Ignore problem for now. . .]

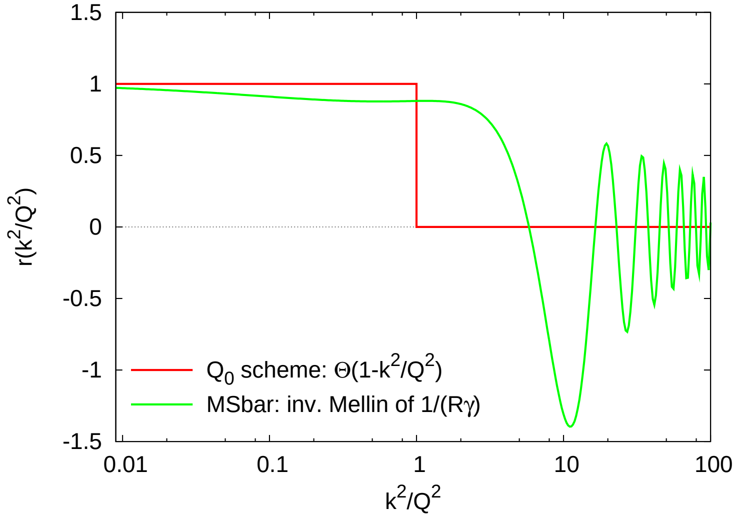
Q_0 v. $\overline{\text{MS}}$



Numerically, $\overline{\text{MS}}$ is much more difficult.

Conceptually, the oscillations are disturbing.

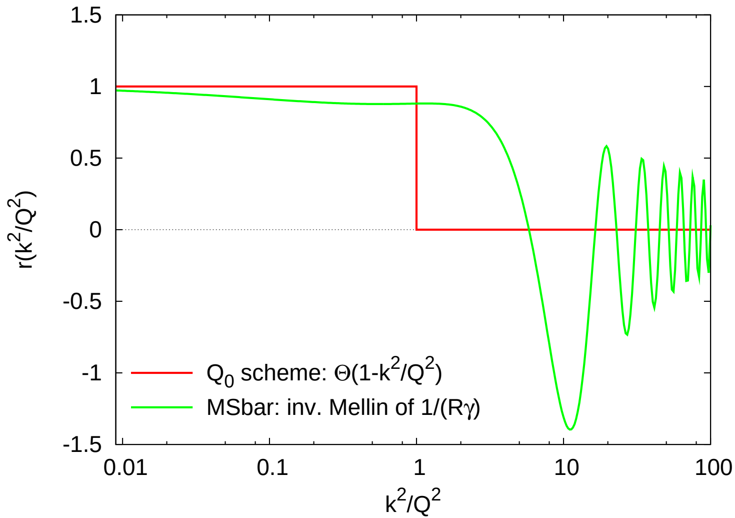
Q_0 v. $\overline{\text{MS}}$



Numerically, $\overline{\text{MS}}$ is much more difficult.

Conceptually, the oscillations are disturbing.

Q_0 v. $\overline{\text{MS}}$



Numerically, $\overline{\text{MS}}$ is much more difficult.

Conceptually, the oscillations are disturbing.