Taking QCD beyond fixed order perturbation theory – systematically

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Today's colliders are QCD machines

Current and forthcoming high-energy colliders:

HERA	Tevatron	LHC
$e^{\pm}p$	$\bar{p}p$	pp

All involve *protons* — understanding what's going on unavoidably involves

QCD

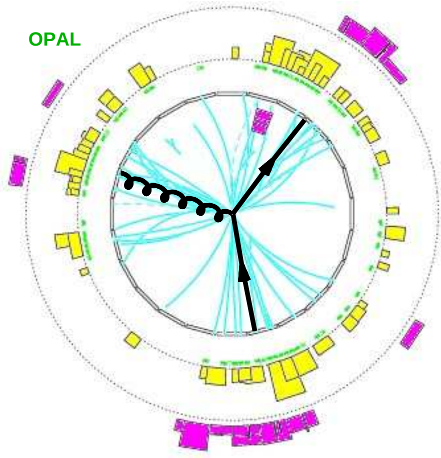
Tevatron: main 'new' object of study is top quark, interest is in checking its couplings and measuring its mass (*e.g.* implications for Higgs).

LHC: don't yet know what 'new' objects will be — but ability to extract them from (QCD) backgrounds and measure their properties will almost certainly be *limited by the quality of our understanding of QCD.*

So where's the problem? It's just Feynman diagrams. . .

Real events

Real events bear superficial resemblance to perturbative picture



But

(a) *Fundamental problem:* want a better understanding of correspondence between (i) the perturbative language used for calculations and (ii) the hadrons that are observed.

(b) To get the most out of QCD events for doing 'other physics' (searches etc.) → understand, quantitatively, how they differ from naive Feynman diags.

E.g. how do you relate the true mass of a new particle to the mass measured by isolating the jets it decays into?

Playground

One way of improving situation is by

Refining our understanding of perturbative QCD

- Next-to-Next-to-Leading-Order (NNLO), multi-leg NLO — Much activity
- Approximations to the behaviour of QCD at all orders — This talk

When discussing new techniques, it's useful to have a *playground:*

- *Simple collider environments:* e^+e^- (LEP), DIS (HERA).
- *Special observables:* event shapes — measures of deviation from idealised lowest order Feynman diagrams.
- Then apply understanding to real analyses at hadron colliders

This talk will examine principles of all-order calculations in the simplest possible environment ($e^+e^- \to 2\,$jets), attempting to illustrate lessons that hold in general.

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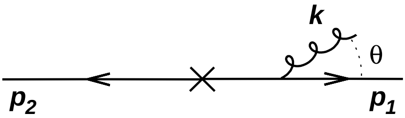
Structure of talk

- Perturbative QCD at fixed orders
  - Soft and collinear divergences
  - Infrared and collinear safety $\leftrightarrow$ (pseudo)-convergent perturbation series
- Fixed-order breakdown, all-order log-enhanced structures
  - fixed orders insufficient for describing most common events
  - understanding of divergences $\leftrightarrow$ all-order rearrangement of perturbation series
- Resummation done systematically
  - issues
  - *recursive* infrared collinear safety
  - automated resummation

Soft, collinear limits

Consider Feynman diagram (c.o.m. energy = Q)



Simplest limit:

- emitted gluon has small energy $E_k \ll Q$ *(soft)*
- is at small angle wrt quark, $\theta \ll 1$ *(collinear)*

Propagator goes **on-shell** ↔ divergence:

$$d\Phi_{q\bar{q}g}|M^2_{q\bar{q}g}| \simeq d\Phi_{q\bar{q}}|M^2_{q\bar{q}}| \cdot \frac{8}{3}\frac{\alpha_s}{\pi} \cdot \frac{dE_k}{E_k}\frac{d\theta}{\theta}$$

Such *soft and collinear divergences* are pivotal in this talk.

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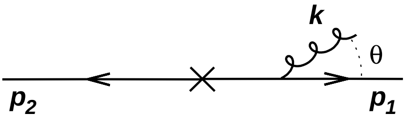

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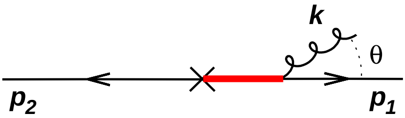

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Probability of emitting 1 gluon

Based on soft-collinear limit, probability for emitting 1 gluon is

$$\text{Prob}(1 \text{ gluon}) \sim \frac{16}{3}\frac{\alpha_s}{\pi}\int_0^Q \frac{dE}{E}\int_0^{\pi/2}\frac{d\theta}{\theta}$$

This is *infinite.* Perhaps integrals should not go below non-perturbative scale Λ?

Put cut-off:

$$\text{Prob}(1 \text{ gluon}) \sim \frac{16}{3}\frac{\alpha_s}{\pi}\int_\Lambda^Q \frac{dE}{E}\int_{\Lambda/Q}^{\pi/2}\frac{d\theta}{\theta} \sim \boxed{\frac{16}{3\pi}\alpha_s \ln^2\frac{Q}{\Lambda}}$$

Two large logarithms, one 'soft', one 'collinear' (both depend on cutoff).

Does small coupling save us? $\alpha_s = 1/(b_0 \ln Q/\Lambda)$:

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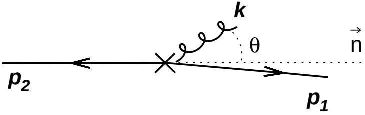
Jet broadening

Instead of calculating ‘flow of gluons’, let’s try and look at *flow of energy.*

E.g. ‘jet broadening’, B_T (transverse momentum flow wrt jet axis)

$$B_T = \frac{1}{2Q}\sum_i |\vec{q}_i \times \vec{n}| \simeq \frac{E_k\theta}{Q} \quad (\theta \ll 1)$$



Do perturbative calculation for mean value of broadening:

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Divergences are cancelled, because ‘observable’ (B_T) vanishes when the gluon is soft or collinear. Result is truly *perturbative.*

Suitable observables are straightforwardly calculable.

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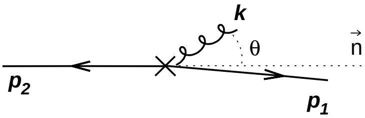

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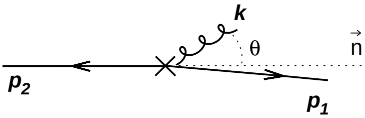

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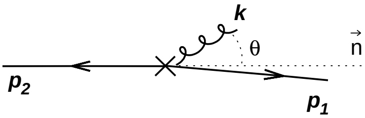

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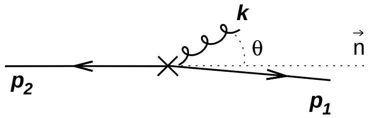

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Infrared (soft) and collinear safety

Crucial property of broadening was that effect of an additional gluon vanished $\propto$ a power of its softness and collinearity.

Infrared and collinear (IRC) safety

Sterman & Weinberg '77

For an observable's distribution to be calculable in perturbation theory, the observable should be infra-red [and collinear] safe, i.e. insensitive to the emission of soft or collinear gluons. In particular if $\vec{p}_i$ is any momentum occurring in its definition, it must be invariant under the branching

$$\vec{p}_i \to \vec{p}_j + \vec{p}_k$$

whenever $\vec{p}_j$ and $\vec{p}_k$ are parallel [collinear] or one of them is small [infrared].

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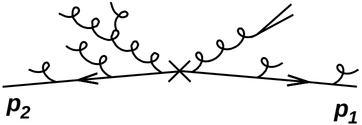
IRC safety and higher orders

Next: calculate higher-order corrections. At each order, probability of emitting gluon $\gg 1 \rightarrow$ complex configurations with many gluons:



But: high multiplicity comes from soft, collinear region – these gluons don't affect observable (IRC safety), and *cancel nearly fully with virtual corrections.*

Field theory:	real-virtual cancellation	$\Rightarrow$
Observable:	IRC safety	

Combination of field theory and observable properties allow us to *pretend* that the real world looks like perturbation theory.

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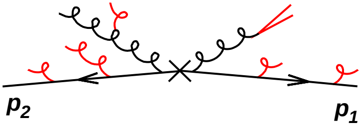

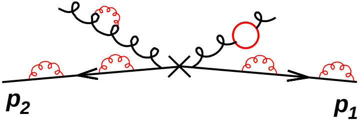

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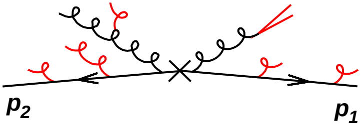

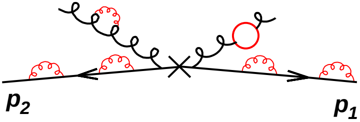

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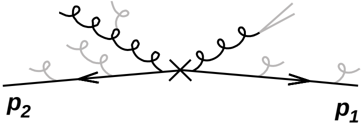

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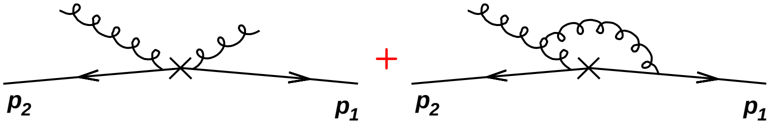
Next-to-leading order (NLO) predictions

Consider pure α_s^2 contributions. Conceptually simple:



In practice

- Physicist calculates matrix elements once → into *computer program.*
- Program generates random configurations (real & virtual), calculates arbitrary IRC-safe observable (subroutine), weights with matrix elements.

Subtlety: how do you combine

- observable in 4-dimensions,
- matrix elements in $4 + 2\epsilon$ dimensions (dim.-reg.)?

General NLO solution: Catani & Seymour '96 + Dittmaier & Trocsanyi '02

First NNLO solution: Gehrmann-De Ridder, Gehrmann & Glover '05

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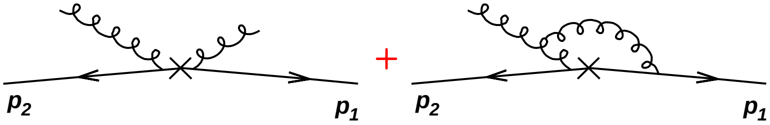

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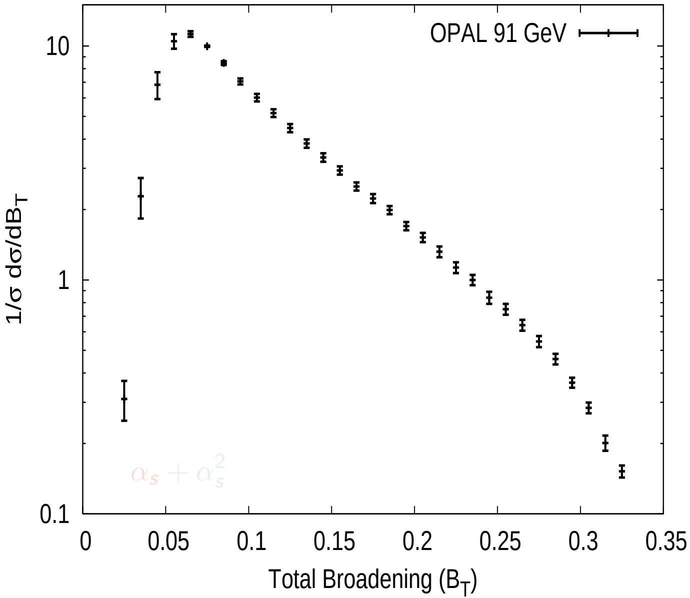

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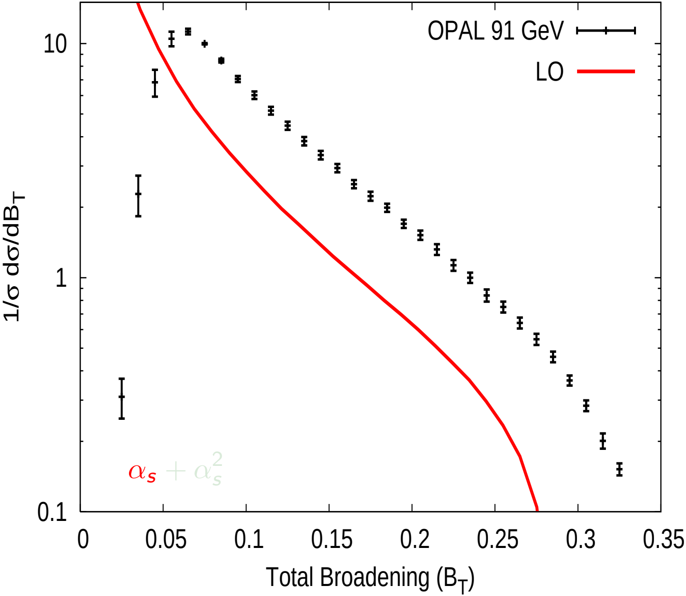

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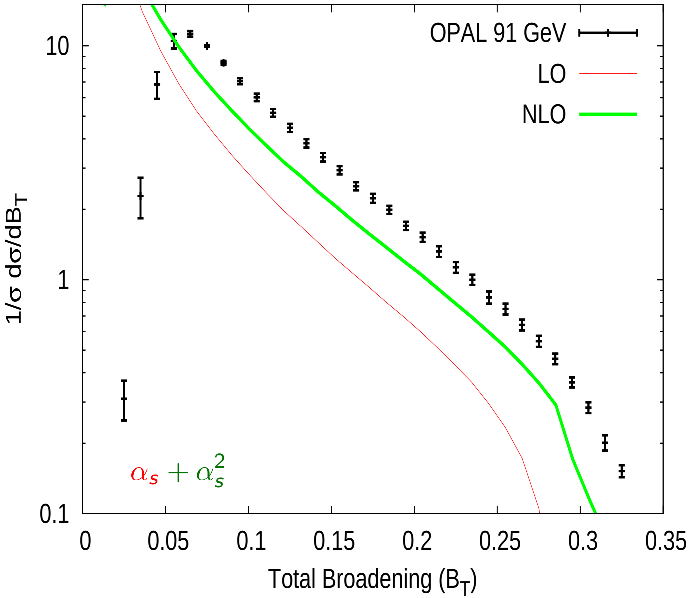

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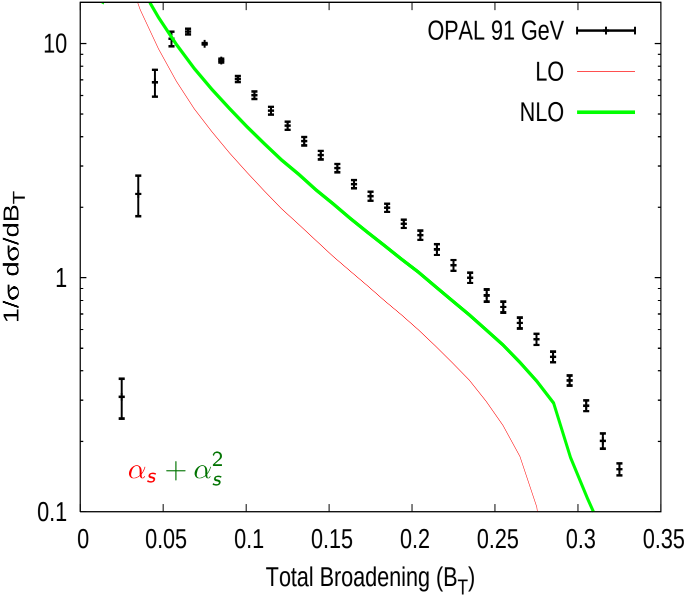

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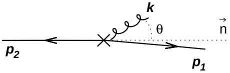
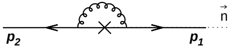
What's happening?

What is probability, $\Sigma(B)$, that broadening $<$ some value B?

$$\Sigma(B) \sim 1 + \underbrace{\frac{16}{3}\frac{\alpha_s}{\pi}\int_0 \frac{dE}{E}\frac{d\theta}{\theta}\Theta(B - \frac{E\theta}{Q})} - \underbrace{\frac{16}{3}\frac{\alpha_s}{\pi}\int_0 \frac{dE}{E}\frac{d\theta}{\theta}}$$





$$\sim 1 - \frac{16}{3}\frac{\alpha_s}{\pi}\int_0 \frac{dE}{E}\frac{d\theta}{\theta}\Theta(\frac{E\theta}{Q} - B) \sim \boxed{1 - \frac{8}{3}\frac{\alpha_s}{\pi}\ln^2 B}$$

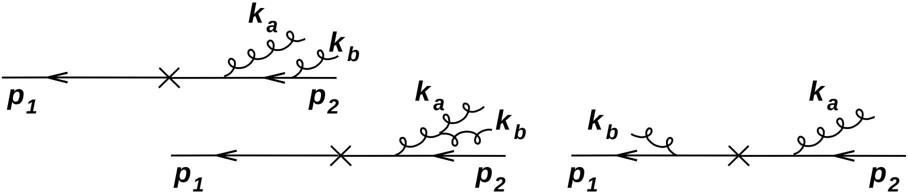
Double logarithm due to **incomplete real-virtual cancellation of soft and collinear divergences,** when considering narrow jets.

NB: resulting distribution diverges

$$\frac{d\Sigma}{dB} \sim \frac{16}{3}\frac{\alpha_s}{\pi}\frac{\ln 1/B}{B}$$

What will happen at next order?

Examine soft-collinear limit of two gluons:



Two propagators nearly on-shell $\leftrightarrow$ 4 divergences $(E_a \ll E_b)$. Can be viewed as two parts (approx.):

- independent emission of two gluons (diags, 1,3)
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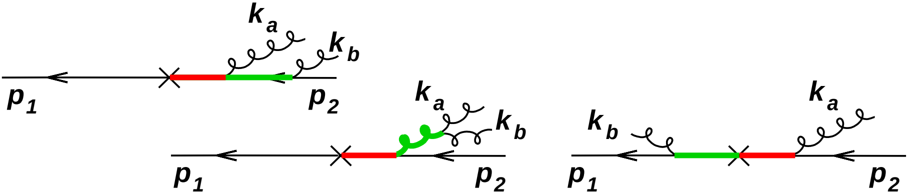

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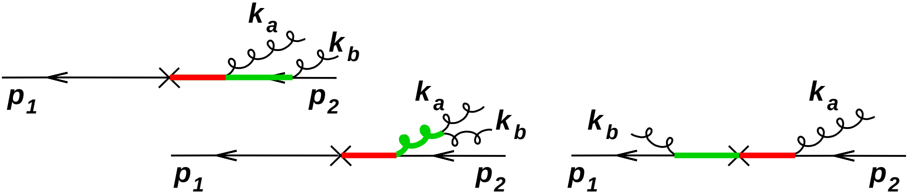

Two propagators nearly on-shell $\leftrightarrow$ 4 divergences ($E_a \ll E_b$) . Can be viewed as two parts (approx.):

- independent emission of two gluons (diags, 1,3)
- nested branching of gluon a (diag. 2)

All diagrams could potentially give us $\boxed{\alpha_s^2 \ln^4 B}$

All orders?

Normal perturbative expansion is fine in formal perturbative $\alpha_s \to 0$ limit

$$\Sigma(B) = 1 + \alpha_s f_1(B) + \alpha_s^2 f_2(B) + \dots \qquad f_n(B) \sim \ln^{2n} B \text{ for } B \ll 1$$

In region where you have most of the data $\ln B \gg 1$ and $\alpha_s^n f_n(B) \sim 1$ — *series does not converge.*

But origin of logs is simple: *residual non-cancellation of real and virtual soft-collinear divergences.* Can imagine calculating them at all orders:

$$\Sigma(B) \simeq \sum_{n=0}^{\infty} H_{n,2n}\, \alpha_s^n \ln^{2n} B + \mathcal{O}\left(\alpha_s^n \ln^{2n-1} B\right)$$

$$= h_1(\alpha_s L^2) + \sqrt{\alpha}_s h_2(\alpha_s L^2) + \dots, \qquad L \equiv \ln \frac{1}{B}$$

This is a **resummation of leading logarithms (LL), $h_1(\alpha_s L^2)$**

Will converge even for large values of the logarithm, $\alpha_s L^2 \sim 1$ since $h_1 \sim 1$, $h_2 \sim 1$ [NB: traded L^{-1} for $\sqrt{\alpha_s}$ in front of h_2]

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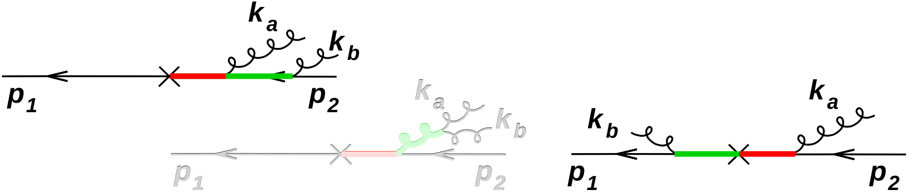
Try broadening resummation

Step 1. Simplify matrix element.

- B measures transverse momentum flow relative to main event ($\sim q\bar{q}$) axis.
- Secondary gluon splitting does not change observable (will cancel fully against virtuals)
- Take only independent emission:

$$d\Phi_n |M^2(k_1, \dots k_n)| \;\rightarrow\; \frac{1}{n!}\prod_{i=1}^{n} \frac{16}{3}\frac{\alpha_s}{\pi}\frac{dE_i}{E_i}\frac{d\theta_i}{\theta_i}$$

Minus corresponding virtual (loop) terms



Try broadening resummation (cont.)

Step 2. Simplify observable

- Calculate observable with *arbitrary number of emissions*. In soft and collinear limit it 'simplifies' to

$$B = \frac{1}{2Q}\left(\sum_{i=1}^{n}|\vec{k}_{ti}| + \left|\sum_{i\in right}^{n}\vec{k}_{ti}\right| + \left|\sum_{i\in left}^{n}\vec{k}_{ti}\right|\right)$$

- For now *approximate this* as

$$B = \frac{1}{Q}\max\{k_{t1}, k_{t2}, \dots k_{tn}\}$$

Since $\ln^2[B \times \mathcal{O}(1)] = \ln^2 B + \mathcal{O}(1) \cdot \ln B$, this does not change LL.

- Translate to limit on all $k_{ti} = E_i\theta_i$:

$$\Sigma(B) \simeq \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\frac{16}{3}\frac{\alpha_s}{\pi}\int\frac{dE_i}{E_i}\frac{d\theta_i}{\theta_i}[\underbrace{\Theta(B - E_i\theta_i)}_{real} - \underbrace{1}_{virt}]$$

$$\simeq \exp\left[-\frac{8}{3}\frac{\alpha_s L^2}{\pi}\right]$$

Exponentiated double logarithms

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Exponentiation

- $\exp[-\alpha_s L^2]$ is typical of *Sudakov suppression* — if you want broadening to be small, pay the price of *suppressing emission* (*i.e.* virtual terms).
- Exponentiated form does not always hold, *e.g.* 'Jade jet resolution,' y_{3J}:

$$\Sigma(y_{3J}) = 1 - \frac{4}{3}\frac{\alpha_s L^2}{\pi} + \frac{5}{12}\left(\frac{4}{3}\frac{\alpha_s L^2}{\pi}\right)^2 + \cdots$$

Brown & Stirling '90

- When it *does* hold, ∃ more powerful reorganisation of logs

$$\Sigma(B) = \exp\left[\sum_{n=1}^{\infty} G_{n,n+1}\,\alpha_s^n L^{n+1} + \mathcal{O}\left(\alpha_s^n L^n\right)\right]$$

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NLL resummations

Next-to-leading-logarithmic (NLL) accuracy is currently *state of the art* for QCD final-state resummations.

Ingredients (in addition to those shown so far):

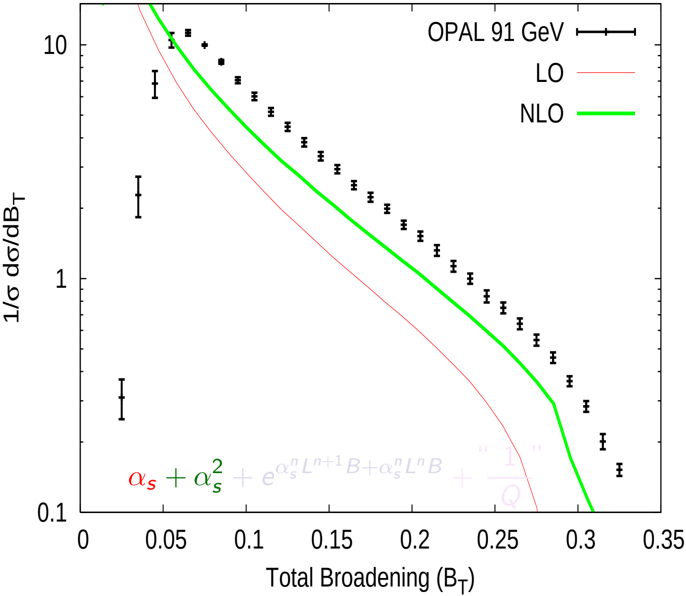
- Full treatment of observable
- Proper coupling (scheme, two-loop running)
- Careful evaluation of sum over emissions

Pioneered: Catani, Trentadue, Turnock, Webber (CTTW) '92

Broadening: CTW '92; Dokshitzer, Lucenti, Marchesini & GPS '98

NB: simple observable (EEC) recently done at NNLL: de Florian & Grazzini '04

Broadening distribution at NLO+NLL



NLL shape OKish!

NB: peak is at $\alpha_s L \sim 1$.

Remaining difference ascribed to parton-hadron transition, *hadronisation*

Only with resummation can hadronisation be separated from perturbative part

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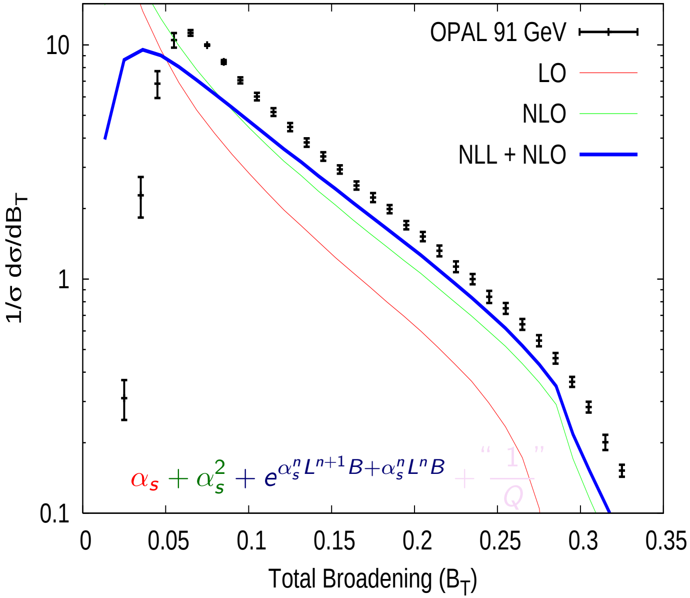

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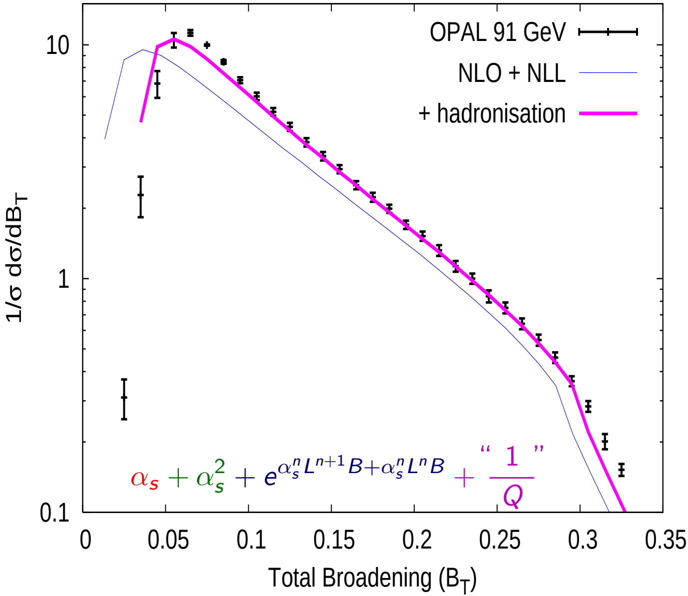

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Manual NLL calculations

Unlike NLO (matrix-element done once, rest done my Monte Carlo), NLL 'event-shape' resummation nearly always been done *manually, analytically.*

e^+e^- 2 jets	ρ_h, ρ_l, ρ_1, τ, B_T, B_W, B_N, y_3^D, y_3^C, T_M, Angularities
DIS 1+1 jets	τ_{zQ}, τ_{zE}, B_{zQ}, B_{tQ}, C_E, ρ_E
Multijet	$e^+e^- \to 3j$ T_m, $T_{m,N}$, D; DIS(1+2) K_{out} $pp \to W + 1j$ K_{out}; $pp \to 2j$ gap-probability (cone,k_t)

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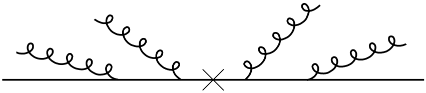
Sources of difficulty (globalness)

Global observable:

e.g. total e^+e^- Broadening, B_T



making $B \ll 1$ restricts emissions everywhere.

Coherence + globalness:

➥ emissions can be resummed as if independent *(proved)*

Answers guaranteed to NLL accuracy

Non-Global observable:

Right-hemisphere Broadening, B_R

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Tempting to *assume* one can:

- ignore left hemisphere ($\mathcal{H}_\mathcal{L}$)
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WRONG AT NLL ACCURACY

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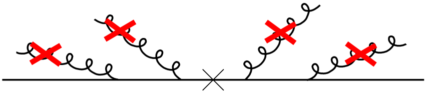

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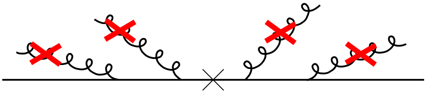

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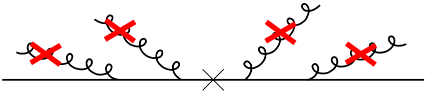

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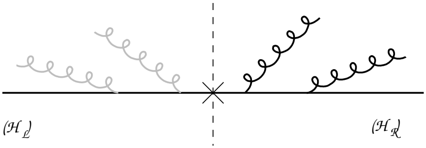

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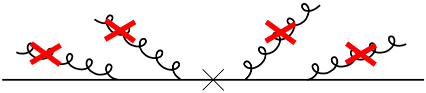

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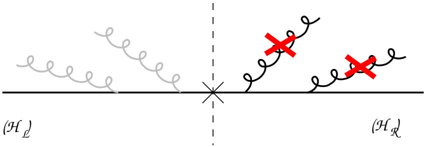

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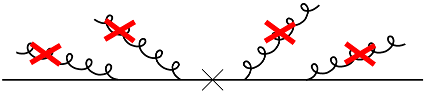

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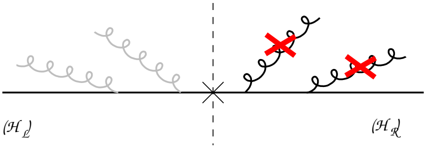

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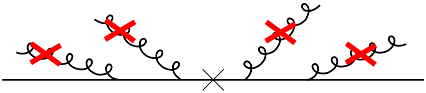

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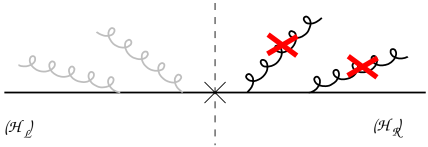

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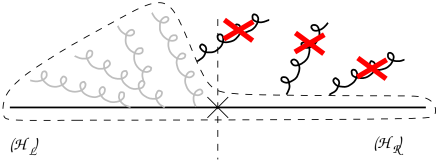
Dasgupta & GPS '01

Resummation of NG observables

All-orders:

Unrestricted semi-soft gluons (left) change pattern of radiation of large-angle soft gluons (right)



➥ $\alpha_s^n \ln^n B_R$

Difficulties, features:

- Logarithms resummed so far only in large-N_c limit
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- In general, boundary between the two regions may have arbitrary shape.
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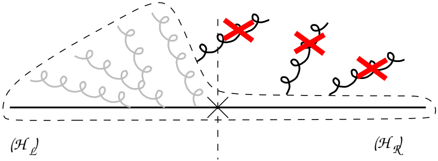

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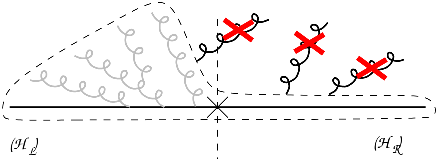

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Sources of difficulty (observable)

Other difficulty is in *handling the soft-collinear limit of the observable:*

- calculate how limit on observable constrains momenta of n particles
- then express constraint in factorised form, if it exists

E.g.

$$\Theta(y_{3C}Q^2 - \max(k_{t1}^2, k_{t2}^2, \dots, k_{tn}^2)) \to \prod_{i=1}^{n} \Theta(y_{3C}Q^2 - k_{ti}^2)$$

y_{3C} = 3-jet resolution, Cambridge algorithm

Most cases are more complex

$$\Theta(\tau Q - k_{t1} - k_{t2} - \dots - k_{tn})) \to \int \frac{d\nu}{2\pi i \nu} e^{\nu\tau Q} \prod_{i=1}^{n} e^{-\nu k_{ti}}$$

τ = any thrust-like observable

Some may even be insoluble analytically

$$\Theta(T_M Q - \max_{\vec{n}}(\vec{k}_{t1} + \vec{k}_{t2} + \dots + \vec{k}_{tn})) \to ???$$

T_M = thrust-major, done numerically Banfi, GPS & Zanderighi '01

Generic resummation?

What we would like:

Something as good as manual analytical resummation

- Guaranteed (verifiable) accuracy, exponentiation
- Separate LL, NLL functions, $g_1(\alpha_s L)$, $g_2(\alpha_s L)$
- Expansions of g_1 and g_2 to fixed order in α_s

Monte Carlo resummation:

Event generators (Herwig, Pythia, ...) generate multiple divergent soft-collinear radiation = powerful automated resummation programs!

✓✓ Observable treated exactly ⇔ very flexible.

✓ Includes hadronisation model

✗ Accuracy sometimes unclear (depends on observable, no NLL for multi-jet processes)

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Strategy

Follow model of fixed order calculations

Identify combination of

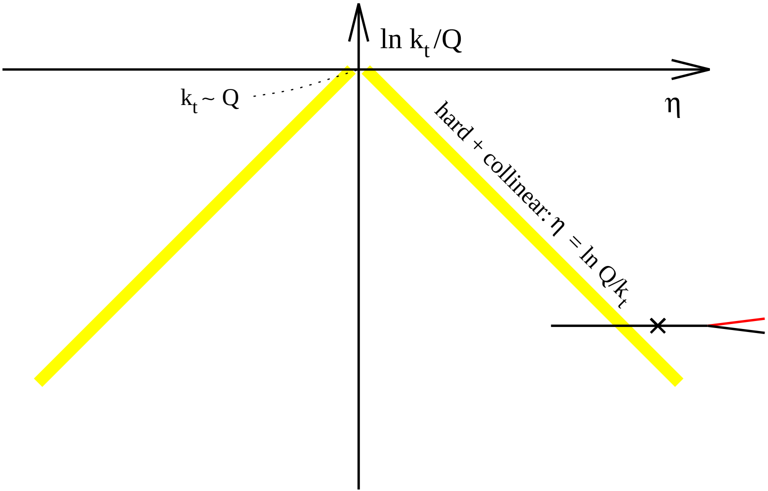
- properties of QCD matrix elements
- requirements on observable

such that a systematic approximation procedure emerges.

NB: will consider only *global* observables, so as to simplify problem.

Phase space ($e^+e^- \to 2\,\text{jets}$)

Use 'Lund' representation of kinematic plane: $\ln k_t$ and $\eta = -\ln\tan\theta/2$



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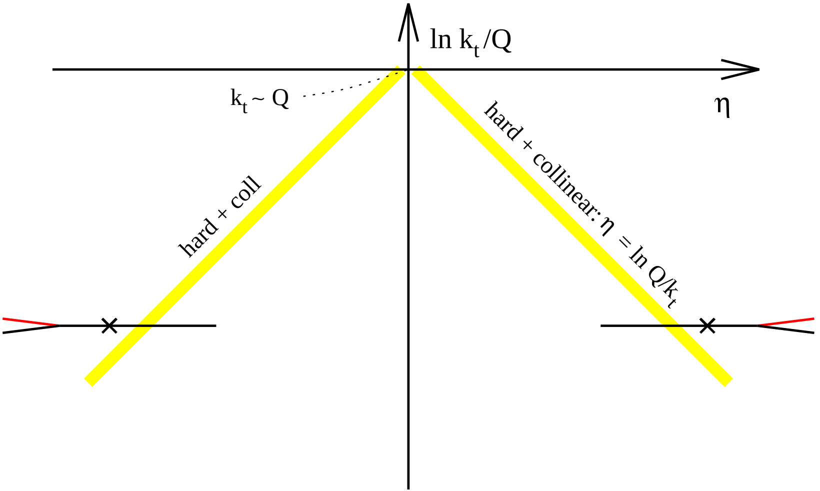

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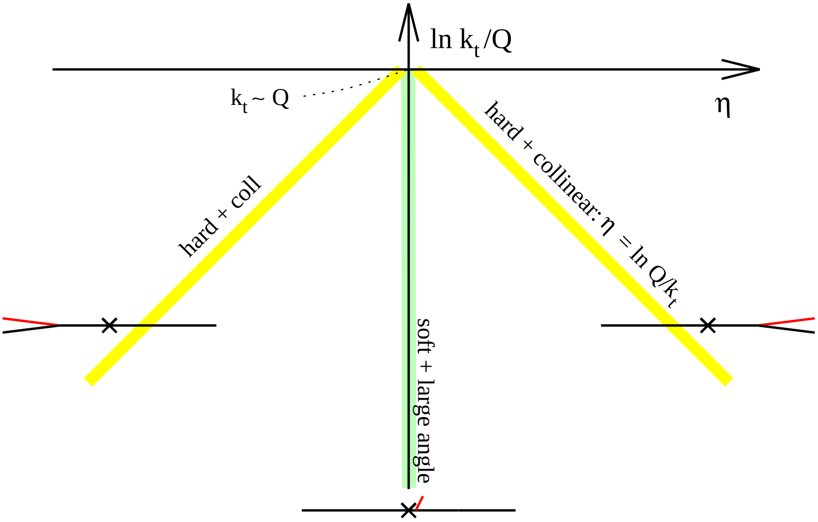

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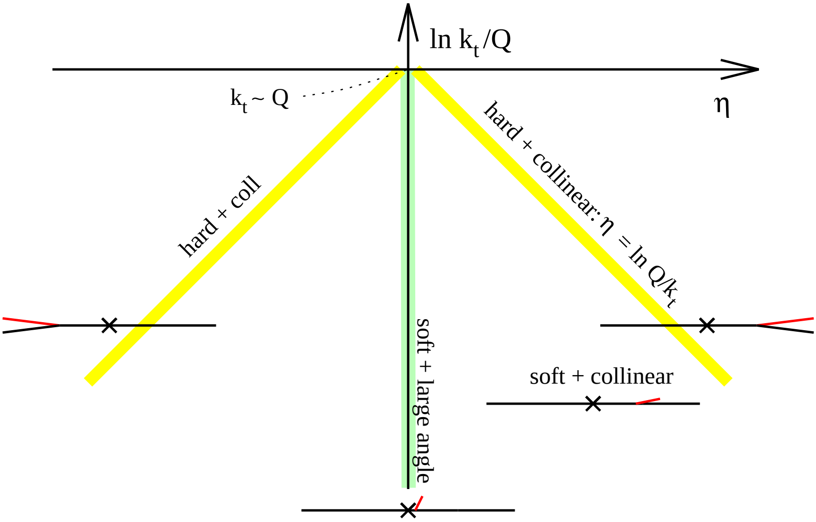
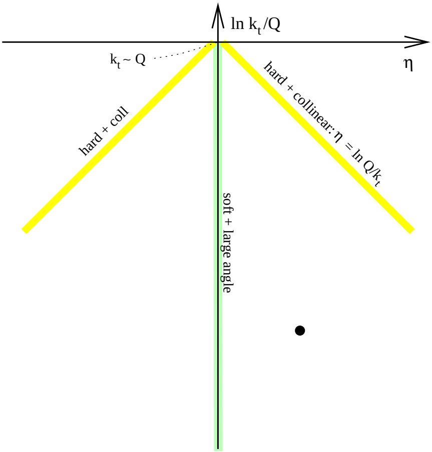


Introduce observable (& 1 emission)



Take *general observable, $V(p_1, \ldots)$.*

Require that it vanish smoothly in soft, collinear limits:

$$V(p_1, p_2, k) \sim (k_t/Q)^a e^{-b|\eta|}$$

Requirement $V(\ldots) < v \to$ boundary of a *vetoed region* for 1 emission

$$\ln v = a \ln \frac{k_t}{Q} - b|\eta|$$

diagram shows $a = b = 1$

Real—virtual cancels *everywhere but vetoed region*, leaving:

$$\Sigma(V < v) = 1 + \underbrace{G_{12}\,\alpha_s L^2}_{\text{Vetoed area}} + \underbrace{G_{11}\,\alpha_s L}_{\text{edges}}$$

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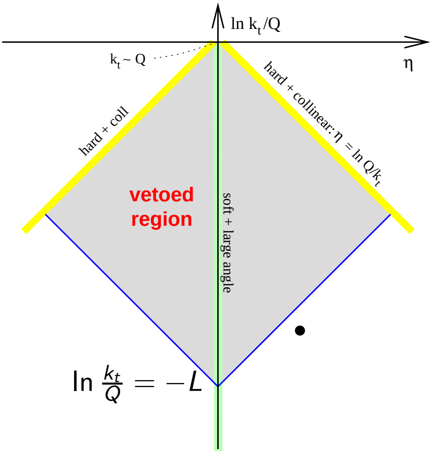

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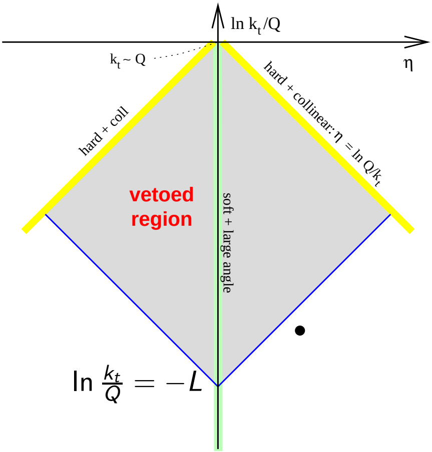

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so take $\alpha_s \to 0$ with *$\alpha_s L$ constant*

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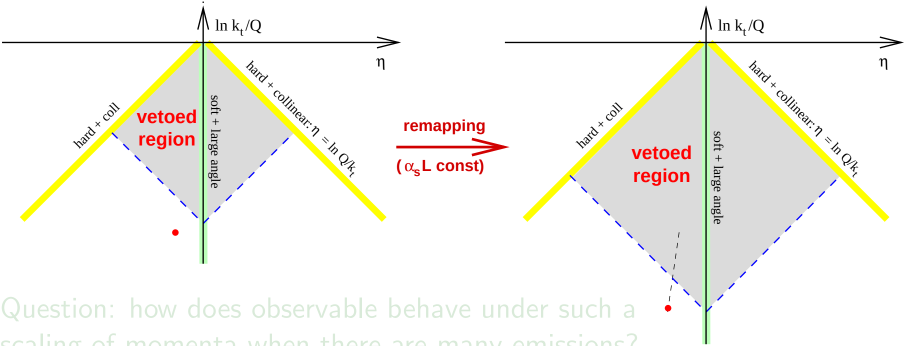

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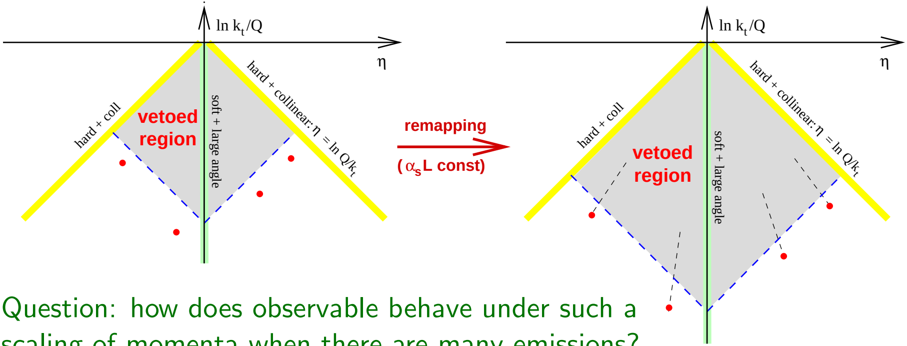

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Scaling requirements

Multiple emission properties

- Parametrise emission momenta by effect on observable:

$\kappa(\bar{v})$ is a momentum such that $V(\{p\},\kappa(\bar{v})) = \bar{v}$

A specific function $\kappa(\bar{v})$ maps out a path in η, $\ln k_t$ space

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$$\lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{p\},\kappa_1(\zeta_1\bar{v}),\kappa_2(\zeta_2\bar{v}),\ldots) = f(\zeta_1,\zeta_2,\ldots)$$

For any $\{\zeta_i\}$, and any set of paths $\{\kappa_i\}$

This allows us to give meaning to the limit $\alpha_s \to 0$ with $\alpha_s L$ fixed, for any number of emissions — *because scaling properties of observable are independent of number of emissions.*

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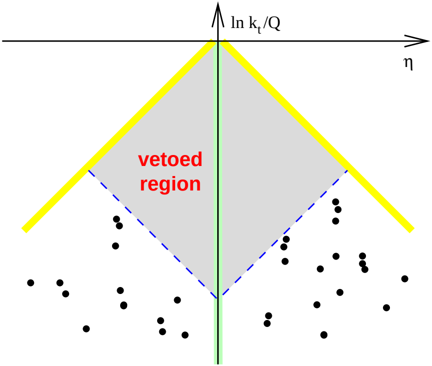

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Need to simplify it (like we simplified fixed-order PT at beginning).

➥Keep just subset of emissions.

But, are we allowed to throw away the remaining emissions?

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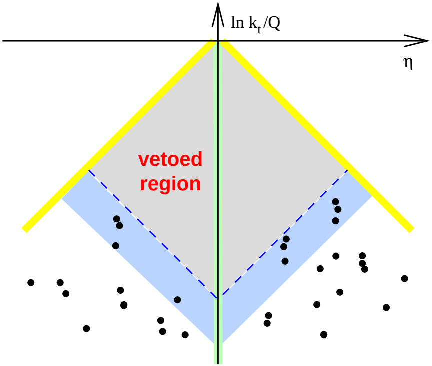

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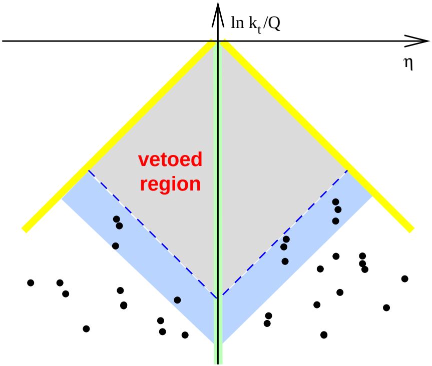

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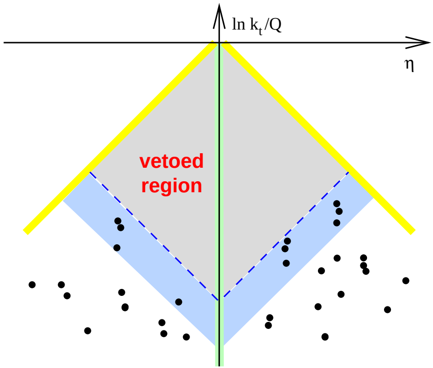

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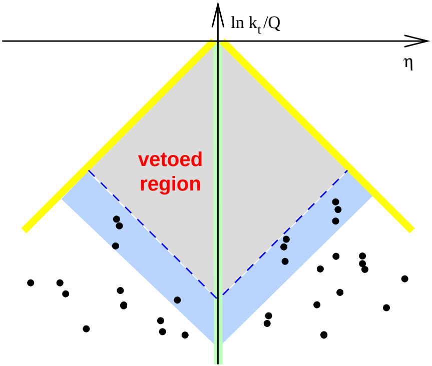

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Matrix element

- anything very soft cancels with corresponding virtual correction
- emissions on disparate angular scales behave independently

QCD coherence

recursive IRC

Recall scaling property

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- Require *recursive* infrared-collinear safety:

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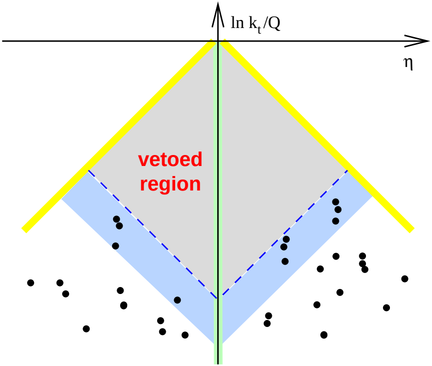
Normal v. recursive IRC safety

	normal IRC safety	*recursive IRC safety*
softness defined relative to	hard scale	soft scale of observable
problem reduces to one of	low *number* of emissions	low *density* of emissions in η
allowing use of	fixed-order PT	independent emission approximation

NB: independent emission approximation results from coherence
$\equiv$ emissions widely separated in angle are independent.

Coherence recently questioned at subleading N_c and high orders ($\alpha_s^4 L^5$) in $pp \to 2$ jets
[Forshaw, Kyrieleis and Seymour '06]

Schematic framework



Sum over real and virtual emissions in blue band and above is sufficient for any resummation accuracy.

- LL: consider just exponential of virtuals in vetoed region:

$$\alpha_s L^2 \to e^{\alpha_s^n L^{n+1}}$$

- NLL: need to account for edges

$$\alpha_s L \to e^{\alpha_s^n L^n}$$

  In blue band: sum over widely separated individual emissions

  low density $\sim \alpha_s$; coherence
  $\to$ treat them as independent

- NNLL: account for corners

$$\alpha_s \to e^{\alpha_s^n L^{n-1}}$$

  and 1 correlated pair of emissions (+ any # of indep. emissions)

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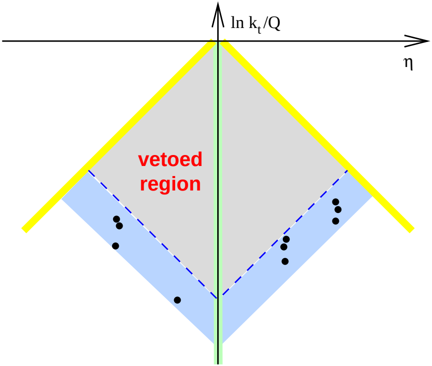

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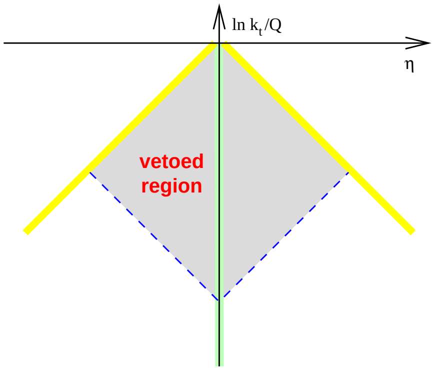

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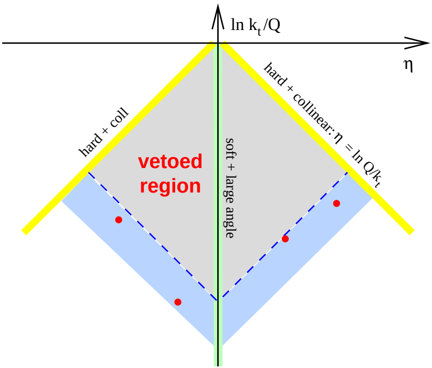

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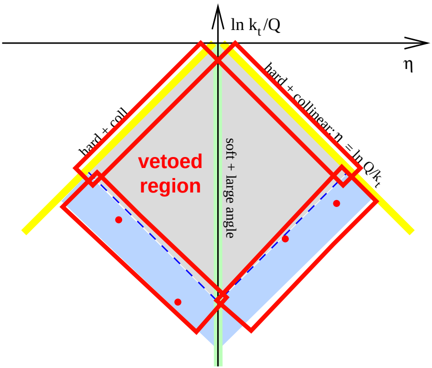

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  In blue band: sum over widely separated individual emissions

  low density $\sim \alpha_s$; coherence
  $\to$ treat them as independent

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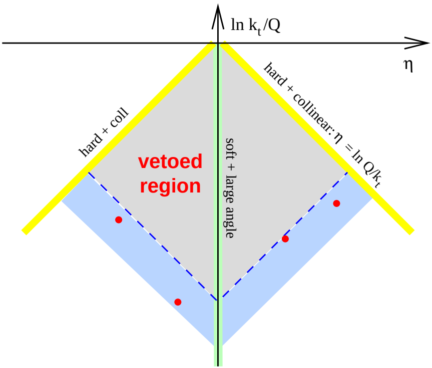
$$\alpha_s \to e^{\alpha_s^n L^{n-1}}$$

  and 1 correlated pair of emissions (+ any # of indep. emissions)

Schematic framework



Sum over real and virtual emissions in blue band and above is sufficient for any resummation accuracy.

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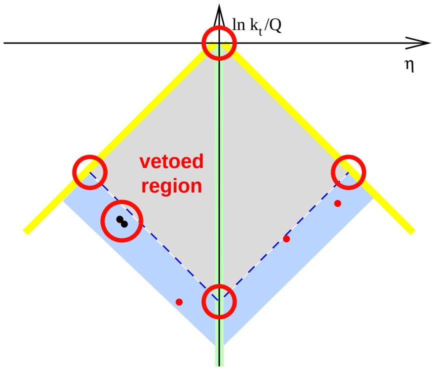

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Analytical work (done once and for all)

A1. formulate exact applicability conditions for the approach (its scope)

A2. derive a master formula for a generic observable in terms of simple properties of the observable

Numerical work (to be repeated for each observable)

N1. let an "expert system" investigate the applicability conditions

N2. it also determines the inputs for a master formula

N3. straightforward evaluation of the master formula, including phase space integration etc.

Note: N1 and N2 are core of automation

a) they will require high precision arithmetic to take asymptotic (soft & collinear) limits

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- Observable must have standard functional form for soft & collinear gluon emission

$$V(\{p\}, k) = d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi)\,.$$

Born momenta   soft collinear emission

- *Determine coefficients* a_ℓ, b_ℓ, d_ℓ and $g_\ell(\phi)$ for emissions close to each hard Born parton (leg) ℓ.
- Require *continuous globalness*, *i.e.* uniform dependence on k_t independently of emission direction ($a_1 = a_2 = \cdots = a$)
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C_ℓ = colour factor; q_ℓ = PDF
$r_\ell(L) \Rightarrow \alpha_s^n L^{n+1}$; $r'_\ell(L)$, $T(L) \Rightarrow \alpha_s^n L^n$

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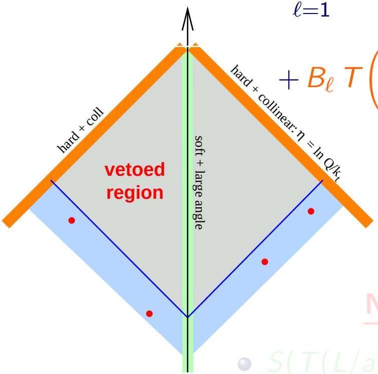

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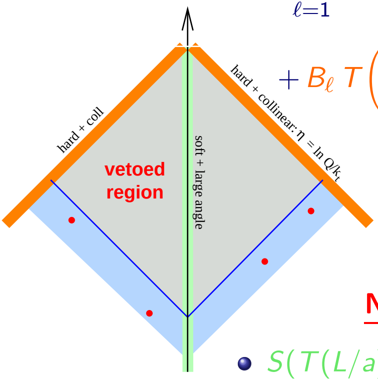

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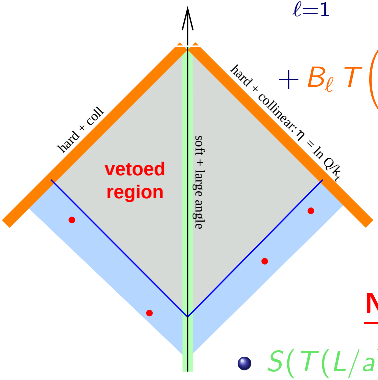

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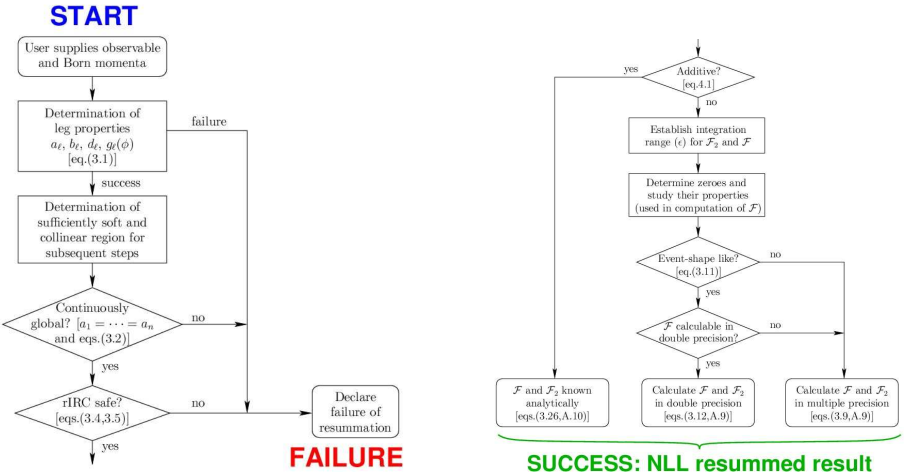
CAESAR flow chart

Computer **A**utomated **E**xpert **S**emi-**A**nalytical **R**esummer

Banfi, GPS, Zanderighi '03–'05



What it doesn't do

- Observables that vanish other than through suppression of radiation (*e.g.* Vector Boson p_t spectrum) have divergence in $g_2(\alpha_s L)$ beyond fixed value of $\alpha_s L$. Rakow & Webber '81; Dasgupta & GPS '02
  - for very-inclusive 2-jet cases analytical resummations are in any case more accurate (NNLL) Higgs p_t: Bozzi et al '03–05 Back-to-back EEC: de Florian & Grazzini '04
  - For less-inclusive cases, this problem is sometimes 'academic' (in region of vanishing X-section).
- Non-global observables are beyond its scope (but perhaps could be included in future).
  - Individual jet properties, or subsets of jets
  - Gap resummations Appleby, Banfi, C. Berger, Dasgupta, Forshaw Kucs, Kyrieleis, Oderda, Seymour, Sterman, . . .
- Threshold resummations not yet thought about in this framework.

What it has done

- Reproduced/verified all known analytical global resummations
- Except for 1 case where it replaces an incomplete result

y_3^D: widely used in fits to α_s
Banfi, GPS & Zanderighi '01

- Correctly identifies cases where it is not able to give correct answer.
- New multi-jet resummations in e^+e^- and DIS
- *First event-shape resummation for hadron-hadron dijet events*

Uses soft-logarithms from Stony Brook group

All results available at `http://qcd-caesar.org`

Program available on request

Some hadron-collider dijet observables

Event-shape	Impact of $\eta_{\max}$	Resummation breakdown	Underlying Event	Jet hadronisation
$\tau_{\perp,g}$	tolerable	none	$\sim \eta_{\max}/Q$	$\sim 1/Q$
$T_{m,g}$	tolerable	none	$\sim \eta_{\max}/Q$	$\sim 1/(\sqrt{\alpha_s} Q)$
y_{23}	tolerable	none	$\sim \sqrt{y_{23}}/Q$	$\sim \sqrt{y_{23}}/Q$
$\tau_{\perp,\mathcal{E}}$, $\rho_{X,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/Q$
$B_{X,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/(\sqrt{\alpha_s} Q)$
$T_{m,\mathcal{E}}$	negligible	serious	$\sim 1/Q$	$\sim 1/(\sqrt{\alpha_s} Q)$
$y_{23,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim \sqrt{y_{23}}/Q$
$\tau_{\perp,\mathcal{R}}$, $\rho_{X,\mathcal{R}}$	none	serious	$\sim 1/Q$	$\sim 1/Q$
$T_{m,\mathcal{R}}$, $B_{X,\mathcal{R}}$	none	tolerable	$\sim 1/Q$	$\sim 1/(\sqrt{\alpha_s} Q)$
$y_{23,\mathcal{R}}$	none	intermediate	$\sim \sqrt{y_{23}}/Q$	$\sim \sqrt{y_{23}}/Q$

The study of such a wide range of observables would have been nearly impossible without automation...

Summary & outlook

Normal QCD perturbation theory relies on *infrared & collinear safety* of observable to allow one to restrict matrix elements for N^{p}LO calculation to $n_{Born} + p$ partons.

In certain (exclusive) regions of phase-space, while formally ($\alpha_s \to 0$) OK, this is practically insufficient: need *all-order resummation* of logarithmically enhanced terms.

New condition: **recursive infrared and collinear safety,** ensures (together with globalness, coherence) that, for NLL resummed accuracy, it is safe to approximate n-parton soft-collinear matrix-element as independent emission.

Enables automation of resummation → CAESAR!

First hadron-hadron dijet event-shape resummations

Many questions for future. Can the automated resummation be made practical beyond NLL accuracy? Are there issues with coherence in processes with incoming hadrons?

EXTRA SLIDES

Hadron collider event shapes

Contradiction?

Theoretical calculations are for global observables.
But experiments only have detectors in limited rapidity range.
(Strictly: series of sub-detectors, of worsening quality as rapidity increases)

Model by cut around beam $|\eta| < \eta_{max}$
➥ Problems with globalness

Take cut as being edge of most forward detector with momentum or energy resolution:

	Tevatron	LHC
η_{max}	3.5	5.0

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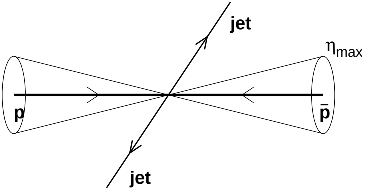

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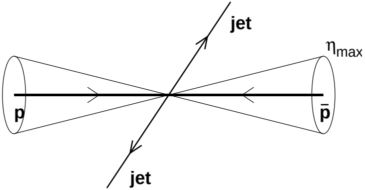

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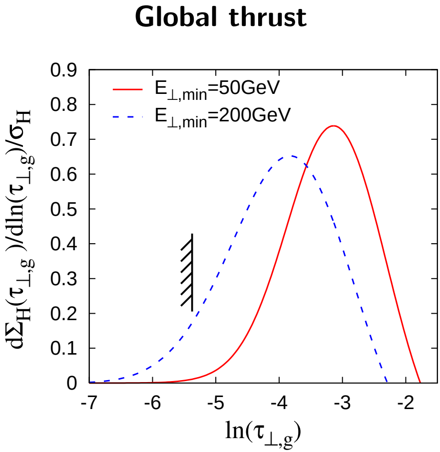
Practical hh event shapes

Particles from beyond max rapidity contribute significantly only for small $V \lesssim e^{-(a+b_\ell)\eta_{\max}}$.

Most of cross section may be *above that limit* — rapidity cut irrelevant.

Banfi et al. '01



Alternative

Measure just centrally & add recoil term (indirect sensitivity to rest of event):

$$\mathcal{R}_{\perp,\mathcal{C}} \equiv \frac{1}{Q_{\perp,\mathcal{C}}} \left| \sum_{i \in \mathcal{C}} \vec{q}_{\perp i} \right| ,$$

Here $g_2(\alpha_s L)$ diverges for $L \sim 1/\alpha_s$ (due to cancellations in vector sum) – study distribution only before divergence.

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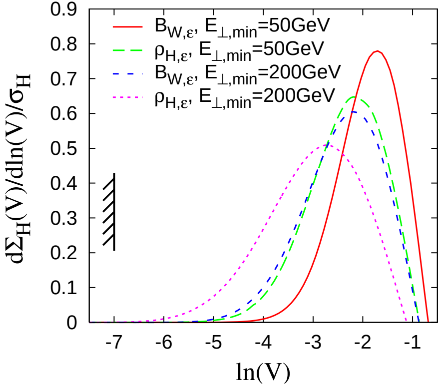
Jet-broadening, jet-mass
$(+k_t/Qe^{-|\eta|})$



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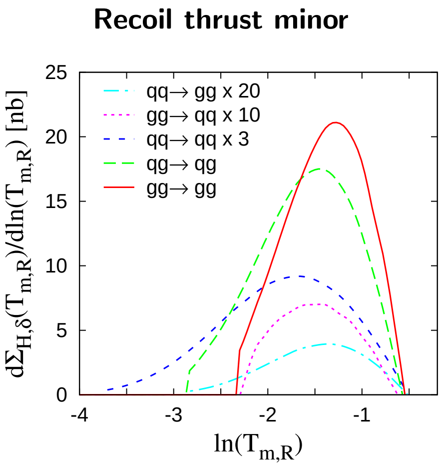

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NB: there may be surprises after more detailed study, *e.g.* matching to NLO...

Grey entries are definitely subject to uncertainty

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