QCD (for LHC)
Lecture 2: Parton Distribution Functions

Gavin Salam

LPTHE, CNRS and UPMC (Univ. Paris 6)

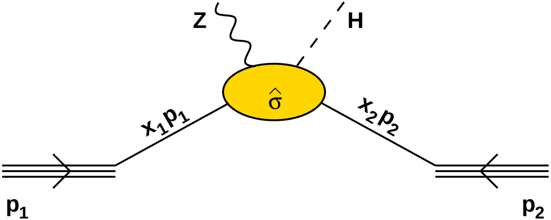
At the 2009 European School of High-Energy Physics
June 2009, Bautzen, Germany

QCD

Lecture 2
(The contents of the proton: Parton Distribution Functions)

Factorization & parton distributions

Cross section for some hard process in hadron-hadron collisions

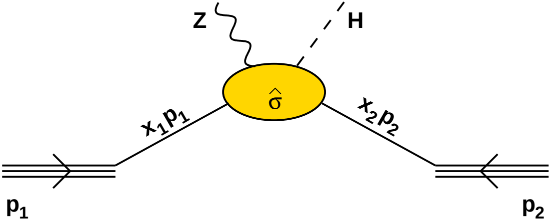


$$\sigma = \int dx_1 f_{q/p}(x_1,\mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2,\mu^2)\,\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)\,, \quad \hat{s} = x_1 x_2 s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section $\leftrightarrow$ *need to know PDFs* to be able to test hard part (e.g. Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

Factorization & parton distributions

Cross section for some hard process in hadron-hadron collisions

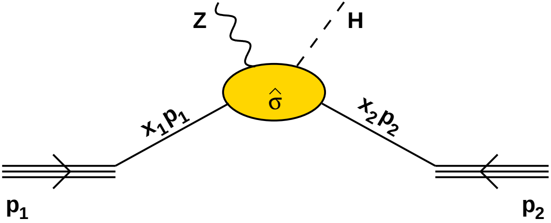


$$\sigma = \int dx_1 f_{q/p}(x_1,\mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2,\mu^2)\, \hat{\sigma}(x_1p_1, x_2p_2, \mu^2)\,, \quad \hat{s} = x_1x_2s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1p_1, x_2p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section ↔ *need to know PDFs* to be able to test hard part (e.g. Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

Factorization & parton distributions

Cross section for some hard process in hadron-hadron collisions

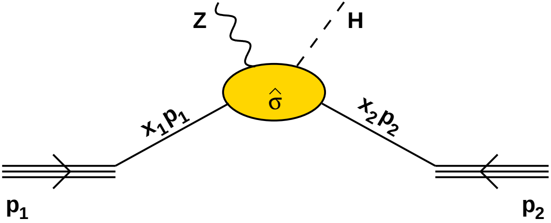


$$\sigma = \int dx_1 f_{q/p}(x_1, \mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2, \mu^2)\, \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)\,, \quad \hat{s} = x_1 x_2 s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section ↔ *need to know PDFs* to be able to test hard part (e.g. Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

Factorization & parton distributions

Cross section for some hard process in hadron-hadron collisions

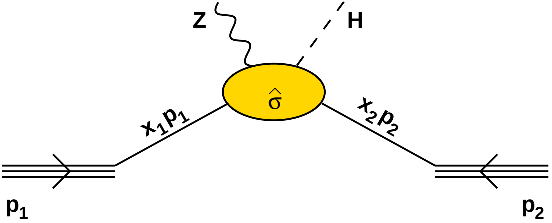


$$\sigma = \int dx_1 f_{q/p}(x_1, \mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2, \mu^2)\, \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)\,, \quad \hat{s} = x_1 x_2 s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section $\leftrightarrow$ *need to know PDFs* to be able to test hard part (e.g. Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

Factorization & parton distributions

Cross section for some hard process in hadron-hadron collisions



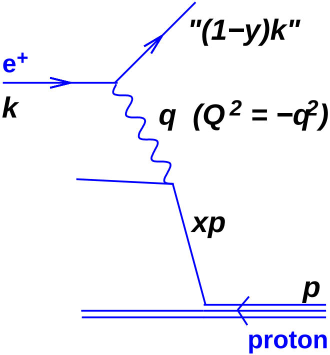
$$\sigma = \int dx_1 f_{q/p}(x_1,\mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2,\mu^2)\, \hat{\sigma}(x_1p_1, x_2p_2, \mu^2)\,, \quad \hat{s} = x_1 x_2 s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1p_1, x_2p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section ↔ *need to know PDFs* to be able to test hard part (e.g. Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs?
  - does picture really stand up to QCD corrections?

NB: non-perturbative

Deep Inelastic Scattering: kinematics

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



Kinematic relations:

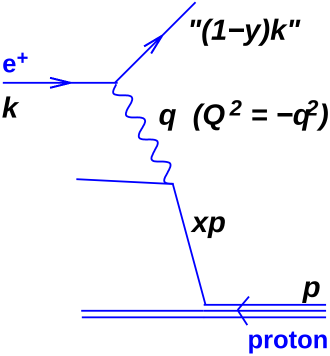
$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

- Q^2 = photon virtuality $\leftrightarrow$ *transverse resolution* at which it probes proton structure
- x = *longitudinal momentum fraction* of struck parton in proton
- y = momentum fraction lost by electron (in proton rest frame)

Deep Inelastic Scattering: kinematics

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



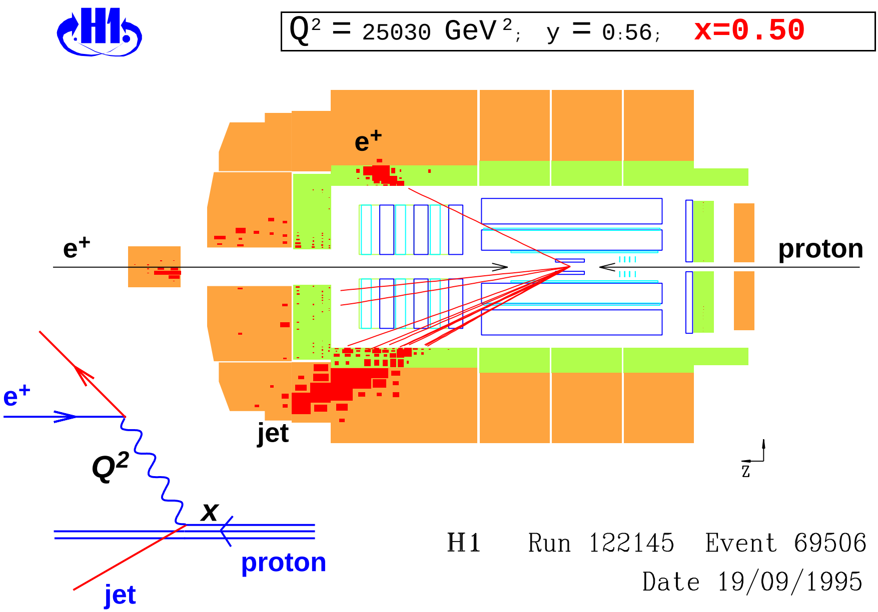
Kinematic relations:

$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

- Q^2 = photon virtuality $\leftrightarrow$ *transverse resolution* at which it probes proton structure
- x = *longitudinal momentum fraction* of struck parton in proton
- y = momentum fraction lost by electron (in proton rest frame)

Deep Inelastic scattering (DIS): example



E.g.: extracting u & d distributions

Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

<u>NB:</u>

- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

E.g.: extracting u & d distributions

Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

NB:

- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

F_2 gives us *combination* of u and d.
How can we extract them separately?

Using neutrons

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:
proton = uud; neutron = ddu

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$
$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

Using neutrons

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:
proton = uud; neutron = ddu

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$
$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

Using neutrons

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:

proton = uud; neutron = ddu

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

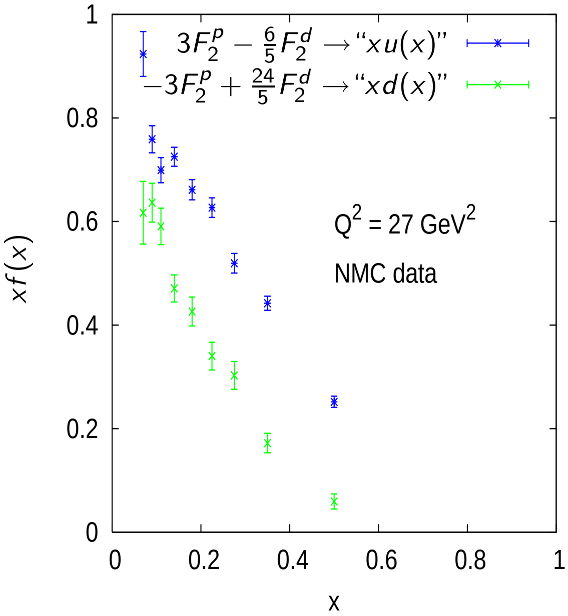
$$F_2^p = \frac{4}{9} u_p(x) + \frac{1}{9} d_p(x)$$

$$F_2^n = \frac{4}{9} u_n(x) + \frac{1}{9} d_n(x) = \frac{4}{9} d_p(x) + \frac{1}{9} u_p(x)$$

Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

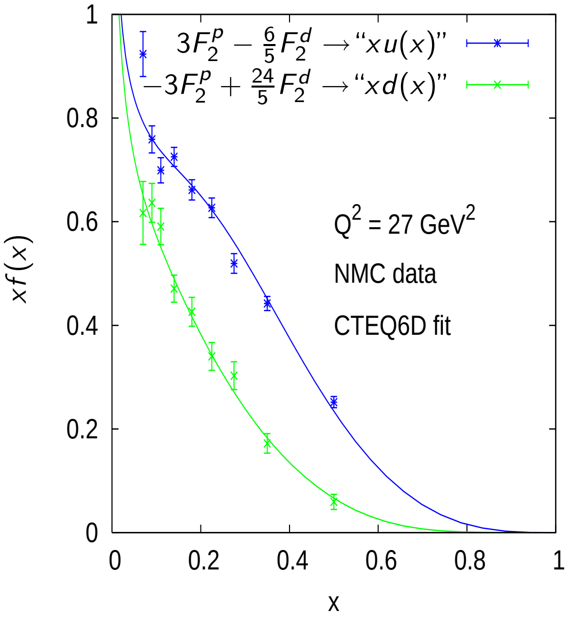
How much u and d?

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

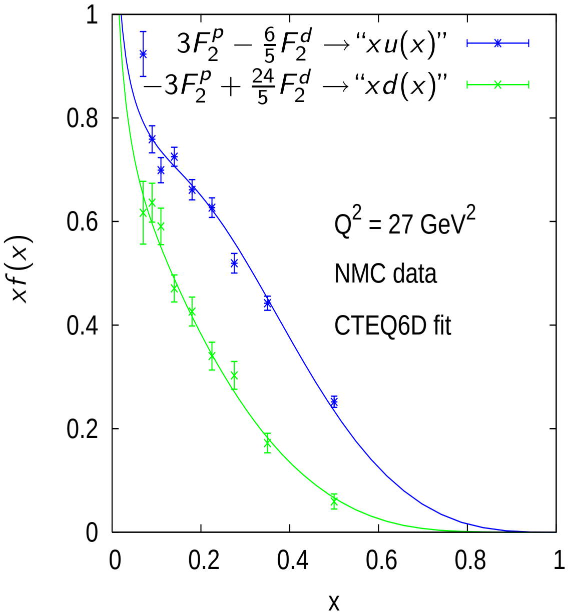
How much u and d?

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

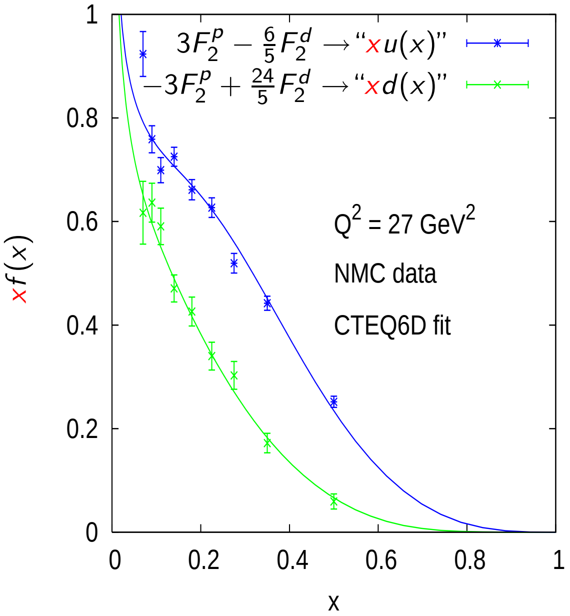
<u>How much u and d?</u>

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

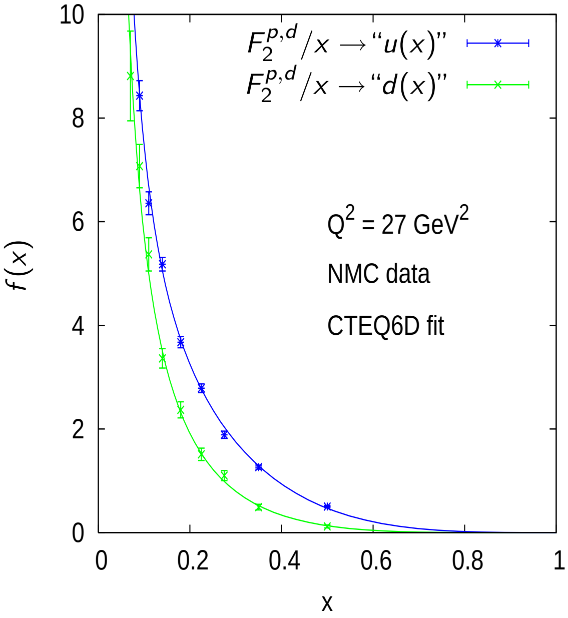
<u>How much u and d?</u>

- Total $U = \int dx\; u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

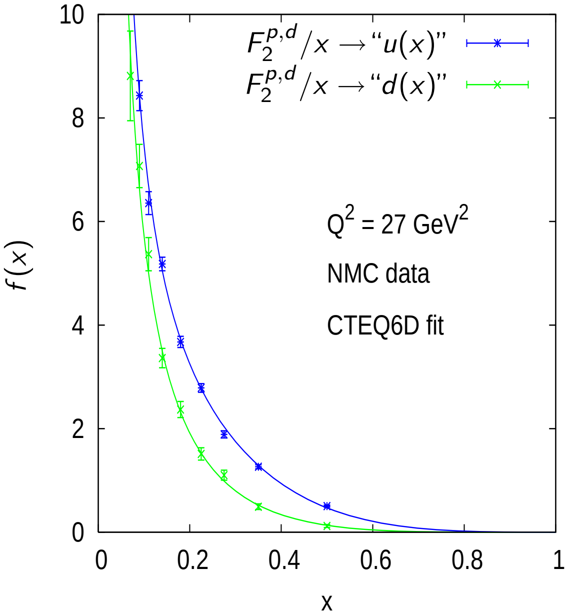
How much u and d?

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

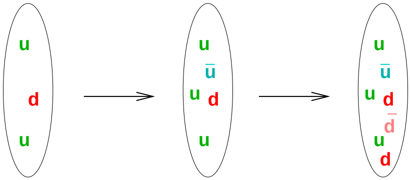
How much u and d?

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

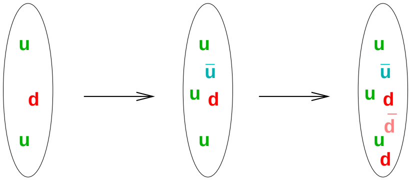
NB: photon interaction $\sim$ square of charge $\to$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

$$\int dx\, (u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

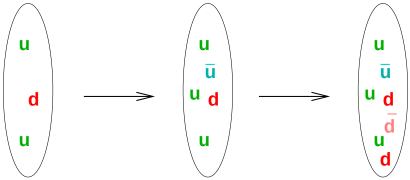
NB: photon interaction $\sim$ square of charge $\rightarrow$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

$$\int dx\,(u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

NB: photon interaction $\sim$ square of charge $\to$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

$$\int dx\,(u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

"Valence" quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both → no good...

Question: what interacts differently with particle & antiparticle?

Answer: W^+ *or* W^-

"Valence" quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both → no good...

Question: what interacts differently with particle & antiparticle?

Answer: W^+ *or* W^-

“Valence” quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

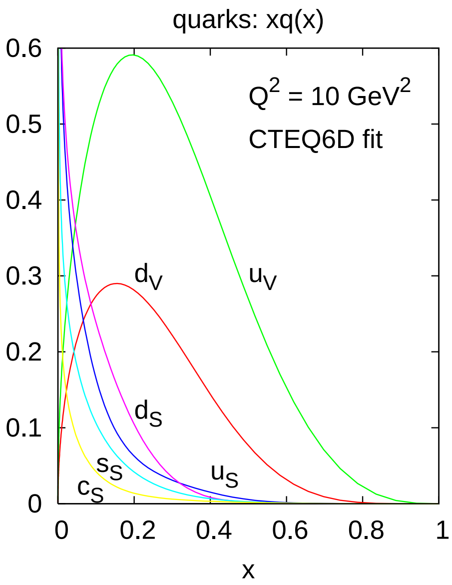
$u - \bar{u} = u_V$ is known as a *valence* distribution.

How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both $\rightarrow$ no good...

Question: what interacts differently with particle & antiparticle?

Answer: W^+ *or* W^-

All quarks

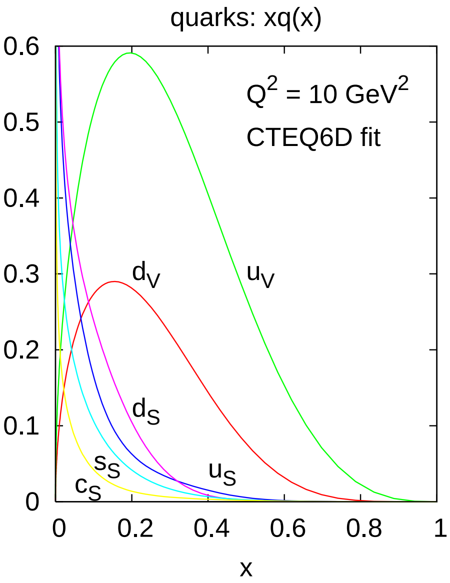


These & other methods $\to$ whole set of quarks & antiquarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*
  $x \to 1 : xq_V(x) \sim (1-x)^3$
  quark counting rules
  $x \to 0 : xq_V(x) \sim x^{0.5}$
  Regge theory
- sea quarks ($u_S = 2\bar{u}, \dots$) fairly *soft* (low-momentum)
  $x \to 1 : xq_S(x) \sim (1-x)^7$
  $x \to 0 : xq_S(x) \sim x^{-0.2}$

All quarks



These & other methods → whole set of quarks & antiquarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*

  $x \to 1 : xq_V(x) \sim (1-x)^3$

  quark counting rules

  $x \to 0 : xq_V(x) \sim x^{0.5}$

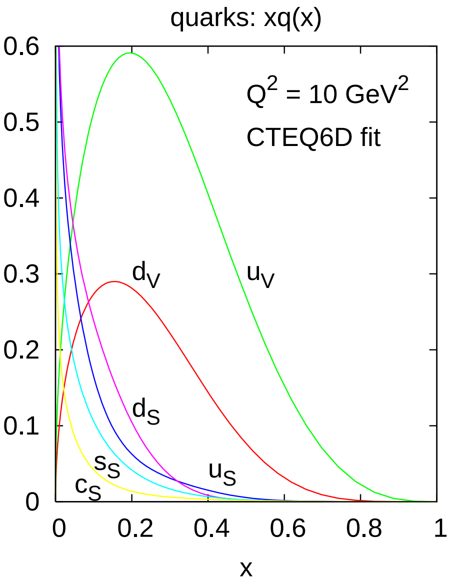
  Regge theory

- sea quarks ($u_S = 2\bar{u}, \ldots$) fairly *soft* (low-momentum)

  $x \to 1 : xq_S(x) \sim (1-x)^7$

  $x \to 0 : xq_S(x) \sim x^{-0.2}$

All quarks



These & other methods $\rightarrow$ whole set of quarks & antiquarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*

  $x \rightarrow 1 : xq_V(x) \sim (1-x)^3$

  quark counting rules

  $x \rightarrow 0 : xq_V(x) \sim x^{0.5}$

  Regge theory

- sea quarks ($u_S = 2\bar{u}, \ldots$) fairly *soft* (low-momentum)

  $x \rightarrow 1 : xq_S(x) \sim (1-x)^7$

  $x \rightarrow 0 : xq_S(x) \sim x^{-0.2}$

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^\pm$.

NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting...

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^{\pm}$.

NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting. . .

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?
Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^{\pm}$.

NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting...

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^\pm$.

NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting...

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^{\pm}$.

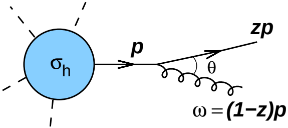
NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting. . .

Recall final-state splitting

Previous lecture: calculated $q \to qg$ $(\theta \ll 1,\ E \ll p)$ for final state of arbitrary hard process (σ_h):

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta^2}{\theta^2}$$



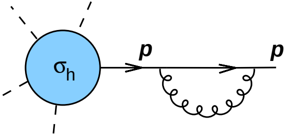
Rewrite with different kinematic variables

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

$$E = (1-z)p$$
$$k_t = E \sin\theta \simeq E\theta$$

If we avoid distinguishing $q+g$ final state from q (infrared-collinear safety), then divergent real and virtual corrections *cancel*

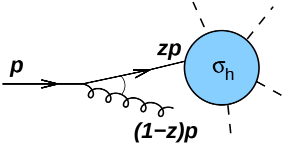
$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



Initial-state splitting

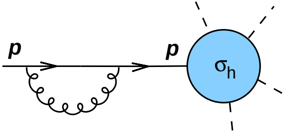
For initial state splitting, hard process occurs *after splitting*, and momentum entering hard process is modified: $p \to zp$.

$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



For virtual terms, momentum entering hard process is unchanged

$$\sigma_{V+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



Total cross section gets contribution with *two different hard X-sections*

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi}\int \frac{dk_t^2}{k_t^2}\frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]$$

NB: We assume σ_h involves momentum transfers $\sim Q \gg k_t$, so ignore extra transverse momentum in σ_h

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \int \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles
So how do we do QCD calculations in such cases?

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \underbrace{\int \frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles
So how do we do QCD calculations in such cases?

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int \frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles

So how do we do QCD calculations in such cases?

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int \frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

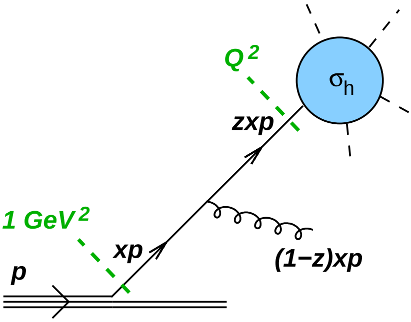
- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.
Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles
So how do we do QCD calculations in such cases?

Collinear cutoff



By what right did we go to $k_t = 0$?

We assumed pert. QCD to be valid for all scales, but *below* 1 GeV *it becomes non-perturbative*.

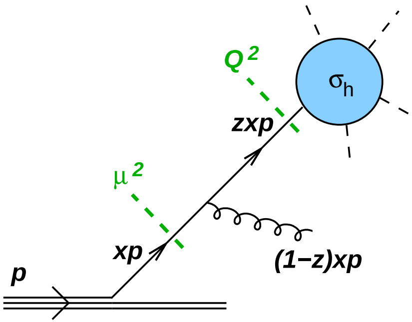
Cut out this divergent region, & instead put non-perturbative quark distribution in proton.

$$\sigma_0 = \int dx\, \sigma_h(xp)\, q(x, 1\ \mathrm{GeV}^2)$$

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_{1\ \mathrm{GeV}^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large)}} \underbrace{\int \frac{dx\, dz}{1-z} [\sigma_h(zxp) - \sigma_h(xp)]\, q(x, 1\ \mathrm{GeV}^2)}_{\text{finite}}$$

In general: replace 1 GeV^2 cutoff with arbitrary *factorization scale* μ^2.

Collinear cutoff



By what right did we go to $k_t = 0$?

We assumed pert. QCD to be valid for all scales, but *below* 1 GeV *it becomes non-perturbative*.

Cut out this divergent region, & instead put non-perturbative quark distribution in proton.

$$\sigma_0 = \int dx\, \sigma_h(xp)\, q(x, \mu^2)$$

$$\sigma_1 \simeq \underbrace{\frac{\alpha_s C_F}{\pi} \int_{\mu^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large)}} \underbrace{\int \frac{dx\, dz}{1-z} \left[\sigma_h(zxp) - \sigma_h(xp)\right] q(x, \mu^2)}_{\text{finite}}$$

In general: replace 1 GeV^2 cutoff with arbitrary *factorization scale* μ^2.

Summary so far

- Collinear divergence for incoming partons *not cancelled* by virtuals.

  Real and virtual have different longitudinal momenta

- Situation analogous to renormalization: need to *regularize* (but in IR instead of UV).

  Technically, often done with dimensional regularization

- Physical sense of regularization is to separate *(factorize)* proton non-perturbative dynamics from perturbative hard cross section.

  Choice of factorization scale, μ^2, is arbitrary between 1 GeV^2 and Q^2

- In analogy with running coupling, we can *vary factorization scale* and get a *renormalization group equation* for parton distribution functions.

  Dokshizer Gribov Lipatov Altarelli Parisi equations (DGLAP)

Summary so far

- Collinear divergence for incoming partons *not cancelled* by virtuals.

  Real and virtual have different longitudinal momenta

- Situation analogous to renormalization: need to *regularize* (but in IR instead of UV).

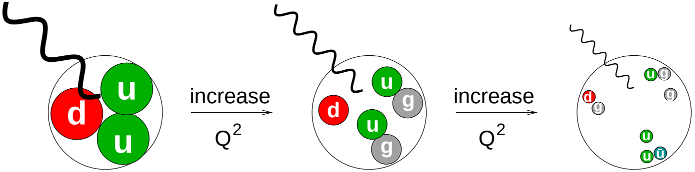
  Technically, often done with dimensional regularization

- Physical sense of regularization is to separate *(factorize)* proton non-perturbative dynamics from perturbative hard cross section.

  Choice of factorization scale, μ^2, is arbitrary between 1 GeV^2 and Q^2

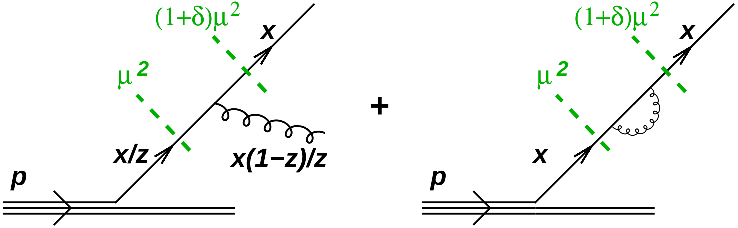
- In analogy with running coupling, we can *vary factorization scale* and get a *renormalization group equation* for parton distribution functions.

  Dokshizer Gribov Lipatov Altarelli Parisi equations (DGLAP)



DGLAP equation ($q \leftarrow q$)

Change convention: (a) now *fix outgoing* longitudinal momentum x; (b) *take derivative* wrt factorization scale μ^2



$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz\, p_{qq}(z)\,\frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu^2)$$

p_{qq} is real $q \leftarrow q$ **splitting kernel:** $p_{qq}(z) = C_F\dfrac{1+z^2}{1-z}$

Until now we approximated it in soft ($z \to 1$) limit, $p_{qq} \simeq \frac{2C_F}{1-z}$

DGLAP rewritten

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\underbrace{\int_x^1 dz\, P_{qq}(z)\,\frac{q(x/z,\mu^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

This involves the *plus prescription:*

$$\int_0^1 dz\,[g(z)]_+\,f(z) = \int_0^1 dz\,g(z)\,f(z) - \int_0^1 dz\,g(z)\,f(1)$$

$z=1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z=1$

DGLAP flavour structure

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

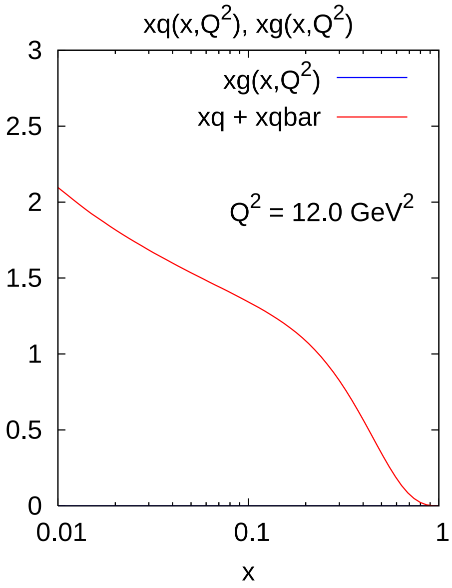
$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- P_{qg}, P_{gg}: *symmetric $z \leftrightarrow 1-z$* (except virtuals)
- P_{qq}, P_{gg}: *diverge for $z \to 1$* soft gluon emission
- P_{gg}, P_{gq}: *diverge for $z \to 0$* Implies PDFs grow for $x \to 0$

Effect of DGLAP (initial quarks)

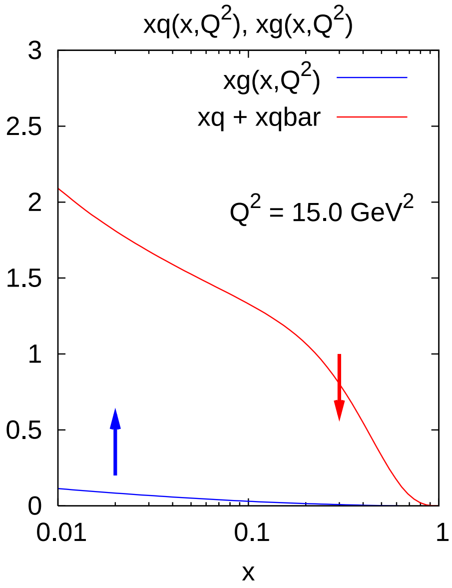


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

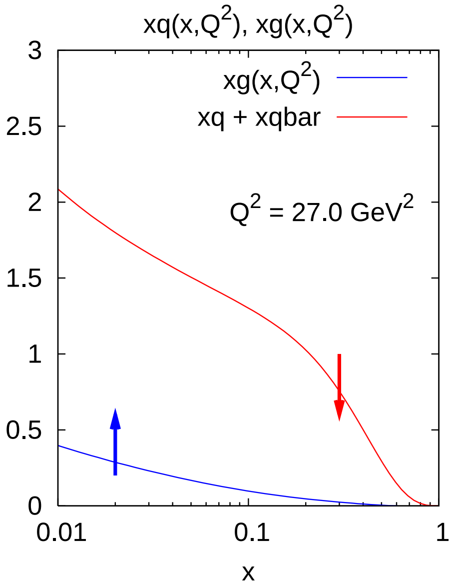


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

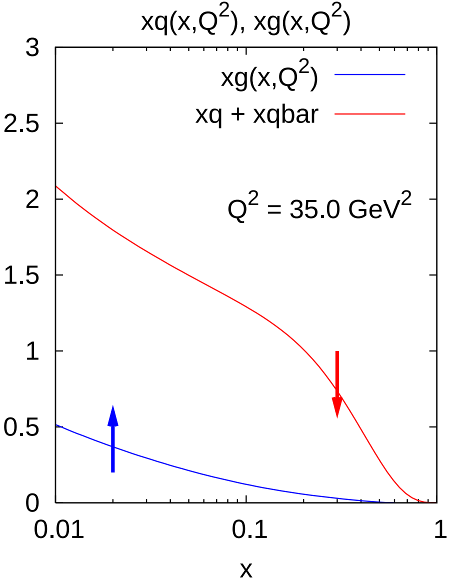


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

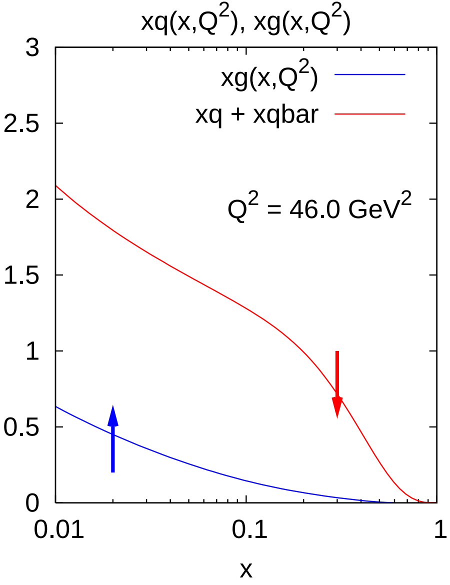


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

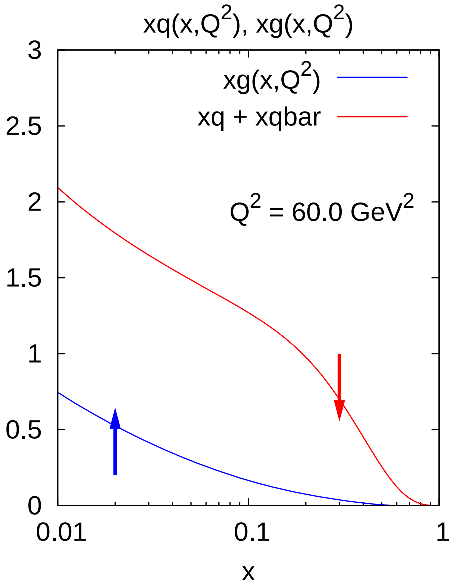


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

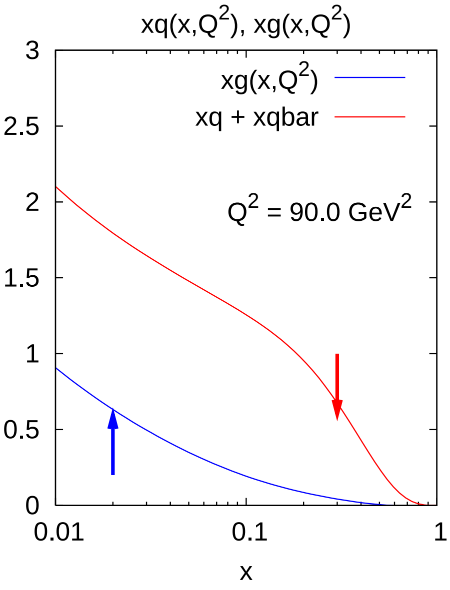


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

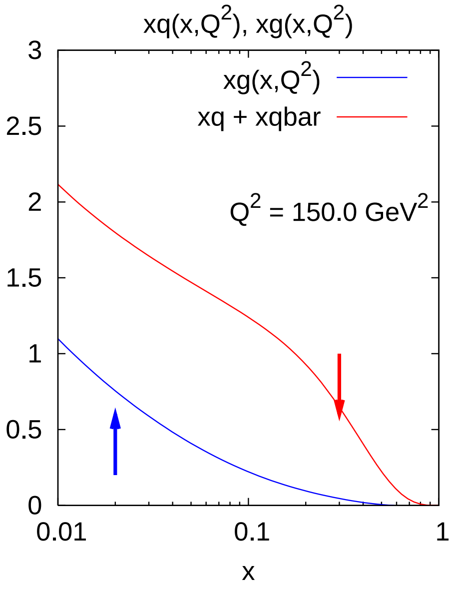


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial quarks)

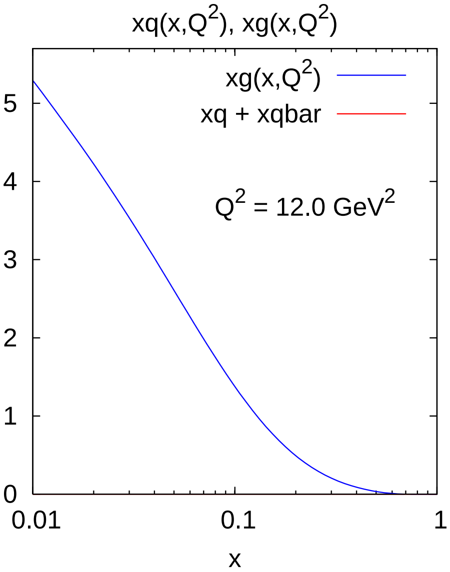


Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q \leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

Effect of DGLAP (initial gluons)

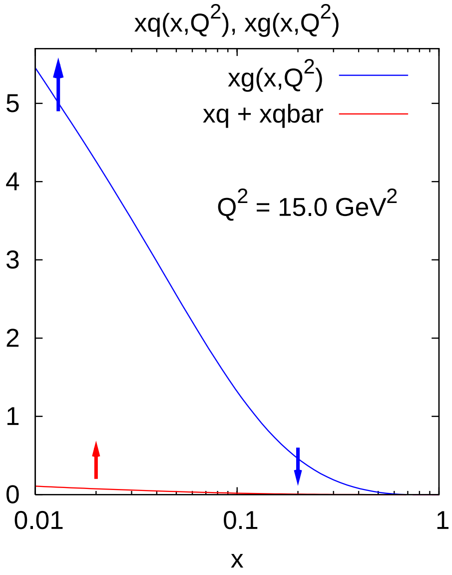


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

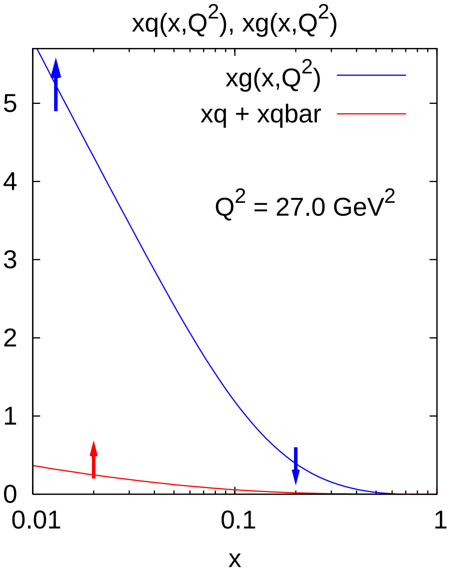


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

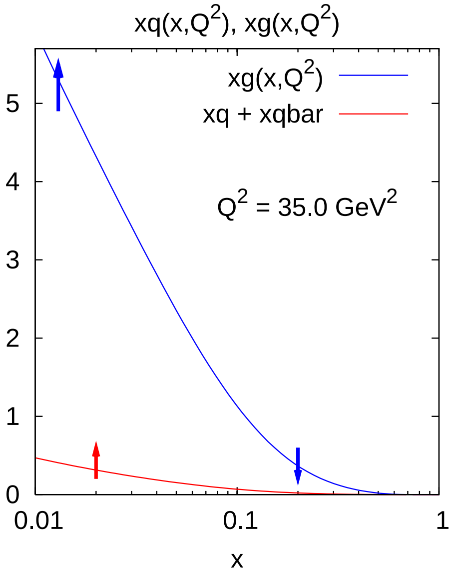


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

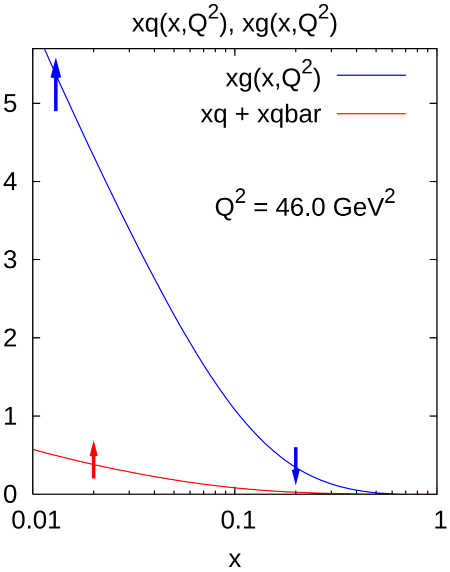


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

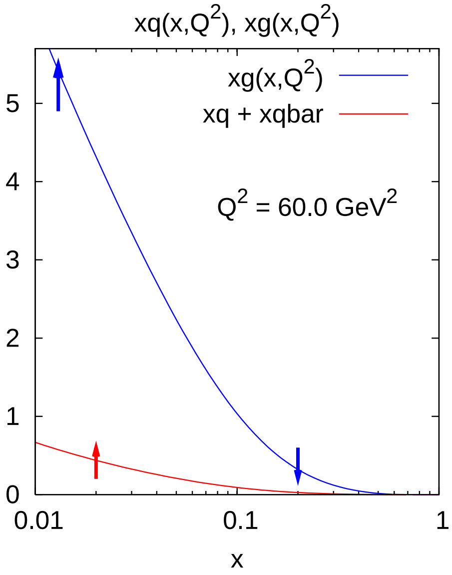


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

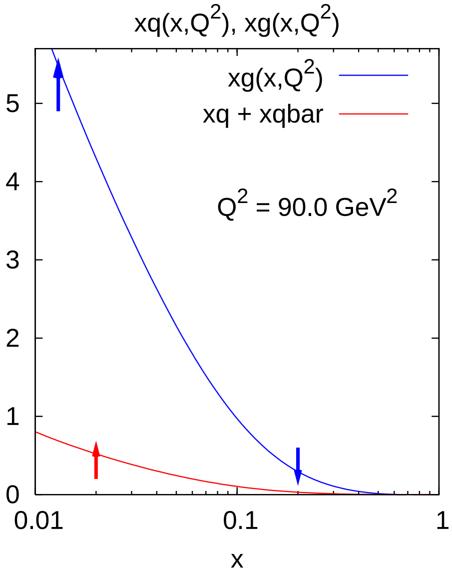


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)

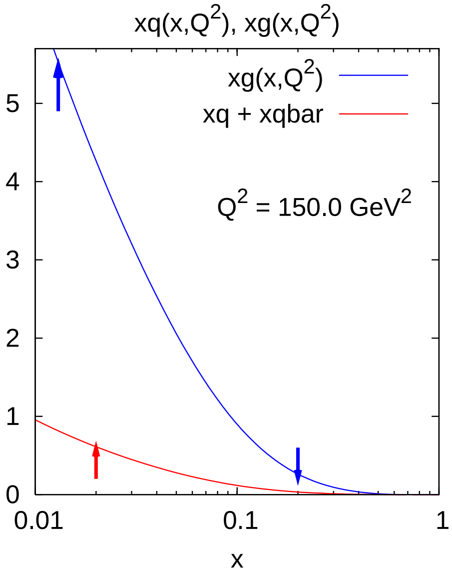


2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)



2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$

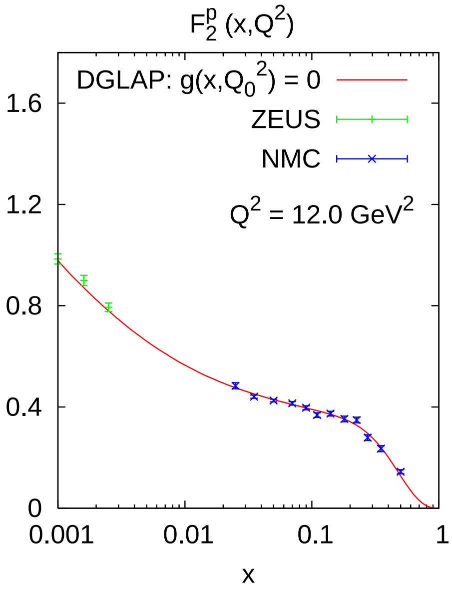
- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

DGLAP evolution

- As Q^2 increases, partons lose longitudinal momentum; distributions all shift to lower x.
- gluons can be seen because they help drive the quark evolution.

Now consider data

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\ \text{GeV}^2$.

NB: Q_0 often chosen lower

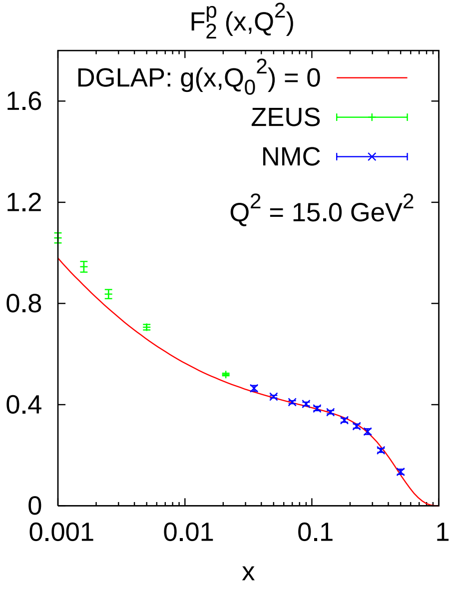
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

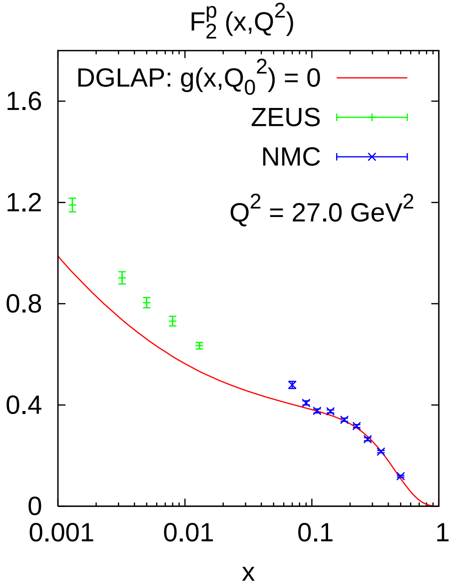
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon $= 0$



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

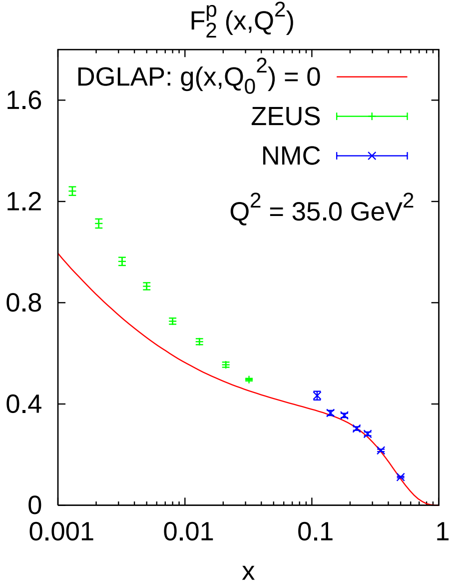
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

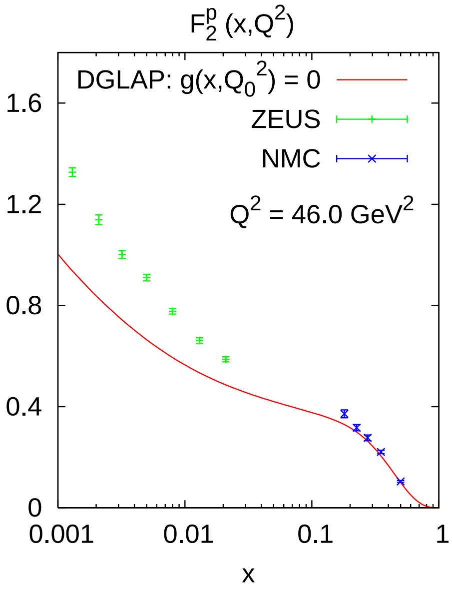
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon $= 0$



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\ \mathrm{GeV}^2$.

NB: Q_0 often chosen lower

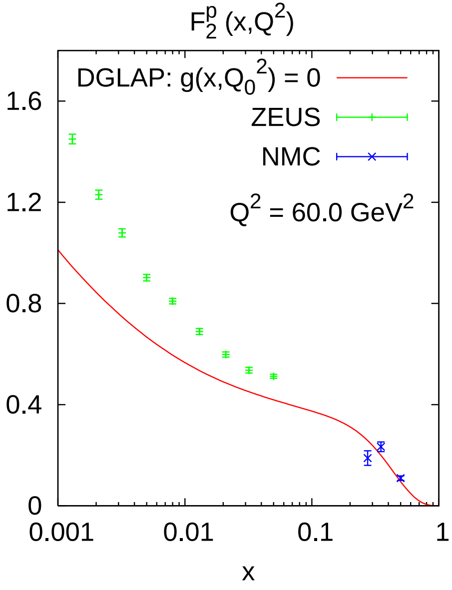
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

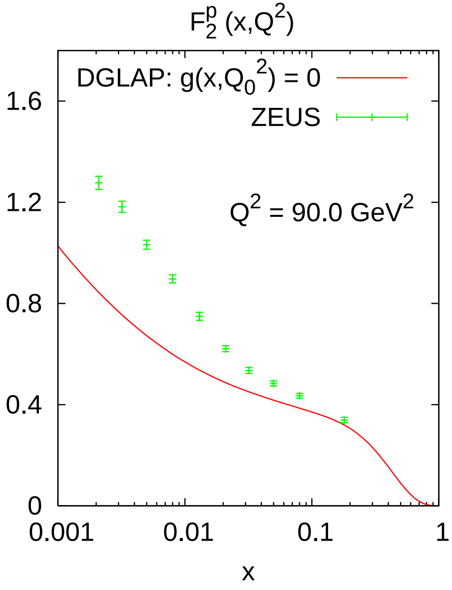
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

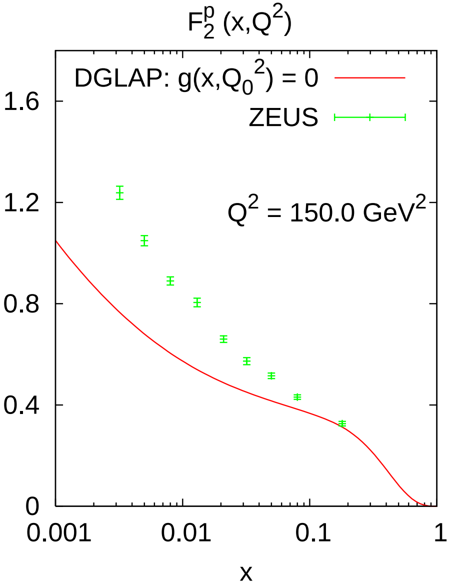
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\ \text{GeV}^2$.

NB: Q_0 often chosen lower

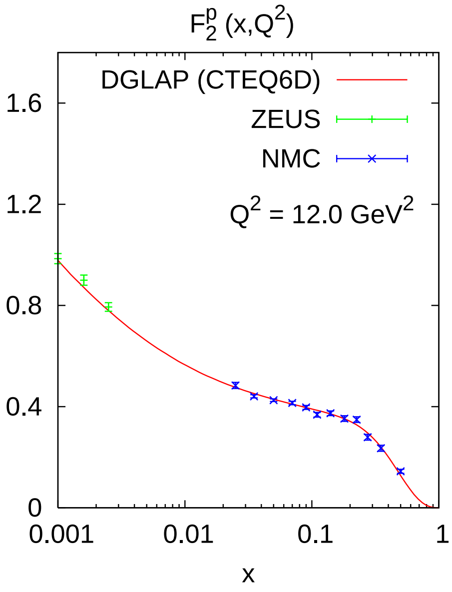
Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

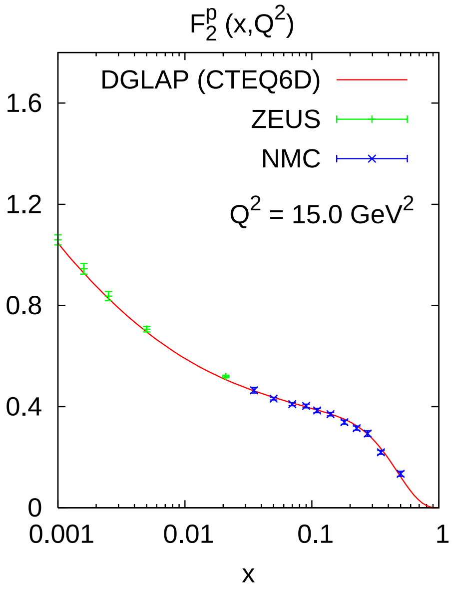
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

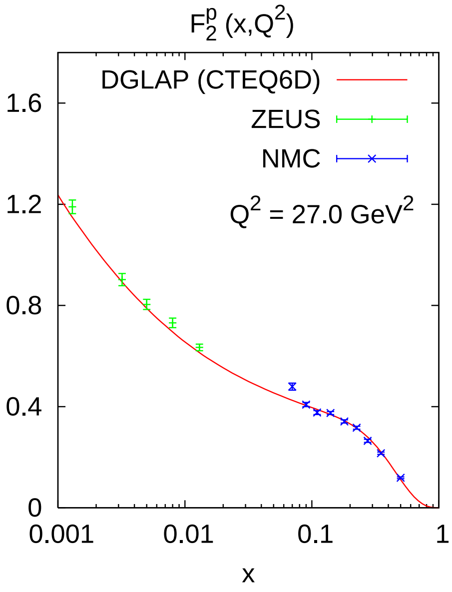
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



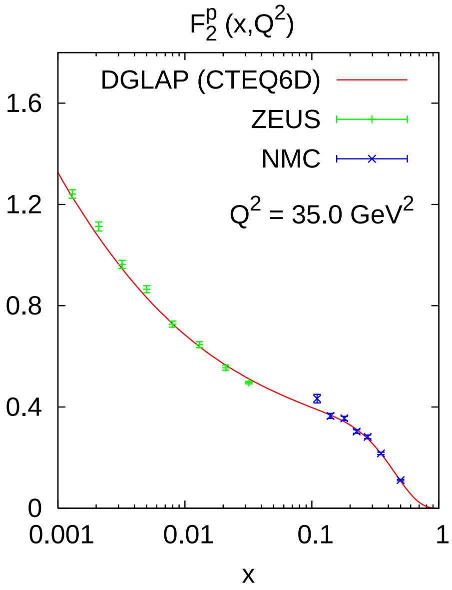
If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, . . . PDF fitting collaborations.

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

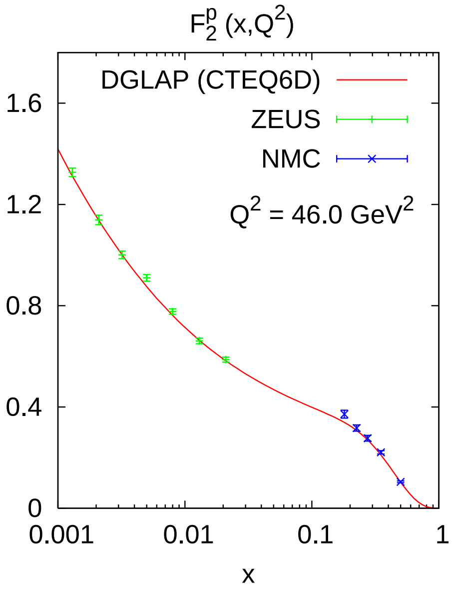
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ... PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

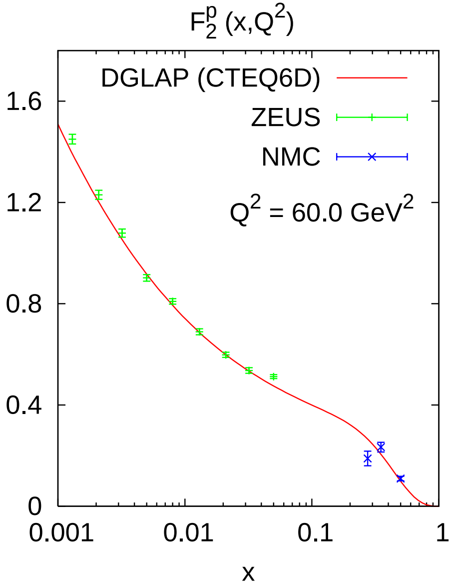
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, . . . PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

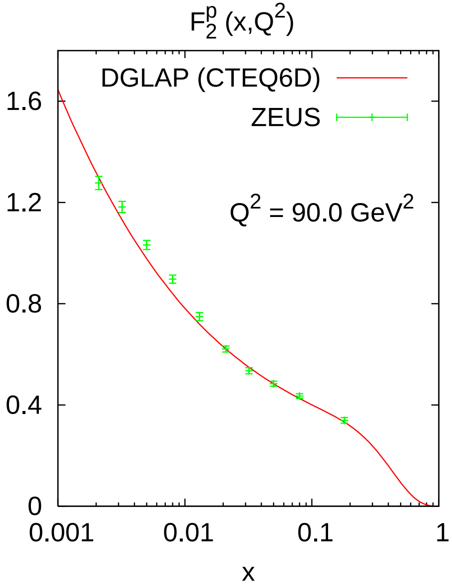
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

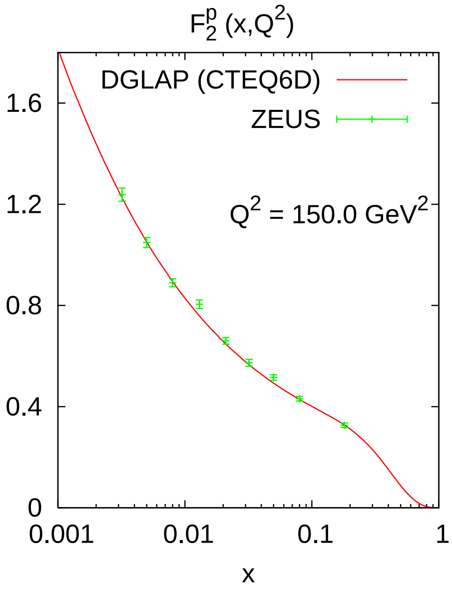
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

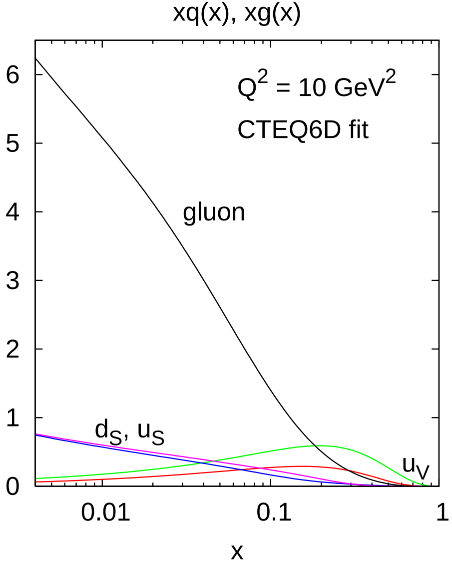
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

Success!

Gluon distribution



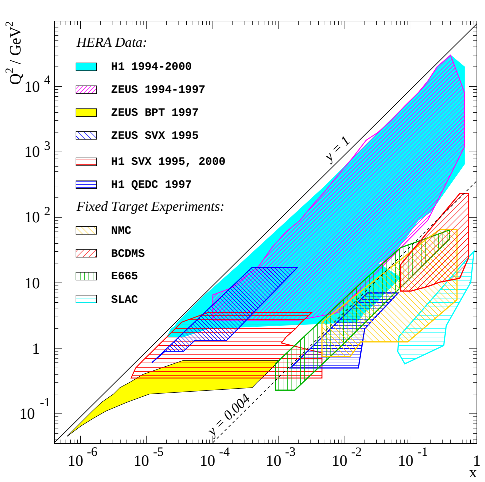
Gluon distribution is **HUGE!**

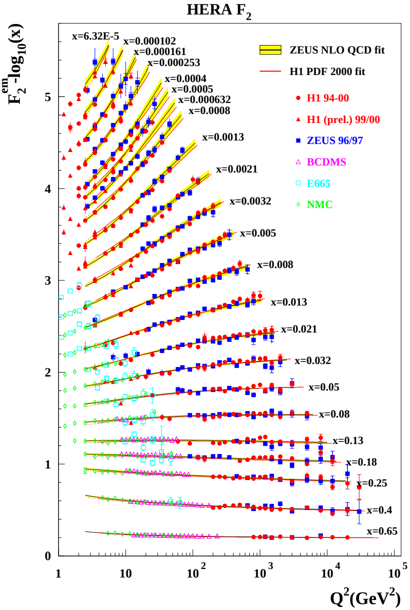
Can we really trust it?

- Consistency: momentum sum-rule is now *satisfied*.

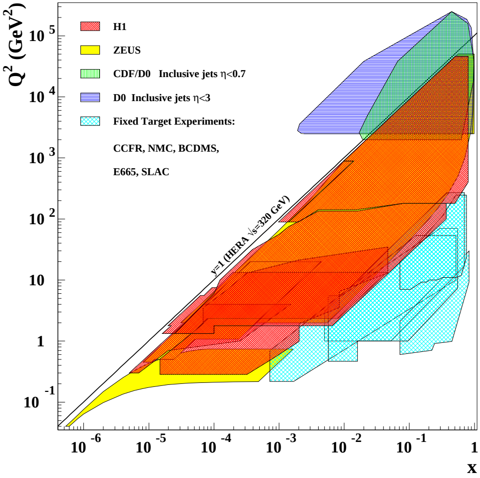
  NB: gluon mostly at small x
- Agrees with vast range of data

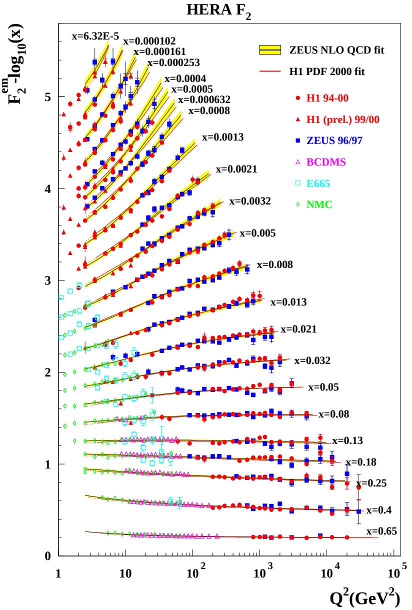
DIS data and global fits





DIS data and global fits

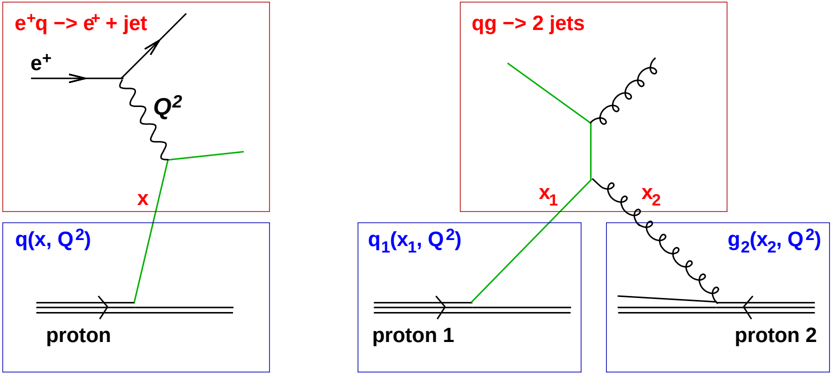




Crucial check: other processes

Factorization of QCD cross-sections into convolution of:

- hard (perturbative) process-dependent **partonic subprocess**
- non-perturbative, process-independent **parton distribution functions**

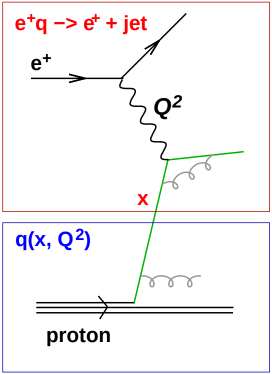


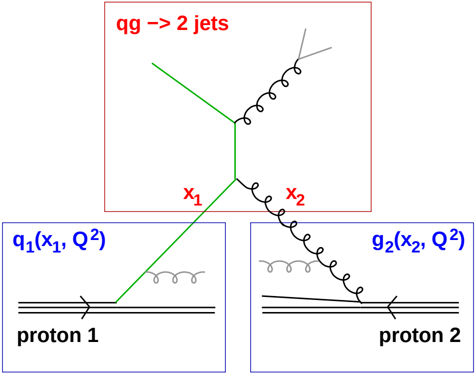
$$\sigma_{ep} = \sigma_{eq} \otimes q \qquad \sigma_{pp\to 2\,jets} = \sigma_{qg\to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$$

Crucial check: other processes

Factorization of QCD cross-sections into convolution of:

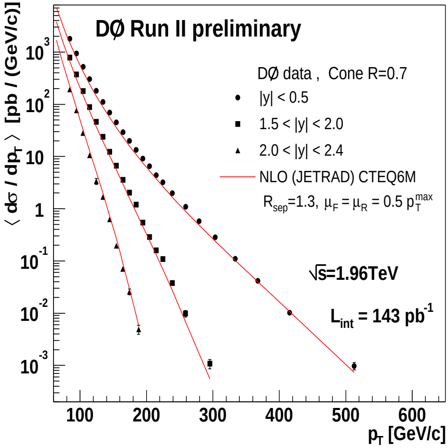
- hard (perturbative) process-dependent **partonic subprocess**
- non-perturbative, process-independent **parton distribution functions**





$$\sigma_{ep} = \sigma_{eq} \otimes q \qquad \sigma_{pp\to 2\,jets} = \sigma_{qg\to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$$

Jet production in $p\bar{p}$



Jet production in proton-antiproton collisions is *good test of large gluon distribution,* since there are large direct contributions from

$$gg \to gg\,, \qquad qg \to qg$$

NB: more complicated to interpret than DIS, since many channels, and x_1, x_2 dependence.

$p_T \sim \sqrt{x_1 x_2 s}$ jet transverse mom.

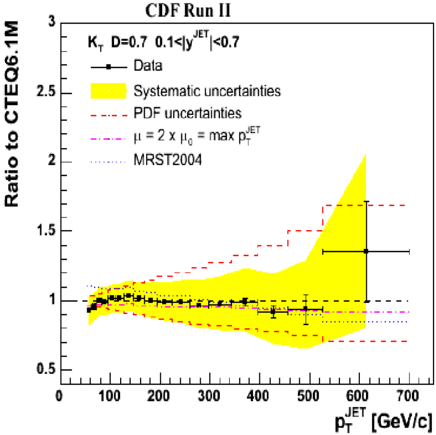
$\sim Q$

$y \sim \frac{1}{2} \log \frac{x_1}{x_2}$ $\quad y = \log \tan \frac{\theta}{2}$

jet angle wrt $p\bar{p}$ beams

Good agreement confirms factorization

Jet production in $p\bar{p}$



Jet production in proton-antiproton collisions is *good test of large gluon distribution,* since there are large direct contributions from

$$gg \to gg\,, \quad qg \to qg$$

NB: more complicated to interpret than DIS, since many channels, and x_1, x_2 dependence.

$p_T \sim \sqrt{x_1 x_2 s}$ jet transverse mom.

$\sim Q$

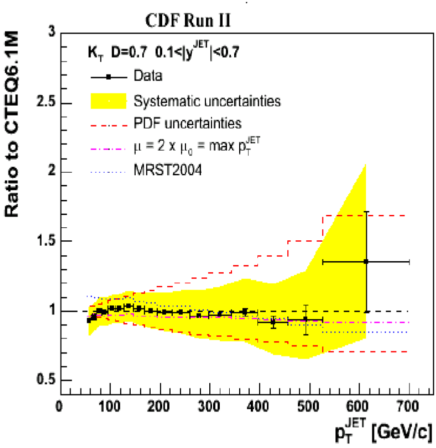
$$y \sim \frac{1}{2} \log \frac{x_1}{x_2}$$

$y = \log \tan \frac{\theta}{2}$

jet angle wrt $p\bar{p}$ beams

Good agreement confirms factorization

Jet production in $p\bar{p}$



Jet production in proton-antiproton collisions is *good test of large gluon distribution,* since there are large direct contributions from

$$gg \to gg\,, \quad qg \to qg$$

NB: more complicated to interpret than DIS, since many channels, and x_1, x_2 dependence.

$p_T \sim \sqrt{x_1 x_2 s}$ jet transverse mom.

$\sim Q$

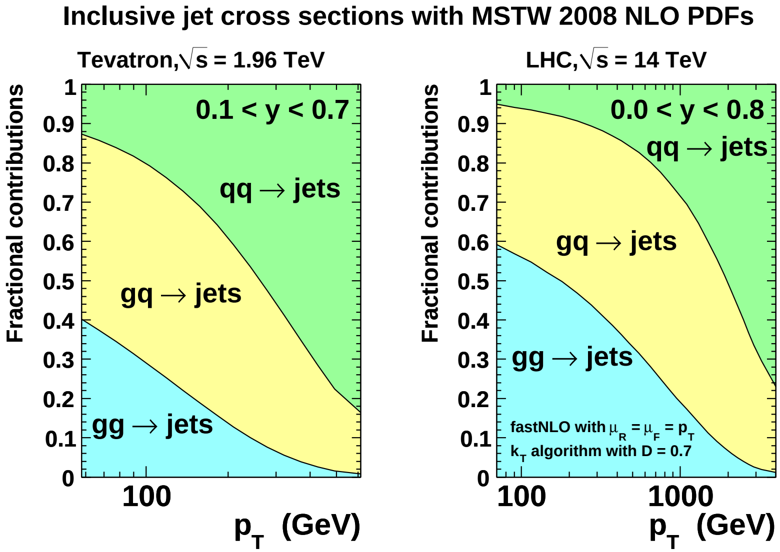
$$y \sim \frac{1}{2}\log\frac{x_1}{x_2}$$

$y = \log \tan \frac{\theta}{2}$

jet angle wrt $p\bar{p}$ beams

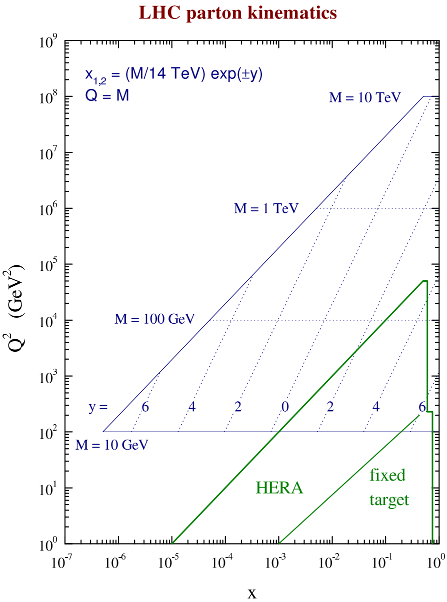
Good agreement confirms factorization

Which PDF channels contribute?



A large fraction of jets are gluon-induced

Taking PDFs from HERA to LHC



Suppose we produce a system of mass M at LHC from partons with momentum fractions x_1, x_2:

- $M = \sqrt{x_1 x_2 s}$
- rapidity $y = \frac{1}{2} \ln \frac{x_1}{x_2}$

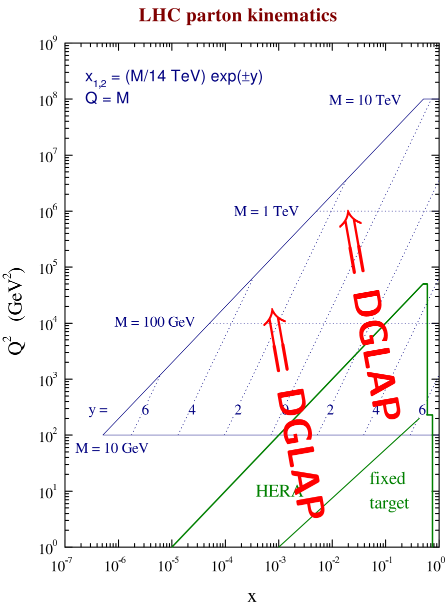
pseudorapidity $\equiv \eta \equiv \ln \tan \frac{\theta}{2}$
$=$ rapidity for massless objects
$\lesssim 5$ at LHC

Are PDFs being used in region where measured?

Only partial kinematic overlap

- DGLAP evolution is **essential** for the prediction of PDFs in the LHC domain.

Taking PDFs from HERA to LHC



Suppose we produce a system of mass M at LHC from partons with momentum fractions x_1, x_2:

- $M = \sqrt{x_1 x_2 s}$
- rapidity $y = \frac{1}{2} \ln \frac{x_1}{x_2}$

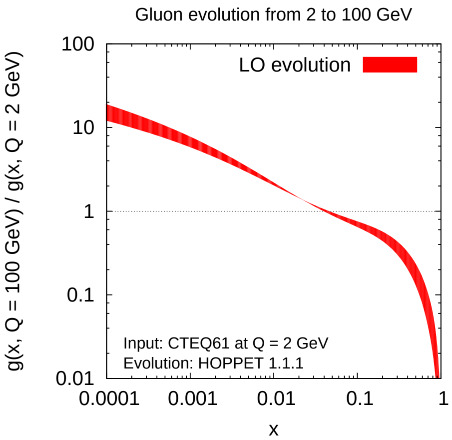
pseudorapidity $\equiv \eta \equiv \ln \tan \frac{\theta}{2}$
$=$ rapidity for massless objects
$\lesssim 5$ at LHC

Are PDFs being used in region where measured?

Only partial kinematic overlap

- DGLAP evolution is **essential** for the prediction of PDFs in the LHC domain.

By how much do PDFs evolve?



Illustrate for the gluon distribution

Here using fixed Q scales

But for HERA $\rightarrow$ LHC

relevant Q range is x-dependent

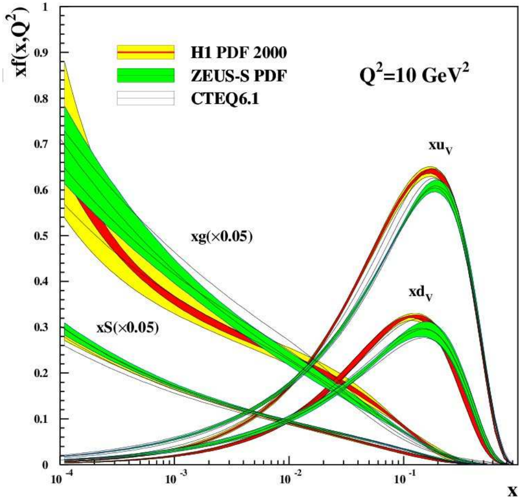
- See factors $\sim 0.1 - 10$
- Remember: LHC involves product of two parton densities

It's crucial to get this right!

Without DGLAP evolution, you couldn't predict anything at LHC

How well do we know the PDFs?

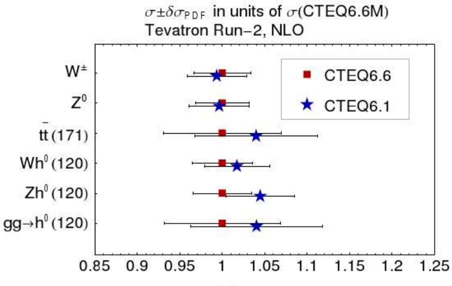
PDF uncertainties



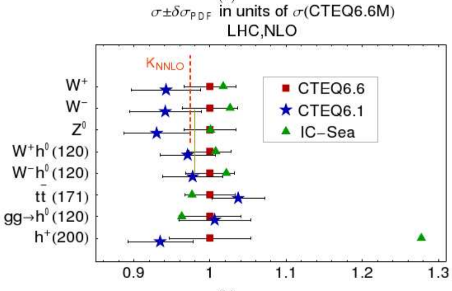
Major activity is translation of experimental errors (and theory uncertainties) into *uncertainty bands on extracted PDFs.*

PDFs with uncertainties allow one to estimate *degree of reliability* of future predictions

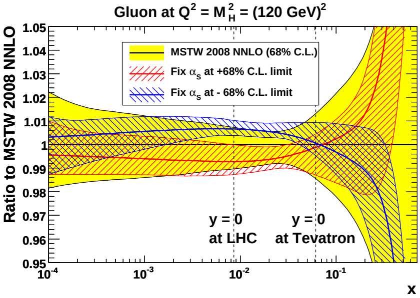
Uncertainties on predictions



(a)



(b)



General message

Data-related errors on PDFs are such that uncertainties are just a few % for many key Tevatron and LHC observables

It's not enough for data-related errors to be small.

DGLAP evolution must also be well constrained.

So evolution must be done with more than just leading-order DGLAP splitting functions

Higher-order calculations

Earlier, we saw leading order (LO) DGLAP splitting functions, $P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)}$:

$$P_{qq}^{(0)}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right] ,$$

$$P_{qg}^{(0)}(x) = T_R \left[x^2 + (1-x)^2 \right] ,$$

$$P_{gq}^{(0)}(x) = C_F \left[\frac{1+(1-x)^2}{x} \right] ,$$

$$P_{gg}^{(0)}(x) = 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \frac{(11 C_A - 4 n_f T_R)}{6} .$$

Higher-order calculations

NLO:

$$P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)} + \frac{\alpha_s^2}{16\pi^2} P^{(1)}$$

Curci, Furmanski & Petronzio '80

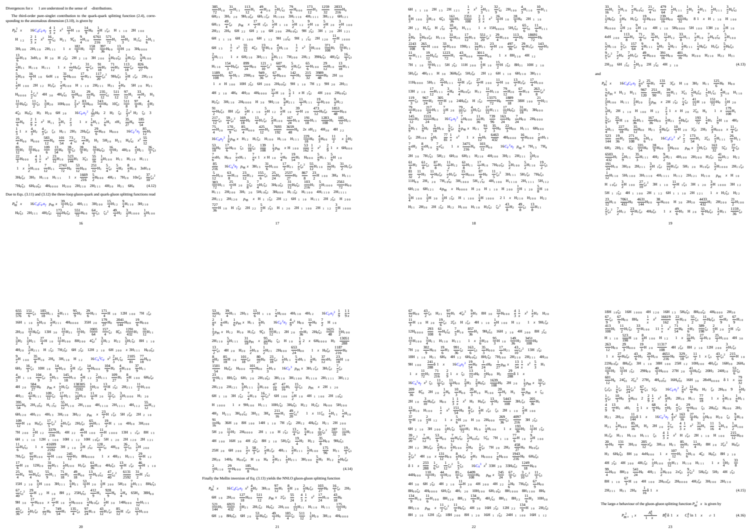
$$P^{(1)}_{\rm ps}(x) = 4\, C_F n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9}\right] + (1+x)\left[5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right]\right)$$

$$\begin{aligned} P^{(1)}_{\rm qg}(x) &= 4\, C_A n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x)\mathrm{H}_{-1,0} - 2p_{\rm qg}(x)\mathrm{H}_{1,1} + x^2\left[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9}\right] \right. \\ &\left. +4(1-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1\right] - 4\zeta_2 x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0\right) + 4\, C_F n_f \left(2p_{\rm qg}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 \right.\right. \\ &\left.\left. -\zeta_2\right] + 4x^2\left[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2}\right] + 2(1-x)\left[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4}\right] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0\right) \end{aligned}$$

$$\begin{aligned} P^{(1)}_{\rm gq}(x) &= 4\, C_A C_F \left(\frac{1}{x} + 2p_{\rm gq}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1\right] - x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9}\right] + 4\zeta_2 - 2 \right. \\ &\left. -7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1 x + (1+x)\left[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9}\right] - 2p_{\rm gq}(-x)\mathrm{H}_{-1,0}\right) - 4\, C_F n_f \left(\frac{2}{3}x \right. \\ &\left. - p_{\rm gq}(x)\left[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9}\right]\right) + 4\, C_F{}^2 \left(p_{\rm gq}(x)\left[3\mathrm{H}_1 - 2\mathrm{H}_{1,1}\right] + (1+x)\left[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0\right] - 3\mathrm{H}_{0,0} \right. \\ &\left. +1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1 x\right) \end{aligned}$$

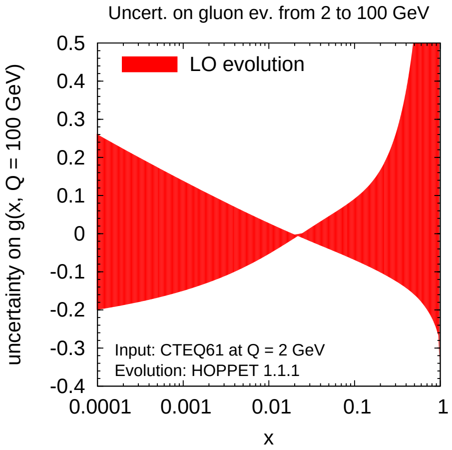
$$\begin{aligned} P^{(1)}_{\rm gg}(x) &= 4\, C_A n_f \left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2\right) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x)\right) + 4\, C_A{}^2 \left(27 \right. \\ &+(1+x)\left[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2}\right] + 2p_{\rm gg}(-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2\right] - \frac{67}{9}\left(\frac{1}{x} - x^2\right) - 12\mathrm{H}_0 \\ &\left. - \frac{44}{3}x^2\mathrm{H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2\right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3\right]\right) + 4\, C_F n_f \left(2\mathrm{H}_0 \right. \\ &\left. + \frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right] - \frac{1}{2}\delta(1-x)\right) . \end{aligned}$$

NNLO splitting functions



NNLO, $P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

Evolution uncertainty

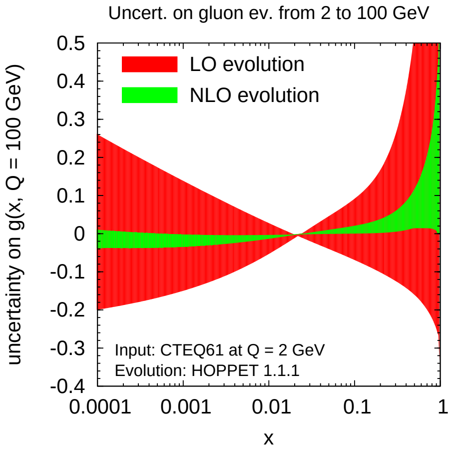


Estimate uncertainties on evolution by changing the scale used for α_s inside the splitting functions

Talk more about such tricks in next lecture

- with LO evolution, uncertainty is $\sim 30\%$
- NLO brings it down to $\sim 5\%$
- NNLO $\rightarrow 2\%$ Commensurate with data uncertainties

Evolution uncertainty

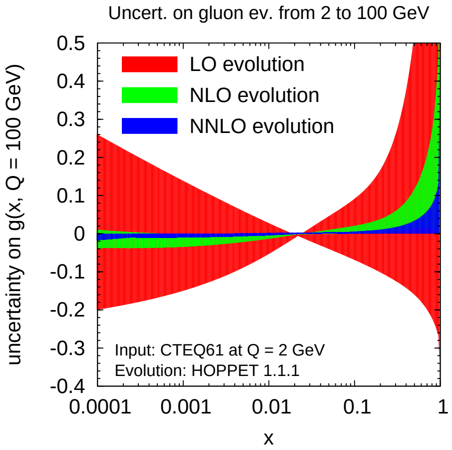


Estimate uncertainties on evolution by changing the scale used for α_s inside the splitting functions

Talk more about such tricks in next lecture

- with LO evolution, uncertainty is $\sim 30\%$
- NLO brings it down to $\sim 5\%$
- NNLO $\rightarrow 2\%$ Commensurate with data uncertainties

Evolution uncertainty



Estimate uncertainties on evolution by changing the scale used for α_s inside the splitting functions

Talk more about such tricks in next lecture

- with LO evolution, uncertainty is $\sim 30\%$
- NLO brings it down to $\sim 5\%$
- NNLO $\rightarrow 2\%$

Commensurate with data uncertainties

Conclusions on PDFs

- Experiments tell us that proton really is what we expected (*uud*)
- Plus lots more: large number of 'sea quarks' ($q\bar{q}$), gluons (50% of momentum)
- *Factorization* is key to usefulness of PDFs
  - Non-trivial beyond lowest order
  - PDFs depend on factorization scale, evolve with *DGLAP equation*
  - Pattern of *evolution gives us info on gluon* (otherwise hard to measure)
  - PDFs really are universal!
- *Precision* of data & QCD calculations is striking.
- Crucial for understanding future signals of *new particles,* e.g. Higgs Boson production at LHC.