

Jets and jet substructure

Gavin Salam (CERN)

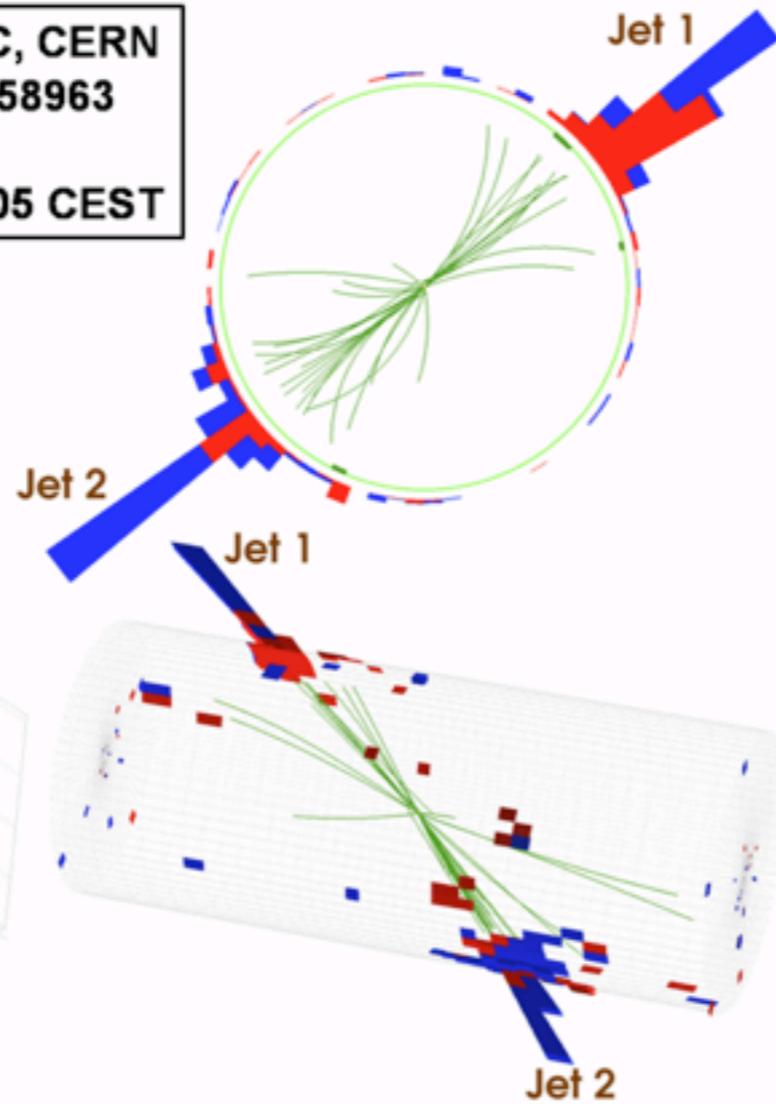
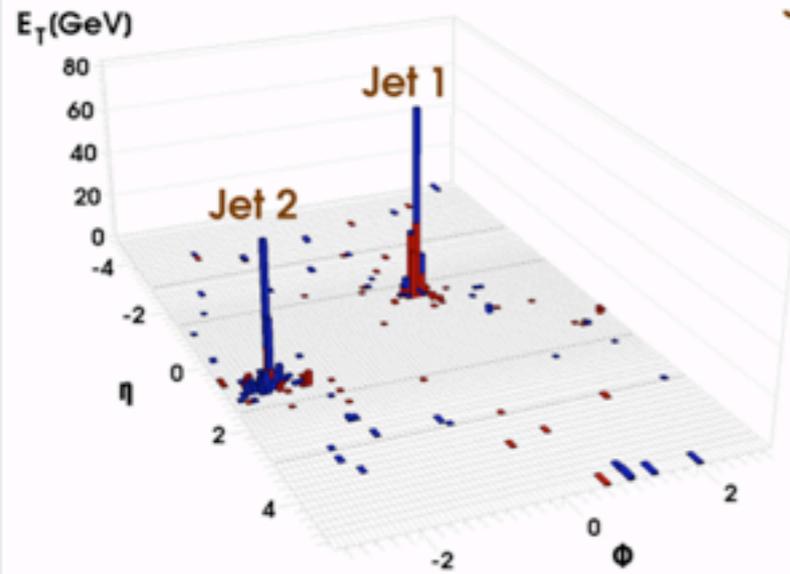
with extensive use of material by Matteo Cacciari
and Gregory Soyez

CFHEP

April 2014



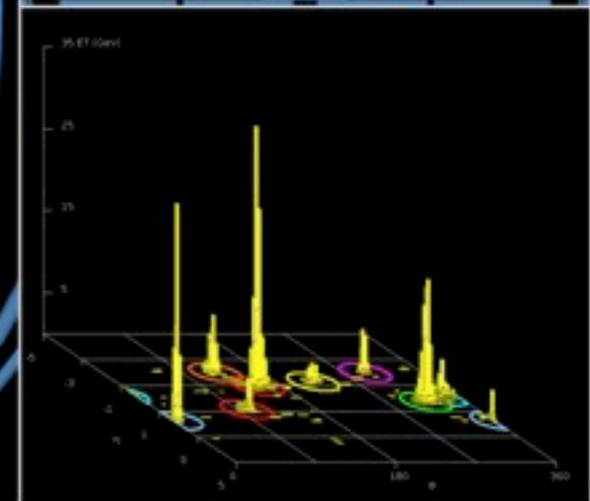
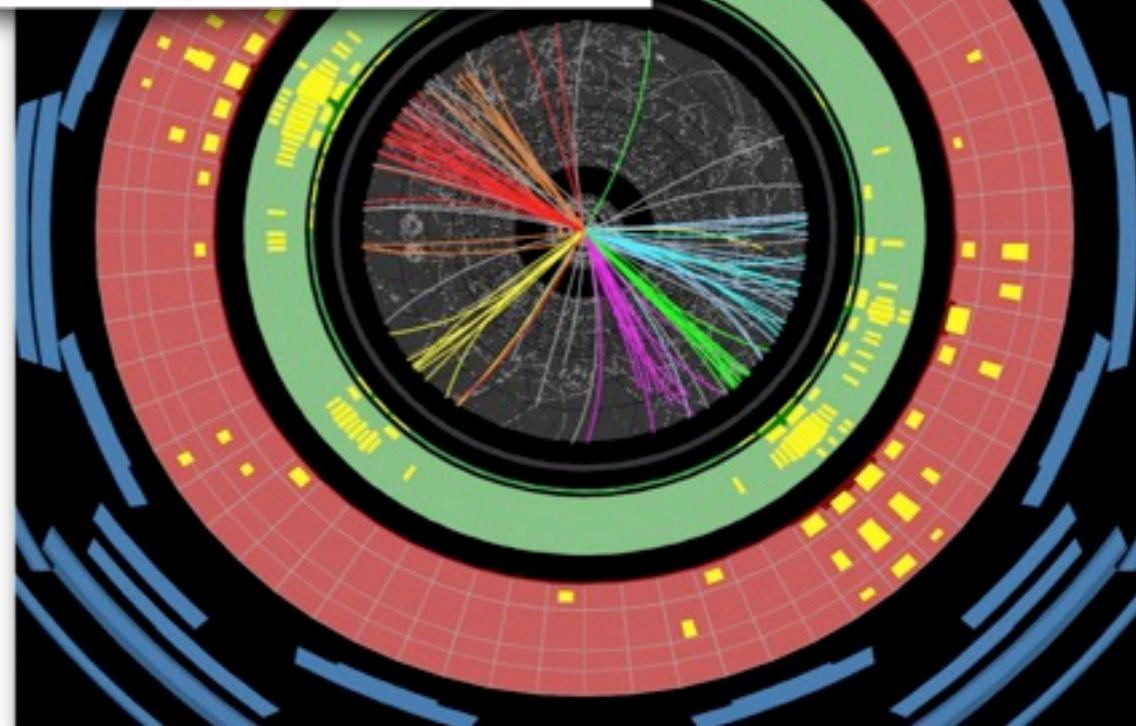
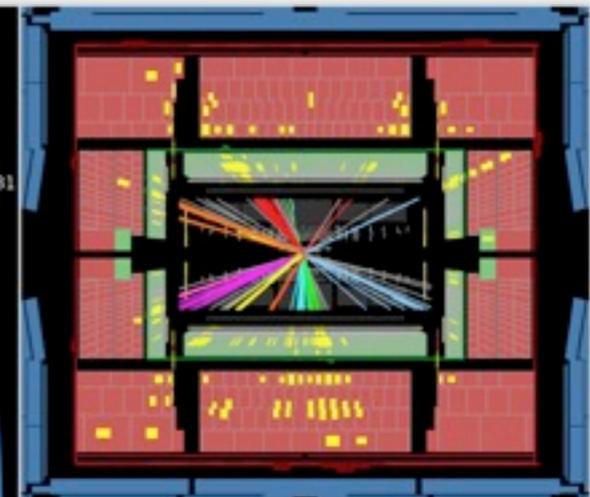
CMS Experiment at LHC, CERN
Run 133450 Event 16358963
Lumi section: 285
Sat Apr 17 2010, 12:25:05 CEST



JETS

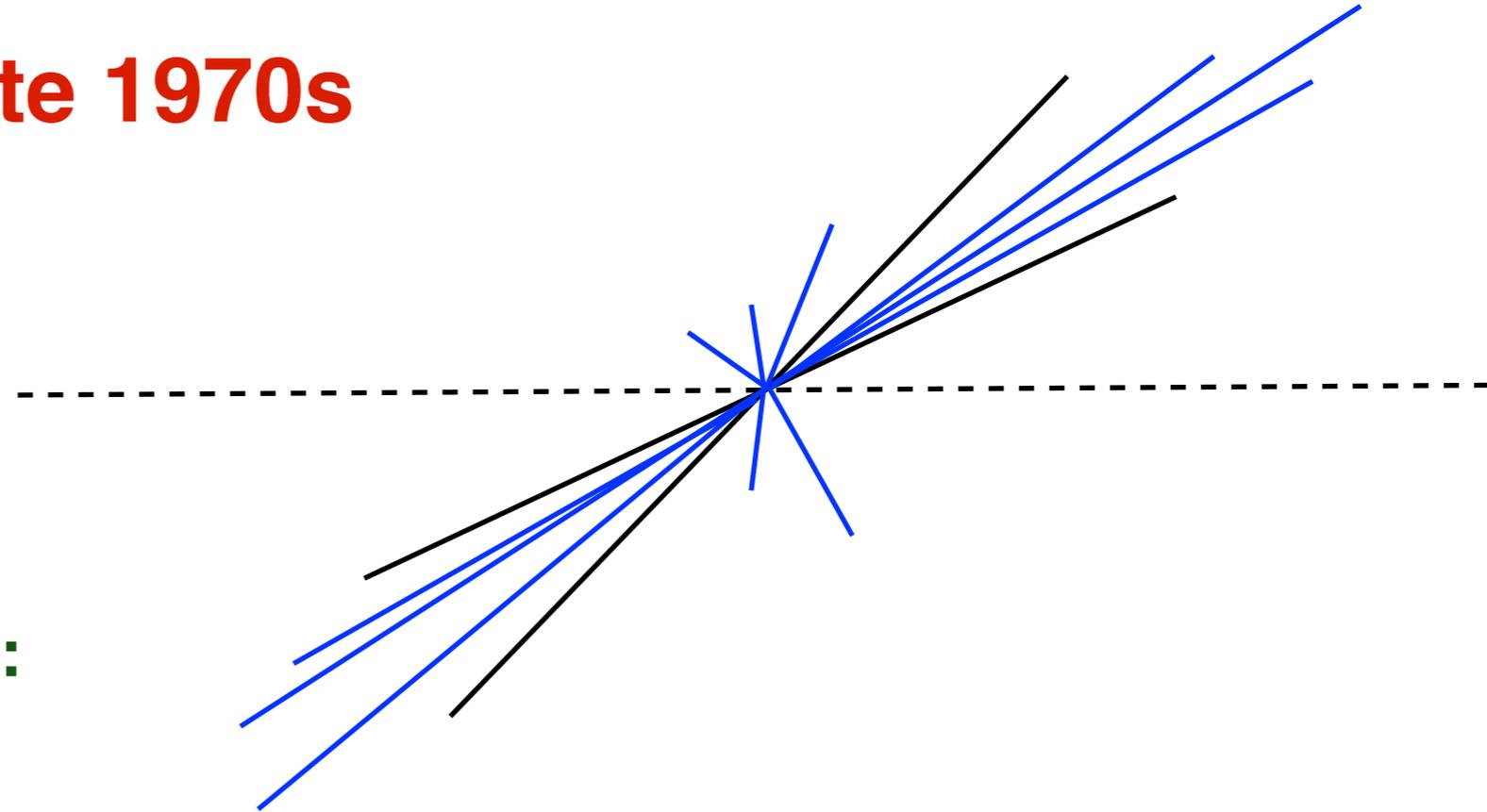
Collimated,
energetic bunches
of particles

ATLAS
EXPERIMENT
Run Number: 166198, Event Number: 100726931
Date: 2010-10-05 03:27:52 CEST



Jets date back to the late 1970s

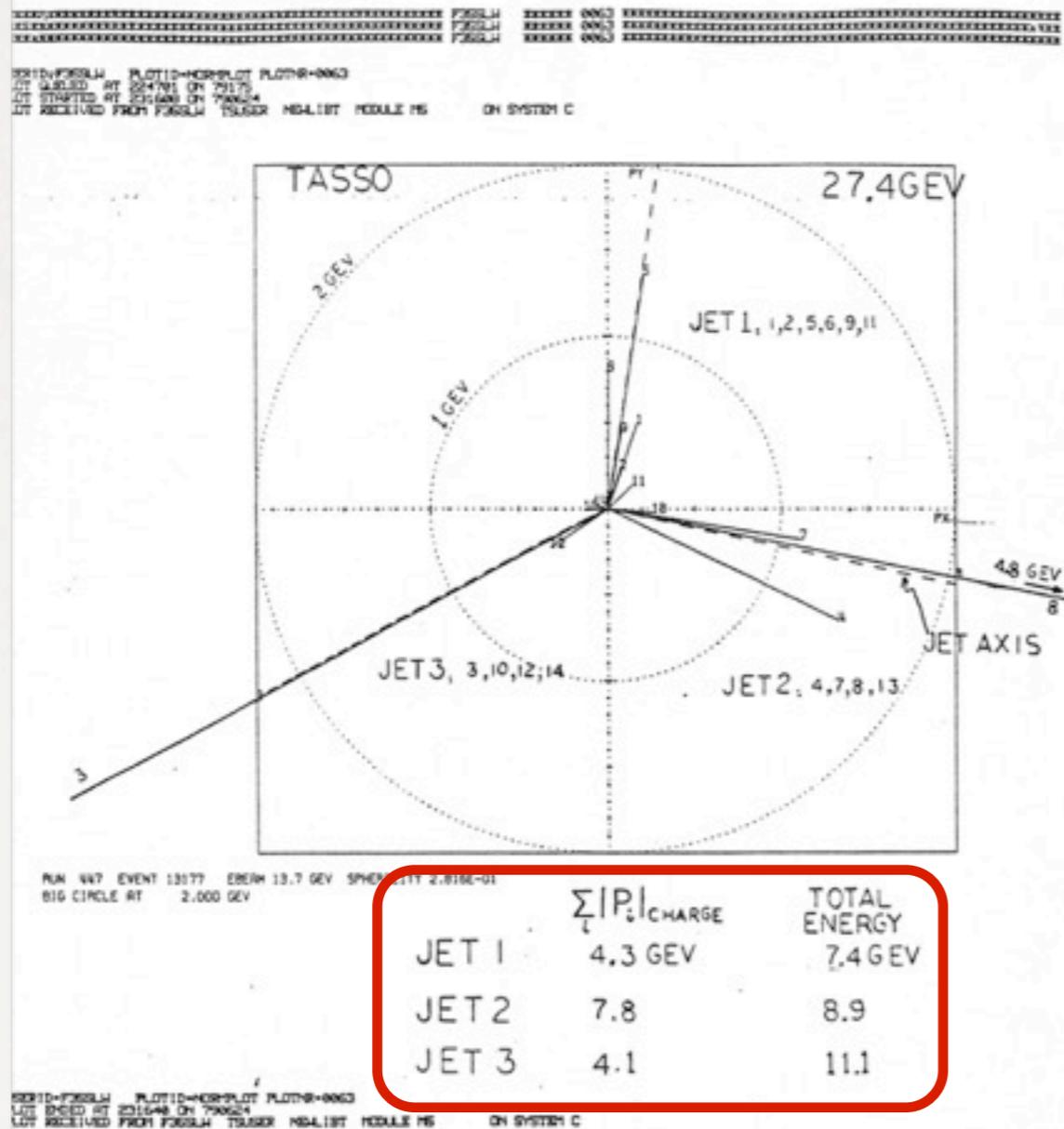
Sterman and Weinberg,
Phys. Rev. Lett. 39, 1436 (1977):



To study jets, we consider the partial cross section $\sigma(E, \theta, \Omega, \epsilon, \delta)$ for e^+e^- hadron production events, in which all but a fraction $\epsilon \ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e^+e^- beam line. We expect this to be measur-

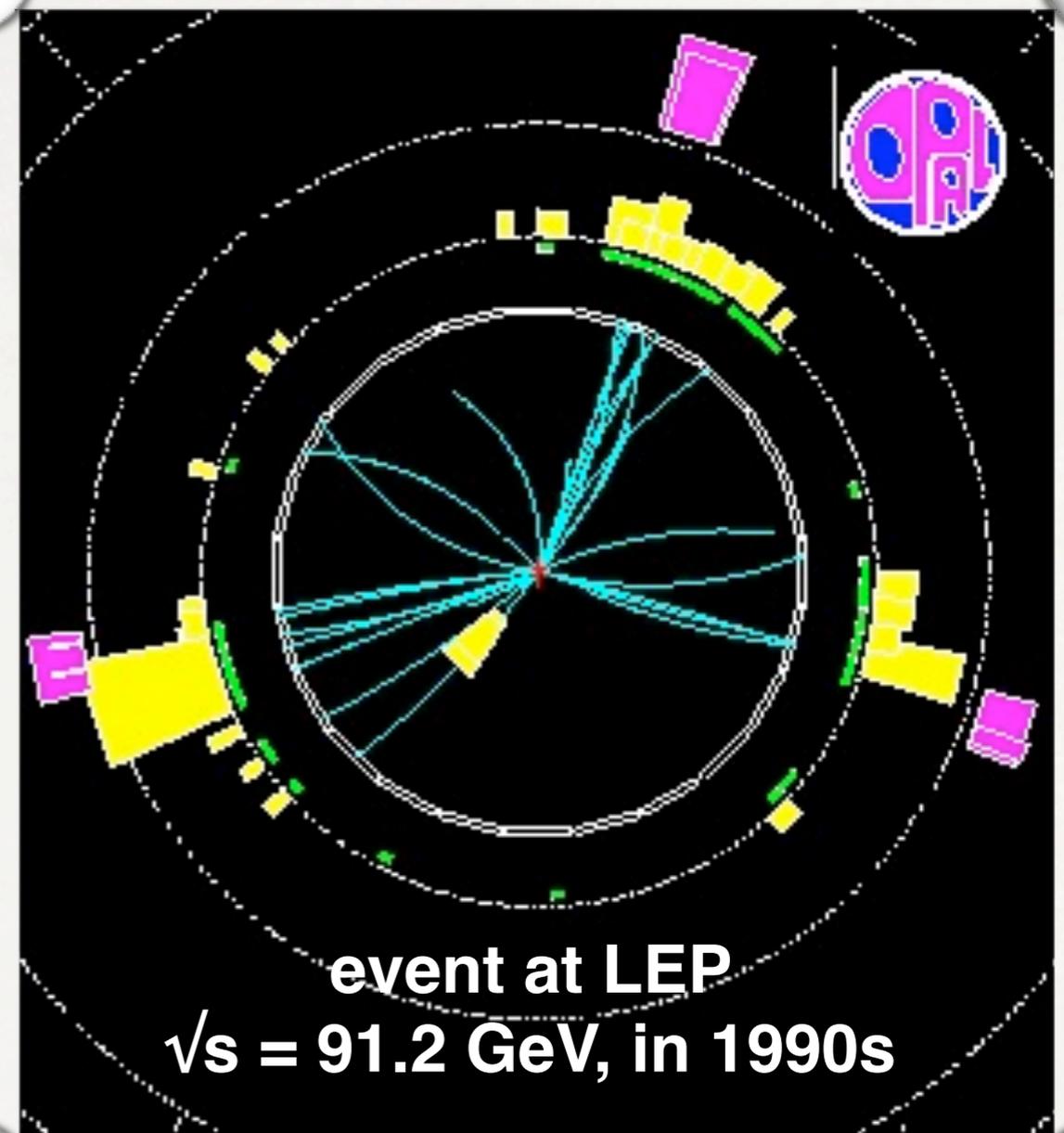
$$\sigma(E, \theta, \Omega, \epsilon, \delta) = (d\sigma/d\Omega)_0 \Omega \left[1 - (g_E^2/3\pi^2) \left\{ 3\ln \delta + 4\ln \delta \ln 2\epsilon + \frac{\pi^3}{3} - \frac{5}{2} \right\} \right]$$

And they've been used and studied at every collider since

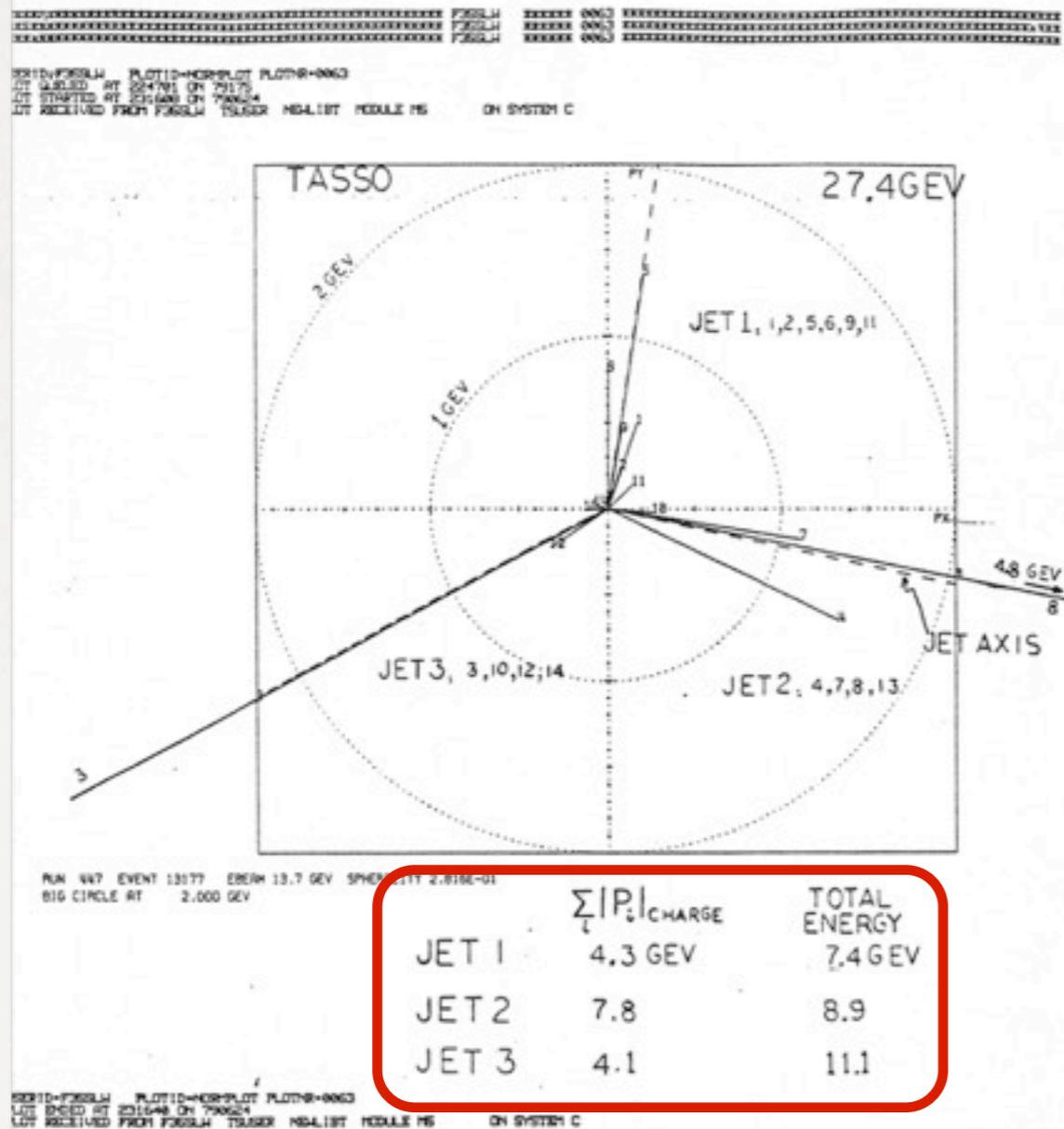


gluon discovery:

event at TASSO
 $\sqrt{s} = 27.4 \text{ GeV}$, in 1979

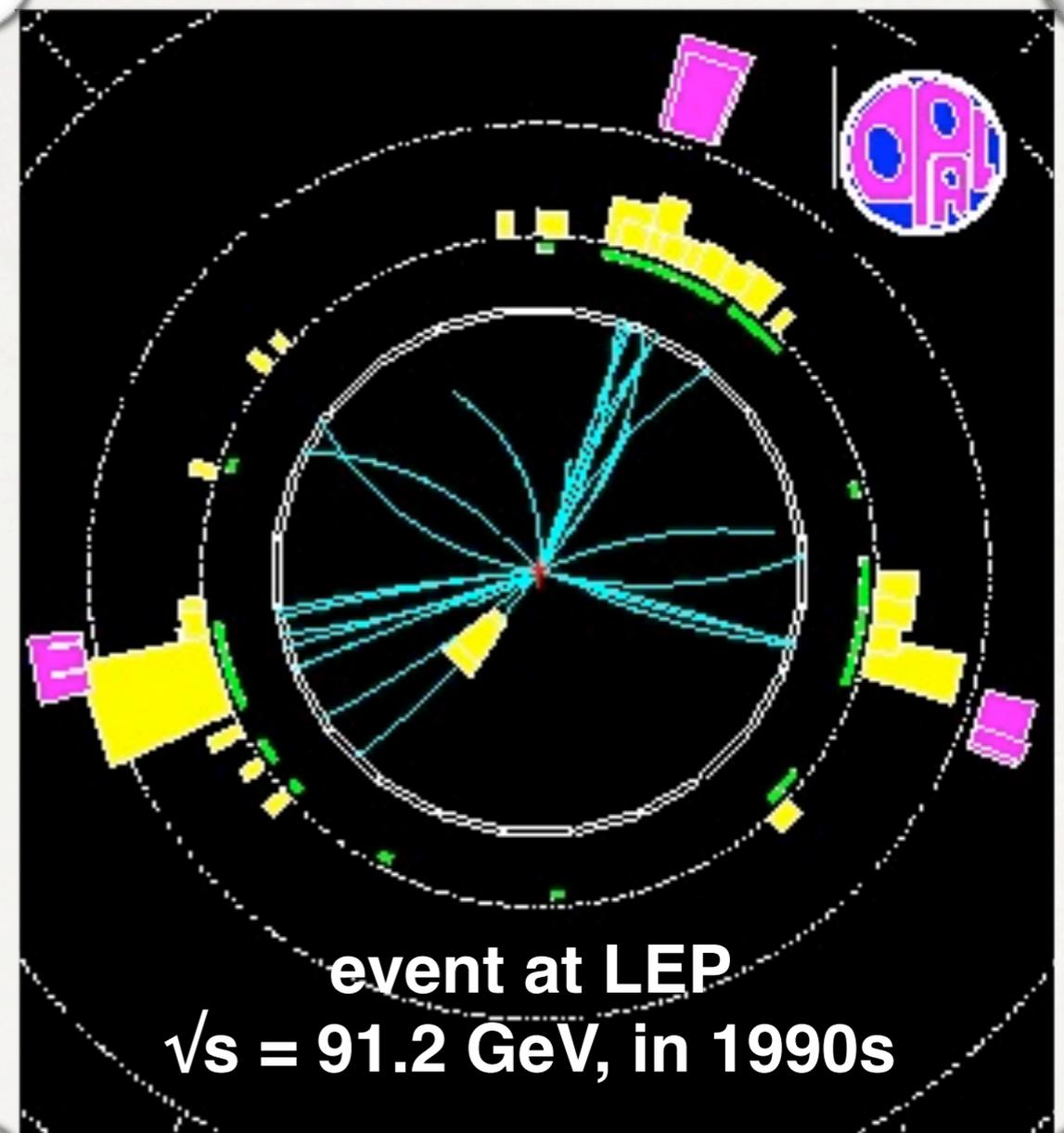


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Find all papers by ATLAS and CMS

578 records found

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1. Measurement of the $W+W^-$ cross section in pp collisions at $\sqrt{s} = 7$ TeV and limits on anomalous WW gamma and WWZ couplings

CMS Collaboration (Serguei Chatrchyan (Yerevan Phys. Inst.) *et al.*). Jun 5, 2013.

CMS-SMP-12-005, CERN-PH-EP-2013-075

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2. Measurement of the hadronic activity in events with a Z and two jets and extraction of the cross section for the electroweak production of a Z with two jets in pp collisions at $\sqrt{s} = 7$ TeV

CMS Collaboration (Serguei Chatrchyan (Yerevan Phys. Inst.) *et al.*). May 31, 2013.

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[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

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Pull out those that refer to one widely used jet-alg
347 records found

reportnumber:CERN-PH-EP (collaboration:atlas or collaboration:CMS) (refersto:recid:779080) Brief format Search Easy Search Advanced Search
find j "Phys.Rev.Lett.,105" :: more

Sort by: Display results:
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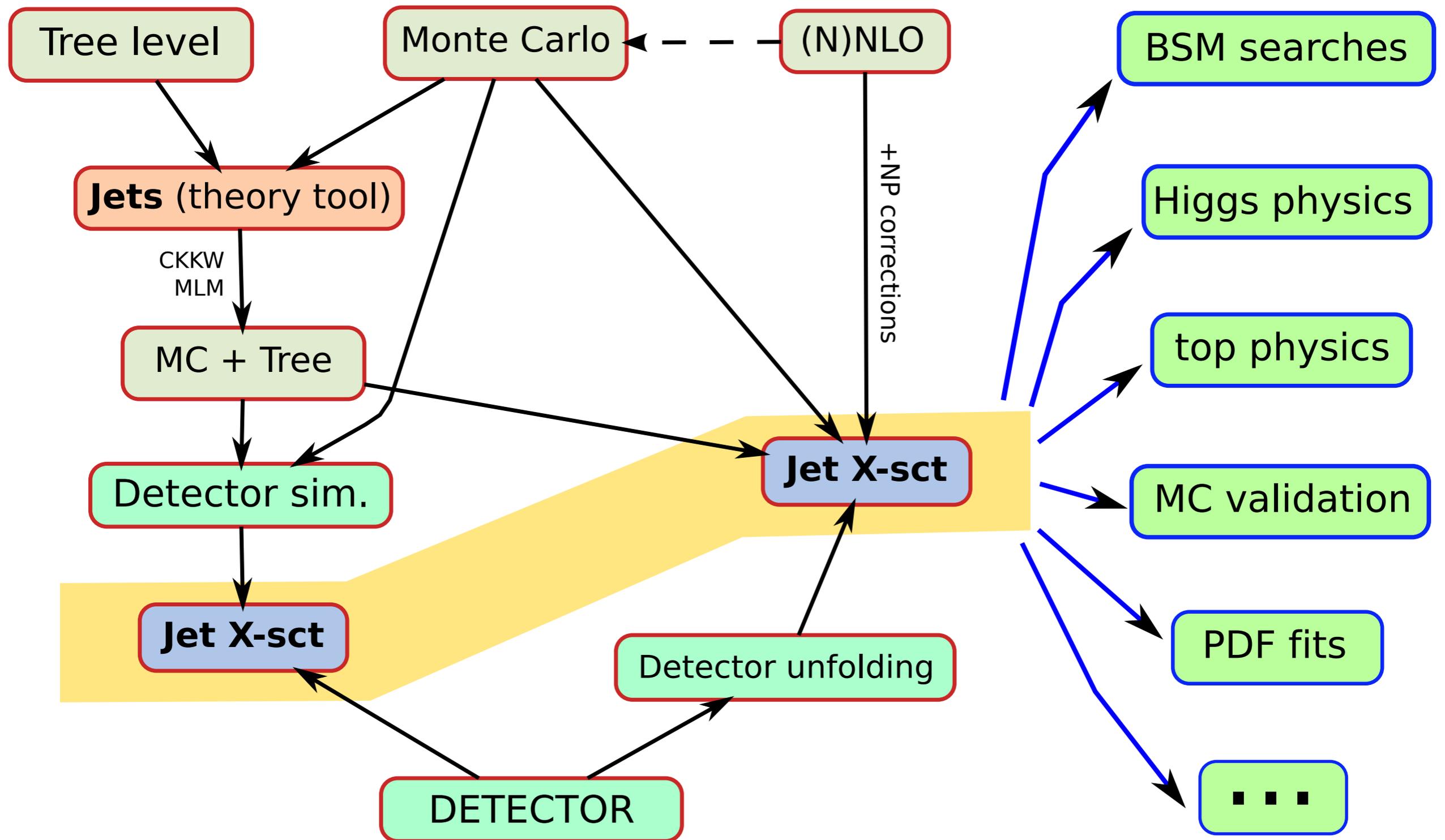
60% of papers use jets!

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HEP 272 records found 1 - 25 ►► jump to record: 1 Search took 0.04 seconds.

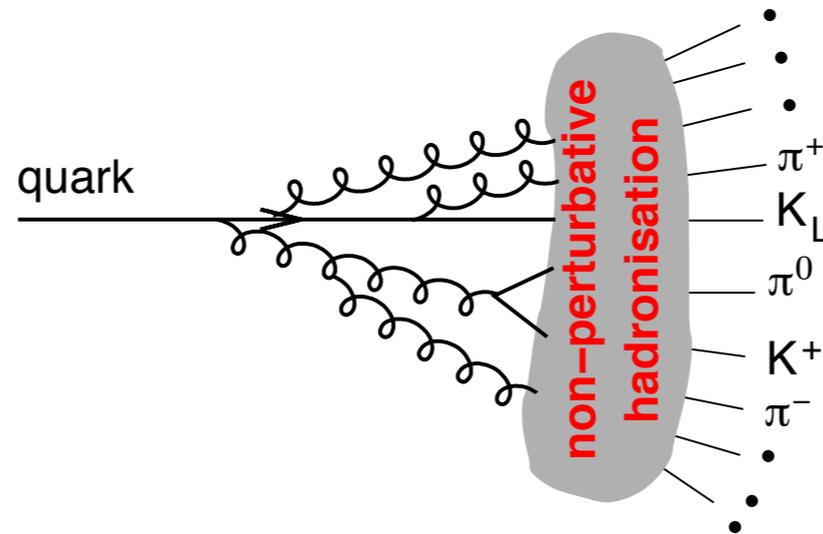
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Jet usage at the LHC

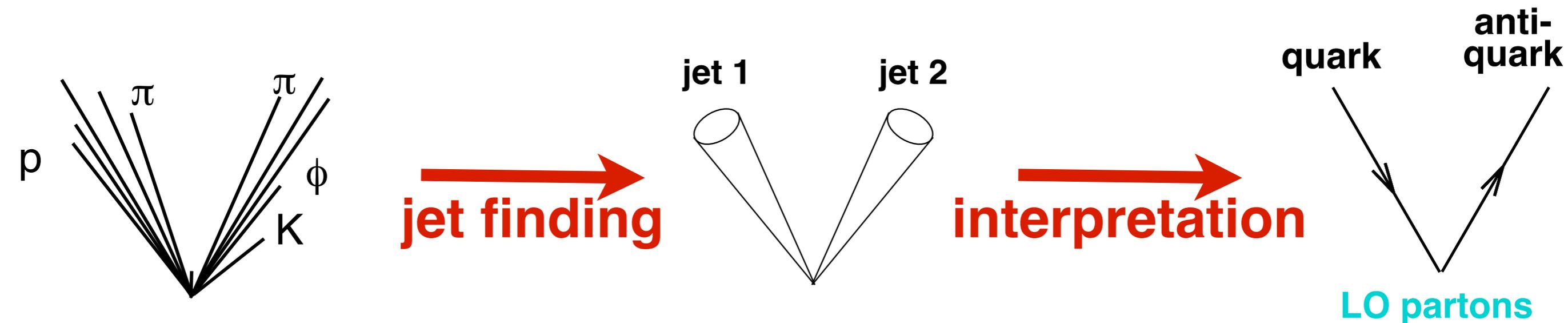


Two key aspects to discussing jets

How jets come to have the structure they do

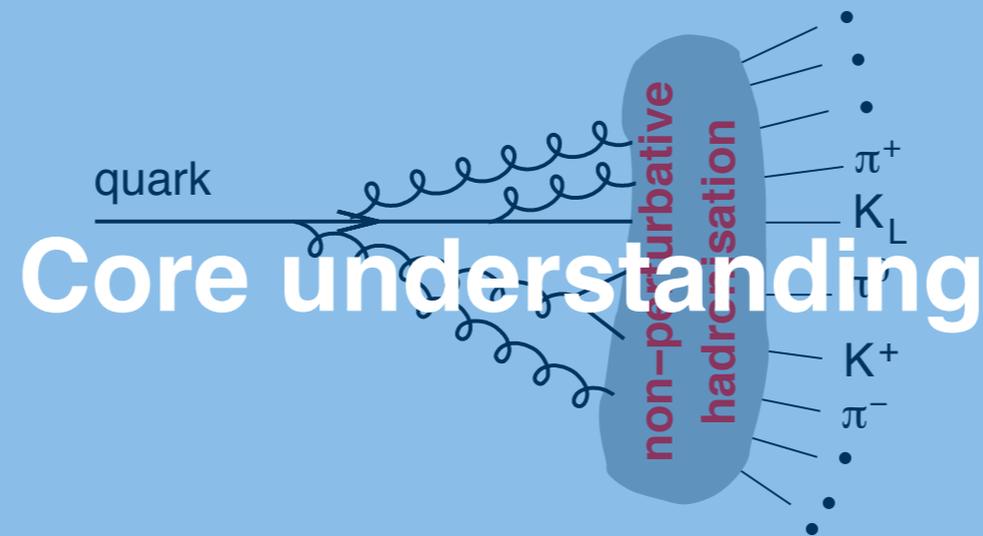


How we “reconstruct” jets

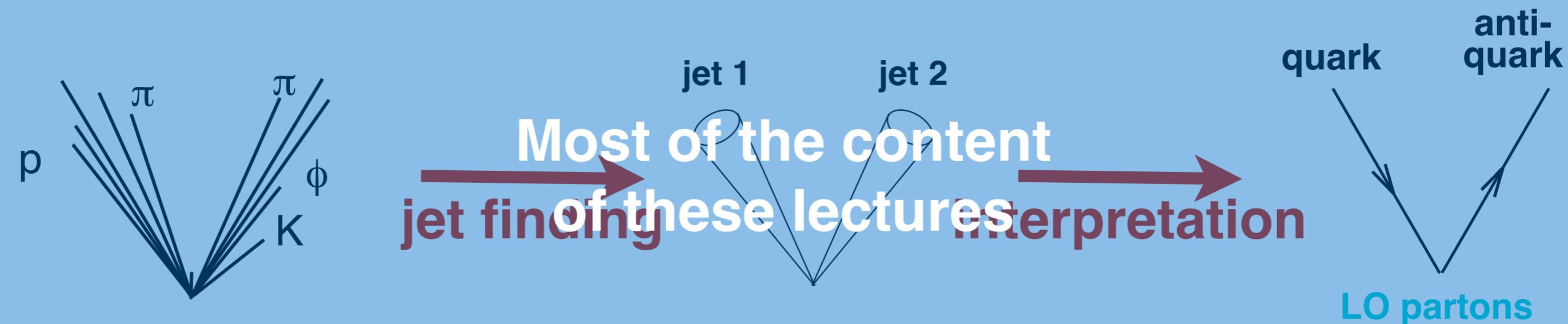


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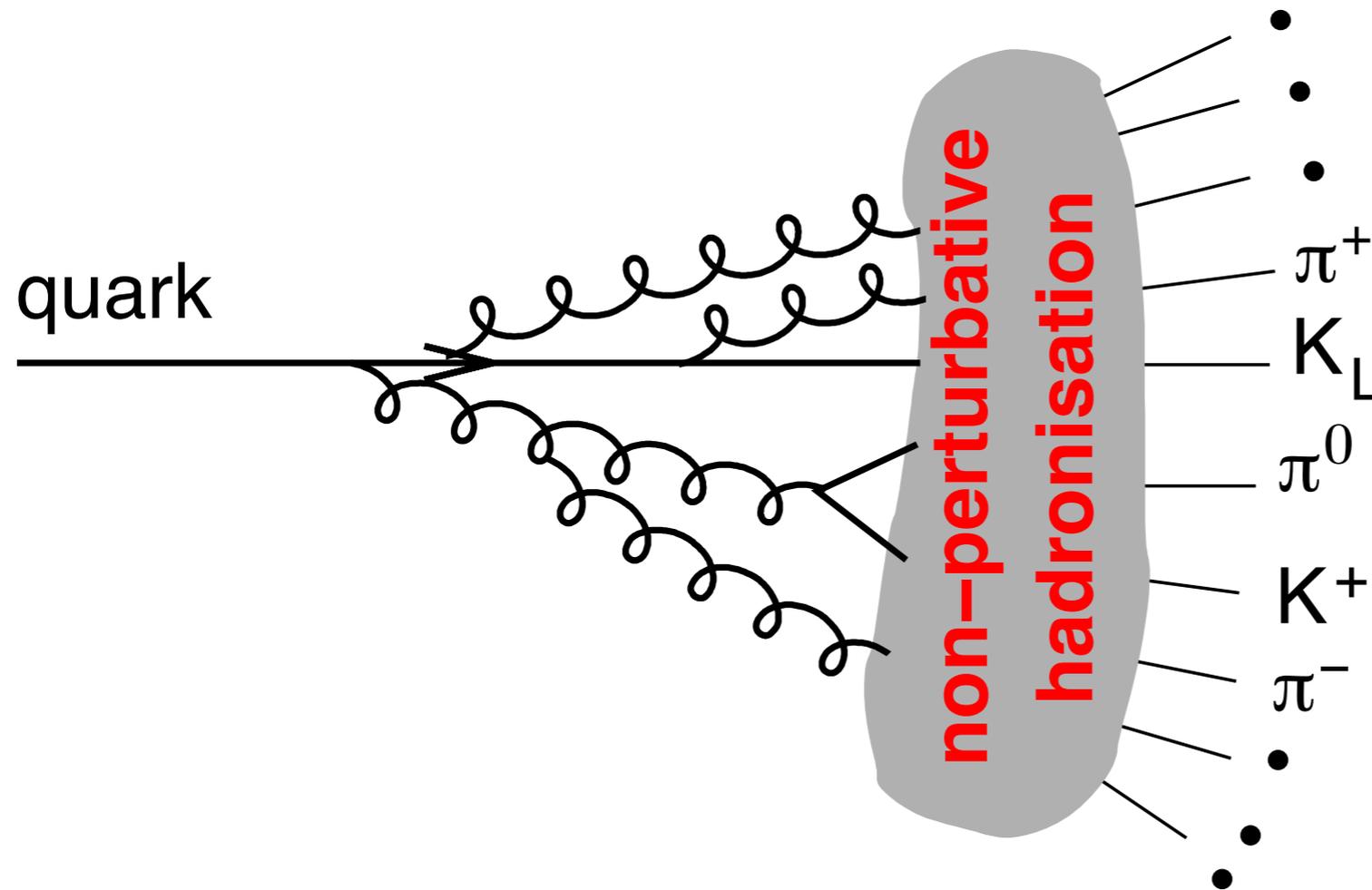
How jets come to have the structure they do



How we “reconstruct” jets



Why do we see jets?



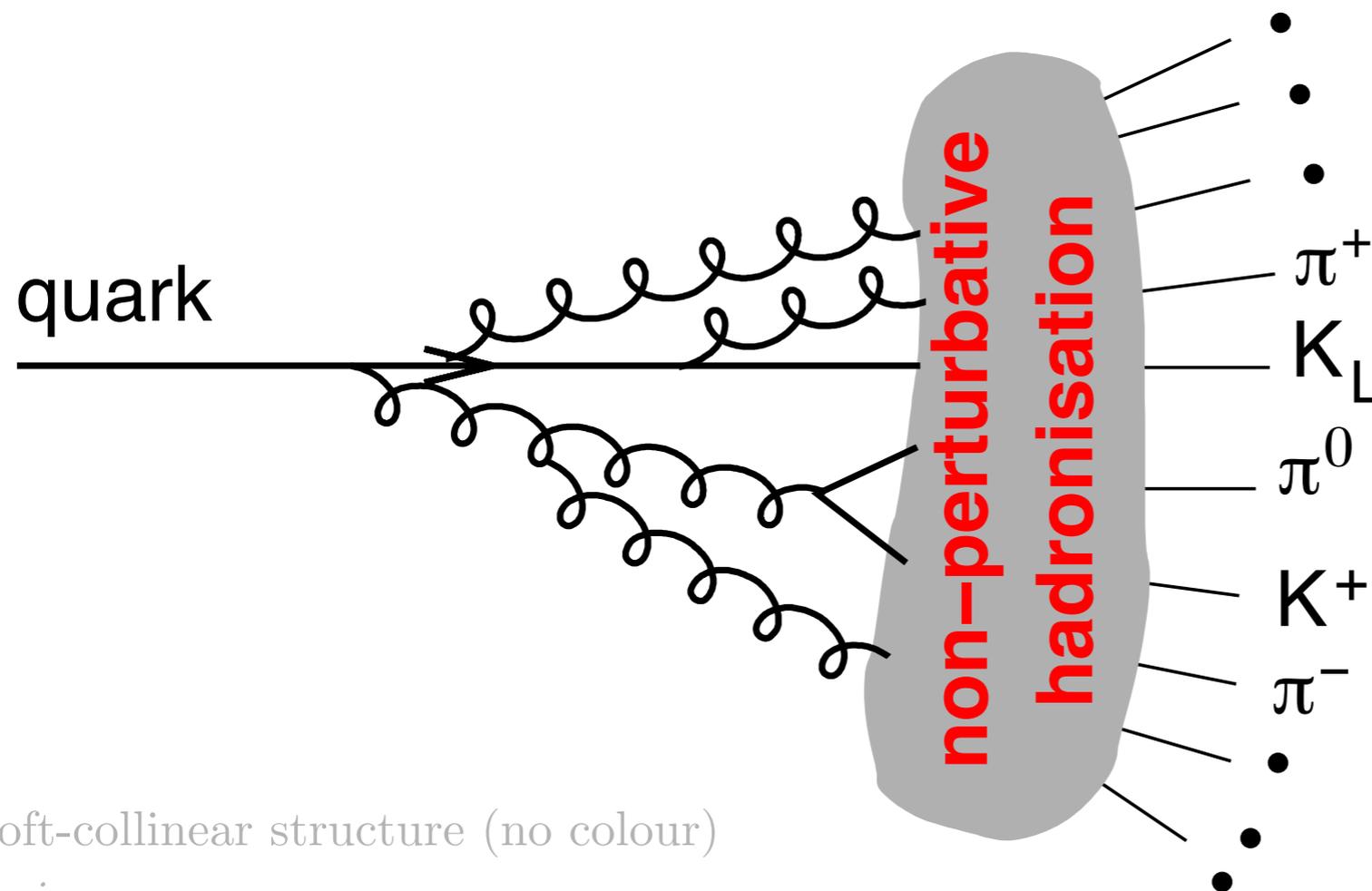
Gluon emission

$$\int \alpha_s \frac{dE}{E} \frac{d\theta}{\theta} \gg 1$$

Non-perturbative physics

$$\alpha_s \sim 1$$

Why do we see jets?



Gluon emission

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Non-perturbative physics

$$\alpha_s \sim 1$$

soft-collinear structure (no colour)

$$\frac{i}{\not{p} + \not{k}} i g_s \not{\epsilon} v(p)$$

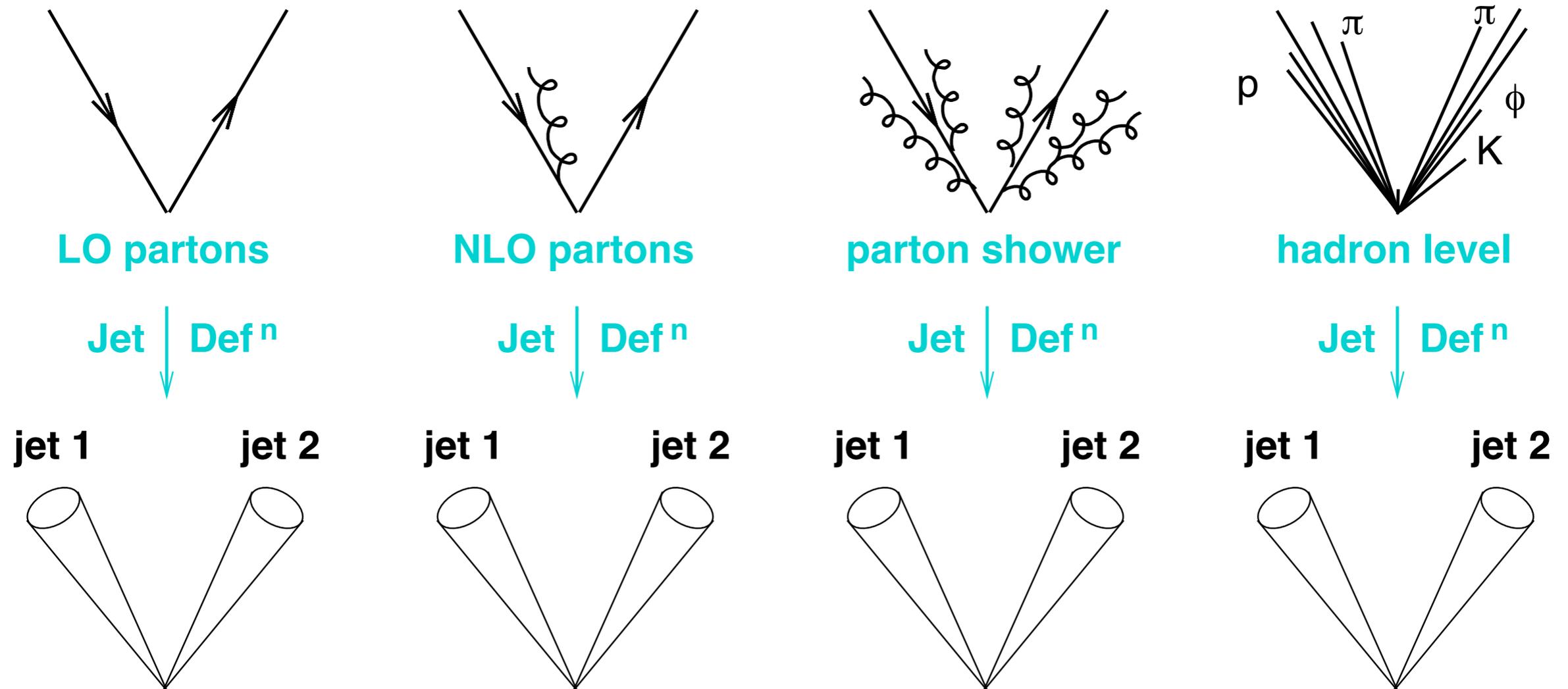
drop \not{k} in numerator, use $\not{p}\not{k} + \not{k}\not{p} = 2p.k$, $\not{p}v(p) = 0$

get $g_s \frac{p.\epsilon}{p.k} v(p)$ take $p = (1, 0, 0, 1)$, $k = E(\cos \theta, \sin \theta, 0, 1)$, $\epsilon = (\sin \theta, -\cos \theta, 0, 0)$

square the amplitude, put in phasespace: $\frac{d^3 k}{2E(2\pi)^3} \rightarrow \frac{E dE d\cos \theta d\phi}{16\pi^3}$

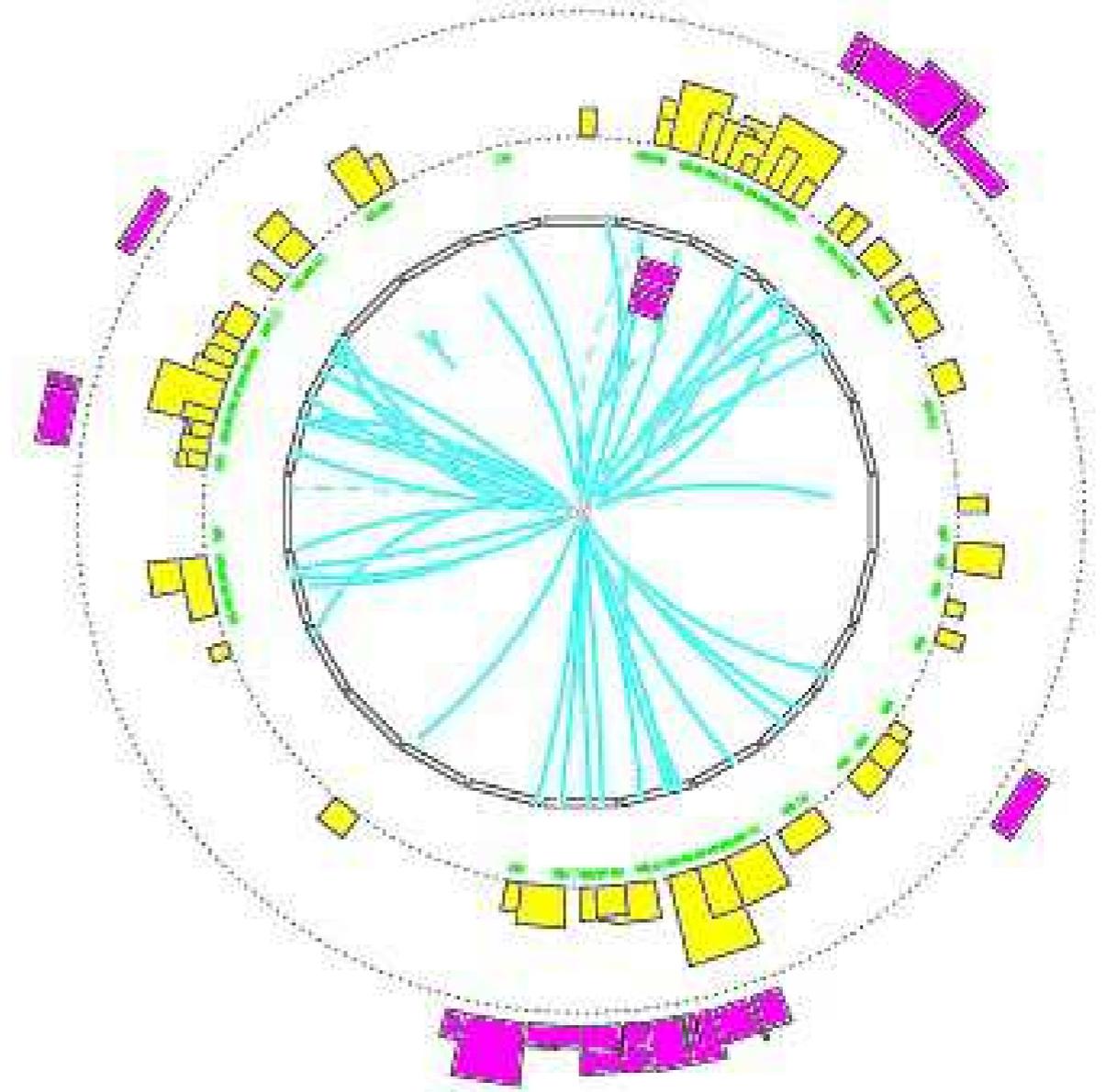
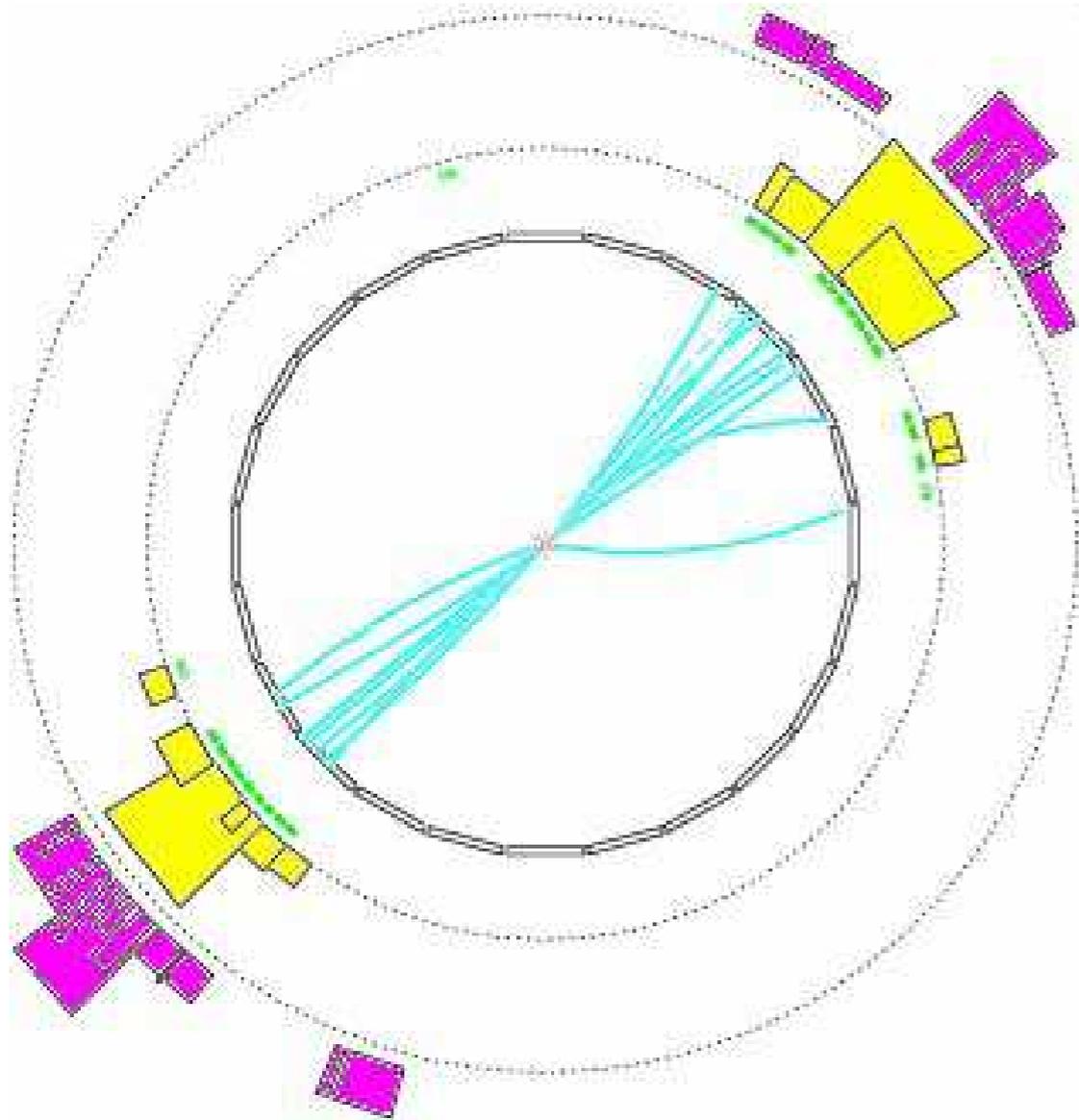
Reconstructing jets

Jet finding as a form of projection

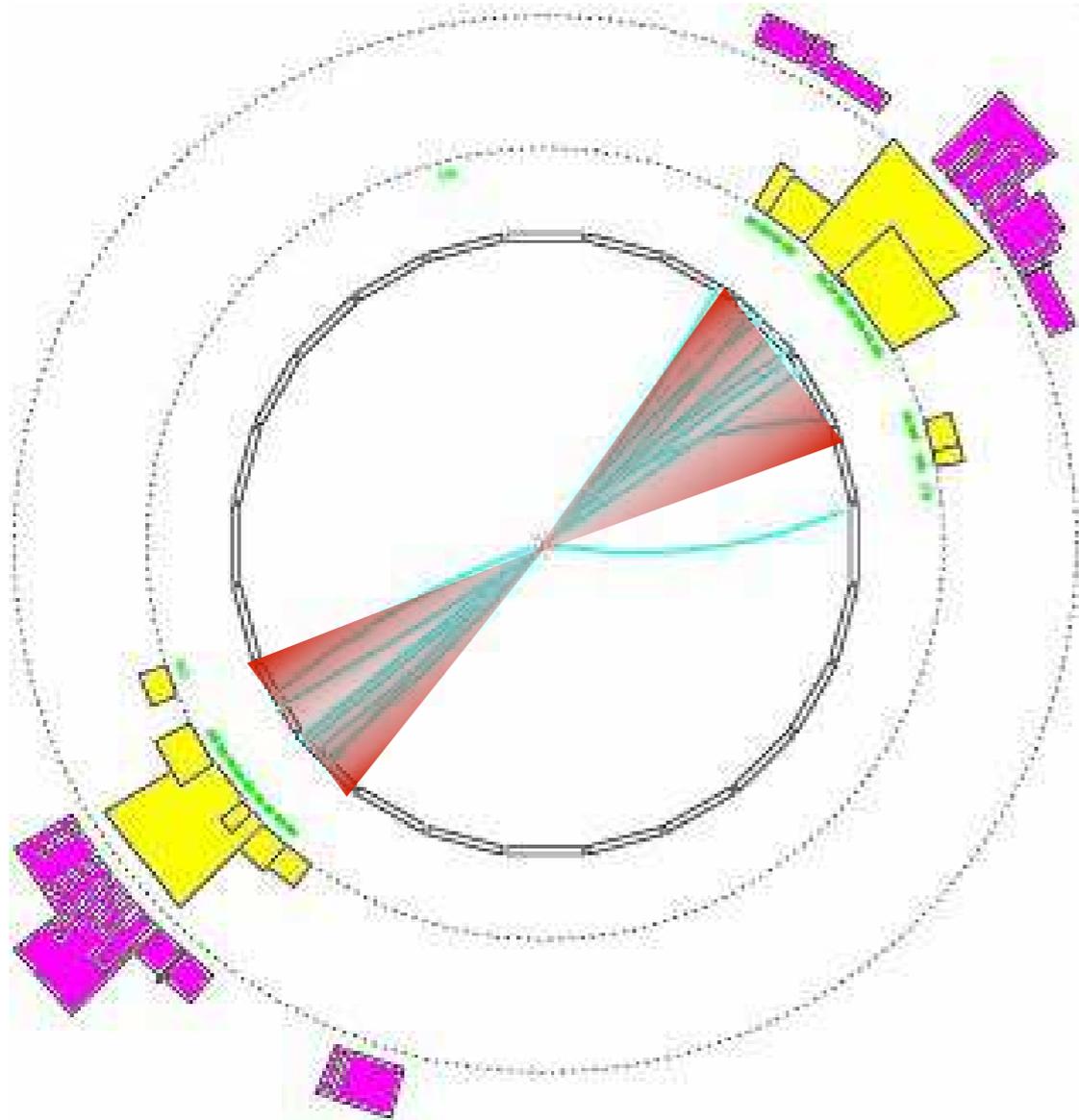


Projection to jets should be resilient to QCD effects

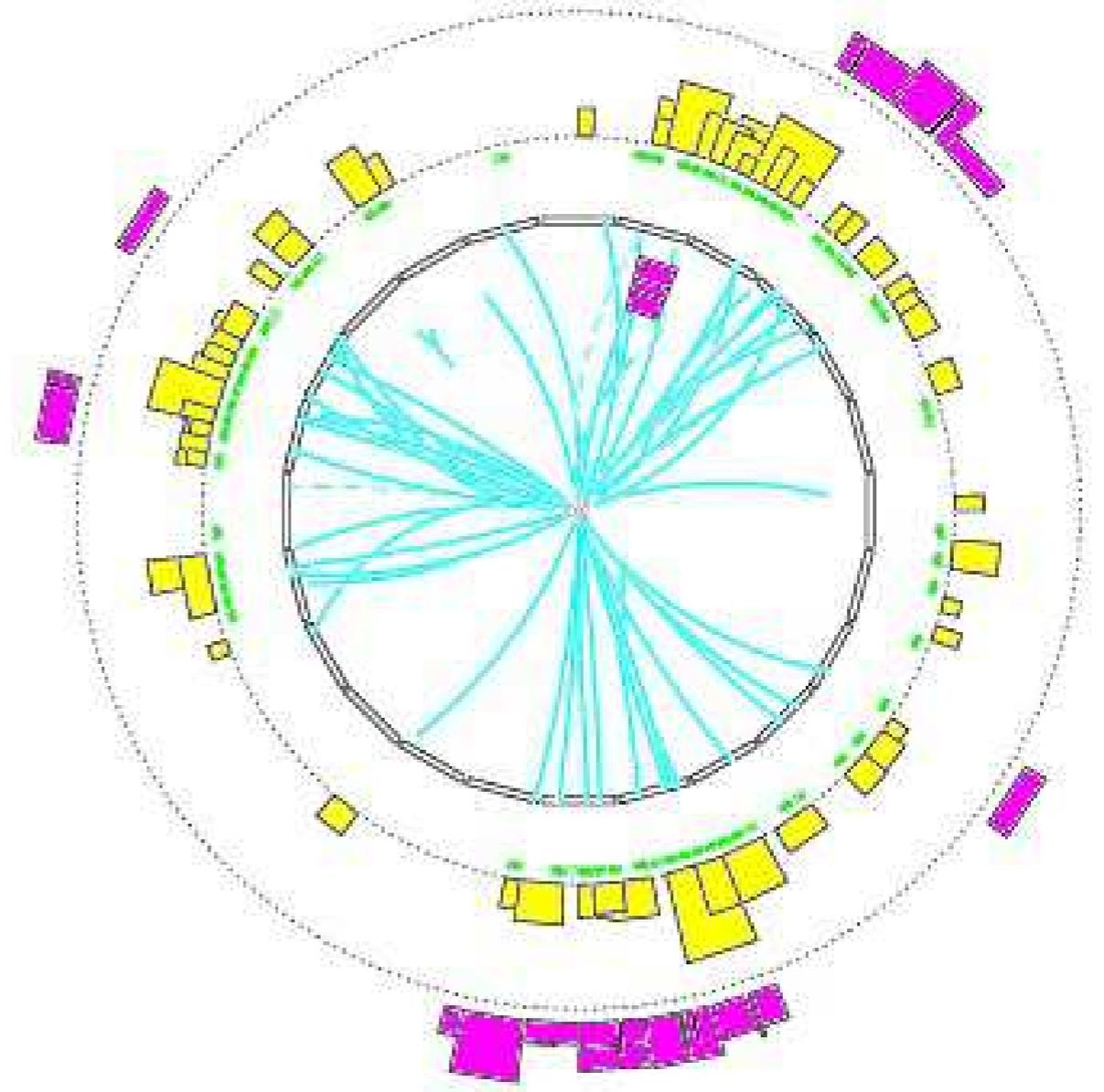
Reconstructing jets is an ambiguous task



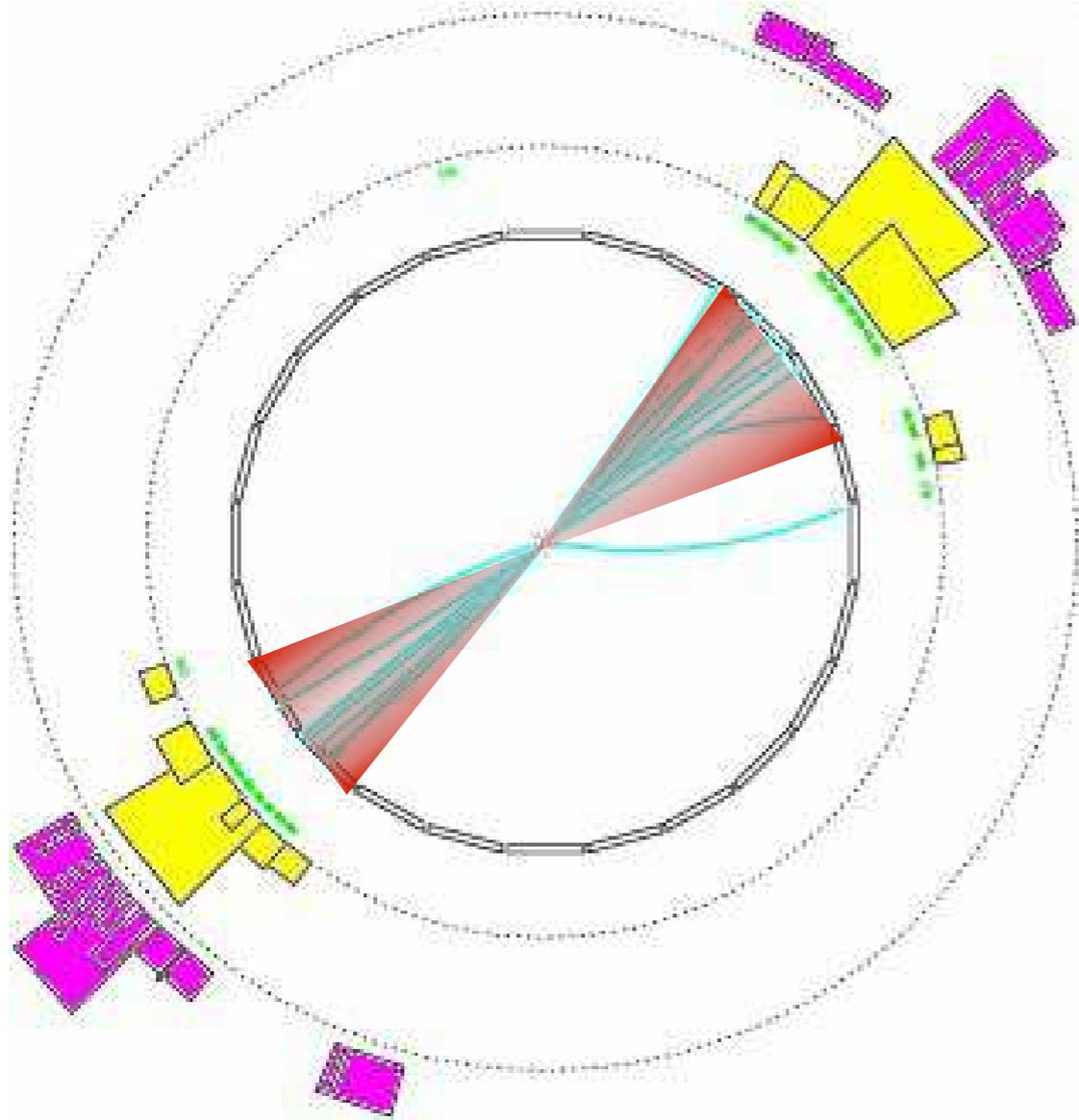
Reconstructing jets is an ambiguous task



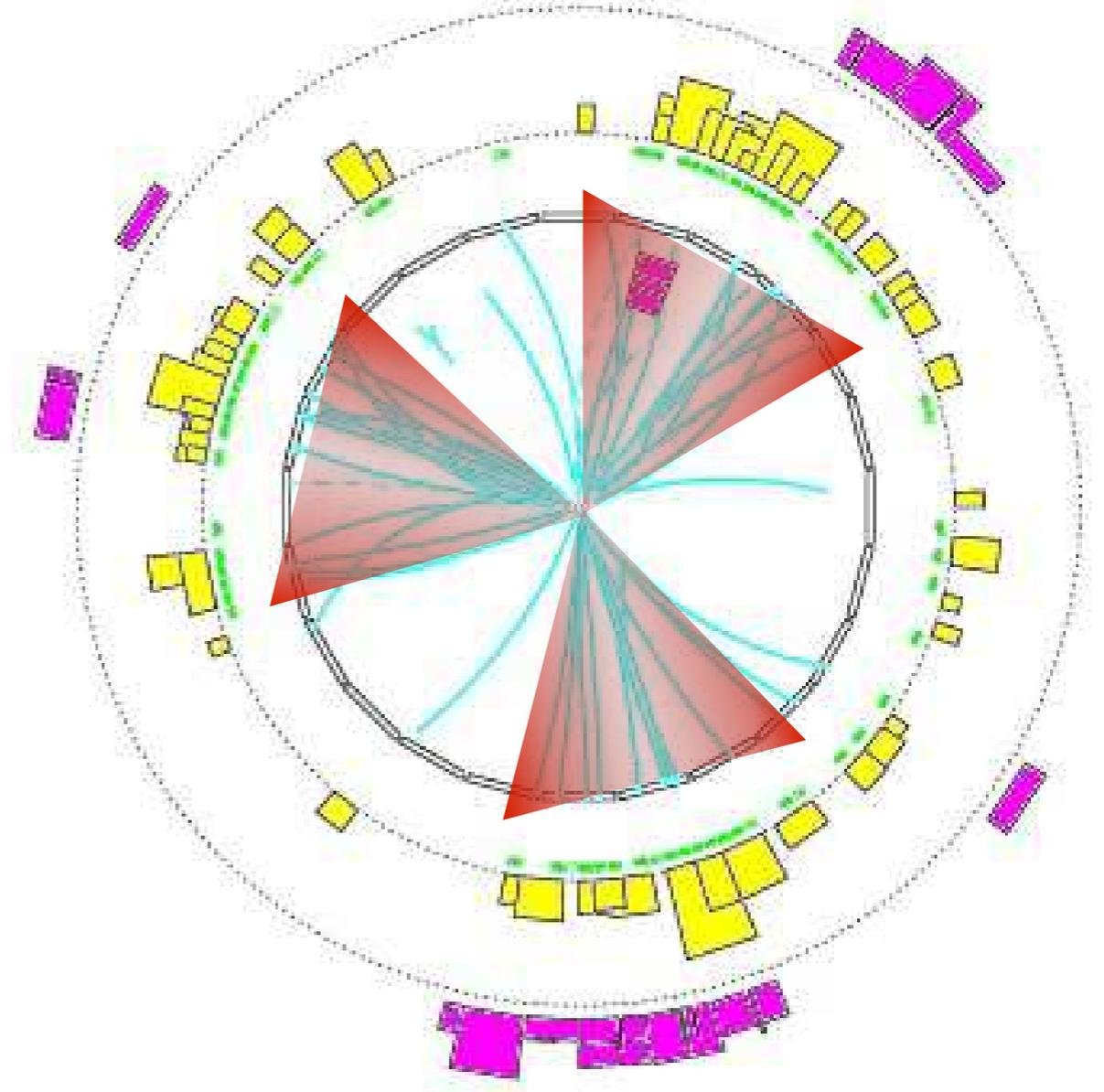
2 clear jets



Reconstructing jets is an ambiguous task

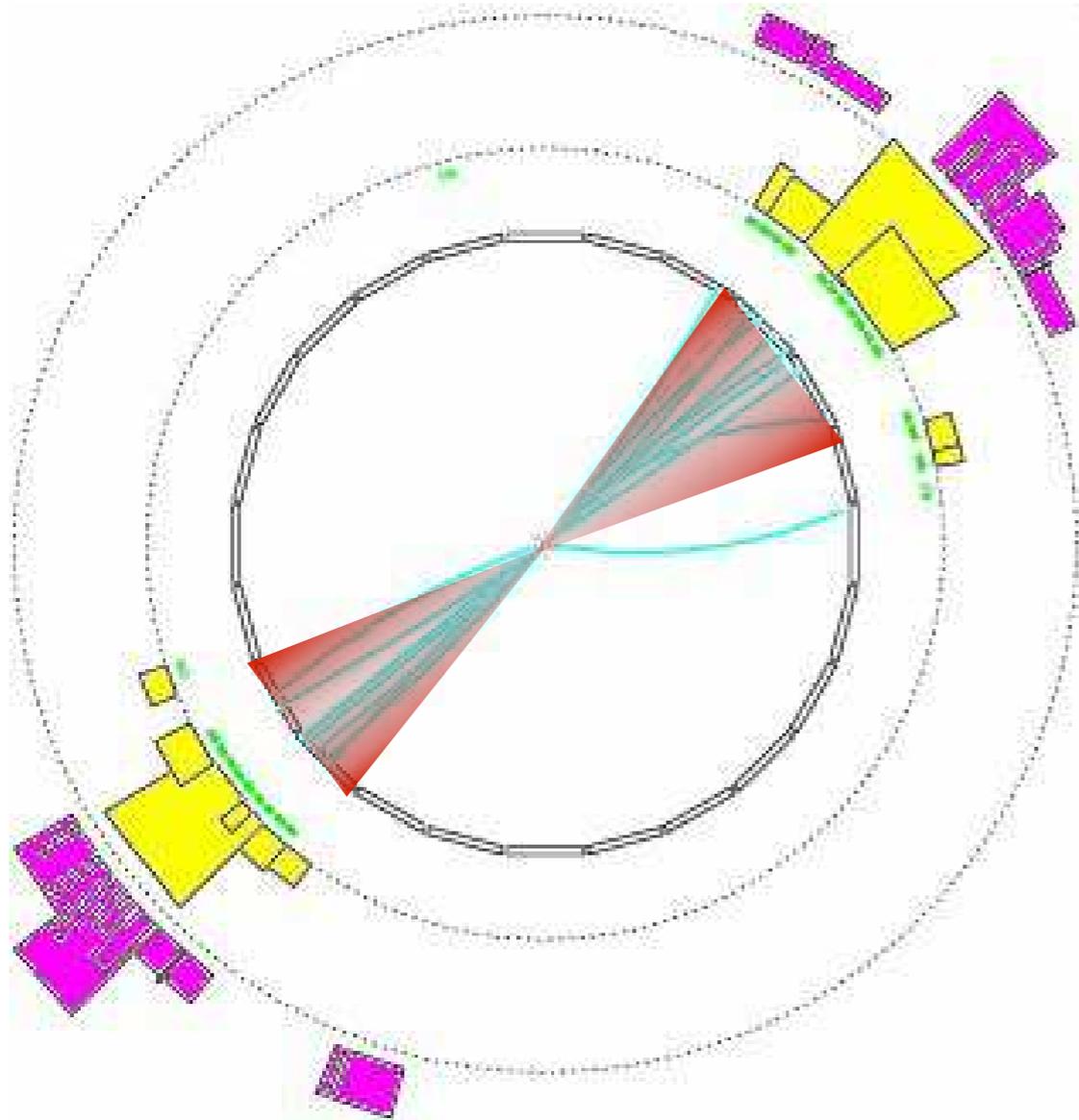


2 clear jets

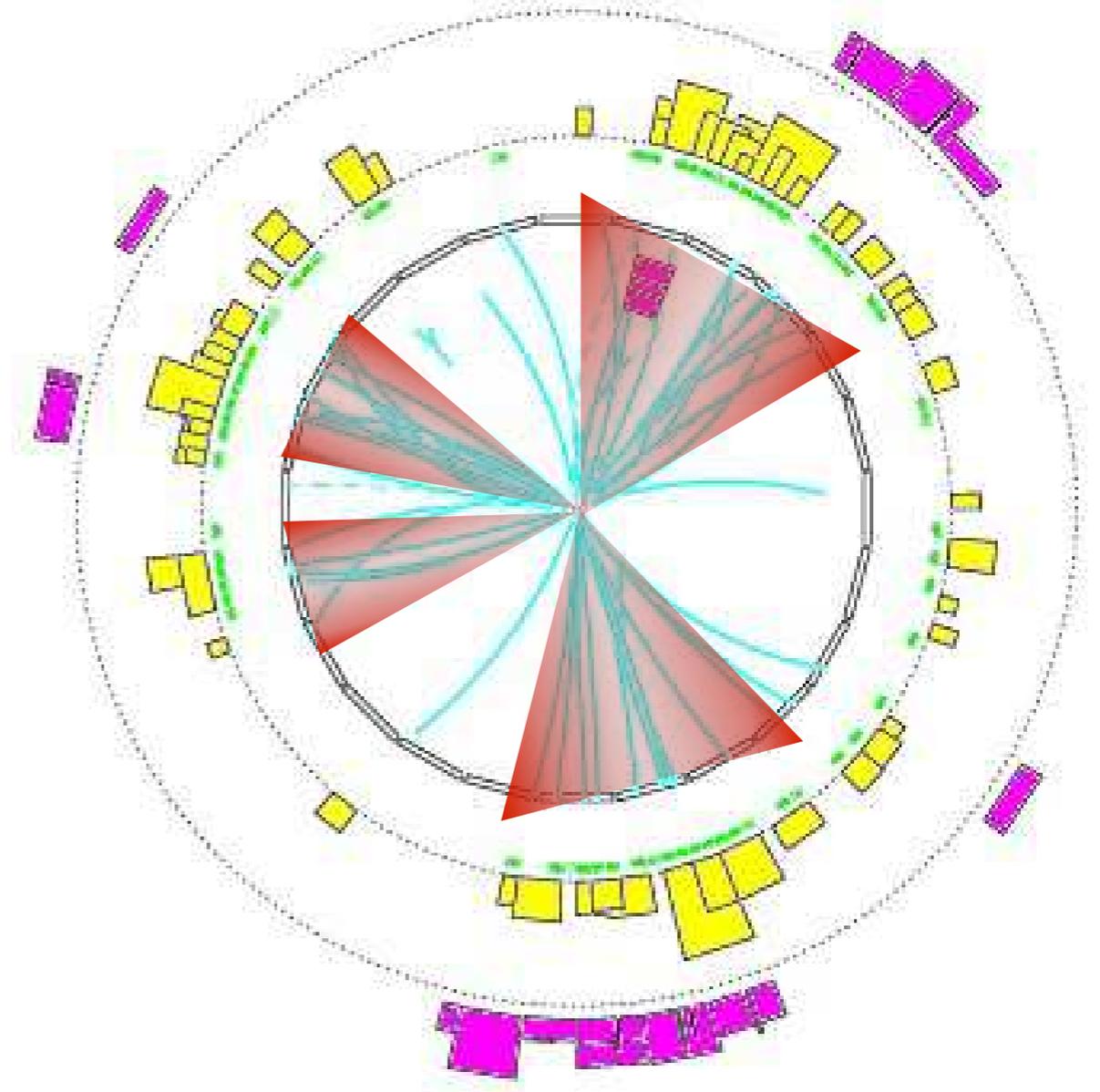


3 jets?

Reconstructing jets is an ambiguous task

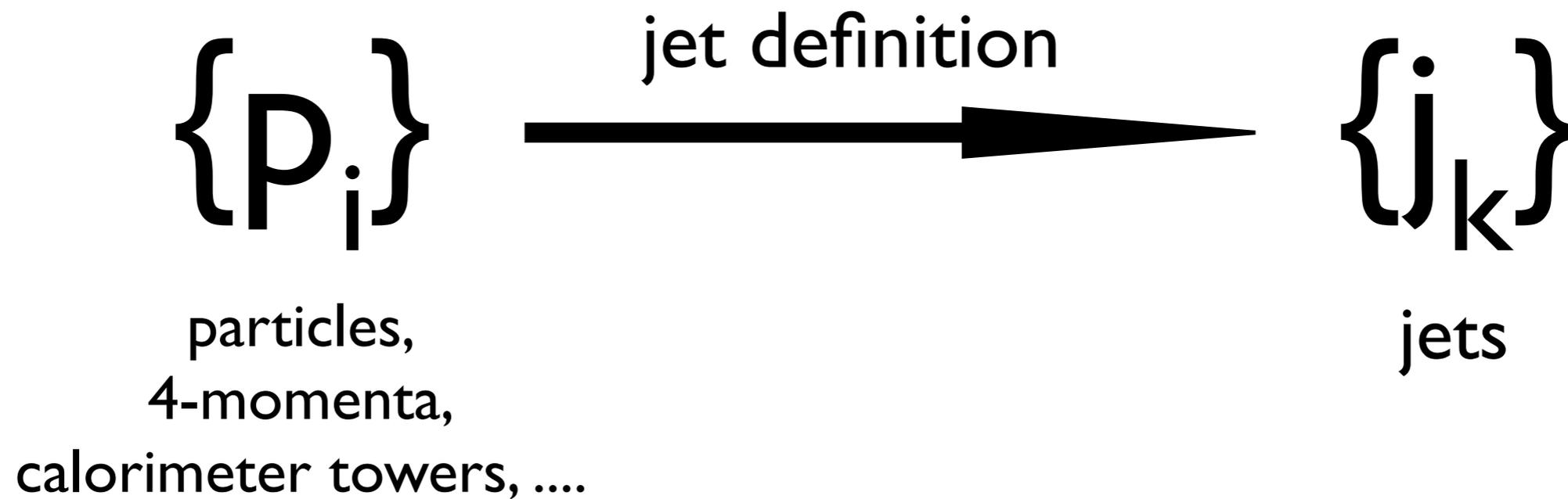


2 clear jets



3 jets?
or 4 jets?

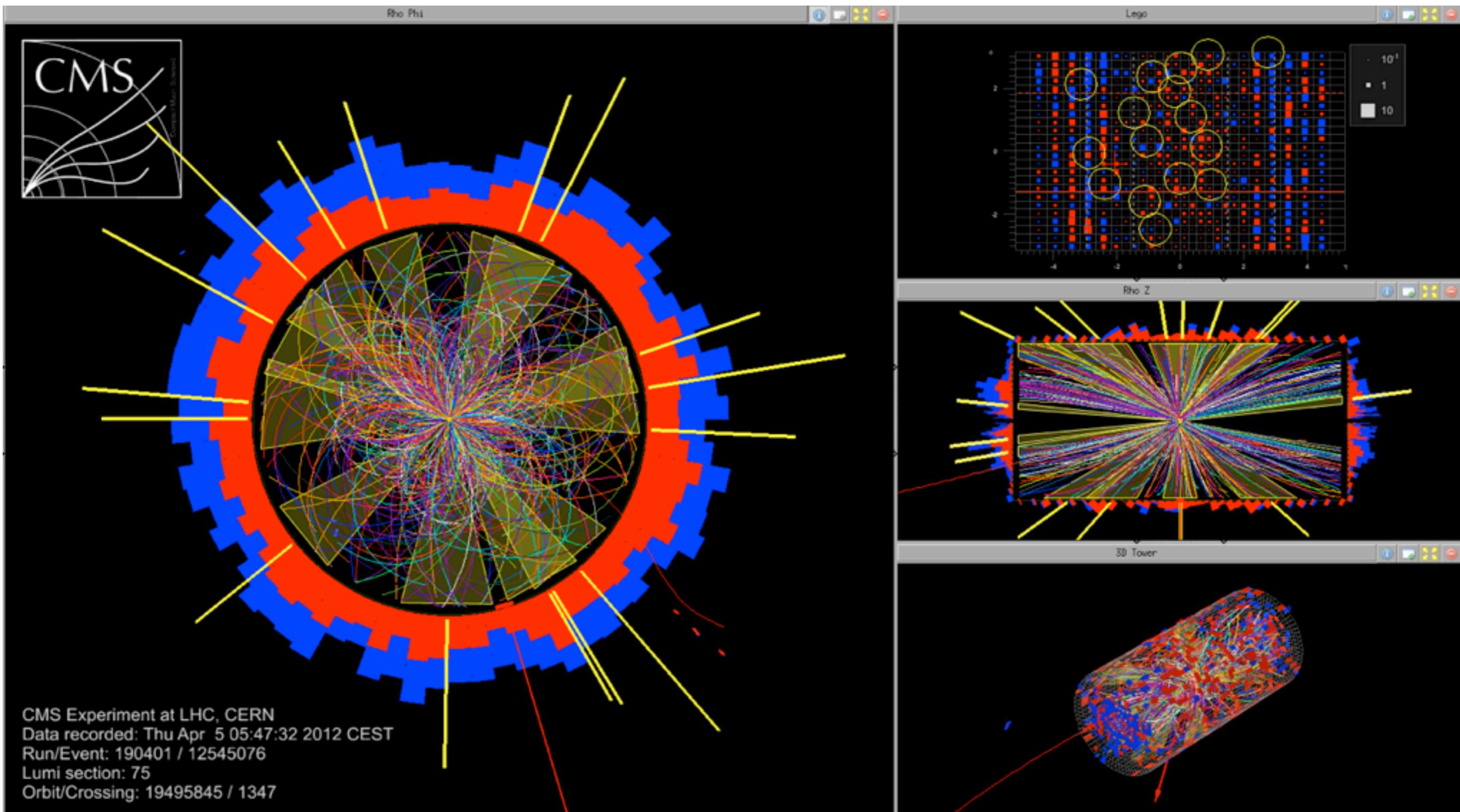
Make a choice: specify a jet definition



- Which particles do you put together into a same jet?
- How do you recombine their momenta (4-momentum sum is the obvious choice, right?)

“Jet [definitions] are legal contracts between theorists and experimentalists”
-- MJ Tannenbaum

They're also a way of organising the information in an event
1000's of particles per events, up to 20.000,000 events per second



Cluster analysis

From Wikipedia, the free encyclopedia

Cluster analysis or **clustering** is the task of grouping a set of objects in such a way that objects in the same group (called **cluster**) are more similar (in some sense or another) to each other than to those in other groups (clusters). It is a main task of exploratory [data mining](#), and a common technique for [statistical data analysis](#) used in many fields, including [machine learning](#), [pattern recognition](#), [image analysis](#), [information retrieval](#), and [bioinformatics](#).

Contents [\[hide\]](#)

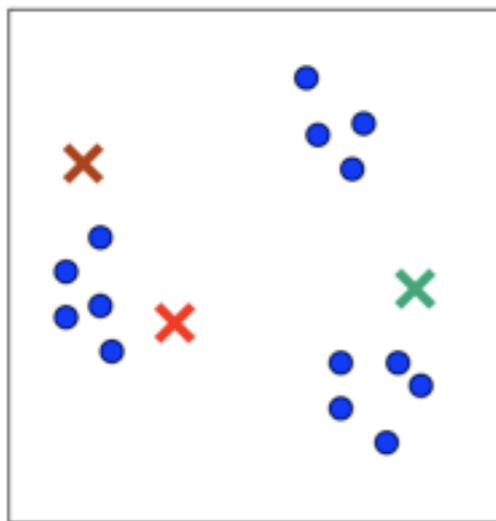
- 1 Clusters and clusterings
- 2 Clustering algorithms
 - 2.1 Connectivity based clustering (hierarchical clustering)
 - 2.2 Centroid-based clustering
 - 2.3 Distribution-based clustering
 - 2.4 Density-based clustering
 - 2.5 Newer developments
- 3 Evaluation of clustering results
 - 3.1 Internal evaluation
 - 3.2 External evaluation
- 4 Applications
- 5 See also
 - 5.1 Related topics
 - 5.2 Related methods
- 6 References

There is no objectively "correct" clustering algorithm, but [...] "clustering is in the eye of the beholder."[\[1\]](#)
The most appropriate clustering algorithm for a particular problem often needs to be chosen experimentally, unless there is a mathematical reason to prefer one cluster model over another.

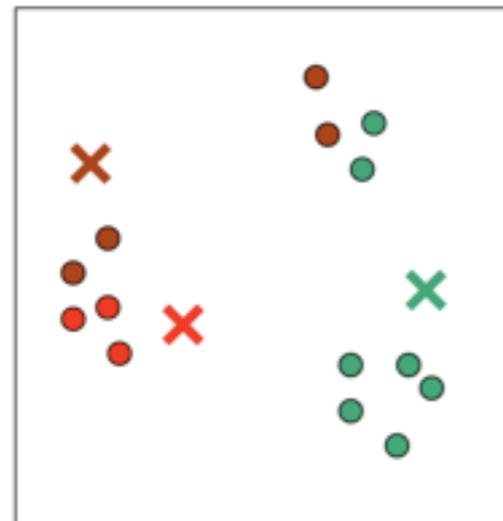
Partitioning / centroid-based clustering [cone algorithms]

Example of a **partitional** algorithm

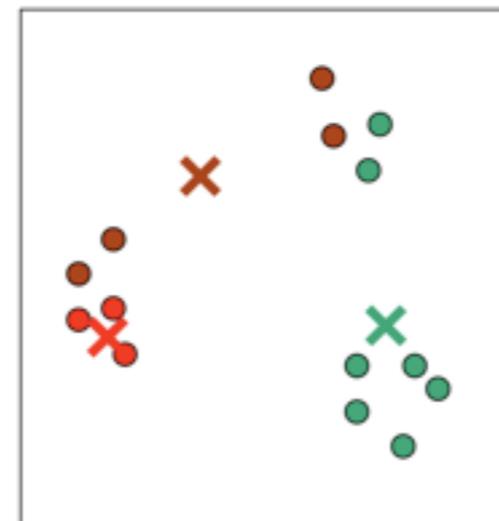
- 1) Choose K centroids at random
- 2) Assign objects to closest centroid, forming K clusters
- 3) Calculate centroid (mean of distances) of each cluster, update centroids
- 4) Check if an object in a cluster is closer to another centroid.
Reallocate in case.
- 5) Repeat from step 3 until no object changes cluster anymore.



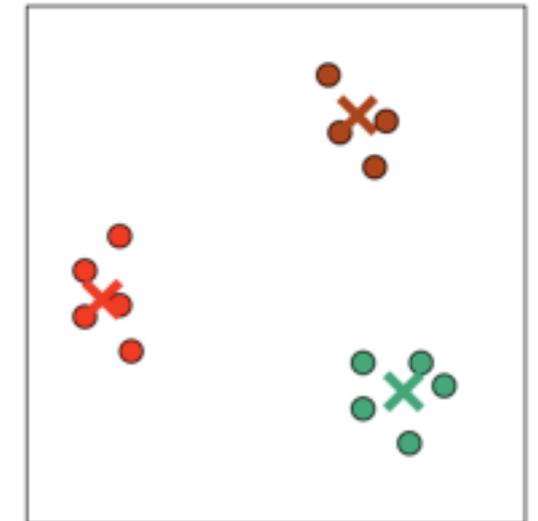
Step 1
(random centroids)



Step 2
(allocate objects)



Step 3
(move centroids)

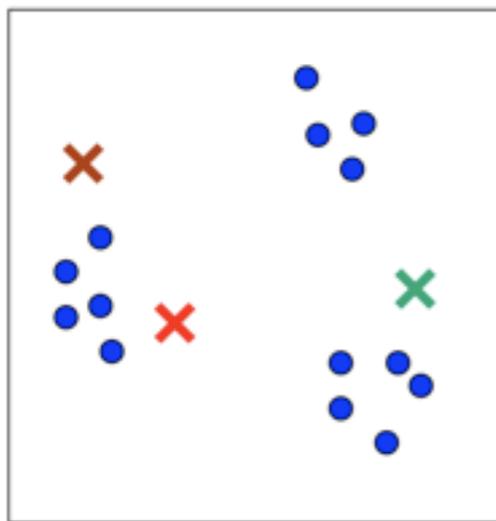


Step 5
(end of iteration)

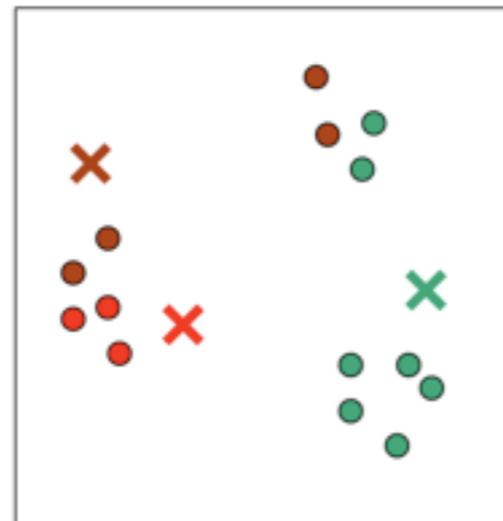
[Figures by E. Garrett-Mayer]

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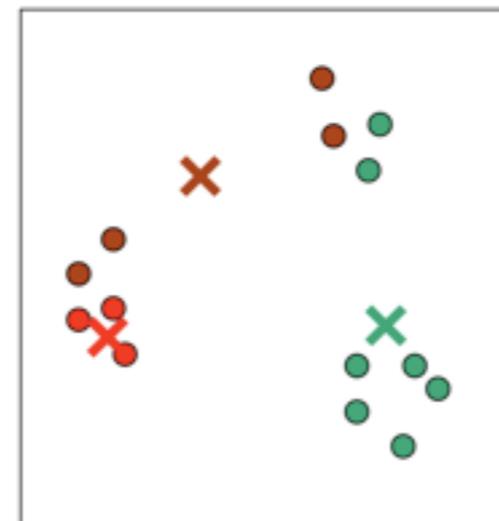
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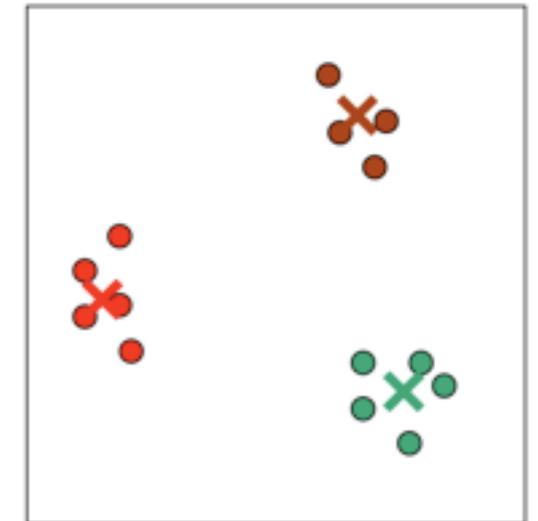
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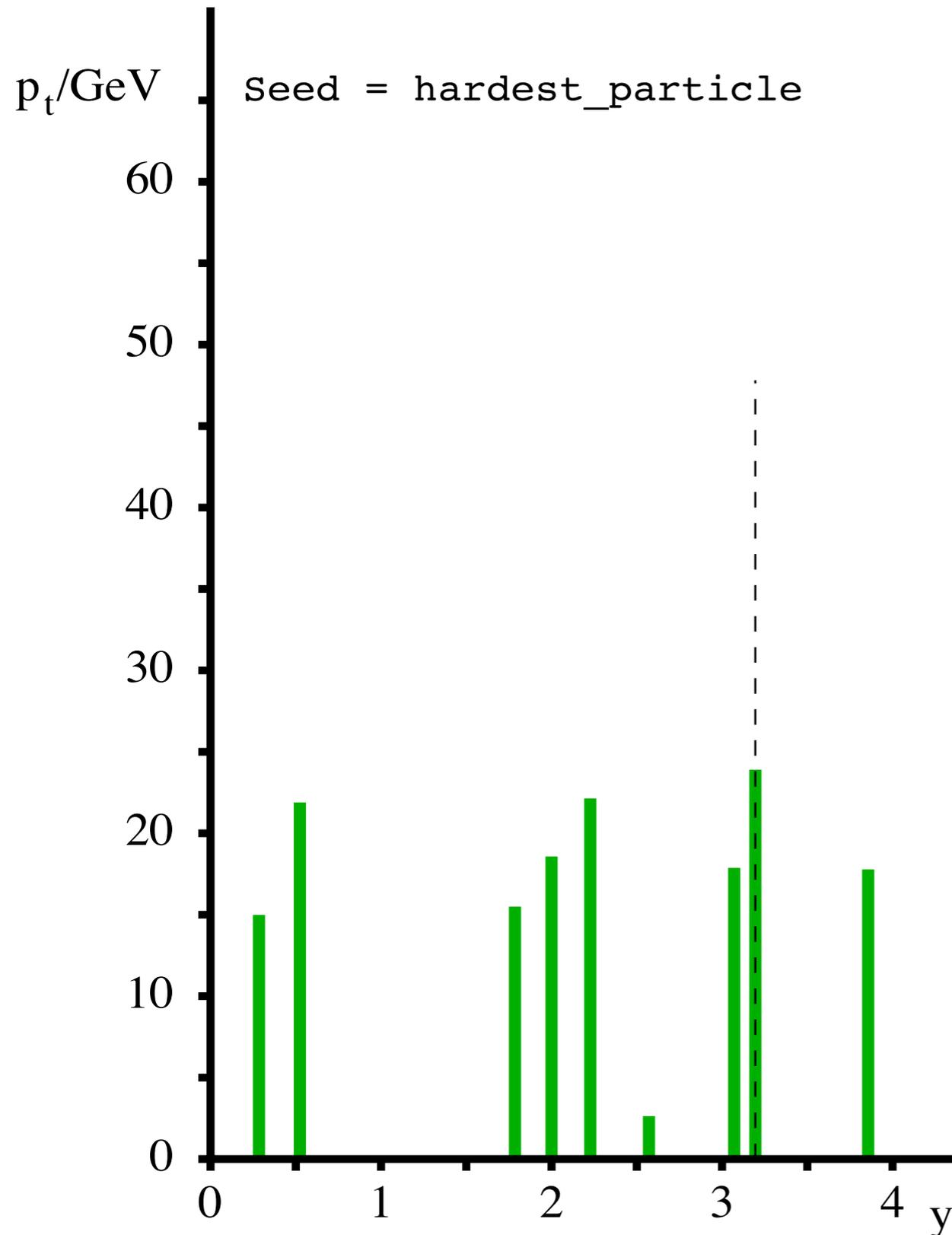
[Figures by E. Garrett-Mayer]

One of the main shortcomings:

result of final convergence can be highly sensitive to choice of initial seeds.

Also, the concept of 'mean distance' (to calculate the centroid) must be defined.

Iterative Cone, Prog Removal (IC-PR)



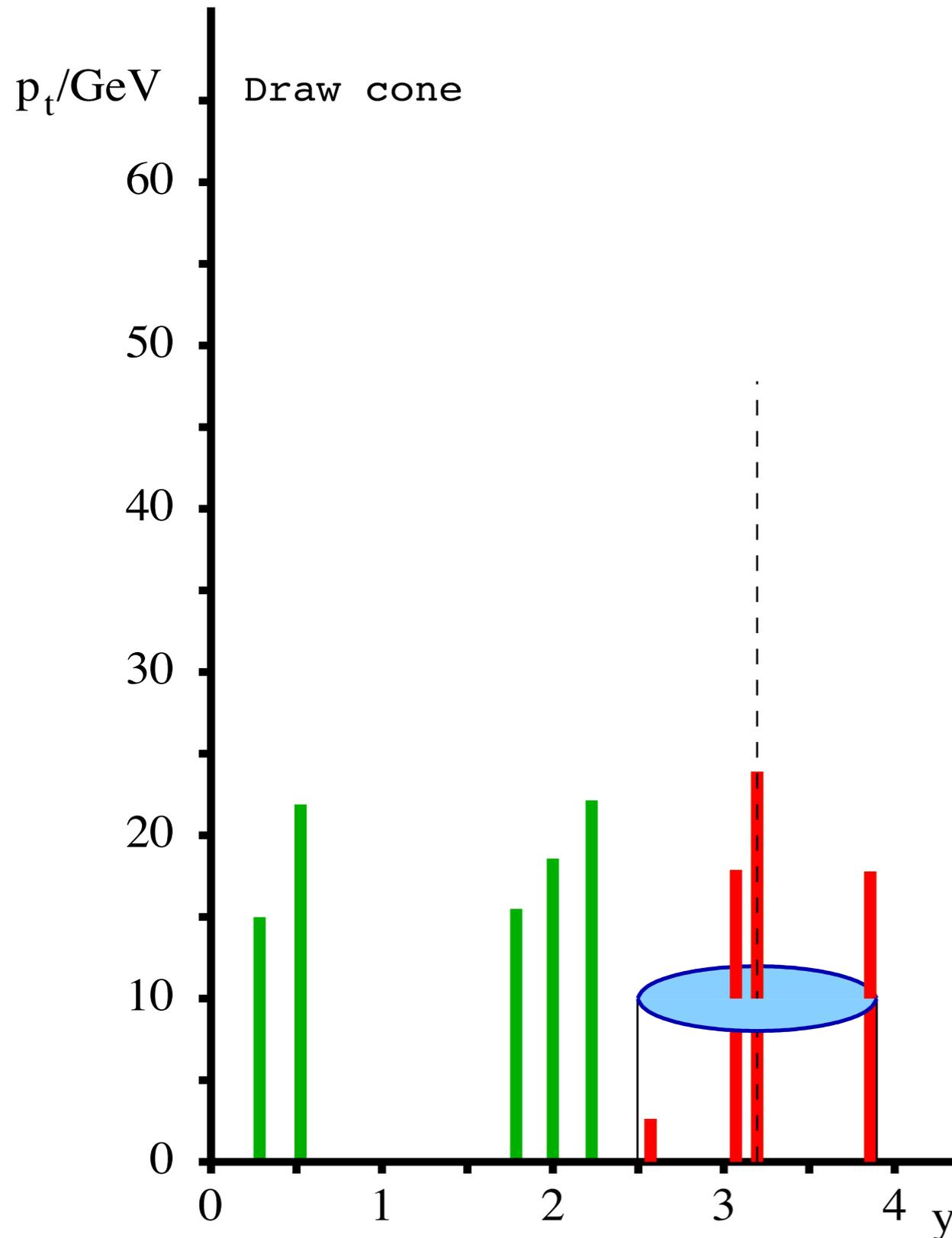
One of the simplest of the cone algs
e.g. CMS iterative cone

- ▶ Take hardest particle as seed for cone axis
- ▶ Draw cone around seed
- ▶ Sum the momenta use as new seed direction, iterate until stable
- ▶ Convert contents into a “jet” and remove from event

Notes

- ▶ “Hardest particle” is collinear unsafe more right away...

Iterative Cone, Prog Removal (IC-PR)



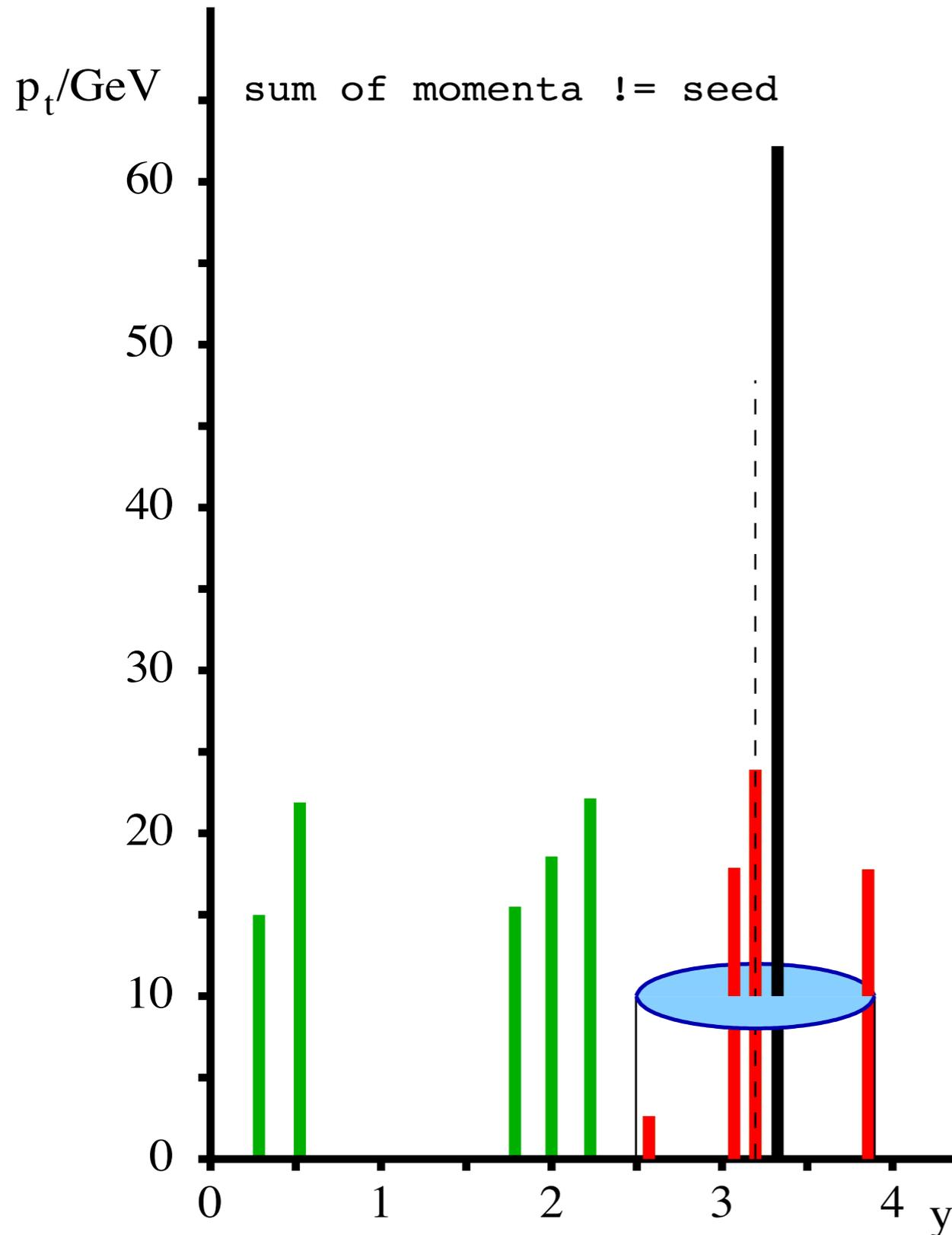
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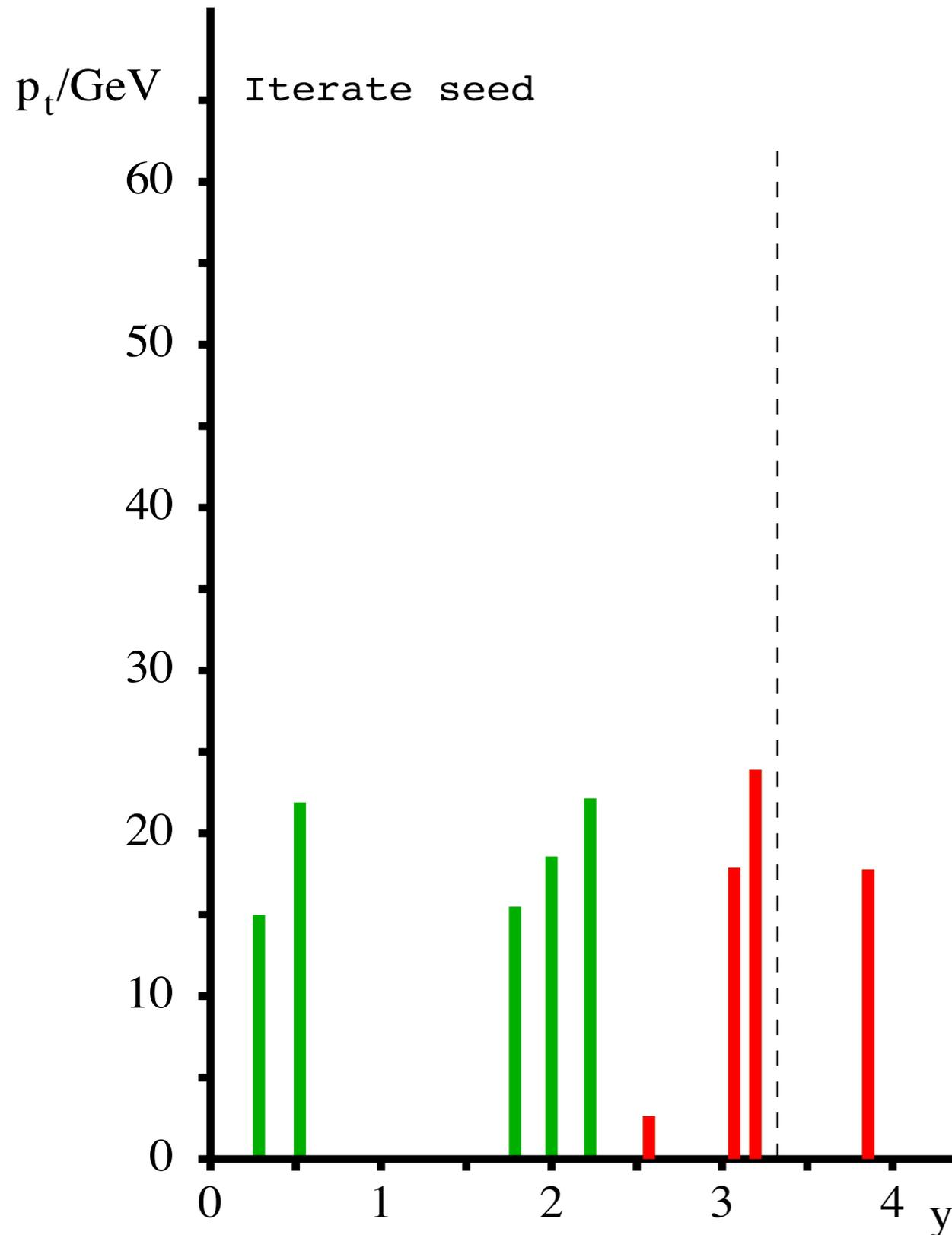
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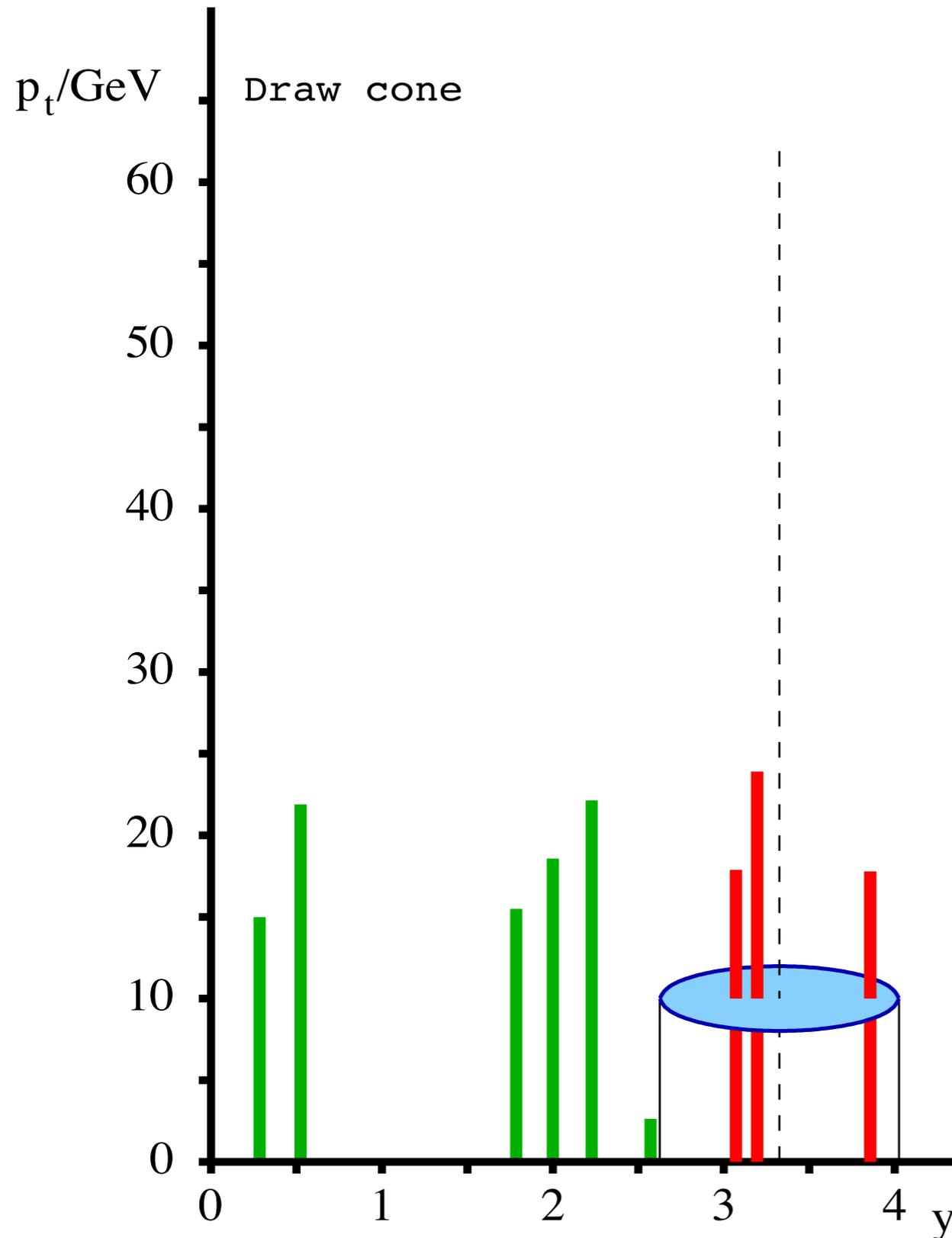
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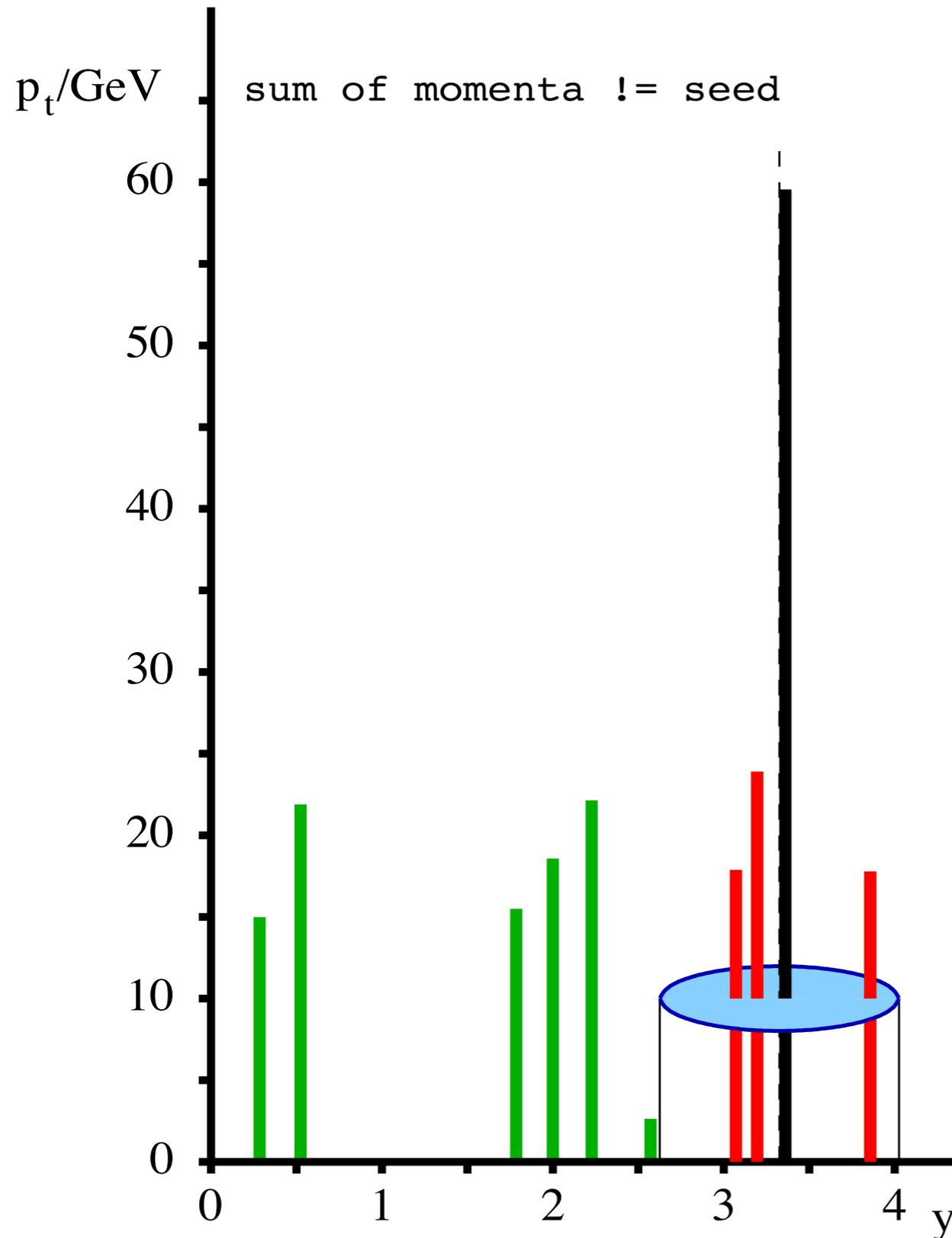
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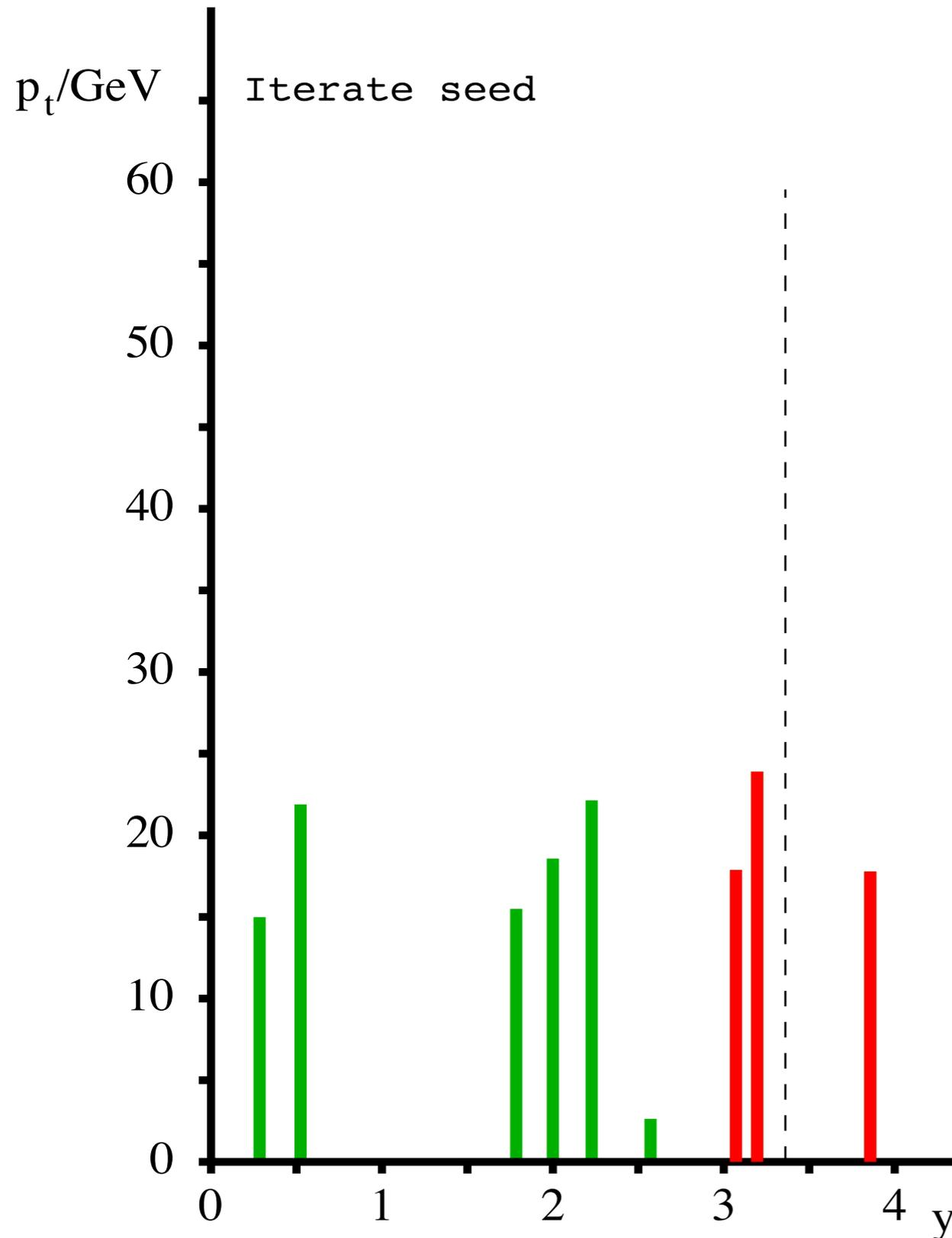
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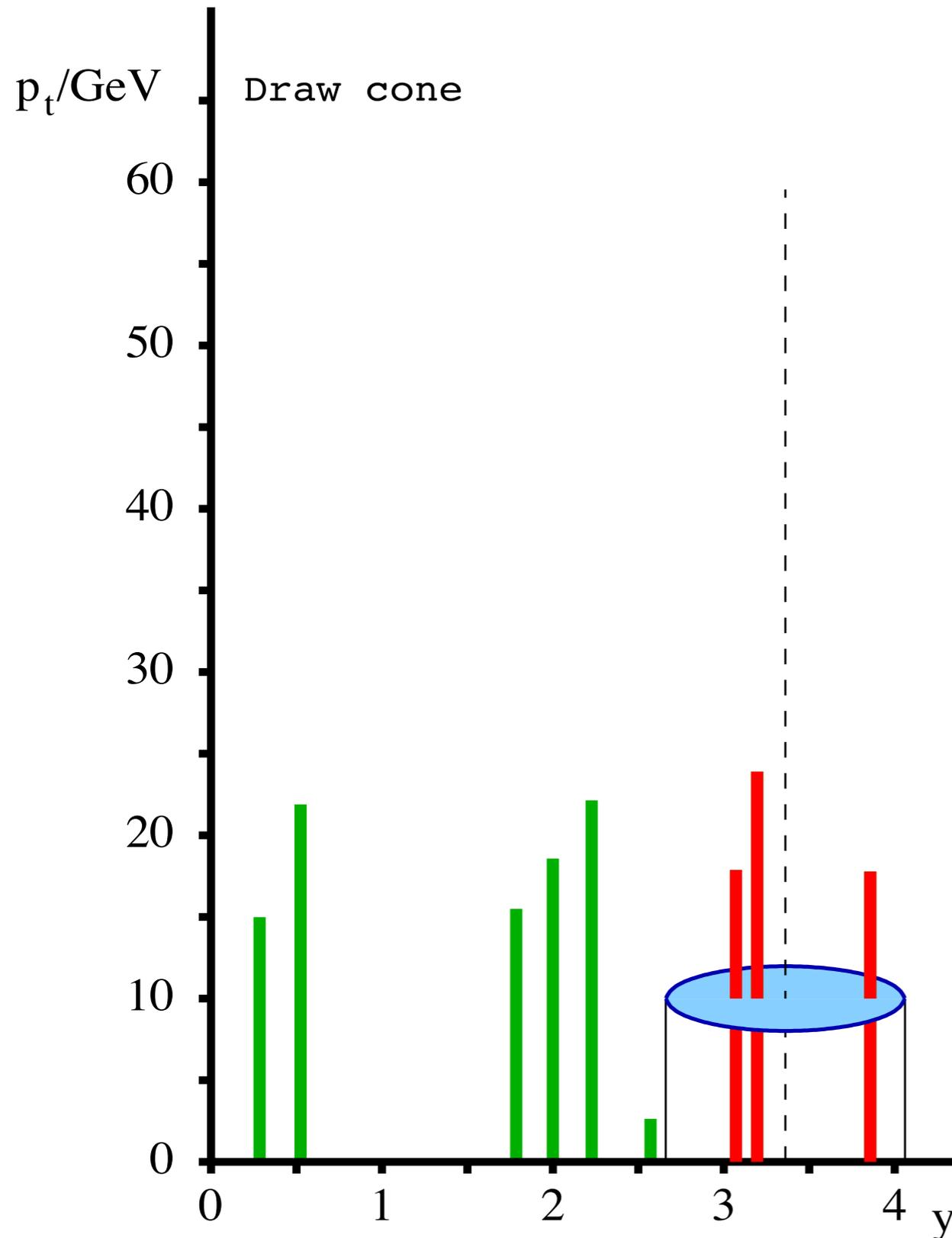
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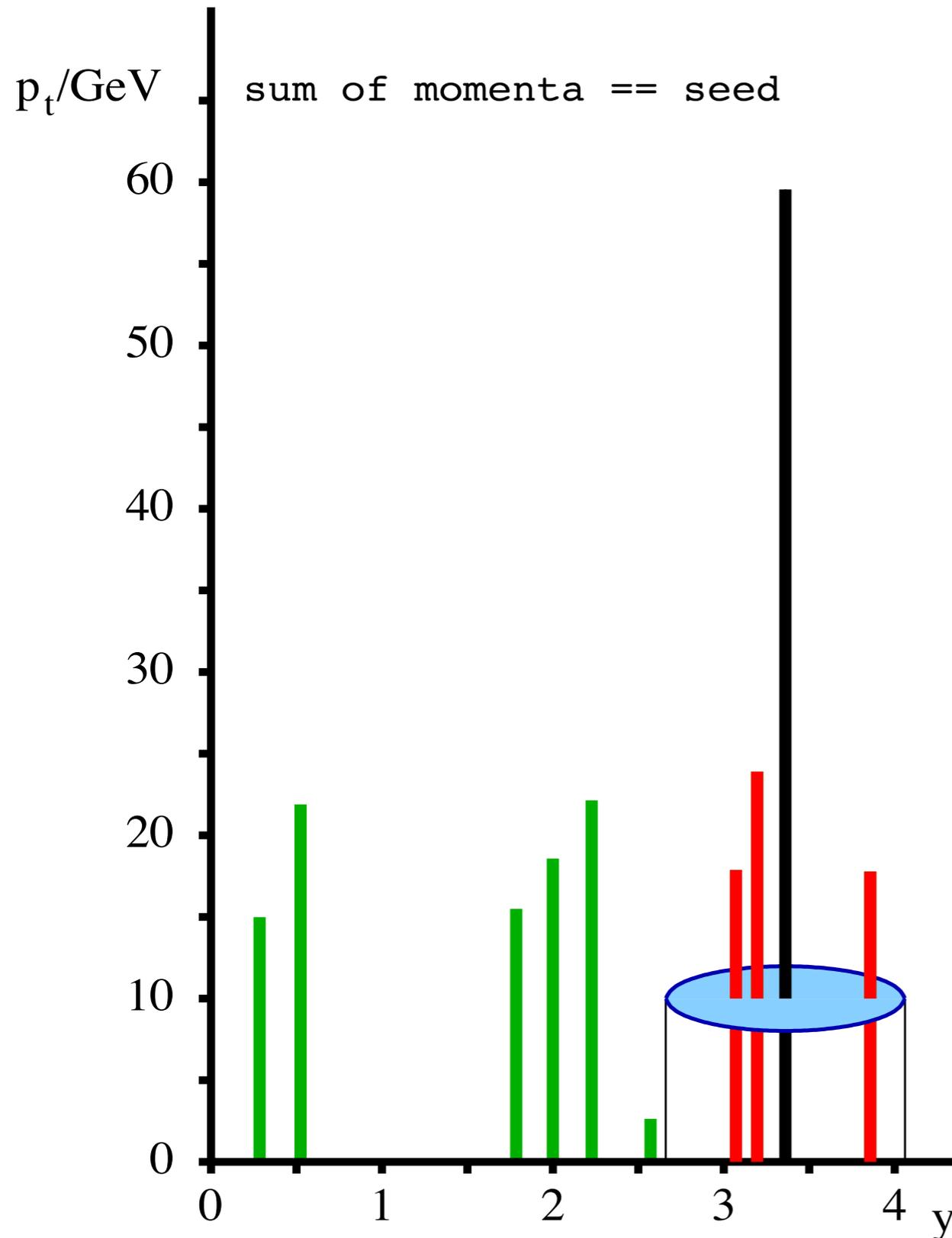
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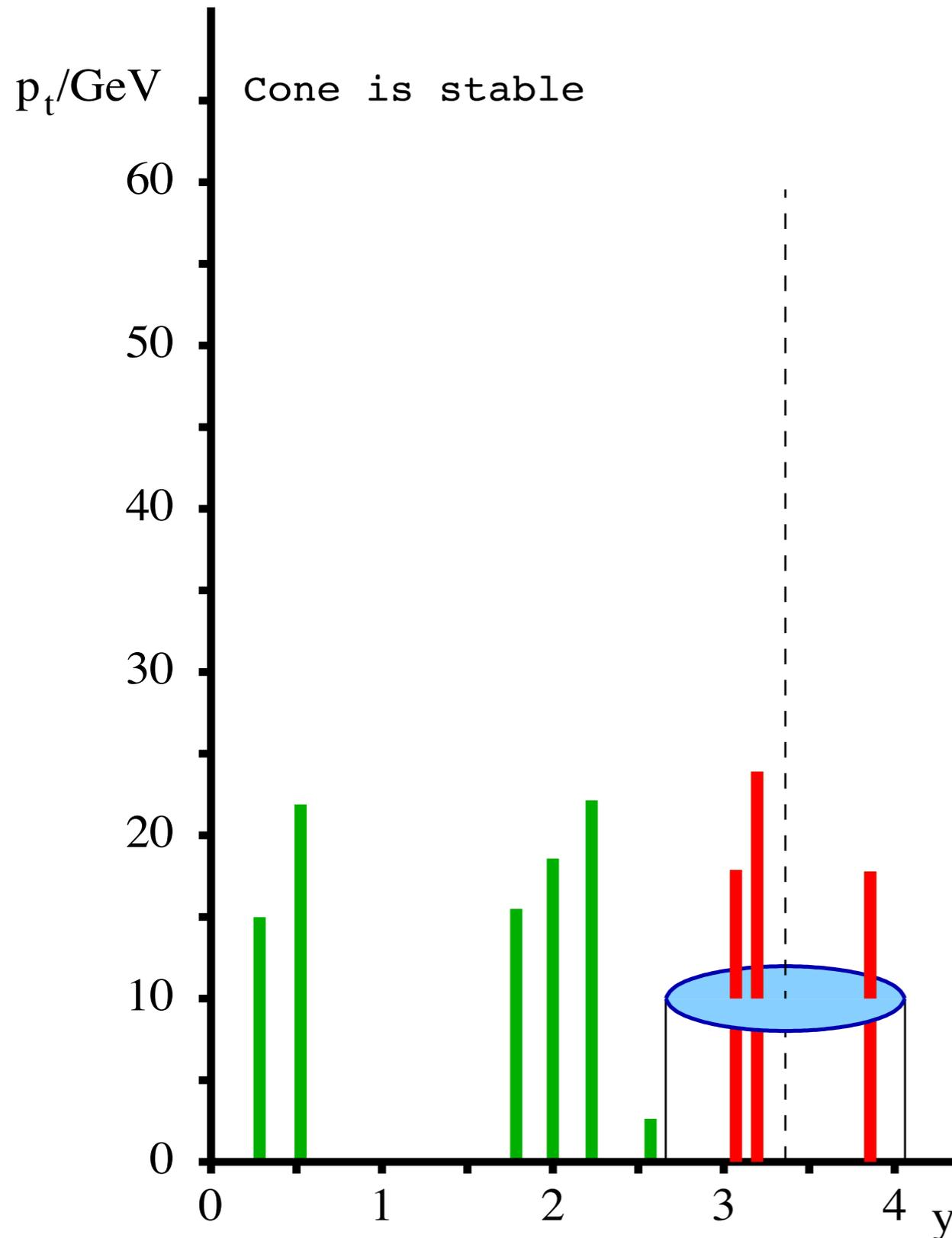
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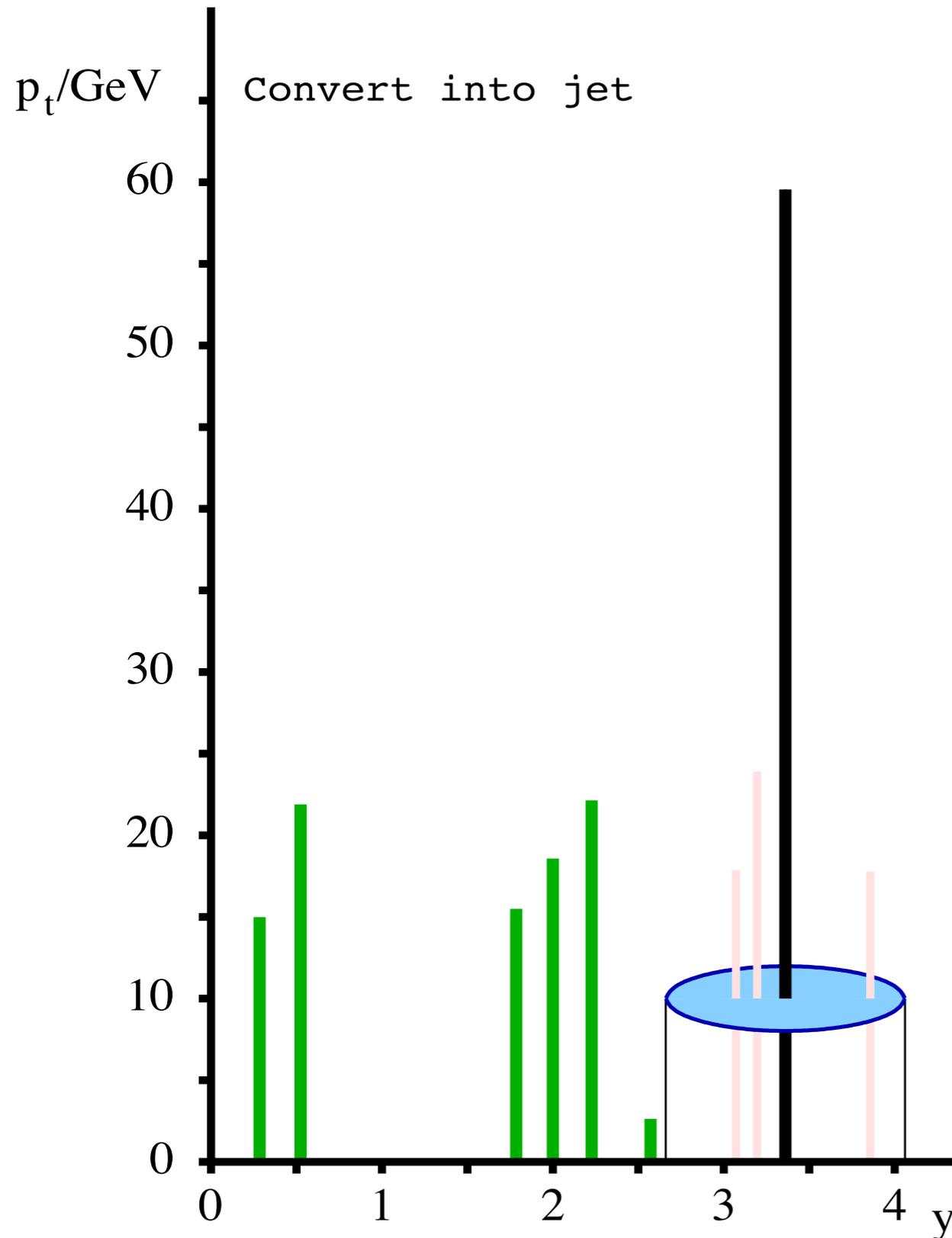
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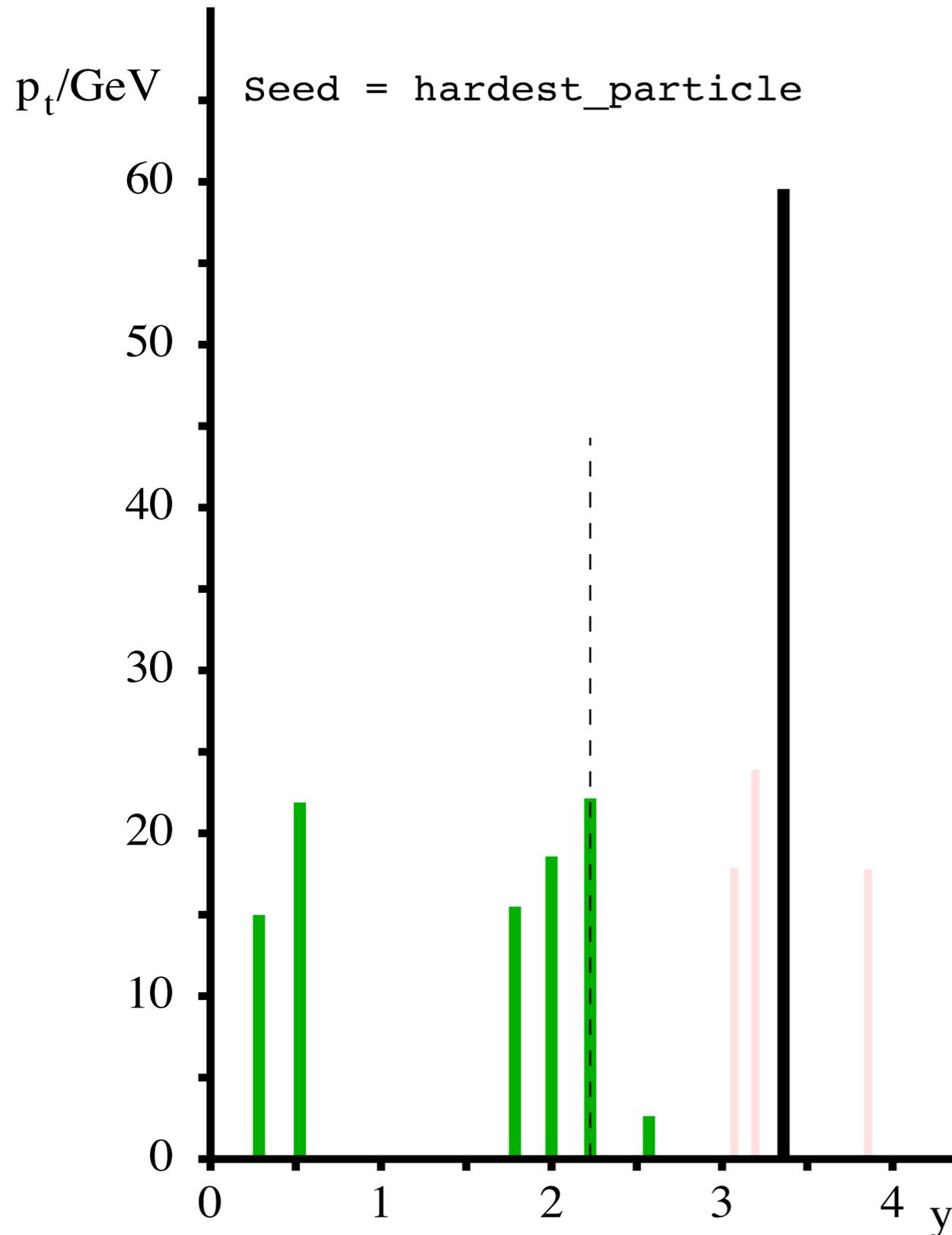
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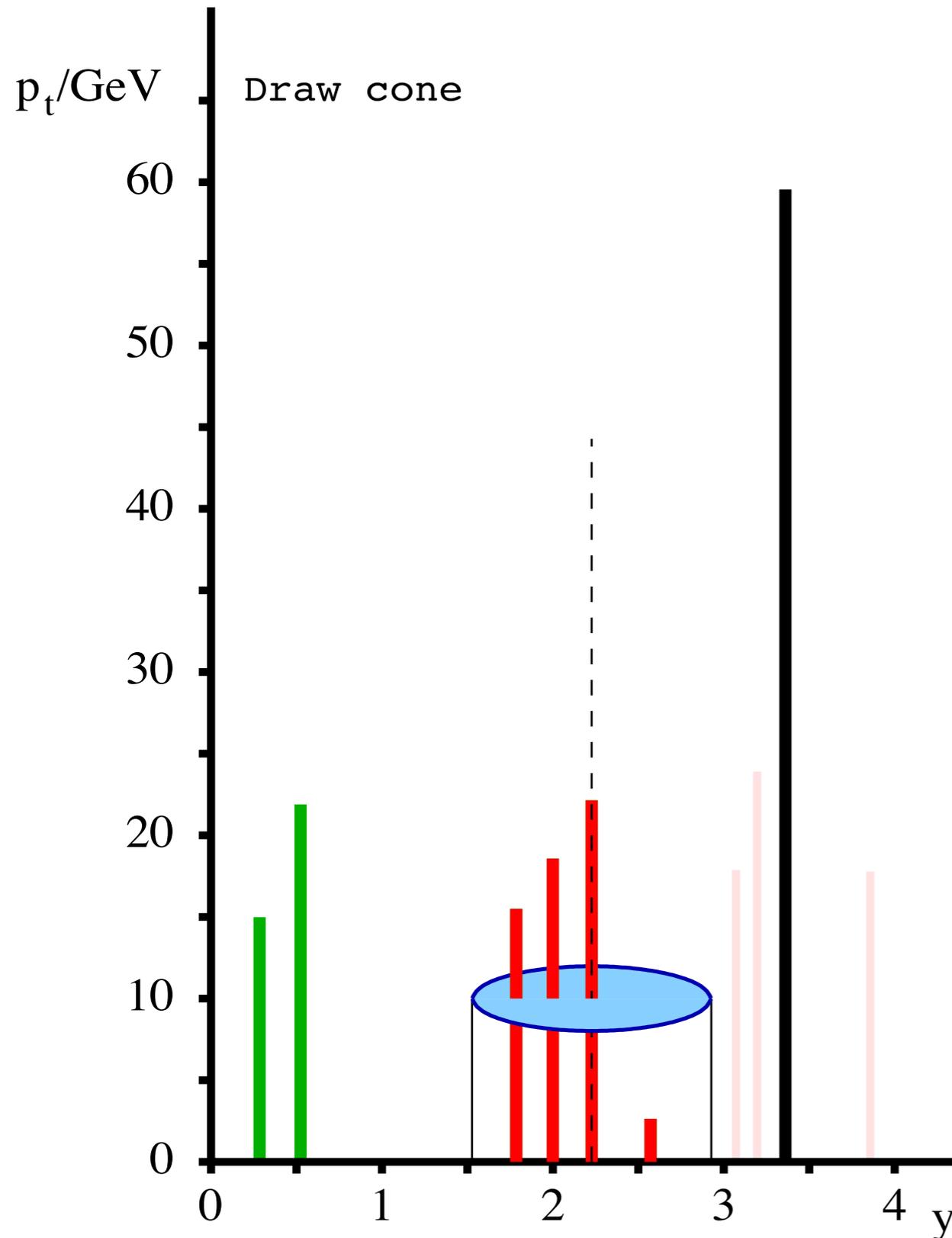
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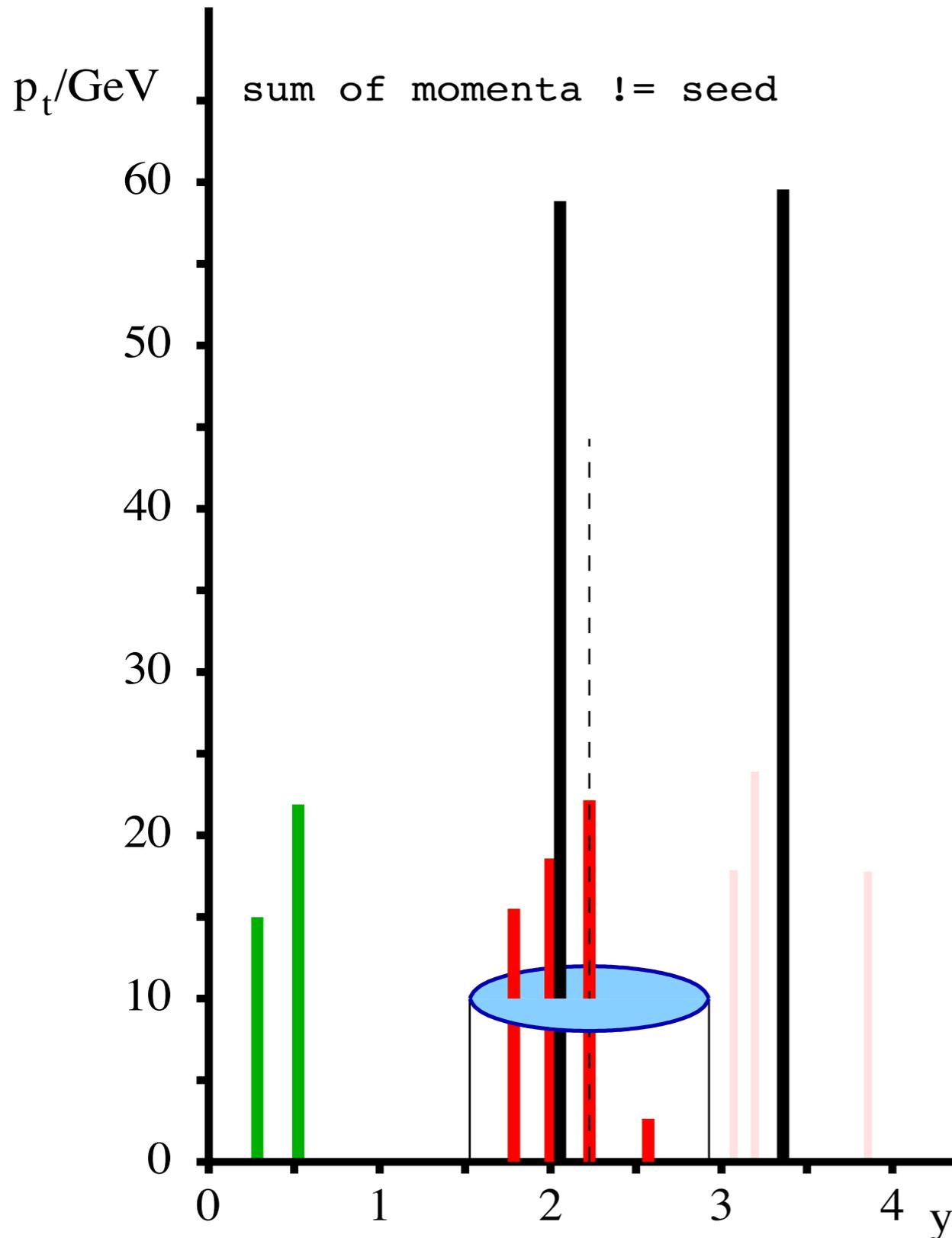
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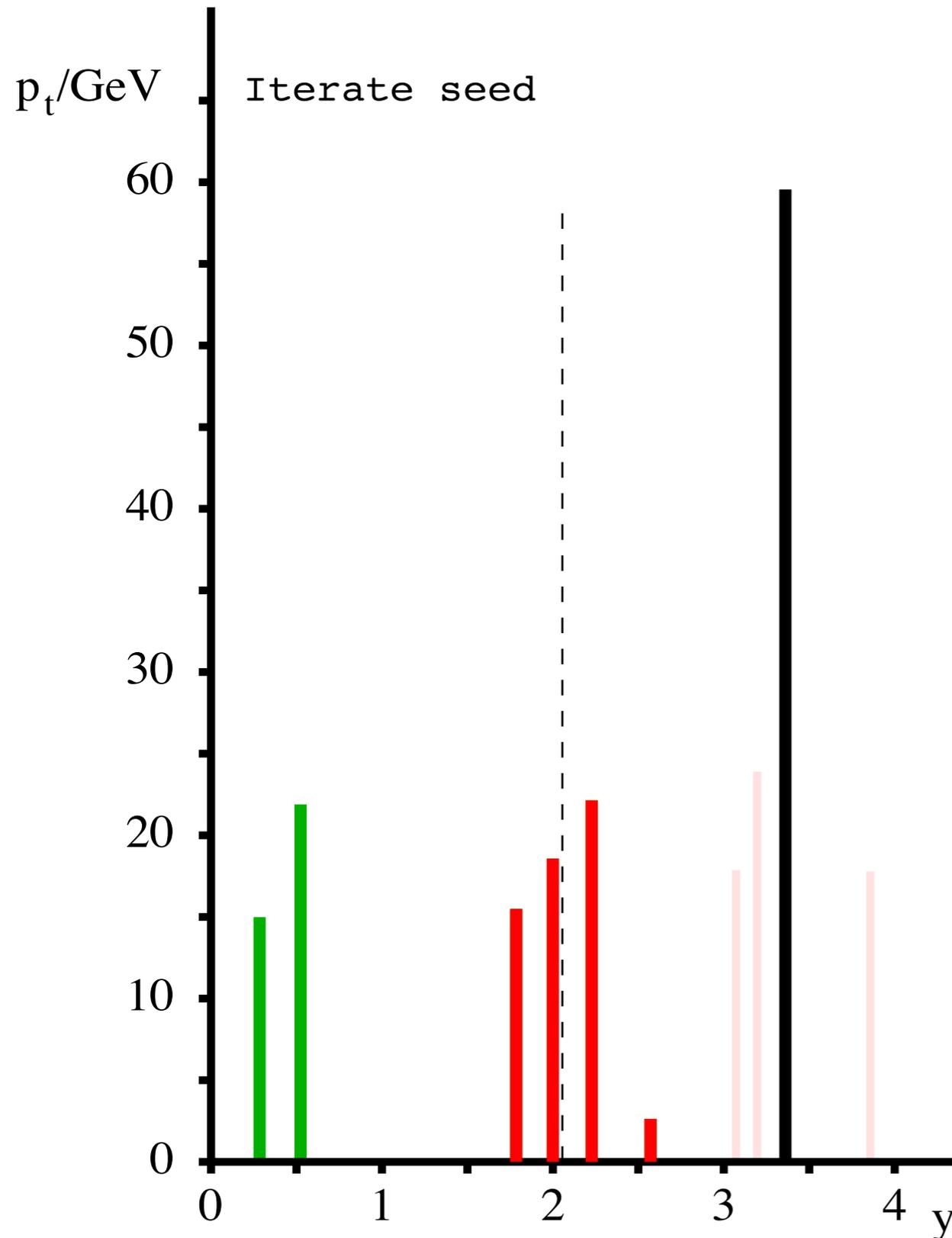
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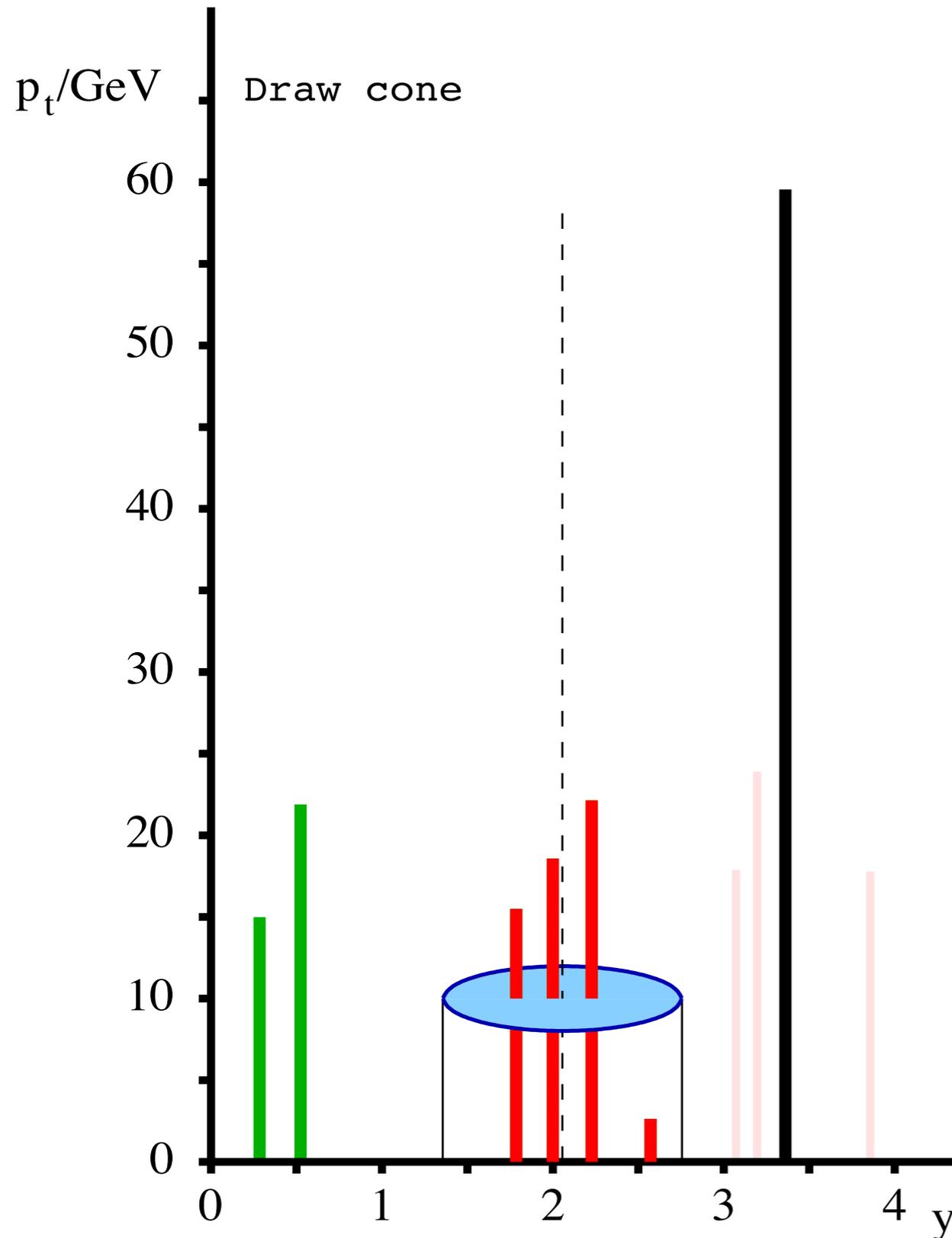
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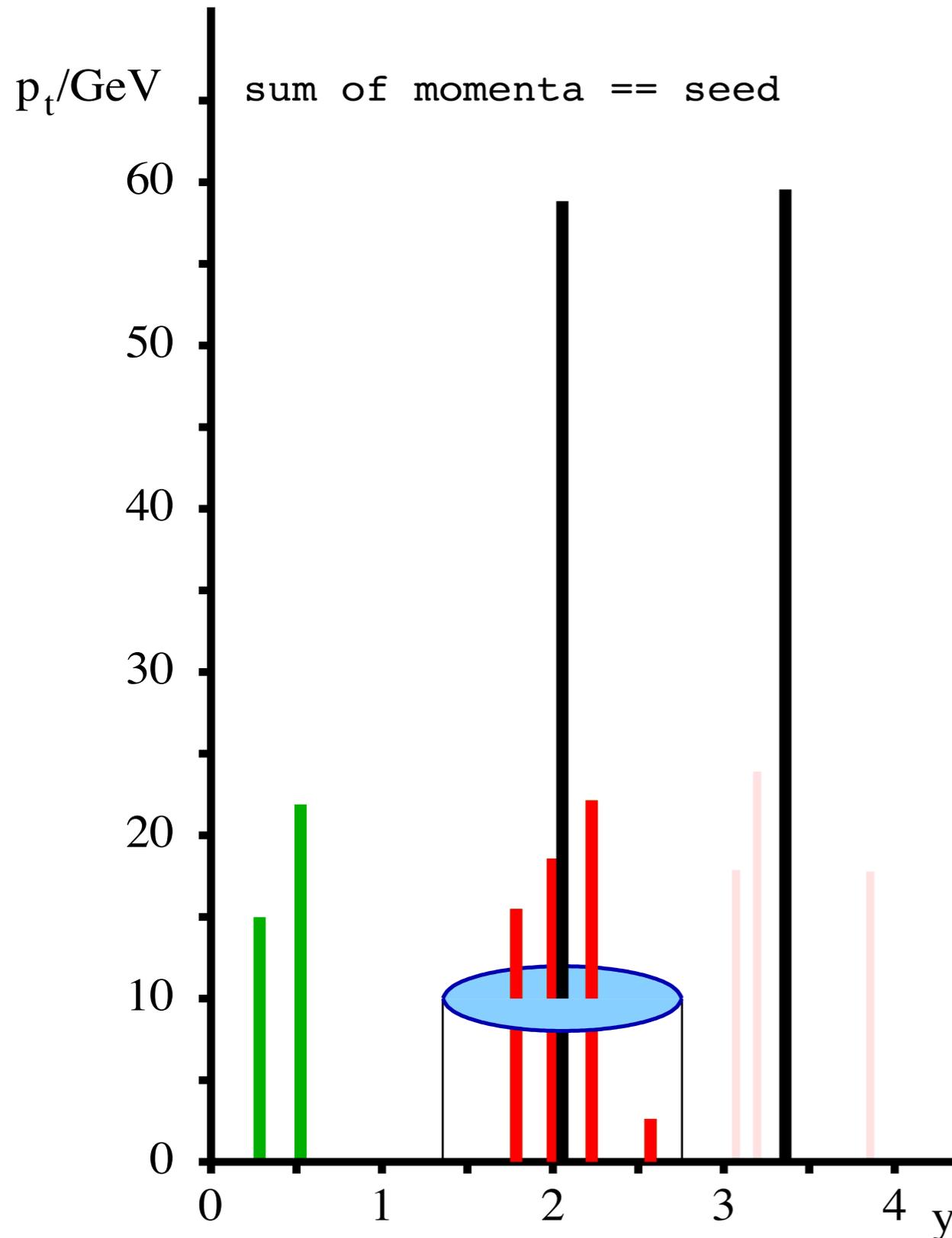
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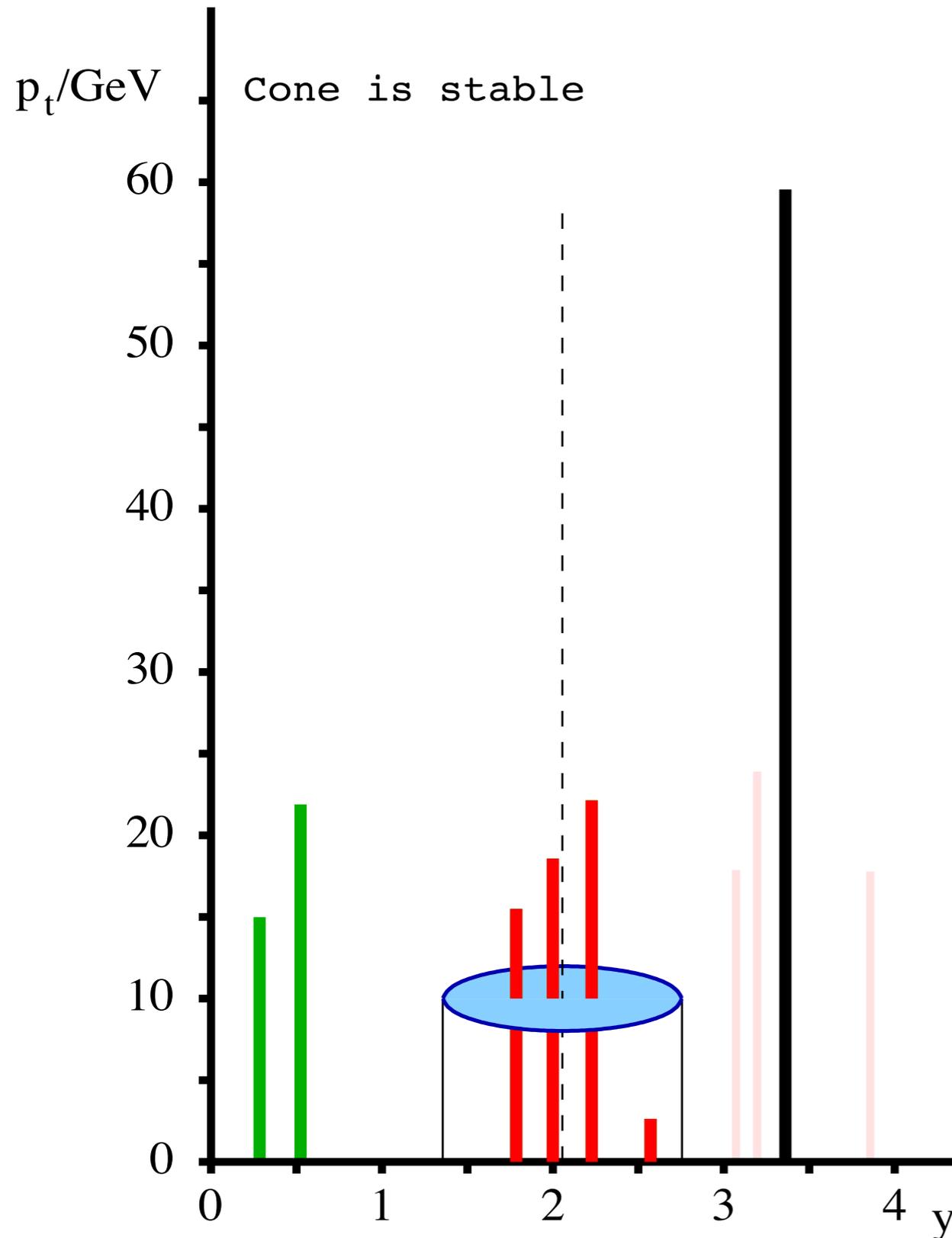
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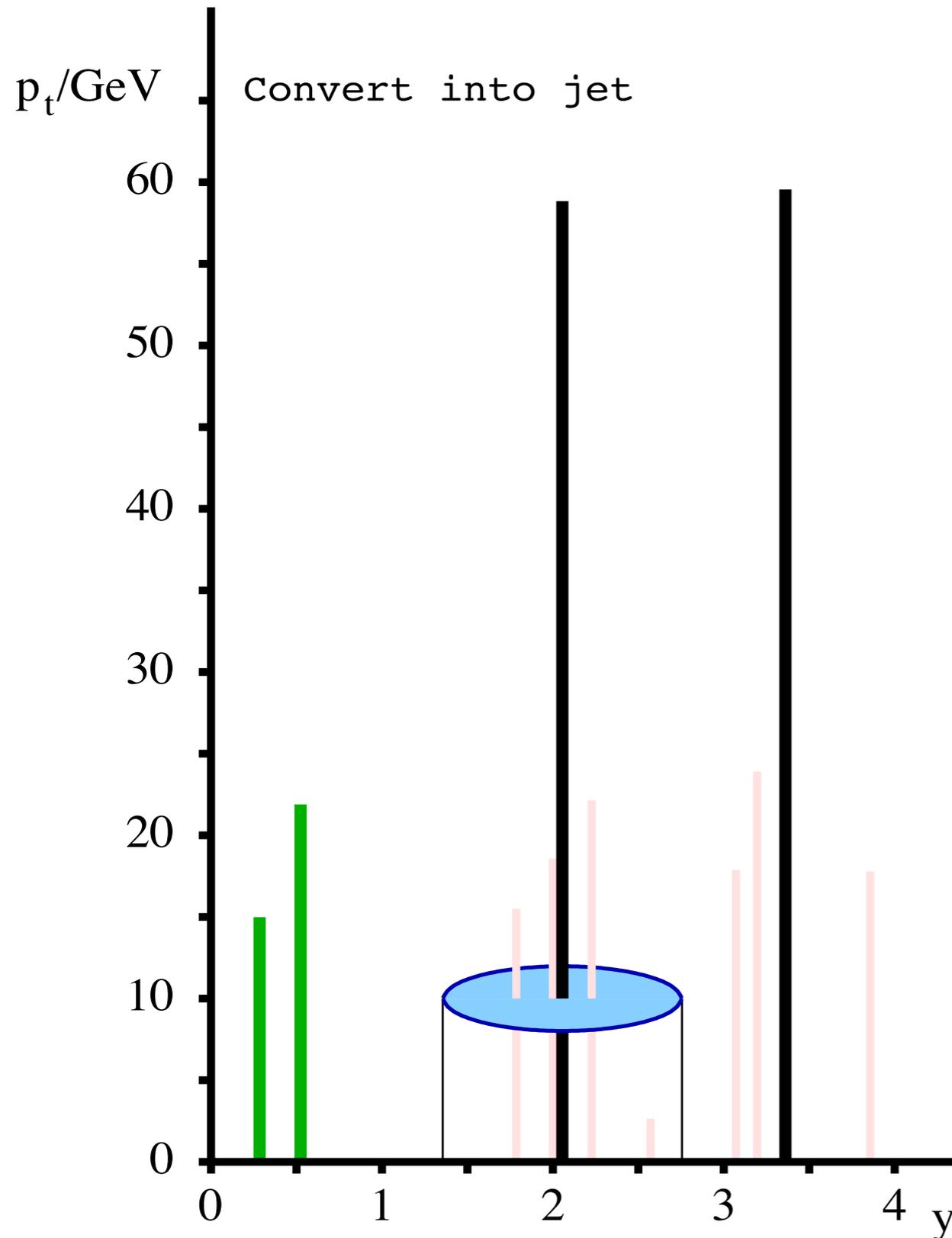
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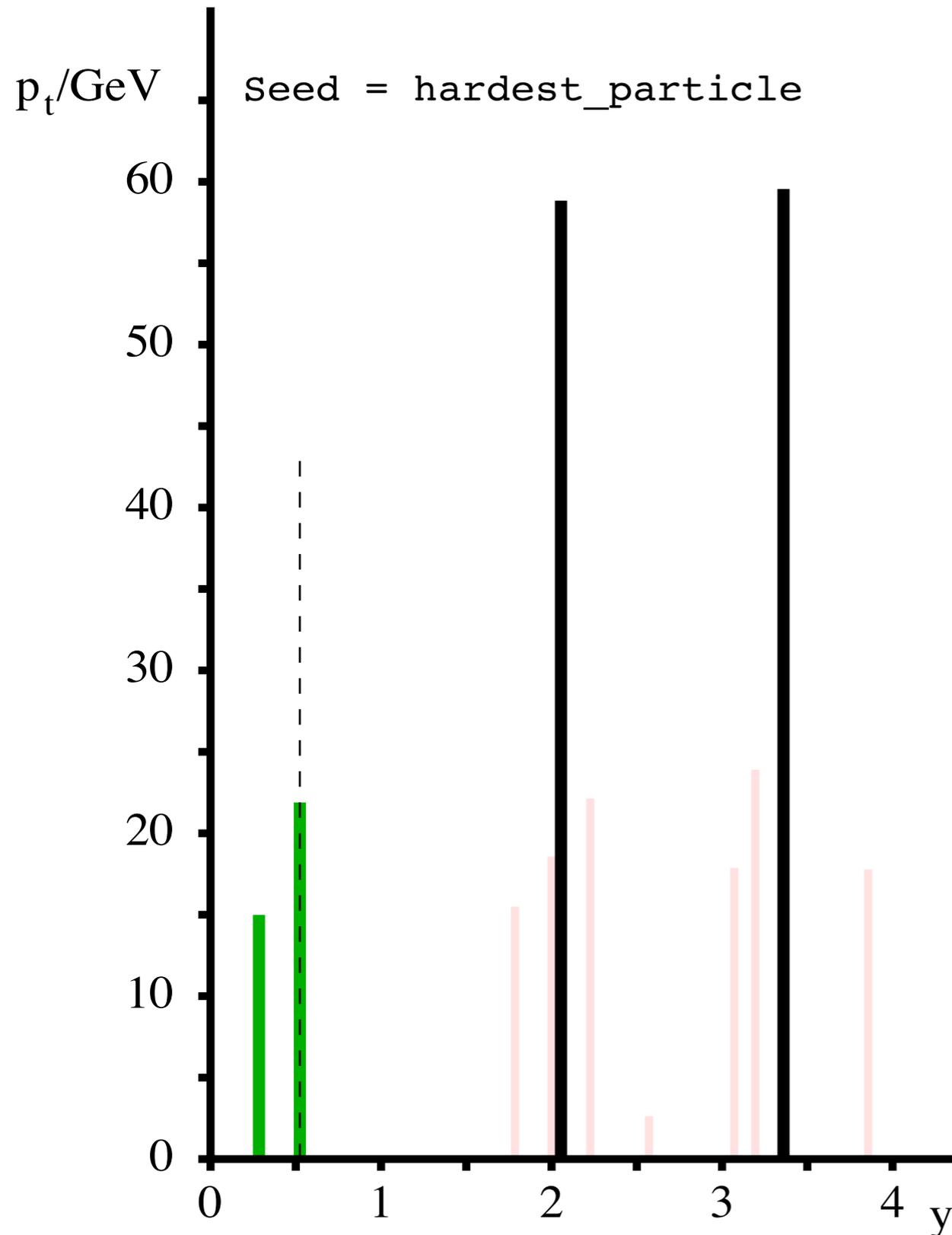
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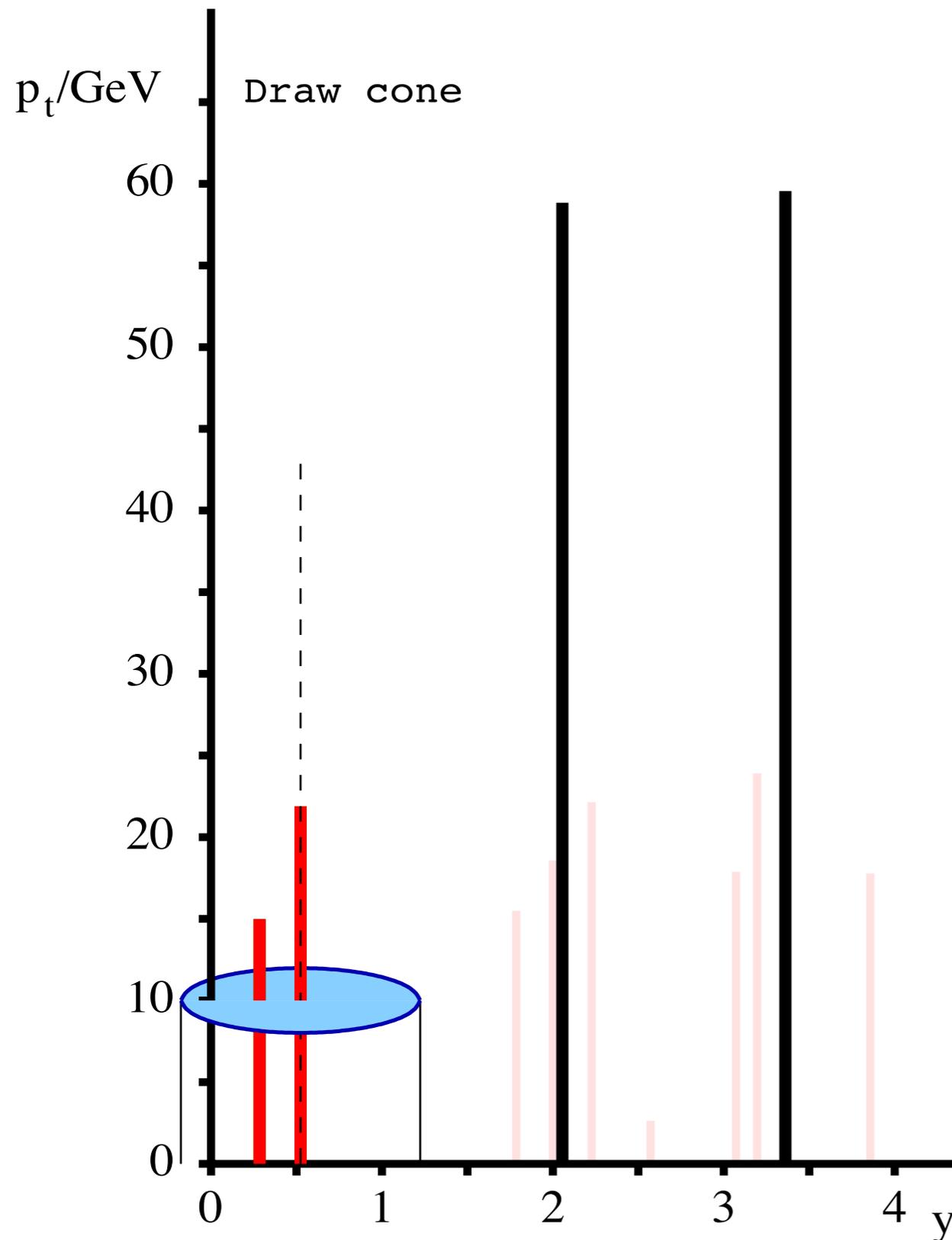
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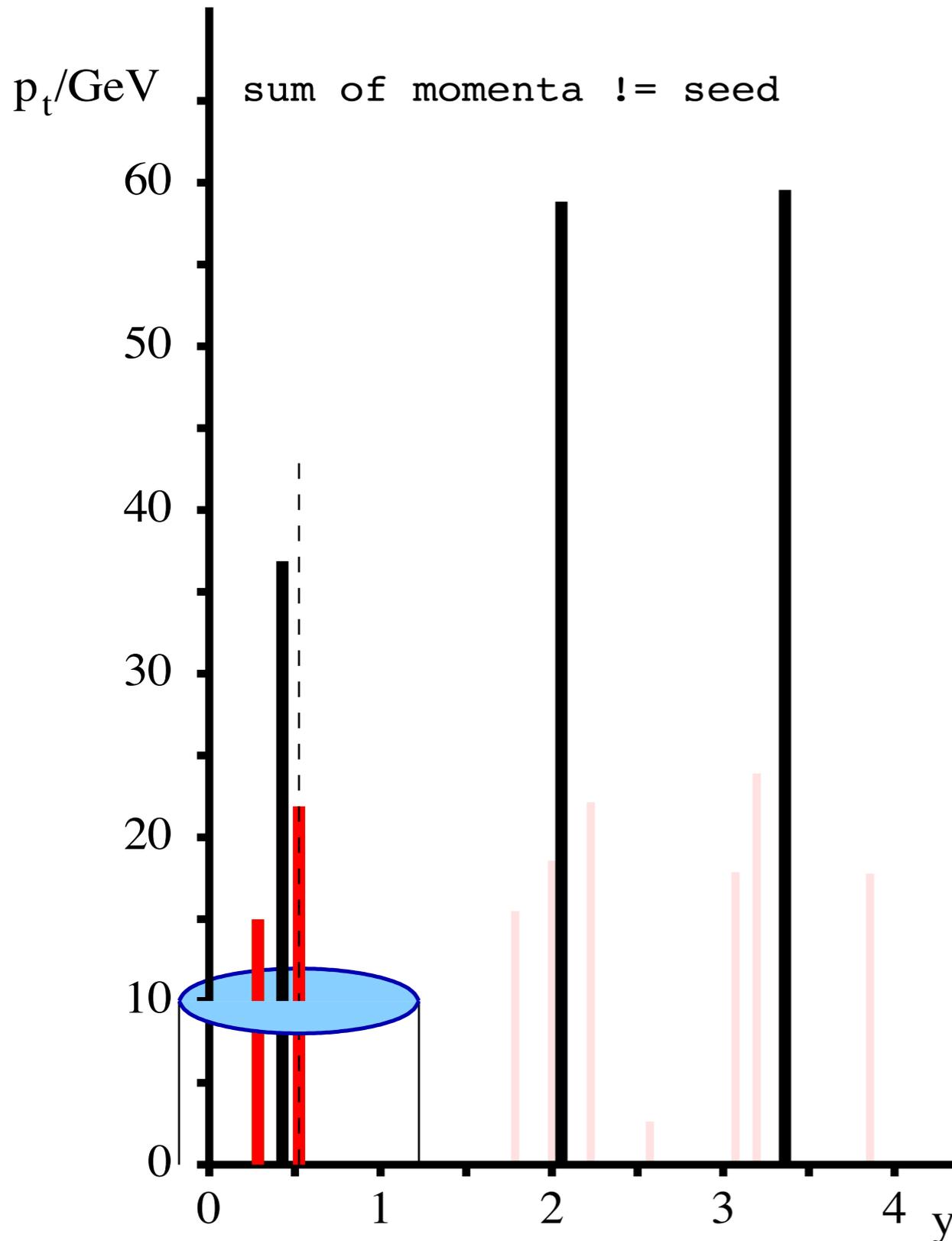
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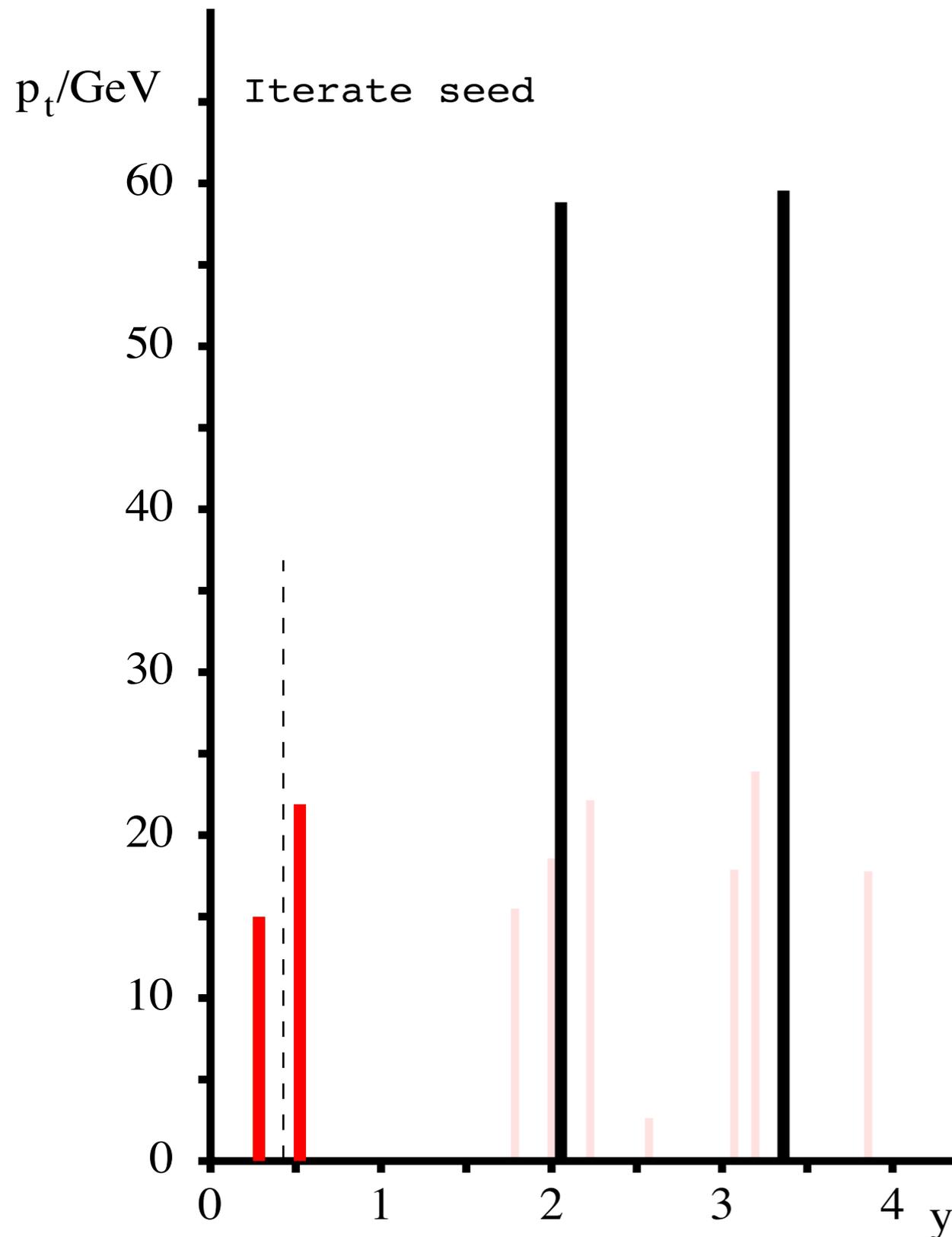
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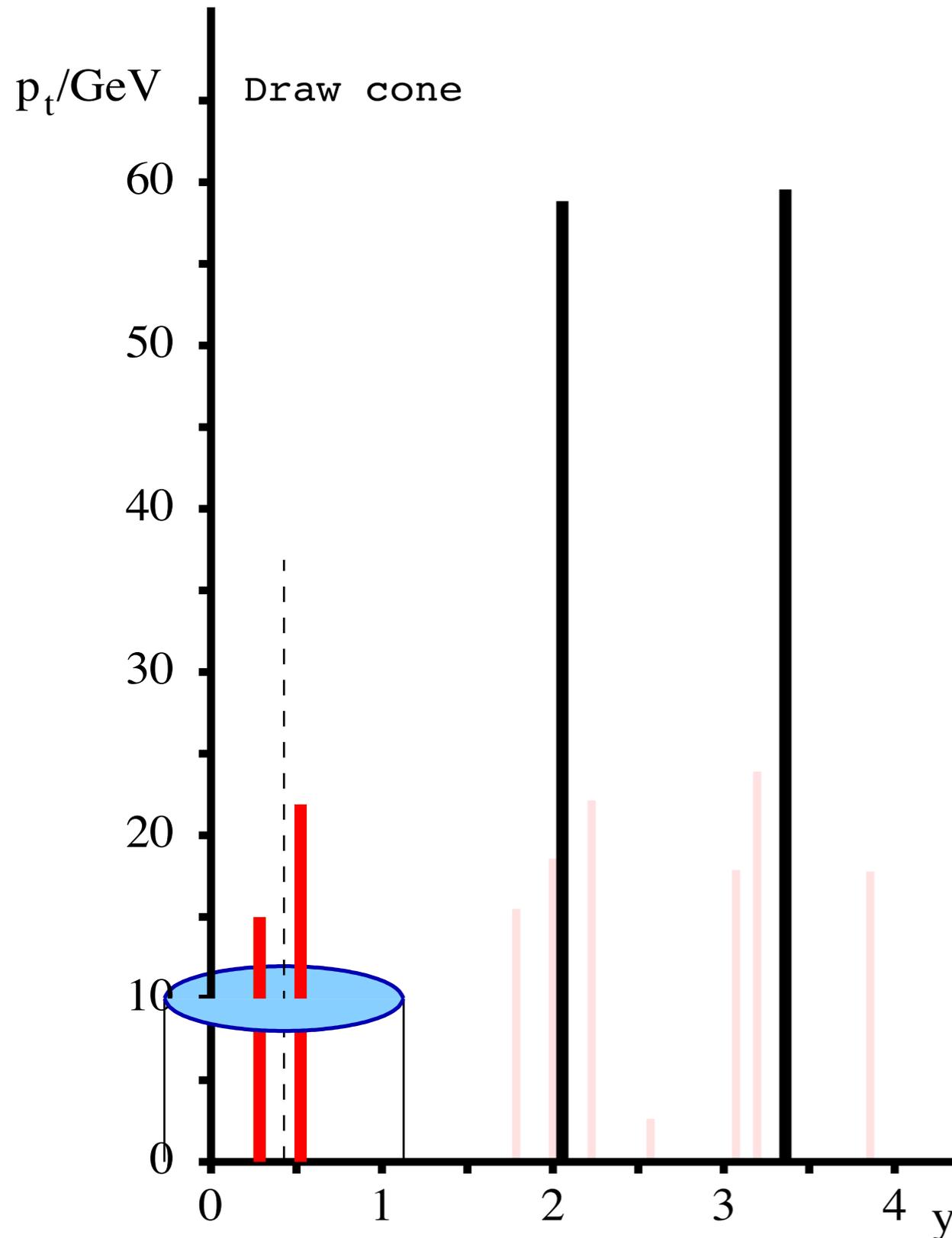
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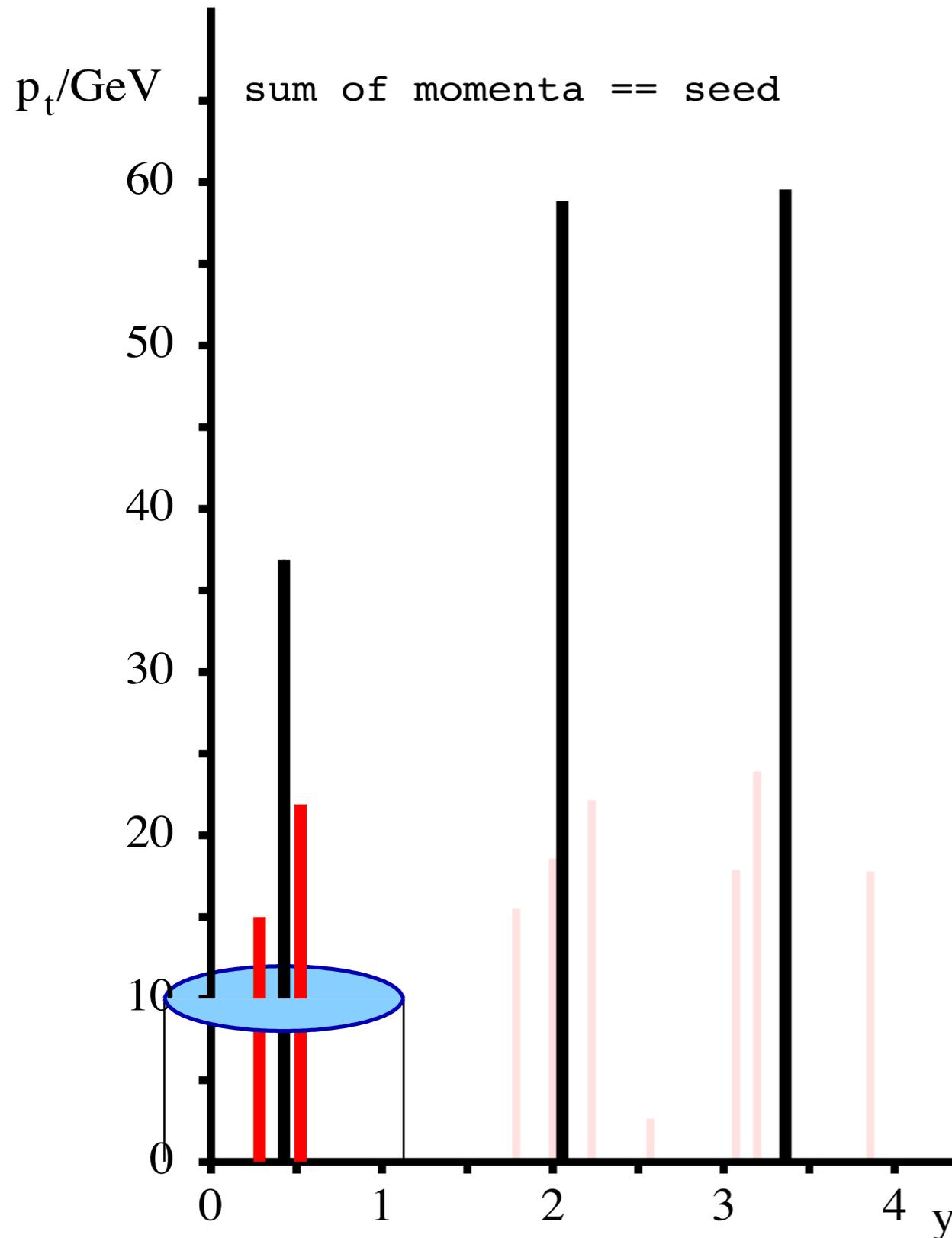
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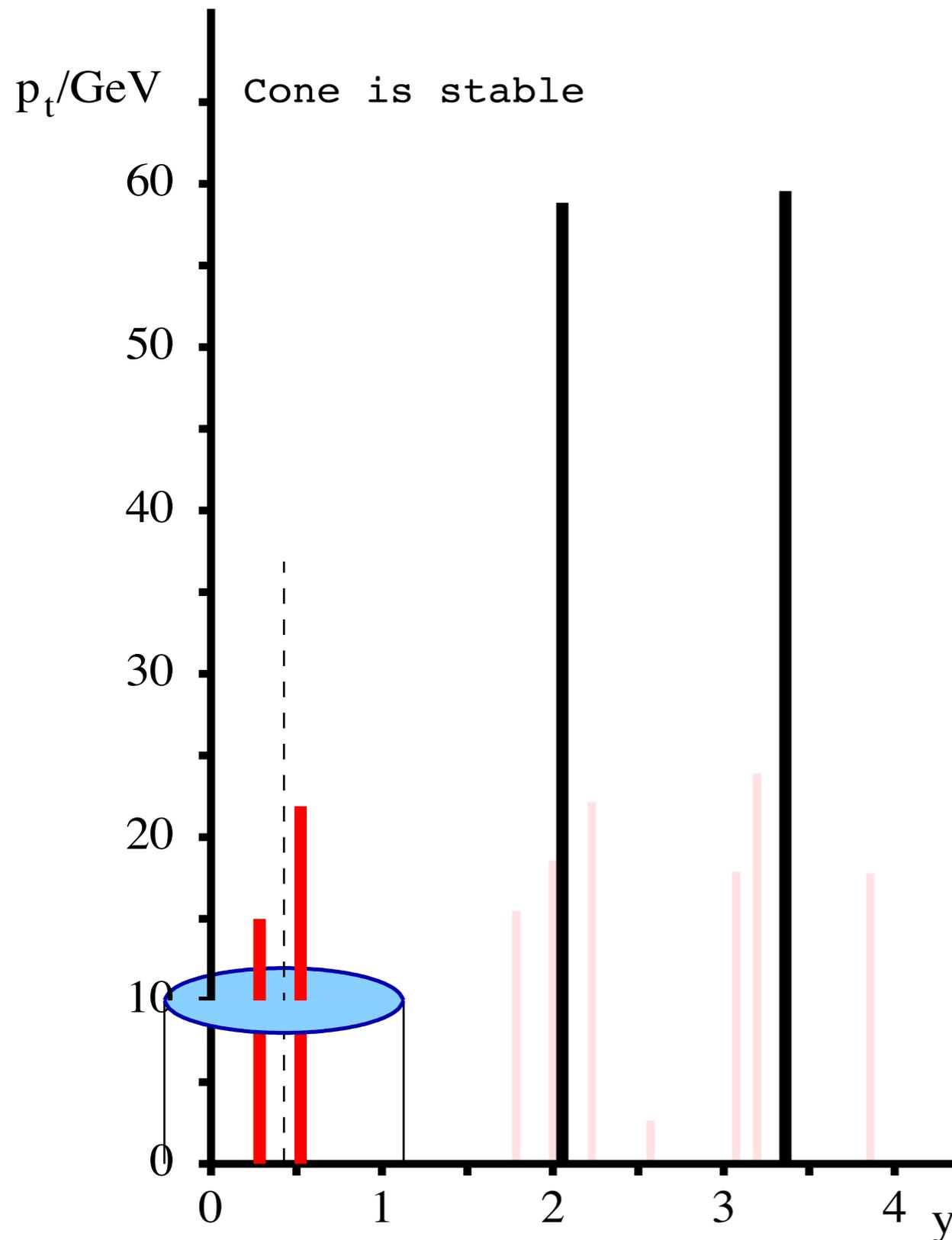
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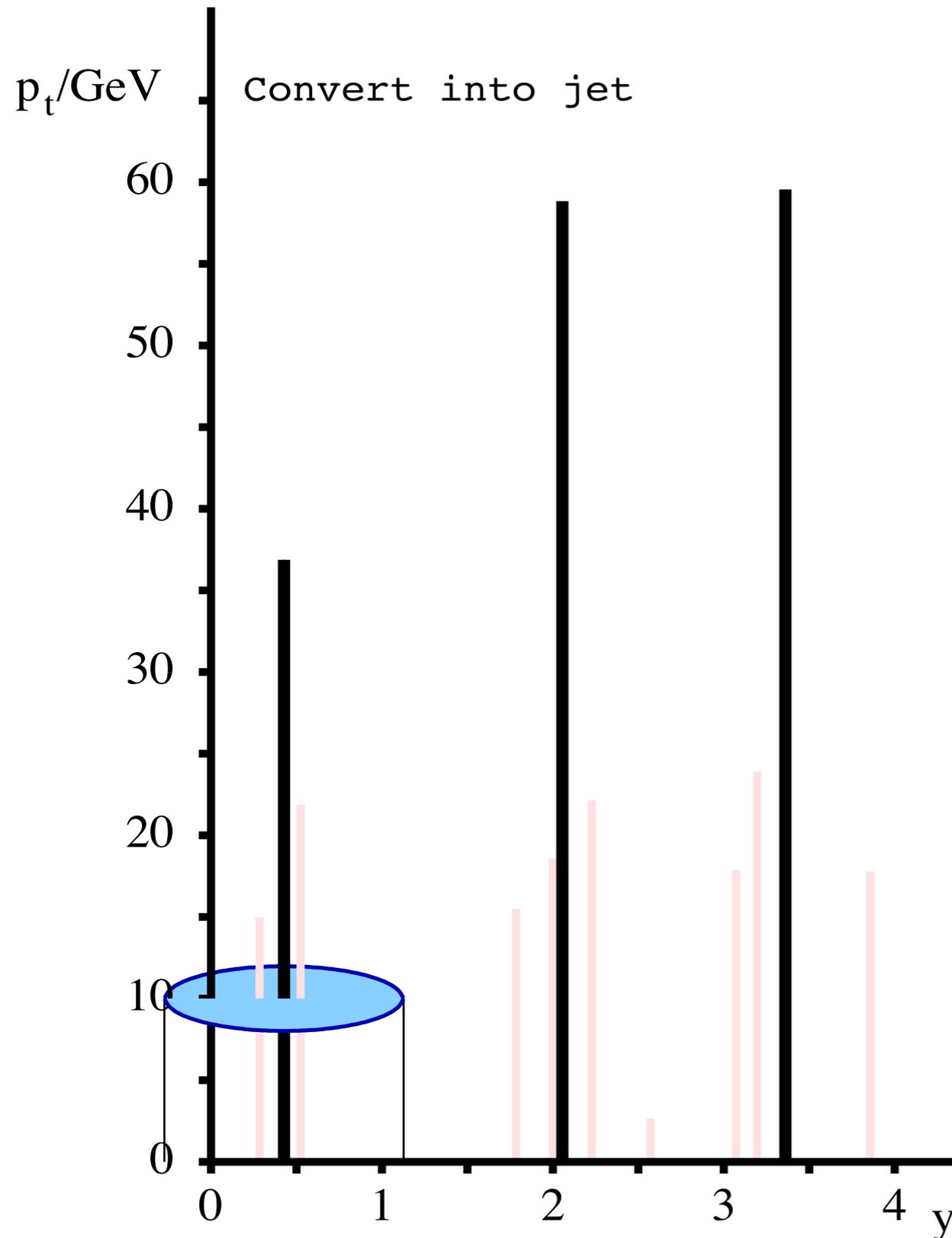
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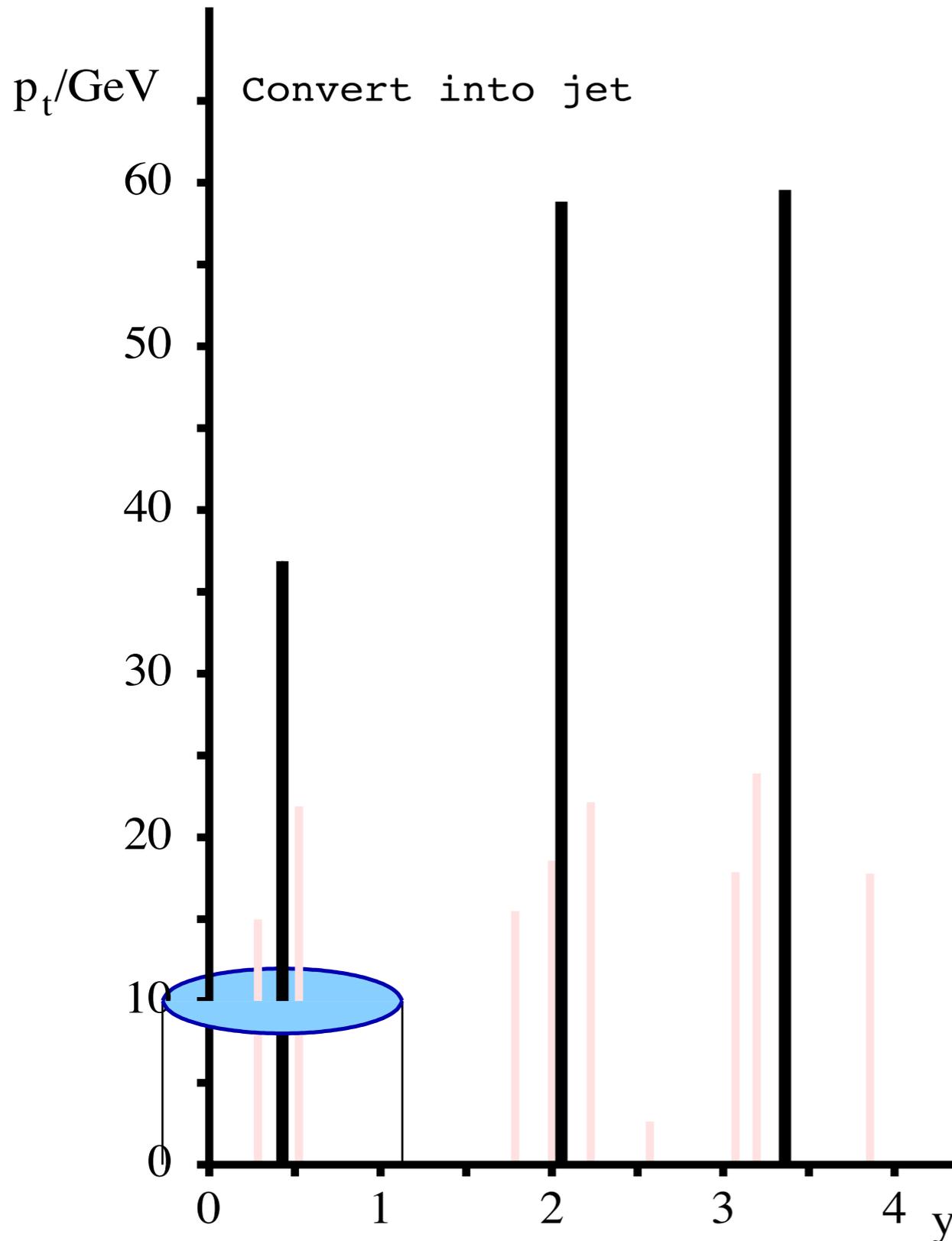
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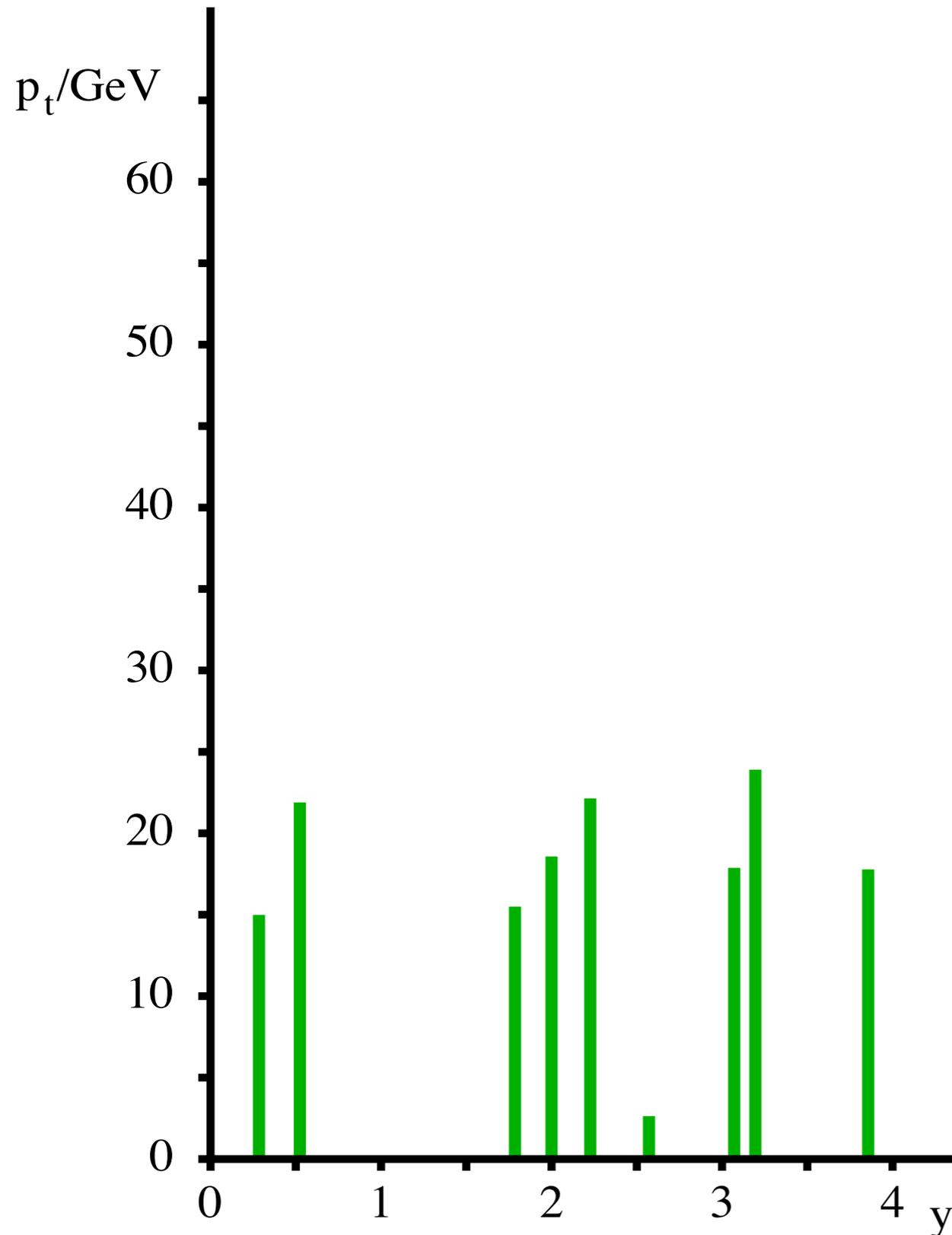
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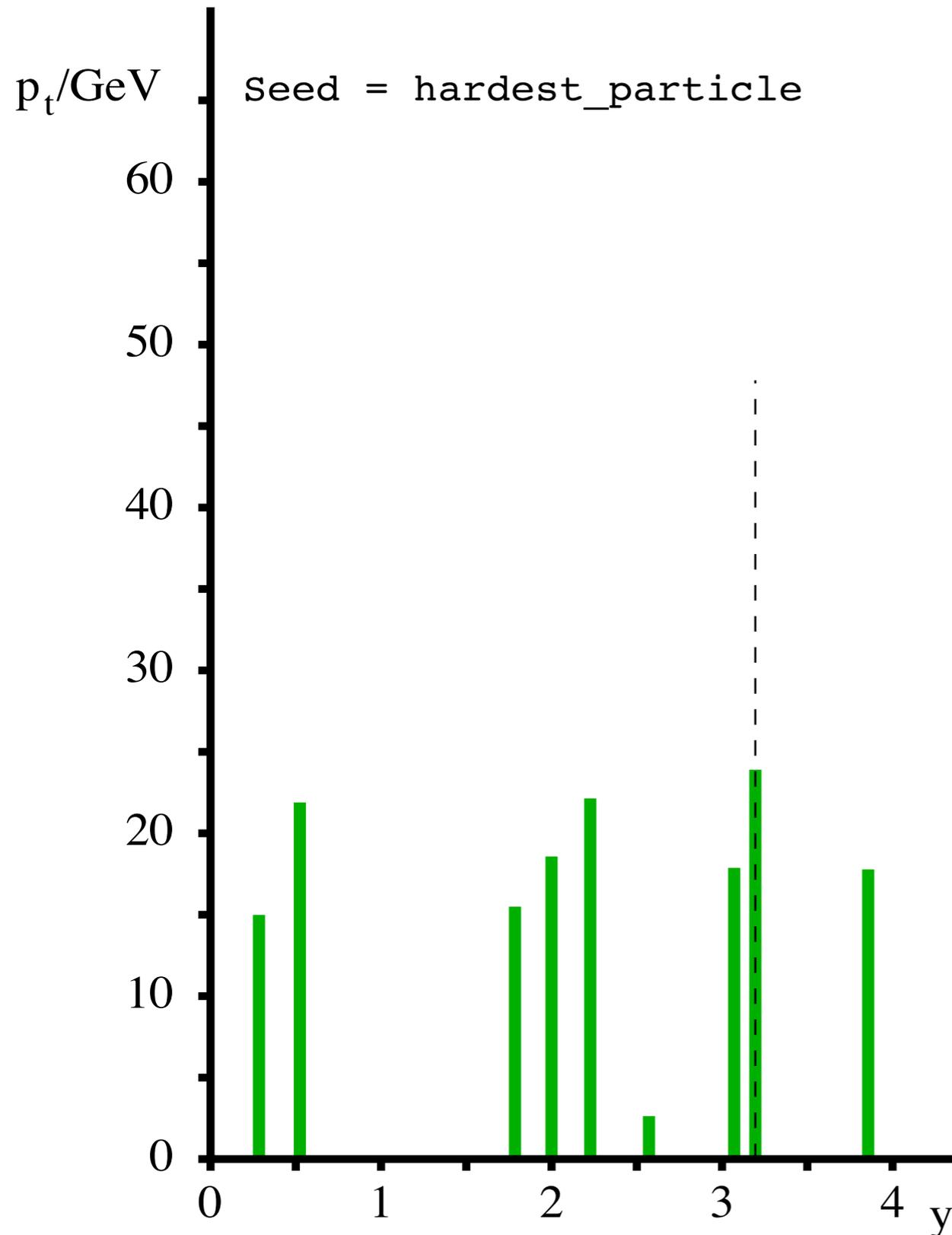
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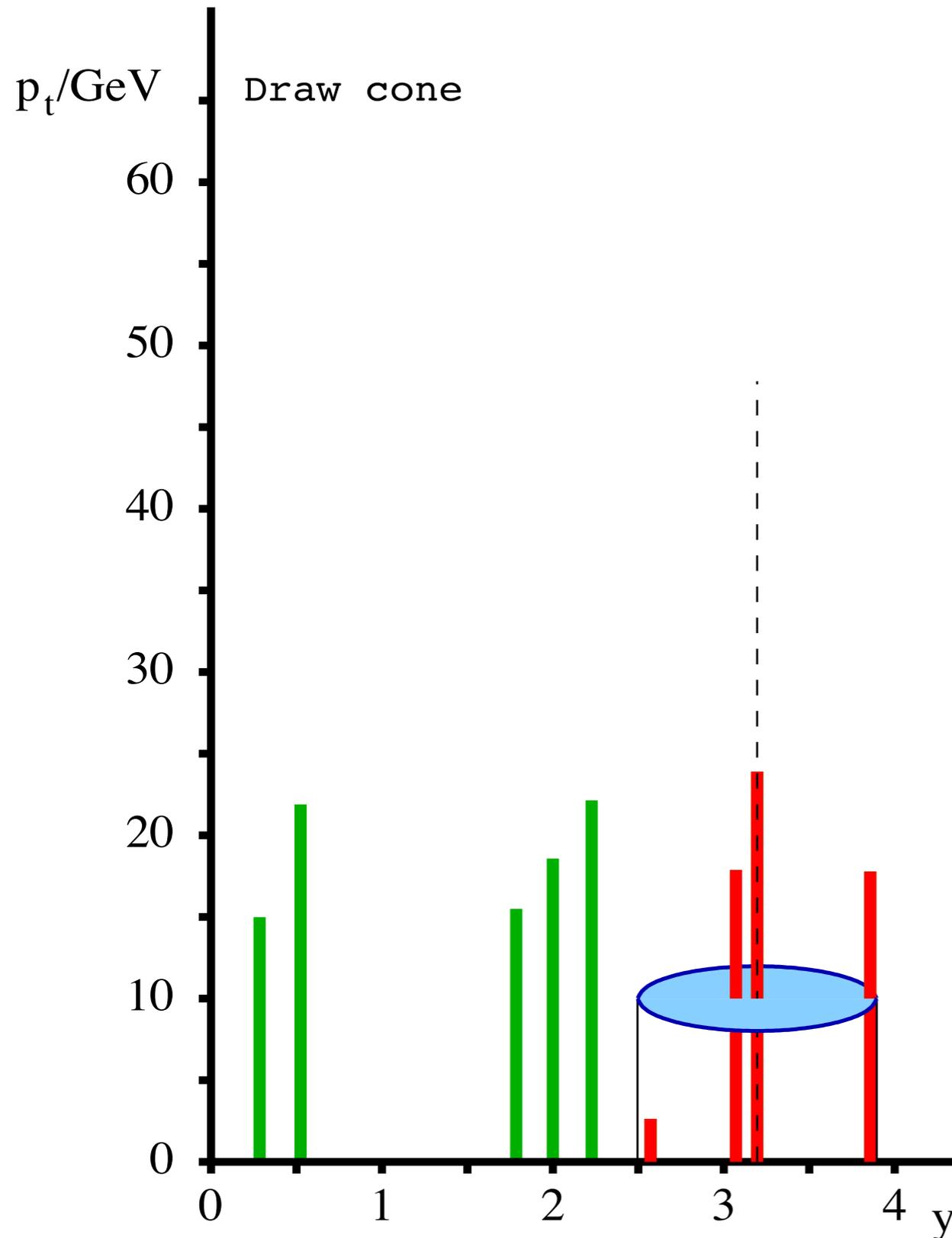
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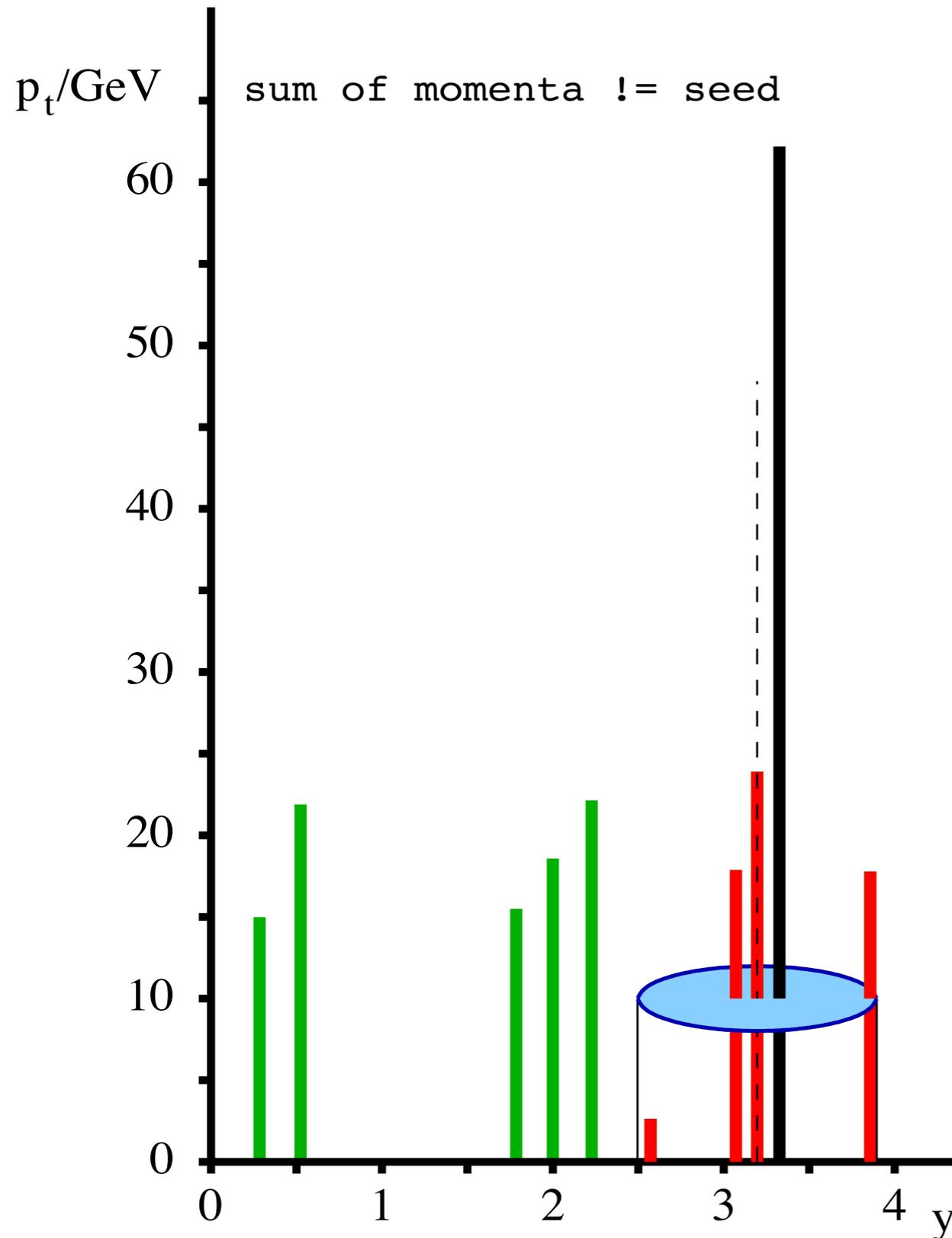
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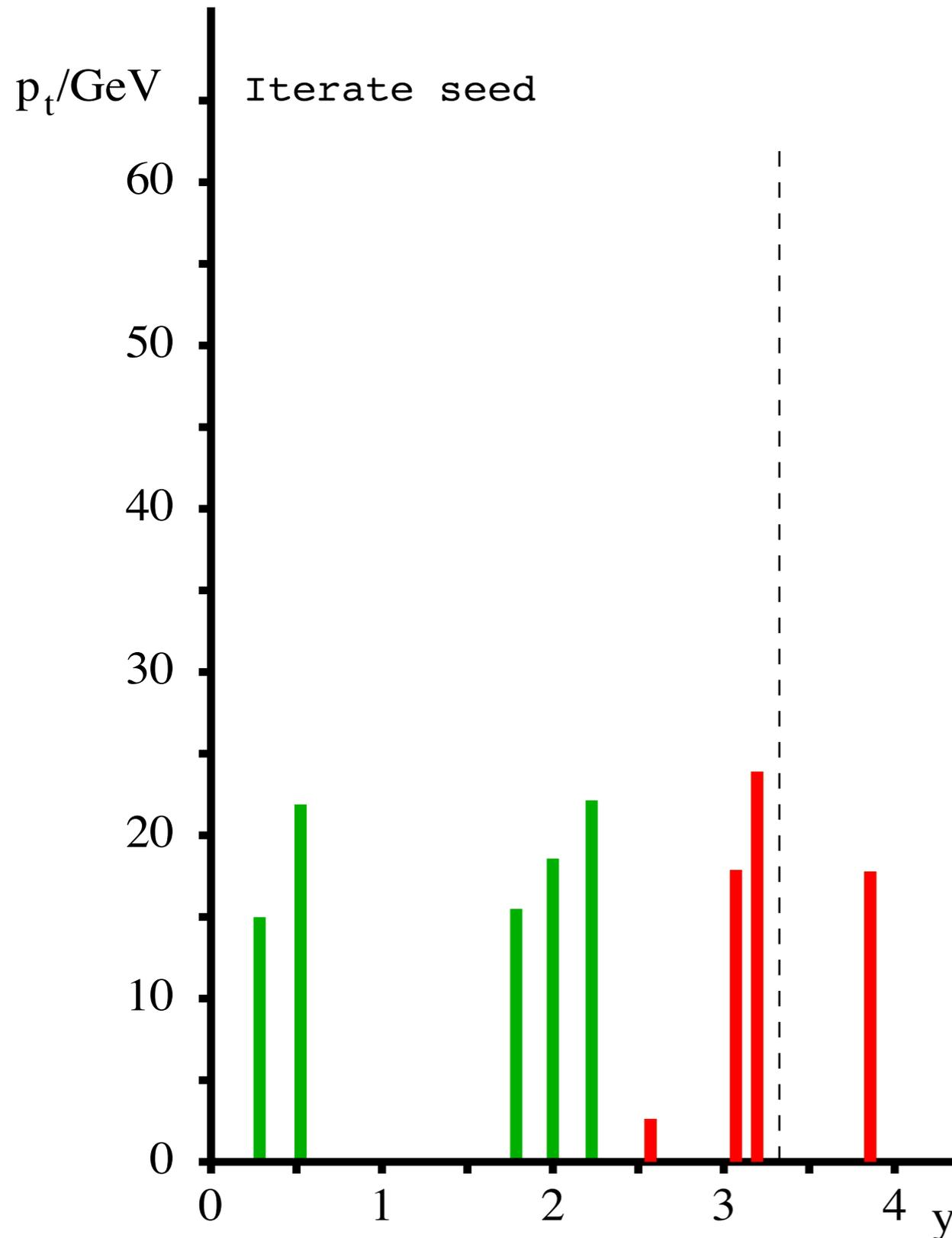
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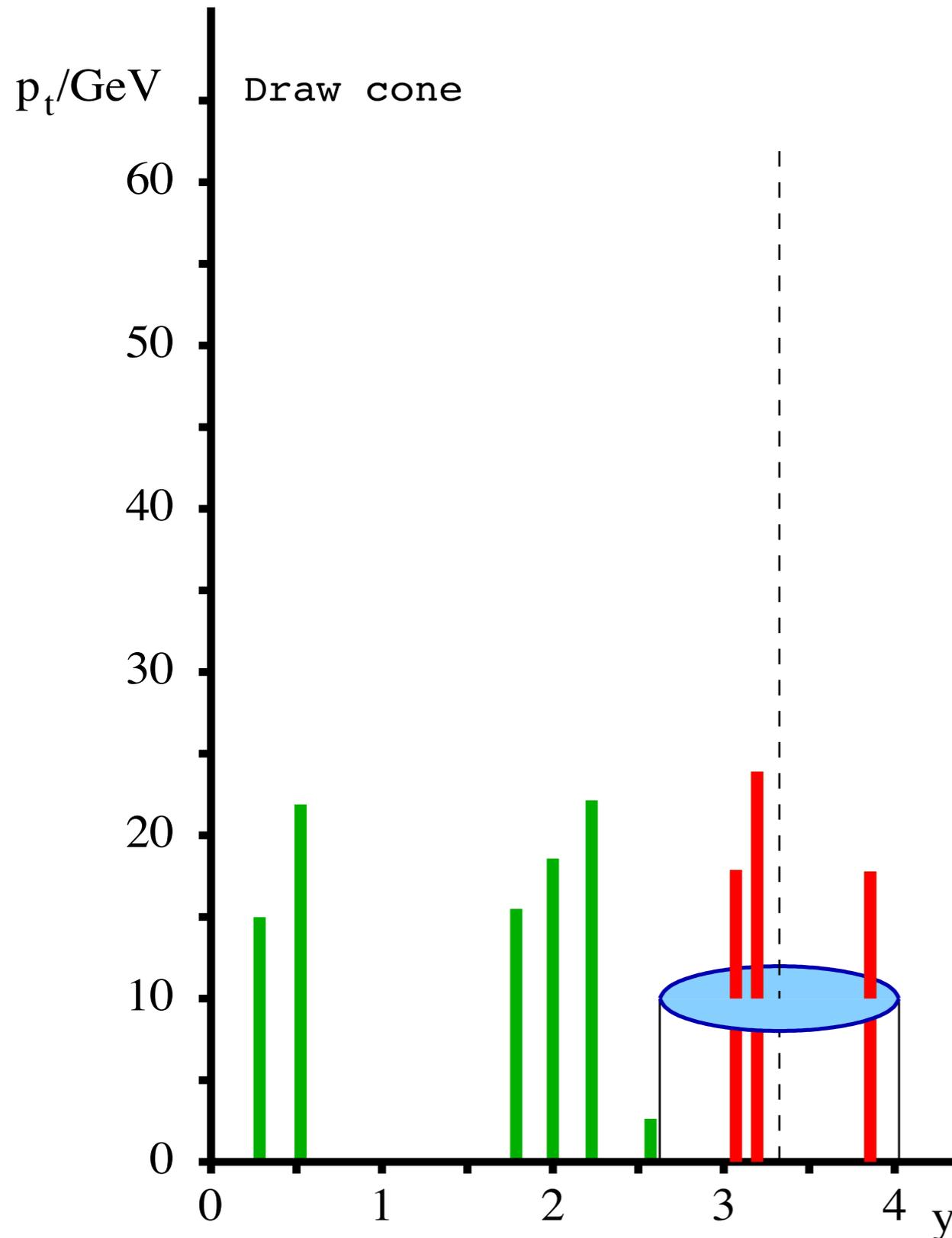
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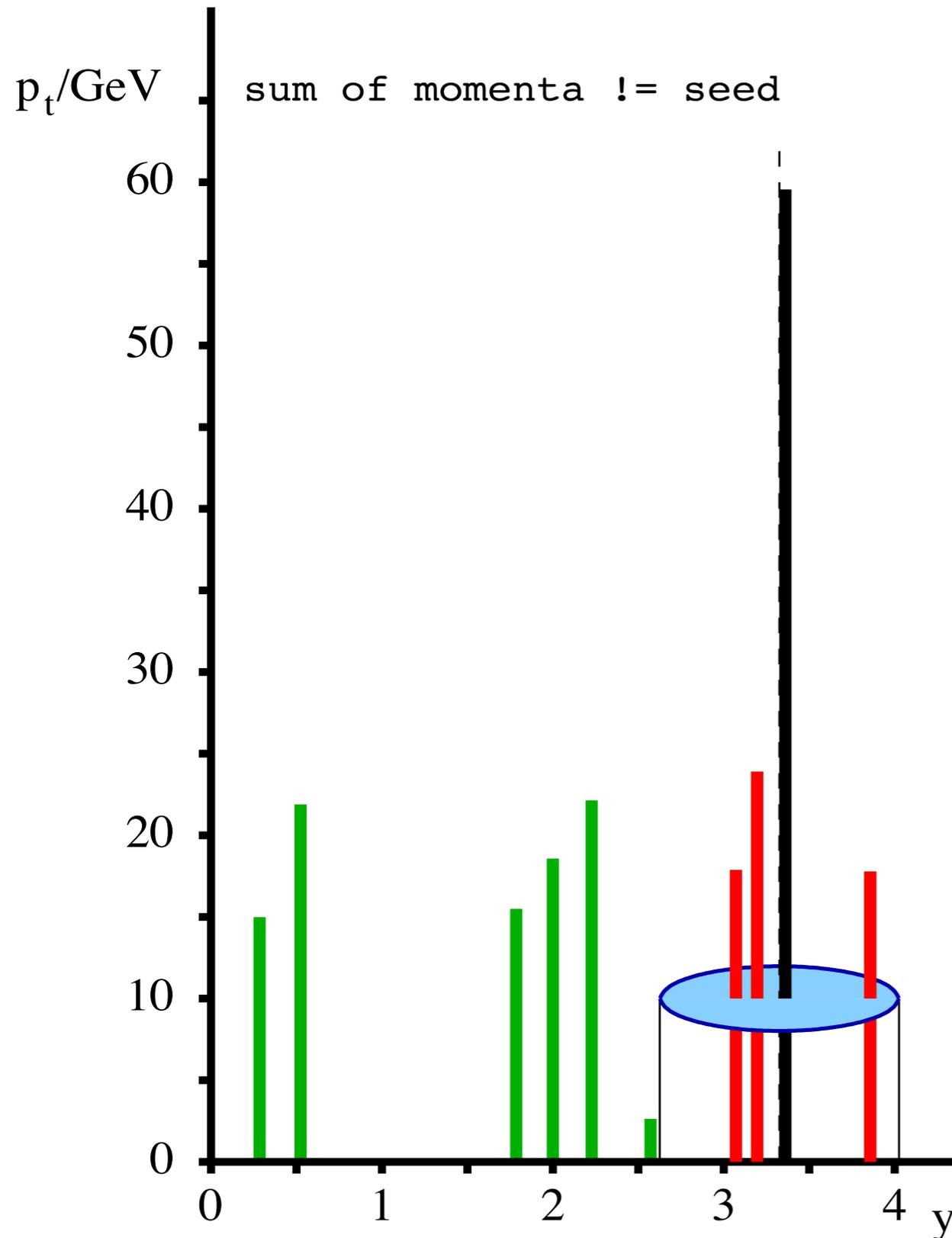
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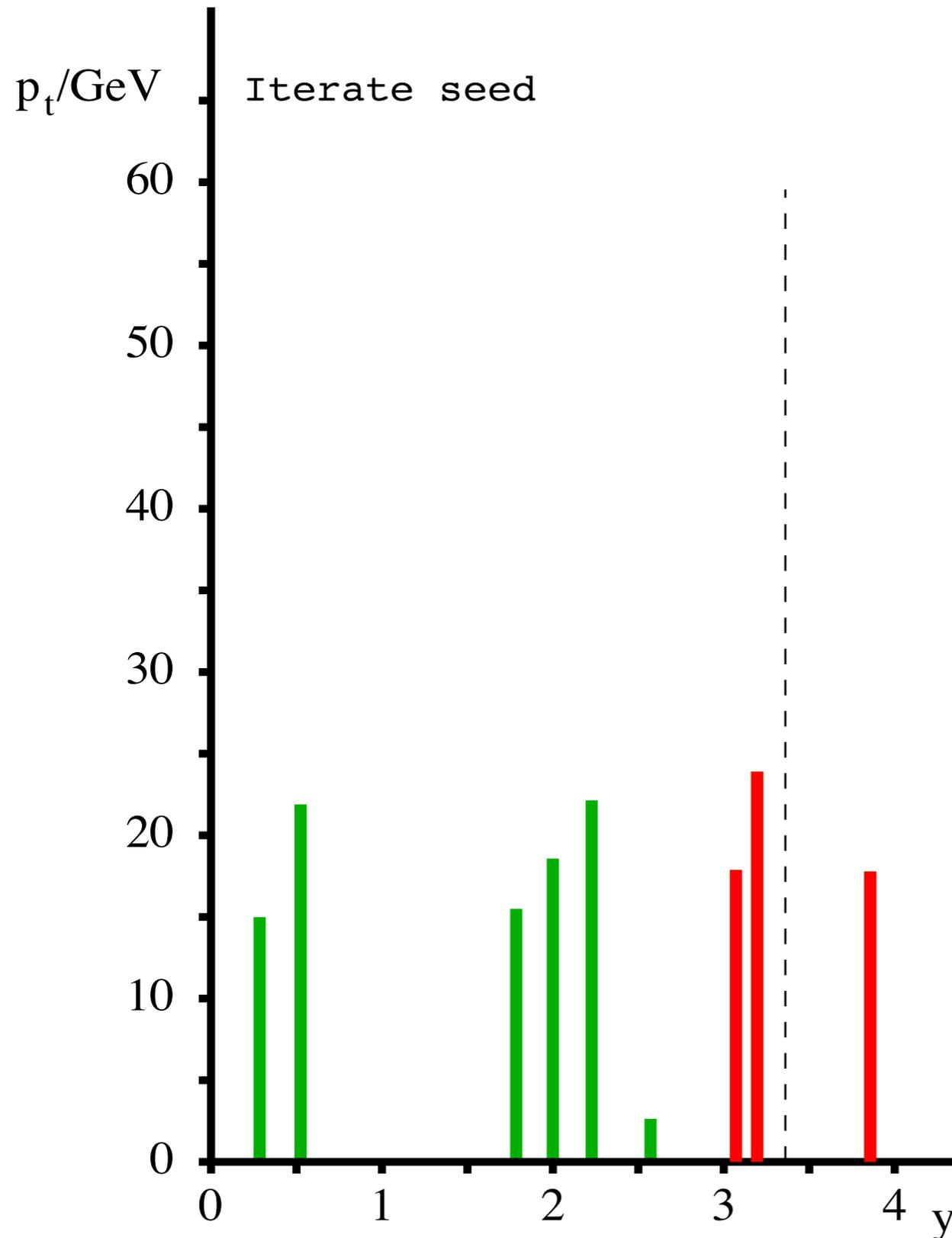
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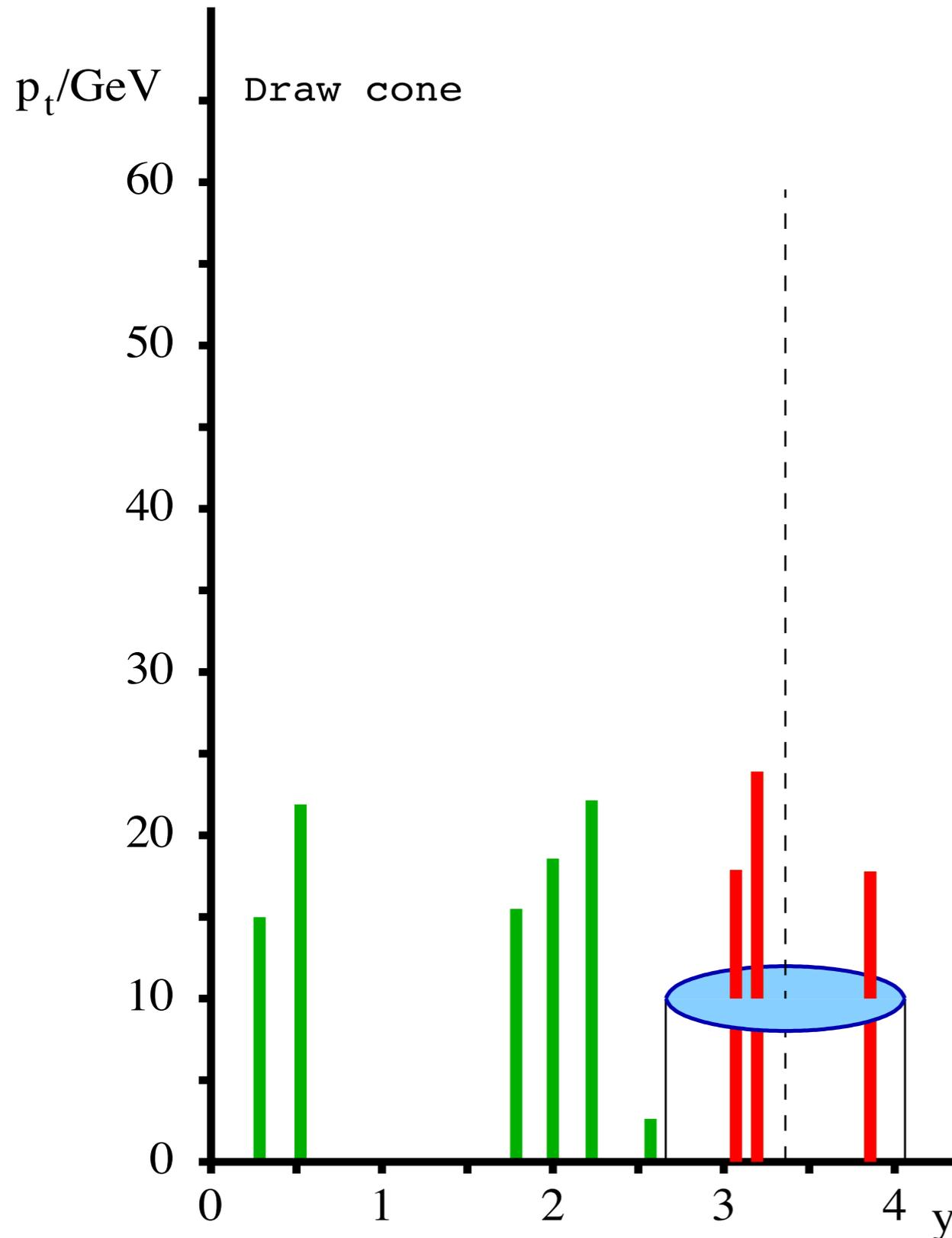
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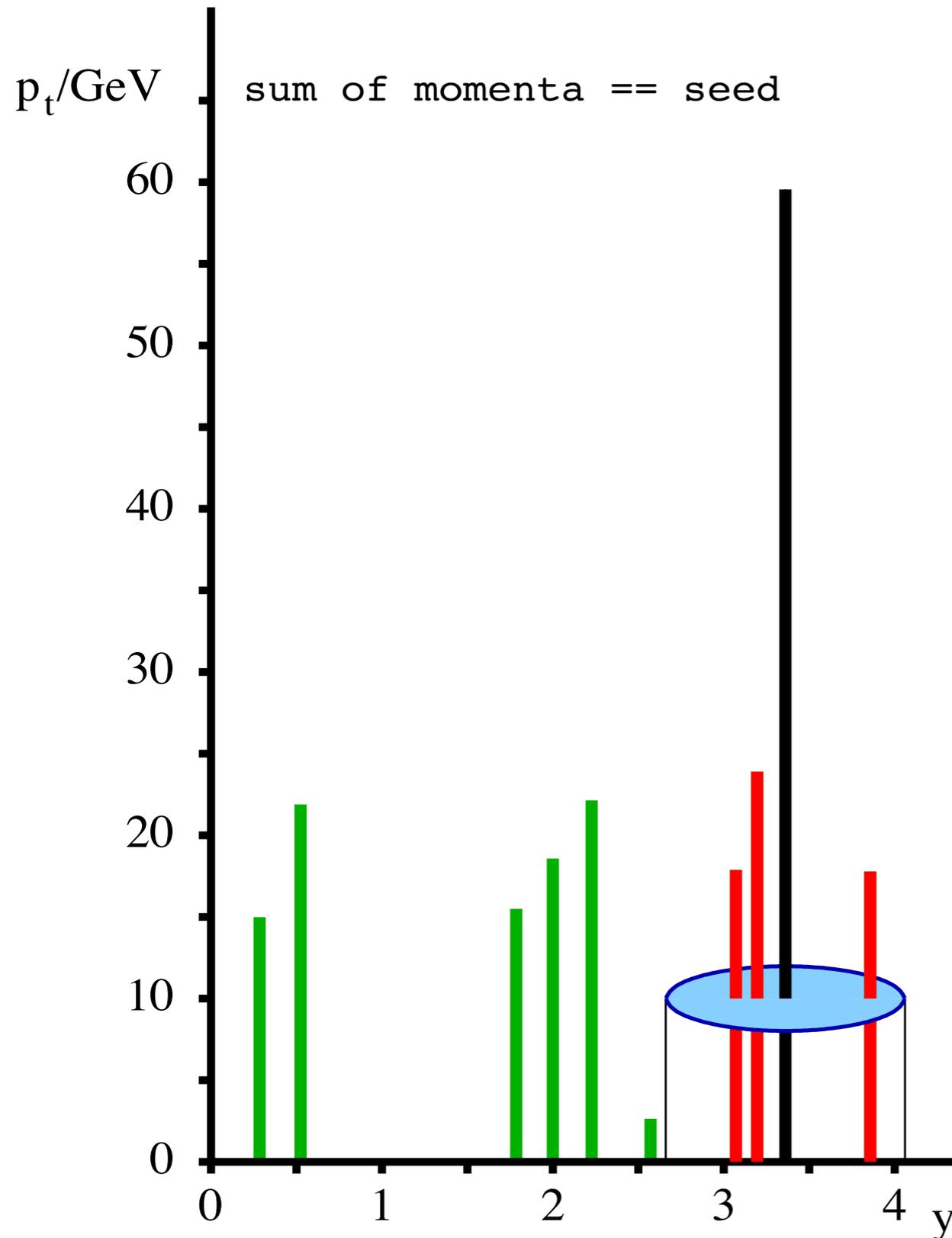
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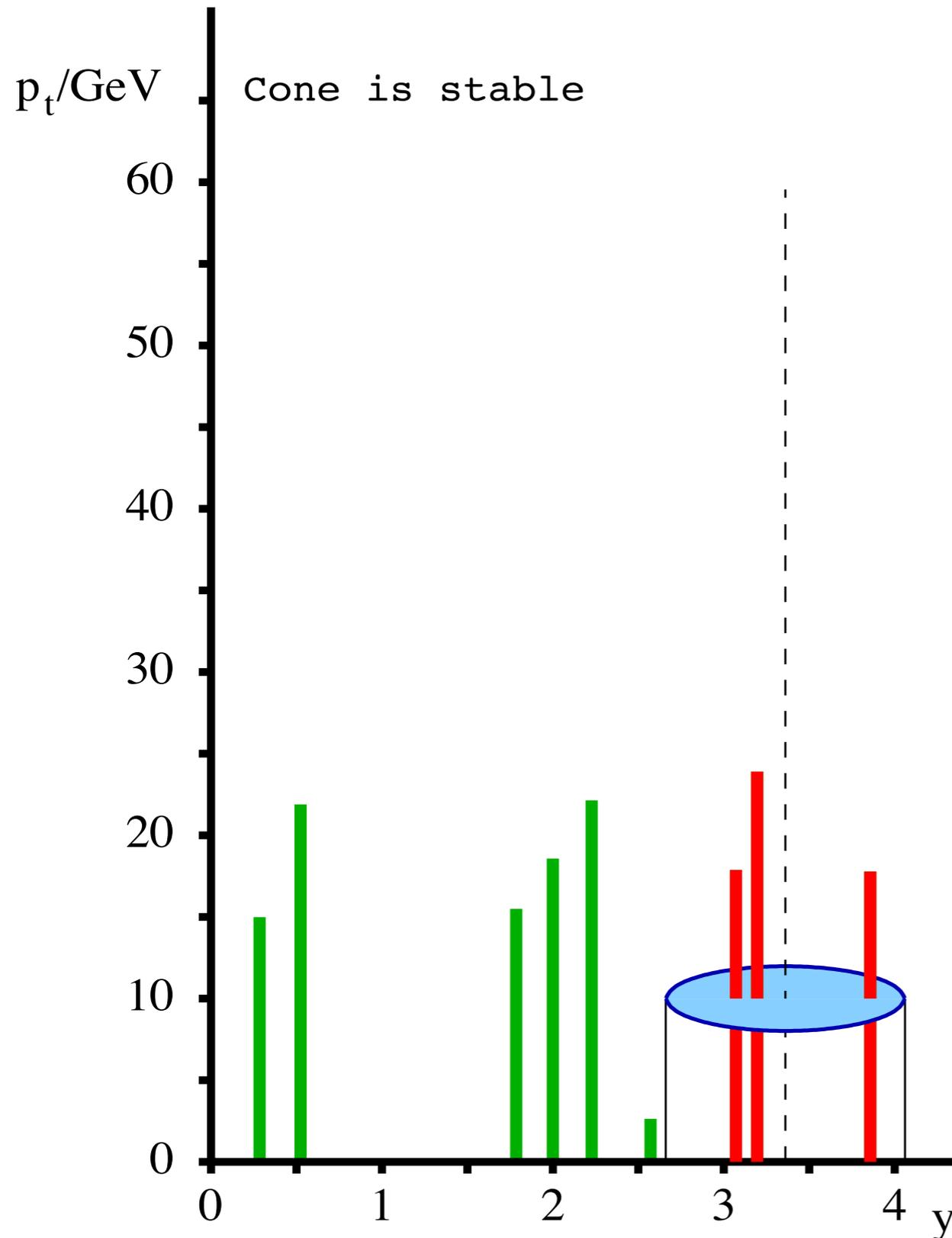
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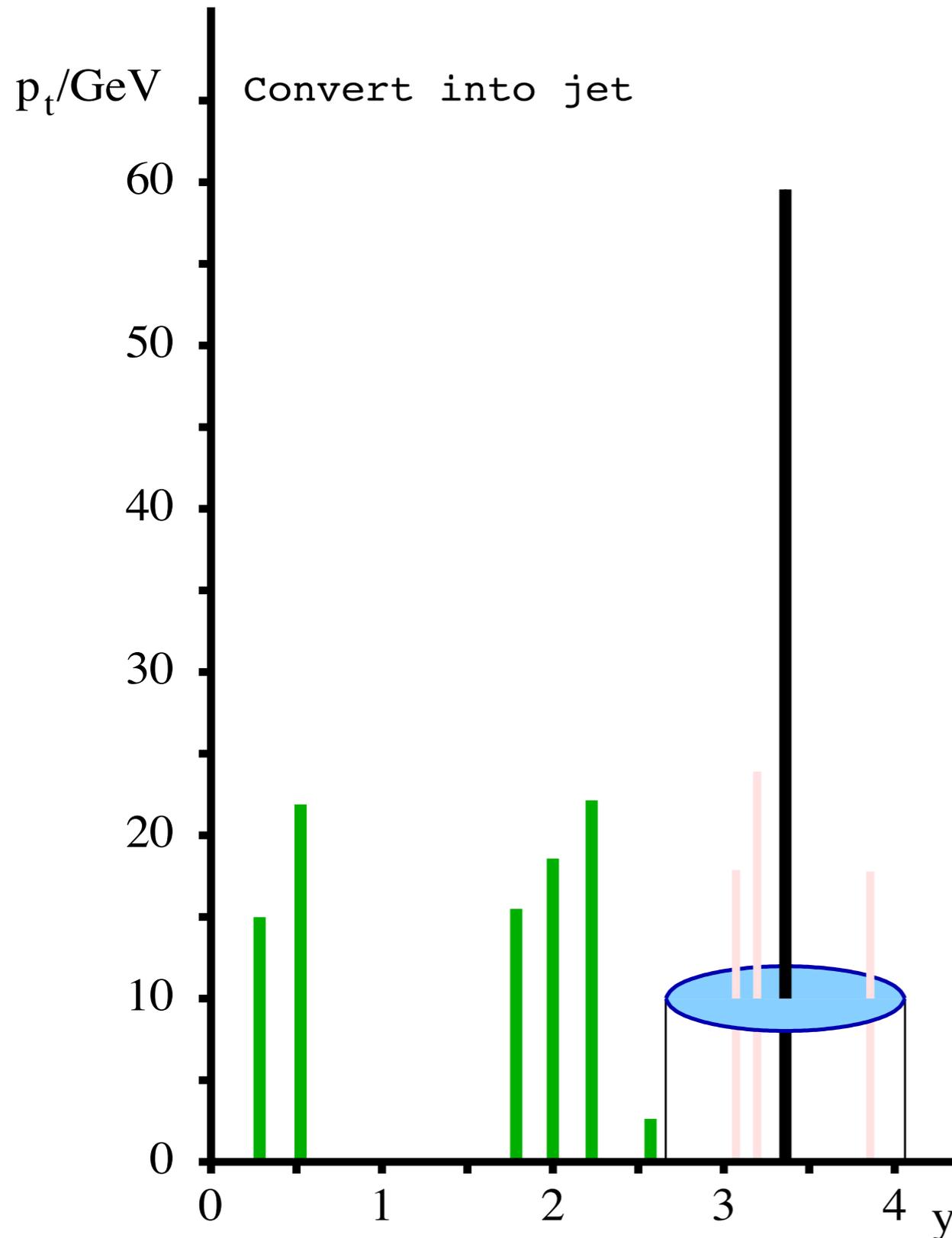
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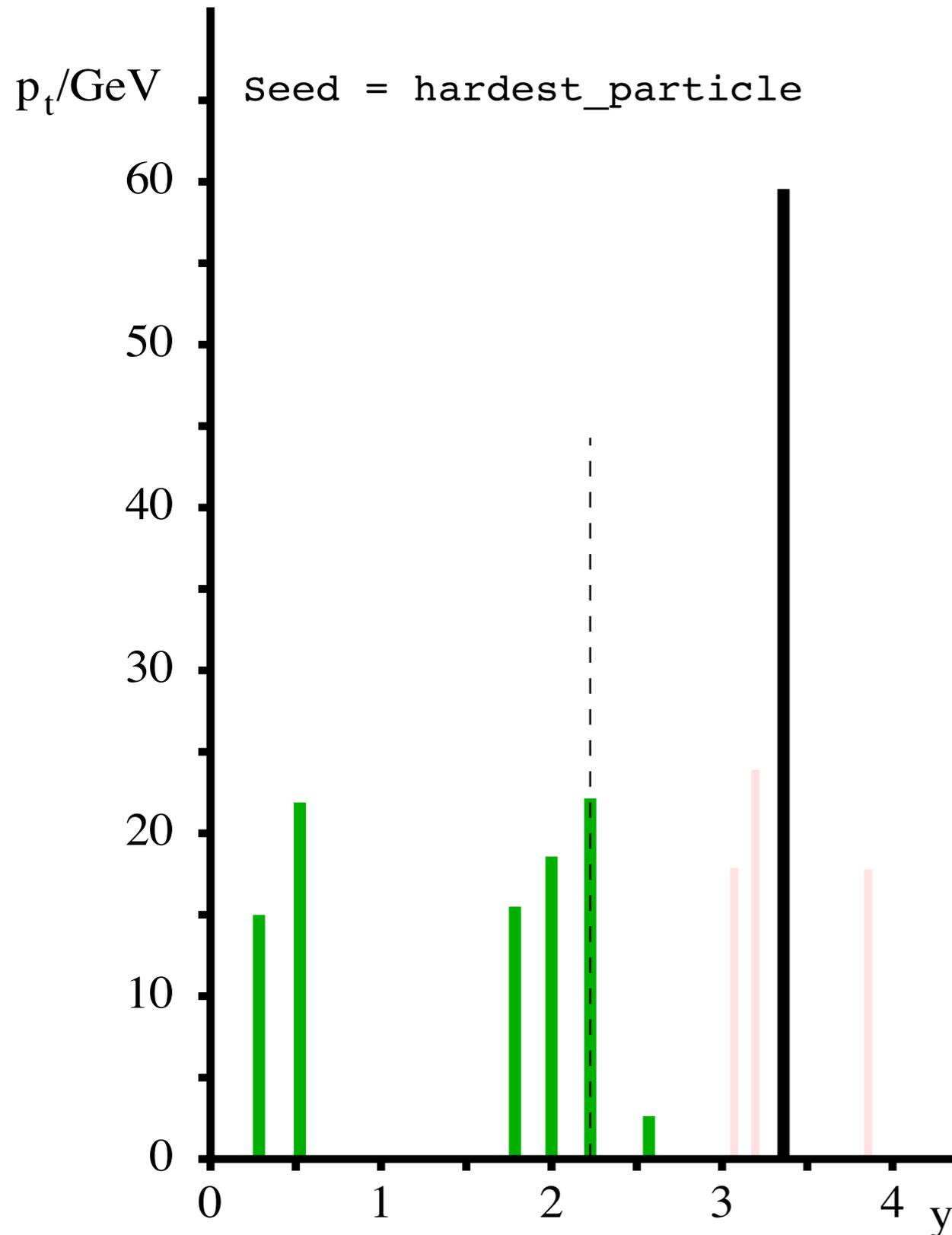
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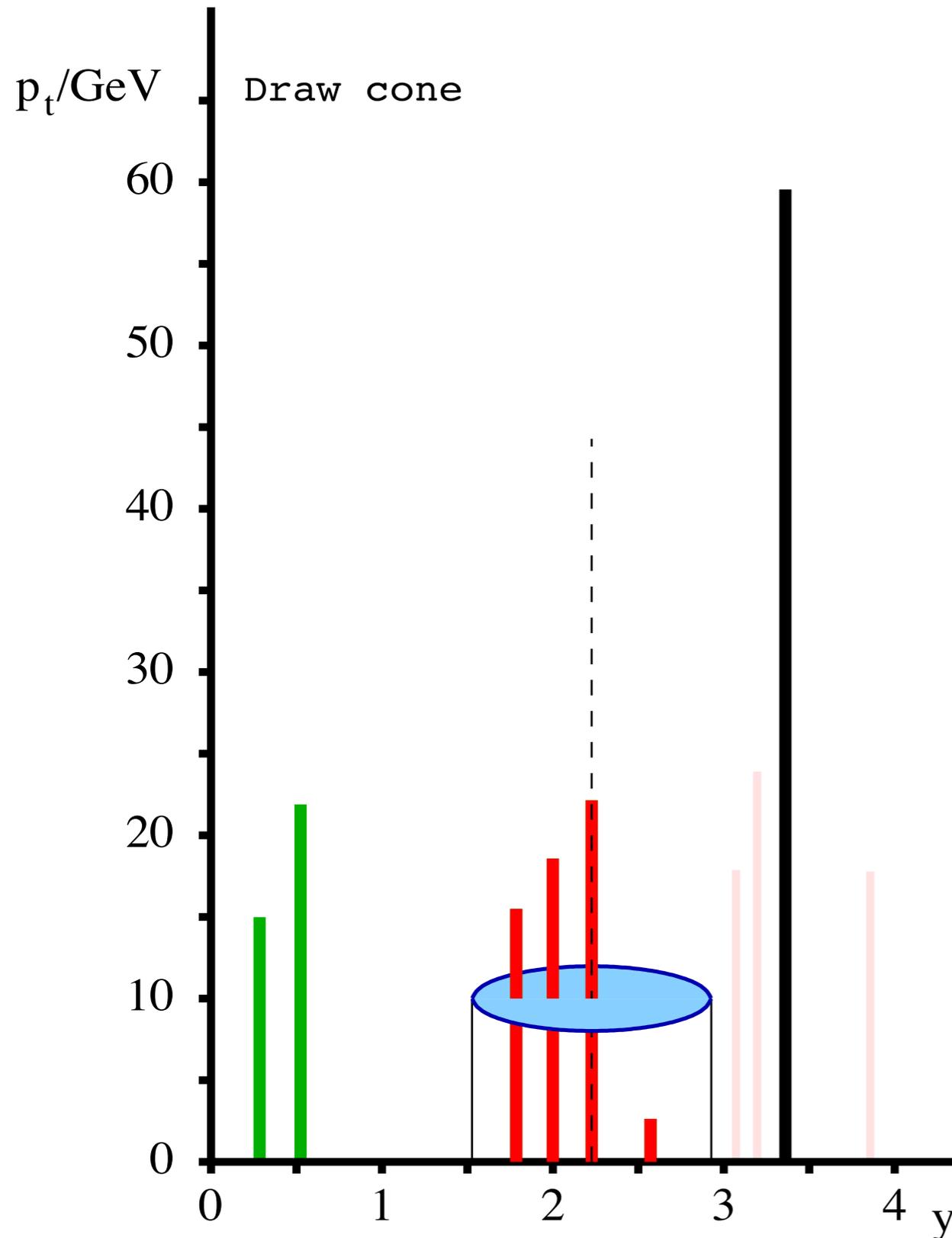
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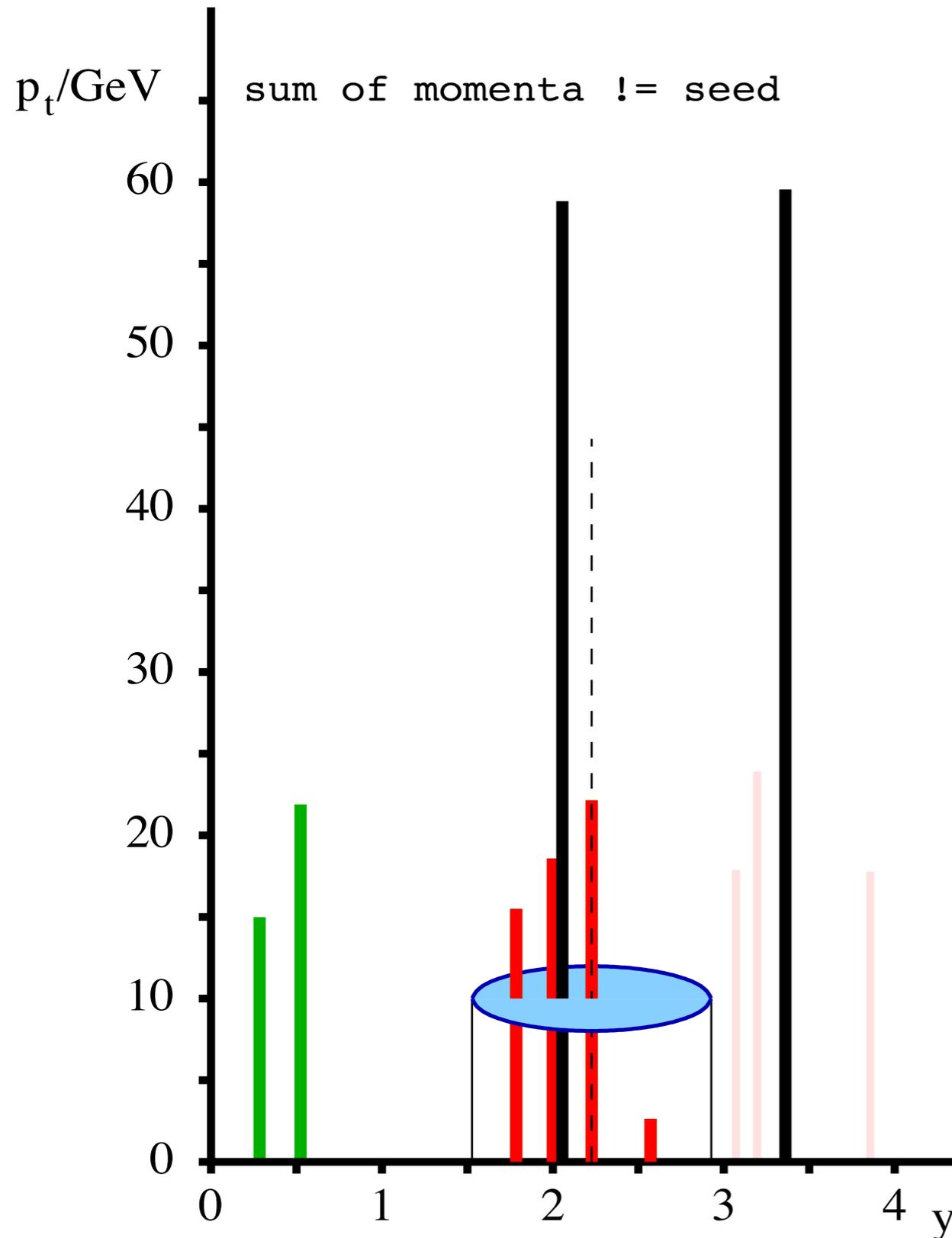
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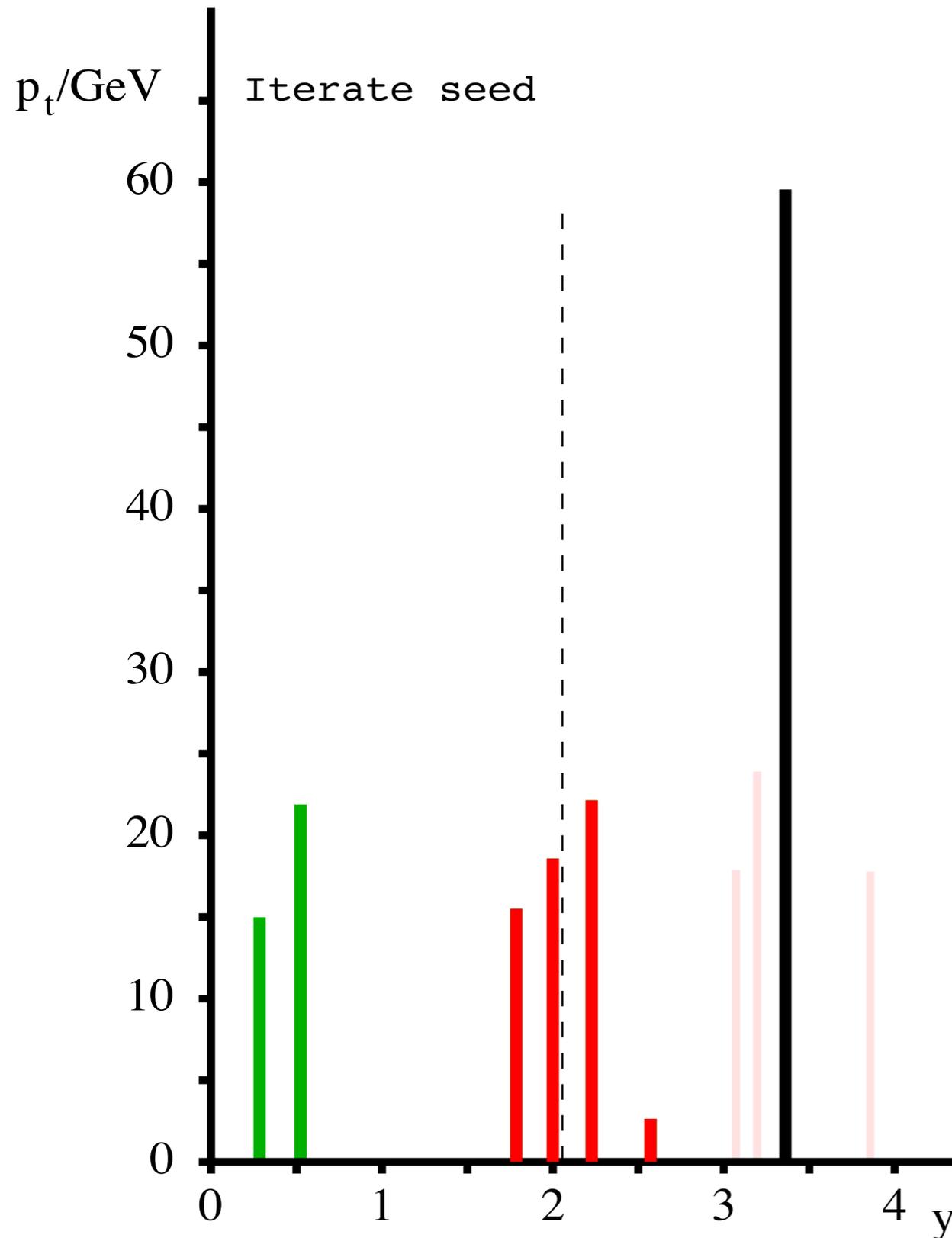
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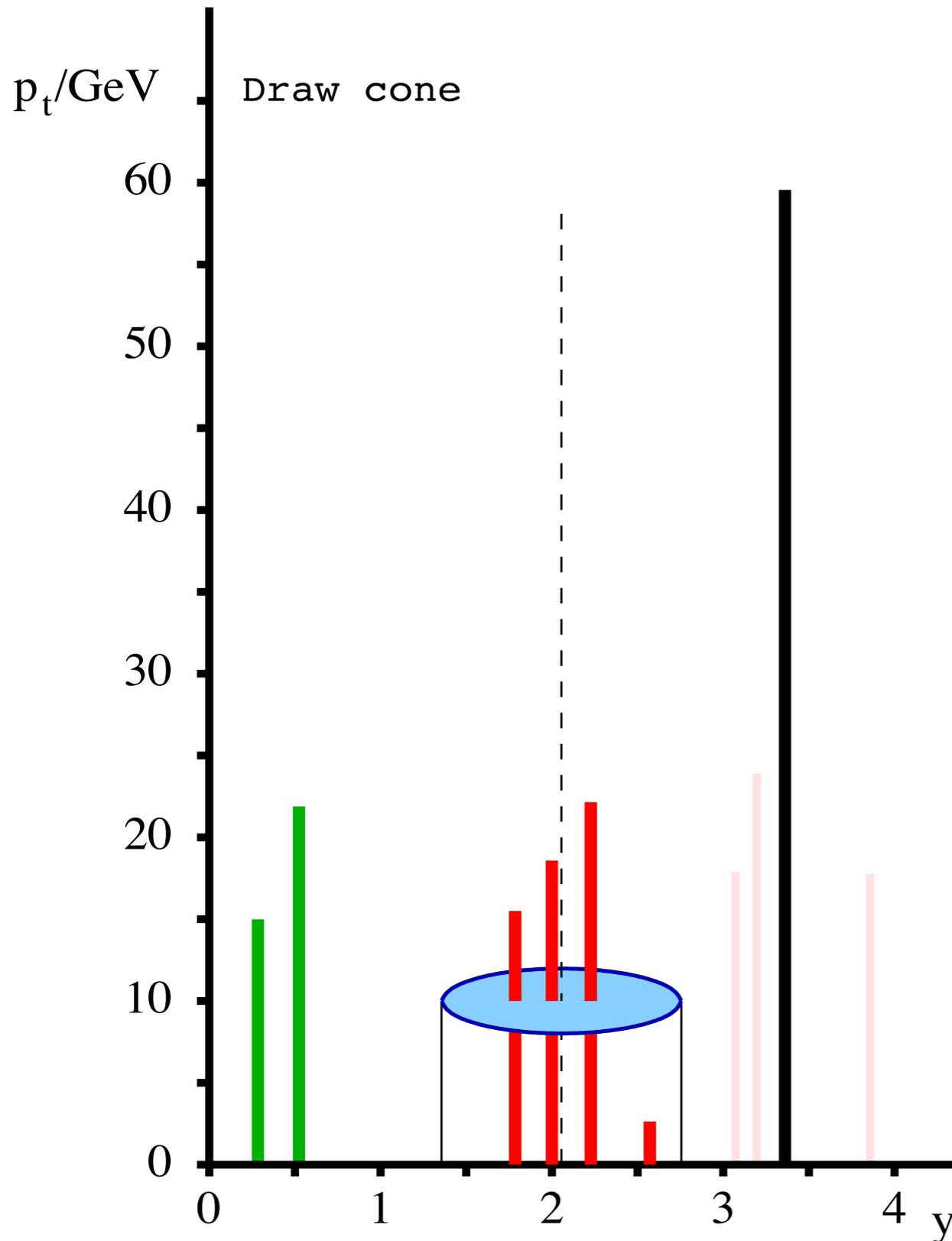
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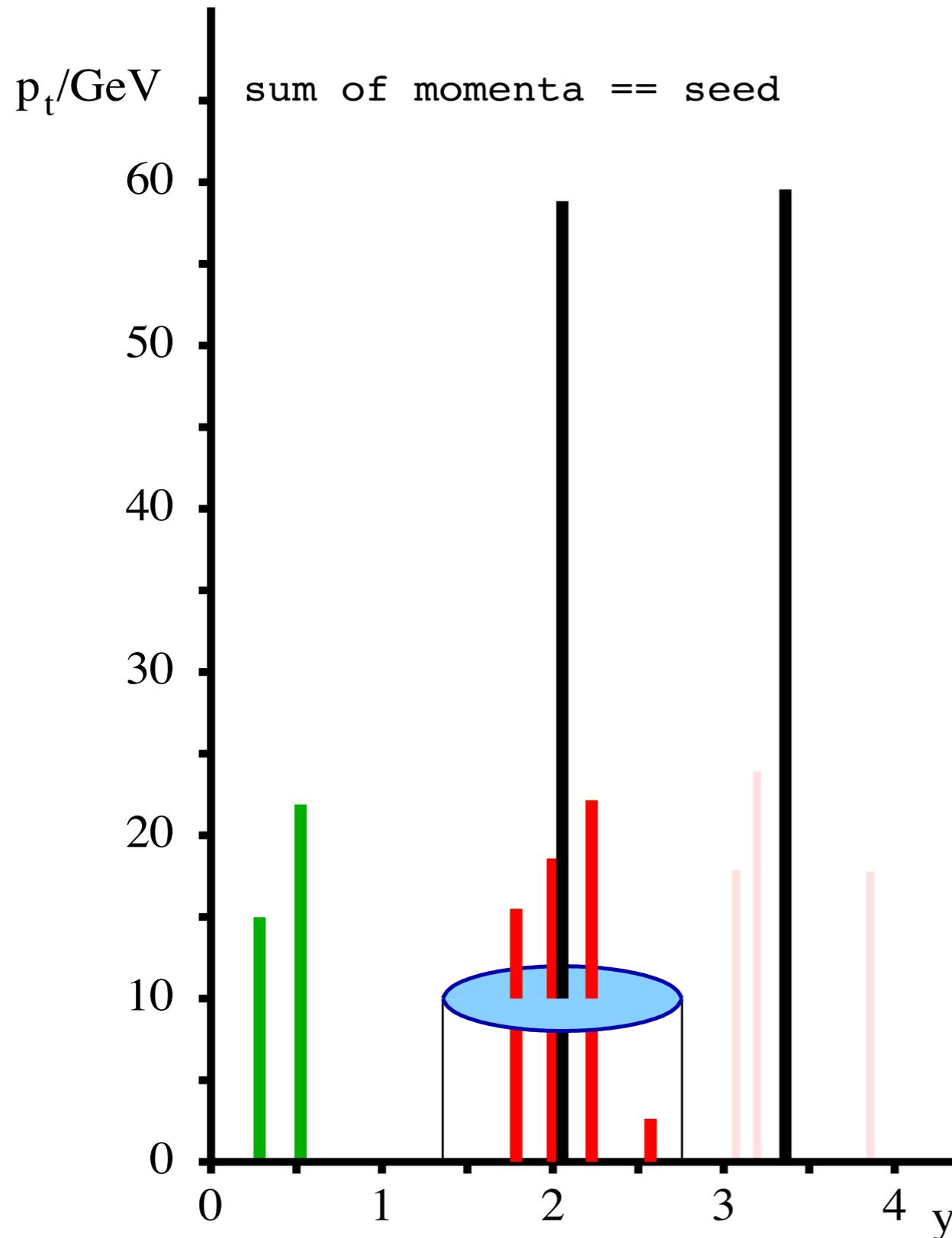
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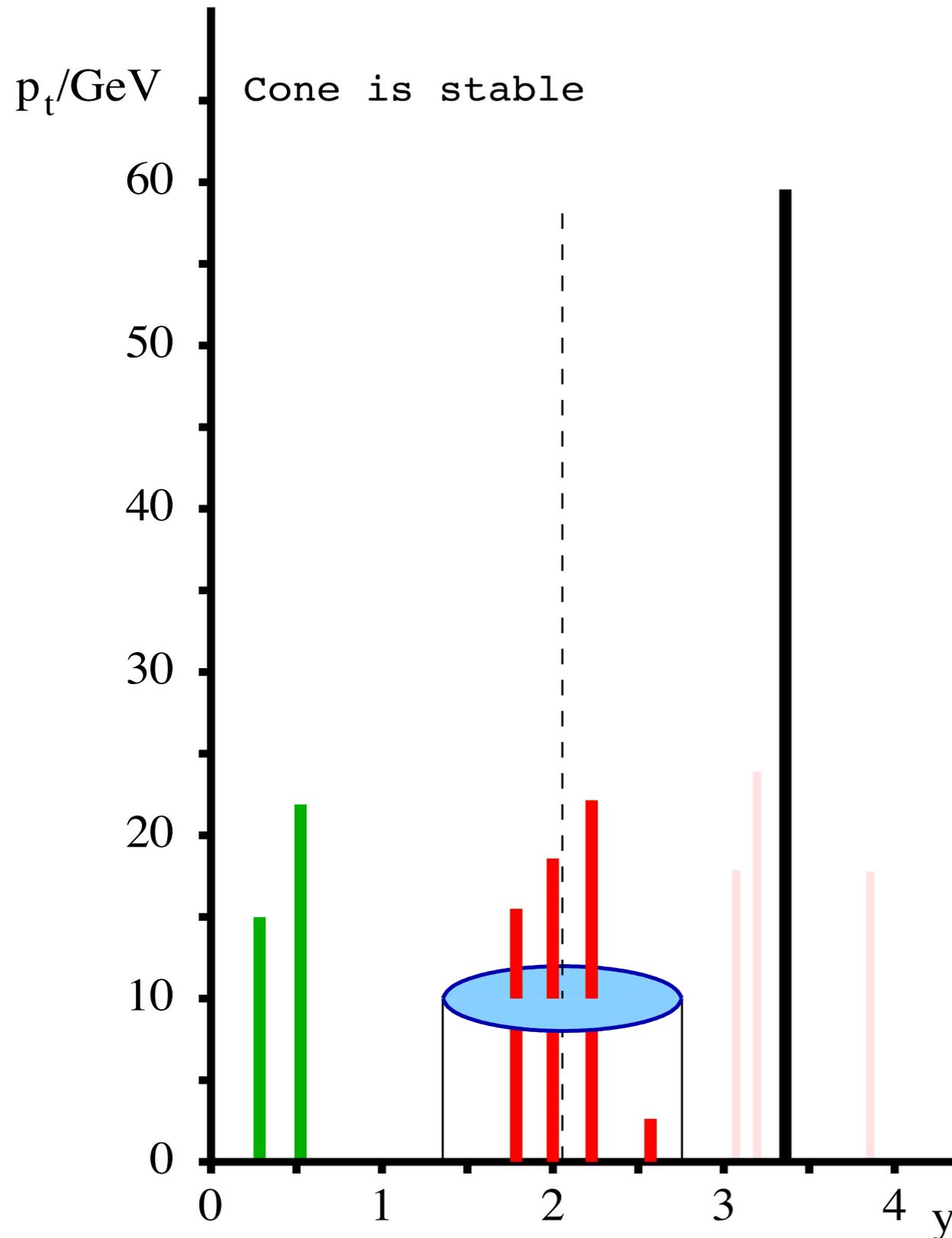
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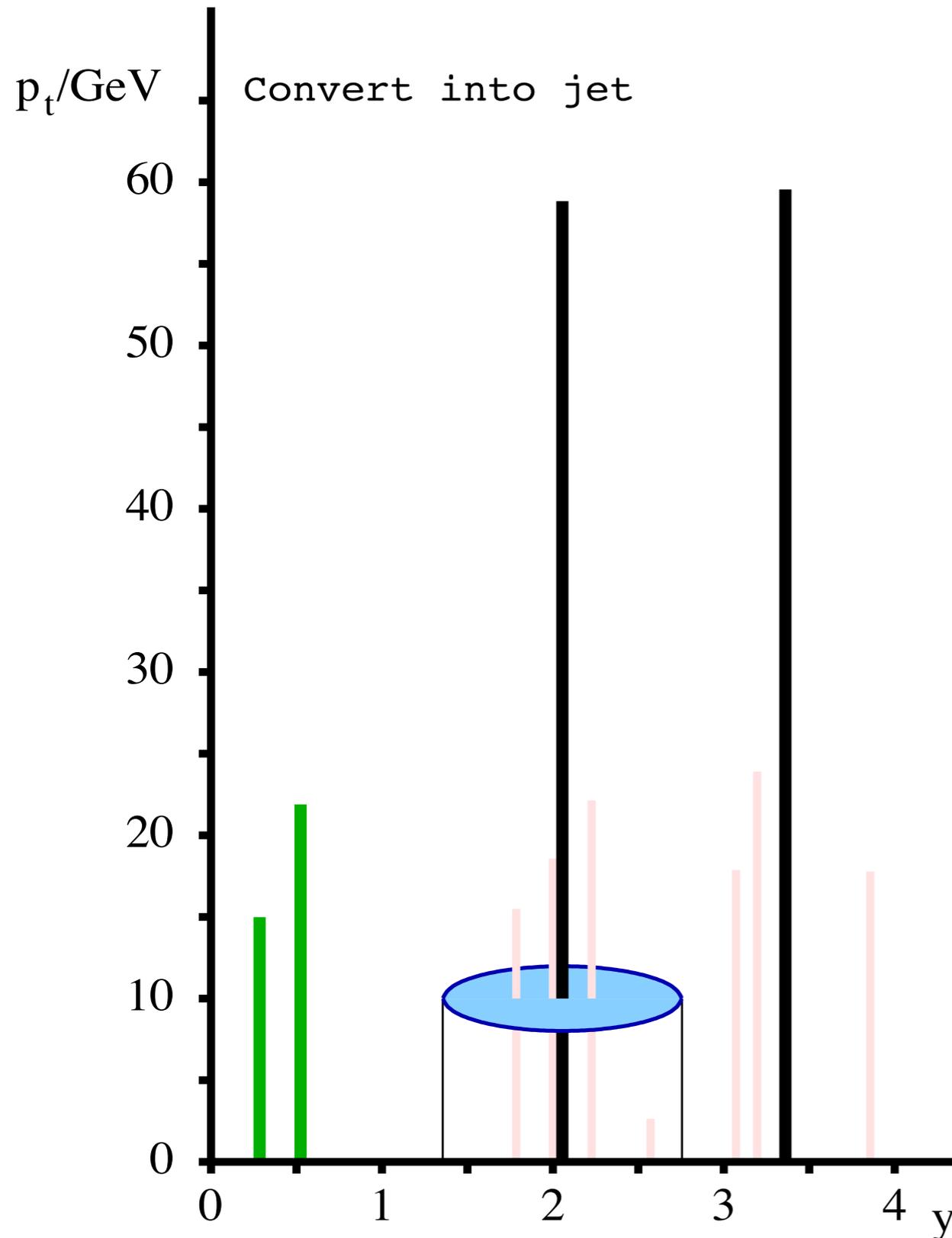
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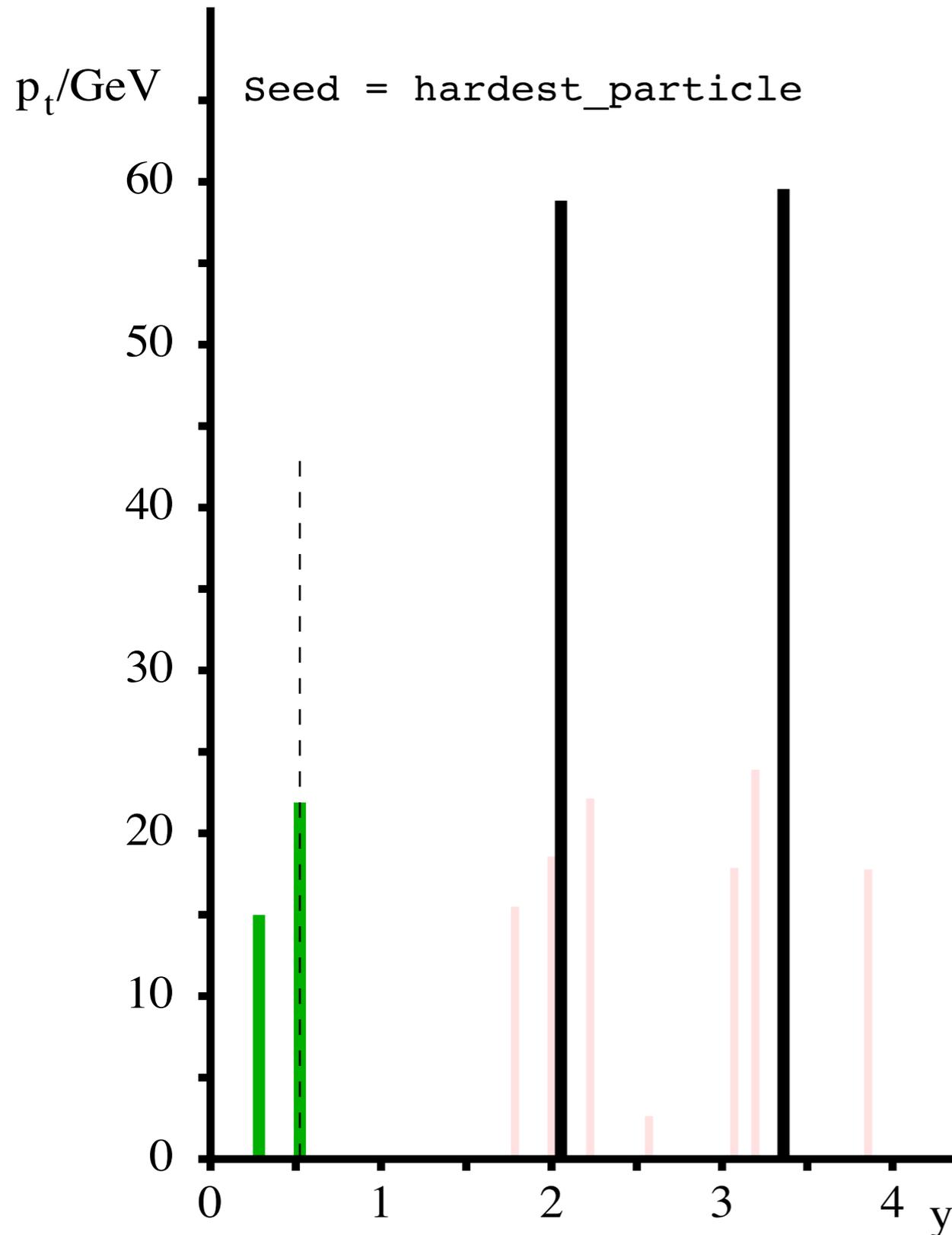
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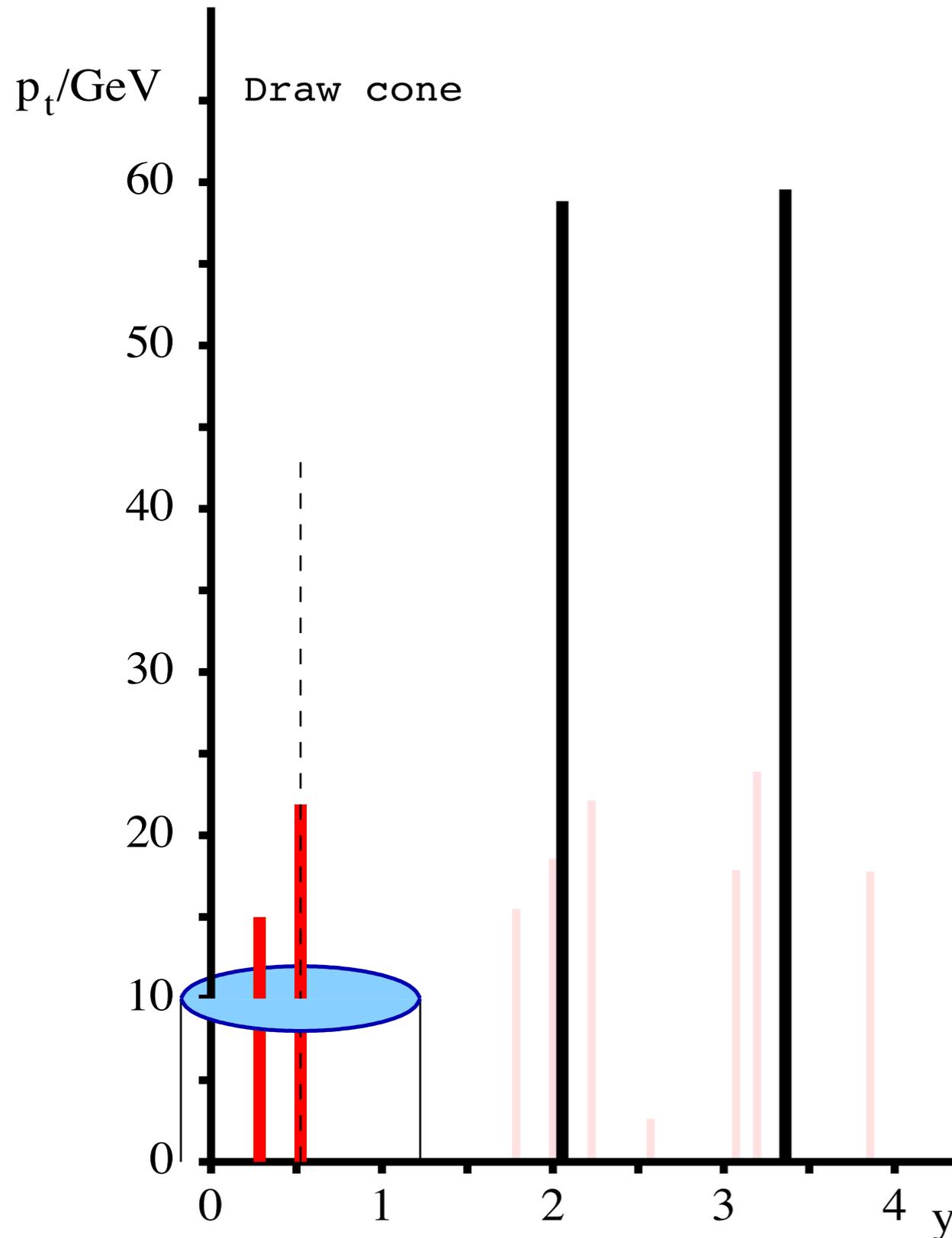
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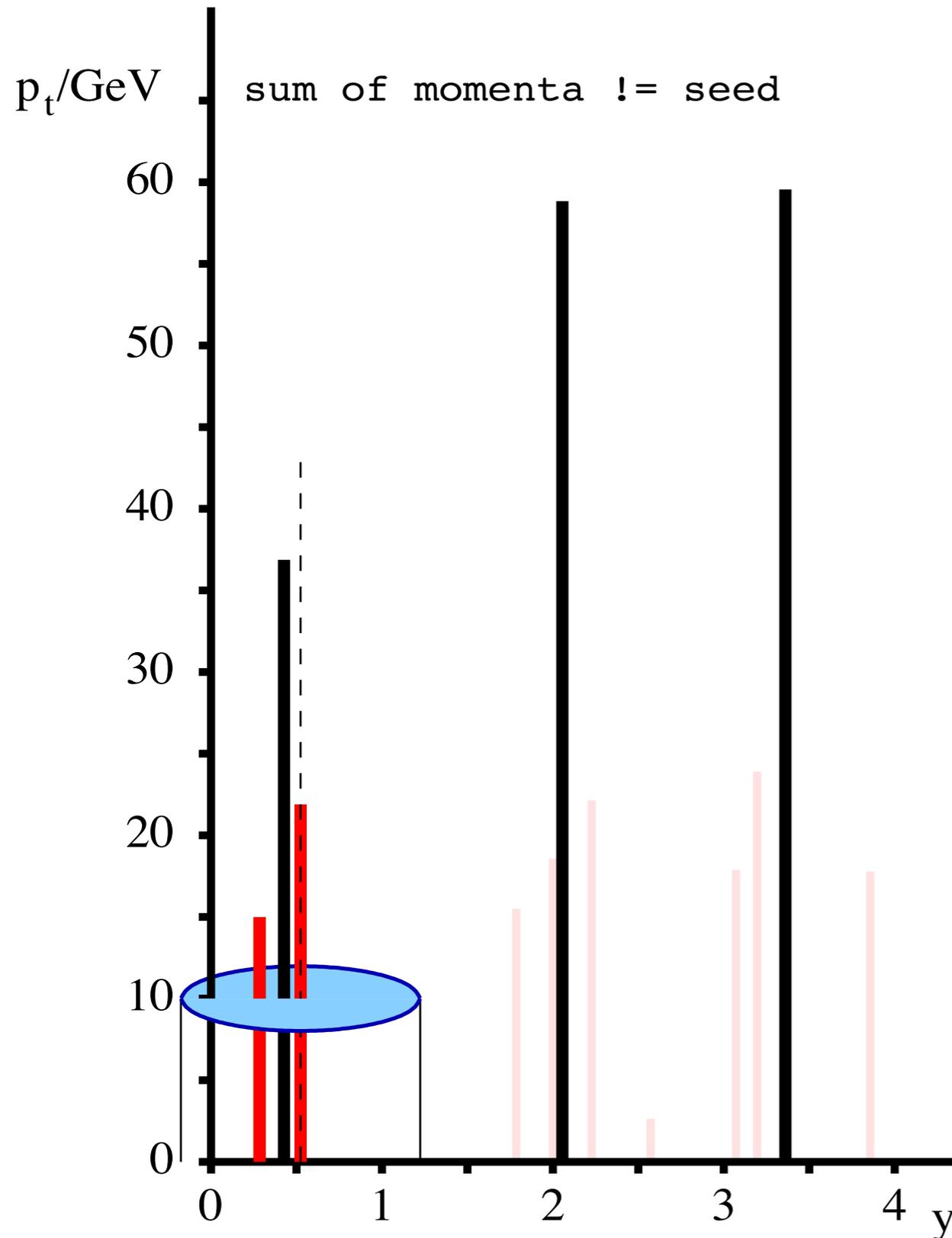
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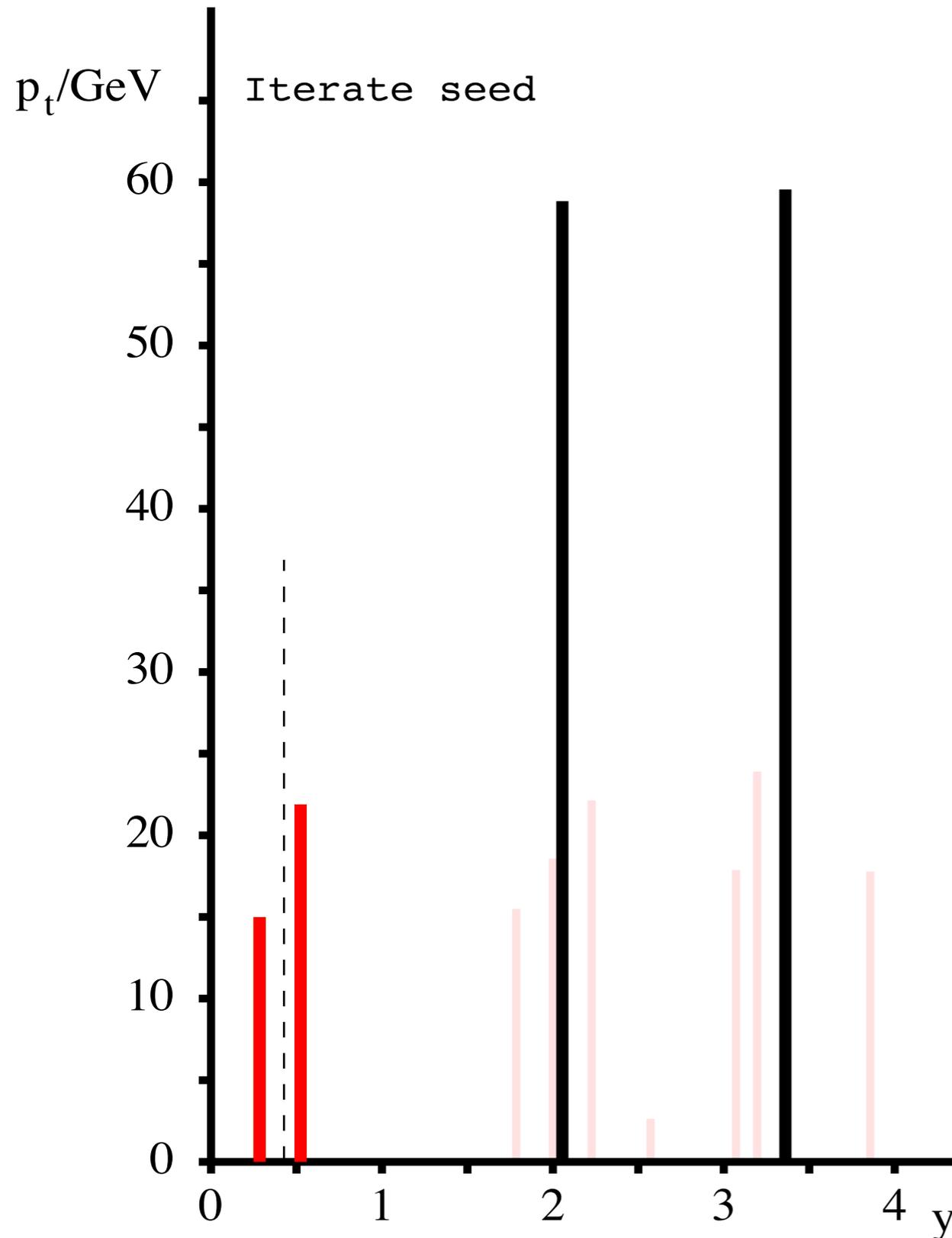
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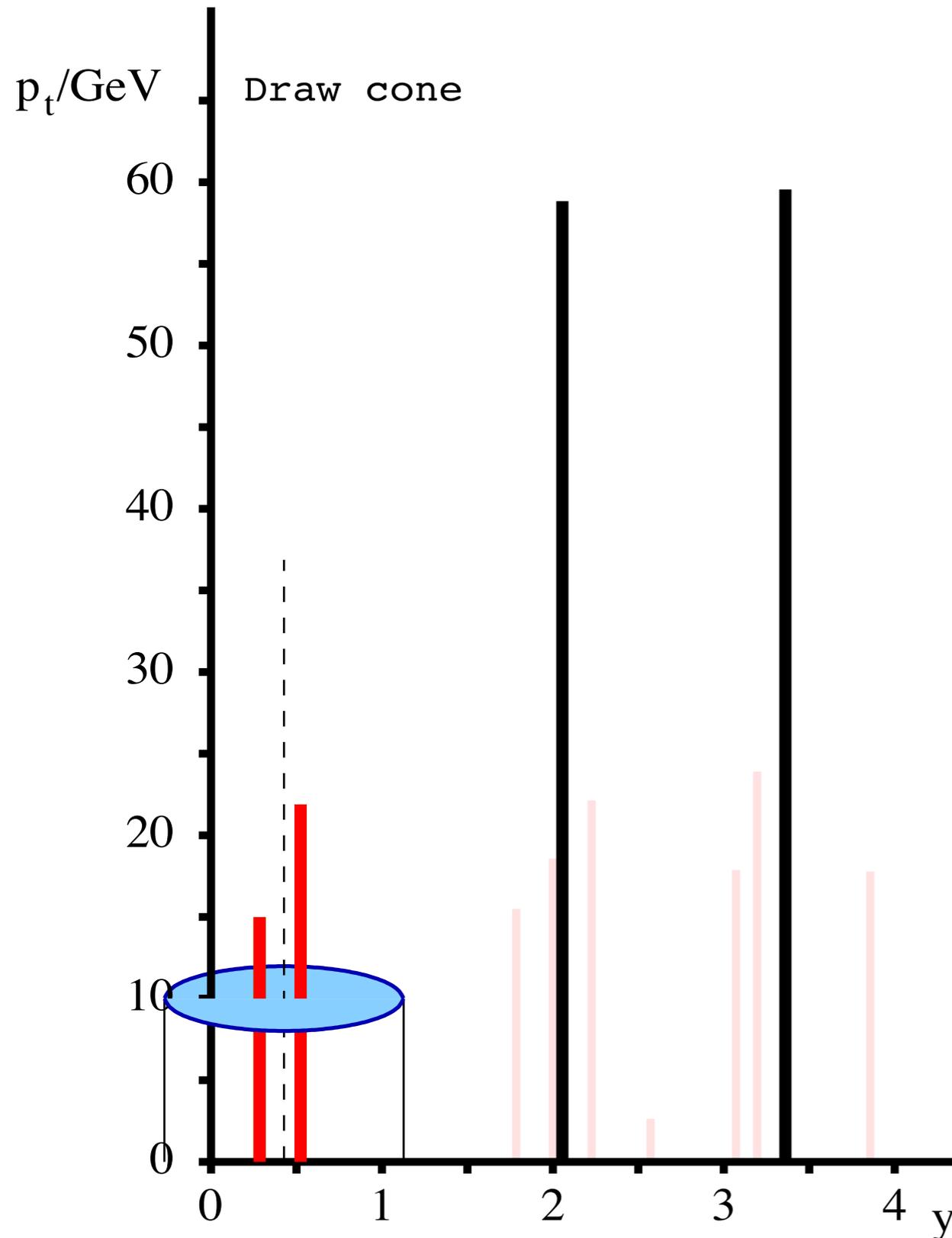
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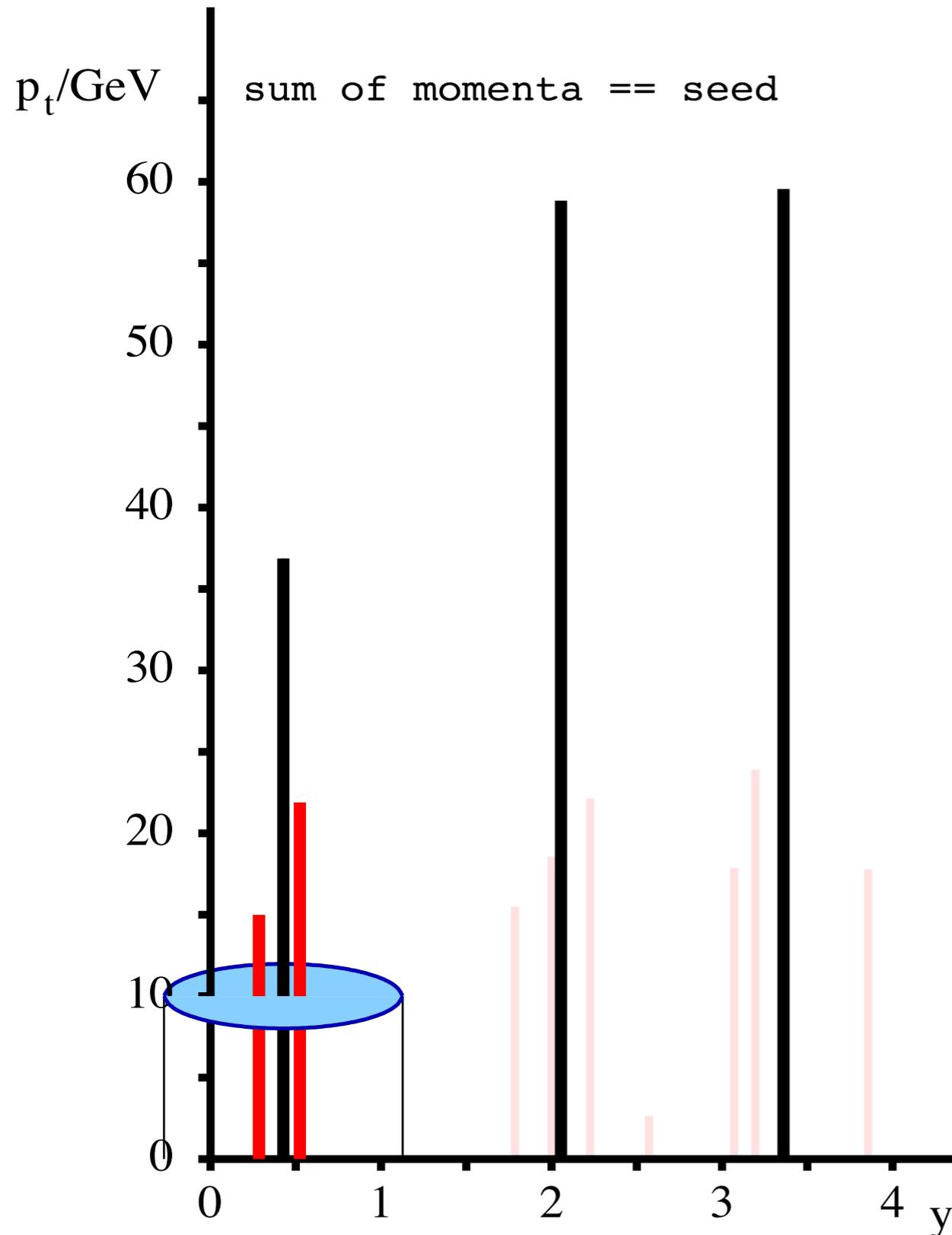
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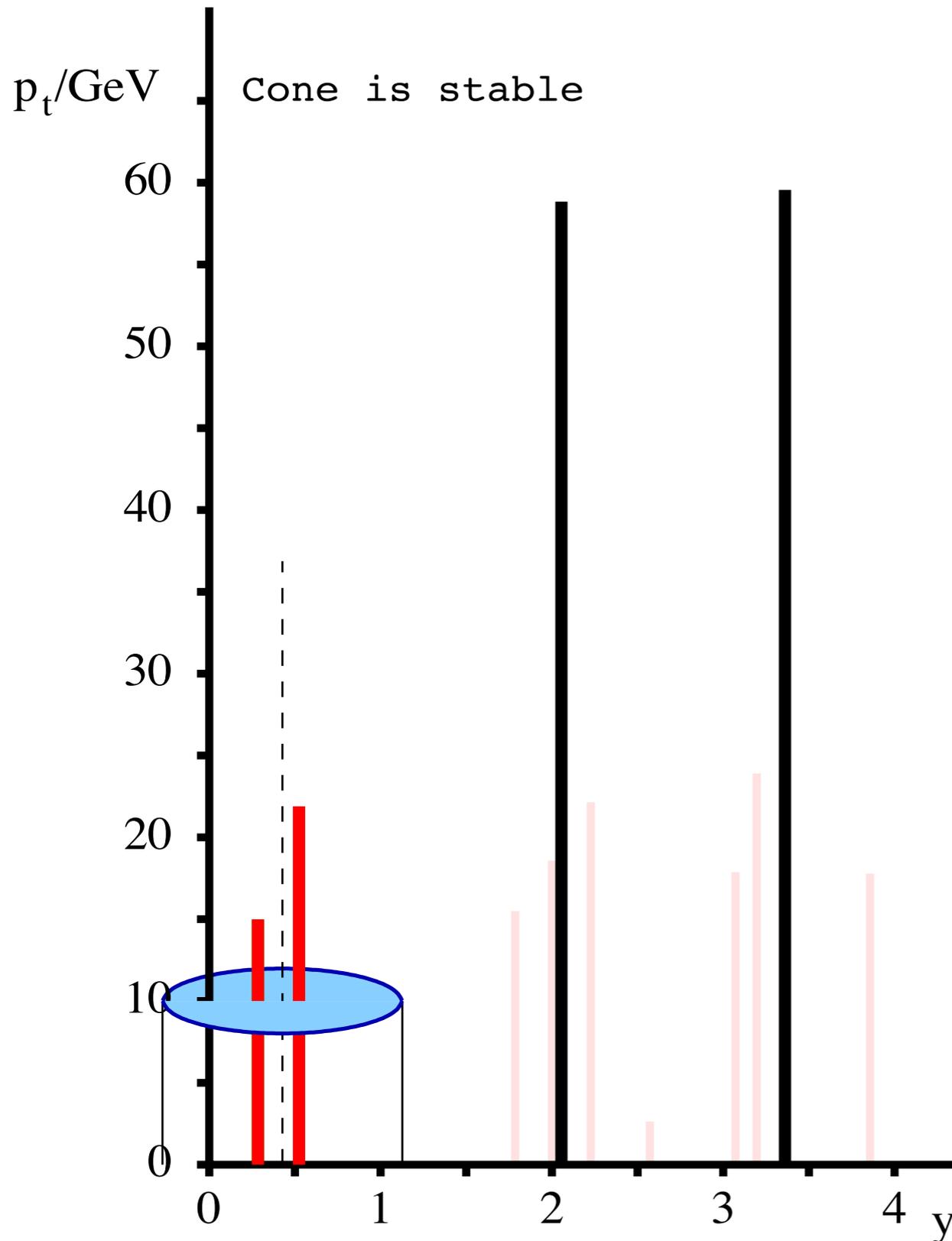
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- ▶ Convert contents into a “jet” and remove from event

Notes

- ▶ “Hardest particle” is collinear unsafe more right away...

Iterative Cone, Prog Removal (IC-PR)

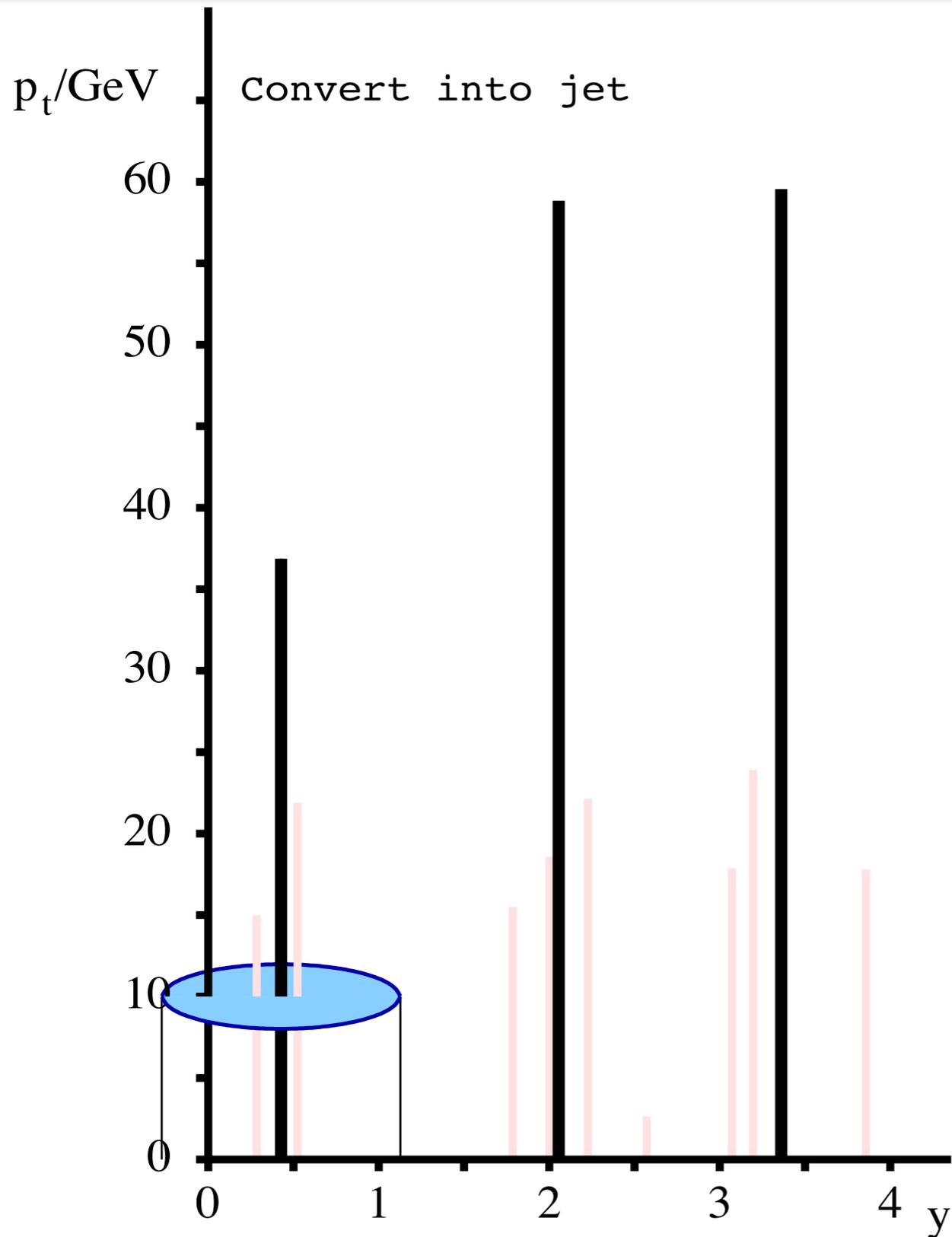


One of the simplest of the cone algs
e.g. CMS iterative cone

- ▶ Take hardest particle as seed for cone axis
- ▶ Draw cone around seed
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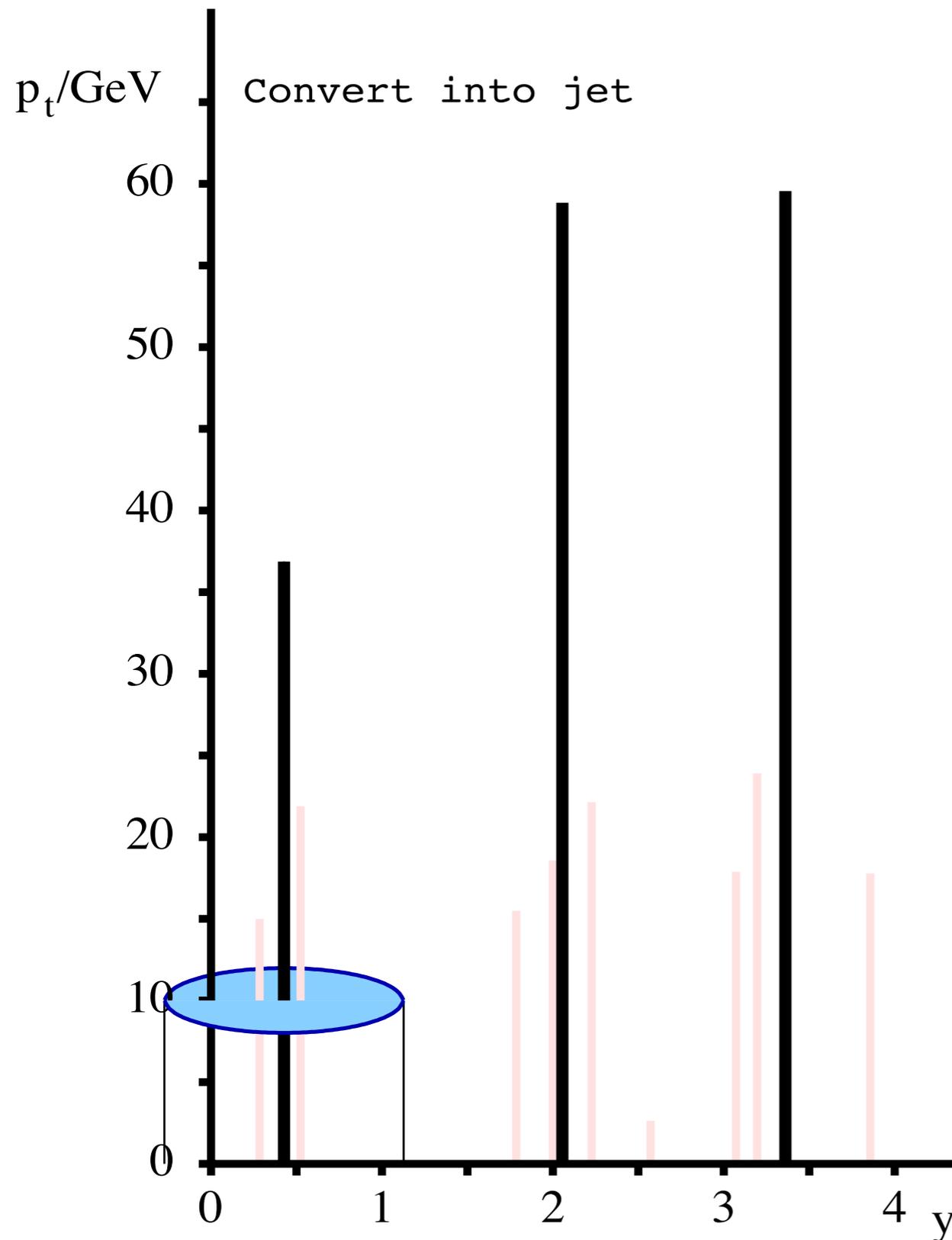
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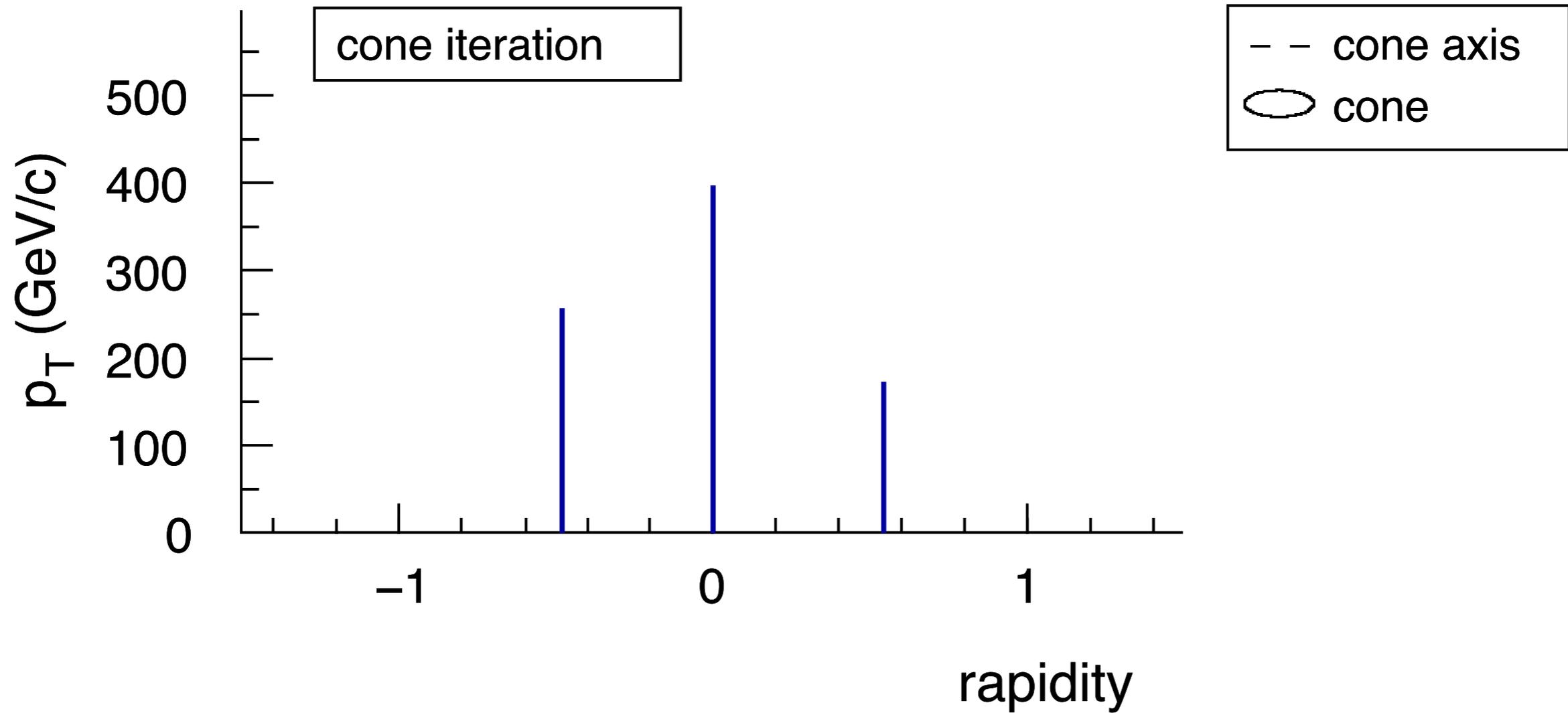


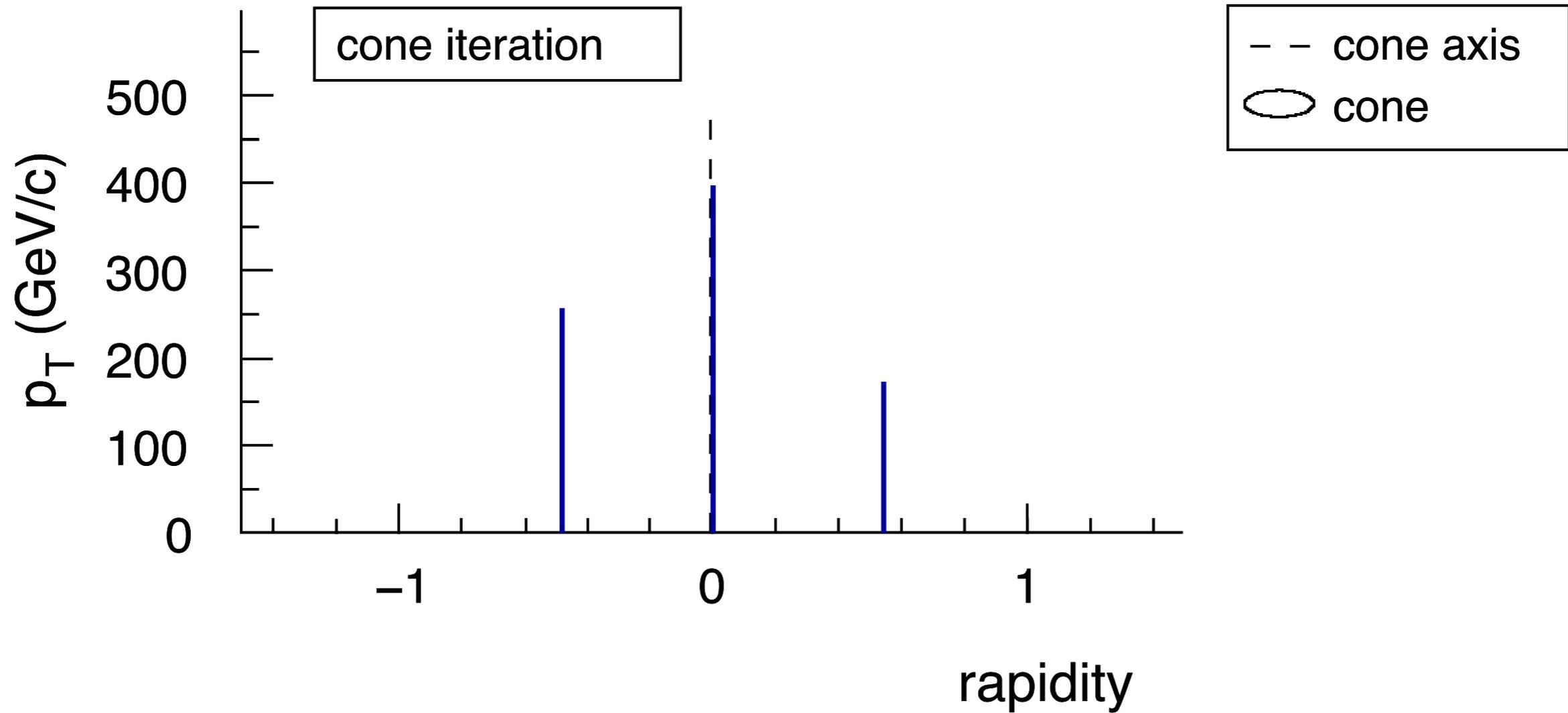
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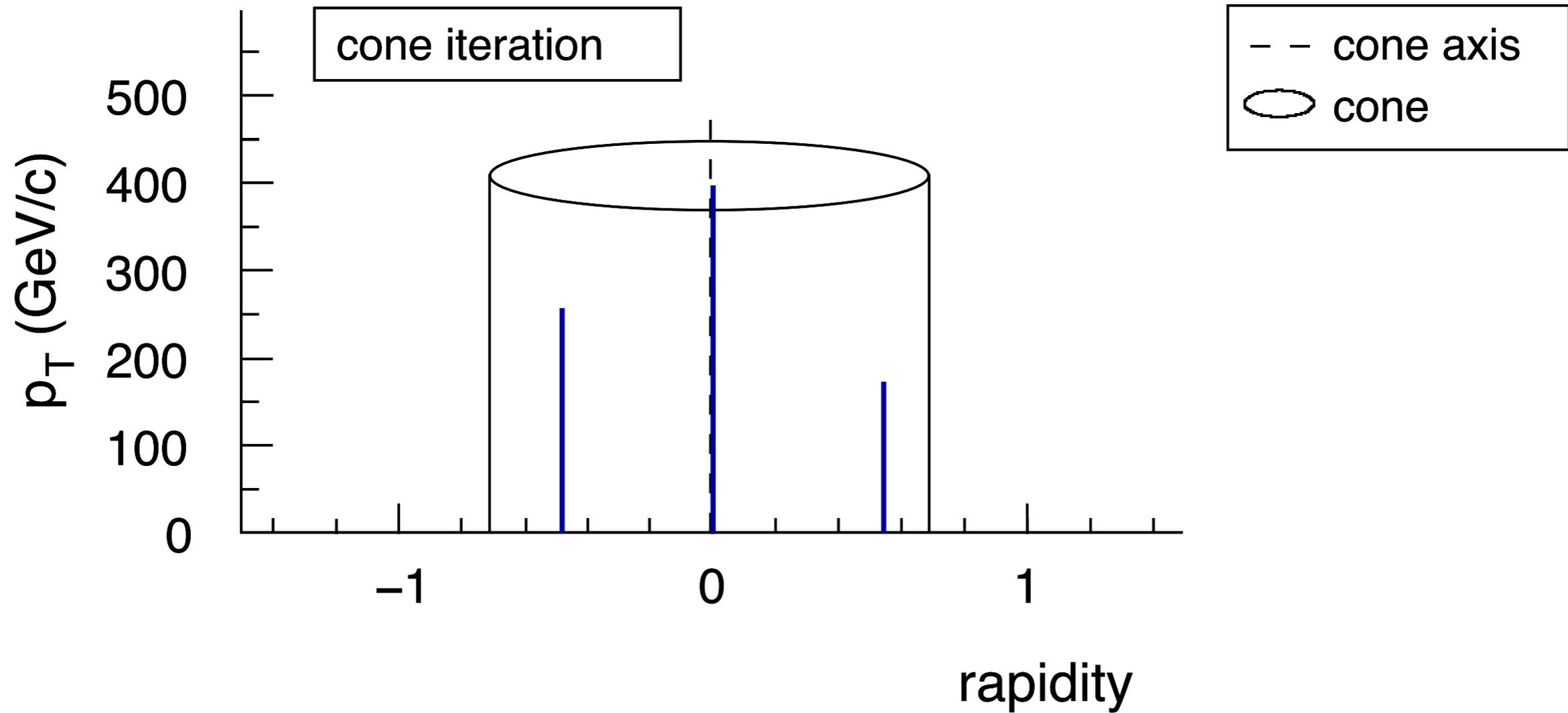
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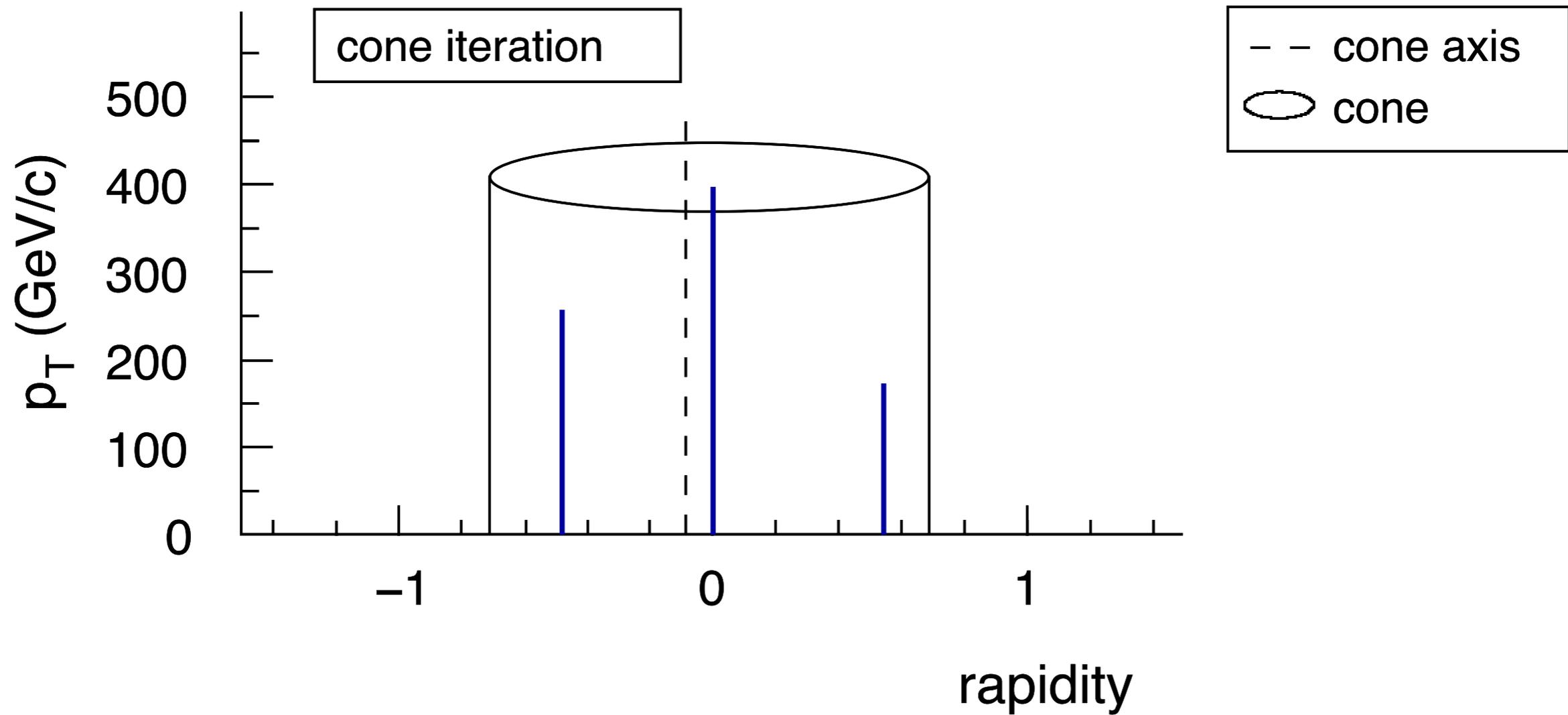




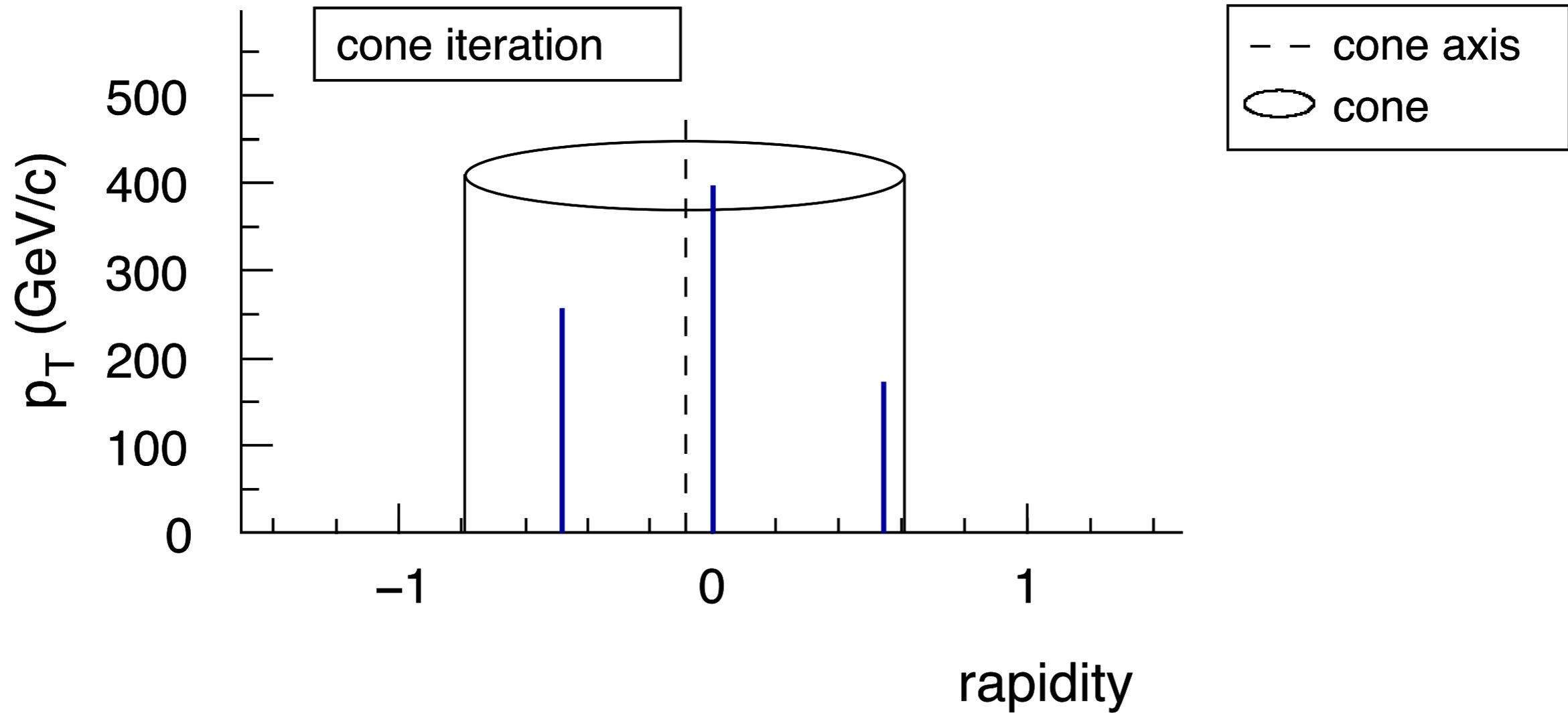
iterative cone issue

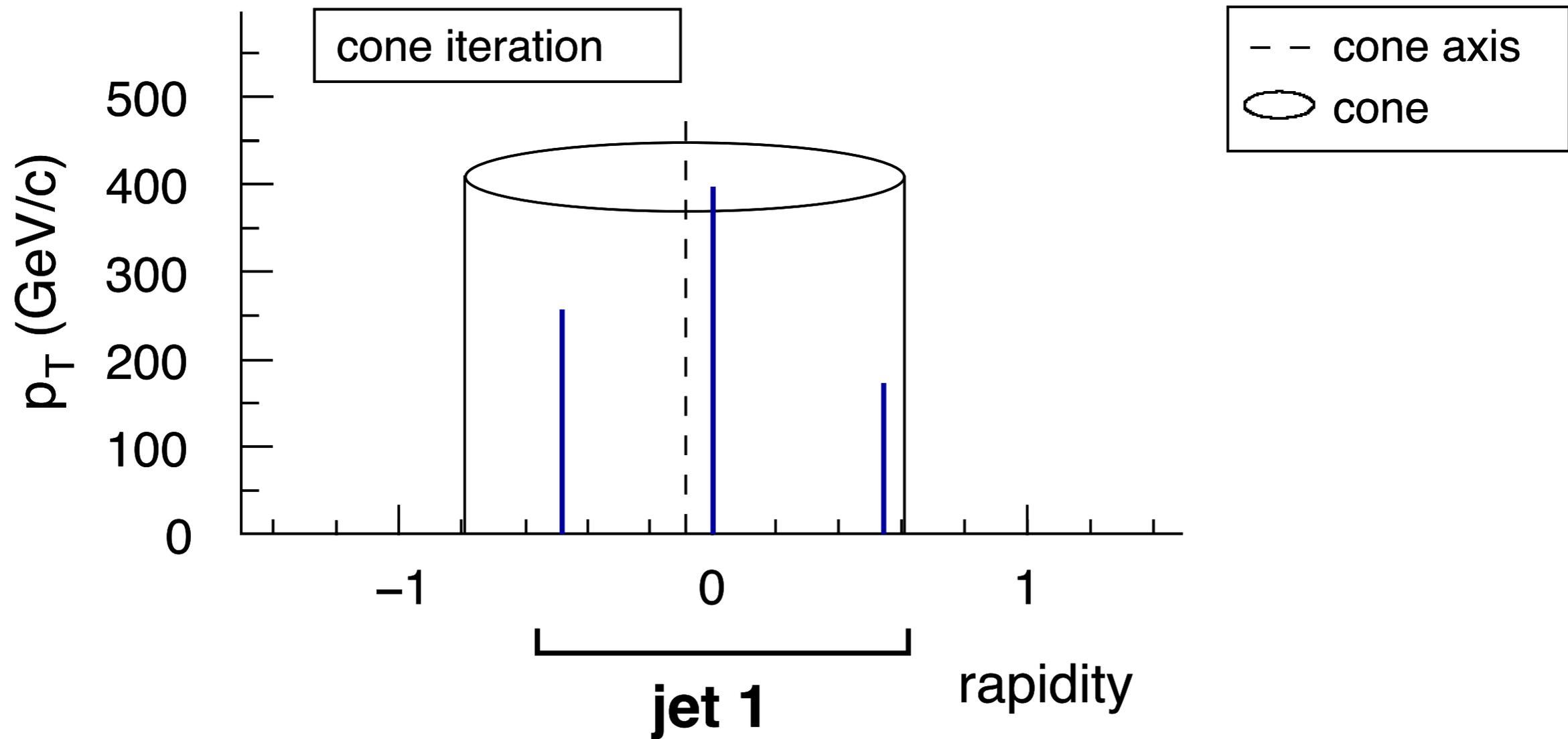


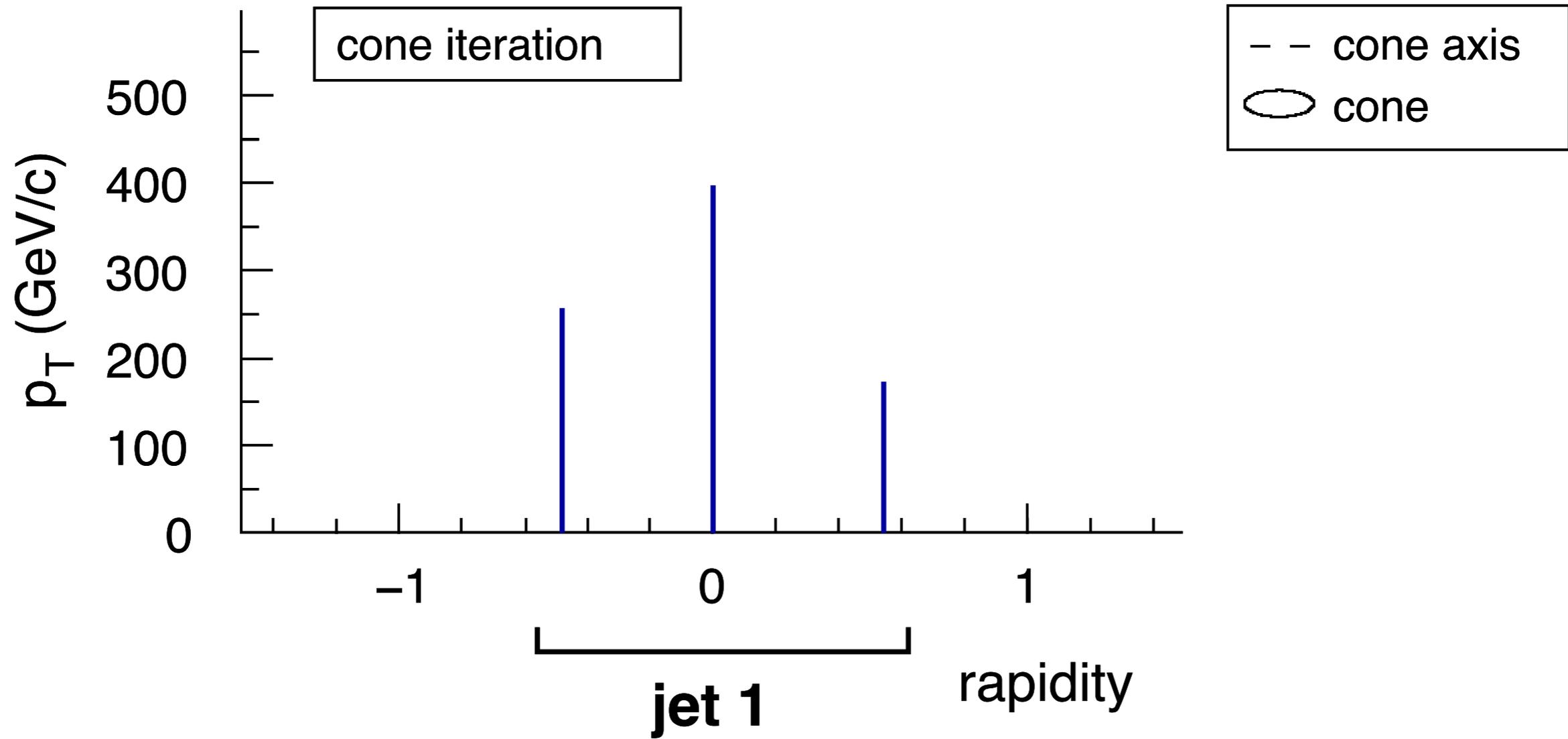
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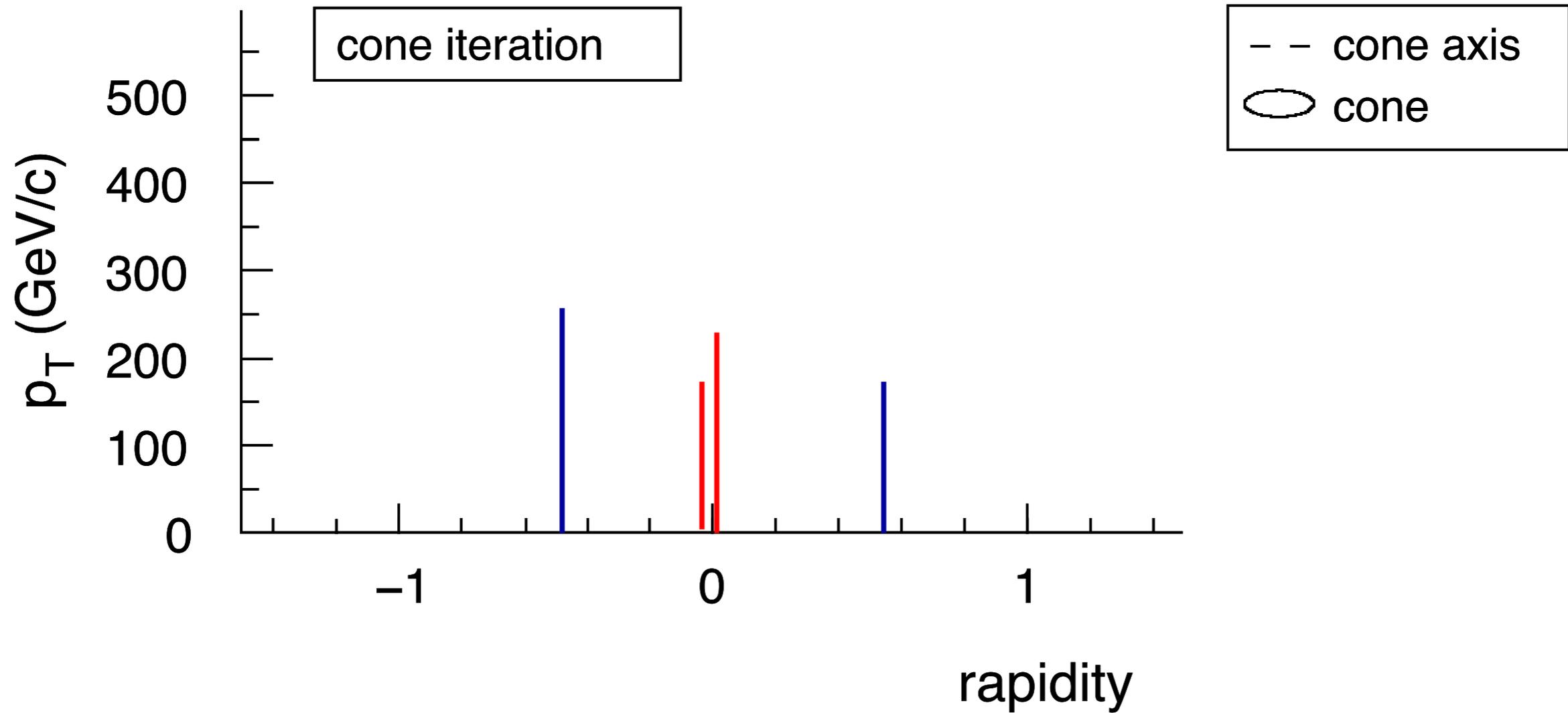


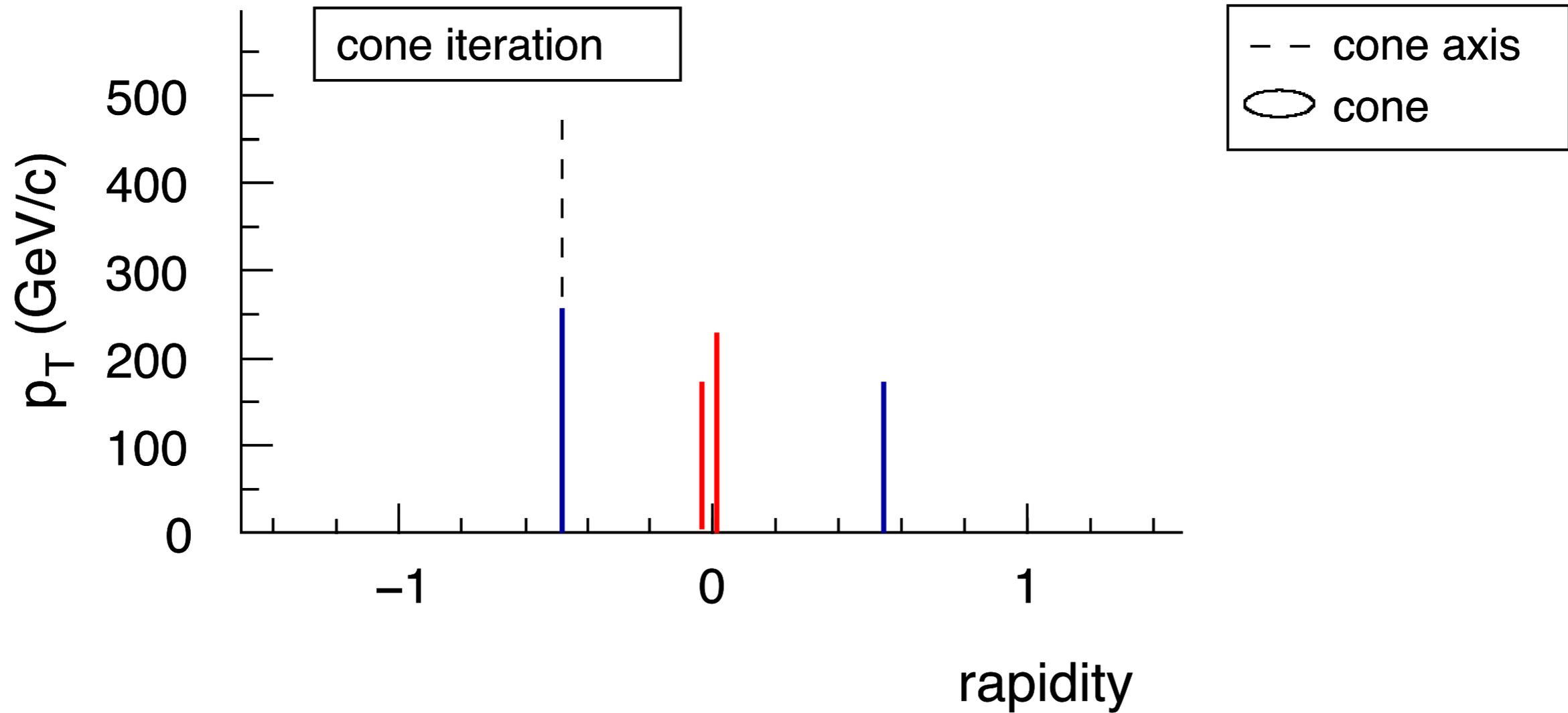
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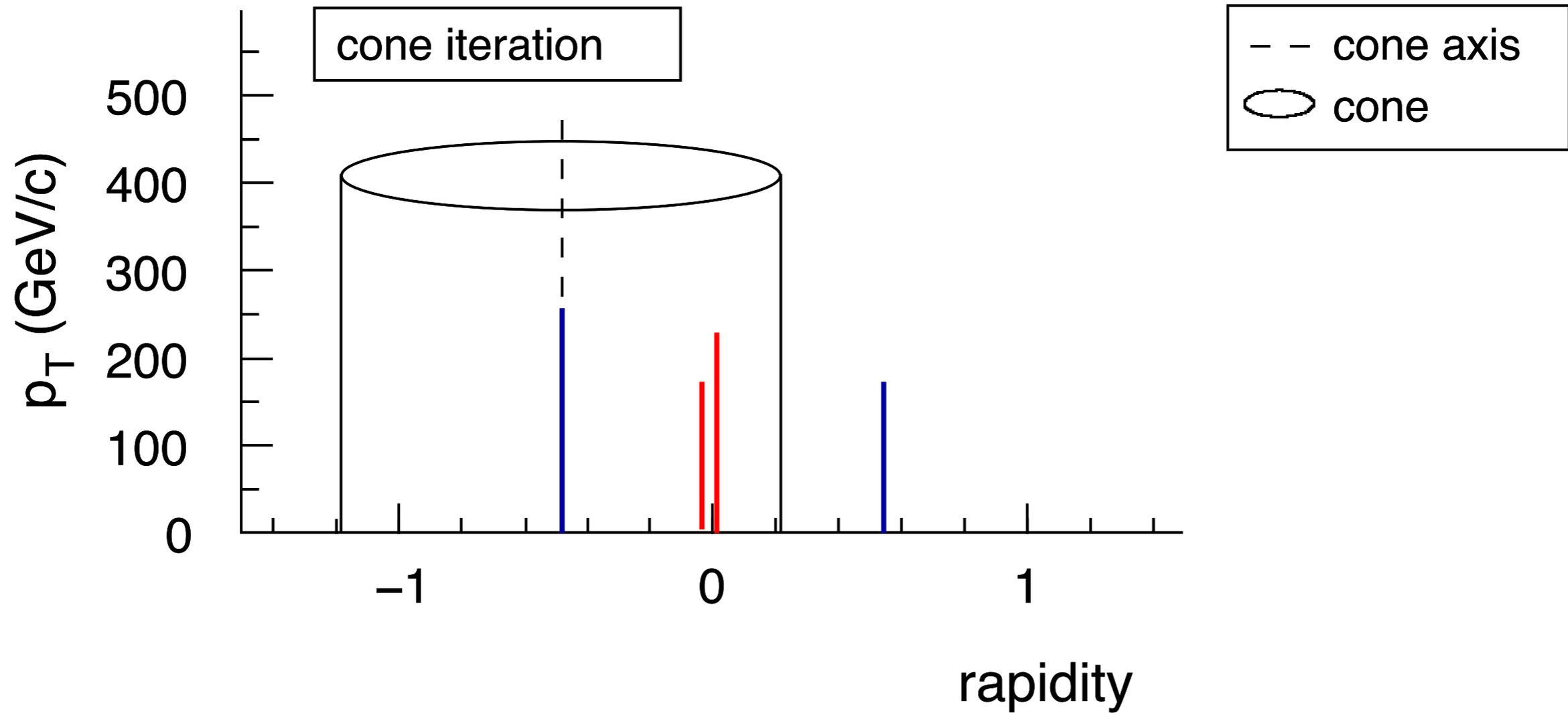


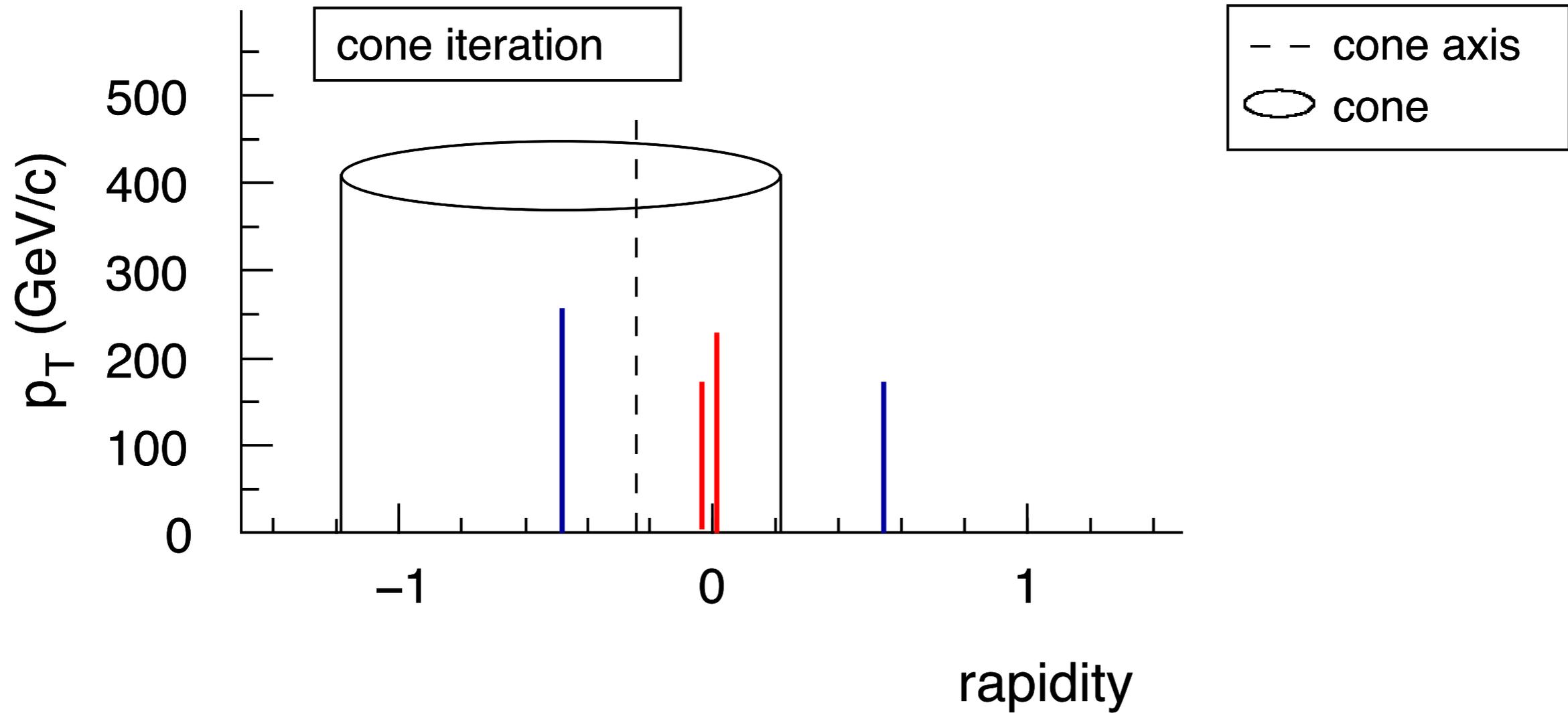


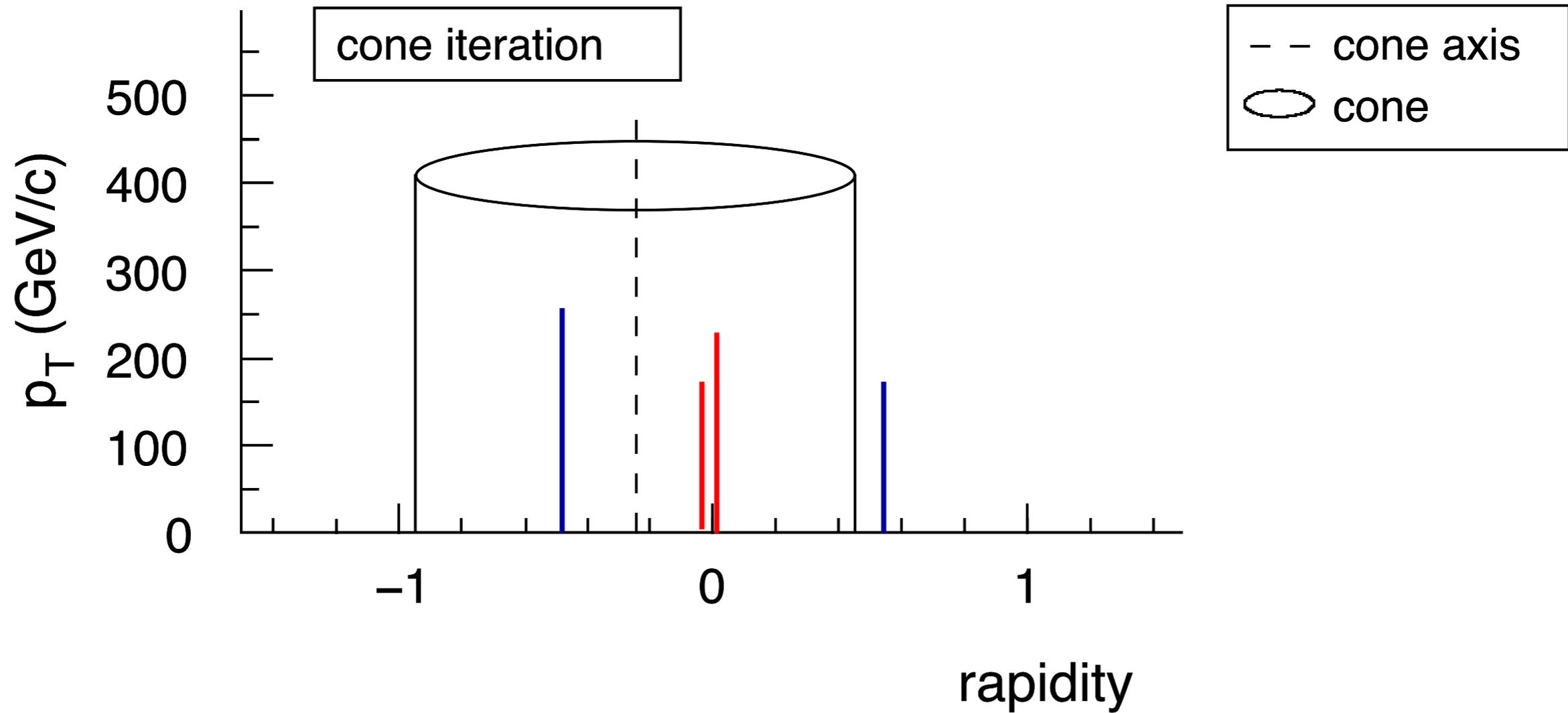


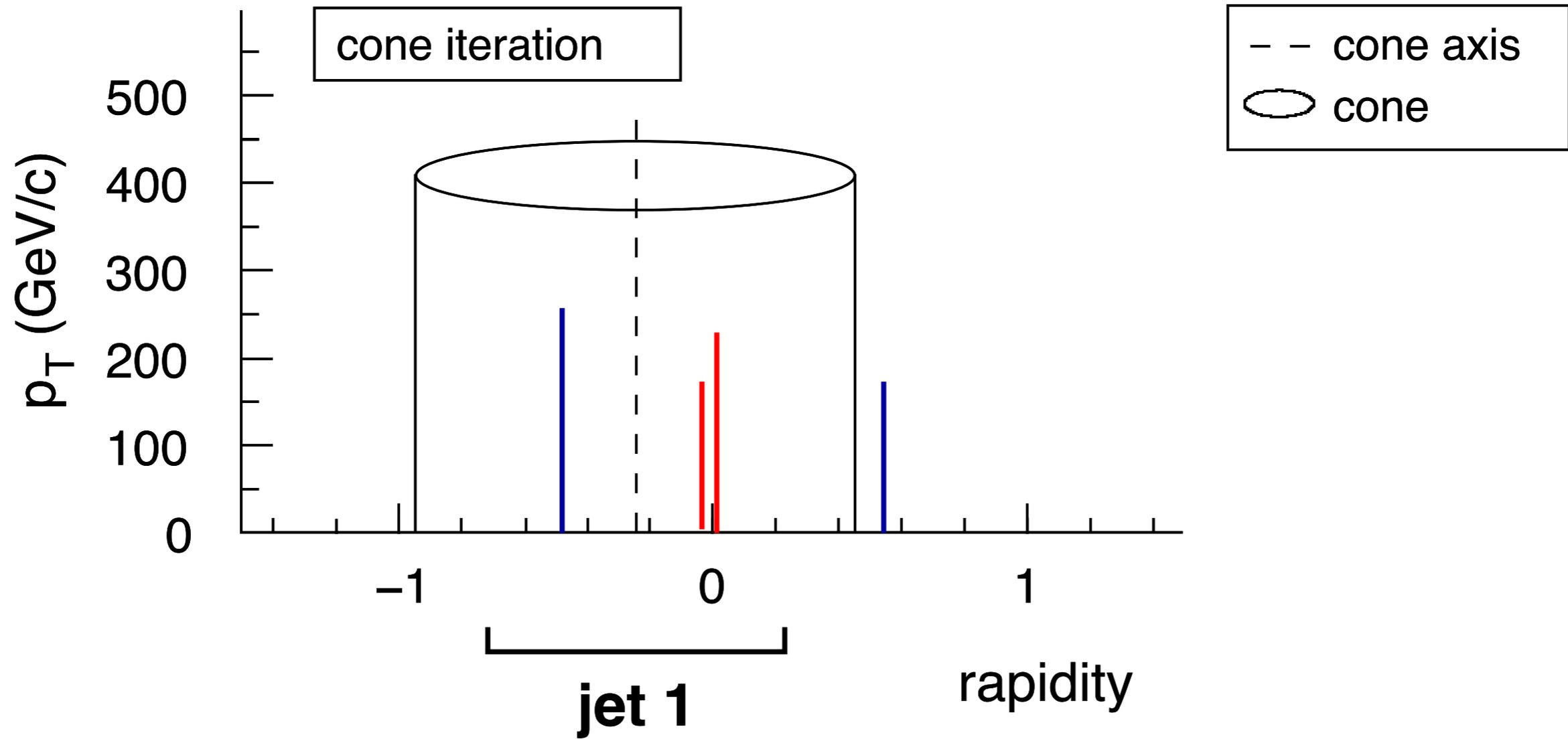


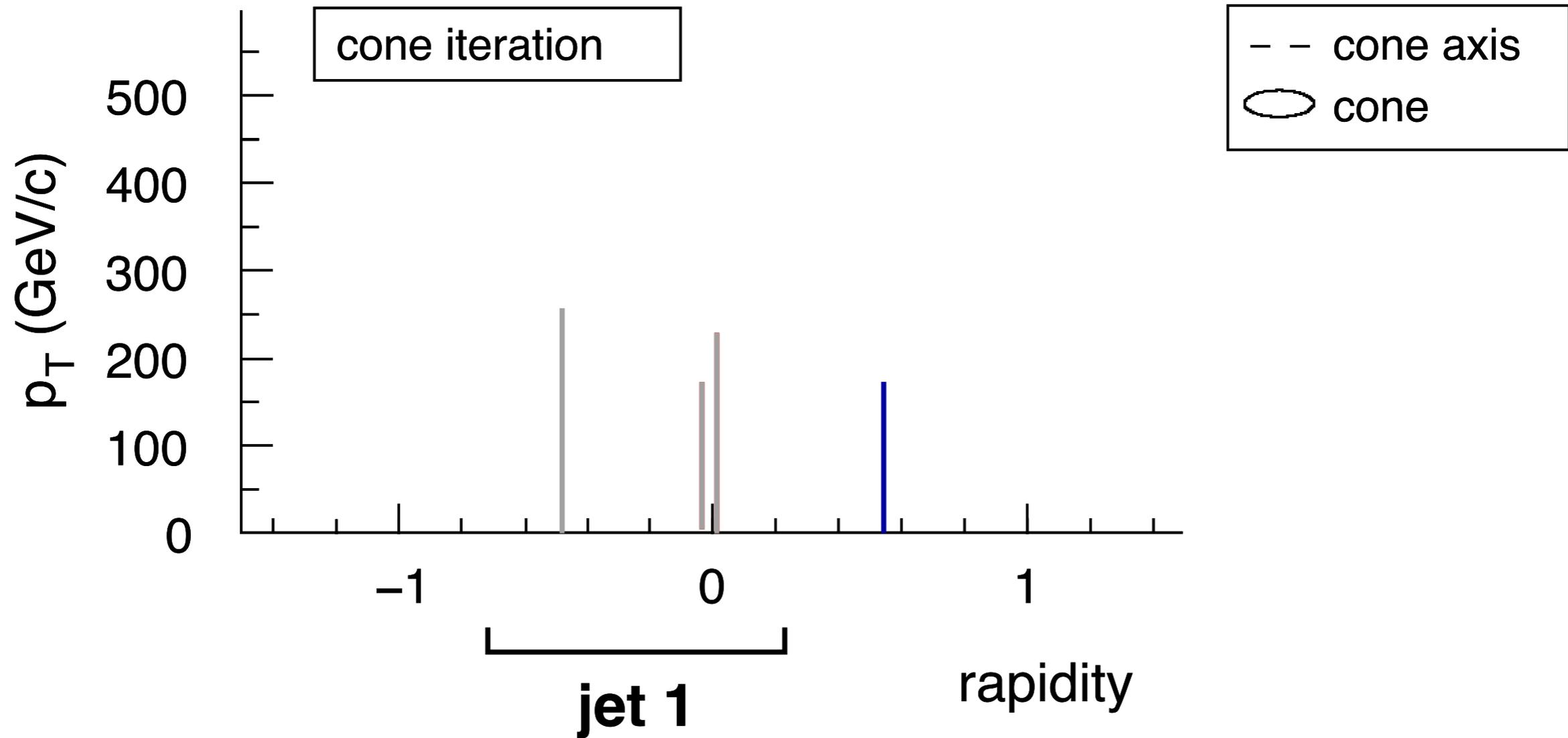




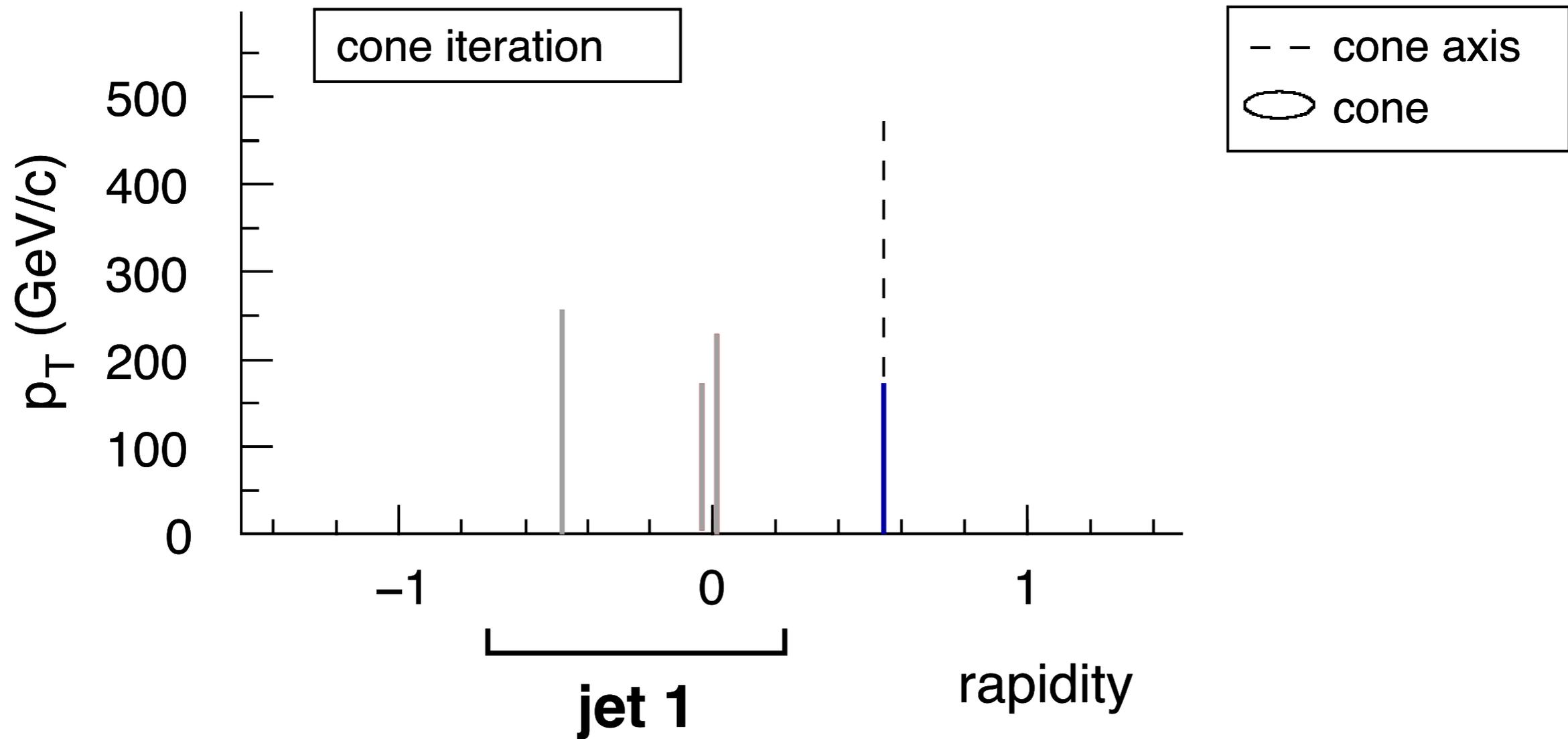


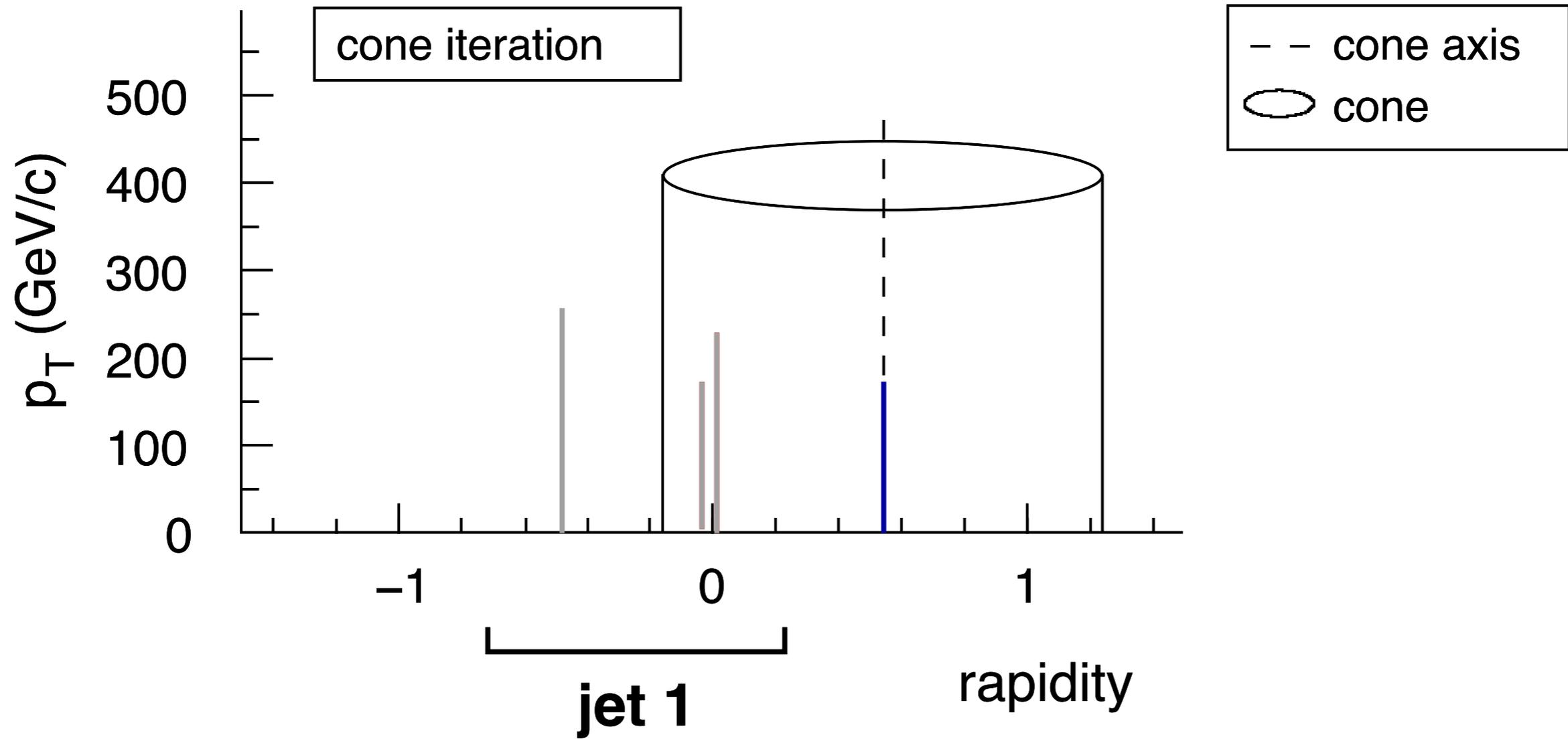


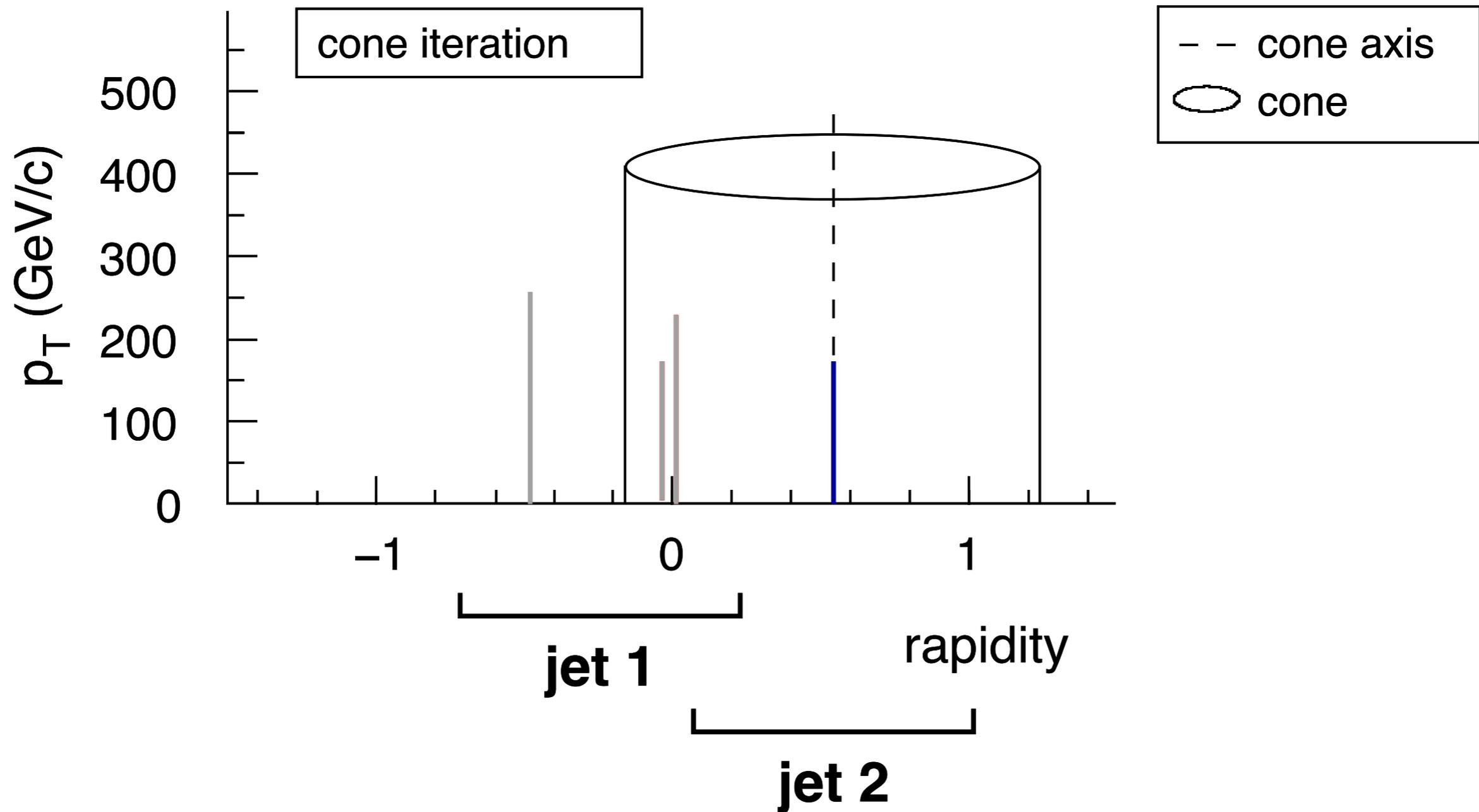


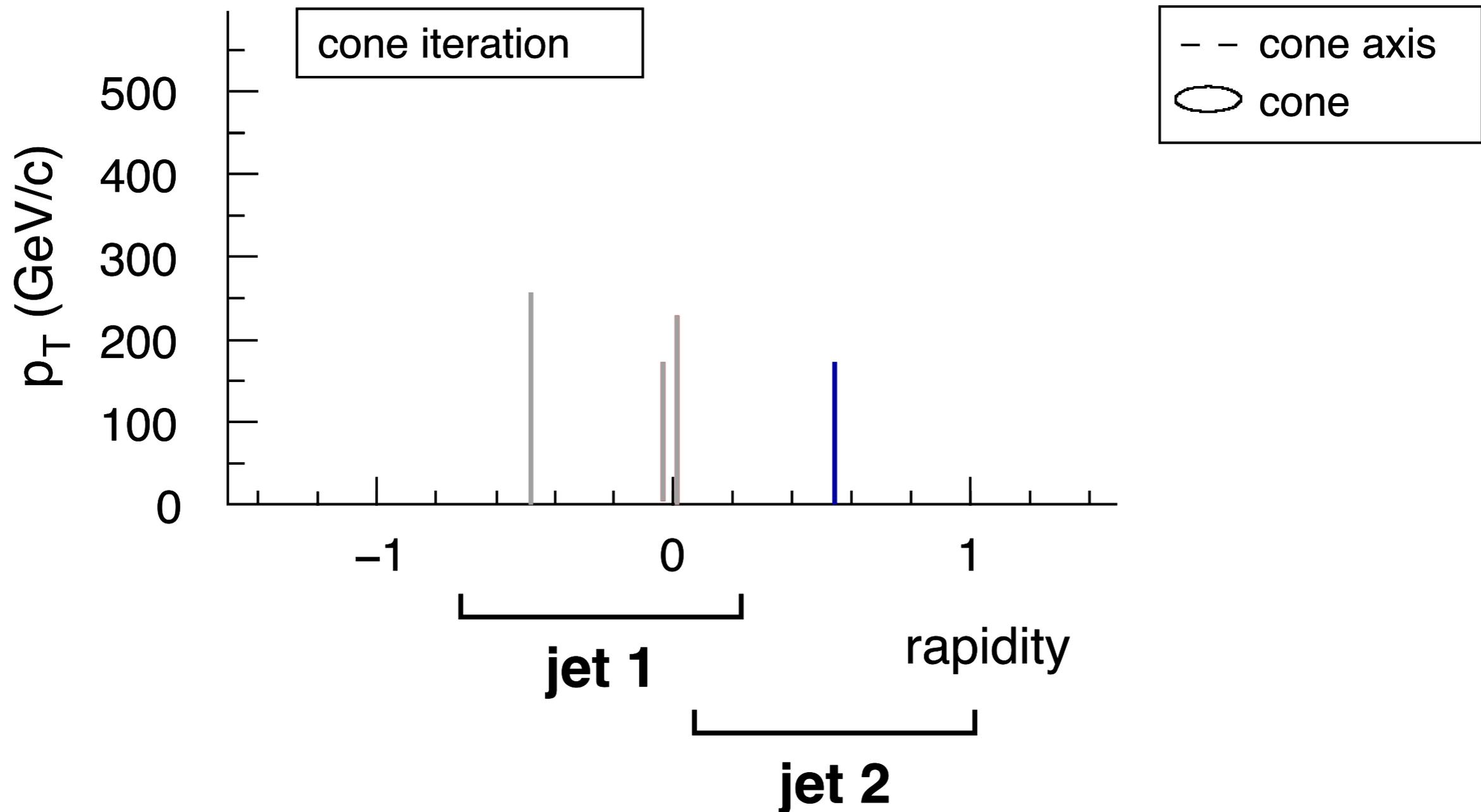


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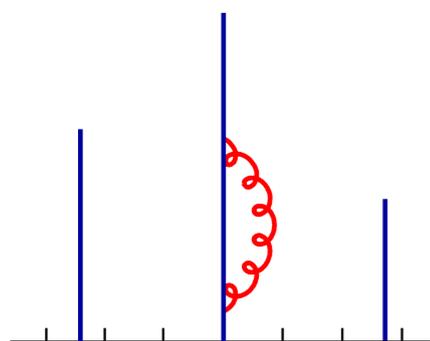






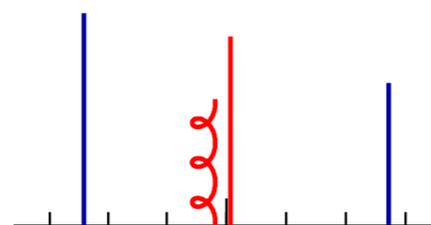
Collinear splitting can modify the final hard jets
 The algorithm is **collinear unsafe**

Collinear Safe



jet 1

$$\alpha_S^n \times (-\infty)$$

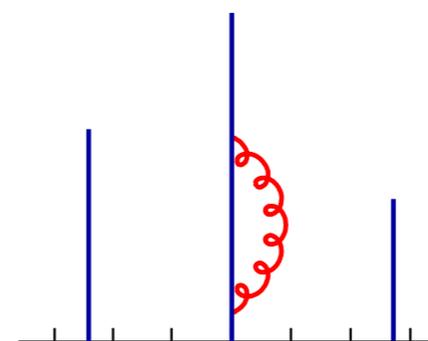


jet 1

$$\alpha_S^n \times (+\infty)$$

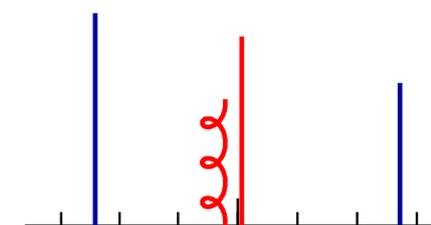
Infinities cancel

Collinear Unsafe



jet 1

$$\alpha_S^n \times (-\infty)$$



jet 1

jet 2

$$\alpha_S^n \times (+\infty)$$

Infinities do not cancel

Invalidates perturbation theory

The full cross section that you measure in experiment should correspond to an expression looking roughly as follows:

$$\sigma^{full} = \sigma_{LO} \left(1 + \alpha_s c_1 + \alpha_s^2 c_2 + \alpha_s^3 c_3 + \dots + \mathcal{O} \left(\frac{\Lambda_{QCD}}{p_t} \right) \right)$$

A perturbative series
plus a non-perturbative contribution, suppressed by a power of Λ_{QCD}/p_t

We don't have the technology to calculate the full series or the non-perturbative part. Typically, one might “just” calculate next-to-leading order

$$\sigma^{NLO} = \sigma_{LO} (1 + \alpha_s c_1)$$

The point to perturbation theory is that the $c_2 \alpha_s^2$, etc. terms are small compared to the ones you have calculated — *hence (e.g.) NLO should be a good approximation.*

Infrared/collinear safety and real life

Real life does not have infinities, but pert. infinity leaves a real-life trace

$$\alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \infty \rightarrow \alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \ln p_t/\Lambda \rightarrow \alpha_s^2 + \underbrace{\alpha_s^3 + \alpha_s^3}_{\text{BOTH WASTED}}$$

Among consequences of IR unsafety:

	<i>Last meaningful order</i>			Known at
	JetClu, ATLAS cone [IC-SM]	MidPoint [IC _{mp} -SM]	CMS it. cone [IC-PR]	
Inclusive jets	LO	NLO	NLO	NLO (→ NNLO)
W/Z + 1 jet	LO	NLO	NLO	NLO (→ NNLO)
3 jets	none	LO	LO	NLO [nlojet++]
W/Z + 2 jets	none	LO	LO	NLO [MCFM]
m _{jet} in 2j + X	none	none	none	NLO [Blackhat/Rocket/...]

NB: 50,000,000\$/£/CHF/€ investment in NLO

Multi-jet contexts much more sensitive: **ubiquitous at LHC**

And LHC relies on QCD for background double-checks
extraction of cross sections, extraction of parameters

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Among consequences of IR unsafety:

There is a way of resolving this kind of problem for cone algorithms

Inclusive jets	LO	NLO	NLO	NLO (→ NNLO)
W/Z + 1 jet	LO	NLO	NLO	NLO (→ NNLO)
3 jets	LO	NLO	NLO	NLO [nlojet++]
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m_{jet} in $2j + X$	LO	NLO	NLO	NLO [Blackhat/Rocket/...]

SISCone

[Seedless Infrared Safe Cone]

But, in the end, this is not the solution that the LHC experiments chose, so let's spend our time on the one they did

Multi-jet con

And LHC relies on QCD for background double-checks extraction of cross sections, extraction of parameters

Hierarchical clustering

[sequential recombination algorithms]

Sequential recombination algorithms (e^+e^-)

[aka hierarchical agglomerative clustering]

- Define a distance measure d_{ij} between all pairs of particles.
- Recombine pair with smallest d_{ij}
- Repeat until all $d_{ij} > d_{cut}$

The algorithm's general behaviour is governed by how one defines the interpair distances d_{ij}

d_{cut} is a **resolution parameter**, which governs how whether jets are coarse or fine objects

JADE algorithm – the original seq. rec. algorithm

$$d_{ij} = m_{ij}^2 \simeq 2E_i E_j (1 - \cos \theta_{ij})$$

The most obvious choice?
But does not give sensible behaviour
[see blackboard]

Classic e^+e^- seq. rec. algorithms (2)

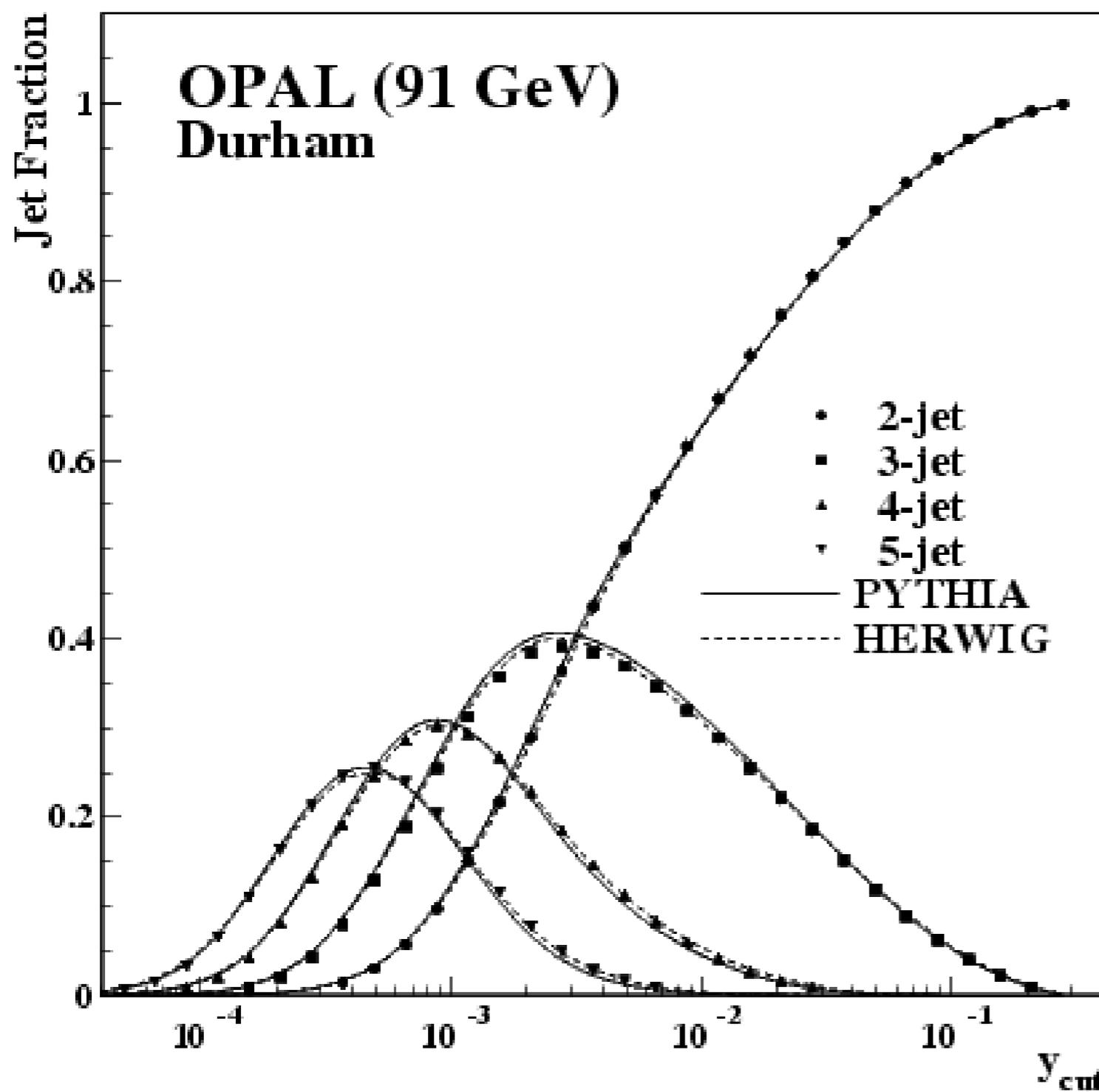
Use QCD divergences to help decide distance measure:

The stronger the divergence between a pair of particles, the more likely it is they should be associated with each other. *[see blackboard]*

k_t / Durham algorithm – the most widely used at LEP

$$d_{ij} = 2 \min(E_i^2, E_j^2) (1 - \cos \theta_{ij})$$

Catani, Dokshitzer, Olsson, Turnock & Webber '91



Fraction of events with n jets, as a function of the resolution parameter



$y_{cut} = d_{cut} / Q^2$

What changes at a hadron collider?

- You have (unseen) beams, introduce a “beam distance”

$$d_{iB} = 2E_i^2(1 - \cos \theta_{iB})$$

squared trans. mom. wrt beam

- You want to use longitudinally invariant variables, i.e. p_t , rapidity (y) and azimuth (φ)

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \Delta R_{ij}^2, \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$$d_{iB} = p_{ti}^2$$

exclusive k_t algorithm

still just one parameter d_{cut}

Catani, Dokshitzer, Seymour & Webber 1993

Two parameters, R and $p_{t,min}$

(These are the two parameters in essentially every widely used hadron-collider jet algorithm)

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \frac{\Delta R_{ij}^2}{R^2}, \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

Reformulate algorithm

1. Find smallest of d_{ij} , d_{iB}
2. If ij , recombine them
3. If iB , call i a jet and remove from list of particles
4. repeat from step 1 until no particles left

Only use jets with $p_t > p_{t,min}$

Inclusive k_t algorithm
S.D. Ellis & Soper, 1993

In what way do the inclusive and exclusive variants' behaviours differ?

*[see ~~black~~**white**board]*

Is one “right”, the other wrong”?

A priori no, maybe we'll come back to this later.

k_t alg.: Find smallest of

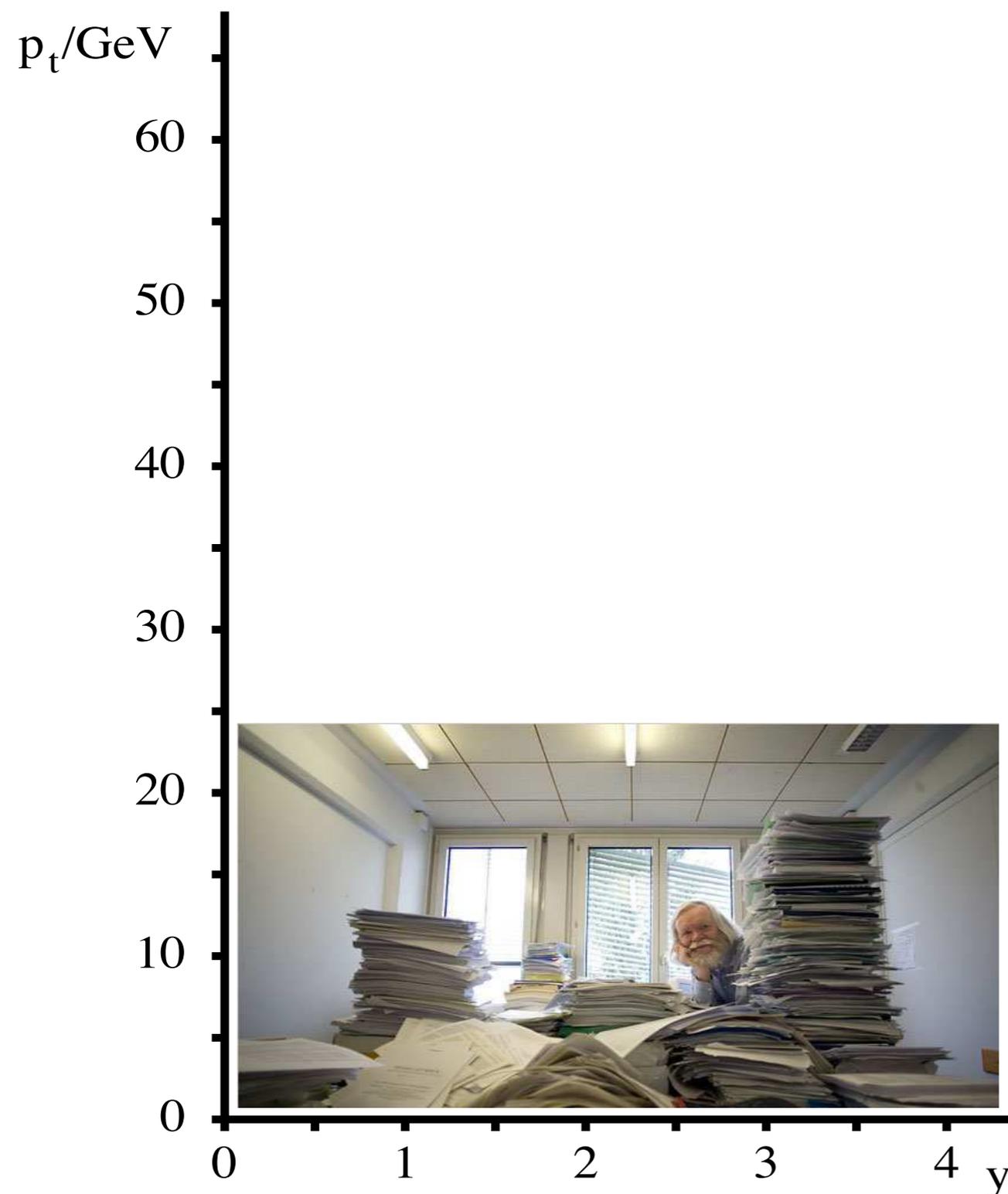
$$d_{ij} = \min(k_{ti}^2, k_{tj}^2) \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = k_{ti}^2$$

- ▶ If d_{ij} recombine
- ▶ if d_{iB} , i is a jet

Example clustering with k_t algorithm, $R = 1.0$

ϕ assumed 0 for all towers





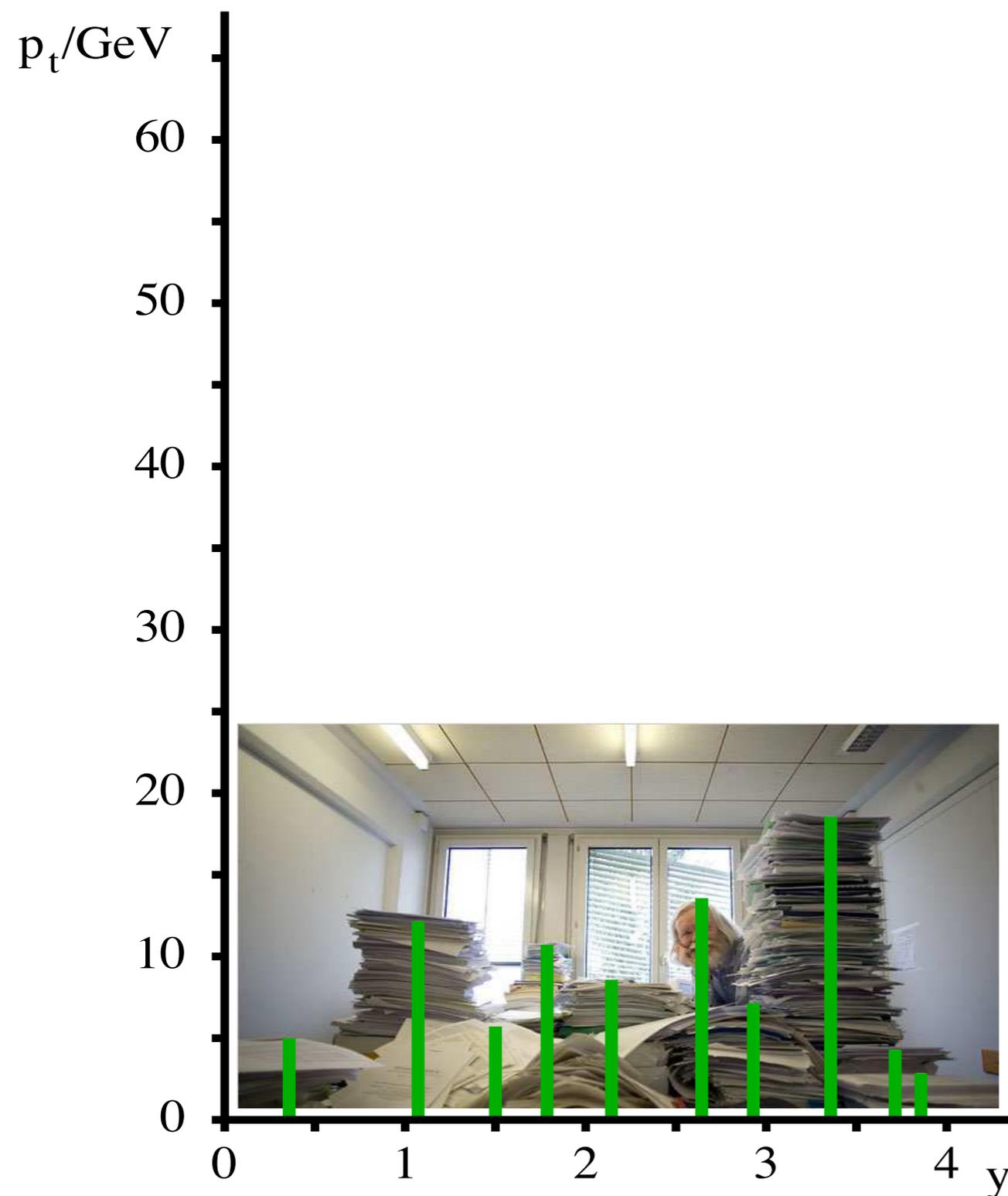
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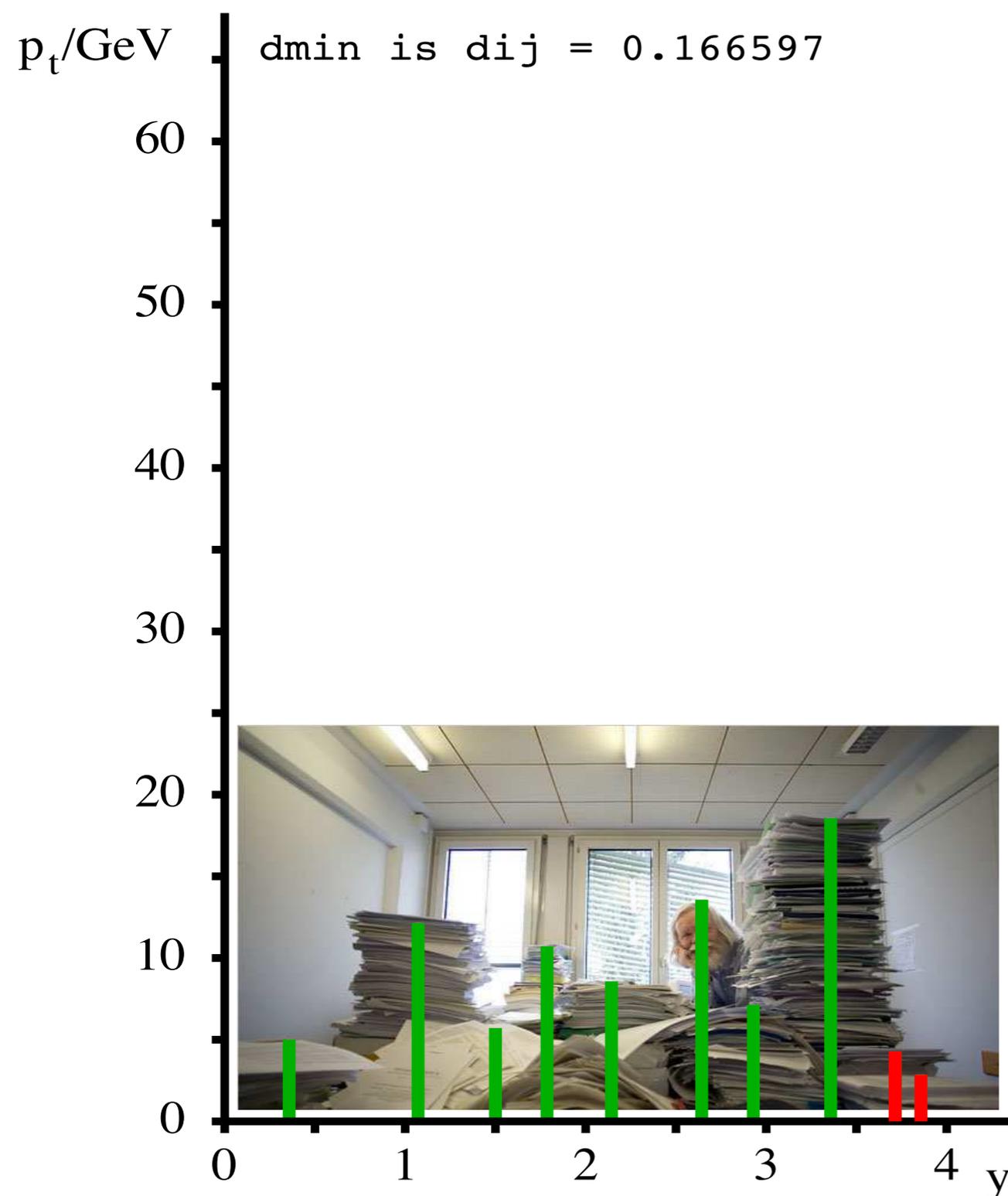
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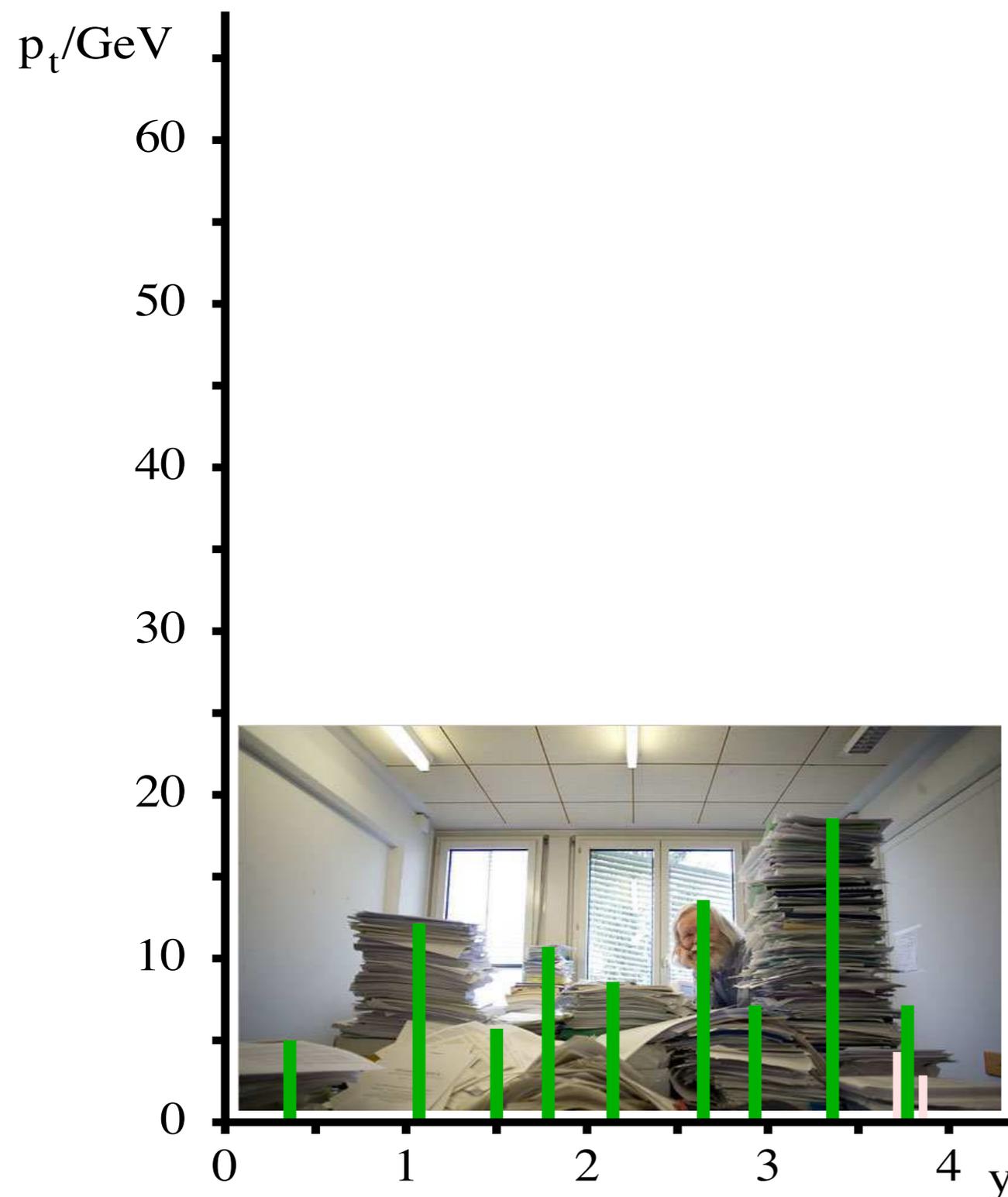
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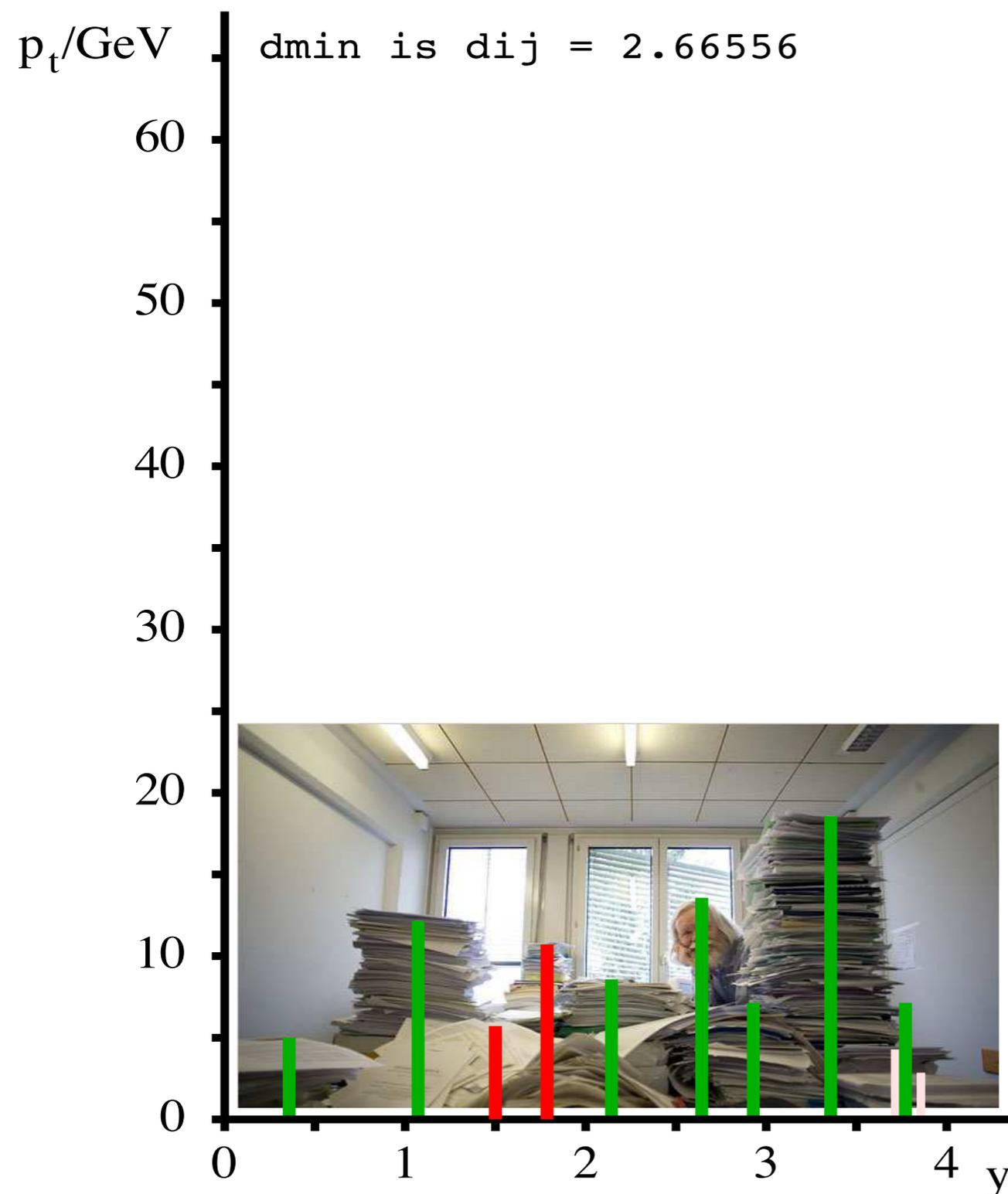
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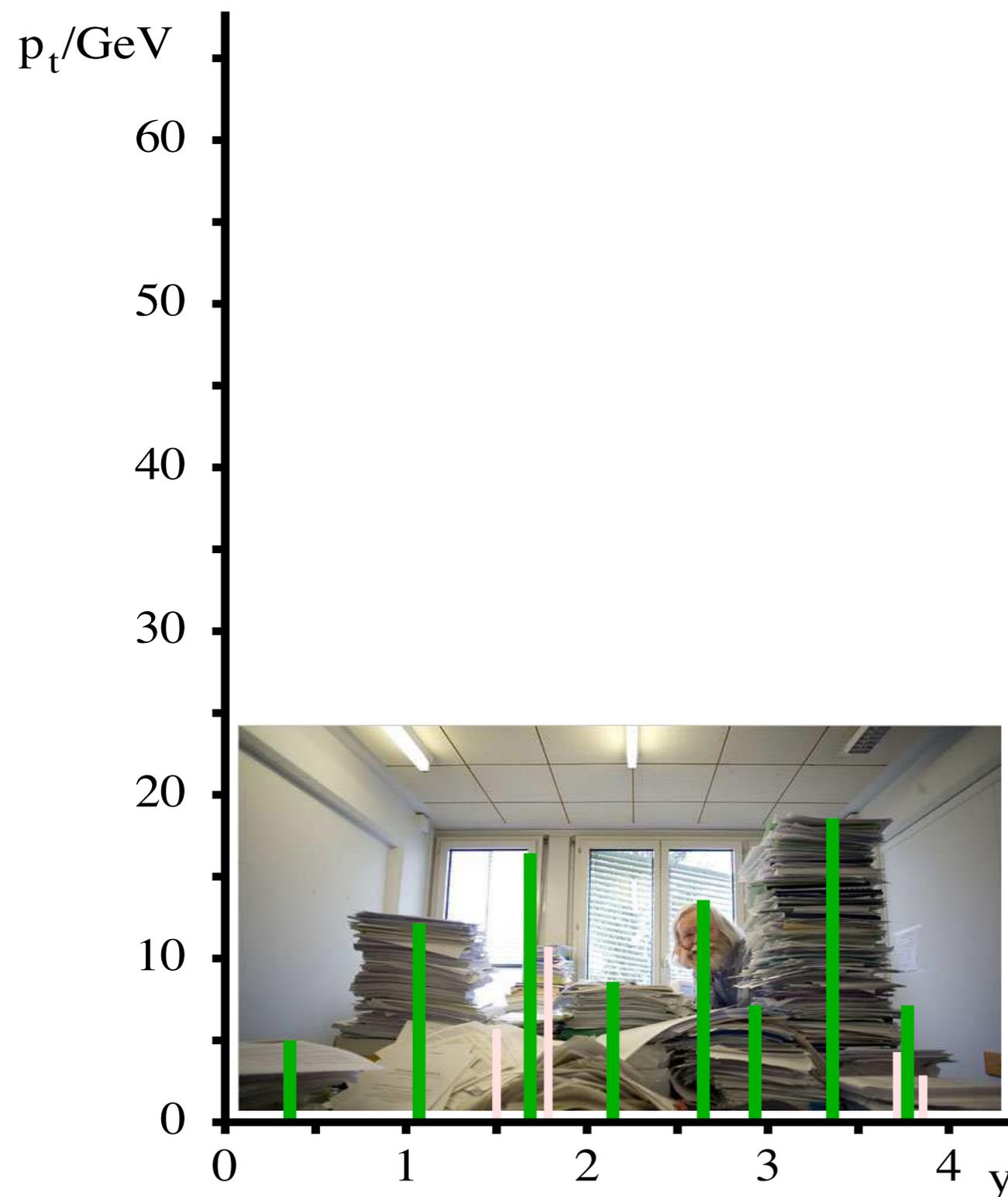
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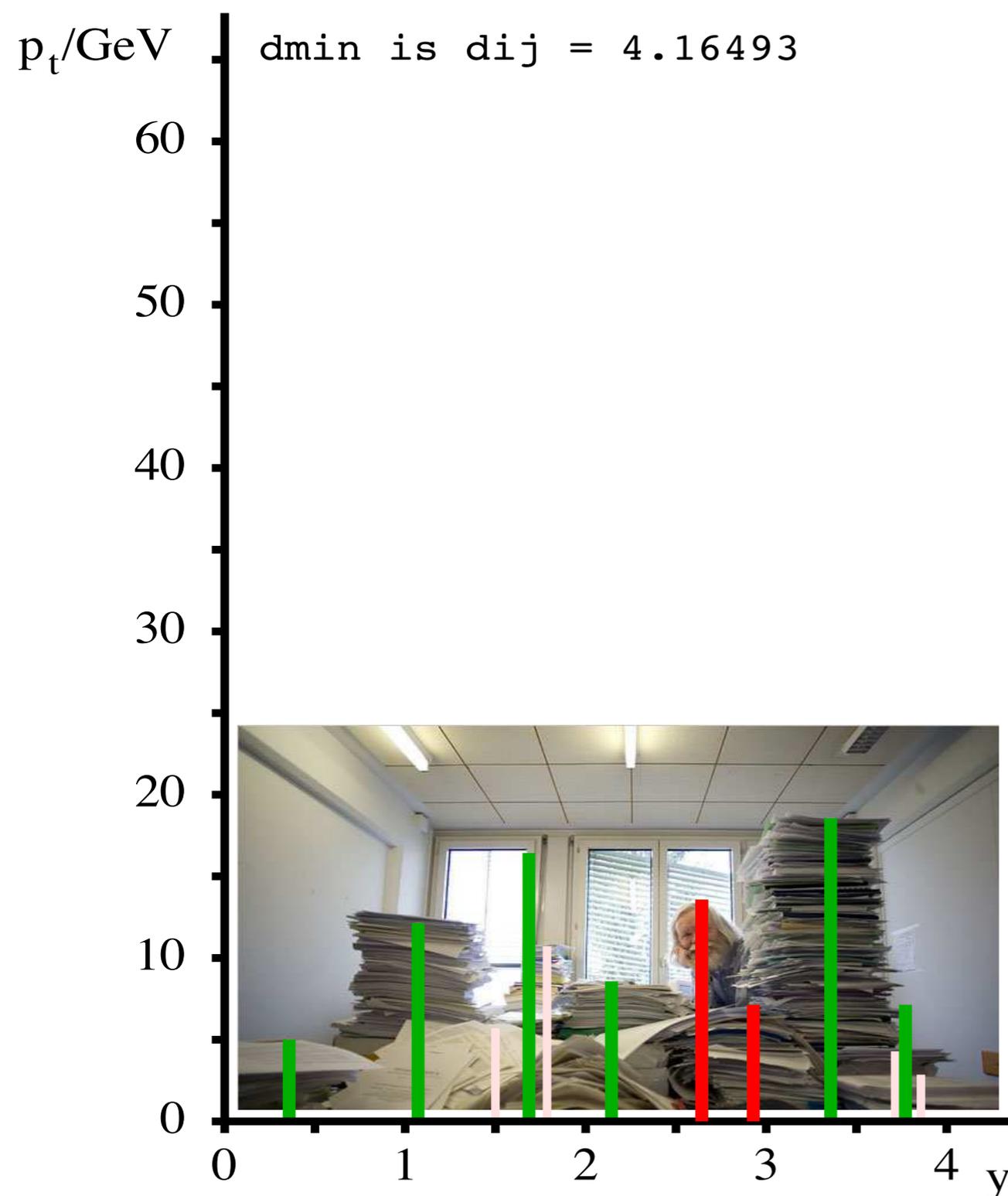
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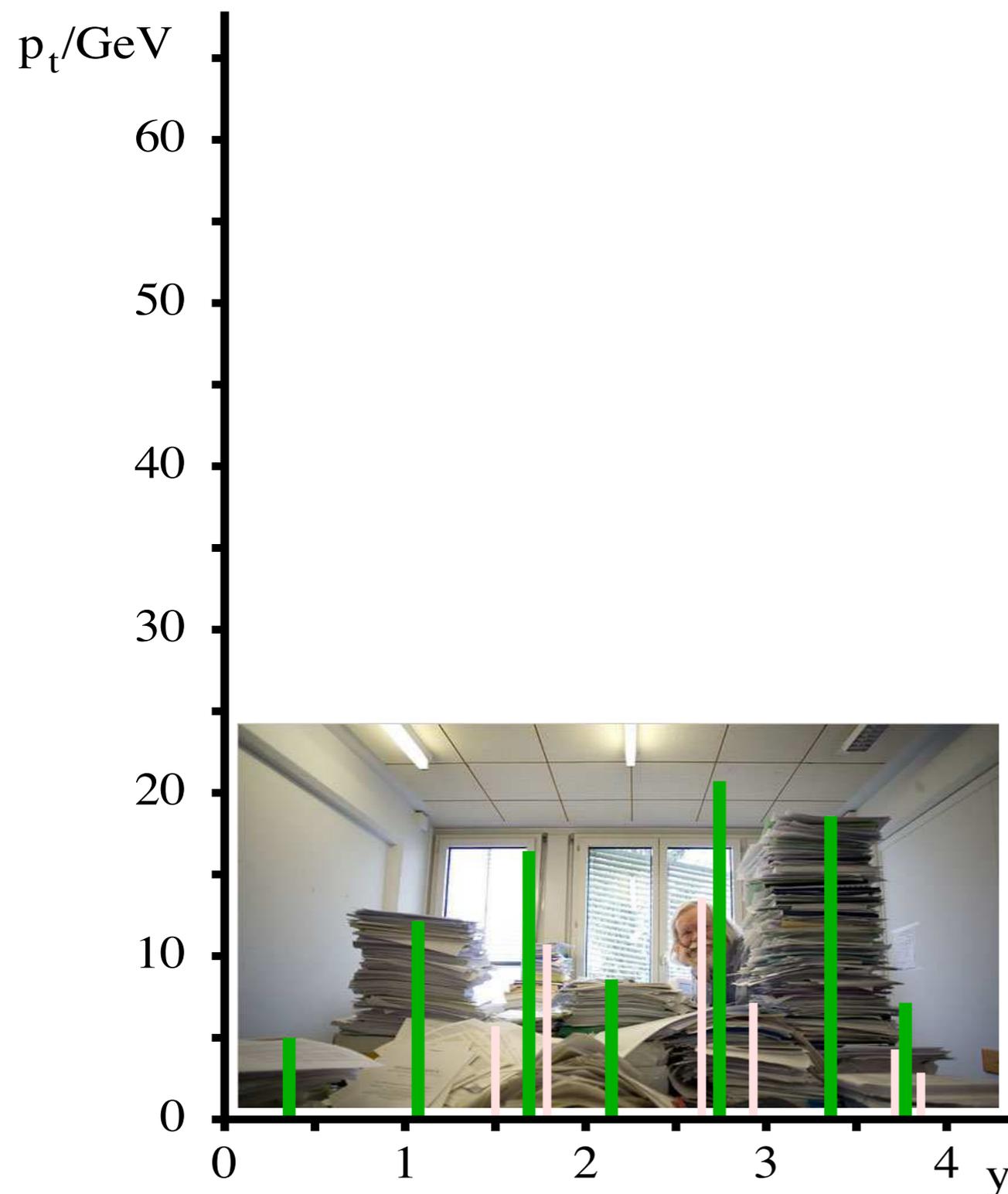
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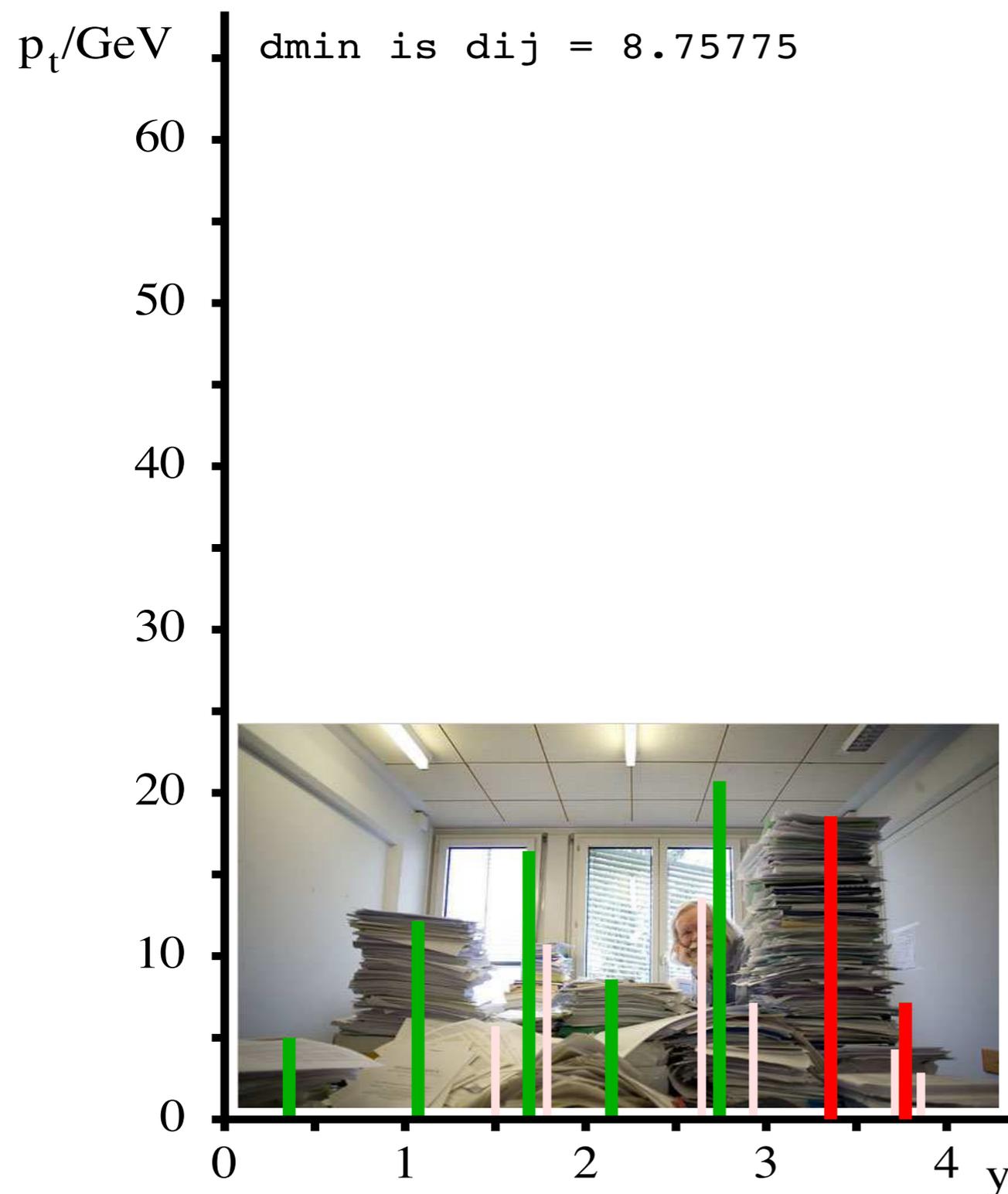
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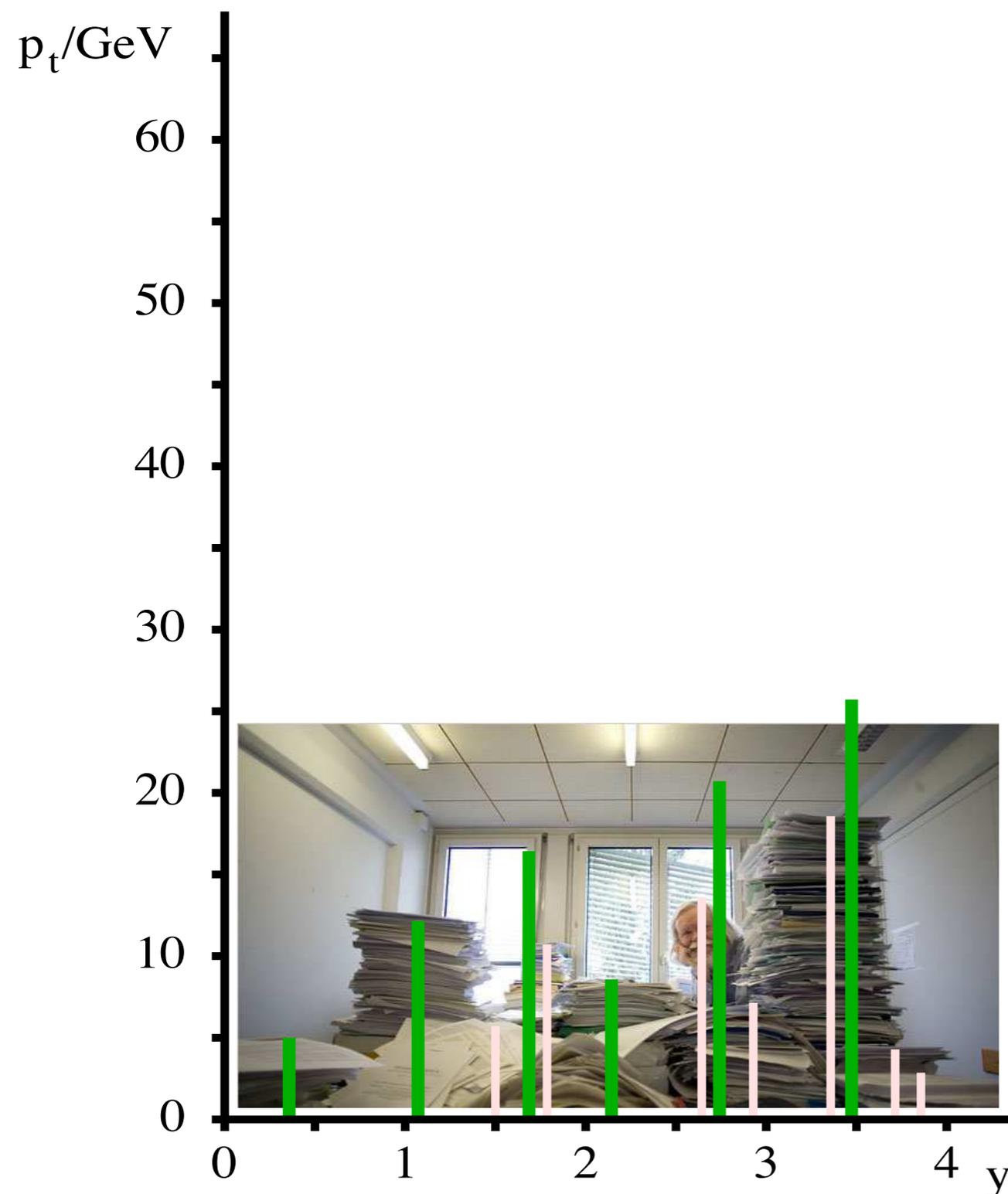
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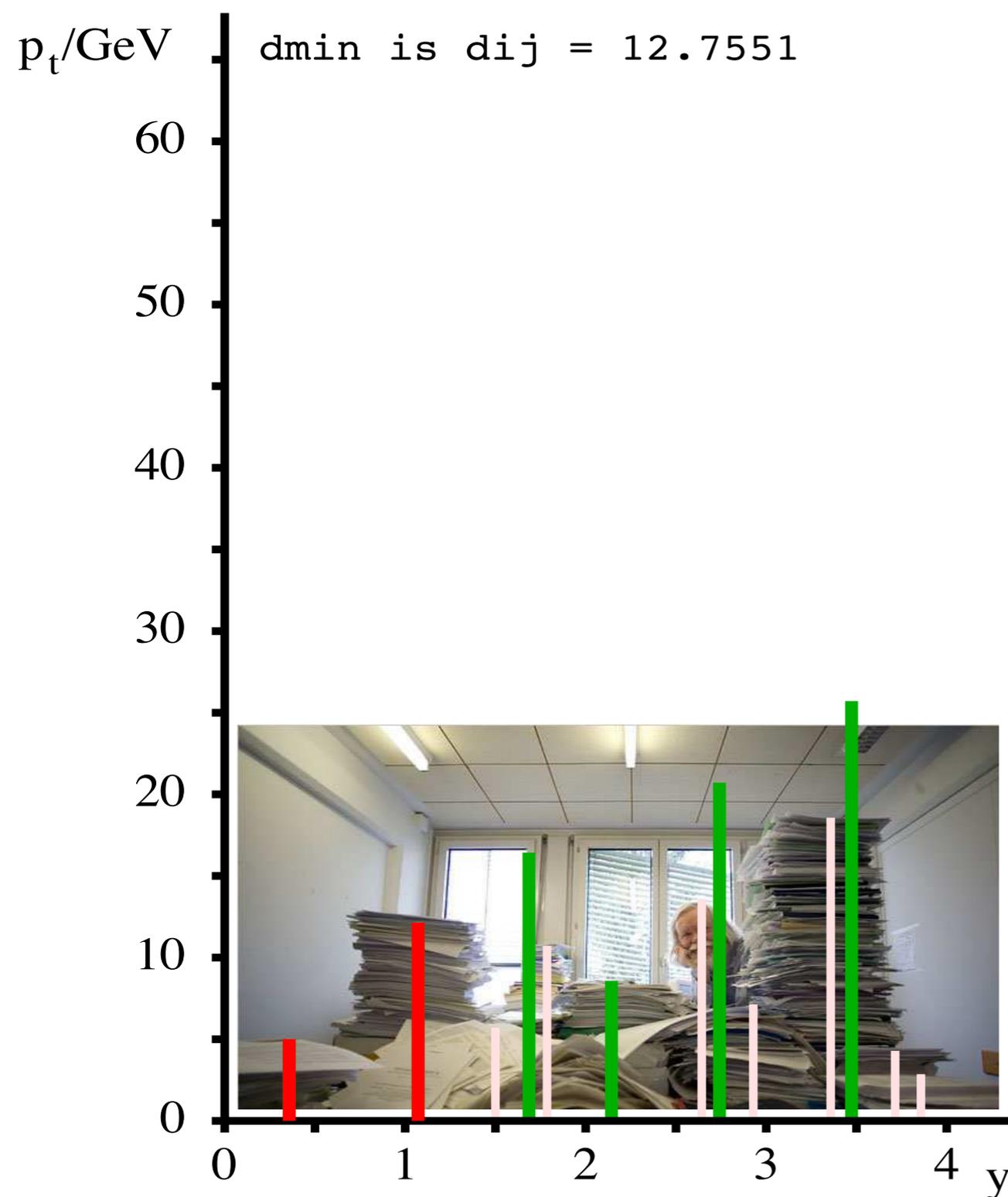
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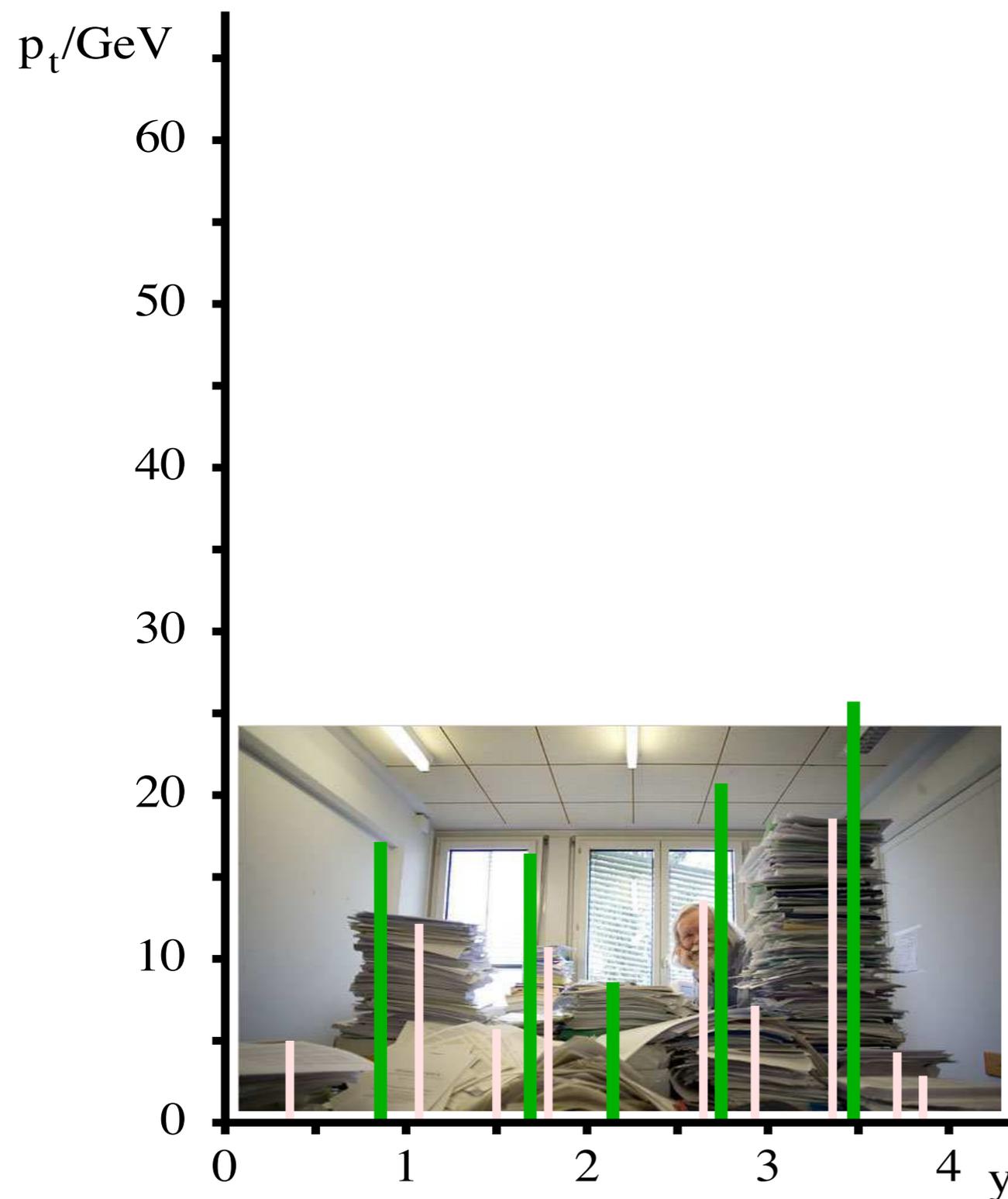
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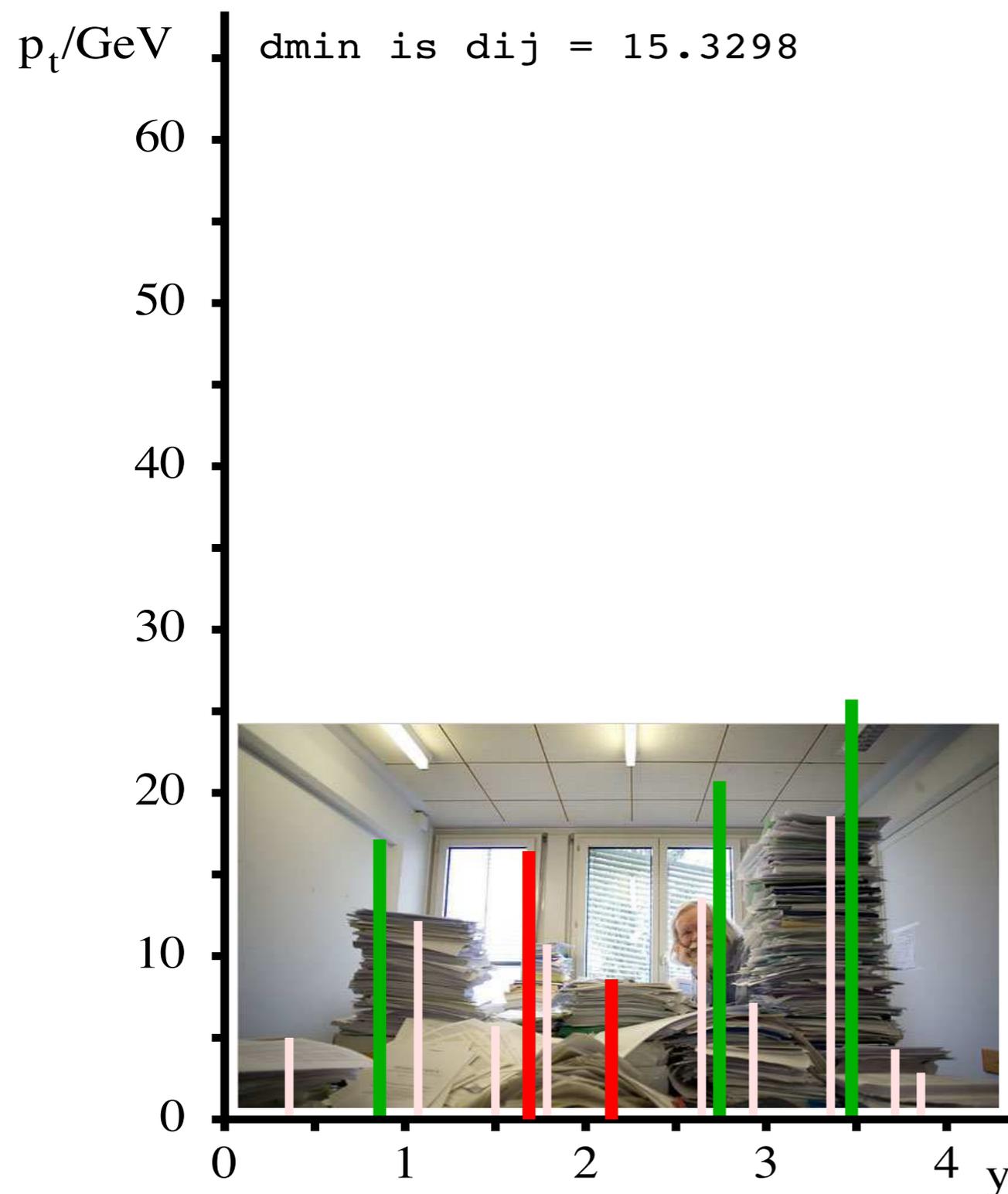
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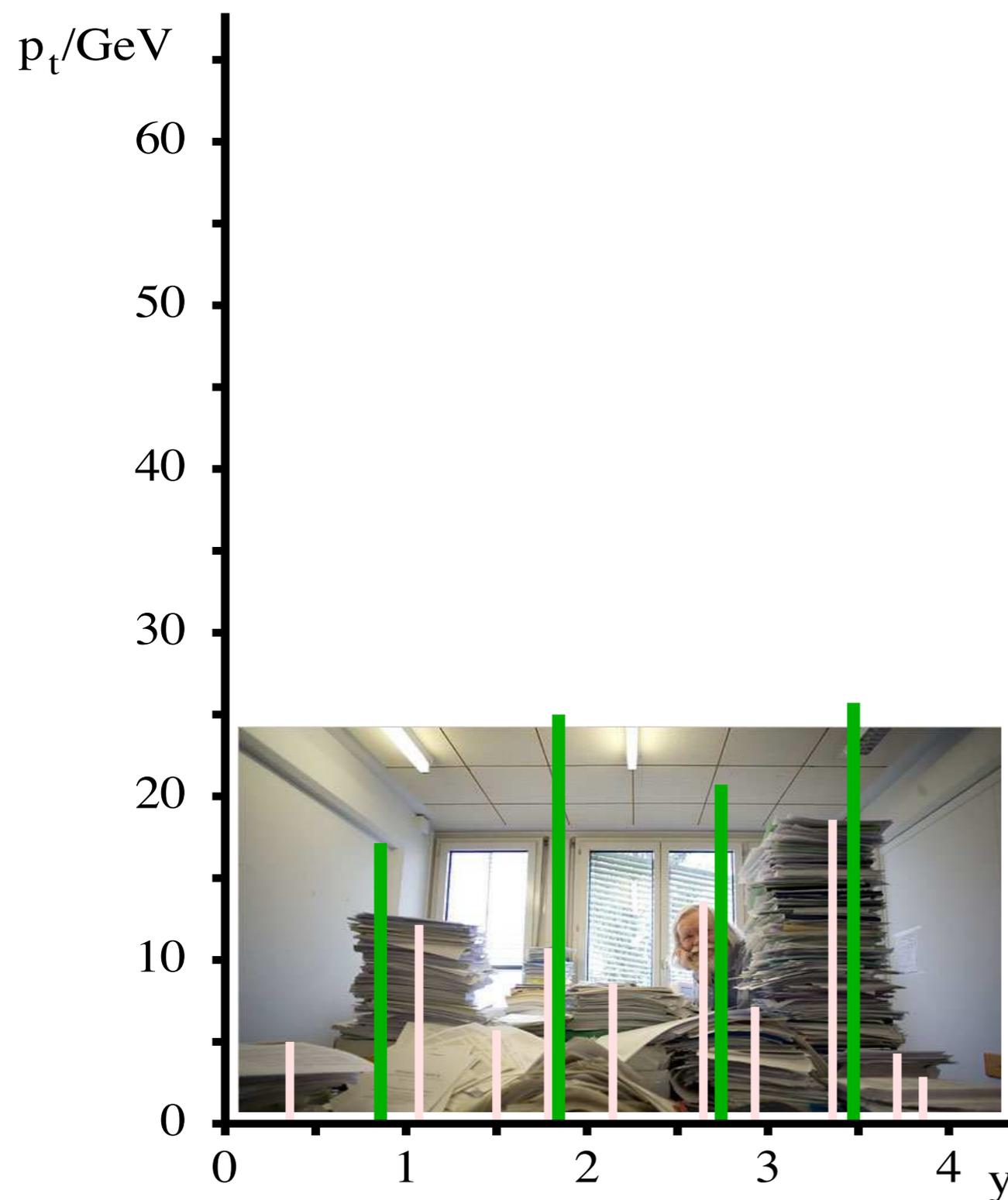
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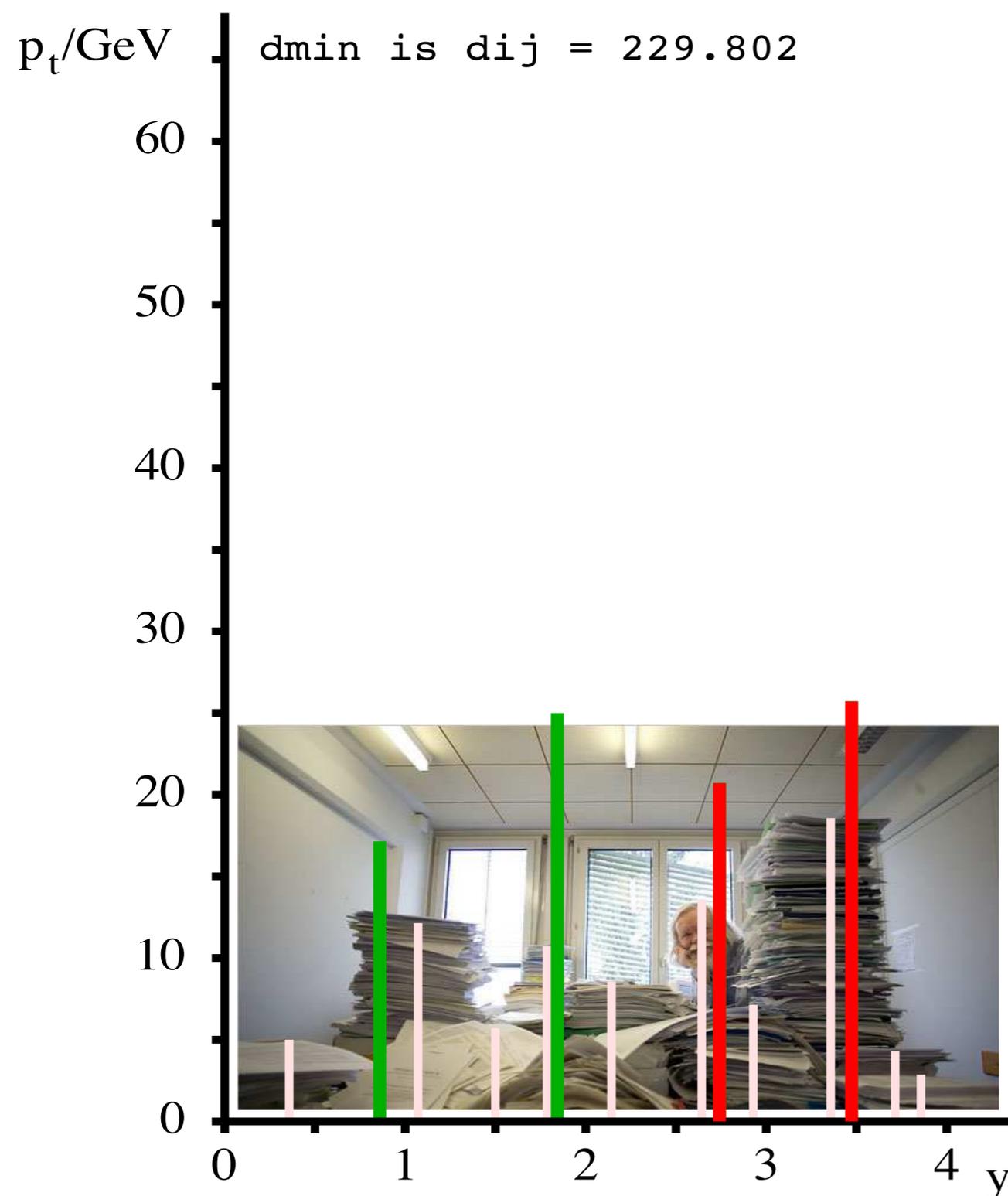
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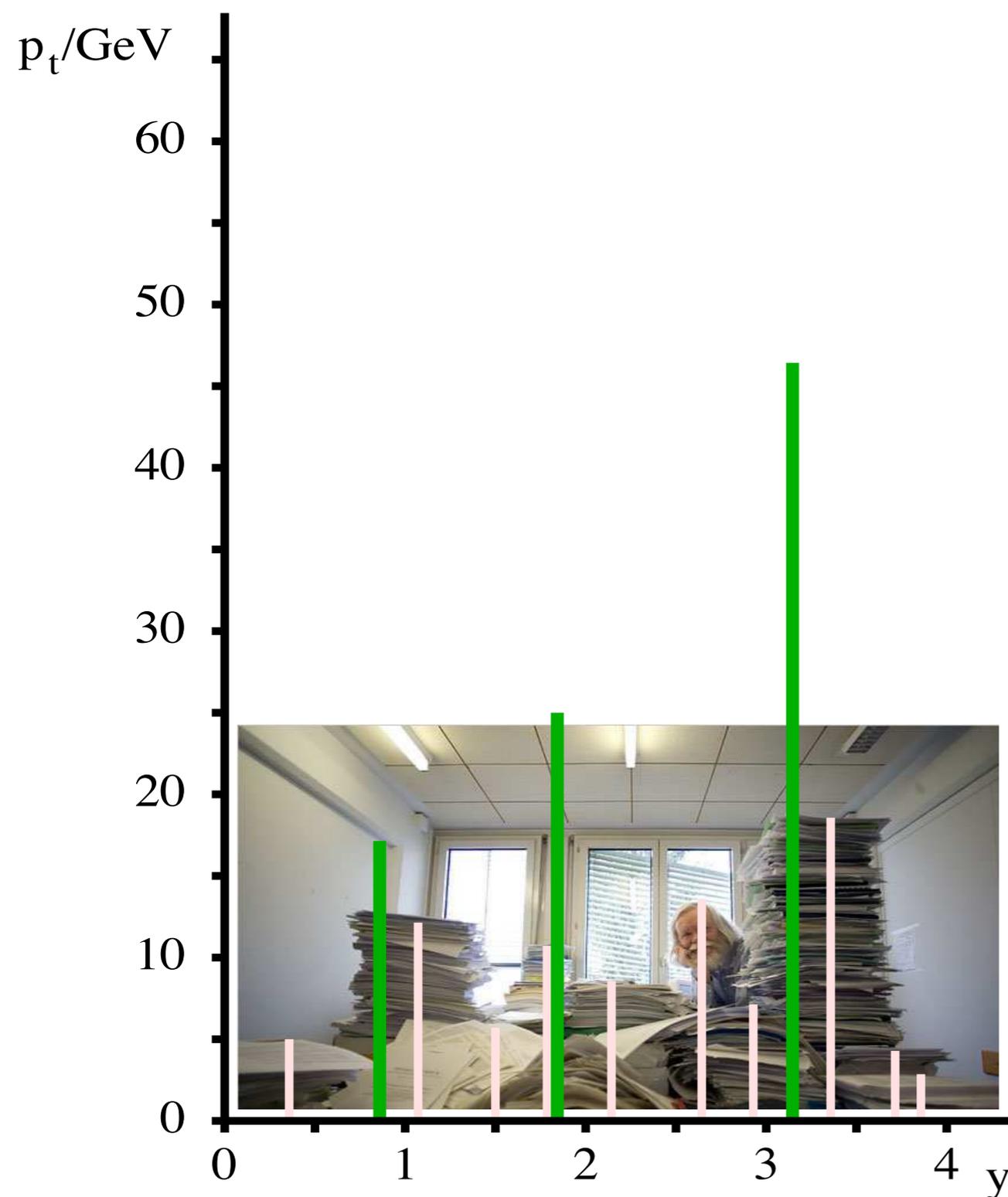
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Example clustering with k_t algorithm, $R = 1.0$

ϕ assumed 0 for all towers



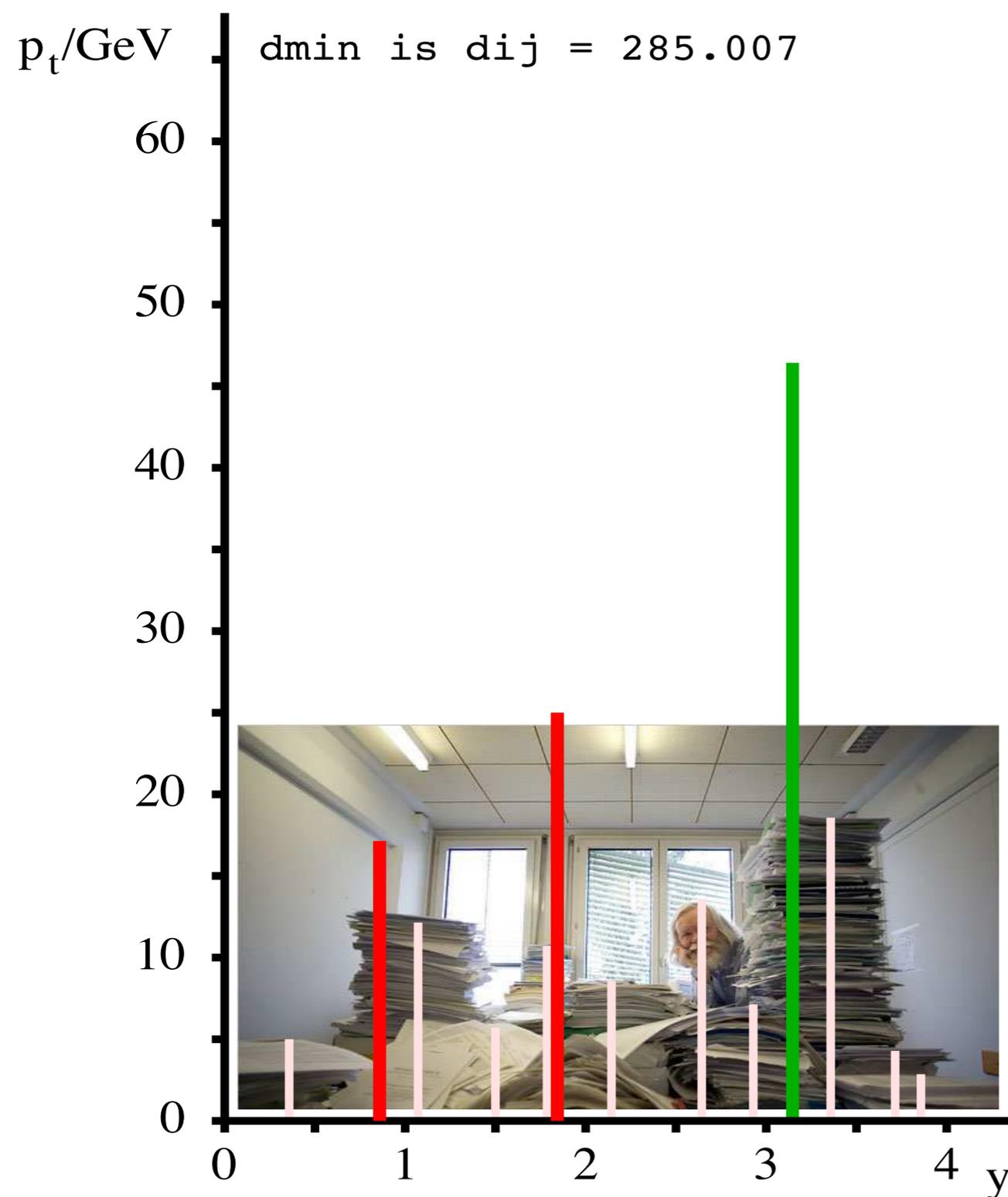
k_t alg.: Find smallest of

$$d_{ij} = \min(k_{ti}^2, k_{tj}^2) \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = k_{ti}^2$$

- ▶ If d_{ij} recombine
- ▶ if d_{iB} , i is a jet

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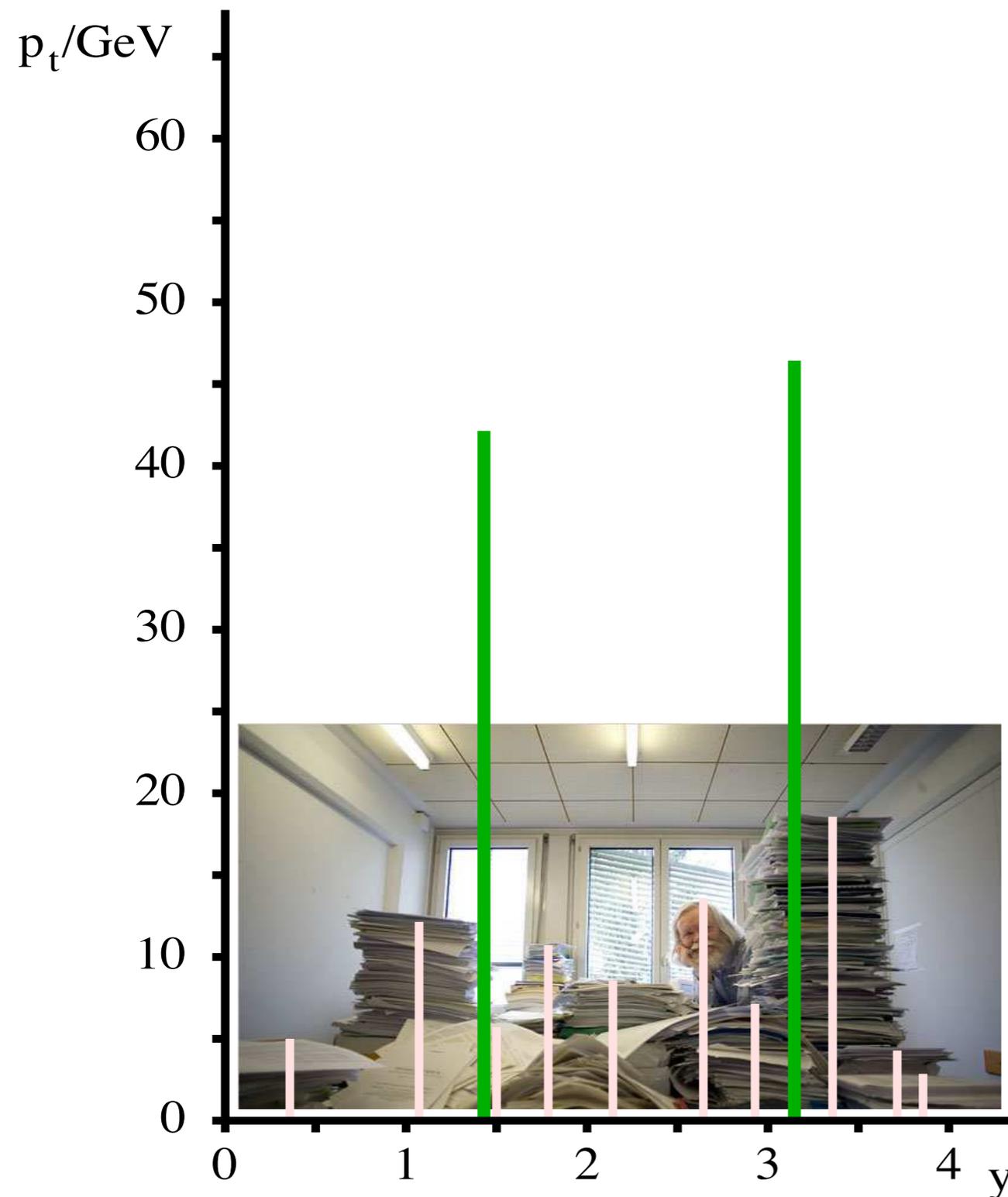
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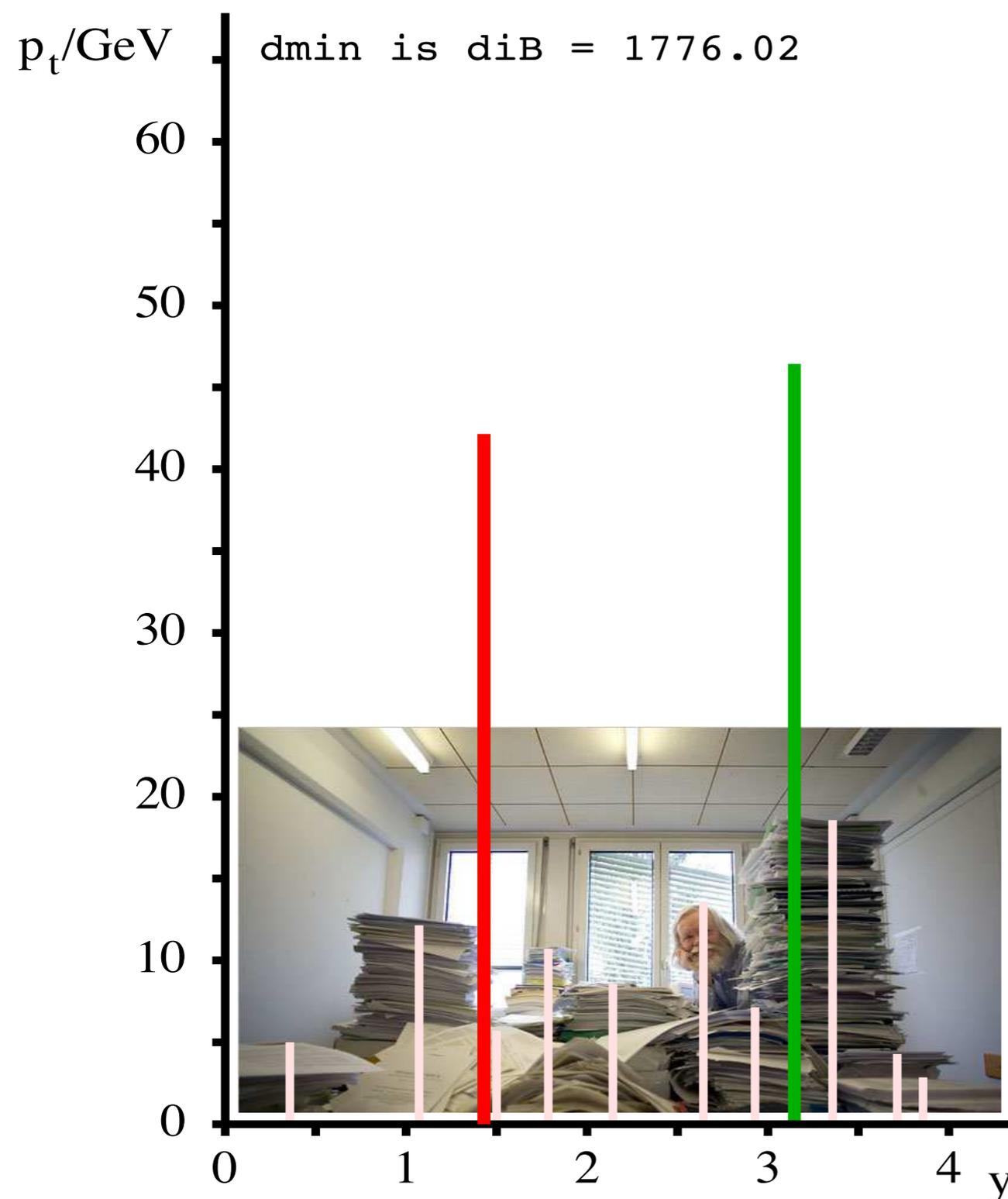
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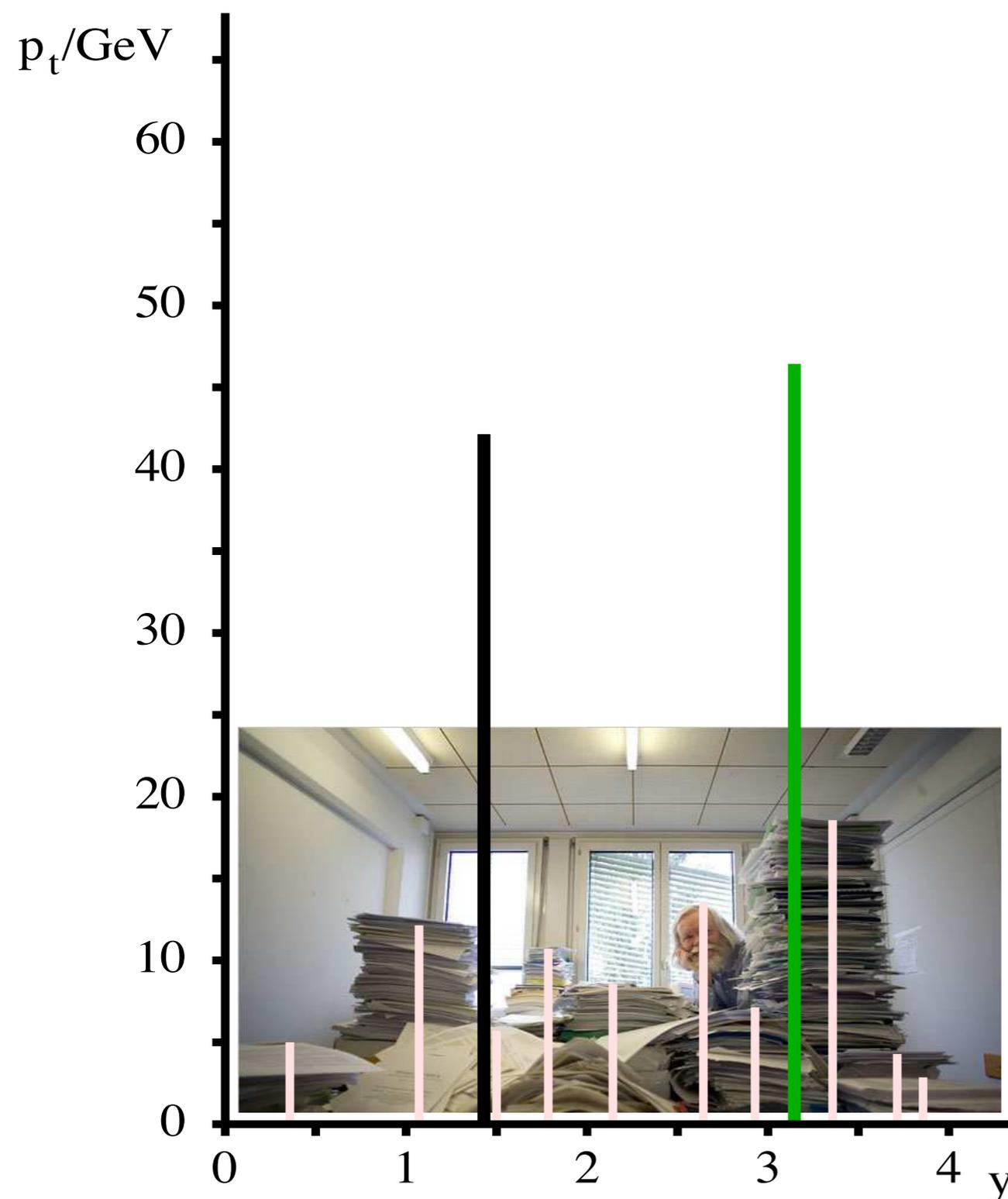
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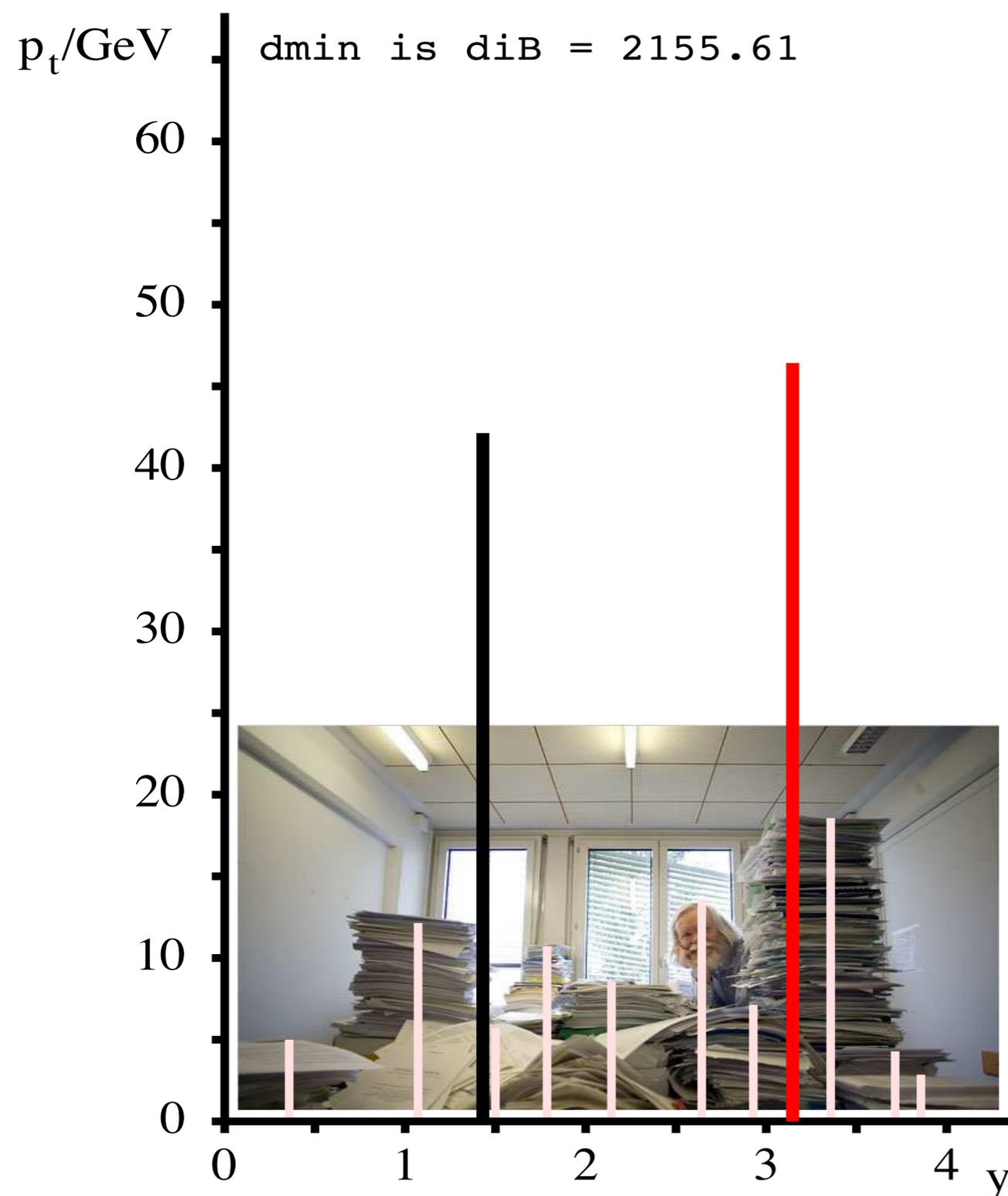
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Example clustering with k_t algorithm, $R = 1.0$

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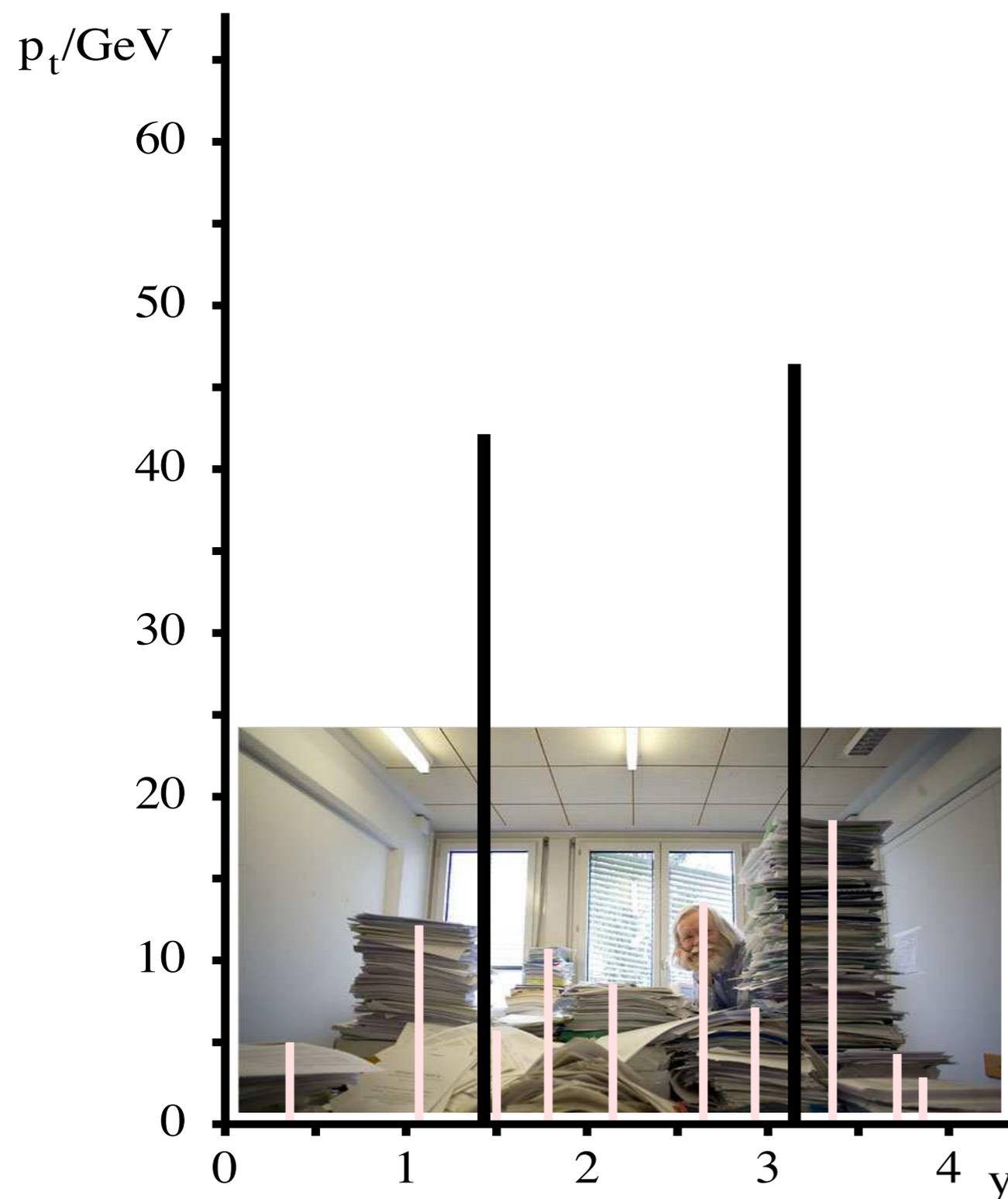
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Example clustering with k_t algorithm, $R = 1.0$

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k_t alg.: Find smallest of

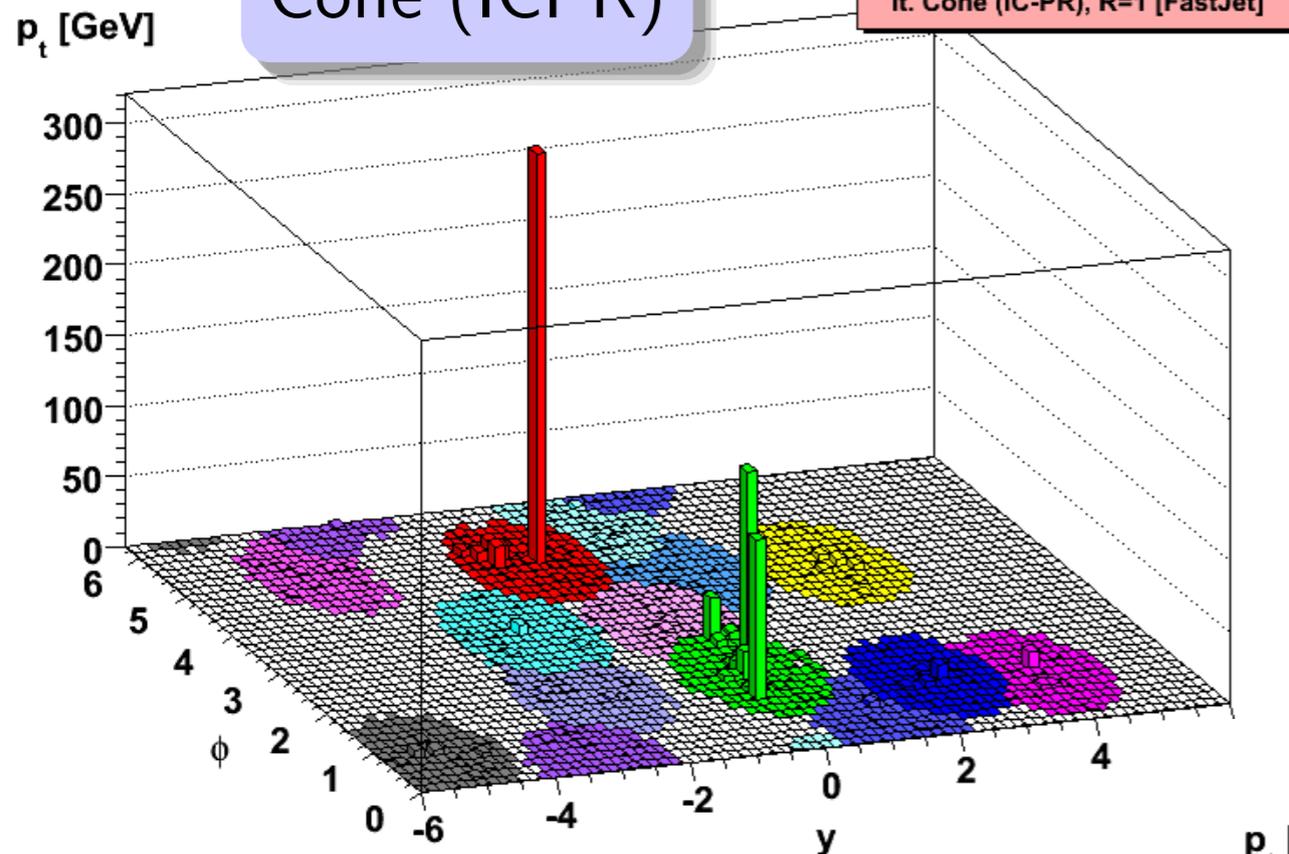
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Example clustering with k_t algorithm, $R = 1.0$

ϕ assumed 0 for all towers

Cone (ICPR)



(Some) cone algorithms give **circular** jets in $y - \phi$ plane

Much appreciated by experiments

e.g. for acceptance corrections

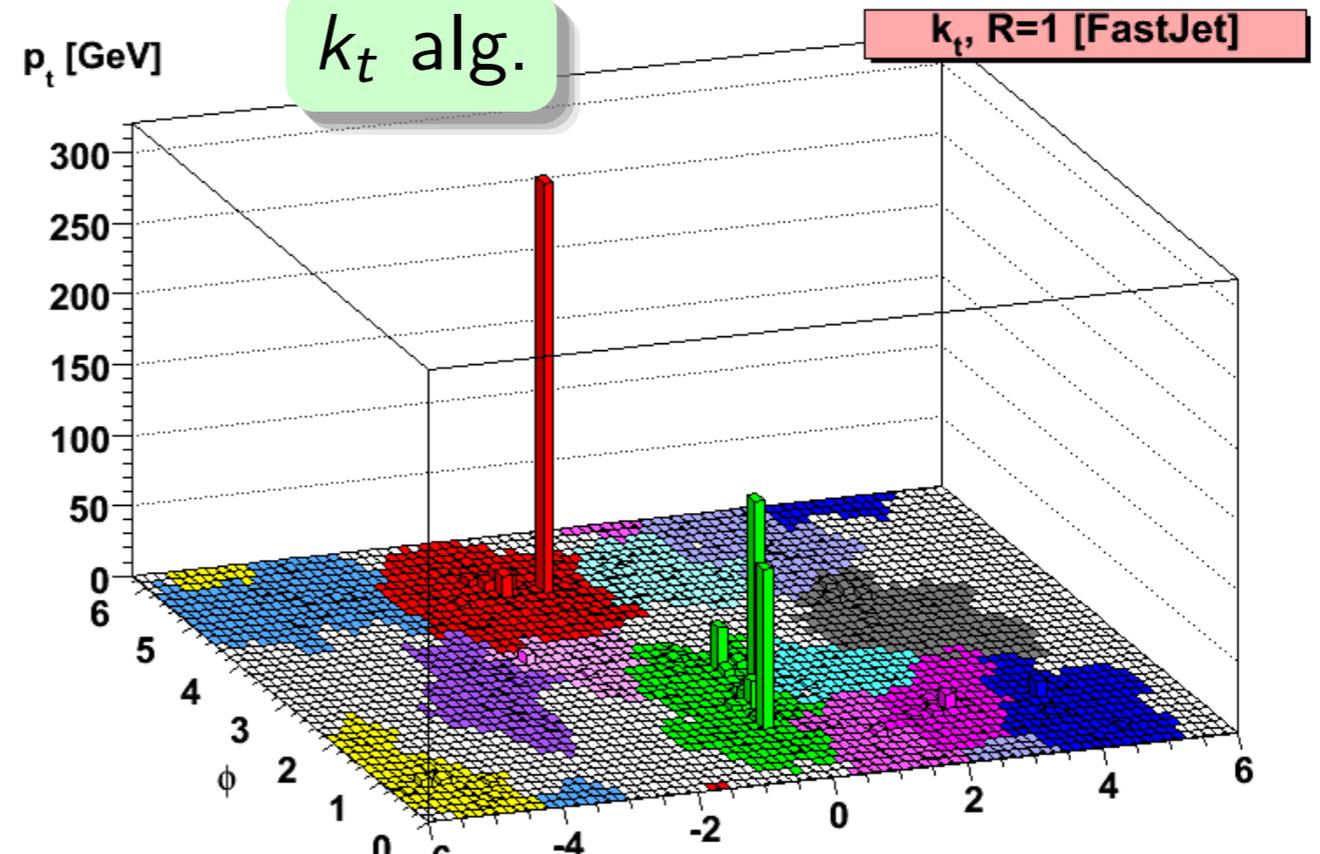
k_t jets are **irregular**

Because soft junk clusters together first:

$$d_{ij} = \min(k_{ti}^2, k_{tj}^2) \Delta R_{ij}^2$$

Regularly held against k_t

k_t alg.



The k_t algorithms form one of several “families” of sequential recombination jet algorithm

Others differ in

1. the choice of distance measure between pairs of particles
[i.e. relative priority given to soft and collinear divergences]
2. using $3 \rightarrow 2$ clustering rather than $2 \rightarrow 1$
[ARCLUS; not used at hadron colliders, so won't discuss it more]

Sequential recombination variants

Cambridge/Aachen: the simplest of hadron-collider algorithms

- Recombine pair of objects closest in ΔR_{ij}
- Repeat until all $\Delta R_{ij} > R$ — remaining objects are jets

Dokshitzer, Leder, Moretti, Webber '97 (Cambridge): more involved e^+e^- form

Wobisch & Wengler '99 (Aachen): simple inclusive hadron-collider form

One still applies a $p_{t,\min}$ cut to the jets, as for inclusive k_t

C/A privileges the collinear divergence of QCD;
it 'ignores' the soft one

Anti- k_t : formulated similarly to inclusive k_t , but with

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$

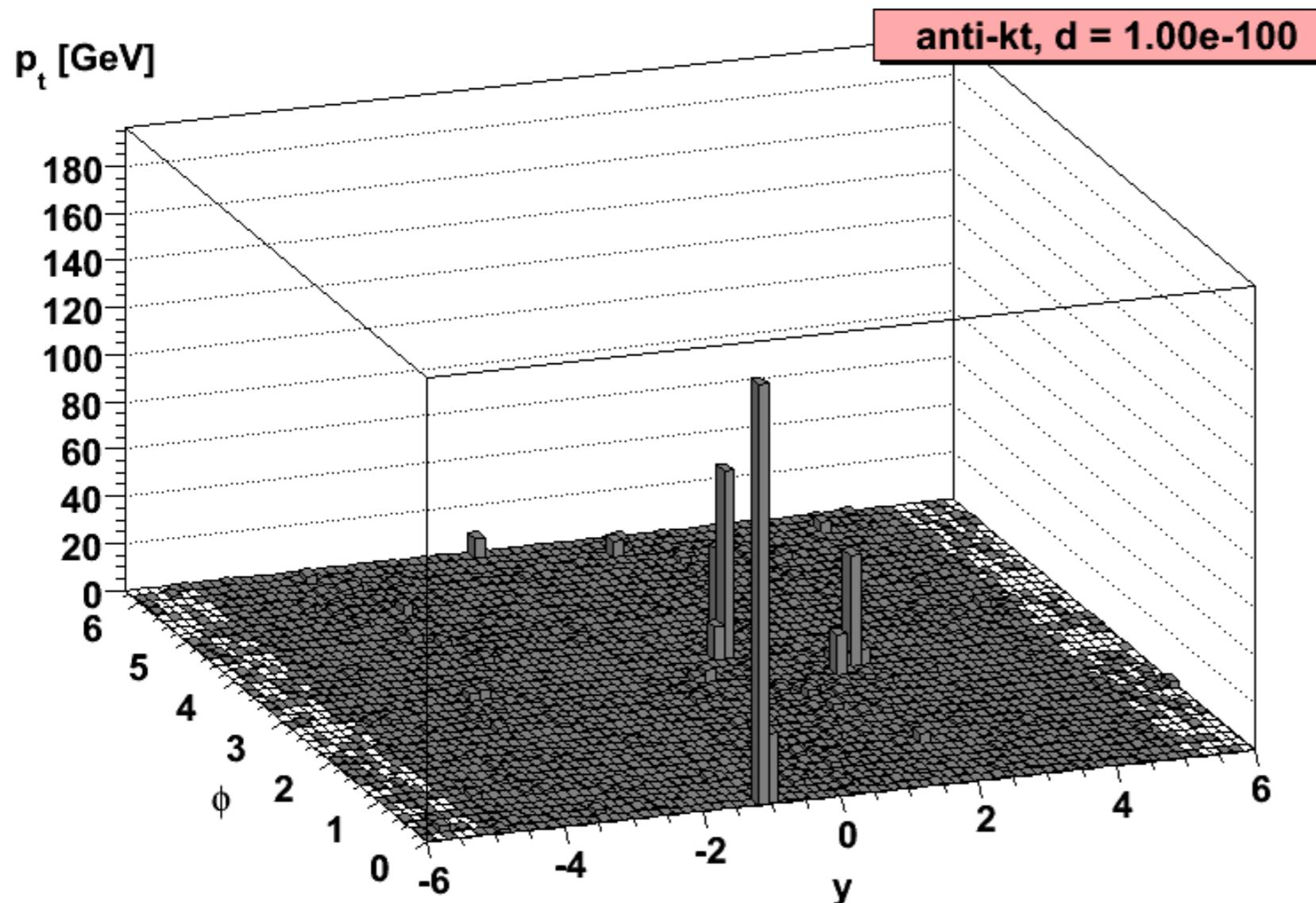
Cacciari, GPS & Soyez '08 [+Delsart unpublished]

Anti- k_t privileges the collinear divergence of QCD and disfavours clustering between pairs of soft particles

Most pairwise clusterings involve at least one hard particle

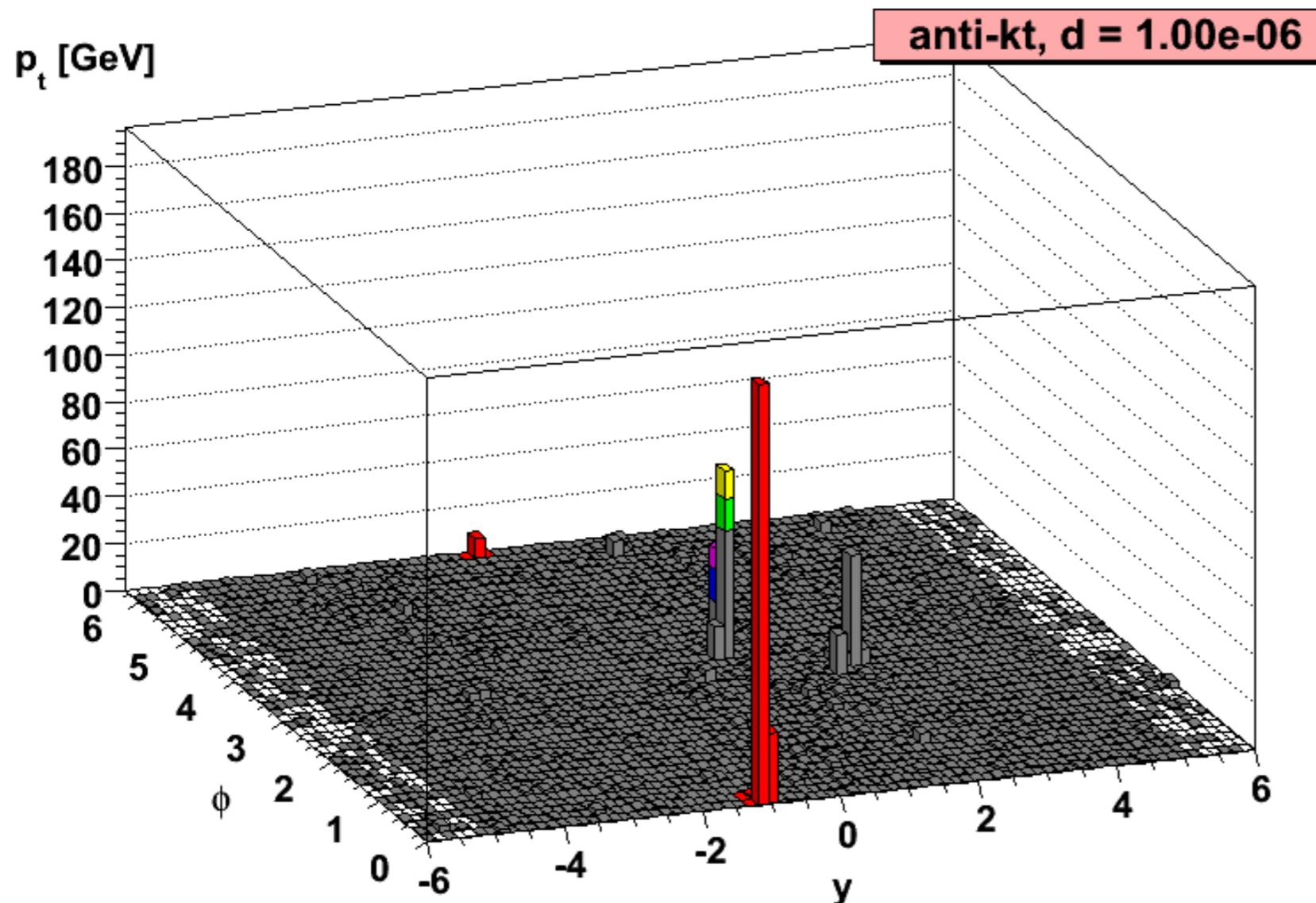
Clustering grows around hard cores

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$



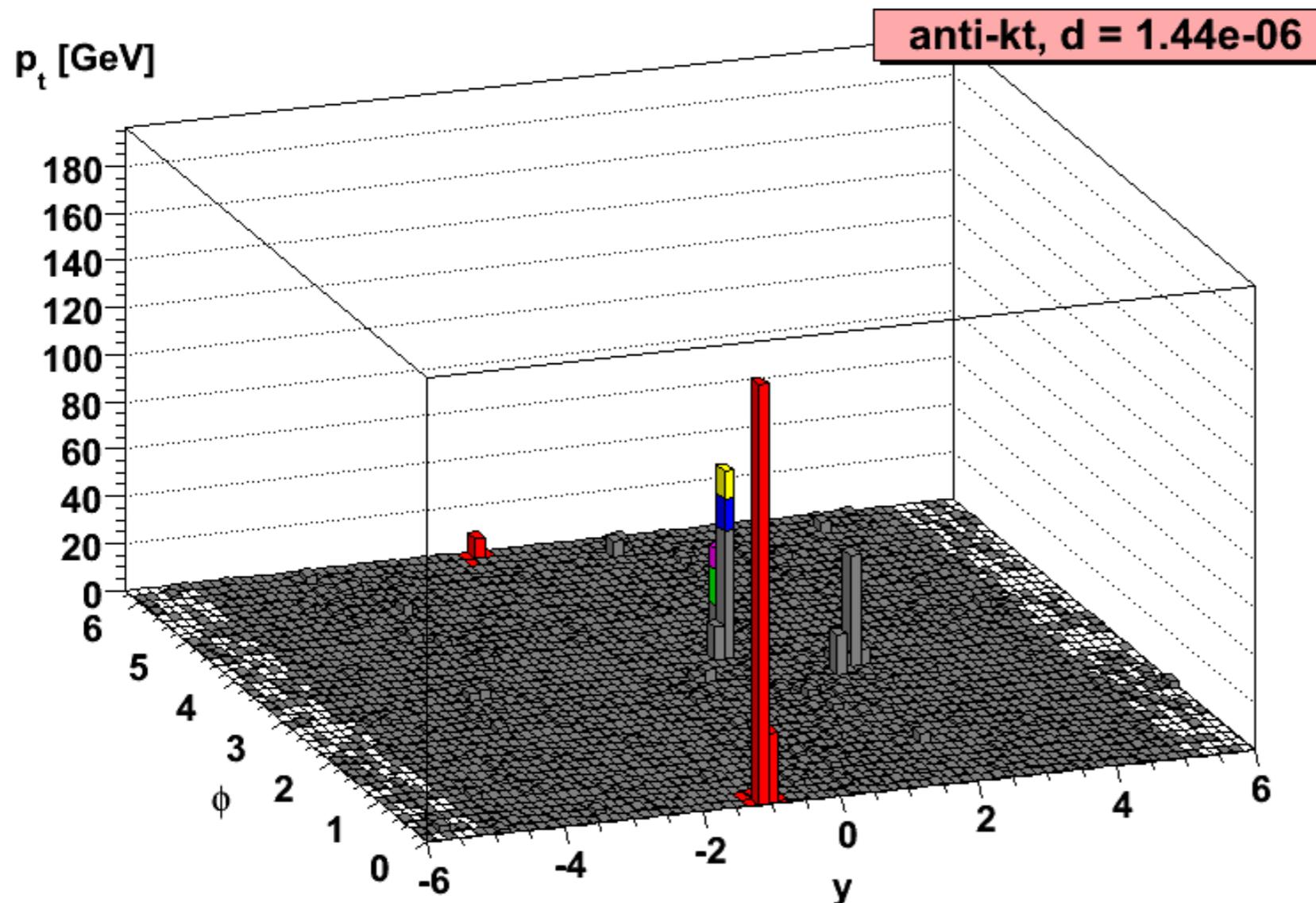
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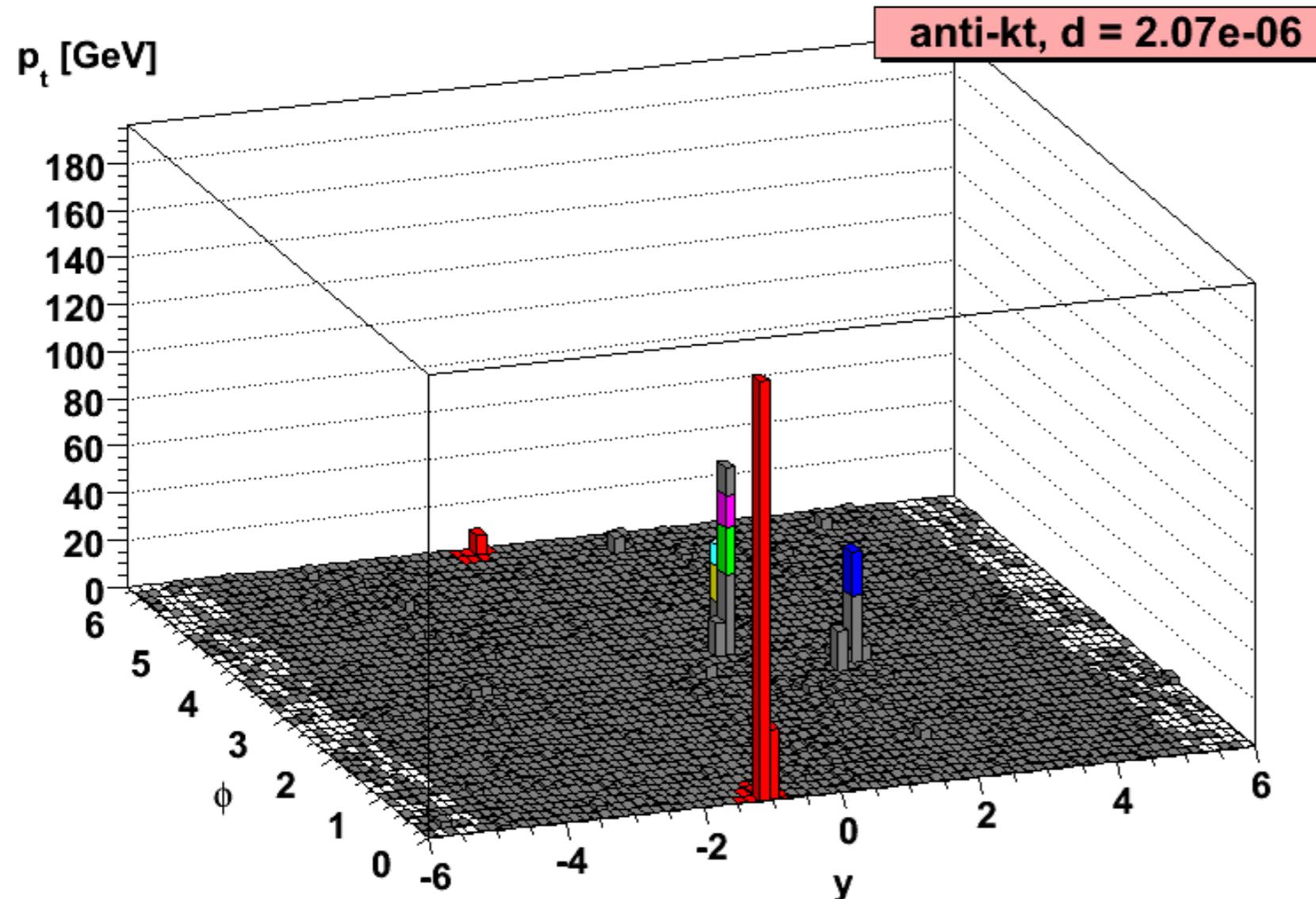
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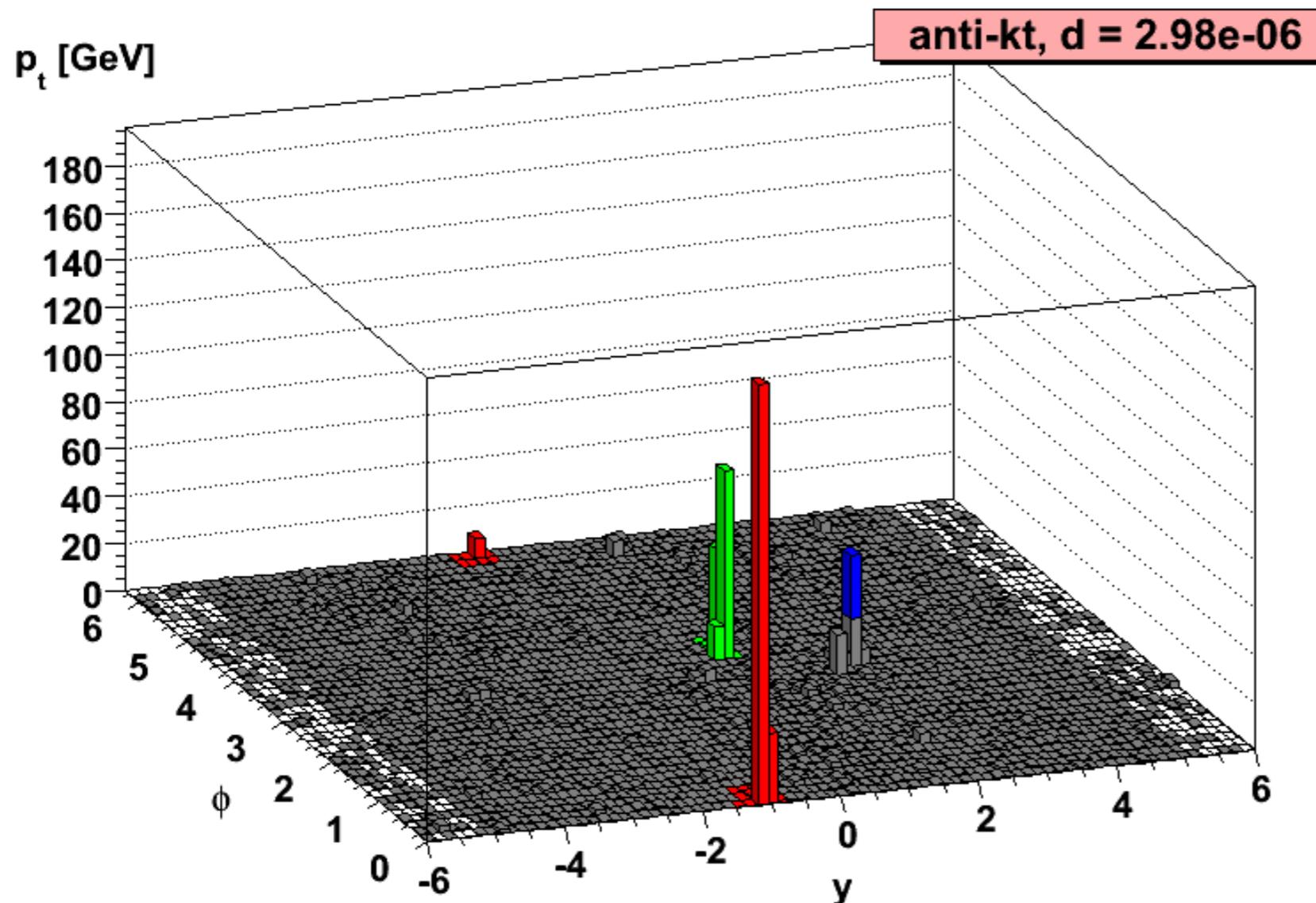
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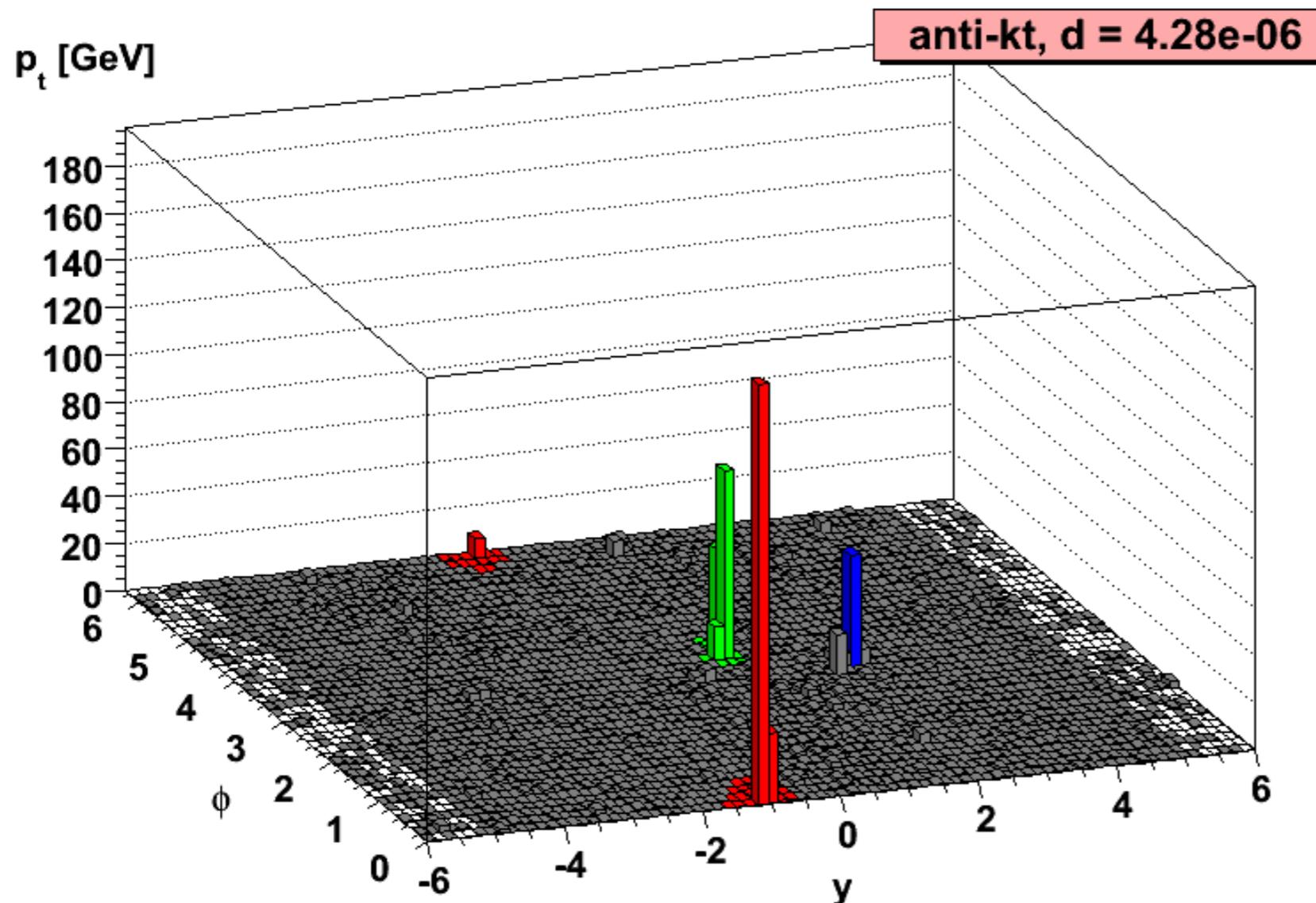
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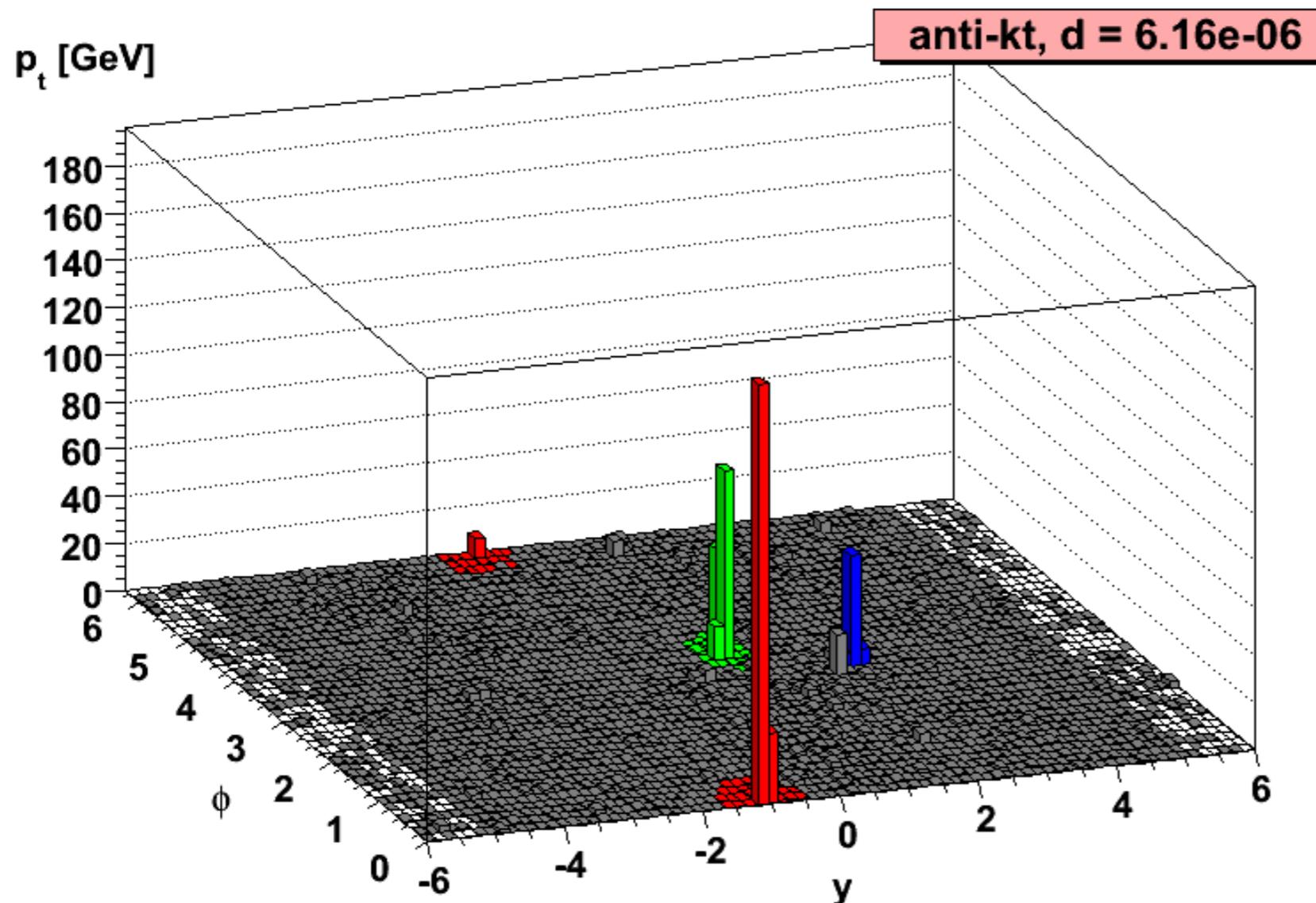
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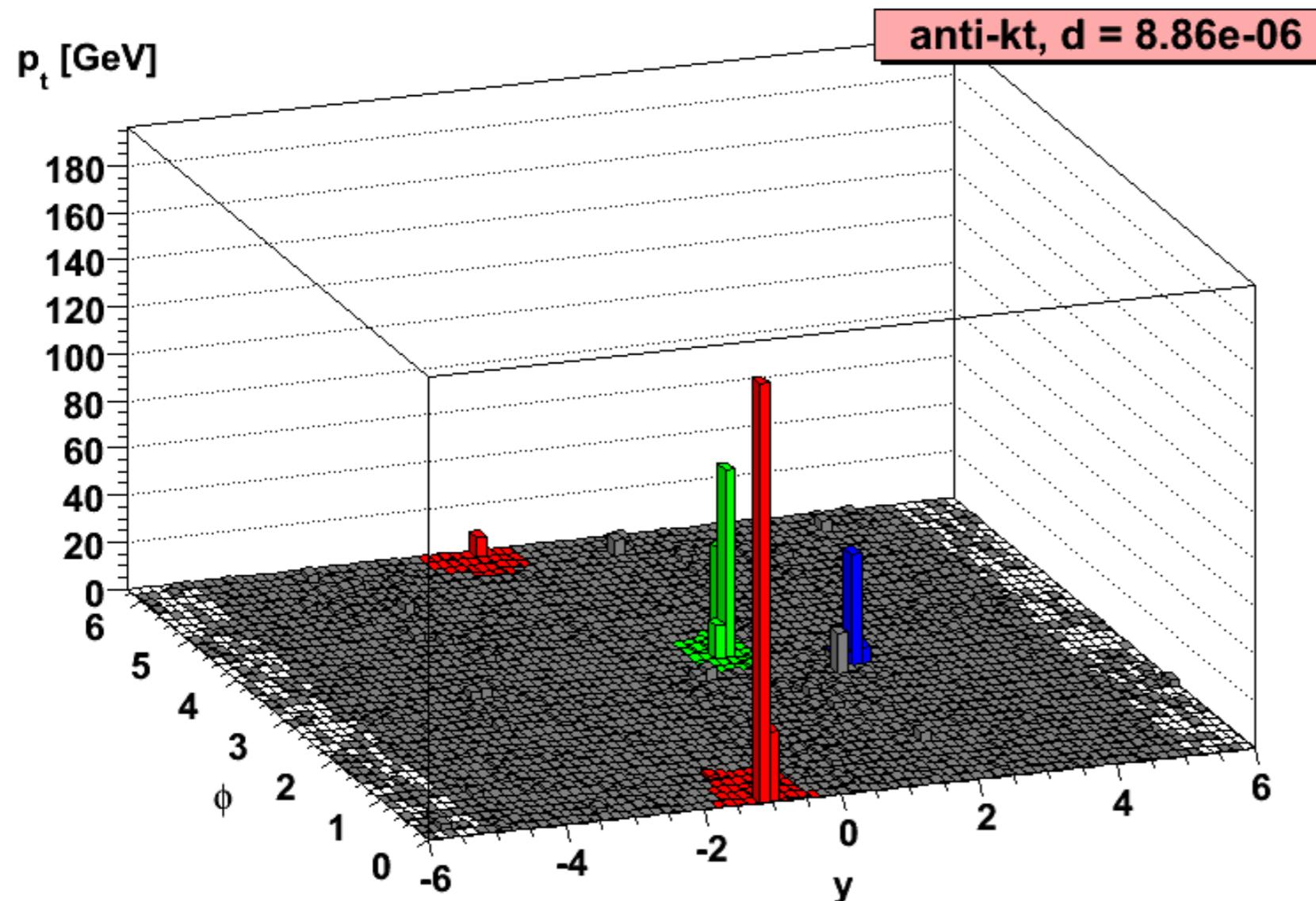
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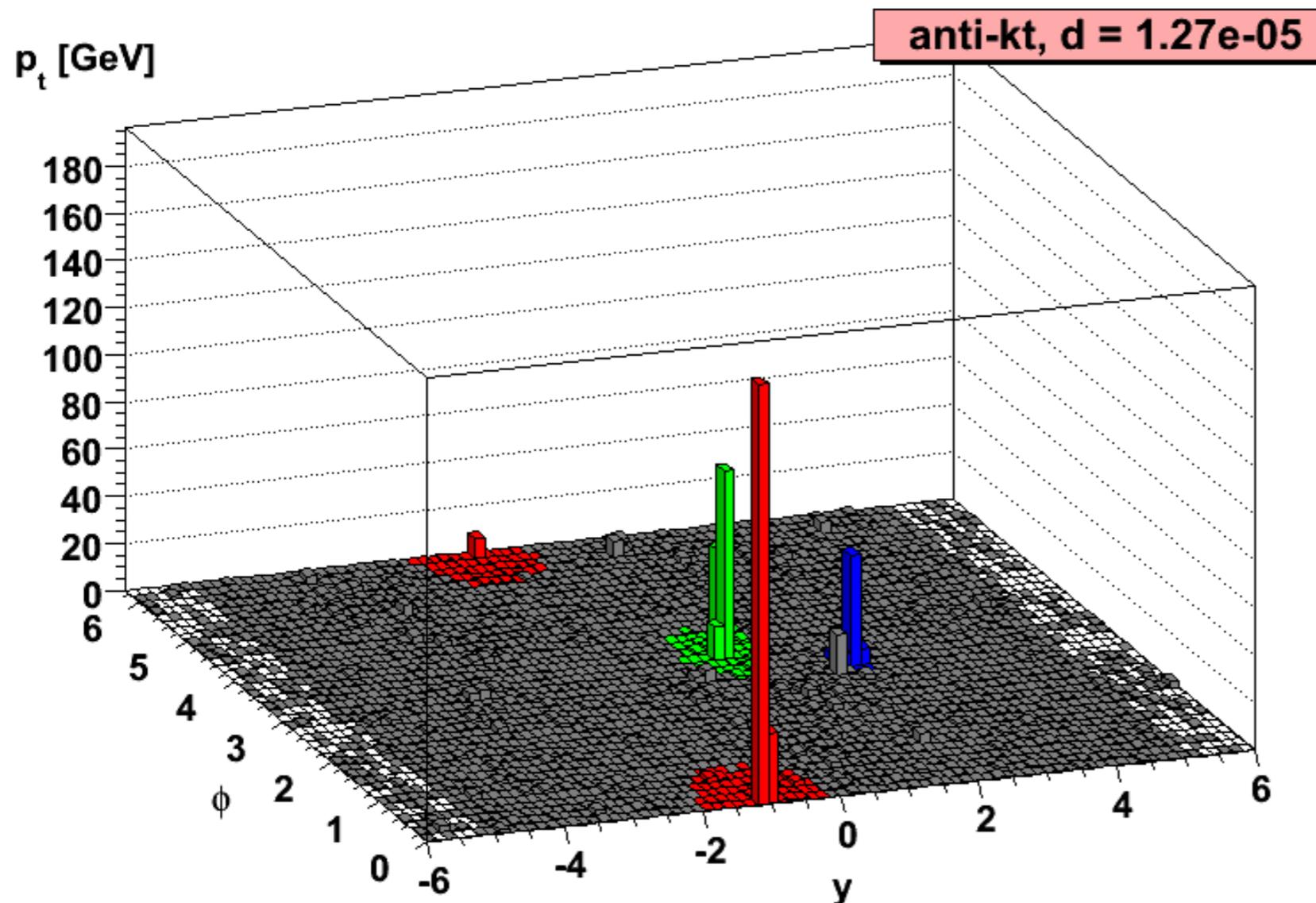
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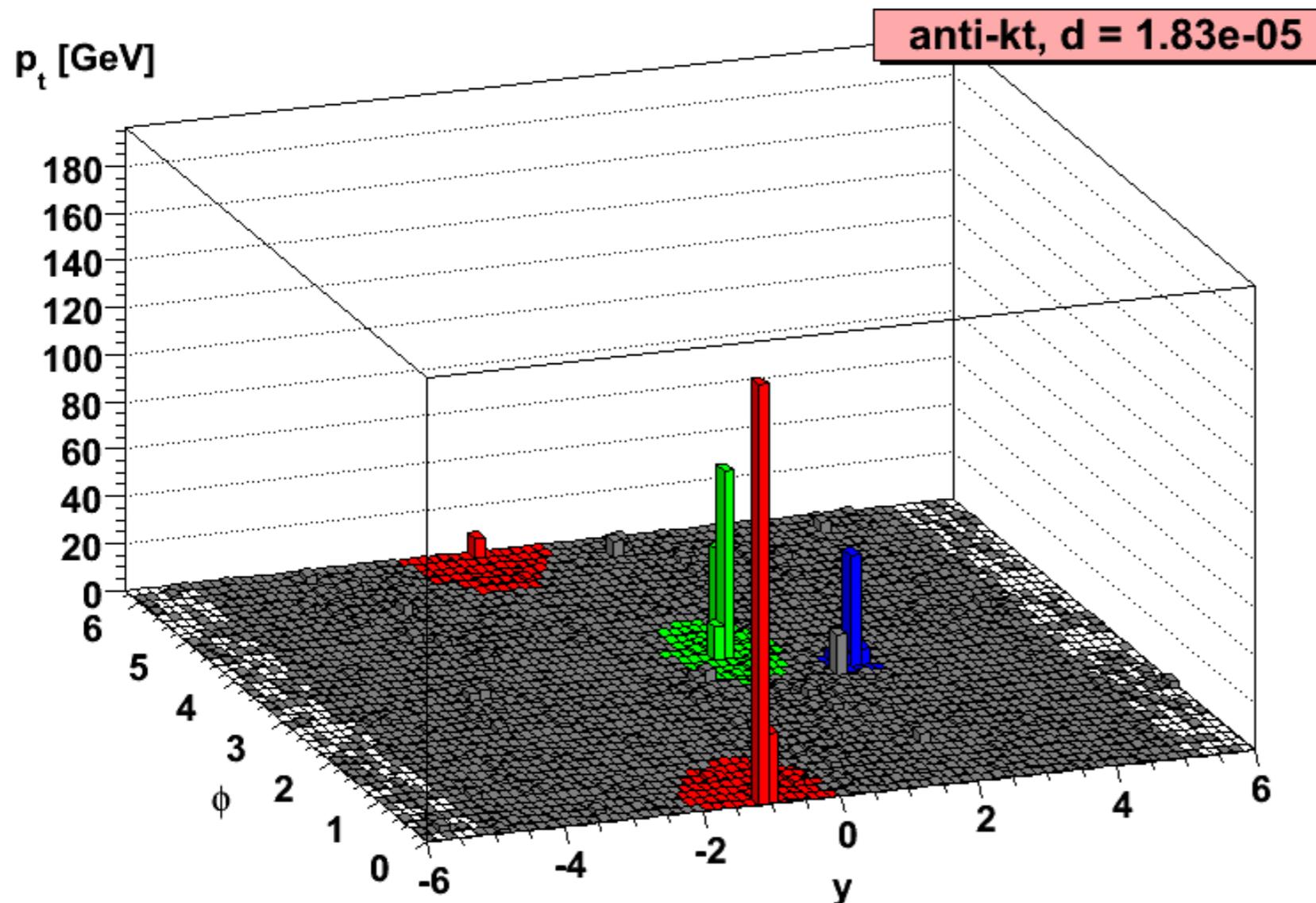
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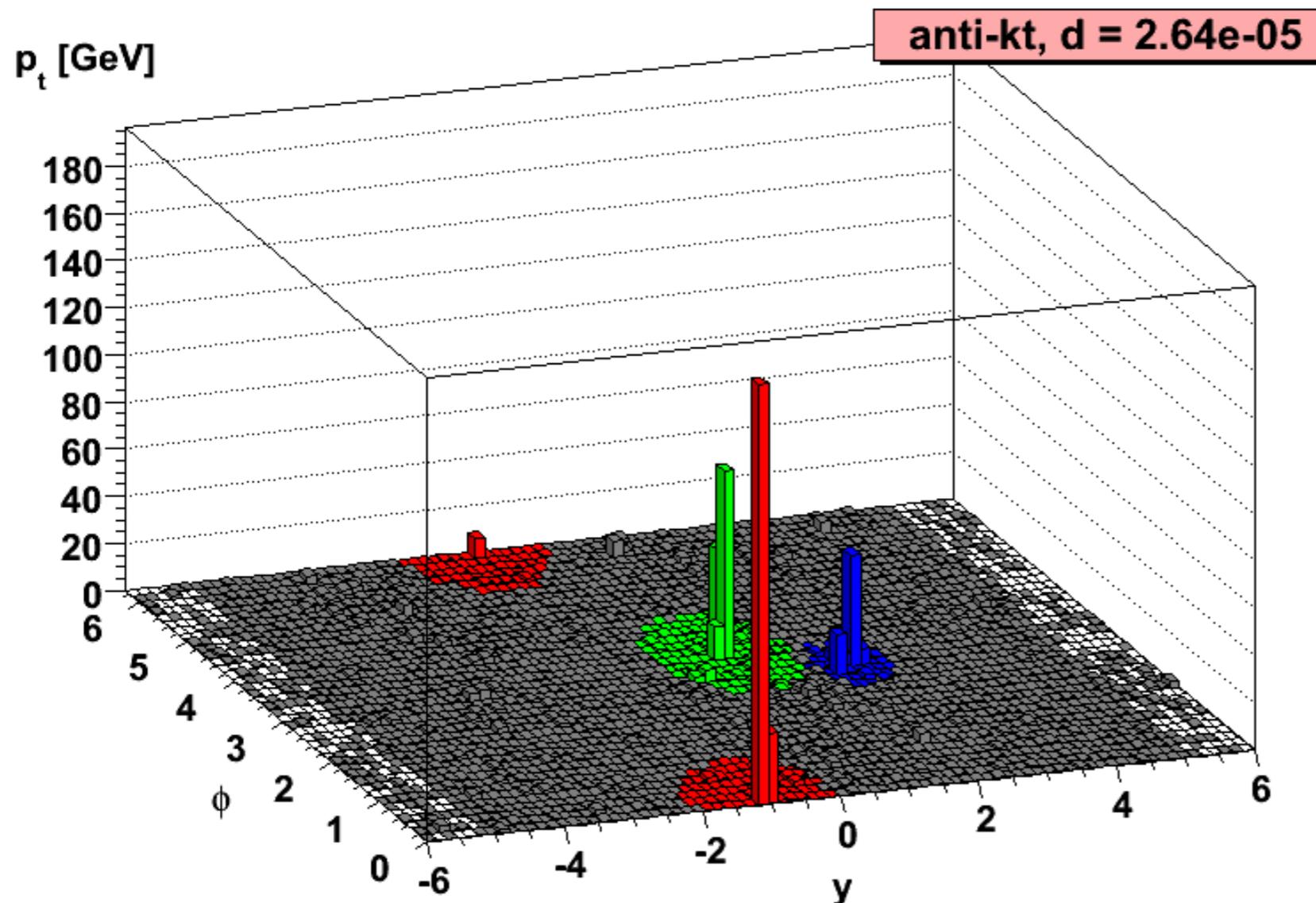
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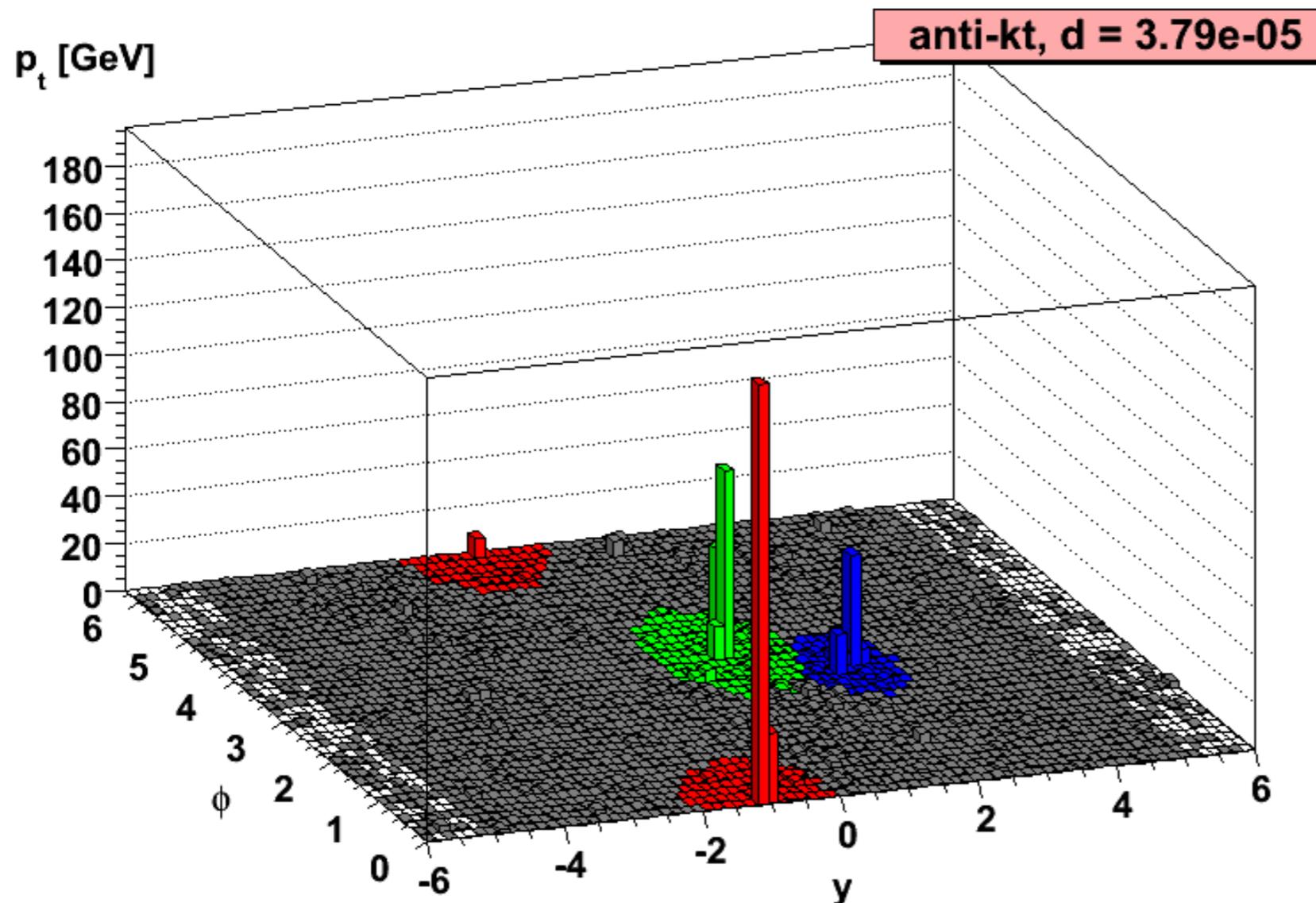
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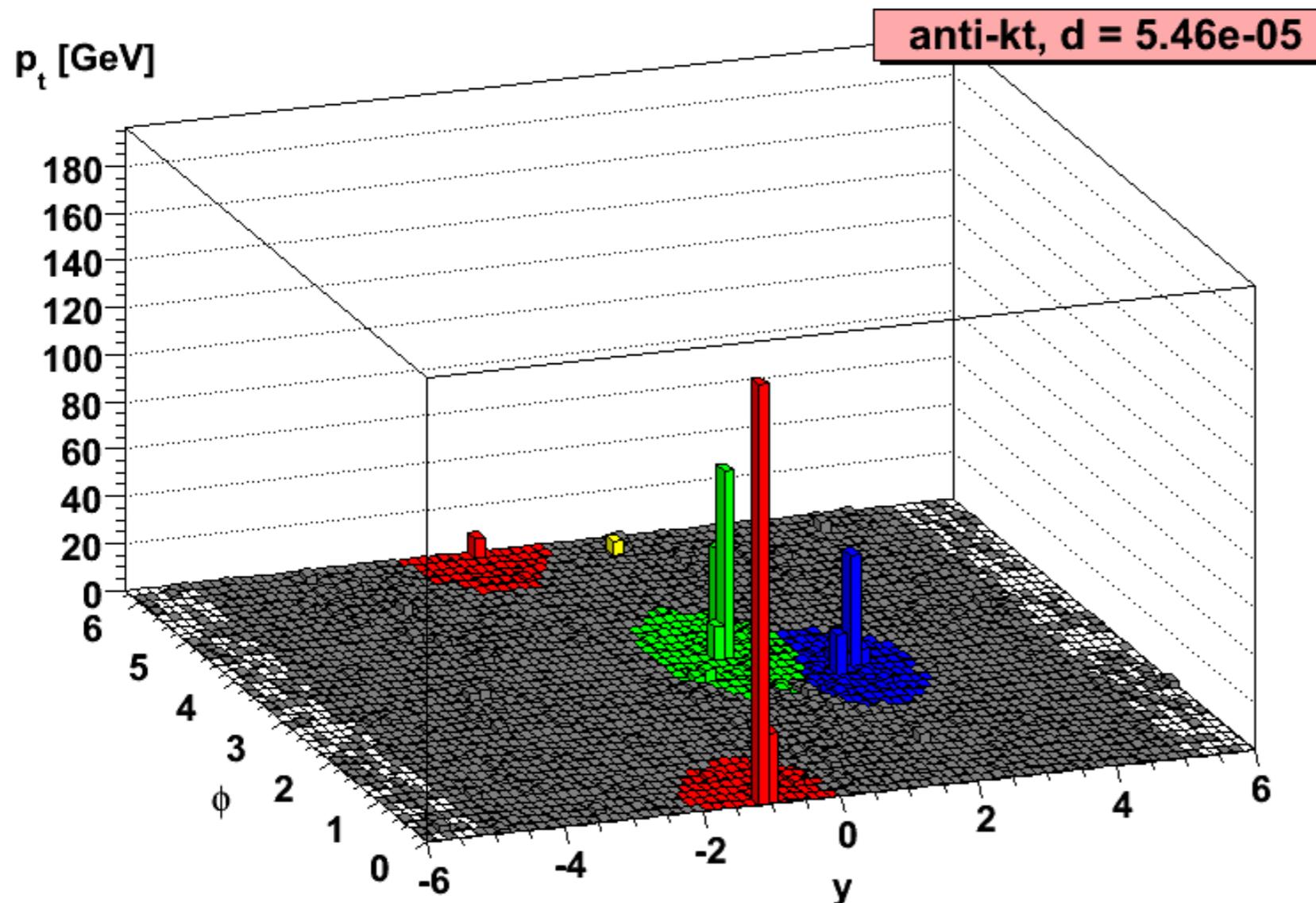
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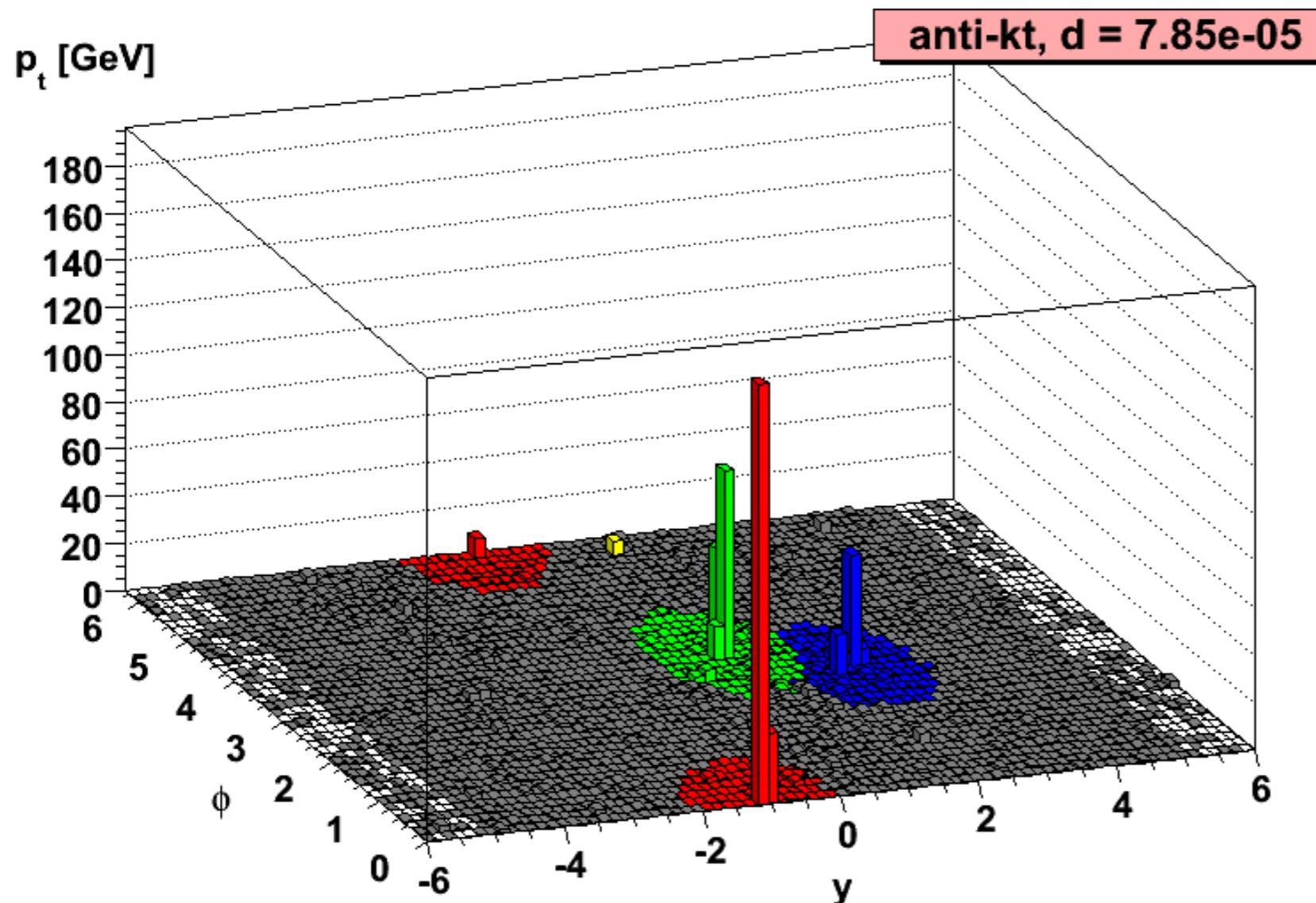
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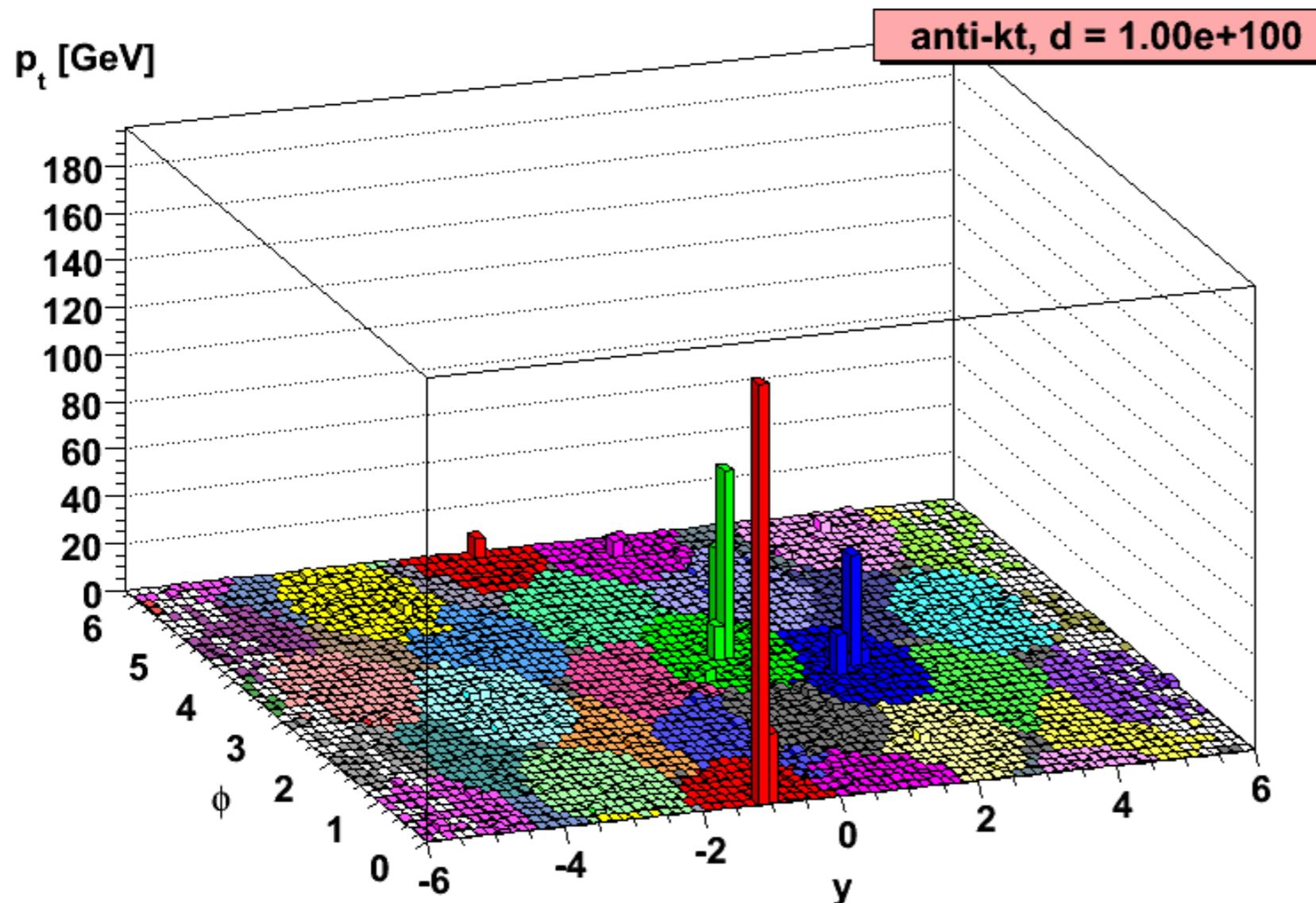
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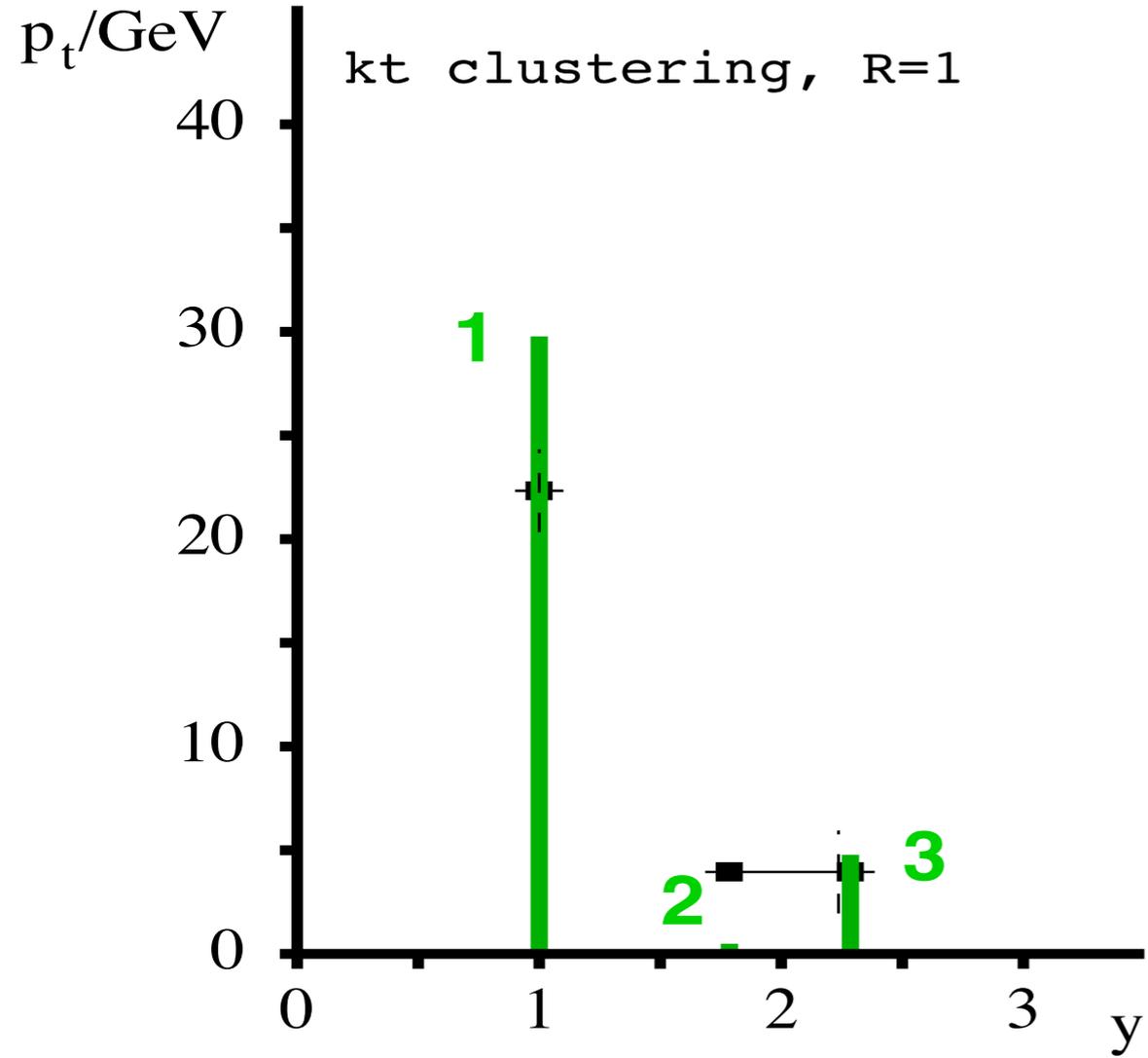
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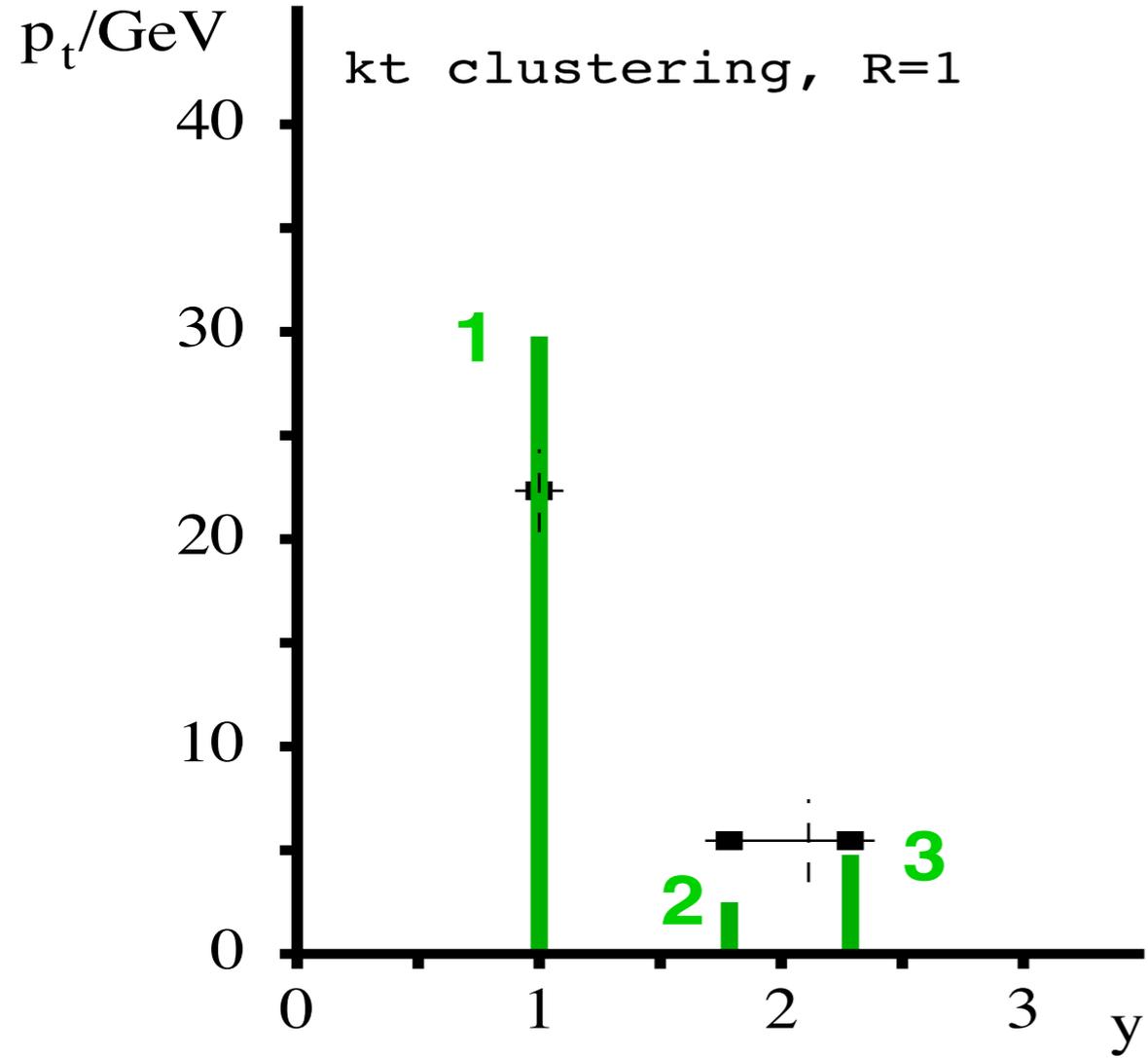
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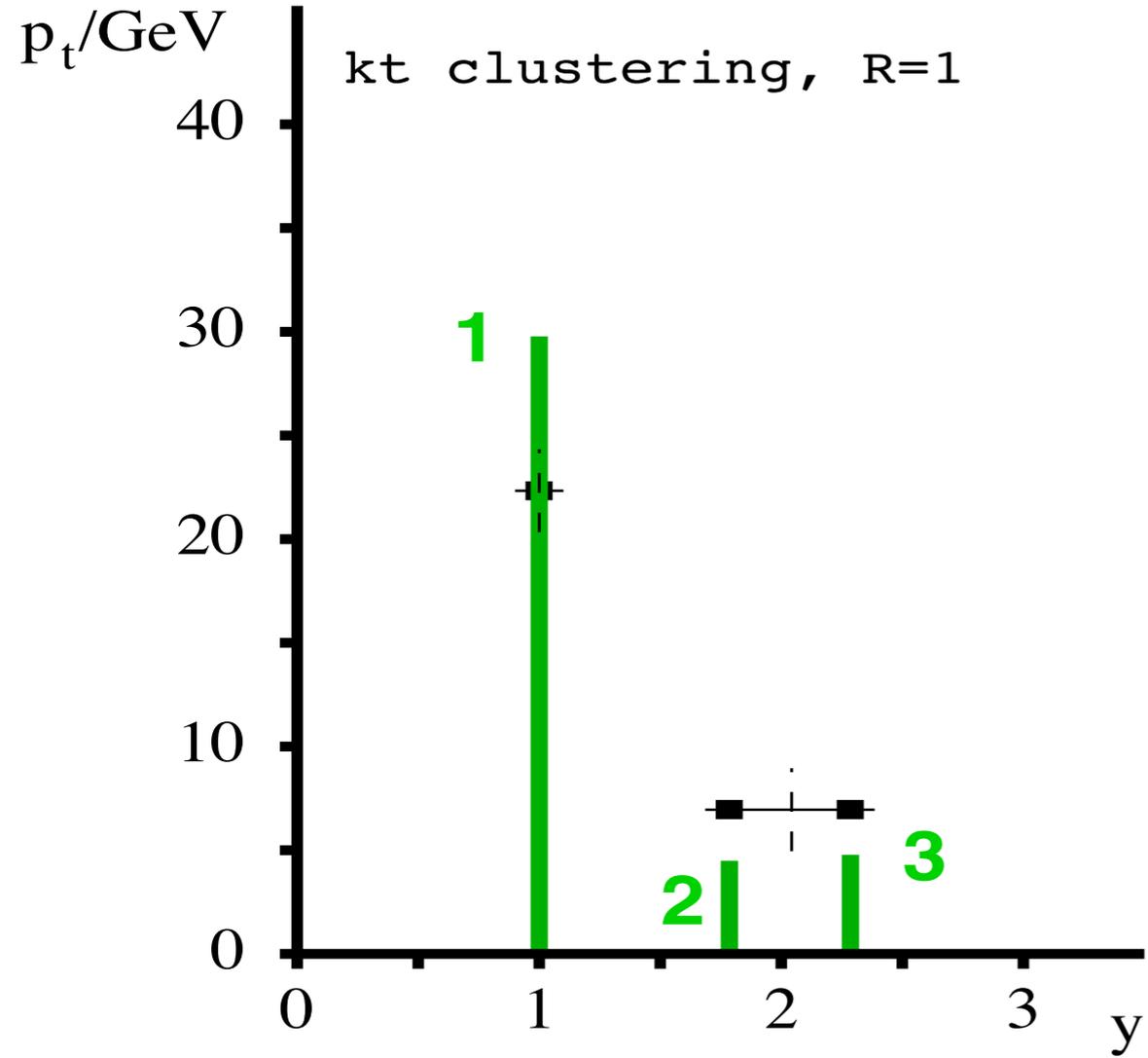


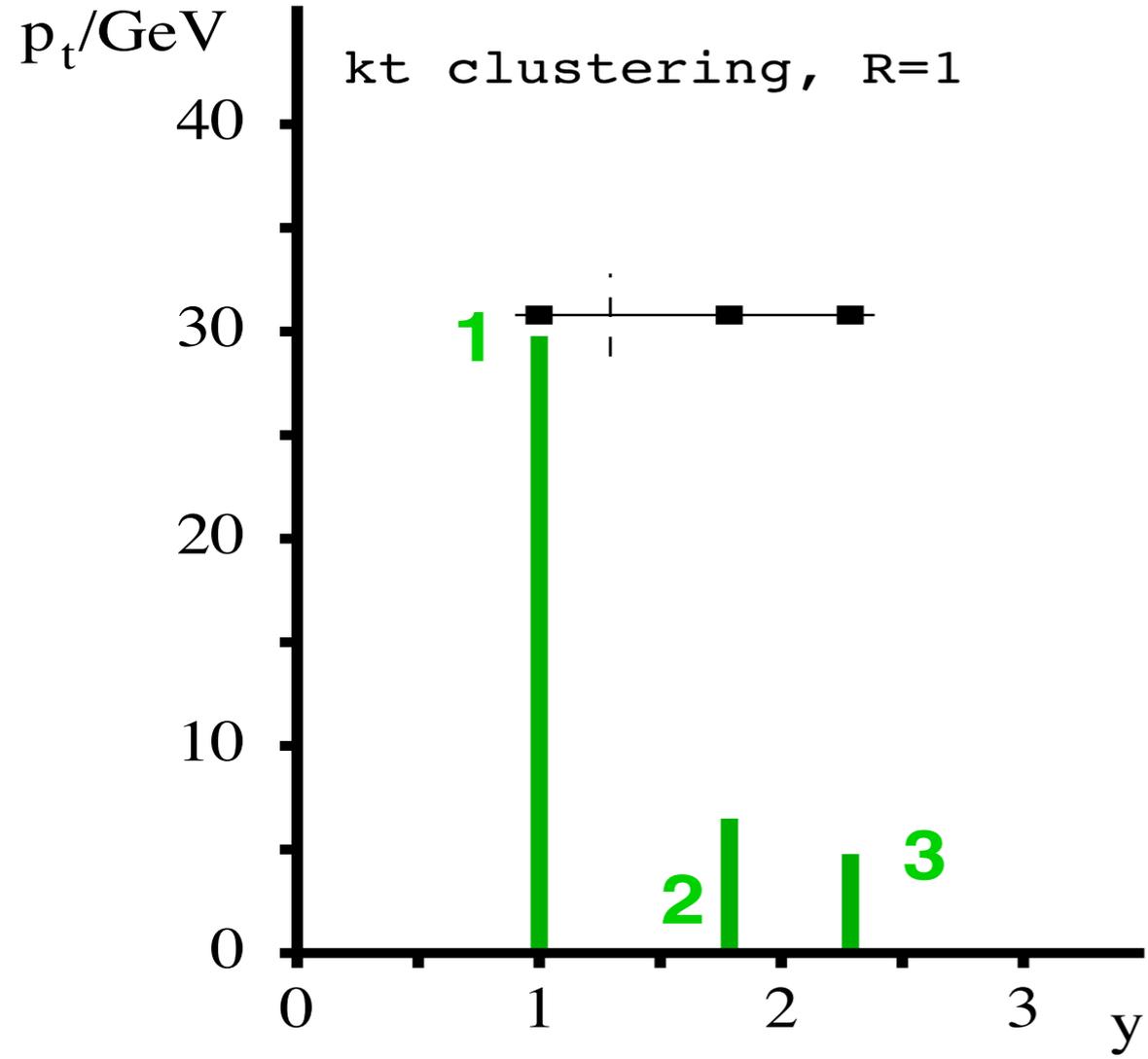
Anti- k_t gives cone-like jets without using stable cones

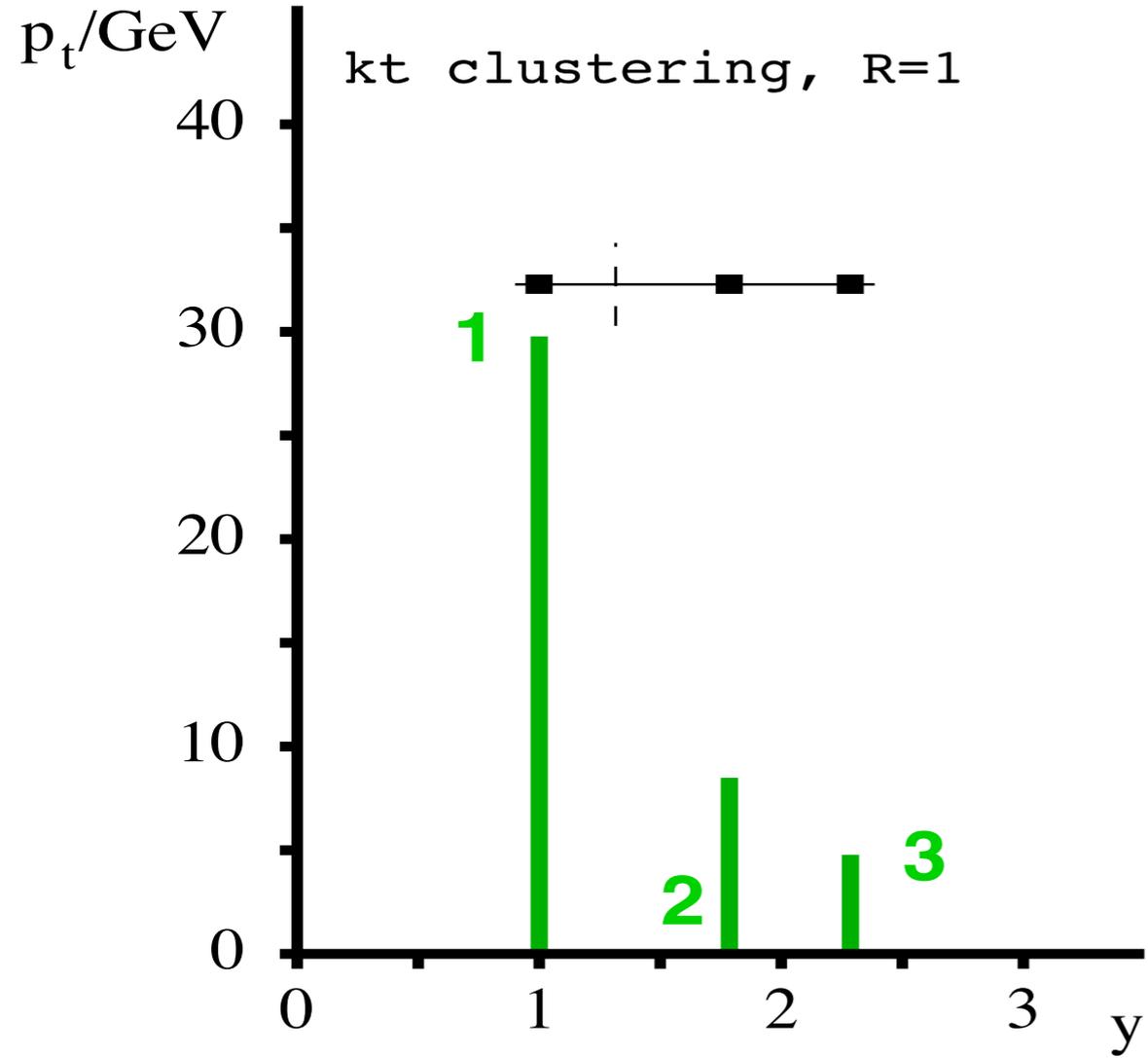
Is it really only about the “circularity”
of the jets’ boundaries?

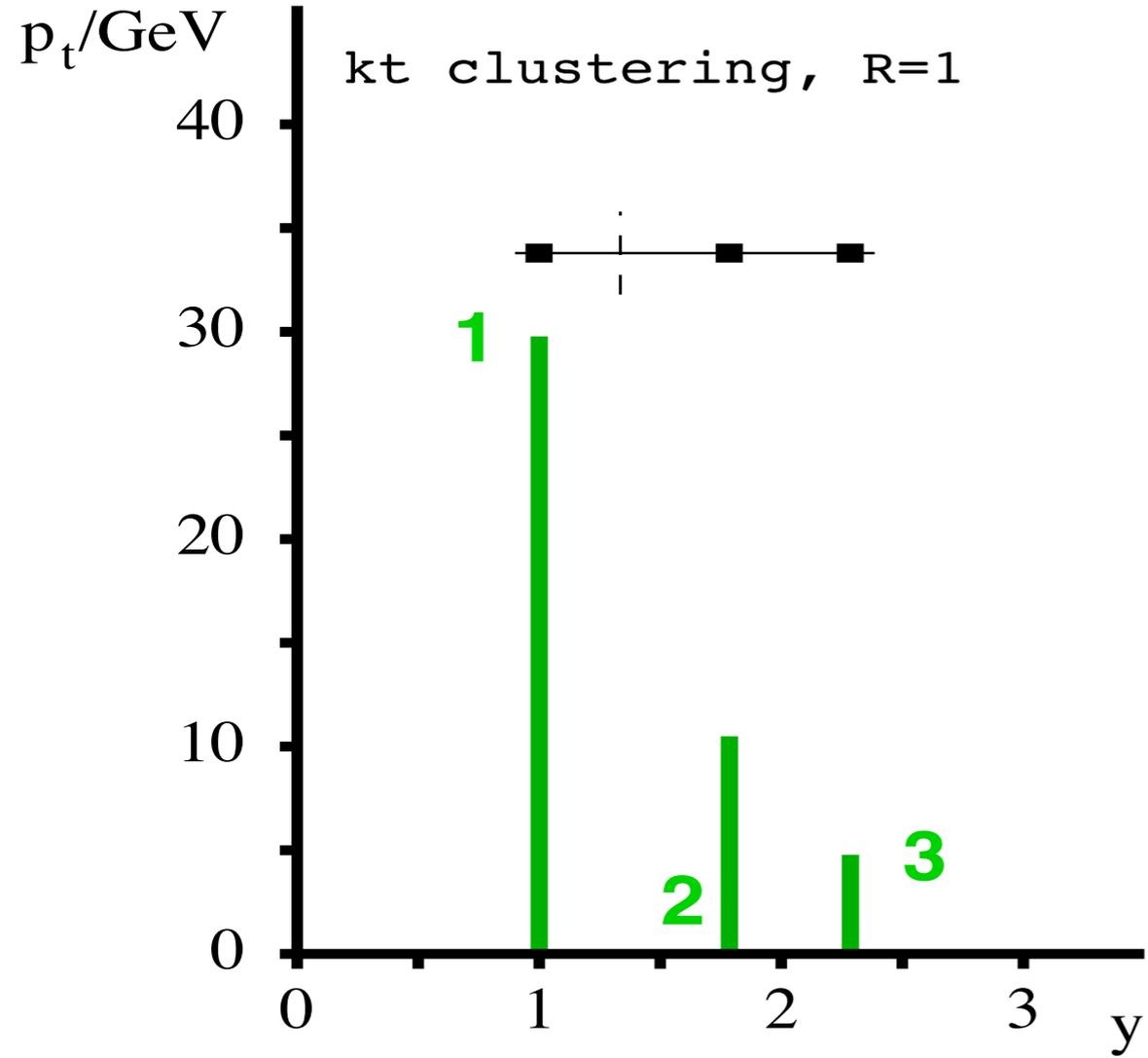


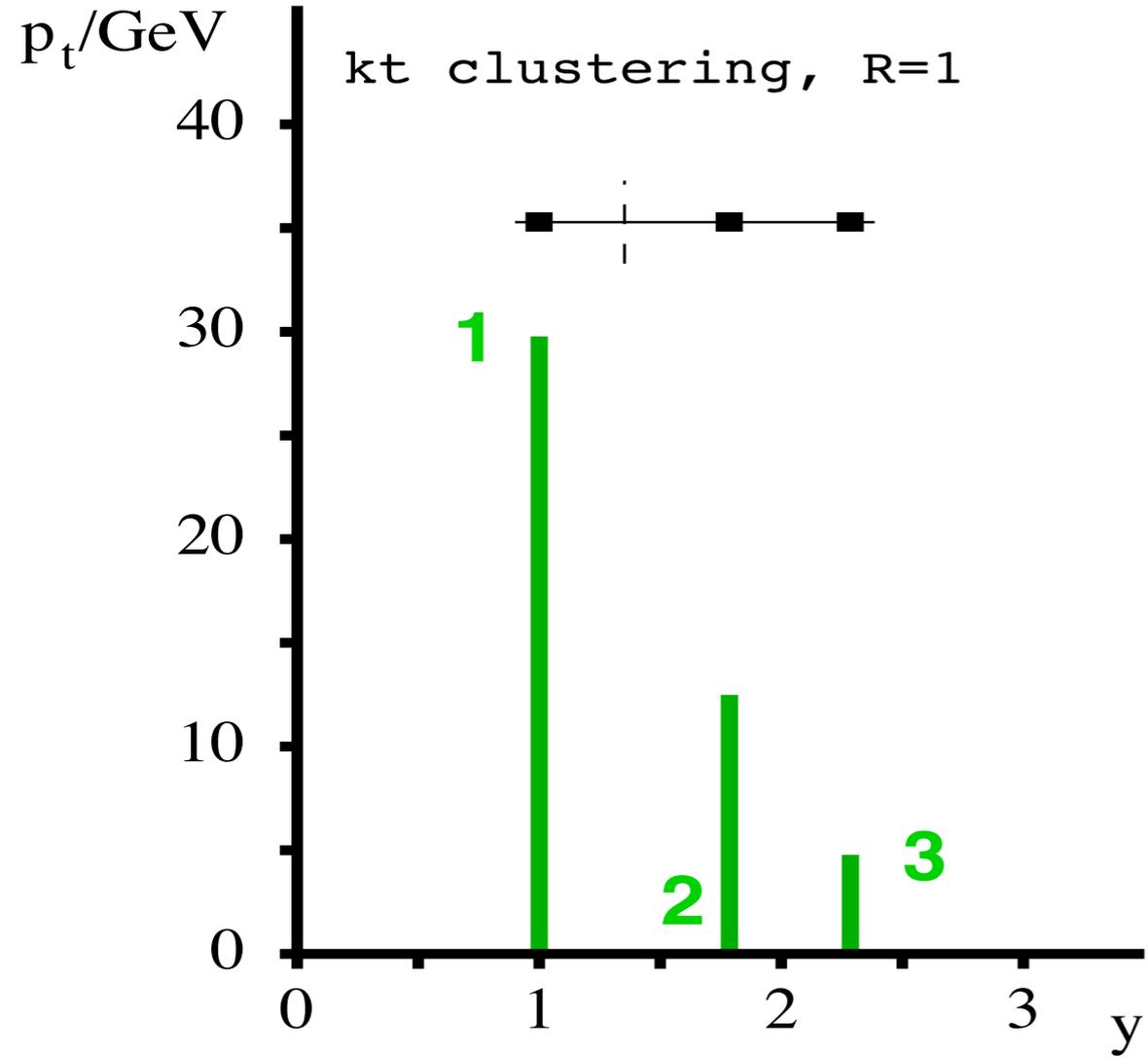


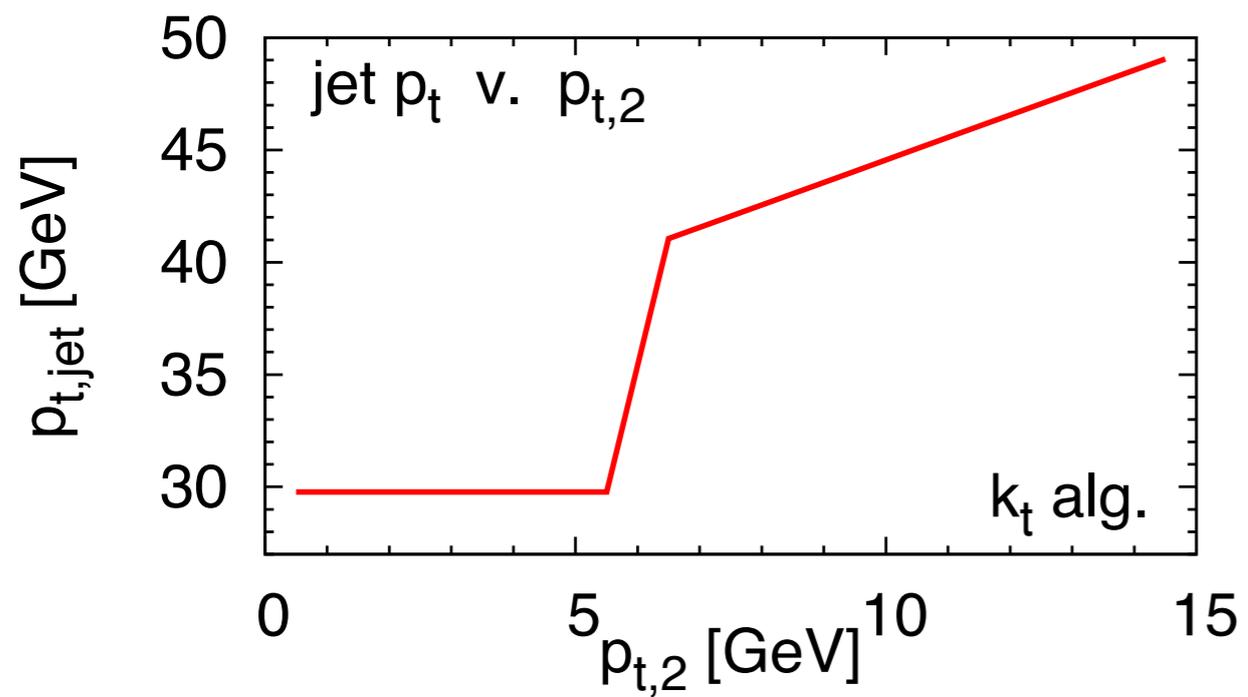
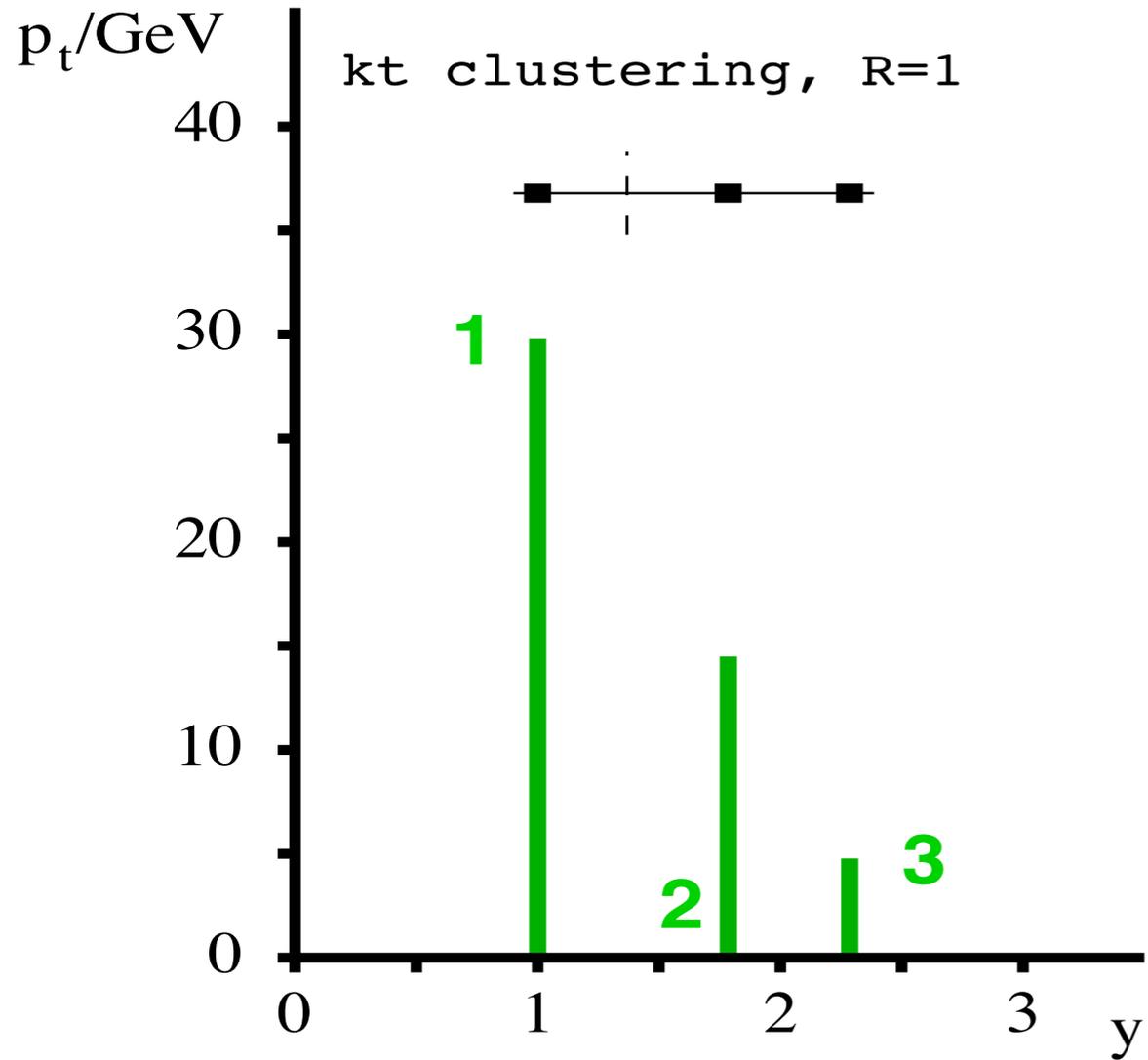




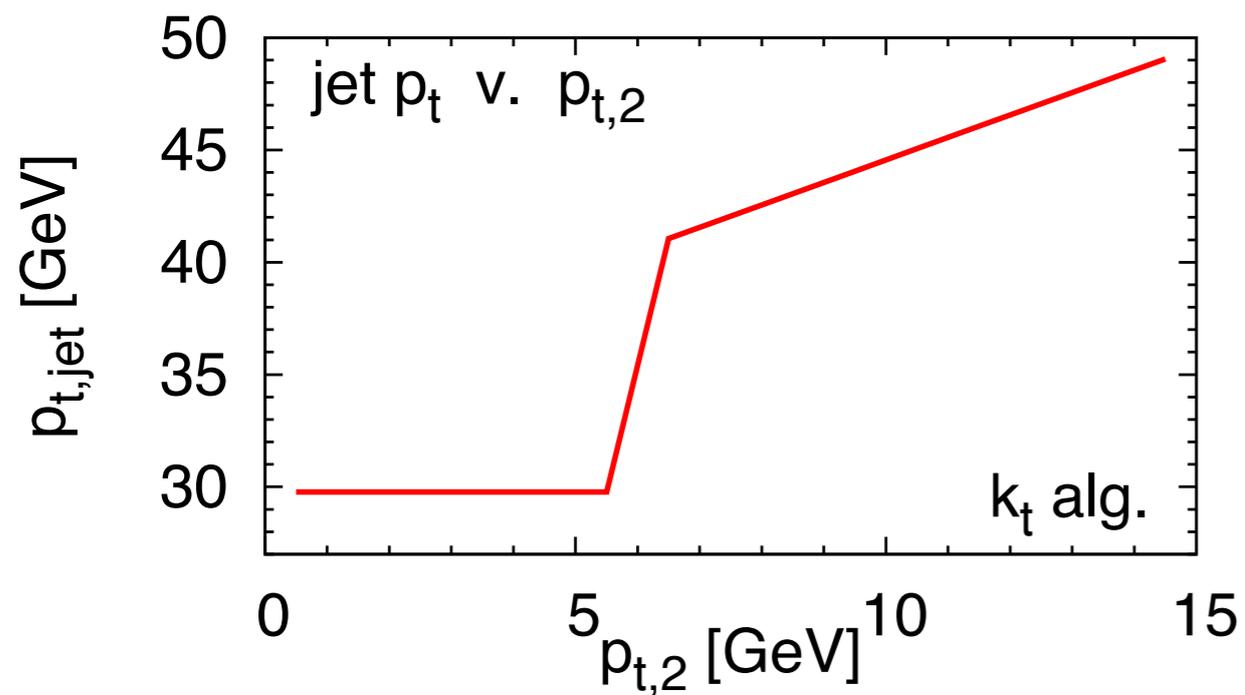
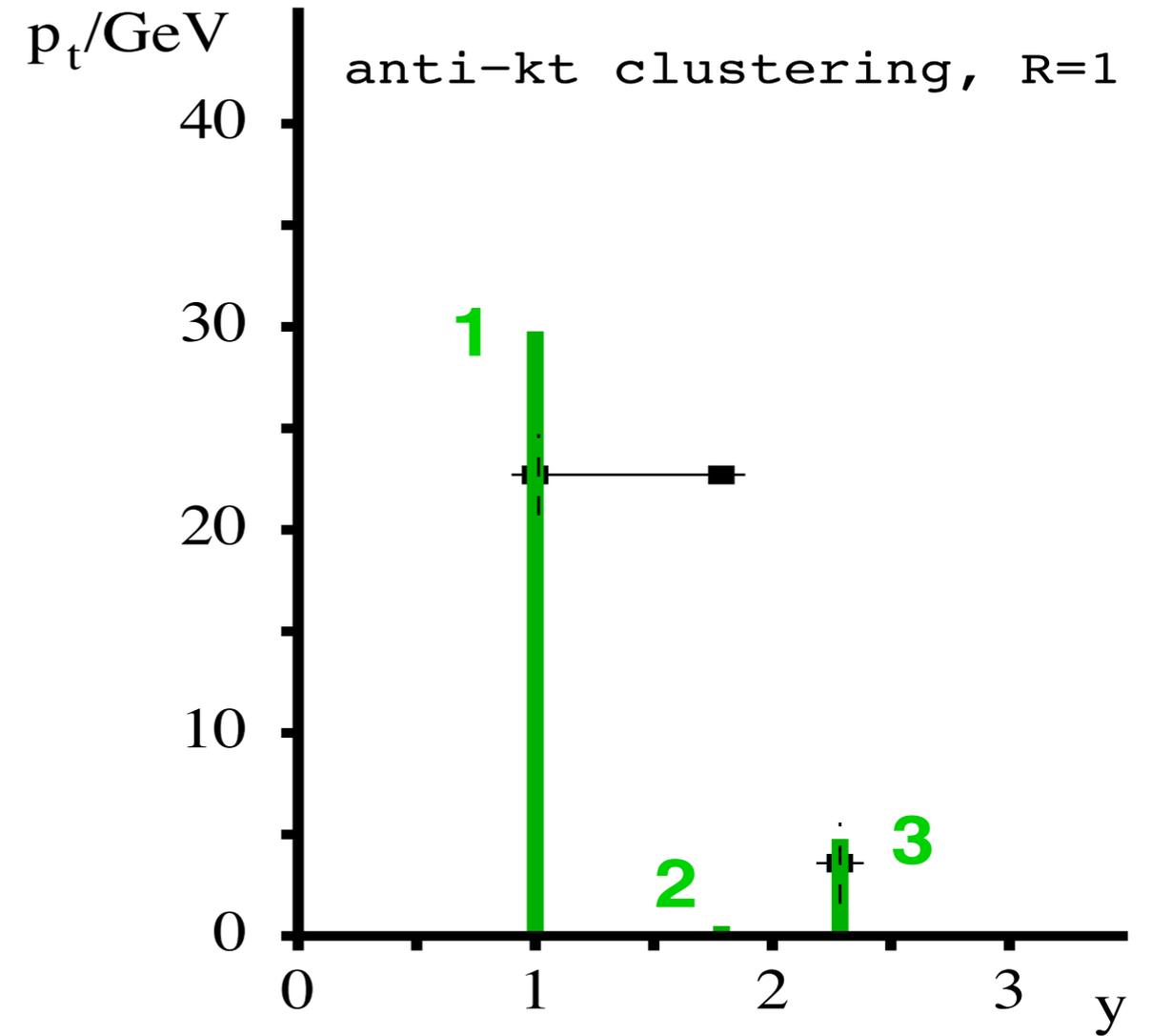
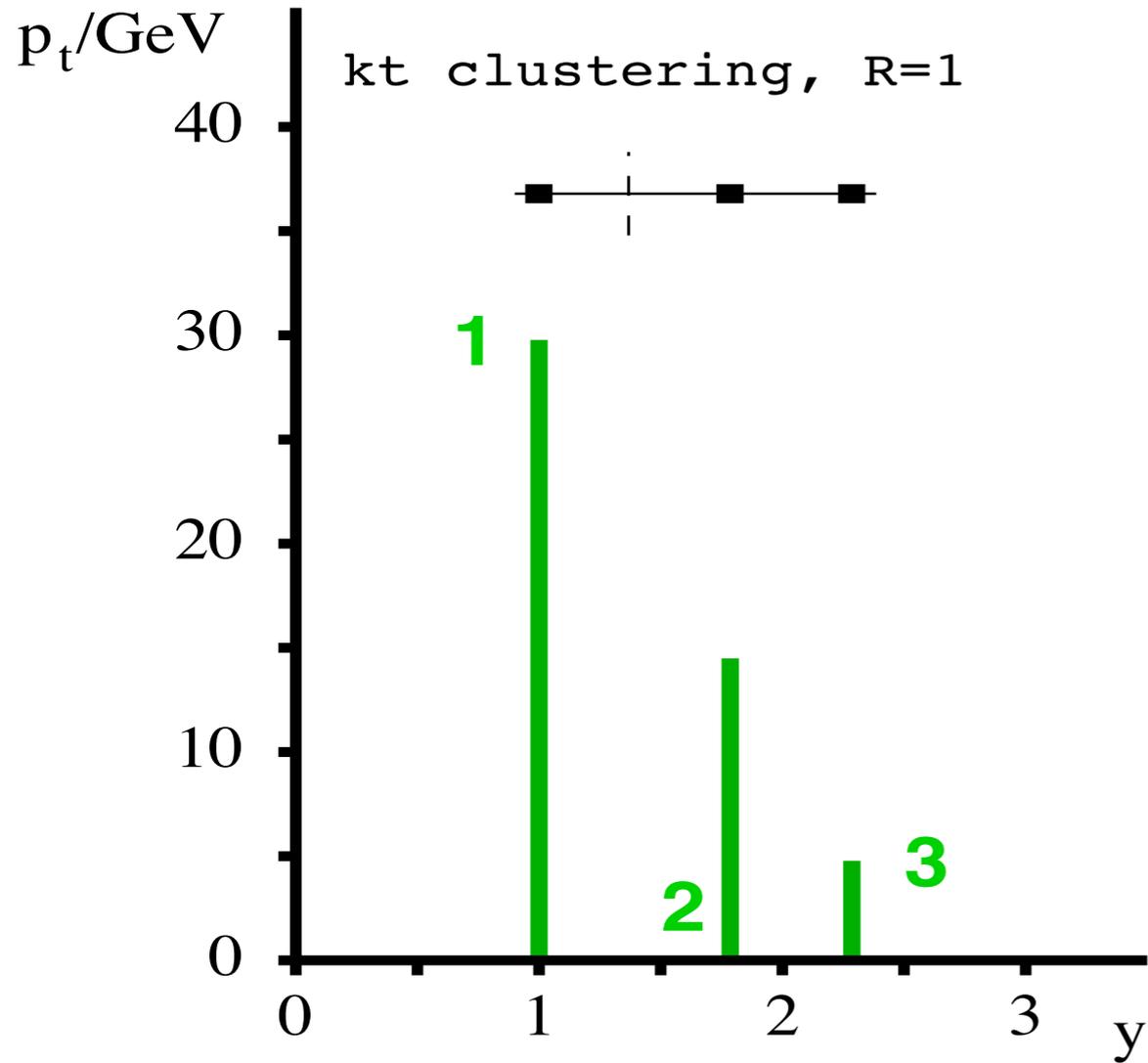




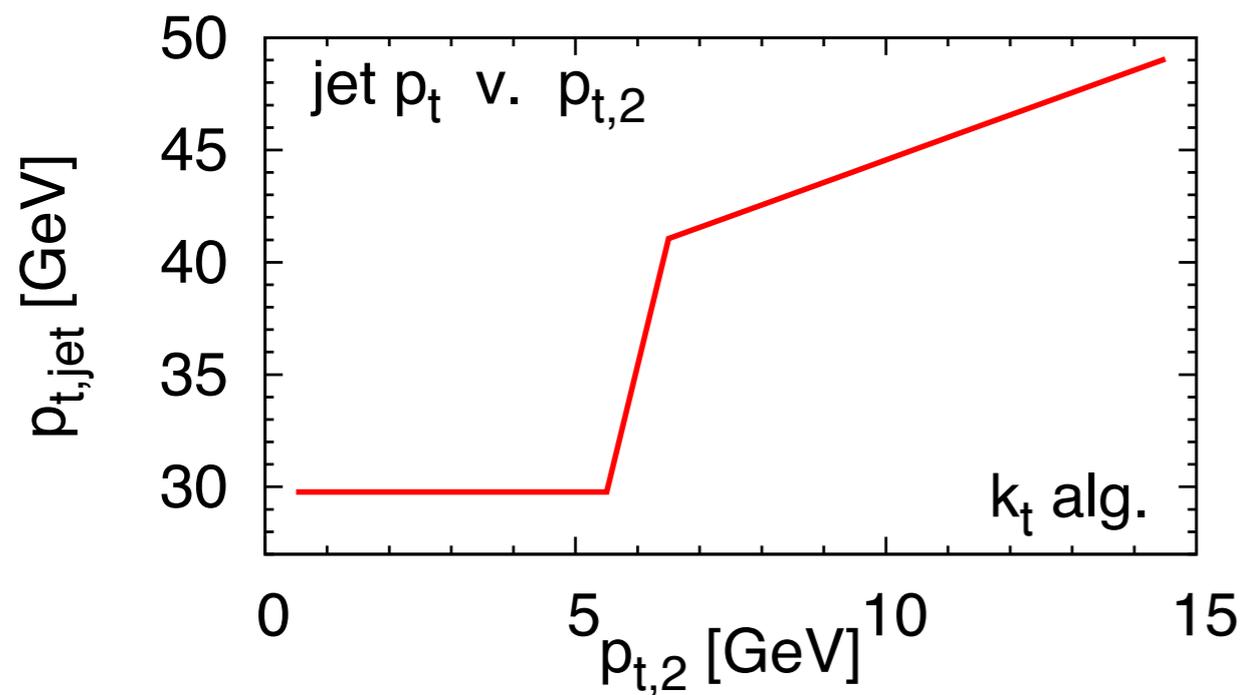
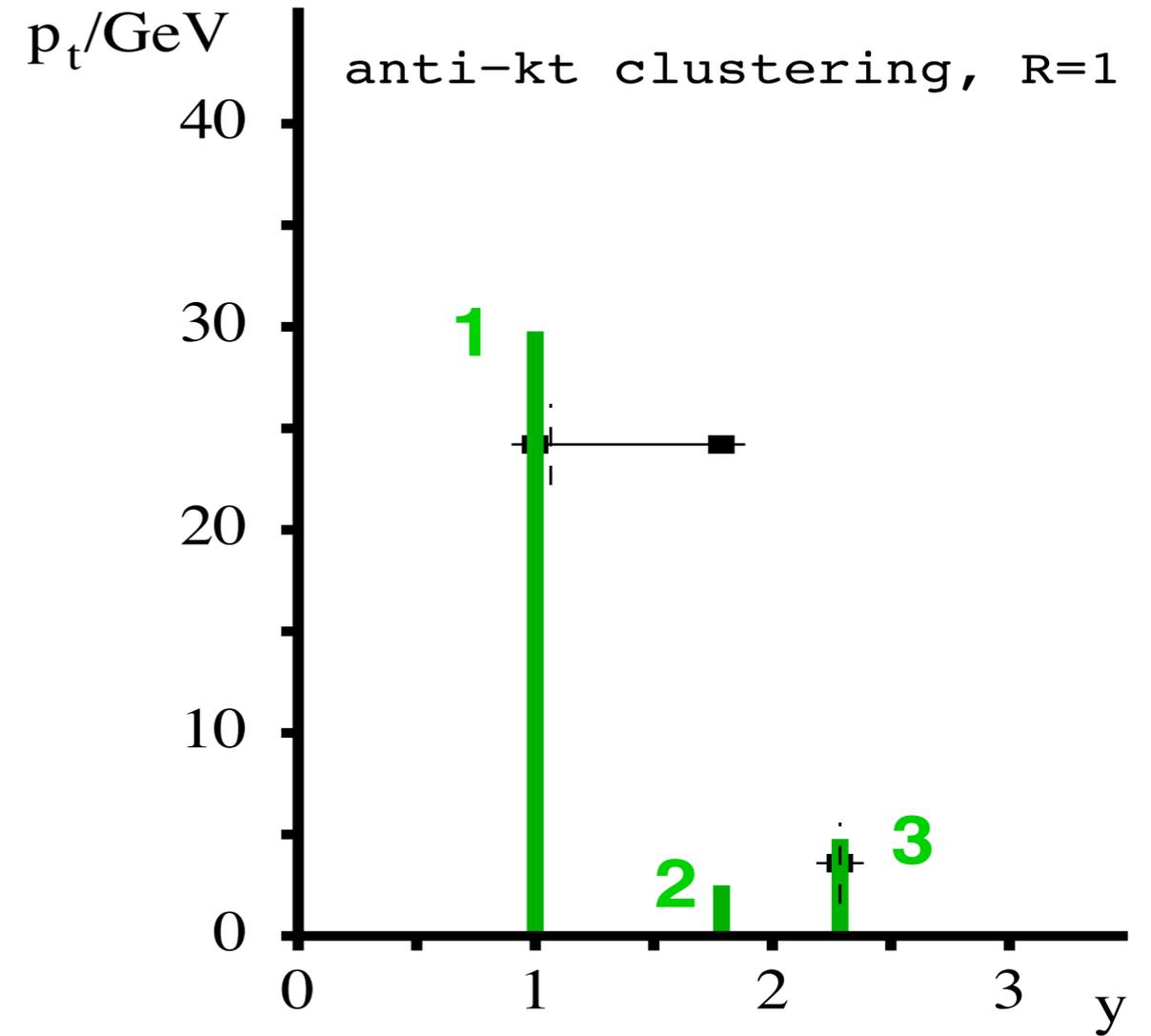
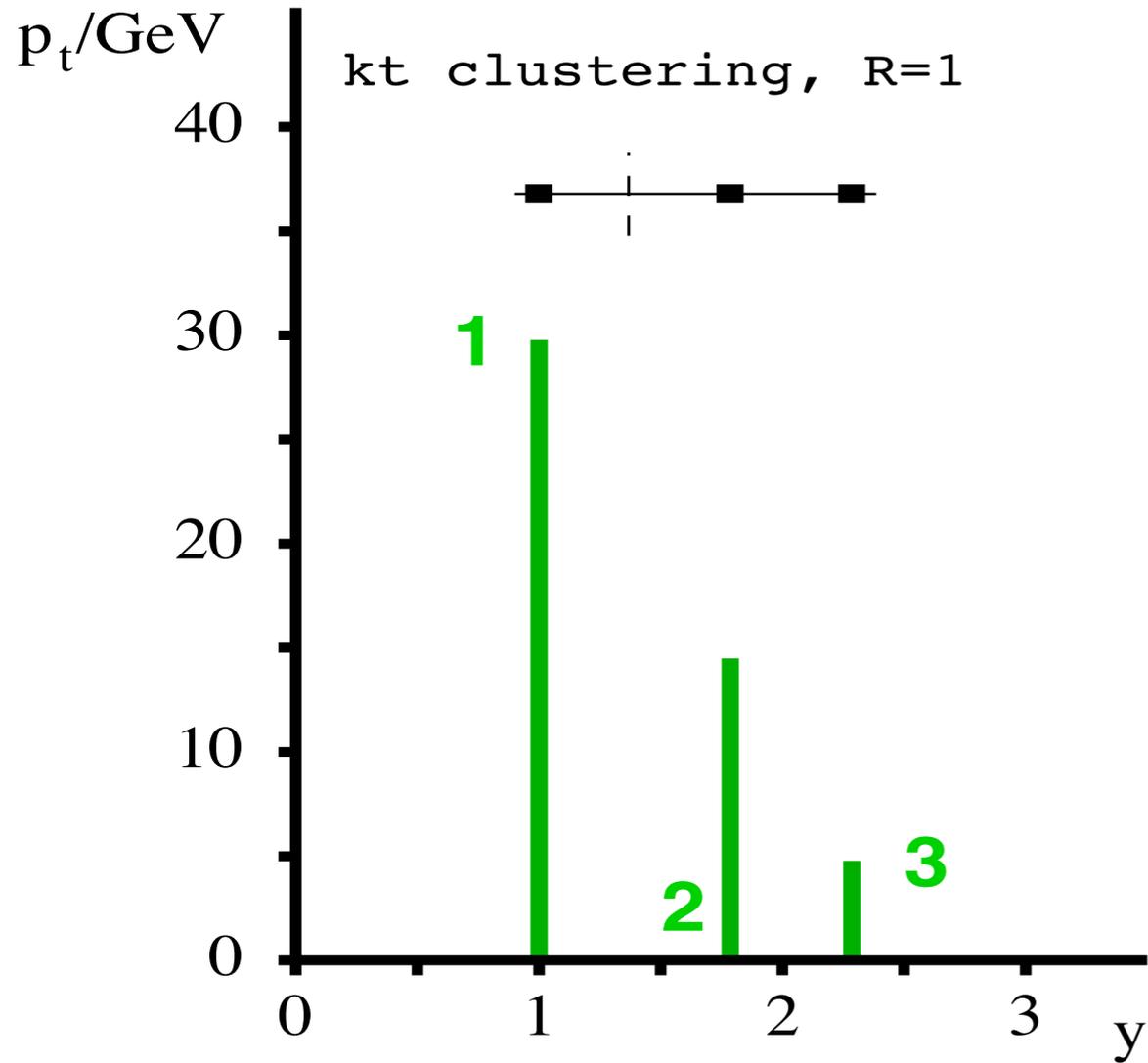




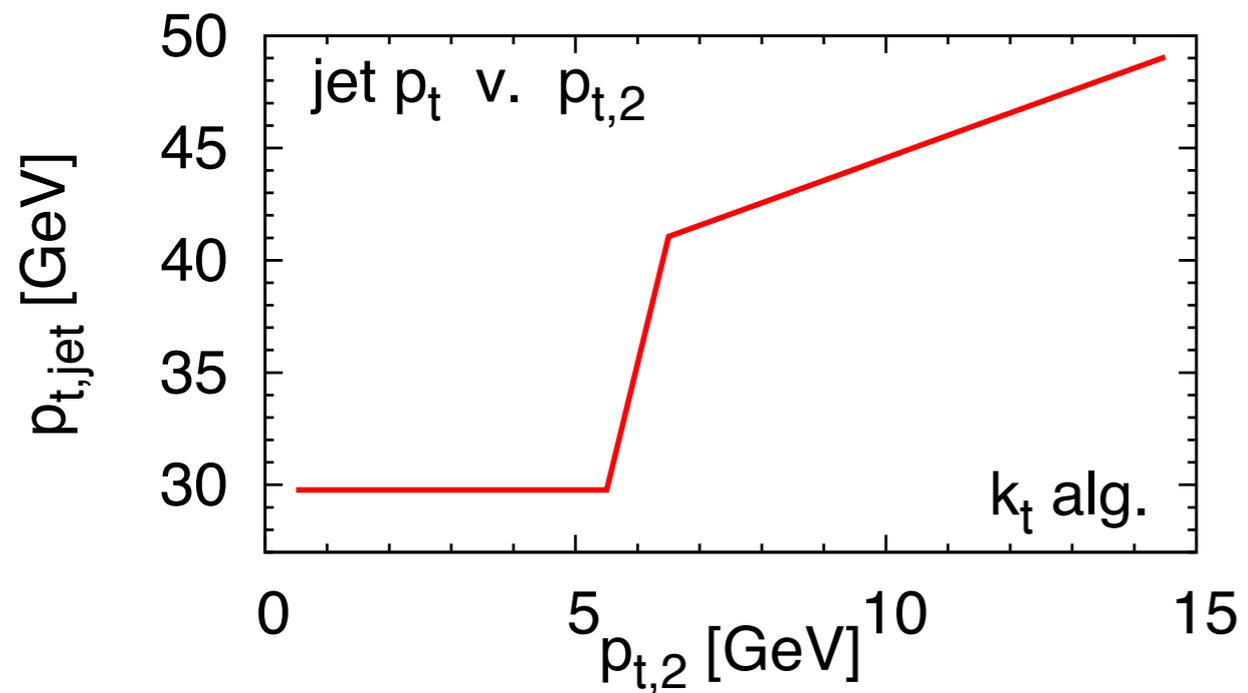
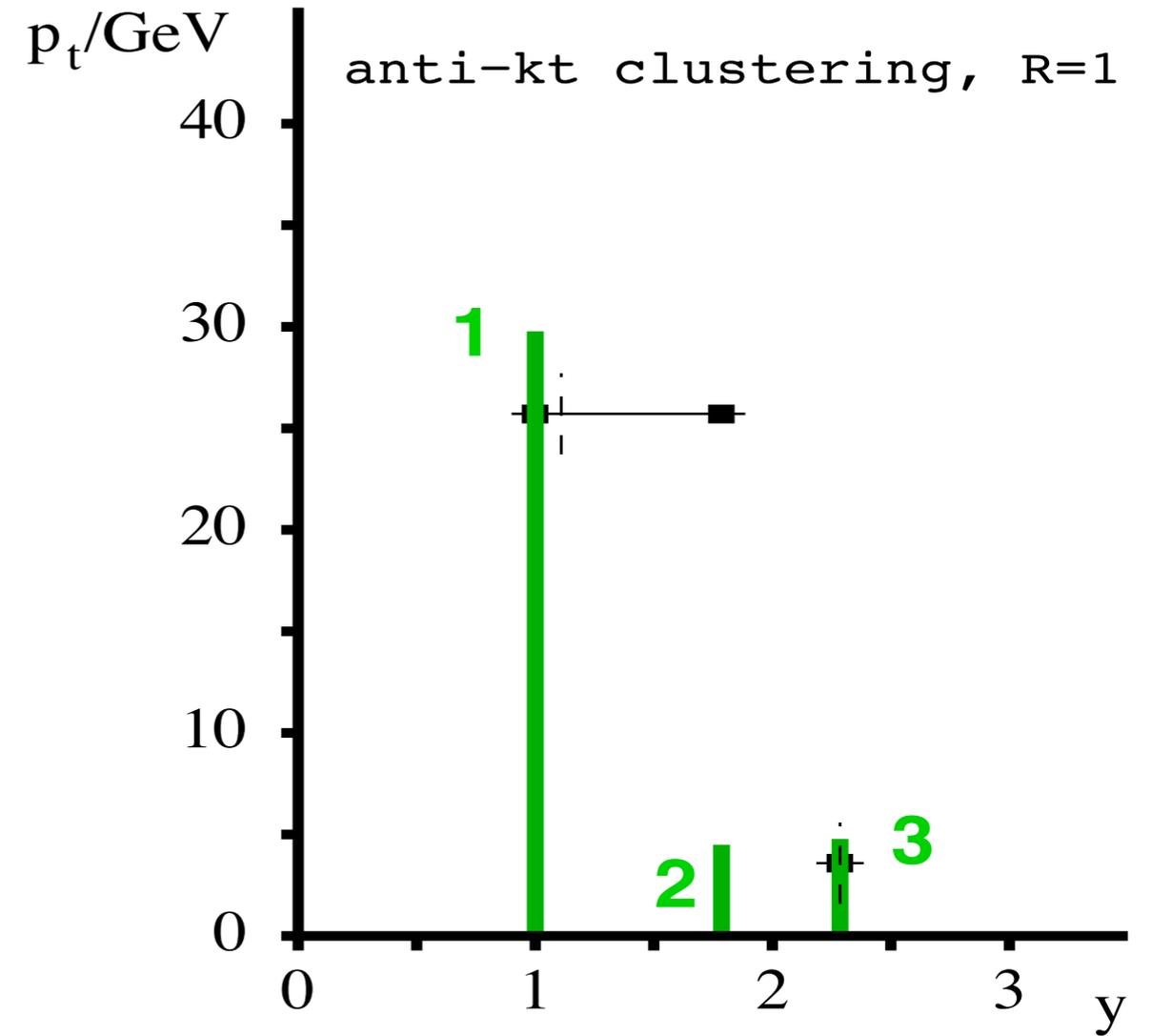
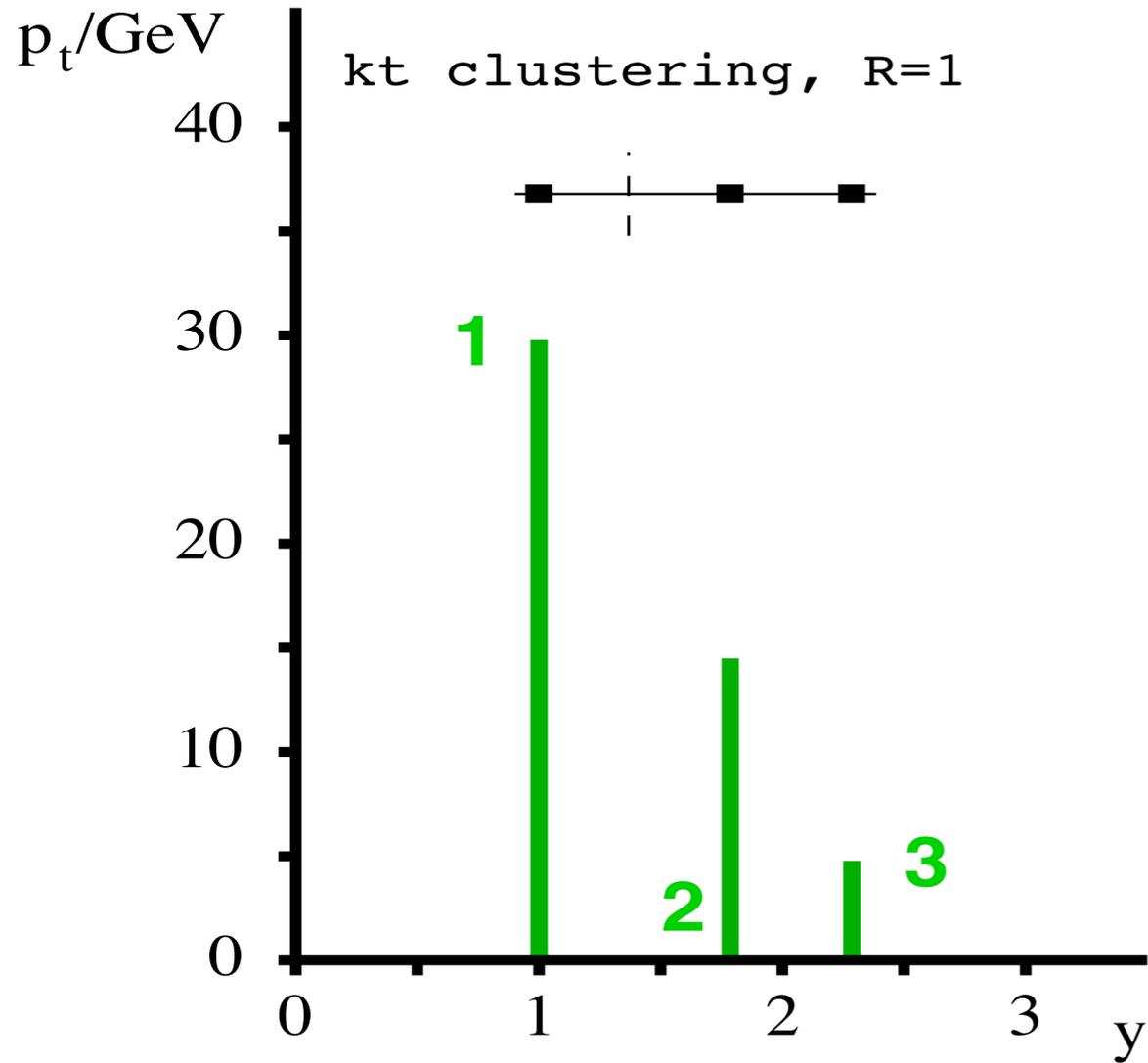
Linearity: k_t v. anti- k_t



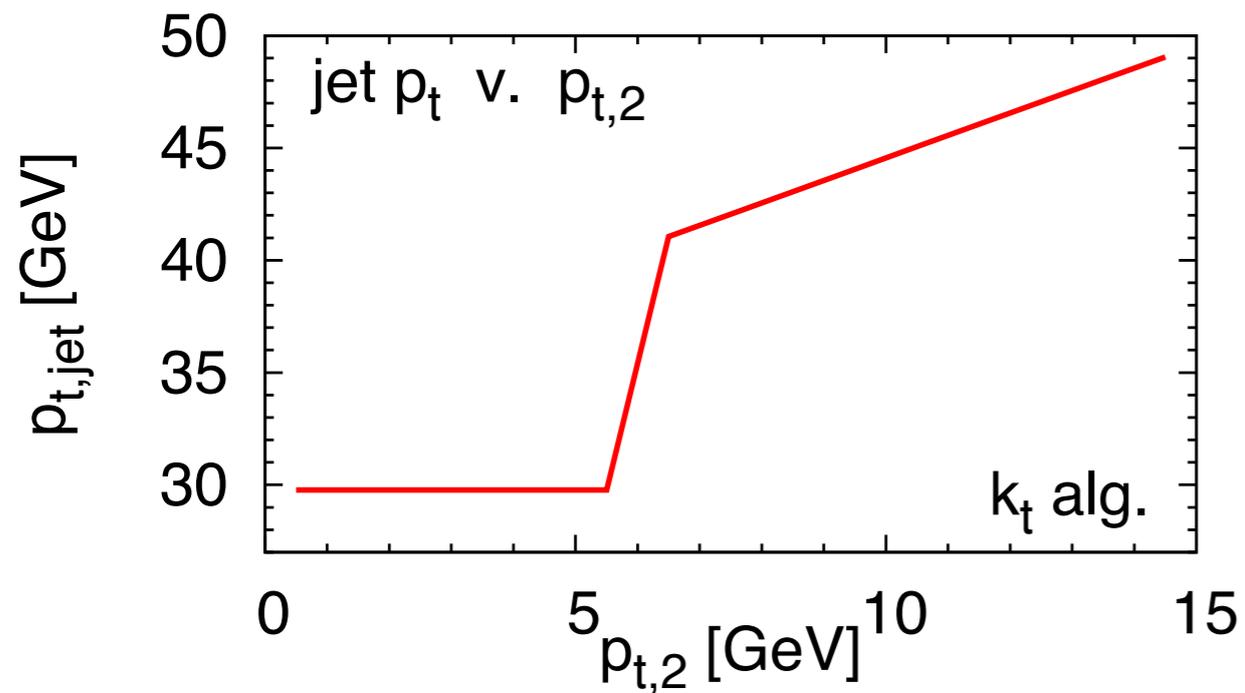
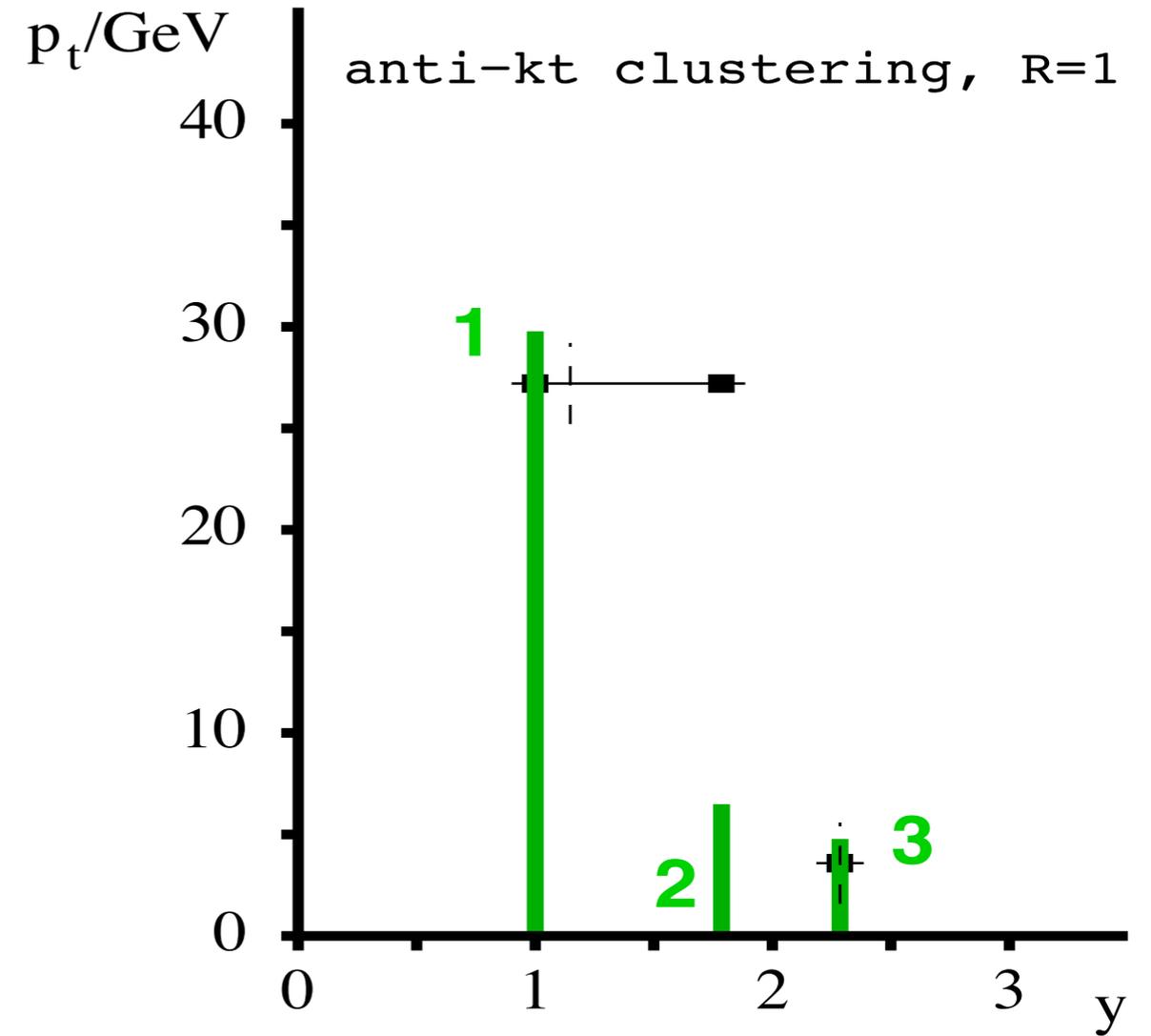
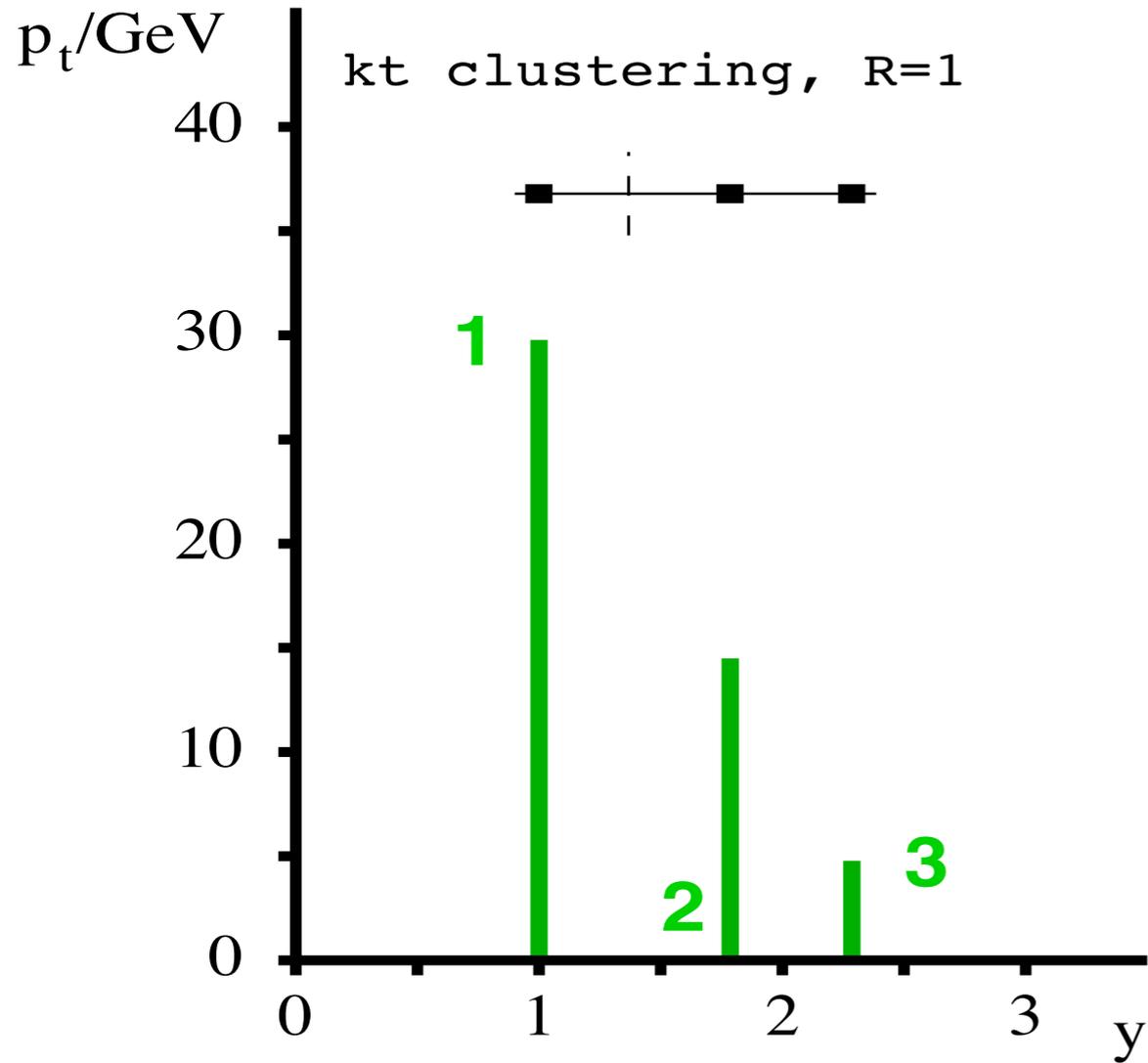
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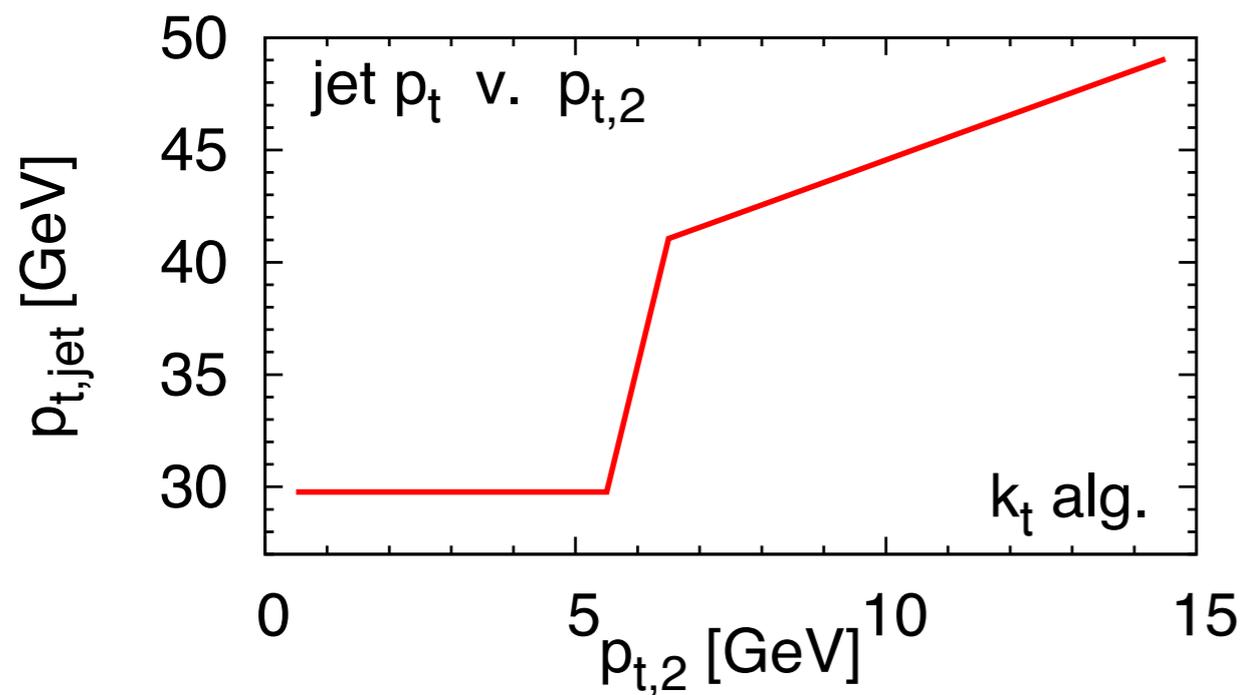
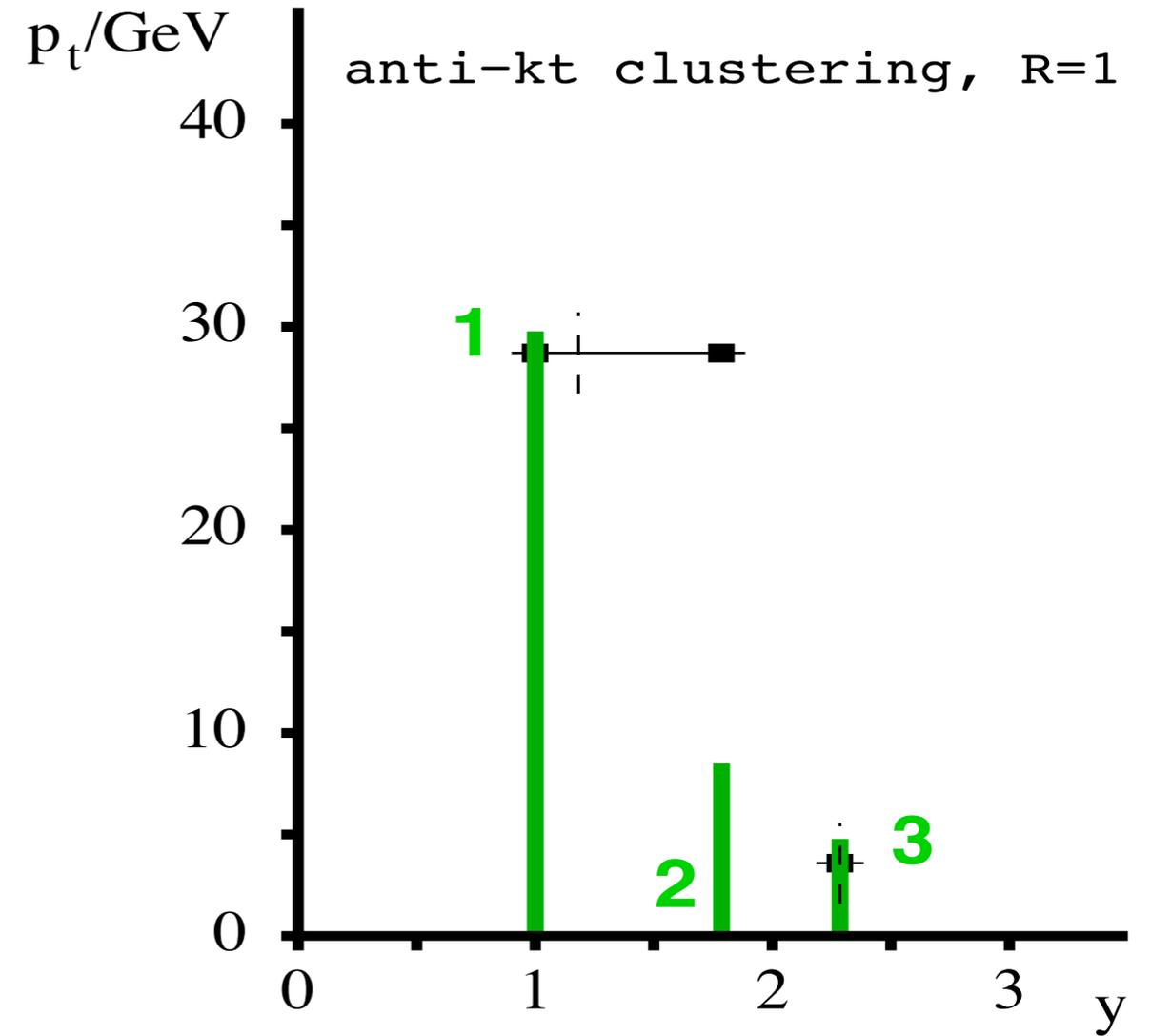
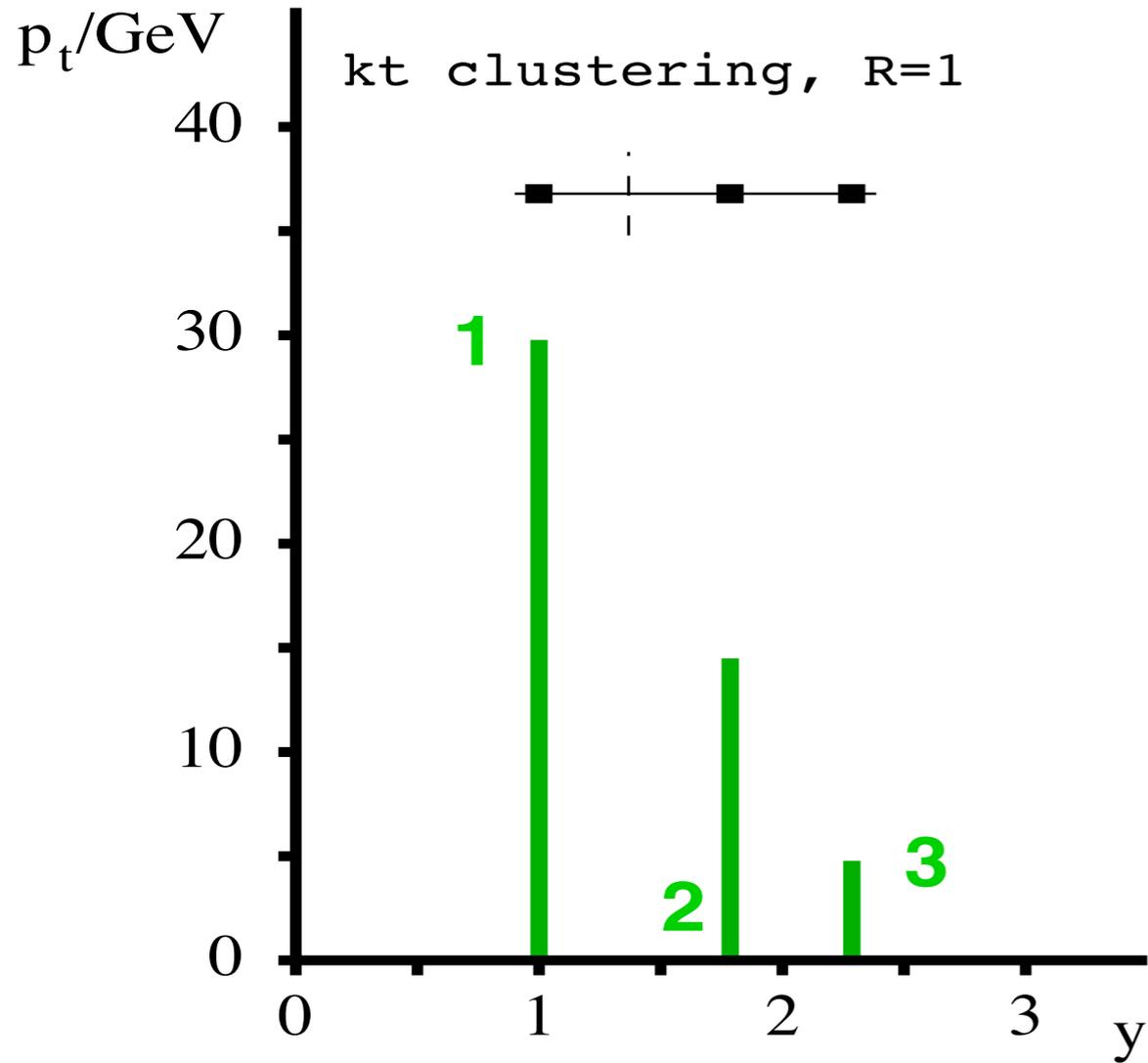
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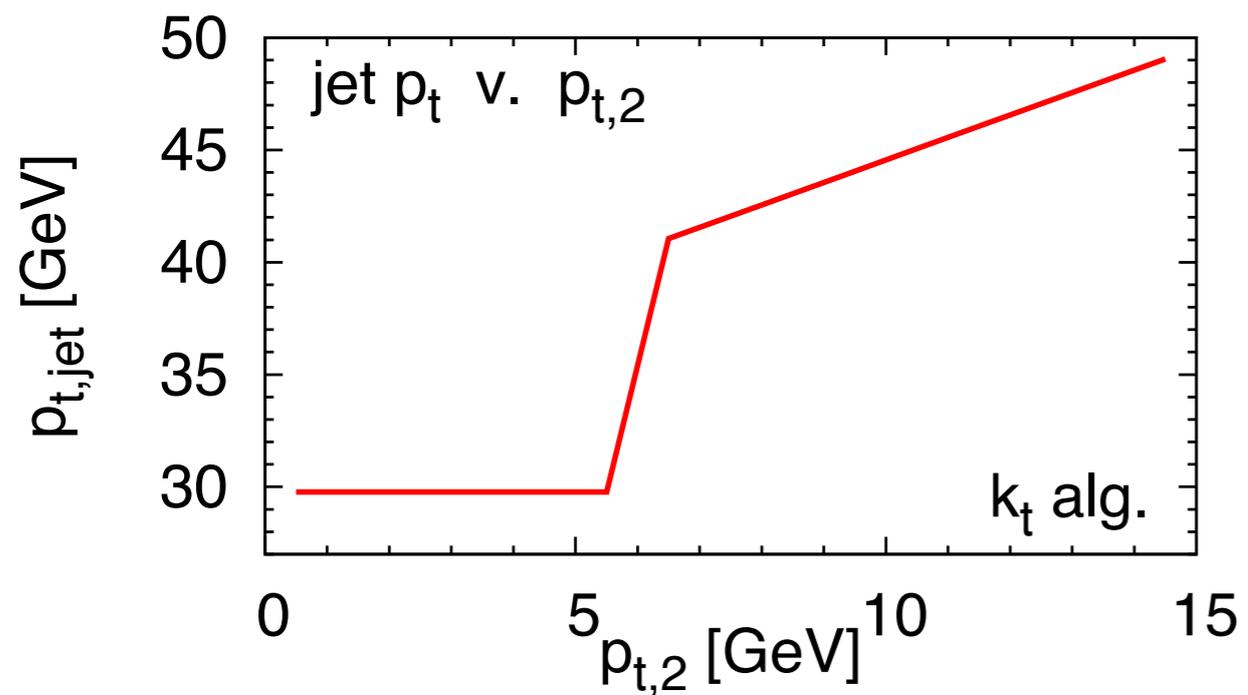
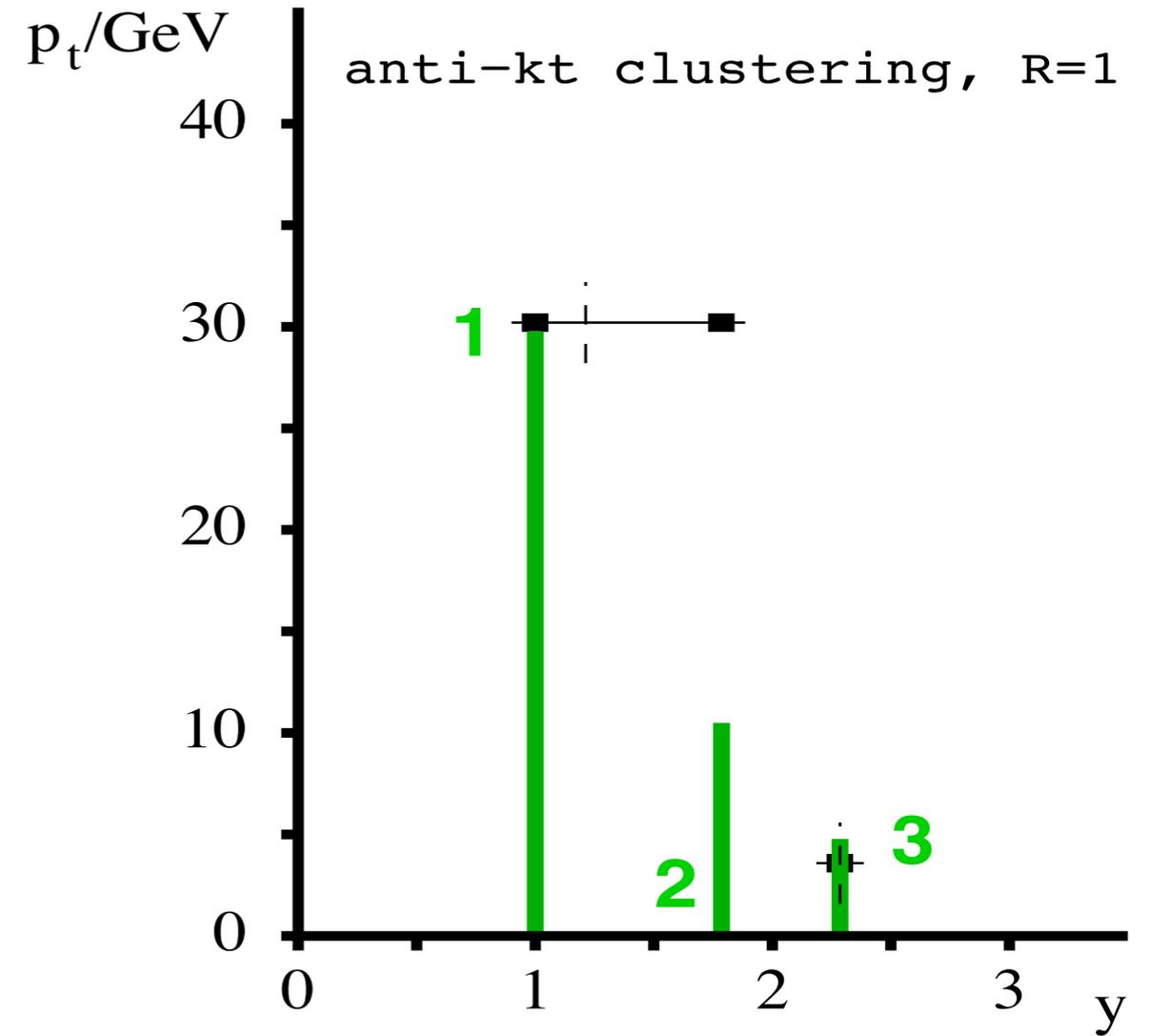
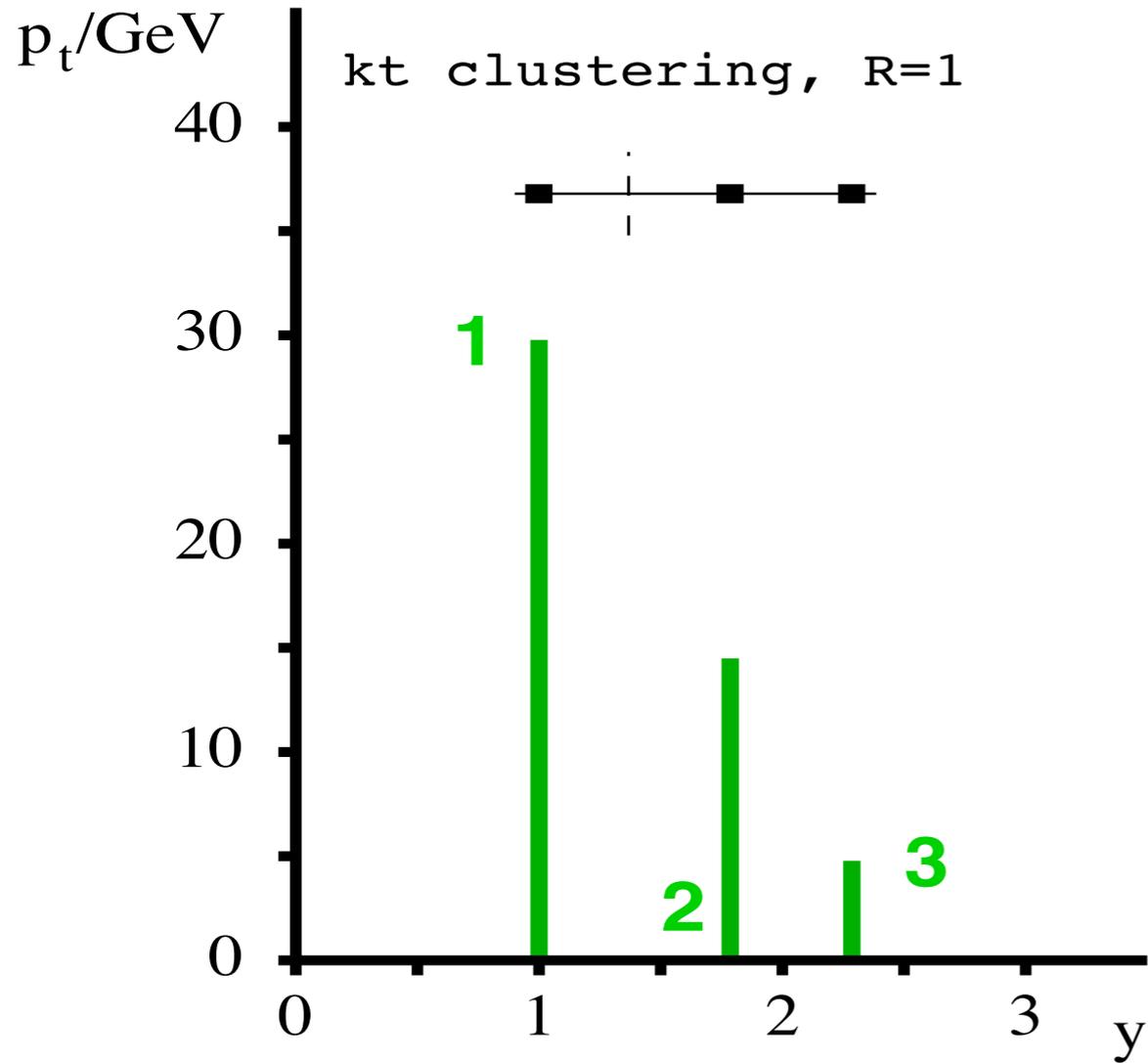
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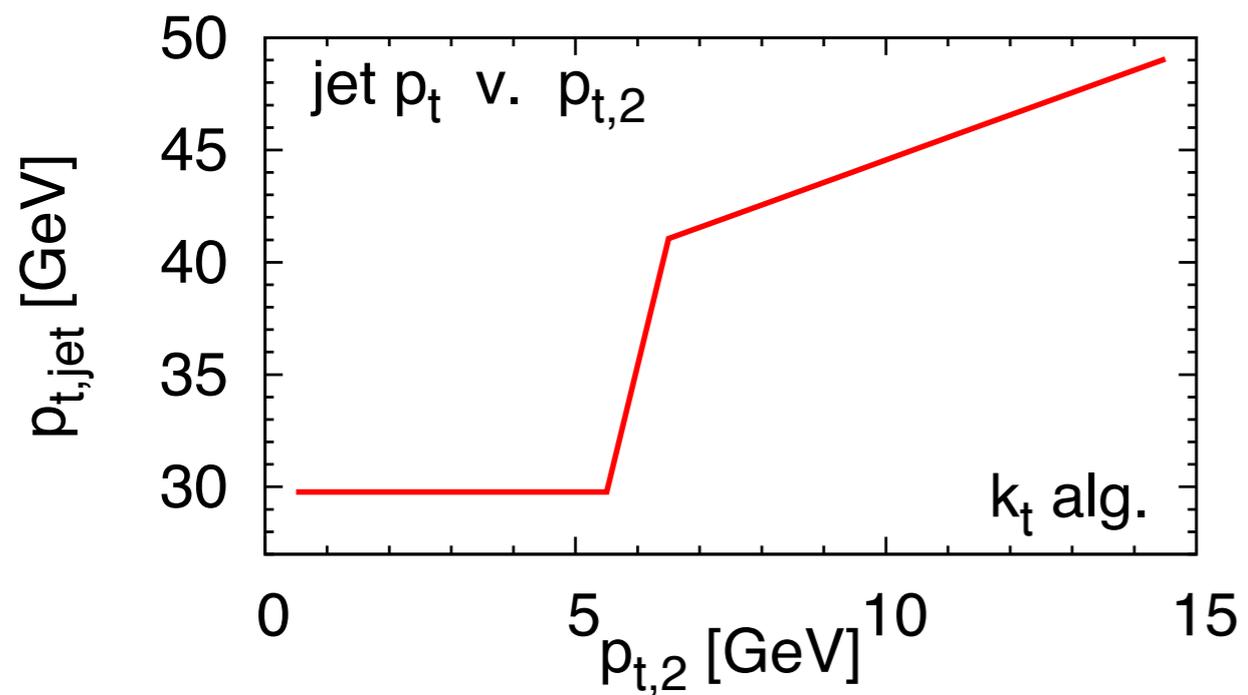
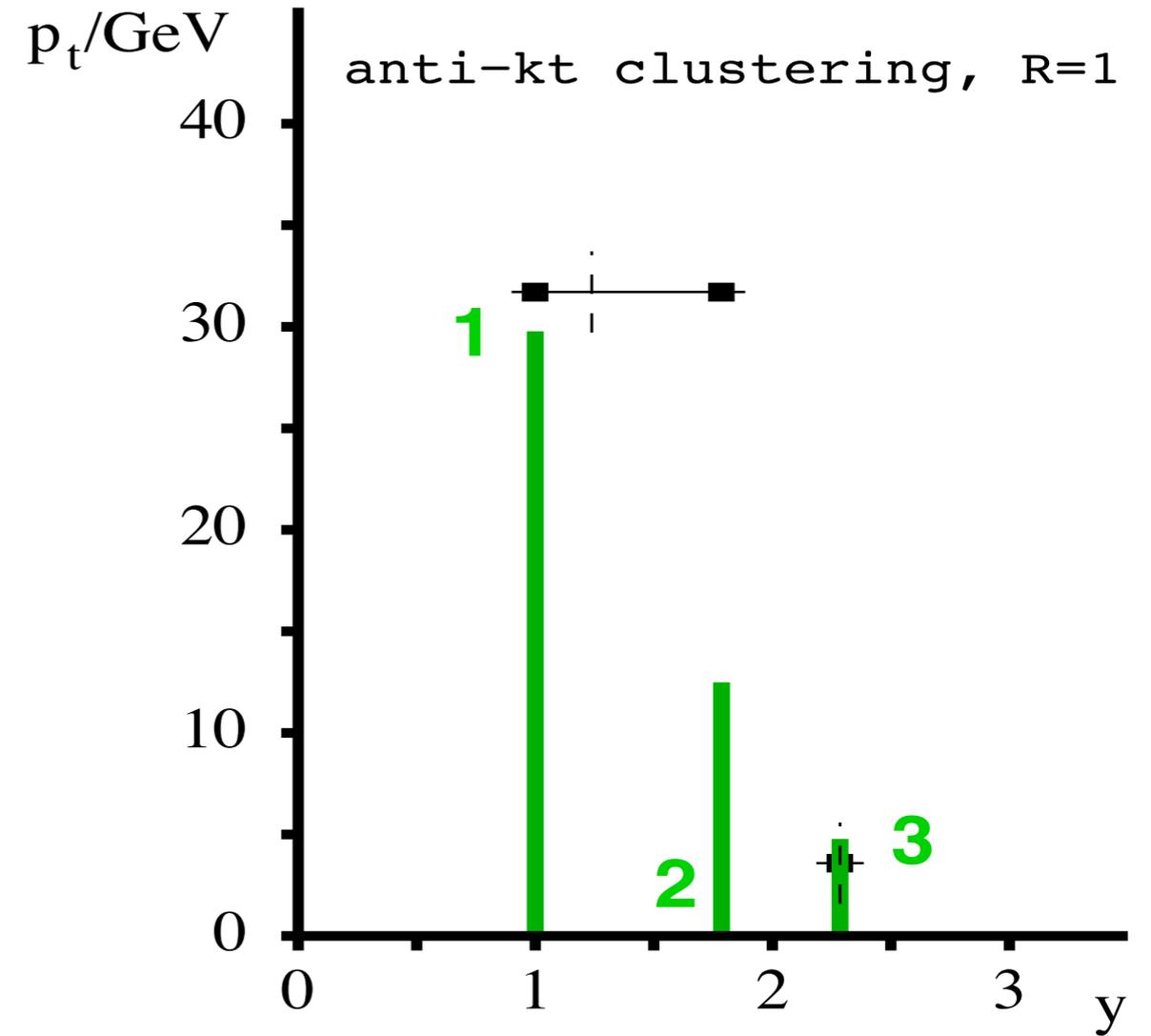
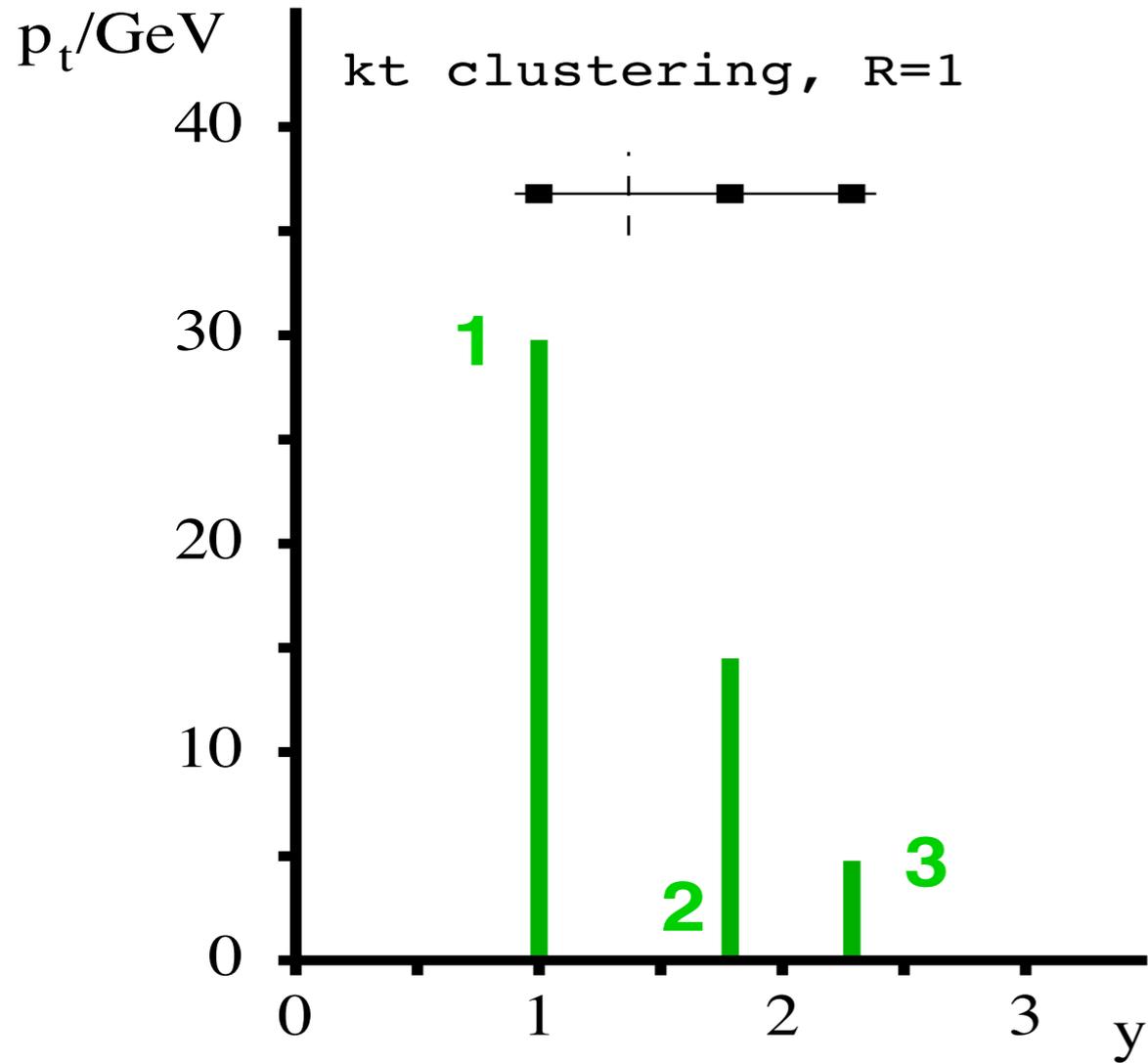
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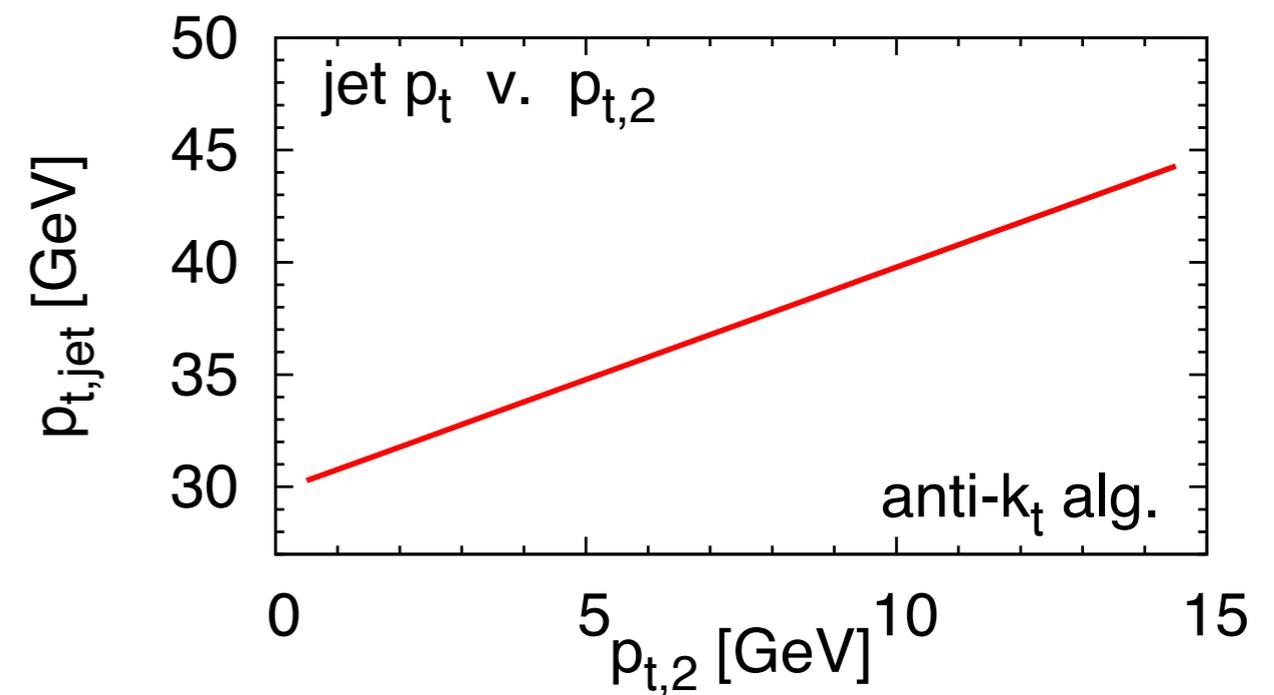
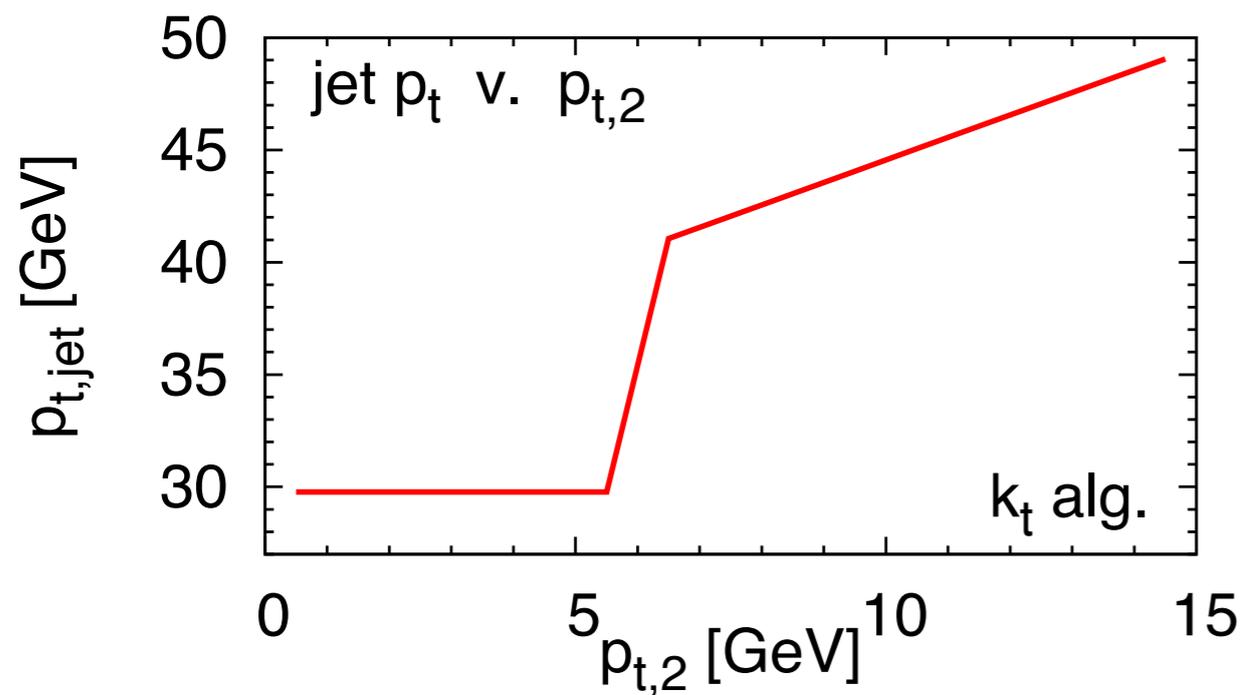
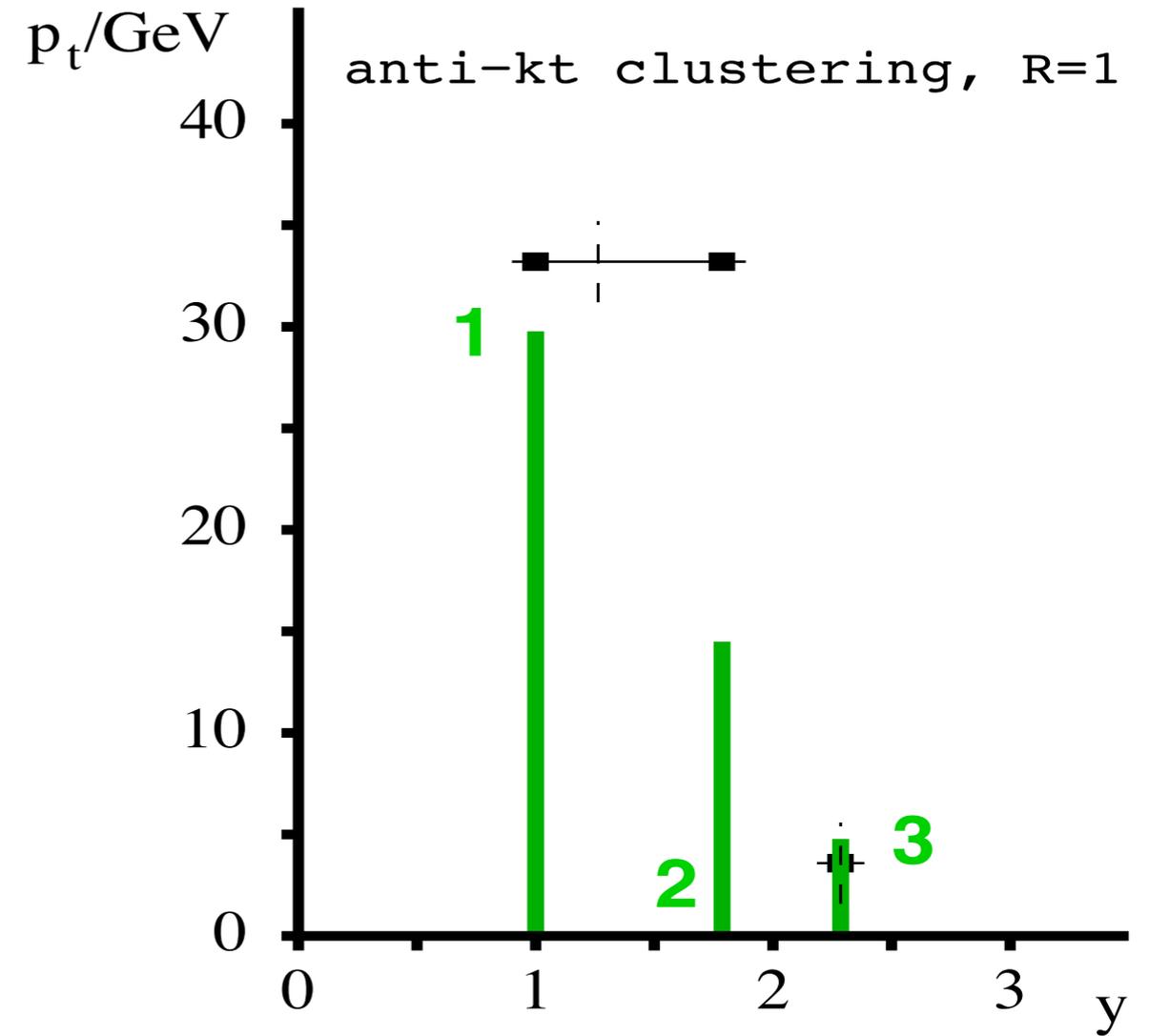
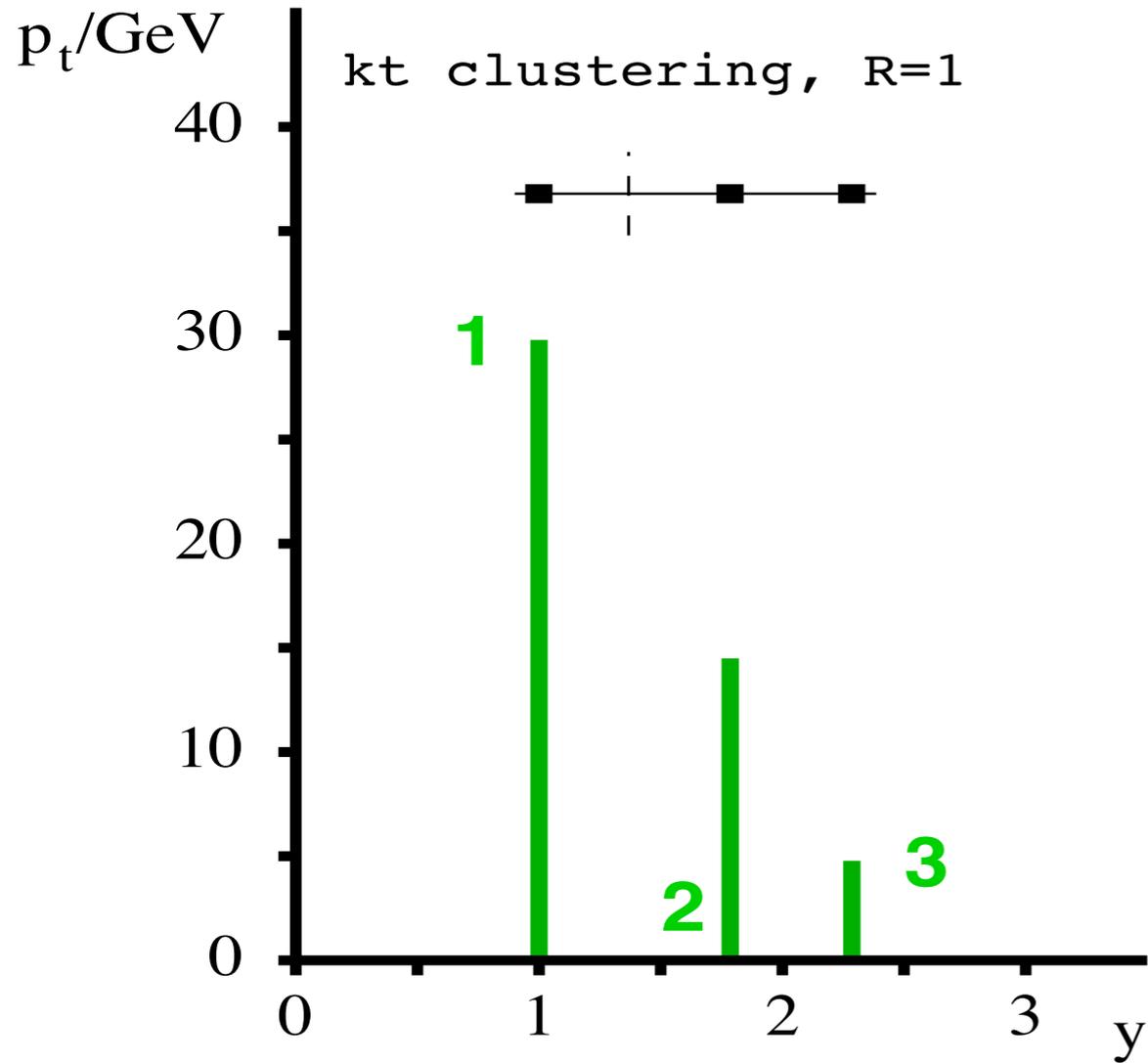
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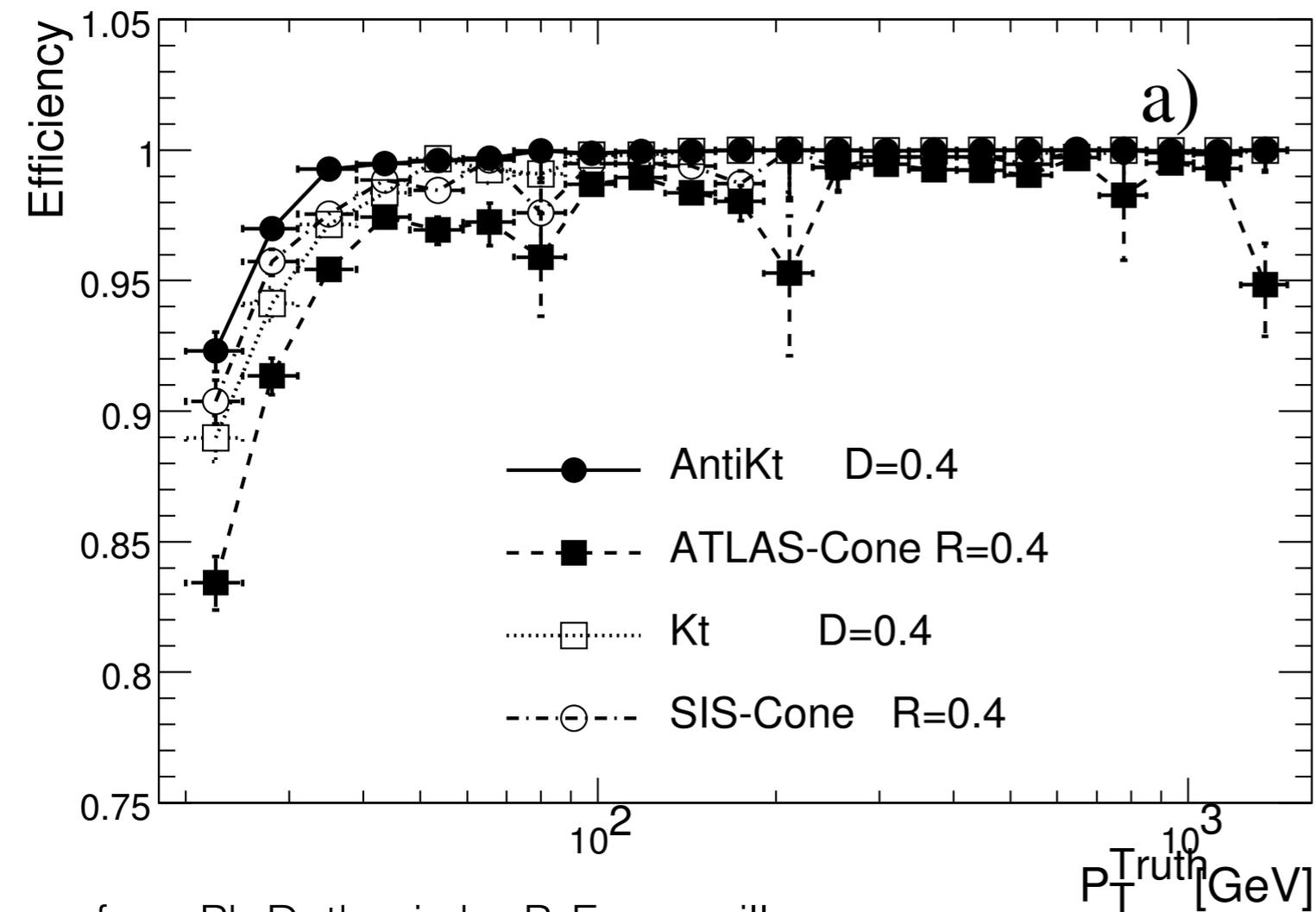


Linearity: k_t v. anti- k_t



Anti- k_t experimental performance

Efficiency for finding detector jet that matches particle jet



from [Ph.D. thesis](#) by P. Francavilla

As good as, or better than all previous experimentally-favoured algorithms.

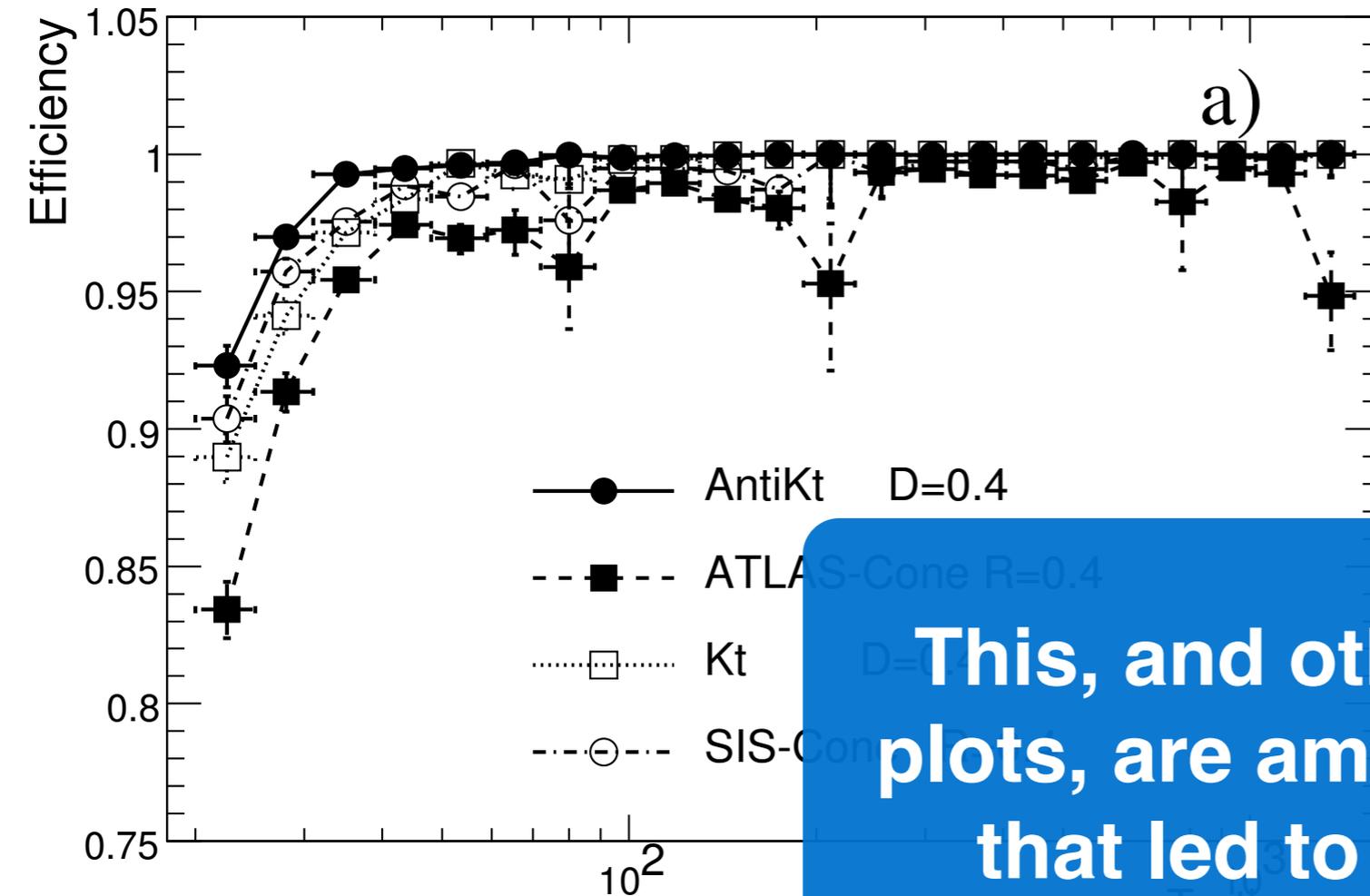
Essentially because anti- k_t has linear response to soft particles.

And it's also infrared and collinear safe (as needed for theory calcs).

[see blackboard for explanation of plot]

Anti- k_t experimental performance

Efficiency for finding detector jet that matches particle jet



As good as, or better than all previous experimentally-favoured algorithms.

Essentially because anti- k_t has linear response to soft particles.

This, and other analogous plots, are among the factors that led to anti- k_t being adopted as the main jet algorithm in ATLAS and CMS

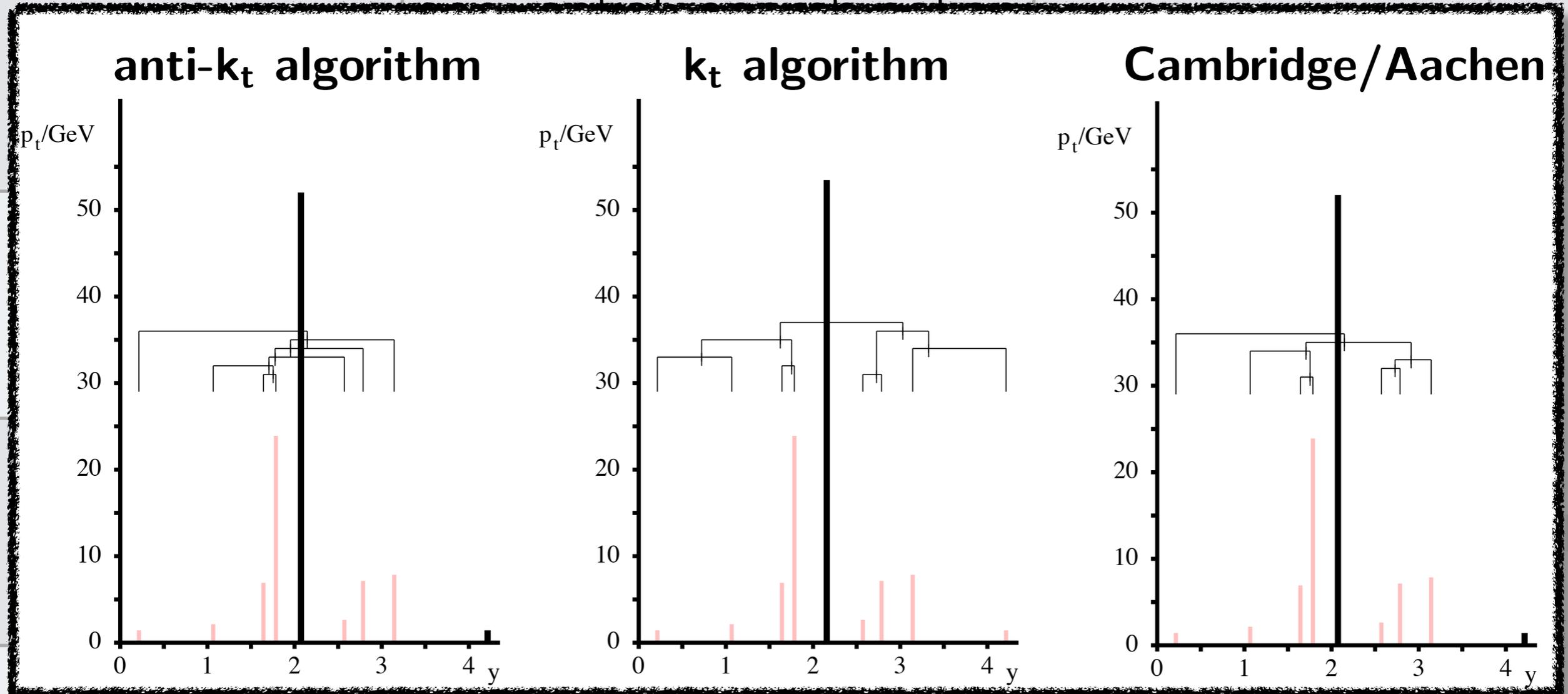
from [Ph.D. thesis](#) by P. Francavilla

[see blackboard for explanation of plot]

Infrared & Collinear safe algorithms

k_t	sequential recombination $d_{ij} = \min(k_{ti}^2, k_{tj}^2) \Delta R_{ij}^2 / R^2$ hierarchical in relative p_t	Catani et al '91 Ellis, Soper '93	$N \ln N$
Cambridge/ Aachen	sequential recombination $d_{ij} = \Delta R_{ij}^2 / R^2$ hierarchical in angle	Dokshitzer et al '97 Wengler, Wobish '98	$N \ln N$
anti- k_t	sequential recombination $d_{ij} = \min(k_{ti}^{-2}, k_{tj}^{-2}) \Delta R_{ij}^2 / R^2$ gives perfectly conical hard jets	MC, Salam, Soyez '08 (Delsart)	$N^{3/2}$
SISCone	Seedless iterative cone with split-merge gives 'economical' jets	Salam, Soyez '07	$N^2 \ln N$

Infrared & Collinear safe algorithms



SISCone

Seedless iterative cone

with split-merge
gives 'economical' jets

Salam, Soyez '07

$N^2 \ln N$

nN

nN

$3/2$

Infrared & Collinear safe algorithms

k_t	sequential recombination $d_{ij} = \min(k_{ti}^2, k_{tj}^2) \Delta R_{ij}^2 / R^2$ hierarchical in relative p_t	Catani et al '91 Ellis, Soper '93	$N \ln N$
Cambridge/ Aachen	sequential recombination $d_{ij} = \Delta R_{ij}^2 / R^2$	Dokshitzer et al '97 Westra, Westra '99	$N \ln N$
anti- k_R	All are available in FastJet, http://fastjet.fr (as well as many IRC unsafe ones)		$N^{3/2}$
SISCone	Other (recent) software SpartyJet (python/root interface to FastJet) SlowJet (in Pythia8) SlowJet (mathematica code by J. Ruderman)		$N^2 \ln N$

Advertisement

All are available in FastJet, <http://fastjet.fr>
(as well as many IRC unsafe ones)

Other (recent) software

SpartyJet (python/root interface to FastJet)
SlowJet (in Pythia8)
SlowJet (mathematica code by J. Ruderman)

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// specify a jet definition  
JetDefinition jet_def(antikt_algorithm, double R);
```

jet_algorithm can be any one of the four IRC safe algorithms, or also most of the old IRC-unsafe ones, for legacy purposes

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vector<PseudoJet> input_particles = . . .;
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```
// extract the jets  
vector<PseudoJet> jets = sorted_by_pt(cs.inclusive(jets));  
  
// pt of hardest jet  
double pt_hardest = jets[0].pt();  
  
// constituents of hardest jet  
vector<PseudoJet> constituents = jets[0].constituents();
```

Time needed to cluster an event with N particles

