Preasymptotics in small-x splitting functions

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HERA LHC Workshop

Geneva, January 2005

Outline

- Introduction
  - Problem of convergence of small-x splitting functions.
  - Quick overview of renorm. group improved small-x approach .

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  - preasymptotics
  - explanation of 'dip'
- $\left[\text{Slow}\right]$ Progress towards phenomenology
  - Toy convolution, $P_{gg} \otimes g$
  - Difficulties with $\overline{\text{MS}}$ scheme

Perturbative structure of P_{gg}

- Small-x gluon splitting function has logarithmic enhancements:

$$xP_{gg}(x) = \sum_{n=1} \alpha_s^n \ln^{n-1} \frac{1}{x}$$

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Leading Logs (LLx):

$$\bar{\alpha}_s + \frac{\zeta(3)}{3} \bar{\alpha}_s^4 \ln^3 \frac{1}{x} + \frac{\zeta(5)}{60} \bar{\alpha}_s^6 \ln^5 \frac{1}{x} + \cdots$$

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Next-to-Leading Logs (NLLx):

$$A_{20}\bar{\alpha}_s^2 + A_{31}\bar{\alpha}_s^3 \ln\frac{1}{x} + A_{42}\bar{\alpha}_s^4 \ln^3\frac{1}{x} + \dots$$

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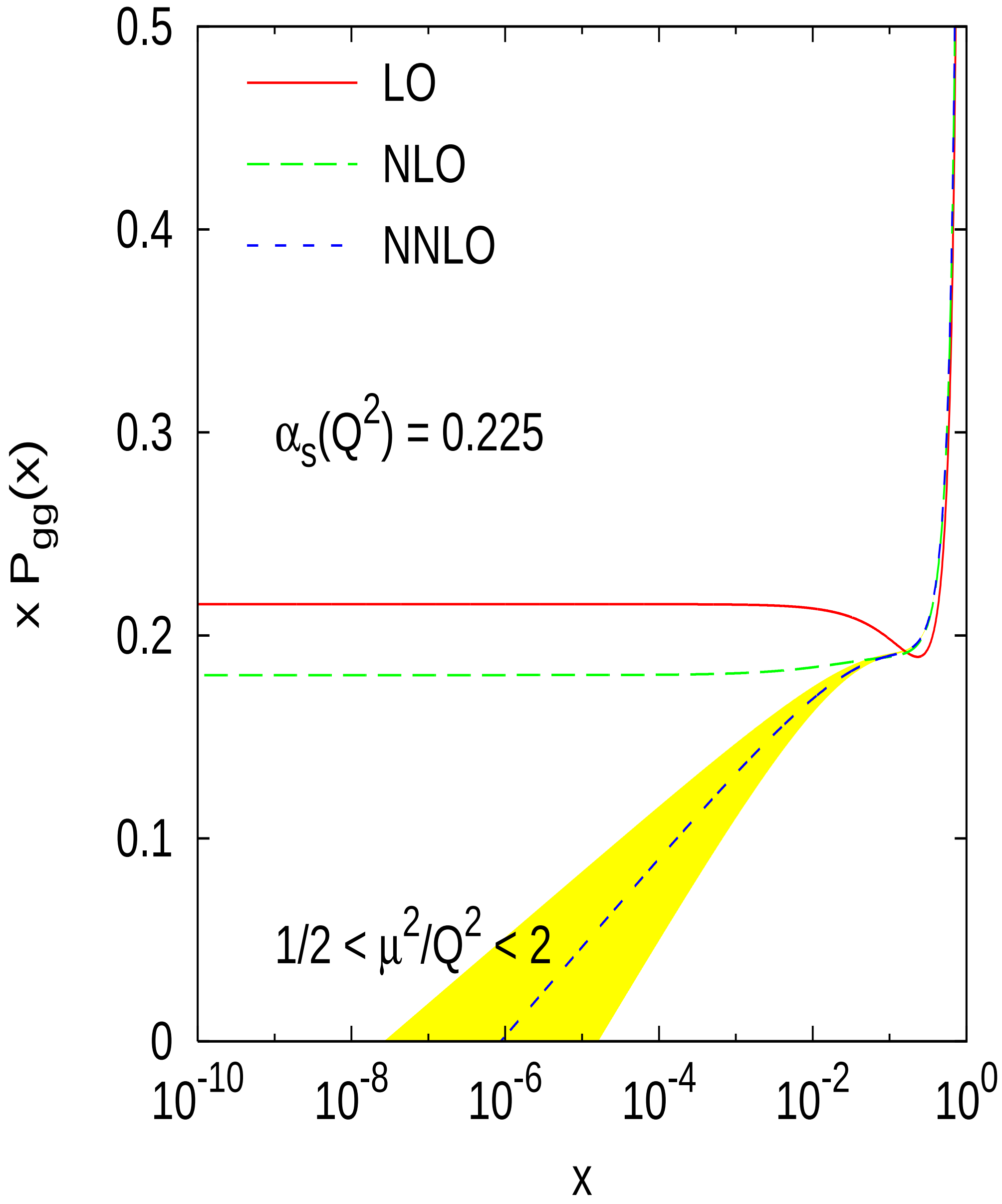


LLx, NLLx

<u>Reminder</u>

- LLx terms rise very fast, $xP_{gg}(x) \sim x^{-0.5}$.

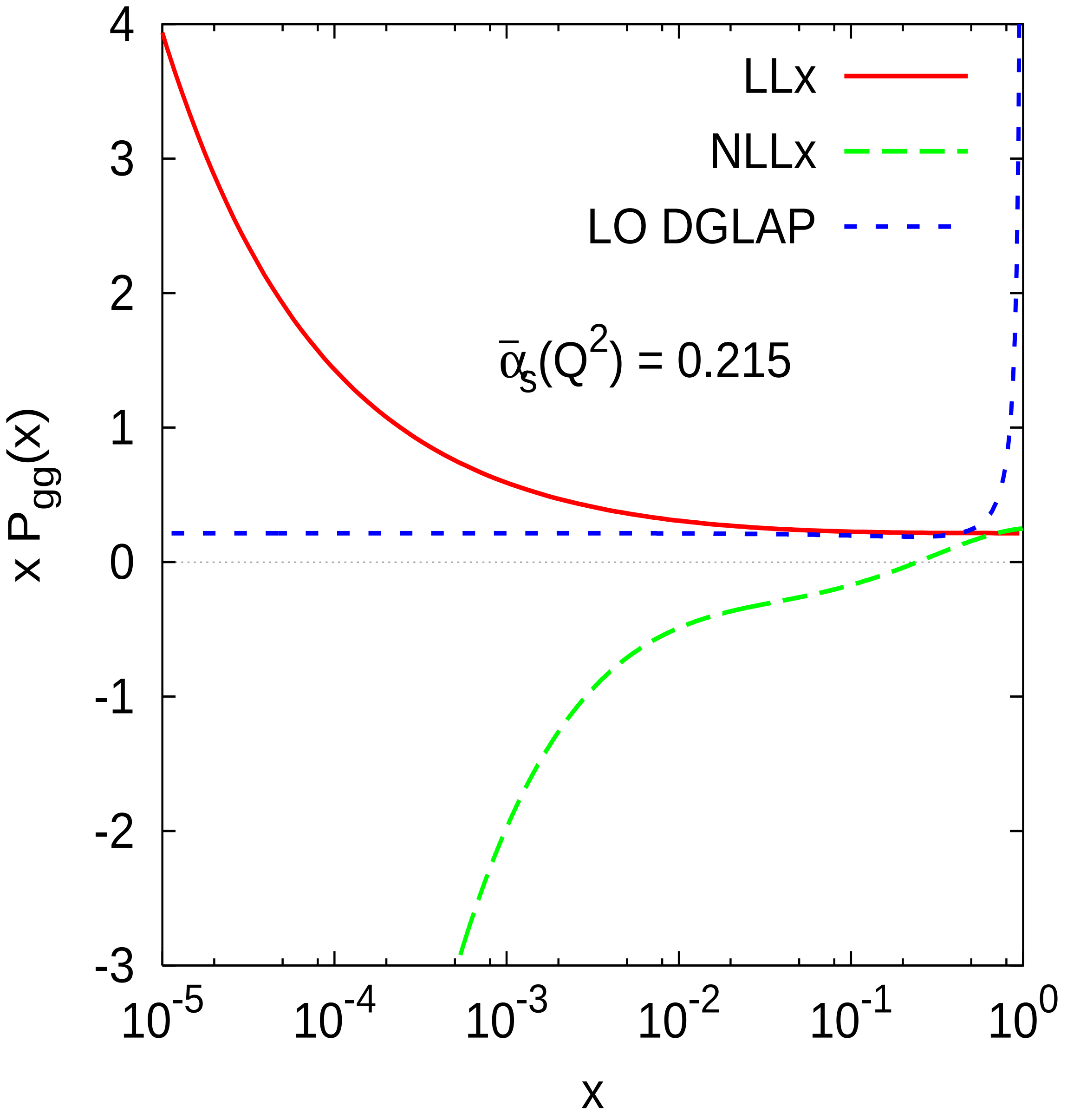
  Incompatible with data.

  Ball & Forte '95

- NLLx terms go negative very fast.

  No one's even tried fitting the data!

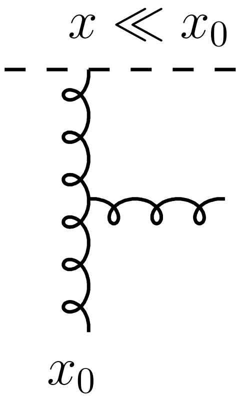
[NB: Taking NLLx terms of P_{gg} is almost the worst possible expansion]



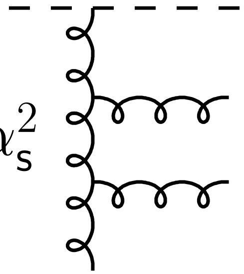
Improved NLLx? Start with kernel. . .

BFKL

$\Big($ α_s



$+ \, \alpha_s^2$



$\Big)$ x $\ln \frac{x_0}{x}$

DGLAP

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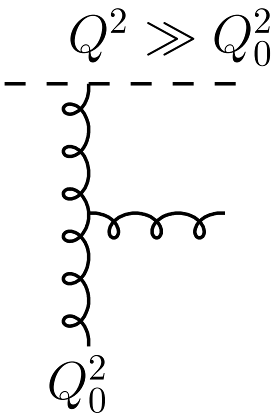

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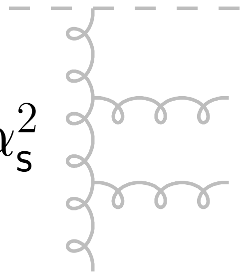

$\Big)$ x $\ln \frac{Q^2}{Q_0^2}$

$+ \, Q^2 \Leftrightarrow Q_0^2$ anti-DGLAP

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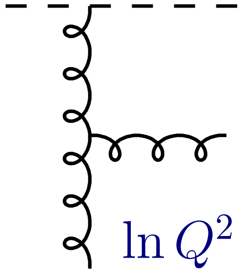
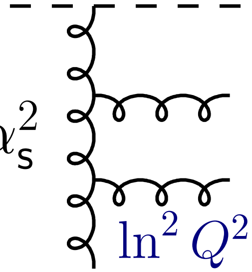
$x \ll x_0$

α_s 

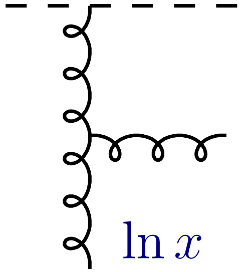
x_0

$+\ \alpha_s^2$ 

$\times \ln \frac{x_0}{x}$

DGLAP

$Q^2 \gg Q_0^2$

α_s 

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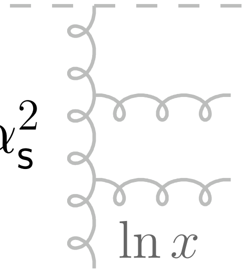
$+\ \alpha_s^2$ 

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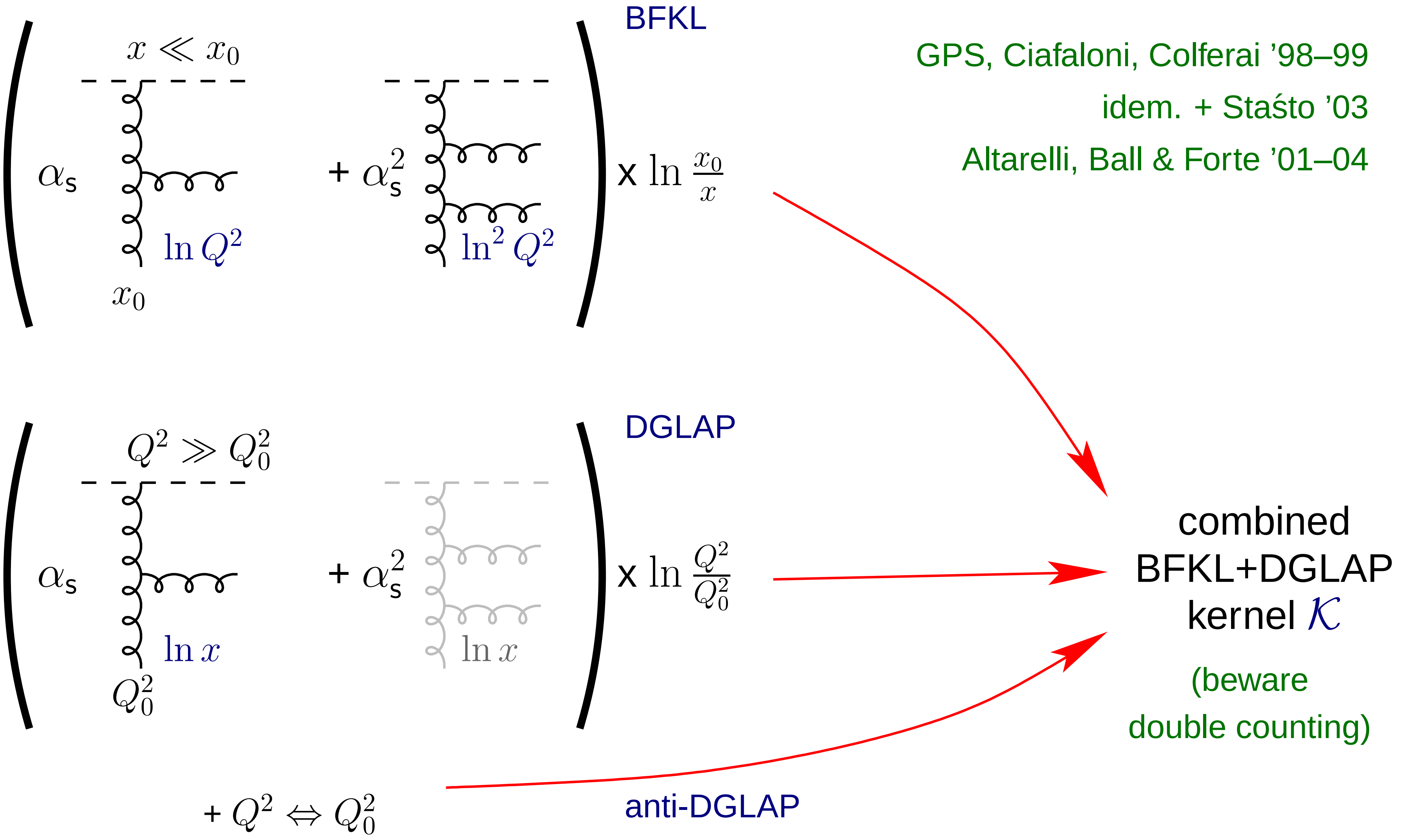
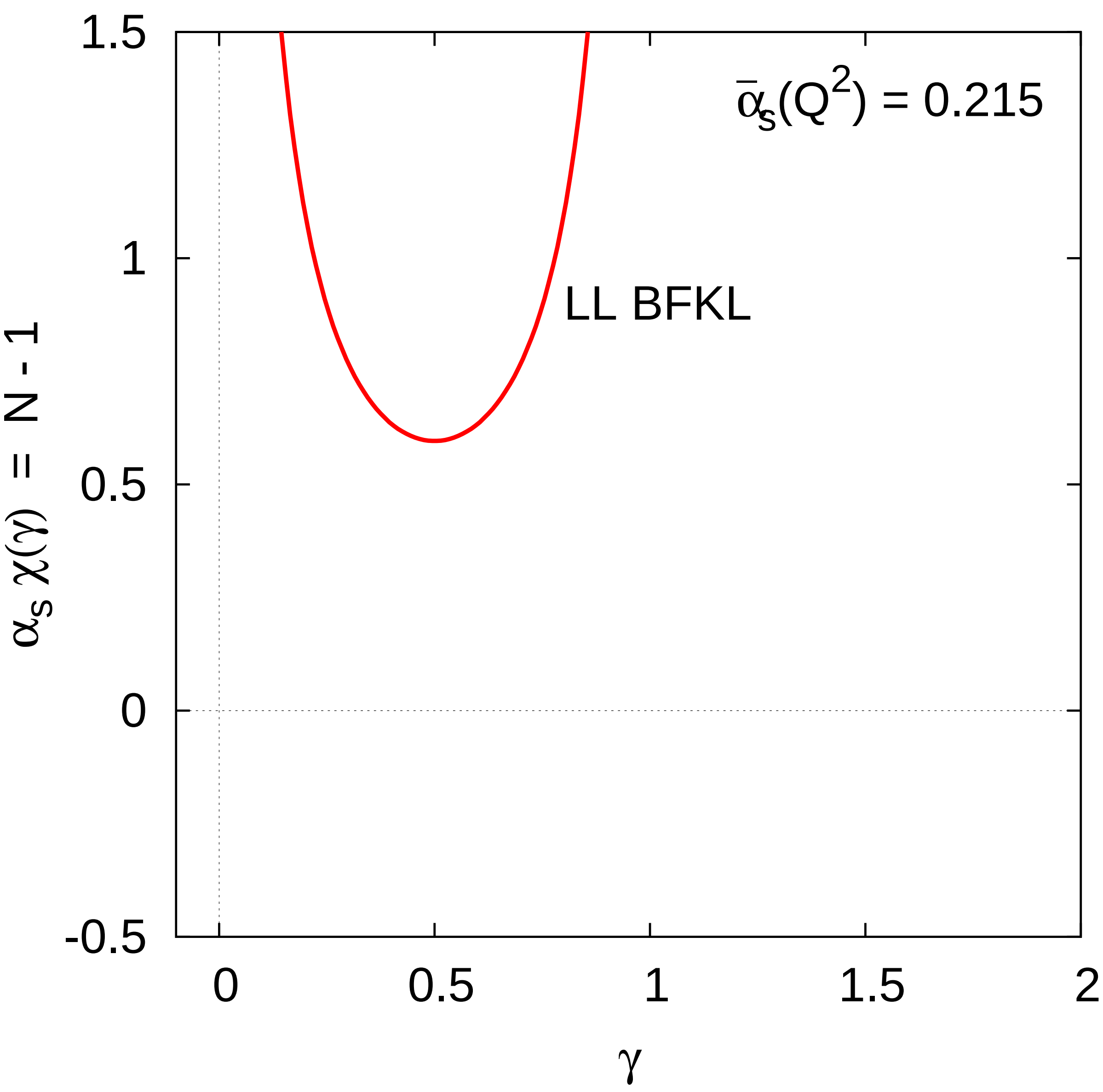


Building up the kernel. . .



Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)

$$\bar{\alpha}_s \chi(\gamma) = $$
$$= \int \frac{dk^2}{k^2} \left(\frac{k^2}{k_0^2}\right)^\gamma \mathcal{K}(k, k_0)$$

Height of minimum is 'BFKL power'

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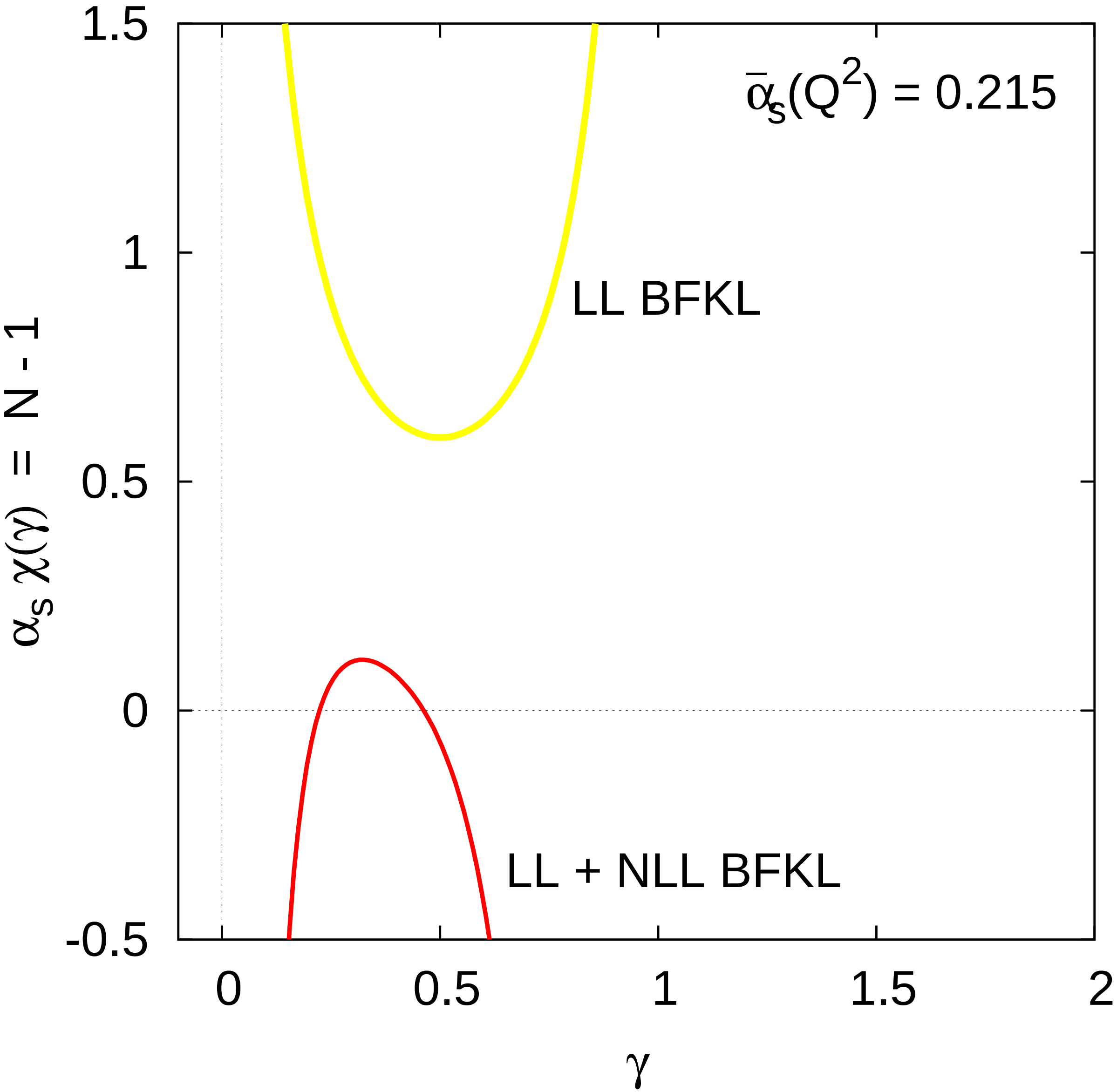

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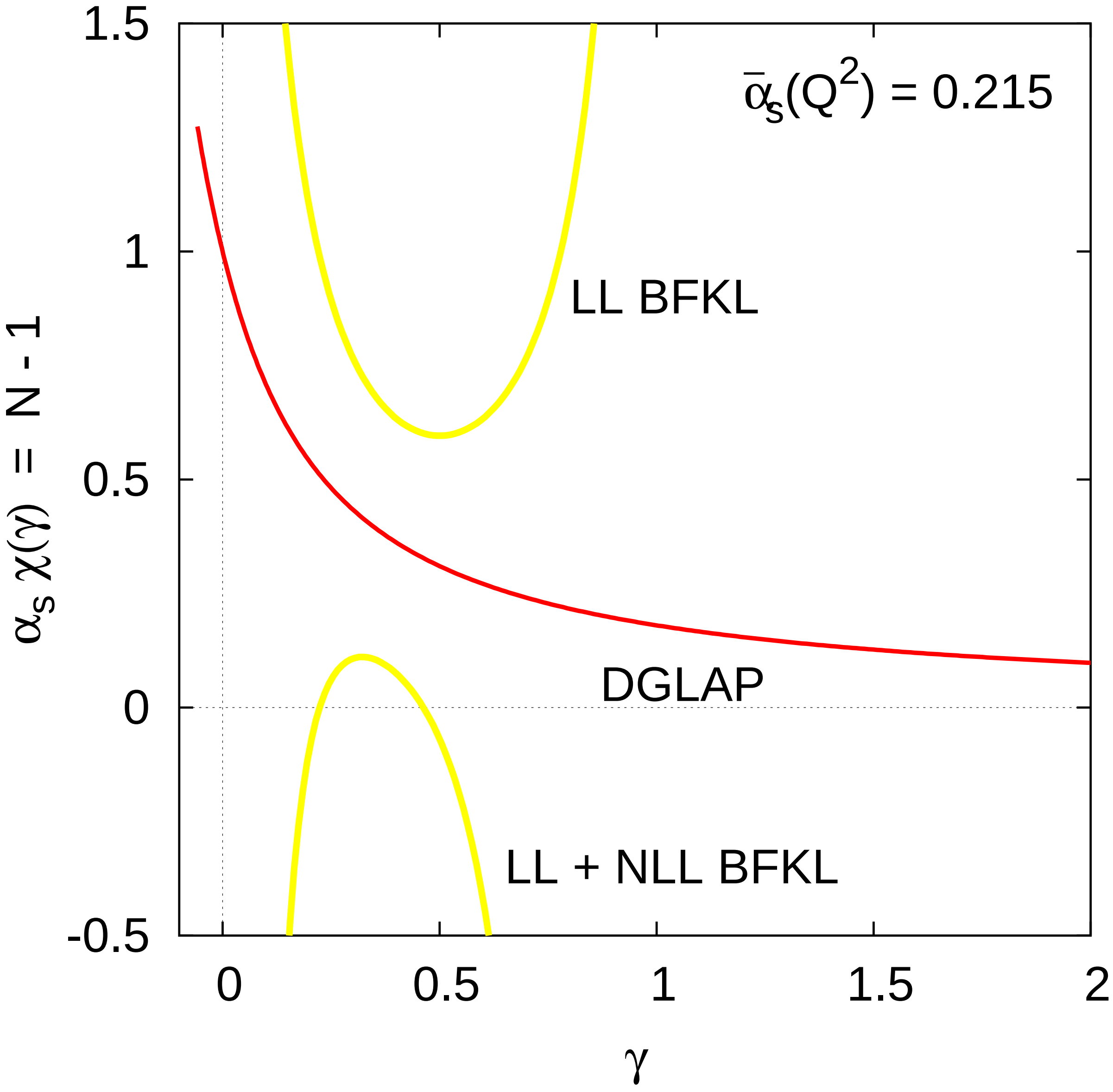

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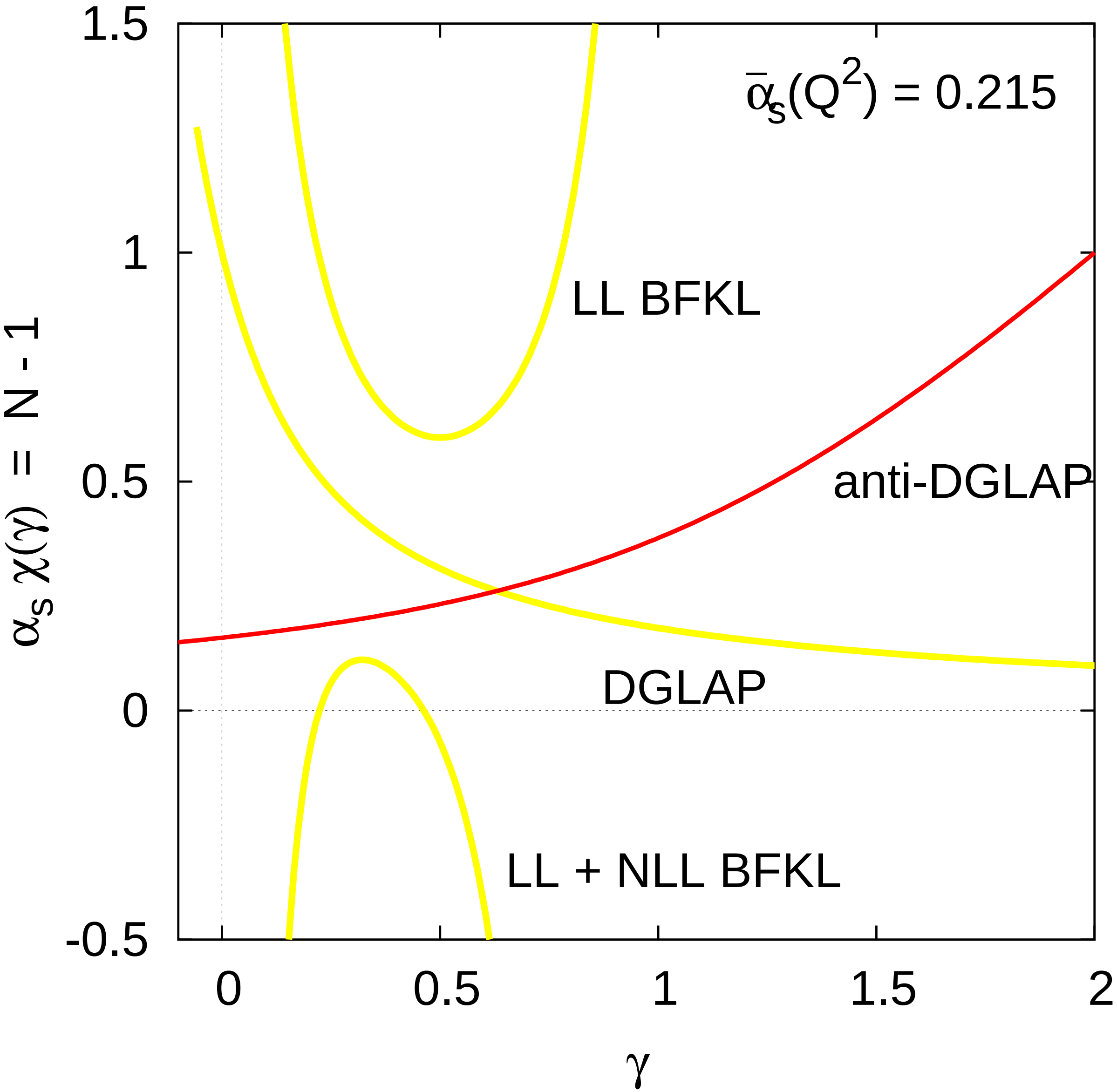

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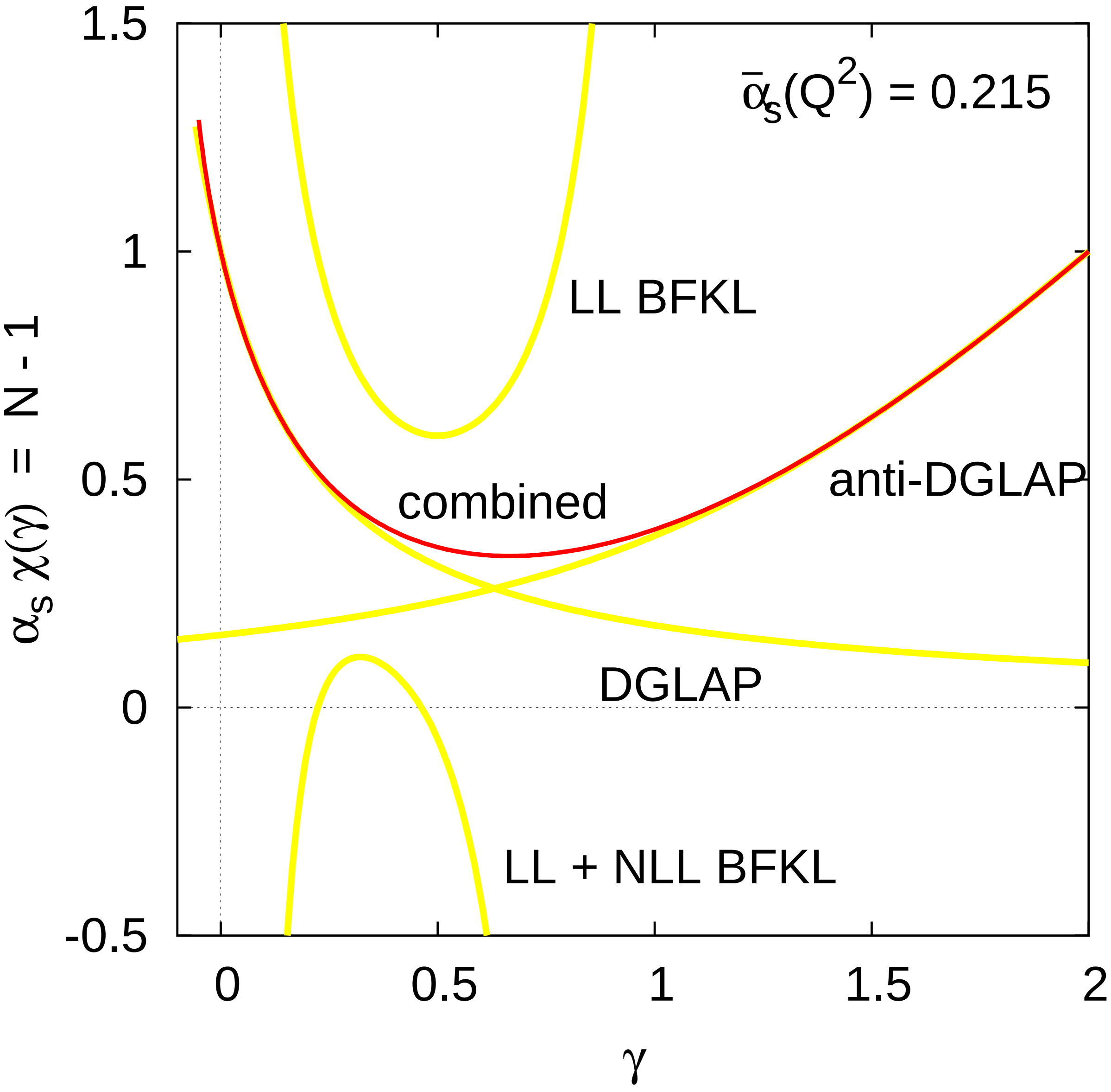

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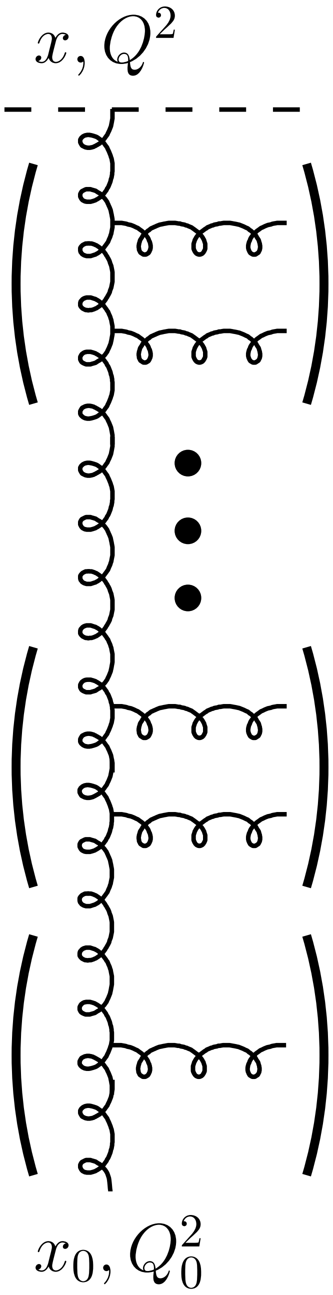

Green function: $G\left(\ln \frac{x}{x_0}; Q_0, Q\right)$

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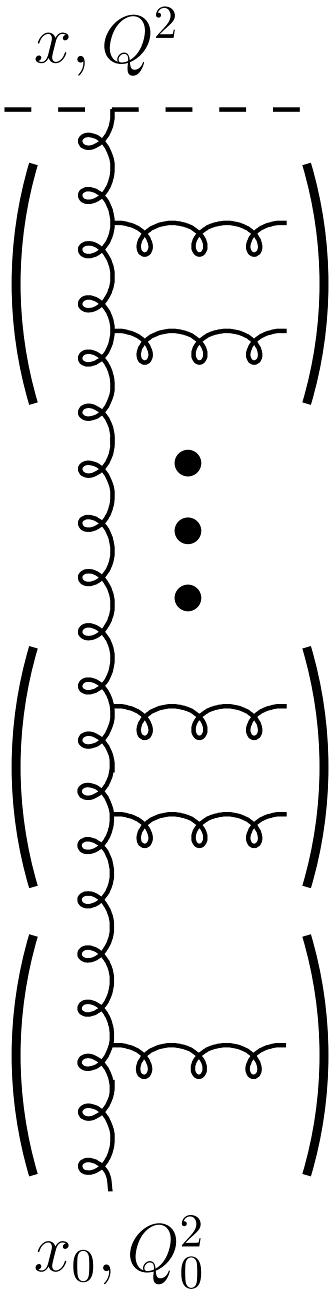

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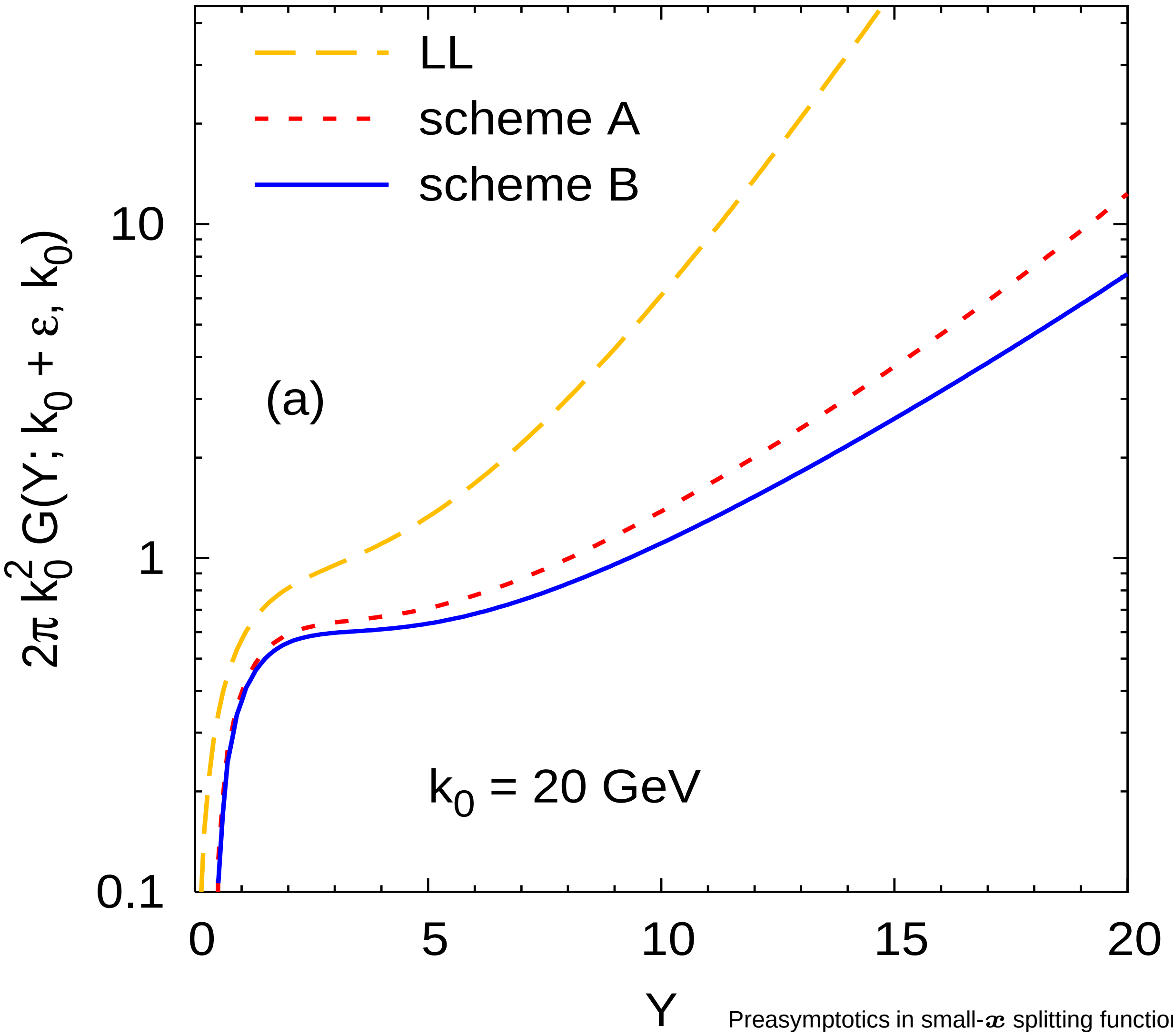

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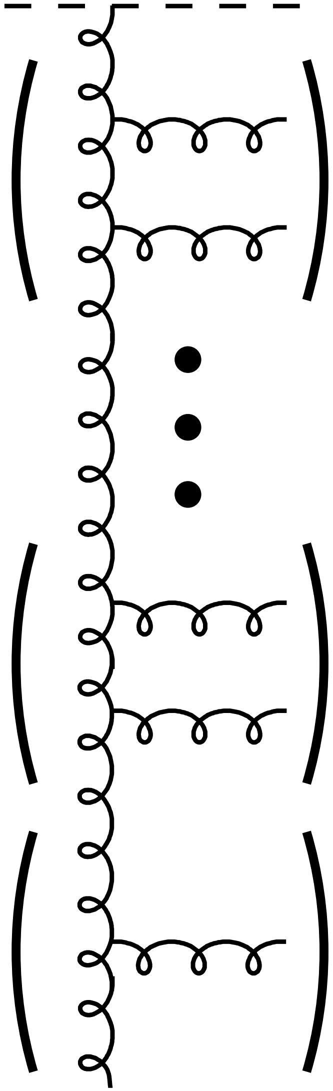

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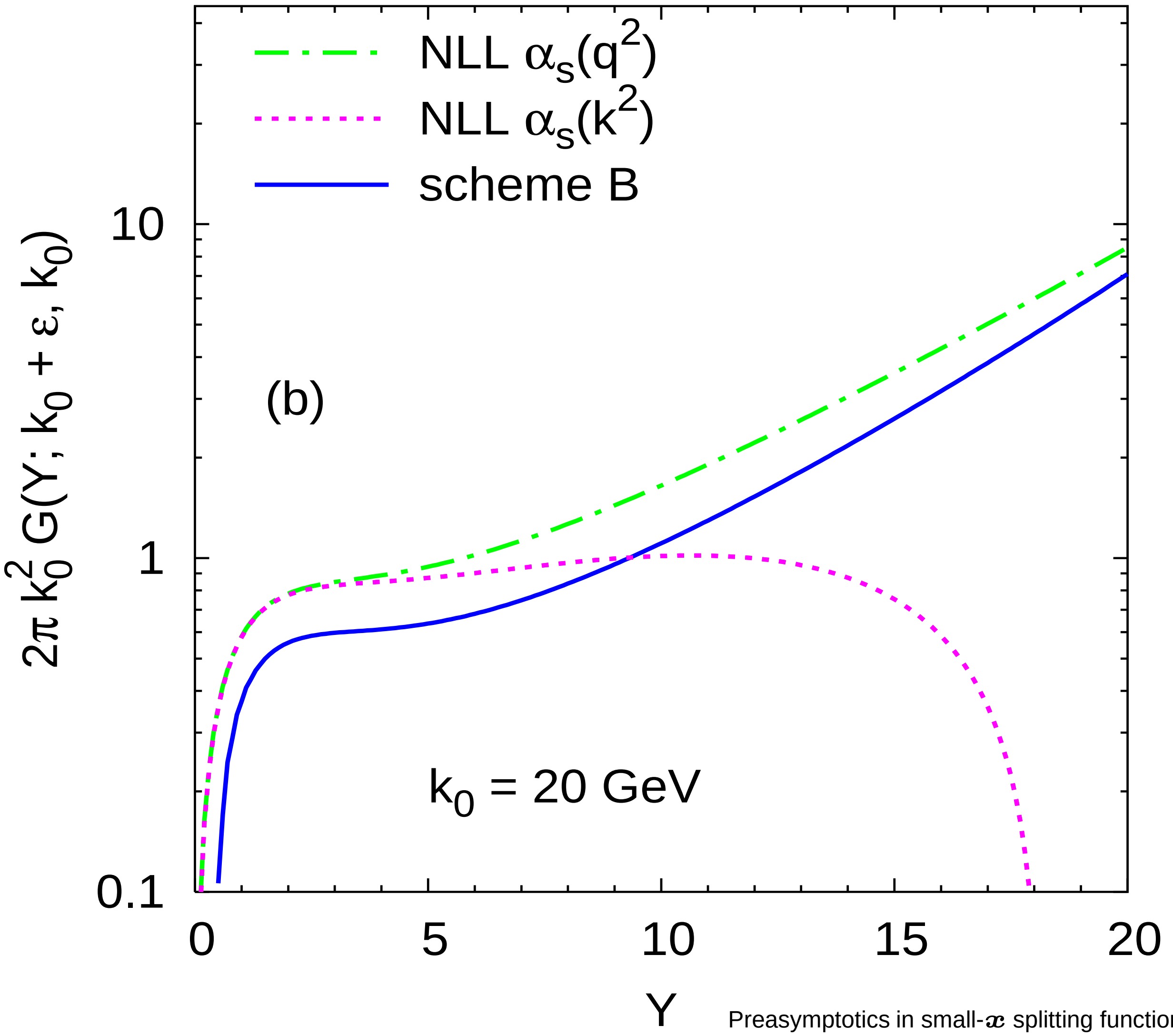

Green function $\Rightarrow$ effective DGLAP splitting function

Construct a gluon density from Green function (take $k \gg k_0$):

$$xg(x, Q^2) \equiv \int^Q d^2k \; G^{(\nu_0 = k^2)}(\ln 1/x, k, k_0)$$

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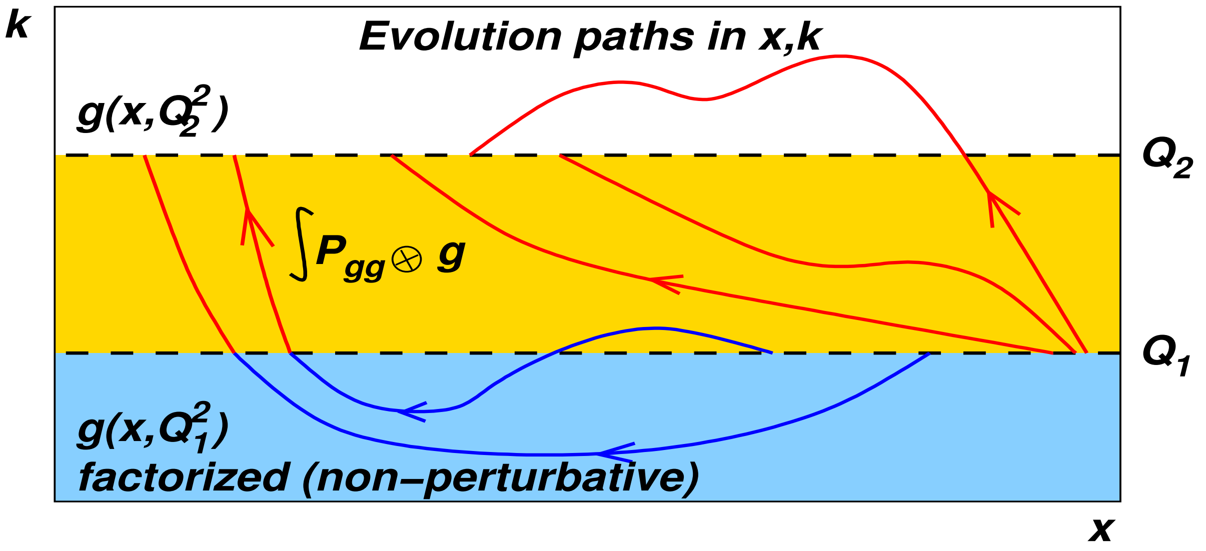
Factorisation

- Splitting function:
  red paths
- Green function:
  all paths



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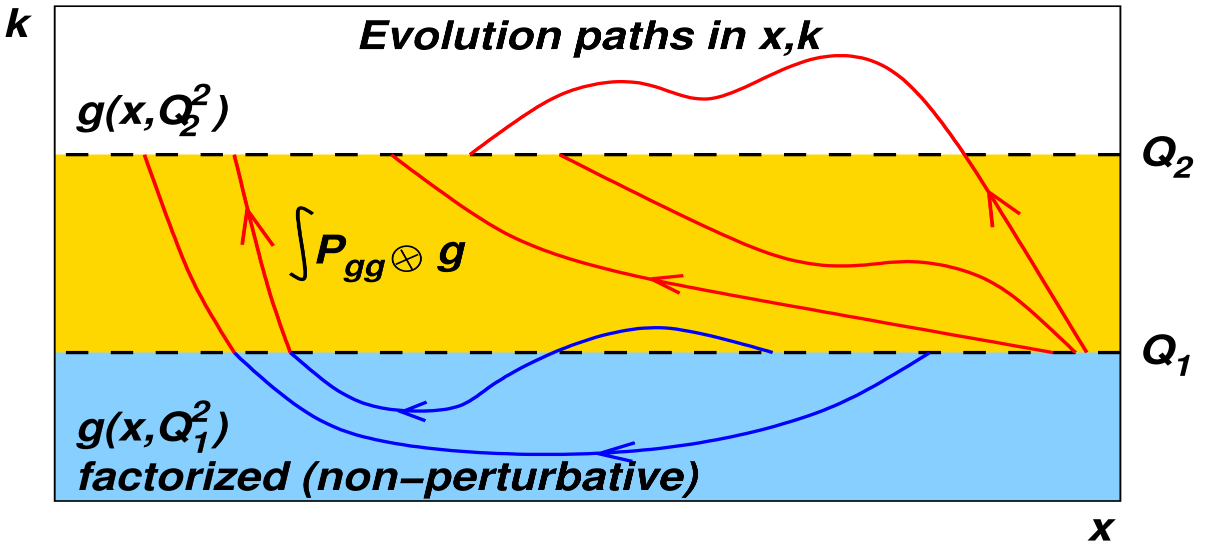
Splitting function $\equiv$ evolution with cutoff



BFKL splitting function 'power'

Two classes of correction, to power growth ω:

$$\omega = 4 \ln 2\, \bar{\alpha}_s(Q^2) \left(1 - \underbrace{6.5\, \bar{\alpha}_s}_{NLL} - \underbrace{4.0\, \bar{\alpha}_s^{2/3}}_{running} + \cdots \right)$$

$$\bar{\alpha}_s = \alpha_s N_c / \pi$$

- NLL piece is *universal*
- running piece appears only in problems with *cutoffs*
  - a consequence of *asymmetry* due to cutoff (only scales higher than cutoff contribute)

$$\alpha_s(Q^2) \longrightarrow \alpha_s(Q^2 e^{-X/(b\alpha_s)^{1/3}})$$

Hancock & Ross '92

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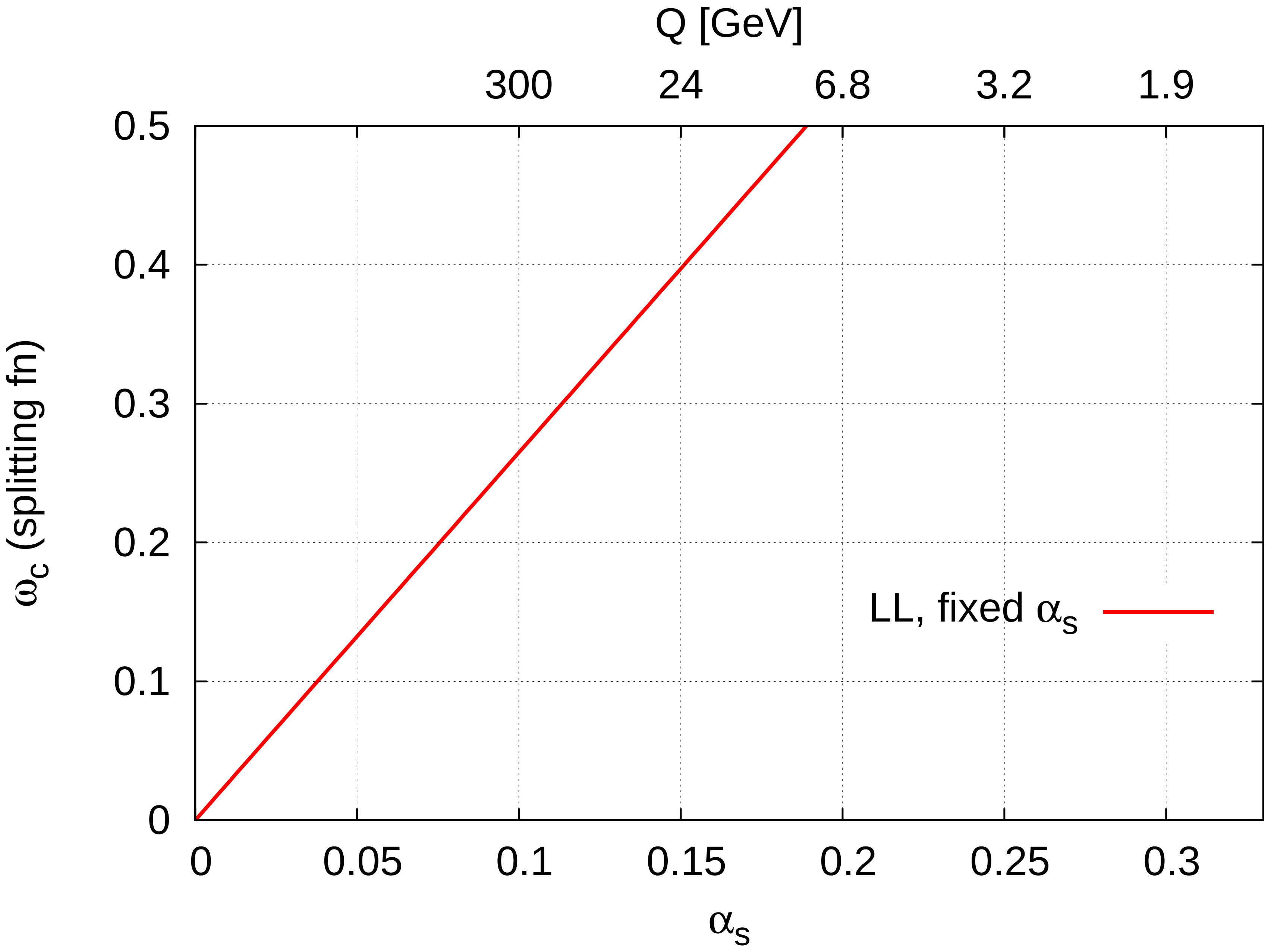
- Beyond first terms, not possible to separate effects of 'pure' higher orders & running coupling

BFKL splitting function power growth – results



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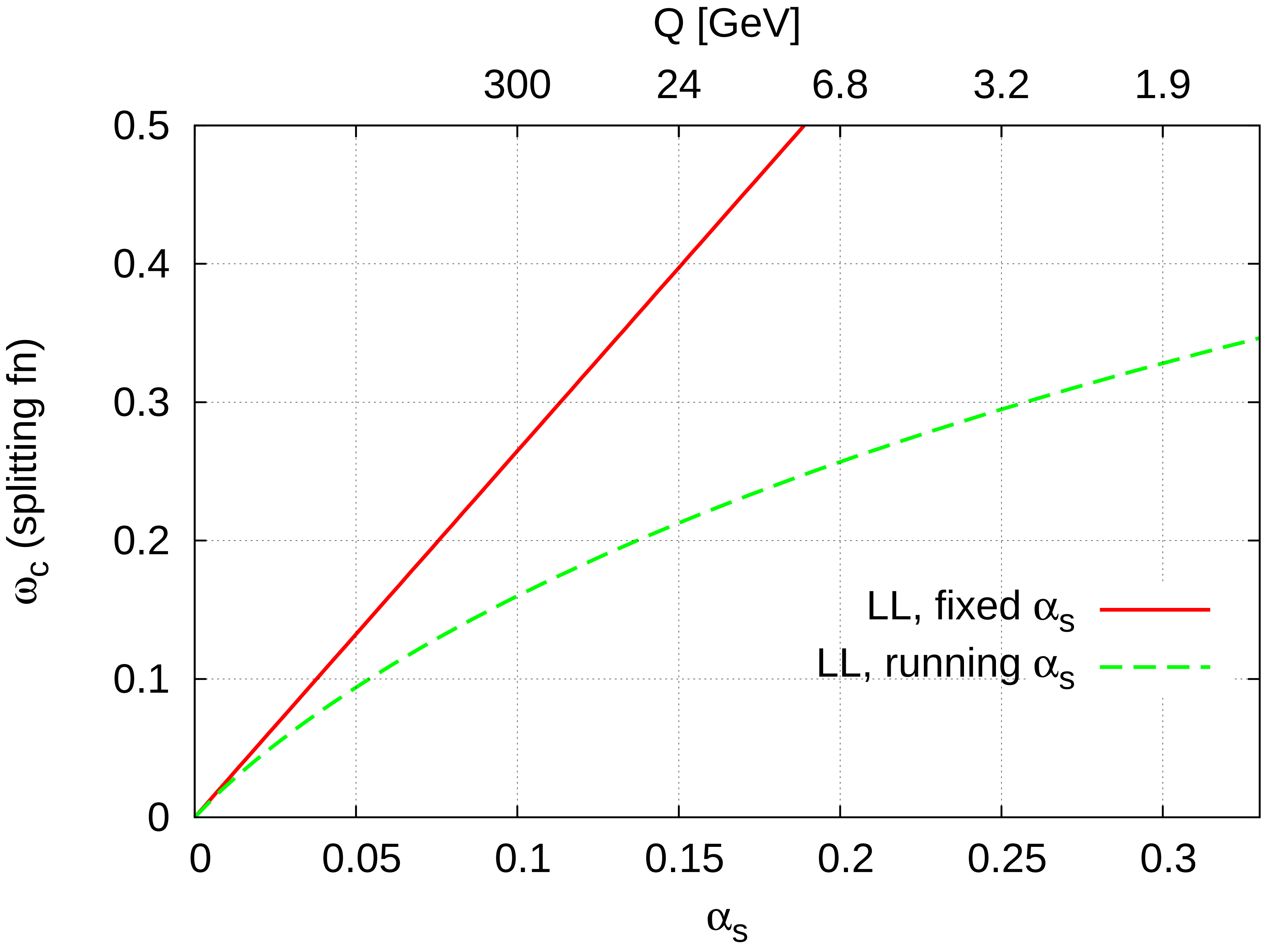

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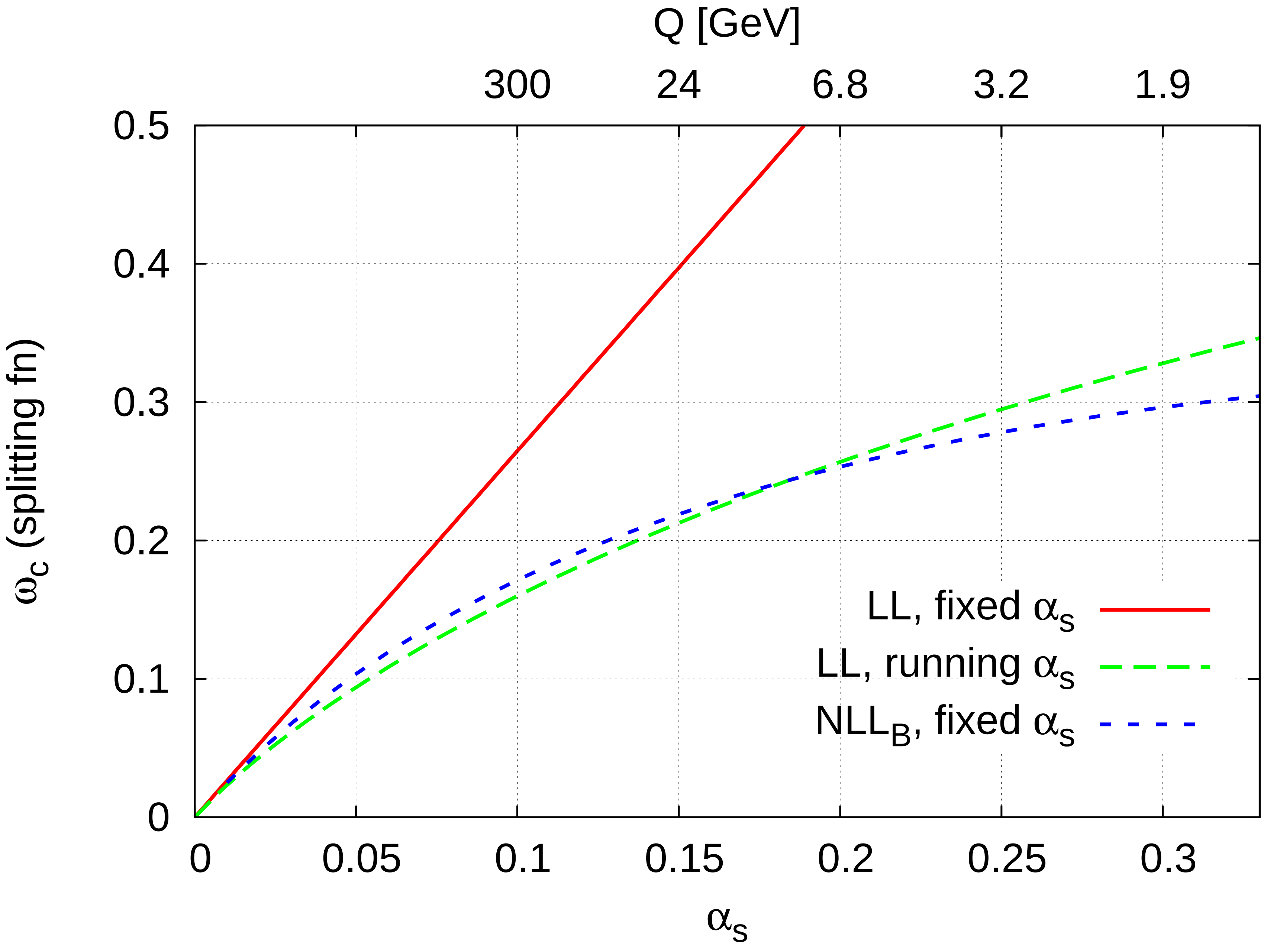

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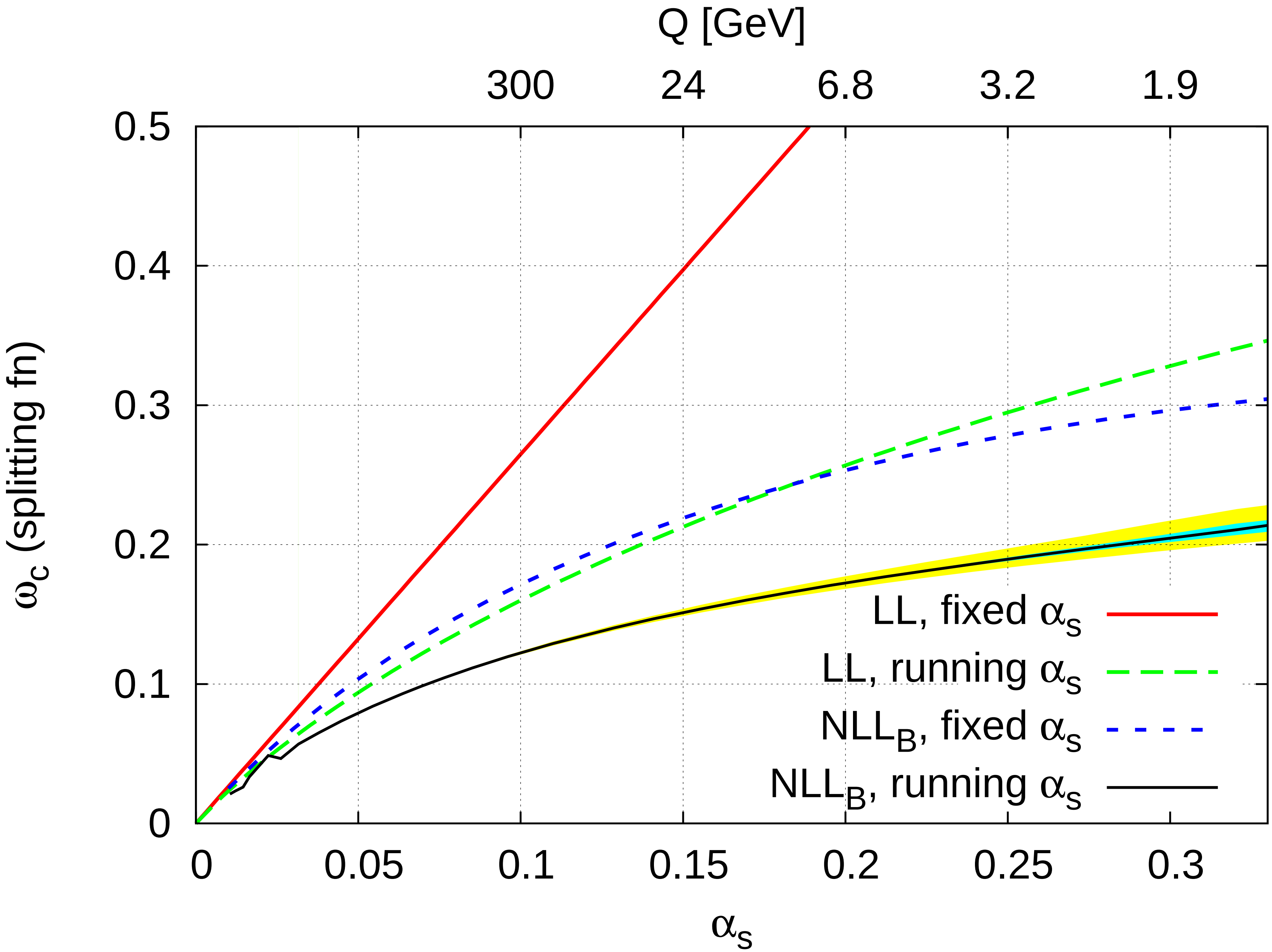
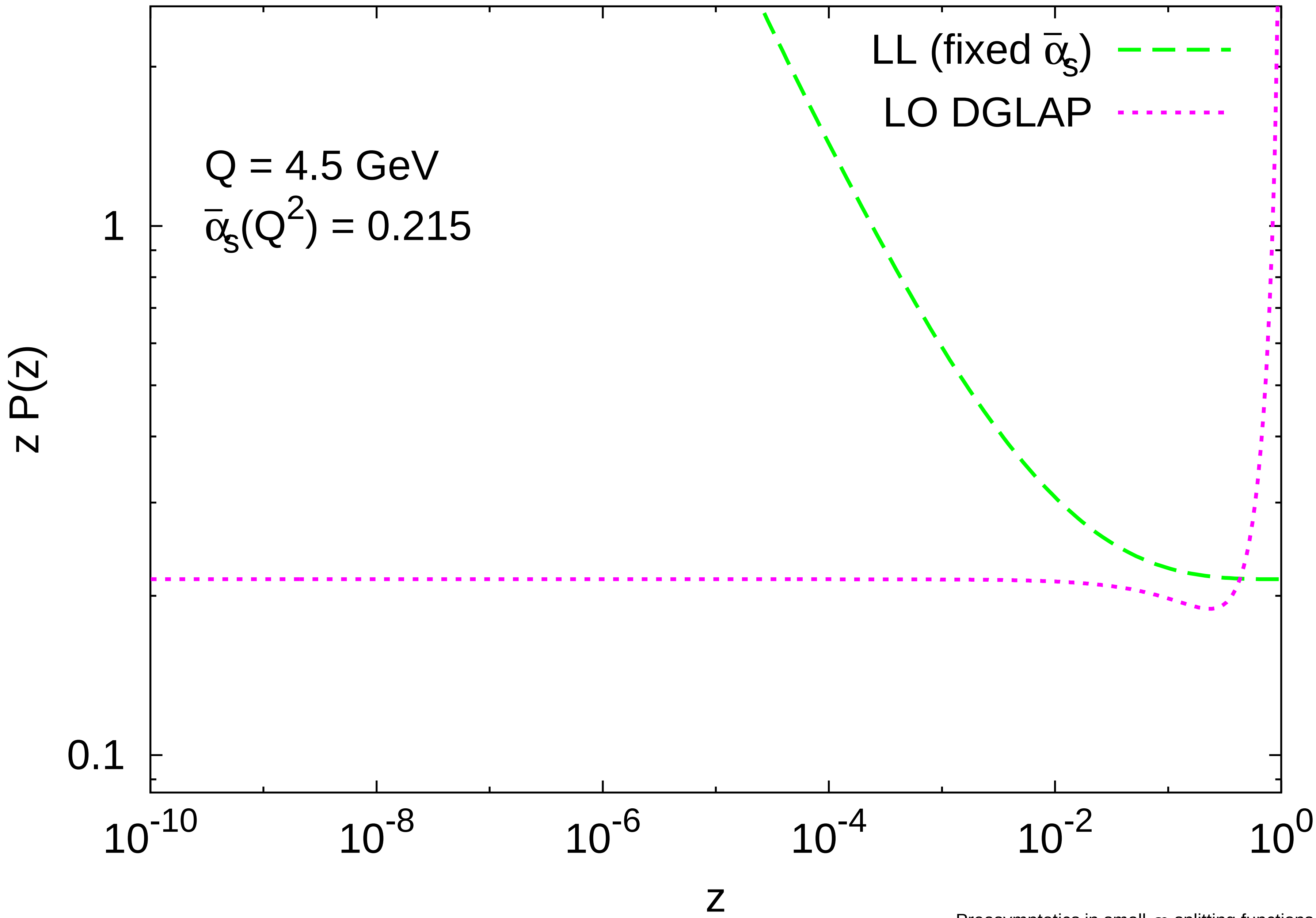


Full $P_{gg}(z)$ splitting fn



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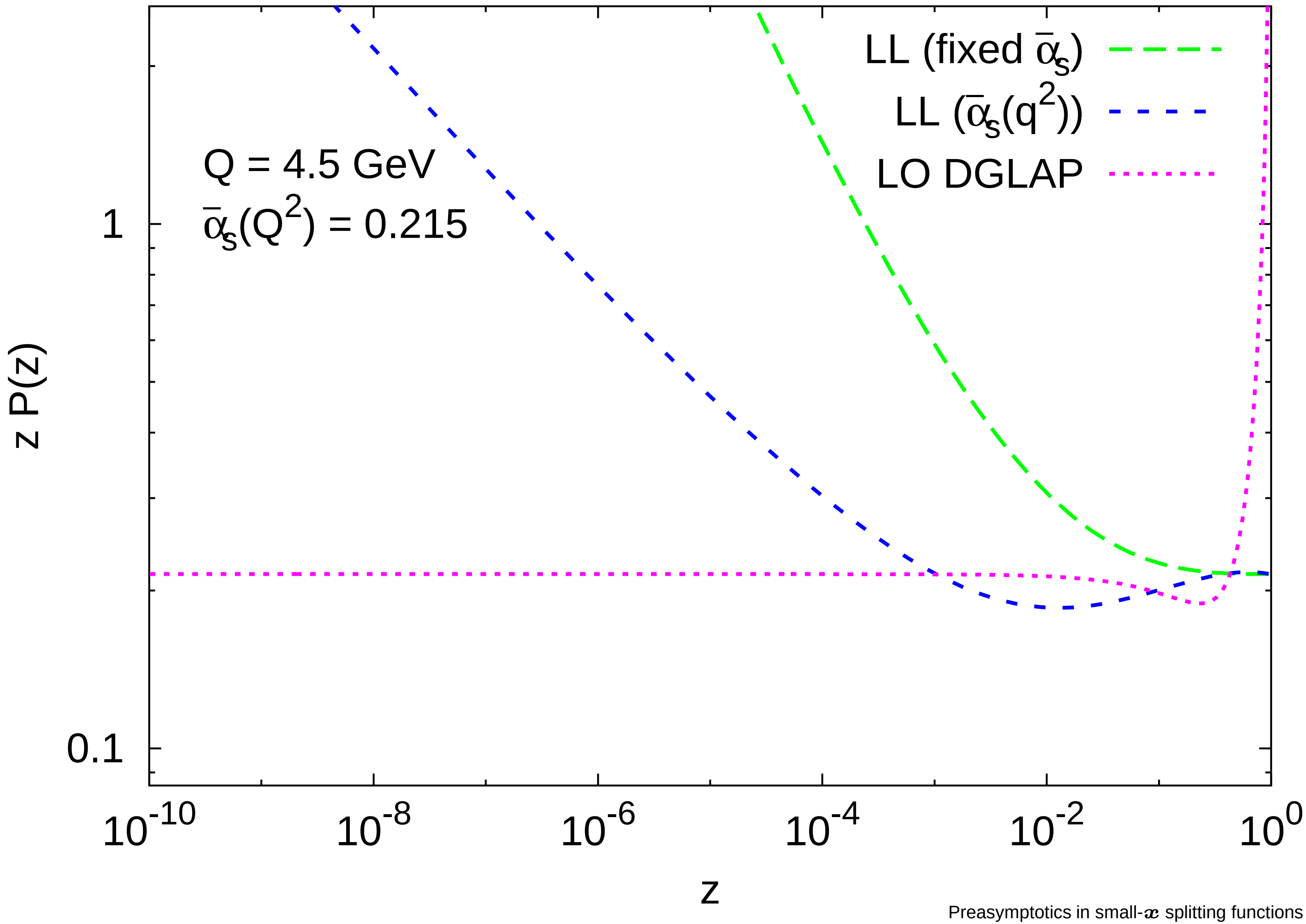

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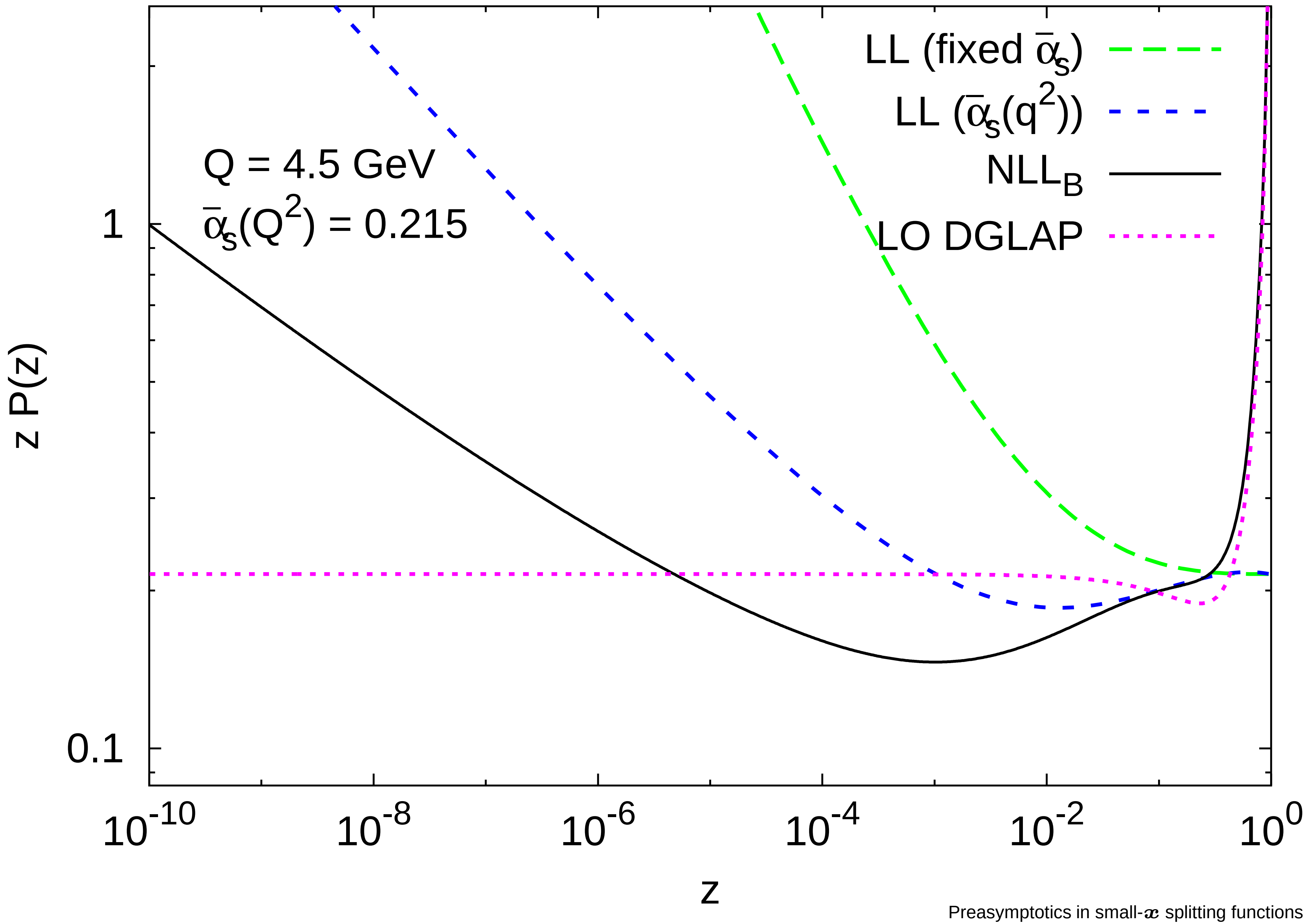
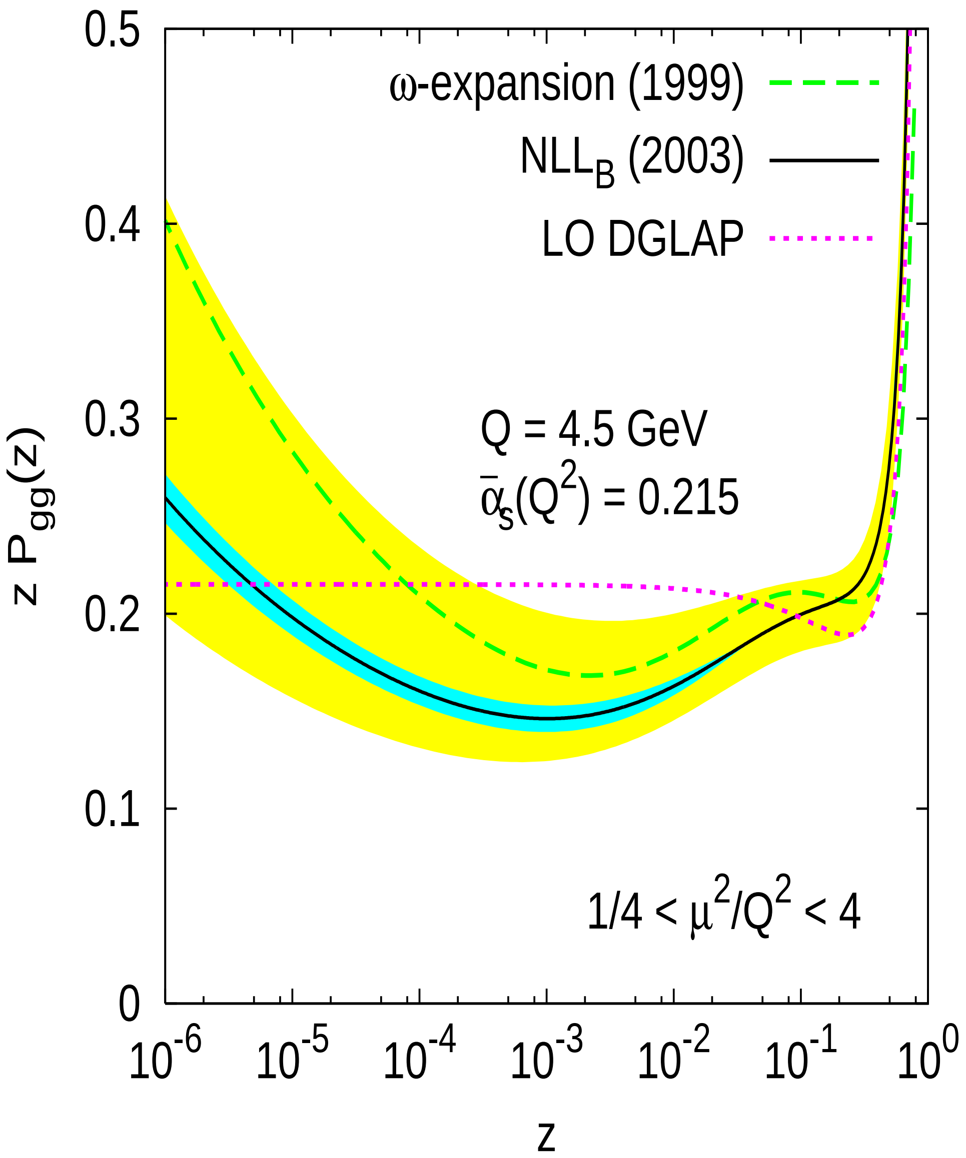


Dominant phenomenological structure in P_{gg} is dip

- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at* $x \sim 10^{-3}$



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  Thorne '99, '01 (running α_s, NLLx)

  ABF '99–'03 (fits, running α_s)

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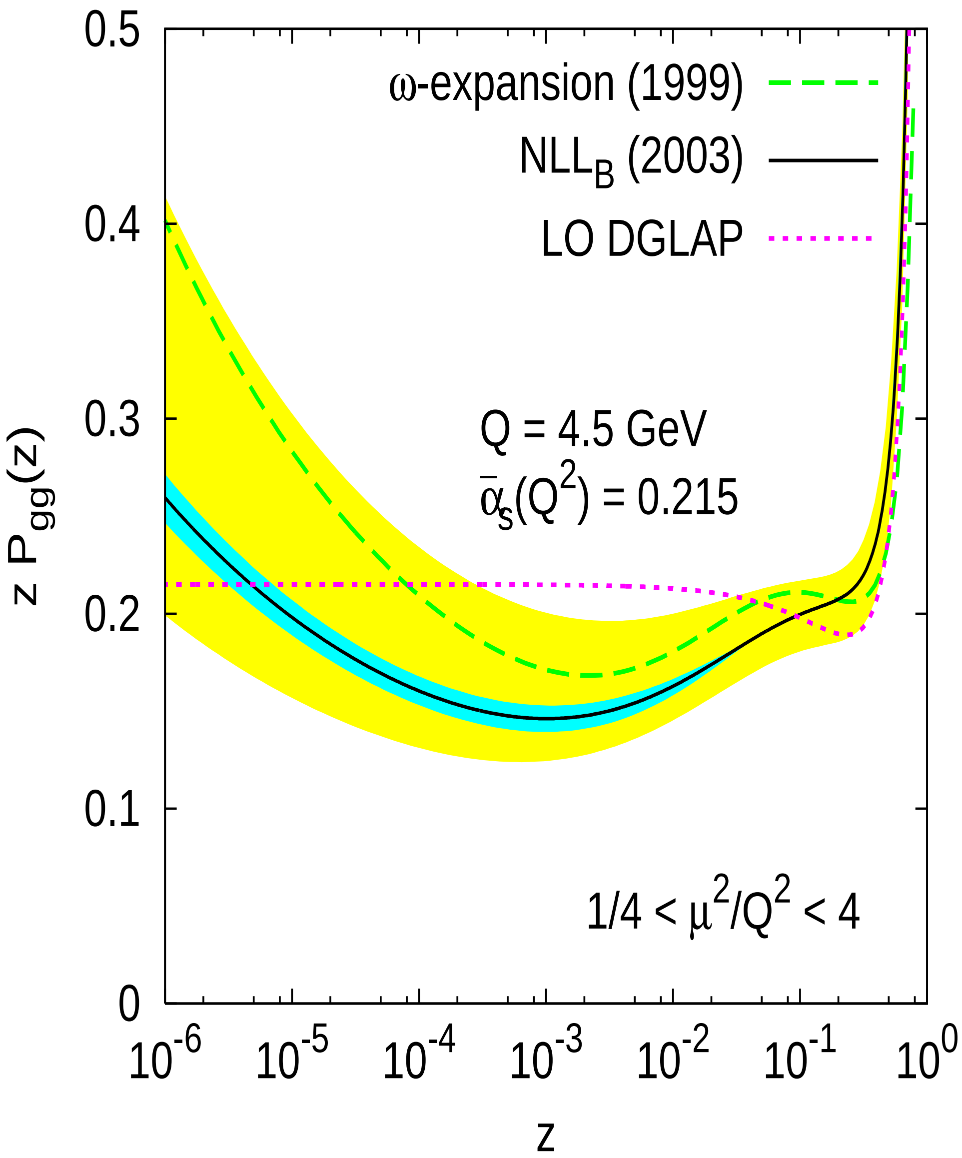
  Is it always the same dip?



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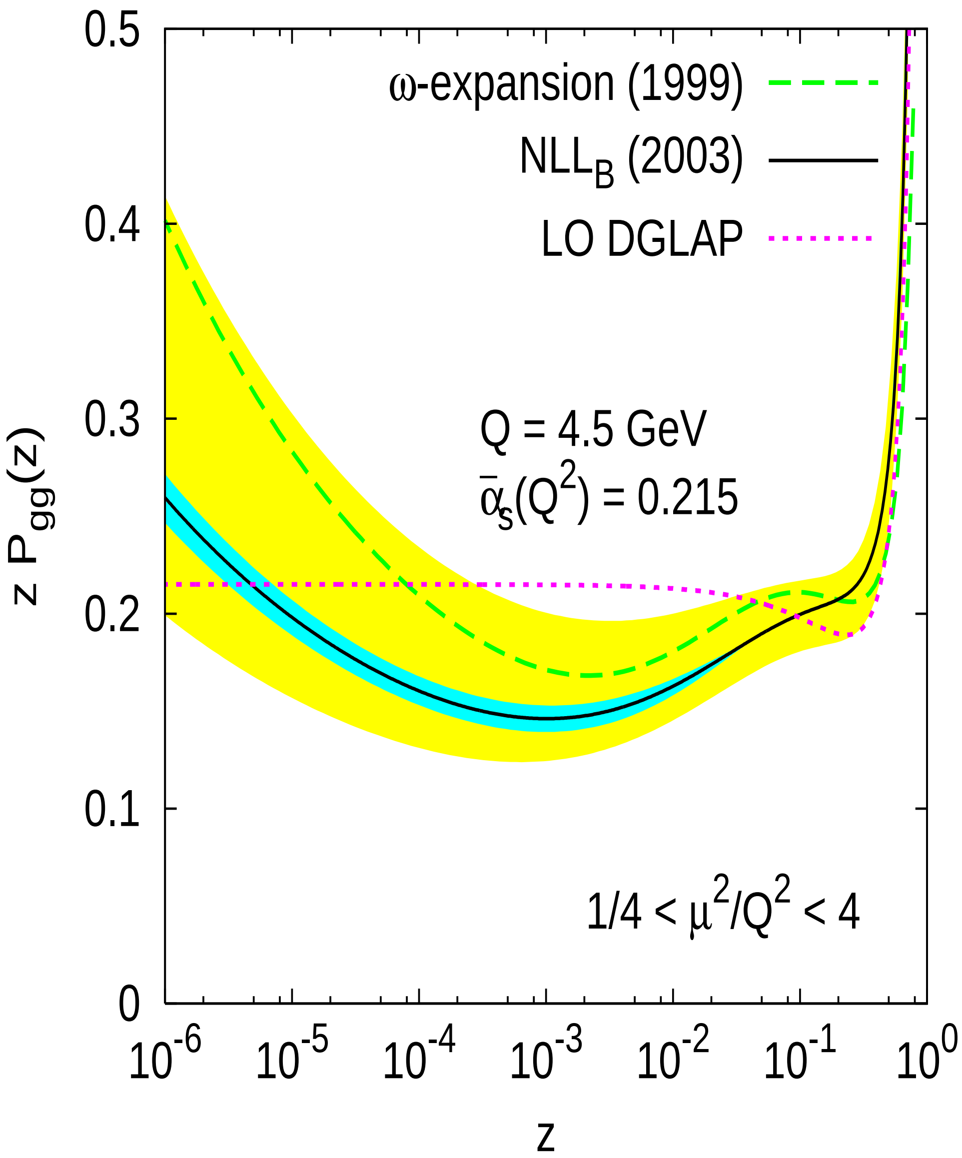

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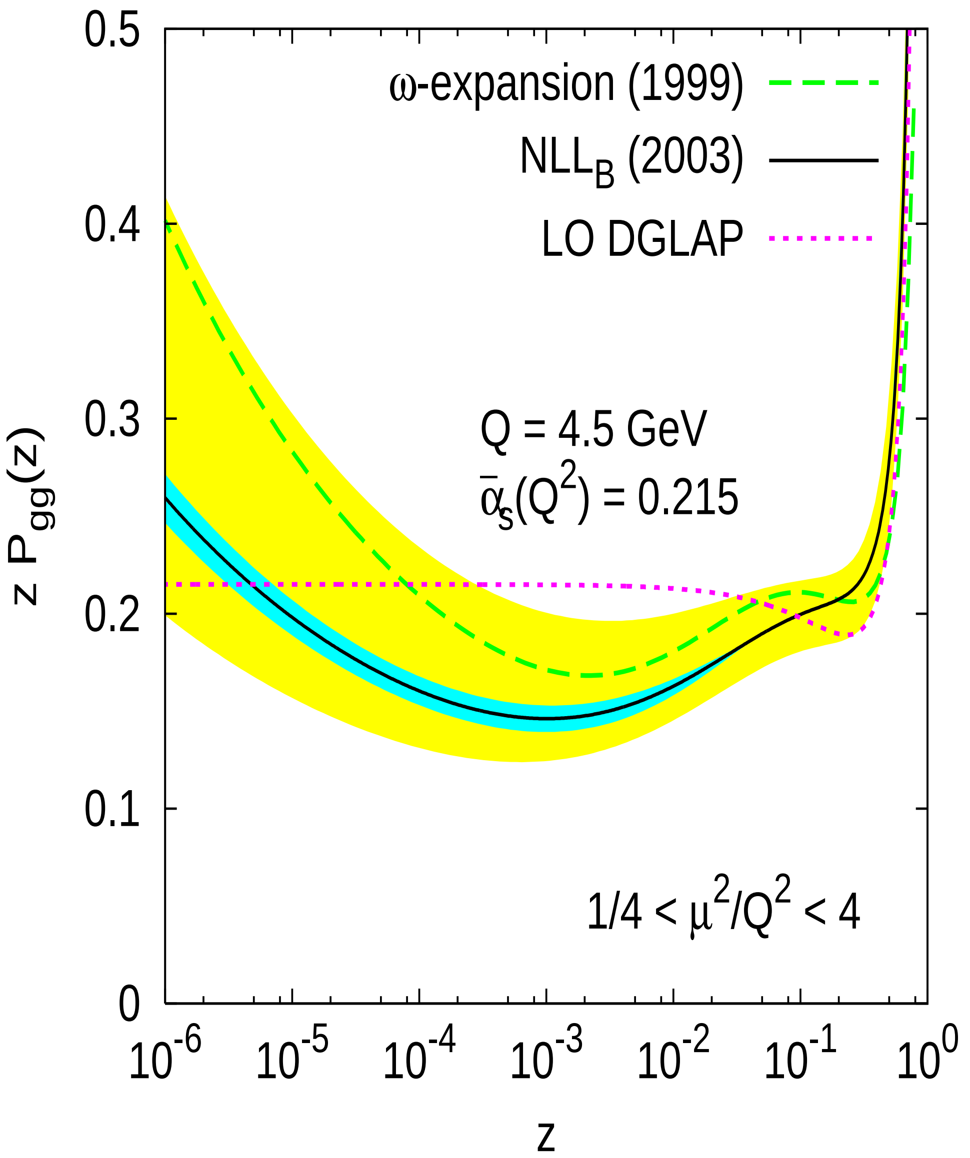
- What is its origin?
  Running α_s, momentum sum rule. . . ?



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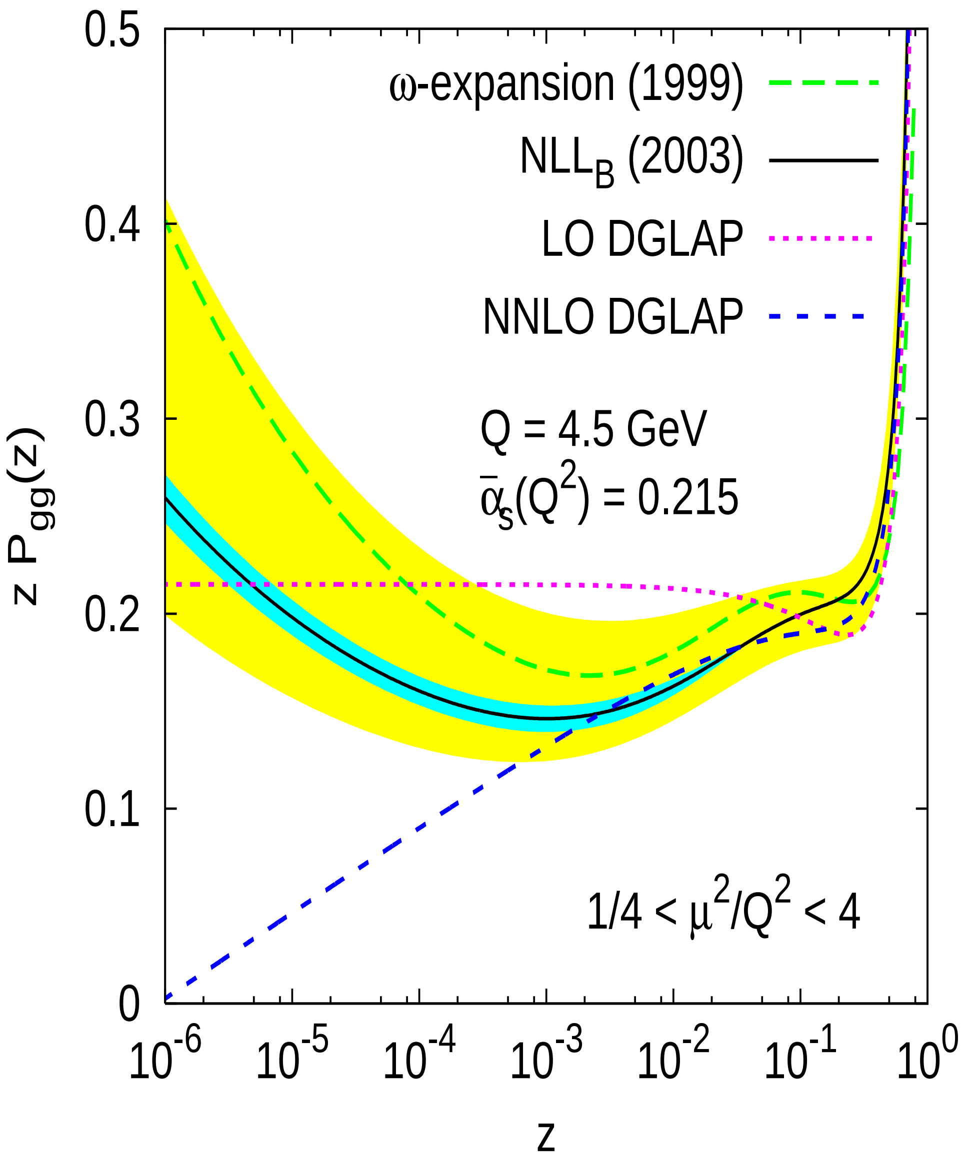
NNLO DGLAP gives a clue. . .

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$



Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
$\vdots$				

const.

$\ln 1/x$

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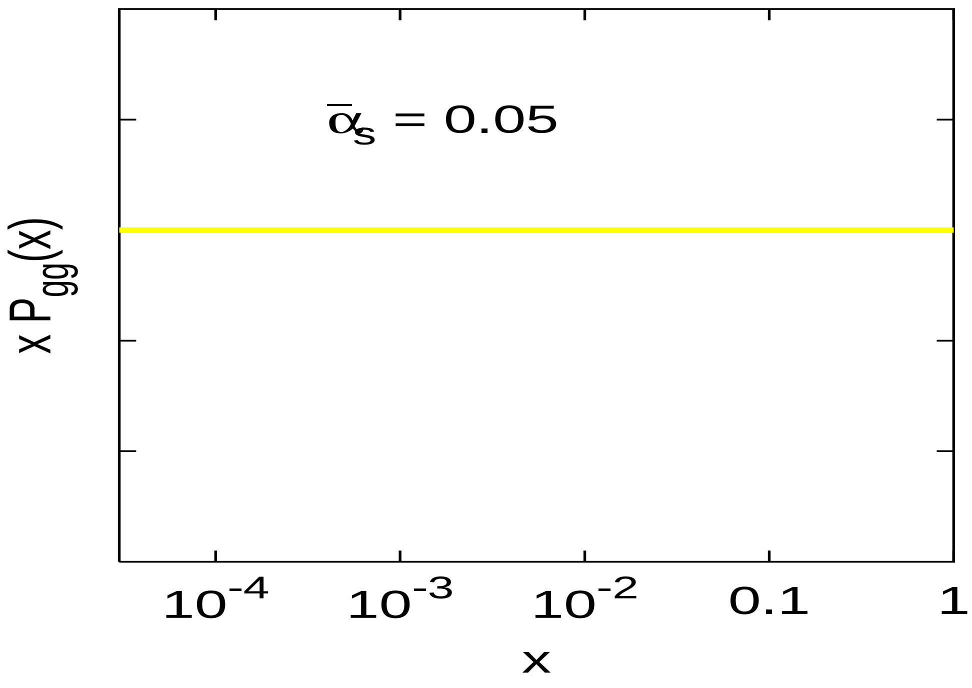


Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	**x**	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
⋮				

const.

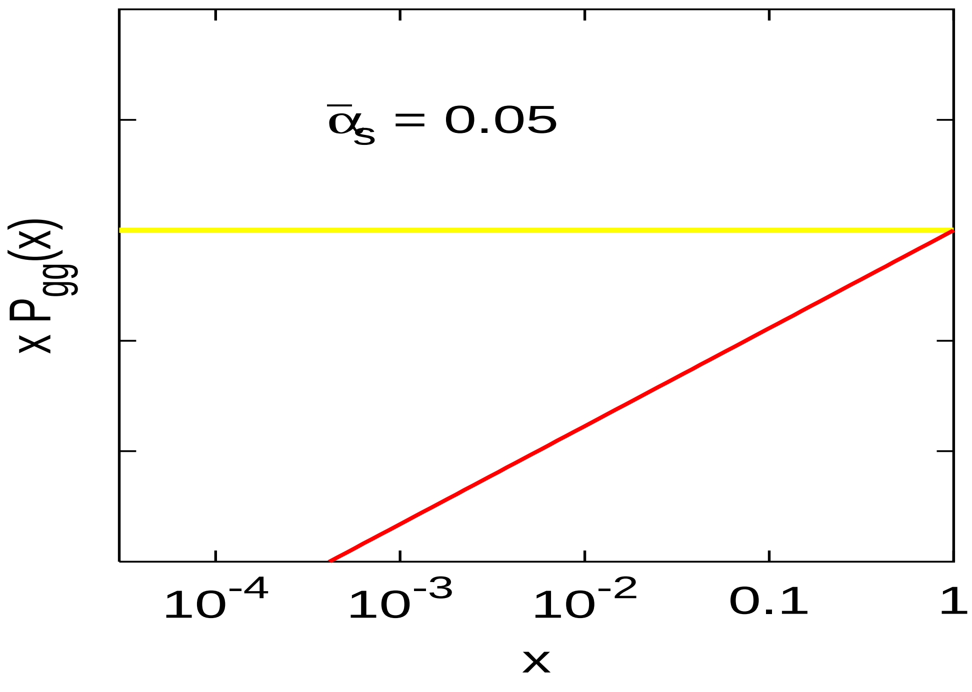
ln 1/x

$\ln^2 1/x$

$\ln^3 1/x$

At moderately small x, first terms with x-dependence are

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$



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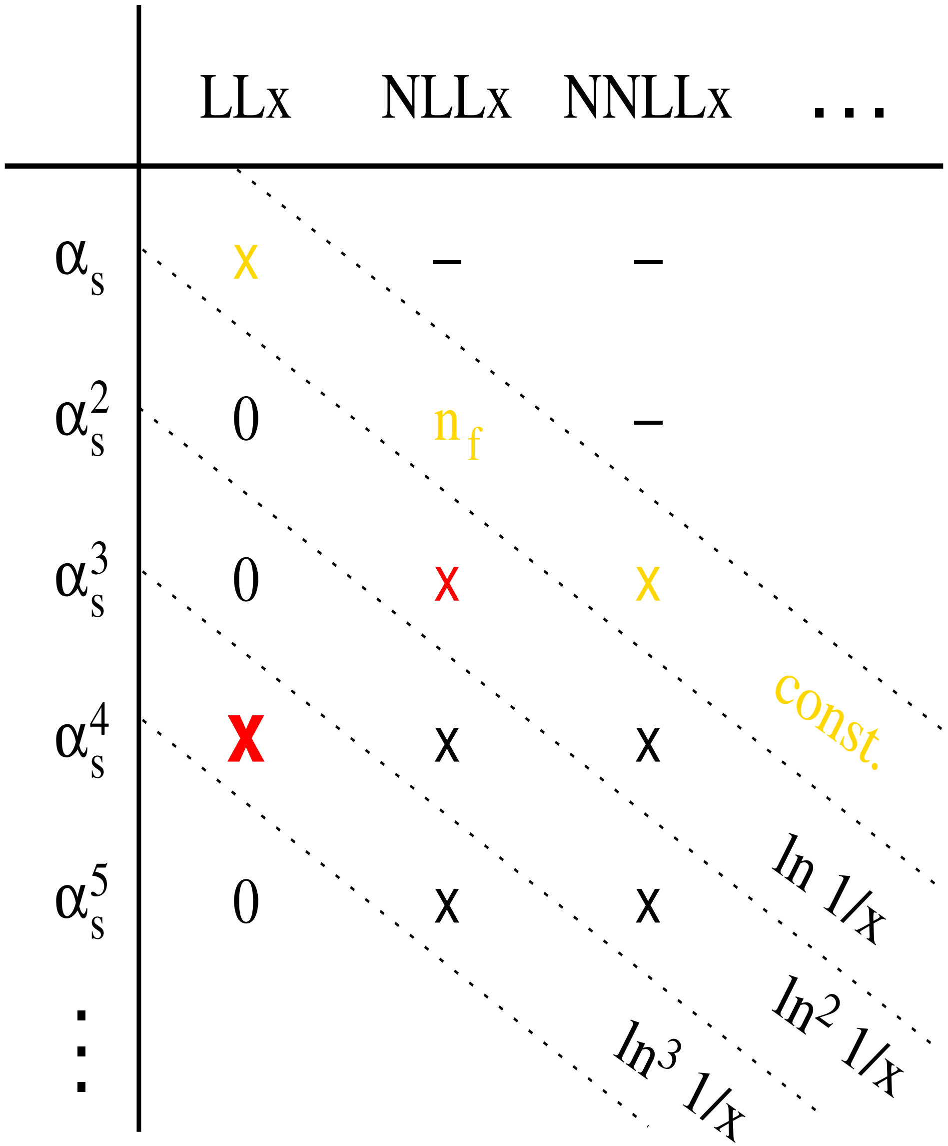
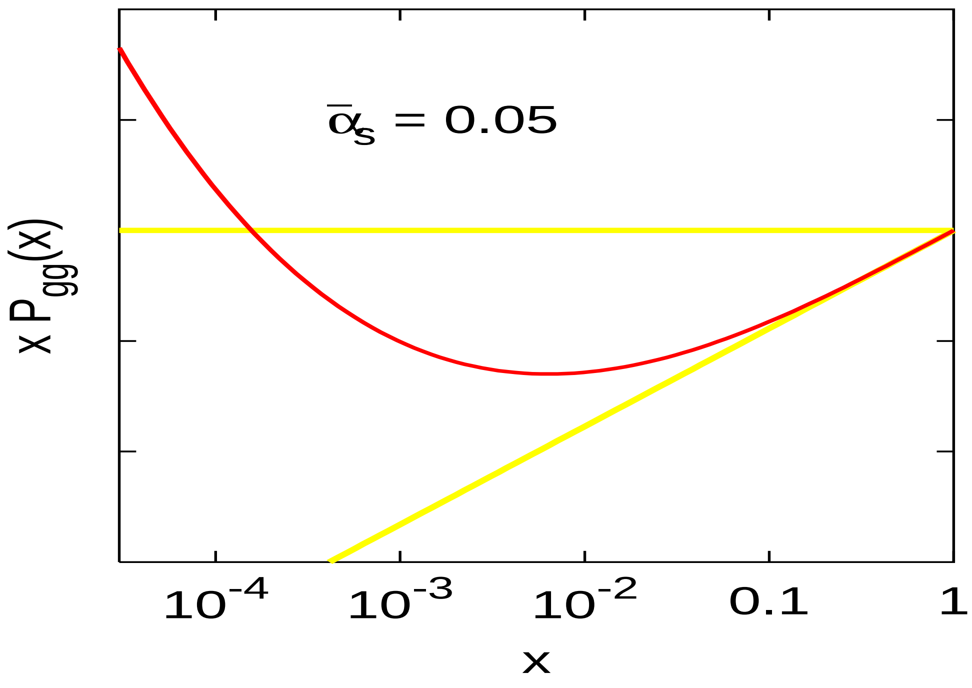


At moderately small x, first terms with x-dependence are

$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x} + \mathbf{0.401}\,\boldsymbol{\bar{\alpha}_s^4 \ln^3 \frac{1}{x}}$$



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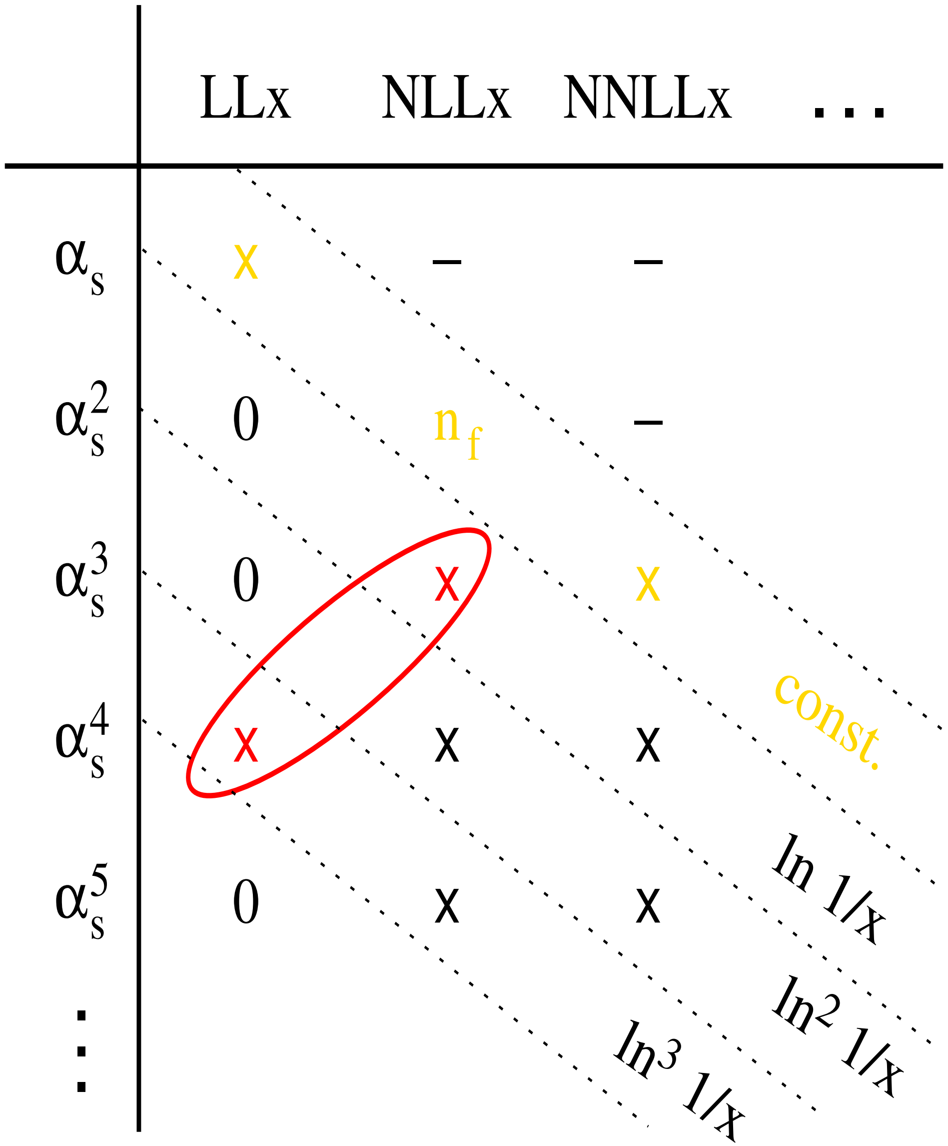

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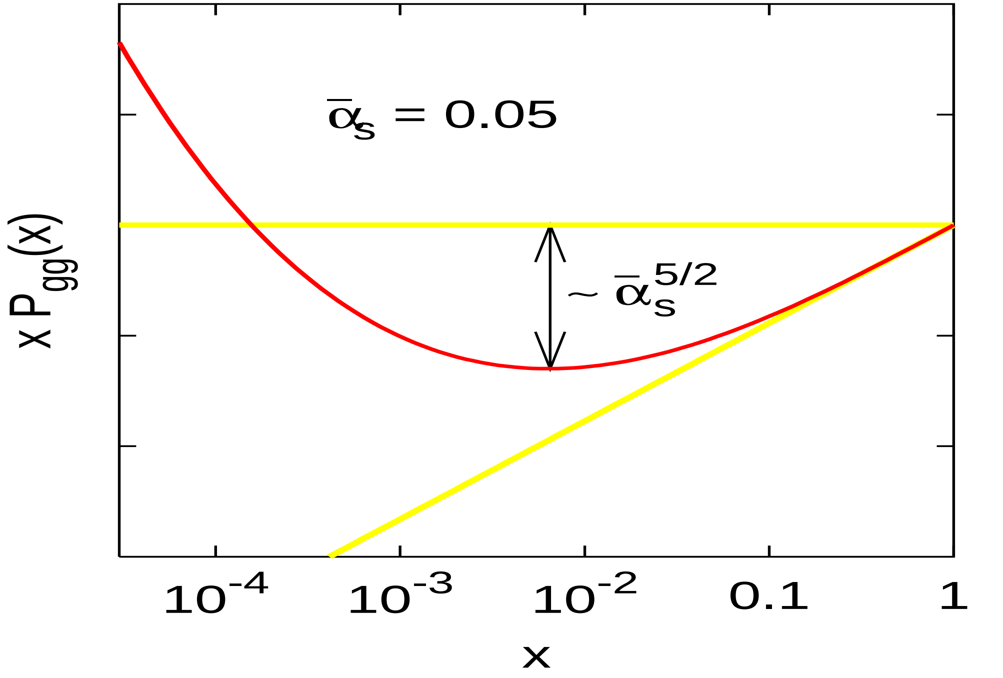
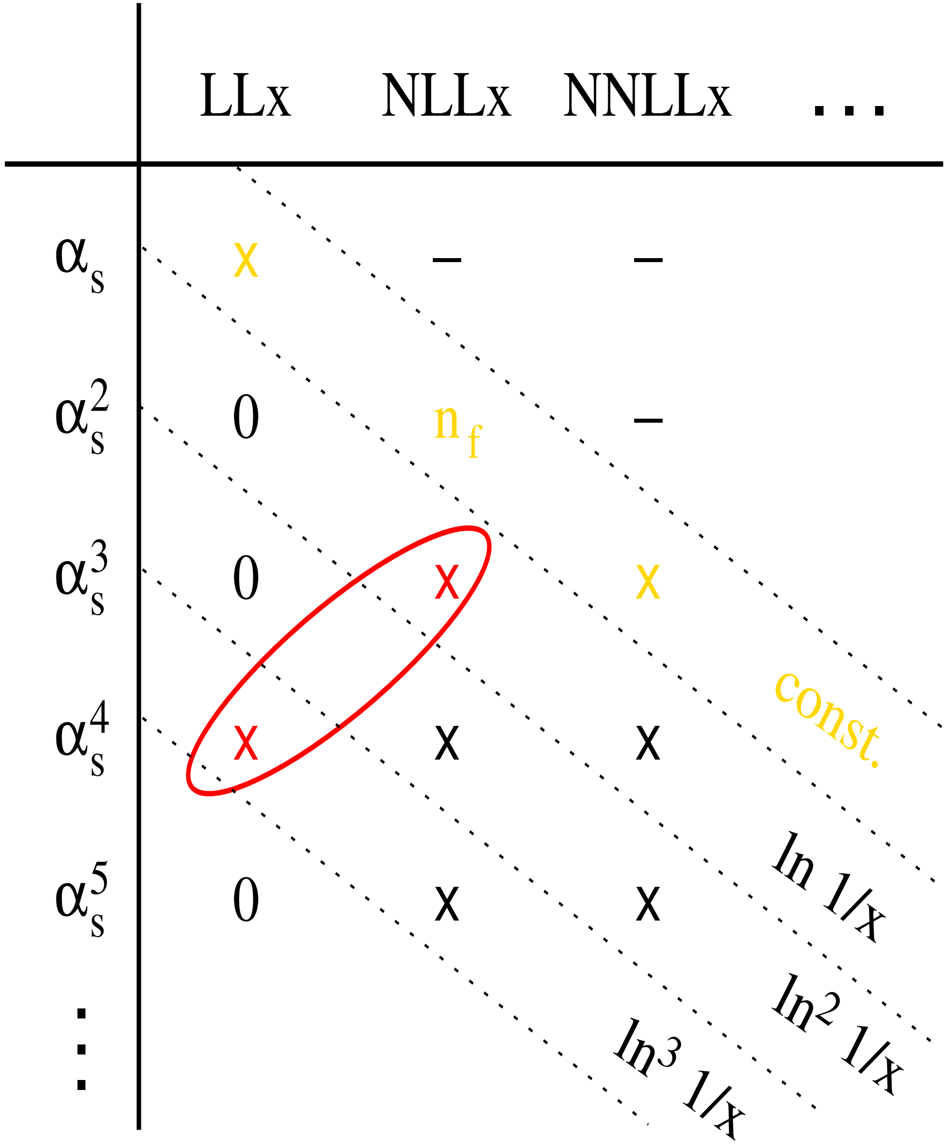
Minimum when

$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln\frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$





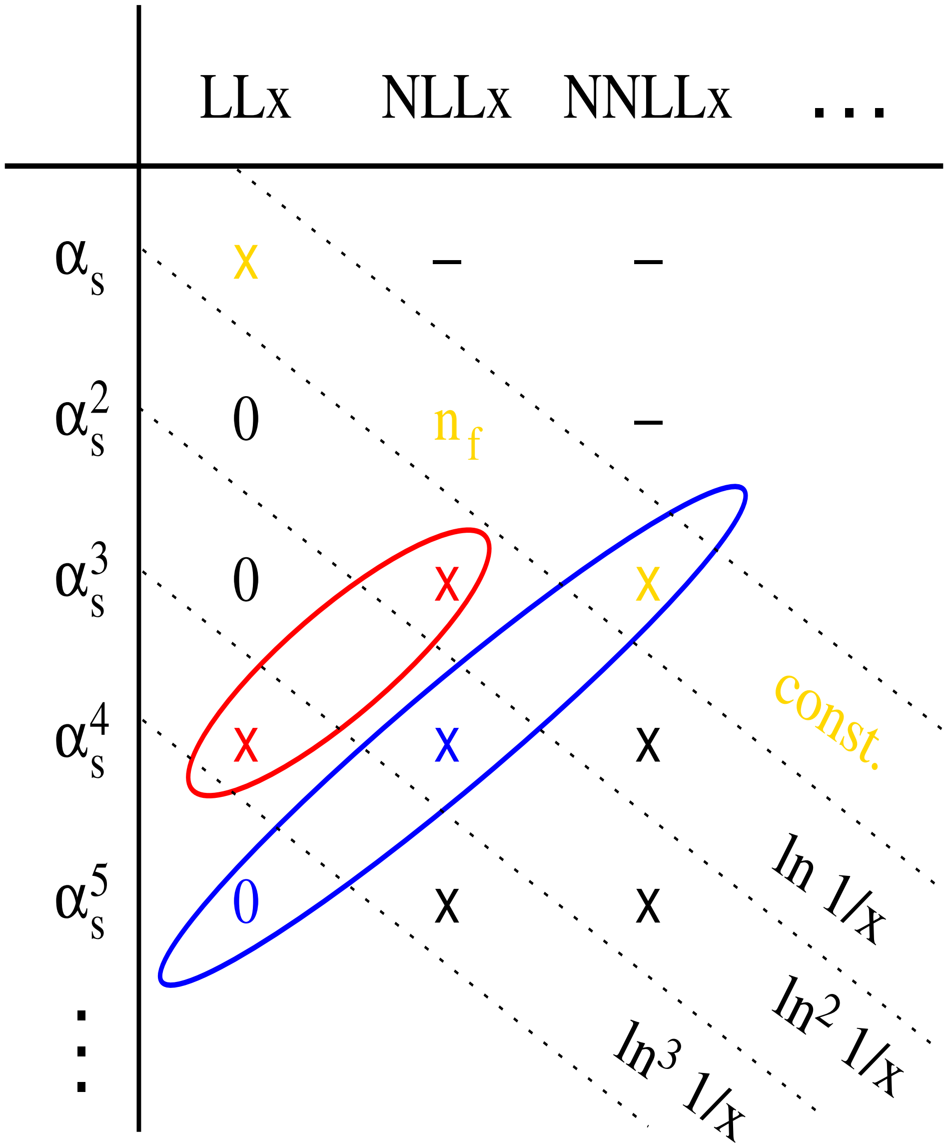
Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}}$$

Depth of dip

$$-d \simeq -1.237 \bar{\alpha}_s^{5/2}$$

Systematic expansion in $\sqrt{\alpha_s}$



Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}} + 6.947$$

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$$-d \simeq -1.237\bar{\alpha}_s^{5/2} - 11.15\bar{\alpha}_s^3$$

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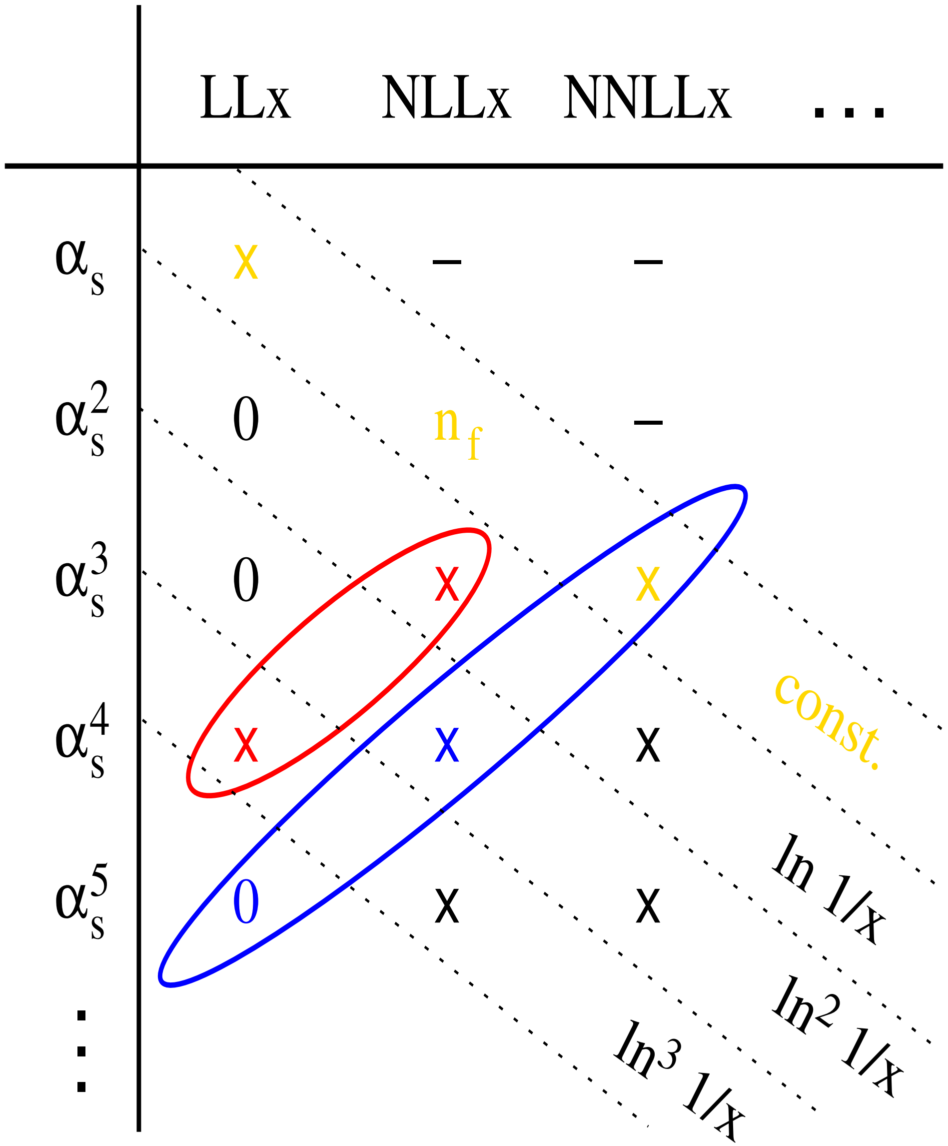

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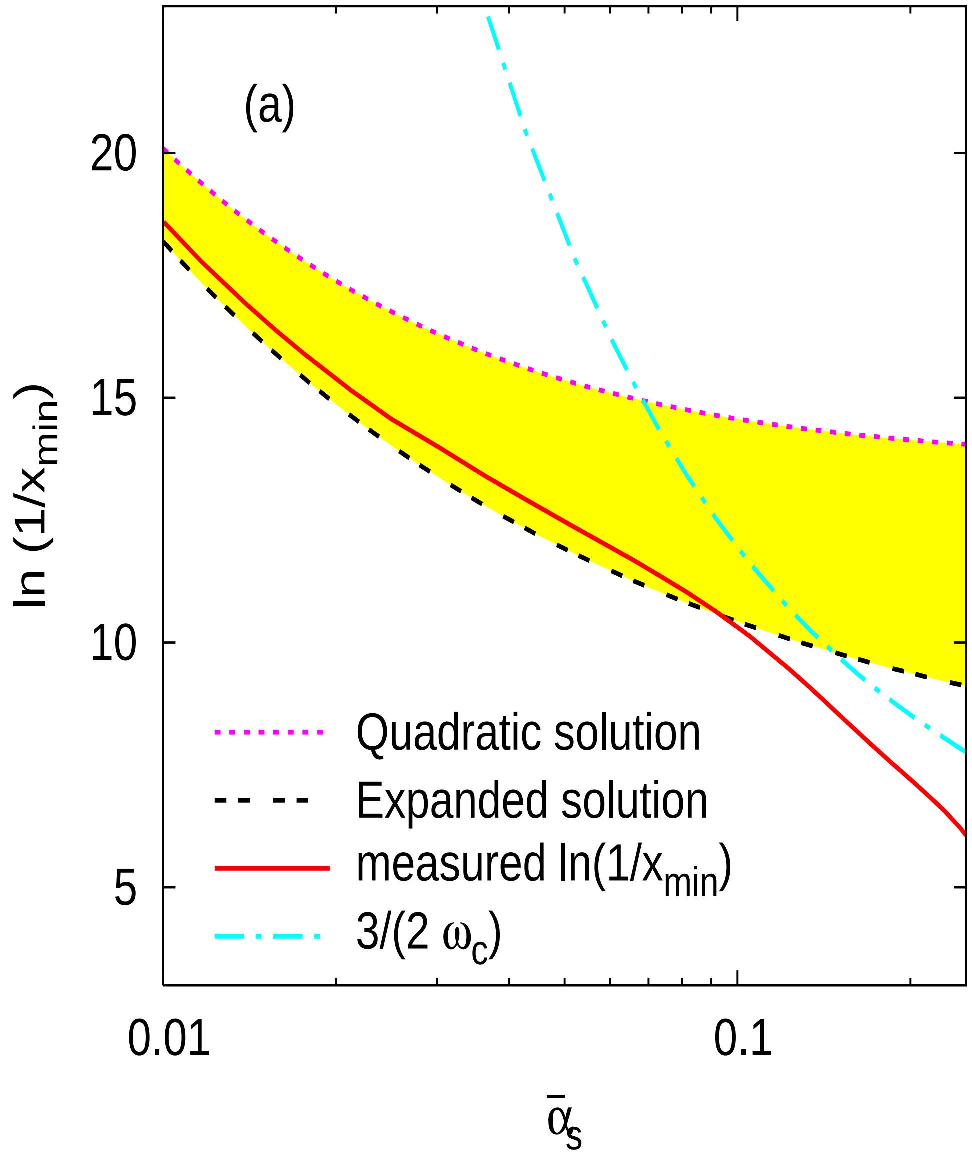
NB:

- convergence is very poor

  As ever at small x!

- higher-order terms in expansion need NNLLx info

Test dip properties v. BFKL+DGLAP resummation



Test position of dip v. α_s

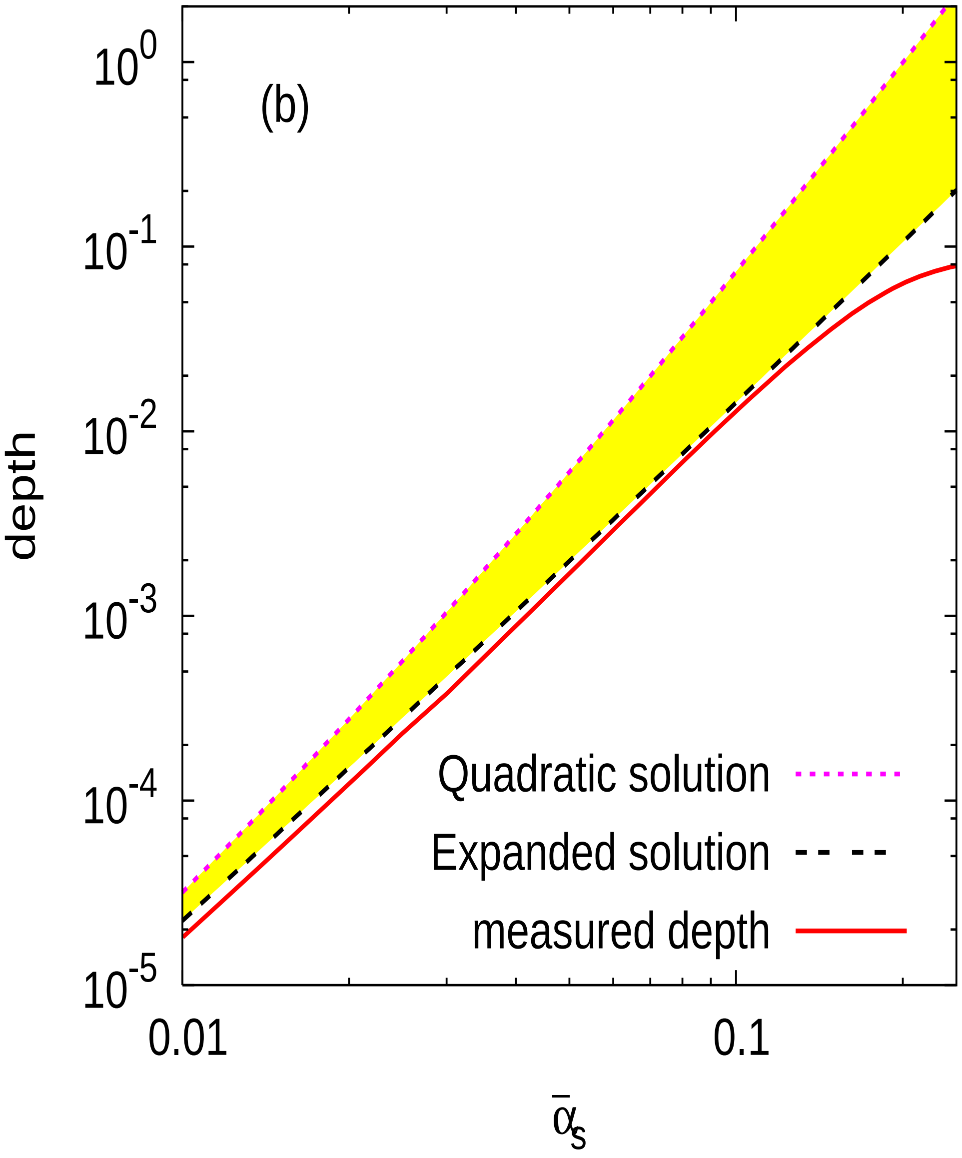
- Band is uncertainty due to higher orders in $\sqrt{\alpha_s}$
- At small α_s, good agreement $\rightarrow$ confirmation of 'dip mechanism'
- At moderate α_s, normal small-x resummation effects 'collide' with dip

$$\ln \frac{1}{x_{\min}} \lesssim \frac{3}{2\omega_c}$$

Dip then comes from interplay between $\alpha_s^3 \ln x$ (NNLO) term and full resummation.

[Actually, story more complex]

Test dip properties v. BFKL+DGLAP resummation



Test depth of dip v. α_s

- similar conclusions!

Phenomenological impact?

Phenomenological relevance comes through impact on growth of small-x gluon with Q^2.

$$\frac{\partial g(x, Q^2)}{d \ln Q^2} = P_{gg} \otimes g + P_{gq} \otimes q$$

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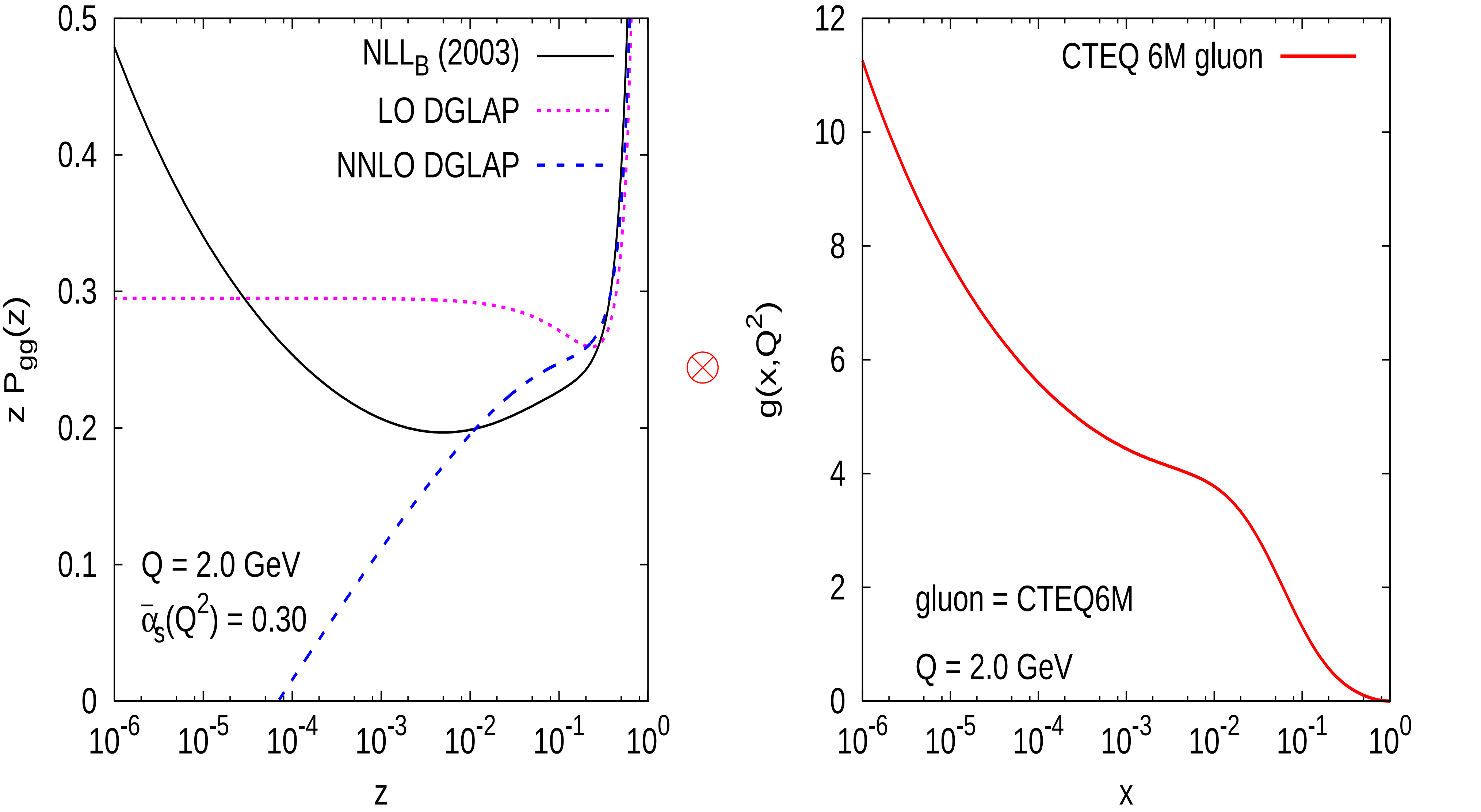
$$\frac{\partial g(x, Q^2)}{d \ln Q^2} = P_{gg} \otimes g + P_{gq} \otimes q$$

At small x, $P_{gg} \otimes g$ dominates.

Take CTEQ6M gluon as ‘test’ case for convolution.

Because it’s nicely behaved at small-x

Phenomenological impact? $P_{gg} \otimes g(x)$



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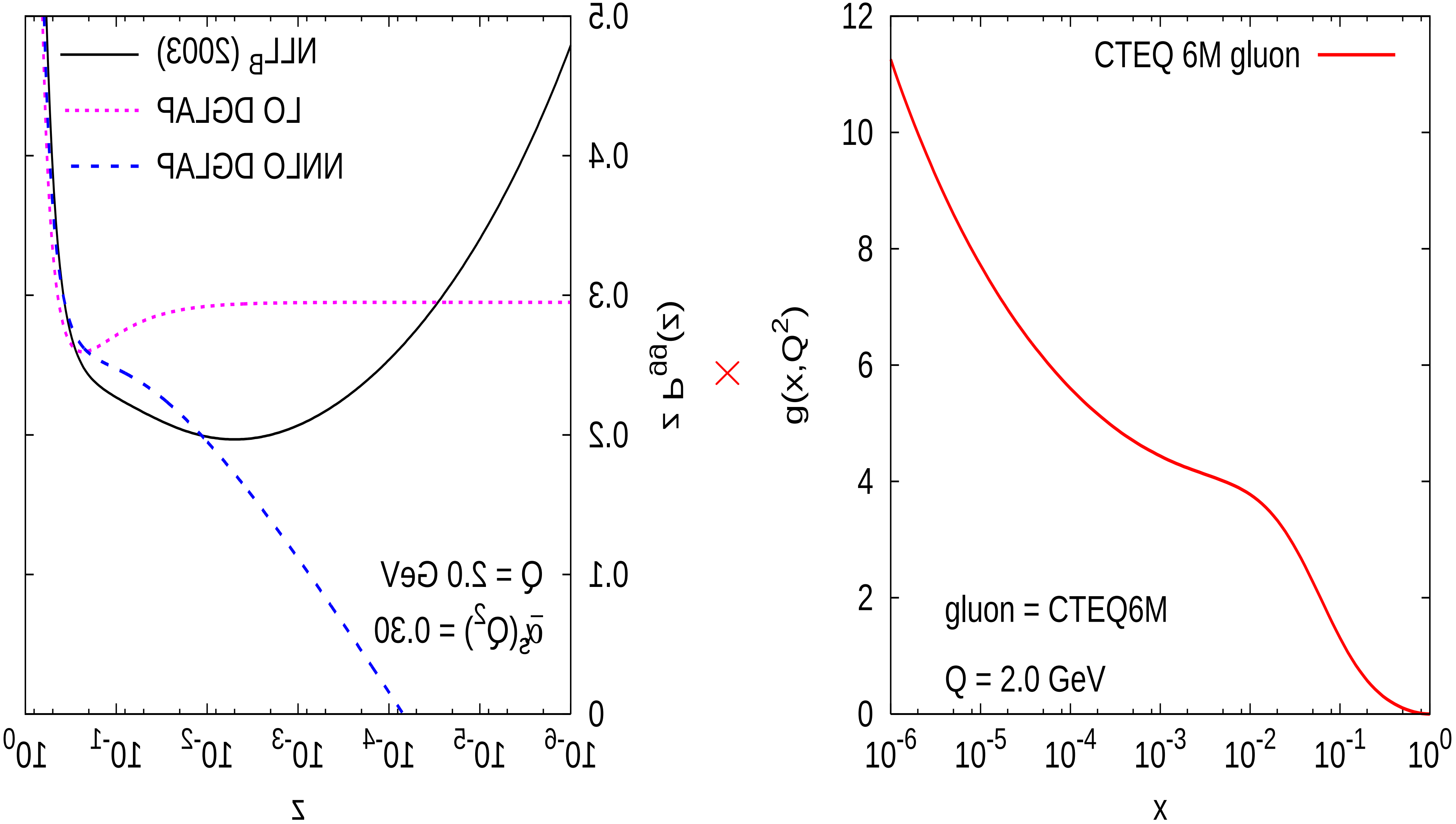

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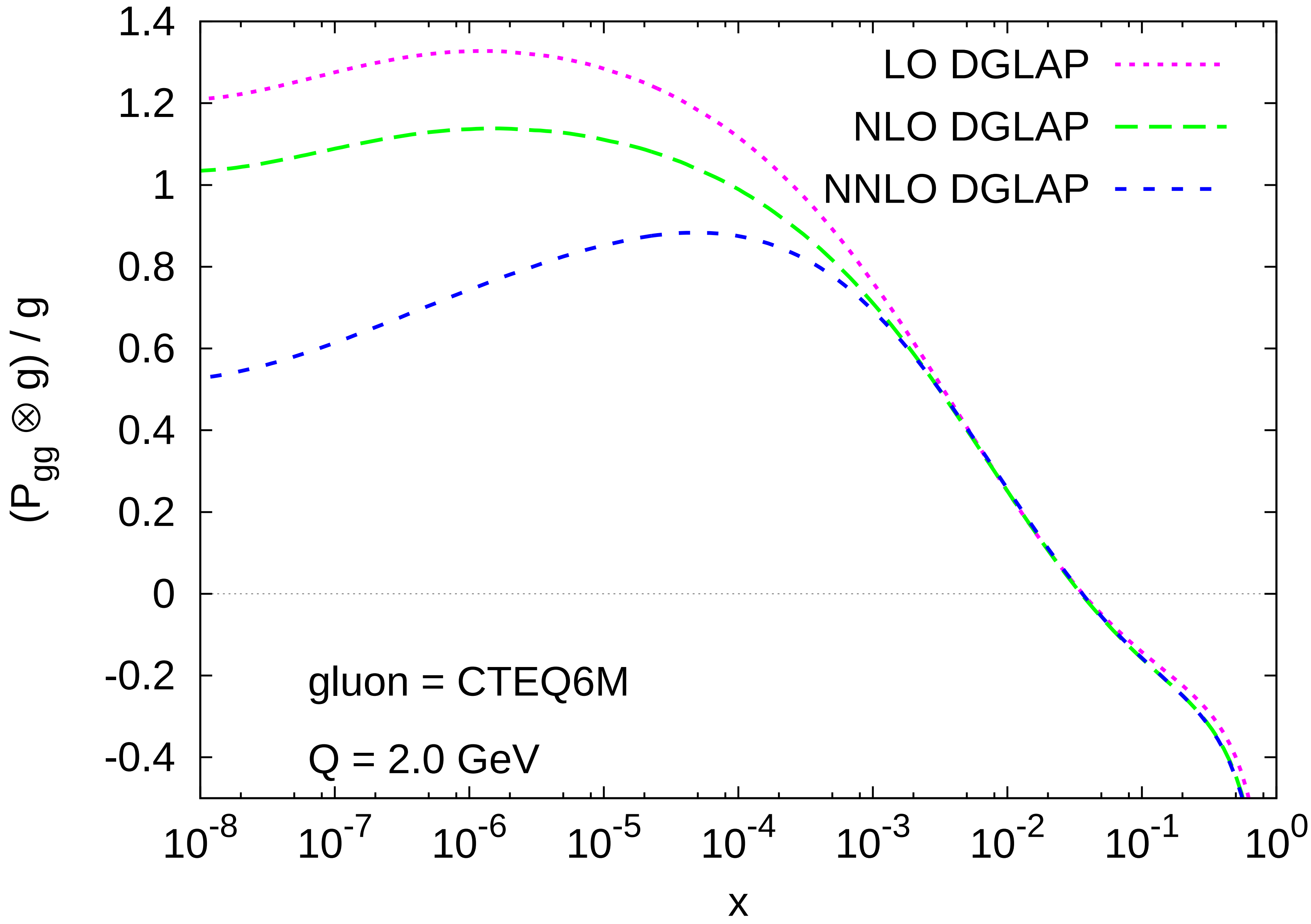

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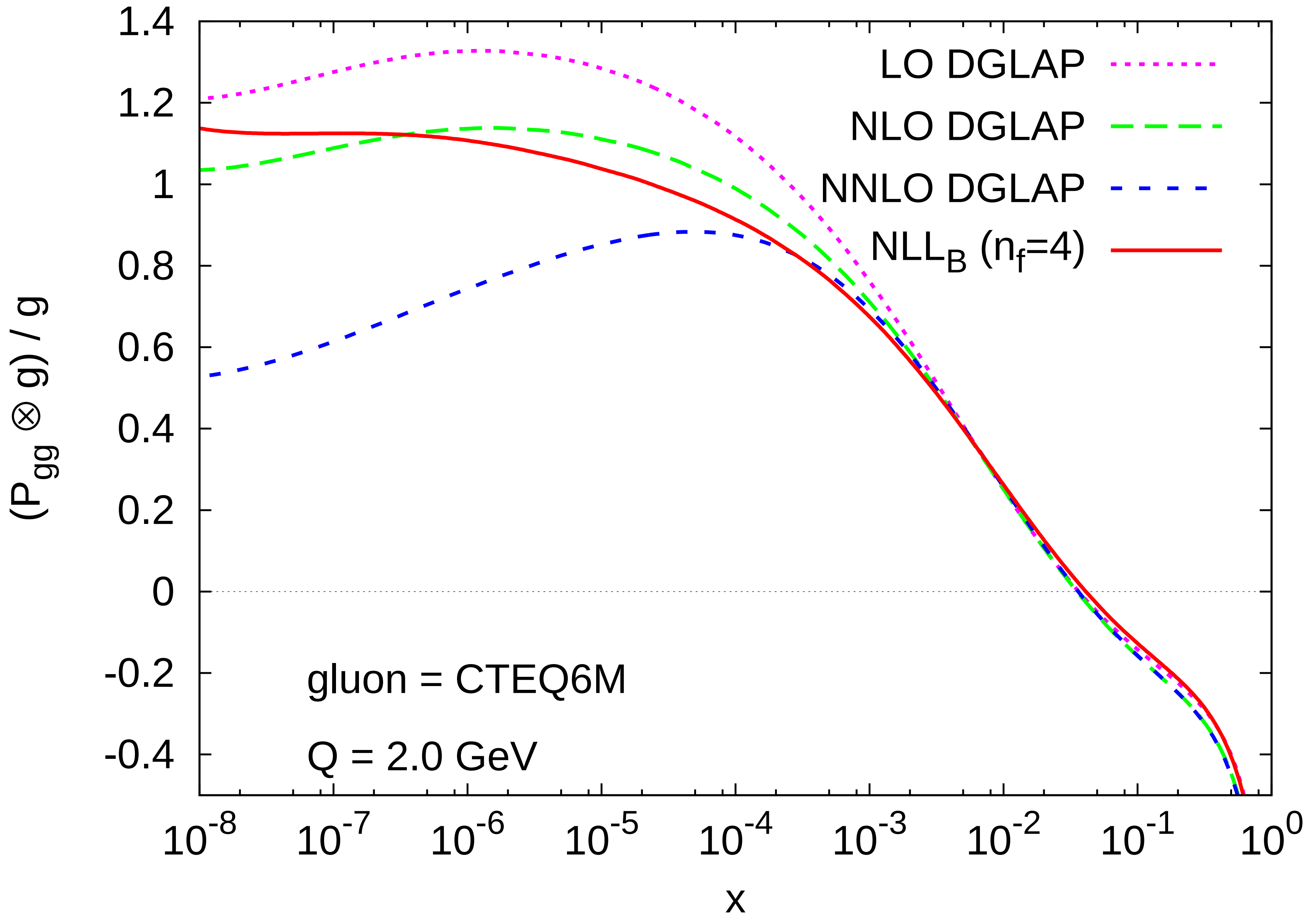

Towards phenomenology?

Steps missing for 'full' phenomenology:

- Resummation of all entries of singlet matrix & coefficient functions.
- Put results in $\overline{\text{MS}}$ factorisation scheme

↪illustrate nature of surprises that arise. . .

Factorisation scheme

Results shown so far in Q_0 scheme. [Catani, Ciafaloni & Hautmann '93]

$$xg(x, Q^2) \equiv \int d^2k\, G(\ln 1/x, k, k_0)\Theta(Q - k) \qquad G^{(0)} = f(x)\delta^2(k - k_0)$$

To translate to $\overline{\text{MS}}$ scheme

$$xg(x, Q^2) \equiv \int d^2k\, G(\ln 1/x, k, k_0) r\left(\frac{k^2}{Q^2}\right), \qquad r\left(\frac{k^2}{Q^2}\right) = \int \frac{d\gamma\, e^{\gamma \ln \frac{Q^2}{k^2}}}{2\pi i\, \gamma R(\gamma)}$$

Should be easy?!

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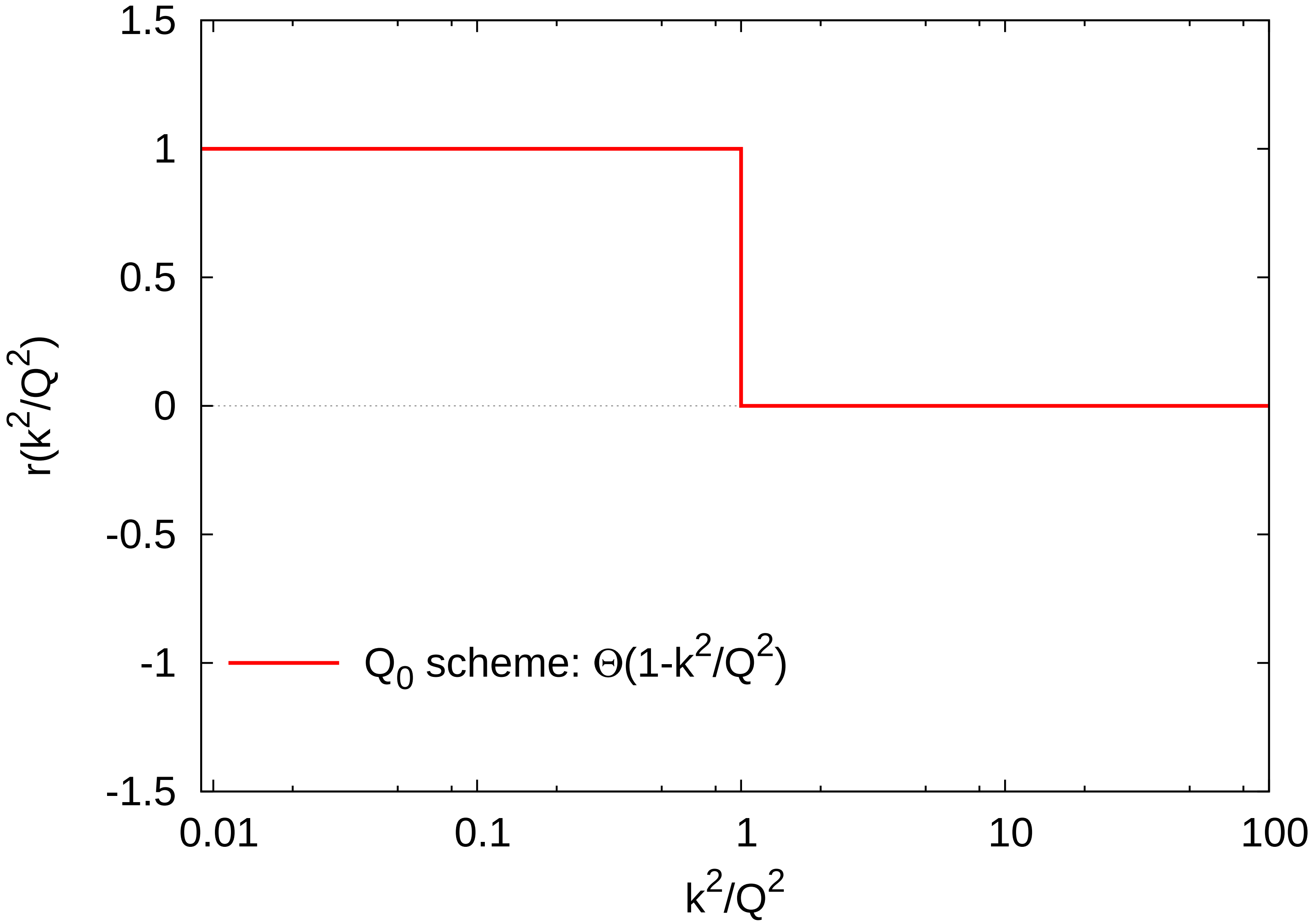
$$R(\gamma) = \left\{\frac{\Gamma(1-\gamma)\chi(\gamma)}{\Gamma(1+\gamma)[-\gamma\chi'(\gamma)]}\right\}^{\frac{1}{2}} \exp\left\{\int_0^\gamma d\gamma' \frac{\psi'(1)-\psi'(1-\gamma')}{\chi(\gamma')}\right\}$$

Catani & Hautmann '94

[NB: involves $\chi(\gamma)$ — does this need to be collinearly improved? Ignore problem for now. . .]

Q_0 v. $\overline{\text{MS}}$



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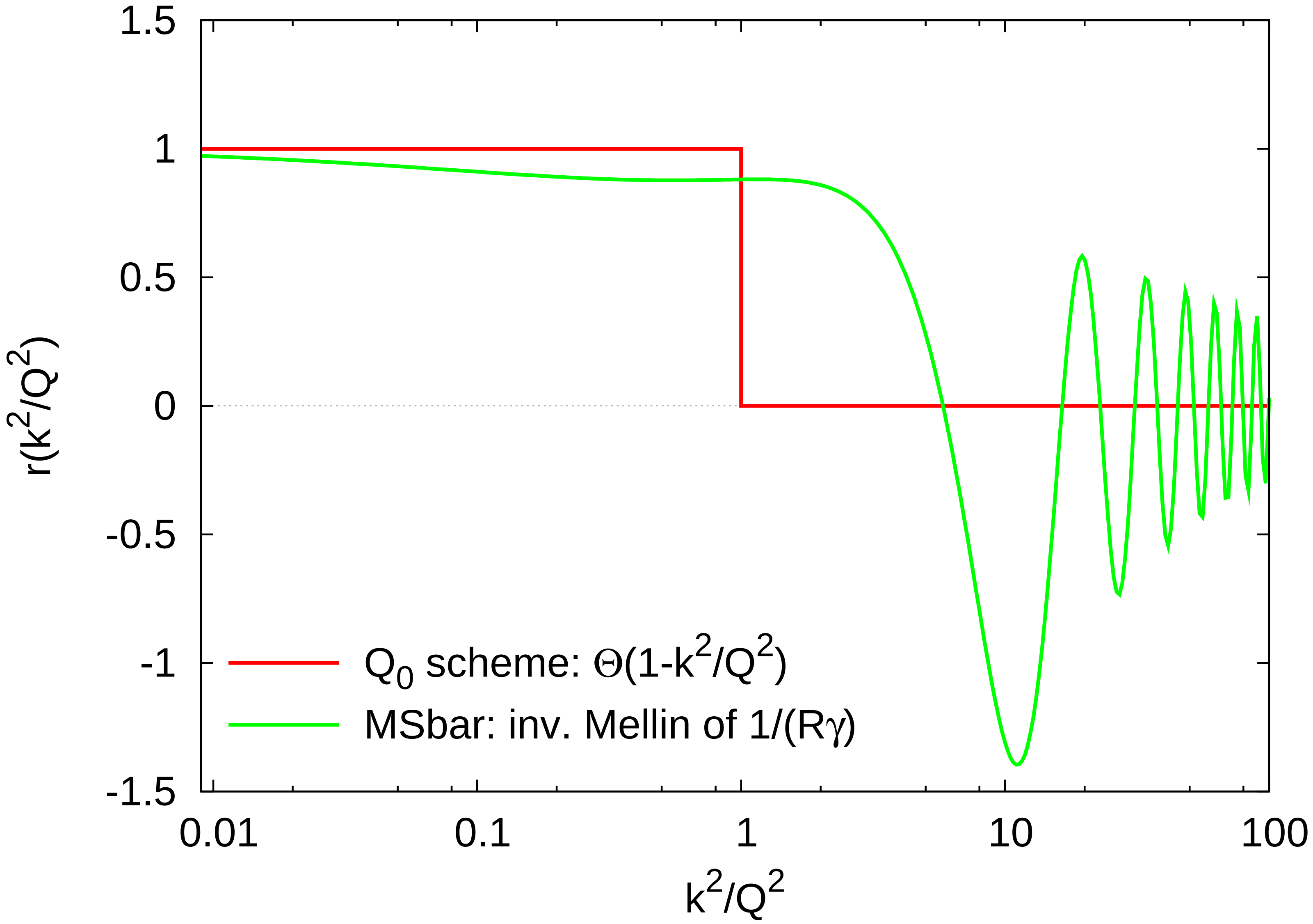

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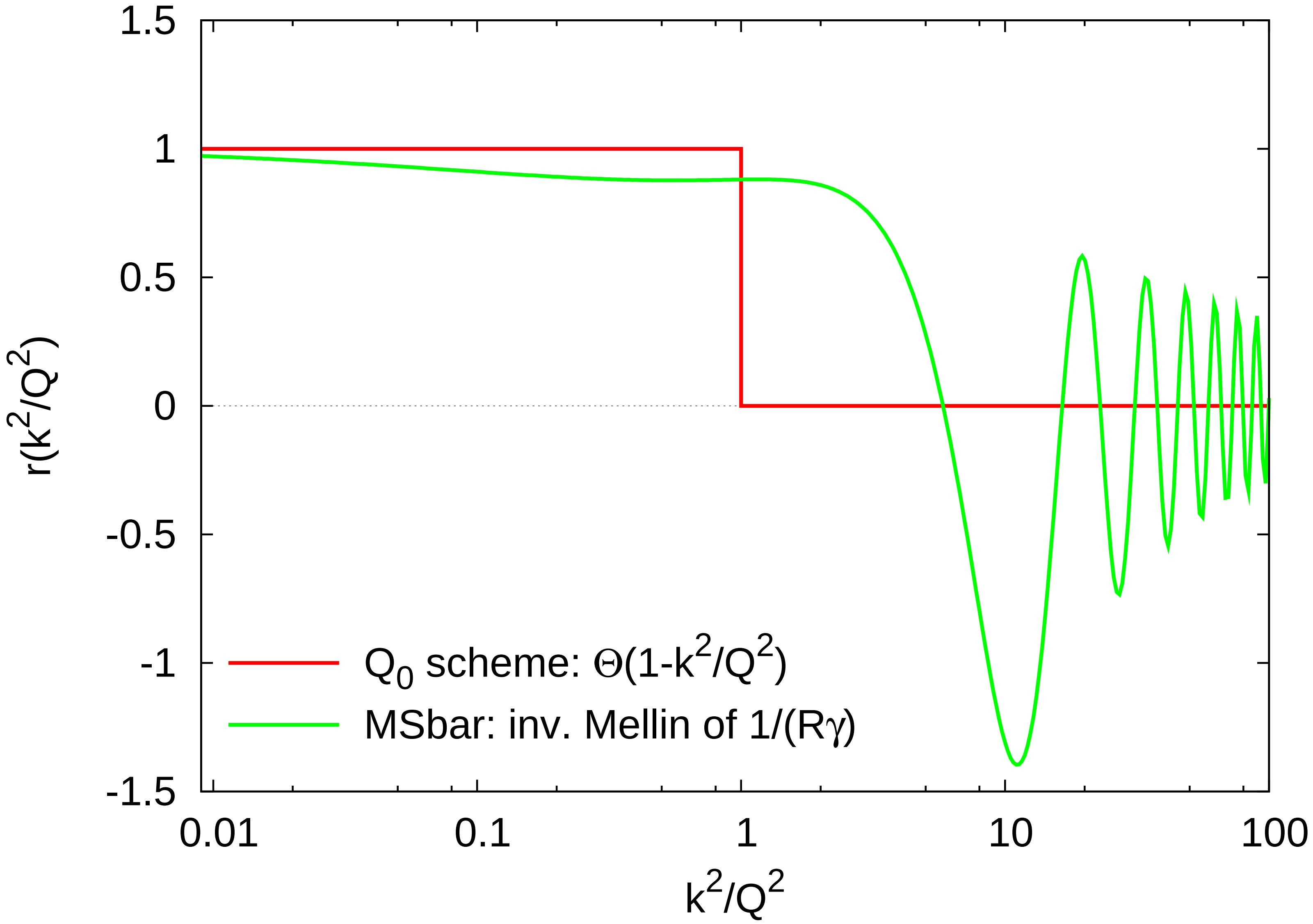

Numerically, $\overline{\text{MS}}$ is much more difficult.

Conceptually, the oscillations are disturbing.

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Detailed phenomenology still needs considerably more work