Jets, our window on partons at the LHC

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Work (in progress) with M. Cacciari,
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Partons — quarks and gluons — are key concepts of QCD.

- It's in terms of quark and gluon fields that we write the Lagrangian
- Perturbative QCD *only* deals with partons
- Concept of parton powerful even beyond perturbation theory
  - hadron classifications
  - exotic states, e.g. colour glass condensate (high gluon densities)

Yet it is surprisingly difficult to ascribe unambiguous meaning to partons.

- Not an asymptotic state of the theory
- Because of confinement
- But also even in perturbation theory

because of collinear divergences
(in massless approx.)

QCD coupling has related problems (probability of emitting a gluon...)

Despite this, there are two decent ways of “seeing” partons;

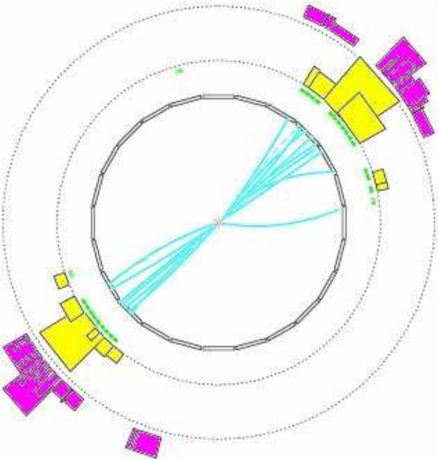
- Scatter some hard probe off them, e.g. a virtual photon
  Deep Inelastic Scattering (DIS)
- See traces of them in the final state
  jets

In each case ill-defined nature of a parton translates into ambiguity in the partonic interpretation of what you see.

In final state, trace of original partons is visible as collimated bunches of energetic hadrons

Picture illustrates $e^+e^- \to Z \to q\bar{q}$

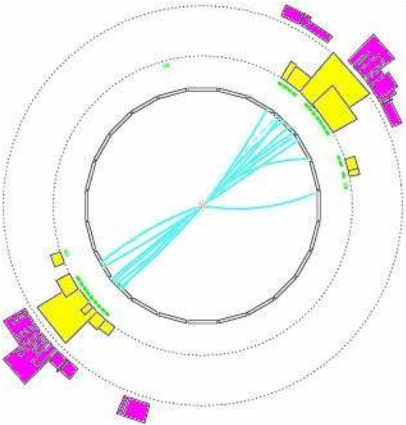
OPAL @ LEP



Information not just visual, but also quantitative

e.g. each bunch has $E \simeq m_Z/2$

Seeing v. defining jets



Jets are what we see.
Clearly(?) 2 jets here

How many jets do you see?
Do you really want to ask yourself this question for 10^8 events?

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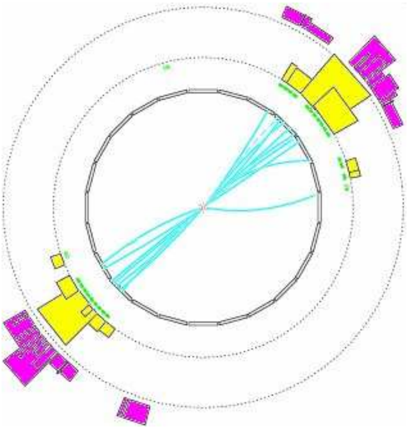

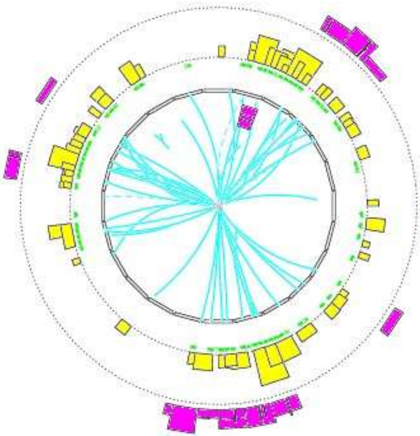

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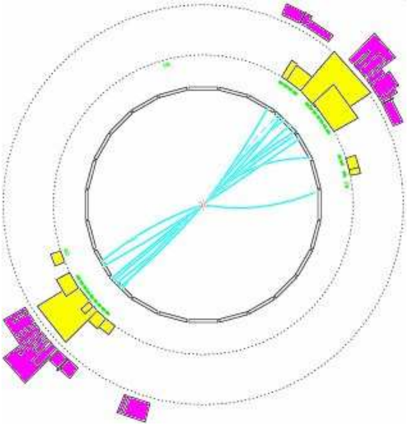

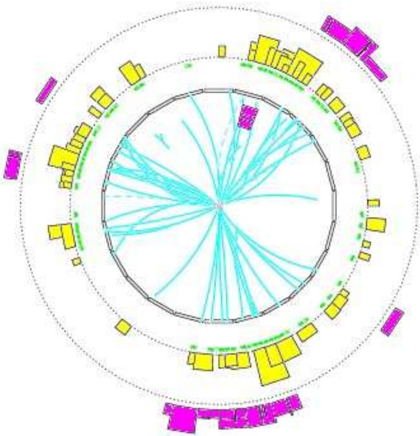

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Jet definition / algorithm

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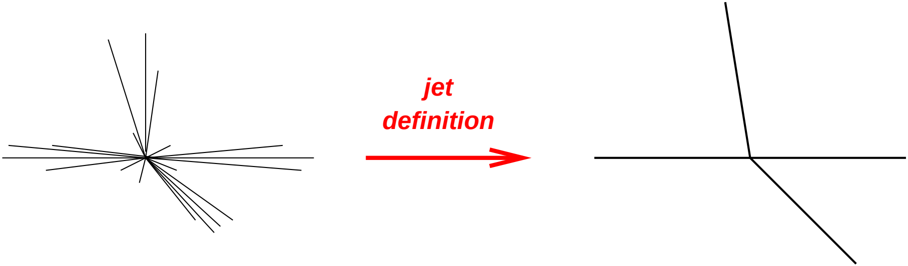

Jets are *as close as we can get to a physical single hard quark or gluon:* with good definitions their properties (multiplicity, energies, [flavour]) are

- finite at any order of perturbation theory
- insensitive to the parton $\rightarrow$ hadron transition

NB: finiteness $\longleftrightarrow$ set of jets depends on jet def.

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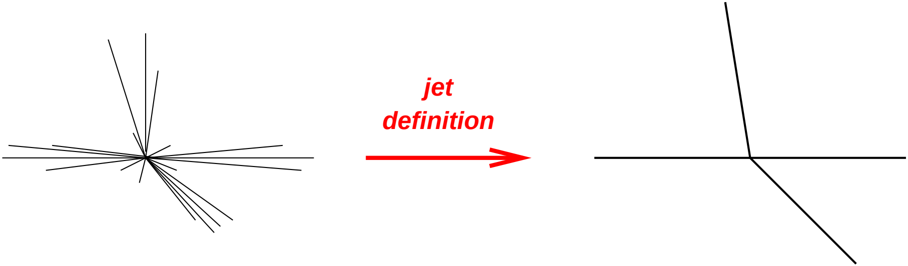

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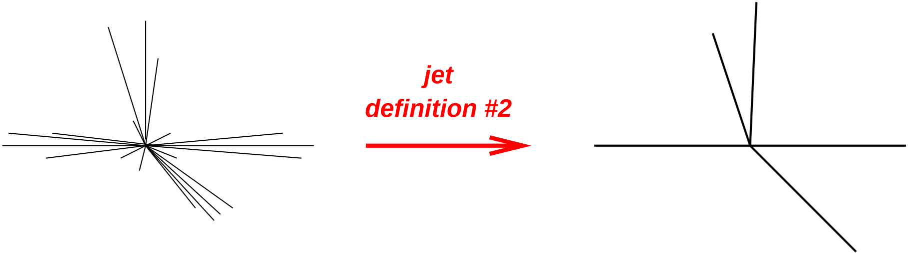

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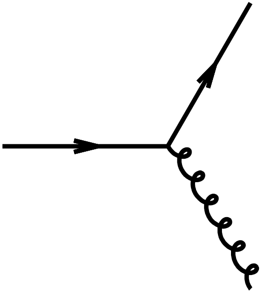

Proper jet definition gives results that are approximately invariant with respect to

- soft and collinear branching

  So divergent real and virtual contributions cancel

  IR & Collinear safety

- local reshuffling of momenta

  Hadronisation

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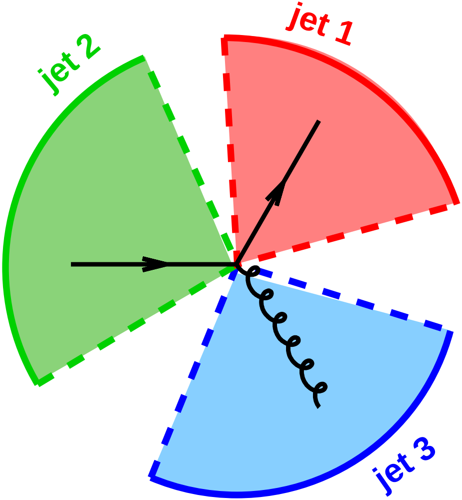

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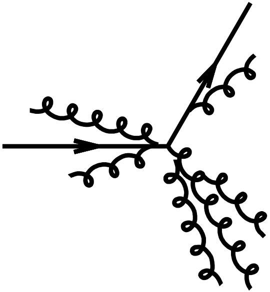

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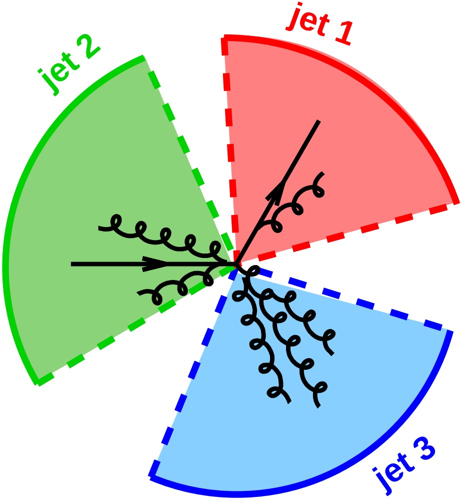

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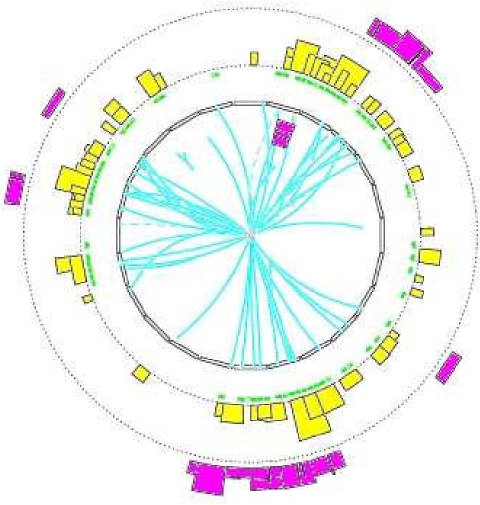

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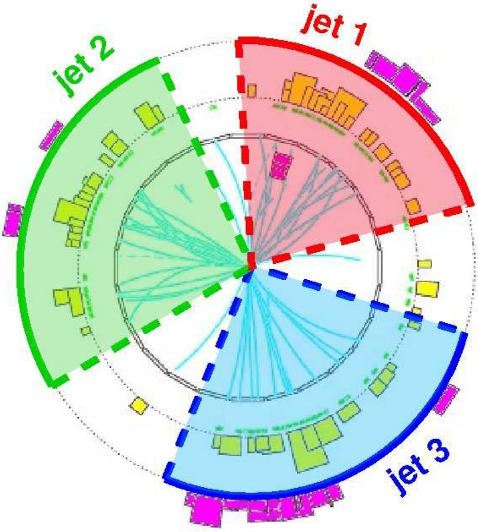

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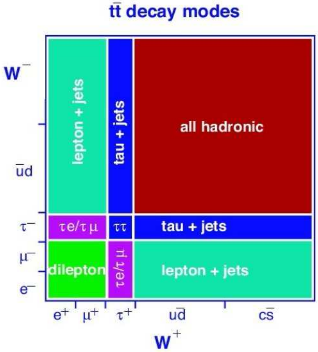
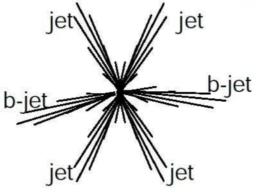
Jets ubiquitous at LHC





All-hadronic
(BR~46%, huge bckg)

picture: Juste LP '05

Heavy objects: multi-jet final-states

- 10^7 $t\bar{t}$ pairs for 10 fb^{-1} (1 year, low-lumi)
- Vast # of QCD multijet events

# jets	# events for 10 fb^{-1}
3	$9 \cdot 10^8$
4	$7 \cdot 10^7$
5	$6 \cdot 10^6$
6	$3 \cdot 10^5$
7	$2 \cdot 10^4$
8	$2 \cdot 10^3$

Tree level
$p_t(\text{jet}) > 60$ GeV, $\theta_{ij} > 30\,\text{deg}$, $|y_{ij}| < 3$
Draggiotis, Kleiss & Papadopoulos '02

Industrial scale calculational effort

Tree-level calculations with many partons / W / Z / H / etc.

- Alpgen
- Madgraph
- Sherpa
- Helas/Helac
- [Twistor-derived rules]

Monte Carlo event generators

- Pythia (f77), Pythia8 (C++)
- Herwig (f77), Herwig++ (C++)
- Ariadne
- Sherpa
- With NLO matching: MC@NLO, POWHEG, (Vincia, GeNeVA, . . .)

Each tool associated with 3–15 people: total of ~ 50

NLO wishlist (2005)

Experimenters' priorities

Les Houches

1. pp → W W + jet
2. pp → H + 2 jets
   - Background to VBF Higgs production
3. pp → $t\bar{t}b\bar{b}$
4. pp → $t\bar{t}$ + 2 jets
   - Background to $t\bar{t}$H
5. pp → W W $b\bar{b}$
6. pp → V V + 2 jets
   - Background to $W\,W \to H \to W\,W$
7. pp → V + 3 jets
   - General background to new physics
8. pp → V V V + jet
   - Background to SUSY trilepton

Currently available

NLOJET++, MCFM, PHOX, ...

http://www.cedar.ac.uk/hepcode/

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NLO wishlist (2007)

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   - Background to VBF Higgs production
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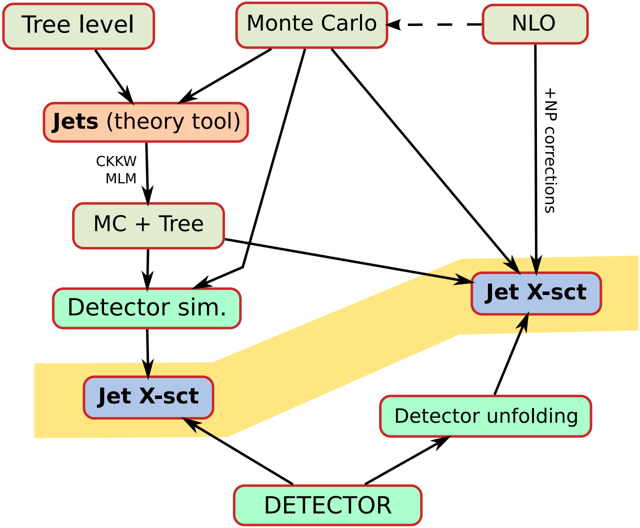
Another 30-50 people active

LHC QCD flowchart



Jet (definitions) provide central link between expt., "theory" and theory

What's new for jets @ LHC?

Number of particles:

Experiment	N
LEP, HERA	50
Tevatron	100–400
LHC low-lumi	800
LHC high-lumi	4000
LHC PbPb	30000

- Range & complexity of signatures (jets, $t\bar{t}$, tj, Wj, Hj, $t\bar{t}j$, WWj, Wjj, SUSY, etc.)
- Theoretical investment

  $\sim$ 100 people $\times$ 10 years

  60 – 100 million $

Physics scales:

Experiment	Physics	Scale
LEP, HERA	Electroweak	100 GeV
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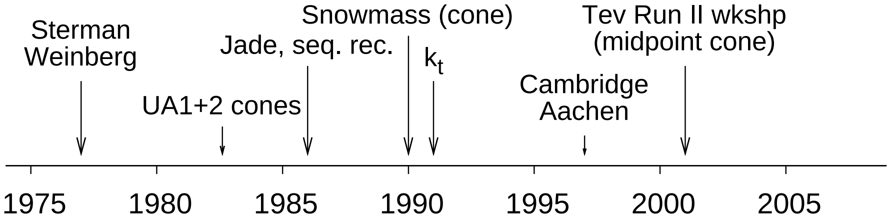
Jet Definition History

- Periodic key developments in jet definitions spurred by ever-increasing experimental/theoretical sophistication.
- Approach of LHC provides motivation for taking a new, fresh, systematic look at jets.
- This talk: **some of the discoveries along the way**



Definitions shown are those with widest exptl. impact

NB: also ARCLUS, OJF, ...

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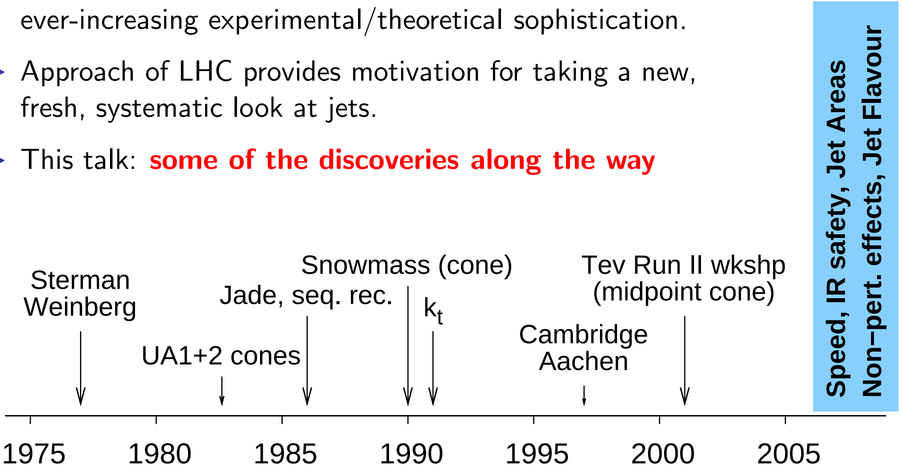

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Two classes of jet algorithm

Sequential recombination	**Cone**
k_t, Jade, Cam/Aachen, ...	UA1, JetClu, Midpoint, ...
Bottom-up: Cluster 'closest' particles repeatedly until few left $\longrightarrow$ jets.	Top-down: Find coarse regions of energy flow (cones), and call them jets.
Works because of mapping: *closeness* $\Leftrightarrow$ *QCD divergence*	Works because *QCD only modifies energy flow on small scales*
Loved by e^+e^-, ep and theorists	Loved by pp and few(er) theorists

Both had serious issues that got in way of practical use and/or physical validity

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Sequential recombination algorithms

k_t algorithm   Catani, Dokshizter, Olsson, Seymour, Turnock, Webber '91–'93
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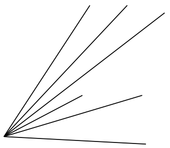
- Find smallest of all $d_{ij} = \min(k_{ti}^2, k_{tj}^2)\Delta R_{ij}^2/R^2$ and $d_{iB} = k_i^2$
- Recombine i, j (if iB: $i \to$ jet)
- Repeat



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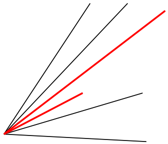

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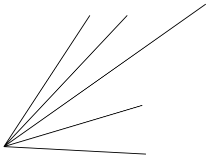

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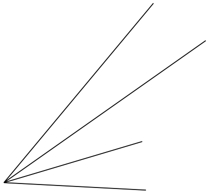

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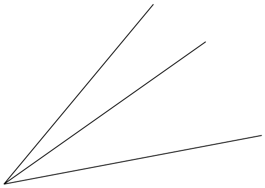

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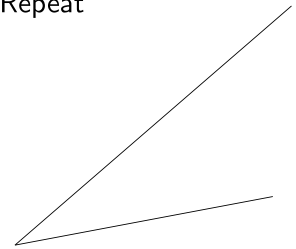
- Find smallest of all $d_{ij} = \min(k_{ti}^2, k_{tj}^2)\Delta R_{ij}^2/R^2$ and $d_{iB} = k_i^2$
- Recombine i, j (if iB: $i \to$ jet)
- Repeat



NB: hadron collider variables

- $\Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (y_i - y_j)^2$
- rapidity $y_i = \frac{1}{2} \ln \frac{E_i + p_{zi}}{E_i - p_{zi}}$
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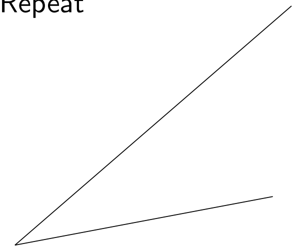

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Why k_t?

k_t distance measures

$$d_{ij} = \min(k_{ti}^2, k_{tj}^2)\Delta R_{ij}^2, \qquad d_{iB} = k_{ti}^2$$

are closely related to structure of divergences for QCD emissions

$$[dk_j]|M^2_{g\to g_i g_j}(k_j)| \sim \frac{\alpha_s C_A}{2\pi}\frac{dk_{tj}}{\min(k_{ti}, k_{tj})}\frac{d\Delta R_{ij}}{\Delta R_{ij}}\,, \qquad (k_{tj} \ll k_{ti}\,,\ \ \Delta R_{ij} \ll 1)$$

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$$[dk_i]|M^2_{Beam\to Beam+g_i}(k_i)| \sim \frac{\alpha_s C_A}{\pi}\frac{dk_{ti}}{k_{ti}}\,d\eta_i\,, \qquad (k_{ti}^2 \ll \{\hat{s}, \hat{t}, \hat{u}\})$$

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Computing...

'Trivial' computational issue:

- for N particles: N^2 d_{ij} searched through N times $= N^3$
- 4000 particles (or calo cells): 1 minute

  NB: often study $10^7 - 10^8$ events (20-200 CPU years)
- Heavy Ions: 30000 particles: 10 hours/event

As far as possible physics choices should not be limited by computing.

Even if we're clever about repeating the full search each time, we still have $\mathcal{O}\left(N^2\right)$ d_{ij}'s to establish

Can we do better than N^2?

There are $N(N-1)/2$ distances d_{ij} — surely we have to calculate them all in order to find smallest?

k_t distance measure is partly *geometrical:*

- Consider smallest $d_{ij} = \min(k_{ti}^2, k_{tj}^2) R_{ij}^2$
- Suppose $k_{ti} < k_{tj}$
- Then: $R_{ij} \le R_{i\ell}$ for any $\ell \neq j$. [If $\exists\ \ell$ s.t. $R_{i\ell} < R_{ij}$ then $d_{i\ell} < d_{ij}$]

In words: if i, j form smallest d_{ij} then j is geometrical nearest neighbour (GNN) of i.

k_t distance need only be calculated between GNNs

Each point has 1 GNN → need only calculate N d_{ij}'s

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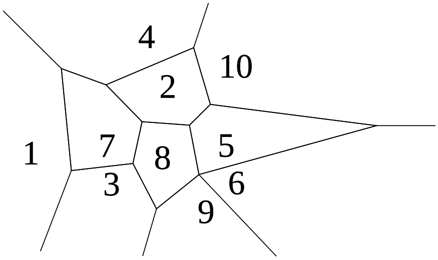
Finding Geom Nearest Neighbours



Given a set of vertices on plane (1...10) a *Voronoi diagram* partitions plane into cells containing all points closest to each vertex

Dirichlet '1850, Voronoi '1908

A vertex's nearest other vertex is always in an adjacent cell.

E.g. GNN of point 7 will be found among 1,4,2,8,3 (it turns out to be 3)

Construction of Voronoi diagram for N points: $N \ln N$ time Fortune '88

Update of 1 point in Voronoi diagram: $\ln N$ time

Devillers '99 [+ related work by other authors]

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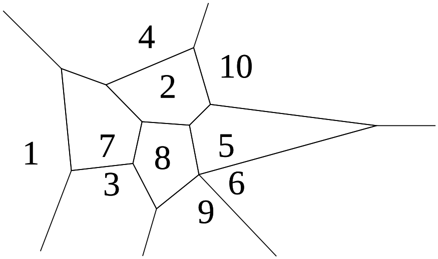

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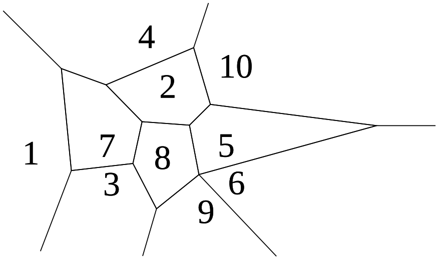

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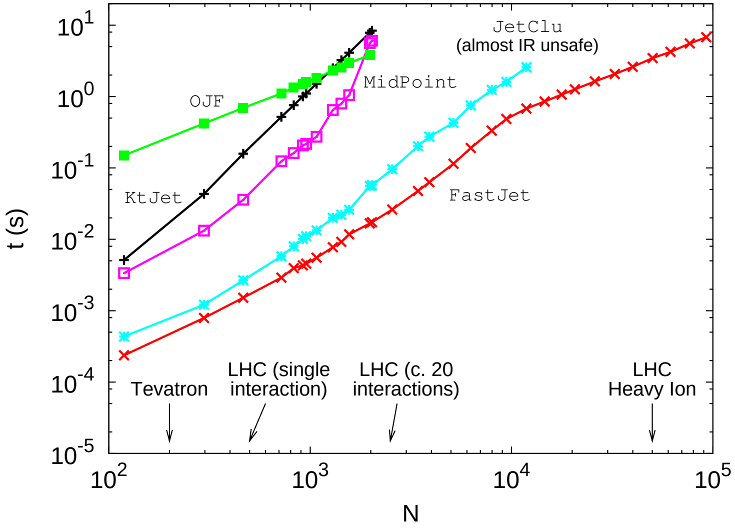
FastJet performance



NB: for $N < 10^4$, FastJet switches to a related geometrical N^2 alg.

Conclusion: speed issues for k_t resolved

Cone basics

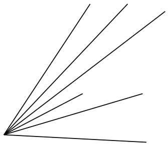
Modern cone algs have two main steps:

- Find some/all stable cones

  ≡ cone pointing in same direction as the momentum of its contents

- Resolve cases of overlapping stable cones

  By running a 'split–merge' procedure



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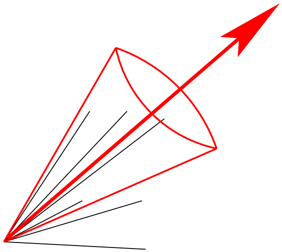

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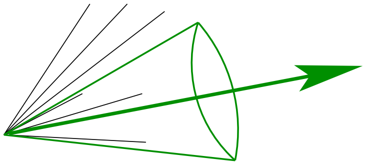

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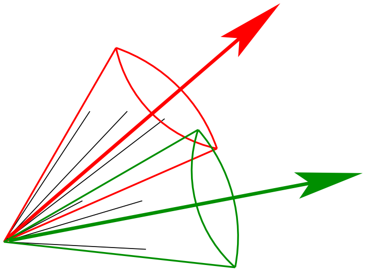

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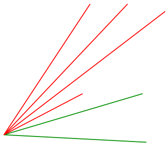

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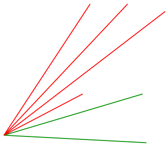


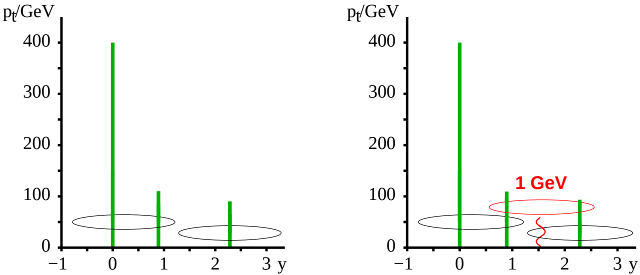
Qu: How do you find the stable cones?

All experiments use iterative methods:

- use each particle as a starting direction for cone; use sum of contents as new starting direction; repeat.
- use additional 'midpoint' starting points between pairs of initial stable cones.

  'Midpoint' algorithm

Midpoint IR problem



Stable cones with midpoint:	{1,2} & {3}	{1,2} & {2,3} & {3}
Jets with midpoint ($f = 0.5$)	{1,2} & {3}	{1,2,3}

Midpoint cone alg. misses some stable cones; extra soft particle → extra starting point → extra stable cone found

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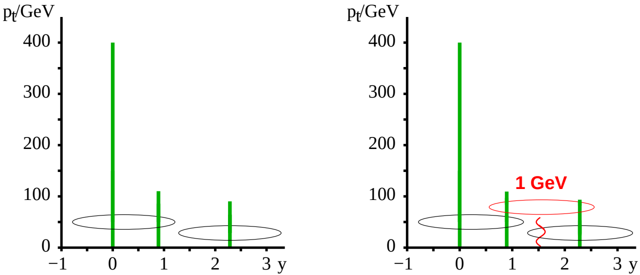

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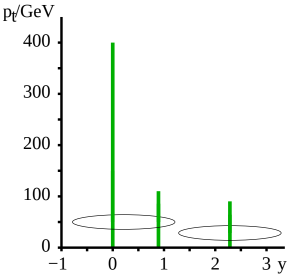

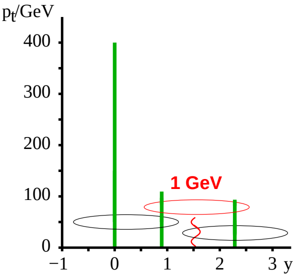

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Midpoint was supposed to solve *just this type of problem.* But worked only at lowest order.

IR/Collinear unsafety is a serious problem!

- Invalidates theorems that ensure finiteness of perturbative QCD
  Cancellation of real & virtual divergences
- Destroys usefulness of (intuitive) partonic picture
  you cannot think in terms of hard partons if adding a 1 GeV gluon changes 100 GeV jets
- 'Pragmatically:' limits accuracy to which it makes sense to calculate

Process	1st miss cones @	Last meaningful order
Inclusive jets	NNLO	NLO [NNLO being worked on]
$W/Z + 1$ jet	NNLO	NLO
3 jets	NLO	LO [NLO in `nlojet++`]
$W/Z + 2$ jets	NLO	LO [NLO in `MCFM`]
jet masses in $2j + X$	LO	**none**

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Rather than define the cone alg. through the *procedure* you use to find cones, define it by the *result you want:*

A cone algorithm should find all stable cones

First advocated: Kidonakis, Oderda & Sterman '97

Guarantees IR safety of the set of stable cones

Only issue: you still need to find the stable cones in practice.

One known exact approach:

- Take each possible subset of particles and see if it forms a stable cone.

  Tevatron Run II workshop, '00 (for fixed-order calcs.)
- There are 2^N subsets for N particles. Computing time $\sim \mathbf{N2^N}$.

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Transform into a geometrical problem

Cones are just *circles* in the $y - \phi$ plane. To find all stable cones:

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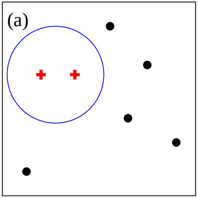
Finding all distinct circular enclosures of a set of points is *geometry:*



Any enclosure can be moved until a pair of points lies on its edge.

Polynomial time recipe for finding all distinct enclosures:

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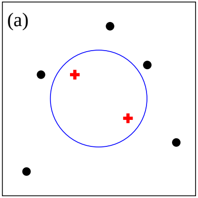

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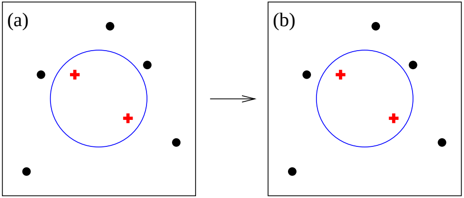

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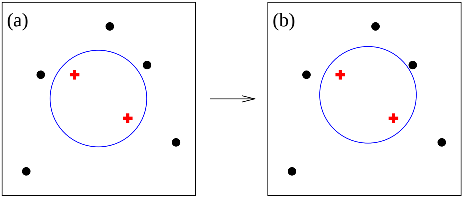

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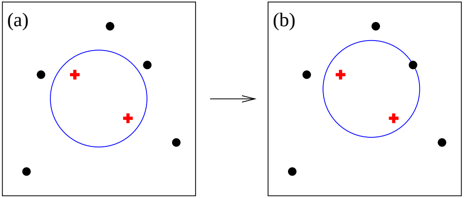

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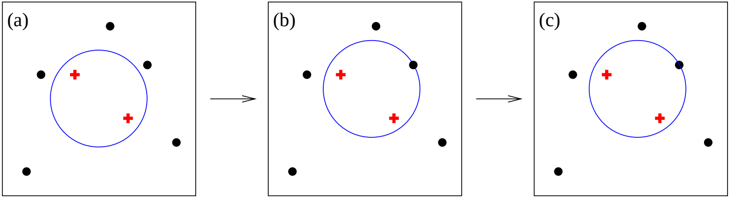

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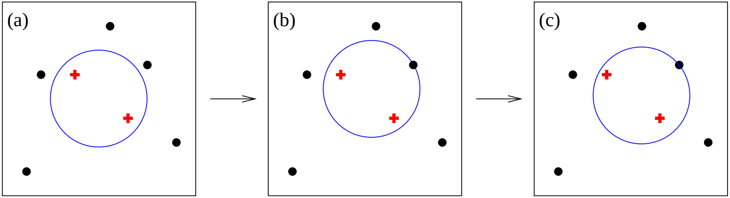

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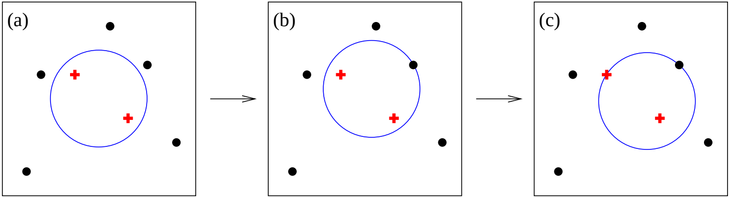

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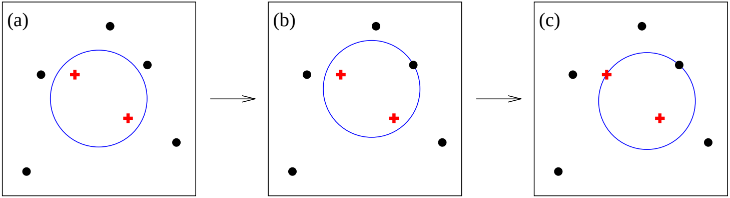

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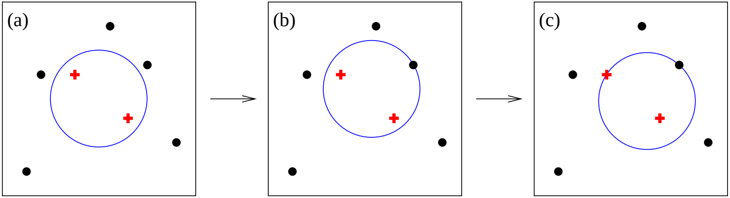
Transform into a geometrical problem

Cones are just *circles* in the $y - \phi$ plane. To find all stable cones:

1. Find all distinct ways of enclosing a subset of particles in a $y - \phi$ circle
2. Check, for each enclosure, if it corresponds to a stable cone

Finding all distinct circular enclosures of a set of points is *geometry:*



Any enclosure can be moved until a pair of points lies on its edge.

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- For each *pair* of points in the plane, draw the two circles that have those two points on their edge.

A Seedless Infrared Safe Cone: SISCone

Naive implementation of this idea would run in N^3 time.

N^2 pairs of points, pay N for each pair to check stability

N^3 is also time taken by midpoint codes (smaller coeff.)

With some thought, this reduces to **$N^2 \ln N$** time.

Traversal order, stability check

checkxor

GPS & Soyez '07

- Much faster than midpoint with no seed threshold

  IR unsafe

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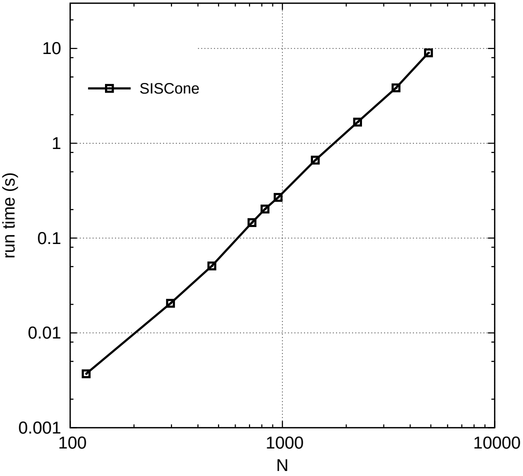

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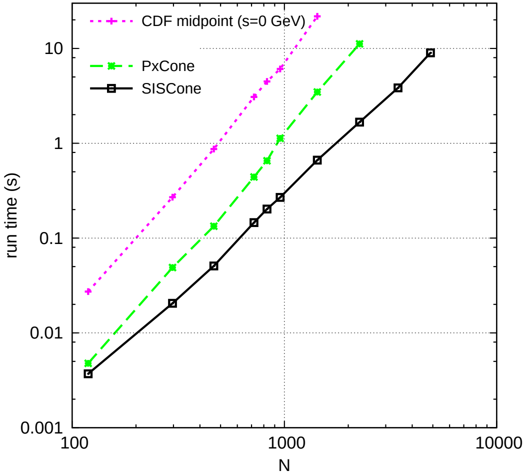

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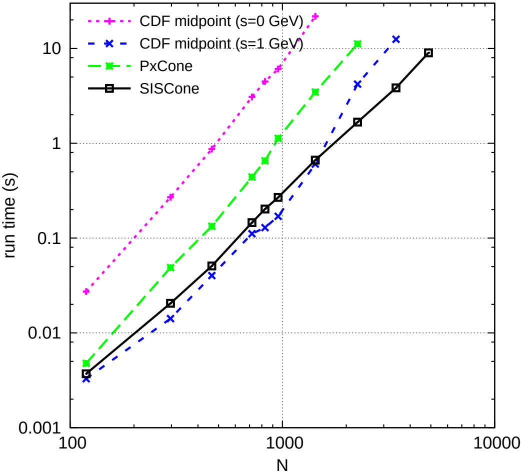

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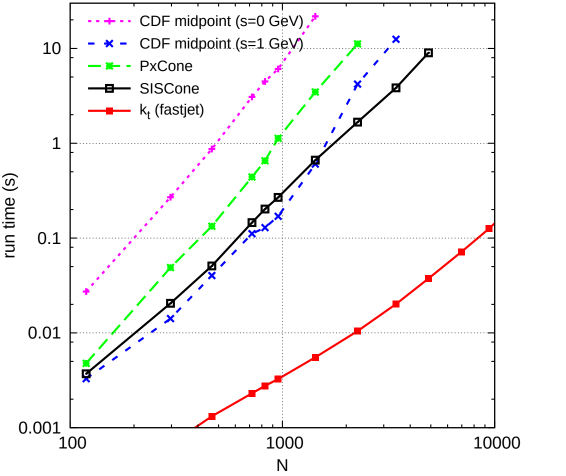

MC cross check of IR safety

- Generate event with $2 < N < 10$ hard particles, find jets
- Add $1 < N_{soft} < 5$ soft particles, find jets again [repeatedly]
- If the jets are different, algorithm is IR unsafe.

Unsafety level	failure rate
2 hard + 1 soft	$\sim 50\%$
3 hard + 1 soft	$\sim 15\%$
SISCone	IR safe !

Be careful with split–merge too

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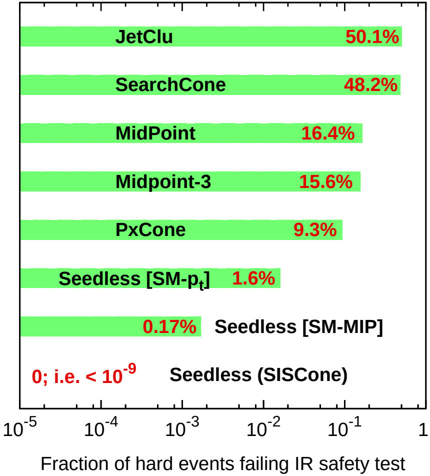


A full set of algs

Complementary set of IR/Collinear safe jet algs $\longrightarrow$ flexbility in studying complex events.

Consider families of jet algs: e.g. sequential recombination with

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \Delta R_{ij}^2 / R^2$$

	Alg. name	Comp. Geometry problem	time
$p = 1$	k_t CDOSTW '91-93; ES '93	Dynamic Nearest Neighbour CGAL (Devillers et al)	$N \ln N$ exp.
$p = 0$	Cambridge/Aachen Dok, Leder, Moretti, Webber '97 Wengler, Wobisch '98	Dynamic Closest Pair T. Chan '02	$N \ln N$
$p = -1$	anti-k_t (cone-like) Cacciari, GPS, Soyez, in prep.	Dynamic Nearest Neighbour CGAL (worst case)	$N^{3/2}$
cone	SISCone GPS Soyez '07 + Tevatron run II '00	All circular enclosures previously unconsidered	$N^2 \ln N$ exp.

All accessible in FastJet

FastJet in software of all (4) LHC collaborations

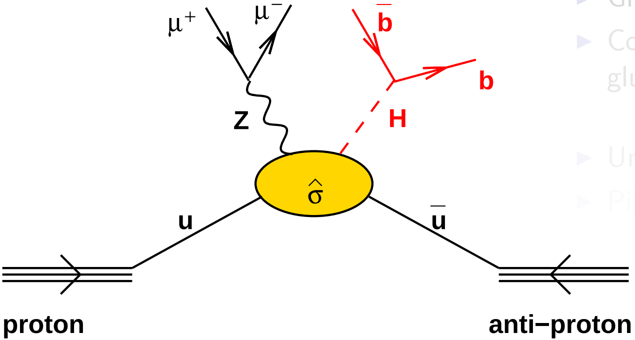
Once you have a decent set of jet algs, *start asking questions about them.*

- They share a common parameter R (angular reach). How do results depend on R?
- In what way do the various algorithms differ?
- How are they to be best used in the challenging LHC environment?

Try to answer questions with Monte Carlo? Gives little understanding of underlying principles.

➥*Supplement with analytical approximations.*

Various contributions



- Gluon emission, $\mathcal{O}(\alpha_s)$
- Conversion of quarks, gluons $\rightarrow \pi^{\pm}$, etc.

  Hadronisation
- Underlying event
- Pileup

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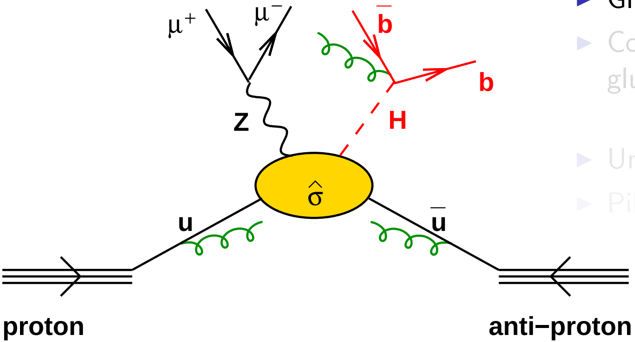

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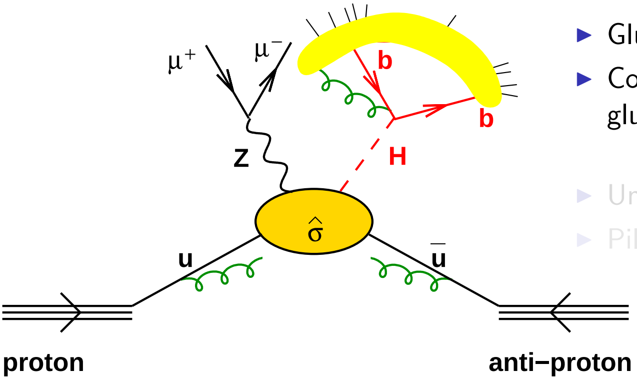

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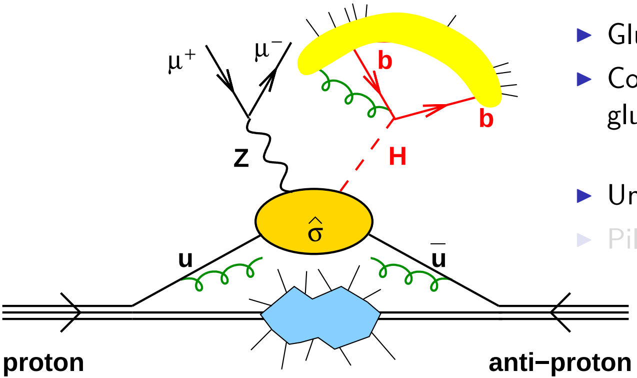

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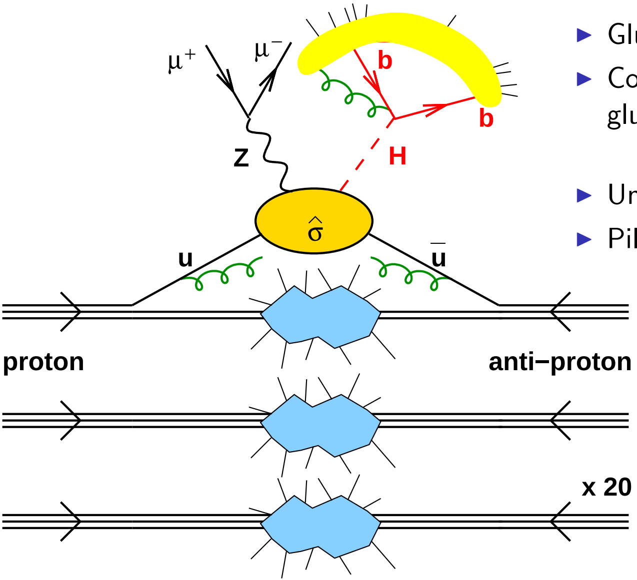

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Perturbative

Start with *quark* with transverse momentum p_t

$$\langle \delta p_t \rangle_{PT} \simeq \frac{1}{\sigma_0} \int d\Phi |M^2| \, \alpha_s(k_{t,rel}) \, (p_{t,jet} - p_t)$$

$$\simeq \frac{\alpha_s C_F}{\pi} \int_R^{\mathcal{O}(1)} \frac{d\theta}{\theta} \int dz \, p_{gq}(z) \cdot ((1-z)p_t - p_t)$$

$$\simeq -1.01 \frac{\alpha_s C_F}{\pi} p_t \ln \frac{1}{R} + \mathcal{O}(\alpha_s p_t)$$

$C_F = 4/3$

Similarly for gluon:

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NB1: $\alpha_s p_t \ln R$ structure & coeff. independent of process
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Simplest form of a trick developed $\sim$ 1995: to establish non-perturbative contribution, replace $\alpha_s(k_{t,rel}) \to \delta\alpha_s(k_{t,rel})$, with support only near Λ_{QCD}.

Dokshitzer & Webber; Korchemsky & Sterman
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E.g.:

$$\frac{2}{\pi}\delta\alpha_s(k_{t,rel}) = \Lambda\delta(k_{t,rel} - \Lambda)$$

$\Lambda = \int dk_{t,rel}\delta\alpha_s(k_{t,rel})$, should be **'universal'**.

Tested for $\sim$ 10 observables in e^+e^- and DIS.

$$\alpha_0 \simeq 0.5 \leftrightarrow \Lambda \simeq 0.4 \text{ GeV}$$

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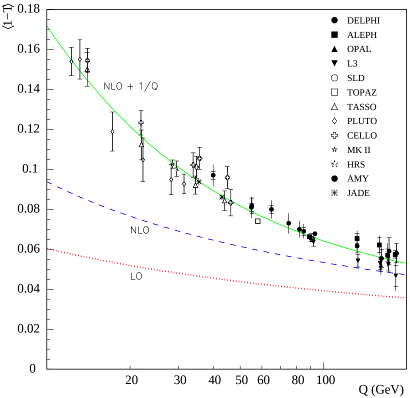

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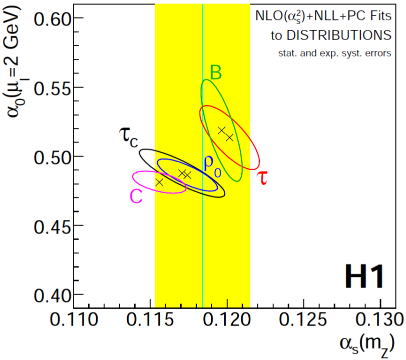

H1 data; Dasgupta & GPS '02

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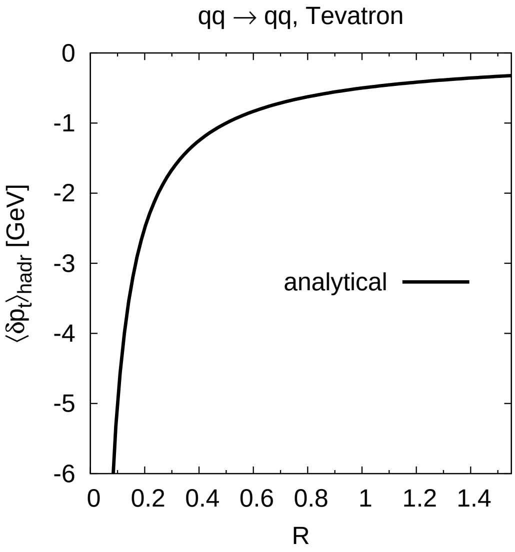

MC hadr. agrees with calc.

- to varying degrees for range of algs
- also in larger gluonic channels

MC UE ≫ naive expectation

- models tuned on same data behave differently
- UE is huge at LHC
- largely indep. of scattering channel

Scale for (non-perturbative!) UE is ∼ 10 GeV

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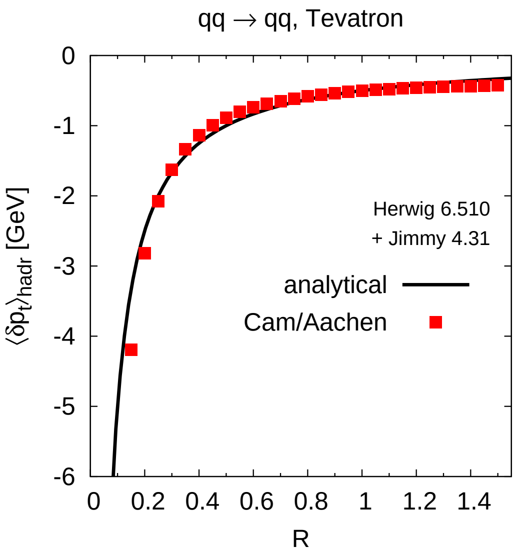

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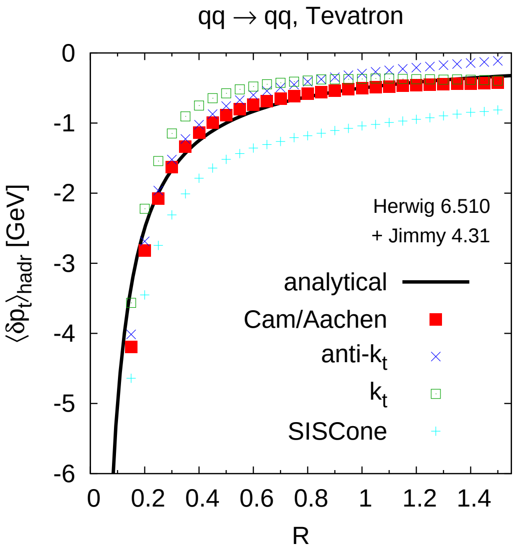

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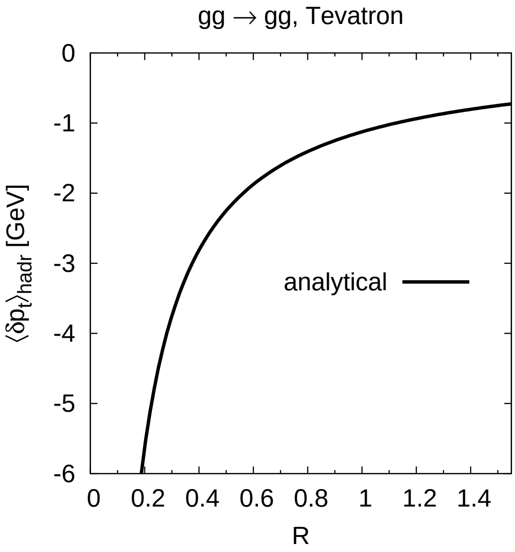

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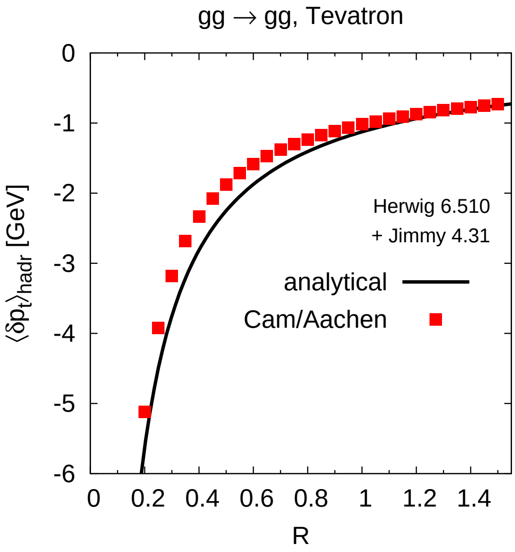

MC hadr. agrees with calc.

- to varying degrees for range of algs
- also in larger gluonic channels

MC UE ≫ naive expectation

- models tuned on same data behave differently
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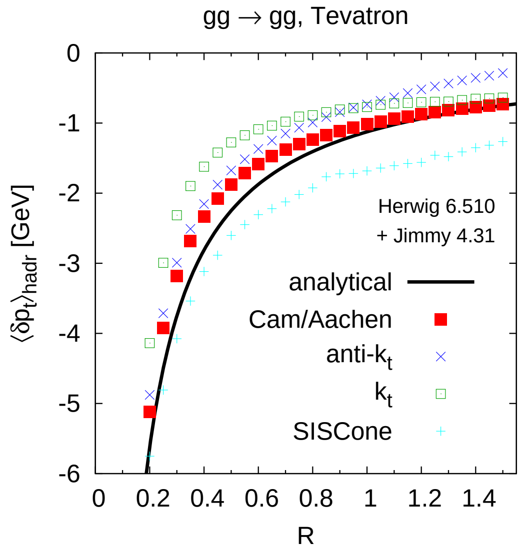

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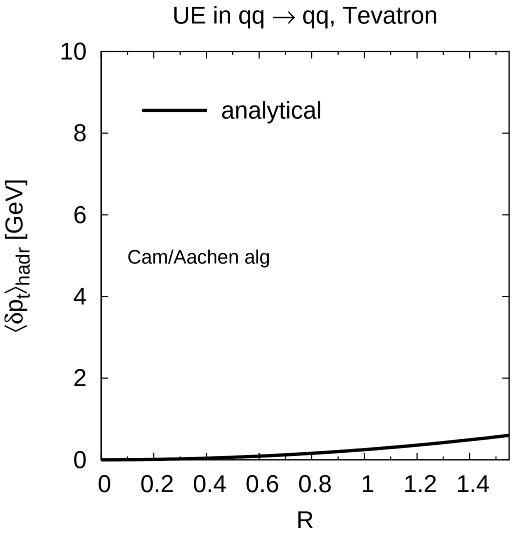

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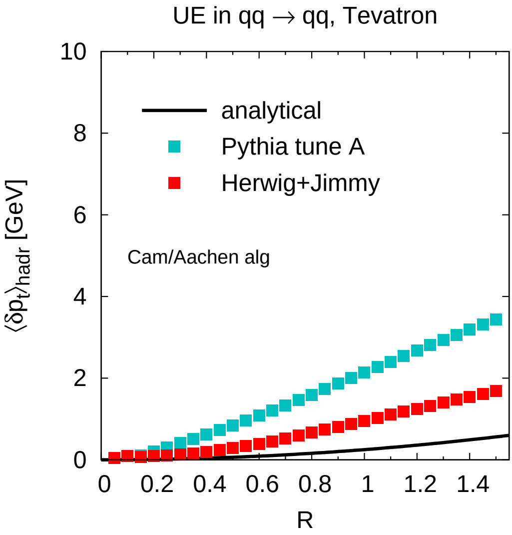

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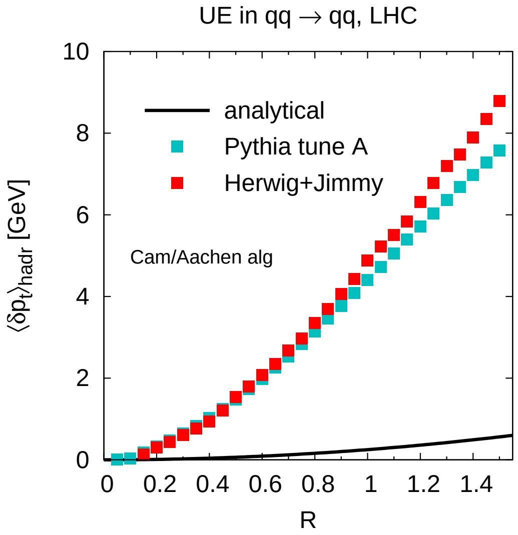

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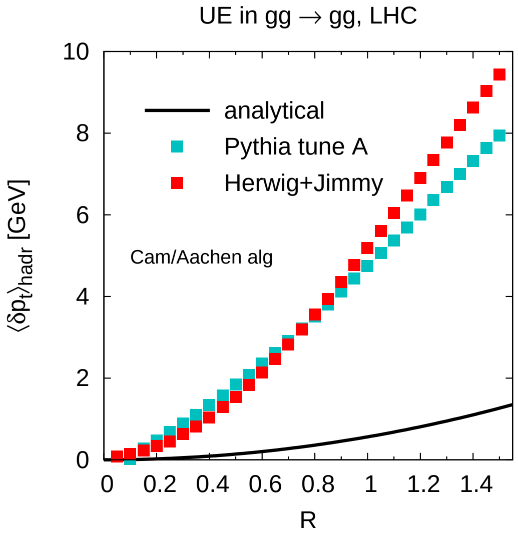

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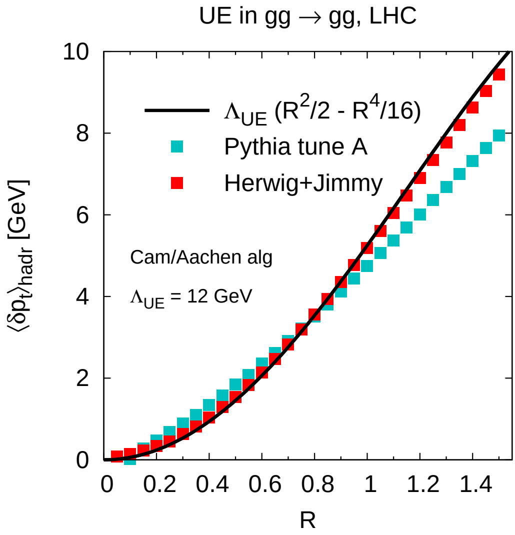

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Optimal R?

	Dependence of jet $\langle \Delta p_t \rangle$ on			
	'partonic' p_t	colour factor	R	$\sqrt{s}$
pert. radiation	$\sim \alpha_s(p_t) p_t$	C_i	$\ln R + \mathcal{O}(1)$	–
hadronization	–	C_i	$-1/R + \mathcal{O}(R)$	–
UE	–	–	$R^2 + \mathcal{O}(R^4)$	s^ω

To get best experimental resolutions, minimise contributions from all 3 components.

Here: sum of squared means

Better still: calculate flucts

NB: this is rough picture, but can still be used to understand general principles.

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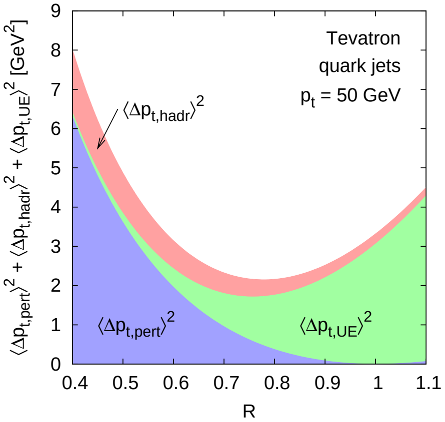

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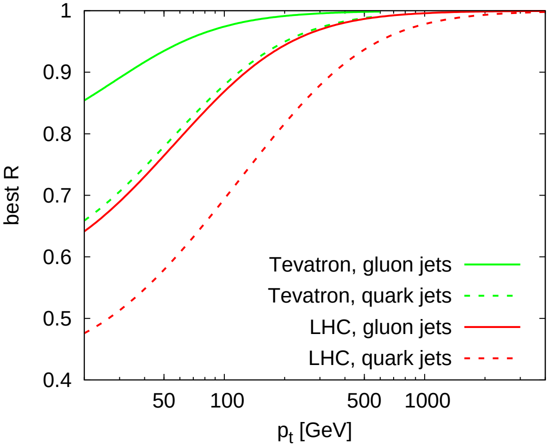
Optimal R v p_t, proc., collider



Basic messages

- higher $p_t \to$ larger R
  Most say opposite
- larger R for gluons than quarks
  Gluon jets wider
- smaller R at LHC than Tevatron
  UE larger

Other results

This last part of talk was an overview of *1 of several* recent jet topics

Others include

- Subtraction of pileup — Cacciari & GPS '07
- Jet areas ↔ sensitivity to UE/pileup — Cacciari, GPS & Soyez prelim
- "Optimising R" — cross checking with MC
  Cacciari, Rojo, GPS & Soyez, for Les Houches
- Jet flavour — e.g. reducing b-jet theory uncertainties from $40-60\%$ to $10-20\%$. — Banfi, GPS & Zanderighi '06, '07

Conclusions / Outlook

- Jets are the closest we can get to seeing and giving meaning to partons
- Play a pivotal role in experimental analyses, comparisons to QCD calculations
- Significant progress in past 2 years towards making them *consistent* (IR/Collinear safe) and *practical* Link with computational geometry
- The physics of how jets behave in a hadron-collider environment is a rich subject — much to be understood, and potential for significant impact in how jets are used at LHC