Giant K factors

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Work performed with Mathieu Rubin and Sebastian Sapeta, arXiv:1006.2144

Workshop on Heavy Particles at the LHC
Pauli Center, Zürich, 5–7 January 2011

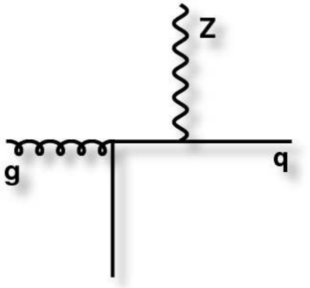
Many searches for New Physics (eg. SUSY) rely on Leading Order predictions for backgrounds.

eg. Z+4jet background to gluino pair production
with NLO technology rapidly becoming mature for such cases

LO often considered good to within a factor of 2
NLO to within 10-20%

This talk is about cases where such "rules of thumb" fail (spectacularly)

The Z+jet process



Use MCFM to examine various properties of such events at LO and NLO.

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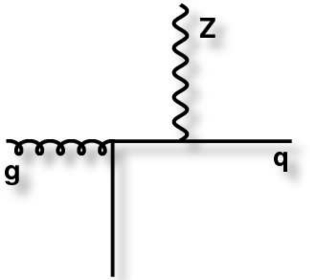

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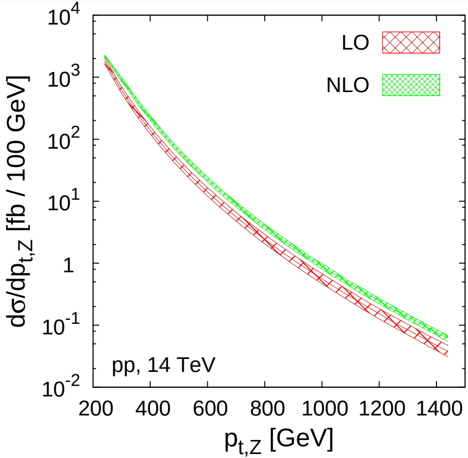
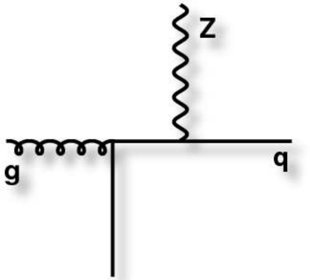


p_t spectrum of **Z boson** gets K-factor of 1.5

Fairly standard kind of occurrence

The Z+jet process



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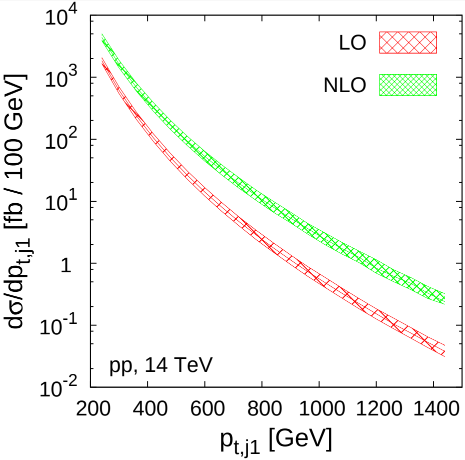
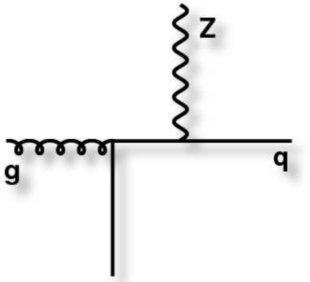


p_t spectrum of **leading jet** gets K-factor of 5–10

related issues in Butterworth, Davison, Rubin & GPS '08
Bauer & Lange '09; Denner, Dittmaier, Kasprzik & Much '09

The Z+jet process



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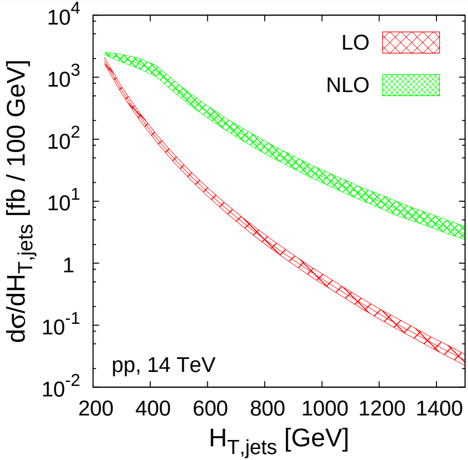
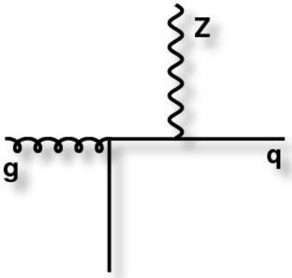


$H_{T,jets} \equiv \sum_{i \in jets} p_{t,i}$ gets K-factor of up to 100

Such things are not supposed to happen with $\alpha_s = 0.1$!

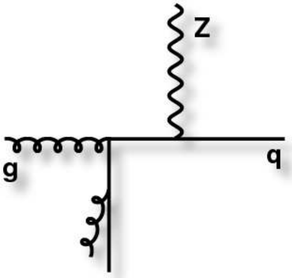
Why the large K-factors?

Leading Order



$\alpha_s \alpha_{EW}$

Next-to-Leading Order



$\alpha_s^2 \alpha_{EW}$

LHC probes scales well above EW scale, $\sqrt{s} \gg M_Z$.
EW bosons are **light**. New log-enhanced topologies appear.

$H_{T,jets}$ is extreme, because at LO $H_{T,jets} \simeq p_{t,jet\,1}$; NLO: $H_{T,jets} \simeq 2p_{t,jet\,1}$

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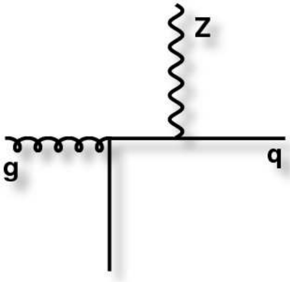

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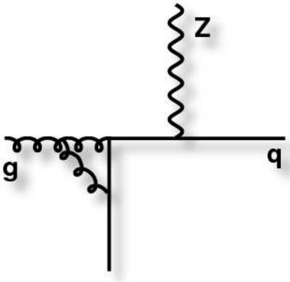

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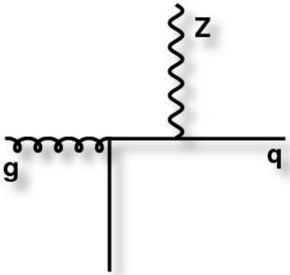

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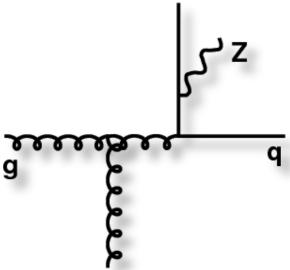

$\alpha_s^2 \alpha_{EW} \ln^2 \frac{p_t}{M_Z}$

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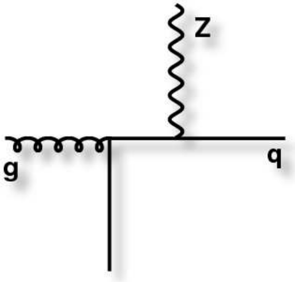	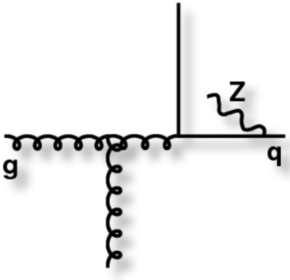
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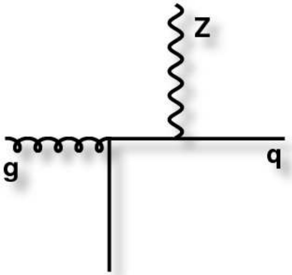

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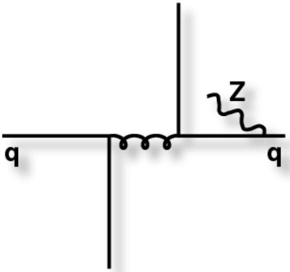

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Though we calculate Z+jet@NLO, giant K-factors really dominated by ("LO") Z+2-parton piece of Z+jet@NLO.

We know LO calculations aren't reliable.

We really want to combine Z+jet@NLO with Z+2-jet@NLO

without double counting

without having to do full Z+jet@NNLO calculation

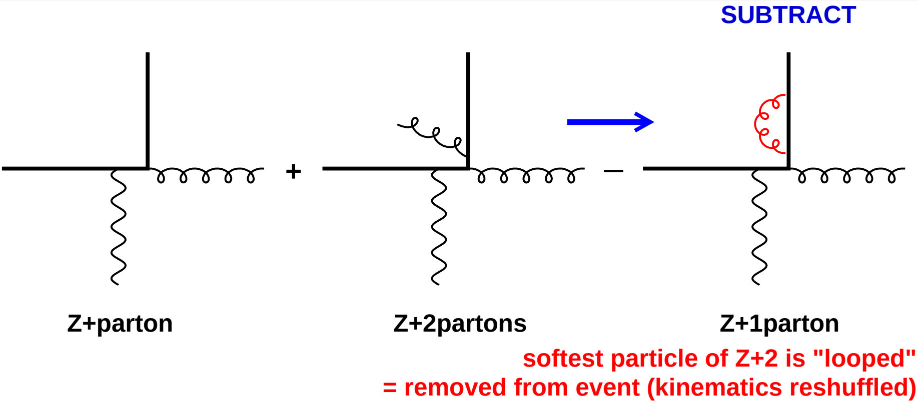
First try the following:

Take the "leading" process
[Z + jet @ LO]

and add in process with one extra jet.
[i.e. include Z + 2 jets @ LO]

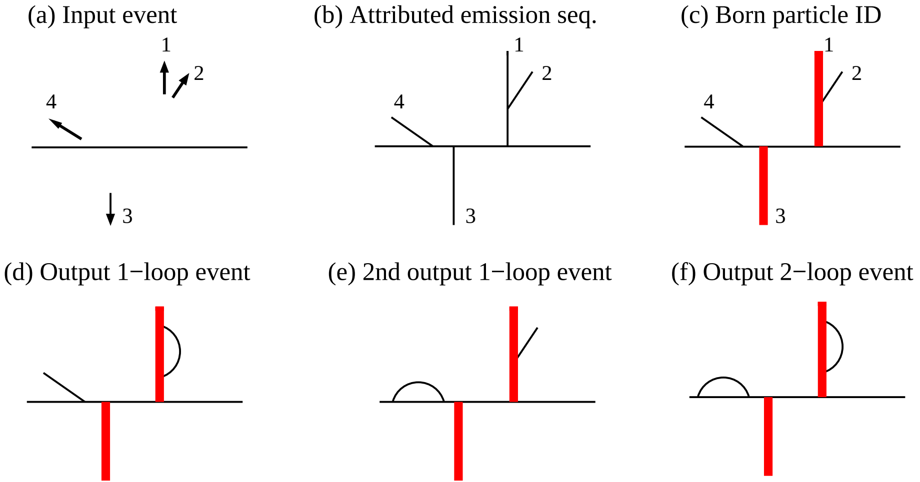
approximate the 1-loop Z+jet term, by requiring cancellation of all divergences

cartoon of the LoopSim idea



- For every $Z + 2$ parton ($2 \to 3$) event, figure out what what $2 \to 2$ event it would really have come from
  "Loop" the softest parton
  [Don't actually explicitly calculate any loop diagrams: simulate the loops]
- Subtract that $2 \to 2$ event
  Unlike MLM, no cutoffs on $2 \to 3$ events
  If done properly, divergences will cancel

cartoon of LoopSim "looping" procedure



- Use jet algorithm to assign a branching structure to event à la CKKW
- The particles that are softest are the ones that will be "looped"

Define operators:

$$U_\ell(\text{event E}) \equiv \text{all simulated } \ell\text{-loop events from E}$$

$$U_\forall(\text{event}) \equiv \sum_{\ell=0} U_\ell(\text{event})$$

"U" stands for unitarisation (cancellation of all divergences)
sum of all diagrams (essentially) adds up to zero

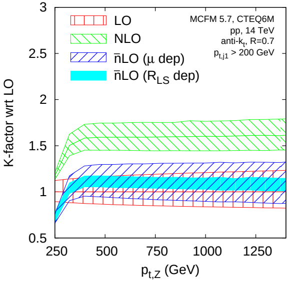
To combine Z+j with Z+2j take

$$\text{Z+j@}\bar{n}\text{LO} \equiv \text{Z+j@LO} + U_\forall(\text{Z+2j@LO})$$

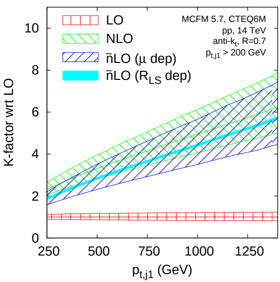
we use "$\bar{n}$LO" to emphasize that this is a crude approximation
to an actual NLO calculation — the exact loops are missing
NB: $U_\forall$ here includes $\ell = 0, 1$

$\bar{n}$LO results (K-factors, normalised to LO)

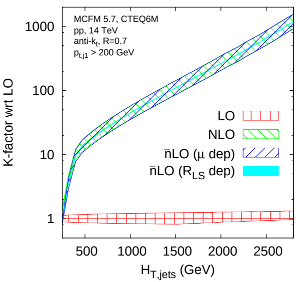
p_t of Z-boson



p_t of jet 1



$H_{T,jets} = \sum_{jets} p_{t,j}$



When the K-factors are large, $\bar{n}$LO agrees well with NLO

MLM matching also does a similar job

Differences between and LoopSim and MLM/CKKW matching:

1. Does not rely on shower (✓: simple; ✗: not easily integrated with shower MCs)

2. Does not need arbitrary separation of Z+1/Z+2/etc. samples with (hard-to-choose) momentum cutoff

3. Can easily be extended beyond LO matching

$\bar{n}$NLO: merging Z+j and Z+2j, both @NLO

Just replace simulated loops with exact loops
Apply LoopSim to exact 1-loop to get (e.g.) simulated 2-loop terms

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$E_{n,\ell} \equiv$ event with n partons and ℓ exact loops
$U_{\forall,\ell} \equiv$ operator to apply when ℓ exact loops known

$$U_{\forall,1}(E_{n,0}) = U_{\forall}(E_{n,0}) - U_{\forall}\left(U_1(E_{n,0})\right)$$
$$U_{\forall,1}(E_{n,1}) = U_{\forall}(E_{n,1})$$

$$\text{Z+j@}\bar{n}\text{NLO} = \text{Z+j@NLO} + U_{\forall,1}(\text{Z+2j@NLO}_{\text{only}})$$

Extension to NLO, NNLO, multi-leg, etc. is almost trivial in LoopSim

Not the case in methods that merge with parton showers too

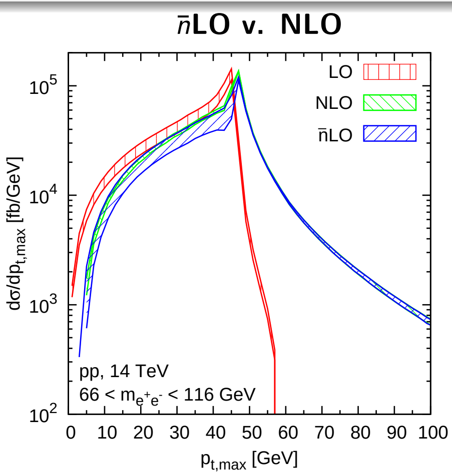
Testing NLO Merging, in 3 processes

1. Z@NLO with Z+j@NLO

2. Z+j@NLO with Z+2j@NLO

3. 2j@NLO with 3j@NLO

Validation: Drell-Yan lepton p_t, $\bar{n}$NLO v. NNLO



Z (i.e. DY) with Z+j from MCFM & LoopSim

For $p_{t\,ell} \gtrsim \frac{1}{2}M_Z + \Gamma_Z$ (giant K-factor!) it had to work
For $p_{t,\ell} \lesssim \frac{1}{2}M_Z + \Gamma_Z$ it's remarkable that it still works

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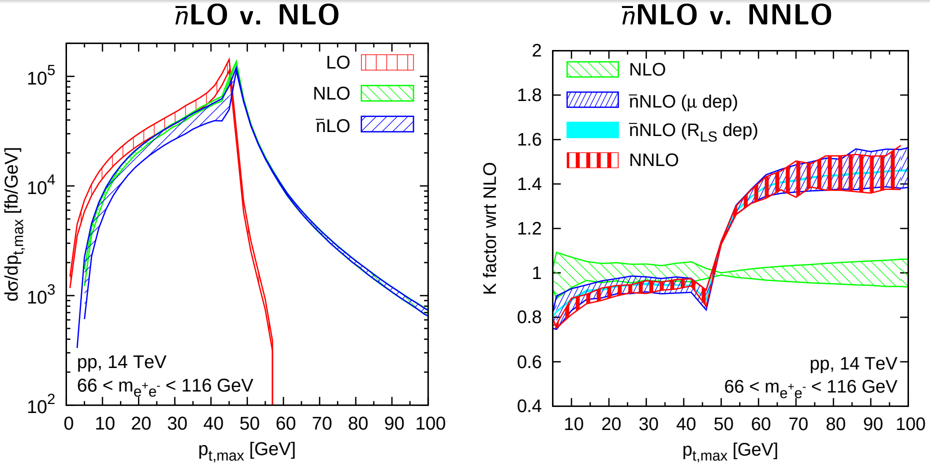

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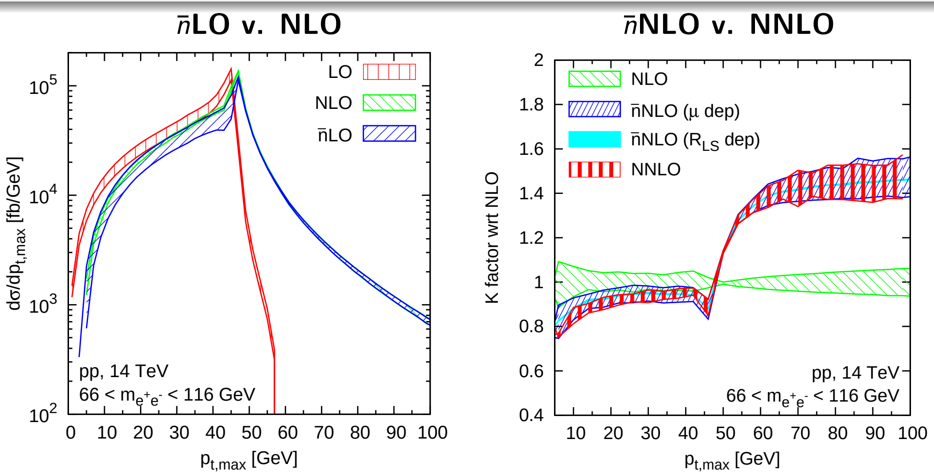

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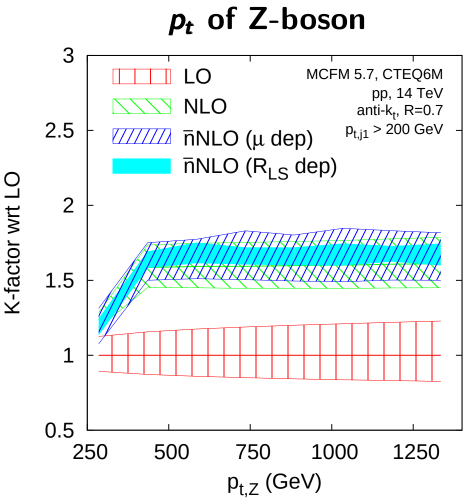
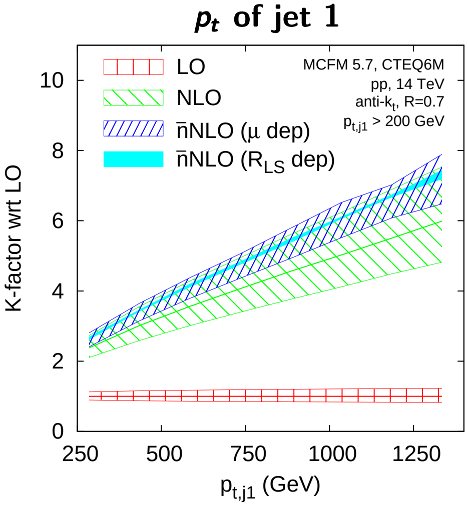
$\bar{n}$NLO for Z+j observables



- p_{tZ} distribution didn't have giant K-factors.
- $\bar{n}$NLO brings no benefit

  To get improvement you would need exact 2-loop terms

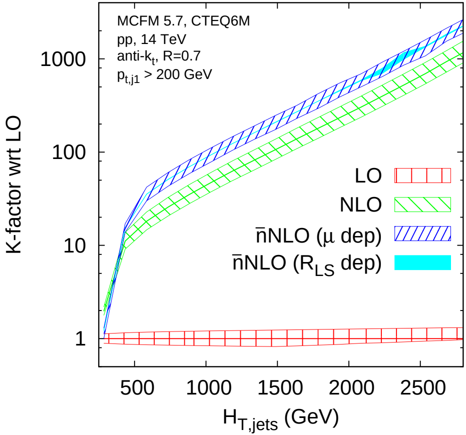
$\bar{n}$NLO for Z+j observables



- p_{tj} distribution seems to converge at $\bar{n}$NLO
- scale uncertainties reduced by $\sim$ factor 2

n̄NLO for Z+j observables

$$H_{T,jets} = \sum_{jets} p_{t,j}$$



- Signficiant further enhancement for $H_{T,jets}$
- n̄NLO brings clear message:

  $H_{T,jets}$ is not a good observable!

What's the problem with H_T?

H_T (effective mass) type observables are widely used in searches

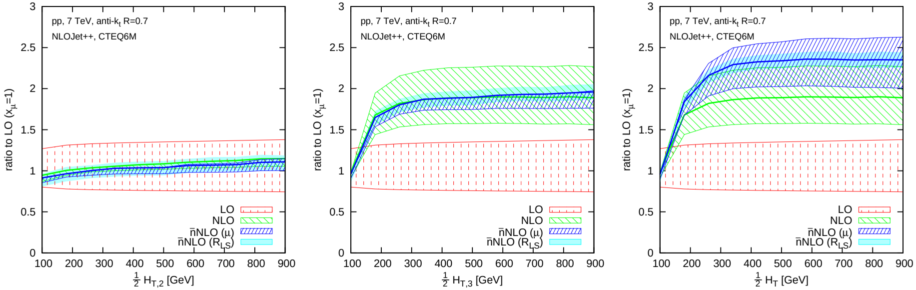
- H_T has a steeply falling distribution (like p_{tj}, p_{tZ})
- At each order (NLO, NNLO), an extra (soft) jet contributes to the H_T sum — e.g. from ISR
- That shifts H_T up, which translates to a substantial increase in the cross section

We can test this hypothesis for plain jet events, using a truncated sum,

$$H_{T,n} = \sum_{i=1}^{n} p_{t,\text{jet}\,i}$$

$H_{T,n}$ in (di)jet events

$H_{T,2}$ $H_{T,3}$ $H_{T,\infty}$



A clear message:
for a process with n objects at lowest order, use $H_{T,n}$

Do you know what gets used in your experiment's searches?
Many writers of LHC SUSY proceedings didn't...

Some take-home messages

Be aware that giant K-factors exist

Always look one order beyond the leading order, for example with MLM/CKKW matching

New tool to get good predictions in such cases: **LoopSim**

Basically an "operator" to generate approximations to unknown loops

Combine Z+j@NLO, Z+2j@NLO to get "$\bar{n}$NLO" Z+jet

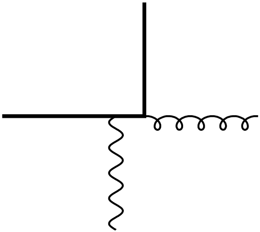
It sometimes works even beyond "giant-K-factor" regions

Watch out for H_T

Even for simple processes, it converges very poorly unless you define it carefully (limit number of objects in sum)

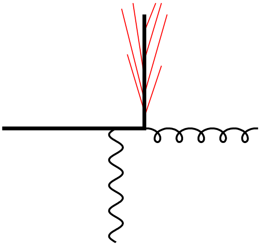
EXTRAS

Add Z+1jet, Z+2jet + shower



Z+parton

Add Z+1jet, Z+2jet + shower



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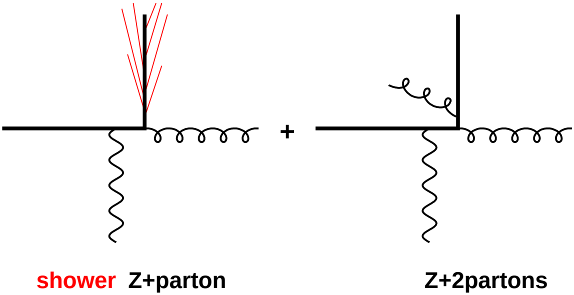

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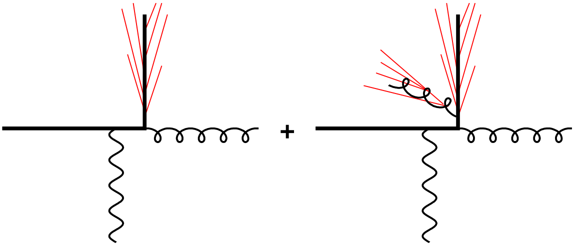
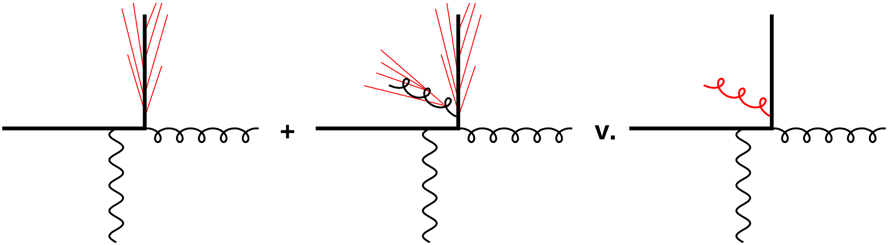


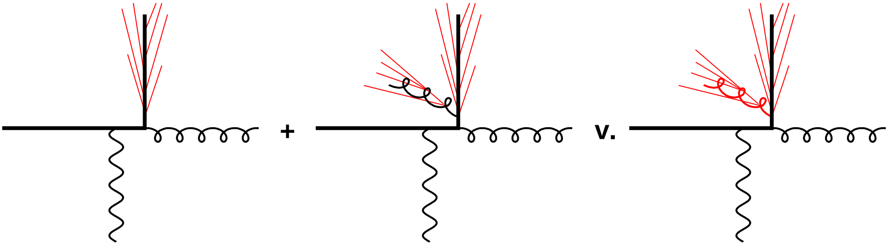
shower Z+parton **shower Z+2partons**

Add Z+1jet, Z+2jet + shower



shower Z+parton — **shower Z+2partons** — **shower of Z+parton generates hard gluon**

Add Z+1jet, Z+2jet + shower



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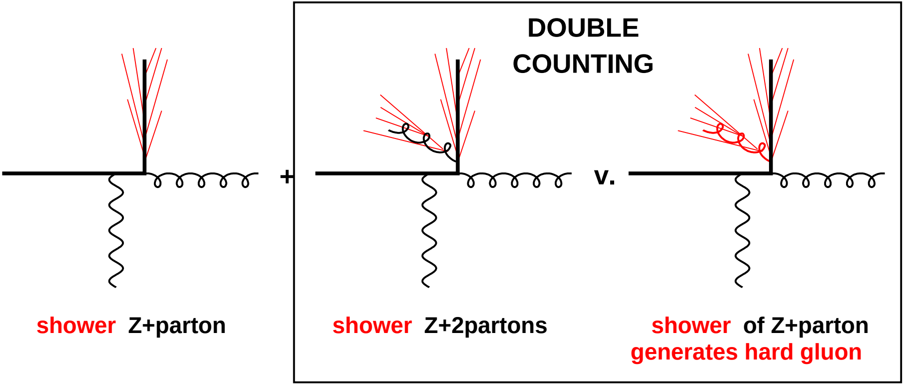
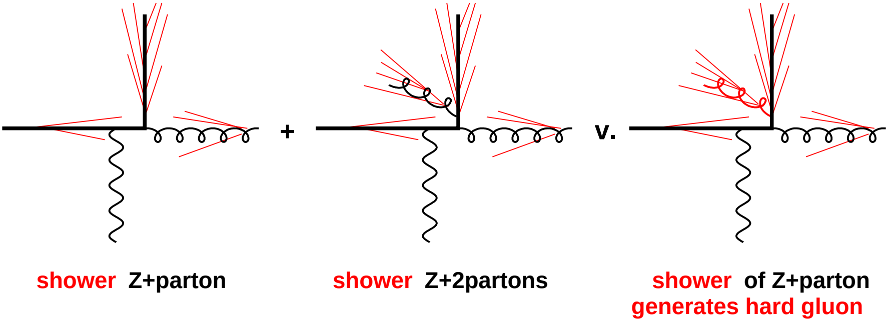


Z + parton implicitly includes part of Z + 2 partons
It's just that the 2nd parton isn't always explicitly "visible"

cartoon of MLM merging of Z+j and Z+2j



- MLM merging relies on parton shower to help figure out what fraction of Z + parton is really Z + 2 partons.
- Our aim is to do that without the parton shower

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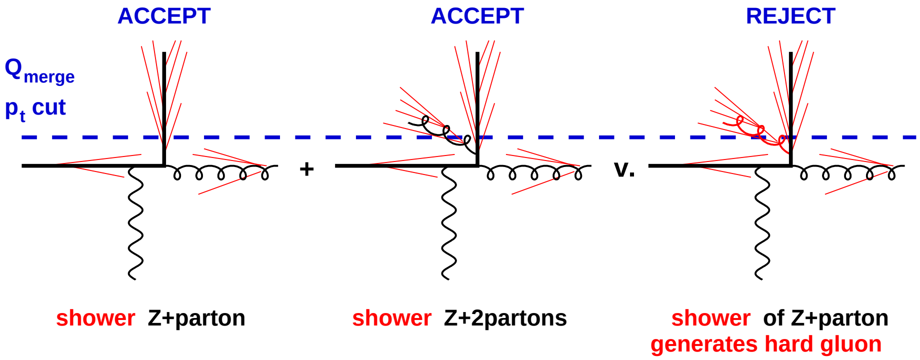

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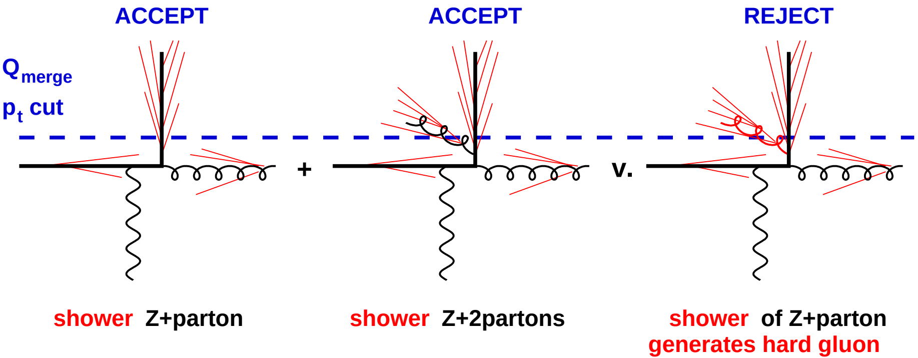

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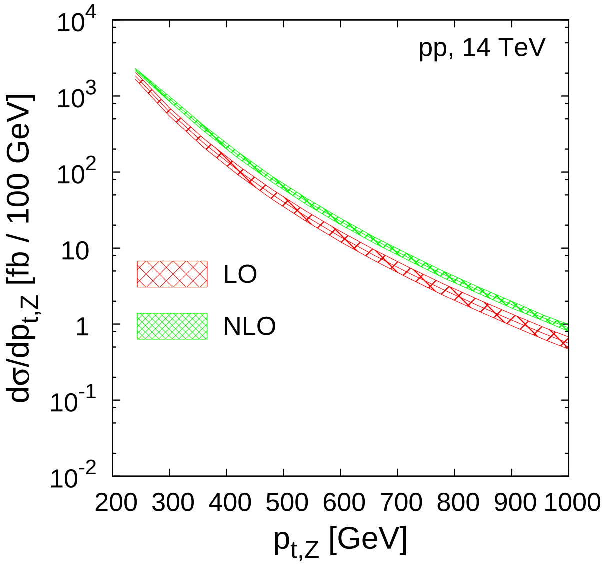
Testing Alpgen + Herwig + MLM Matching

p_t of Z-boson



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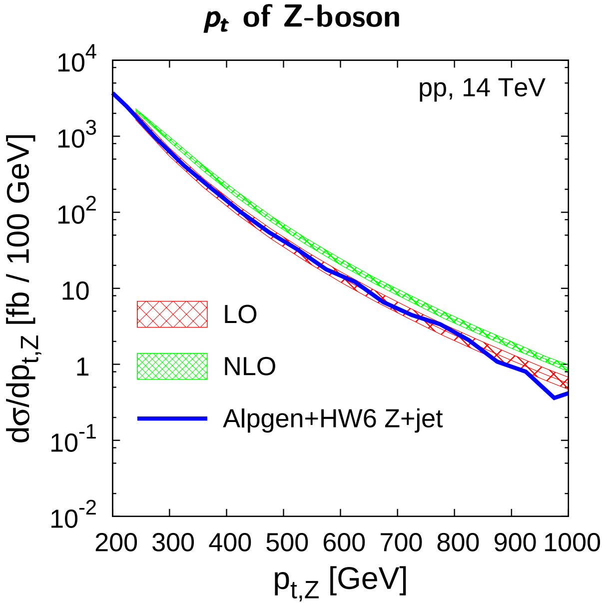

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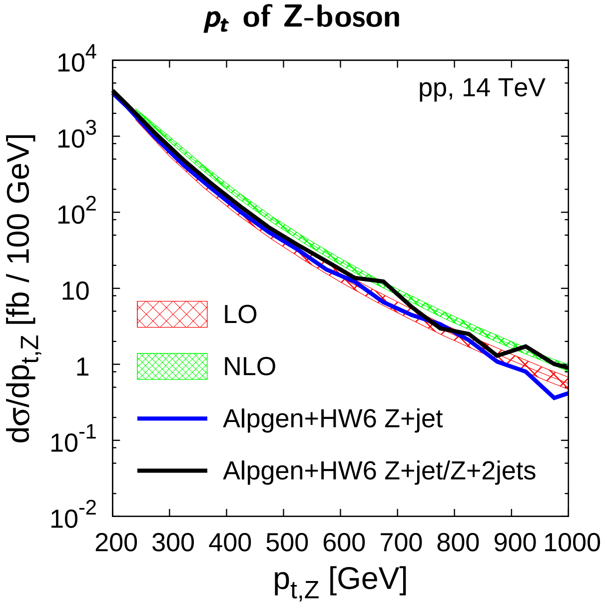
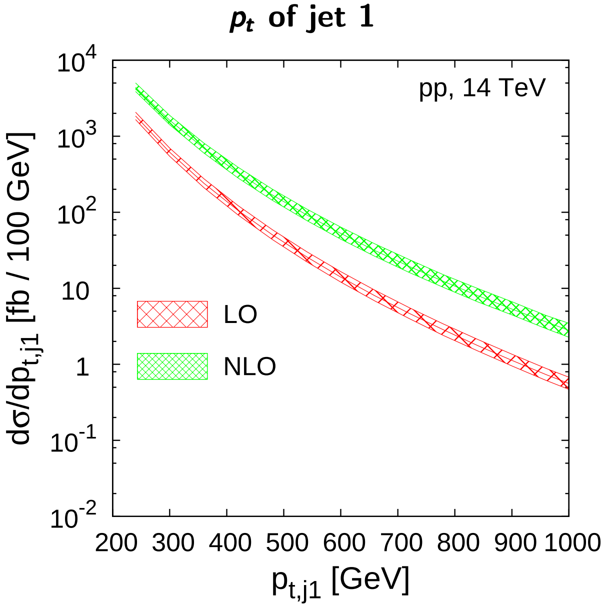


All predictions similar and stable

Testing Alpgen + Herwig + MLM Matching



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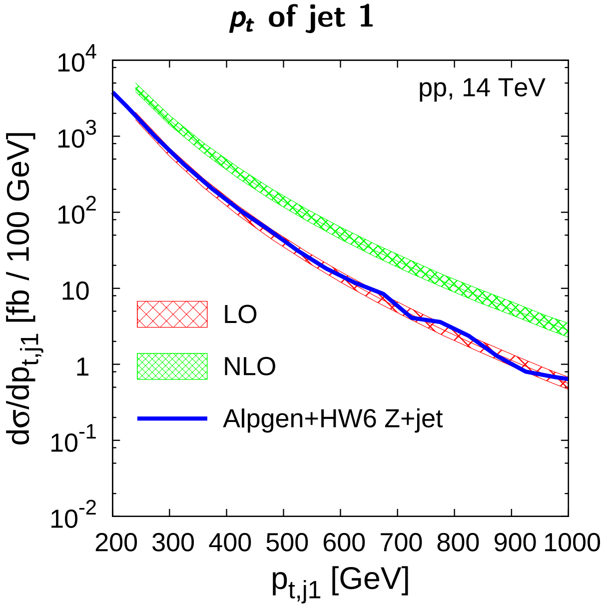
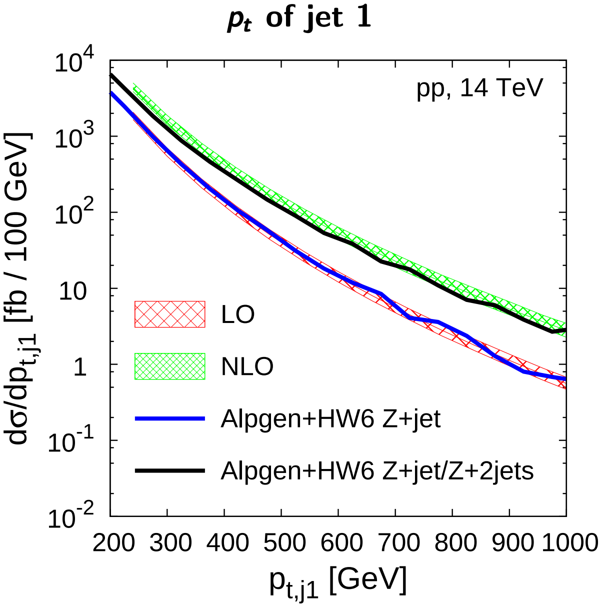


Showered Z+j $\simeq$ LO

Testing Alpgen + Herwig + MLM Matching

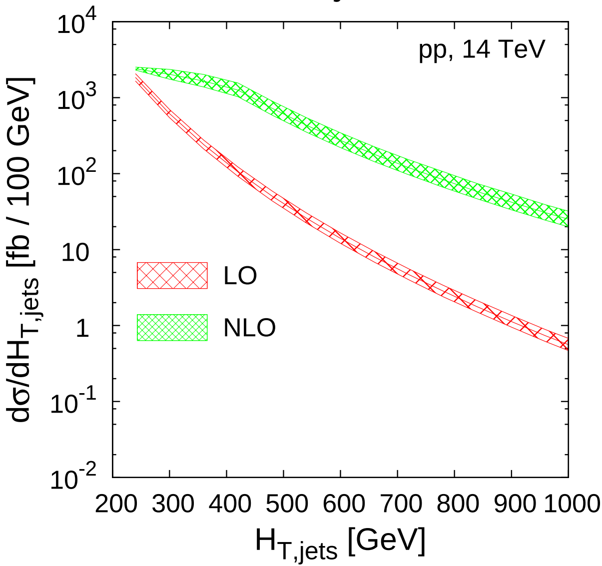


Showered Z+j $\simeq$ LO

Showered Z+j/Z+2j $\simeq$ NLO

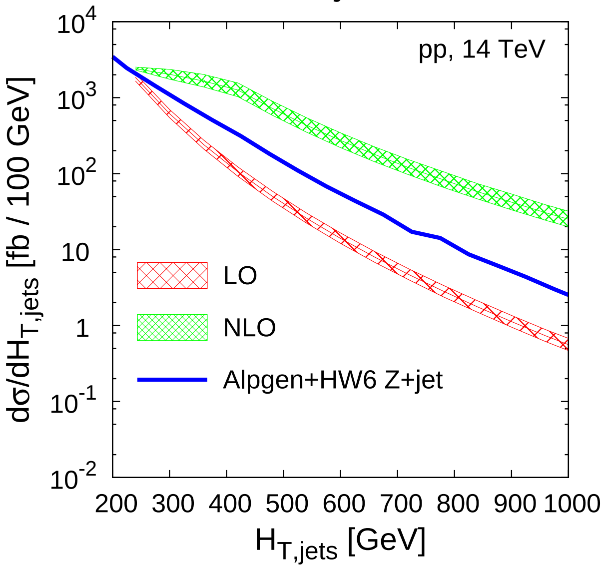
Testing Alpgen + Herwig + MLM Matching

$$H_{T,jets} = \sum_{jets} p_{t,j}$$



Testing Alpgen + Herwig + MLM Matching

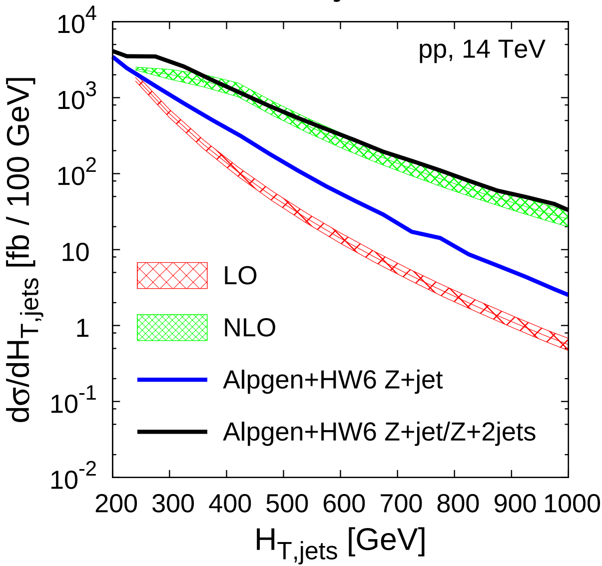
$H_{T,jets} = \sum_{jets} p_{t,j}$



Showered Z+j $\neq$ LO

Testing Alpgen + Herwig + MLM Matching

$H_{T,jets} = \sum_{jets} p_{t,j}$



Showered Z+j $\neq$ LO

Showered Z+j/Z+2j $\simeq$ NLO