CUTS FOR 2-BODY DECAYS AT COLLIDERS

Snowmass Energy Frontier Workshop, 1 September 2021

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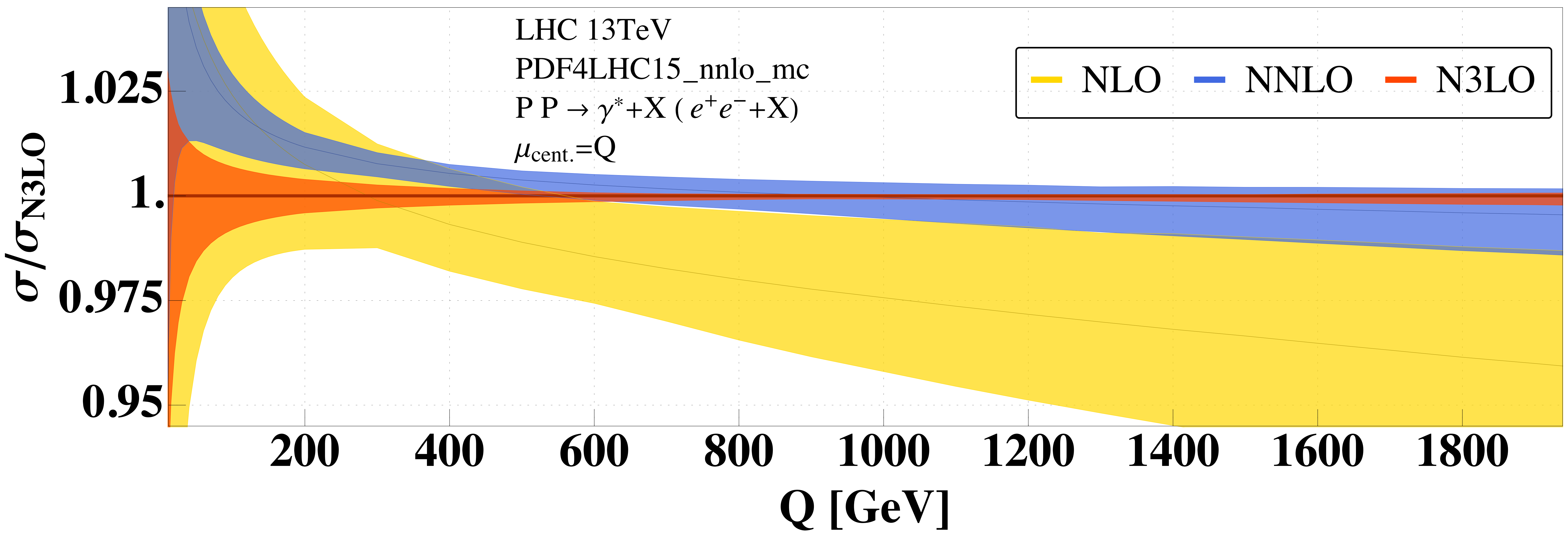
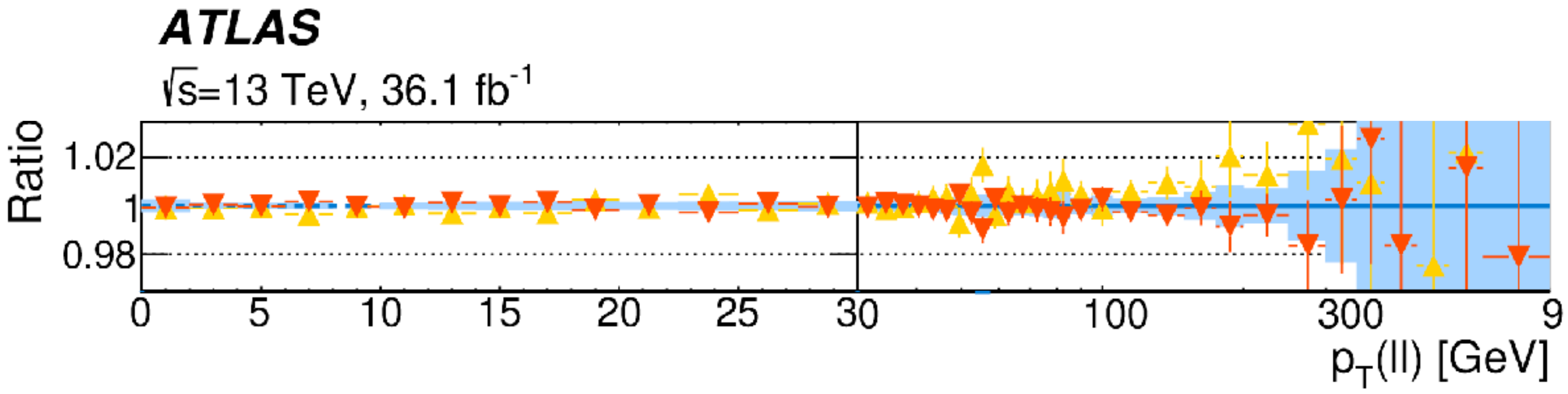
Precision is a developing frontier at the energy frontier



Theoretically

e.g. N3LO DY mass-spectrum from Duhr, Dulat & Mistlberger, Phys.Rev.Lett. 2020



Experimentally

e.g. ATLAS Drell-Yan p_t spectrum, EPJC 2020

Precision is crucial part of LHC programme: e.g. establishing the Higgs sector

Over the next 15 years

Today's 10–20% → 2% at HL-LHC

We wouldn't consider QED established if it had only been tested at 10% accuracy

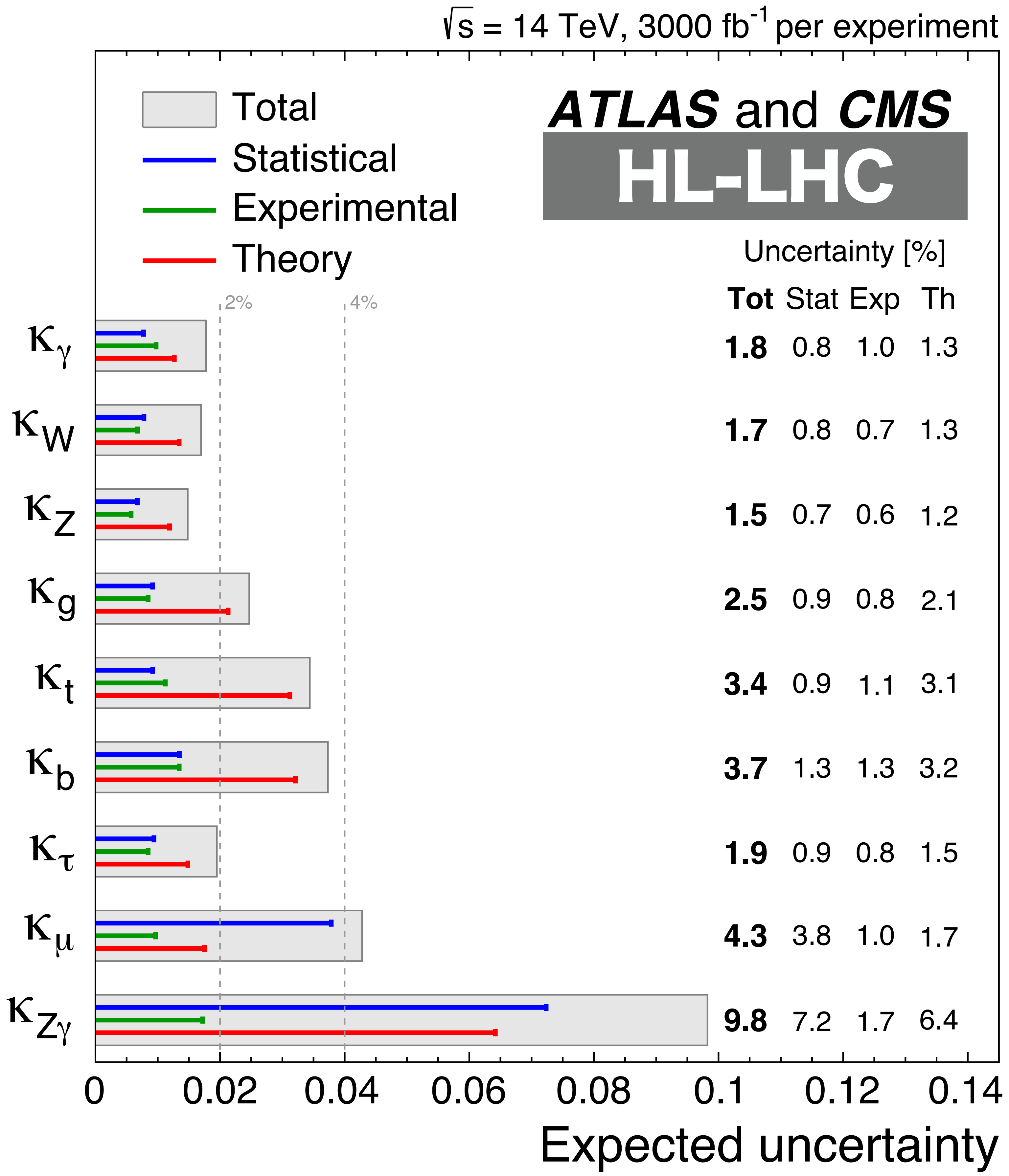


Figure 1. Projected uncertainties on κ_i, combining ATLAS and CMS: total (grey box), statistical (blue), experimental (green) and theory (red). From Ref. [2].

Starting point for any hadron-collider analysis: acceptance (fiducial) cuts

E.g. ATLAS/CMS $H \to \gamma\gamma$ cuts

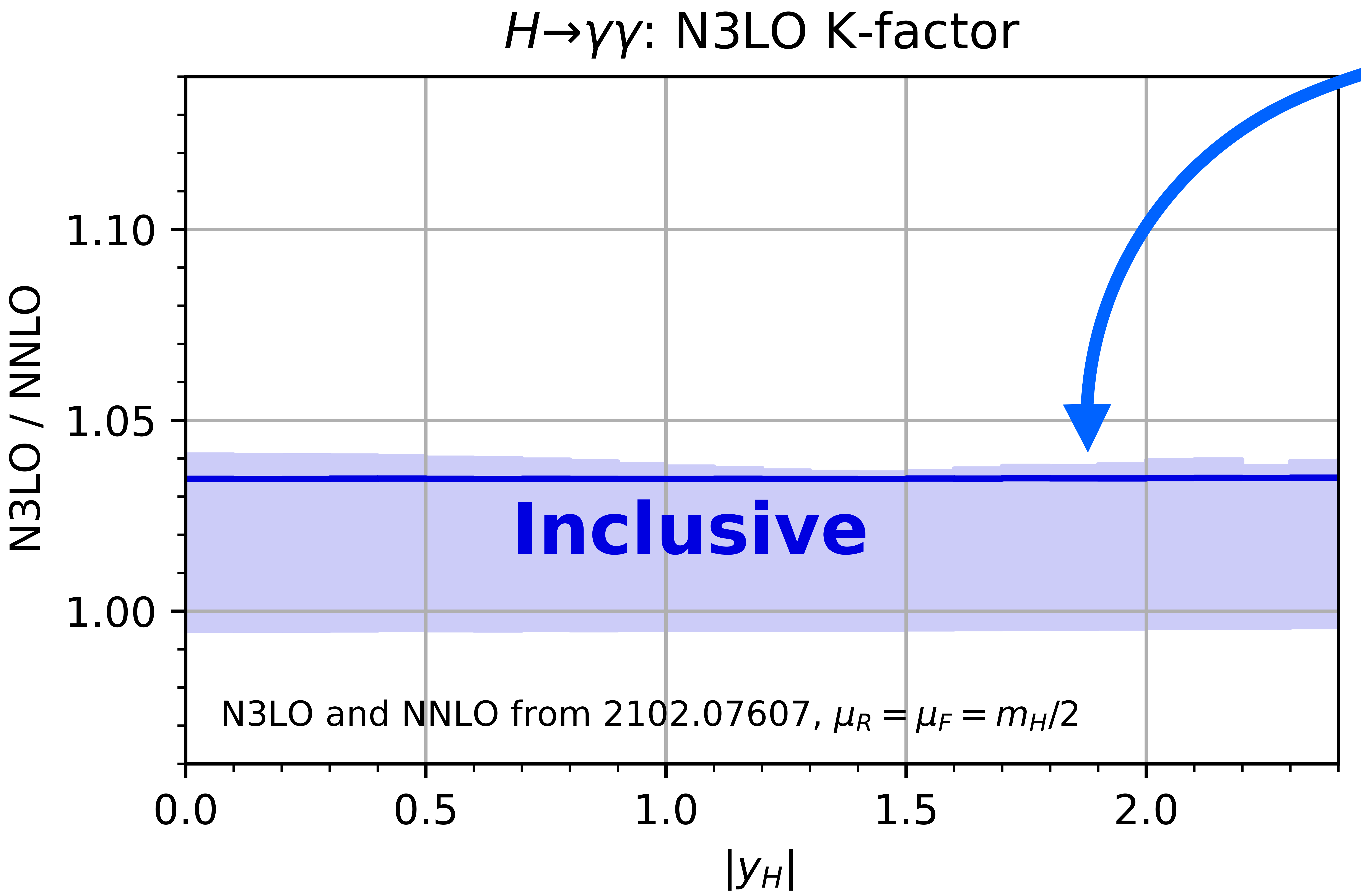
- Higher-p_t photon: $p_{t,\gamma} > 0.35 m_{\gamma\gamma}$ (ATLAS) or $m_{\gamma\gamma}/3$ (CMS)
- Lower-p_t photon: $p_{t,\gamma} > 0.25 m_{\gamma\gamma}$
- Both photons: additional rapidity and isolation cuts

Essential for good reconstruction of the photons and for rejecting large low-p_t backgrounds.

Theory-experiment comparisons with identical "fiducial" cuts often considered
the Gold Standard of collider physics

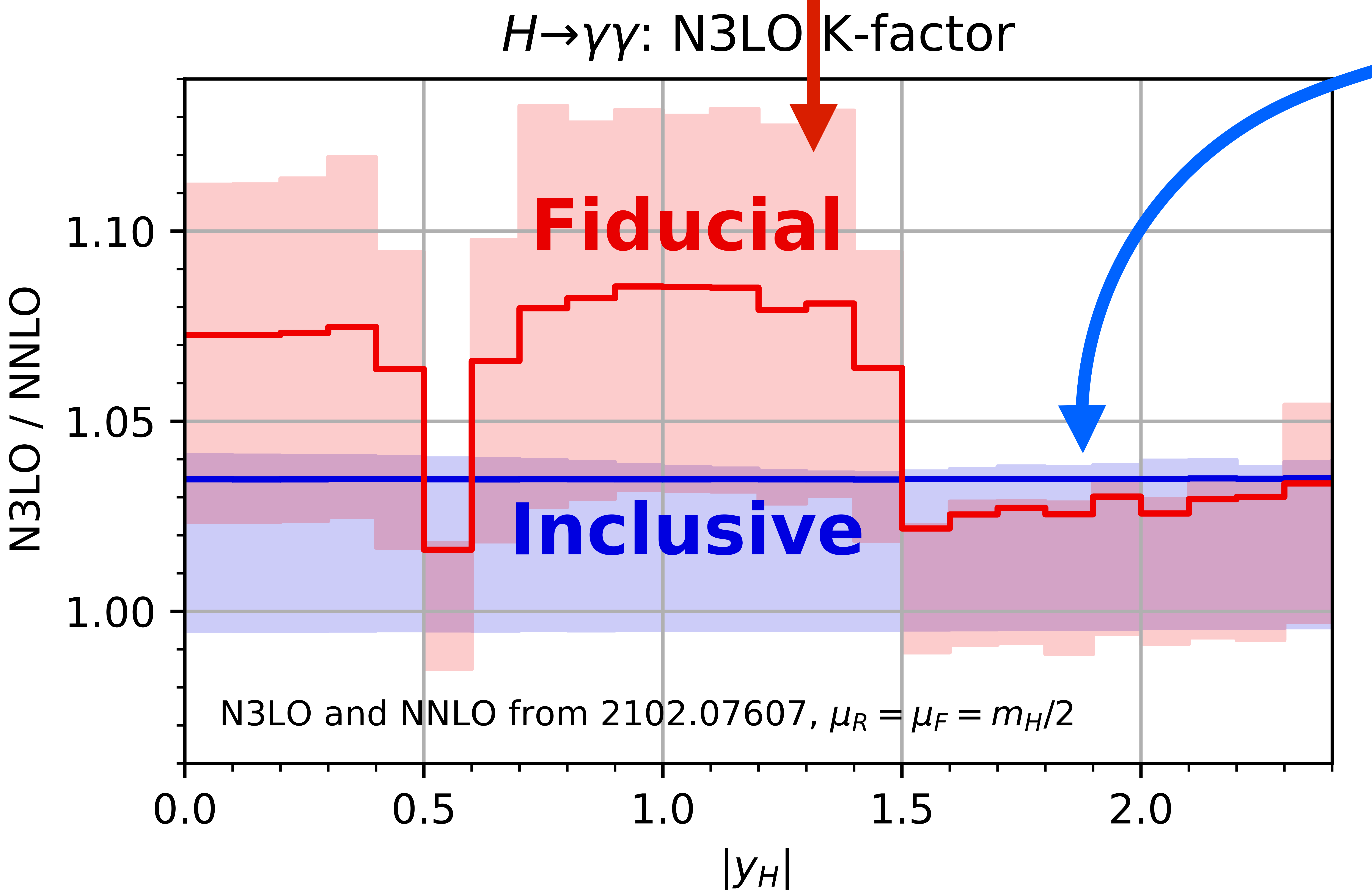
Recent surprise: H→γγ

inclusive N3LO σ uncertaities



Chen, Gehrmann, Glover, Huss, Mistlberger & Pelloni, 2102.07607

Recent surprise: H→γγ fiducial N3LO σ uncertainties ~2× greater than inclusive N3LO σ uncertaities

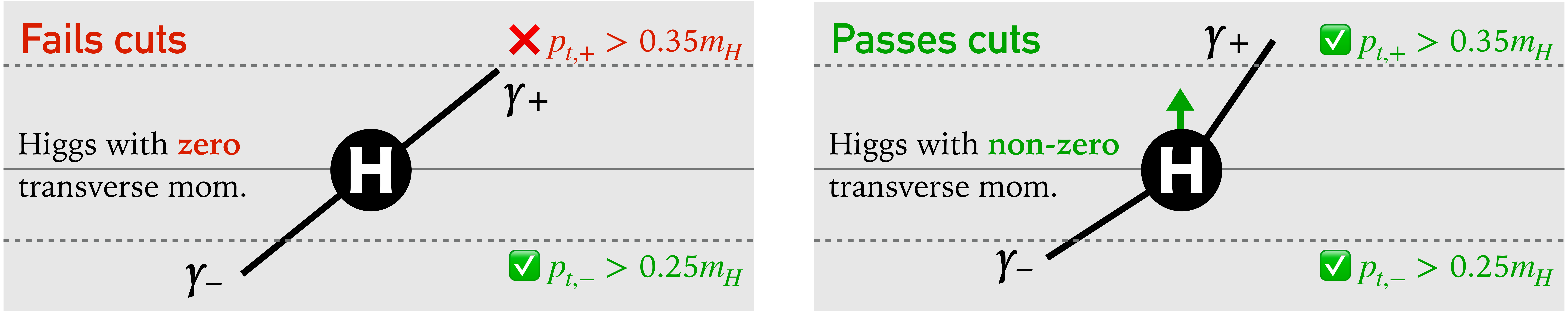


Chen, Gehrmann, Glover, Huss, Mistlberger & Pelloni, 2102.07607

"Gold standard" fiducial cross section gives much worse prediction

Why?
And can this be solved?

Standard $p_{t,\gamma}$ cuts → Higgs p_t dependence of acceptance

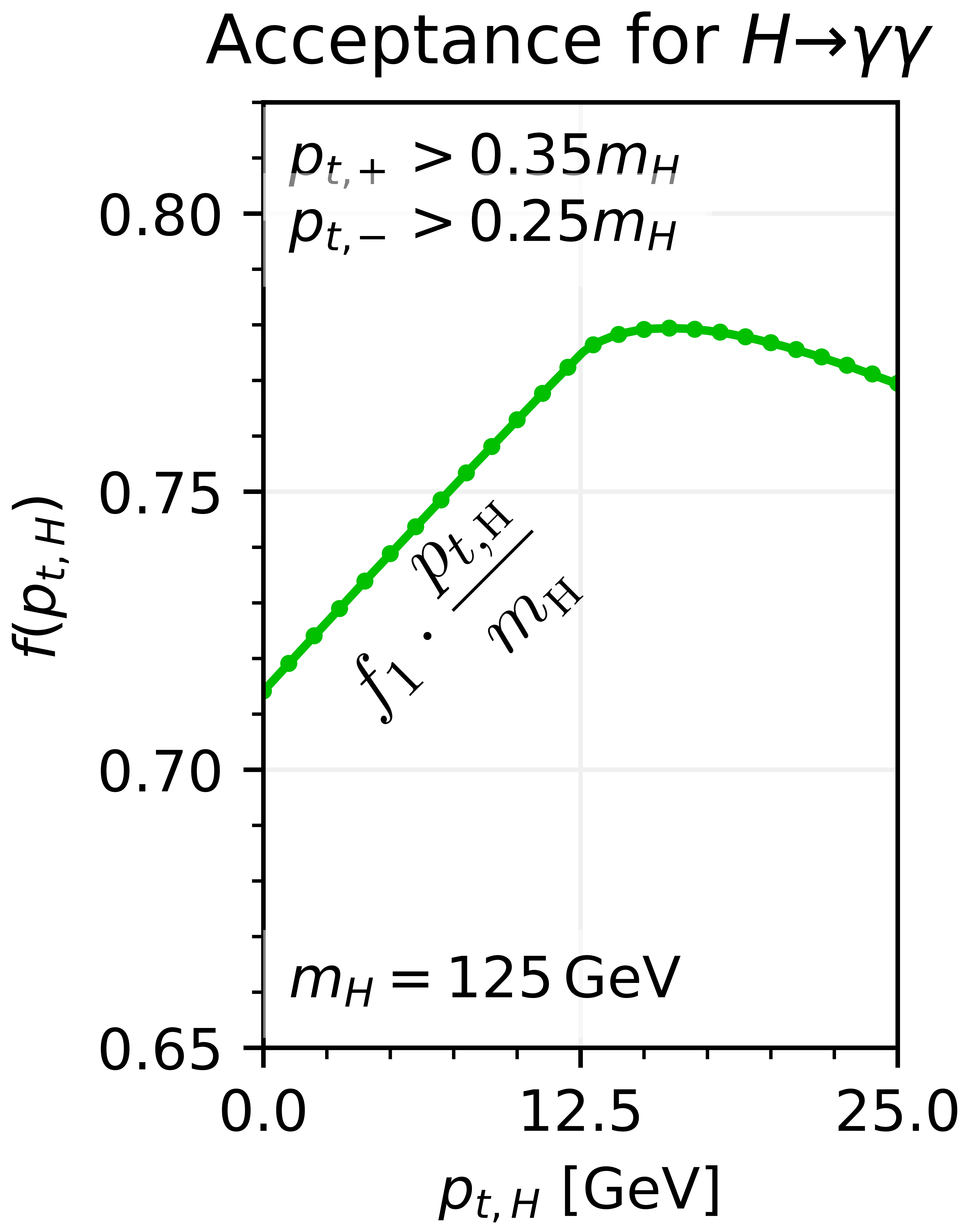


Numbers are for ATLAS $H \to \gamma\gamma$ p_t cuts, CMS cuts are similar

$$p_{t,\pm}(p_{t,\mathrm{H}}, \theta, \phi) = \frac{m_\mathrm{H}}{2} \sin\theta \pm \frac{1}{2} p_{t,\mathrm{H}} |\cos\phi| + \frac{p_{t,\mathrm{H}}^2}{4 m_\mathrm{H}} \left(\sin\theta \cos^2\phi + \csc\theta \sin^2\phi\right) + \mathcal{O}_3 \,,$$

$$p_{t,\mathrm{prod}}(p_{t,\mathrm{H}}, \theta, \phi) = \sqrt{p_{t,+} p_{t,-}} = \frac{m_\mathrm{H}}{2} \sin\theta + \frac{p_{t,\mathrm{H}}^2}{4 m_\mathrm{H}} \frac{\sin^2\phi - \cos^2\theta \cos^2\phi}{\sin\theta} + \mathcal{O}_4$$

Linear p_{tH} dependence of H acceptance, $f(p_{tH})$ → impact on perturbative series



$$f(p_{t,\mathrm{H}}) = f_0 + f_1 \cdot \frac{p_{t,\mathrm{H}}}{m_\mathrm{H}} + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

See e.g. Frixione & Ridolfi '97
Ebert & Tackmann '19
idem + Michel & Stewart '20
Alekhin et al '20

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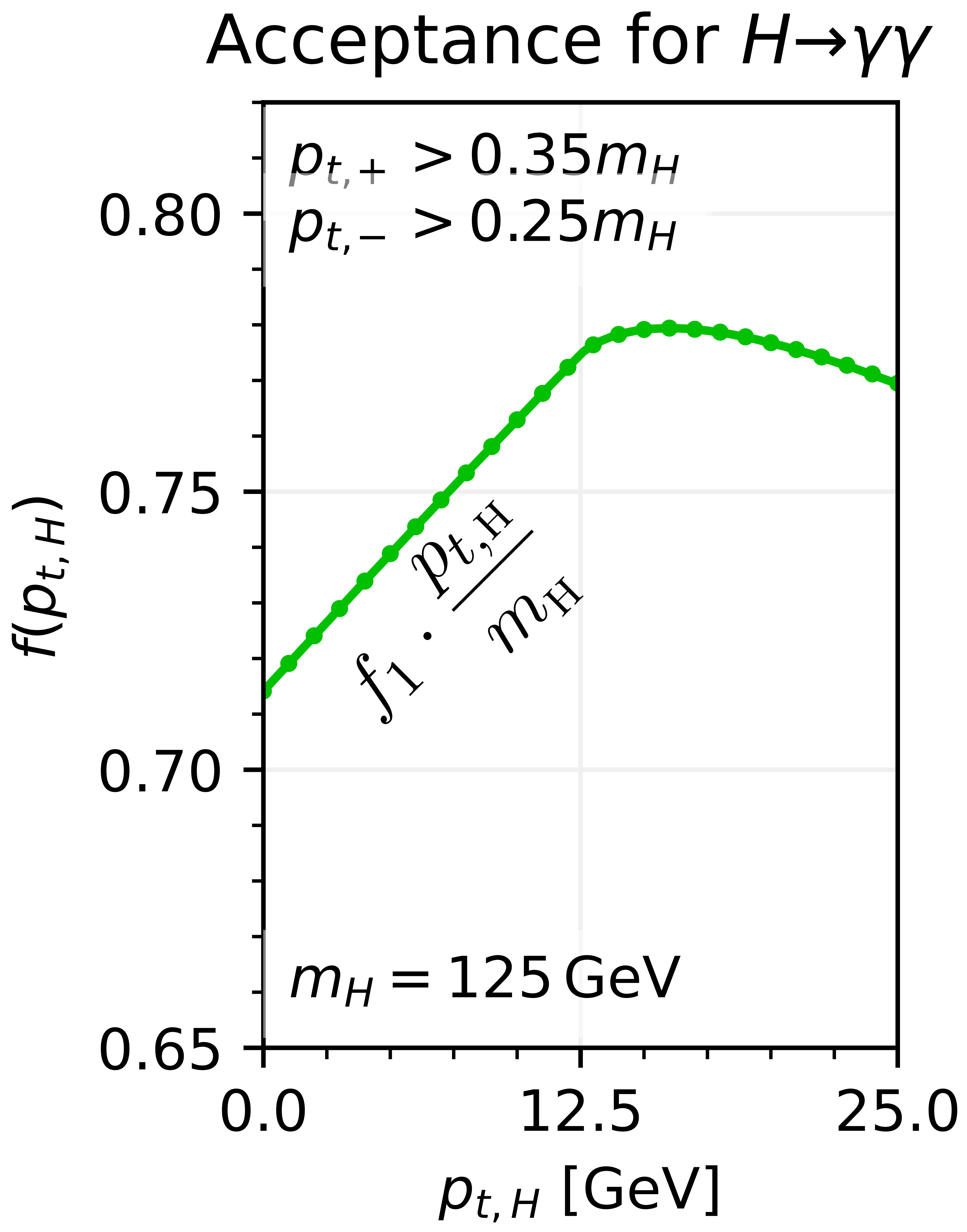

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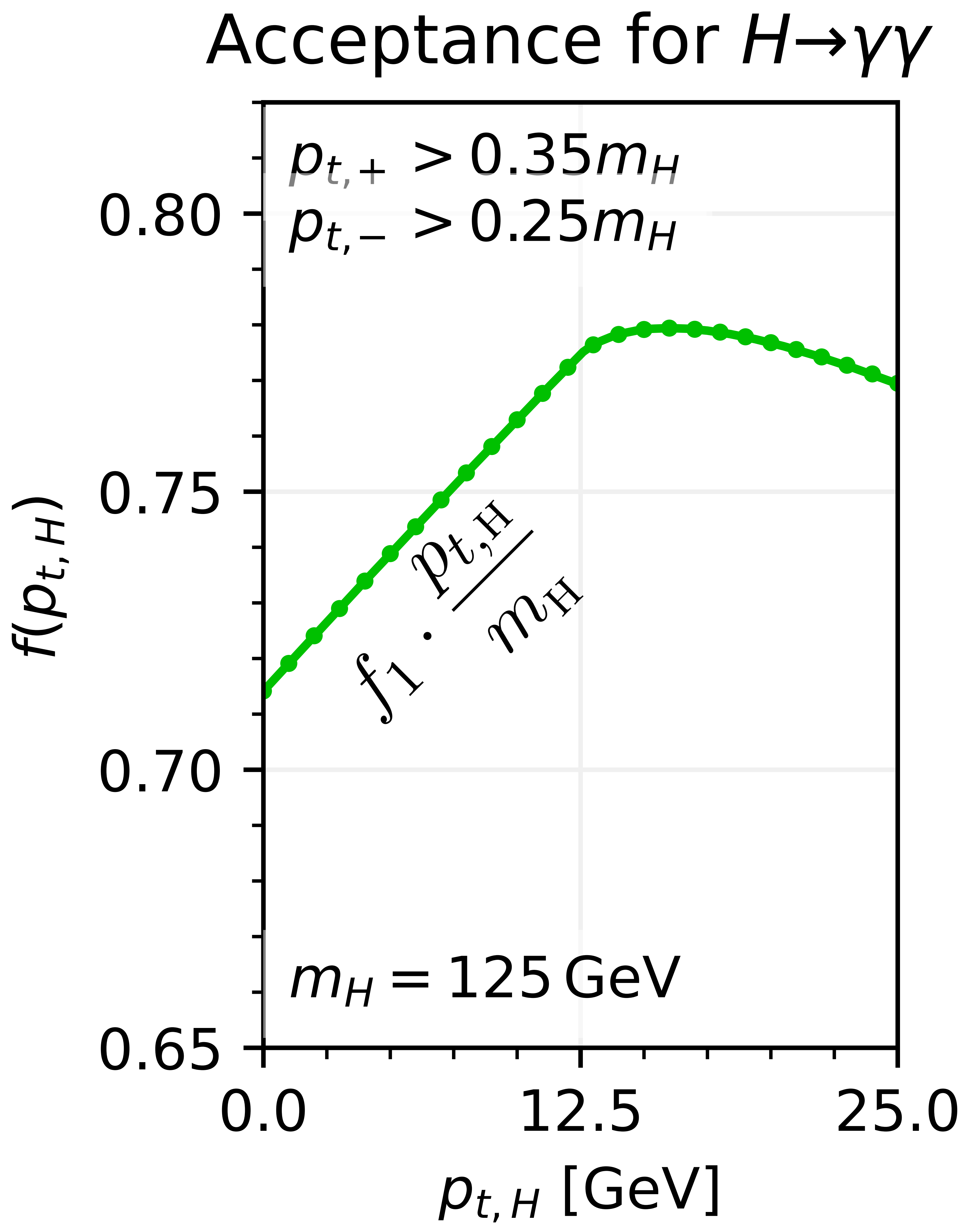
$$\frac{d\sigma^{\mathrm{DL}}}{dp_{t,\mathrm{H}}} = \frac{\sigma_{\mathrm{tot}}}{p_{t,\mathrm{H}}} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2 \log^{2n-1} \frac{m_\mathrm{H}}{2p_{t,\mathrm{H}}}}{(n-1)!} \left(\frac{2C_A \alpha_s}{\pi}\right)^n$$

Linear p_{tH} dependence of H acceptance, $f(p_{tH})$ → impact on perturbative series



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$$\sigma_{\mathrm{fid}} = \int \frac{d\sigma^{\mathrm{DL}}}{dp_{t,\mathrm{H}}} f(p_{t,\mathrm{H}})\, dp_{t,\mathrm{H}}$$

$$= \sigma_{\mathrm{tot}} \left[f_0 + f_1 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2n)!}{2(n!)} \left(\frac{2C_A\alpha_s}{\pi}\right)^n + \cdots \right]$$

Growth ∝ n!

Behaviour of perturbative series in various log approximations

Resummed results

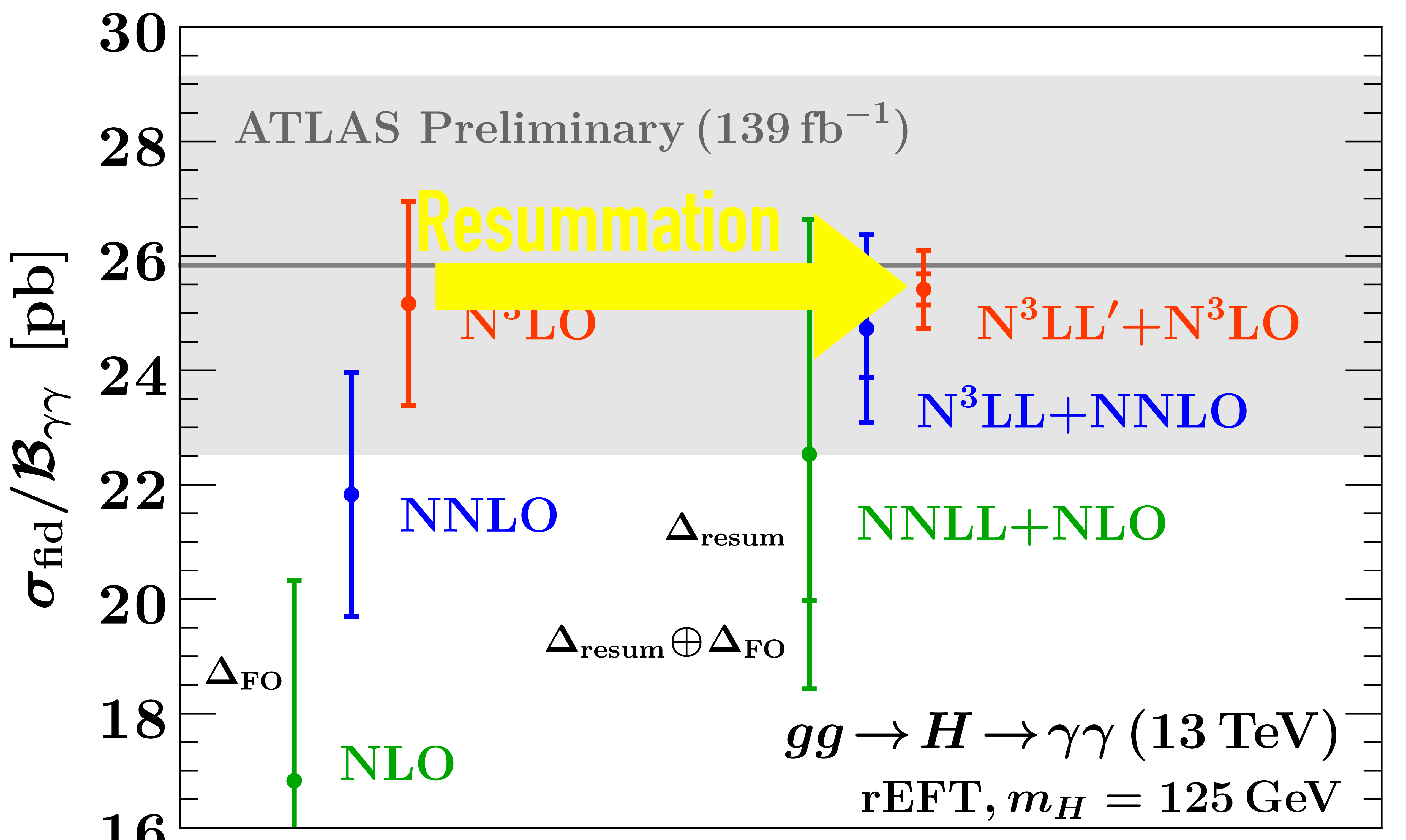
$$
\begin{aligned}
\frac{\sigma_{\text{asym}} - f_0\sigma_{\text{inc}}}{\sigma_0 f_0} &\simeq 0.15_{\alpha_s} - 0.29_{\alpha_s^2} + 0.71_{\alpha_s^3} - 2.39_{\alpha_s^4} + 10.26_{\alpha_s^5} + \ldots &\simeq 0.06 \quad \text{@DL},\\
&\simeq 0.15_{\alpha_s} - 0.23_{\alpha_s^2} + 0.44_{\alpha_s^3} - 1.15_{\alpha_s^4} + 3.83_{\alpha_s^5} + \ldots &\simeq 0.06 \quad \text{@LL},\\
&\simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.29_{\alpha_s^3} + \ldots &\simeq 0.10 \quad \text{@NNLL},\\
&\simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.31_{\alpha_s^3} + \ldots &\simeq 0.12 \quad \text{@N3LL}.
\end{aligned}
$$

Thanks to Pier Monni & RadISH for supplying NN(N)LL distributions & expansions, $\mu = m_H/2$

- At DL & LL (DL+running coupling) **factorial divergence sets in from first orders**
- Poor behaviour of N3LL is qualitatively similar to that seen by Billis et al '21
- At N3LO, there is extreme sensitivity to unphysically low p_{tH} (down to ~ 1 MeV, see backup)
- **Is the only solution to do resummation?**

Solution #1: use p_{tH} resummation

- Billis, Dehnadi, Ebert, Michel & Tackmann, 2102.08039, argue you should evaluate the fiducial cross section only after resummation of the p_{tH} distribution.
- For legacy measurements, resummation is only viable solution
- Our view: not an ideal solution
  - Fiducial σ is a hard cross section and shouldn't need resummation
  - losing the ability to use fixed order on its own would be a big blow to the field (e.g. flexibility; robustness of seeing fixed-order & resummation agree)



Billis, Dehnadi, Ebert, Michel & Tackmann, 2102.08039

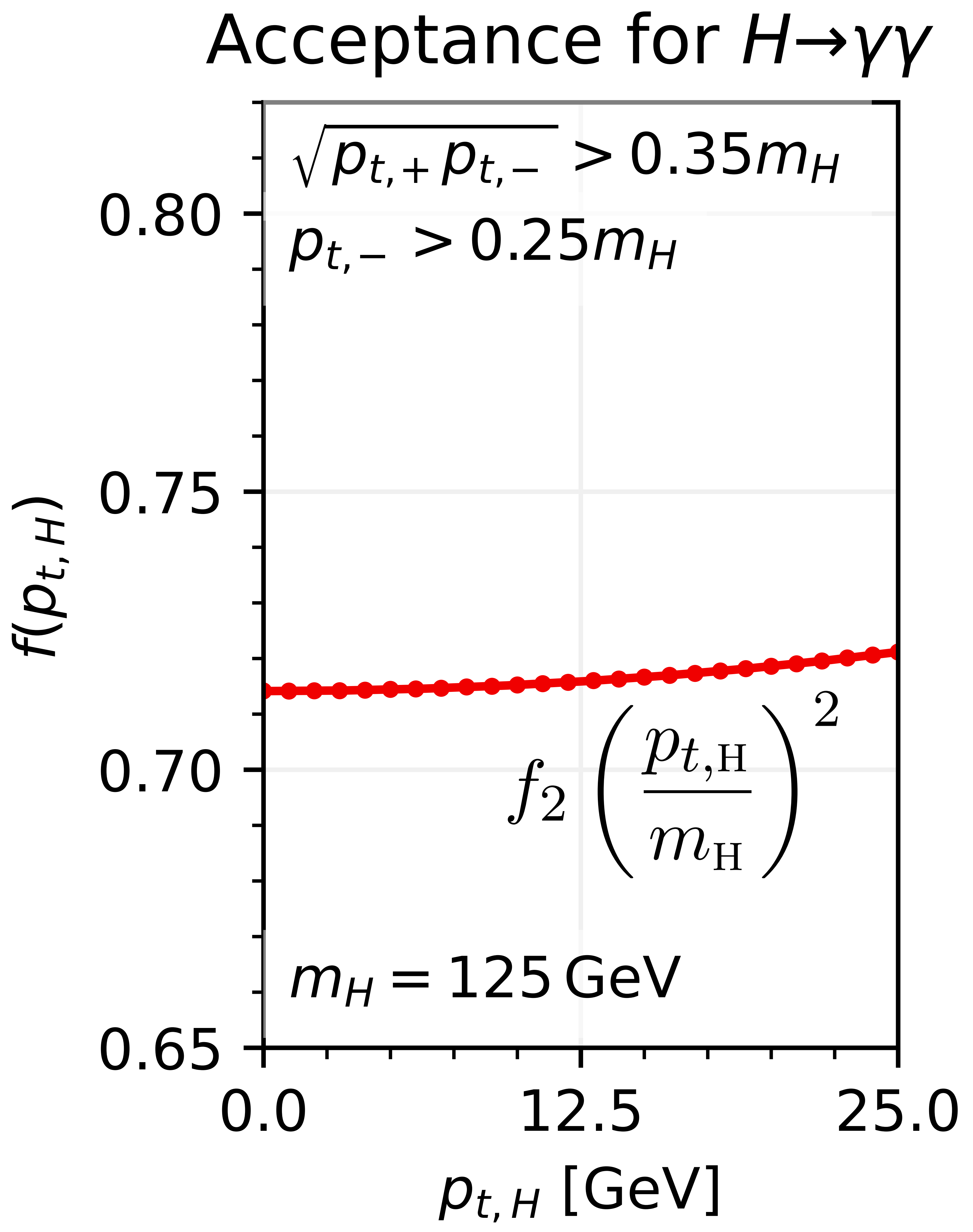
Solution #2: for future measurements, change the cuts

- Simplest option is to replace the cut on the leading photon with a cut on the product of the two photon p_t's
- E.g. $p_{t,\gamma+}\, p_{t,\gamma-} > (0.35 m_H)^2$ (and still keep softer photon cut $p_{t,\gamma-} > 0.25 m_H$)
- The product has no linear dependence on $p_{t,H}$

$$p_{t,\text{prod}}(p_{t,\text{H}}, \theta, \phi) = \sqrt{p_{t,+} p_{t,-}} = \frac{m_\text{H}}{2} \sin\theta + \frac{p_{t,\text{H}}^2}{4 m_\text{H}} \frac{\sin^2\phi - \cos^2\theta \cos^2\phi}{\sin\theta} + \mathcal{O}_4$$

[Several other options are possible, but this combines simplicity and good performance]

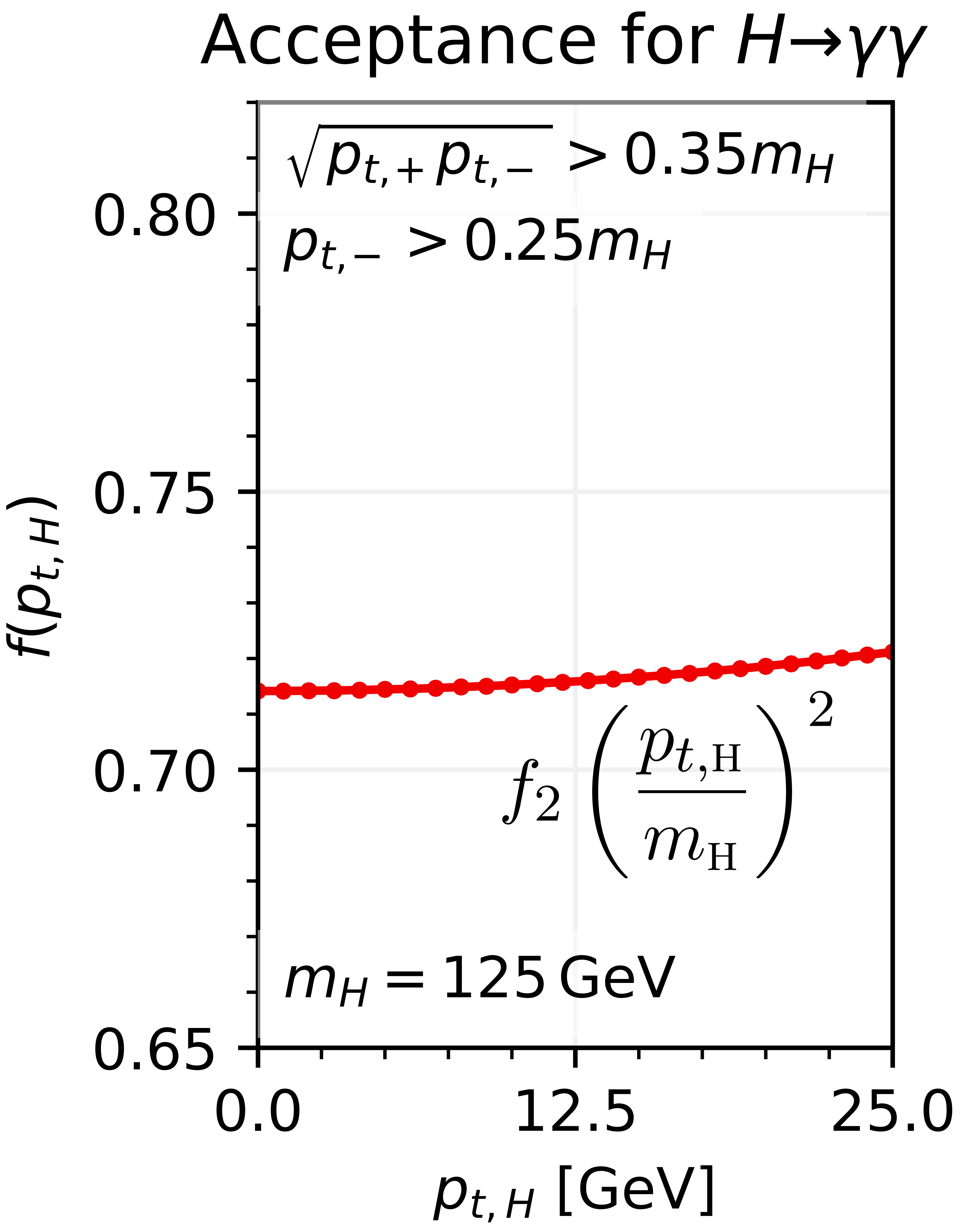
Replace cut on leading photon → cut on product of photon p_t's



$$f(p_{t,\mathrm{H}}) = f_0 + f_2 \left(\frac{p_{t,\mathrm{H}}}{m_\mathrm{H}}\right)^2 + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

linear → quadratic

Replace cut on leading photon → cut on product of photon p_t's



$$f(p_{t,\mathrm{H}}) = f_0 + f_2\left(\frac{p_{t,\mathrm{H}}}{m_\mathrm{H}}\right)^2 + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

linear → quadratic

$$\frac{(2n)!}{2(n!)}\left(\frac{2C_A\alpha_s}{\pi}\right)^n \to \frac{1}{4^n}\frac{(2n)!}{4(n!)}\left(\frac{2C_A\alpha_s}{\pi}\right)^n$$

Using product cuts dampens the factorial divergence

Behaviour of perturbative series with product cuts

$$\frac{\sigma_{\rm prod} - f_0\sigma_{\rm inc}}{\sigma_0 f_0} \simeq 0.005_{\alpha_s} - 0.002_{\alpha_s^2} + 0.002_{\alpha_s^3} - 0.001_{\alpha_s^4} + 0.001_{\alpha_s^5} + \ldots \simeq 0.003 \quad @\text{DL},$$

$$\simeq 0.005_{\alpha_s} - 0.002_{\alpha_s^2} + 0.000_{\alpha_s^3} - 0.000_{\alpha_s^4} + 0.000_{\alpha_s^5} + \ldots \simeq 0.003 \quad @\text{LL},$$

$$\simeq 0.005_{\alpha_s} + 0.002_{\alpha_s^2} - 0.001_{\alpha_s^3} + \ldots \simeq 0.005 \quad @\text{NNLL},$$

$$\simeq 0.005_{\alpha_s} + 0.002_{\alpha_s^2} - 0.001_{\alpha_s^3} + \ldots \simeq 0.006 \quad @\text{N3LL}.$$

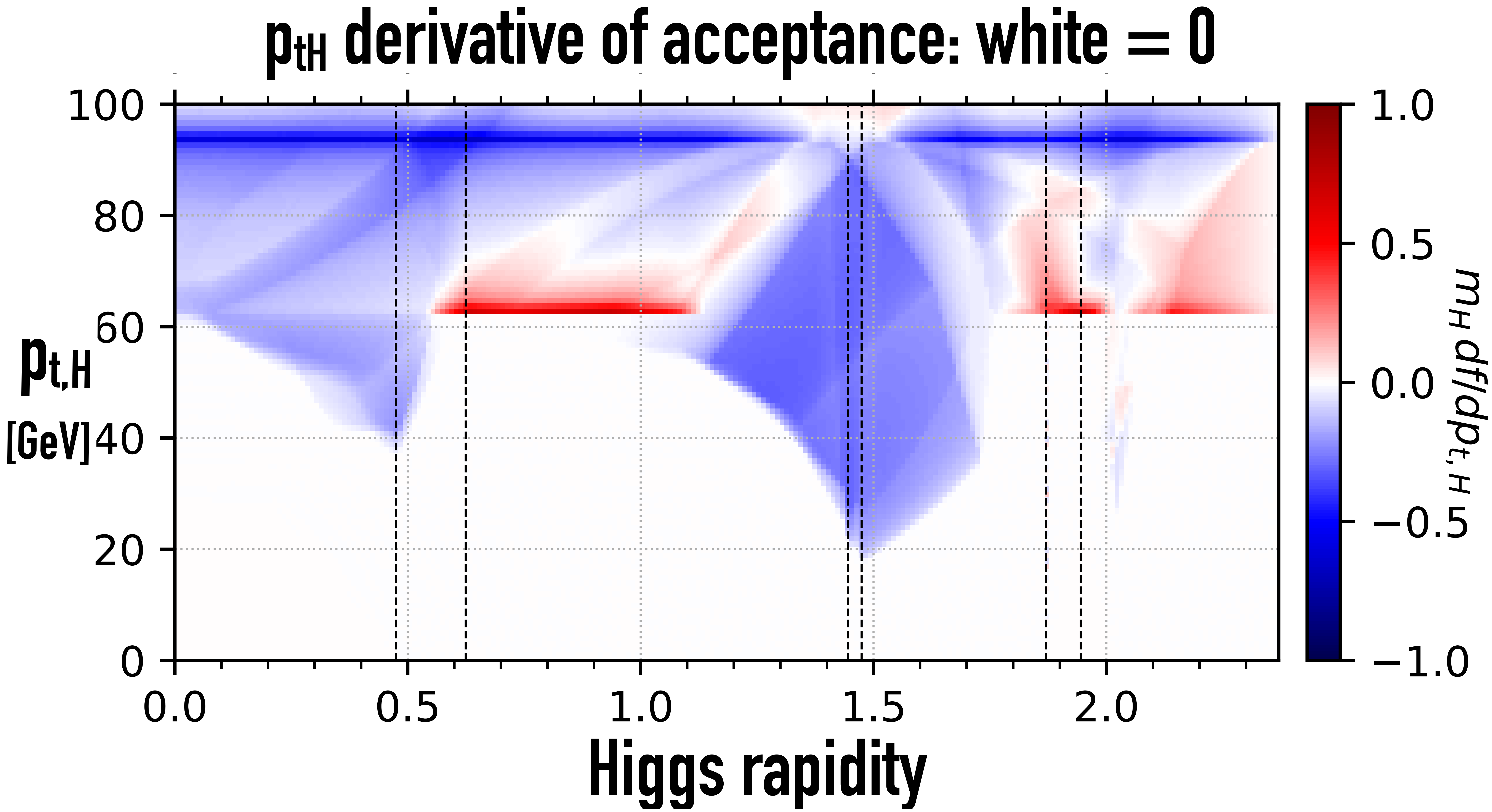
Resummed results

Thanks to Pier Monni & RadISH for supplying NN(N)LL distributions & expansions, $\mu = m_H/2$

- Factorial growth of series strongly suppressed
- **N3LO truncation agrees well with all-order result**
- Per mil agreement between fixed-order and resummation **gives confidence that all is under control**

More in arXiv:2106.08329 + code at https://github.com/gavinsalam/two-body-cuts

- interplay with rapidity cuts (product cuts basically remain OK)
- cuts where the **acceptance is independent of p_{tH}** at small p_{tH} (should have perturbative properties of a high-p_{tH} cross section; uses Collins-Soper type variables)
- outline of extension to Drell-Yan



hardness and rapidity compensating (CBI_{HR}) cuts

$p_{t,-} > 0.25 m_H$, $|y_\gamma| < 2.37$
not $1.37 < |y_\gamma| < 1.52$

Conclusions

- Fixed-order perturbation theory can be badly compromised by existing (2-body) cuts ($\rightarrow$ intriguing questions about asymptotics of QCD perturbative series)
- In simple cases (e.g. $H \rightarrow \gamma\gamma$), can be solved by resummation. But physics will be more robust if we can reliably use both fixed-order and resummed+FO results (and both yield similar central values & uncertainties)
- A better long-term solution may be to **revisit experimental cuts**:
  - product and boost-invariant cuts give much better perturbative series
  - Likely relevant also for other processes (see backup for DY: effects at the 1%-level)
- Cuts with little p_{tH} dependence may be useful also, e.g., for extrapolating measurements to STXS or more inclusive cross sections, with limited dependence on BSM or non-pert effects.
- **Needs addressing in future LHC measurements for robust accuracy in Run 3 & HL-LHC**

Backup

Cut Type	cuts on	small-$p_{t,\mathrm{H}}$ dependence	f_n coefficient	$p_{t,\mathrm{H}}$ transition
symmetric	$p_{t,-}$	linear	$+2s_0/(\pi f_0)$	none
asymmetric	$p_{t,+}$	linear	$-2s_0/(\pi f_0)$	Δ
sum	$\frac{1}{2}(p_{t,-} + p_{t,+})$	quadratic	$(1+s_0^2)/(4f_0)$	2Δ
product	$\sqrt{p_{t,-} + p_{t,+}}$	quadratic	$s_0^2/(4f_0)$	2Δ
staggered	$p_{t,1}$	quadratic	$s_0^4/(4f_0^3)$	Δ
Collins-Soper	$p_{t,\mathrm{CS}}$	none	—	2Δ
CBI_H	$p_{t,\mathrm{CS}}$	none	—	$2\sqrt{2}\Delta$
rapidity	y_γ	quadratic	$f_0 s_0^2/2$	

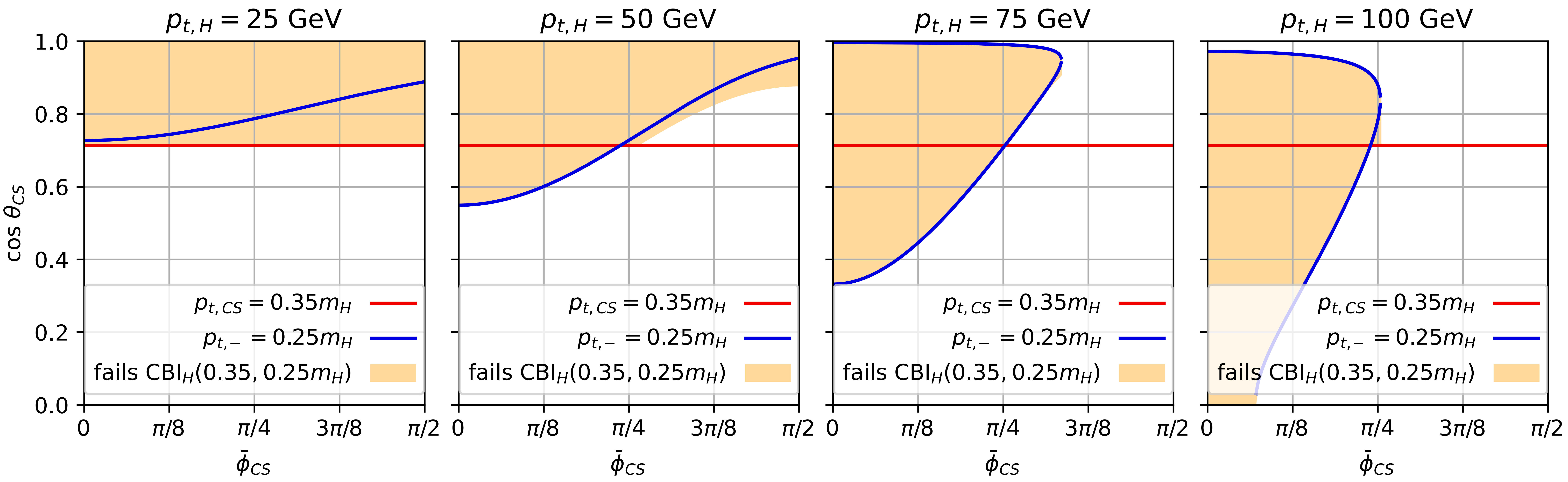
Table 1: Summary of the main hardness cuts, the variable they cut on at small $p_{t,\mathrm{H}}$, and the small-$p_{t,\mathrm{H}}$ dependence of the acceptance. For linear cuts $f_n \equiv f_1$ multiplies $p_{t,\mathrm{H}}/m_\mathrm{H}$, while for quadratic cuts $f_n \equiv f_2$ multiplies $(p_{t,\mathrm{H}}/m_\mathrm{H})^2$ (in all cases there are additional higher order terms that are not shown). For a leading threshold of $p_{t,\mathrm{cut}}$, $s_0 = 2p_{t,\mathrm{cut}}/m_\mathrm{H}$ and $f_0 = \sqrt{1-s_0^2}$, while for the rapidity cut $s_0 = 1/\cosh(y_\mathrm{H} - y_\mathrm{cut})$. For a cut on the softer lepton's transverse momentum of $p_{t,-} > p_{t,\mathrm{cut}} - \Delta$, the right-most column indicates the $p_{t,\mathrm{H}}$ value at which the $p_{t,-}$ cut starts to modify the behaviour of the acceptance (additional $\mathcal{O}\left(\Delta^2/m_\mathrm{H}\right)$ corrections not shown). For the interplay between hardness and rapidity cuts, see sections 4.2, 4.3 and 5.2.

Hardness [and rapidity] compensating boost invariant cuts (CBI_H and CBI_{HR})

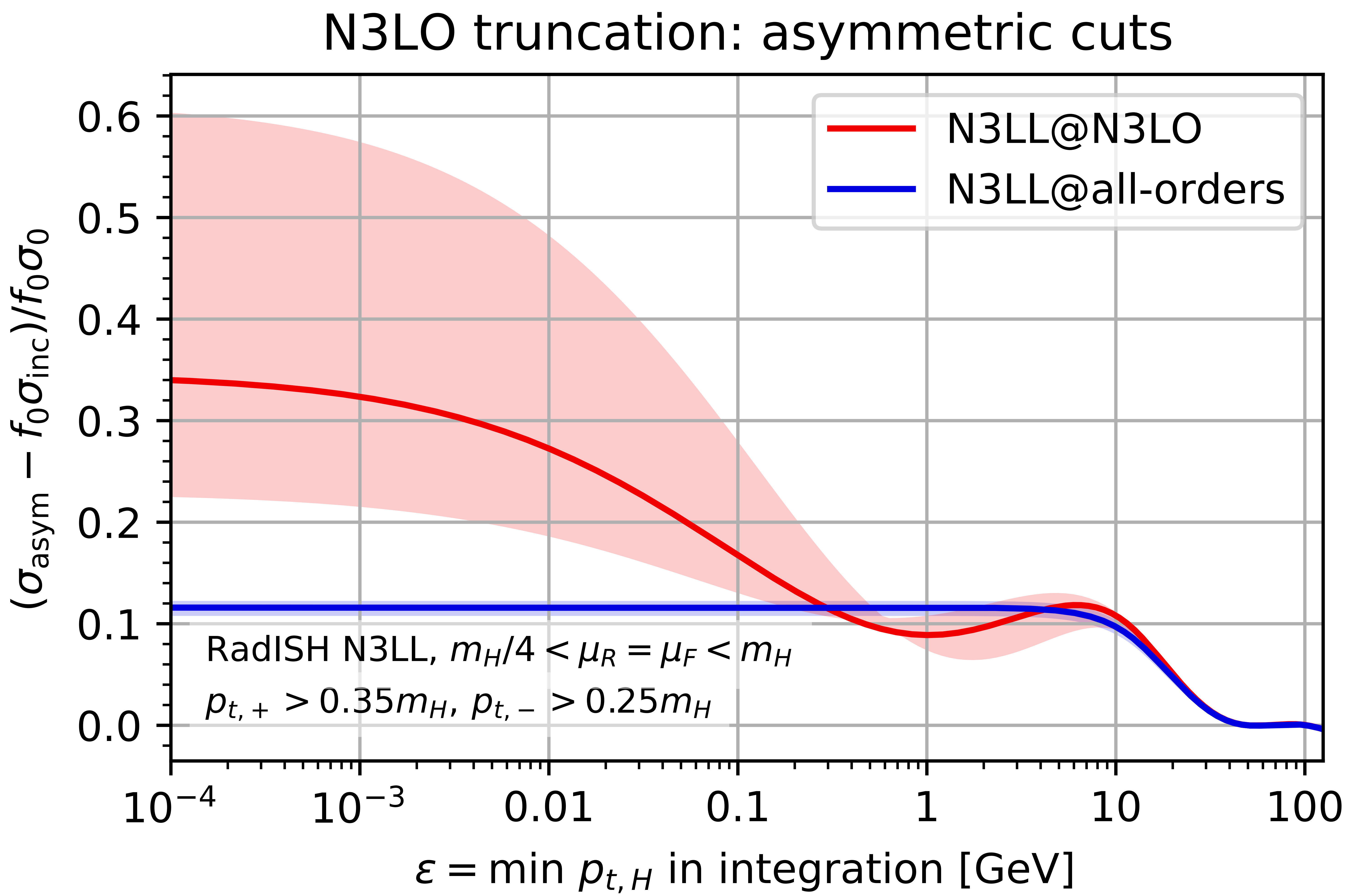
Core idea 1: cut on decay p_t in Collins-Soper frame

$$\vec{p}_{t,\text{CS}} = \frac{1}{2}\left[\vec{\delta}_t + \frac{\vec{p}_{t,12}.\vec{\delta}_t}{p_{t,12}^2}\left(\frac{m_{12}}{\sqrt{m_{12}^2+p_{t,12}^2}}-1\right)\vec{p}_{t,12}\right], \qquad \vec{\delta}_t = \vec{p}_{t,1} - \vec{p}_{t,2}$$

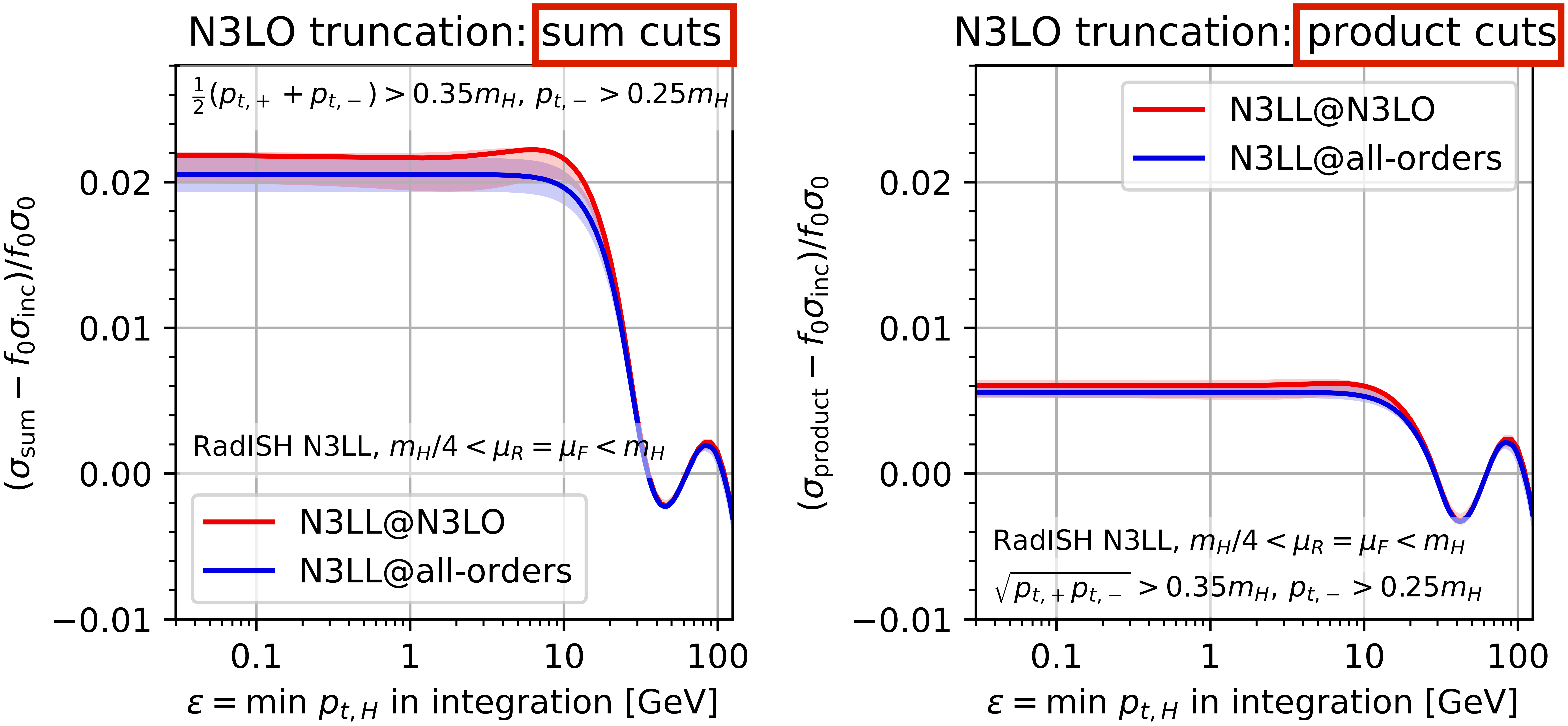
Core idea 2: relax $p_{t,CS}$ cut at higher $p_{t,H}$ values to maintain constant / maximal acceptance



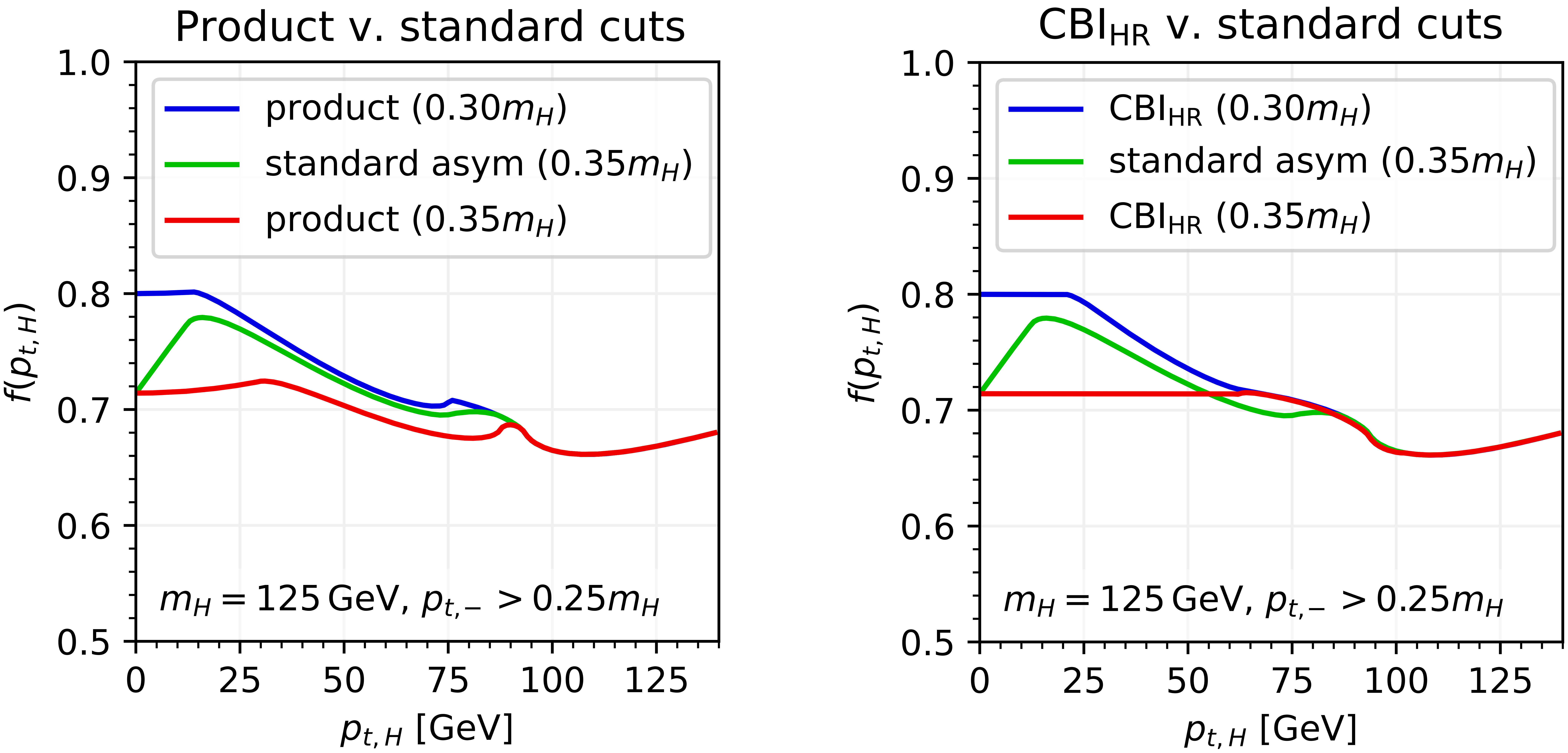
Sensitivity to low Higgs p_t (and also scale bands): standard cuts



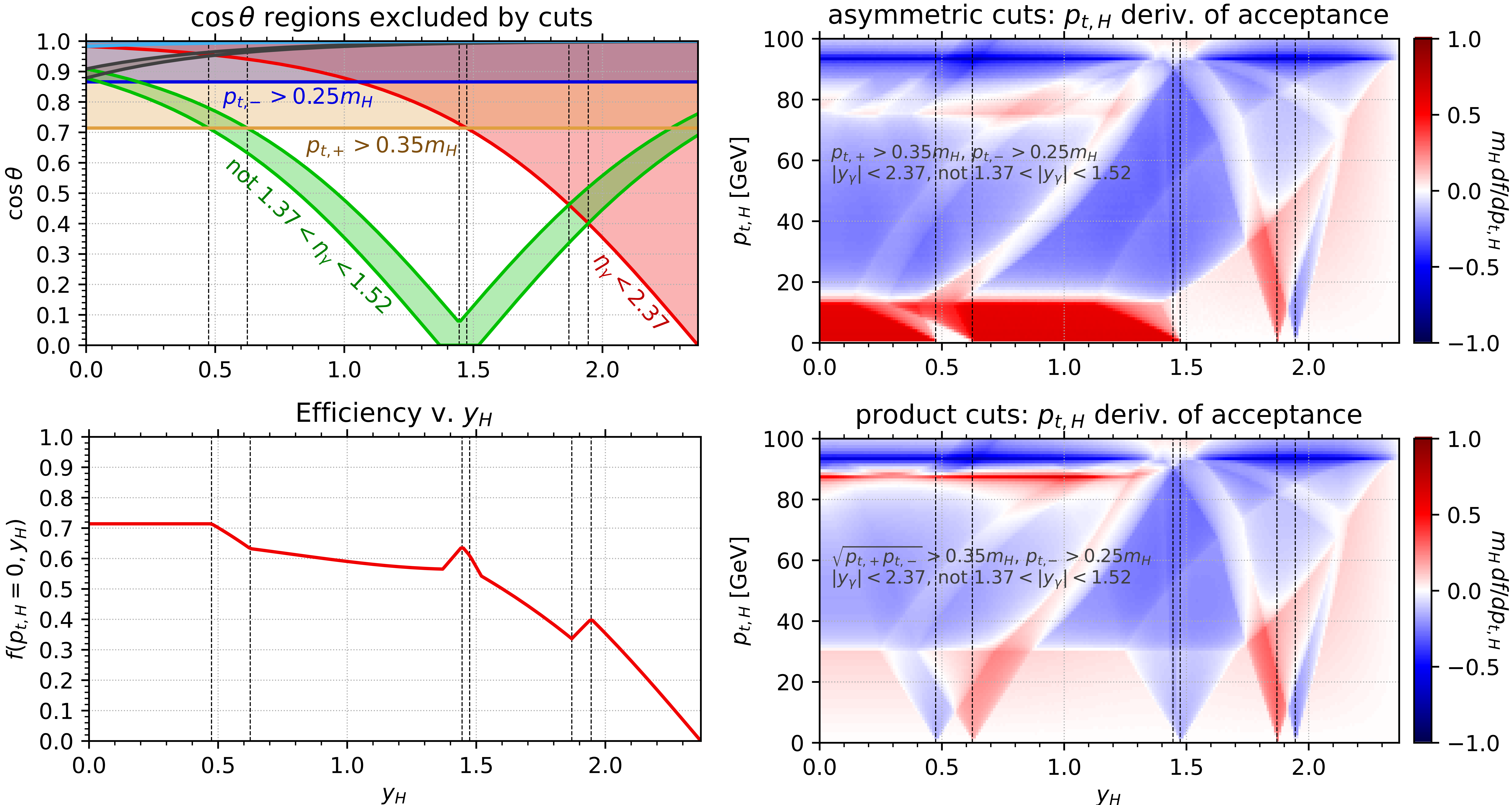
Sensitivity to low Higgs p_t (and also scale bands): sum & product cuts



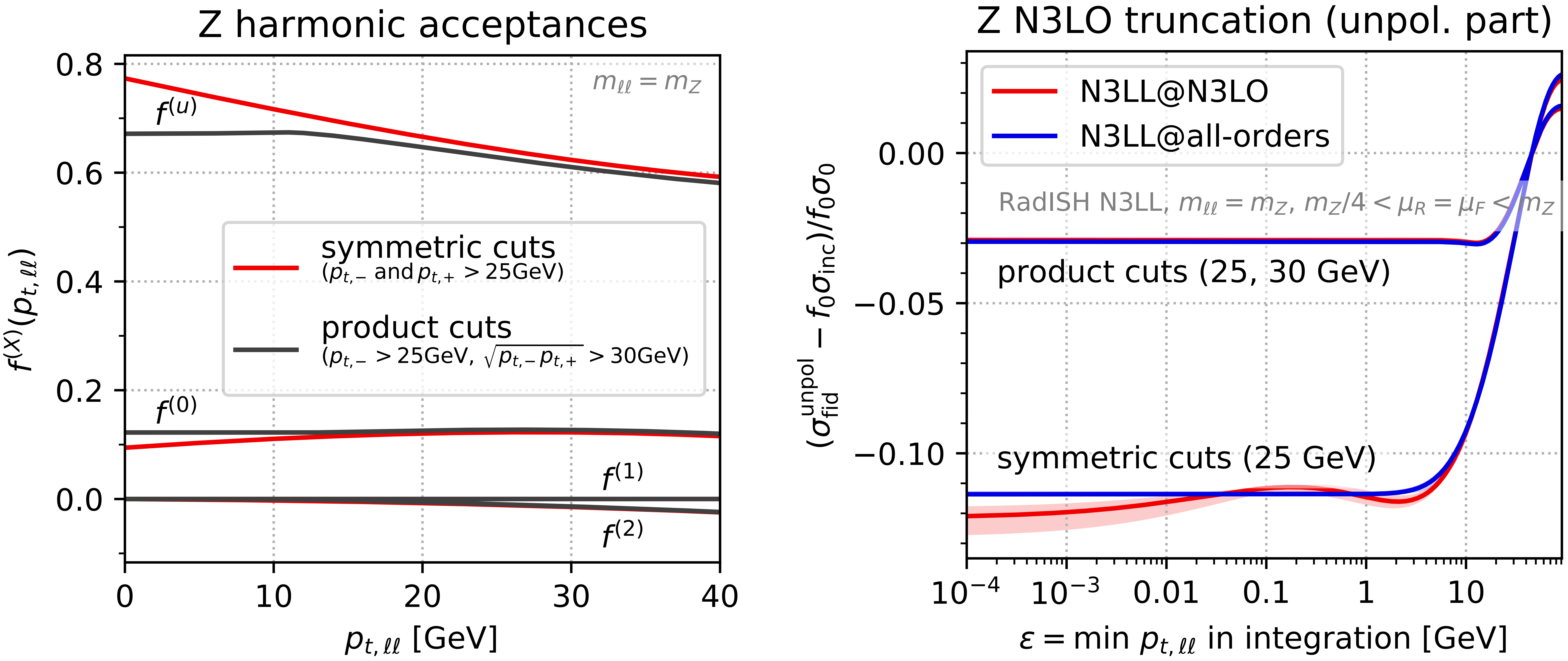
Option of changing thresholds



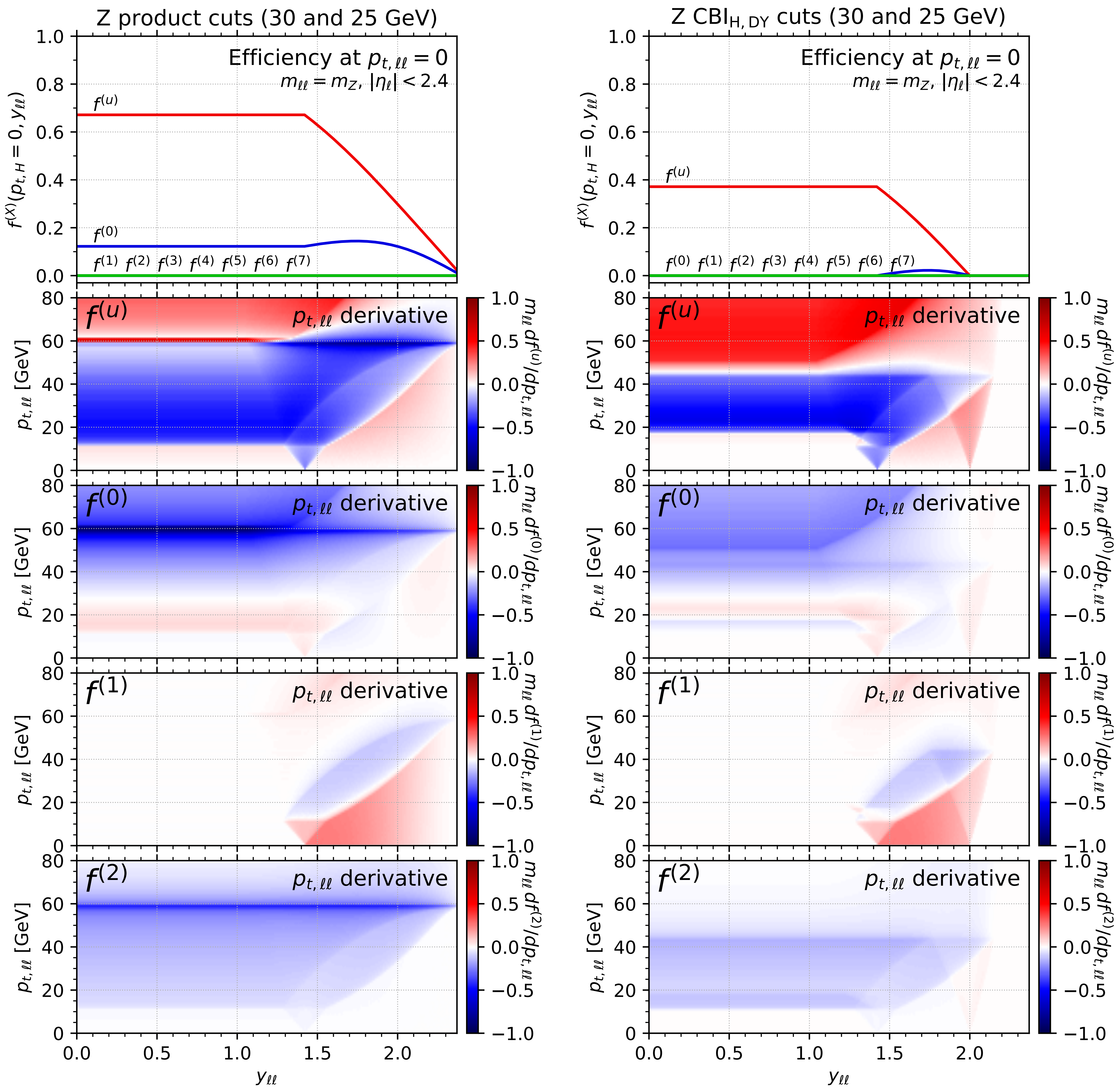
Interplay with rapidity cuts



Example in Drell-Yan case



DY p_t dependence of harmonic acceptances with product and boost invariant cuts



Getting identically zero pt dependence for all harmonic acceptances requires an extra cut

$$\cos\theta > \bar{c} = \frac{-c_0 + \sqrt{4 - 3c^2}}{2}$$