CUTS FOR 2-BODY DECAYS AT COLLIDERS

18th workshop of the LHC Higgs Working Group, 3 December 2021

Gavin Salam, with Emma Slade, arXiv:2106.08329
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Precision is crucial part of LHC programme: e.g. establishing the Higgs sector

Over the next 15 years

Today's ~8% (on H→γγ)

→ ~2% at HL-LHC

We wouldn't consider QED established if it had only been tested at O(10%) accuracy

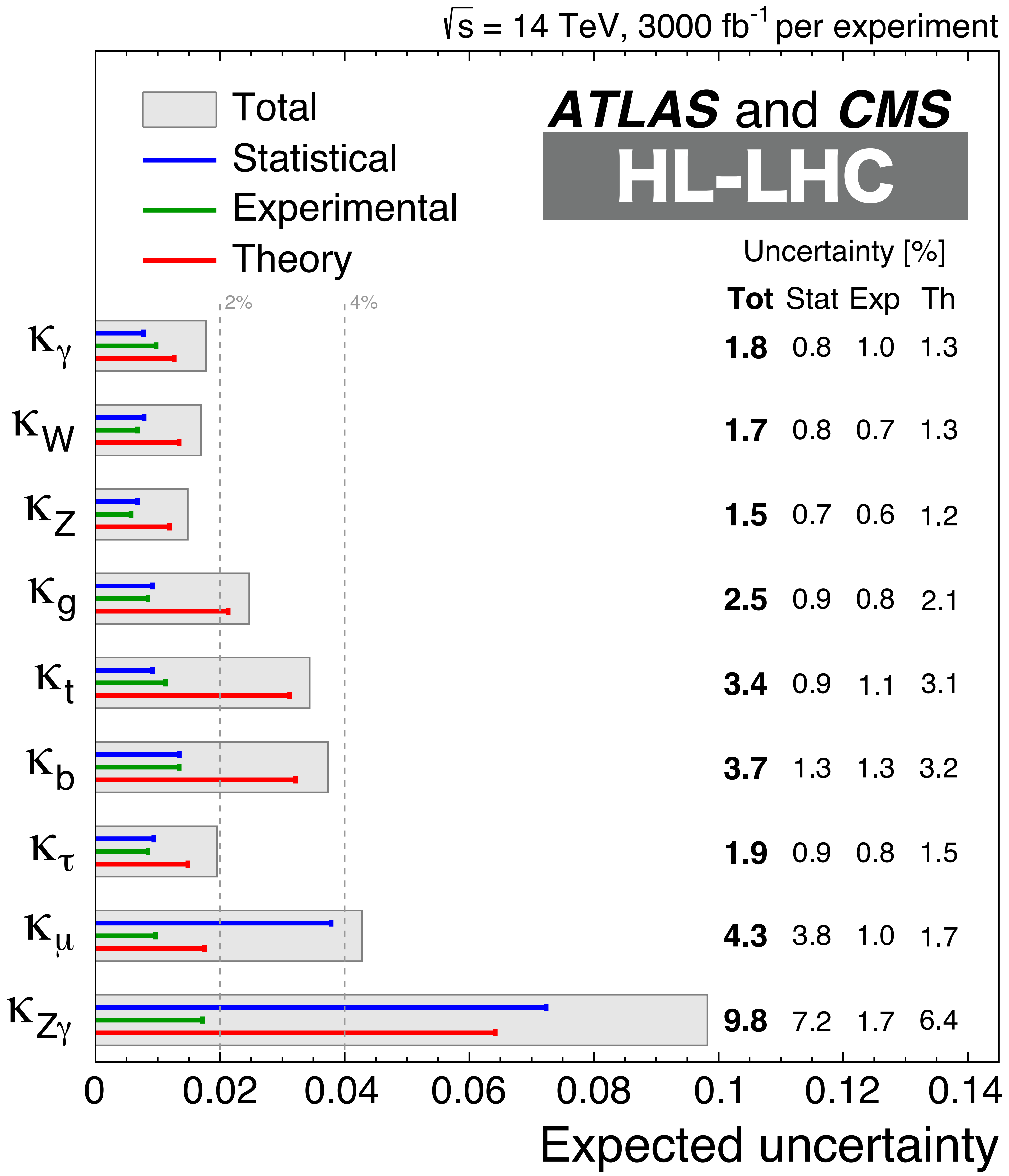


Figure 1. Projected uncertainties on κ_i, combining ATLAS and CMS: total (grey box), statistical (blue), experimental (green) and theory (red). From Ref. [2].

Starting point for any hadron-collider analysis: acceptance (fiducial) cuts

E.g. ATLAS/CMS $H \to \gamma\gamma$ cuts

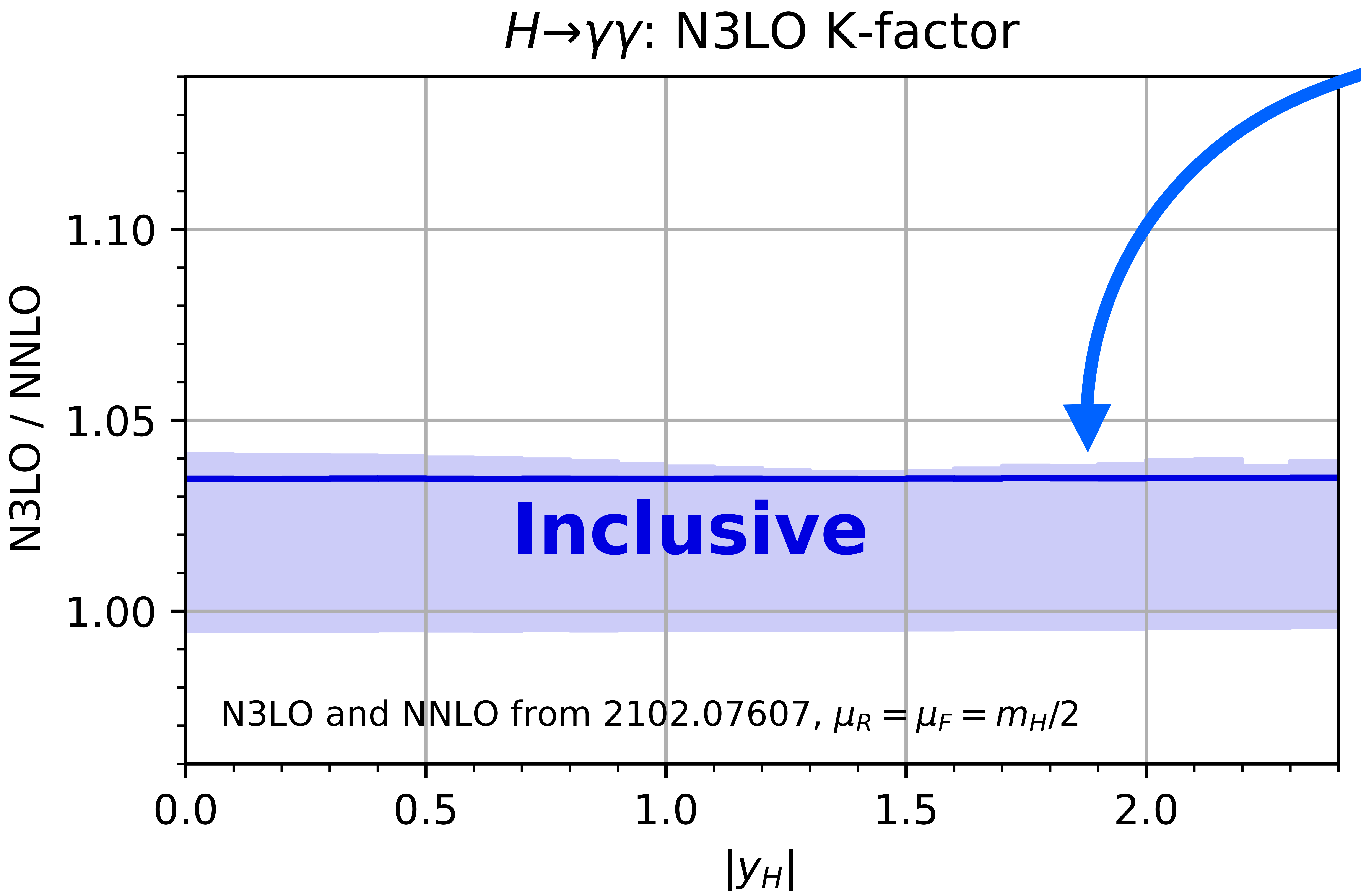
- Higher-p_t photon: $p_{t,\gamma} > 0.35 m_{\gamma\gamma}$ (ATLAS) or $m_{\gamma\gamma}/3$ (CMS)
- Lower-p_t photon: $p_{t,\gamma} > 0.25 m_{\gamma\gamma}$
- Both photons: additional rapidity and isolation cuts

Essential for good reconstruction of the photons and for rejecting large low-p_t backgrounds.

> Theory-experiment comparisons with identical "fiducial" cuts often considered the Gold Standard of collider physics

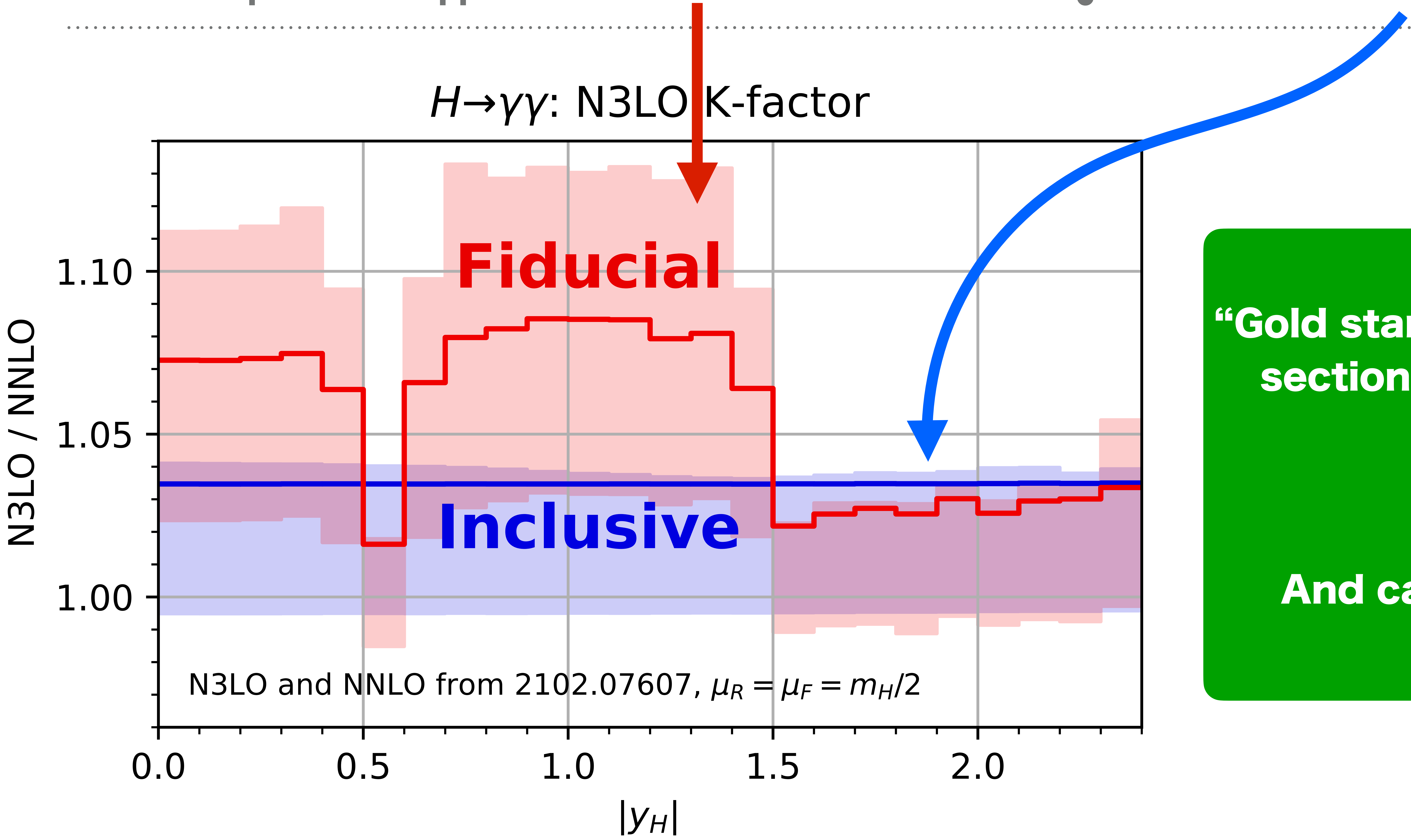
Recent surprise: H→γγ

inclusive N3LO σ uncertaities



Chen, Gehrmann, Glover, Huss, Mistlberger & Pelloni, 2102.07607

Recent surprise: H→γγ fiducial N3LO σ uncertainties ~2× greater than inclusive N3LO σ uncertaities

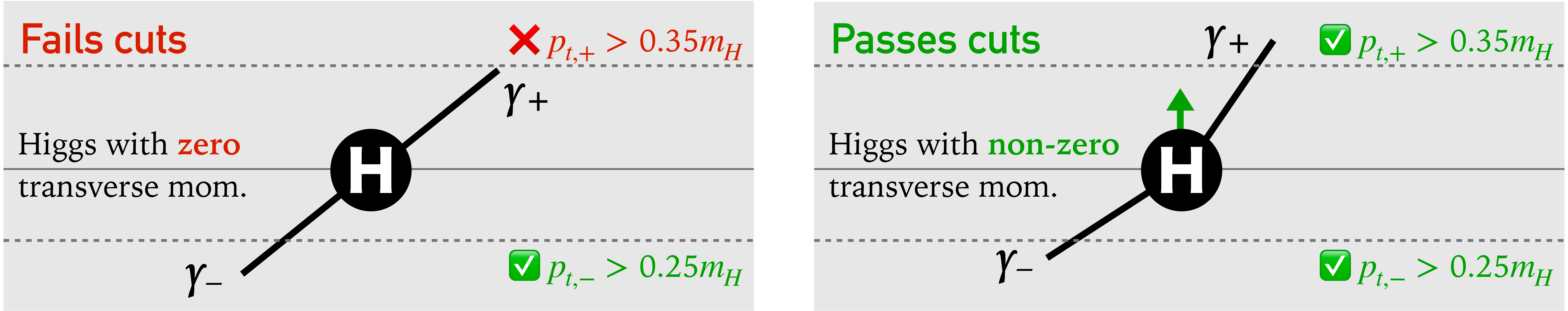


Chen, Gehrmann, Glover, Huss, Mistlberger & Pelloni, 2102.07607

"Gold standard" fiducial cross section gives much worse prediction

Why?
And can this be solved?

Standard $p_{t,\gamma}$ cuts → Higgs p_t dependence of acceptance

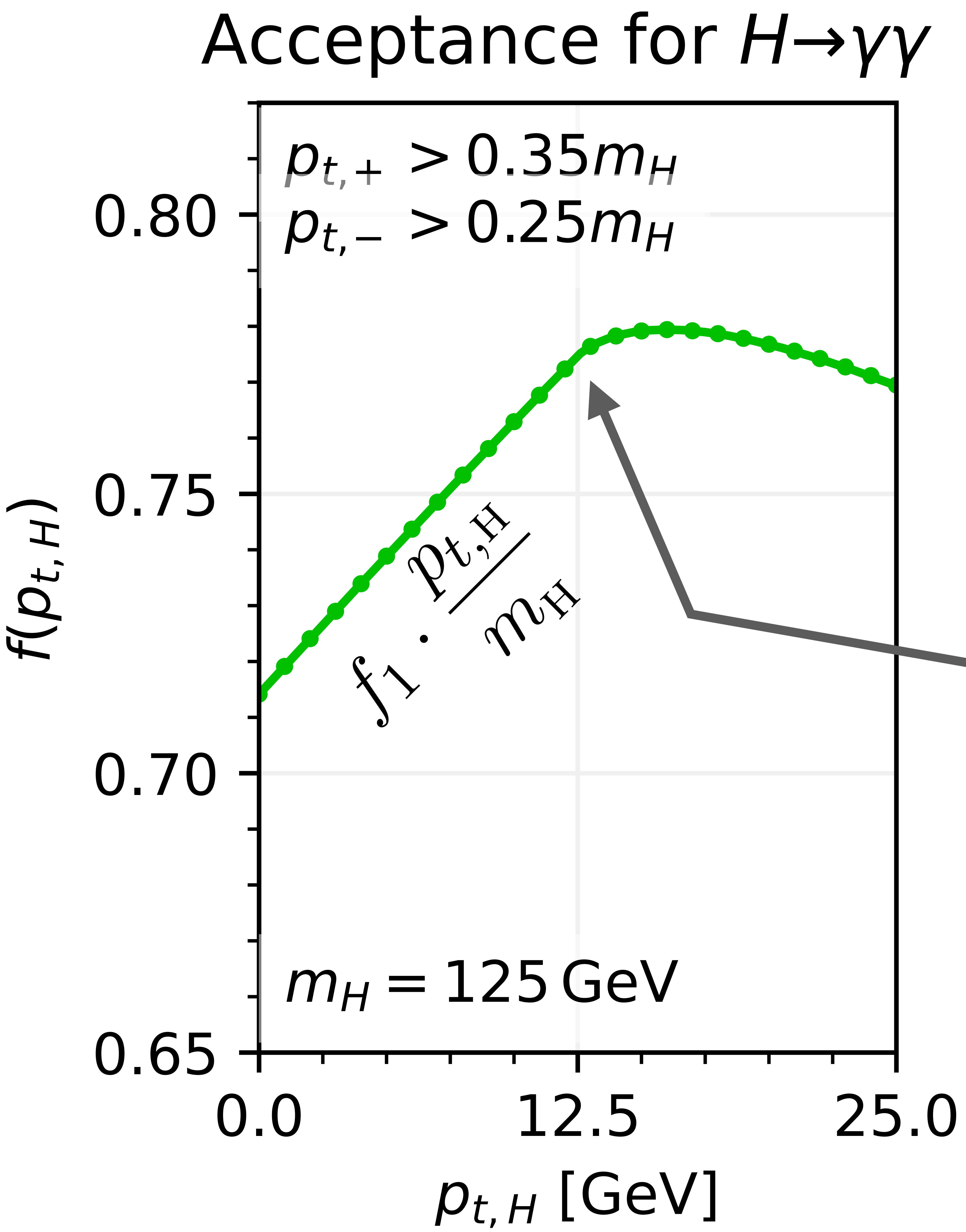


Numbers are for ATLAS H→ γγ p_t cuts, CMS cuts are similar

Expect acceptance to **increase with increasing $p_{t,\mathrm{H}}$**

$$p_{t,\pm}(p_{t,\mathrm{H}}, \theta, \phi) = \frac{m_\mathrm{H}}{2} \sin\theta \pm \frac{1}{2} p_{t,\mathrm{H}} |\cos\phi| + \frac{p_{t,\mathrm{H}}^2}{4 m_\mathrm{H}} \left(\sin\theta \cos^2\phi + \csc\theta \sin^2\phi\right) + \mathcal{O}_3\,,$$

Linear p_{tH} dependence of H acceptance ≡ $f(p_{tH})$



$$f(p_{t,\mathrm{H}}) = f_0 + f_1 \cdot \frac{p_{t,\mathrm{H}}}{m_\mathrm{H}} + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

See e.g. Frixione & Ridolfi '97
Ebert & Tackmann '19
idem + Michel & Stewart '20
Alekhin et al '20

f_0 and f_1 are coefficients whose values depend on the cuts

effect of $p_{t,-}$ cut sets in at $0.1 m_\mathrm{H}$

perturbative series for fiducial cross sections

$$f(p_{t,\mathrm{H}}) = f_0 + f_1 \cdot \frac{p_{t,\mathrm{H}}}{m_\mathrm{H}} + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

Fiducial cross section depends on acceptance and Higgs p_t distribution

$$\sigma_{\mathrm{fid}} = \int \frac{d\sigma}{dp_{t,\mathrm{H}}} f(p_{t,\mathrm{H}}) dp_{t,\mathrm{H}}$$

To understand qualitative perturbative behaviour consider simple (double-log) approx for p_t distribution

$$\frac{d\sigma^{\mathrm{DL}}}{dp_{t,\mathrm{H}}} = \frac{\sigma_{\mathrm{tot}}}{p_{t,\mathrm{H}}} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2 \log^{2n-1} \frac{m_\mathrm{H}}{2p_{t,\mathrm{H}}}}{(n-1)!} \left(\frac{2C_A \alpha_s}{\pi}\right)^n$$

Integrate to get result.
Observe pathological perturbative behaviour

$$\sigma_{\mathrm{fid}} = \sigma_{\mathrm{tot}} \left[f_0 + f_1 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2n)!}{2(n!)} \left(\frac{2C_A \alpha_s}{\pi}\right)^n + \cdots \right]$$

Growth $\propto n!$

Behaviour of perturbative series in various log approximations

Resummed results

$$\frac{\sigma_{\text{asym}} - f_0\sigma_{\text{inc}}}{\sigma_0 f_0} \simeq 0.15_{\alpha_s} - 0.29_{\alpha_s^2} + 0.71_{\alpha_s^3} - 2.39_{\alpha_s^4} + 10.31_{\alpha_s^5} + \dots \simeq 0.06 \quad \text{@DL},$$

$$\simeq 0.15_{\alpha_s} - 0.23_{\alpha_s^2} + 0.44_{\alpha_s^3} - 1.15_{\alpha_s^4} + 3.86_{\alpha_s^5} + \dots \simeq 0.06 \quad \text{@LL},$$

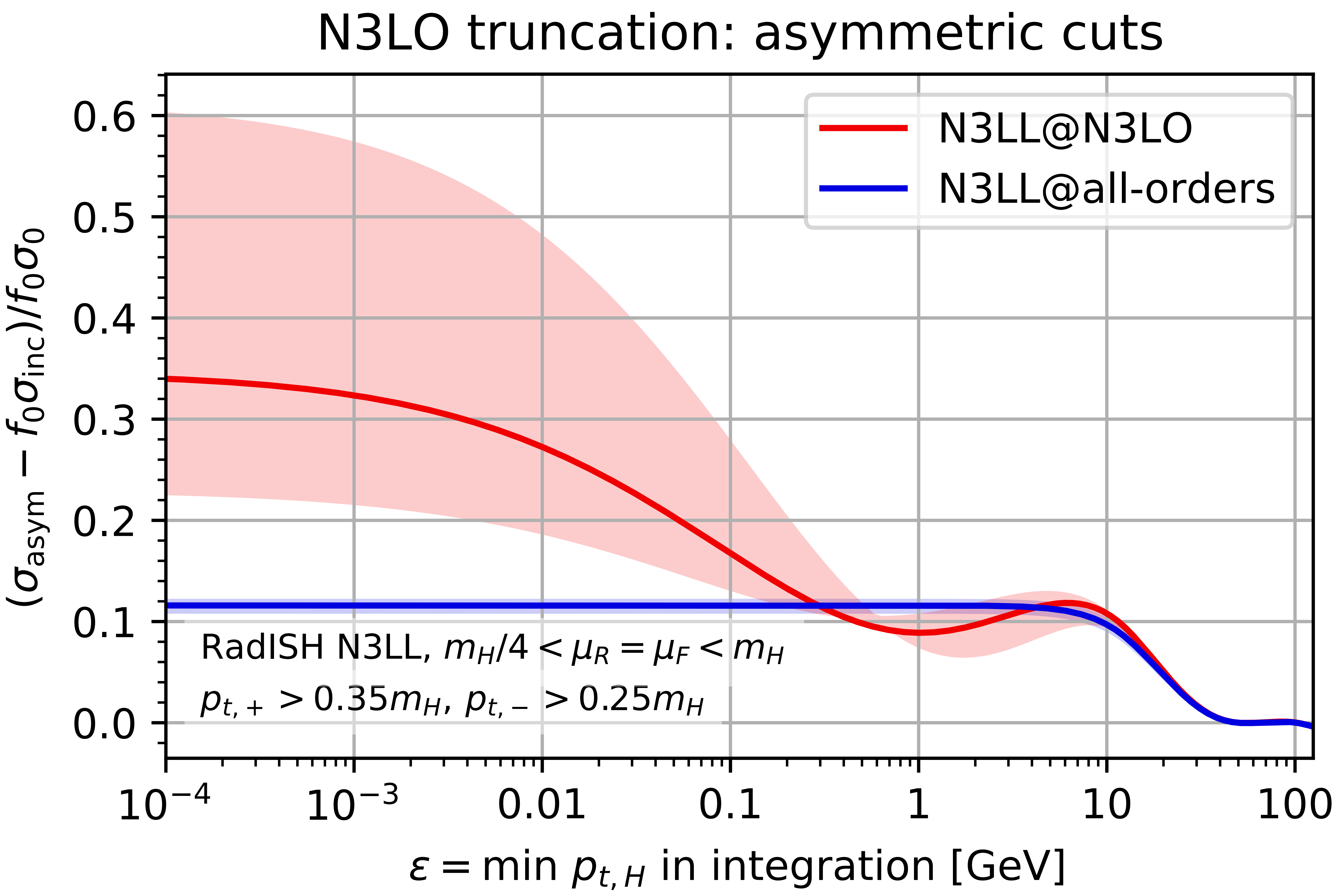
$$\simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.29_{\alpha_s^3} + \dots \simeq 0.10 \quad \text{@NNLL},$$

$$\simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.31_{\alpha_s^3} + \dots \simeq 0.12 \quad \text{@N3LL}.$$

Thanks to Pier Monni & RadISH for supplying NN(N)LL distributions & expansions, $\mu = m_H/2$

- At DL & LL (DL+running coupling) **factorial divergence sets in from first orders**
- Poor behaviour of N3LL is qualitatively similar to that seen by Billis et al '21
- Theoretically similar to a power-suppressed ambiguity $\sim (\Lambda_{\text{QCD}}/m_{\text{H}})^{0.205}$ [inclusive cross sections expected to have Λ^2/m^2]

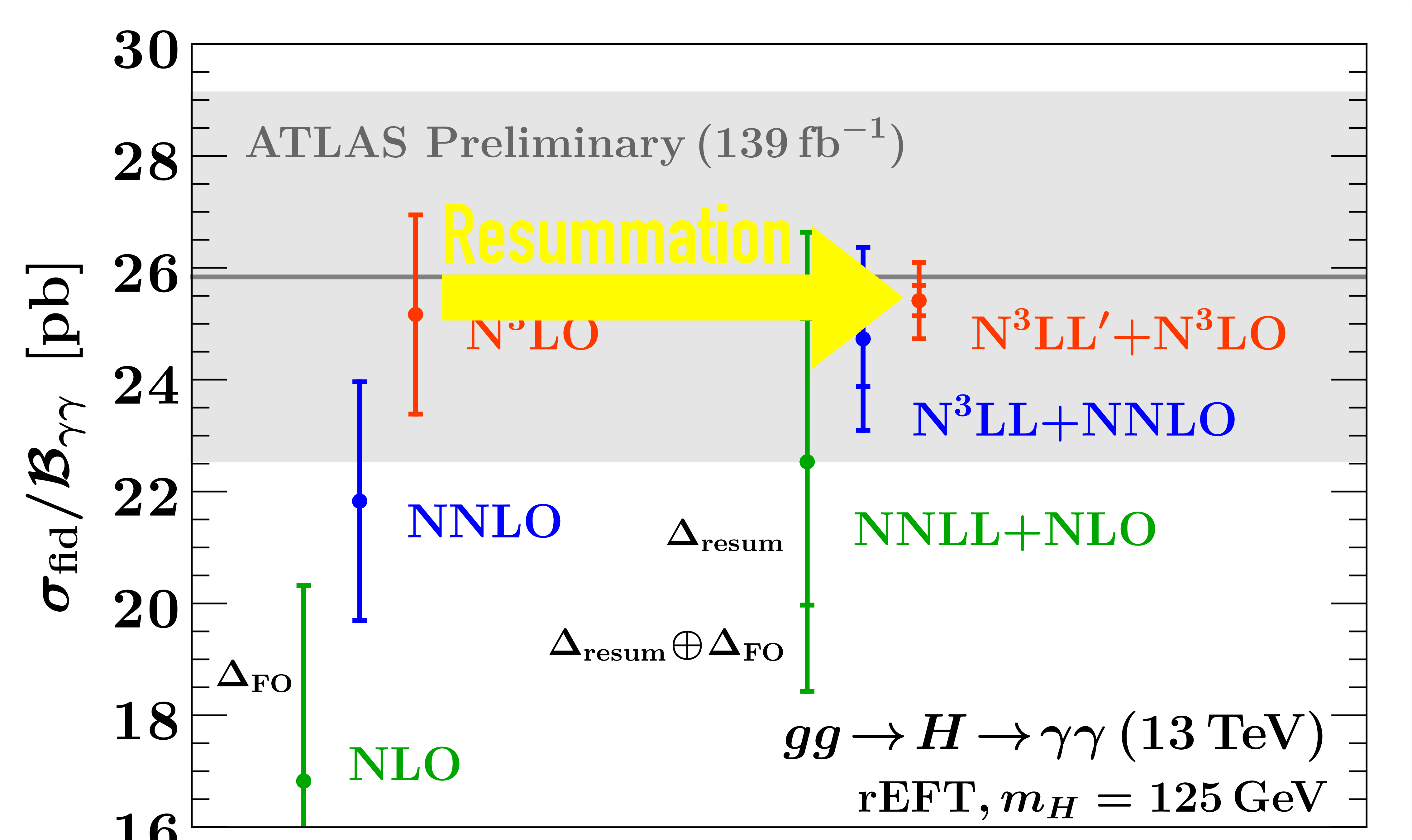
Sensitivity to low Higgs p_t (and also scale bands): standard cuts



- ➤ fixed-order result very sensitive to minimum $p_{t,\mathrm{H}}$ value explored in phase-space integration
- ➤ only converges once you explore down to $p_{t,\mathrm{H}} \sim 1\,\mathrm{MeV}$
- ➤ i.e. extremely difficult to get reliable fixed-order result and once you have it, it is of dubious physical meaning

Solution #1: only ever calculate σ_{fid} with help of p_{tH} resummation

- Billis, Dehnadi, Ebert, Michel & Tackmann, 2102.08039, argue you should evaluate the fiducial cross section only after resummation of the p_{tH} distribution.
- For legacy measurements, resummation is only viable solution
- Our view: not an ideal solution
  - Fiducial σ is a hard cross section and shouldn't need resummation
  - losing the ability to use fixed order on its own would be a big blow to the field (e.g. flexibility; robustness of seeing fixed-order & resummation agree)
  - sensitivity to variation of acceptance at low $p_{t,H}$ → complications (e.g. sensitivity to heavy-quark effects in resummation and PDFs — not consistently treated in any N3LL resummation today)



Billis, Dehnadi, Ebert, Michel & Tackmann, 2102.08039

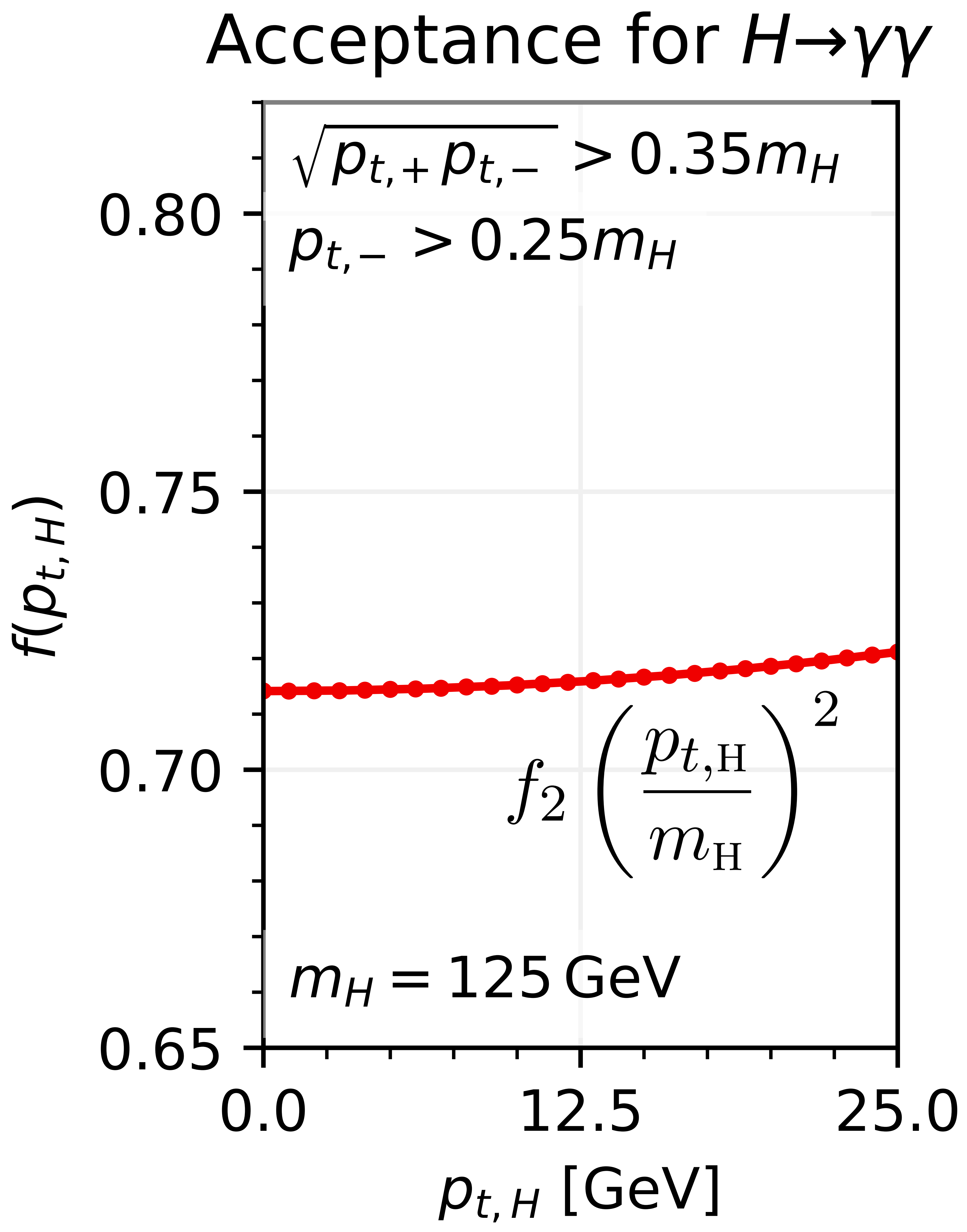
Solution #2a: for future measurements, make simple changes to the cuts

- Simplest option is to replace the cut on the leading photon with a cut on the product of the two photon p_t's
- E.g. $p_{t,\gamma+} \times p_{t,\gamma-} > (0.35m_H)^2$ (and still keep softer photon cut $p_{t,\gamma-} > 0.25m_H$)
- The product has no linear dependence on $p_{t,H}$

$$p_{t,\text{prod}}(p_{t,\text{H}}, \theta, \phi) = \sqrt{p_{t,+}p_{t,-}} = \frac{m_\text{H}}{2}\sin\theta + \frac{p_{t,\text{H}}^2}{4m_\text{H}}\frac{\sin^2\phi - \cos^2\theta\cos^2\phi}{\sin\theta} + \mathcal{O}_4$$

[Several other options are possible, but this combines simplicity and good performance]

Replace cut on leading photon → cut on **product of photon p_t's**

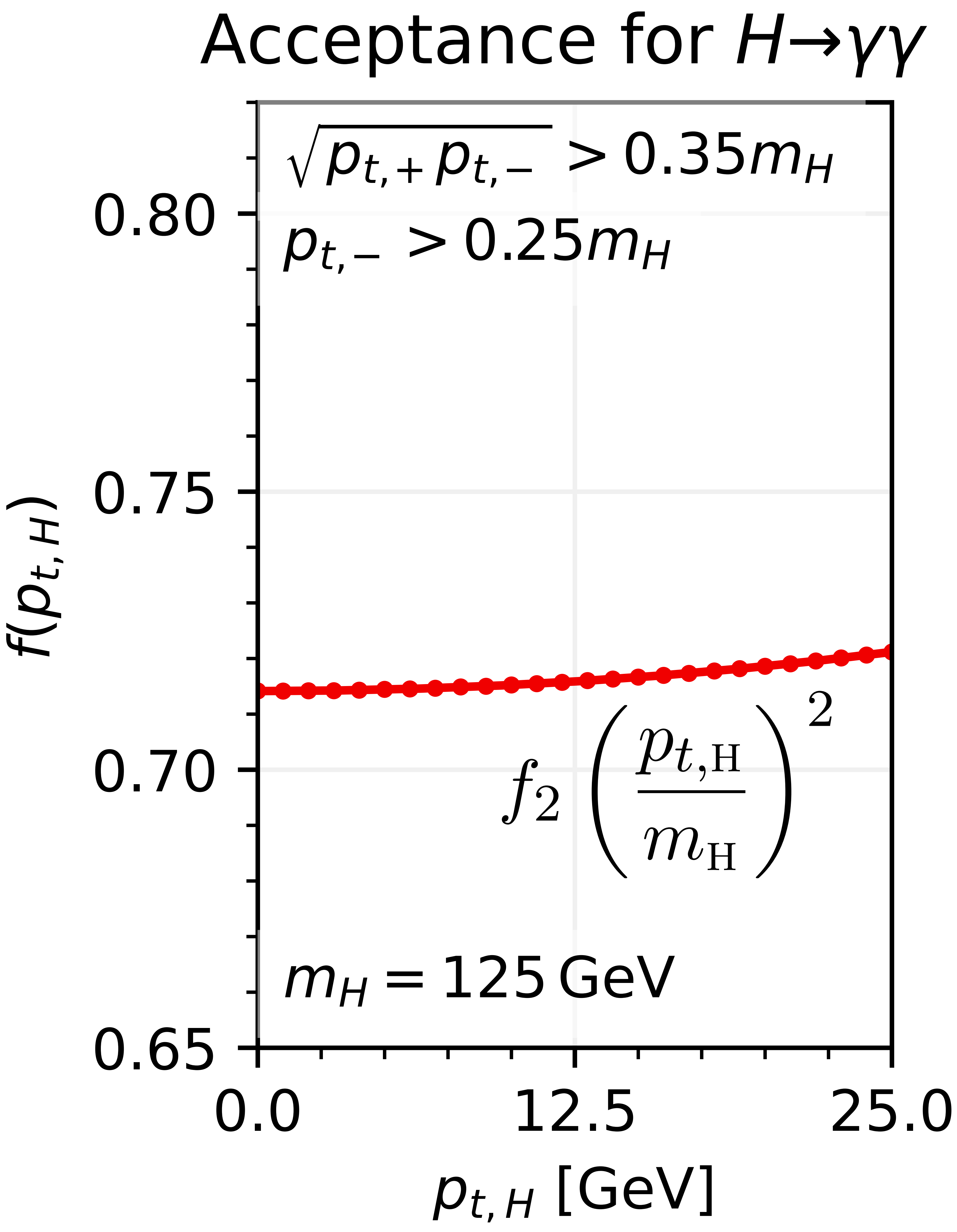


$$f(p_{t,\mathrm{H}}) = f_0 + f_2 \left(\frac{p_{t,\mathrm{H}}}{m_\mathrm{H}}\right)^2 + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

linear → quadratic

NB: the cut on the softer photon is still maintained

Replace cut on leading photon → cut on **product of photon p_t's**



$$f(p_{t,\mathrm{H}}) = f_0 + f_2\left(\frac{p_{t,\mathrm{H}}}{m_\mathrm{H}}\right)^2 + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

linear → quadratic

$$\frac{(2n)!}{2(n!)}\left(\frac{2C_A\alpha_s}{\pi}\right)^n \rightarrow \frac{1}{4^n}\frac{(2n)!}{4(n!)}\left(\frac{2C_A\alpha_s}{\pi}\right)^n$$

Using product cuts dampens the factorial divergence

NB: the cut on the softer photon is still maintained

Behaviour of perturbative series with product cuts

$$
\begin{aligned}
\frac{\sigma_{\text{prod}} - f_0\sigma_{\text{inc}}}{\sigma_0 f_0} &\simeq 0.005_{\alpha_s} - 0.002_{\alpha_s^2} + 0.002_{\alpha_s^3} - 0.001_{\alpha_s^4} + 0.001_{\alpha_s^5} + \ldots \\
&\simeq 0.005_{\alpha_s} - 0.002_{\alpha_s^2} + 0.000_{\alpha_s^3} - 0.000_{\alpha_s^4} + 0.000_{\alpha_s^5} + \ldots \\
&\simeq 0.005_{\alpha_s} + 0.002_{\alpha_s^2} - 0.001_{\alpha_s^3} + \ldots \\
&\simeq 0.005_{\alpha_s} + 0.002_{\alpha_s^2} - 0.001_{\alpha_s^3} + \ldots
\end{aligned}
$$

Resummed results

$\simeq 0.003$ @DL,

$\simeq 0.003$ @LL,

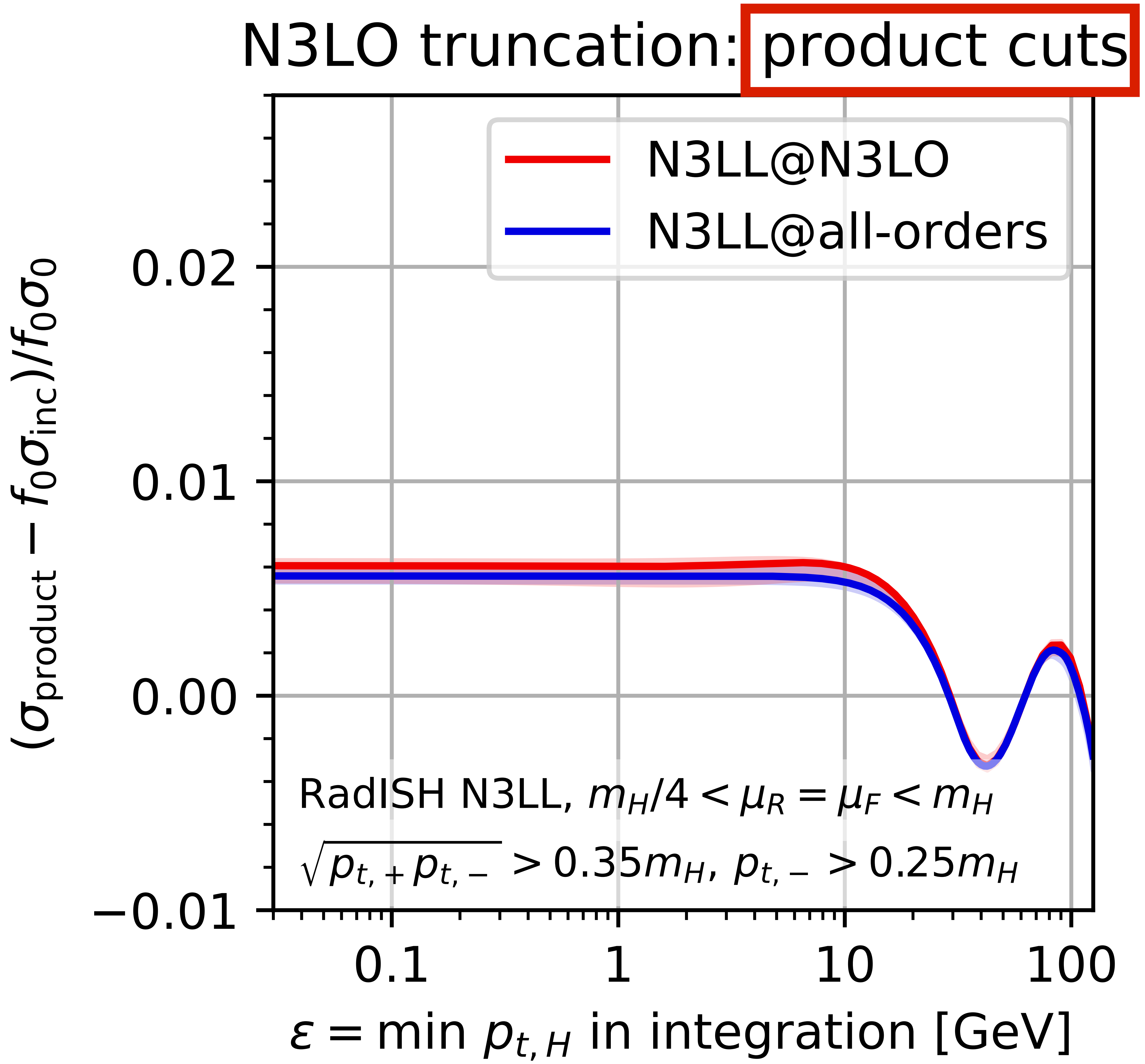
$\simeq 0.005$ @NNLL,

$\simeq 0.006$ @N3LL.

Thanks to Pier Monni & RadISH for supplying NN(N)LL distributions & expansions, $\mu = m_H/2$

- Factorial growth of series strongly suppressed
- **N3LO truncation agrees well with all-order result**
- Per mil agreement between fixed-order and resummation **gives confidence that all is under control**

fixed-order sensitivity to low p_{tH} is gone

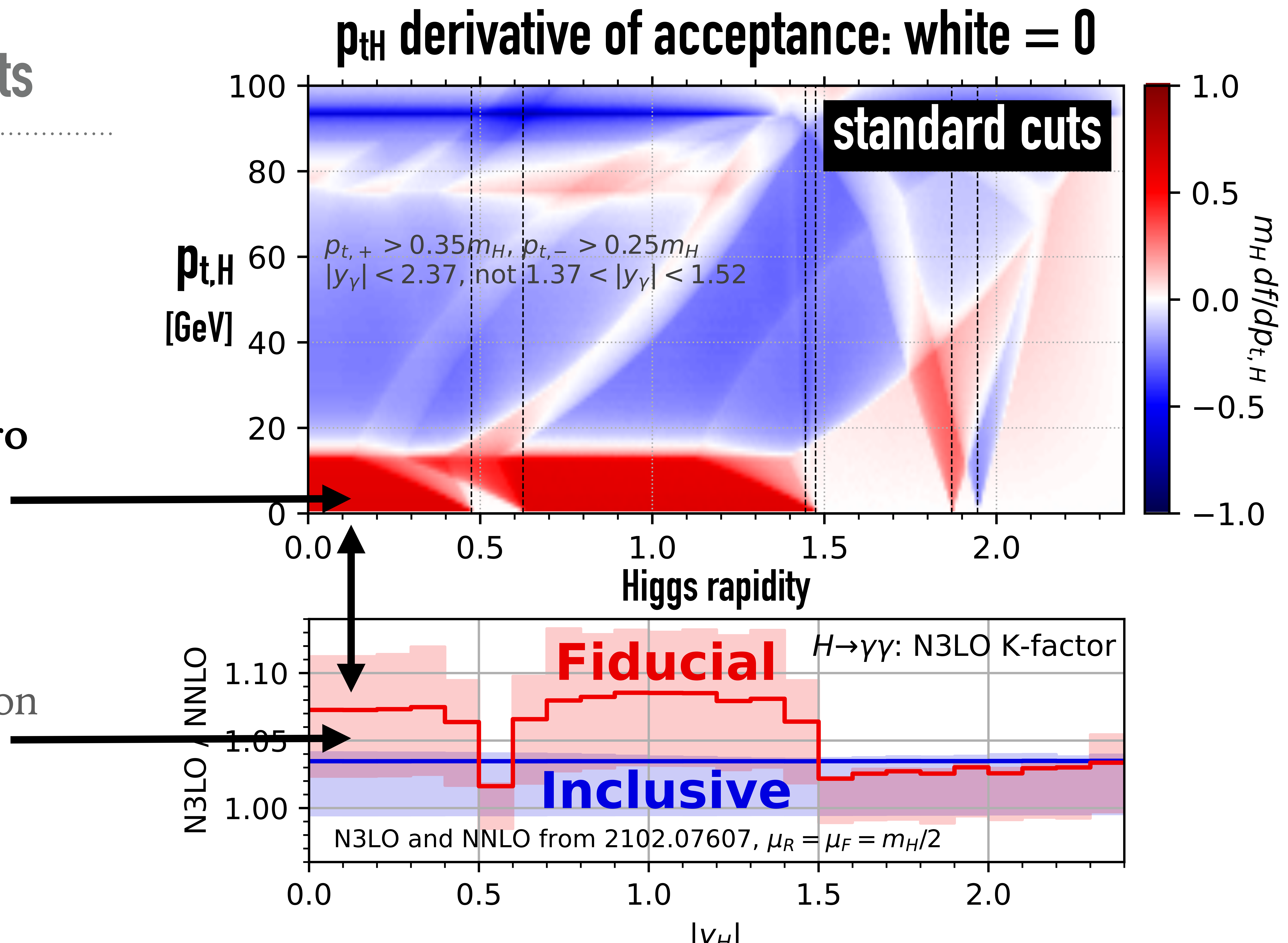


- fixed-order becomes insensitive to $p_{t,\mathrm{H}}$ values below a few GeV
- overall size of (non-Born part of) fiducial acceptance corrections much smaller
- resummation and fixed order agree at per-mil level

interplay with η_γ cuts

$f(p_{t,\mathrm{H}}, y_\mathrm{H})$ has **non-zero** linear $p_{t,\mathrm{H}}$ derivative at $p_{t,\mathrm{H}} = 0$

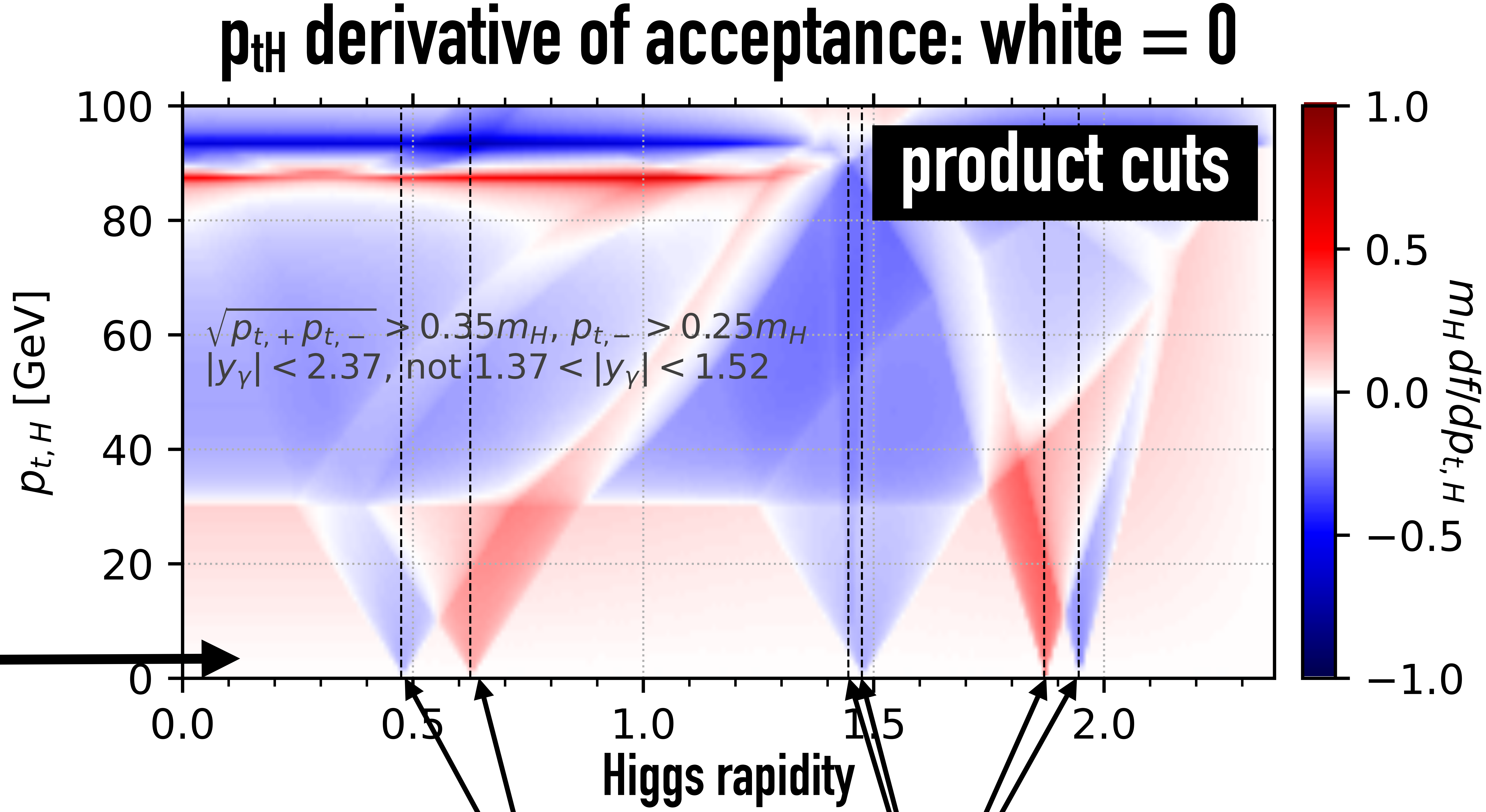
fixed-order perturbation theory has trouble



interplay with η_γ cuts

$f(p_{t,\mathrm{H}}, y_\mathrm{H})$ has **zero** linear $p_{t,\mathrm{H}}$ derivative at $p_{t,\mathrm{H}} = 0$

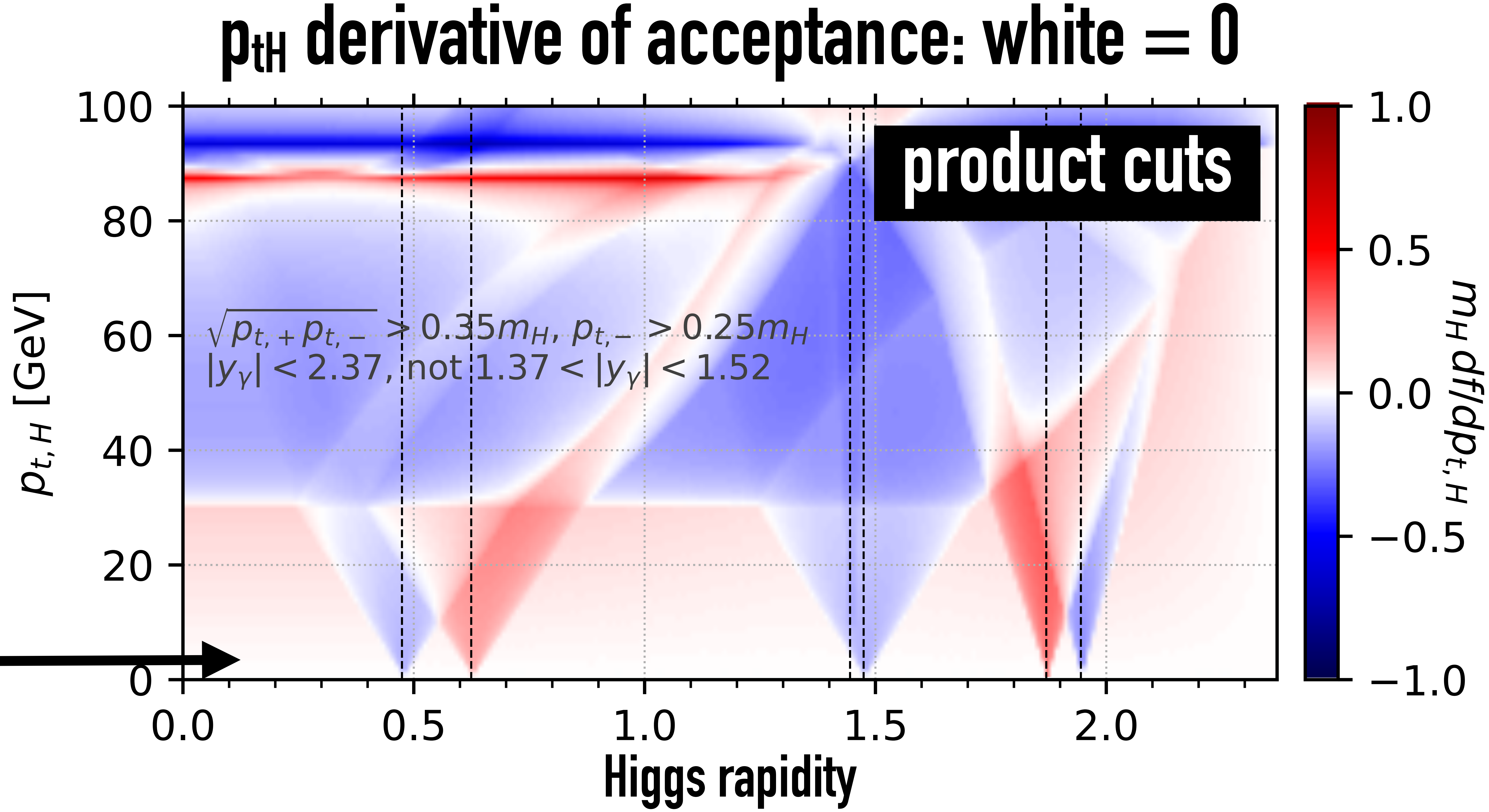
fixed-order perturbation theory will be fine



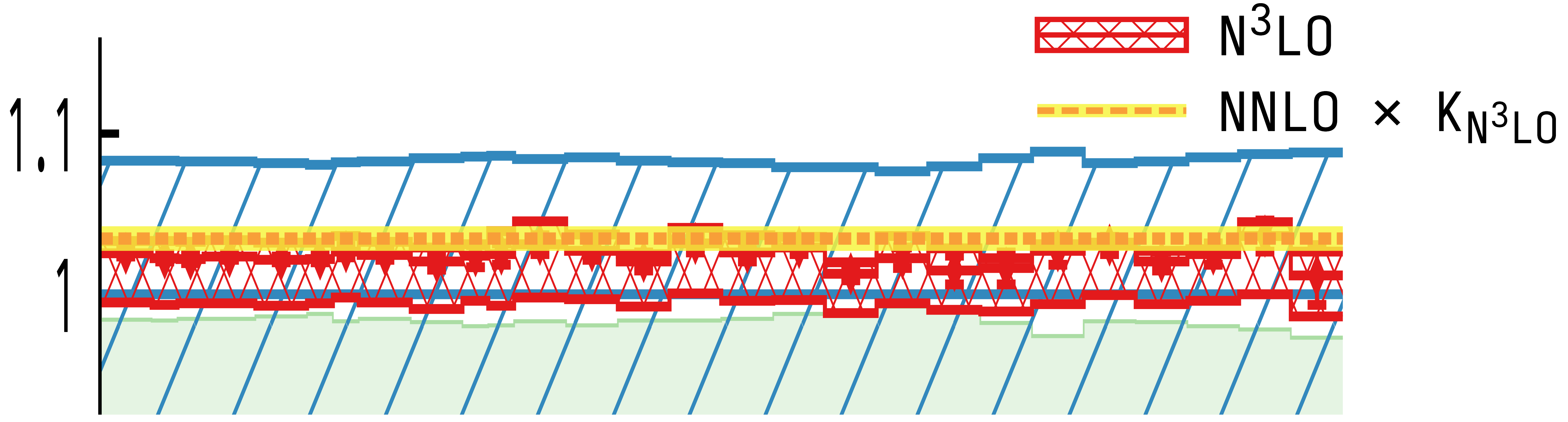
NB: at these points Born η_γ and $p_{t,\gamma}$ cuts are degenerate. If doing rapidity binning, choose bins that are not too narrow (e.g. ±0.1 around them)

interplay with η_γ cuts

$f(p_{t,\mathrm{H}}, y_\mathrm{H})$ has **zero** linear $p_{t,\mathrm{H}}$ derivative at $p_{t,\mathrm{H}} = 0$



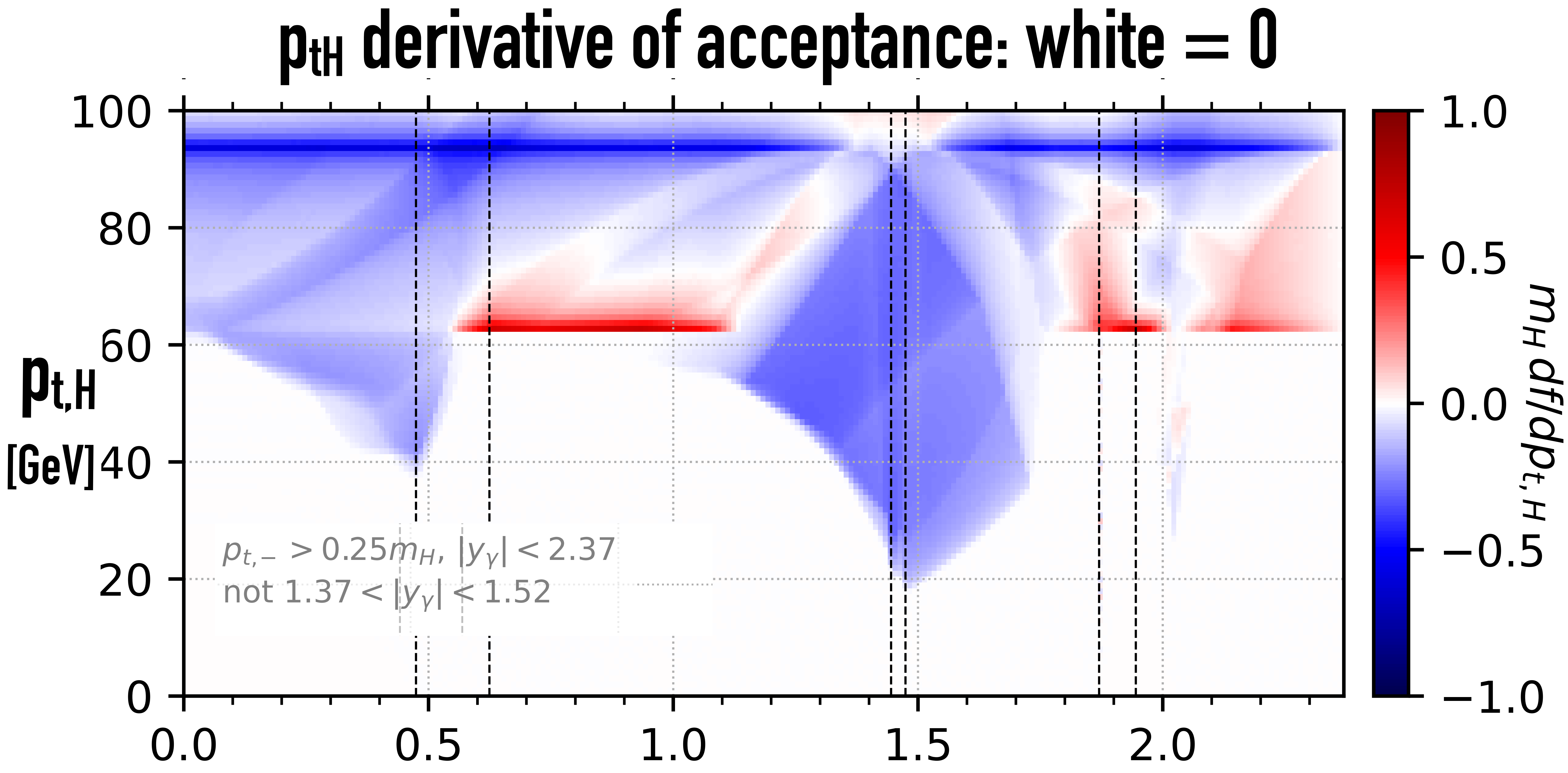
fixed-order perturbation theory will be fine



Huss et al preliminary @ Higgs 2021

Solution #2b: design cuts whose acceptance is independent of p_{tH} (at small p_{tH})

- keep standard cuts on softer photon p_t and on photon rapidities
- replace harder-photon p_t cut with Collins-Soper angle cut (transverse boost-invariant)
- selectively loosen CS angle cut to keep p_{tH}-independent acceptance as far as possible



hardness and rapidity compensating (CBI_{HR}) cuts

details in arXiv:2106.08329 + code at https://github.com/gavinsalam/two-body-cuts

Solution #3: defiducialise (cf. Glazov 2001.02933 for DY)

- **Option 3a**: divide out both $p_{t,\mathrm{H}}$ and y_H dependence of acceptance from fiducial differential cross section

$$\sigma_\mathrm{defid} = \int_{-y_\mathrm{H}^\mathrm{max}}^{+y_\mathrm{H}^\mathrm{max}} dy_\mathrm{H} \int_0^{p_{t,\mathrm{H}}^\mathrm{max}} dp_{t,\mathrm{H}} \frac{d\sigma^\mathrm{fid}}{dy_\mathrm{H}dp_{t,\mathrm{H}}} \frac{1}{f(y_\mathrm{H}, p_{t,\mathrm{H}})}\,, \equiv \int_{-y_\mathrm{H}^\mathrm{max}}^{+y_\mathrm{H}^\mathrm{max}} dy_\mathrm{H} \int_0^{p_{t,\mathrm{H}}^\mathrm{max}} dp_{t,\mathrm{H}} \frac{d\sigma}{dy_\mathrm{H}dp_{t,\mathrm{H}}}\,,$$

- **Option 3b**: divide out just $p_{t,\mathrm{H}}$ dependence of acceptance from fiducial differential cross section (adapted from suggestion by referee of paper)

$$\sigma_{\mathrm{defid},p_{t,\mathrm{H}}} = \int_{-y_\mathrm{H}^\mathrm{max}}^{+y_\mathrm{H}^\mathrm{max}} dy_\mathrm{H} \int_0^{p_{t,\mathrm{H}}^\mathrm{max}} dp_{t,\mathrm{H}} \frac{d\sigma^\mathrm{fid}}{dy_\mathrm{H}dp_{t,\mathrm{H}}} \frac{f(y_\mathrm{H},0)}{f(y_\mathrm{H}, p_{t,\mathrm{H}})}\,, \equiv \int_{-y_\mathrm{H}^\mathrm{max}}^{+y_\mathrm{H}^\mathrm{max}} dy_\mathrm{H} \int_0^{p_{t,\mathrm{H}}^\mathrm{max}} dp_{t,\mathrm{H}} \frac{d\sigma}{dy_\mathrm{H}dp_{t,\mathrm{H}}} f(y_\mathrm{H},0)\,,$$

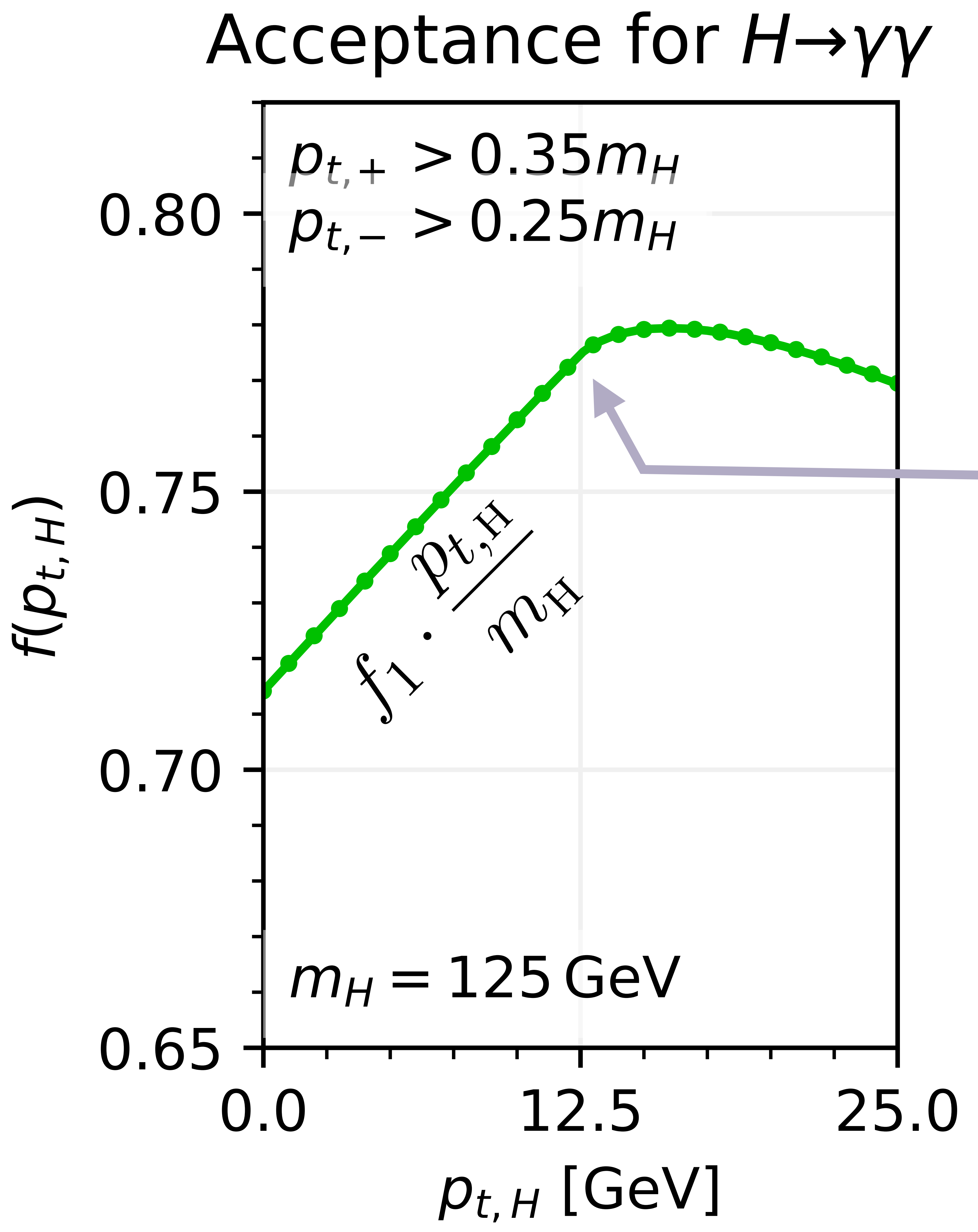
NB1: some care needed in choice of integration limits, to avoid division by zero (or, for 3a, by small numbers for $y_\mathrm{H} \gtrsim 2$)
NB2: defiducialisation is theoretically robust for a scalar particle (in a way that it is not for DY)
NB3: code at https://github.com/gavinsalam/two-body-cuts can also help with defiducialisation for Higgs

Conclusions

- Fixed-order perturbation theory can be badly compromised by existing (2-body) cuts (→ intriguing questions about asymptotics of QCD perturbative series)
- In simple cases (e.g. $H \to \gamma\gamma$), can be solved by resummation. But physics will be more robust if we can reliably use both fixed-order and resummed+FO results (and both yield similar central values & uncertainties)
- A better long-term solution may be to **revisit experimental cuts**:
  - product and boost-invariant cuts give much better perturbative series
  - Likely relevant also for other processes (see backup for DY: effects at the 1%-level)
- Alternatively: in Higgs case, you can **defiducialise**
- Cuts with little p_{tH} dependence (or defiducialisation) may be useful also, e.g., for extrapolating measurements to STXS or more inclusive cross sections, with limited dependence on BSM or non-perturbative effects.
- **Needs addressing in future LHC measurements for robust accuracy in Run 3 & HL-LHC**

Backup

Linear p_{tH} dependence of H acceptance, $f(p_{tH})$



$$f(p_{t,\mathrm{H}}) = f_0 + f_1 \cdot \frac{p_{t,\mathrm{H}}}{m_\mathrm{H}} + \mathcal{O}\left(\frac{p_{t,\mathrm{H}}^2}{m_\mathrm{H}^2}\right)$$

See e.g. Frixione & Ridolfi '97
Ebert & Tackmann '19
idem + Michel & Stewart '20
Alekhin et al '20
f_0 and f_1 are coefficients whose values depend values of cuts

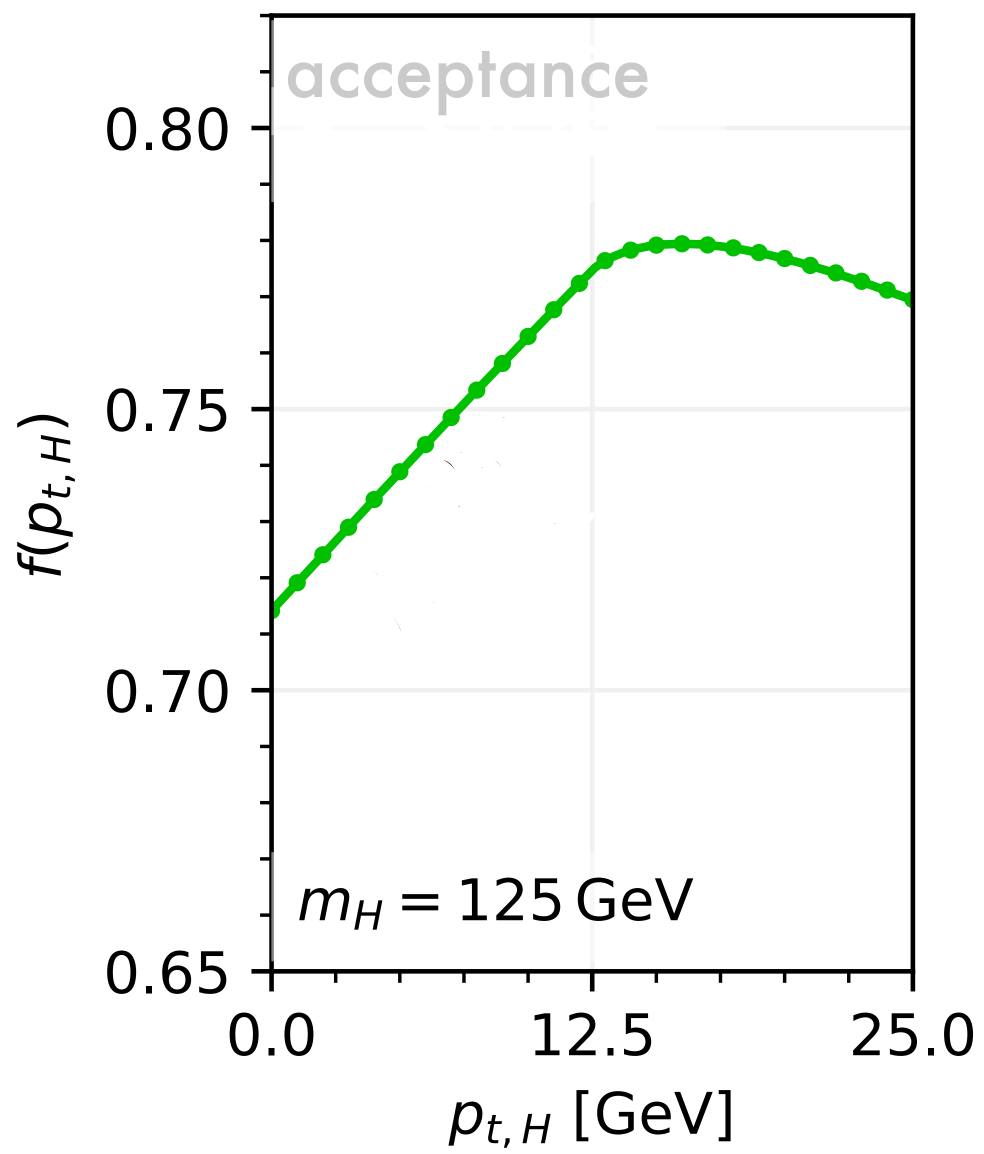
effect of $p_{t,-}$ cut sets in at $0.1 m_\mathrm{H}$

$p_{t,\mathrm{H}}$ dependence of acceptance (at 10% level) → relating measured cross section and total cross section requires info about the $p_{t,\mathrm{H}}$ distribution.

Cut Type	cuts on	small-$p_{t,\text{H}}$ dependence	f_n coefficient	$p_{t,\text{H}}$ transition
symmetric	$p_{t,-}$	linear	$+2s_0/(\pi f_0)$	none
asymmetric	$p_{t,+}$	linear	$-2s_0/(\pi f_0)$	Δ
sum	$\frac{1}{2}(p_{t,-} + p_{t,+})$	quadratic	$(1 + s_0^2)/(4f_0)$	2Δ
product	$\sqrt{p_{t,-} + p_{t,+}}$	quadratic	$s_0^2/(4f_0)$	2Δ
staggered	$p_{t,1}$	quadratic	$s_0^4/(4f_0^3)$	Δ
Collins-Soper	$p_{t,\text{CS}}$	none	—	2Δ
CBI$_H$	$p_{t,\text{CS}}$	none	—	$2\sqrt{2}\Delta$
rapidity	y_γ	quadratic	$f_0 s_0^2/2$	

Table 1: Summary of the main hardness cuts, the variable they cut on at small $p_{t,\text{H}}$, and the small-$p_{t,\text{H}}$ dependence of the acceptance. For linear cuts $f_n \equiv f_1$ multiplies $p_{t,\text{H}}/m_\text{H}$, while for quadratic cuts $f_n \equiv f_2$ multiplies $(p_{t,\text{H}}/m_\text{H})^2$ (in all cases there are additional higher order terms that are not shown). For a leading threshold of $p_{t,\text{cut}}$, $s_0 = 2p_{t,\text{cut}}/m_\text{H}$ and $f_0 = \sqrt{1 - s_0^2}$, while for the rapidity cut $s_0 = 1/\cosh(y_\text{H} - y_\text{cut})$. For a cut on the softer lepton's transverse momentum of $p_{t,-} > p_{t,\text{cut}} - \Delta$, the right-most column indicates the $p_{t,\text{H}}$ value at which the $p_{t,-}$ cut starts to modify the behaviour of the acceptance (additional $\mathcal{O}\left(\Delta^2/m_\text{H}\right)$ corrections not shown). For the interplay between hardness and rapidity cuts, see sections 4.2, 4.3 and 5.2.

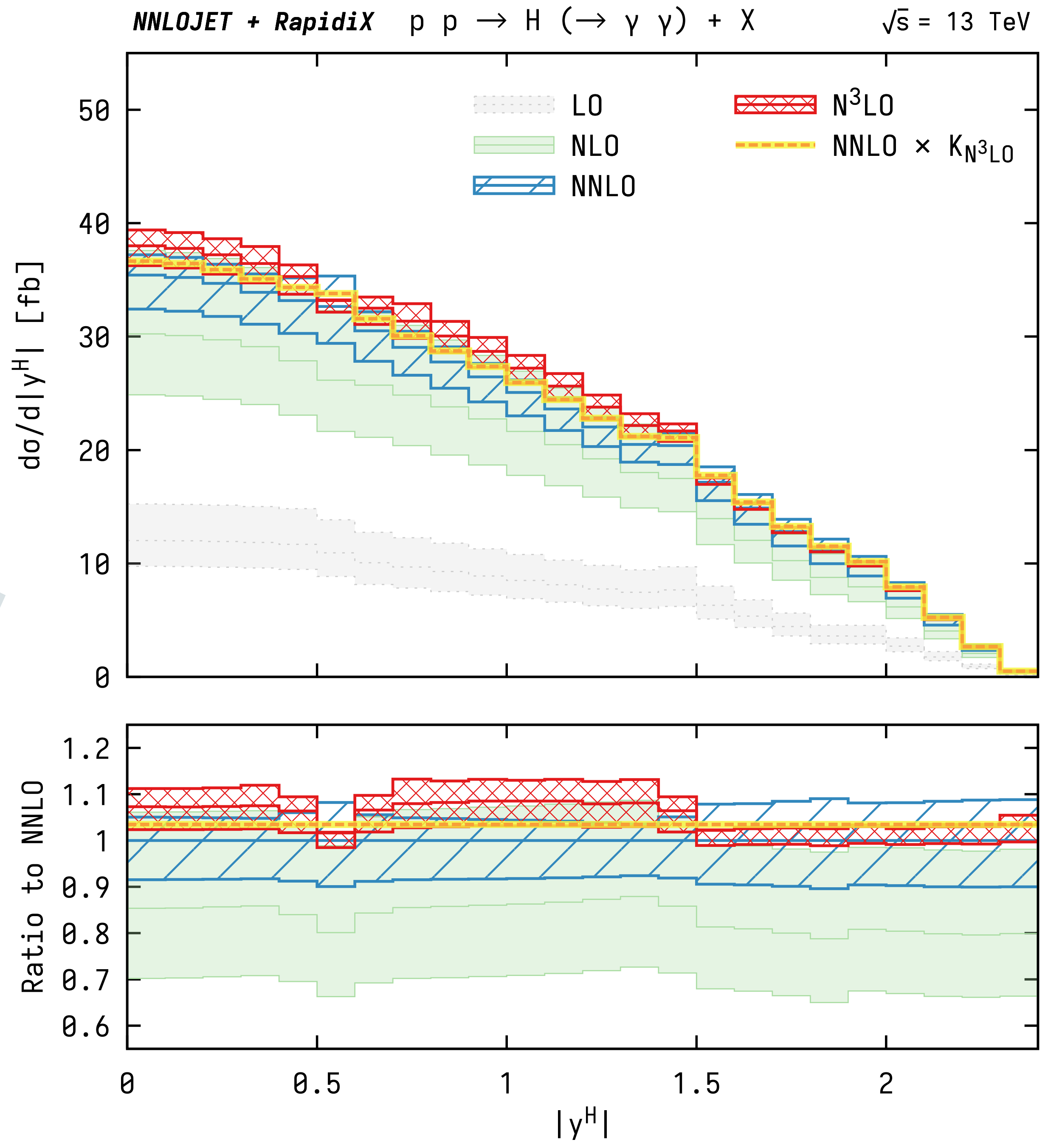
Cuts to remove the IR sensitivity



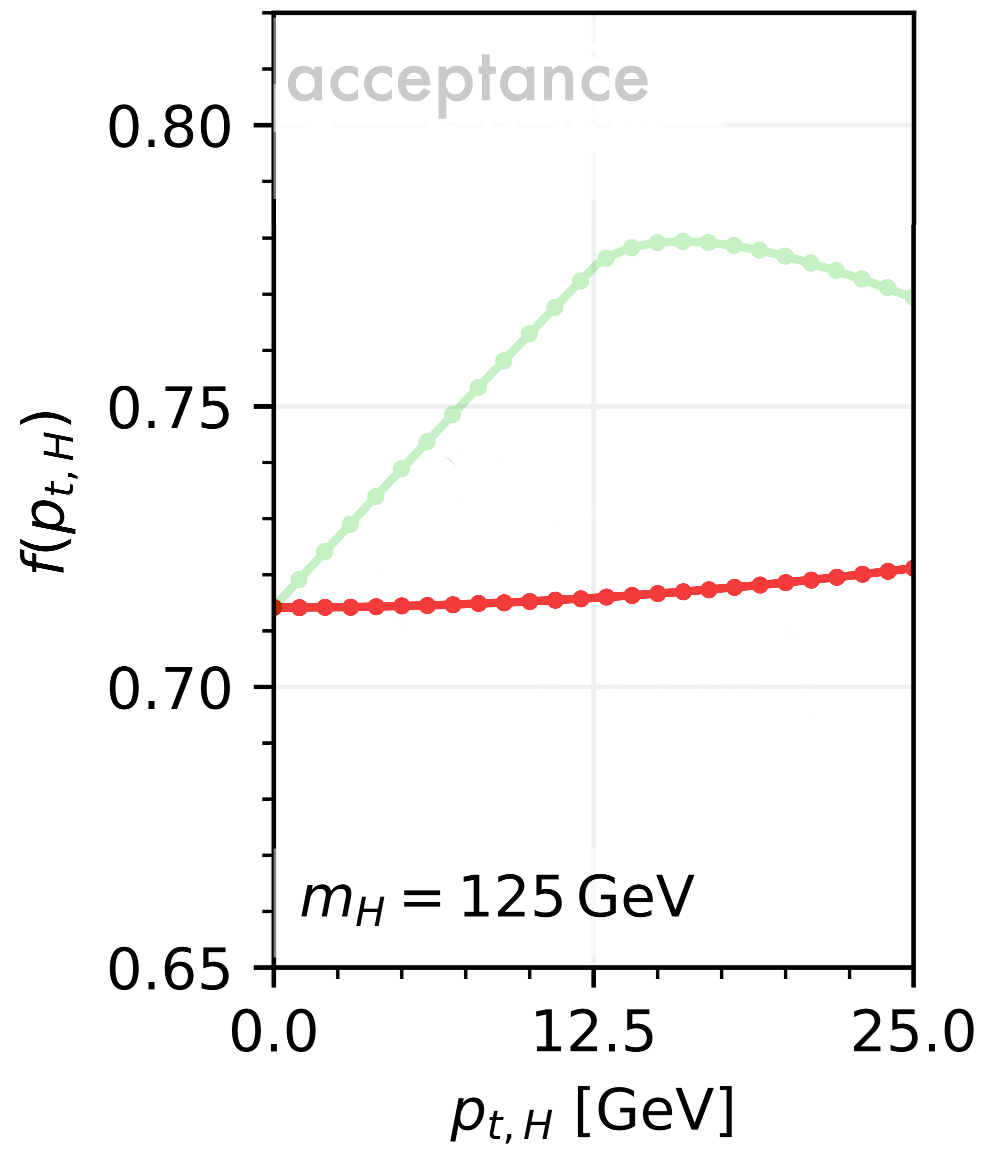
ATLAS

$$p_T^{\gamma_1} \geq 0.35 \cdot M_H$$
$$p_T^{\gamma_2} \geq 0.25 \cdot M_H$$

$$f(p_T^H) = f_0 + f_1 \cdot p_T^H + \mathcal{O}((p_T^H)^2)$$



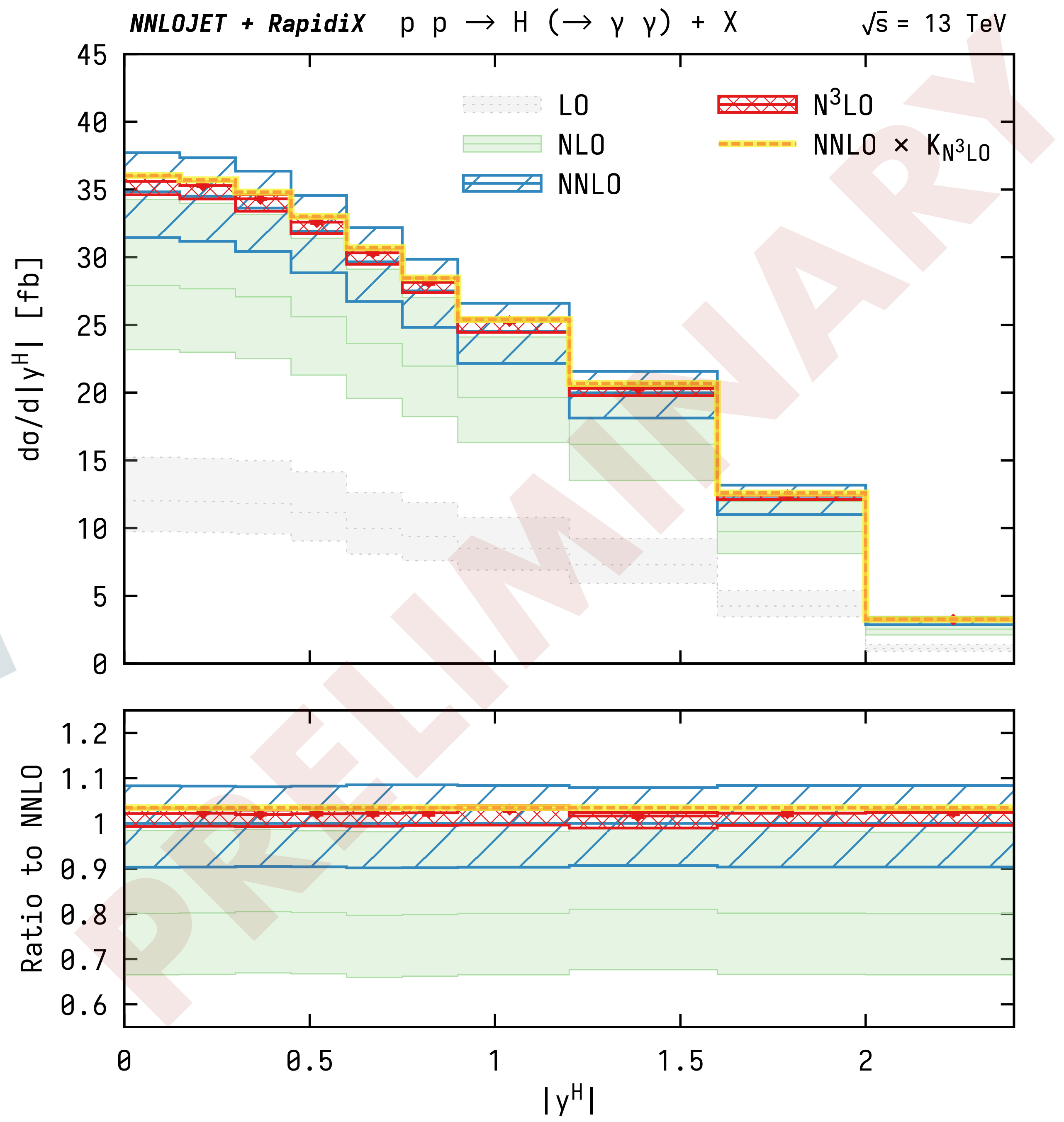
Alex Huss @ Higgs 2021



Product cuts [Salam, Slade '21]

$$\sqrt{p_T^{\gamma_1} p_T^{\gamma_2}} \geq 0.35 \cdot M_H$$
$$p_T^{\gamma_2} \geq 0.25 \cdot M_H$$

$$f(p_T^H) = f_0 + f_1 \cdot p_T^H + f_2 \cdot (p_T^H)^2 + \mathcal{O}((p_T^H)^3)$$



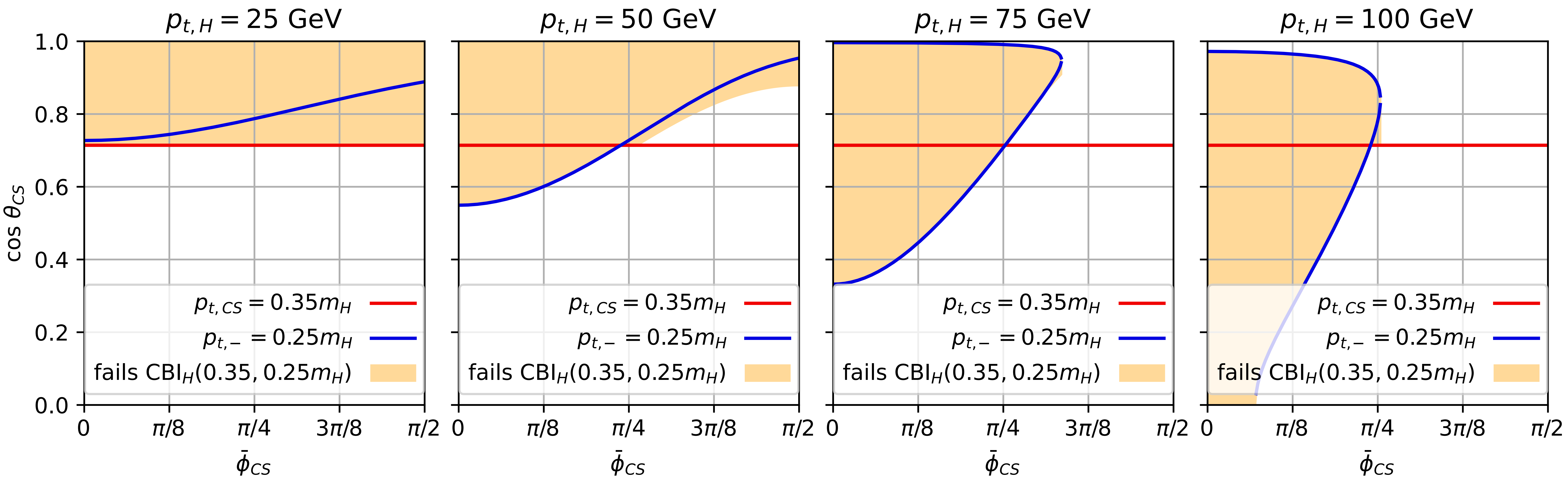
- $\text{NNLO} \times K_{\text{N}^3\text{LO}}$ $\approx$ N^3LO
- very flat
- no "features"
- robust (v.s. resummation)

Hardness [and rapidity] compensating boost invariant cuts (CBI_H and CBI_{HR})

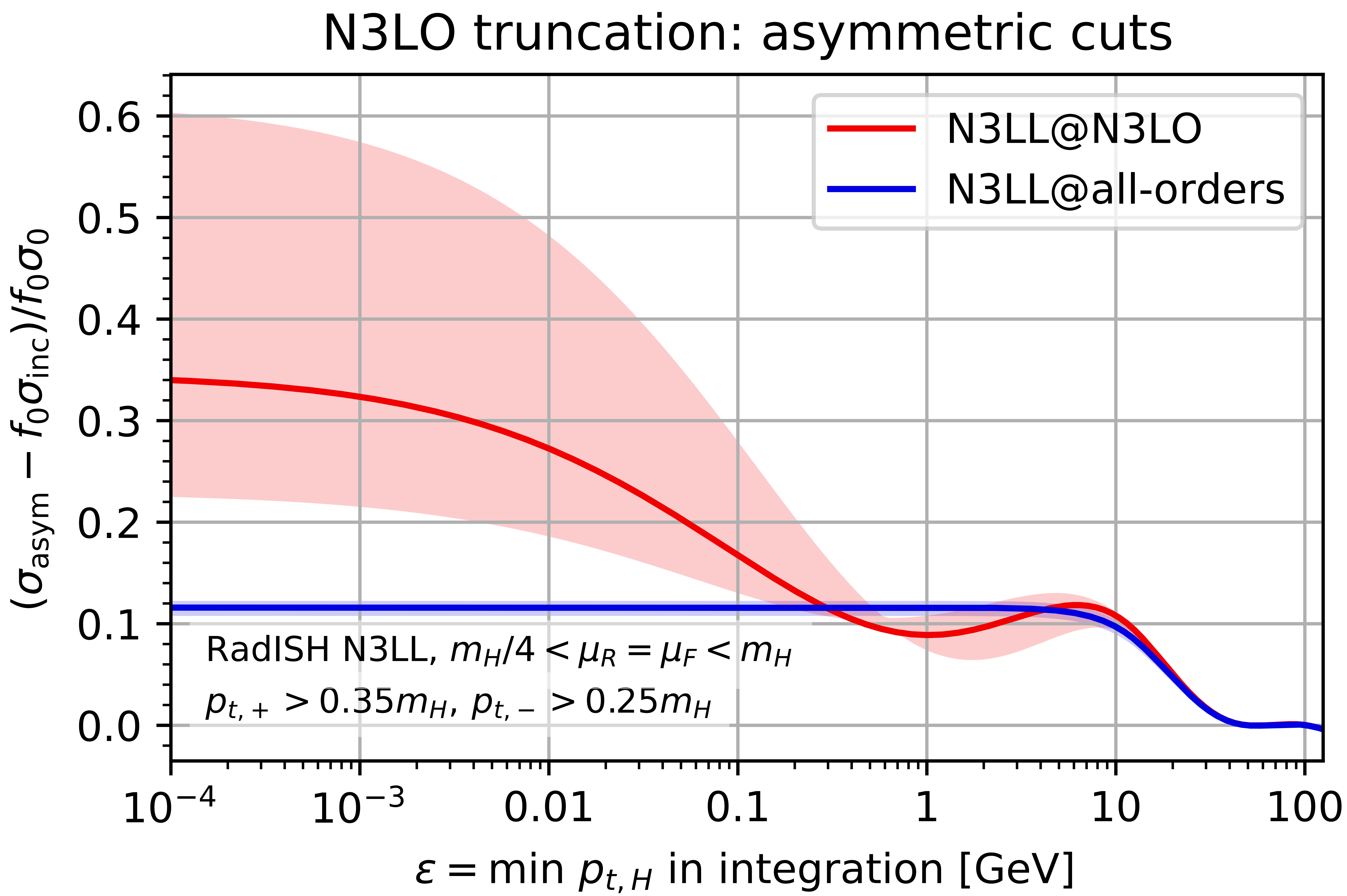
Core idea 1: cut on decay p_t in Collins-Soper frame

$$\vec{p}_{t,\mathrm{CS}} = \frac{1}{2}\left[\vec{\delta}_t + \frac{\vec{p}_{t,12}.\vec{\delta}_t}{p_{t,12}^2}\left(\frac{m_{12}}{\sqrt{m_{12}^2 + p_{t,12}^2}} - 1\right)\vec{p}_{t,12}\right], \qquad \vec{\delta}_t = \vec{p}_{t,1} - \vec{p}_{t,2}$$

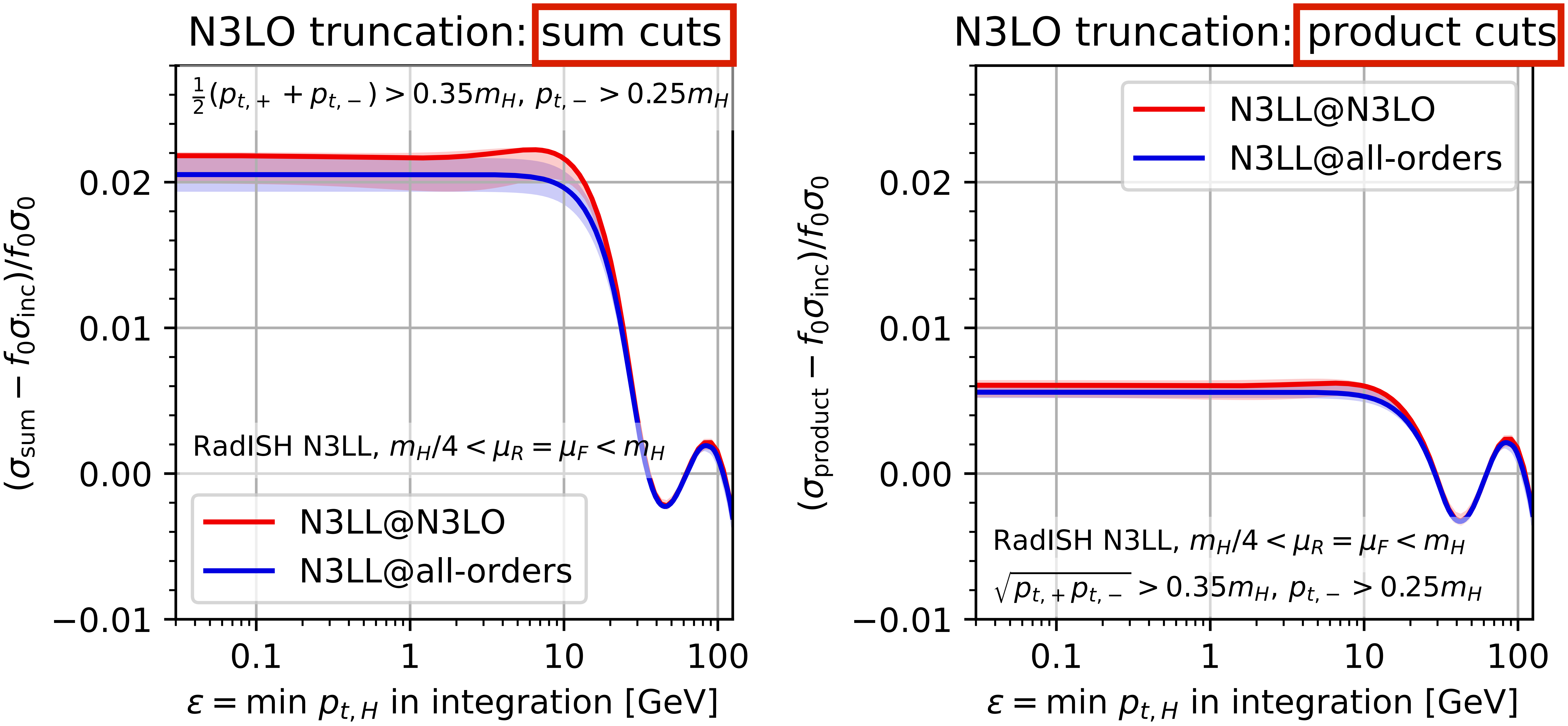
Core idea 2: relax $p_{t,\mathrm{CS}}$ cut at higher $p_{t,\mathrm{H}}$ values to maintain constant / maximal acceptance



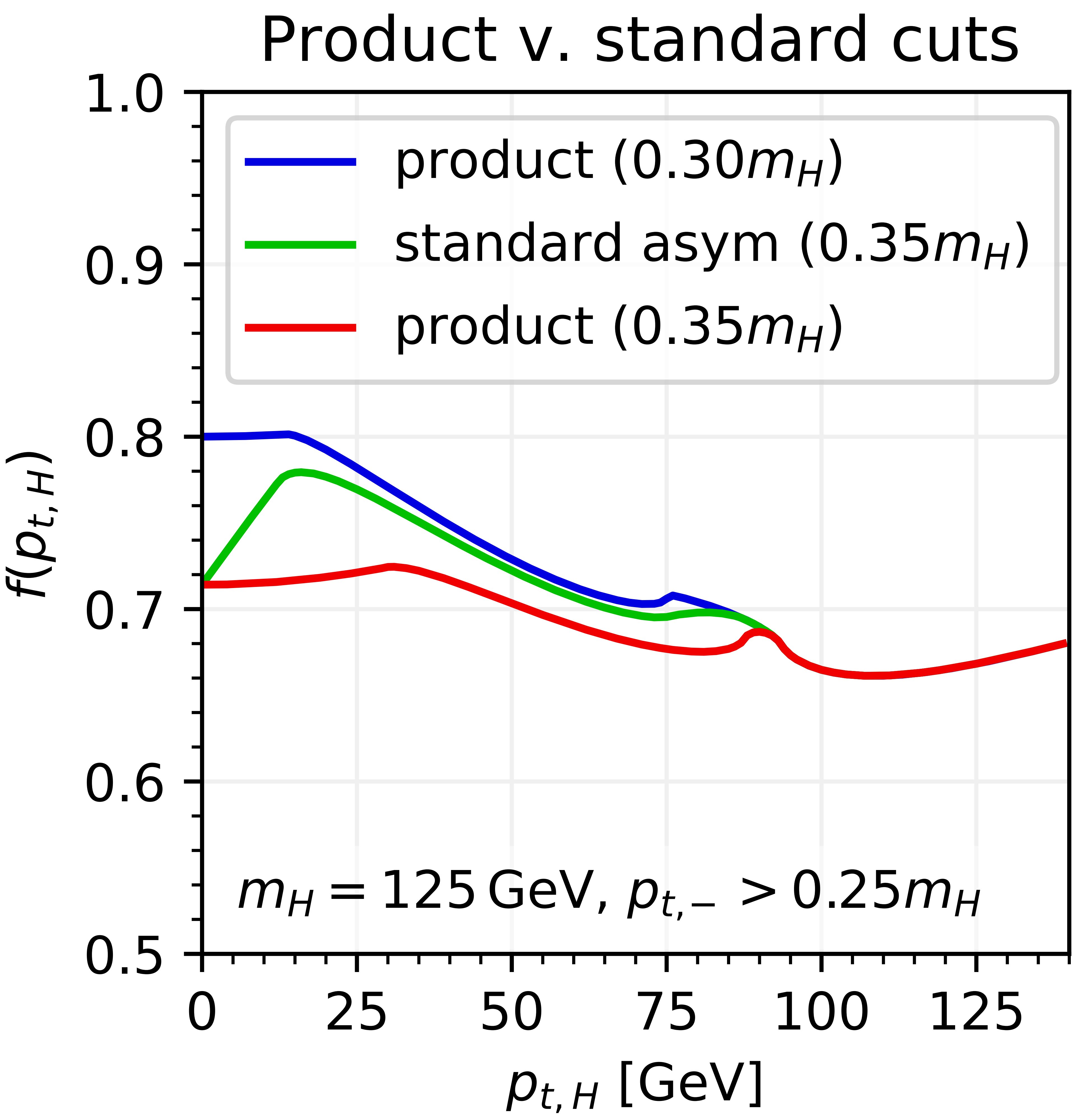
Sensitivity to low Higgs p_t (and also scale bands): standard cuts

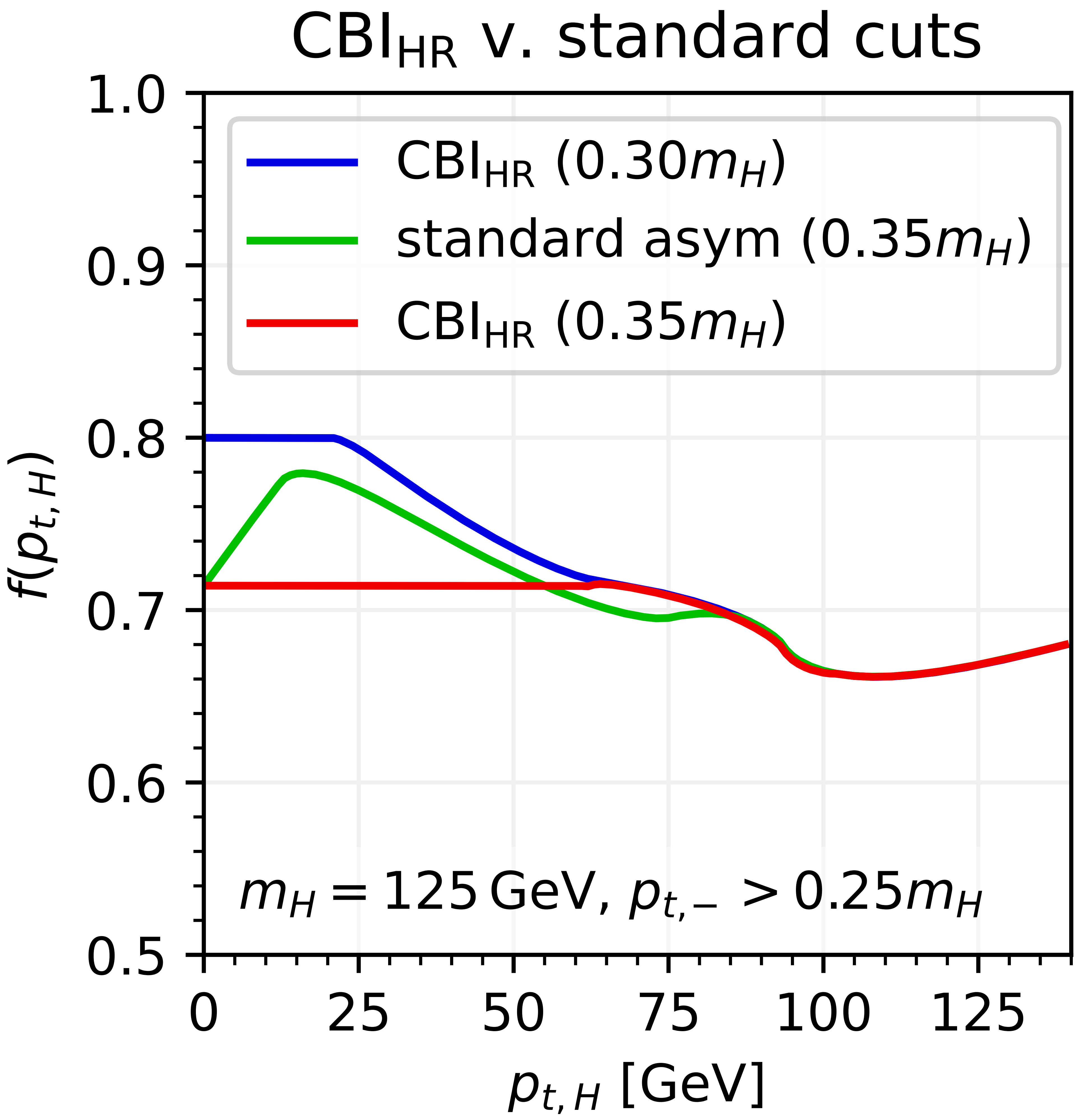


Sensitivity to low Higgs p_t (and also scale bands): sum & product cuts

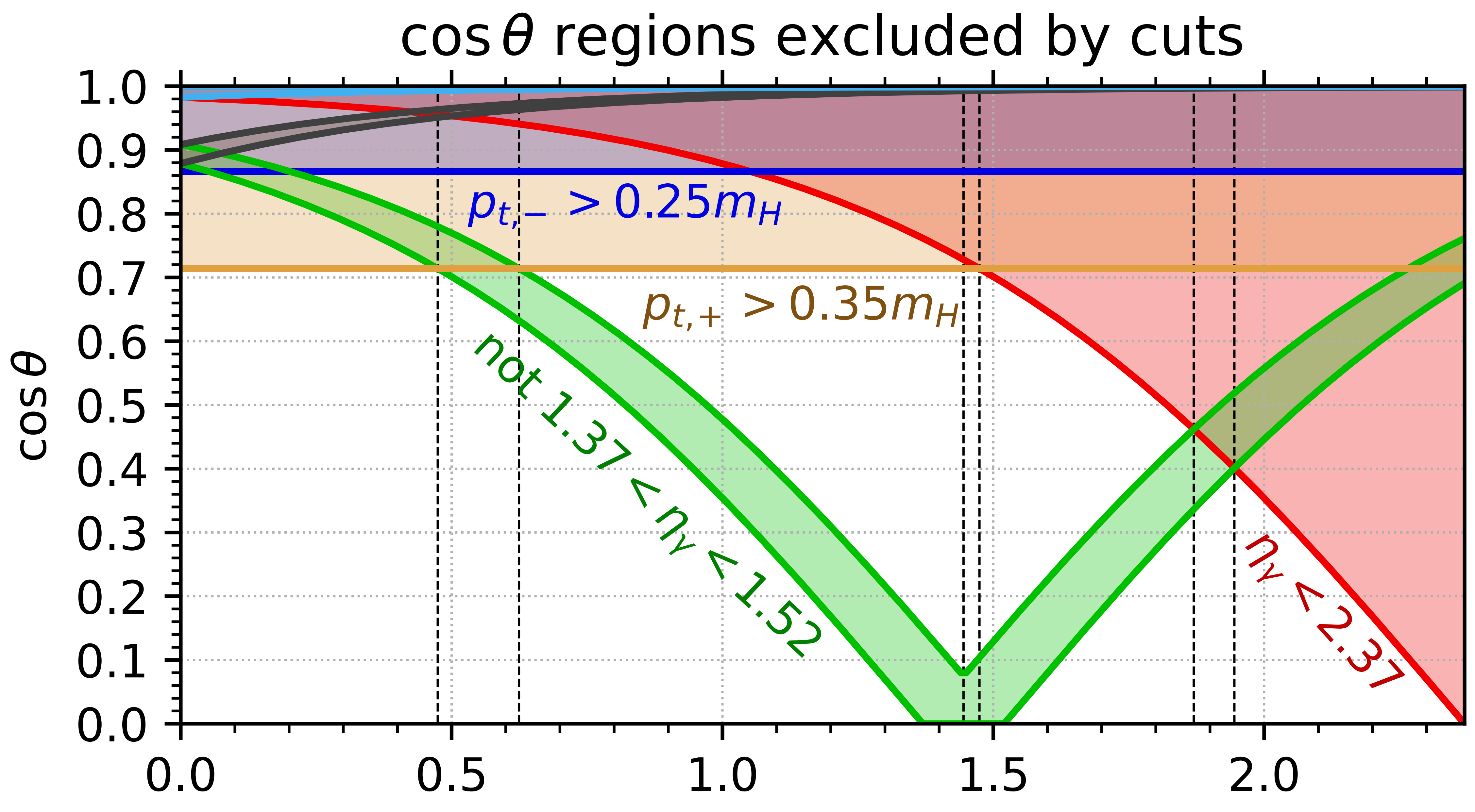


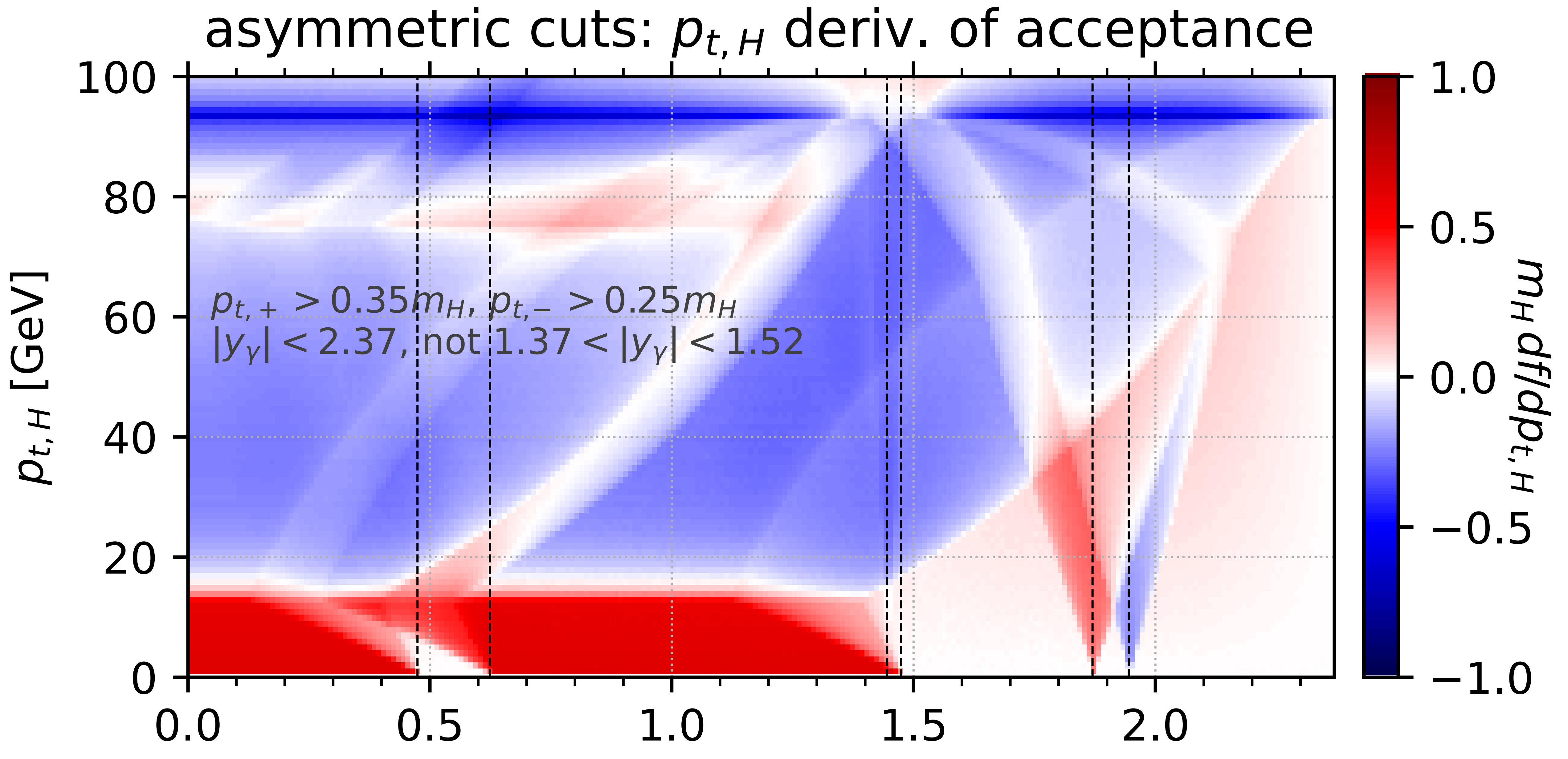
Option of changing thresholds

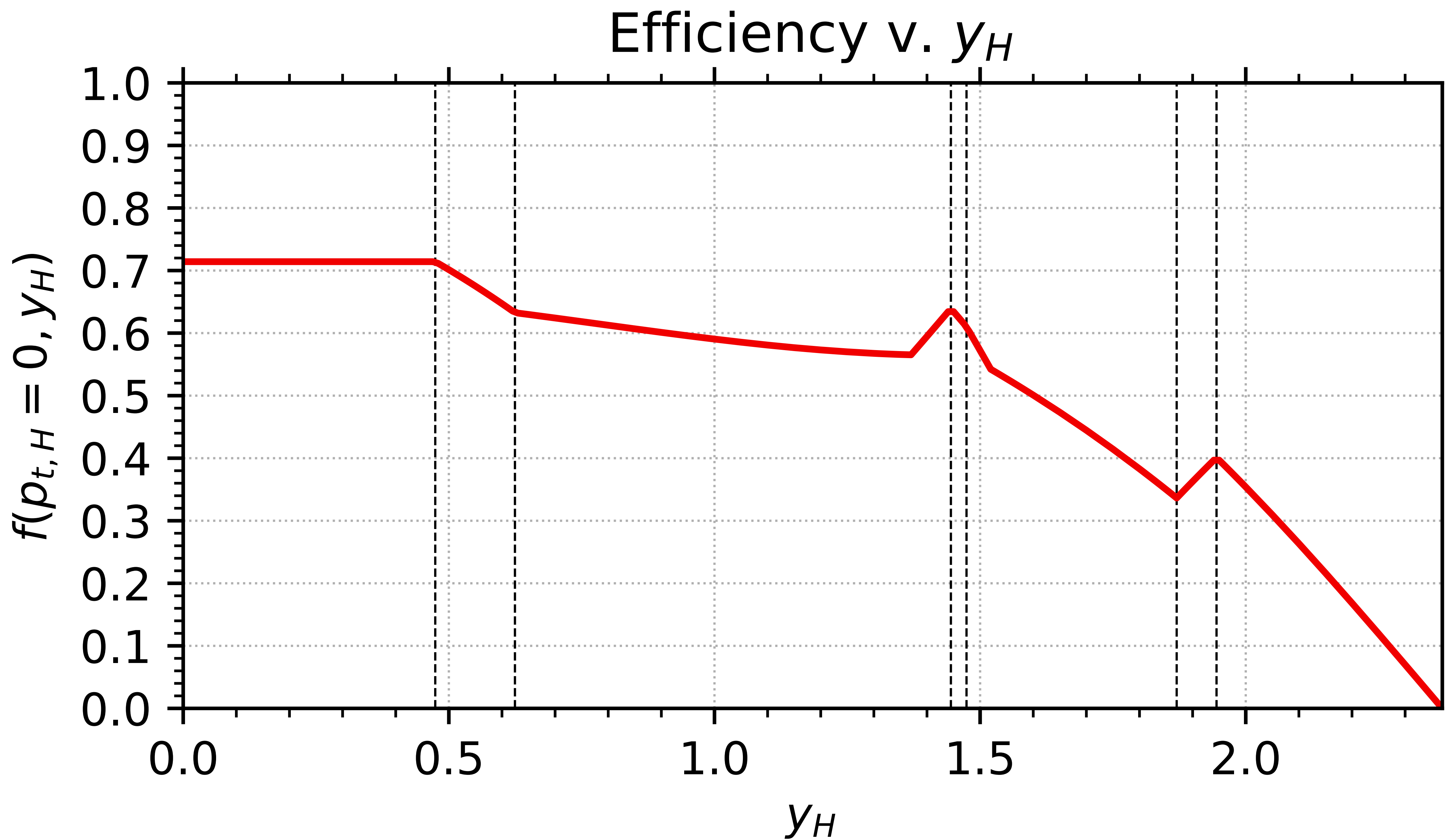


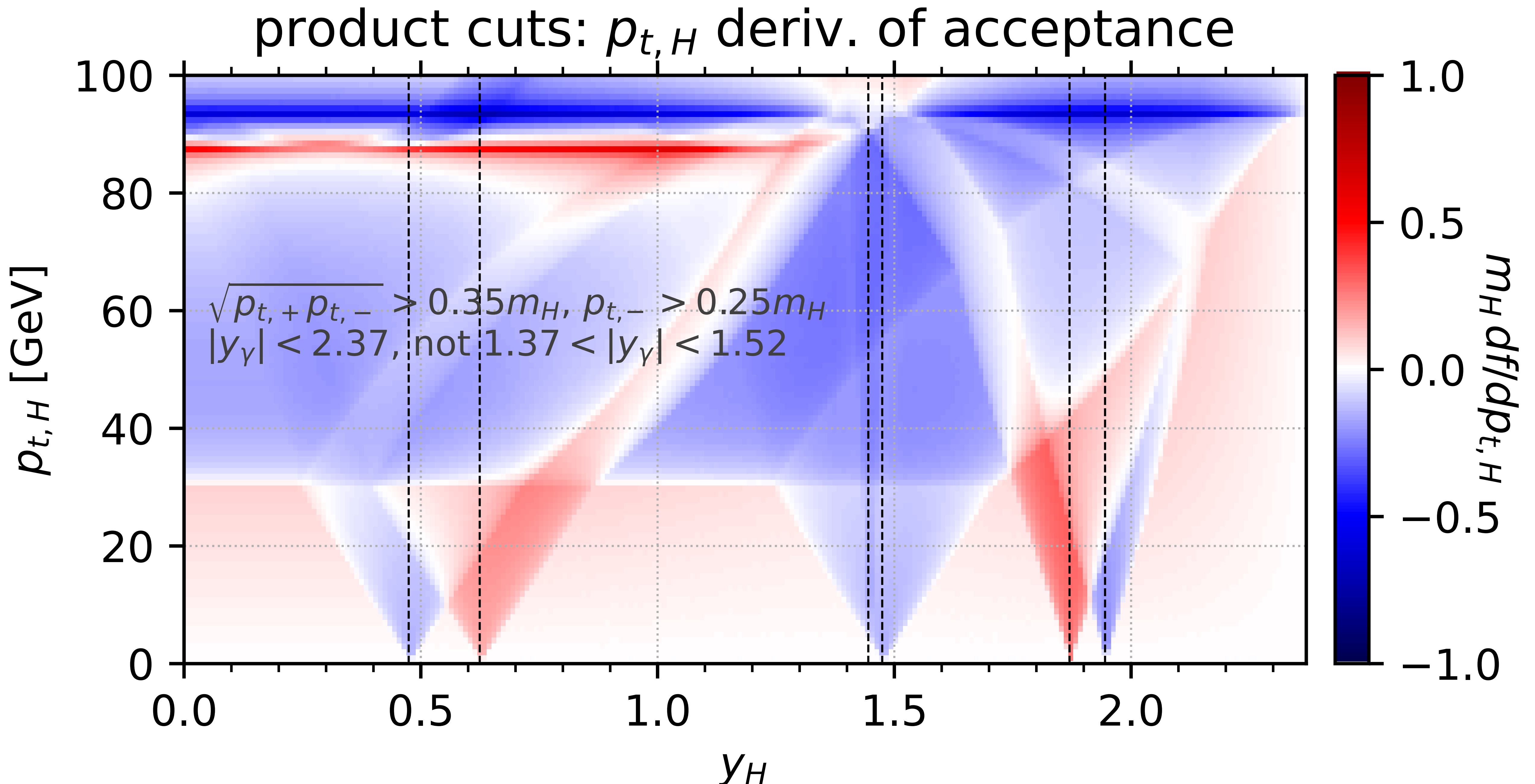


Interplay with rapidity cuts

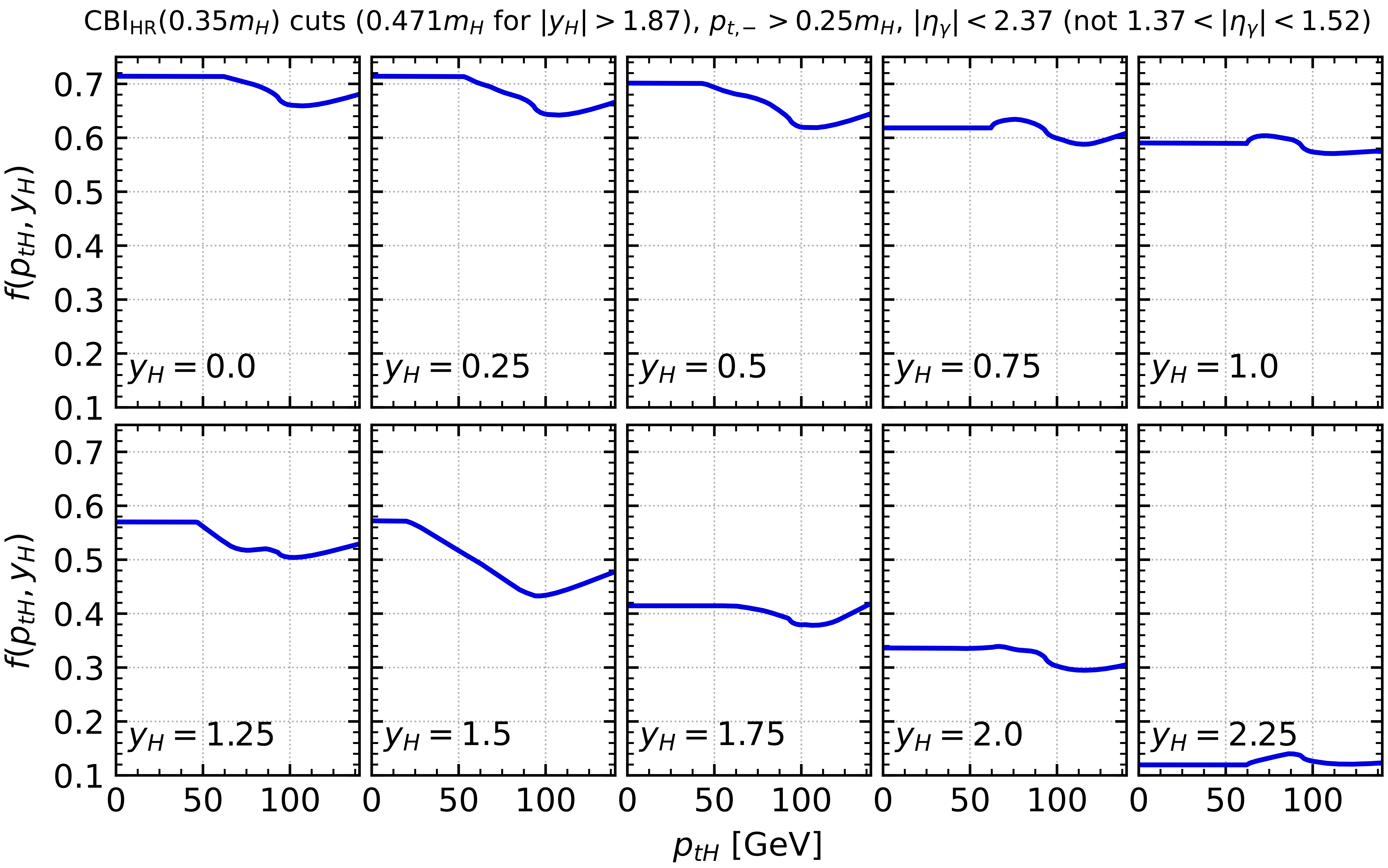




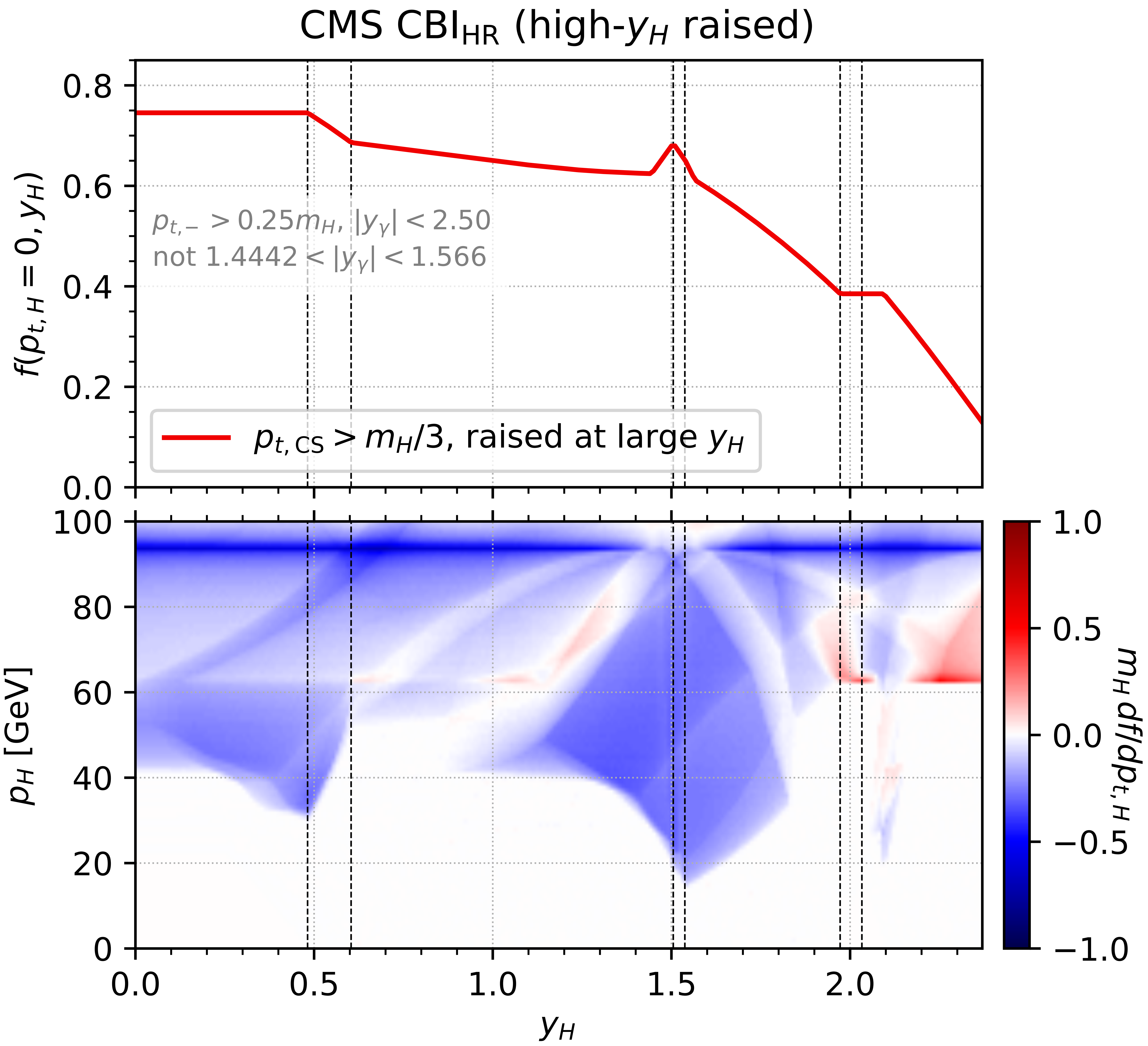




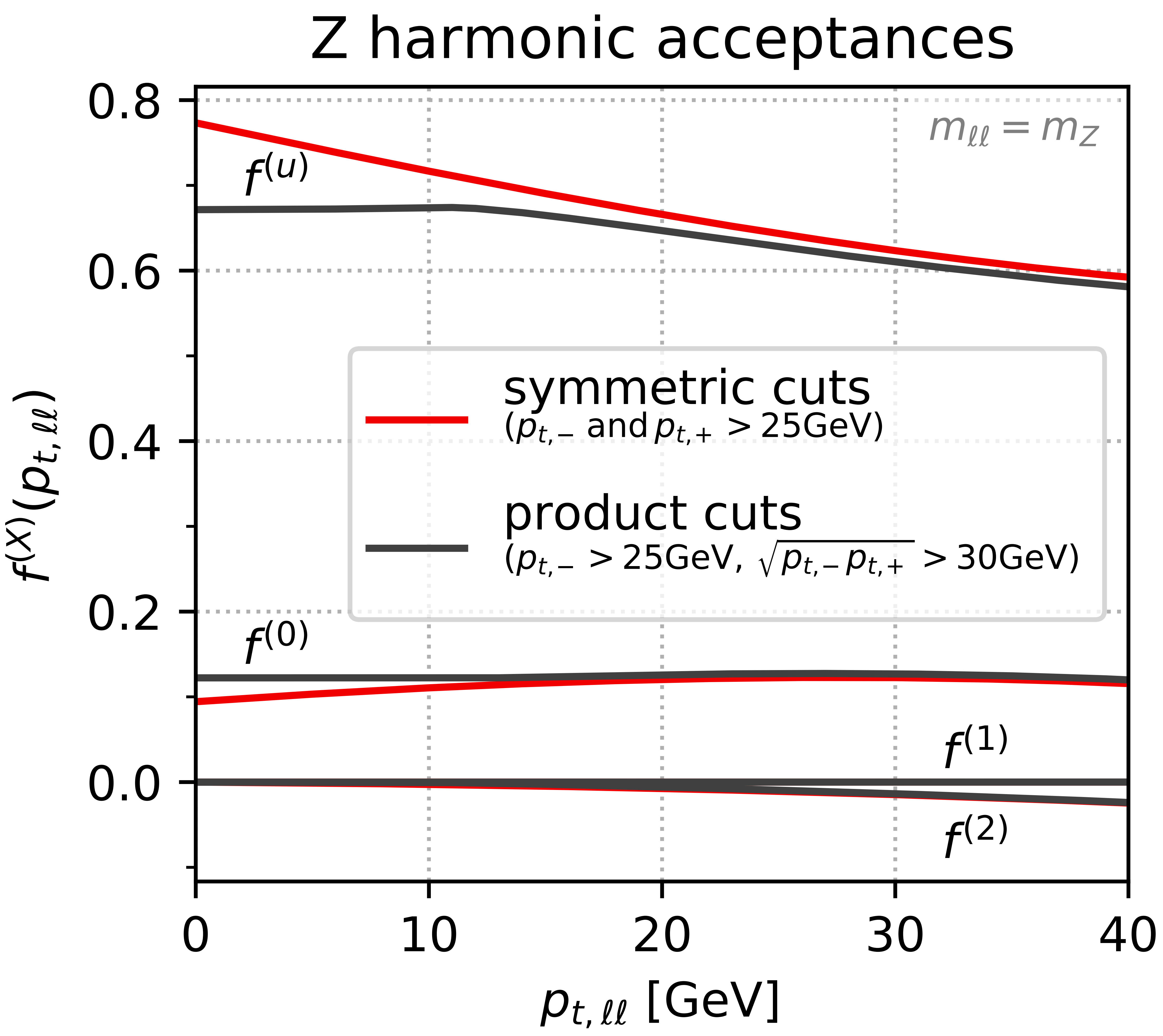
CBI_{HR} cuts: acceptance v. p_{tH} at several y_H values

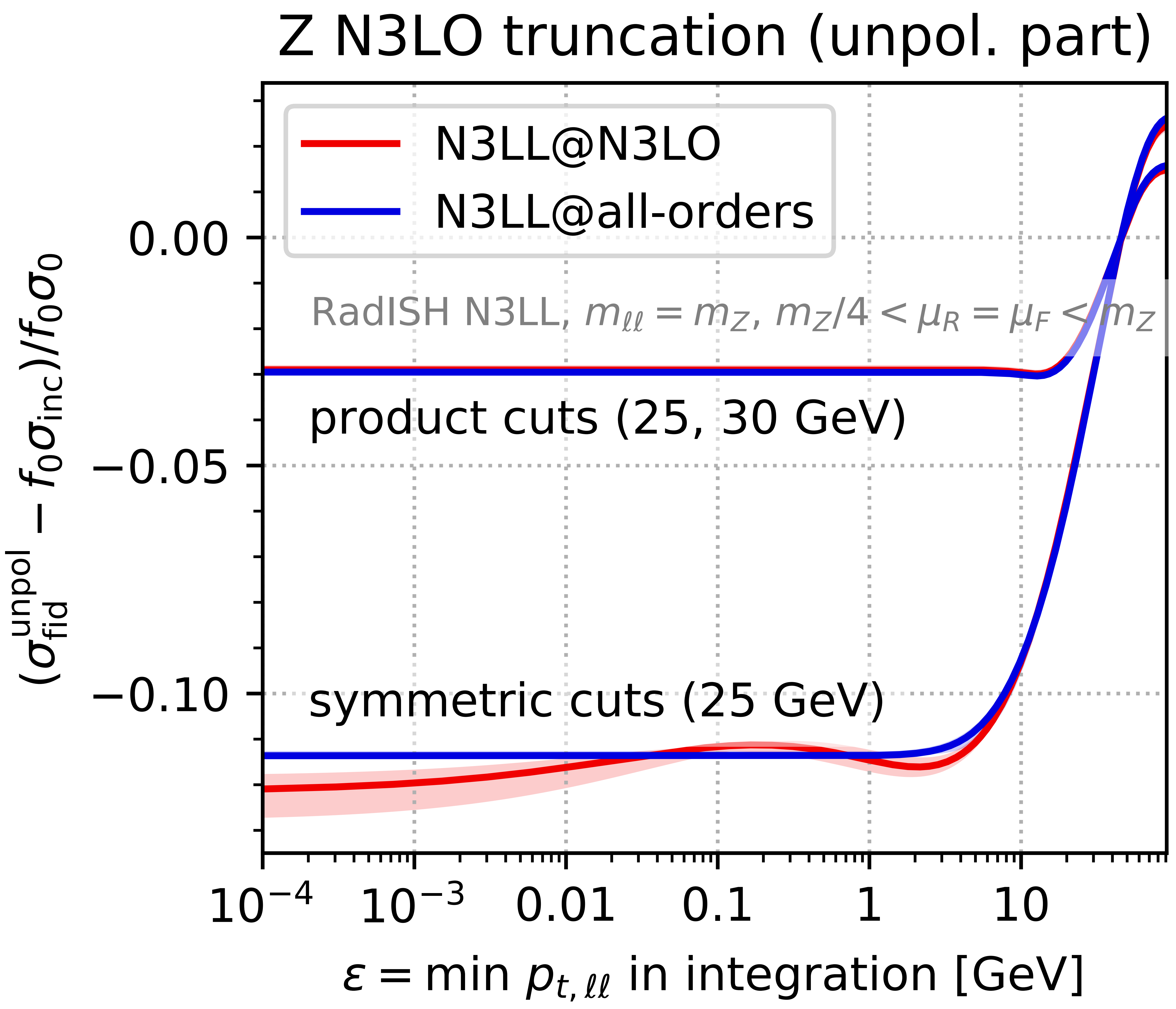


CBI_{HR} w. CMS rapidity cuts

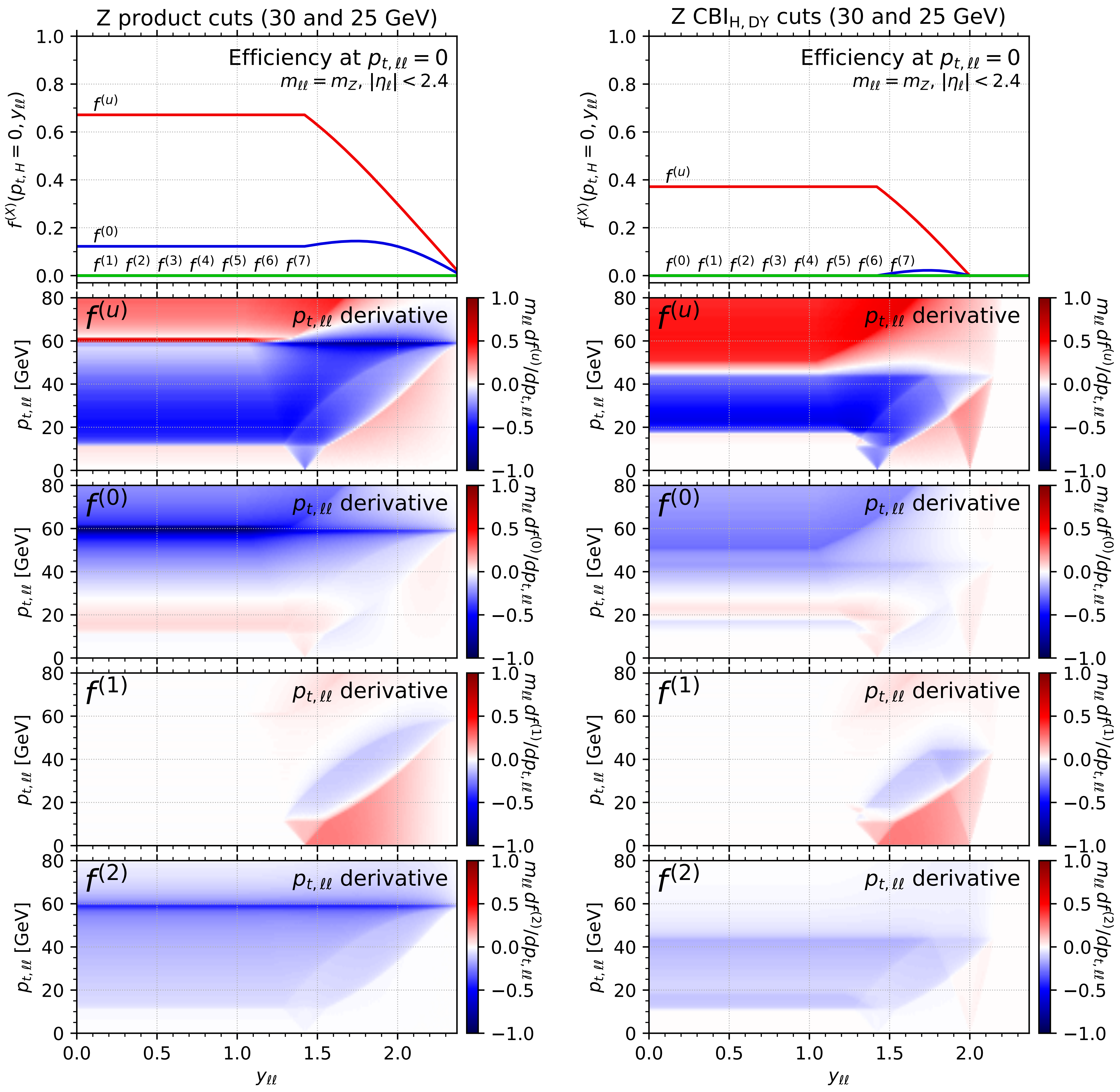


Example in Drell-Yan case





DY p_t dependence of harmonic acceptances with product and boost invariant cuts



Getting identically zero pt dependence for all harmonic acceptances requires an extra cut

$$\cos\theta > \bar{c} = \frac{-c_0 + \sqrt{4 - 3c^2}}{2}$$