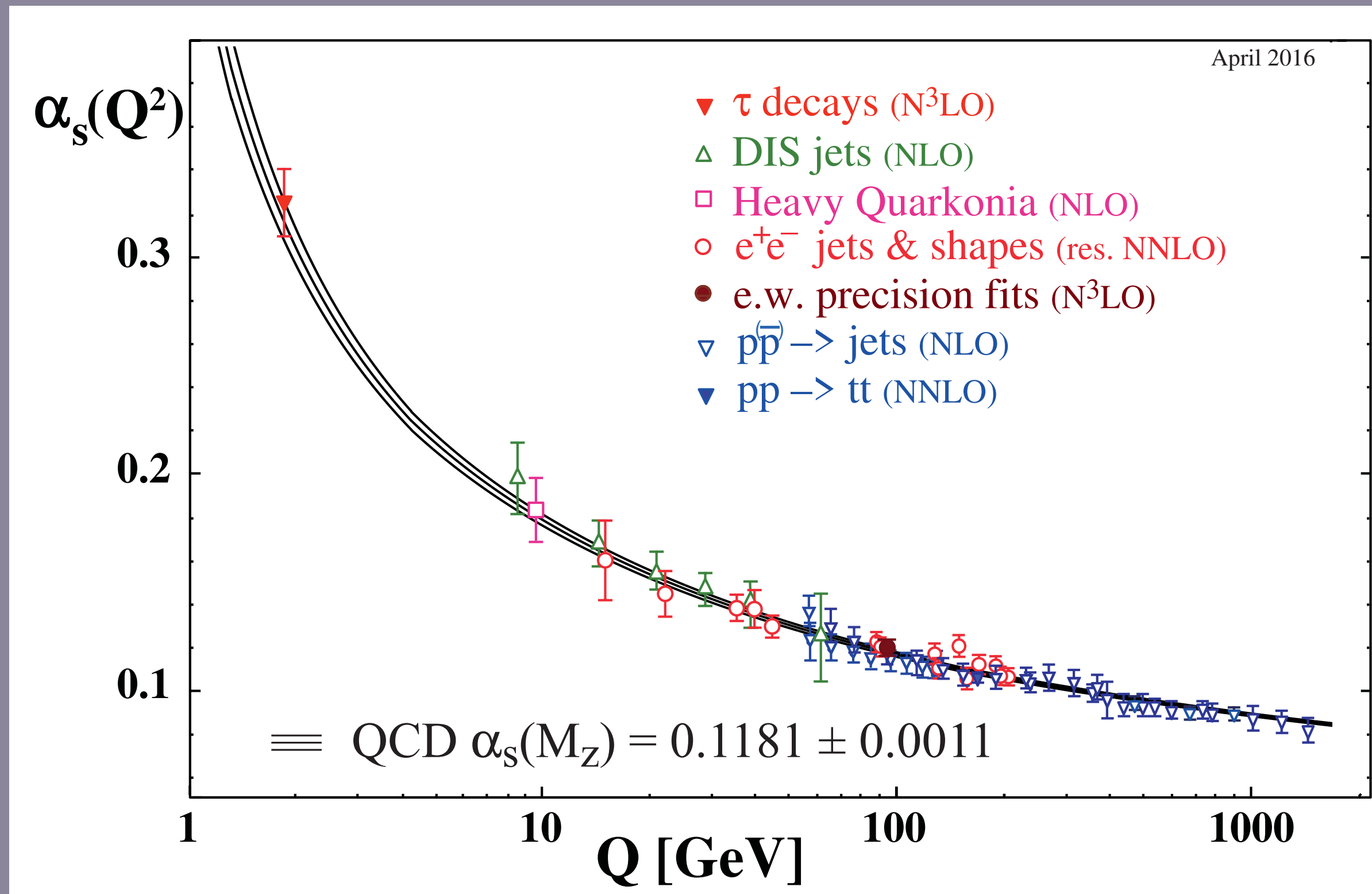


# QCD across colliders and energy scales

Special colloquium in honour of the retirement of Siggie Bethke



**Gavin Salam**  
Rudolf Peierls Centre for  
Theoretical Physics  
& All Souls College, Oxford



# particle physics

---

## “big unanswered questions”

about fundamental particles & their interactions  
(dark matter, matter-antimatter asymmetry,  
nature of dark energy, hierarchy of scales...)

v.

## “big answerable questions”

and how we go about answering them  
(nature of Higgs interactions, validity of SM up to high scales,  
lepton flavour universality, pattern of neutrino mixing, ...)

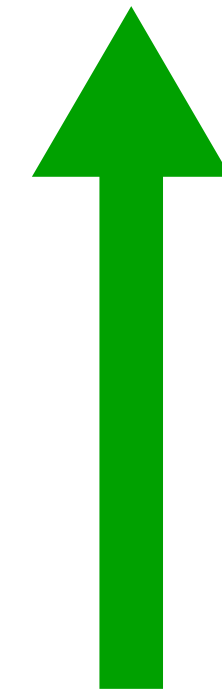
# The Lagrangian and Higgs interactions: two out of three qualitatively new!

---

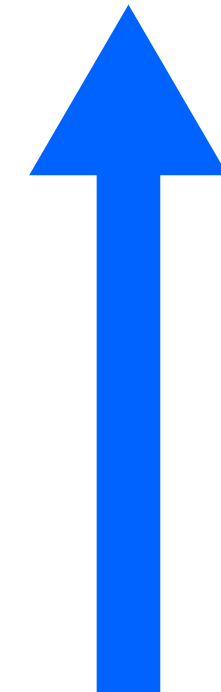
$$\mathcal{L}_{\text{SM}} = \dots + |D_\mu \phi|^2 + \psi_i y_{ij} \psi_j \phi - V(\phi)$$



Gauge interactions, structurally like those in QED, QCD, EW, **studied for many decades** (but now with a scalar)

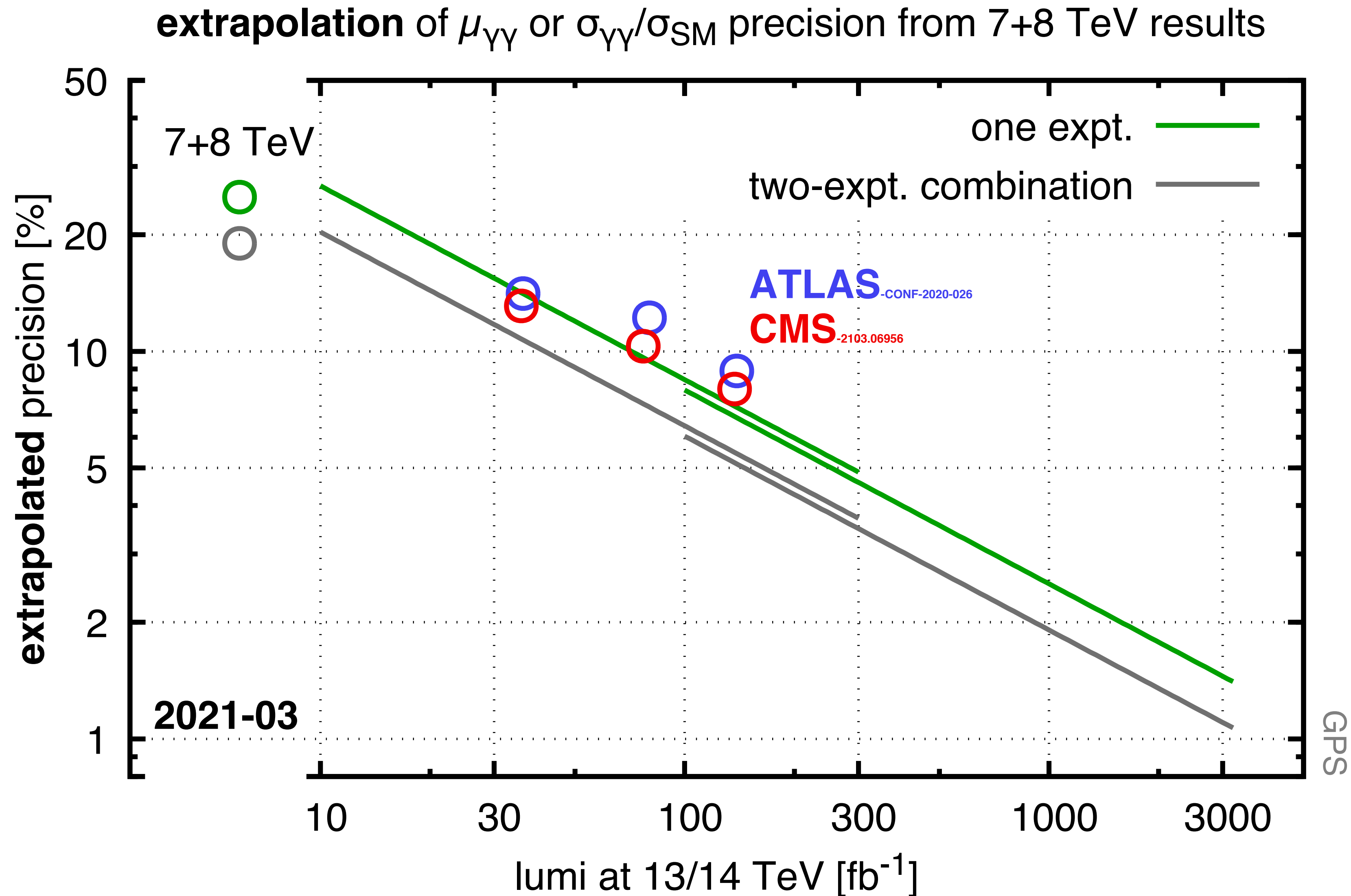


Yukawa interactions. Responsible for fermion masses, and induces “fifth force” between fermions. **Direct study started only in 2018!**



Higgs potential → self-interaction  
Holds the SM together.  
**Unobserved**

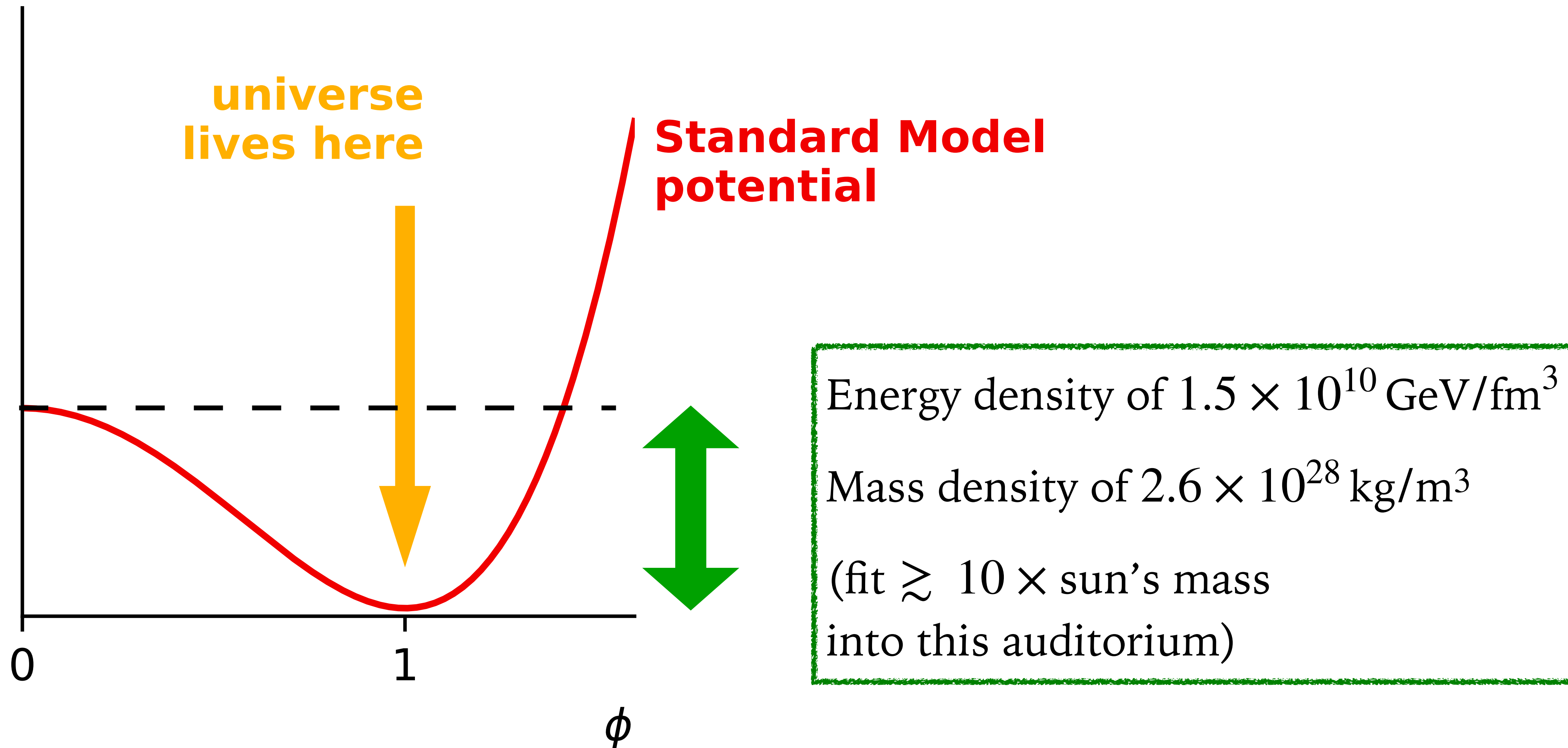
# The LHC is increasingly a precision machine, even for Higgs physics



1% uncertainty on  $\alpha_s$   
→ 2% uncertainty on  
Higgs cross section

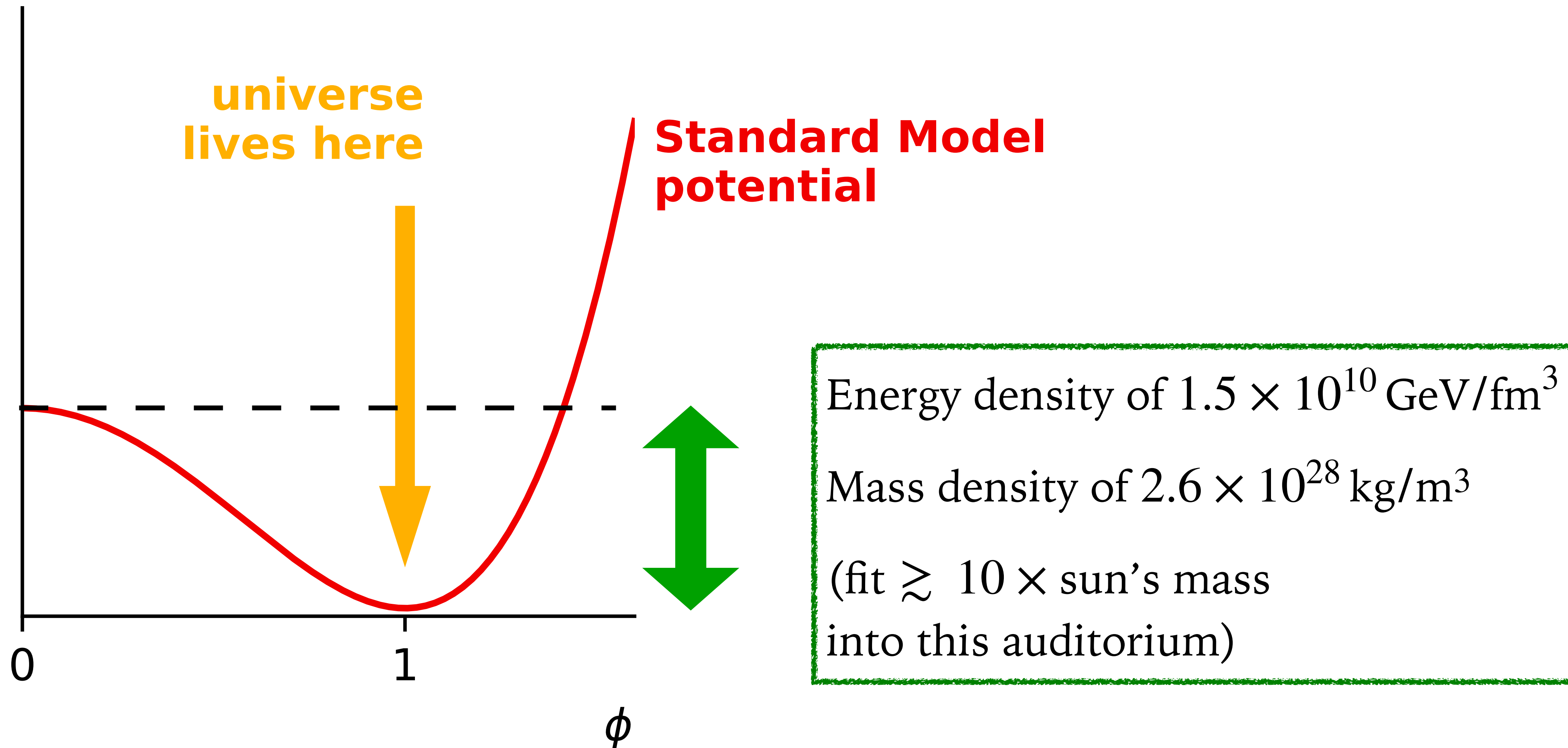
# Higgs potential — huge energy densities

$V(\phi)$ , SM



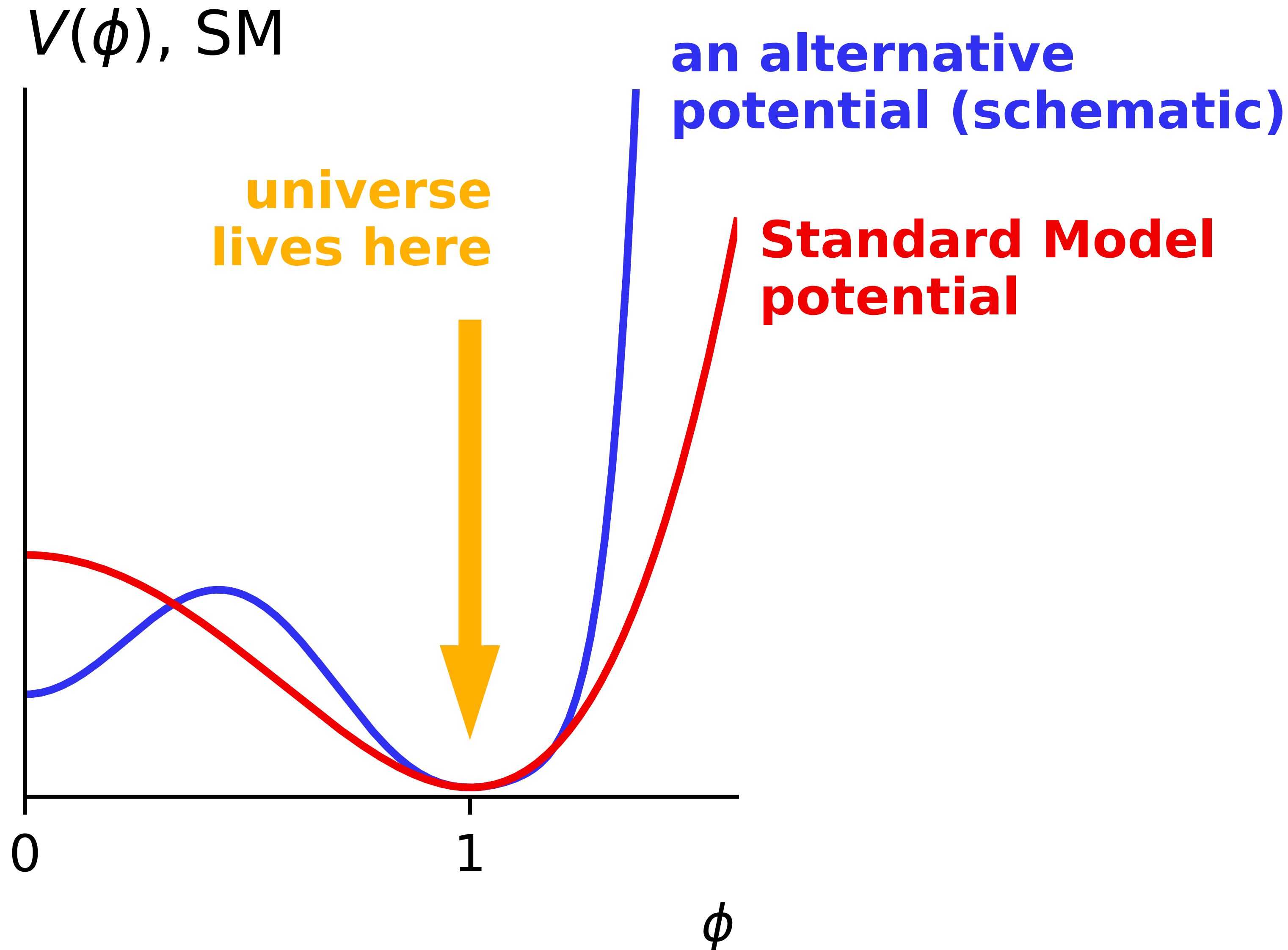
# Higgs potential — huge energy densities

$V(\phi)$ , SM

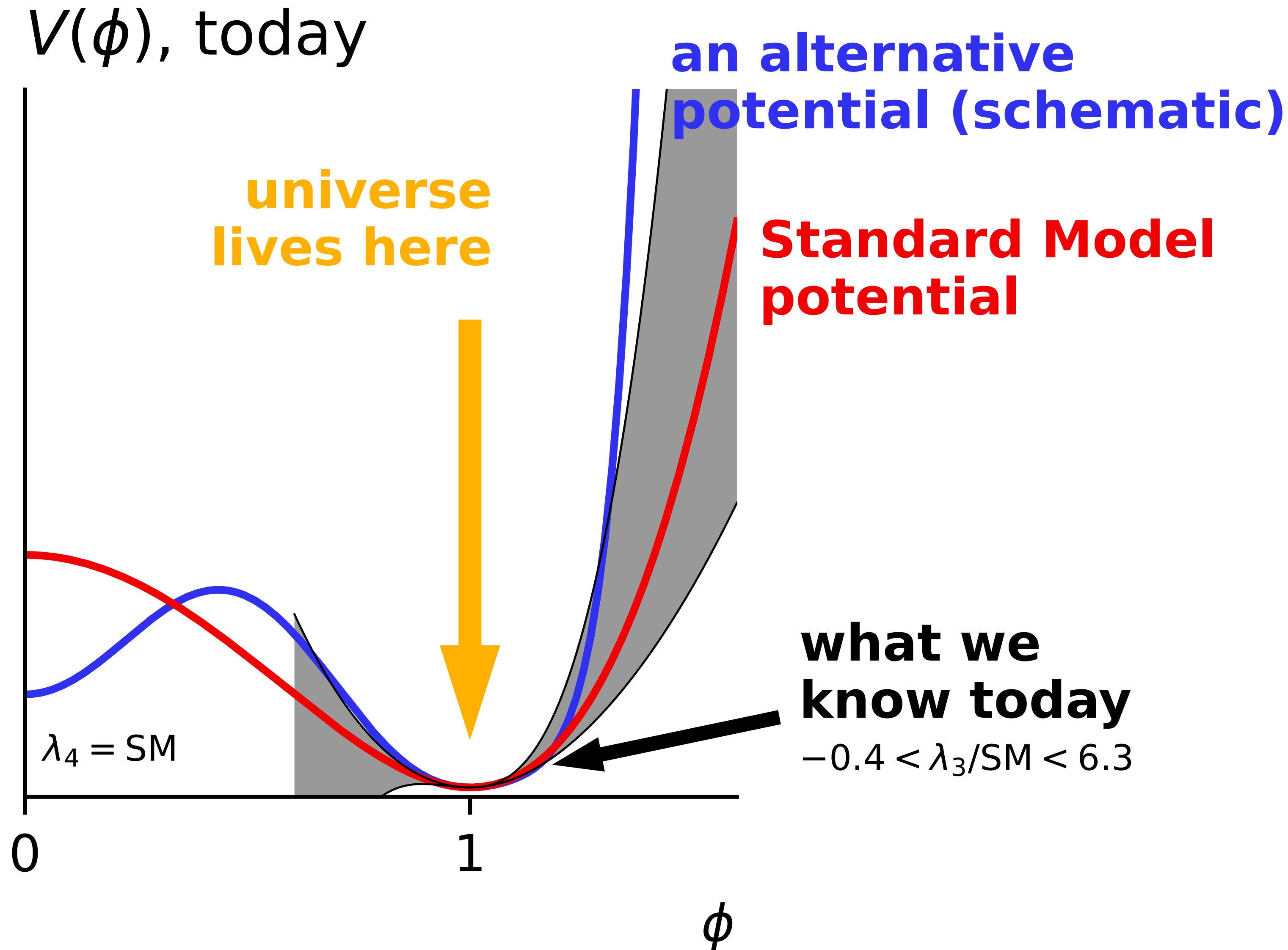


# Higgs potential — huge energy densities — yet to be experimentally confirmed

---

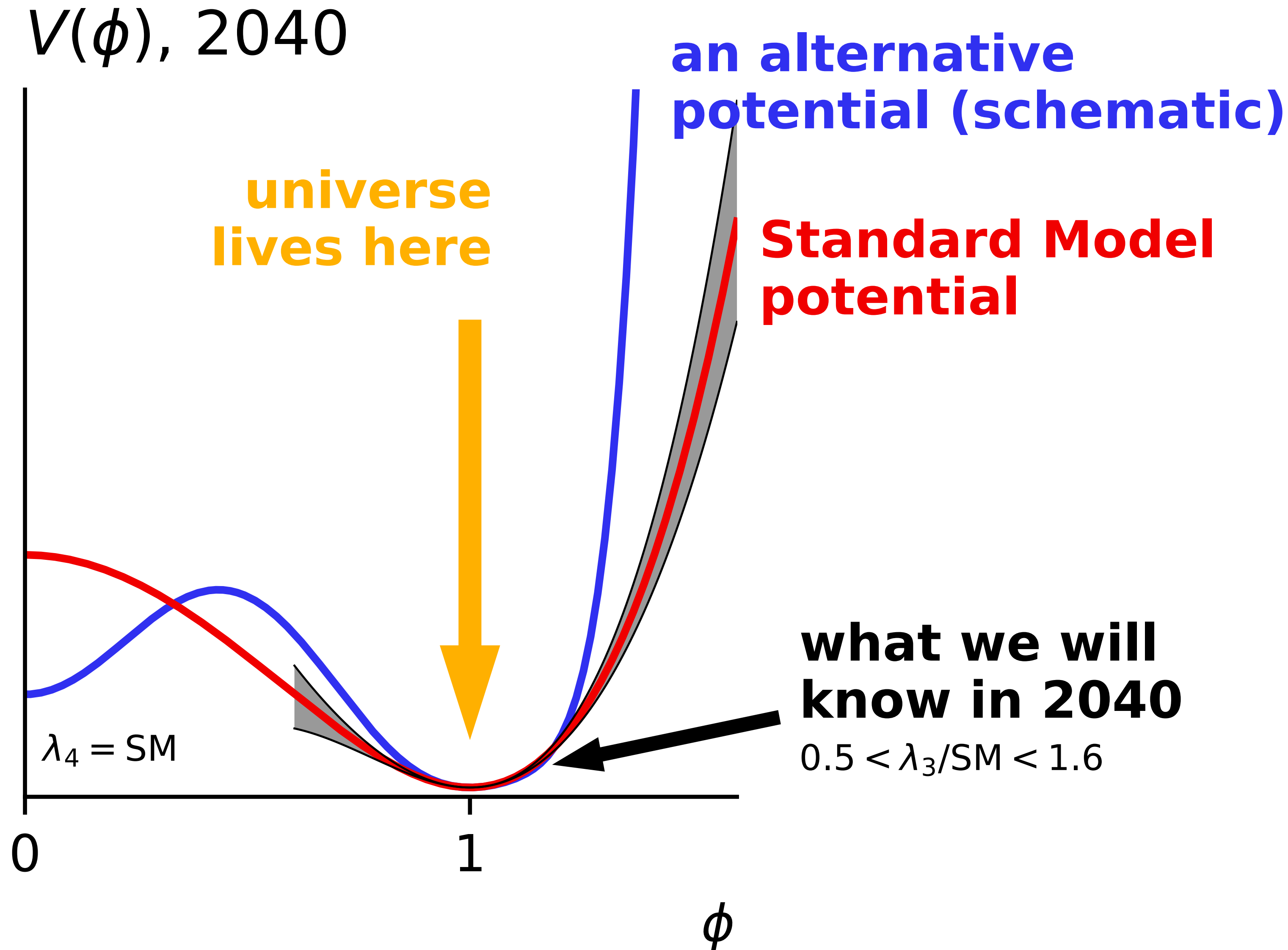


# Higgs potential — huge energy densities — yet to be experimentally confirmed

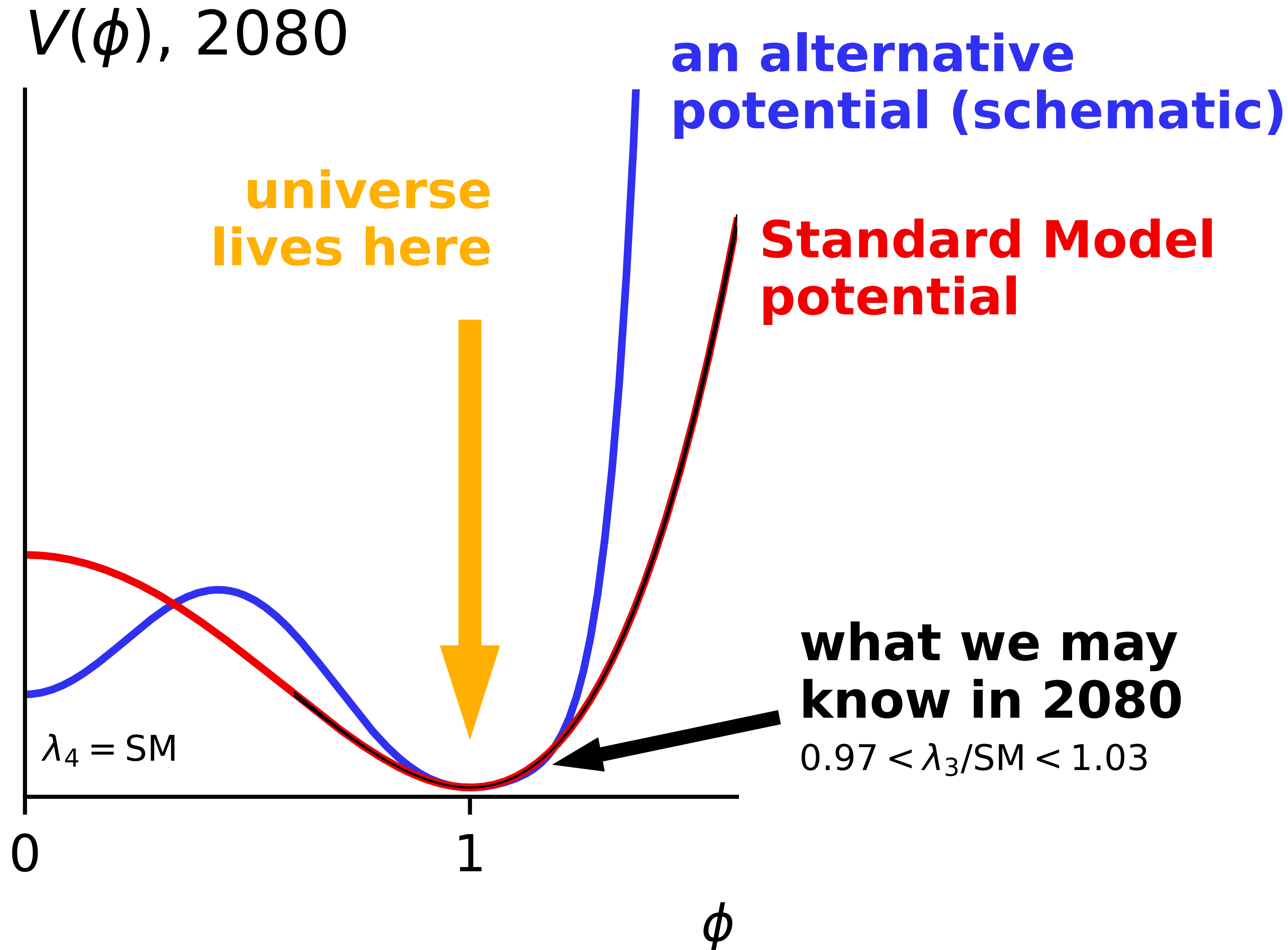




# Higgs potential — huge energy densities — yet to be experimentally confirmed



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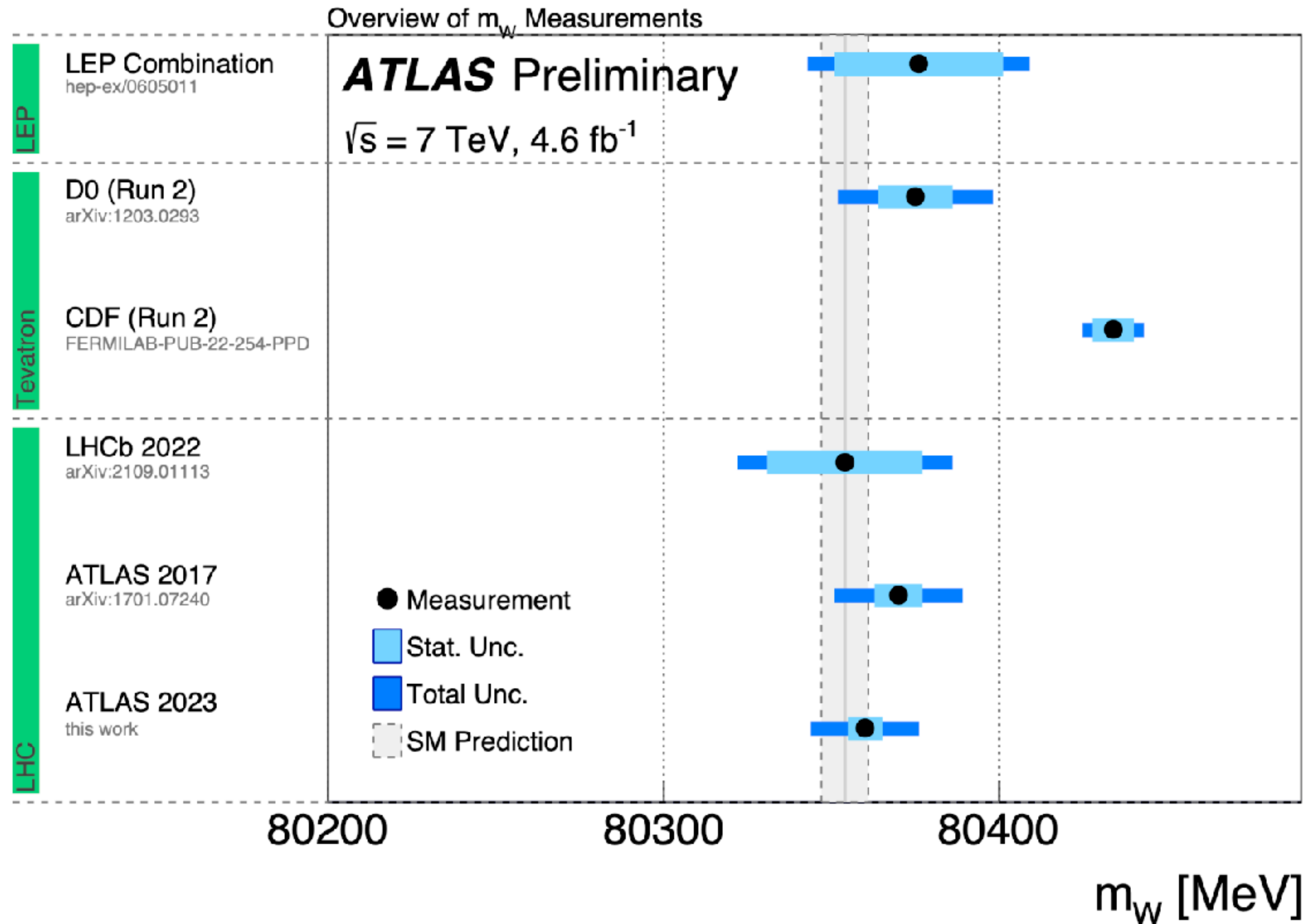


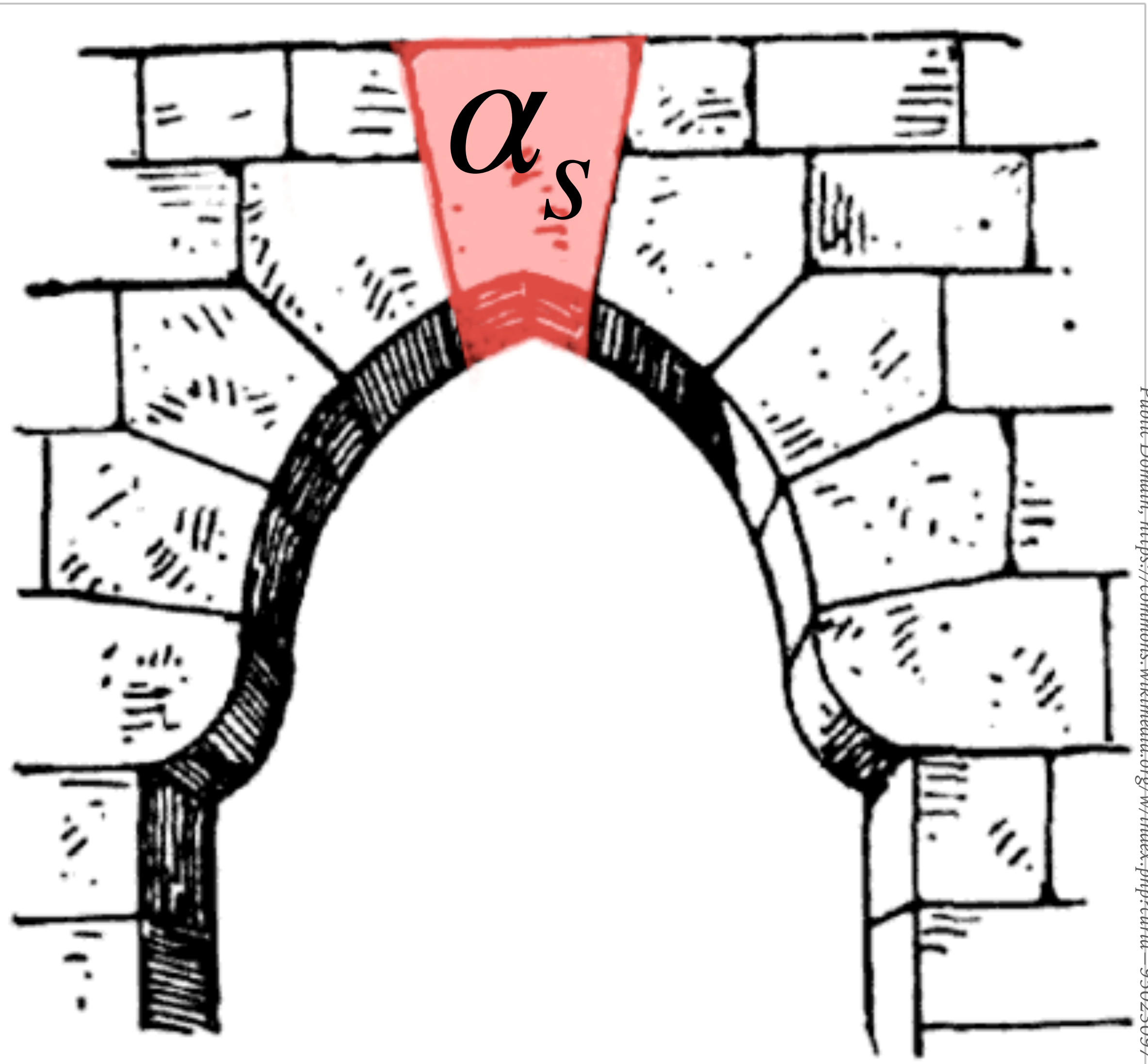
# ATLAS 2209.10910 ( $HH \rightarrow bb\tau\tau$ ) systematics [highest expected sensitivity]

Table 4: Breakdown of the relative contributions to the uncertainty in the extracted signal cross-sections, as determined in the likelihood fit (described in Section 8) to data. They are obtained by fixing the relevant nuisance parameters in the likelihood fit, subtracting the square of the obtained uncertainty in the fitted signal cross-section from the square of the total uncertainty, taking the square root, and then dividing by the total uncertainty. The sum in quadrature of the individual components differs from the total uncertainty due to correlations between uncertainties in the different groups.

Uncertainty source	Non-resonant $HH$	Resonant $X \rightarrow HH$		
		300 GeV	500 GeV	1000 GeV
<b>Data statistical + floating normalisation</b>	81%	76%	90%	93%
Data statistical	81%	76%	90%	93%
$t\bar{t}$ and $Z$ + HF normalisations	4%	8%	3%	5%
<b>Systematic</b>	58%	65%	43%	37%
MC statistical	28%	44%	33%	18%

# Do we know how to do precision physics at hadron colliders?

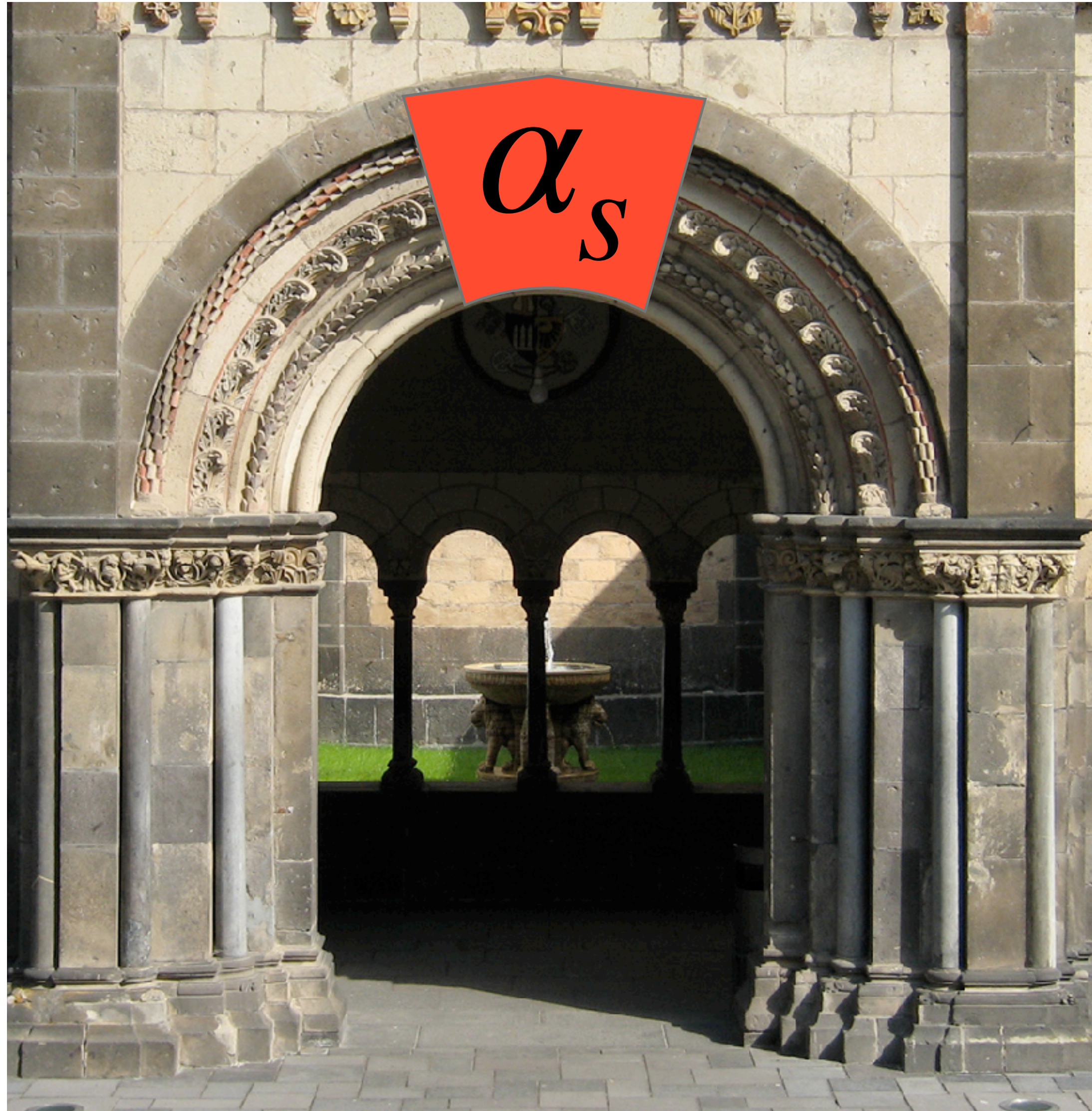




Public Domain, <https://commons.wikimedia.org/w/index.php?curid=95023097>

# the strong coupling

# Maria Laach



[https://de.wikipedia.org/wiki/Abtei\\_Maria\\_Laach#/media/Datei:Maria\\_Laach\\_05.jpg](https://de.wikipedia.org/wiki/Abtei_Maria_Laach#/media/Datei:Maria_Laach_05.jpg)

# the strong coupling

# TESTS OF QCD \*

HD-PY 92/13

OPAL-CR093

October 23, 1992

Siegfried Bethke §

Physikalisches Institut, University of Heidelberg

Philosophenweg 12

D-6900 Heidelberg, Germany

DETERMINATIONS OF $\alpha_s$ .....	16
$\alpha_s$ from $e^+e^-$ Annihilations .....	18
$\alpha_s$ from Deep Inelastic Scattering .....	23
$\alpha_s$ from Hadron Collisions .....	24
$\alpha_s$ from Heavy Quarkonia Decays .....	24
$\alpha_s$ from Mass Splitting of Charmonium States .....	25
Summary of $\alpha_s$ Measurements .....	25

The final world average is thus quoted to be

$$\alpha_s(M_{Z^0}) = 0.118 \pm 0.007,$$

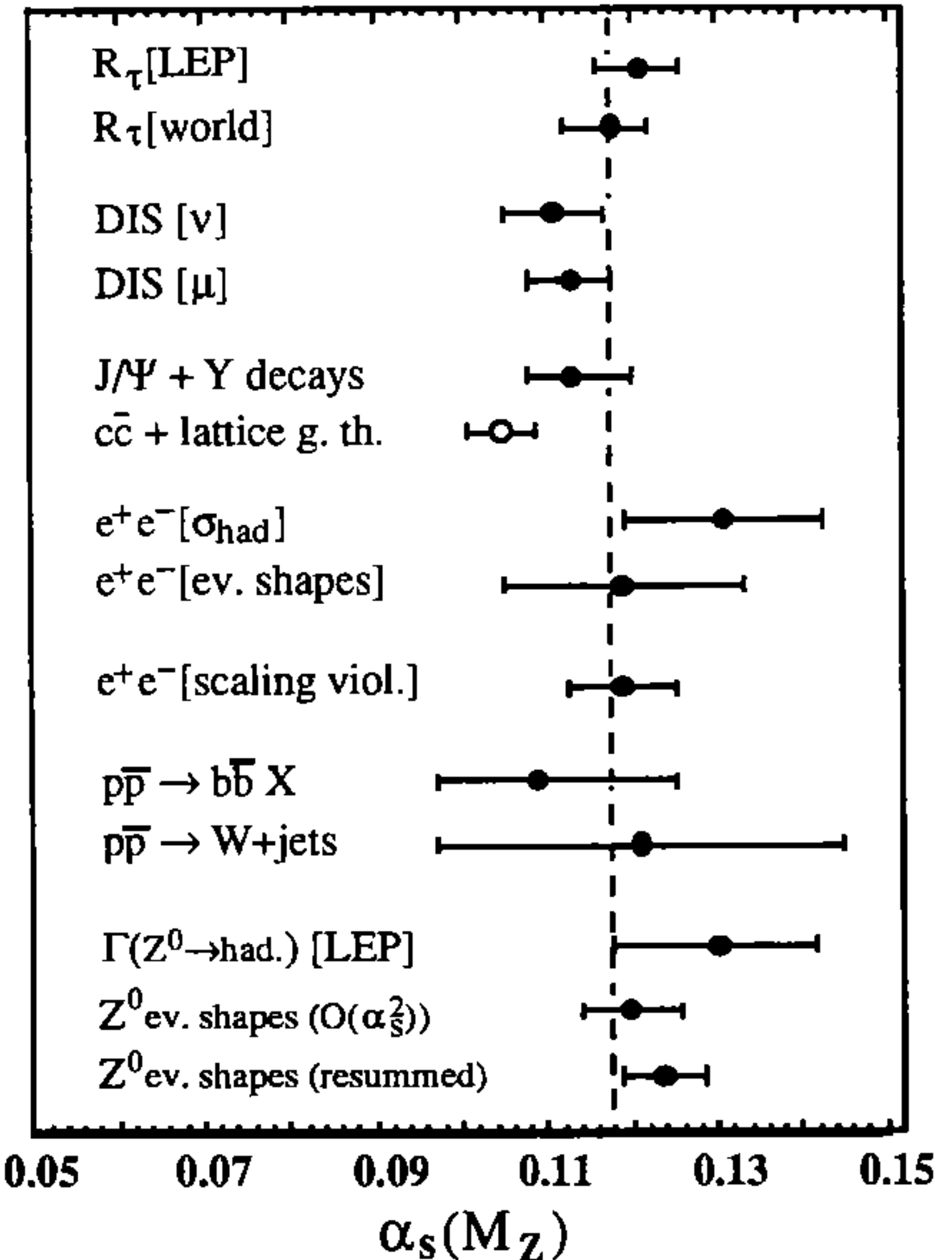


Fig. 31. Summary of measurements of  $\alpha_s(M_{Z^0})$ .

# TESTS OF QCD \*

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$\alpha_s$ from Mass Splitting of Charmonium States .....	25
Summary of $\alpha_s$ Measurements .....	25

Gerne hätte ich fortgeschrieben  
aber es ist liegen geblieben.

*Johann Wolfgang Goethe*

The final world average is thus quoted to be

$$\alpha_s(M_{Z^0}) = 0.118 \pm 0.007 ,$$



### A Summary of alpha-s measurements

Siegfried Bethke (Heidelberg U.), Stefano Catani (CERN) (1992)

Contribution to: [27th Rencontres de Moriond: QCD and High-energy Hadronic Interacti](#)

[pdf](#) [cite](#) [claim](#) [reference search](#) [1 citation](#)

# 1992

### Summary of alpha-s measurements

Siegfried Bethke (Aachen, Tech. Hochsch.) (Oct, 1994)

Published in: *Nucl.Phys.B Proc.Suppl.* 39BC (1995) 198 - Contribution to: [Summer Sc Aspects of Collider Physics, High-energy Physics: QCD Workshop 94, Summer School on Hadronic Aspects of Collider Physics](#), 105-121

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [31 citations](#)

# 1994

### Status of alpha-s measurements

Siegfried Bethke (Aachen, Tech. Hochsch.) (1995)

Contribution to: [30th Rencontres de Moriond: QCD and High-energy Hadronic Interactions](#)

[pdf](#) [links](#) [cite](#) [claim](#) [reference search](#) [0 citations](#)

# 1995

### Experimental tests of asymptotic freedom

Siegfried Bethke (Aachen, Tech. Hochsch.) (Sep, 1996)

Published in: *Nucl.Phys.B Proc.Suppl.* 54 (1997) 314-326 - Contribution to: [High-energy I Euroconference on Quantum Chromodynamics \(QCD 96\)](#), 314-324 - e-Print: [hep-ex/9609014](#) [hep-ex]

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [39 citations](#)

# 1996

### QCD tests at e+ e- colliders

Siegfried Bethke (Aachen, Tech. Hochsch.) (Sep, 1997)

Published in: *Nucl.Phys.B Proc.Suppl.* 64 (1998) 54-62 - Contribution to: [High-Energy Phys Euroconference on Quantum Chromodynamics: QCD 97: 25th Anniversary of QCD](#) - e-Print: [hep-ex/9710030](#) [hep-ex]

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [32 citations](#)

# 1997

### Jet physics at LEP and world summary of alpha(s)

S. Bethke (Aachen, Tech. Hochsch.) (Sep, 1998)

Contribution to: [RADCOR 1998](#), 243-260 - e-Print: [hep-ex/9812026](#) [hep-ex]

[pdf](#) [cite](#) [claim](#) [reference search](#) [36 citations](#)

# 1998

### Determination of the QCD coupling alpha\_s

S. Bethke (Munich, Max Planck Inst.) (Apr, 2000)

Published in: *J.Phys.G* 26 (2000) R27 - e-Print: [hep-ex/0004021](#) [hep-ex]

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [261 citations](#)

# 2000

### alpha\_s 2002

Siegfried Bethke (Munich, Max Planck Inst.) (Nov, 2002)

Published in: *Nucl.Phys.B Proc.Suppl.* 121 (2003) 74-81 - Contribution to: [QCD 02, 74 ex/0211012](#) [hep-ex]

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [101 citations](#)

# 2002

### alpha\_s at Zinnowitz 2004

Siegfried Bethke (Munich, Max Planck Inst.) (Jul, 2004)

Published in: *Nucl.Phys.B Proc.Suppl.* 135 (2004) 345-352 - Contribution to: [7th DESY Elementary Particle Theory: Loops and Legs in Quantum Field Theory](#), 345-352 - e-Print: [hep-ex/0407021](#) [hep-ex]

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [131 citations](#)

# 2004

### Experimental tests of asymptotic freedom

Siegfried Bethke (Munich, Max Planck Inst.) (Jun, 2006)

Published in: *Prog.Part.Nucl.Phys.* 58 (2007) 351-386 - e-Print: [hep-ex/0608035](#) [hep-ex]

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [388 citations](#)

# 2006

### The 2009 World Average of alpha(s)

Siegfried Bethke (Munich, Max Planck Inst.) (Aug, 2009)

Published in: *Eur.Phys.J.C* 64 (2009) 689-703 - e-Print: [0908.1135](#) [hep-ph]

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [561 citations](#)

# 2009

### Review of Particle Physics (RPP)

Particle Data Group - J. Beringer (LBL, Berkeley) et al. (2012)

Published in: *Phys.Rev.D* 86 (2012) 010001

[DOI](#) [cite](#) [claim](#) [reference search](#) [9,068 citations](#)

# 2011

### Review of Particle Physics

Particle Data Group - K.A. Olive (Minnesota U.) et al. (2014)

Published in: *Chin.Phys.C* 38 (2014) 090001

[links](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [9,091 citations](#)

# 2013

### Review of Particle Physics

Particle Data Group - C. Patrignani (Bologna U. and INFN, Bologna) et al. (Oct 3, 2016)

Published in: *Chin.Phys.C* 40 (2016) 10, 100001

[pdf](#) [links](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [6,794 citations](#)

# 2015

### Determination of the strong coupling constant alpha\_s(m\_Z) from measurement of the total cross section for top-antitop quark production

Thomas Kljnsma (Zurich, ETH), Siegfried Bethke (Munich, Max Planck Inst.), Günther Dis Gavin P. Salam (CERN and Paris, LPTHE) (Aug 24, 2017)

Published in: *Eur.Phys.J.C* 77 (2017) 11, 778 - e-Print: [1708.07495](#) [hep-ph]

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [36 citations](#)

# 2017

### Pre-2019 summaries of alpha\_s

Siegfried Bethke (Munich, Max Planck Inst.) (Jul 9, 2019)

Published in: *PoS ALPHAS2019* (2019) 001 - Contribution to: [alphas-2019](#), 7-12, [alpl](#)

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [0 citations](#)

# 2019

# Three decades of the strong coupling

Uncertainty has gone down by an order of magnitude to  $\sim 0.8\%$

central value has stayed stable, today

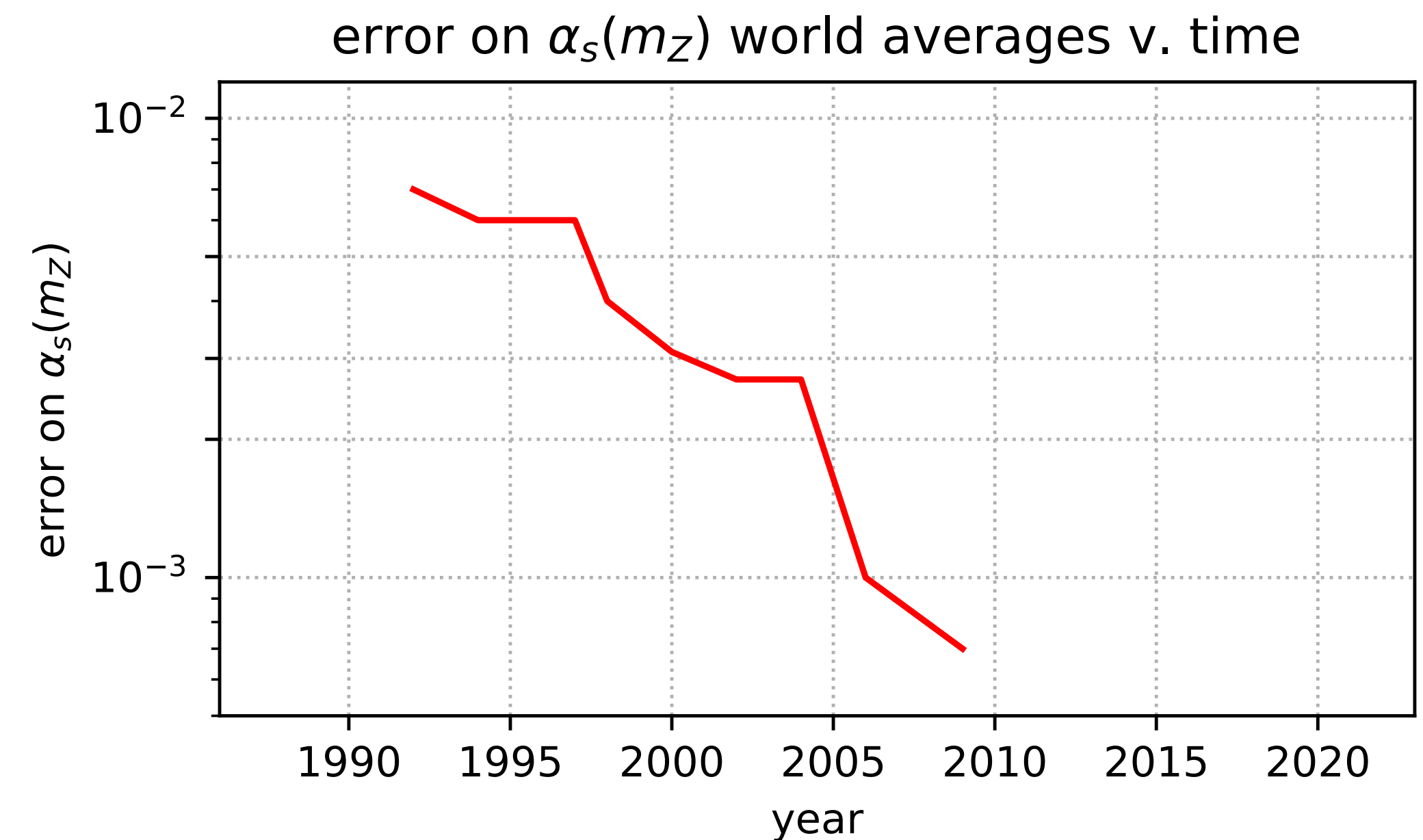
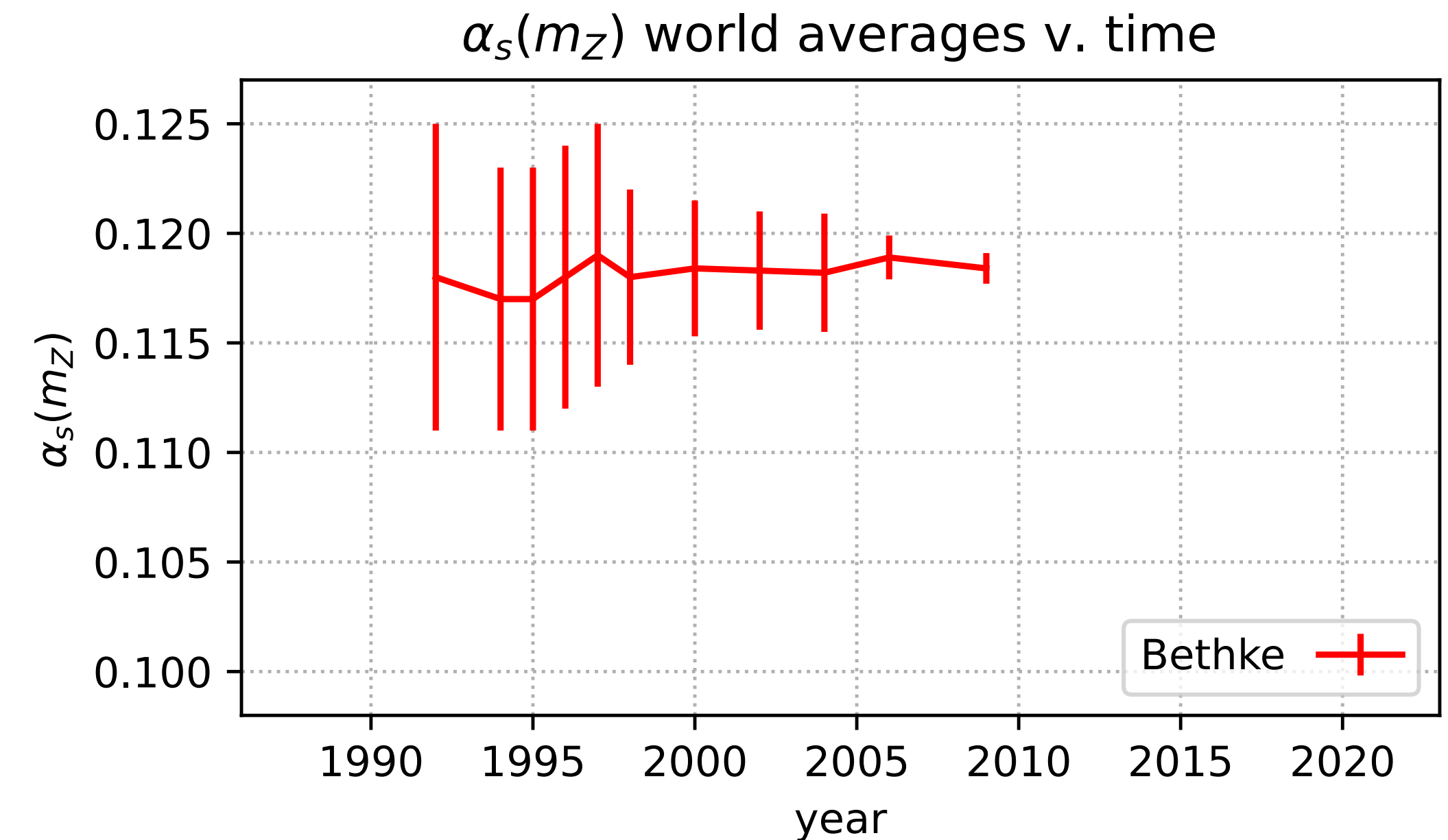
$$\alpha_s(m_Z) = 0.1179 \pm 0.0009$$

## Sources of improvement

- data (LEP, DIS,  $\sim$ LHC)
- better theory (e.g. NNLO, N3LL)
- better computers (e.g. for lattice)

## Challenges

- how to handle spread of error estimates (e.g. when systematic dominated)



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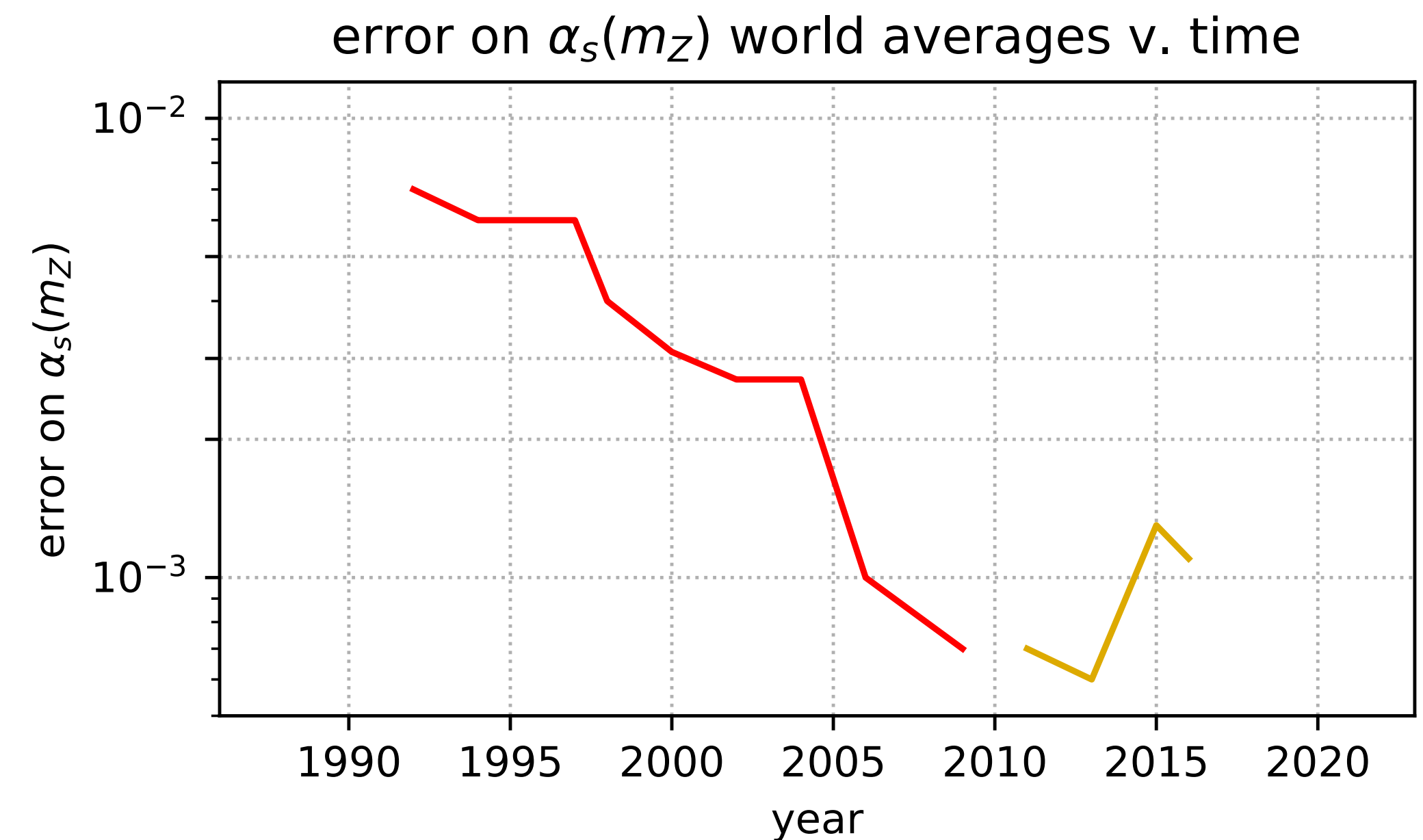
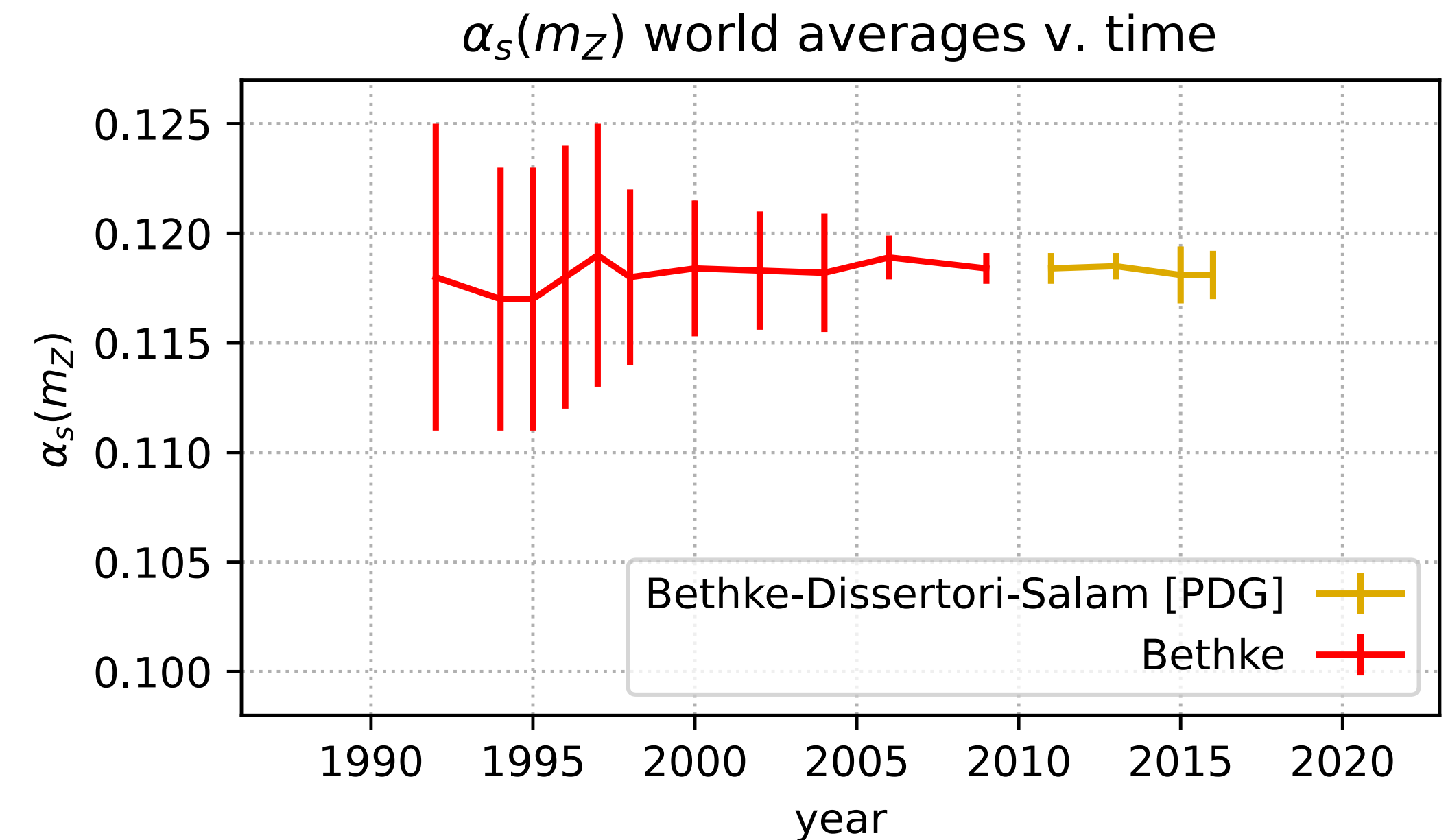
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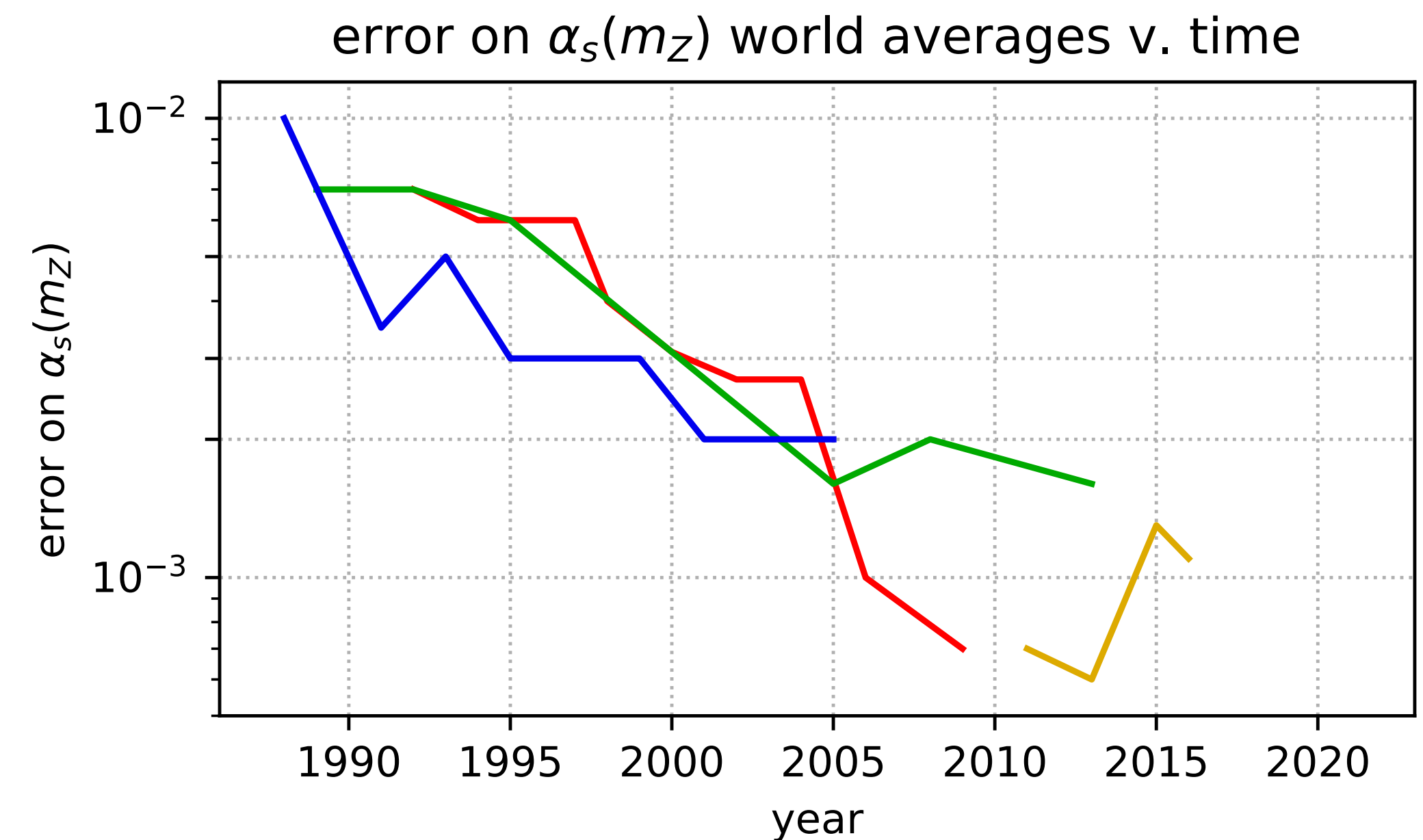
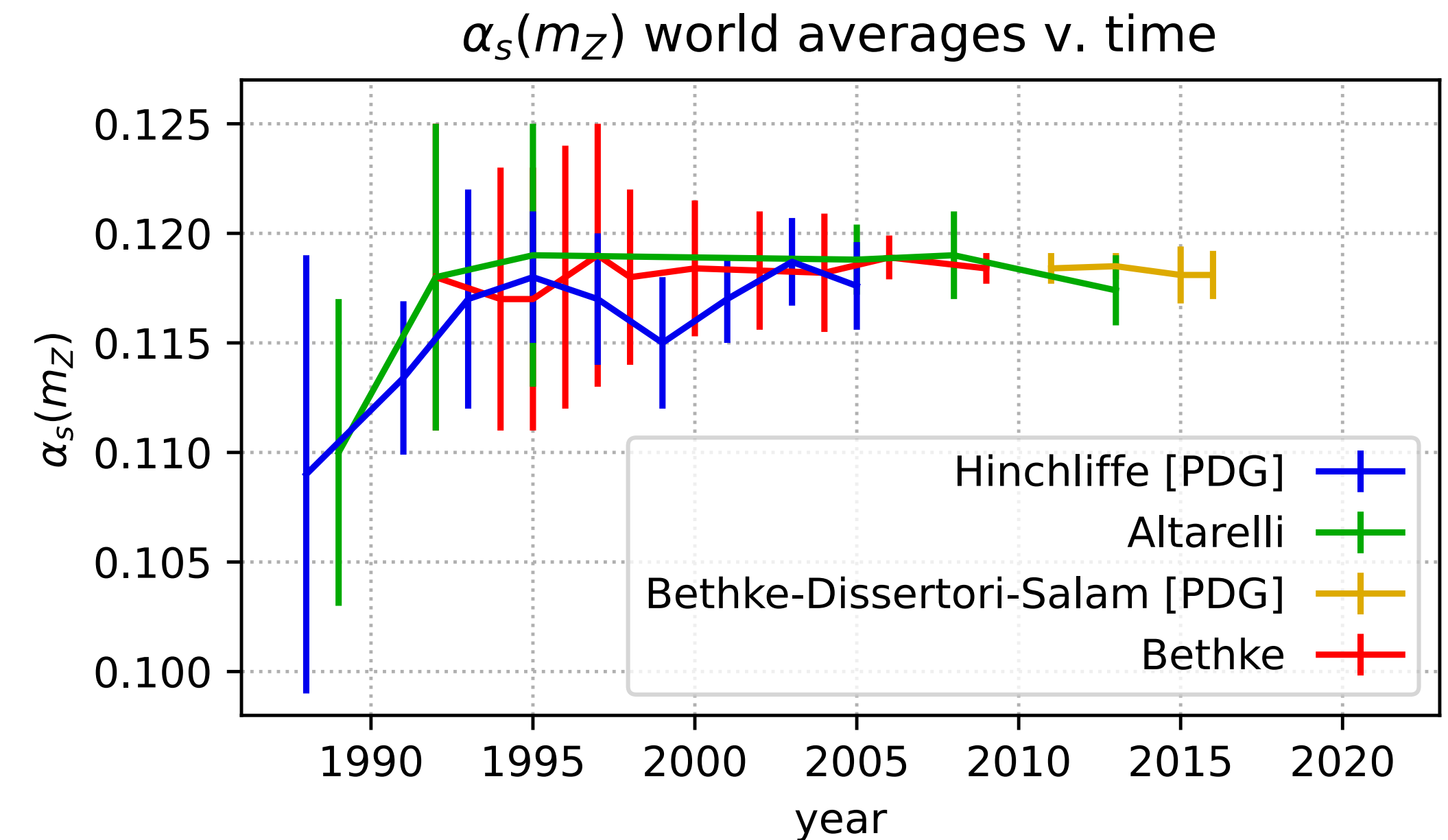
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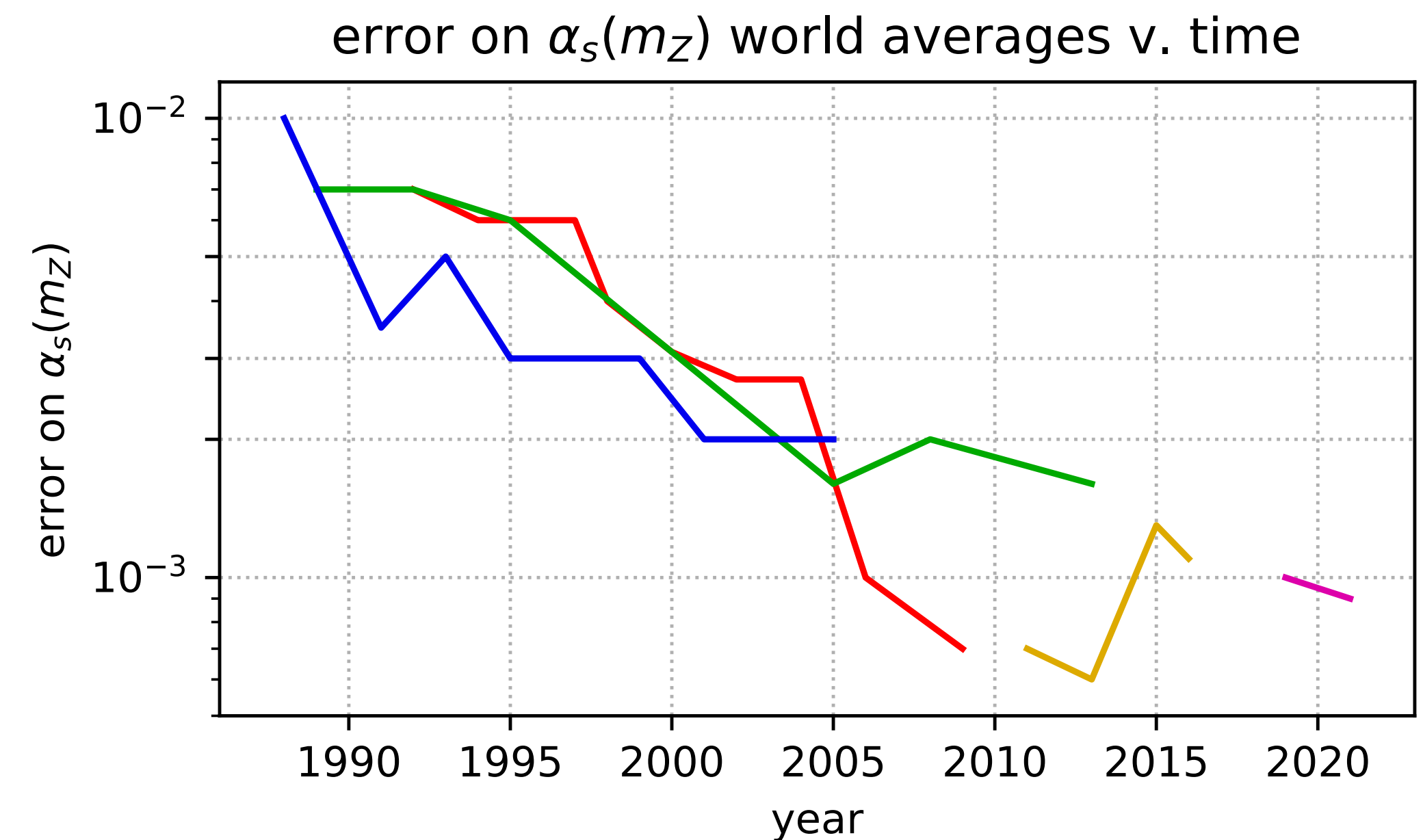
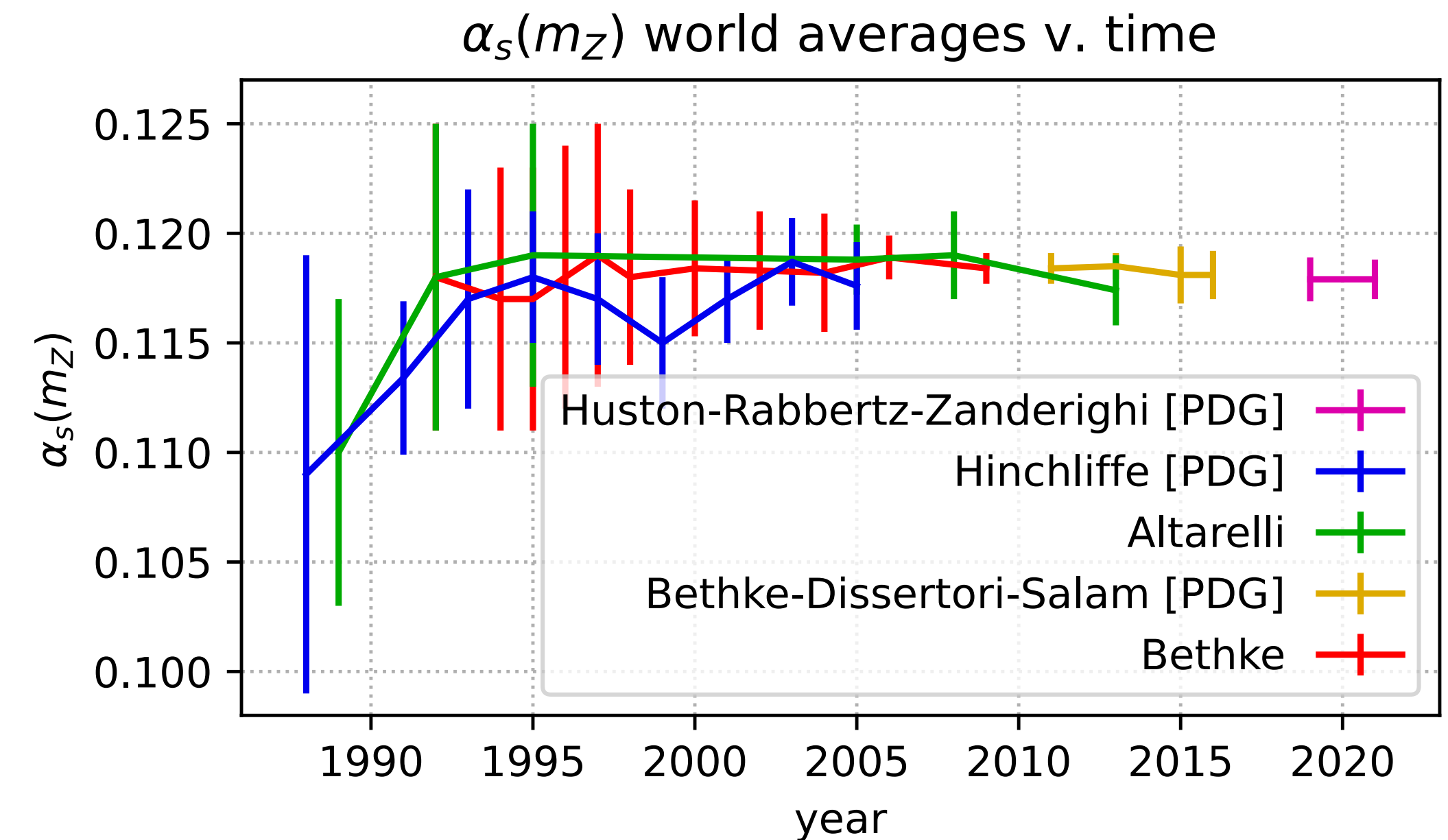
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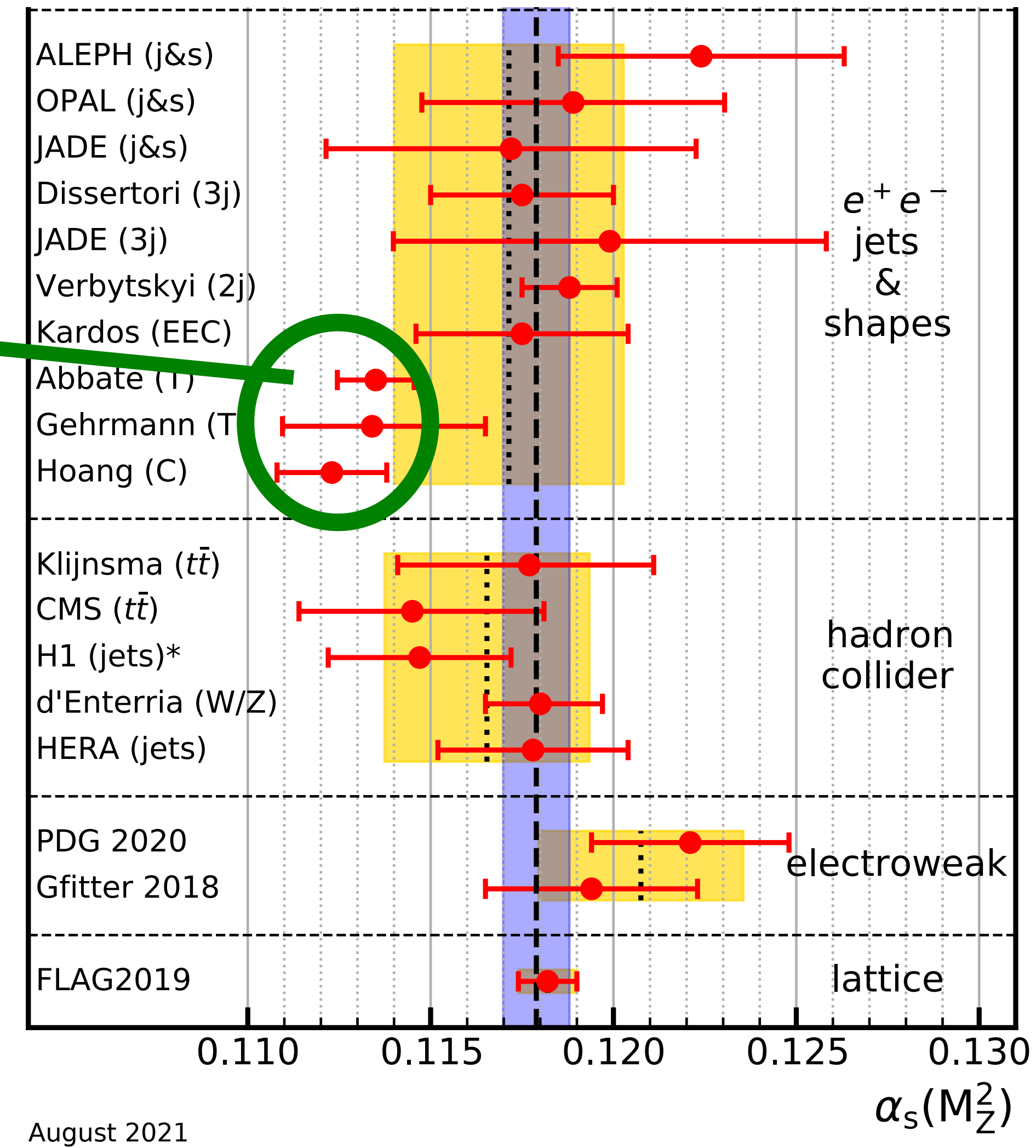
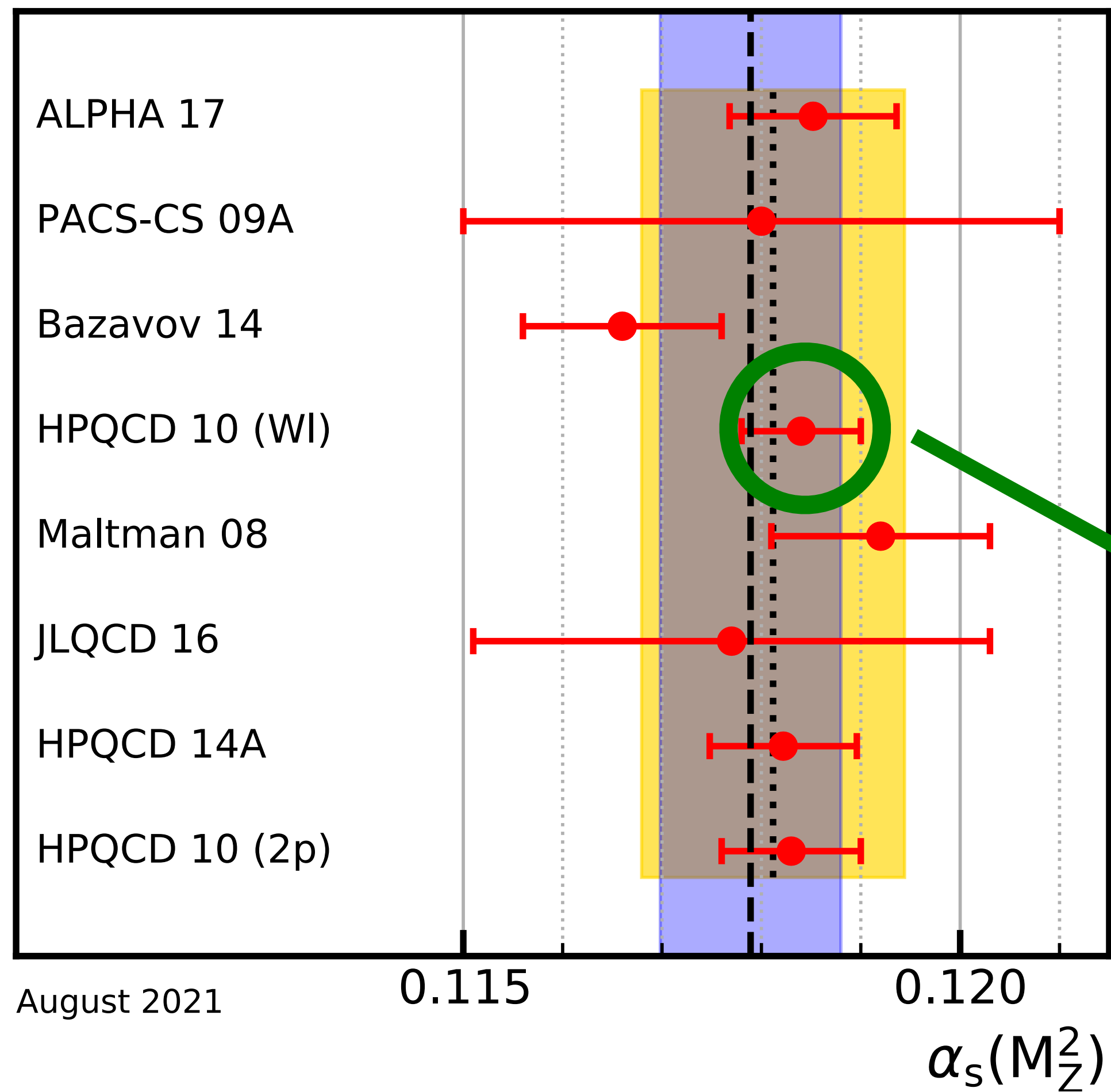
## Sources of improvement

- data (LEP, DIS,  $\sim$ LHC)
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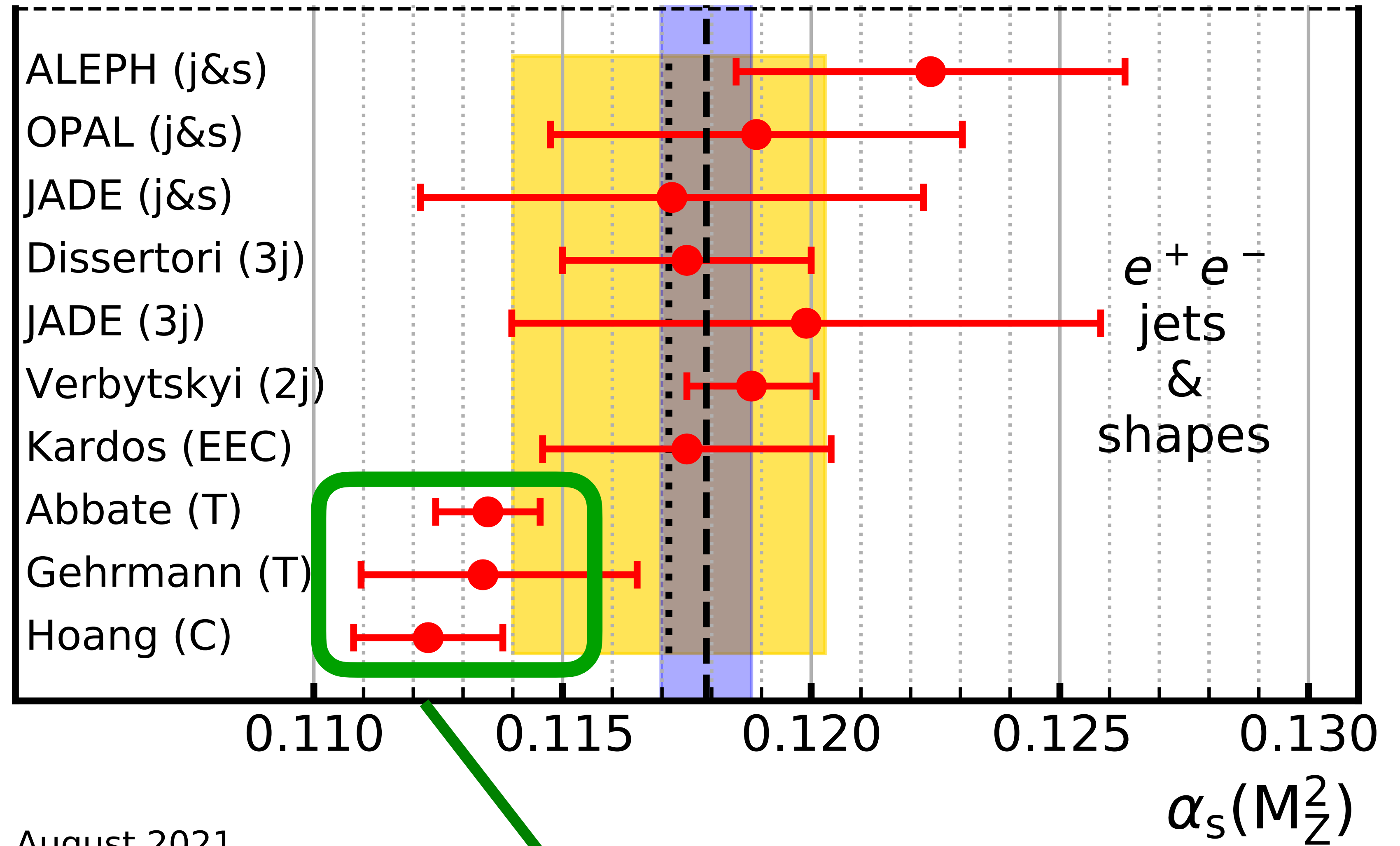
- how to handle spread of error estimates (e.g. when systematic dominated)





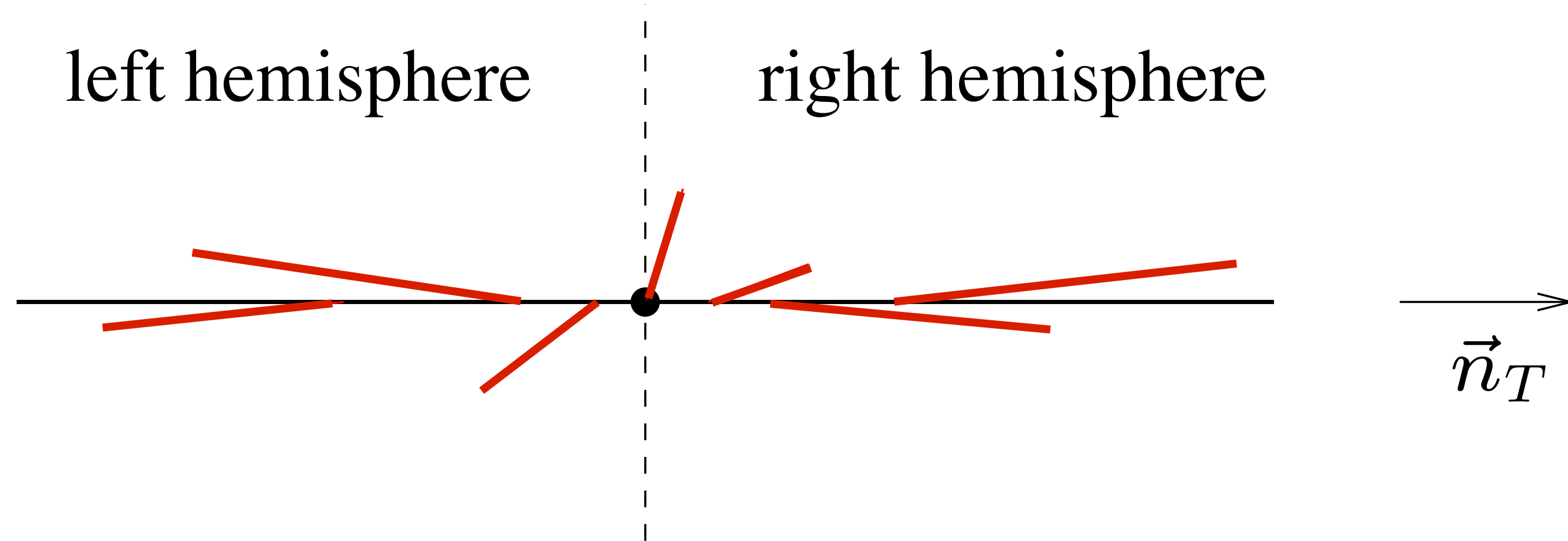
**Figure 9.5:** Lattice determinations that enter the FLAG2019 average. The yellow (light shaded) band and dotted line indicates the average value for this sub-field. The dashed line and blue (dark shaded) band represent the final world average value of  $\alpha_s(M_Z^2)$ .<sup>a</sup>

# Event shapes



August 2021

outliers and/or  
small errors



thrust  $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}, \quad \tau = 1 - T,$

$C$ -parameter  $C = \frac{3 \sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{2 (\sum_i |\vec{p}_i|)^2},$

jet-mass  $\rho = \frac{\left( \sum_{i \in \text{hemisphere}} p_i \right)^2}{(\sum_i E_i)^2},$

broadening  $B_T = \frac{\sum_i p_{Ti}}{\sum_i |\vec{p}_i|}.$

event shapes  
measure amount of  
radiation relative to

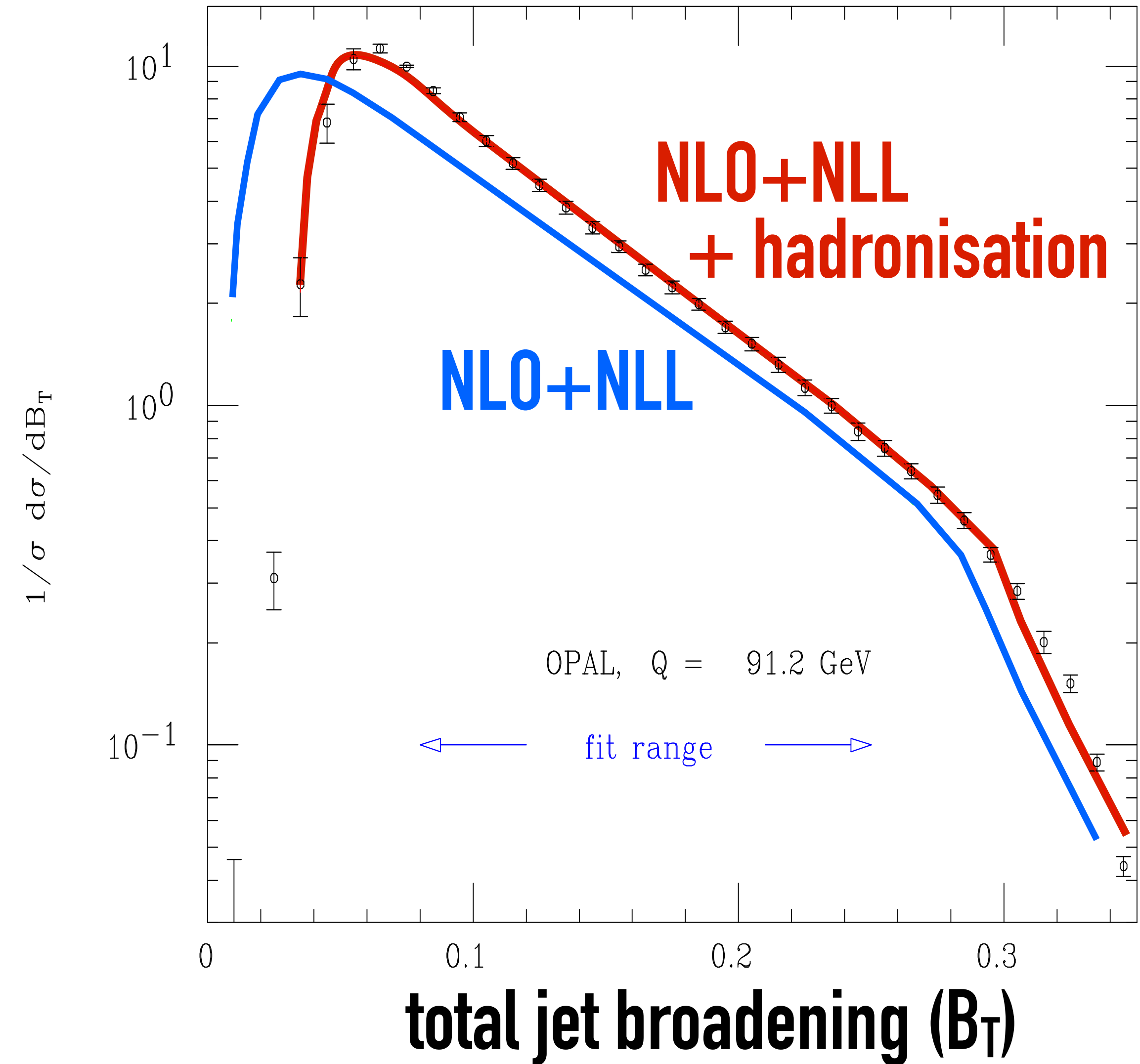
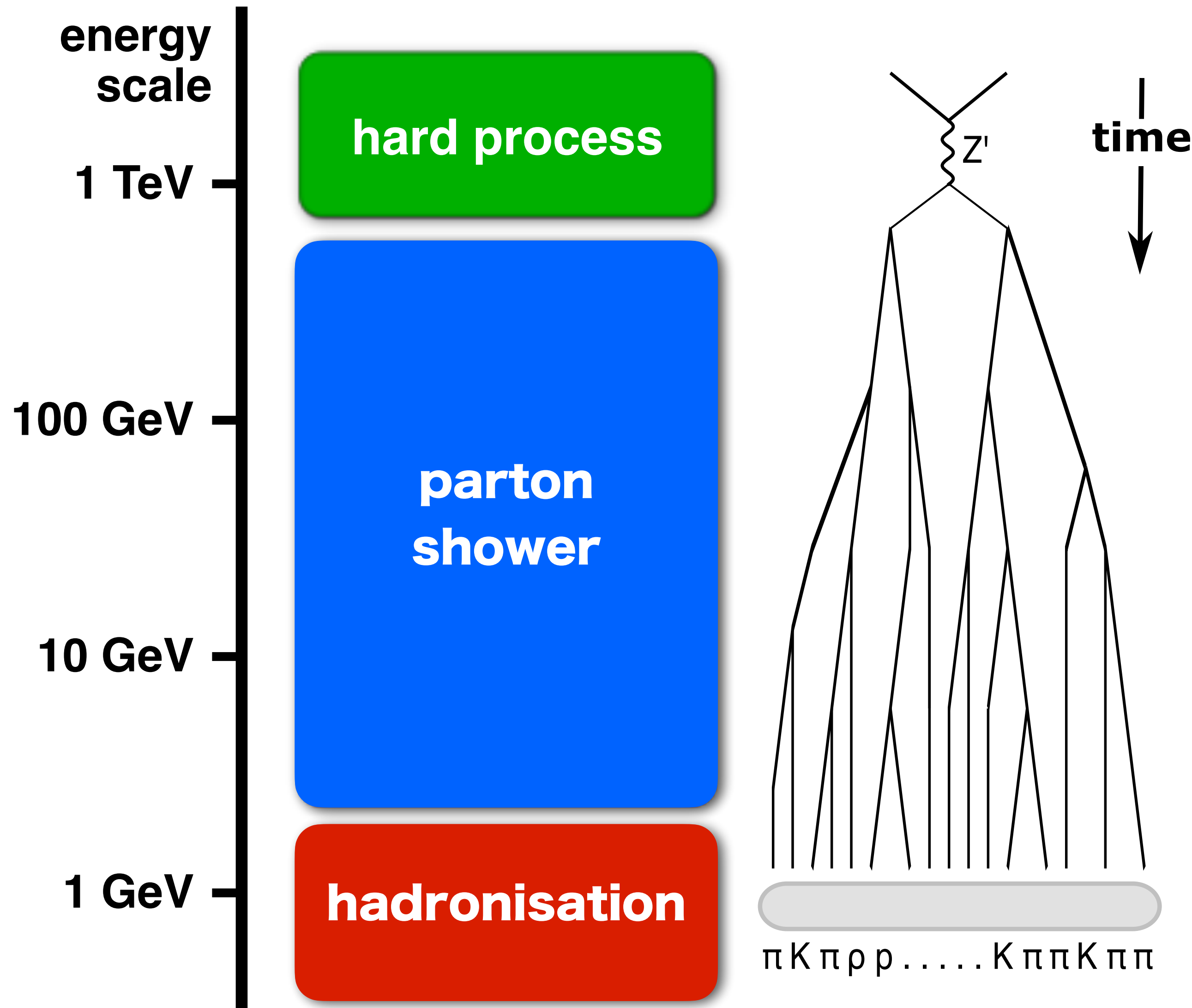
simple

$$e^+ e^- \rightarrow q \bar{q}$$

event



# non-perturbative physics & hadronisation: the bane of quantitative collider QCD



# non-perturbative physics & hadronisation: the bane of quantitative collider QCD

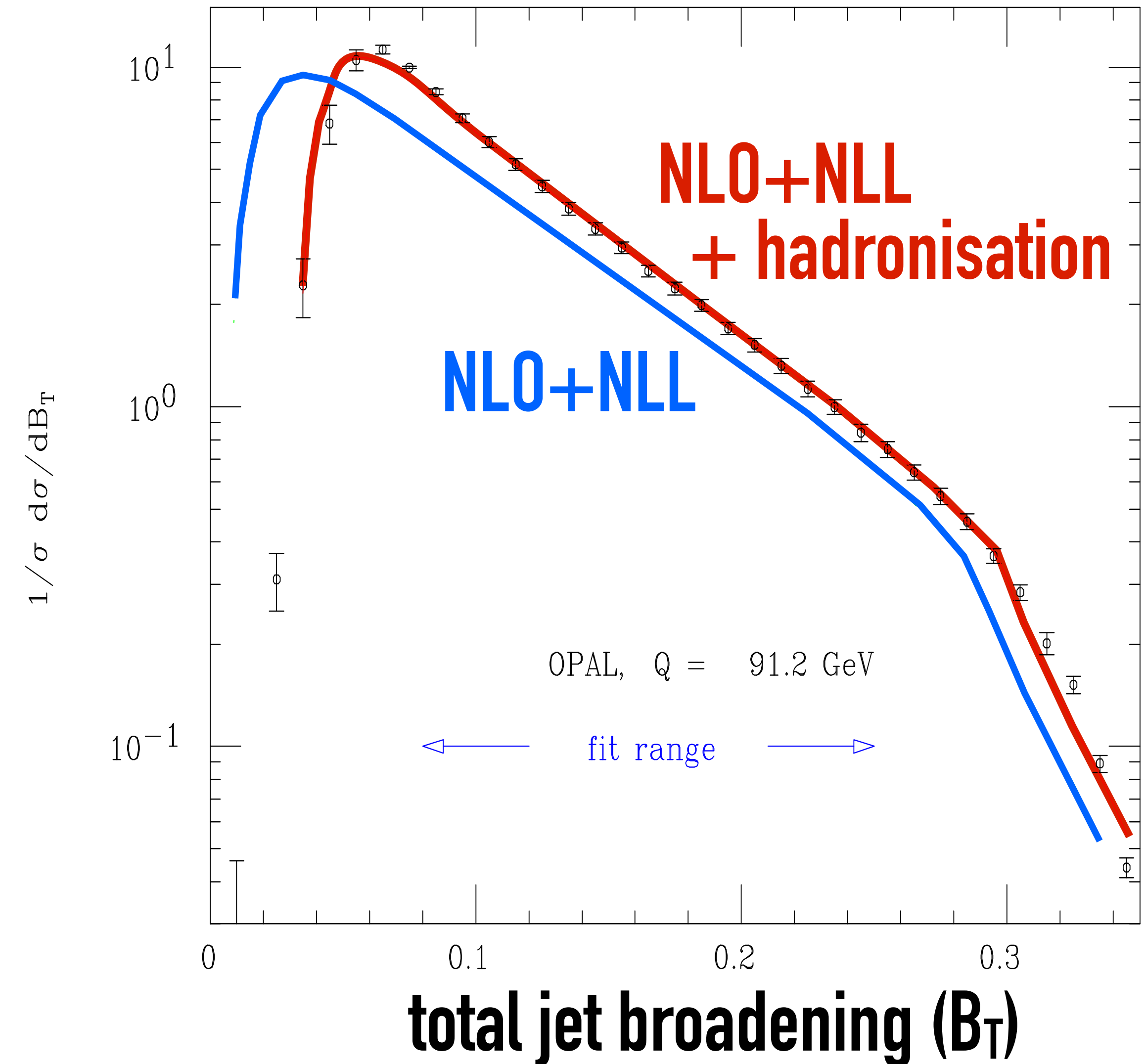
c. 1995, theorists proposed analytical approaches to quantifying hadronisation (Dokshitzer, Marchesini & Webber; Beneke & Braun; Manohar & Wise; Korchemsky & Sterman).

$$\delta V \sim \frac{c_V \alpha_0}{Q}$$

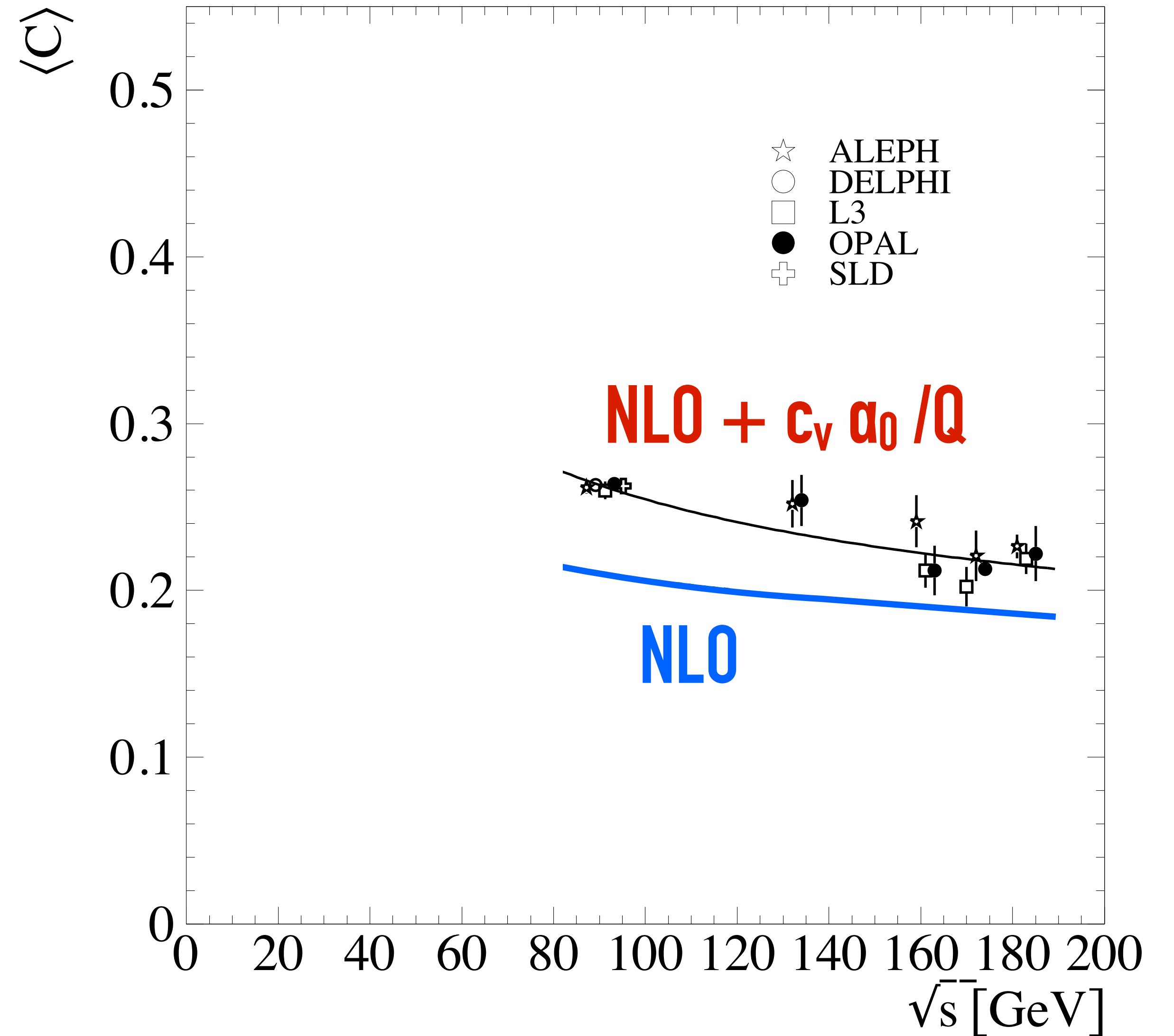
Did they match data?

Two key features to check:

- universality of  $\alpha_0$  across many shapes
- scaling with centre-of-mass energy  $Q$



**Only LEP had measured full  
range of event shapes  
but LEP did not have enough  
lever-arm in Q**



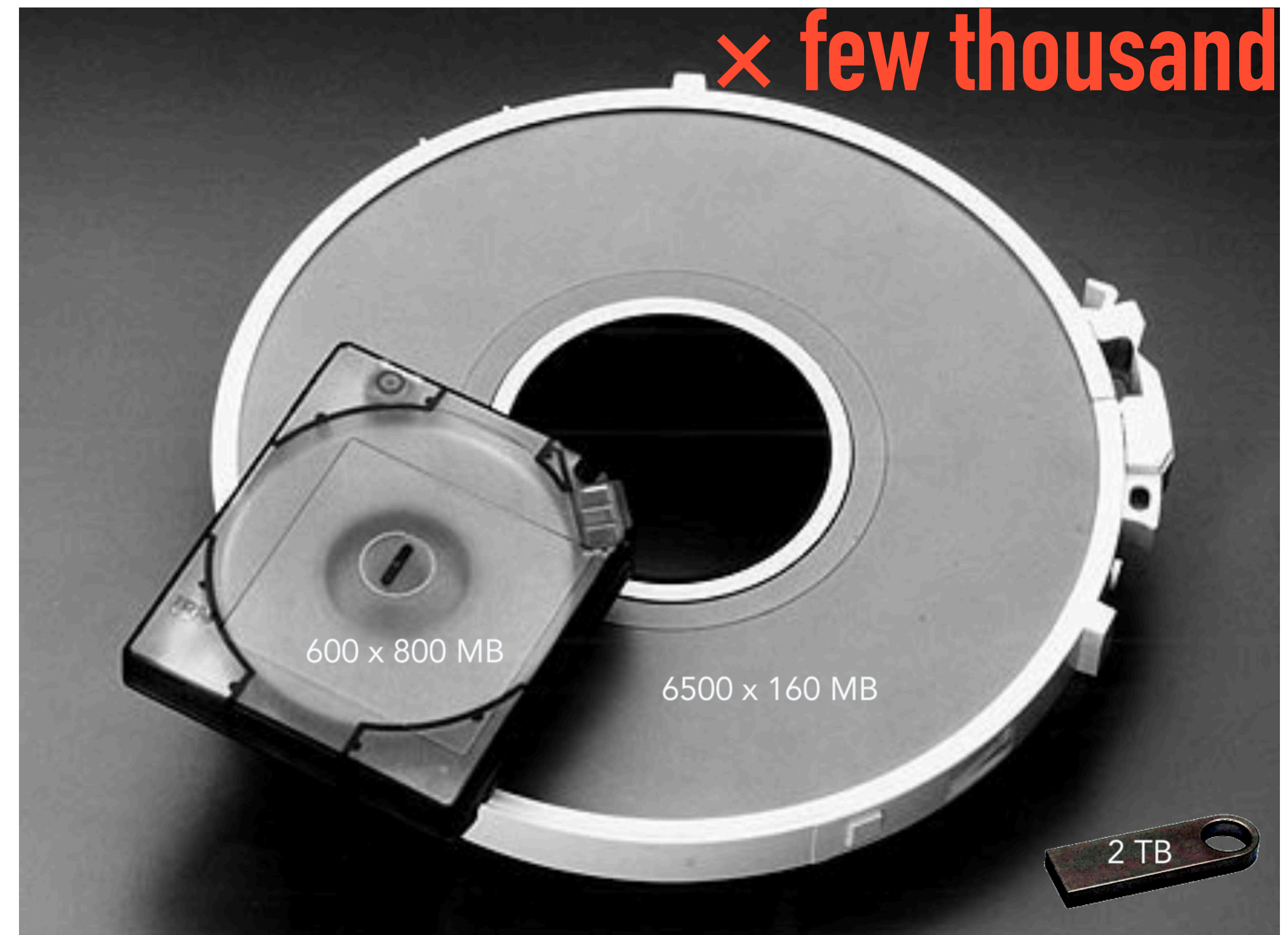
# The JADE Experiment at the PETRA $e^+e^-$ collider - history, achievements and revival

S. Bethke<sup>1\*</sup> and A. Wagner<sup>2\*</sup>

JADE experiment: 1979 – 1986 at DESY  
[Japan-Deutschland-England]

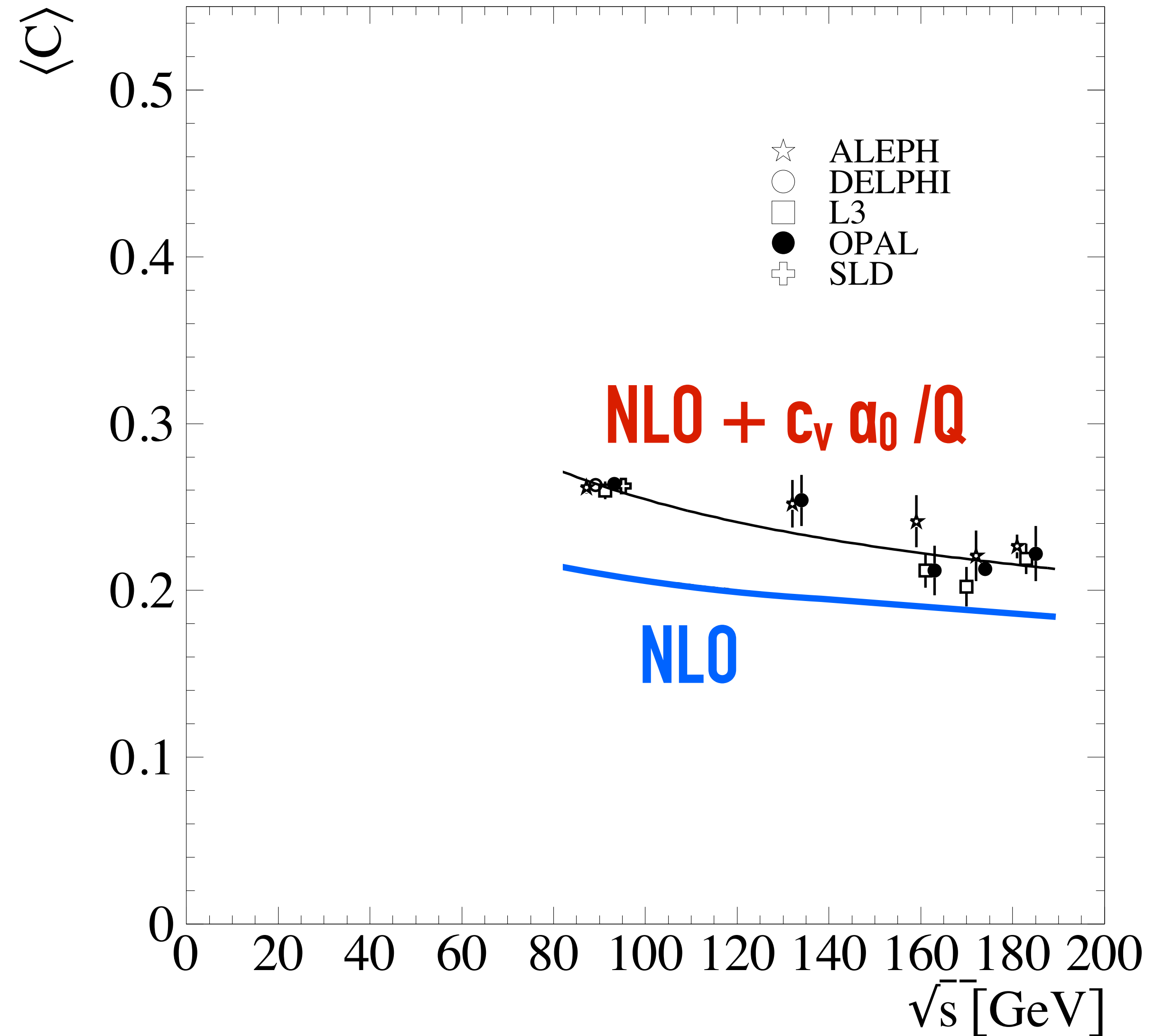
“So the original JADE data were preserved **[except] the JADE luminosity files.** [...] A worldwide search [...] found a printed version [...] **on green recycling paper and too faint for scanning** [...] the numbers had to be typed in a tedious effort into a text file. Only 5 typing errors were found and corrected by a checksum routine.”

[Bethke & Wagner, 2208.11076](#)

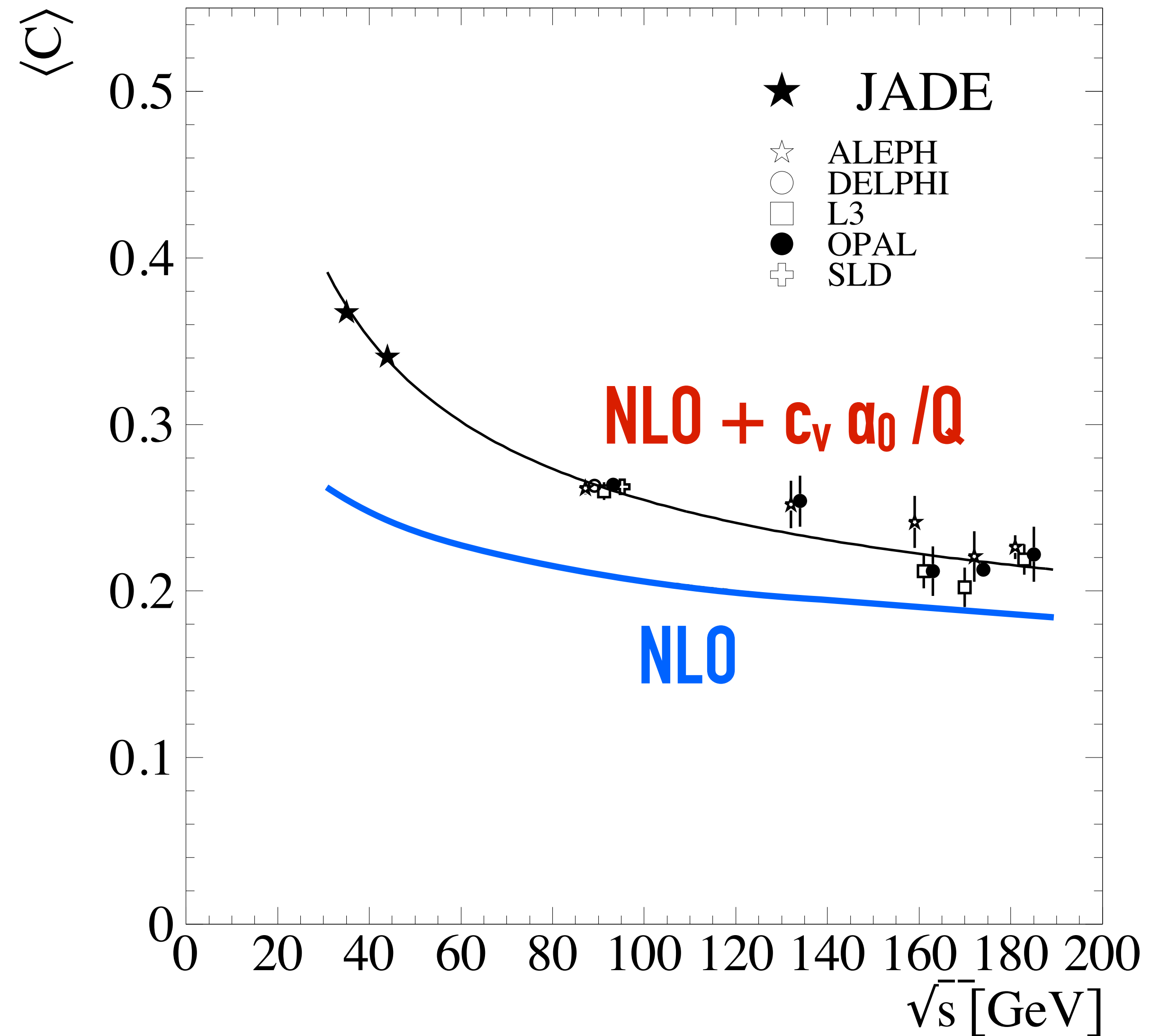


**Fig. 22** The JADE data were originally stored on about 6500 IBM tapes, later converted and written to about 600 IBM3490 cartridges, and nowadays conveniently fit onto a single 2 TB USB memory stick.

**Only LEP had measured full  
range of event shapes  
but LEP did not have enough  
lever-arm in Q**



Adding in the JADE data  
played major role in  
confirming the simple  
theoretical picture



# JADE data + fits

---

(a)	$\langle 1 - T \rangle$	$\langle M_H^2/s \rangle$	$\langle B_T \rangle$	$\langle B_W \rangle$	$\langle C \rangle$	average
$\alpha_S(M_{Z^0})$	<b>0.1198</b>	<b>0.1141</b>	<b>0.1183</b>	<b>0.1190</b>	<b>0.1176</b>	<b>0.1177</b>
$Q$ range [GeV]	13-183	14-183	35-183	35-183	35-183	
$\bar{\alpha}_0(2 \text{ GeV})$	<b>0.509</b>	<b>0.614</b>	<b>0.442</b>	<b>0.392</b>	<b>0.451</b>	<b>0.473</b>

[JADE collab., hep-ex/9903009](#)

“In March 2022, the members of the JADE collaboration unanimously decided to release all JADE data and software to be publicly accessible as “open data” and maintained within the CERN open data initiative [104]. The implementation of JADE data, software and documentation into this environment is currently in progress.”

[Bethke & Wagner, 2208.11076](#)

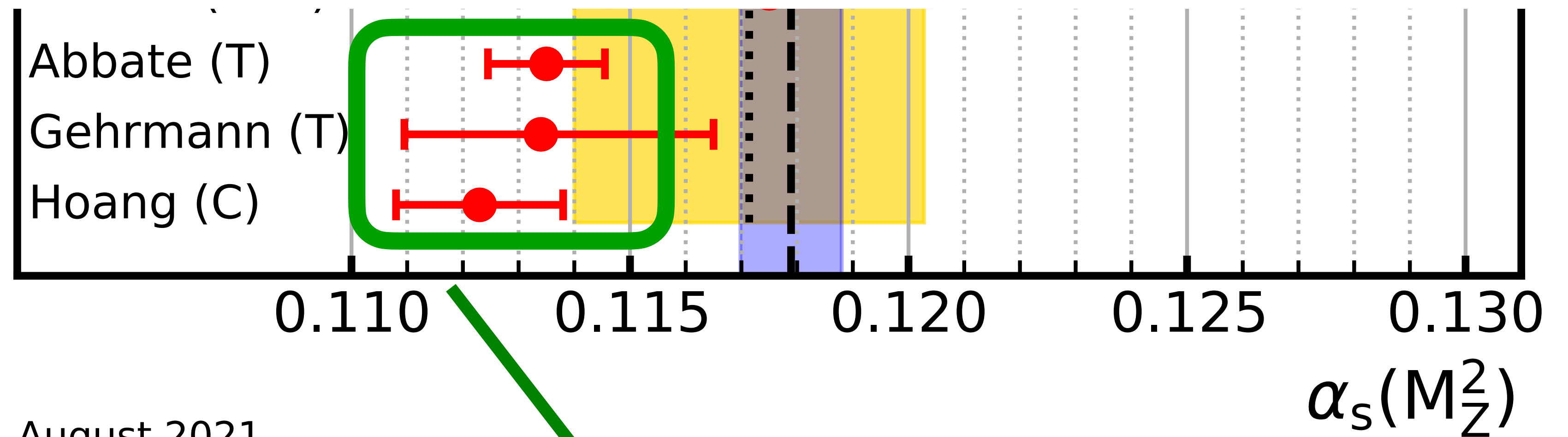
*cf. ongoing work by Verbytskyi @ MPI*

World average:  $\alpha_s(m_Z) = 0.1179 \pm 0.0009$

Thrust:  $\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{hadr}} \pm 0.0009_{\text{pert}}$  [1006.3080](#)

C-parameter:  $\alpha_s(m_Z) = 0.1119 \pm 0.0006_{\text{exp+had}} \pm 0.0013_{\text{pert}}$  [1501.04111](#)

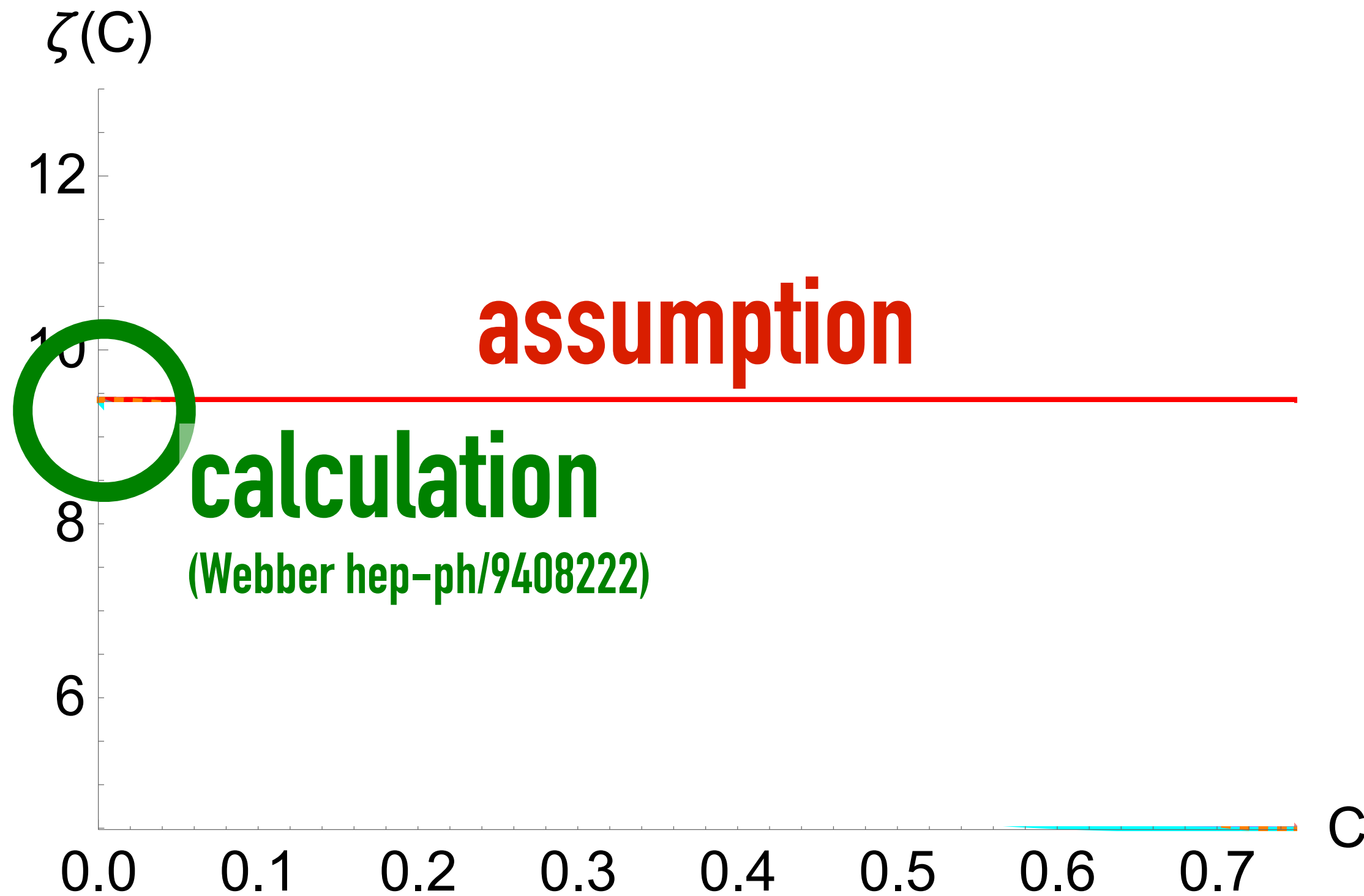
NNLO + N3LL + **1/Q**



**outliers and/or  
small errors**



# non-perturbative shift as $f^n$ of $C$



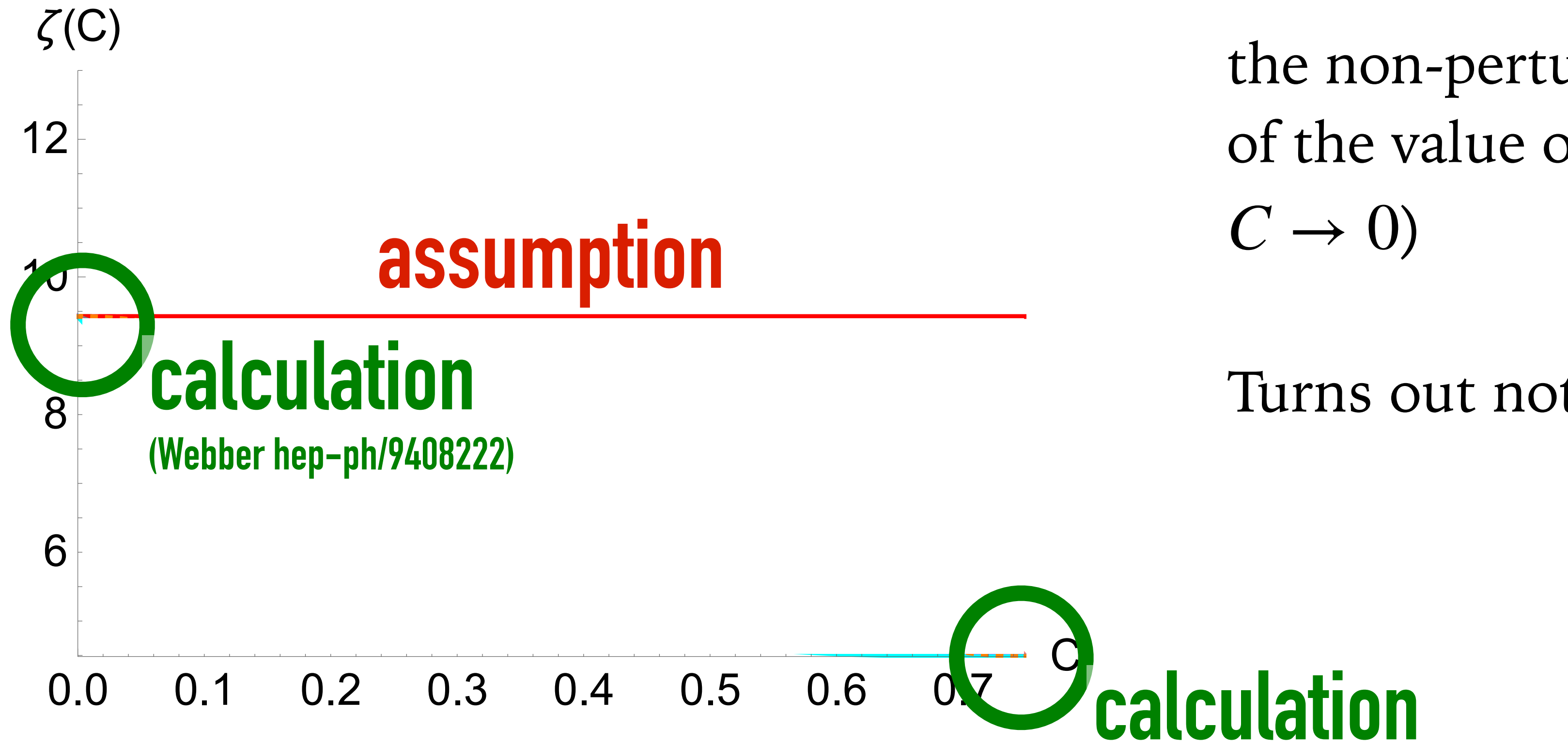
**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ .

critical assumption in those high-precision fits:

the non-perturbative shift is independent of the value of the observable (valid when  $C \rightarrow 0$ )

Turns out not be true

# non-perturbative shift as $f^n$ of $C$



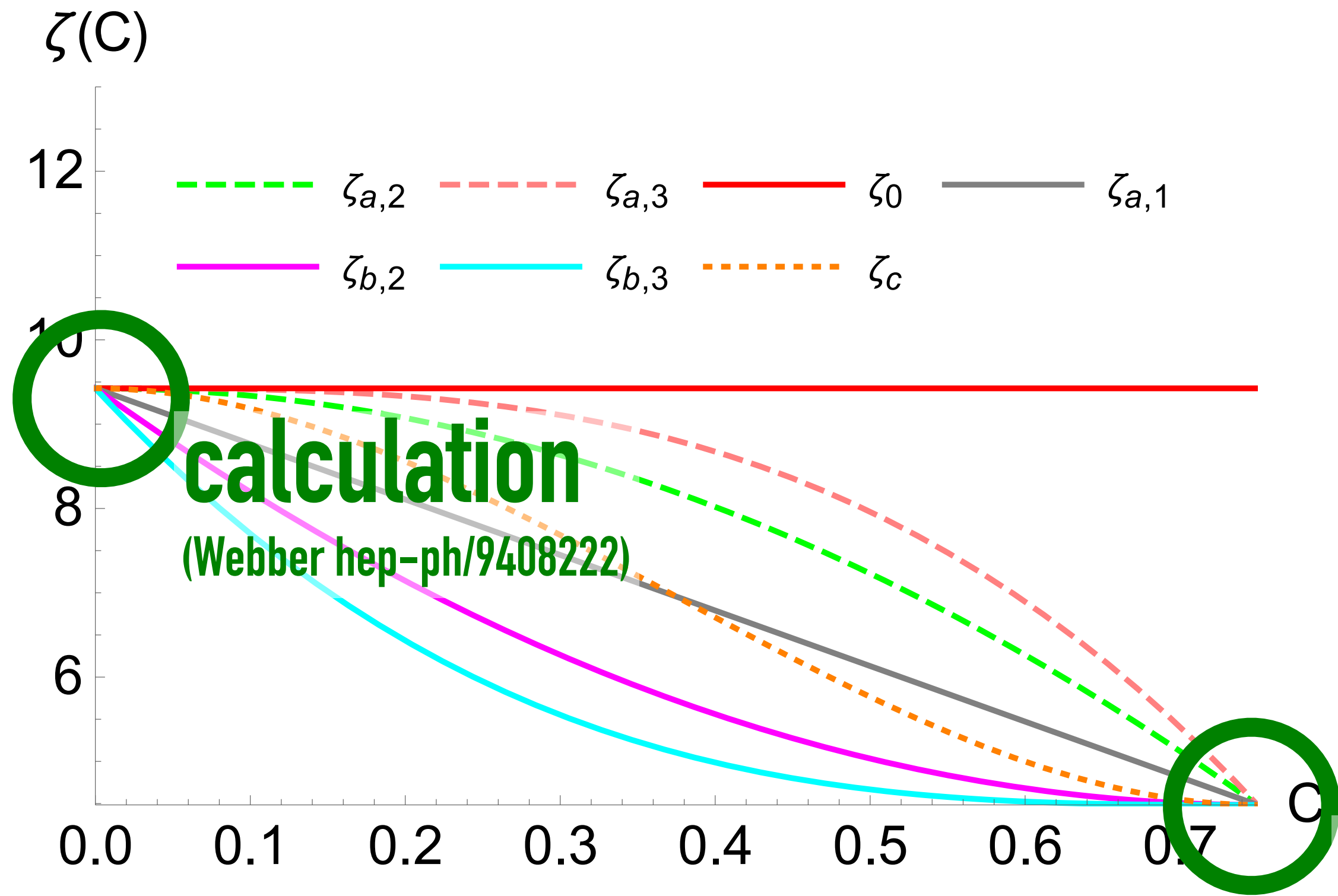
**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ . (Luisoni, Monni, GPS, 2012.00622)

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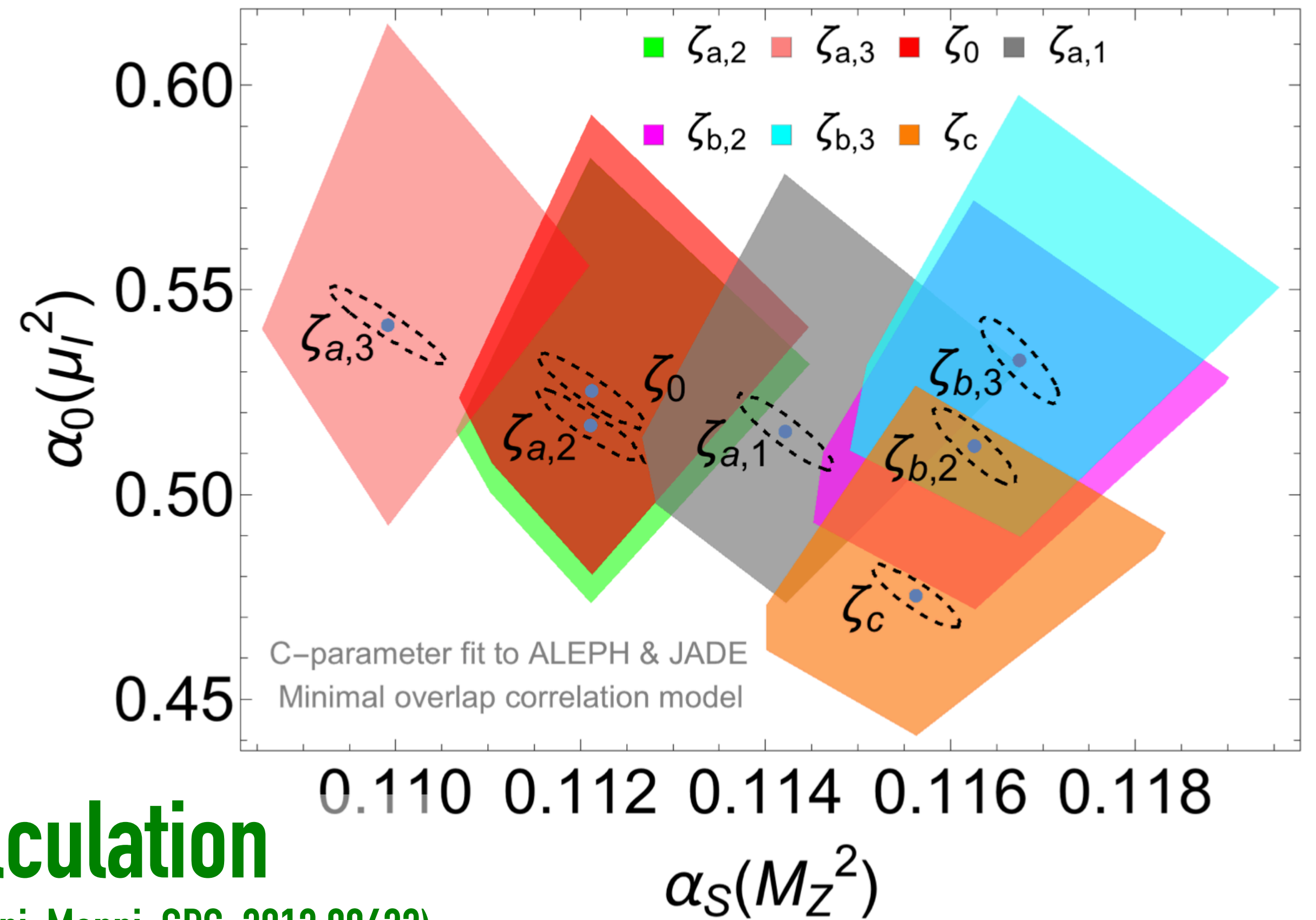
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Turns out not be true

# non-perturbative shift as f<sup>n</sup> of C

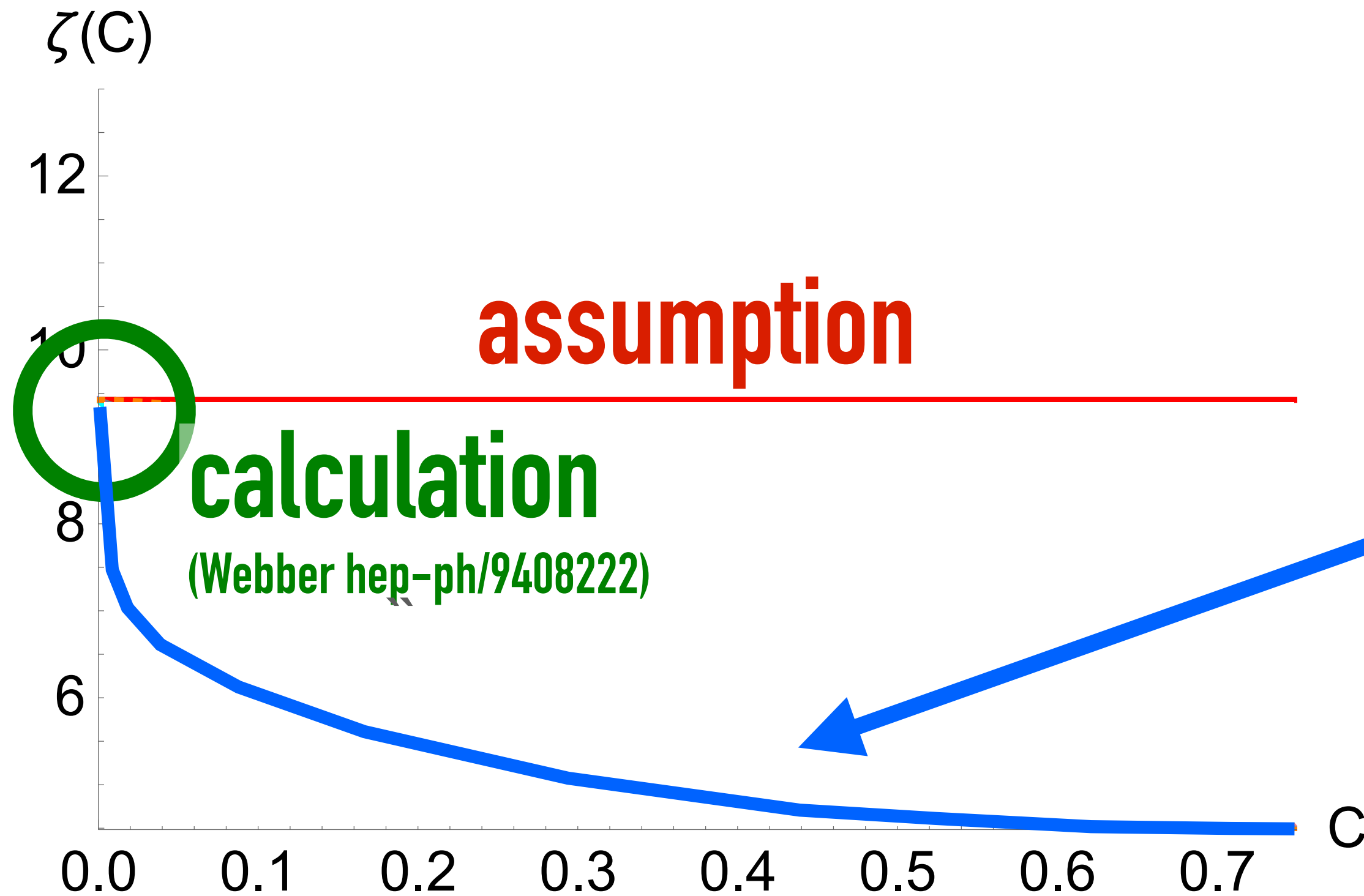


# fit results with different interpolations



**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ . (Luisoni, Monni, GPS, 2012.00622)

# non-perturbative shift as $f^n$ of $C$

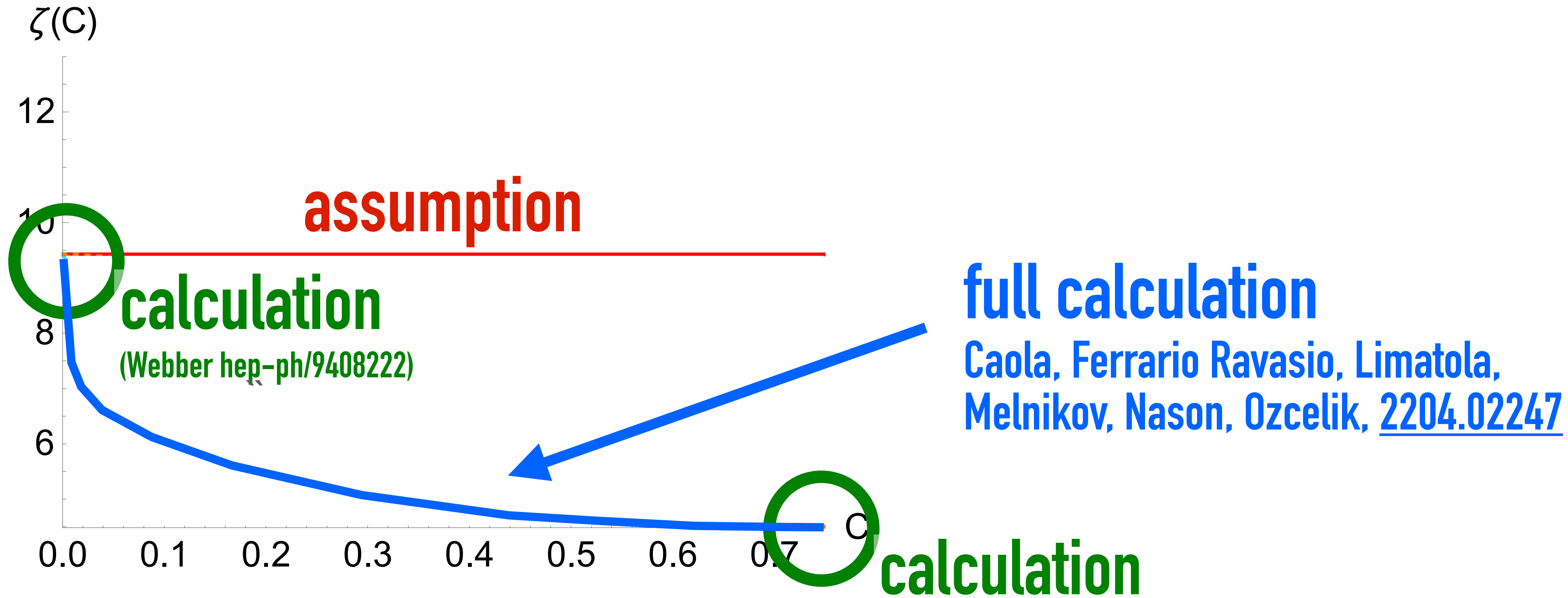


**full calculation**

Caola, Ferrario Ravasio, Limatola,  
Melnikov, Nason, Ozcelik, [2204.02247](https://arxiv.org/abs/2204.02247)

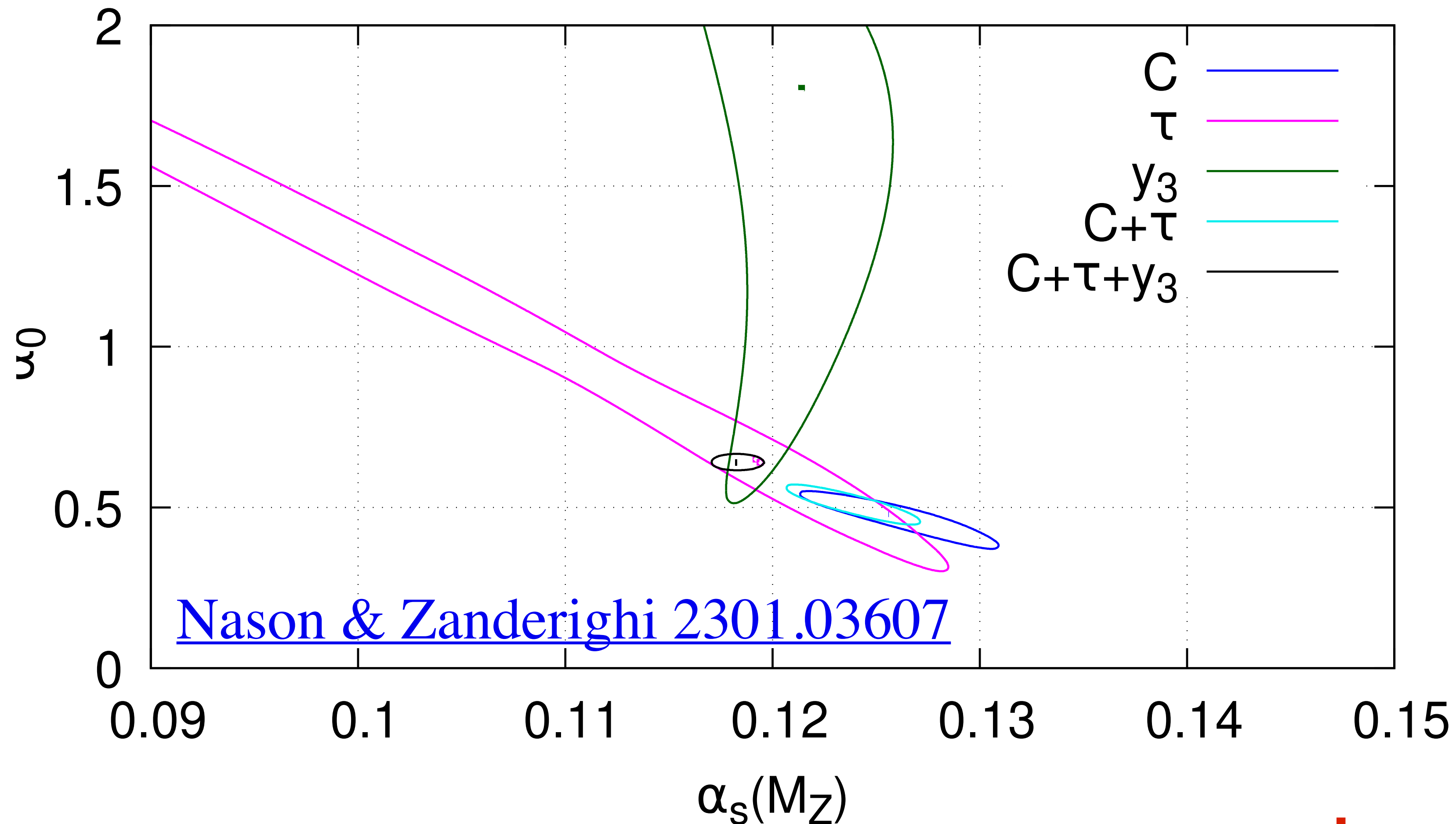
**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ .

# non-perturbative shift as $f^n$ of $C$



**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ .

# Fits with full (1st-order) non-perturbative correction



fits restricted to 3-jet region, NNLO + 1/Q

fixed 1/Q:  $\alpha_s = 0.1132$

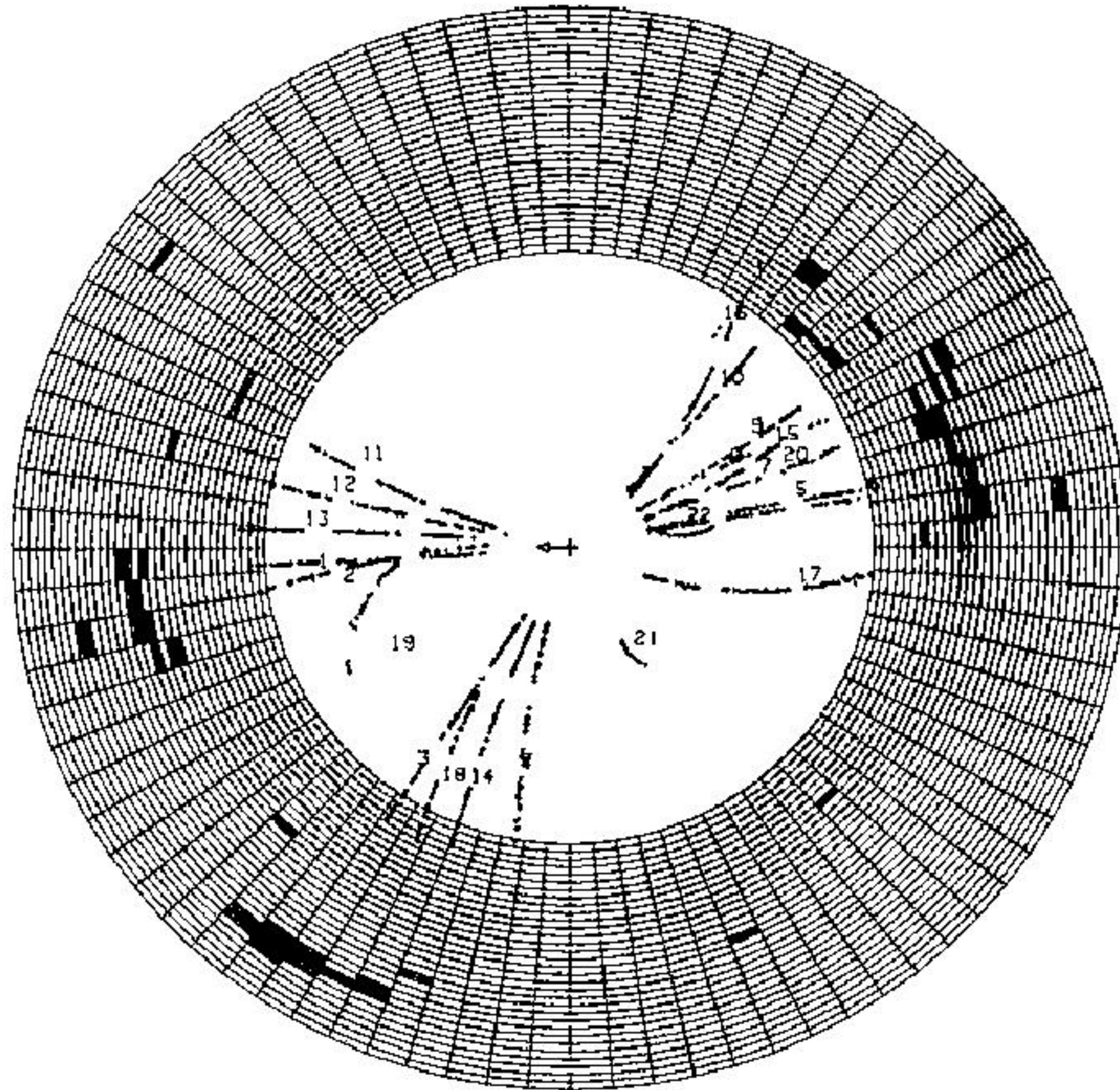
full 1/Q:  $\alpha_s = 0.1182$

*“variations of our procedure can lead easily to differences of the order of a percent”*

**resolves a long-standing tension**

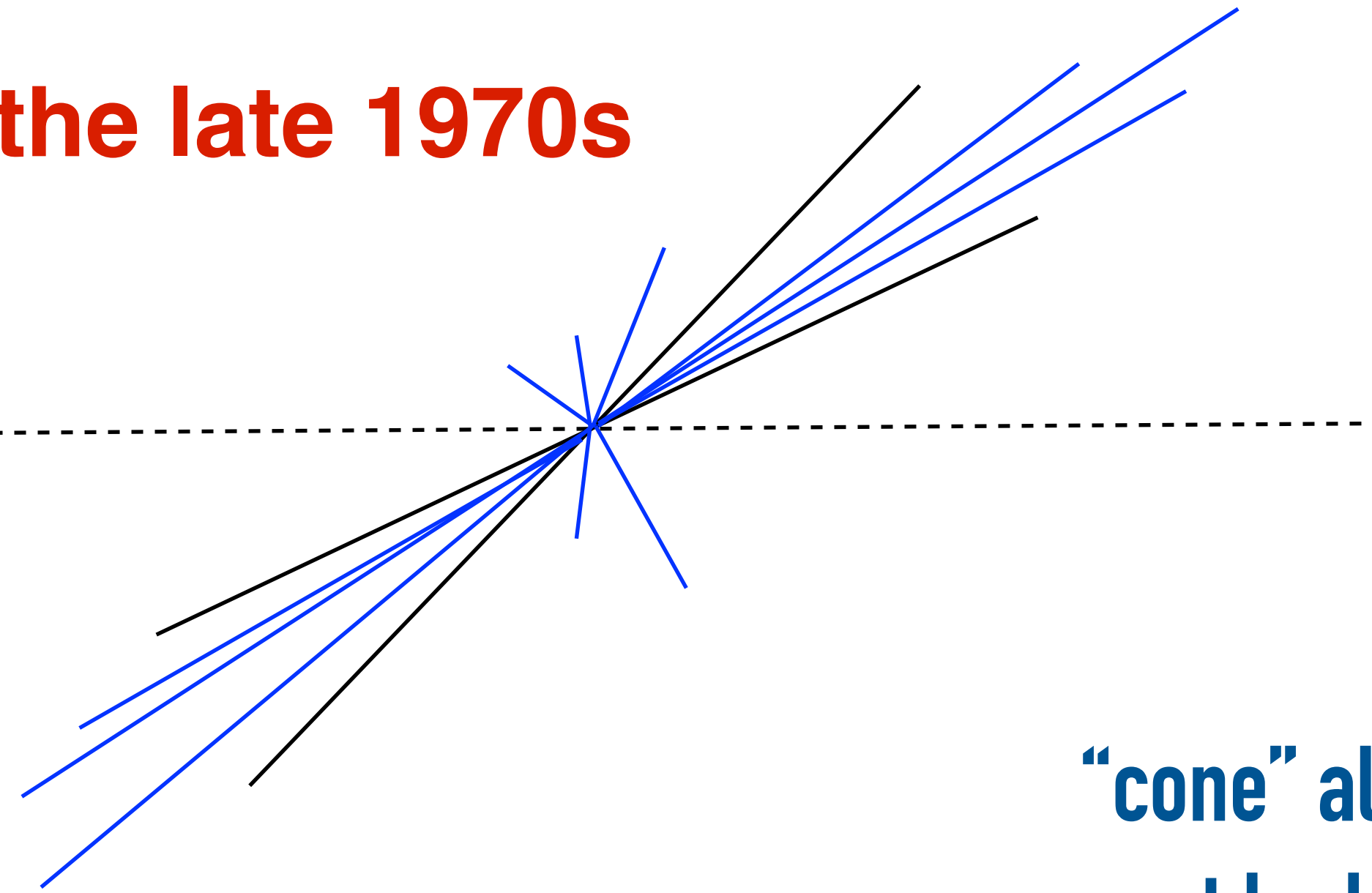
# QCD jets

*how do you project particles into “jets” in a way that makes sense experimentally & in perturbative QCD?*



**Fig. 10** 3-jet event recorded at  $E_{cm} = 33$  GeV, displayed as projection of hits in the central Jet-chamber to the plane perpendicular to the beam axis (central cross), and in a perspective view of the lead-glass counters. Those counters hit by particles are filled in black.

# Jet definitions dated back to the late 1970s



Sterman and Weinberg,  
Phys. Rev. Lett. 39, 1436 (1977):

“cone” algorithms were favourite  
at hadron colliders until late  
2000’s, but very difficult to make  
infrared safe

To study jets, we consider the partial cross section  
 $\sigma(E, \theta, \Omega, \epsilon, \delta)$  for  $e^+e^-$  hadron production events, in which all but  
a fraction  $\epsilon \ll 1$  of the total  $e^+e^-$  energy  $E$  is emitted within  
some pair of oppositely directed cones of half-angle  $\delta \ll 1$ ,  
lying within two fixed cones of solid angle  $\Omega$  (with  $\pi\delta^2 \ll \Omega \ll 1$ )  
at an angle  $\theta$  to the  $e^+e^-$  beam line. We expect this to be measur-

$$\sigma(E, \theta, \Omega, \epsilon, \delta) = (d\sigma/d\Omega)_0 \Omega \left[ 1 - (g_E^2/3\pi^2) \left\{ 3\ln \delta + 4\ln \delta \ln 2\epsilon + \frac{\pi^3}{3} - \frac{5}{2} \right\} \right]$$



# SISCone: first infrared/collinear safe cone algorithm [Soyez & GPS, [0704.0292](#)]

---

**Algorithm 2** Procedure for establishing the list of all stable cones (protojets). For simplicity, parts related to the special case of multiple cocircular points (see footnote 7) are not shown. They are a straightforward generalisation of steps 6 to 13.

---

- 1: For any group of collinear particles, merge them into a single particle.
  - 2: **for** particle  $i = 1 \dots N$  **do**
  - 3: Find all particles  $j$  within a distance  $2R$  of  $i$ . If there are no such particles,  $i$  forms a stable cone of its own.
  - 4: Otherwise for each  $j$  identify the two circles for which  $i$  and  $j$  lie on the circumference. For each circle, compute the angle of its centre  $C$  relative to  $i$ ,  $\zeta = \arctan \frac{\Delta\phi_{iC}}{\Delta y_{iC}}$ .
  - 5: Sort the circles found in steps 3 and 4 into increasing angle  $\zeta$ .
  - 6: Take the first circle in this order, and call it the current circle. Calculate the total momentum and checkxor for the cones that it defines. Consider all 4 permutations of edge points being included or excluded. Call these the “current cones”.
  - 7: **repeat**
  - 8: **for** each of the 4 current cones **do**
  - 9: If this cone has not yet been found, add it to the list of distinct cones.
  - 10: If this cone has not yet been labelled as unstable, establish if the in/out status of the edge particles (with respect to the cone momentum axis) is the same as when defining the cone; if it is not, label the cone as unstable.
  - 11: **end for**
  - 12: Move to the next circle in order. It differs from the previous one either by a particle entering the circle, or one leaving the circle. Calculate the momentum for the new circle and corresponding new current cones by adding (or removing) the momentum of the particle that has entered (left); the checkxor can be updated by XORing with the label of that particle.
  - 13: **until** all circles considered.
  - 14: **end for**
  - 15: **for** each of the cones not labelled as unstable **do**
  - 16: Explicitly check its stability, and if it is stable, add it to the list of stable cones (protojets).
  - 17: **end for**
- 

---

**Algorithm 3** The disambiguated, scalar  $\tilde{p}_t$  based formulation of a Tevatron Run-II type split–merge procedure [6], with overlap threshold parameter  $f$  and transverse momentum threshold  $p_{t,\min}$ . To ensure boost invariance and IR safety, for the ordering variable and the overlap measure, it uses of  $\tilde{p}_{t,\text{jet}} = \sum_{i \in \text{jet}} |p_{t,i}|$ , *i.e.* a scalar sum of the particle transverse momenta (as in a ‘ $p_t$ ’ recombination scheme).

---

- 1: **repeat**
  - 2: Remove all protojets with  $p_t < p_{t,\min}$ .
  - 3: Identify the protojet ( $i$ ) with the highest  $\tilde{p}_t$ .
  - 4: Among the remaining protojets identify the one ( $j$ ) with highest  $\tilde{p}_t$  that shares particles (overlaps) with  $i$ .
  - 5: **if** there is such an overlapping jet **then**
  - 6: Determine the total  $\tilde{p}_{t,\text{shared}} = \sum_{k \in i \& j} |p_{t,k}|$  of the particles shared between  $i$  and  $j$ .
  - 7: **if**  $\tilde{p}_{t,\text{shared}} < f\tilde{p}_{t,j}$  **then**
  - 8: Each particle that is shared between the two protojets is assigned to the one to whose axis it is closest. The protojet momenta are then recalculated.
  - 9: **else**
  - 10: Merge the two protojets into a single new protojet (added to the list of protojets, while the two original ones are removed).
  - 11: **end if**
  - 12: If steps 7–11 produced a protojet that coincides with an existing one, maintain the new protojet as distinct from the existing copy(ies).
  - 13: **else**
  - 14: Add  $i$  to the list of final jets, and remove it from the list of protojets.
  - 15: **end if**
  - 16: **until** no protojets are left.
- 

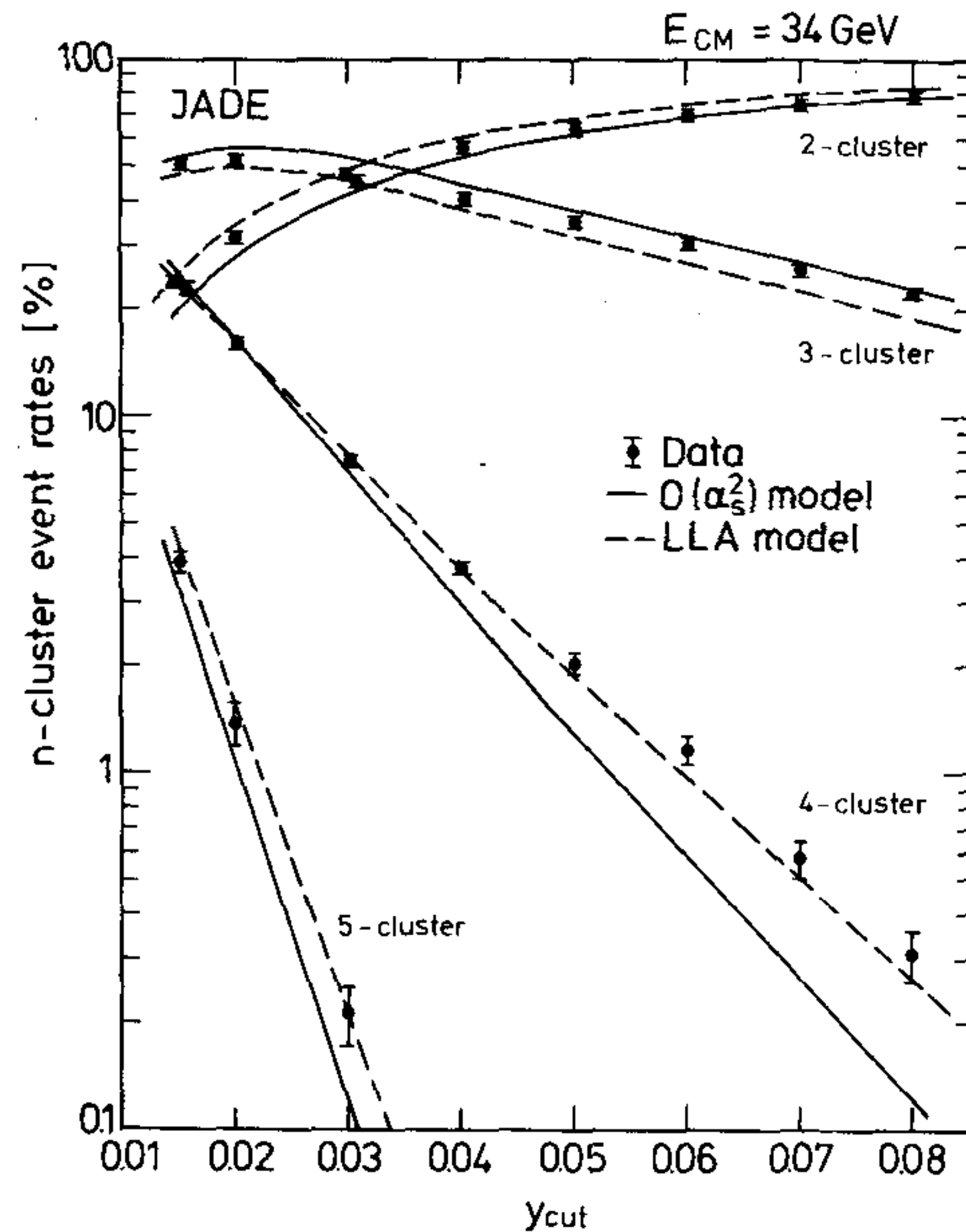
**unappealingly complex**

# Experimental Studies on Multijet Production in $e^+e^-$ Annihilation at PETRA Energies

Z. Phys. C – Particles and Fields 33, 23–31 (1986)

[978 citations](#)

JADE Collaboration



## the JADE algorithm

For all pairs of particles  $k$  and  $l$  of an event, the scaled invariant mass squared  $y_{kl} = M_{kl}^2/E_{vis}^2$  is calculated, where  $E_{vis}$  is the total visible energy of an event\*. The two particles with the smallest value of  $y_{kl}$  are replaced by a pseudoparticle or “cluster” of four-momentum  $(p_k + p_l)$ . This procedure is repeated until all  $y_{kl}$  exceed a certain threshold value  $y_{cut}$ ,

**the first complete, infrared & collinear safe jet algorithm — remarkably simple**

## EXPERIMENTAL INVESTIGATION OF THE ENERGY DEPENDENCE OF THE STRONG COUPLING STRENGTH

JADE Collaboration

S. BETHKE<sup>a,1</sup>, J. ALLISON<sup>b</sup>, K. AMBRUS<sup>a</sup>, R.J. BARLOW<sup>b</sup>, W. BARTEL<sup>c</sup>, C.K. BOWDERY<sup>d</sup>, S.L. CARTWRIGHT<sup>e,2</sup>, J. CHRIN<sup>b</sup>, D. CLARKE<sup>e</sup>, A. DIECKMANN<sup>a</sup>, I.P. DUERDOTH<sup>b</sup>, G. ECKERLIN<sup>a</sup>, E. ELSER<sup>a</sup>, R. FELST<sup>c</sup>, A.J. FINCH<sup>d</sup>, F. FOSTER<sup>d</sup>, T. GREENSHAW<sup>f</sup>, J. HAGEMANN<sup>f</sup>, D. HAIDT<sup>c</sup>, J. HEINTZE<sup>a</sup>, G. HEINZELMANN<sup>f</sup>, K.H. HELLENBRAND<sup>a,3</sup>, P. HILL<sup>g</sup>, G. HUGHES<sup>d</sup>, H. KADO<sup>2</sup>, K. KAWAGOE<sup>h</sup>, C. KLEINWORT<sup>f</sup>, G. KNIES<sup>c</sup>, T. KOBAYASHI<sup>h</sup>, S. KOMAMIYA<sup>a,4</sup>, H. KREHBIEL<sup>c</sup>, J. von KROGH<sup>a</sup>, M. KUHLEN<sup>f</sup>, F.K. LOEBINGER<sup>b</sup>, A.A. MACBETH<sup>b</sup>, N. MAGNUSSEN<sup>c,5</sup>, R. MARSHALL<sup>e</sup>, R. MEINKE<sup>c</sup>, R.P. MIDDLETON<sup>e</sup>, M. MINOWA<sup>h</sup>, P.G. MURPHY<sup>b</sup>, B. NAROSKA<sup>c</sup>, J.M. NYE<sup>d</sup>, J. OLSSON<sup>c</sup>, F. OULD-SAAD<sup>f</sup>, A. PETERSEN<sup>f,6</sup>, R. RAMCKE<sup>f</sup>, H. RIESEBERG<sup>3</sup>, D. SCHMIDT<sup>c,5</sup>, H. v.d. SCHMITT<sup>a</sup>, L. SMOLIK<sup>a</sup>, U. SCHNEEKLOTH<sup>f,2</sup>, J.A.J. SKARD<sup>g,7</sup>, J. SPITZER<sup>a</sup>, P. STEFFEN<sup>c</sup>, K. STEPHENS<sup>b</sup>, A. WAGNER<sup>a</sup>, I.W. WALKER<sup>d</sup>, P. WARMING<sup>b,8</sup>, G. WEBER<sup>f</sup>, M. ZIMMER<sup>a</sup> and G.T. ZORN<sup>g</sup>

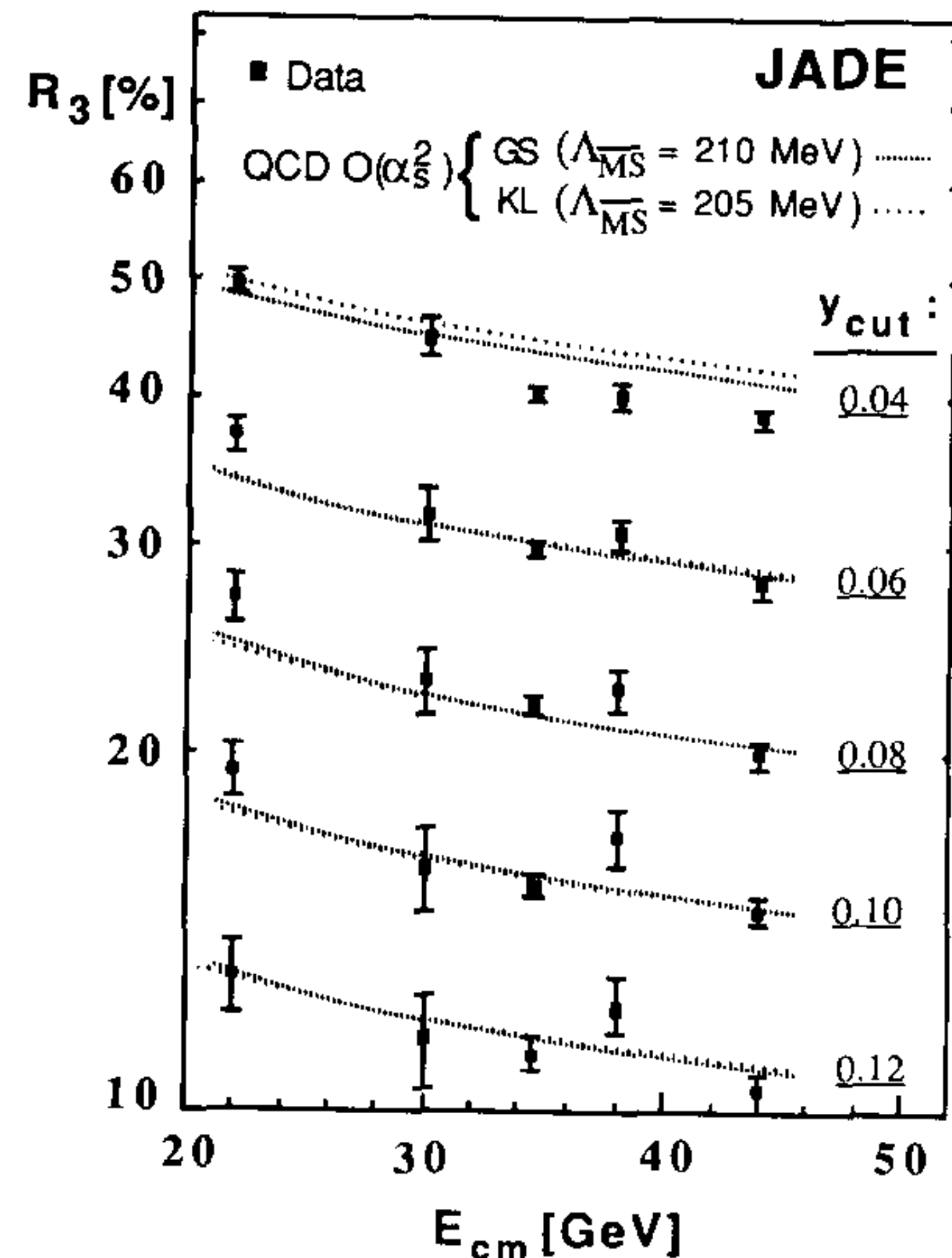


Fig. 2. Three-jet event rates at different centre of mass energies for various values of  $y_{cut}$ , together with the direct predictions of the complete second-order perturbative QCD calculations of Gottschalk and Shatz (GS) and of Kramer and Lampe (KL).

# characteristics of the JADE algorithm

---

## Jet cross sections at leading double logarithm in $e^+ e^-$ annihilation

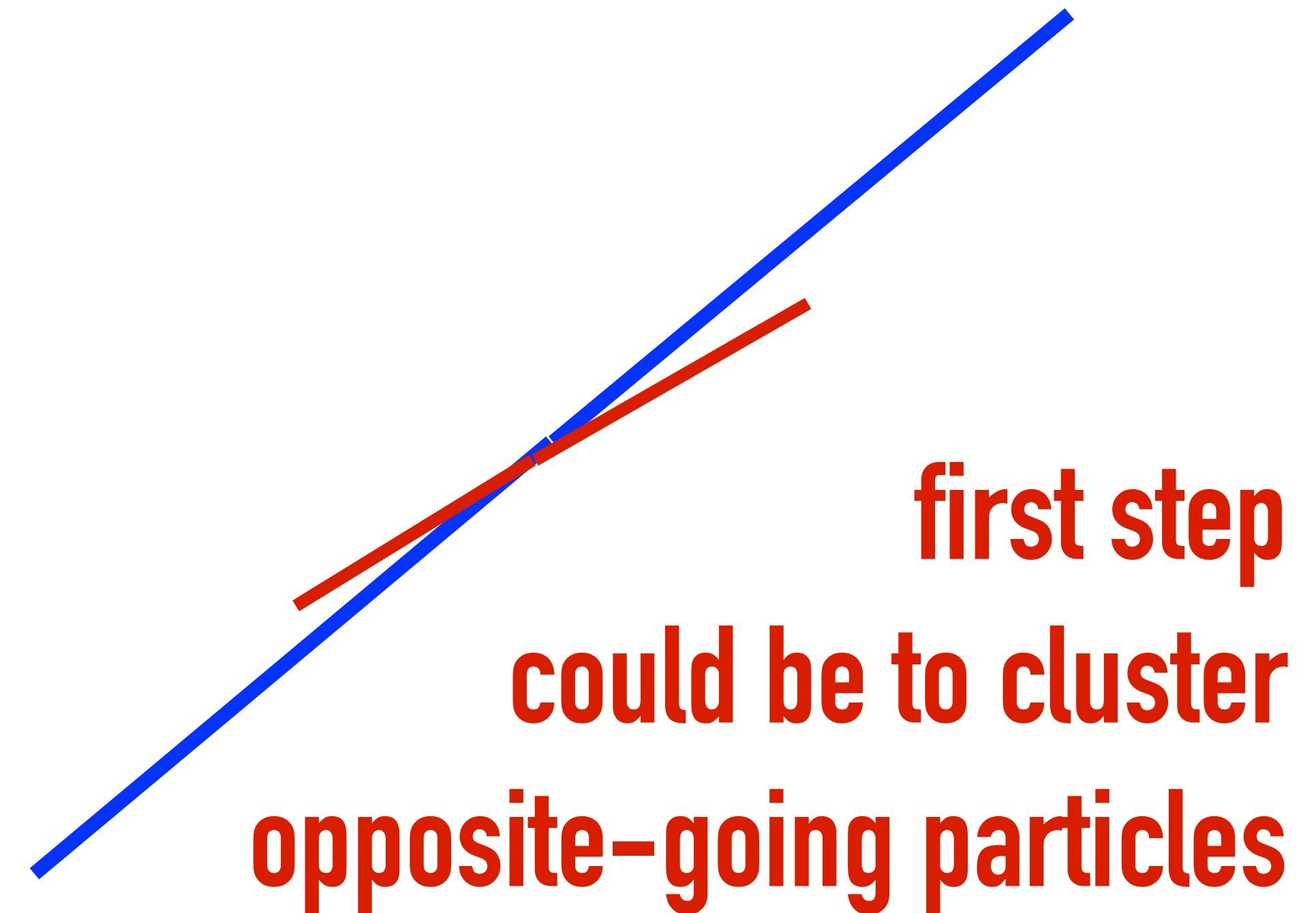
Brown & Stirling, PLB 1990

It was, however, conjectured by Smilga [8] that the two-jet fraction would exponentiate:

$$f_2 = \exp\left(-\frac{C_F \alpha_s}{\pi} \ln^2 y\right). \quad (37)$$

[carrying out the explicit calculation] we obtain

$$f_2 = 1 - \frac{C_F \alpha_s}{\pi} \ln^2 y + \frac{1}{2!} \left(\frac{C_F \alpha_s}{\pi}\right)^2 \ln^4 y \boxed{\left(\frac{5}{6}\right)}.$$





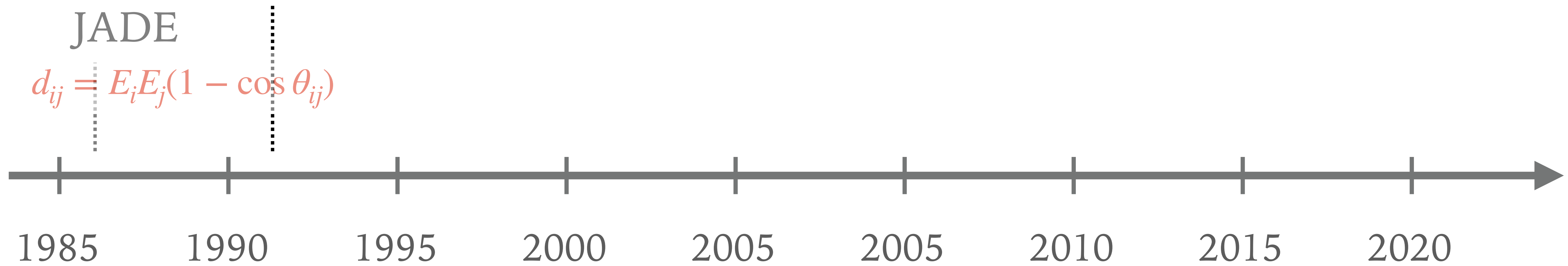
**JET STUDIES WORKSHOP  
ST. JOHN'S COLLEGE, DURHAM.  
DECEMBER 1990**

# JADE algorithm descendants: **modify pairwise distance**

---

Durham  $k_t$ -algorithm:  $d_{ij} = \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})$

Catani, Dokshitzer, Olsson, Turnock, Webber

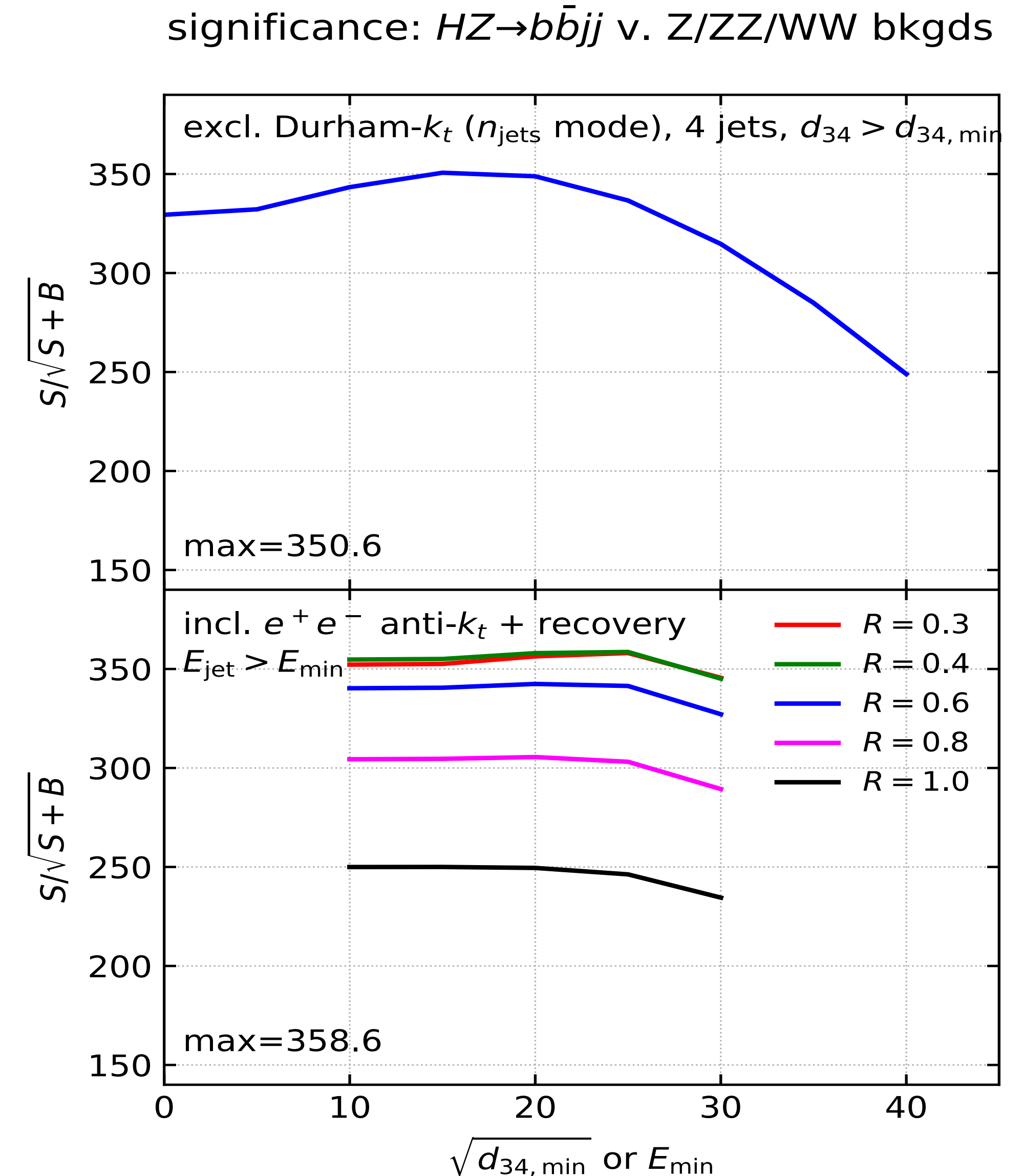


# Durham- $k_t$ algorithm: widely used at LEP, good performance for FCCee

## FCC $e^+e^- \rightarrow HZ \rightarrow b\bar{b}jj$ analysis illustration of impact of $d_{34}$ cut

- too small a limit on  $d_{34}$  leads to enhanced background
- too large a limit cuts out large fraction of signal
- One can scan over  $d_{34}$  cut to optimise  $S/\sqrt{S+B}$
- A modern analysis might use the event-by event  $d_{34}$  value as a ML input (or full jet momenta)

*Cacciari, GPS  
& Soyez 2022*



# JADE algorithm descendants: **modify pairwise distance**

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Catani, Dokshitzer, Olsson, Turnock, Webber

JADE

$$d_{ij} = E_i E_j (1 - \cos \theta_{ij})$$

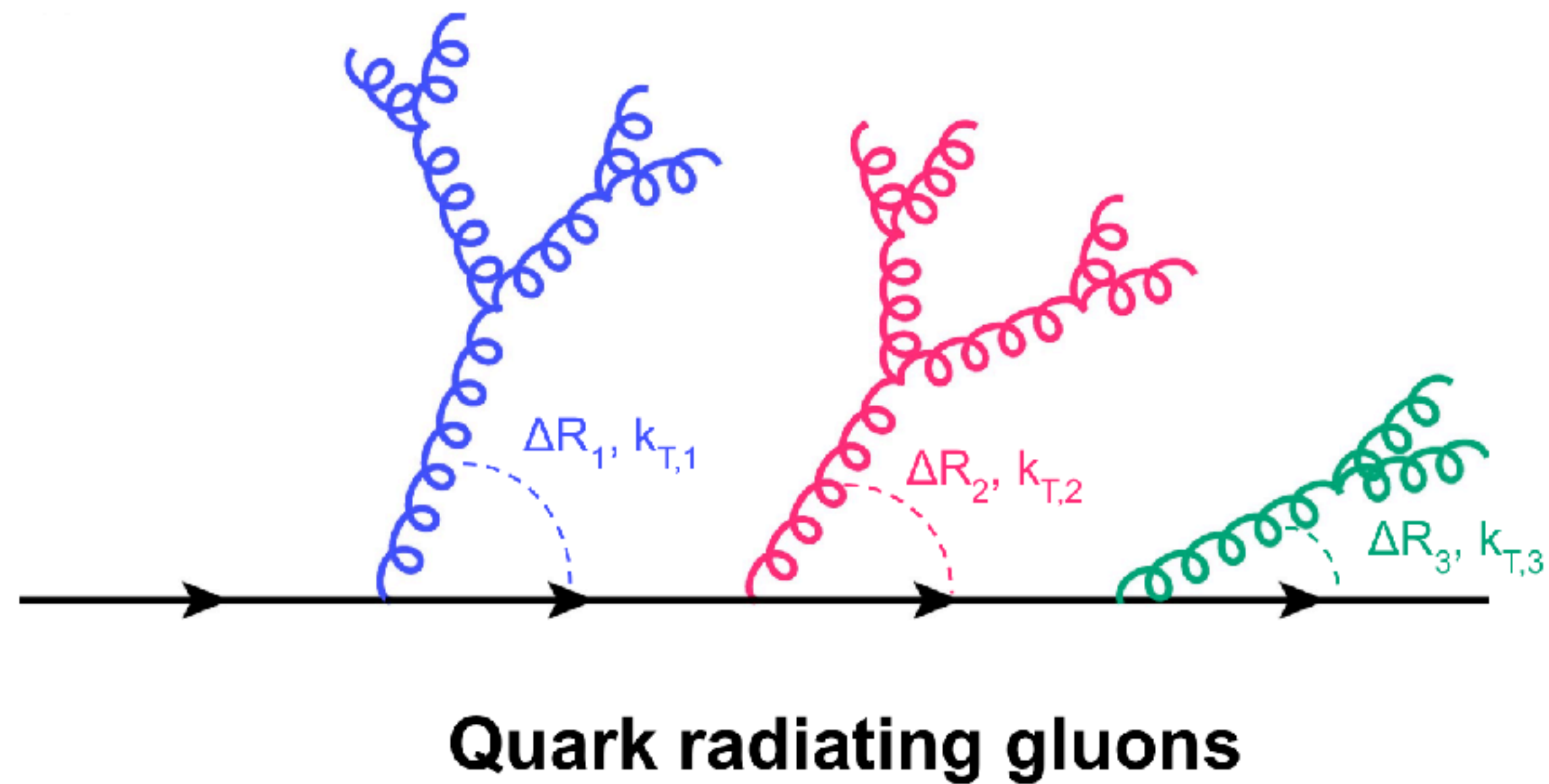
Cambridge-algorithm:  $v_{ij} = (1 - \cos \theta_{ij})$

Dokshitzer, Leder, Moretti, Webber; simplified by Wobisch & Wengler



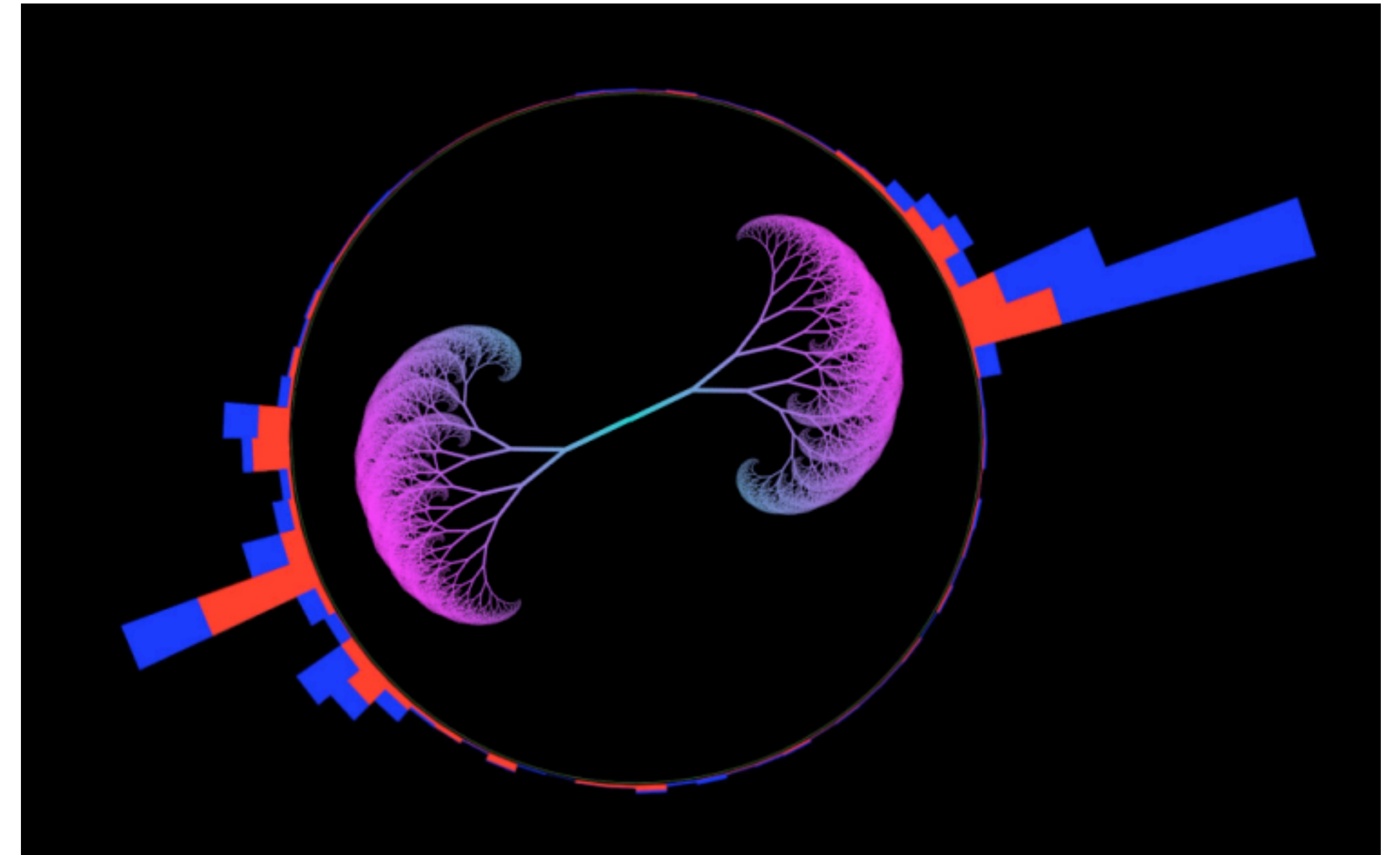


# Cambridge/Aachen algorithm: **best for substructure**

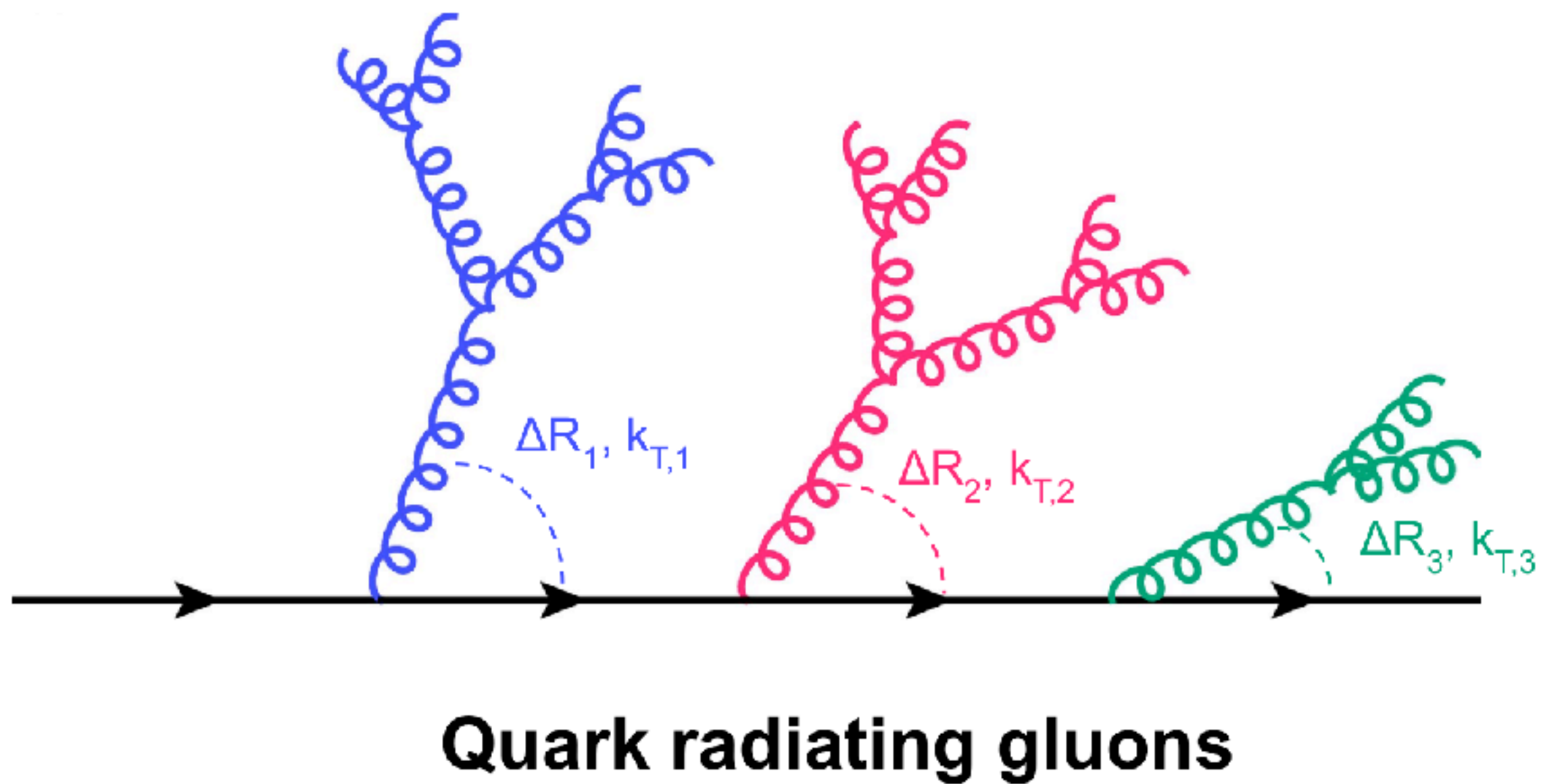


<https://cms.cern/news/fractal-tree-quarks-and-gluons>

based on “Lund plane” concept for declustering and analysing the C/A sequence  
Dreyer, GPS & Soyez, 1807.04758



# Cambridge/Aachen algorithm: **best for substructure**

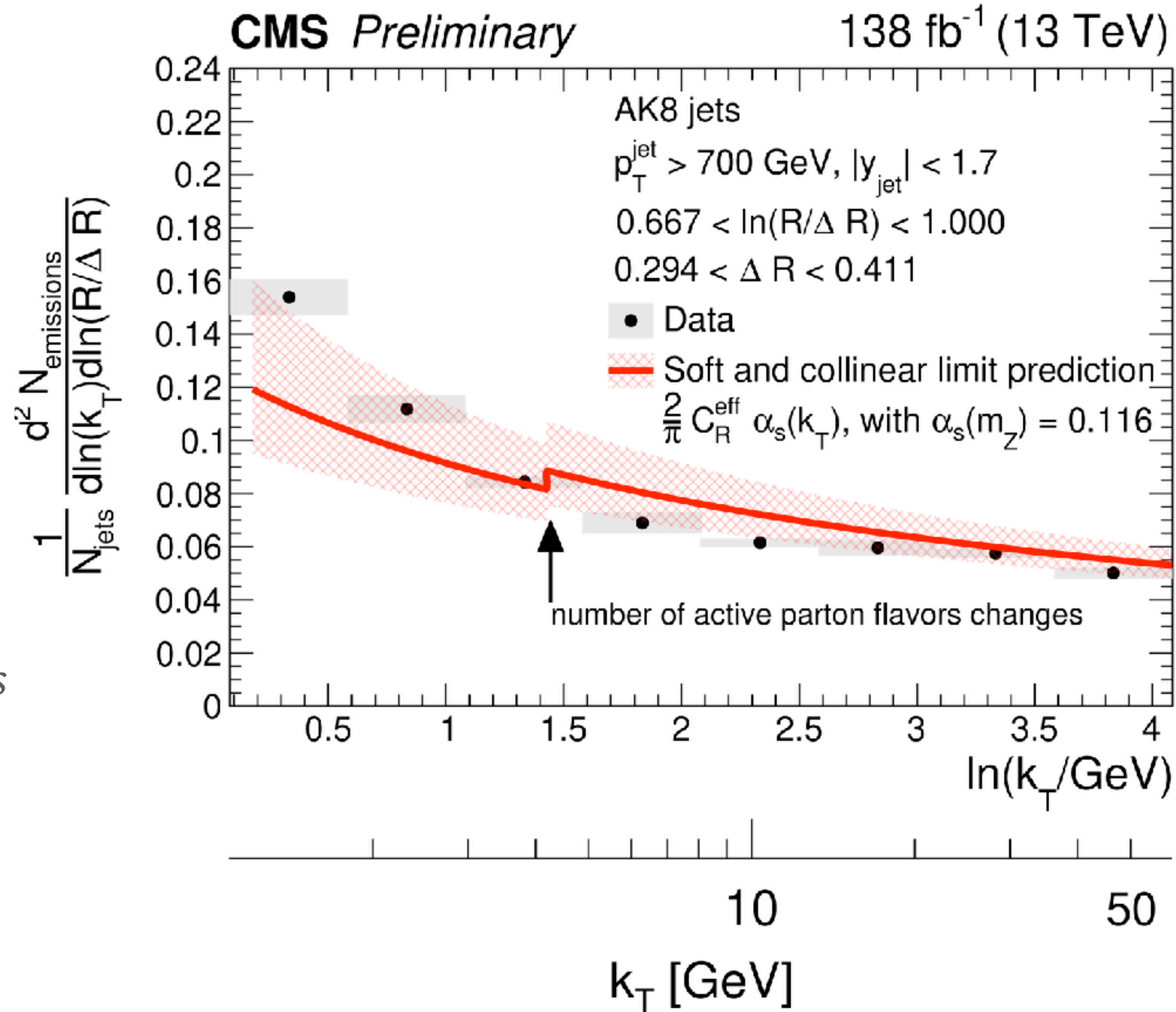


<https://cms.cern/news/fractal-tree-quarks-and-gluons>

based on “Lund plane” concept,

Dreyer, GPS & Soyez, [1807.04758](#)

measurements also by ALICE and ATLAS



# JADE algorithm descendants: **modify pairwise distance**

---

Durham  $k_t$ -algorithm:  $d_{ij} = \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})$

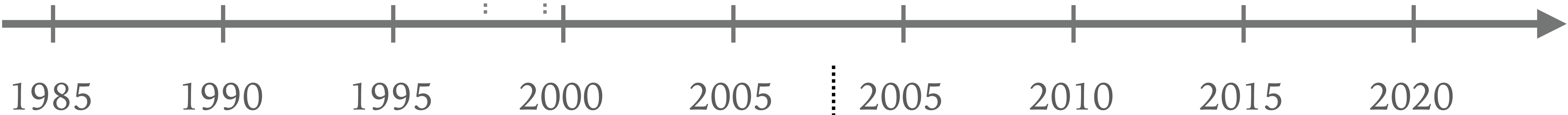
Catani, Dokshitzer, Olsson, Turnock, Webber

JADE

$$d_{ij} = E_i E_j (1 - \cos \theta_{ij})$$

Cambridge-algorithm:  $v_{ij} = (1 - \cos \theta_{ij})$

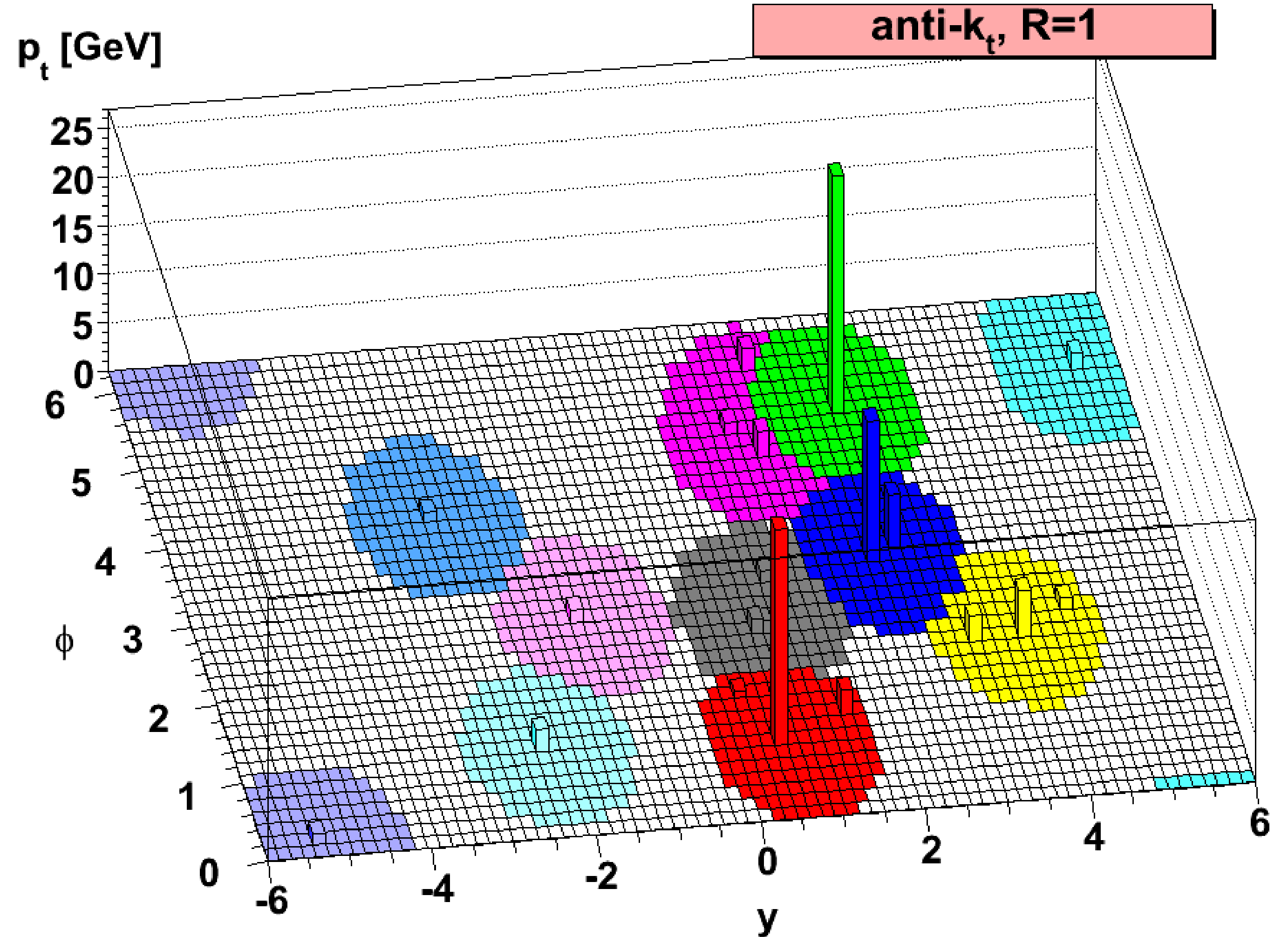
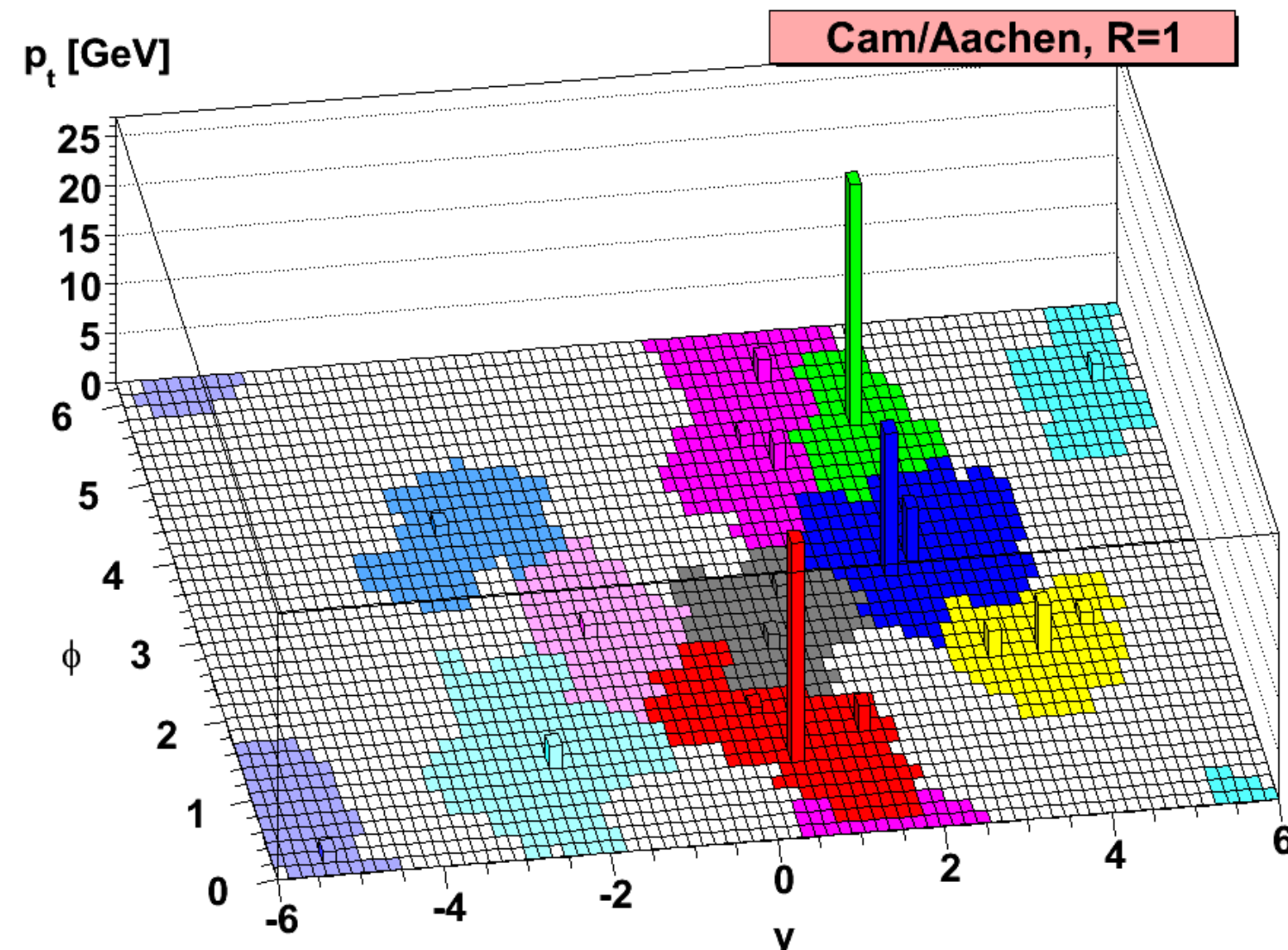
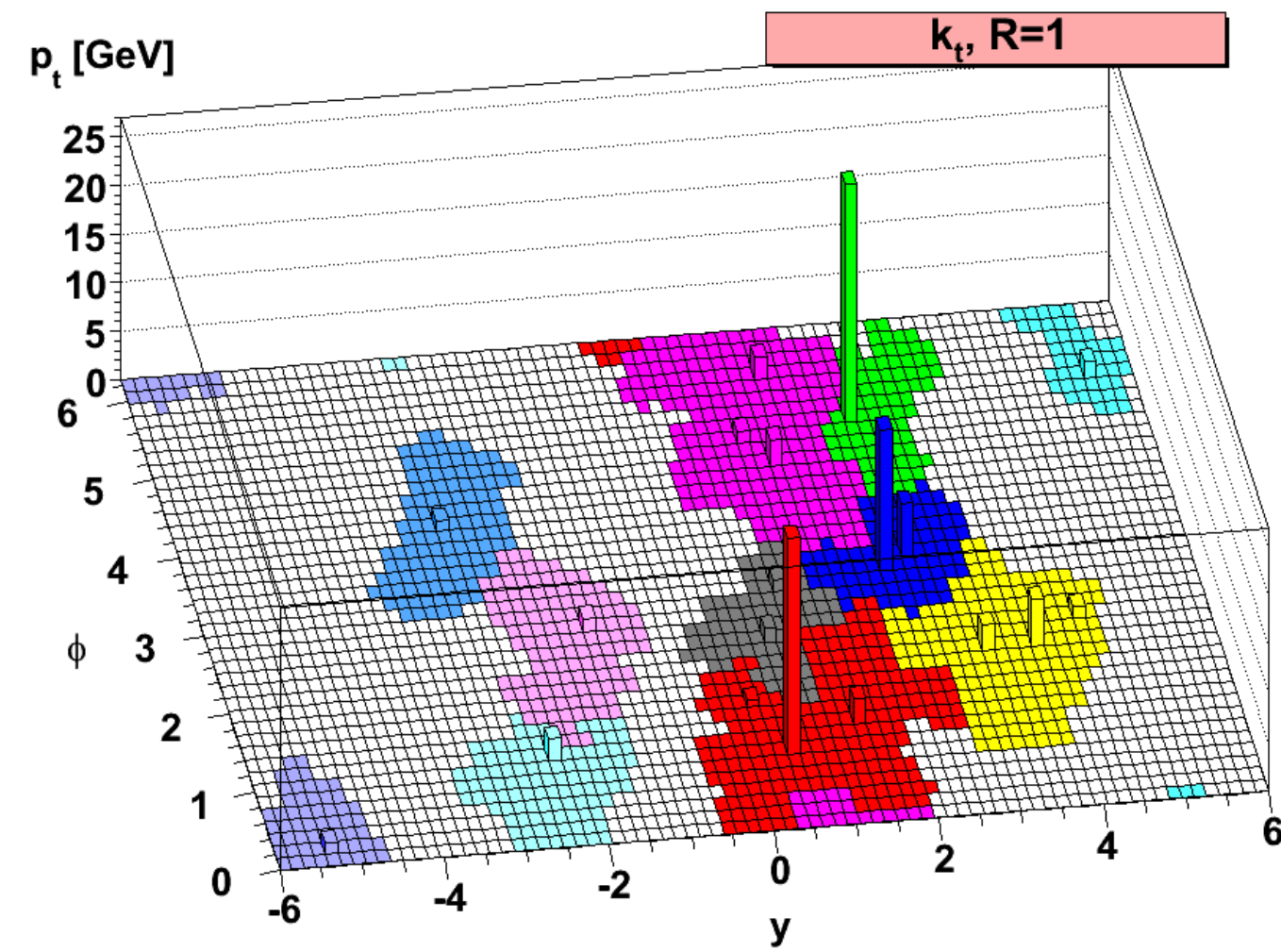
Dokshitzer, Leder, Moretti, Webber; simplified by Wobisch & Wengler



anti- $k_t$  algorithm:  $d_{ij} = (1 - \cos \theta_{ij}) / \max(E_i^2, E_j^2)$

Cacciari, Soyez, GPS

# anti- $k_t$ algorithm: circular jets made it default LHC choice



# JADE algorithm descendants: **modify pairwise distance**

Durham  $k_t$ -algorithm:  $d_{ij} = \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})$

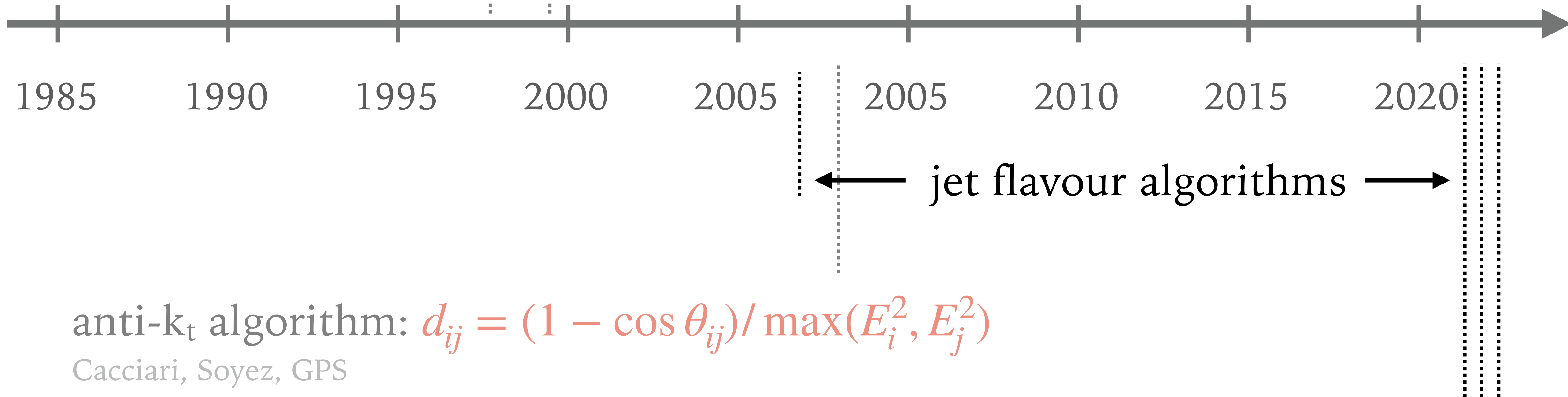
Catani, Dokshitzer, Olsson, Turnock, Webber

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Cacciari, Soyez, GPS

# jet flavour algorithms: one of today's frontiers in jet finding

---

- can you make the “flavour” infrared and collinear safe (e.g. is it a quark-jet or a gluon-jet) & keep other good properties of standard jet algorithms
- early work: Banfi, GPS & Zanderighi, [hep-ph/0601139](#)
- Recent work:
  - Caletti, Larkoski, Marzani, Reichelt, [2205.01109](#) & [2205.01117](#)
  - Czakon, Mitov, Poncelet, [2205.11879](#)
  - Gauld, Huss, Stagnitto, [2208.11138](#)
  - Caola, Grabarczyk, Hutt, GPS & Scyboz, Thaler, [to appear soon](#)
- **common theme: “undesirable” JADE property of clustering particles going in opposite directions turns out to be essential for flavoured pairs in order to make flavour IRC safe.**

# conclusions

# Concluding remarks

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- Siggis efforts & ideas pervade particle physics
  - Value of data preservation
  - Simplicity of pairwise clustering in jet physics
  - Care in bringing together different elements in the field as with strong-coupling world averages
- We should look forward to his wisdom continuing to advance the field!