Developments in perturbative QCD

Gavin Salam
LPTHE, Universities of Paris VI and VII and CNRS

Lepton Photon 2005
30 June – 5 July 2005

Perturbative QCD?

QCD 'engineering'
[Well-defined goals]

- Measuring real fundamentals — Couplings, masses
- Measuring non-perturbative 'pseudo-fundamentals' — PDFs
- Predicting signals and backgrounds — Tevatron & LHC

QCD for its own sake
[exploration]

- New structures in field theory — Twistors
- New phases of QCD — Colour glass condensate
- Parton-hadron interface — e.g. underlying events

Perturbative QCD?

QCD 'engineering'
[Well-defined goals]

⟺

QCD for its own sake
[exploration]

- Measuring real fundamentals — Couplings, masses
- Measuring non-perturbative 'pseudo-fundamentals' — PDFs
- Predicting signals and backgrounds — Tevatron & LHC

- New structures in field theory — Twistors
- New phases of QCD — Colour glass condensate
- Parton-hadron interface — e.g. underlying events

Perturbative QCD?

QCD 'engineering'
[Well-defined goals]

⟺

QCD for its own sake
[exploration]

- Measuring real fundamentals
  Couplings, masses
- Measuring non-perturbative 'pseudo-fundamentals' PDFs
- Predicting signals and backgrounds Tevatron & LHC

- New structures in field theory
  Twistors
- New phases of QCD
  Colour glass condensate
- Parton-hadron interface
  e.g. underlying events

This talk:

- Follow some of the 'engineering' progress since last Lepton-Photon
  Getting the most of out the Tevatron & gearing up for LHC
- Detour via some unexpected discoveries

Outline

Fixed order	Tree level	NLO	NNLO
Hard properties of rare events	Many jets Low accuracy	A few jets Fair accuracy	Fewest jets Best accuracy

All orders	Monte Carlo	Analytical Rsmns
Details of common events	Multi−purpose Good job + hadrons	Case−by−case High acc., partons

Saturation

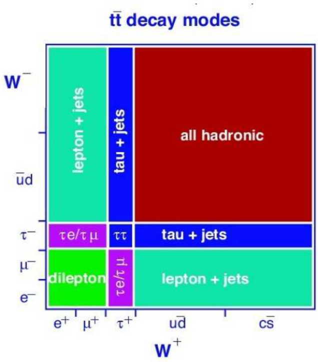
Murky

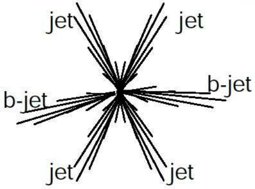
Fun!

Fixed order	**Tree level**	**NLO**	**NNLO**
Hard properties of rare events	Many jets Low accuracy	A few jets Fair accuracy	Fewest jets Best accuracy

All orders	**Monte Carlo**	**Analytical Rsmns**	**Saturation**
Details of common events	Multi−purpose Good job + hadrons	Case−by−case High acc., partons	Murky Fun!

Multi-jet final-states





All-hadronic
(BR~46%, huge bckg)

Juste

Heavy objects: multi-jet final-states

- Need to understand QCD multi-jet production (background)
- Best we can do: tree level ≤ 8 jets

# jets	# events for $10\,\mathrm{fb}^{-1}$
3	$9 \cdot 10^8$
4	$7 \cdot 10^7$
5	$6 \cdot 10^6$
6	$3 \cdot 10^5$
7	$2 \cdot 10^4$
8	$2 \cdot 10^3$

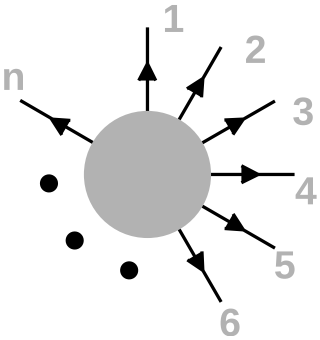
Tree level
$p_t(\text{jet}) > 60$ GeV, $\theta_{ij} > 30\,\text{deg}$, $|y_{ij}| < 3$
Draggiotis, Kleiss & Papadopoulos '02

Tree level: example simplifications

$$\mathcal{A}^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{perms} \underbrace{\text{Tr}(T_1 T_2 \ldots T_n)}_{\text{colour struct.}} \underbrace{A^{\text{tree}}(1,2,\ldots,n)}_{\text{colour ordered amp.}}$$

Tree level: example simplifications

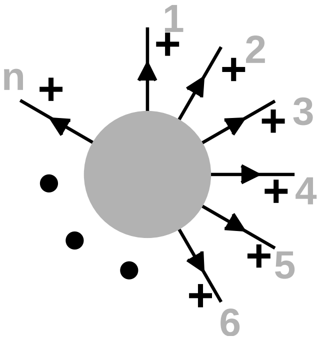
$$\mathcal{A}^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{perms} \underbrace{\text{Tr}(T_1 T_2 \ldots T_n)}_{\text{colour struct.}} \underbrace{A^{\text{tree}}(1,2,\ldots,n)}_{\text{colour ordered amp.}}$$



n	# diags	# col-ord diags
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
10	10525900	7335

Tree level: example simplifications

$$\mathcal{A}^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{perms} \underbrace{\text{Tr}(T_1 T_2 \ldots T_n)}_{\text{colour struct.}} \underbrace{A^{\text{tree}}(1,2,\ldots,n)}_{\text{colour ordered amp.}}$$

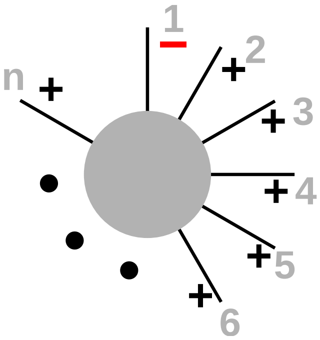


Helicity amplitude: simplifies!

$$A^{\text{tree}}(+ + + + \ldots) = 0$$

Tree level: example simplifications

$$\mathcal{A}^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{perms} \underbrace{\text{Tr}(T_1 T_2 \ldots T_n)}_{\text{colour struct.}} \underbrace{A^{\text{tree}}(1,2,\ldots,n)}_{\text{colour ordered amp.}}$$

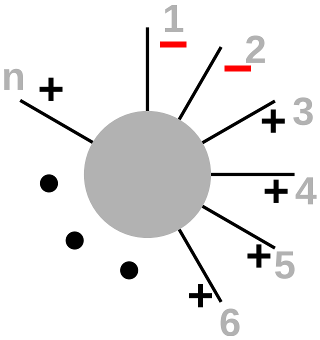


Helicity amplitude: simplifies!

$$A^{\text{tree}}(-+++\ldots) = 0$$

Tree level: example simplifications

$$\mathcal{A}^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{perms} \underbrace{\text{Tr}(T_1 T_2 \ldots T_n)}_{\text{colour struct.}} \; \underbrace{A^{\text{tree}}(1,2,\ldots,n)}_{\text{colour ordered amp.}}$$



Maximal Helicity Violating (MHV)

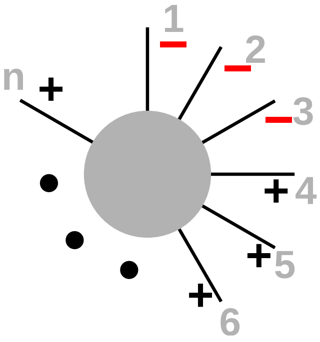
Helicity amplitude: simplifies!

$$A^{\text{tree}}(--++\ldots) = \frac{i\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

Parke & Taylor, Kunszt '85
Berends & Giele '88

Tree level: example simplifications

$$\mathcal{A}^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{perms} \underbrace{\text{Tr}(T_1 T_2 \ldots T_n)}_{\text{colour struct.}} \; \underbrace{A^{\text{tree}}(1,2,\ldots,n)}_{\text{colour ordered amp.}}$$



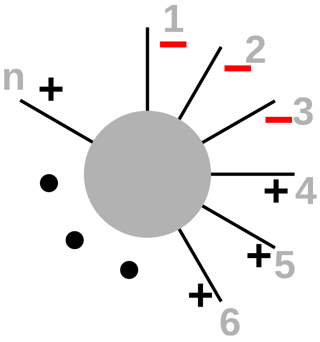
$$\begin{aligned}
&A_n(-+\cdots+--) = \\
&= \frac{i}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle (n-2)\,(n-1)\rangle\,[(n-2)\,(n-1)]\,[(n-1)\,n]\,[n\,1]\,[1\,2]} \\
&\times\Bigg(\frac{(n-1\,n)\,(1\,2)\,[(n-1)\,(n-2)]\,\langle (n-1)^-|\,K_-\,|2^-\rangle^2}{S_{3,n-1}} + \frac{(1\,n)\,\langle (n-1)\,(n-2)\rangle\,[1\,2]\,\langle 1^-|\,K_-\,|(n-2)^-\rangle^2}{S_{1,n-3}} \\
&+\frac{\langle n\,(n-1)\rangle\langle 1\,(n-1)\rangle\langle 1\,(n-2)\rangle\,[1\,n]\,[1\,2]\,[(n-1)\,(n-2)]\,\langle 1^-|\,K_-\,|(n-2)^-\rangle}{S_{1,n-3}} \\
&+\frac{\langle n\,1\rangle\langle (n-1)\,1\rangle\langle (n-1)\,2\rangle\,[(n-1)\,n]\,[1\,2]\,[(n-1)\,(n-2)]\,\langle (n-1)^-|\,K_-\,|2^-\rangle}{S_{3,n-1}} \\
&-\langle 1\,(n-1)\rangle^2 S_{2,n-2}\,[1\,2]\,[(n-1)\,(n-2)] - \frac{(n-1\,n)\,(1\,n)\,\langle 1\,(n-1)\rangle\langle 1^-|\,K_-\,|(n-2)^-\rangle\,[1\,2]}{S_{1,n-3}} \\
&-[n\,1]\,[n\,(n-1)]\,[1\,2]\,[(n-1)\,(n-2)] \\
&\quad\times\sum_{l=3}^{n-3}\Bigg[\frac{\langle n\,(n-1)\rangle^2\langle (n-1)\,1\rangle\langle 1^-|\,\not{K}_{1,l-1}\not{k}_l\,|1^+\rangle}{S_{1,l-1}S_{1,l}} \\
&\qquad+\frac{\langle n\,1\rangle^2\langle 1\,(n-1)\rangle\langle (n-1)^-|\,\not{K}_{l+1,n-1}\not{k}_l\,|(n-1)^+\rangle}{S_{l+1,n-1}S_{l,n-1}} \\
&\qquad-\frac{\langle n\,1\rangle\langle n\,(n-1)\rangle\langle (n-1)\,1\rangle\langle (n-1)^-|\,\not{K}_{l+1,n}\not{k}_l\,|1^+\rangle}{S_{1,l}S_{l,n-1}} \\
&\qquad-\frac{\langle n\,1\rangle\langle n\,(n-1)\rangle^2\langle (n-1)^-|\,\not{K}_{l+1,n}\not{k}_l\,|1^+\rangle\langle 1^-|\,\not{K}_{l,n}\,|n^-\rangle}{S_{1,l-1}S_{1,l}S_{l,n-1}} \\
&\qquad-\frac{\langle n\,1\rangle^2\langle n\,(n-1)\rangle\langle (n-1)^-|\,\not{K}_{l+1,n-1}\not{k}_l\,|1^+\rangle\langle (n-1)^-|\,\not{K}_{l+1,n}\,|n^-\rangle}{S_{1,l}S_{l+1,n-1}S_{l,n-1}}\Bigg]\Bigg)
\end{aligned} \tag{5.2}$$

NEXT to Maximal Helicity Violating (NMHV)

Kosower, '90

Tree level: example simplifications

$$\mathcal{A}^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{perms} \underbrace{\text{Tr}(T_1 T_2 \ldots T_n)}_{\text{colour struct.}} \underbrace{A^{\text{tree}}(1,2,\ldots,n)}_{\text{colour ordered amp.}}$$



NEXT to Maximal Helicity Violating (NMHV)

Helicity amplitude: simplifies!

$$A^{\text{tree}}(---++\ldots) = \frac{1}{F_{3,1}} \sum_{j=4}^{n-1} \frac{\langle 1|P_{2,j} P_{j+1,2}|3\rangle}{P_{2,j}^2 P_{j+1,2}^2} \times \frac{\langle j+1\; j\rangle}{[2|P_{2,j}|j+1\rangle\langle j|P_{j+1,2}|2]}$$

Britto et al., hep-th/0503198

Just one of vast array of new results obtained with *Twistor* techniques.

Twistors & MHV rules

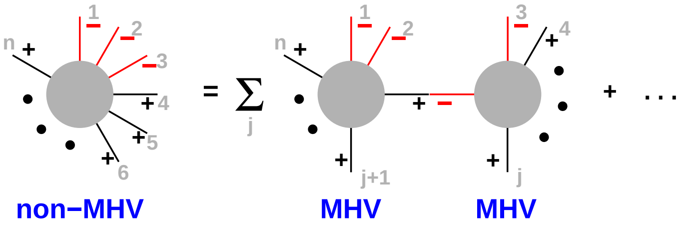
Twistor space: Fourier trans. wrt +ve helicity spinors ($\sim \sqrt{\text{Fourier trans.}}$)

Penrose 1967

Twistor space reveals new properties of helicity amplitudes ⇔ proposal of *'weak-weak' duality* with a topological string theory.

Witten hep-th/0312171

Deduce systematic rules for building non-MHV amplitudes by joining MHV ones with off-shell scalar progator: Cachazo, Svrcek & Witten hep-th/0403047



Alternative rules (off-shell → complex mom.) from field-theory + analyticity

Britto, Cachazo & Feng hep-th/0412265; *idem.* + Witten hep-th/0501052

Twistor Applications

Very active field: 140 articles in 18 months ($\sim$ 30 by 'QCD people')

Tree level

- Specific compact results, including for NNMHV — Kosower; Roiban et al
  - *Hints of yet deeper simplifications* — Luo & Wen; Britto et al
- Efficient (recursive) formulations — Bena, Bern, Kosower
  - NB: recall $\exists$ 'standard' numerical methods for tree-level calculations: — Berends-Giele; 'Alpha'
- Massless quarks, gluinos — Georgiou, Glover & Khoze; Wu & Zhu
- External Higgs boson — Dixon/Badger, Glover & Khoze
- External weak boson — Bern, Forde, Kosower & Mastrolia
- Collinear limits — Birthwright et al

Amazing progress in short time. . .

Fixed order Hard properties of rare events	**Tree level** Many jets Low accuracy	**NLO** A few jets Fair accuracy	**NNLO** Fewest jets Best accuracy
All orders Details of common events	**Monte Carlo** Multi−purpose Good job + hadrons	**Analytical Rsmns** Case−by−case High acc., partons	**Saturation** Murky Fun!

NLO Wish lists

Experimenters' priorities

Les Houches

1. pp → WW + jet
2. pp → H + 2 jets
   - Background to VBF Higgs production
3. pp → $t\bar{t}b\bar{b}$
4. pp → $t\bar{t}$ + 2 jets
   - Background to $t\bar{t}$H
5. pp → $WW\ b\bar{b}$
6. pp → VV + 2 jets
   - Background to $WW \to H \to WW$
7. pp → V + 3 jets
   - General background to new physics
8. pp → VVV + jet
   - Background to SUSY trilepton

Currently available

NLOJET++, MCFM, PHOX, ...

http://www.cedar.ac.uk/hepcode/

Theorist's list (G. Heinrich)

- 2 → 3 (OK for a good student!)
  - pp → WW + jet
  - pp → VVV + jet
  - pp → H + 2 jets
- 2 → 4 (Beyond today's means)
  - pp → 4 jets
  - pp → $t\bar{t}$ + 2 jets
  - pp → $t\bar{t}b\bar{b}$
  - pp → V + 3 jets
  - pp → VV + 2 jets
  - pp → VVV + jet
  - pp → $WW\ b\bar{b}$

NLO Wish lists

Experimenters' priorities

Les Houches

1. pp → WW + jet
2. pp → H + 2 jets
   - Background to VBF Higgs production
3. pp → $t\bar{t}b\bar{b}$
4. pp → $t\bar{t}$ + 2 jets
   - Background to $t\bar{t}$H
5. pp → $WW\ b\bar{b}$
6. pp → VV + 2 jets
   - Background to $WW \to H \to WW$
7. pp → V + 3 jets
   - General background to new physics
8. pp → VVV + jet
   - Background to SUSY trilepton

Currently available

NLOJET++, MCFM, PHOX, ...

`http://www.cedar.ac.uk/hepcode/`

Theorist's list (G. Heinrich)

- 2 → 3 (OK for a good student!)
  - pp → WW + jet
  - pp → VVV + jet
  - pp → H + 2 jets
- 2 → 4 (Beyond today's means)
  - pp → 4 jets
  - pp → $t\bar{t}$ + 2 jets
  - pp → $t\bar{t}b\bar{b}$
  - pp → V + 3 jets
  - pp → VV + 2 jets
  - pp → VVV + jet
  - pp → $WW\ b\bar{b}$

NLO Wish lists

Experimenters' priorities

Les Houches

1. pp → WW + jet
2. pp → H + 2 jets
   - Background to VBF Higgs production
3. pp → $t\bar{t}b\bar{b}$
4. pp → $t\bar{t}$ + 2 jets
   - Background to $t\bar{t}$H
5. pp → $WW\ b\bar{b}$
6. pp → VV + 2 jets
   - Background to $WW \to H \to WW$
7. pp → V + 3 jets
   - General background to new physics
8. pp → VVV + jet
   - Background to SUSY trilepton

Currently available

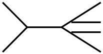
NLOJET++, MCFM, PHOX, ...

`http://www.cedar.ac.uk/hepcode/`

Theorist's list (G. Heinrich)

- 2 → 3 (OK for a good student!)
  - pp → WW + jet
  - pp → VVV + jet
  - pp → H + 2 jets
- 2 → 4 (Beyond today's means)
  - pp → 4 jets
  - pp → $t\bar{t}$ + 2 jets
  - pp → $t\bar{t}b\bar{b}$
  - pp → V + 3 jets
  - pp → VV + 2 jets
  - pp → VVV + jet
  - pp → $WW\ b\bar{b}$

NLO bottleneck: loop calc.

2 → 3 @ NLO ~  +  + **Tricks to cancel divergences**

2 → 4 @ Tree **2 → 3 @ 1−loop** **(dipole subtraction)**

Traditionally: 1-loop term calculated all-analytically (1–2 years per 2 → 3 proc.)

Bern, Dixon, Kosower, Kunszt, Signer, Trócsanyi, ...

Instead: attempt to *automate*

1. Get expressions for all Feynman graphs (QGRAF, FeynArts)
2. Evaluate resulting integrals
   - Subtract out divergences *before integrating*, do rest numerically.

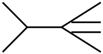
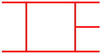
     Nagy, Soper '03
   - Reduce all loop integrals to combinations of known *basis set*.

     Binoth, Guillet, Heinrich, Pilon, Schubert '05

     Ellis, Giele, Glover, Zanderighi '04-05; + others

Promising, but still early days!

NLO bottleneck: loop calc.

2 → 3 @ NLO ~  +  + **Tricks to cancel divergences**

2 → 4 @ Tree | **2 → 3 @ 1−loop** | **(dipole subtraction)**

Bottleneck

Traditionally: 1-loop term calculated all-analytically (1–2 years per 2 → 3 proc.)

Bern, Dixon, Kosower, Kunszt, Signer, Trócsanyi, ...

Instead: attempt to *automate*

1. Get expressions for all Feynman graphs (QGRAF, FeynArts)
2. Evaluate resulting integrals
   - Subtract out divergences *before integrating*, do rest numerically.

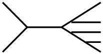
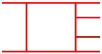
     Nagy, Soper '03
   - Reduce all loop integrals to combinations of known *basis set*.

     Binoth, Guillet, Heinrich, Pilon, Schubert '05

     Ellis, Giele, Glover, Zanderighi '04-05; + others

Promising, but still early days!

NLO bottleneck: loop calc.

2 -> 4 @ NLO ~  +  + **Tricks to cancel divergences**

2 -> 5 @ Tree | **2 -> 4 @ 1−loop** | **(dipole subtraction)**

Impossible so far!

Traditionally: 1-loop term calculated all-analytically (1–2 years per $2 \to 3$ proc.)

Bern, Dixon, Kosower, Kunszt, Signer, Trócsanyi, ...

Instead: attempt to *automate*

1. Get expressions for all Feynman graphs (QGRAF, FeynArts)
2. Evaluate resulting integrals
   - Subtract out divergences *before integrating*, do rest numerically.

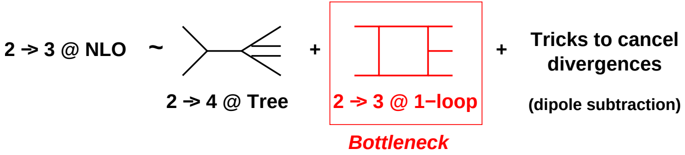
     Nagy, Soper '03
   - Reduce all loop integrals to combinations of known *basis set*.

     Binoth, Guillet, Heinrich, Pilon, Schubert '05

     Ellis, Giele, Glover, Zanderighi '04-05; + others

Promising, but still early days!

NLO bottleneck: loop calc.



Traditionally: 1-loop term calculated all-analytically (1–2 years per $2 \to 3$ proc.)

Bern, Dixon, Kosower, Kunszt, Signer, Trócsanyi, ...

Instead: attempt to *automate*

1. Get expressions for all Feynman graphs (QGRAF, FeynArts)
2. Evaluate resulting integrals
   - Subtract out divergences *before integrating*, do rest numerically.

     Nagy, Soper '03
   - Reduce all loop integrals to combinations of known *basis set*.

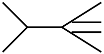
     Binoth, Guillet, Heinrich, Pilon, Schubert '05

     Ellis, Giele, Glover, Zanderighi '04-05; + others

Promising, but still early days!

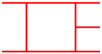
NLO bottleneck: loop calc.

2 → 3 @ NLO ~



2 → 4 @ Tree

+



2 → 3 @ 1−loop

Bottleneck

+ **Tricks to cancel divergences**

(dipole subtraction)

Traditionally: 1-loop term calculated all-analytically (1–2 years per $2 \to 3$ proc.)

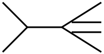
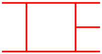
Bern, Dixon, Kosower, Kunszt, Signer, Trócsanyi, ...

Instead: attempt to *automate*

1. Get expressions for all Feynman graphs (QGRAF, FeynArts)
2. Evaluate resulting integrals
   - Subtract out divergences *before integrating*, do rest numerically.
     Nagy, Soper '03
   - Reduce all loop integrals to combinations of known *basis set.*
     Binoth, Guillet, Heinrich, Pilon, Schubert '05
     Ellis, Giele, Glover, Zanderighi '04-05; + others

Promising, but still early days!

NLO bottleneck: loop calc.

2 → 3 @ NLO ~  +  + **Tricks to cancel divergences**

2 → 4 @ Tree | **2 → 3 @ 1−loop** | **(dipole subtraction)**

Bottleneck

Traditionally: 1-loop term calculated all-analytically (1–2 years per $2 \to 3$ proc.)

Bern, Dixon, Kosower, Kunszt, Signer, Trócsanyi, ...

Instead: attempt to *automate*

1. Get expressions for all Feynman graphs (QGRAF, FeynArts)
2. Evaluate resulting integrals
   - Subtract out divergences *before integrating*, do rest numerically.
     Nagy, Soper '03
   - Reduce all loop integrals to combinations of known *basis set.*

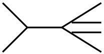
Binoth, Guillet, Heinrich, Pilon, Schubert '05

Ellis, Giele, Glover, Zanderighi '04-05; + others

Promising, but still early days!

NLO bottleneck: loop calc.

2 → 3 @ NLO ~



2 → 4 @ Tree

+

2 → 3 @ 1−loop

Bottleneck

+ **Tricks to cancel divergences**

(dipole subtraction)

Traditionally: 1-loop term calculated all-analytically (1–2 years per $2 \to 3$ proc.)

Bern, Dixon, Kosower, Kunszt, Signer, Trócsanyi, ...

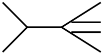
Instead: attempt to *automate*

1. Get expressions for all Feynman graphs (QGRAF, FeynArts)
2. Evaluate resulting integrals
   - Subtract out divergences *before integrating*, do rest numerically.
     Nagy, Soper '03
   - Reduce all loop integrals to combinations of known *basis set.*
     Binoth, Guillet, Heinrich, Pilon, Schubert '05
     Ellis, Giele, Glover, Zanderighi '04-05; + others

Promising, but still early days!

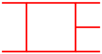
NLO bottleneck: loop calc.

2 -> 3 @ NLO ~



2 -> 4 @ Tree

+



2 -> 3 @ 1−loop

Bottleneck

+ **Tricks to cancel divergences**

(dipole subtraction)

Traditionally: 1-loop term calculated all-analytically (1–2 years per $2 \to 3$ proc.)

Bern, Dixon, Kosower, Kunszt, Signer, Trócsanyi, ...

Instead: attempt to *automate*

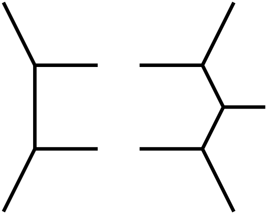
1. Get expressions for all Feynman graphs (QGRAF, FeynArts)
2. Evaluate resulting integrals
   - Subtract out divergences *before integrating*, do rest numerically.
     Nagy, Soper '03
   - Reduce all loop integrals to combinations of known *basis set.*
     Binoth, Guillet, Heinrich, Pilon, Schubert '05
     Ellis, Giele, Glover, Zanderighi '04-05; + others

Or maybe twistors will solve it all first?

Twistors @ 1-loop?

- Twistors are good for tree-level diagrams
- 'Sew' tree-level diagrams $\rightarrow$ loops

➥ Sew twistors to get loops?



'Sewing' only works well for SUSY Yang-Mills (simpler structures)

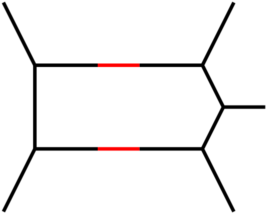
$\mathcal{N} = 4$ SUSY	$\mathcal{N} = 1$ SUSY	QCD
all-n NMHV	subset of all-n NMHV	all-n finite $(\pm + + + + \cdots)$
full 7-gluon	full 6-gluon	full 4-gluon
2-loop 5, 6-gluon!		

Many authors...

Twistors @ 1-loop?

- Twistors are good for tree-level diagrams
- 'Sew' tree-level diagrams → loops

➥ Sew twistors to get loops?



'Sewing' only works well for SUSY Yang-Mills (simpler structures)

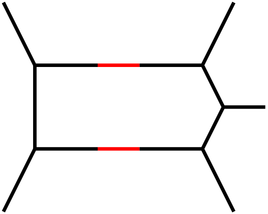
$\mathcal{N}=4$ SUSY	$\mathcal{N}=1$ SUSY	QCD
all-n NMHV	subset of all-n NMHV	all-n finite $(\pm++++\cdots)$
full 7-gluon	full 6-gluon	full 4-gluon
2-loop 5, 6-gluon!		

Many authors...

Twistors @ 1-loop?

- Twistors are good for tree-level diagrams
- 'Sew' tree-level diagrams → loops

➥ Sew twistors to get loops?



'Sewing' only works well for SUSY Yang-Mills (simpler structures)

$\mathcal{N} = 4$ SUSY	$\mathcal{N} = 1$ SUSY	QCD
all-n NMHV	subset of all-n NMHV	all-n finite $(\pm + + + + \cdots)$
full 7-gluon	full 6-gluon	full 4-gluon
2-loop 5, 6-gluon!		

Many authors...

Also promising, but still early days!

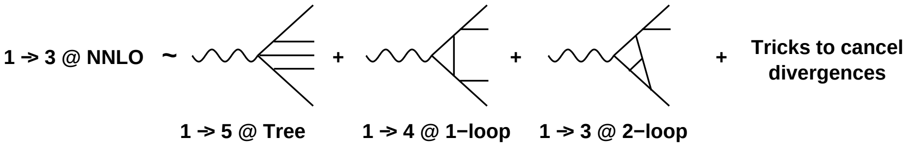
Fixed order Hard properties of rare events	**Tree level** Many jets Low accuracy	**NLO** A few jets Fair accuracy	**NNLO** Fewest jets Best accuracy
All orders Details of common events	**Monte Carlo** Multi−purpose Good job + hadrons	**Analytical Rsmns** Case−by−case High acc., partons	**Saturation** Murky Fun!

NNLO status & road-map

- Processes **with two QCD partons** are mostly done
  - $e^+e^- \to$ hadrons, $\tau \to \nu +$ hadrons
  - DIS coeff. fns., sum rules
  - $pp \to W, Z, \gamma^*, H, WH, ZH$
- **Next in line:** $e^+e^- \to 3\,$jets?
  - simplest!
  - α_s & other measurements at LEP are theory limited

    theory uncertainty $\sim 3 - 4\times$ exp. error
  - useful for studying perturbative/ non-perturbative interface.
- Then DIS $\to 2 + 1$ and $pp \to 2\,$jets...

NNLO bottleneck

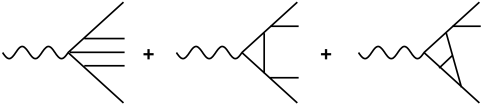


NNLO bottleneck

$4 + 2\varepsilon$ dim:
J is observable

$\int d\Phi_5 \; J(p_{1..5})$ $\quad \int d\Phi_4 \, \varepsilon^{-2} J(p_{1..4})$ $\quad \int d\Phi_3 \, \varepsilon^{-4} J(p_{1..3})$

1 -> 3 @ NNLO ~



+ Tricks to cancel divergences

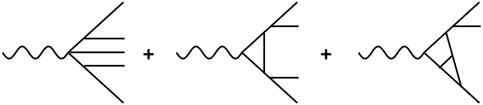
1 -> 5 @ Tree **1 -> 4 @ 1−loop** **1 -> 3 @ 2−loop**

NNLO bottleneck

$4+2\varepsilon$ dim:
J is observable

$\int d\Phi_5 \; J(p_{1..5})$ $\int d\Phi_4 \; \varepsilon^{-2} J(p_{1..4})$ $\int d\Phi_3 \; \varepsilon^{-4} J(p_{1..3})$

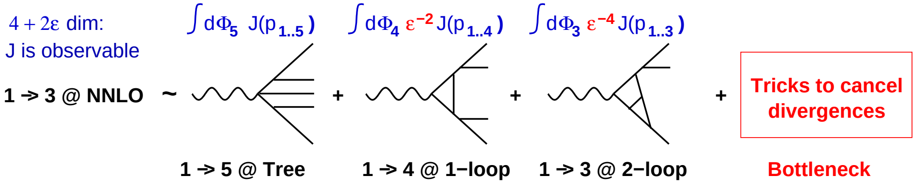
1 -> 3 @ NNLO ~



+ **Tricks to cancel divergences**

1 -> 5 @ Tree **1 -> 4 @ 1−loop** **1 -> 3 @ 2−loop** **Bottleneck**

NNLO bottleneck



"You have to do the integral, but you don't know the integrand"

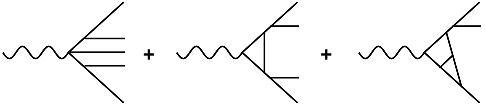
Anastasiou (KITP LoopFest III)

NNLO bottleneck

$4+2\varepsilon$ dim:
J is observable

$\int d\Phi_5\ J(p_{1..5})$ $\int d\Phi_4\ \varepsilon^{-2} J(p_{1..4})$ $\int d\Phi_3\ \varepsilon^{-4} J(p_{1..3})$

1 → 3 @ NNLO ~



Tricks to cancel divergences

1 → 5 @ Tree **1 → 4 @ 1−loop** **1 → 3 @ 2−loop** **Bottleneck**

How to get cancellations?

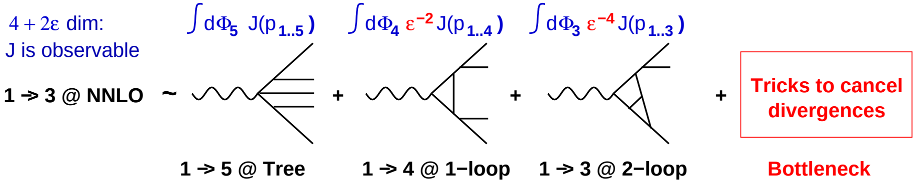
1. Subtraction method: Standard approach @ NLO
   - Tested in $e^+e^- \to 2$ jets; applied to C_F^3 colour part of $e^+e^- \to 3$ jets

     $(\alpha_s C_F/2\pi)^3$ piece of $\langle 1 - T\rangle = -20.4 \pm 4$

     Gehrmann-de Ridder, Gehrmann & Glover '04
   - Full 'antenna' subtraction formulae recently published idem. '05
2. Sector decomposition for isolating divergences Binoth & Heinrich '00
   - Tested in $e^+e^- \to 2$ jets Anastasiou, Melnikov & Petriello '04; B & H '04
   - Applied to $pp \to W, Z, H$ (fully differential)

     Anastasiou, Dixon, Melnikov & Petriello '03–04

NNLO bottleneck



How to get cancellations?

1. Subtraction method: Standard approach @ NLO
   - Tested in $e^+e^- \to 2\,\text{jets}$; applied to C_F^3 colour part of $e^+e^- \to 3\,\text{jets}$

     $$(\alpha_s C_F/2\pi)^3 \text{ piece of } \langle 1 - T \rangle = -20.4 \pm 4$$

     Gehrmann-de Ridder, Gehrmann & Glover '04
   - Full 'antenna' subtraction formulae recently published — idem. '05
2. Sector decomposition for isolating divergences — Binoth & Heinrich '00
   - Tested in $e^+e^- \to 2\,\text{jets}$ — Anastasiou, Melnikov & Petriello '04; B & H '04
   - Applied to $pp \to W, Z, H$ (fully differential)

     Anastasiou, Dixon, Melnikov & Petriello '03–04

NNLO DGLAP Splitting functions

Landmark calculation:

Unpolarised NNLO DGLAP splitting functions and α_s^3 electromagnetic DIS coefficient functions (F_2, F_L).

Moch, Vermaseren, Vogt '04-

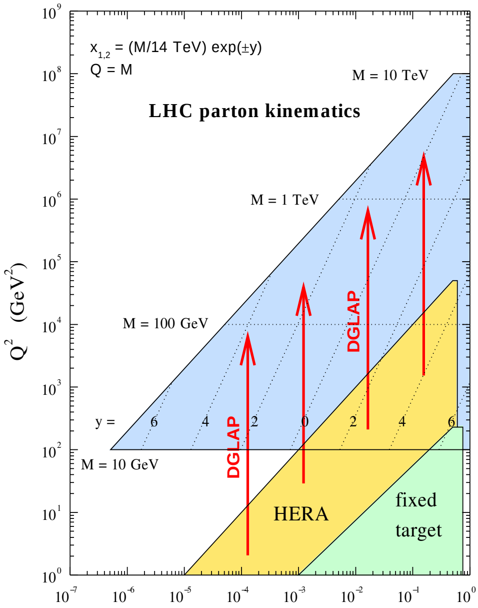
DGLAP splitting functions are **pivotal** in going from HERA to Tevatron and LHC.

NNLO DGLAP Splitting functions

Landmark calculation:
Unpolarised NNLO DGLAP splitting functions and α_s^3 electromagnetic DIS coefficient functions (F_2, F_L).

Moch, Vermaseren, Vogt '04-

DGLAP splitting functions are **pivotal** in going from HERA to Tevatron and LHC.



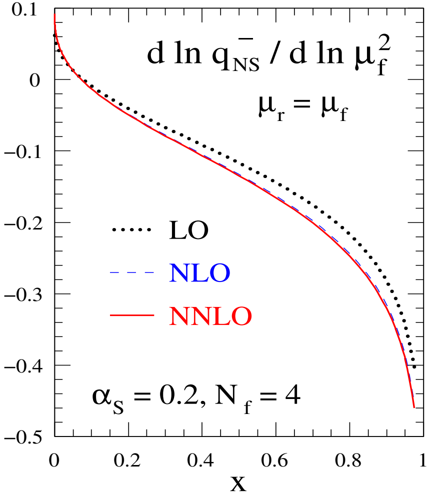
NNLO DGLAP result

NNLO DGLAP stability

DGLAP NNLO Properties:

- Generally very good convergence
- Potentially dangerous small-x and large-x regions are fairly tame and can be controlled

Ciafaloni et al '03
Altarelli, Ball & Forte '04
Corcella & Magnea '05
Moch, Vermaseren & Vogt '05



NNLO DGLAP stability

DGLAP NNLO Properties:

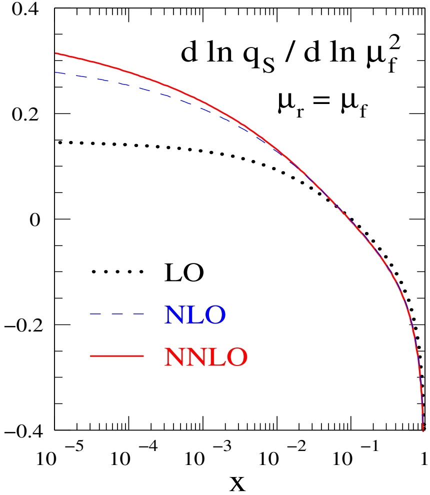
- Generally very good convergence
- Potentially dangerous small-x and large-x regions are fairly tame and can be controlled

Ciafaloni et al '03

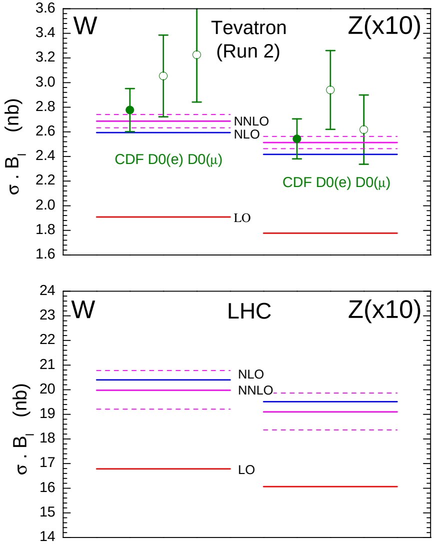
Altarelli, Ball & Forte '04

Corcella & Magnea '05

Moch, Vermaseren & Vogt '05



Example NNLO predictions



Martin et al

Combine

- NNLO splitting functions
- NNLO PDF fits
- NNLO Drell-Yan X-sct

➥ High-precision predictions for *W* production.

4% – 5.5% total error

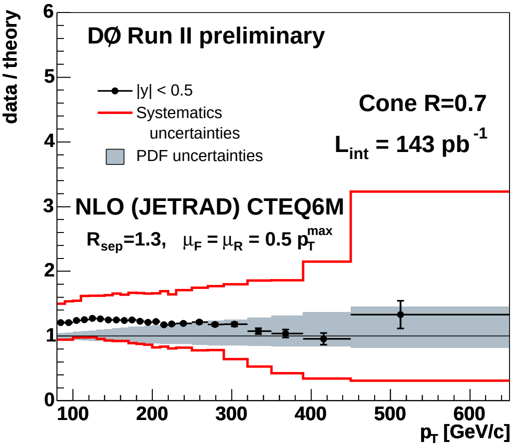
MRST → 4%, CTEQ → 5.5%

Possible use as *standard candle* for luminosity measurements.

Needs good understanding of PDF uncertainties

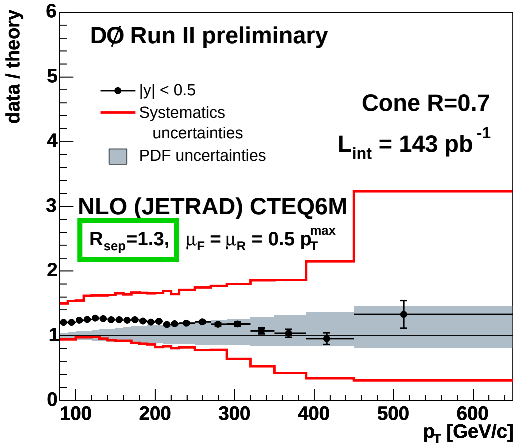
CTEQ, MRST, Alekhin, H1, ZEUS

Calculate what you measure...



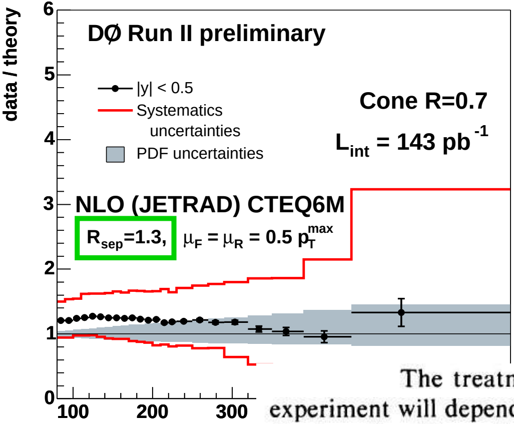
E.g.: CDF, D0 cone jet X-sect

Calculate what you measure...



E.g.: CDF, D0 cone jet X-sect

Calculate what you measure...

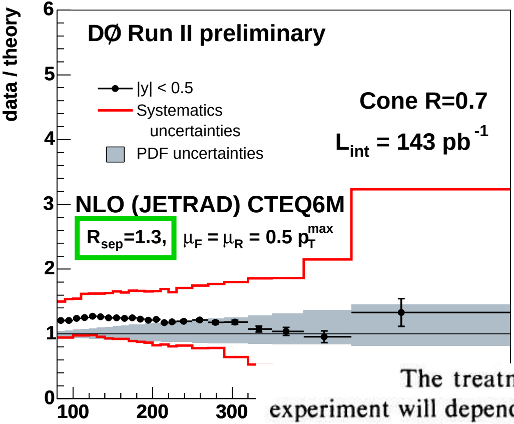


E.g.: CDF, D0 cone jet X-sect

Ellis, Kunszt & Soper PRL 69, 3615 (1992):

The treatment of this configuration in a real experiment will depend in detail on the implementation of the jet algorithm. To simulate the experimental algorithm in a simple way we add an extra constraint in our theoretical jet algorithm. When two partons, a and b, are separated by more than $R_{sep}(\leq 2R)$, $R_{ab} = [(\eta_a - \eta_b)^2 + (\phi_a - \phi_b)^2]^{1/2} \geq R_{sep}$, we no longer merge them into a single jet.

Calculate what you measure...



E.g.: CDF, D0 cone jet X-sect

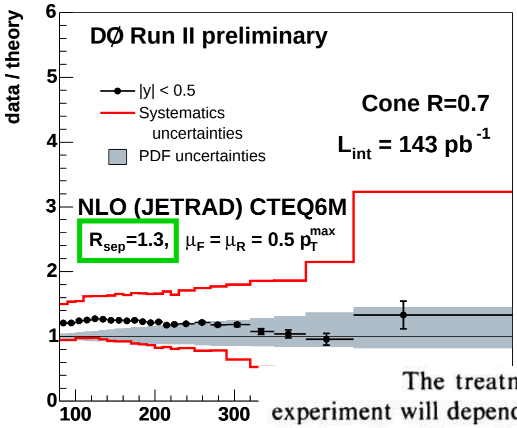
$R_{sep} = 1.3$ is a **fudge factor**

moderate $\sim 5 - 10\%$ effect

Ellis, Kunszt & Soper PRL 69, 3615 (1992):

The treatment of this configuration in a real experiment will depend in detail on the implementation of the jet algorithm. To simulate the experimental algorithm in a simple way we add an extra constraint in our theoretical jet algorithm. When two partons, a and b, are separated by more than $R_{\text{sep}}(\leq 2R)$, $R_{ab} = [(\eta_a - \eta_b)^2 + (\phi_a - \phi_b)^2]^{1/2} \geq R_{\text{sep}}$, we no longer merge them into a single jet.

Calculate what you measure...



E.g.: CDF, D0 cone jet X-sect

$R_{sep} = 1.3$ is a **fudge factor**

moderate $\sim 5 - 10\%$ effect

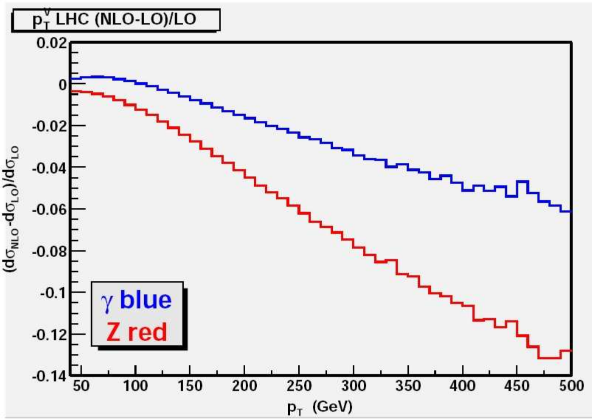
- Don't fudge!
- Use **identical** algorithms for NLO and exp.

(Cone or Kt algorithms)

Ellis, Kunszt & Soper PRL 69, 3615 (1992):

The treatment of this configuration in a real experiment will depend in detail on the implementation of the jet algorithm. To simulate the experimental algorithm in a simple way we add an extra constraint in our theoretical jet algorithm. When two partons, a and b, are separated by more than $R_{sep}(\leq 2R)$, $R_{ab} = [(\eta_a - \eta_b)^2 + (\phi_a - \phi_b)^2]^{1/2} \geq R_{sep}$, we no longer merge them into a single jet.

EW is not so weak



Widely discussed for ILC. How about *pp*?

e.g. NLO EW corrections to $pp \to Z+$jet

These are *significant* (even NNLO $\sim$ few %)

Maina Moretti Ross '04

Kulesza et al '04

QED effects $\lesssim 1\%$

Martin et al.

Glosser et al

Fixed order Hard properties of rare events	**Tree level** Many jets Low accuracy	**NLO** A few jets Fair accuracy	**NNLO** Fewest jets Best accuracy
All orders Details of common events	**Monte Carlo** Multi−purpose Good job + hadrons	**Analytical Rsmns** Case−by−case High acc., partons	**Saturation** Murky Fun!

MC status & plans

Fortran

- Matching to multi-parton LO matrix elements now widespread CKKW
- New, better shower in Pythia ($k_\perp$ ordered)
- Underlying event models much improved / more practical
- Reaching end of line soon! Pythia, Jimmy (Herwig)

C++

based on ThePEG			Independent	
Herwig++	Pythia 7	Ariadne/LDC	Pythia 8	Sherpa
ready for e^+e^- in prog. for pp	*cancelled*	planned	being coded	ready for e^+e^- and pp

Includes new, improved angular-ordered shower

New player! Dresden group

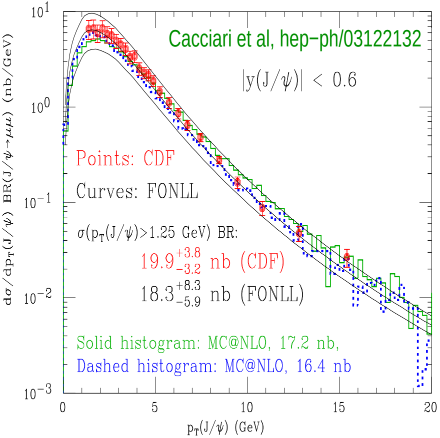
MC@NLO

Limitation of MC's: contain only soft-collinear part of NLO ($\mathrm{NLO_{MC}}$)

Much work done on combining full NLO with MCs.

One approach applied to multiple processes: MC@NLO (with HERWIG)

Frixione, Webber & Nason



$$\mathrm{MC@NLO} = \mathrm{MC} + \underbrace{(\mathrm{NLO} - \mathrm{NLO_{MC}})}_{\sim\text{finite}}$$

- Applied to $pp \to$ $H,\ V,\ V\,V,\ Q\bar{Q},\ \ell^+\ell^-,\ H\,V$
  many with spin correl.
- Compares well with
  - data
  - analytical calculations
- *But:* to calculate $\mathrm{NLO_{MC}}$ needs deep understanding of MC.

MC with lots of NLO

Issues

- Complexity of adding new processes (determination of $\mathrm{NLO}_{\mathrm{MC}}$)
- Desire for 'overlapping' processes, *each* at NLO
  - $pp \to W$ — NLO $\to W+1$; parton shower $\to W+2, 3, \ldots$
  - $pp \to W + \mathrm{jet}$
  - $pp \to W + 2\mathrm{jets}$

Recent proposal

Nagy-Soper

- Classify event according to number of *narrow jets* (m)

  avoid overlap between 1-jet, 2-jet phase space, etc. (cf. CKKW)
- Generate $(m+1)^{\mathrm{th}}$ emission *outside MC framework*

  but with correct Sudakov's etc. $\equiv$ a well-controlled mini-MC
- *Correct* ***this*** *with NLO*
- Let usual MC deal with $(m+2)\ldots n$ parton showering *inside* narrow jets

Interesting ideas, but still to be tested in practice. . .

Fixed order	Tree level	NLO	NNLO
Hard properties of rare events	Many jets Low accuracy	A few jets Fair accuracy	Fewest jets Best accuracy

All orders	Monte Carlo	Analytical Rsmns	Saturation
Details of common events	Multi−purpose Good job + hadrons	Case−by−case High acc., partons	Murky Fun!

NNLL resummations

Best accuracies (NNLL) for most inclusive observables

Higgs transverse-mom. distribution.

First differential NNLL resummation

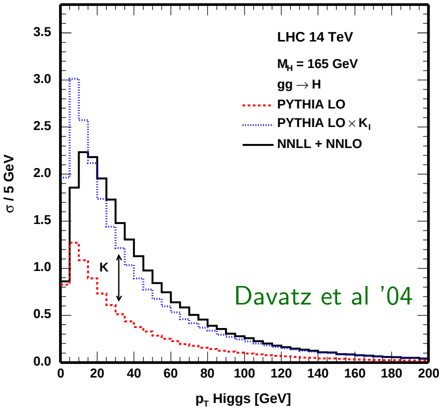
Resums $L \simeq \ln \frac{M_H}{Q_t}$ (for $gg \to H$)

$$\exp[\alpha_s^n L^{n+1} + \alpha_s^n L^n + \alpha_s^n L^{n-1}]$$

Bozzi et al '03

- NNLL uncertainty $\sim 7\%$ ($\sim$NLL/2)
- Shape quite different from plain parton showering (Pythia) — relevant for Higgs searches ($gg \to H \to WW \to \ell\nu\ell\nu$)?

Davatz et al '04



Automating NLL resummations

Final-state resummations (less inclusive, NLL) are *labour intensive* and *easily got wrong*.

In 15 years ∼ 20 event-shapes, jet rates!

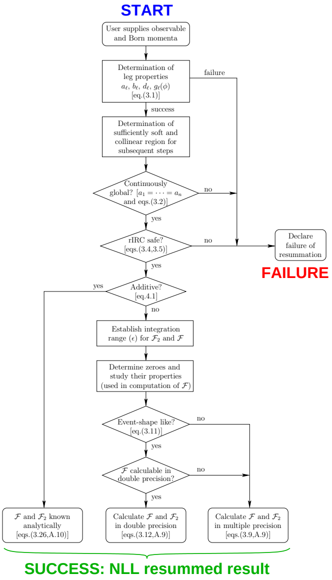
Automation Banfi, GPS & Zanderighi '03–04

- User provides subroutine for an observable
- Program
  - probes it with range of 'trial events'
  - deduces various *analytical characteristics*
  - establishes if it is within supported *class*
  - iff so, *calculates full NLL resummation*

Applications

- multi-jet event-shapes in e^+e^- and DIS
- event shapes for hadron-hadron colliders

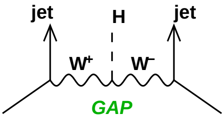
Banfi & co.'04; Berger '05

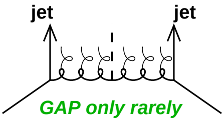


Gap resummations

How rare are gaps in $pp \to 2\,$jets with big ΔY?

Answer needs advanced tools



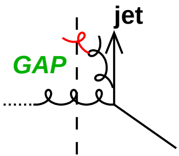


Non-global logarithms

- Appear for measurements of *part* of phase space

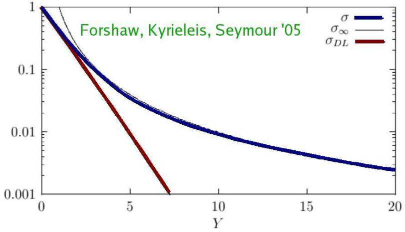
  Also e.g. dijet properties, Banfi & Dasgupta '03
- Only in large-N_c limit! Not automated!

  Connections to BFKL: Marchesini-Mueller '03; Weigert '03



Multi-jet structure

- Stony Brook soft-colour evolution
- Breakdown of 'probabilistic radiation'



Are Monte Carlos up to the job?
Unknown. . .

Fixed order Hard properties of rare events	**Tree level** Many jets Low accuracy	**NLO** A few jets Fair accuracy	**NNLO** Fewest jets Best accuracy
All orders Details of common events	**Monte Carlo** Multi−purpose Good job + hadrons	**Analytical Rsmns** Case−by−case High acc., partons	**Saturation** Murky Fun!

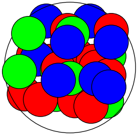
Minimum bias?

Mostly we've looked at *rare* occurrences.

Now turn to *typical* $p\bar{p}$ interactions:

- Exclusively in the realm of models? Pythia, Herwig minimum bias models?

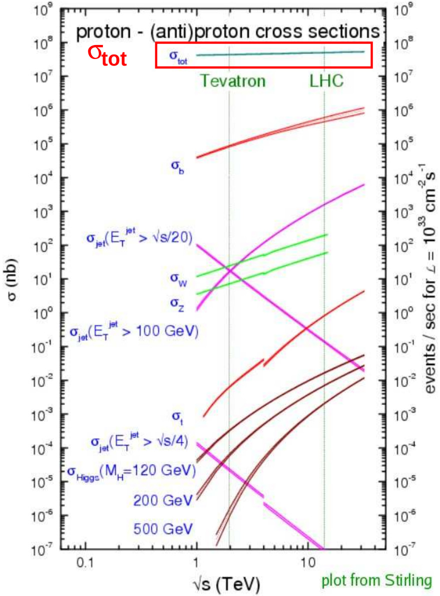
Gluon density may be so high that new perturbative scale arises: **saturation scale**, Q_s



$$Q_s^2 \sim \alpha_s(Q_s^2)\rho_g(Q_s^2)$$

Q_s increases with $\sqrt{s}$

If $Q_s(\sqrt{s}) \gtrsim 1$ GeV, it *sets scale of minijets in minimum-bias & underlying events*



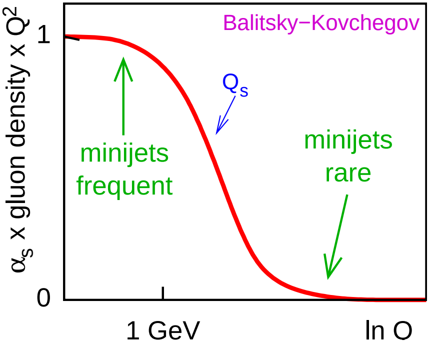
plot from Stirling

Saturation

Relevance to LHC is *speculative*

HERA: $Q_s \sim 1$ GeV?
LHC: ~ 2 GeV?

Gluon density $\leftrightarrow$ minijet rate?
Balitsky-Kovchegov (BK) eqn.



2 intriguing new results

1. BK equation is like widely studied (genetics, populations, stat. phys.)
   Fisher-Kolmogorov-Petrovsky-Piscounov equation
   Munier & Peschanski '03
2. BK is $\sim$ mean field. Real QCD *might* be more like
   stochastic FKPP: finite coupling $\leftrightarrow$ (finite population)$^{-1}$
   Iancu, Mueller & Munier '04

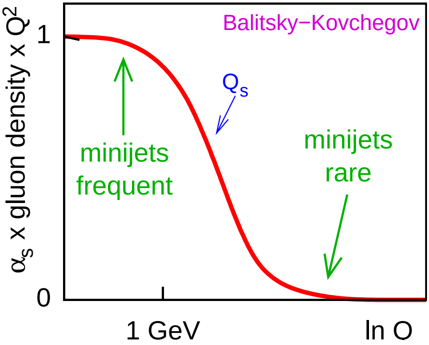
Keep an eye on this: $\sim$ 30 papers since Jan. 05

Saturation

Relevance to LHC is *speculative*

HERA: $Q_s \sim 1$ GeV?
LHC: ~ 2 GeV?

Gluon density $\leftrightarrow$ minijet rate?
Balitsky-Kovchegov (BK) eqn.



2 intriguing new results

1. BK equation is like widely studied (genetics, populations, stat. phys.) *Fisher-Kolmogorov-Petrovsky-Piscounov* equation
   Munier & Peschanski '03
2. BK is $\sim$ mean field. Real QCD *might* be more like *stochastic FKPP*: finite coupling $\leftrightarrow$ (finite population)$^{-1}$
   Iancu, Mueller & Munier '04

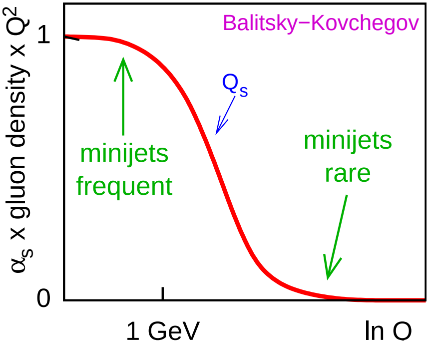
Keep an eye on this: $\sim$ 30 papers since Jan. 05

Saturation

Relevance to LHC is *speculative*

HERA: $Q_s \sim 1$ GeV?
LHC: ~ 2 GeV?

Gluon density $\leftrightarrow$ minijet rate?
Balitsky-Kovchegov (BK) eqn.



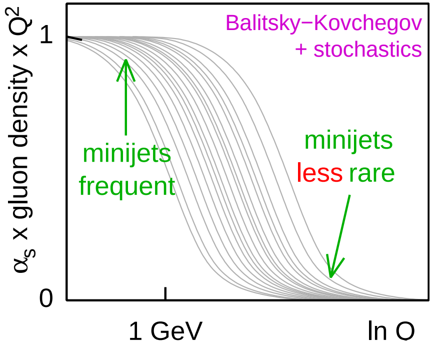
2 intriguing new results

1. BK equation is like widely studied (genetics, populations, stat. phys.) *Fisher-Kolmogorov-Petrovsky-Piscounov* equation
Munier & Peschanski '03

2. BK is $\sim$ mean field. Real QCD *might* be more like *stochastic FKPP*: finite coupling $\leftrightarrow$ (finite population)$^{-1}$
Iancu, Mueller & Munier '04

Keep an eye on this: $\sim$ 30 papers since Jan. 05

Saturation



Relevance to LHC is *speculative*

HERA: $Q_s \sim 1$ GeV?
LHC: ~ 2 GeV?

Gluon density $\leftrightarrow$ minijet rate?
Balitsky-Kovchegov (BK) eqn.

2 intriguing new results

1. BK equation is like widely studied (genetics, populations, stat. phys.)
   Fisher-Kolmogorov-Petrovsky-Piscounov equation
   Munier & Peschanski '03
2. BK is $\sim$ mean field. Real QCD *might* be more like
   stochastic FKPP: finite coupling $\leftrightarrow$ (finite population)$^{-1}$
   Iancu, Mueller & Munier '04

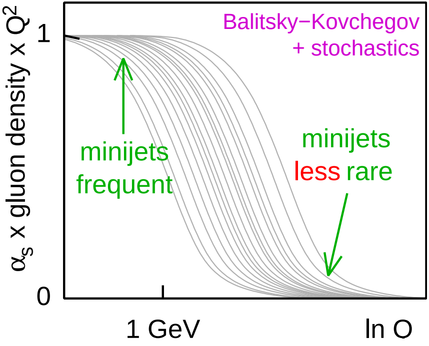
Keep an eye on this: $\sim$ 30 papers since Jan. 05

Saturation

Relevance to LHC is *speculative*

HERA: $Q_s \sim 1$ GeV?
LHC: ~ 2 GeV?

Gluon density $\leftrightarrow$ minijet rate?
Balitsky-Kovchegov (BK) eqn.



2 intriguing new results

1. BK equation is like widely studied (genetics, populations, stat. phys.)
Fisher-Kolmogorov-Petrovsky-Piscounov equation
Munier & Peschanski '03

2. BK is $\sim$ mean field. Real QCD *might* be more like
stochastic FKPP: finite coupling $\leftrightarrow$ (finite population)$^{-1}$
Iancu, Mueller & Munier '04

Keep an eye on this: $\sim$ 30 papers since Jan. 05

Conclusions

Is QCD on track for LHC?

- Mostly yes
- Twistors: major theory advance — full impact cannot yet be gauged
  - ✓ Many string theorists are now thinking about QCD-related problems
  - Some phenomenologists are now thinking about string theory!
- Other aspects ($N^{(2)}LO$, MC, resummations,...) progressing 'steadily'
  - new ideas around. E.g. NNLO: sector decomposition, antenna subtraction
  - systematic automation is high on priority lists
  - phenomenology: PDF fitting, targeted resummations
- Perturbative QCD is not *just* 'engineering'!
  - Saturation — unexpected links to statistical physics
  - Hadronisation
  - Exclusive QCD
  - ...

Thanks to: Butterworth, P. Ciafaloni, Comelli, Dokshitzer, R.K. Ellis, Kosower, Lonnblad, Marchesini, Moretti, Seymour, Webber

Conclusions

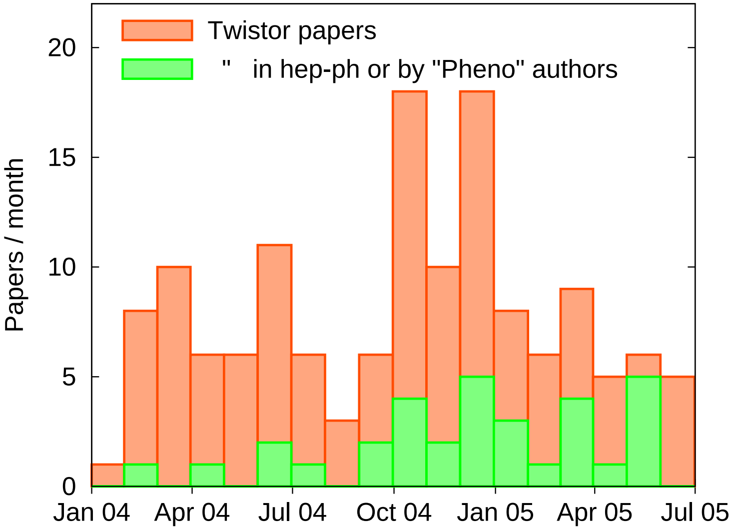
Is QCD on track for LHC?

- Mostly yes
- Twistors: major theory advance — full impact cannot yet be gauged
  - ✓ Many string theorists are now thinking about QCD-related problems
  - Some phenomenologists are now thinking about string theory!
- Other aspects ($N^{(2)}LO$, MC, resummations,...) progressing 'steadily'
  - new ideas around. E.g. NNLO: sector decomposition, antenna subtraction
  - systematic automation is high on priority lists
  - phenomenology: PDF fitting, targeted resummations
- Perturbative QCD is not *just* 'engineering'!
  - Saturation — unexpected links to statistical physics
  - Hadronisation
  - Exclusive QCD
  - ...

Thanks to: Butterworth, P. Ciafaloni, Comelli, Dokshitzer, R.K. Ellis, Kosower, Lonnblad, Marchesini, Moretti, Seymour, Webber

EXTRA SLIDES

Twistor Papers



1-loop example

Example of Giele-Glover method

$$\int \frac{d^D\ell \; \ell^{\mu_1}\ell^{\mu_2}}{(\ell+q_1)^2(\ell+q_2)^2(\ell+q_3)^2(\ell+q_4)^2}$$
$$= \tfrac{1}{2} g^{\mu_1\mu_2} I(D+2;1,1,1,1) \; + 2q_1^{\mu_1} 2q_1^{\mu_2} I_4(D+4;3,1,1,1) + \dots$$

Then

$$2I_4(8;3,1,1,1) = -2\left(\textstyle\sum_i S_{1i}^{-1}\right) I_4(8;2,1,1,1)$$
$$- S_{11}^{-1} I_4(6;1,1,1,1) - S_{12}^{-1} I_3(6;1,0,1,1)$$
$$- S_{13}^{-1} I_3(6;1,1,0,1) - S_{14}^{-1} I_3(6;1,1,1,0)$$

The $I_n(D;1,1,1,1)$ etc. are the basis integrals. S_{ij} is kinematical matrix, $S_{ij} = (q_i - q_j)^2$.

Reduction procedure done numerically for each kinematic configuration.

Loops: $\mathcal{A}^{QCD} = \mathcal{A}^{\mathcal{N}=4} - 4\mathcal{A}^{\mathcal{N}=1} + \mathcal{A}^{\text{scalar}}$

$\mathcal{N} = 4$ SUSY QCD: Because(!) of extra particles, simpler than plain QCD.

- 1-loop: methods for *n*-gluon NMHV, all explicit results for 7-gluon
  Britto, Cachazo & Feng; Bern et al
- 2-loop: non-MHV amplitudes for 5 & 6 gluons. Buchbinder & Cachazo

$\mathcal{N} = 1$ SUSY QCD (1-loop)

- Reproduce all MHV results of Bern, Dixon, Dunbar & Kosower
  Quigley & Rozali; Bedford et al
- Results for some all-*n* NMHV; full results for 6-gluons
  Bidder et al; Britto et al

Real QCD ('traditional' limit: 5 legs)

- All finite 1-loop contributions ($+ + + \cdots$, $- + + + \cdots$, quarks)
  Bern, Dixon & Kosower
- Reproduce some divergent 1-loop diagrams (4 gluons). Brandhuber et al

Likely that much progress still to come!

summary of NNLO calculations (~1990 →)

ep

- DIS pol. and unpol. structure function coefficient functions
- Sum Rules (GLS, Bj, …)
- DGLAP splitting functions Moch Vermaseren Vogt (2004)

e^+e^-

- total hadronic cross section, and $Z \to$ hadrons, $\tau \to \nu$ + hadrons
- heavy quark pair production near threshold
- C_F^3 part of σ(3 jet) Gehrmann-De Ridder, Gehrmann, Glover(2004)

pp

- inclusive W,Z,γ^* van Neerven et al, Harlander and Kilgore corrected (2002)
- inclusive γ^* polarised Ravindran, Smith, Van Neerven (2003)
- W,Z,γ^* differential rapidity dis^n Anastasiou, Dixon, Melnikov, Petriello (2003)
- H^0, A^0 Harlander and Kilgore; Anastasiou and Melnikov; Ravindran, Smith, Van Neerven (2002-3)
- WH, ZH Brein, Djouadi, Harlander (2003)

HQ

- QQ onium and Qq meson decay rates

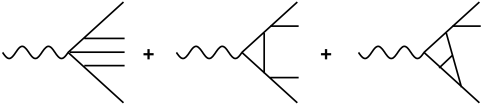
+ other partial/approximate results (e.g. soft, collinear) and NNLL improvements

Cancelling NNLO divergences

$4+2\varepsilon$ dim:
J is observable

$\int d\Phi_5 \; J(p_{1..5})$ $\quad \int d\Phi_4 \; \varepsilon^{-2} J(p_{1..4})$ $\quad \int d\Phi_3 \; \varepsilon^{-4} J(p_{1..3})$

1 -> 3 @ NNLO ~



\+ **Tricks to cancel divergences**

1 -> 5 @ Tree | **1 -> 4 @ 1−loop** | **1 -> 3 @ 2−loop** | **Bottleneck**

How to get cancellations?

1. Subtraction method:

$$\int d^D\Phi_5 M_5 J(p_{1..5}) + \int d^D\Phi_4 M_4 J(p_{1..4}) + \dots$$

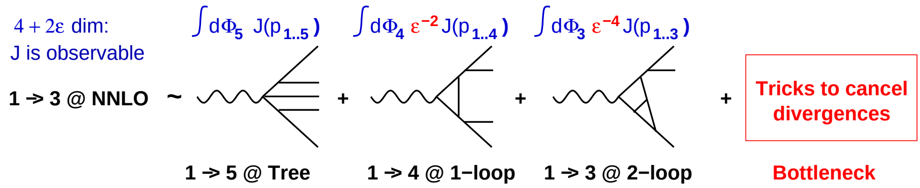
Applied to $e^+e^- \to 2$ jets and C_F^3 colour part of $e^+e^- \to 3$ jets:

$$(\alpha_s C_F/2\pi)^3 \text{ piece of } \langle 1-T \rangle = -20.4 \pm 4$$

Gehrmann-De Ridder, Gehrmann & Glover

In principle all $e^+e^- \to 3$ jet ‘antenna’ subtraction pieces are ready — ‘just’ need to be coded!

Cancelling NNLO divergences



How to get cancellations?

1. Subtraction method:

$$\int d^4\Phi_5[M_5 J(p_{1..5}) - \mathcal{S}_5 J(\tilde{p}_{1..3})] + \int d^4\Phi_4[M_4 J(p_{1..4}) + \mathcal{S}_4 J(\tilde{p}_{1..3})] + \dots$$

Applied to $e^+e^- \to 2$ jets and C_F^3 colour part of $e^+e^- \to 3$ jets:

$(\alpha_s C_F/2\pi)^3$ piece of $\langle 1 - T\rangle = -20.4 \pm 4$

Gehrmann-De Ridder, Gehrmann & Glover

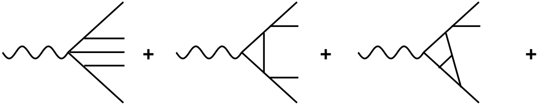
In principle all $e^+e^- \to 3$ jet 'antenna' subtraction pieces are ready — 'just' need to be coded!

Cancelling NNLO divergences

$4+2\varepsilon$ dim:
J is observable

$\int d\Phi_5 \; J(p_{1..5})$ $\int d\Phi_4 \, \varepsilon^{-2} J(p_{1..4})$ $\int d\Phi_3 \, \varepsilon^{-4} J(p_{1..3})$

1 -> 3 @ NNLO ~



Tricks to cancel divergences

1 -> 5 @ Tree | **1 -> 4 @ 1−loop** | **1 -> 3 @ 2−loop** | **Bottleneck**

How to get cancellations?

1. Subtraction method:

$$\int d^4\Phi_5 [M_5 J(p_{1..5}) - \mathcal{S}_5 J(\tilde{p}_{1..3})] + \int d^4\Phi_4 [M_4 J(p_{1..4}) + \mathcal{S}_4 J(\tilde{p}_{1..3})] + \dots$$

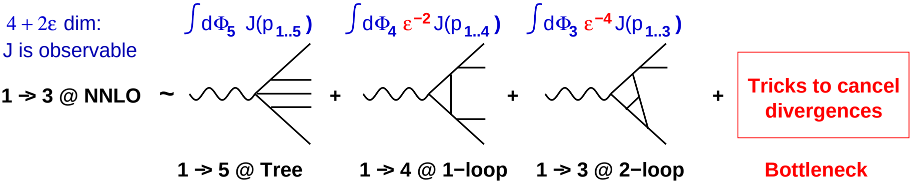
Applied to $e^+e^- \to 2$ jets and C_F^3 colour part of $e^+e^- \to 3$ jets:

$$(\alpha_s C_F/2\pi)^3 \text{ piece of } \langle 1 - T \rangle = -20.4 \pm 4$$

Gehrmann-De Ridder, Gehrmann & Glover

In principle all $e^+e^- \to 3$ jet ‘antenna’ subtraction pieces are ready — ‘just’ need to be coded!

Cancelling NNLO divergences



How to get cancellations?

2. Sector decomposition for isolating divergences — Binoth & Heinrich

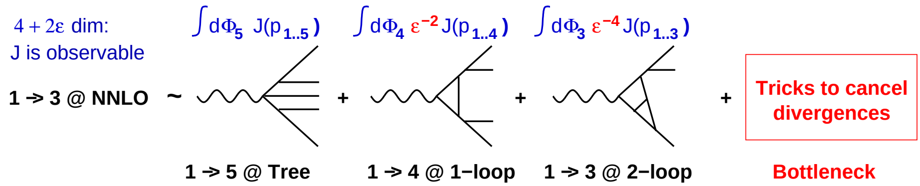
$$\int d^D\Phi_5 M_5 J(p_{1..5}) = \epsilon^{-4}\int d^4\Phi_5 f_{-4} M_5 J(p_{1..5}) + \cdots + \int d^4\Phi_5 f_0 M_5 J(p_{1..5})$$

The f_{-i} involve plus-distributions of kinematic invariants. Each integral finite.

Applied to

- $e^+e^- \to 2\,\text{jets}$ — Binoth & Heinrich; Anastasiou, Melnikov & Petriello
- $pp \to W, Z, H$ (fully differential) — Anastasiou, Dixon, Melnikov & Petriello

Cancelling NNLO divergences



How to get cancellations?

2. Sector decomposition for isolating divergences — Binoth & Heinrich

$$\int d^D\Phi_5 M_5 J(p_{1..5}) = \epsilon^{-4} \int d^4\Phi_5 f_{-4} M_5 J(p_{1..5}) + \cdots + \int d^4\Phi_5 f_0 M_5 J(p_{1..5})$$

The f_{-i} involve plus-distributions of kinematic invariants. Each integral finite.

Applied to

- $e^+e^- \to 2\,\text{jets}$ — Binoth & Heinrich; Anastasiou, Melnikov & Petriello
- $pp \to W, Z, H$ (fully differential) — Anastasiou, Dixon, Melnikov & Petriello