Impact of higher orders in the high-energy limit of QCD

[OR: Is BFKL predictive?]

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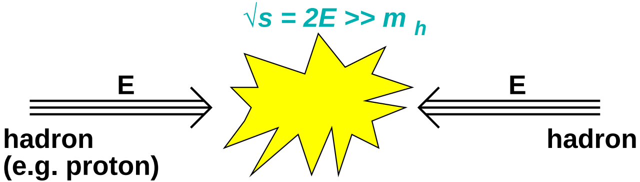
LPTHE, Universities of Paris VI and VII and CNRS

BNL Riken lunch seminar
Upton, NY, May 2005

High-energy limit

One of the major unsolved problems of QCD (and Yang-Mills theory in general) is the understanding of its *high-energy limit.*

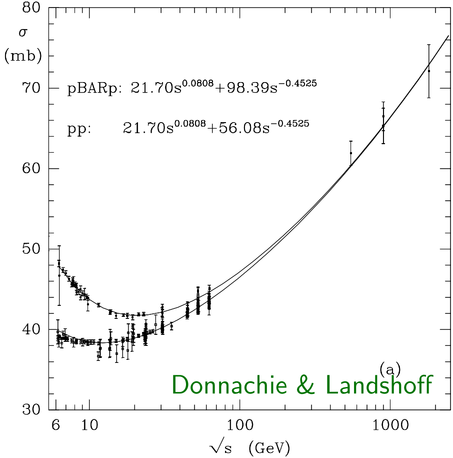
I.e. the limit in which C.O.M. energy ($\sqrt{s}$) is much larger than *all other scales* in the problem.



Want to understand:

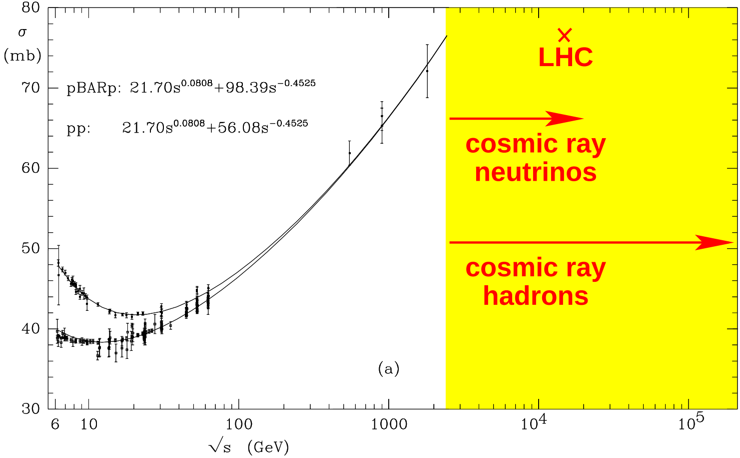
- asymptotic behaviour of cross section, $\sigma_{hh}(s) \sim ??$
- properties of final states for large s.

Experimental knowledge



- Some knowledge exists about behaviour of cross section experimentally
- Slow rise as energy increases
- Data insufficient to make reliable statements about functional form
  - $\sigma \sim s^{0.08}$?
  - $\sigma \sim \ln^2 s$?
- Understanding of final-states is $\sim$ inexistent
- Would like theoretical predictions. . .

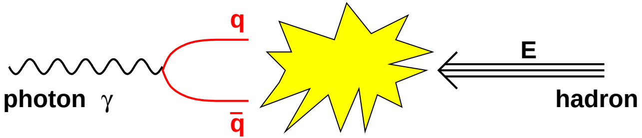
Experimental knowledge



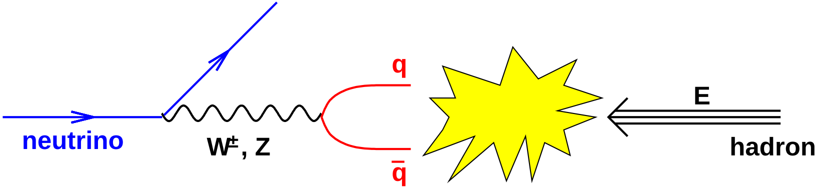
Future experiments go to much higher energies.

Not just for hadrons

Problem is must more general than just for hadrons. E.g. photon can *fluctuate* into a quark-antiquark (hadronic!) state:



Even a neutrino can behave like a hadron



Hadronic component dominates high-energy cross section

Outline

- Perturbative, leading-logarithmic (LL), calculation of cross-section growth — Using just classical field theory
- Failure of comparison to data
- Higher-order corrections
  - NLL corrections
  - Problems & solutions
- Splitting functions

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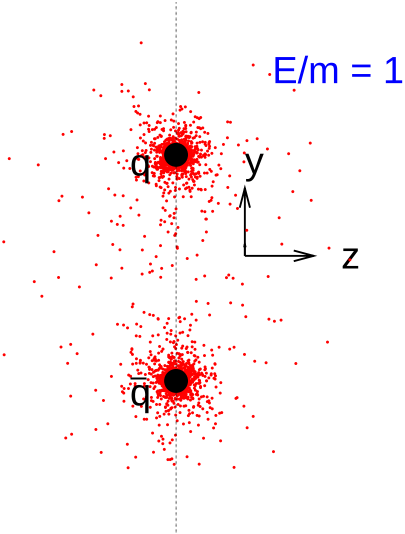

Look at density of *gluons* from dipole field ($\sim$ energy density).

QCD $\simeq$ QED

- *Large energy* $\equiv$ *large boost* (along z axis), by factor
- Fields flatten into *pancake*.
  - simple longitudinal structure

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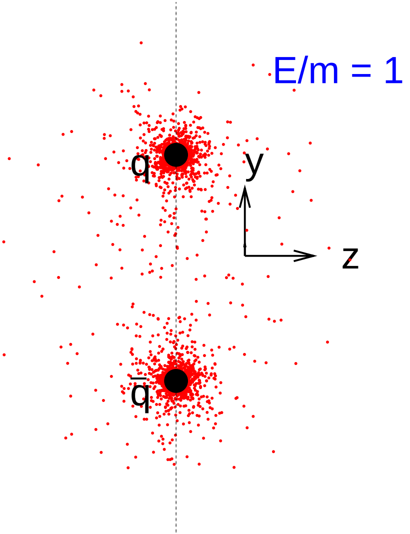

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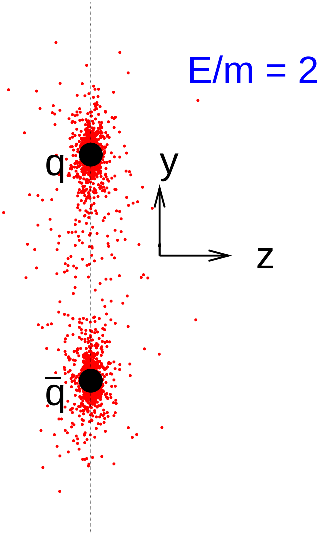

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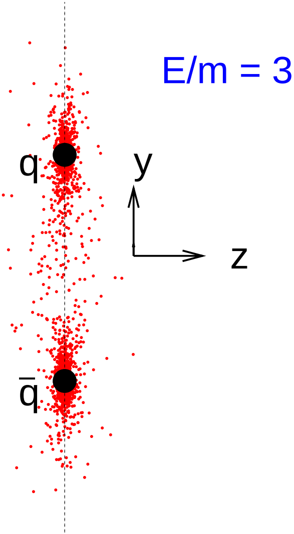

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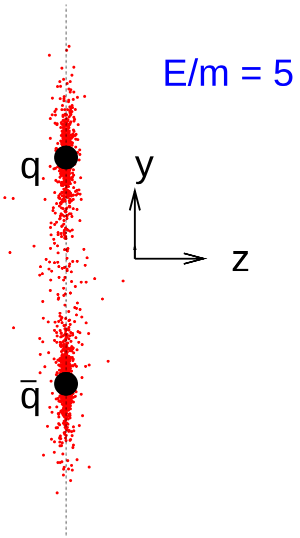

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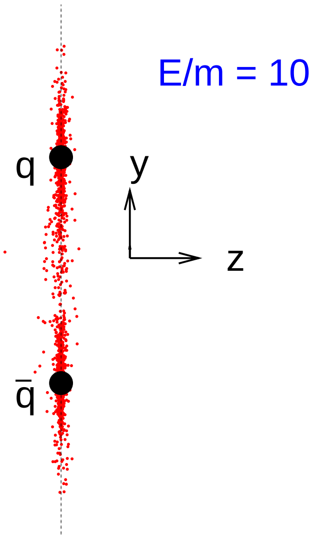

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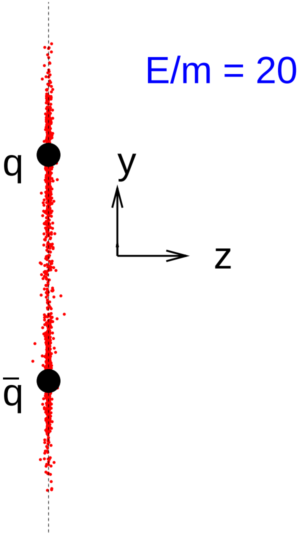

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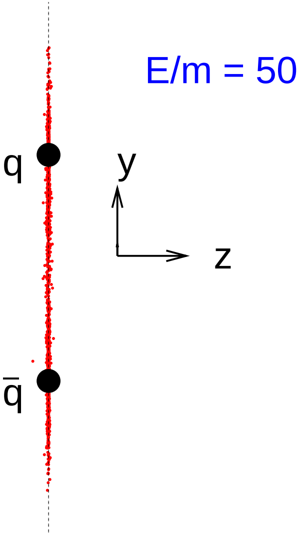

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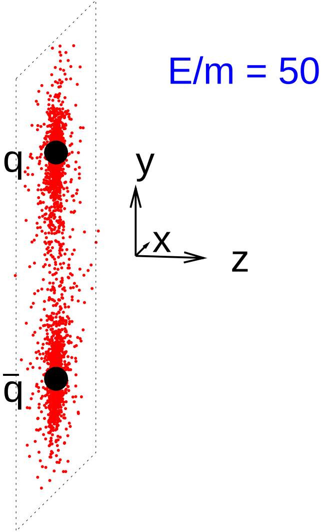

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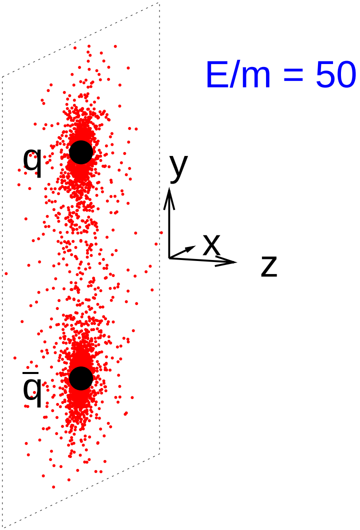

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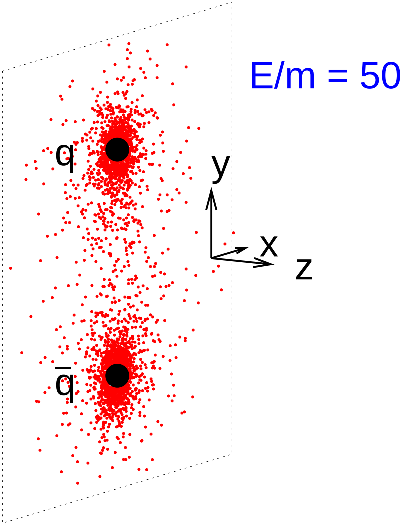

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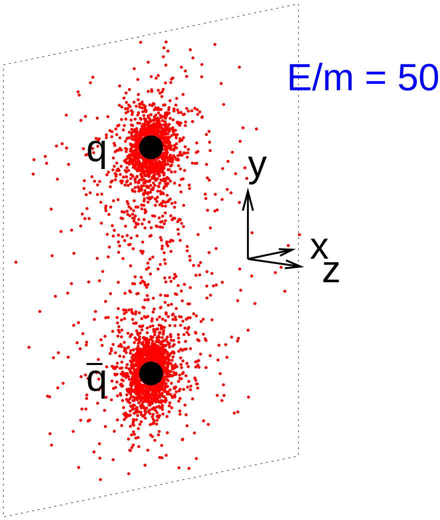

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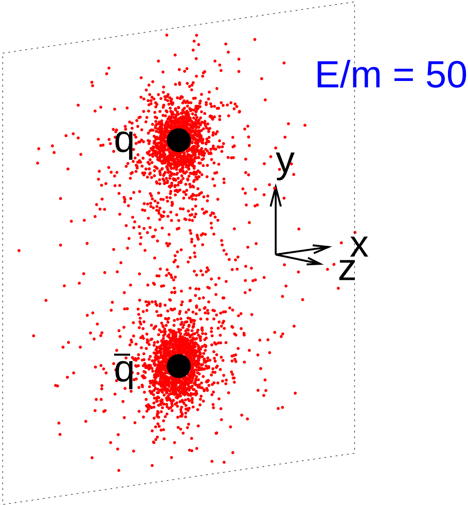

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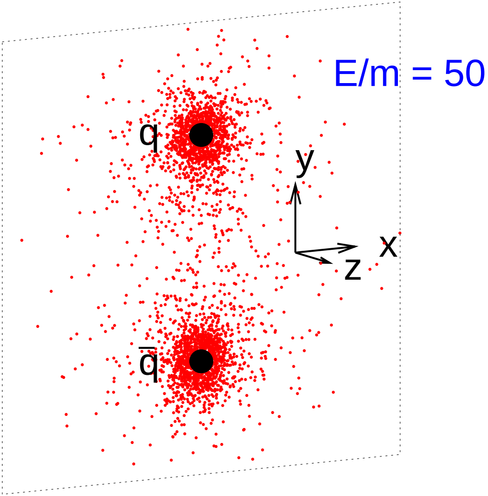

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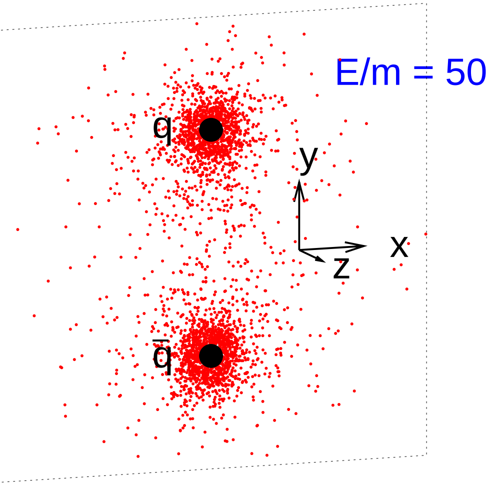

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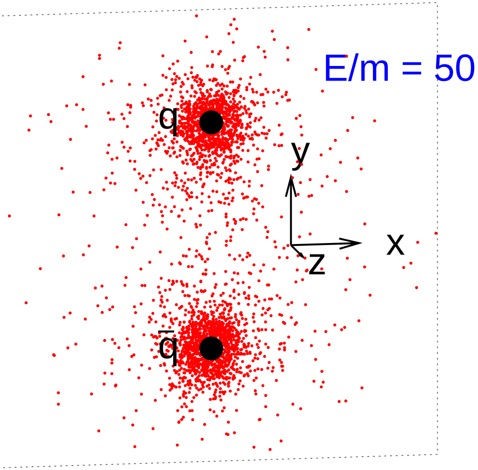

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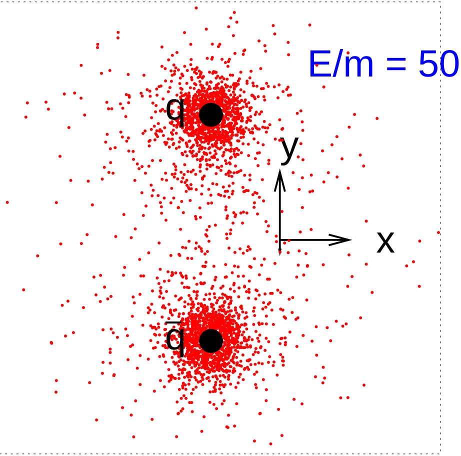

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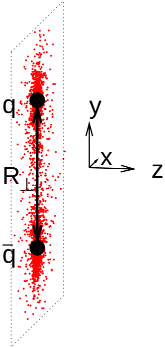
Total number of gluons

Longitudinal structure of energy density (N_c = # of colours):

$$\frac{d\epsilon}{dz} \sim \frac{\alpha_s N_c}{\pi} \times E\delta(z) \times \text{transverse}$$



Fourier transform → energy density in field per unit of long. momentum (p_z)

$$\frac{d\epsilon}{dp_z} \sim \frac{\alpha_s N_c}{\pi} \times \text{transverse}\,, \qquad m \ll p_z \ll E$$

→ number (n) of gluons (each gluon has energy p_z):

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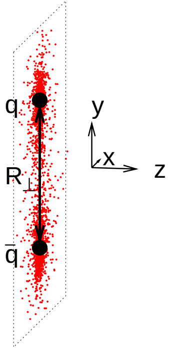

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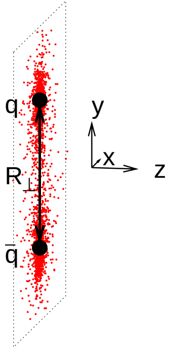

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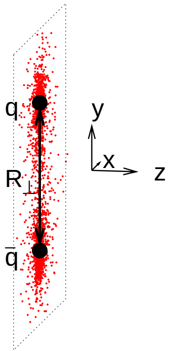

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High-energy limit $\sqrt{s}$, $E \to \infty$

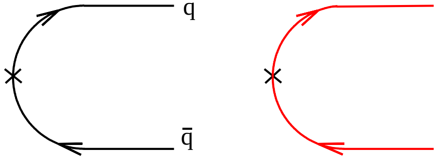
- Calculation so far is first-order perturbation theory.
- Fixed order perturbation theory is reliable if series converges quickly.
- At high energies, $n \sim \alpha_s \ln E \sim 1$.
- What happens with higher orders?

$$(\alpha_s \ln E)^n?$$

Leading Logarithms (LL). Any fixed order potentially non-convergent...

Multiple gluon emission

Start with bare $q\bar{q}$ dipole:



Emission of 1 gluon is like QED case — modulo additional colour factor (number of different ways to repaint quark):

$$\alpha \to \alpha_s N_c/2$$ (approx)

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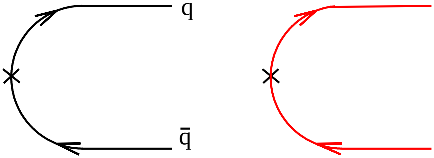
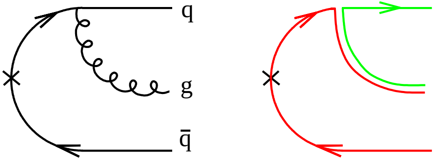


Emit a gluon:



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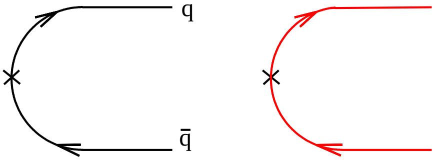
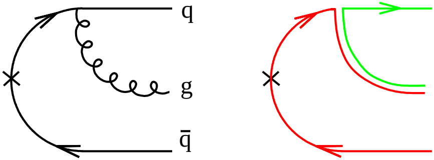


Emit a gluon:



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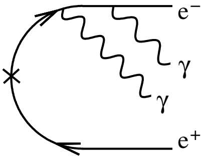

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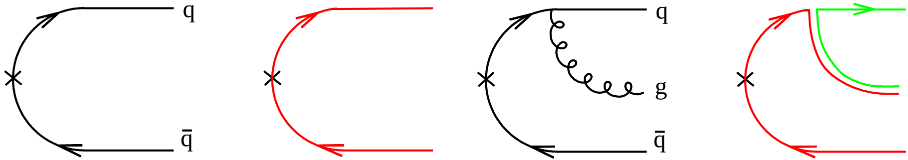

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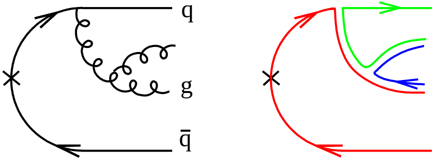

Iterating gluon emission

Problem is self-similar: dipole $\to$ 2 dipoles $\to$ 4 dipoles $\to \ldots$

Number of dipoles (or gluons) grows *exponentially:*

$$n \sim \exp\left[\frac{\alpha_s N_c}{\pi} \ln E \times \mathrm{transverse}\right] \sim E^{\frac{\alpha_s N_c}{\pi} \times \mathrm{transverse}}$$

Tranverse part $\to$ many complications/interest

- transverse part is *conformally invariant* ➥Extensive mathematical studies
- In high-energy limit it reduces to a pure number: $4 \ln 2$

$$n \sim E^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} \sim E^{0.5}$$

Balitsky Fadin Kuraev Lipatov Pomeron (1976)

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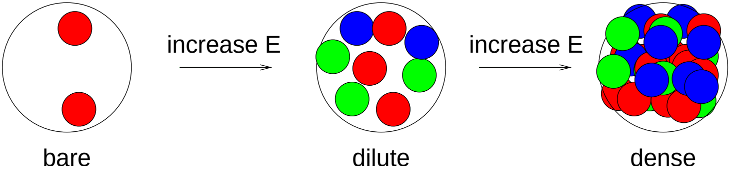
BFKL: rising cross sections

$$n_{\text{dipoles}}(E) \sim E^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} \quad \Leftrightarrow \quad \sigma_{hh} \sim s^{\frac{\alpha_s N_c}{\pi} 4 \ln 2}, \qquad \frac{\alpha_s N_c}{\pi} 4 \ln 2 \simeq 0.5$$

- Completely incompatible with rise of $p\bar{p}$ cross section ($\sim s^{0.08}$)
  - $p\bar{p}$ is simply beyond perturbation theory
- experimentally spectactular — if observable in some process. . .
- Raises many theoretical issues — high gluon densities should lead to non-linear effects: *high fields, but still perturbative*

Colour Glass Condensate



BFKL: rising cross sections

$$n_{\text{dipoles}}(E) \sim E^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} \quad \Leftrightarrow \quad \sigma_{hh} \sim s^{\frac{\alpha_s N_c}{\pi} 4 \ln 2}, \qquad \frac{\alpha_s N_c}{\pi} 4 \ln 2 \simeq 0.5$$

- Completely incompatible with rise of $p\bar{p}$ cross section ($\sim s^{0.08}$)
  - $p\bar{p}$ is simply beyond perturbation theory
- experimentally spectactular — if observable in some process. . .
- Raises many theoretical issues — high gluon densities should lead to non-linear effects: *high fields, but still perturbative*

Colour Glass Condensate

How can we search for BFKL experimentally?

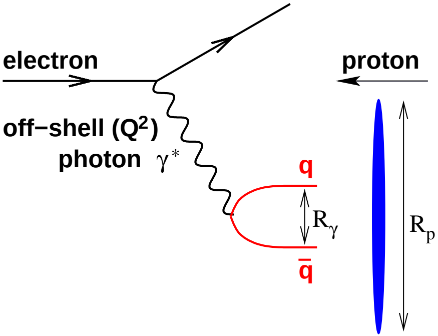
- Need to ensure we are in regime where perturbation theory can be applied
  - Choose appropriate hadronic scales (small R)

Deep Inelastic Scattering (DIS)

Getting small transverse sizes (needed for $\alpha_s \ll 1$) *and* asymptotically large collision energies is experimentally difficult.

In general collide two hadronic probes — try a compromise: *make one of them small*

$$R_\gamma \sim \frac{1}{Q} \ll R_p \sim \frac{1}{m_p}$$



- $q\bar{q}$ probe measures (roughly) number of gluons in proton up to scale Q
- NB: DIS more usually viewed as photon hitting quarks in proton

- Some of physics perturbative ($Q \gtrsim p_t \gg m_p$)
- But if $\ln Q^2 \gtrsim \ln s$ we have *competition* between

$$(\alpha_s \ln s)^n \quad \text{v.} \quad (\alpha_s \ln s \ln Q^2)^n$$

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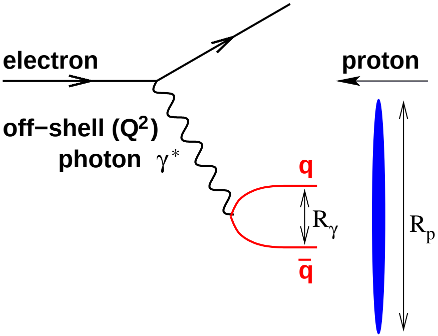

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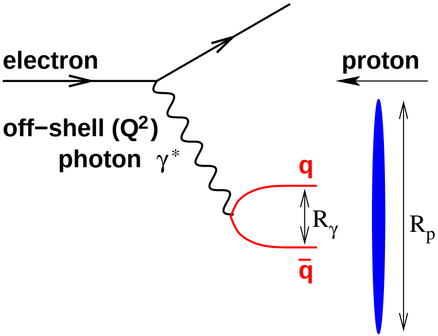

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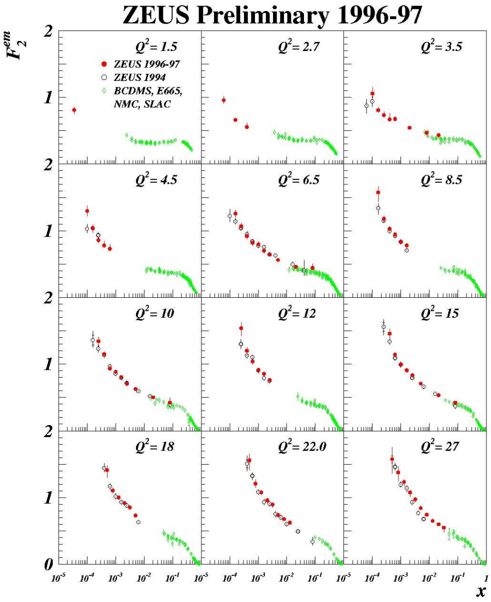

- F_2 is rescaled cross section
- $x = \frac{p_z}{p_{z,\text{proton}}} \sim \frac{1}{s}$
- Clear rise of cross section at high energies (low x).

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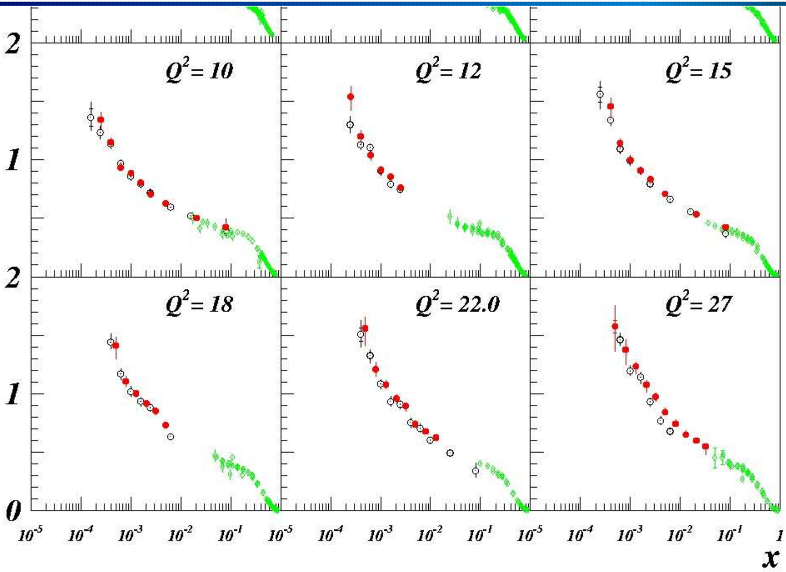

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- Cle
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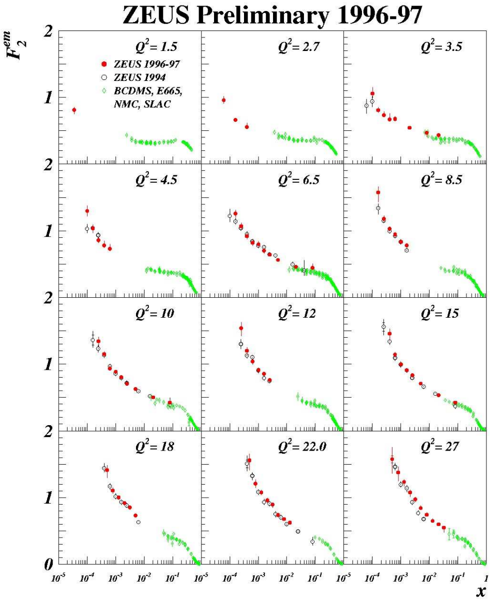

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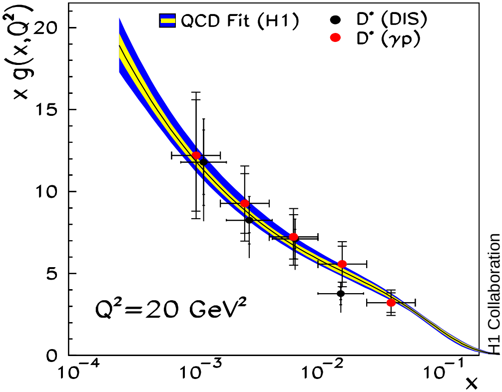
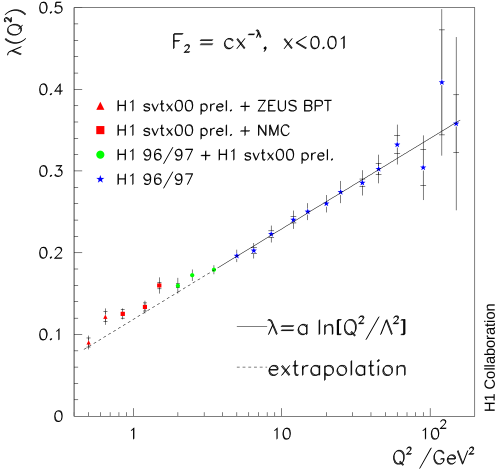
Large number of gluons



- Convert cross sections into estimate of number of gluons
- Various independent extractions
- *Up to 20 gluons per unit* $\ln x$ *(or unit* $\ln p_z$*)!*

Is it BFKL?



- Check if BFKL by looking at power (λ) of x
- For BFKL, expect $\lambda \simeq 0.5$
- *Definitely not LL BFKL!*

There is some growth — where does it come from?

It is due to combination of $x \ll 1$ and $Q^2 \gg m_p^2$ — resummation of terms $(\alpha_s \ln \frac{1}{x} \ln Q^2)^n$:

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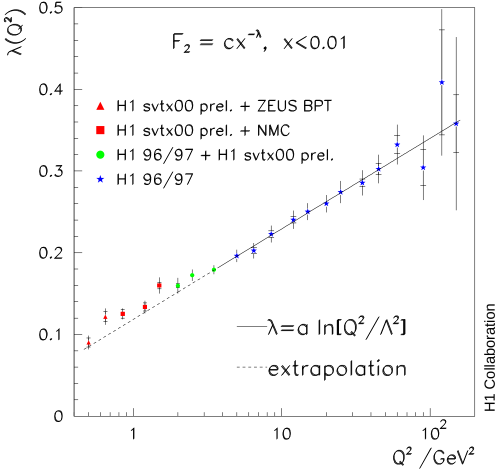

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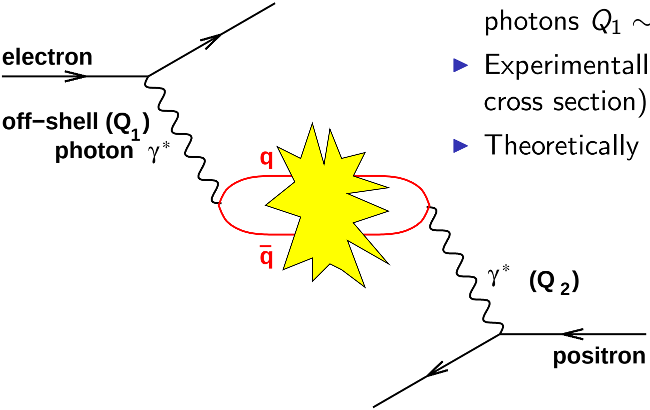
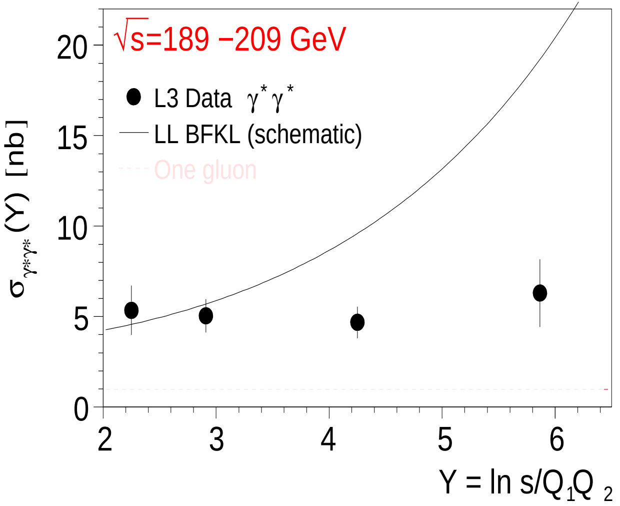
$\gamma^* \gamma^*$ collisions



- Eliminate ratios of transverse scales by colliding two virtual photons $Q_1 \sim Q_2$
- Experimentally difficult (small cross section)
- Theoretically clean

Results from LEP



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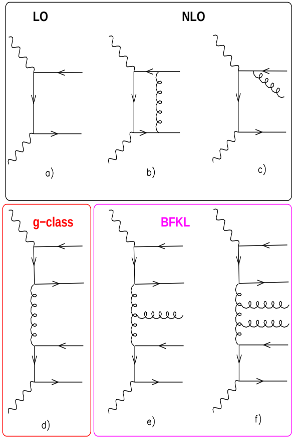

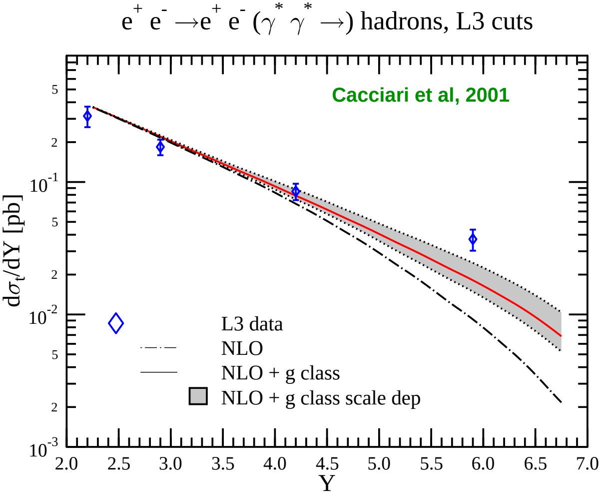

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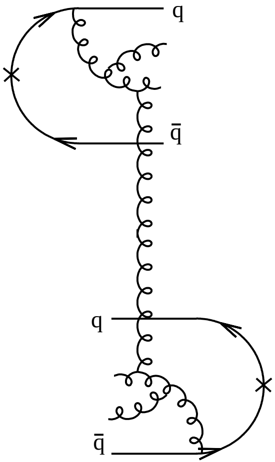
Wavefunction v. ladder graphs



evolution in wavefunctions

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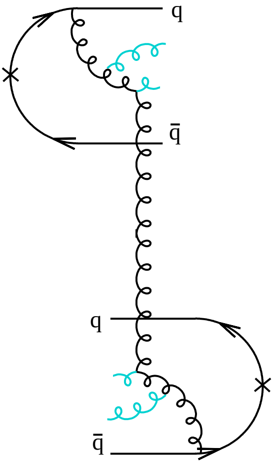

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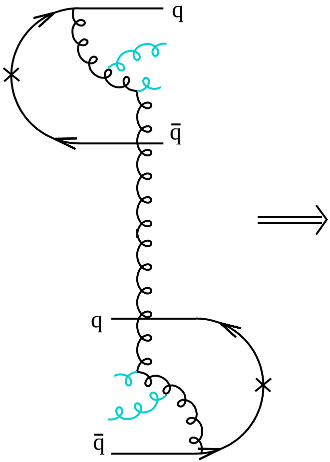

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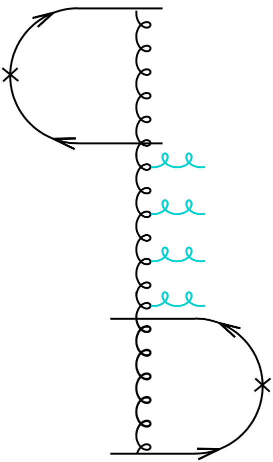
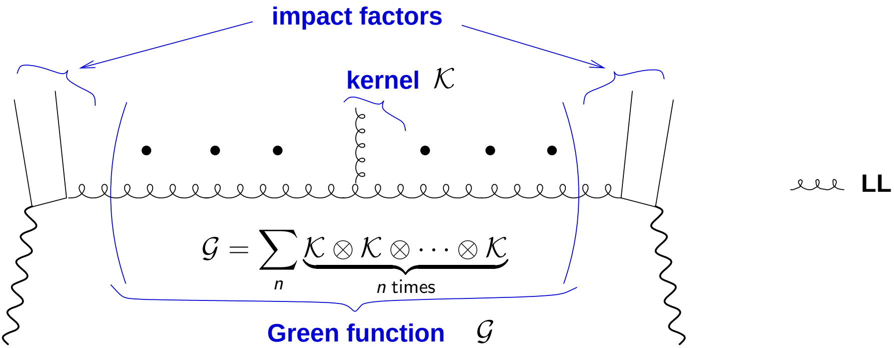


evolution as ladder diagram

NLL ingredients

Label various parts of cross-section calculation

NLL: include relative $\mathcal{O}(\alpha_s)$ corrections to each



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Associated with power growth

<u>Impact factors (proc.-dependent):</u>

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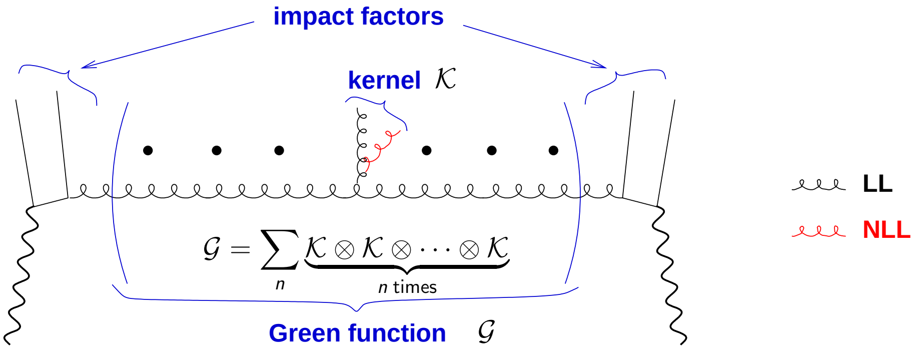

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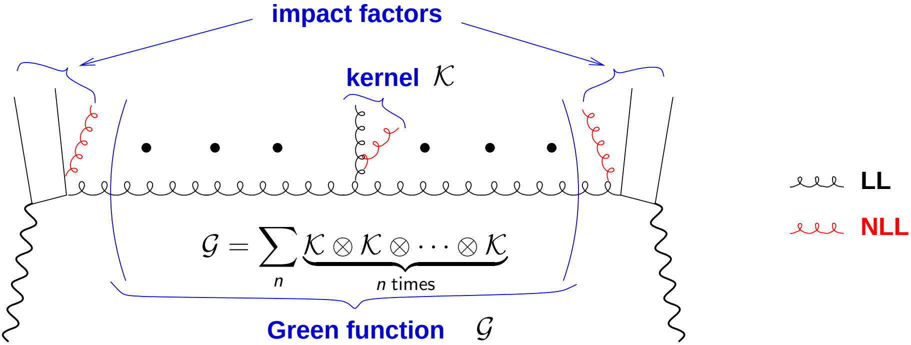

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NLL power

Cast NLL corrections to kernel as modification of power:

$$\sigma \sim G(Y,k,k) \sim \exp\left[\,4\ln 2\bar{\alpha}_s(1-6.5\bar{\alpha}_s)\;Y\,\right]$$

NB: k = transv. mom. scale

- Very *poorly convergent* ($\bar{\alpha}_s = \alpha_s N_c/\pi \simeq 0.15\cdots 0.2$)
- Unstable perturbative hierarchy: *expansion of power has limited sense*

➥ Instead, try solving BFKL equation with full NLL kernel (including running coupling)

$$G(Y,k,k_0) = \frac{\delta(k-k_0)}{2\pi k_0} + \int_0^Y dy \int d k'^2\, \mathcal{K}(k,k')\, G(Y-y,k,k')$$

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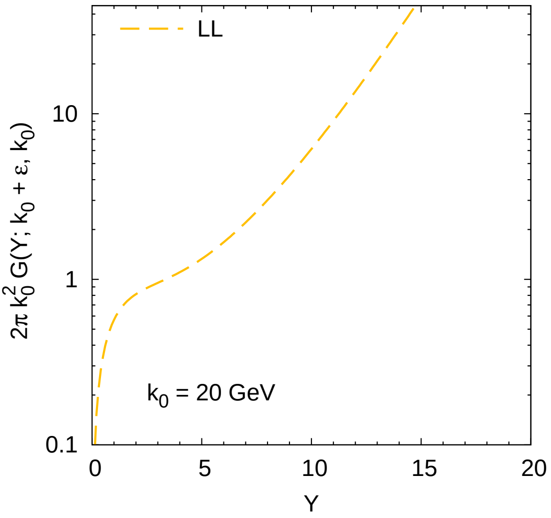
NLL Green function solution



Various convention choices affect higher orders (NNLLx):

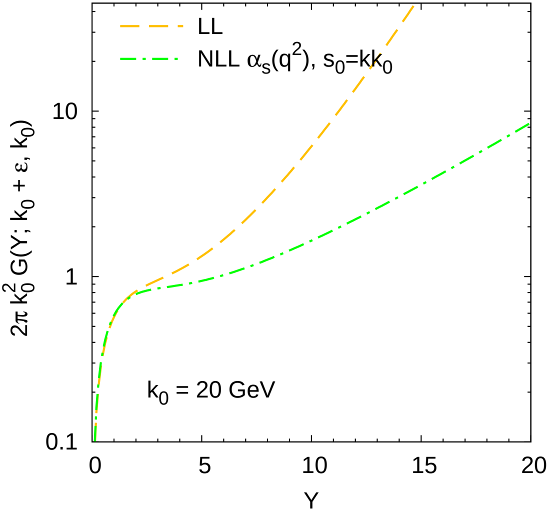
- scale of α_s
- 'energy-scale' s_0 ($Y = \ln s/s_0$).

Extreme sensitivity to choice of convention $\Leftrightarrow$ poor perturbative convergence.

NB: Andersen & Sabio Vera solutions $\sim$ green curve

Need to understand origin of instability

NLL Green function solution



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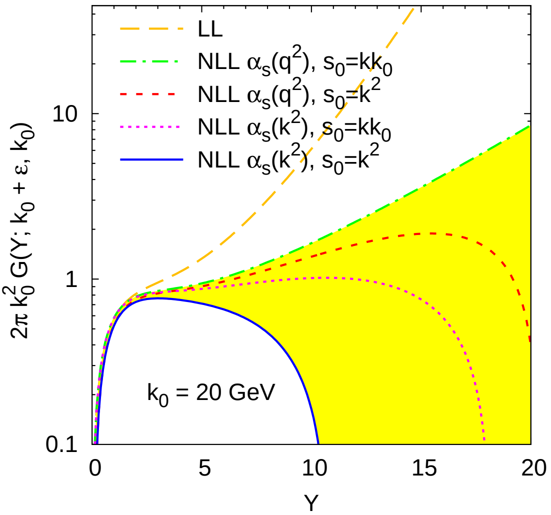

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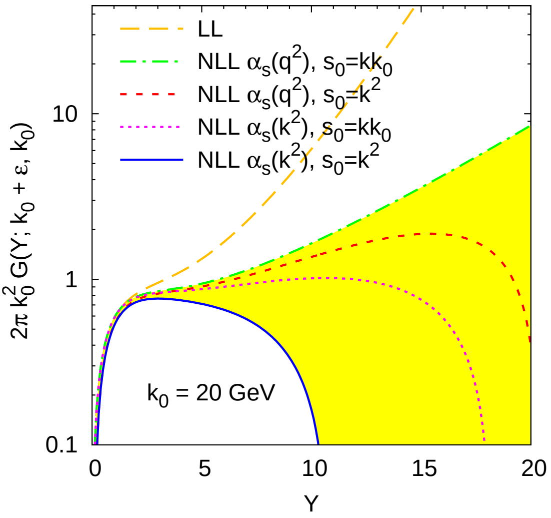

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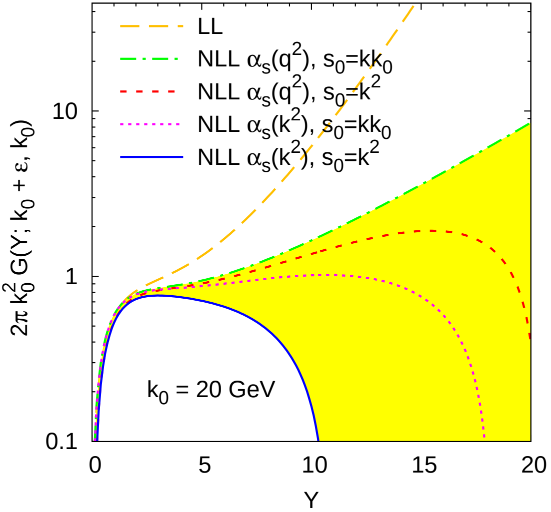

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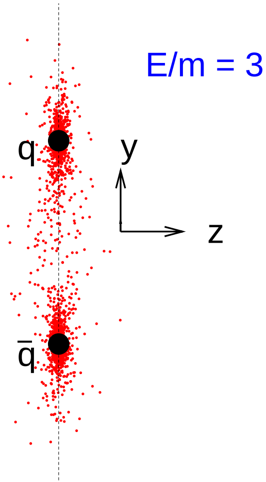
NLL: why so bad?



- First branching occurs for $Y \sim \frac{c}{\alpha_s}$
- In practice c is small: $e^Y \sim 2-5$
- Energy-distribution $\neq$ perfect $\delta(z)$
- 'degree of imperfection' depends on transverse position

  Ciafaloni '88

  Andersson et al; Kwiecinski et al '96
- Dominant part $\equiv$ double & single $\perp$ logs
  - Responsible for $\sim 90\%$ of NLL corrections
  - Can be used to supplement NLL at all orders

  GPS; Ciafaloni & Colferai, '98–99

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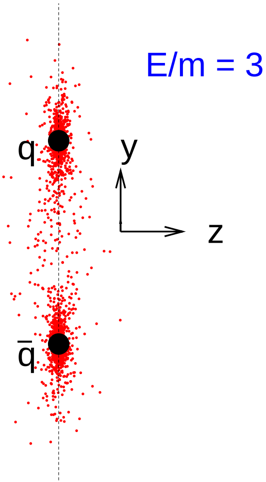

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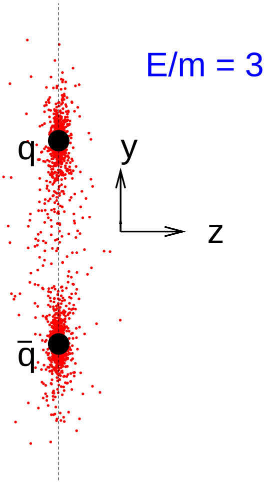

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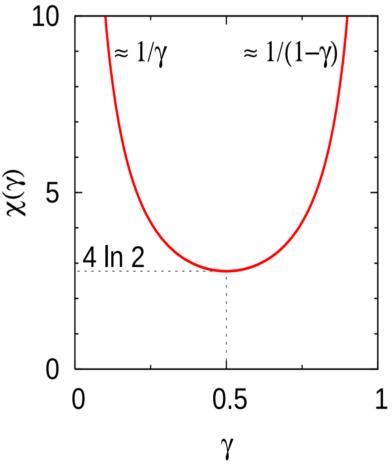
Characteristic function



Eigenvalues of kernel:

$$\mathcal{K} \otimes (k^2)^\gamma = \bar{\alpha}_s \chi(\gamma) \cdot (k^2)^\gamma$$

$\chi(\gamma)$ is *characteristic function*

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

$\rightarrow$ high energy evolution, $\sigma \sim e^{\bar{\alpha}_s \chi(\gamma) Y}$.

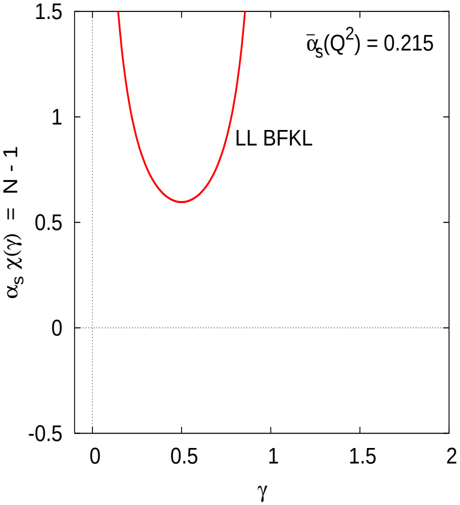
- dominant part at high energies is *minimum* (only stable solution)

$$\sigma \sim e^{4 \ln 2 \bar{\alpha}_s Y} \sim e^{0.5 Y}$$

$\alpha_s \simeq 0.2$

- pole $(1/\gamma)$ corresponds to $\perp$ logarithms $\rightarrow$ DL terms $\alpha_s Y \ln Q^2$

Building up the kernel...



Examine $\bar{\alpha}_s \chi(\gamma)$

minimum = BFKL power

$$\chi(\gamma) = \underbrace{\chi_0(\gamma)}_{LL} + \underbrace{\bar{\alpha}_s \chi_1(\gamma)}_{NLL} + \dots$$

- NLL terms *pathologically large.* minimum → max. (unstable)
  oscillating X-sctns, ...
- Culprit: ⊥ DGLAP logs

  $$\frac{\bar{\alpha}_s}{\gamma} - \frac{11}{12}\frac{\bar{\alpha}_s^2}{\gamma^2} + \dots$$

  $[\gamma^{-1} \Leftrightarrow \ln Q^2]$
- Known at *all orders* ($\gamma \to 0$)

  $$\approx \frac{\bar{\alpha}_s}{\bar{\alpha}_s + \gamma}$$

  'Rotated $\gamma(N)$'
- Symmetry $\gamma \leftrightarrow N - \gamma$

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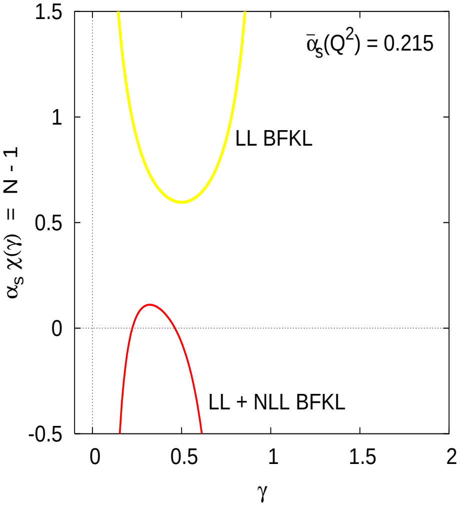

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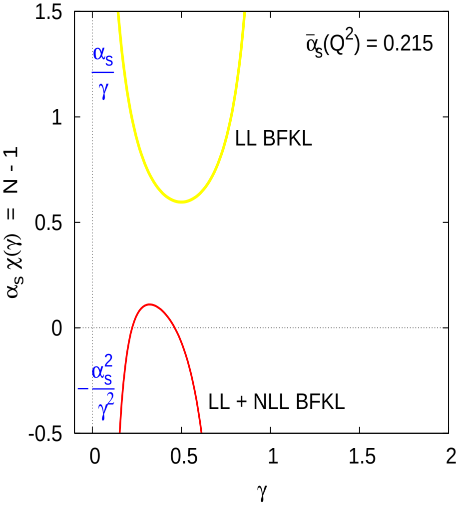

Examine $\bar{\alpha}_s\chi(\gamma)$

minimum = BFKL power

$$\chi(\gamma) = \underbrace{\chi_0(\gamma)}_{LL} + \underbrace{\bar{\alpha}_s\chi_1(\gamma)}_{NLL} + \dots$$

- NLL terms *pathologically large.* minimum → max. (unstable)
  oscillating X-sctns, ...
- Culprit: ⊥ DGLAP logs
  $$\frac{\bar{\alpha}_s}{\gamma} - \frac{11}{12}\frac{\bar{\alpha}_s^2}{\gamma^2} + \dots$$
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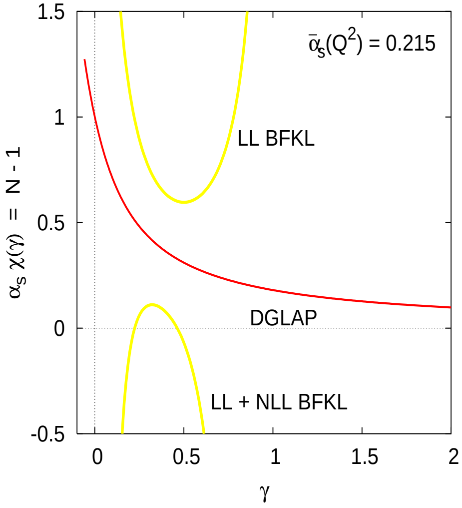

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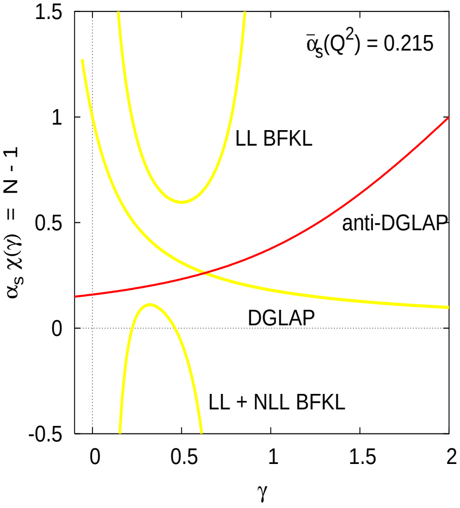

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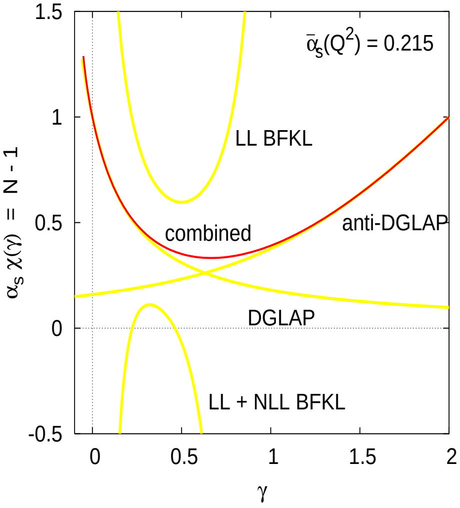

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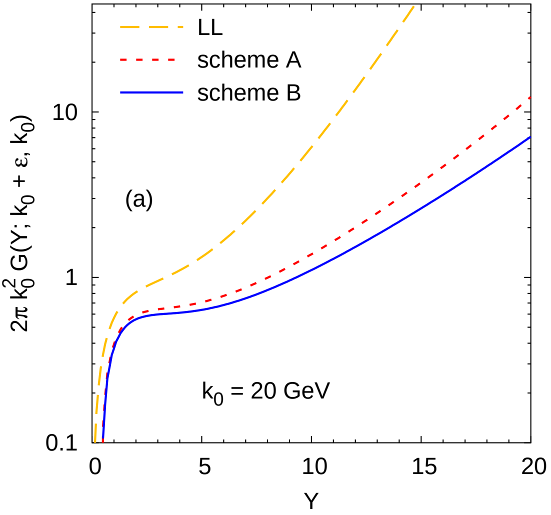
Assemble all constraints:

stable, sensible kernel

Ciafaloni, Colferai, GPS & Staśto;

Altarelli, Ball & Forte; '99–'05

Green fn. from improved kernel



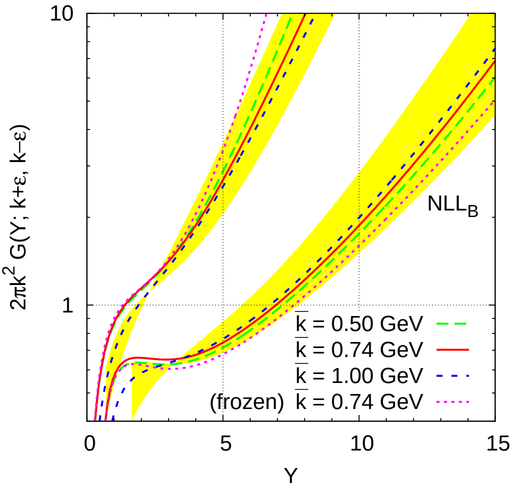
Various schemes for combining NLLx BFKL with DGLAP:

- scheme A ($\mathrm{NLL_A}$) violates mom. sum-rule at $\mathcal{O}(\alpha_s^2)$
- scheme B ($\mathrm{NLL_B}$) satisfies it at all orders

Different schemes → similar results

more

Check stability of results



Check stability wrt:

- renorm. scale variation $1/2 < x_\mu < 2$
- change of infrared cutoff

Uncertainties seem under control

Growth is

- suppressed for $Y \lesssim 5$
- slowed beyond $Y \lesssim 5$

Should be consistent with data

full tests need impact factors

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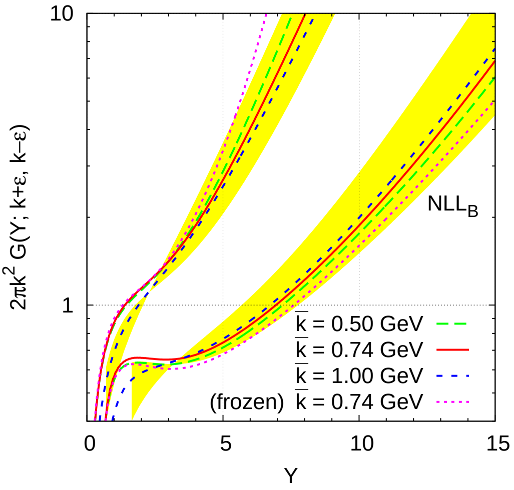

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Limitations of Green function

Green function

$G(Y, k, k_0)$ perturbatively calculable for $k, k_0 \gg \Lambda_{QCD}$.

- Fine for $\gamma^*\gamma^*$, Mueller-Navelet jets (hadron-hadron), Forward jets (DIS).

But: rare processes – of interest mainly for testing BFKL

Recall:

We were interested in proton (e.g. $F_2(x, Q^2)$ structure fn in DIS).

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- these also get small-x enhancements

➥ *Calculate them!*

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$$\frac{dg(x, Q^2)}{d \ln Q^2} = \int \frac{dz}{z} P_{gg,\mathrm{eff}}(z, Q^2)\, g\left(\frac{x}{z}, Q^2\right)$$

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  red paths

- Green function:

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Splitting function ≡ evolution with cutoff

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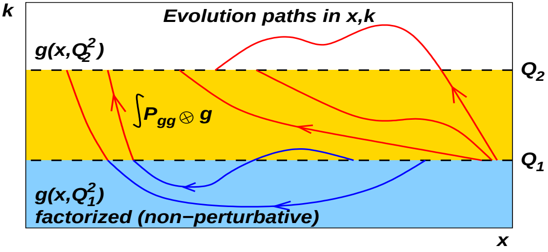

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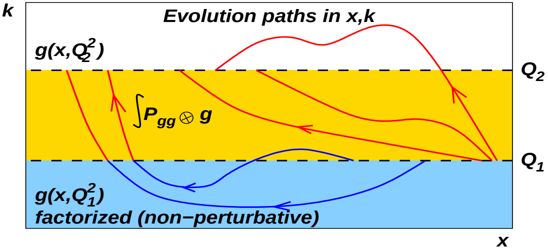

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Perturbative structure of P_{gg}

- Small-x gluon splitting function has logarithmic enhancements:

$$xP_{gg}(x) = \sum_{n=1} \alpha_s^n \ln^{n-1}\frac{1}{x} + \sum_{n=2} \alpha_s^n \ln^{n-2}\frac{1}{x} + \ldots$$

- NNLO (α_s^3): first small-x enhancement in gluon splitting function.

Leading Logs (LLx)

$$\bar{\alpha}_s + \frac{\zeta(3)}{3}\bar{\alpha}_s^4 \ln^3\frac{1}{x} + \frac{\zeta(5)}{60}\bar{\alpha}_s^6 \ln^5\frac{1}{x} + \cdots$$

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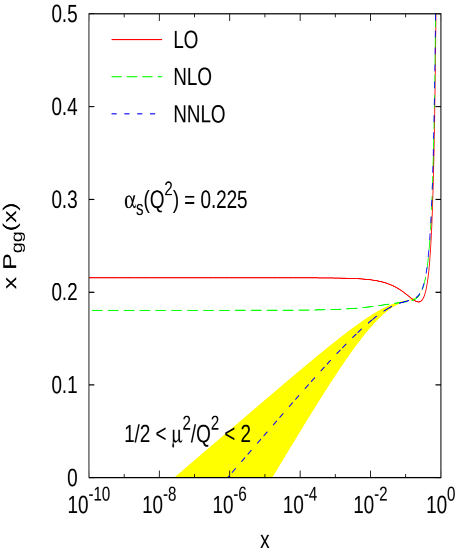


Moch, Vermaseren & Vogt

LLx, NLLx

Problem:

- LLx terms rise very fast, $xP_{gg}(x) \sim x^{-0.5}$.

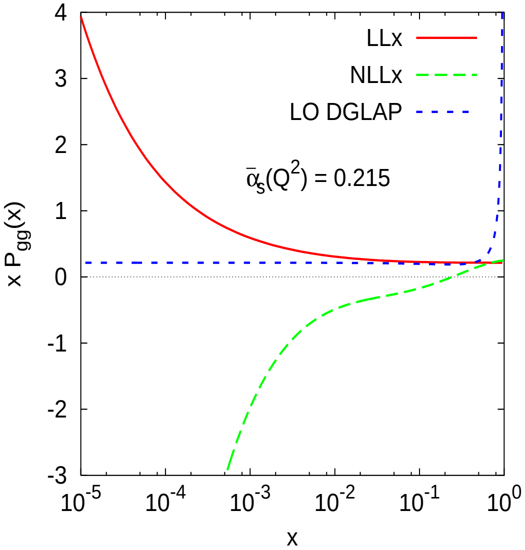
  Incompatible with data.

  Ball & Forte '95

- NLLx terms go negative very fast.

  No one's even tried fitting the data!

[NB: Taking NLLx terms of P_{gg} is almost the worst possible expansion]



BFKL splitting function 'power'

Two classes of correction, to power growth ω:

$$\omega = 4\ln 2\,\bar{\alpha}_s(Q^2)\left(1 - \underbrace{6.5\,\bar{\alpha}_s}_{NLL} - \underbrace{4.0\,\bar{\alpha}_s^{2/3}}_{running} + \cdots\right)$$

- NLL piece is *universal*

As before, add approximate higher orders via $\mathrm{NLL_B}$ kernel

- running piece appears only in problems with *cutoffs*
  - a consequence of *asymmetry* due to cutoff (only scales higher than cutoff contribute)

$$\alpha_s(Q^2) \to \alpha_s(Q^2 e^{-X/(b\alpha_s)^{1/3}})$$

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- Beyond first terms, not possible to separate effects of 'pure' higher orders & running coupling

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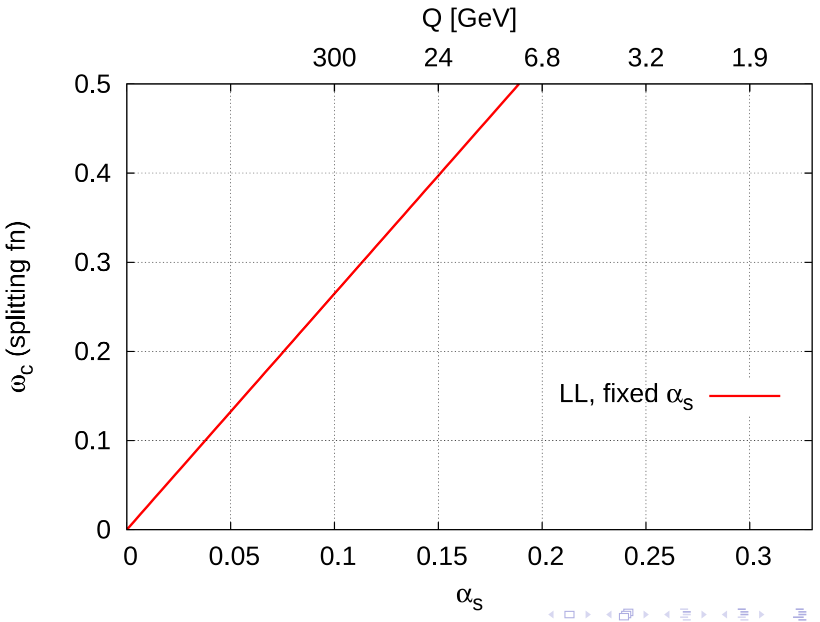

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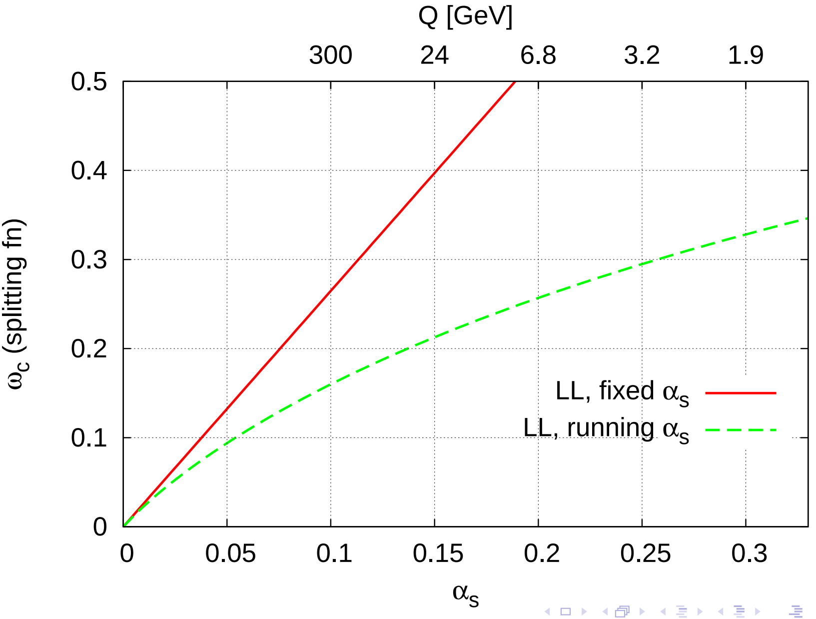

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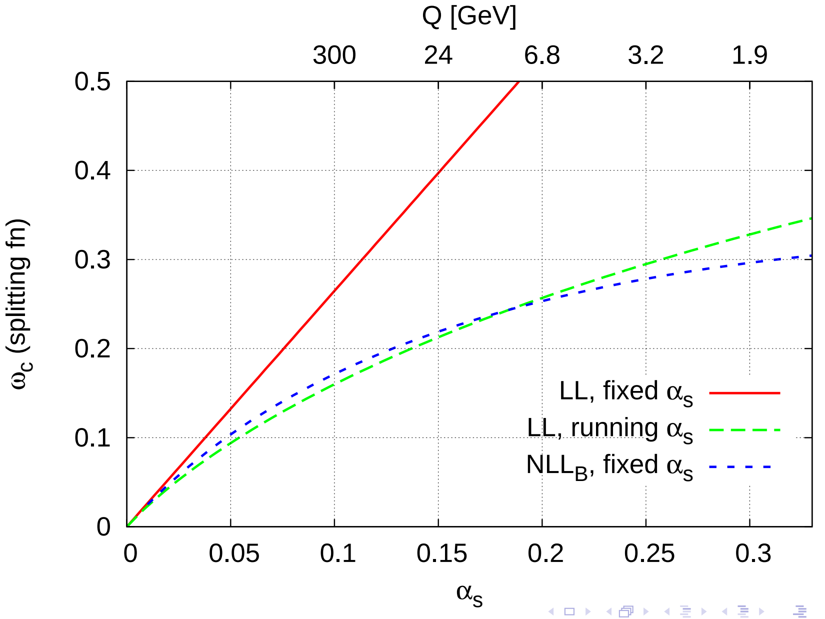

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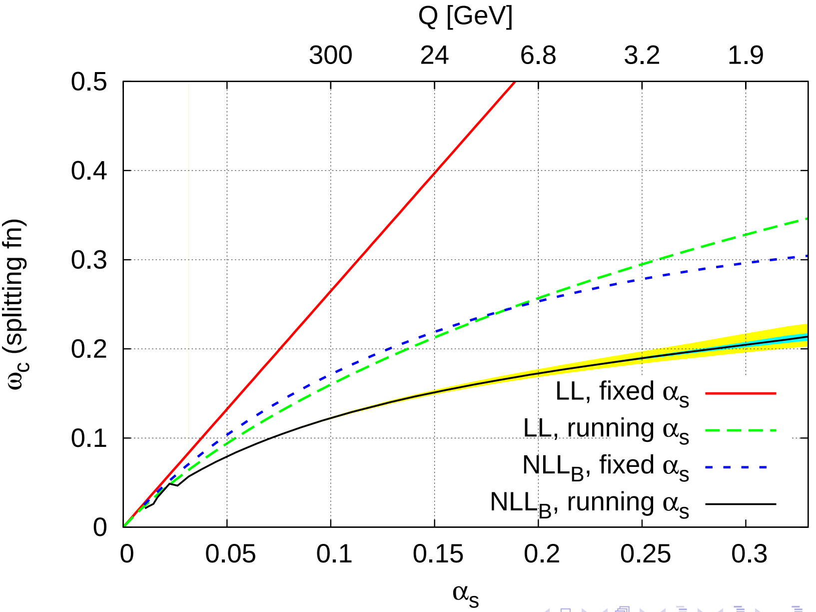
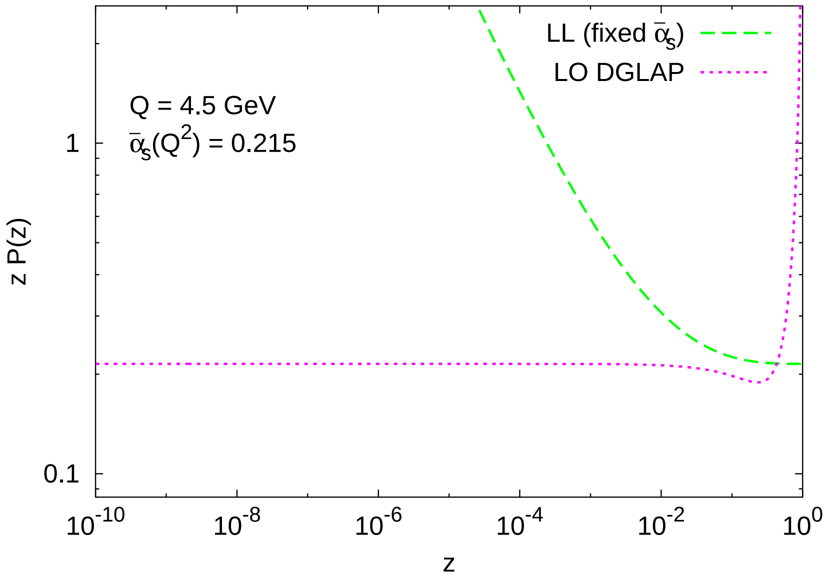


Full $P_{gg}(z)$ splitting fn



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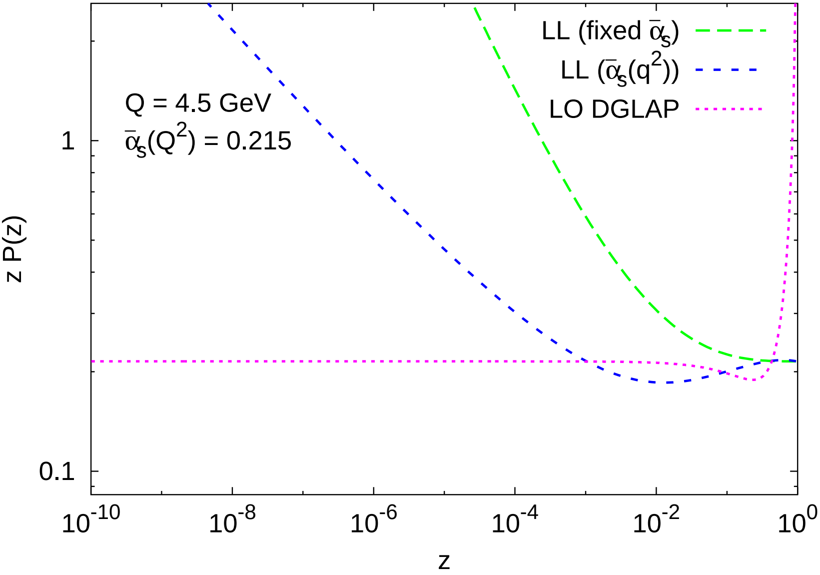

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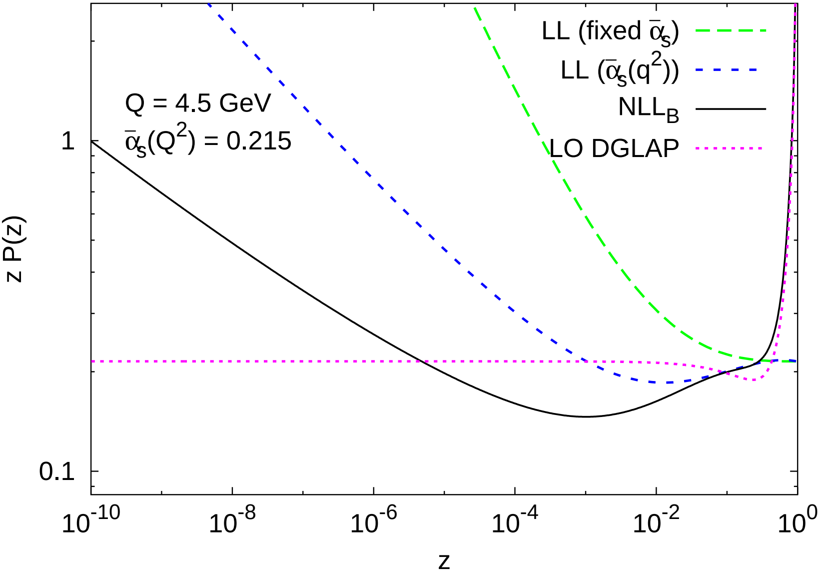

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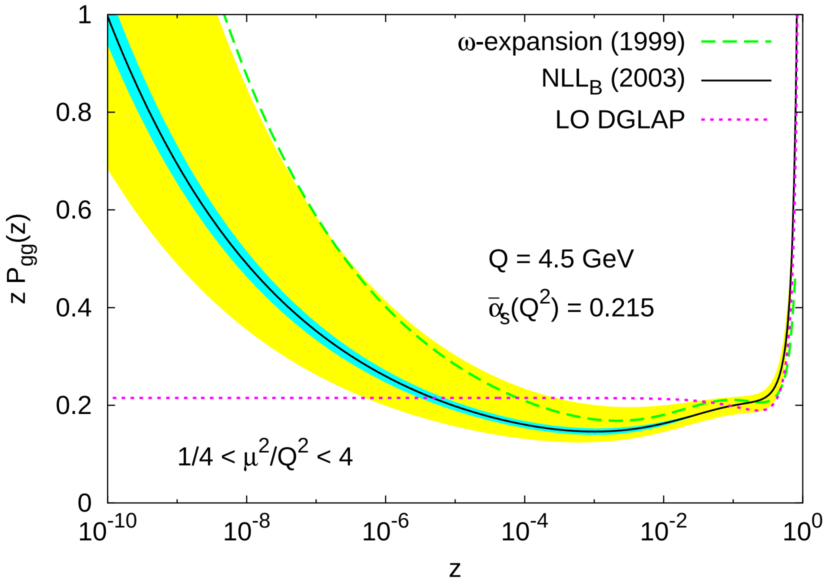

Phenomenology: dip dominates P_{gg}

- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at $x \sim 10^{-3}$*

Questions:

- Is the dip a rigorous prediction?
- What is its origin?

Running α_s, mom. sum rule. . . ?

NNLO DGLAP gives a clue. . .

$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$

Interaction between $-\bar{\alpha}_s^3 \ln 1/x$ and BFKL growth

$$\ln \frac{1}{x_{dip}} \sim \frac{1}{\sqrt{\alpha_s}}$$

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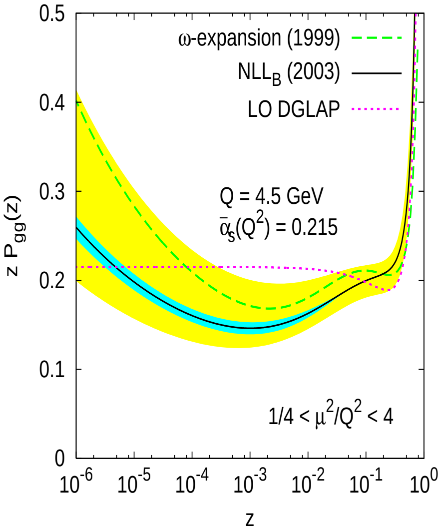

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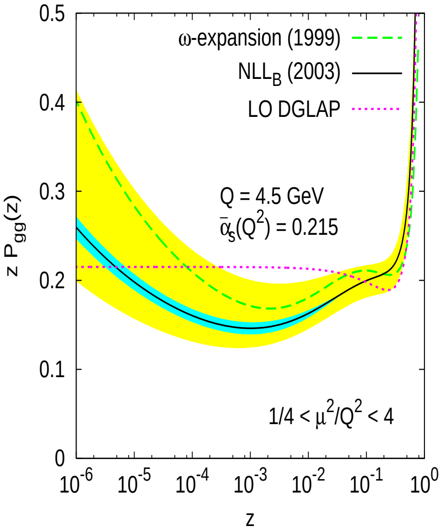

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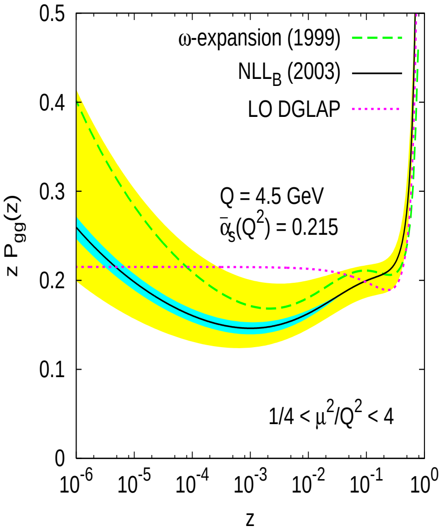

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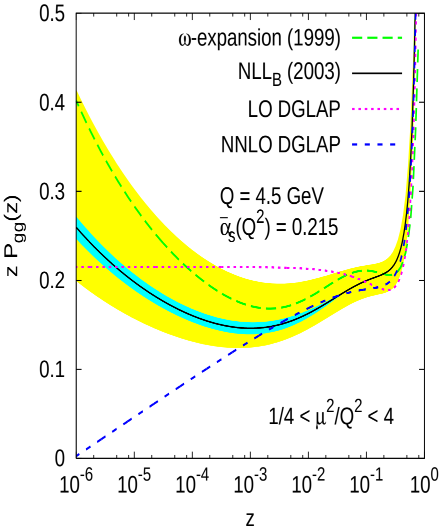

more

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- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at $x \sim 10^{-3}$*

Questions:

- Is the dip a rigorous prediction?
- What is its origin?

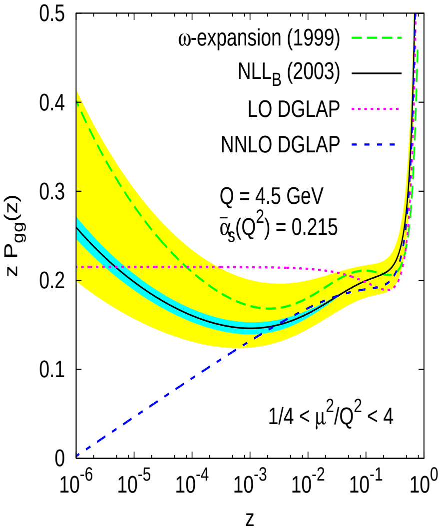
Running α_s, mom. sum rule. . . ?

NNLO DGLAP gives a clue. . .

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$

Interaction between $-\bar{\alpha}_s^3 \ln 1/x$ and BFKL growth

$$\ln \frac{1}{x_{dip}} \sim \frac{1}{\sqrt{\alpha_s}}$$



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Phenomenological impact?

Phenomenological relevance comes through impact on growth of small-x gluon with Q^2.

$$\frac{\partial g(x,Q^2)}{d\ln Q^2} = P_{gg}\otimes g + P_{gq}\otimes q$$

At small x, $P_{gg}\otimes g$ dominates.

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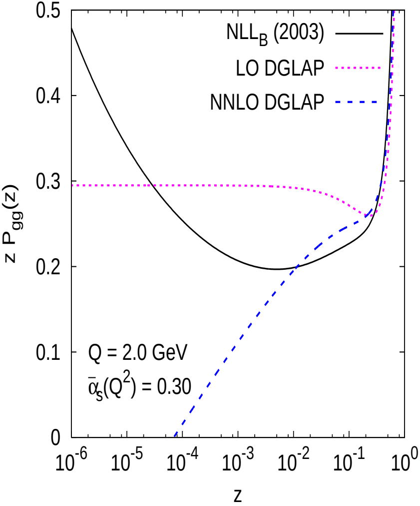
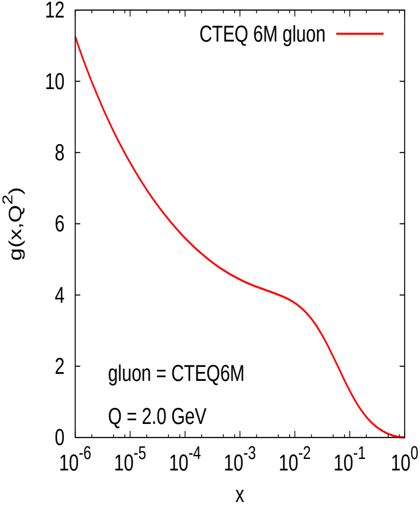


$\otimes$



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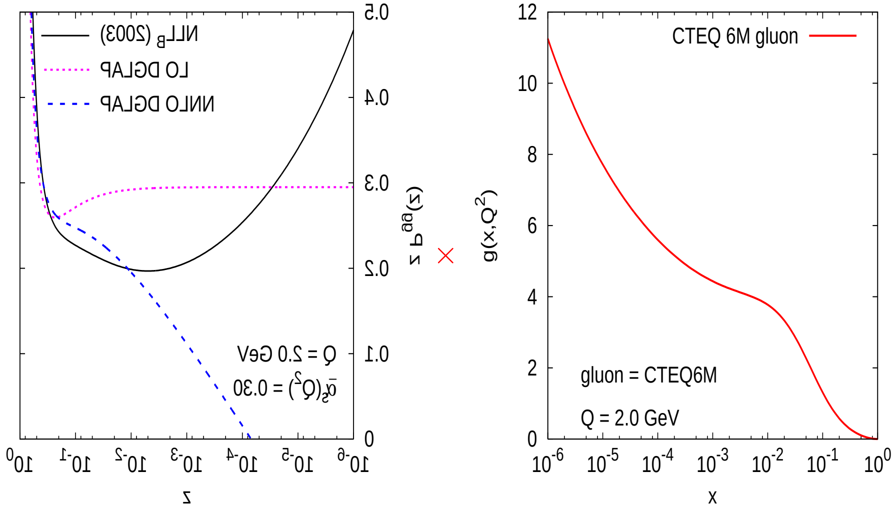

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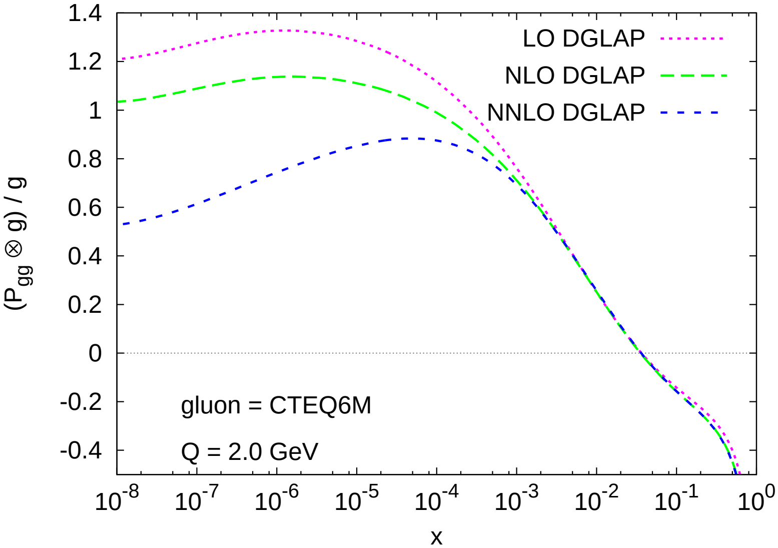

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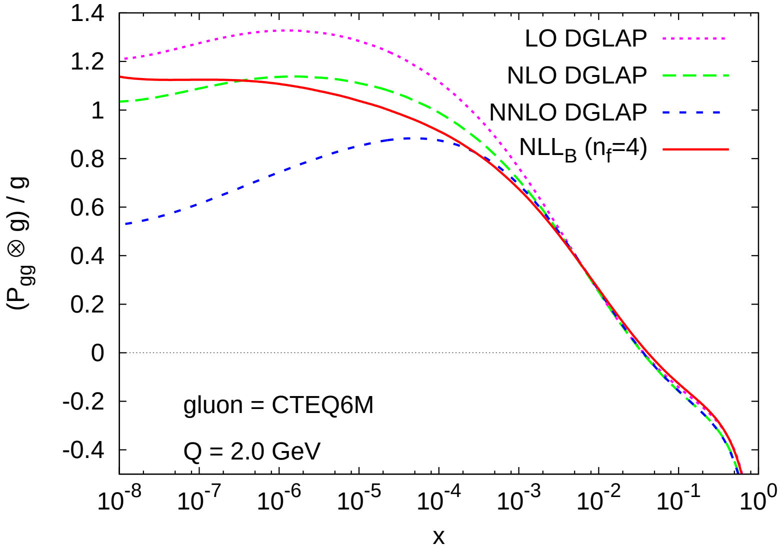

Conclusions

- High-energy limit is one of most challenging problems of QCD.
- Much is now understood about some central elements of small-x resummations:
  - gluon *Green* function
  - gluon *splitting* function

  It seems both can be predicted with confidence
- Phenomenological tests are *essential*
  - Mueller-Navelet jets at LHC, $\gamma^*\gamma^*$ at ILC
  - Structure functions from HERA
- Some ingredients still missing
  - NLL Impact factors
  - Full singlet matrix for splitting functions (not just P_{gg}).
- Big, active question not touched on:

Saturation & limit of high gluon density

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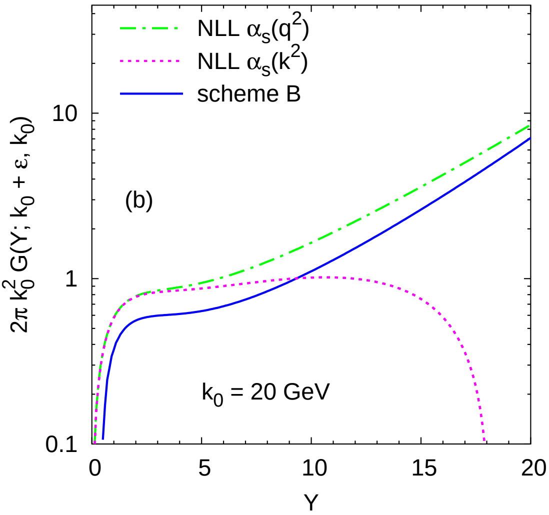

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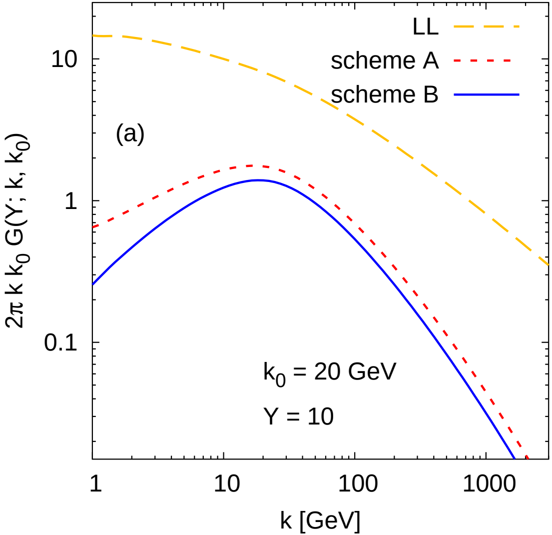

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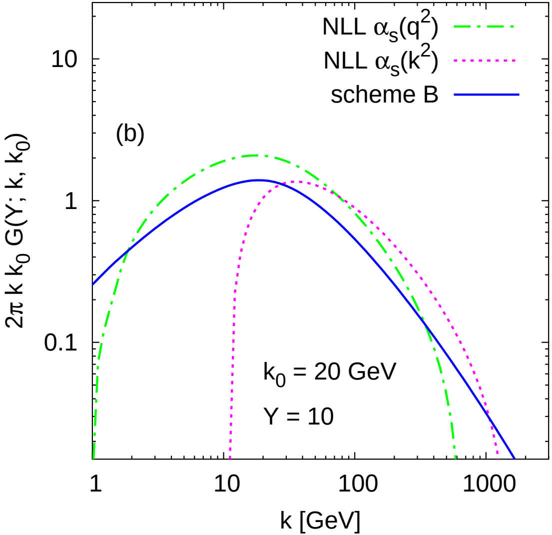

Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
⋮				

const.

$\ln 1/x$

$\ln^2 1/x$

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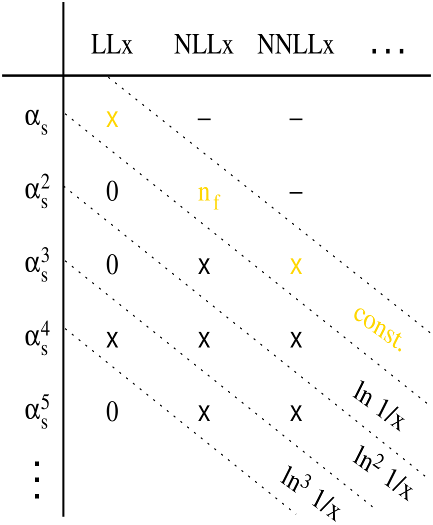

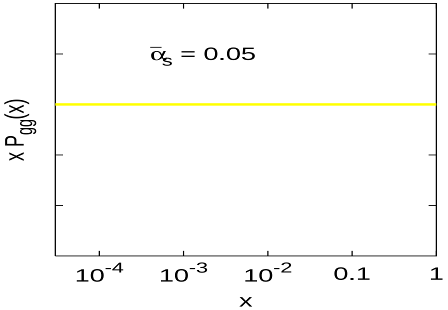

At moderately small x, first terms with x-dependence are

$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x} + 0.401\,\bar{\alpha}_s^4 \ln^3\frac{1}{x}$$

Minimum when

$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln\frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$

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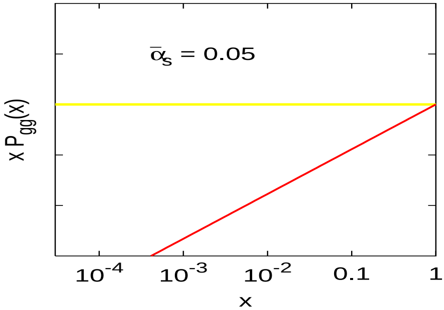

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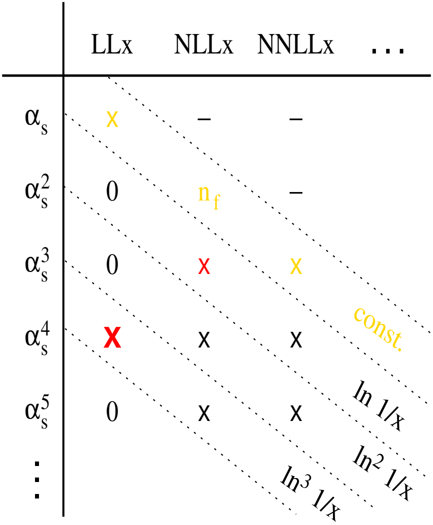

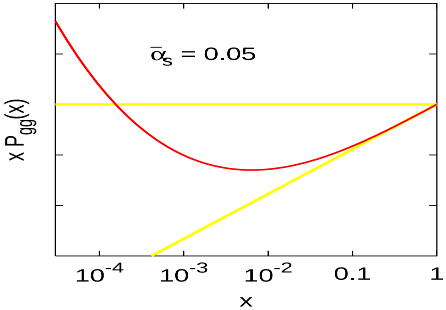

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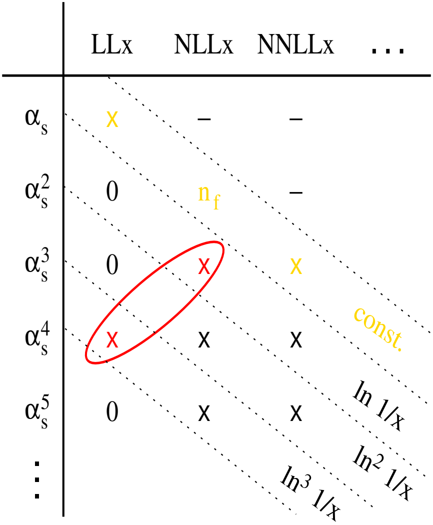

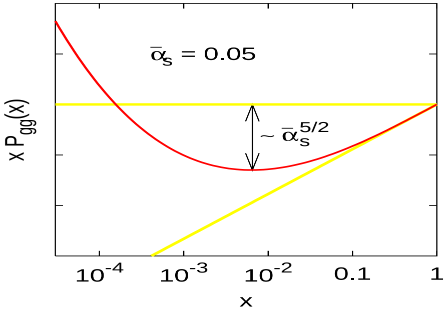

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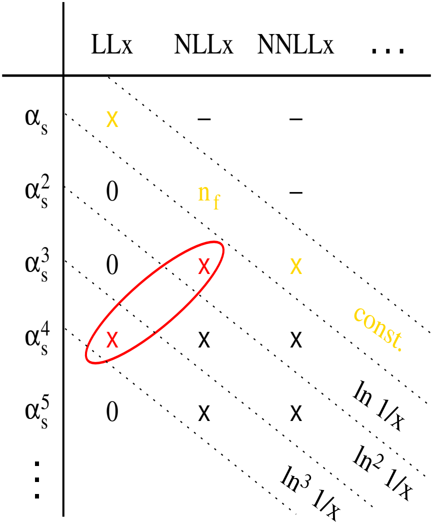
Systematic expansion in $\sqrt{\alpha_s}$



Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}} + 6.947 + \cdots$$

Depth of dip

$$-d \simeq -1.237\bar{\alpha}_s^{5/2} - 11.15\bar{\alpha}_s^3 + \cdots$$

NB:

- convergence is very poor

  As ever at small x!

- higher-order terms in expansion need NNLLx info

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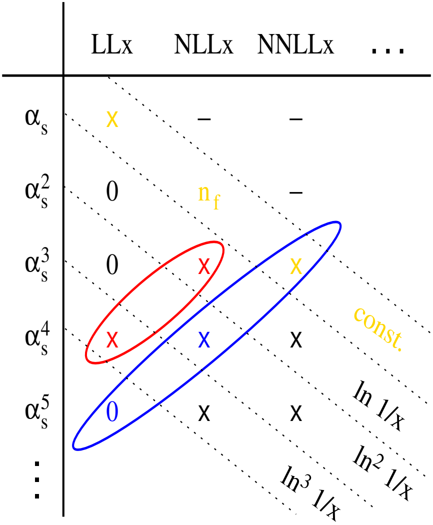

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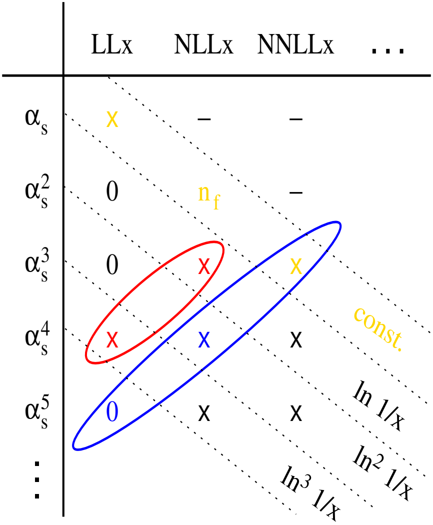

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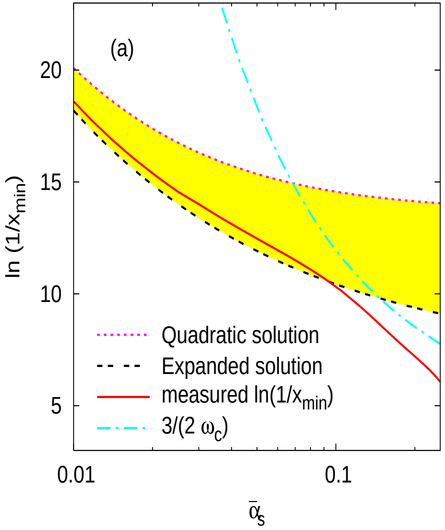

Test *position* of dip v. α_s

- Band is uncertainty due to higher orders in $\sqrt{\alpha_s}$
- At small α_s, good agreement $\to$ confirmation of 'dip mechanism'
- At moderate α_s, normal small-x resummation effects 'collide' with dip

$$\ln \frac{1}{x_{\min}} \lesssim \frac{3}{2\omega_c}$$

Dip then comes from interplay between $\alpha_s^3 \ln x$ (NNLO) term and full resummation.

[Actually, story more complex]

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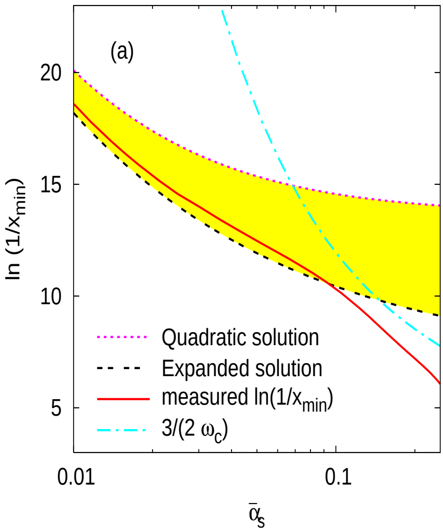

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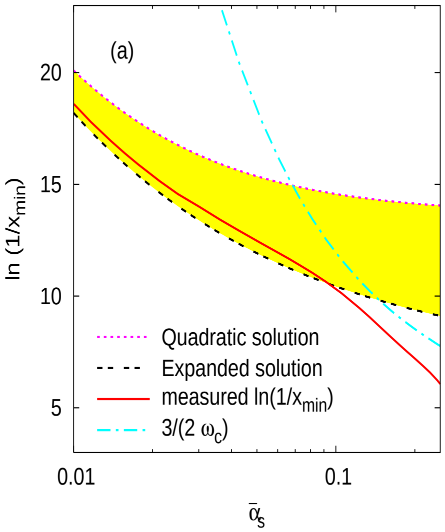

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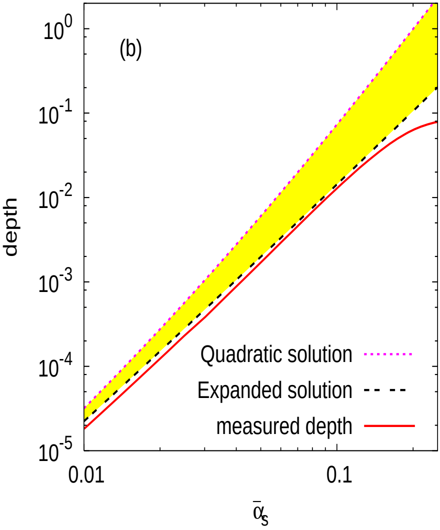

Test *depth* of dip v. α_s

- similar conclusions!

back

Towards phenomenology?

Steps missing for 'full' phenomenology:

- Resummation of all entries of singlet matrix & coefficient functions.
- Put results in $\overline{\text{MS}}$ factorisation scheme

↪illustrate nature of surprises that arise. . .

Factorisation scheme

Results shown so far in Q_0 scheme. [Catani, Ciafaloni & Hautmann '93]

$$xg(x,Q^2) \equiv \int d^2k \; G(\ln 1/x, k, k_0)\Theta(Q-k) \qquad G^{(0)} = f(x)\delta^2(k-k_0)$$

To translate to $\overline{\text{MS}}$ scheme

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Should be easy?!

$$R(\gamma) = \left\{\frac{\Gamma(1-\gamma)\chi(\gamma)}{\Gamma(1+\gamma)[-\gamma\chi'(\gamma)]}\right\}^{\frac{1}{2}} \exp\left\{\int_0^\gamma d\gamma' \frac{\psi'(1)-\psi'(1-\gamma')}{\chi(\gamma')}\right\}$$

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[NB: involves $\chi(\gamma)$ — does this need to be collinearly improved? Ignore problem for now. . .]

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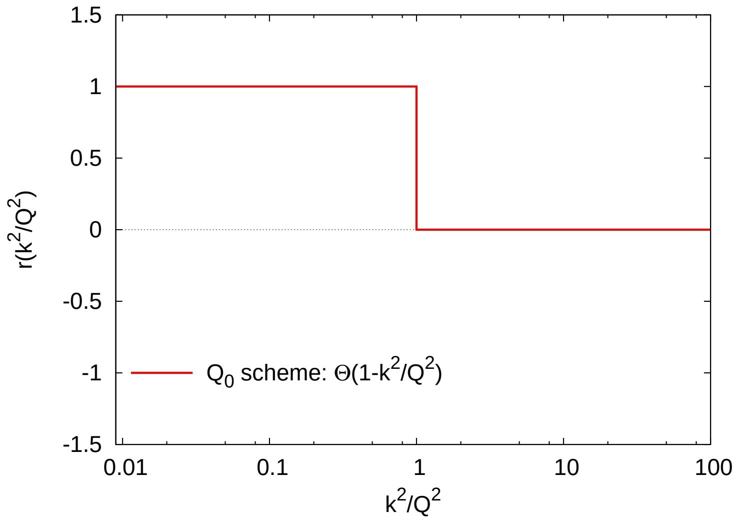

Numerically, $\overline{\mathrm{MS}}$ is much more difficult.

Conceptually, the oscillations are disturbing.

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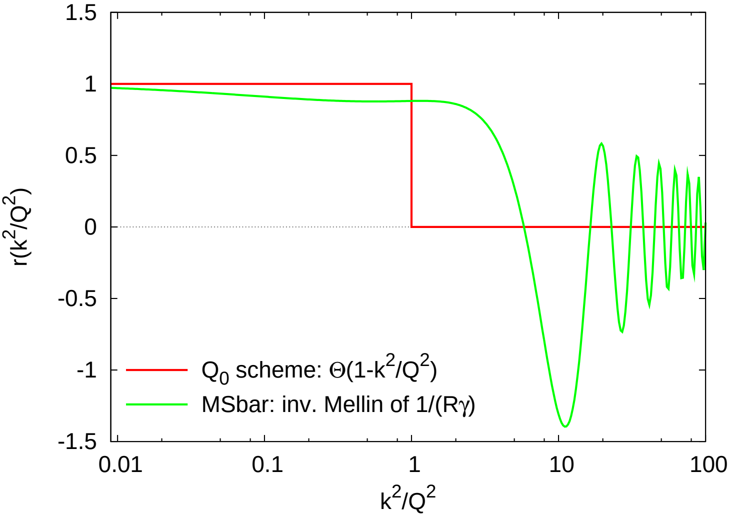

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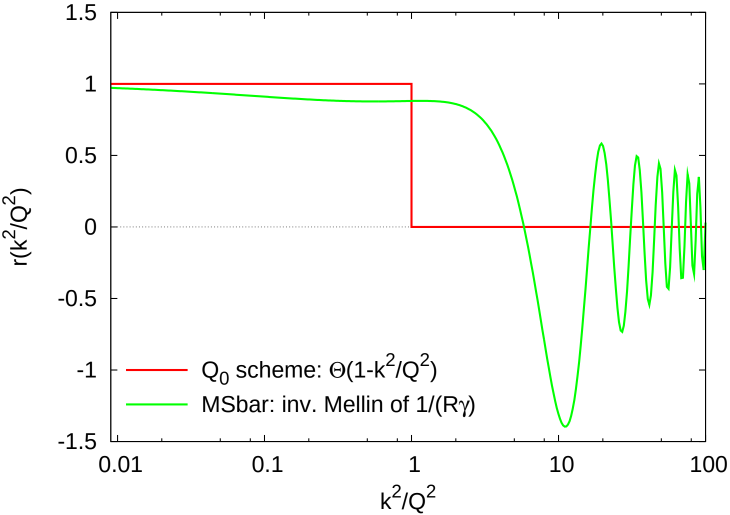

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