

# QCD for LHC

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*Rudolf Peierls Centre for Theoretical Physics*

*All Souls College*

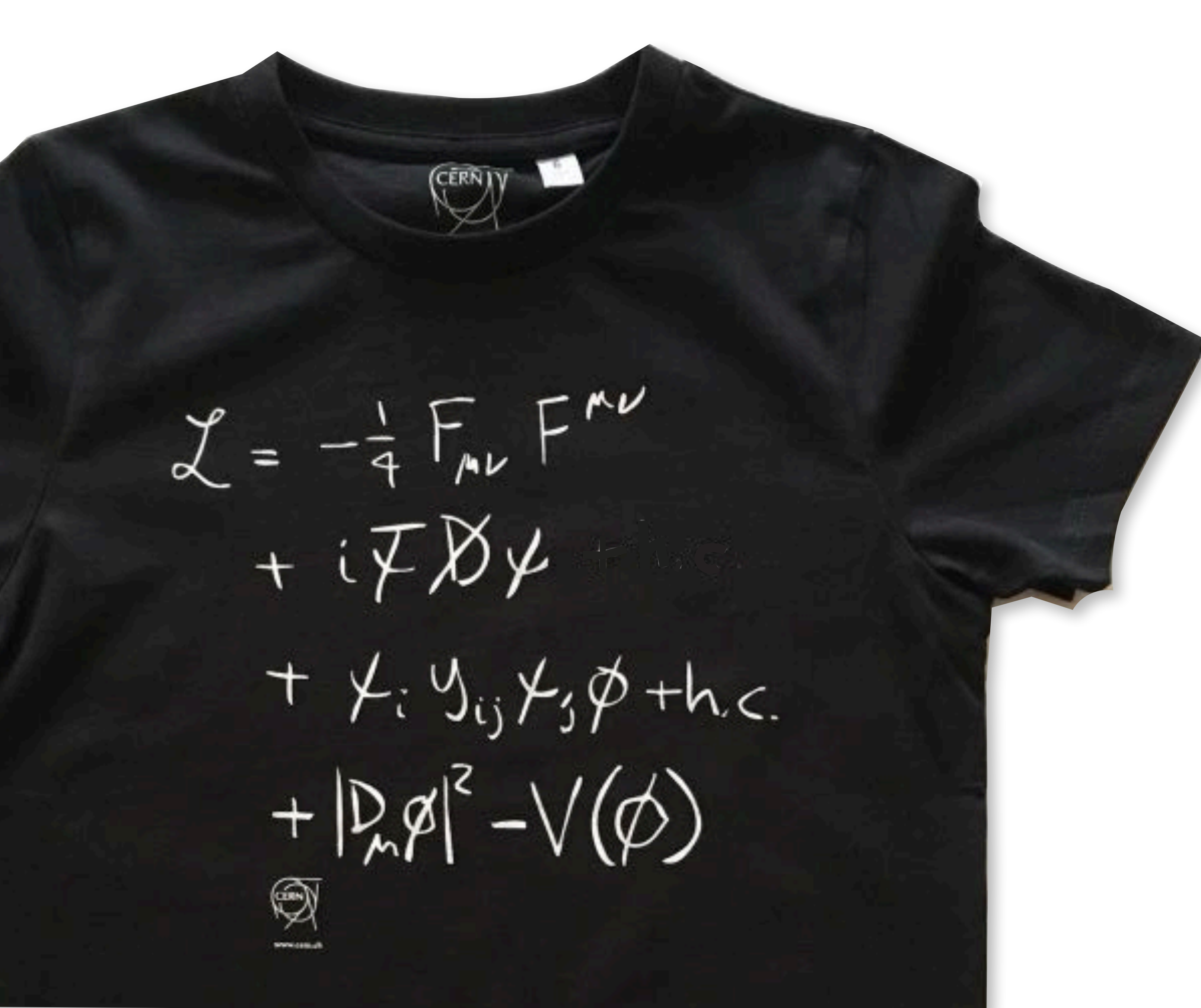
*\* on leave from CERN and CNRS*



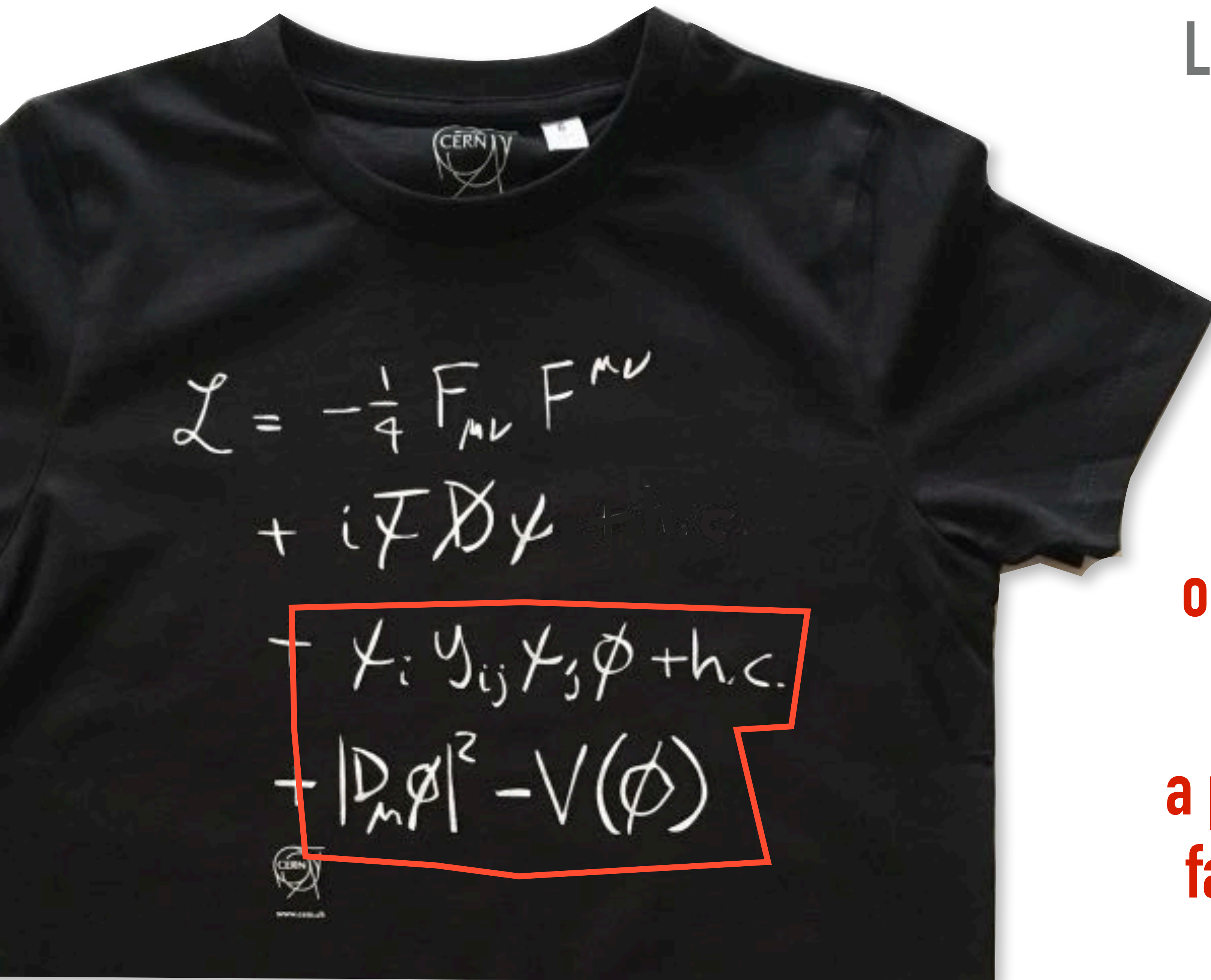
THE ROYAL SOCIETY

**International School of Subnuclear physics  
57th course, In Search of the Unexpected  
Erice, June 2019**

**what are we trying to  
learn at the LHC?**


$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i\bar{\psi} \not{D} \psi \\ & + \bar{\psi}_i Y_{ij} \psi_j \phi + \text{h.c.} \\ & + |D_\mu \phi|^2 - V(\phi)\end{aligned}$$

**what is the underlying  
Lagrangian of particle  
physics?**



LHC is first machine to directly  
access Higgs sector

Is it the minimal version  
hypothesised in the SM?

**origin of mass for W/Z**

**origin of mass for fermions via  
Yukawa couplings**

**a potential  $V(\phi)$  that is theorists'  
favourite toy ( $\phi^4$ ), but yet to be  
confirmed in nature**

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i \bar{\psi} \not{D} \psi \\ & + \psi_i y_{ij} \psi_j \phi + \text{h.c.} \\ & + |D_\mu \phi|^2 - V(\phi) \end{aligned}$$

+ ????

Is there anything else at the  
~TeV scale?

**If not, then many people  
worry about fine tuning**

what are the values of the  
parameters of the SM?

**couplings**

(esp. strong coupling)

**masses**

(e.g. top & W masses)

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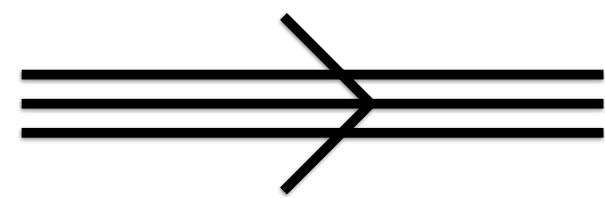
(e.g. top & W masses)

$$\frac{M_h}{\text{GeV}} > 124.2 - \frac{190}{\log_{10}^2 \frac{T_{\text{RH}}}{\text{GeV}}} + 2.0 \left( \frac{M_t}{\text{GeV}} - 173.34 \right) - 0.6 \left( \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.$$

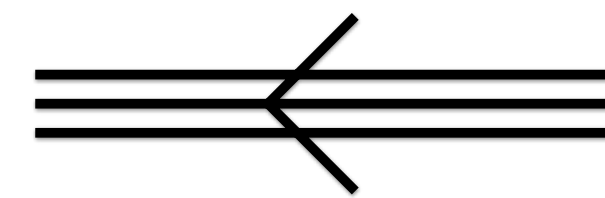
[arXiv:1505.04825](https://arxiv.org/abs/1505.04825)

# A proton-proton collision: INITIAL STATE

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**proton**

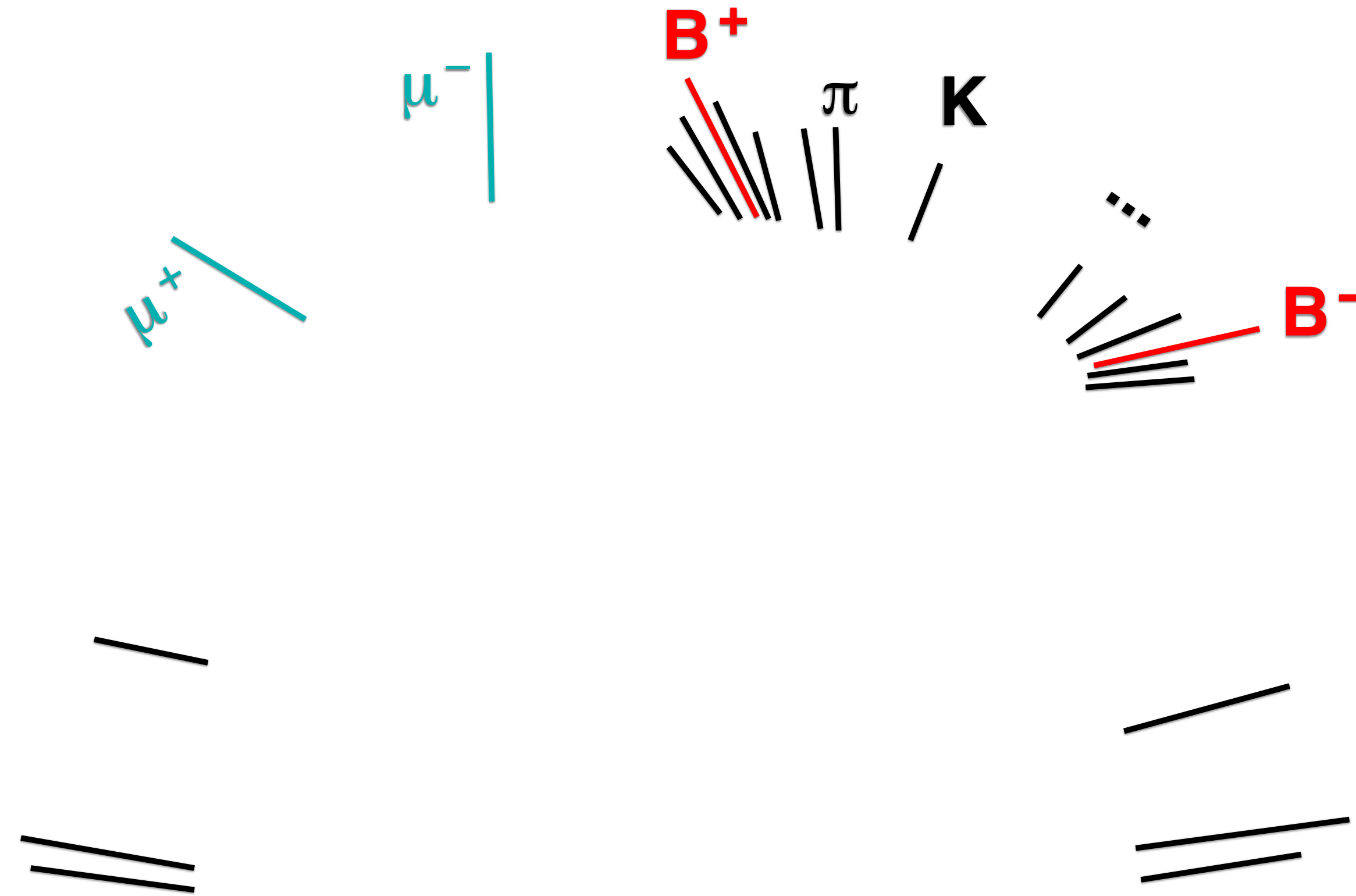


**proton**



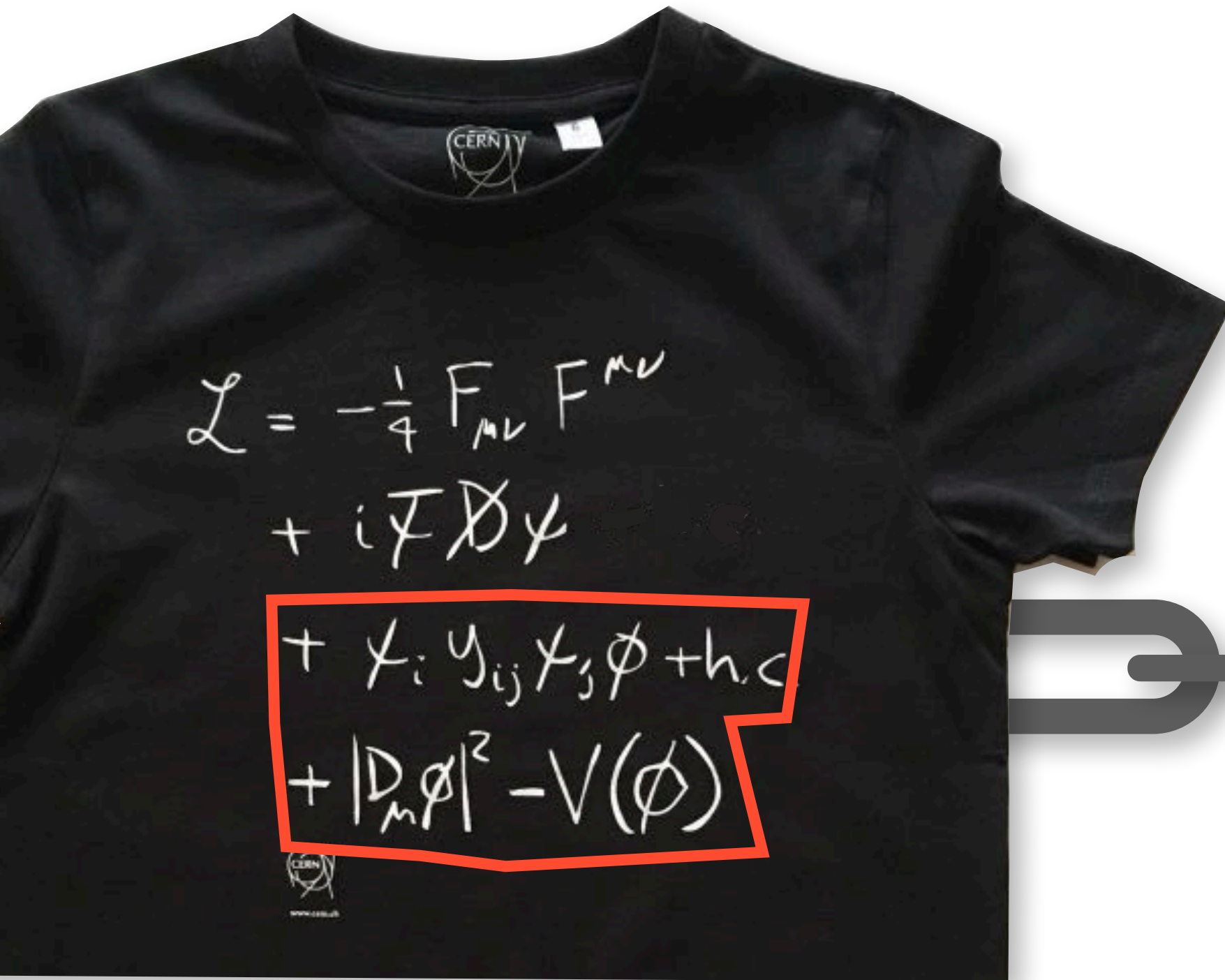
# A proton-proton collision: FINAL STATE

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*(actual final-state multiplicity  $\sim$  several hundred hadrons)*

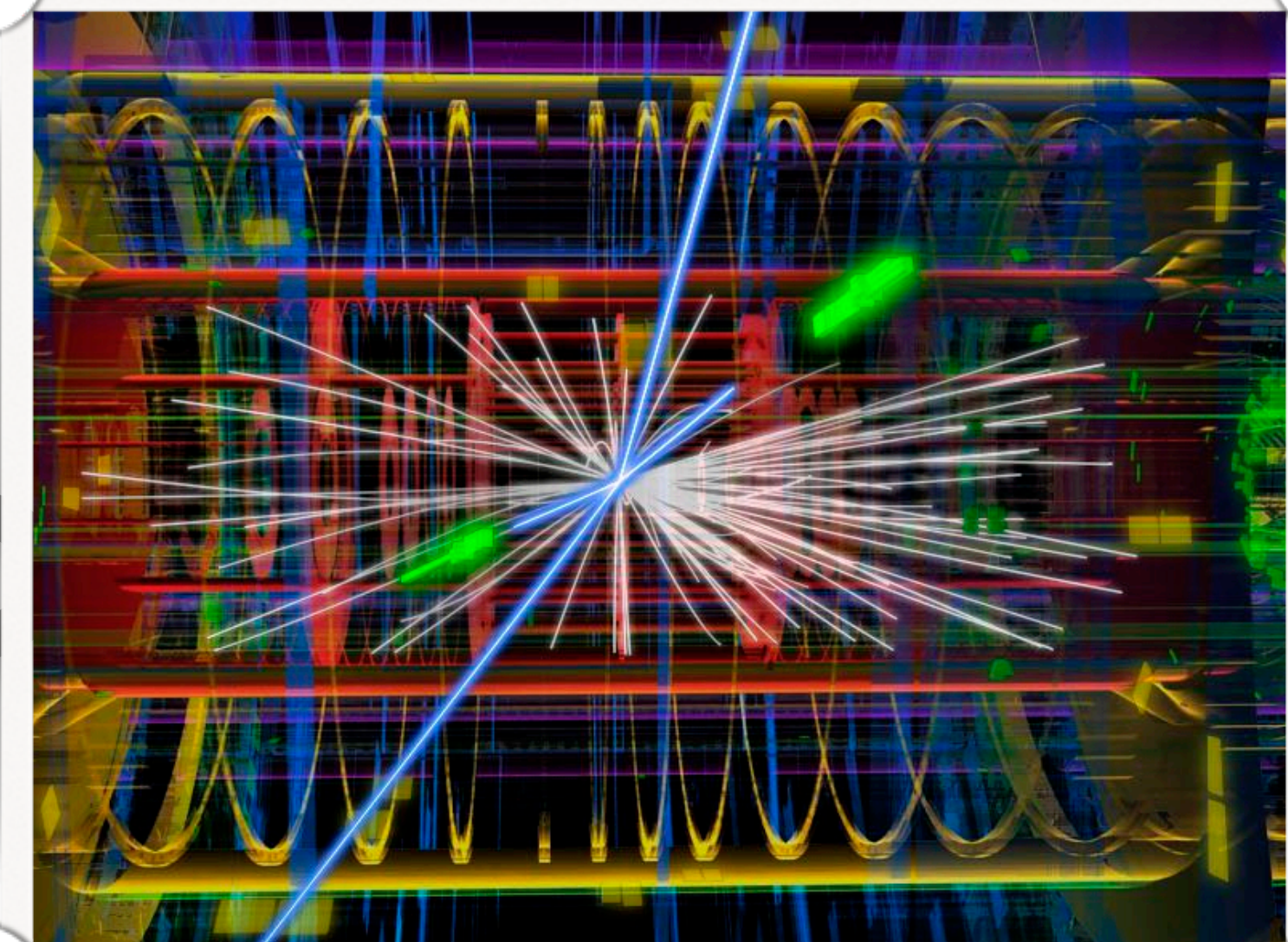
# UNDERLYING THEORY



*how do you make  
quantitative  
connection?*

*through a chain  
of experimental  
and theoretical links*

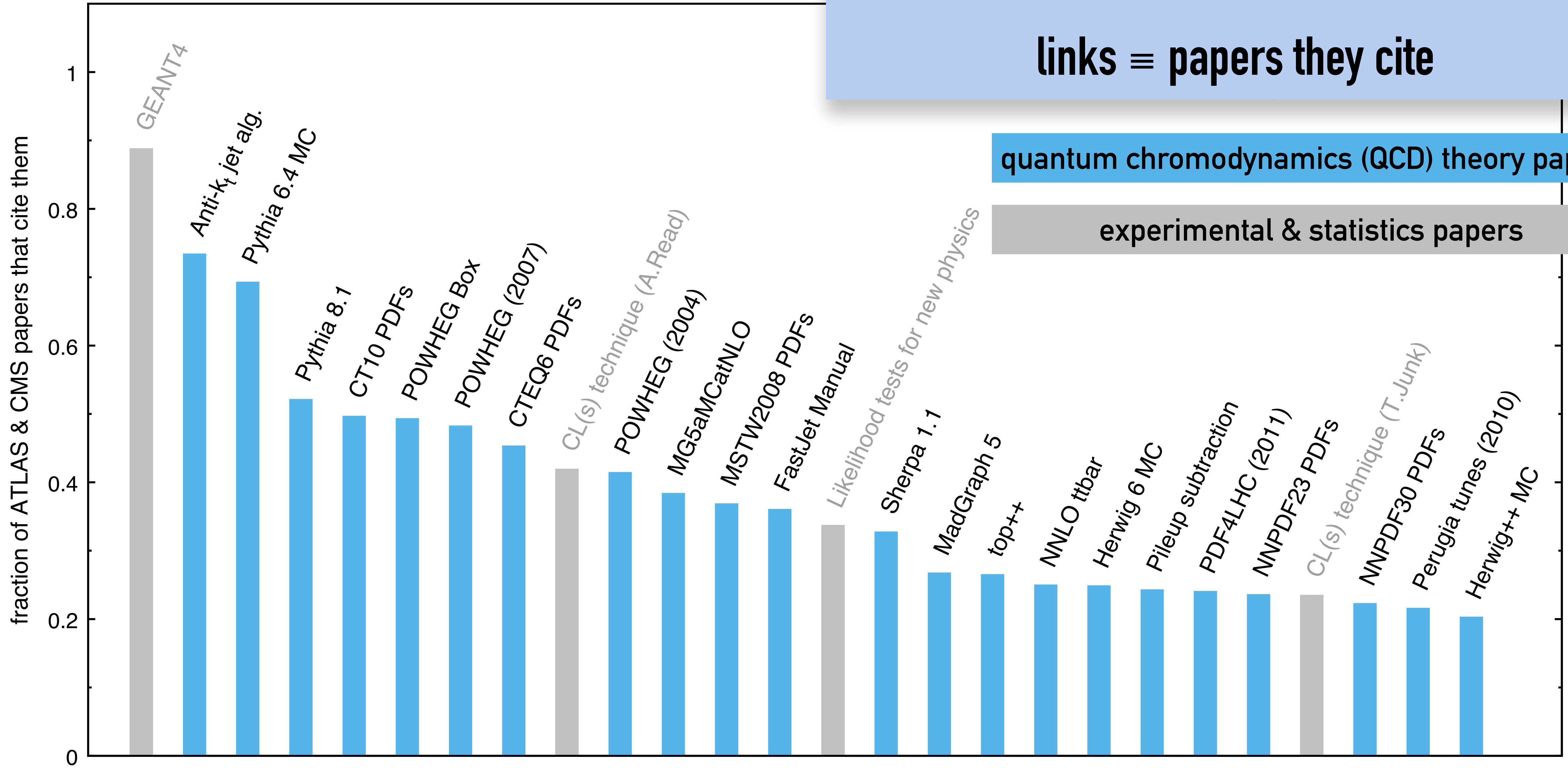
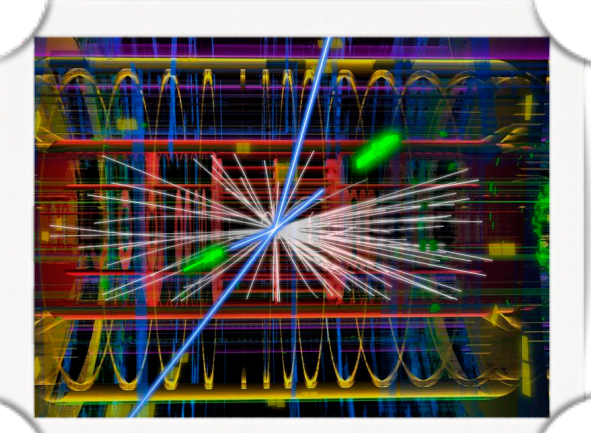
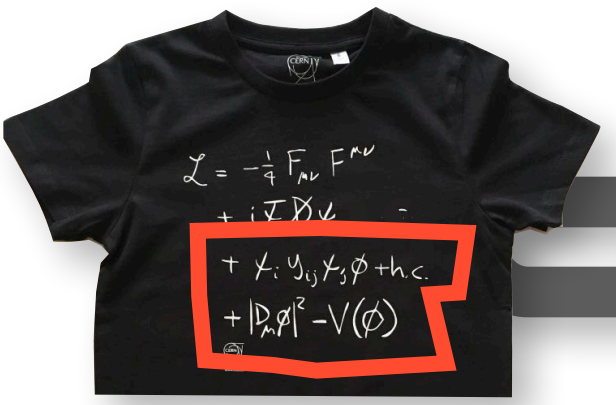
# EXPERIMENTAL DATA



# What are the links?

ATLAS and CMS (big LHC expts.) have written ~1000 articles since 2014

links  $\equiv$  papers they cite



Plot by GP Salam based on data from InspireHEP

## this lecture: 7 small parts

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1. structure of QCD Lagrangian
2. a master formula
3. the strong coupling
4. parton distribution functions
5. fixed order calculations
6. Monte Carlo event generators
7. jets

# the QCD lagrangian

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*and lattice QCD*

# quantum chromodynamics (QCD)

Quarks — 3 colours:  $\psi_a = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

Quark part of Lagrangian:

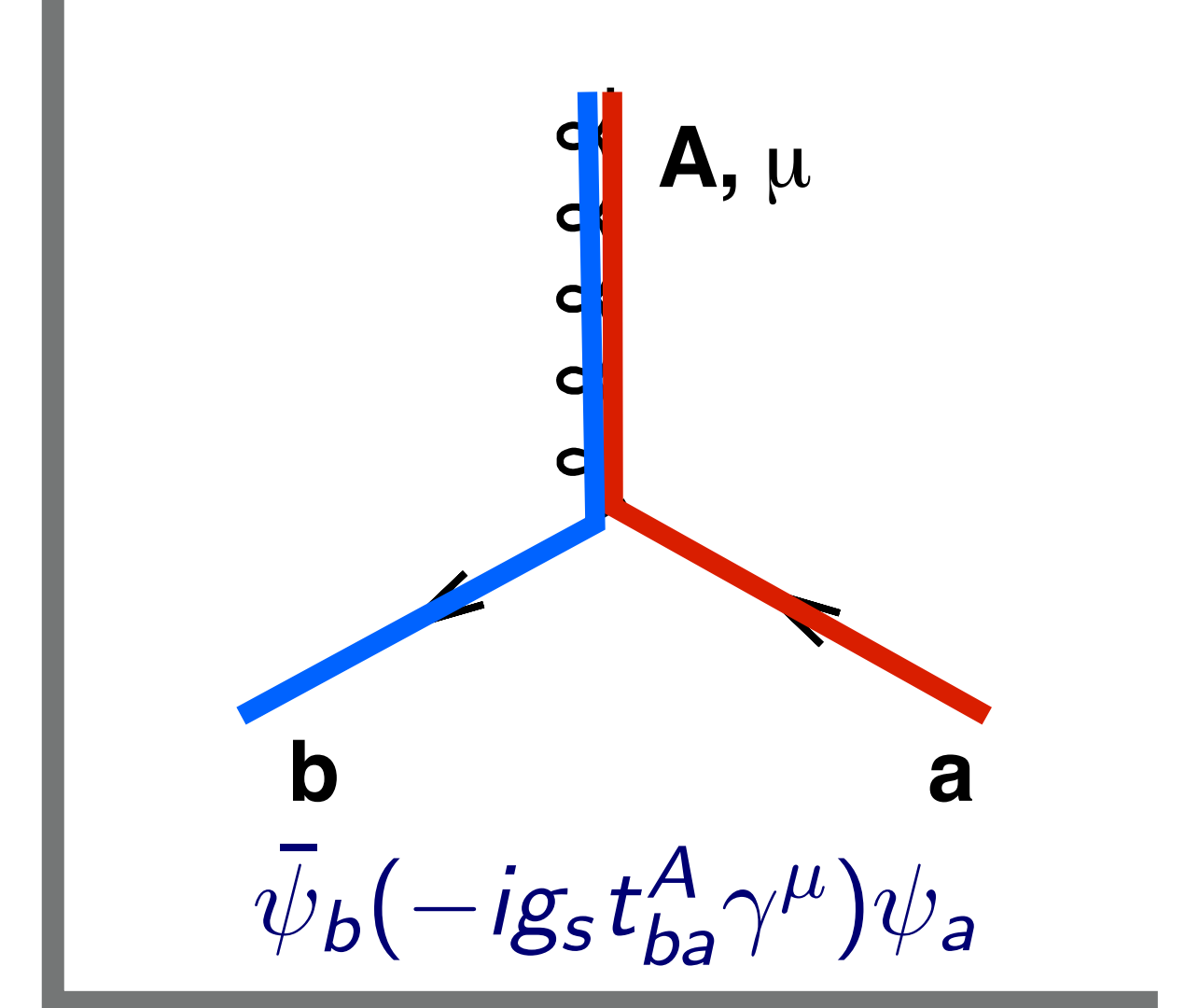
$$\mathcal{L}_q = \bar{\psi}_a (i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t_{ab}^C \mathcal{A}_\mu^C - m) \psi_b$$

$SU(3)$  local gauge symmetry  $\leftrightarrow$  8 ( $= 3^2 - 1$ ) generators  $t_{ab}^1 \dots t_{ab}^8$  corresponding to 8 gluons  $\mathcal{A}_\mu^1 \dots \mathcal{A}_\mu^8$ .

A representation is:  $t^A = \frac{1}{2} \lambda^A$ ,

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix},$$



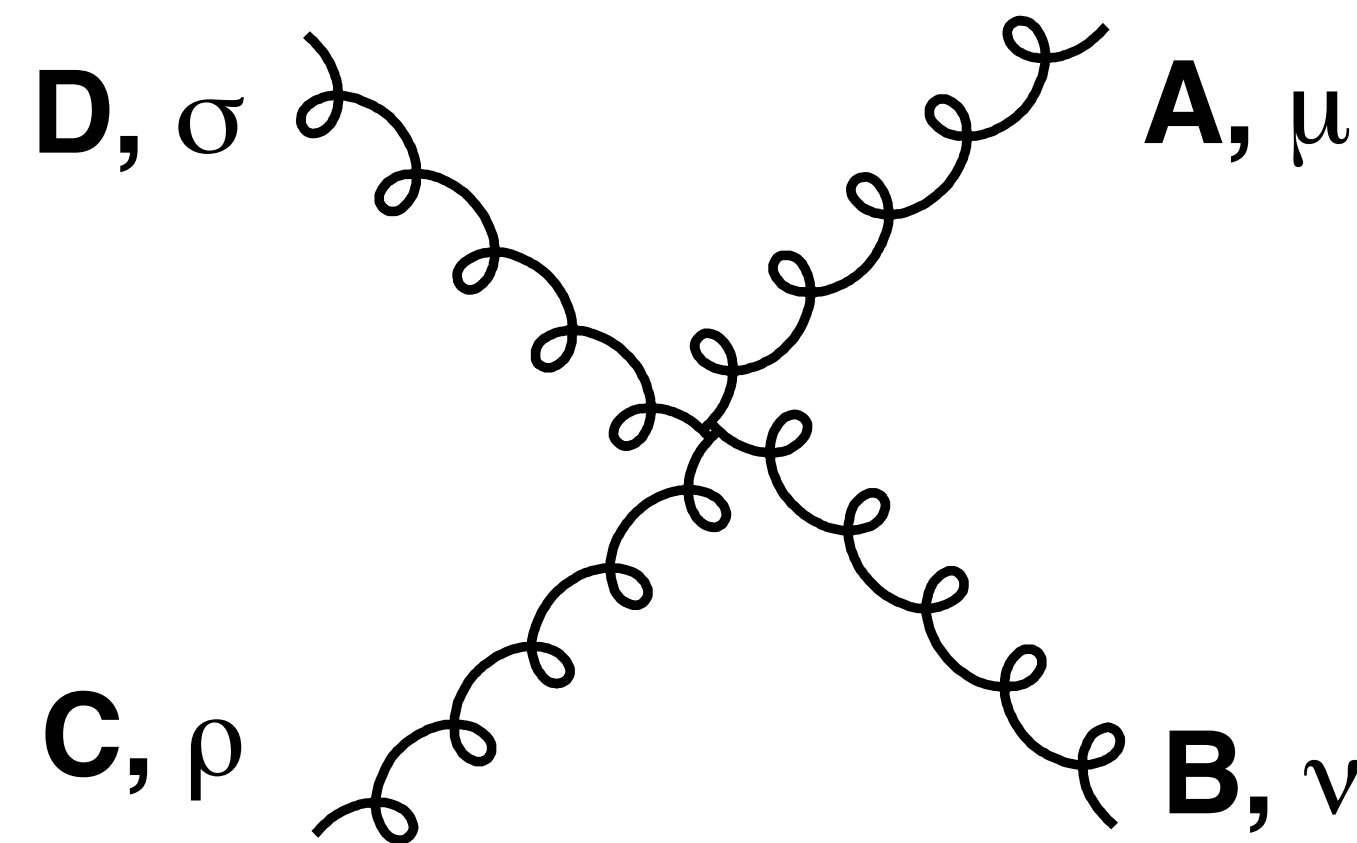
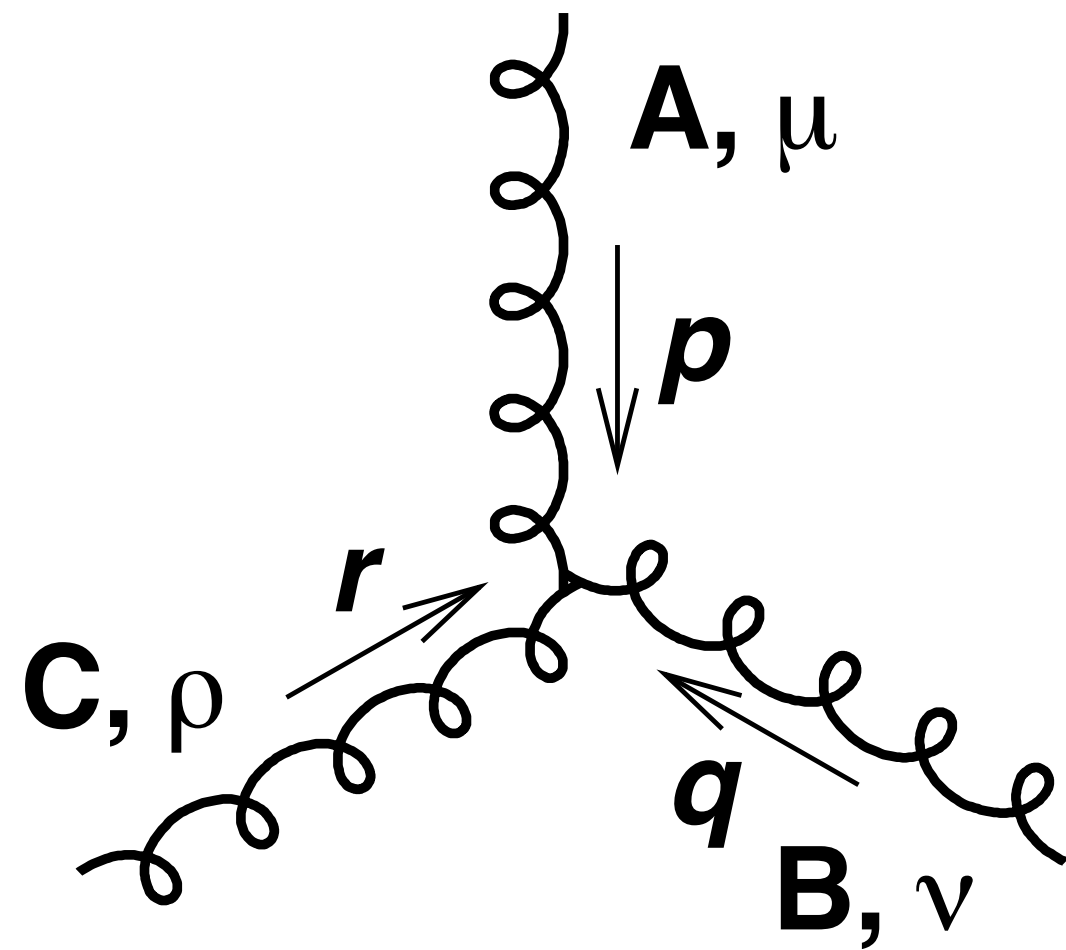
# quantum chromodynamics (QCD)

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Field tensor:  $F_{\mu\nu}^A = \partial_\mu \mathcal{A}_\nu^A - \partial_\nu \mathcal{A}_\mu^A - g_s f_{ABC} \mathcal{A}_\mu^B \mathcal{A}_\nu^C$       $[t^A, t^B] = if_{ABC} t^C$

$f_{ABC}$  are structure constants of  $SU(3)$  (antisymmetric in all indices —  $SU(2)$  equivalent was  $\epsilon^{ABC}$ ). Needed for gauge invariance of gluon part of Lagrangian:

$$\mathcal{L}_G = -\frac{1}{4} F_A^{\mu\nu} F^A_{\mu\nu}$$

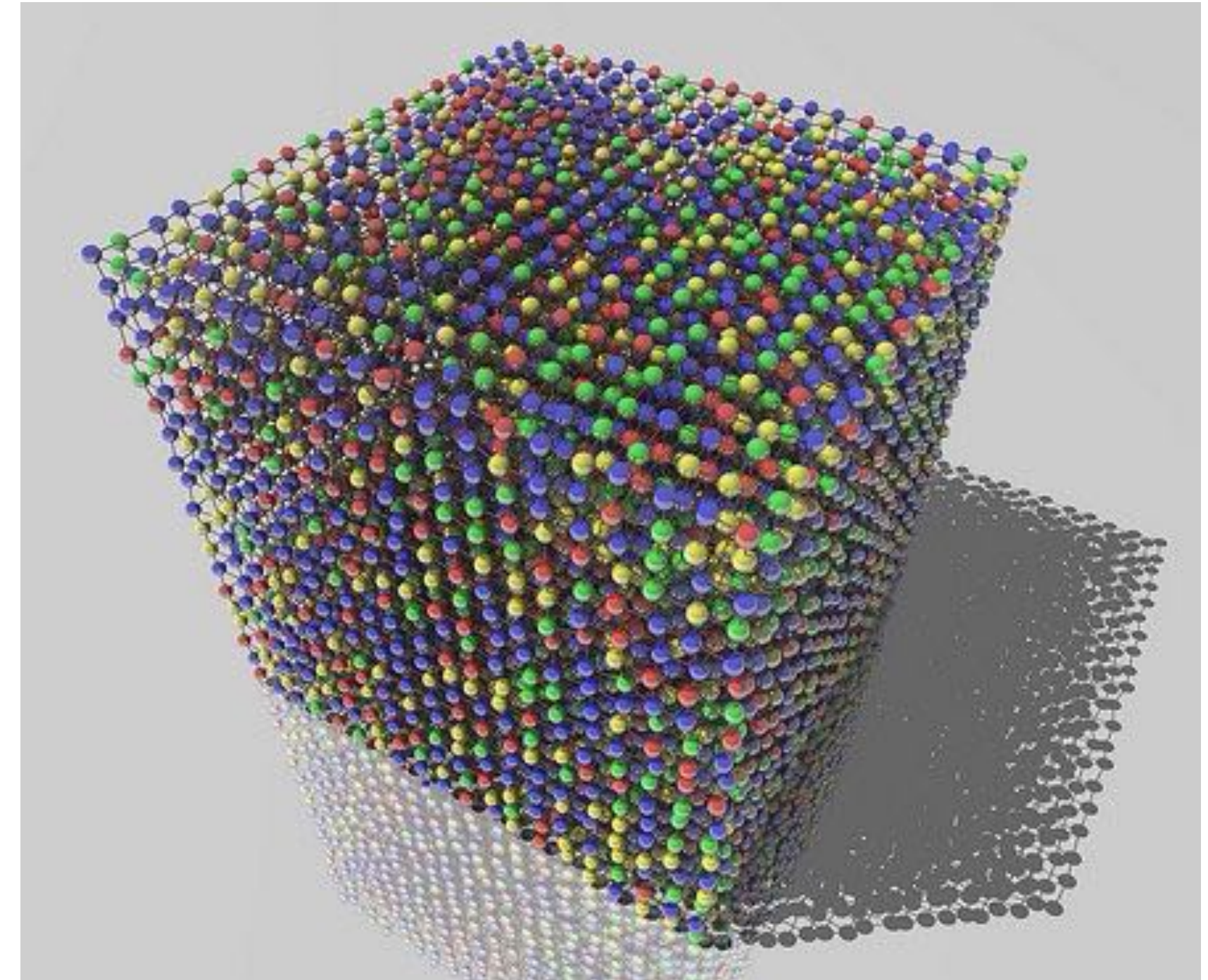


# quantum chromodynamics (QCD)

---

The only complete solution uses **lattice QCD**

- put all quark & gluon fields on a 4d lattice  
(NB: imaginary time)
- Figure out most likely configurations  
(Monte Carlo sampling)



*image credit [fdecomite](#) [[flickr](#)]*

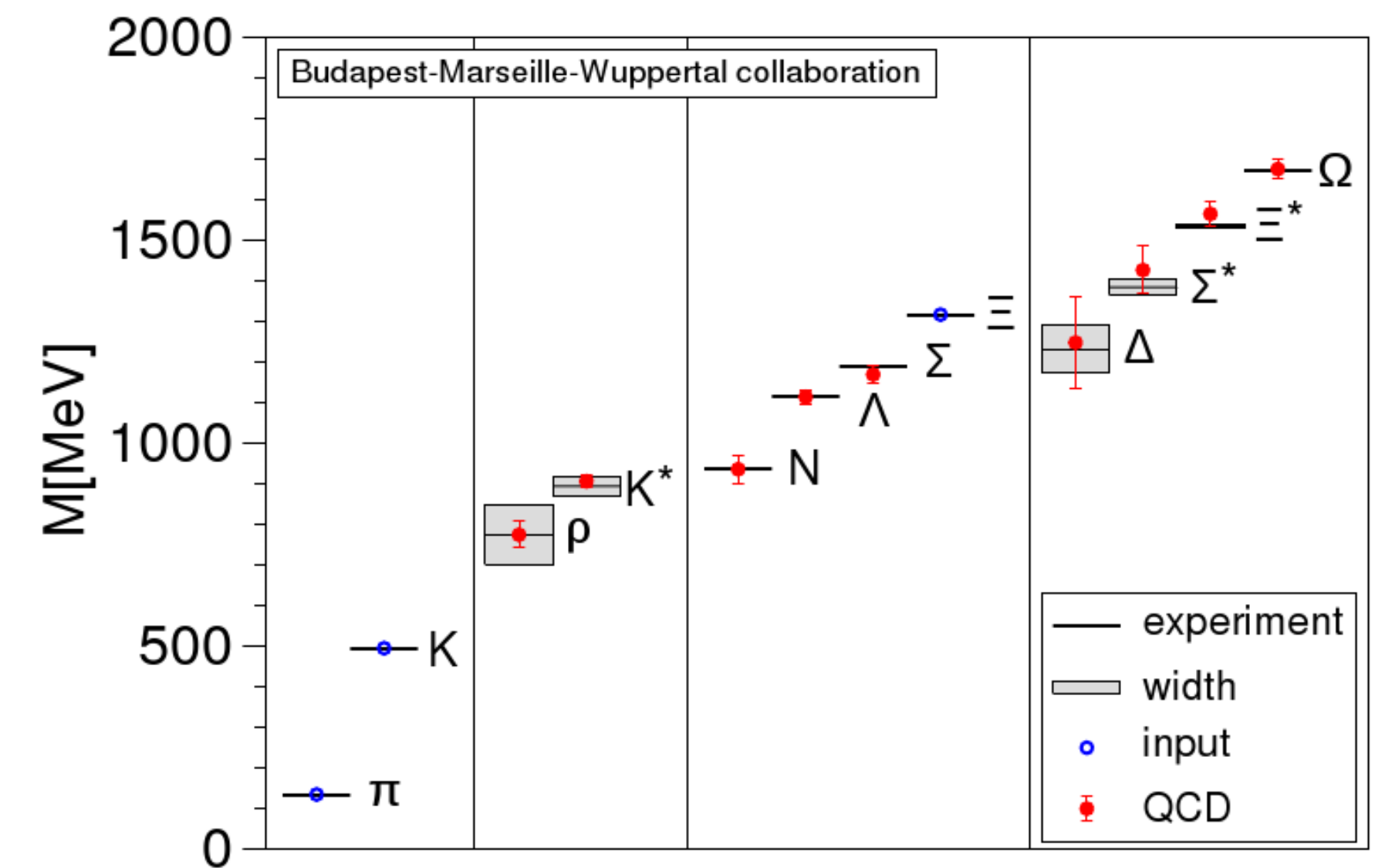


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## hadron spectrum from lattice QCD



*Durr et al, arXiv:0906.3599*

For LHC reactions, lattice would have to

- Resolve smallest length scales ( $2 \text{ TeV} \sim 10^{-4} \text{ fm}$ )
- Contain whole reaction (pion formed on timescale  $\sim 1 \text{ fm}$ , with boost of  $10^4$  — i.e.  $10^4 \text{ fm}$ )

That implies  $10^8$  nodes in each dimension, i.e.  $10^{32}$  nodes — **inconceivable**

# the strong coupling, $\alpha_s$

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*it feeds into everything else in collider QCD*

*for more info see [arXiv:1712.05165](https://arxiv.org/abs/1712.05165), [arXiv:1902.08191](https://arxiv.org/abs/1902.08191)*

# All couplings run: the QCD coupling runs fastest

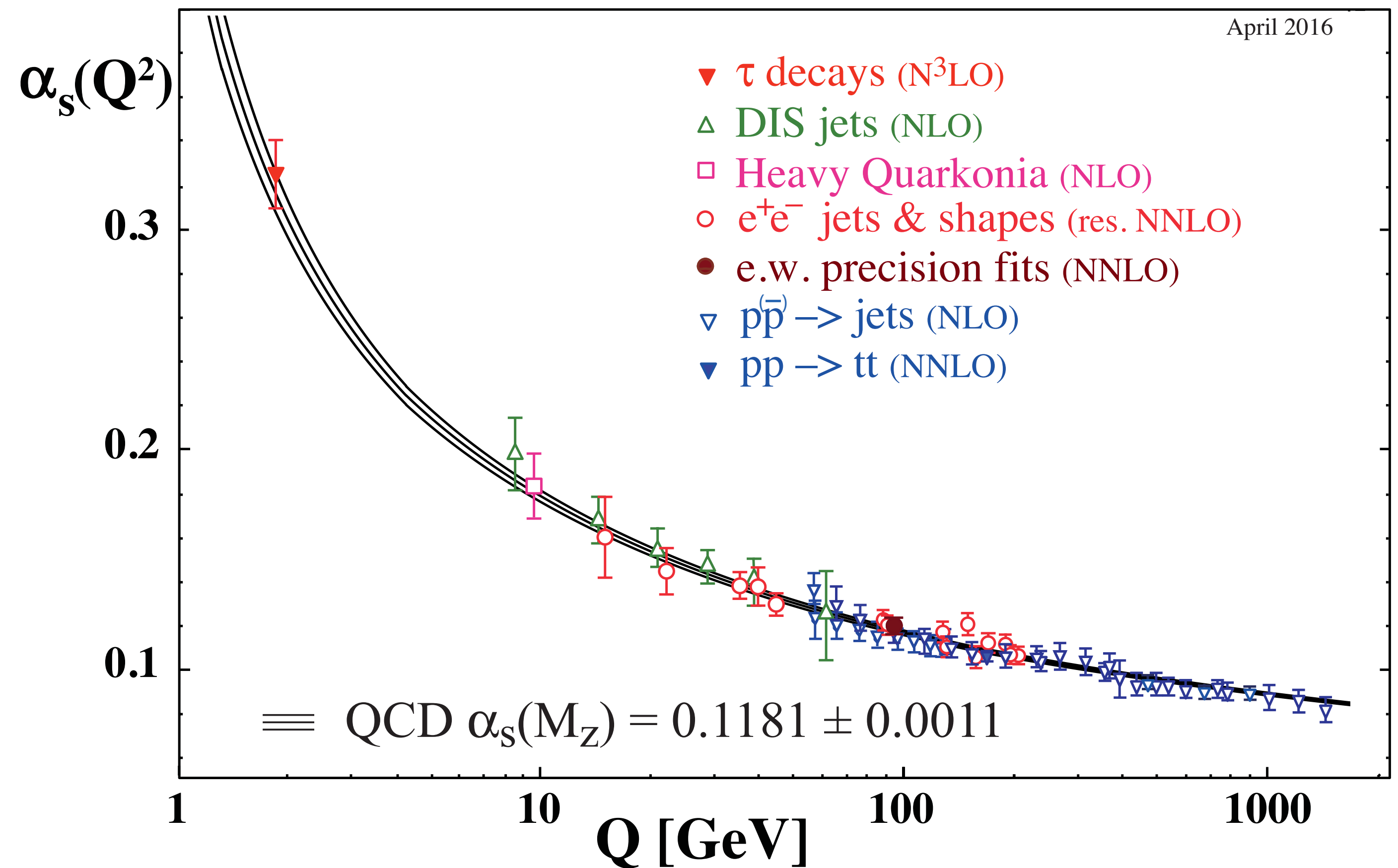
$$\text{Solve } Q^2 \frac{d\alpha_s}{dQ^2} \simeq -b_0 \alpha_s^2 \Rightarrow \alpha_s(Q^2) \simeq \frac{\alpha_s(Q_0^2)}{1 + b_0 \alpha_s(Q_0^2) \ln \frac{Q^2}{Q_0^2}} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$$

$\Lambda \approx 0.2 \text{ GeV}$  (aka  $\Lambda_{\text{QCD}}$ ) is the fundamental scale of QCD, at which perturbative coupling blows up.

- it sets the mass scale for most hadrons
- perturbation theory only valid for  $Q \gg \Lambda$ , where  $\alpha_s$  is small

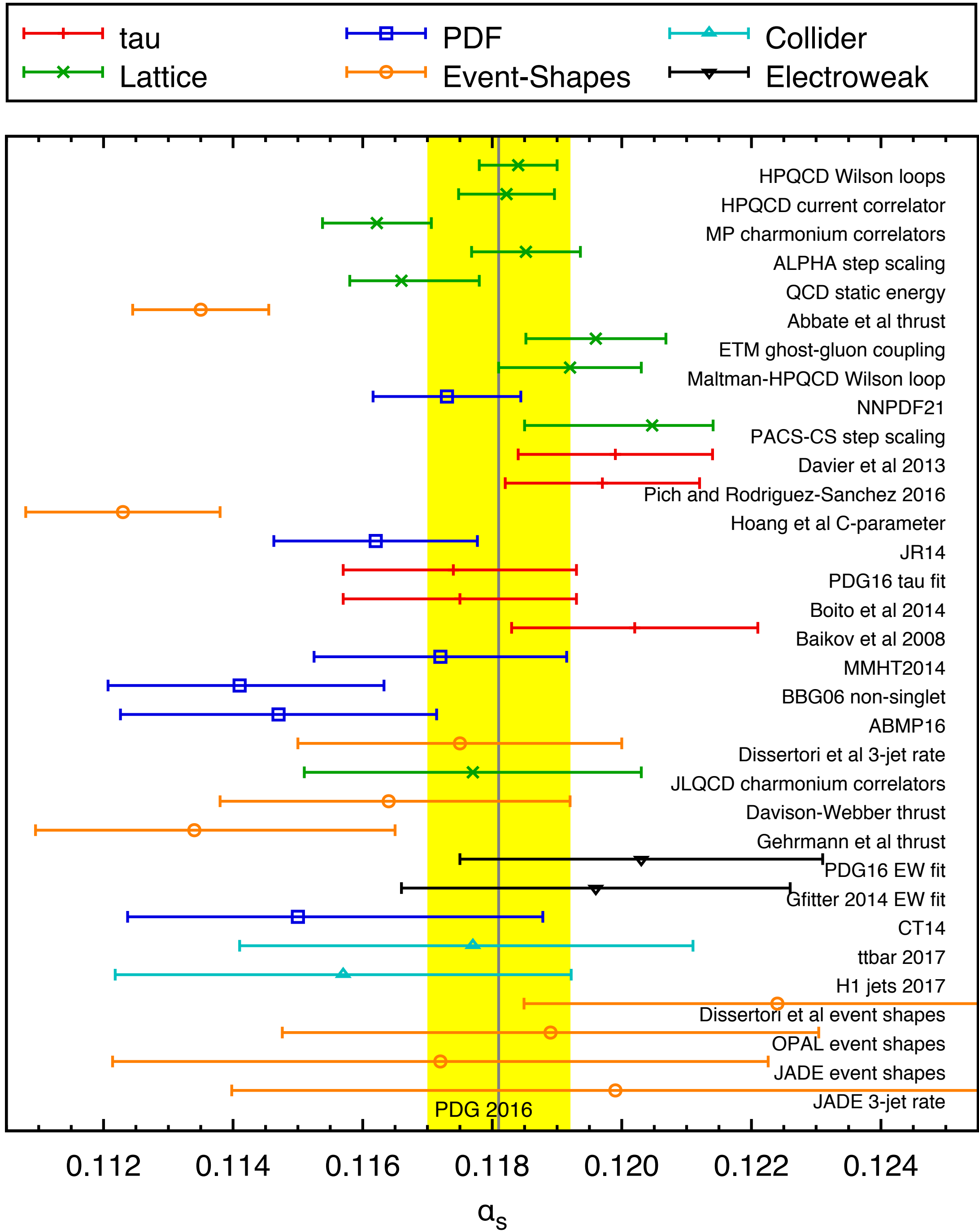
**PDG World Average:**

$$\alpha_s(M_Z) = 0.1181 \pm 0.0011 \text{ (0.9\%)}$$



# strong-coupling determinations

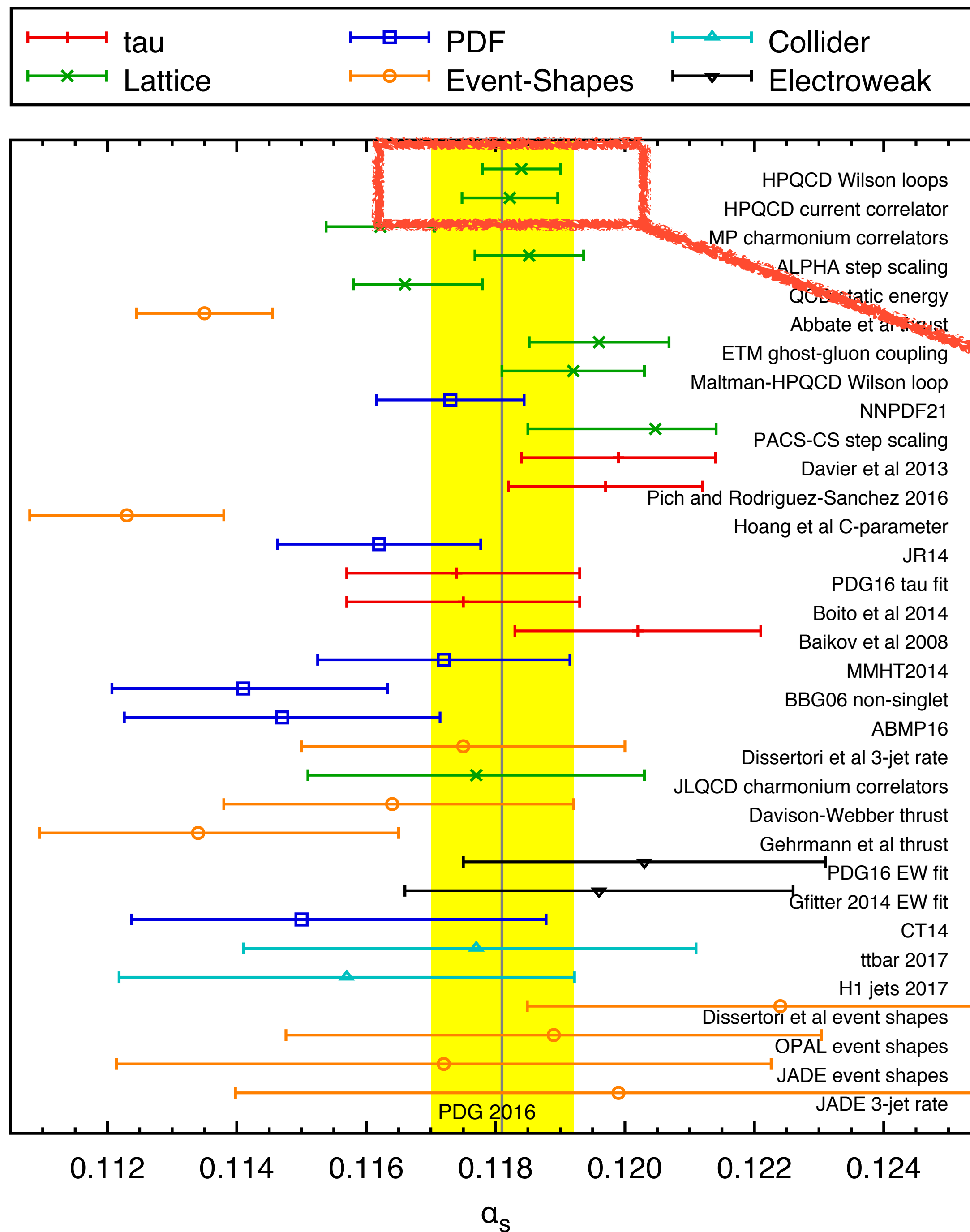
*Bethke, Dissertori & GPS in PDG '16*



- Most consistent set of independent determinations is from lattice
- Two determinations with smallest errors are from same group (HPQCD, 1004.4285, 1408.4169)
  - $\alpha_s(M_Z) = 0.1183 \pm 0.0007 (0.6\%)$  [heavy-quark correlators]
  - $\alpha_s(M_Z) = 0.1183 \pm 0.0007 (0.6\%)$  [Wilson loops]
- Many determinations quote small uncertainties ( $\approx 1\%$ ). Most are disputed!
- Most robust is perhaps ALPHA lattice result
  - $\alpha_s(M_Z) = 0.1185 \pm 0.00084 (0.7\%)$
- Some determinations quote anomalously small central values ( $\sim 0.113$  v. world avg. of  $0.1181 \pm 0.0011$ ). Also disputed

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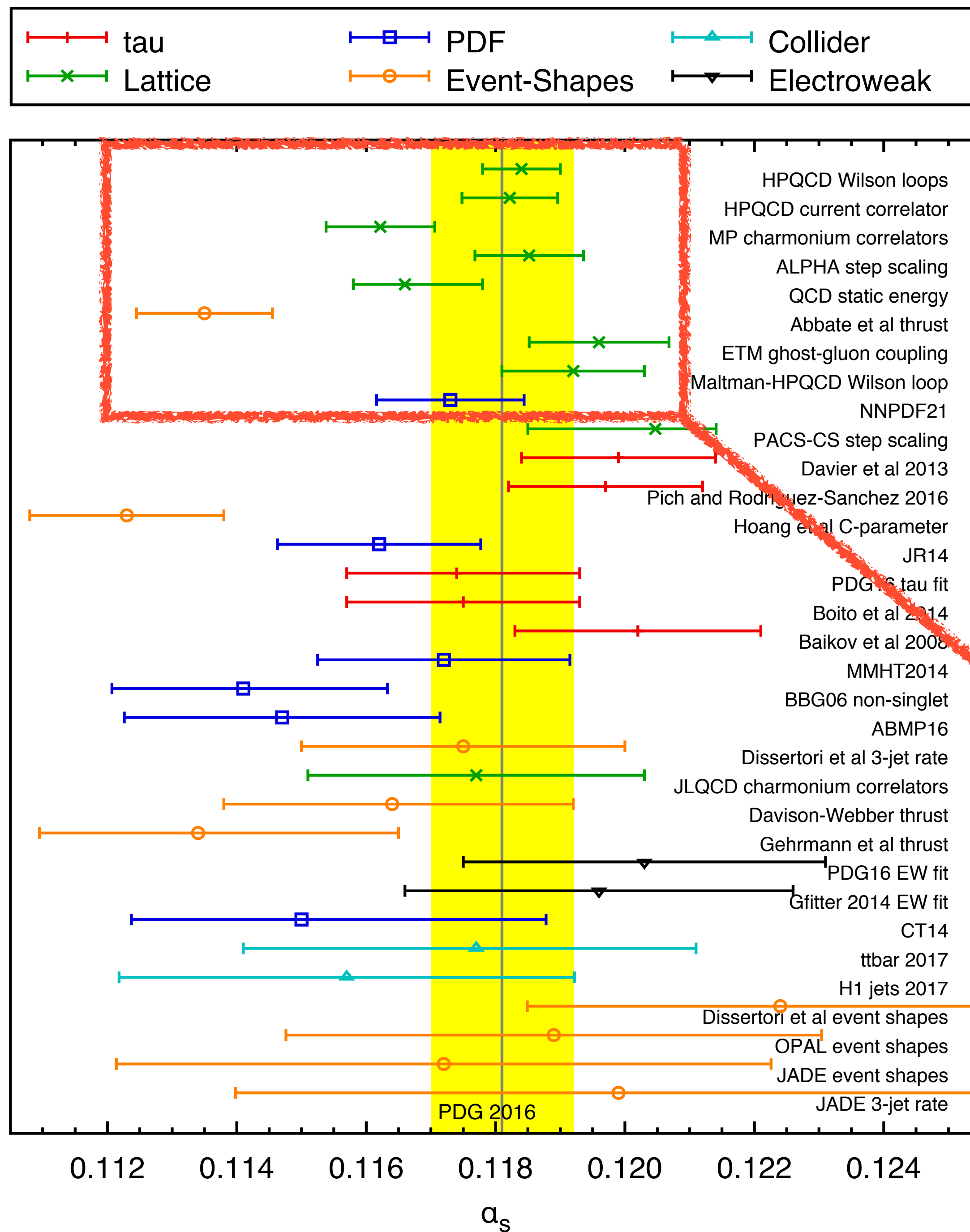
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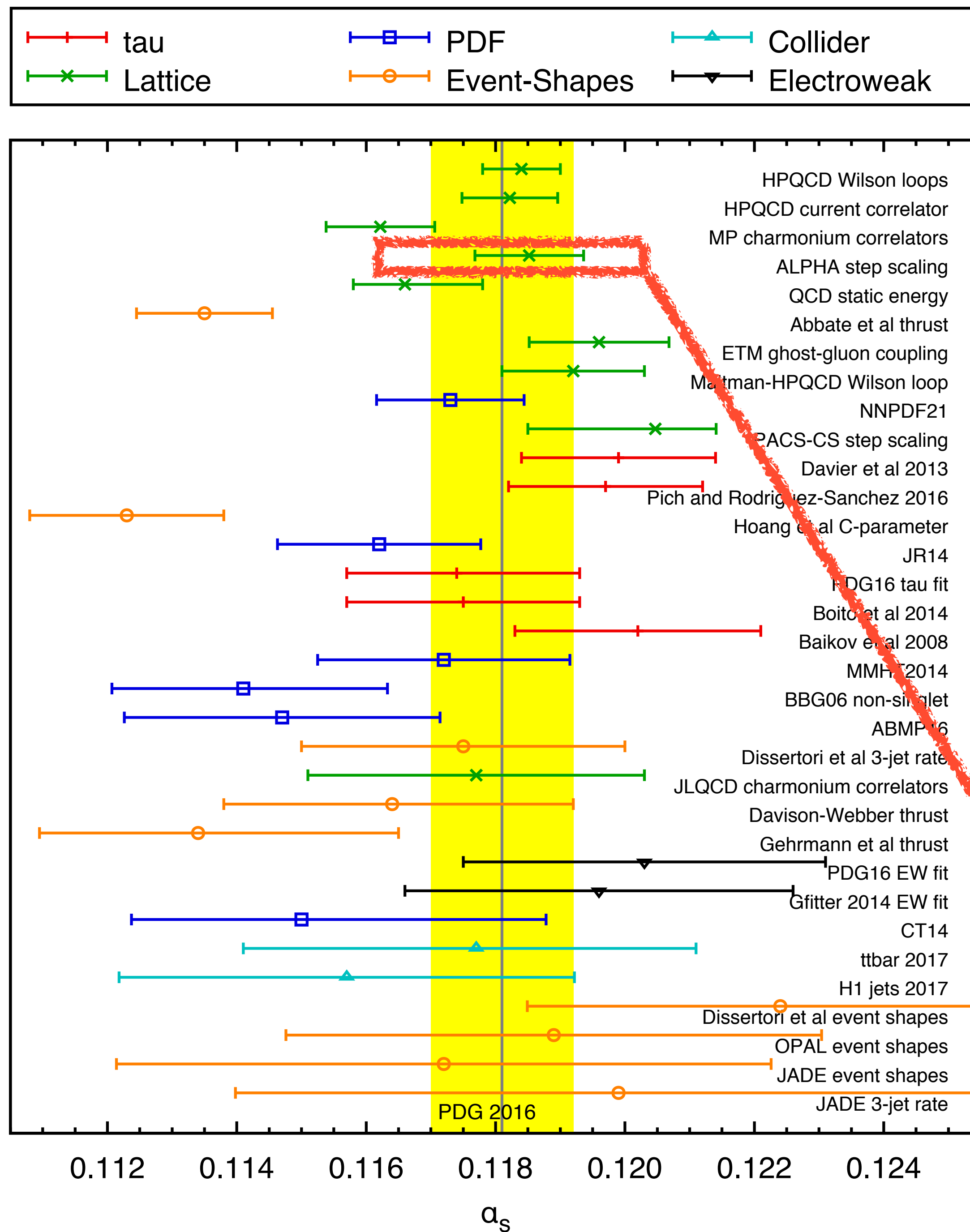
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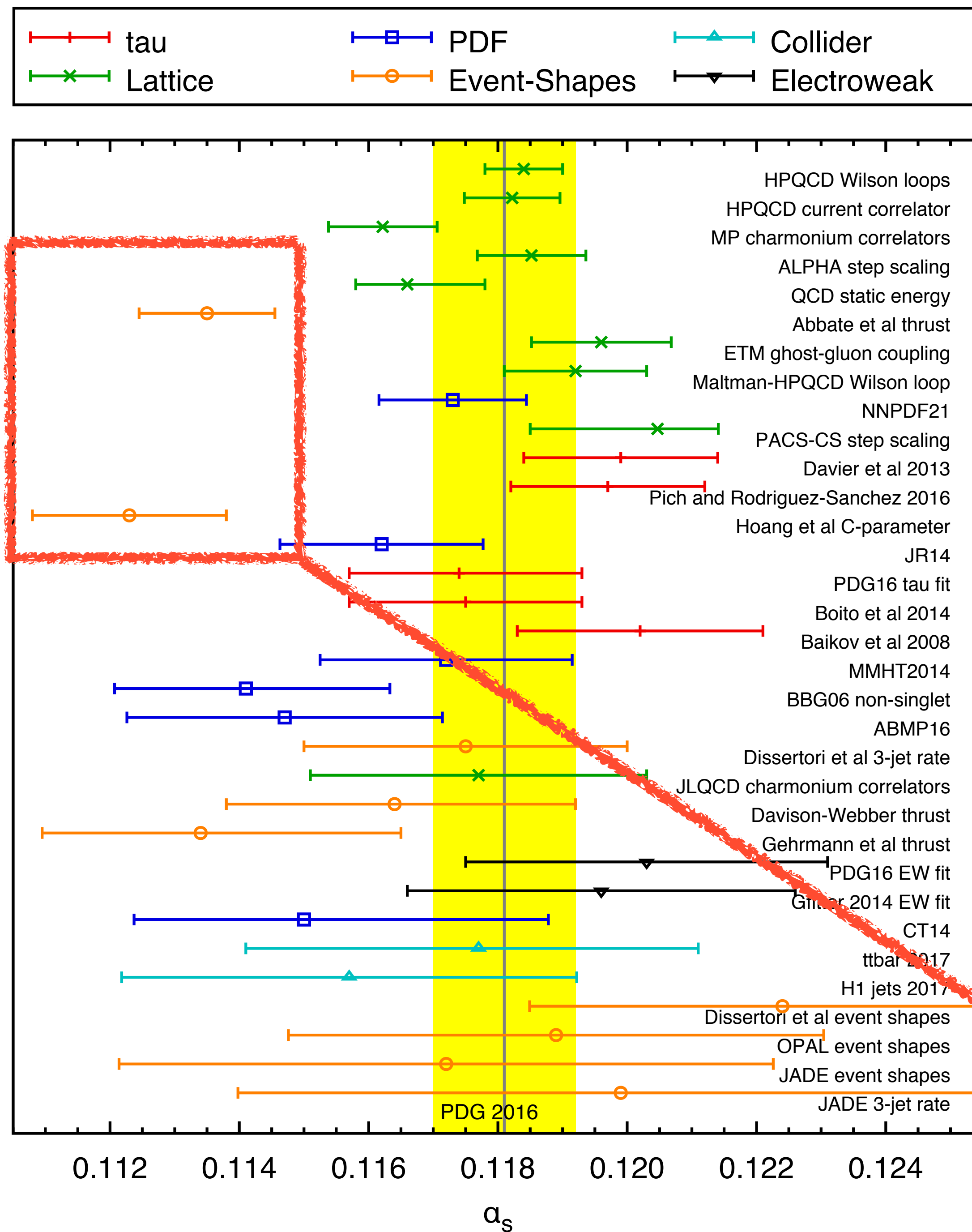
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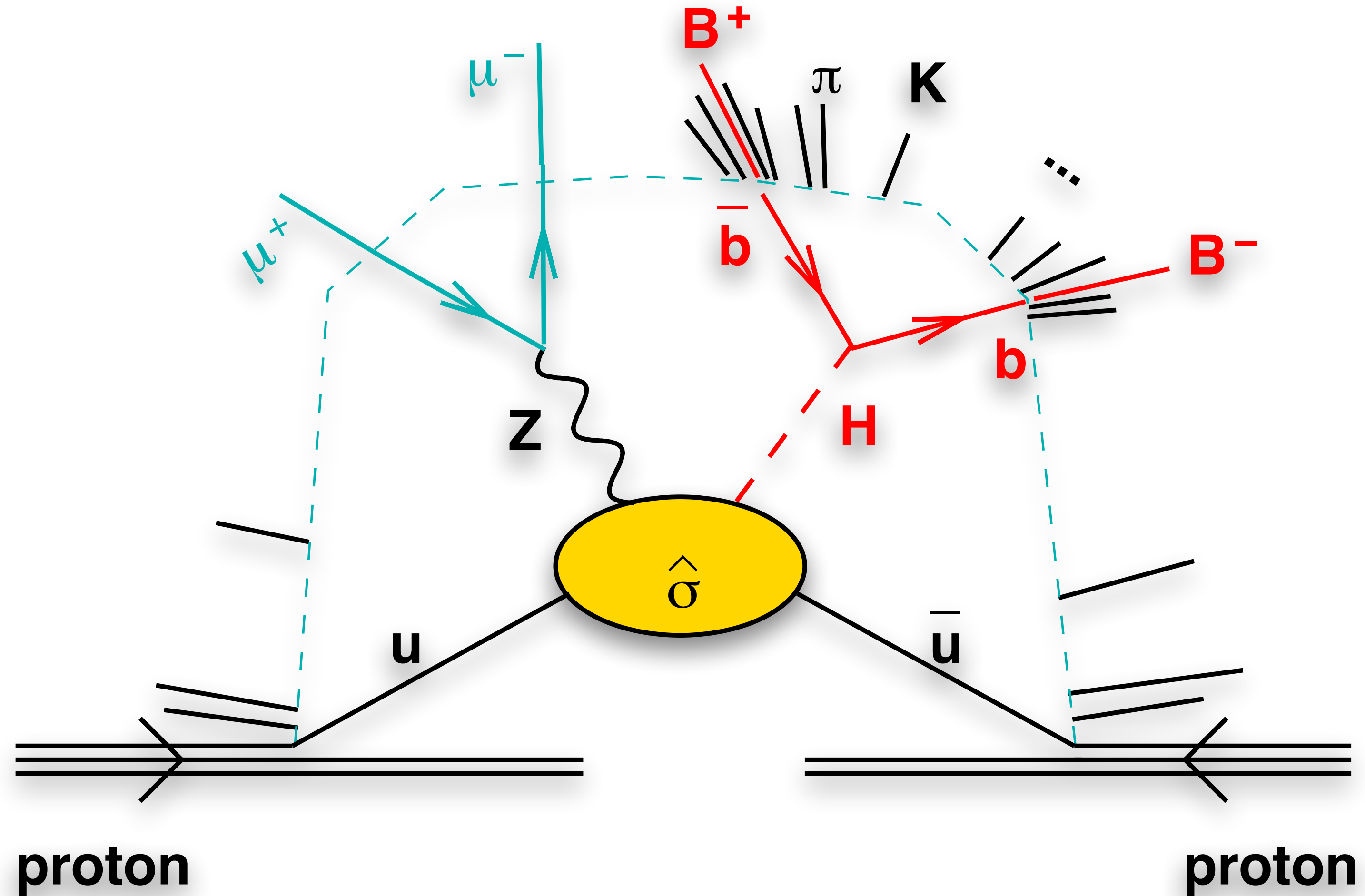
# factorisation

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*and perturbative expansions*

# a proton-proton collision: FILLING IN THE PICTURE

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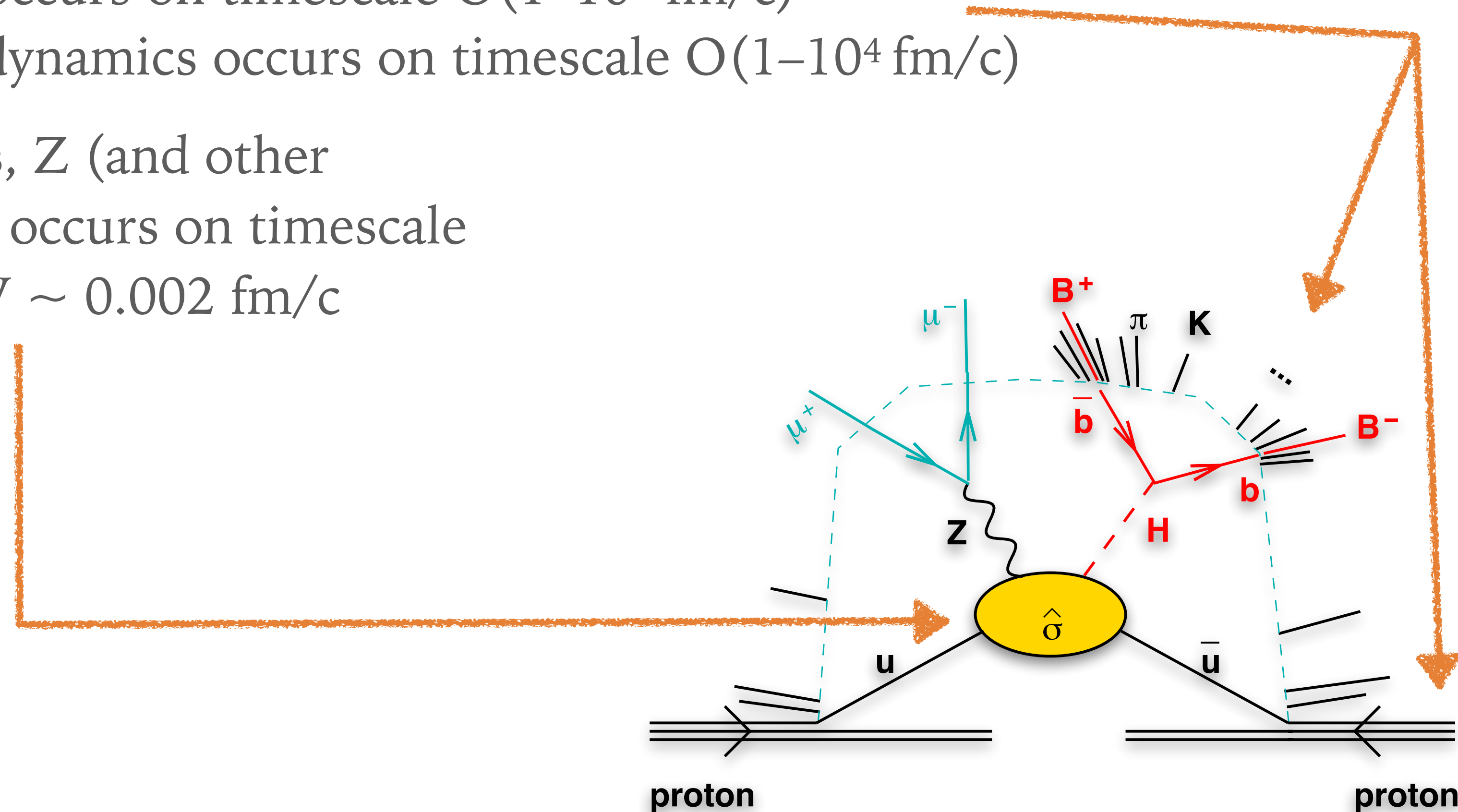


# Why is simplification “allowed”?

## key idea #1

## FACTORISATION

- ▶ Proton’s dynamics occurs on timescale  $O(1-10^4 \text{ fm}/c)$   
Final-state hadron dynamics occurs on timescale  $O(1-10^4 \text{ fm}/c)$
- ▶ Production of Higgs, Z (and other “**hard processes**”) occurs on timescale  $1/M_H \sim 1/125 \text{ GeV} \sim 0.002 \text{ fm}/c$



That means we can separate — “**factorise**” — the hard process, i.e. treat it as independent from all the hadronic dynamics

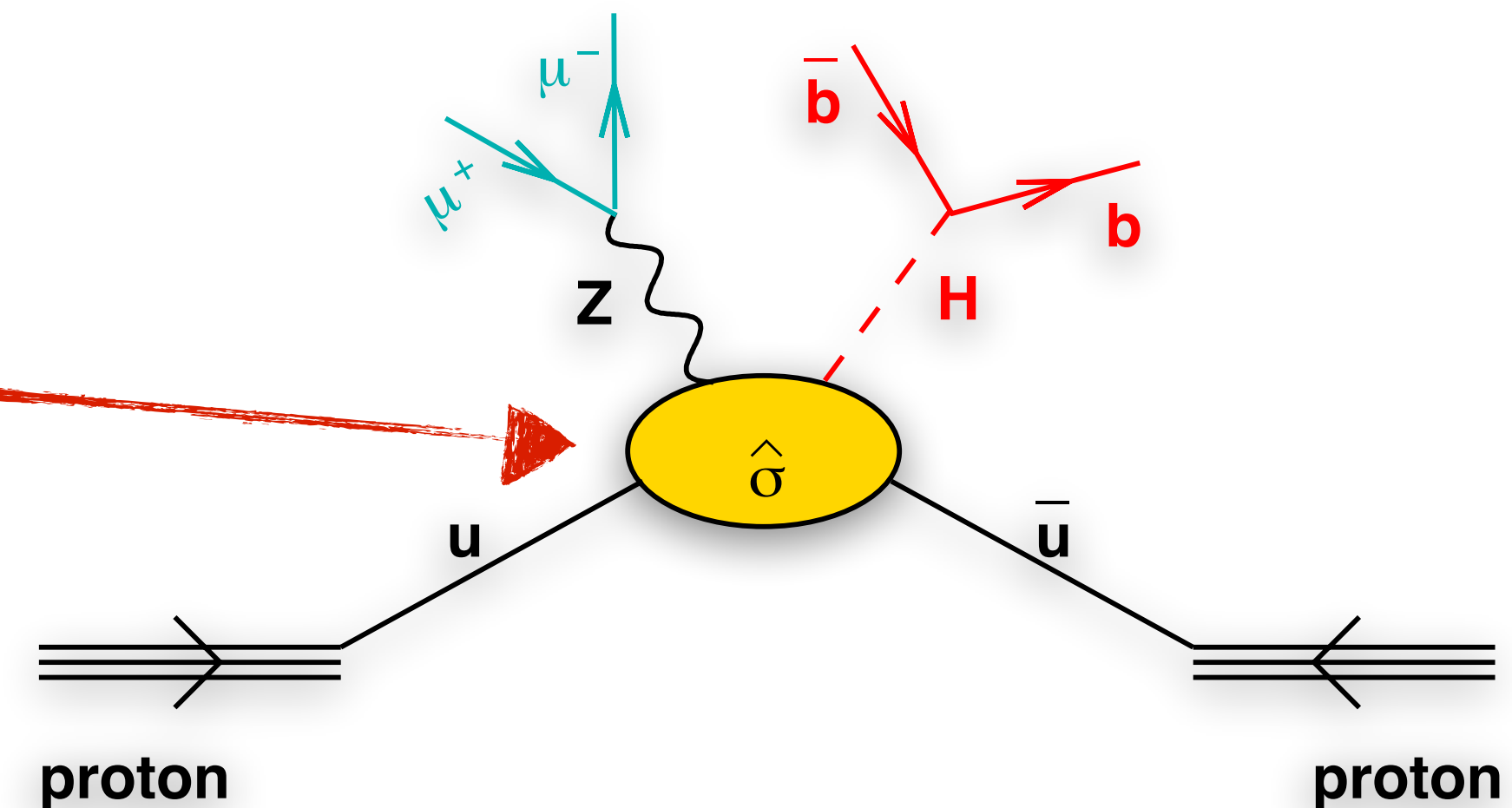
# Why is simplification “allowed”?

# key idea #2 USE PERTURBATION THEORY

- ▶ On timescales  $1/M_H \sim 1/125 \text{ GeV} \sim 0.002 \text{ fm}$  you can take advantage of **asymptotic freedom**
- ▶ i.e. you can write results in terms of an expansion in the (*not so*) strong coupling constant  $\alpha_s(125 \text{ GeV}) \sim 0.11$

$$\hat{\sigma} = \hat{\sigma}_0 (\mathbf{1} + c_1 \alpha_s + c_2 \alpha_s^2 + \dots)$$

**LO**  
(Leading Order)



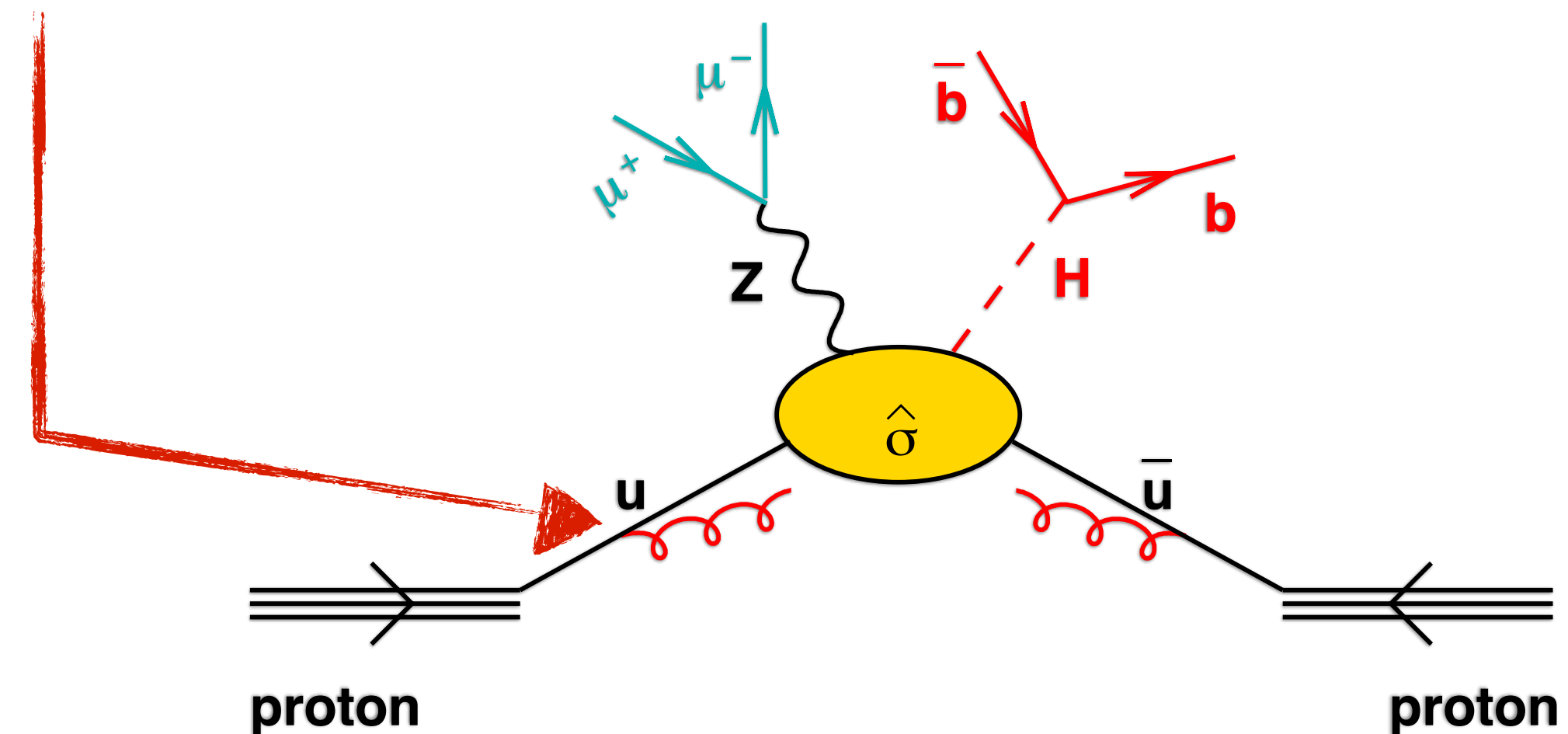
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# key idea #2 short-distance QCD corrections are perturbative

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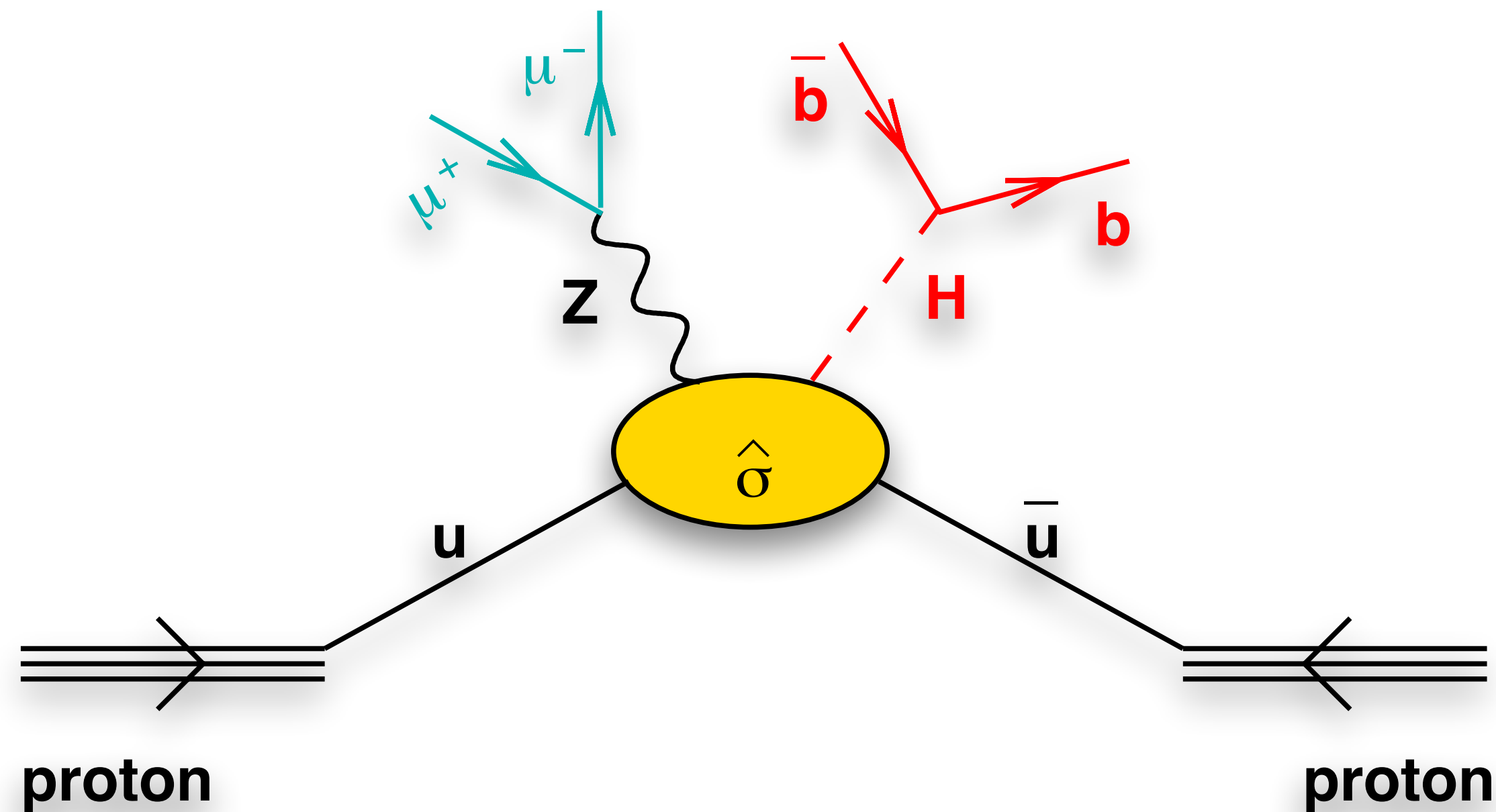
**NNLO**  
(Next-to-next-to-Leading Order)



# the master equation

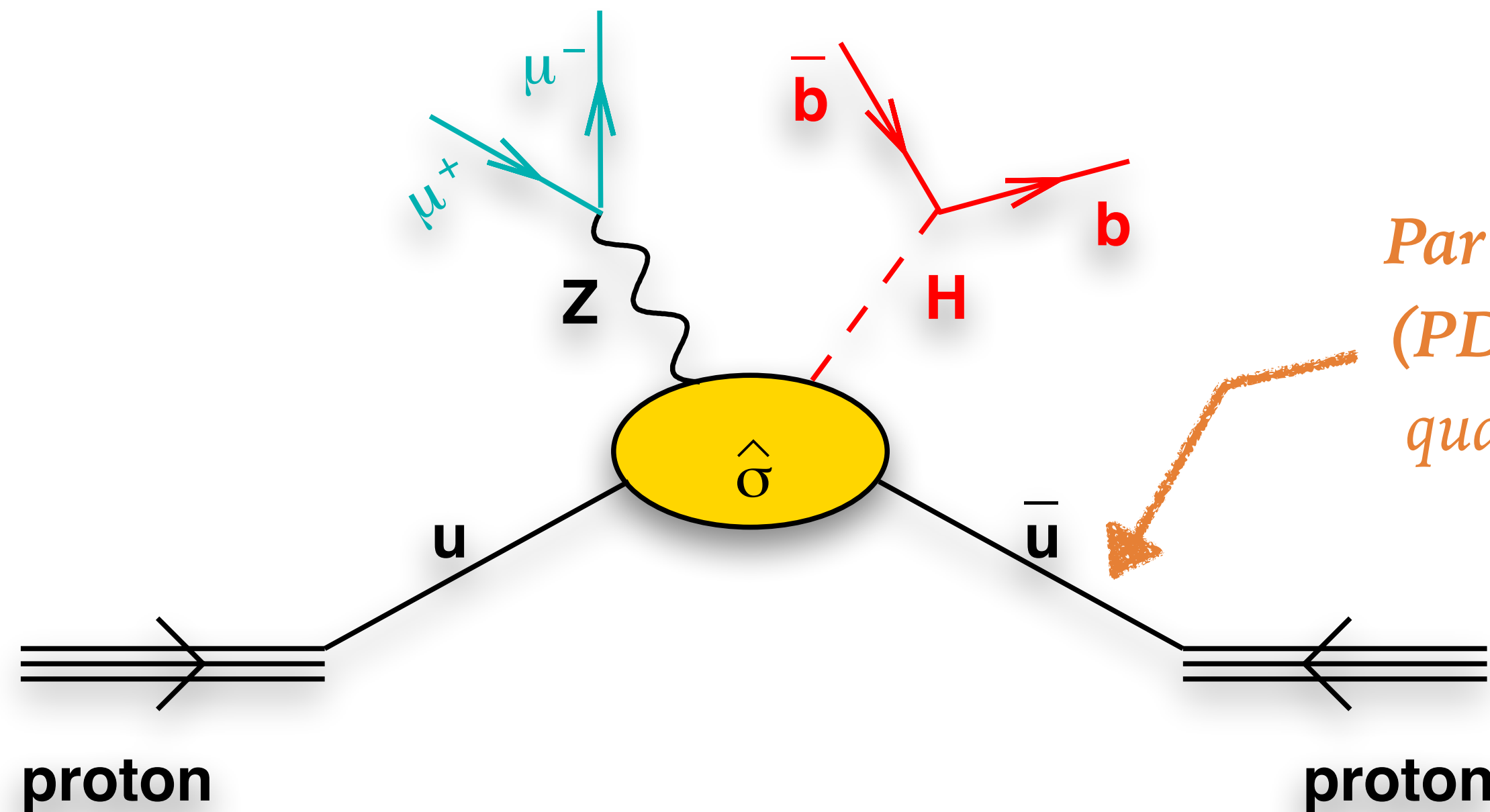
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$$\sigma(h_1 h_2 \rightarrow ZH + X) = \sum_{n=0}^{\infty} \alpha_s^n(\mu_R^2) \sum_{i,j} \int dx_1 dx_2 f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \times \hat{\sigma}_{ij \rightarrow ZH+X}^{(n)}(x_1 x_2 s, \mu_R^2, \mu_F^2) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right),$$



# the master equation

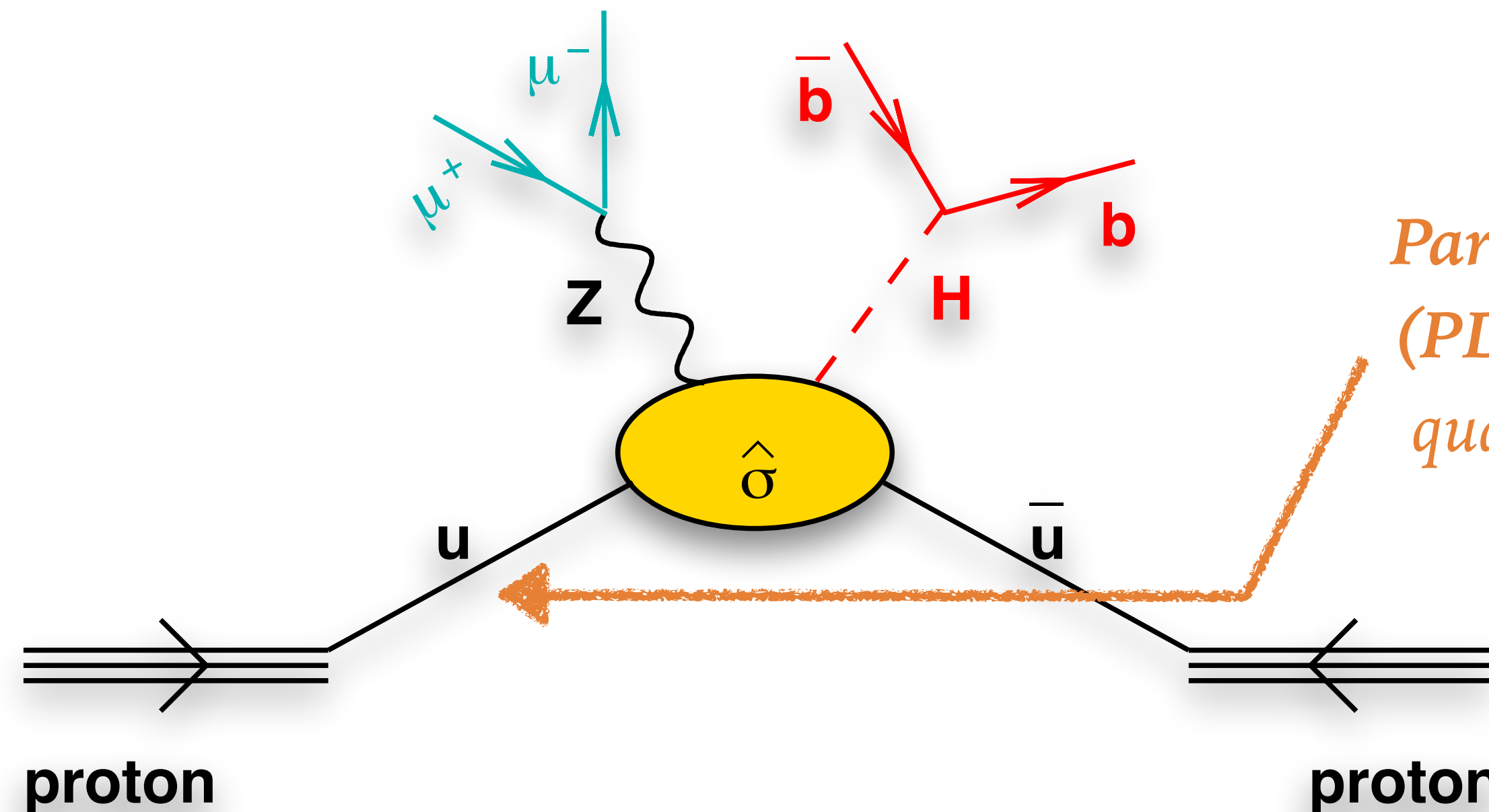
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*Parton distribution function (PDF): e.g. number of up anti-quarks carrying fraction  $x_2$  of proton's momentum*

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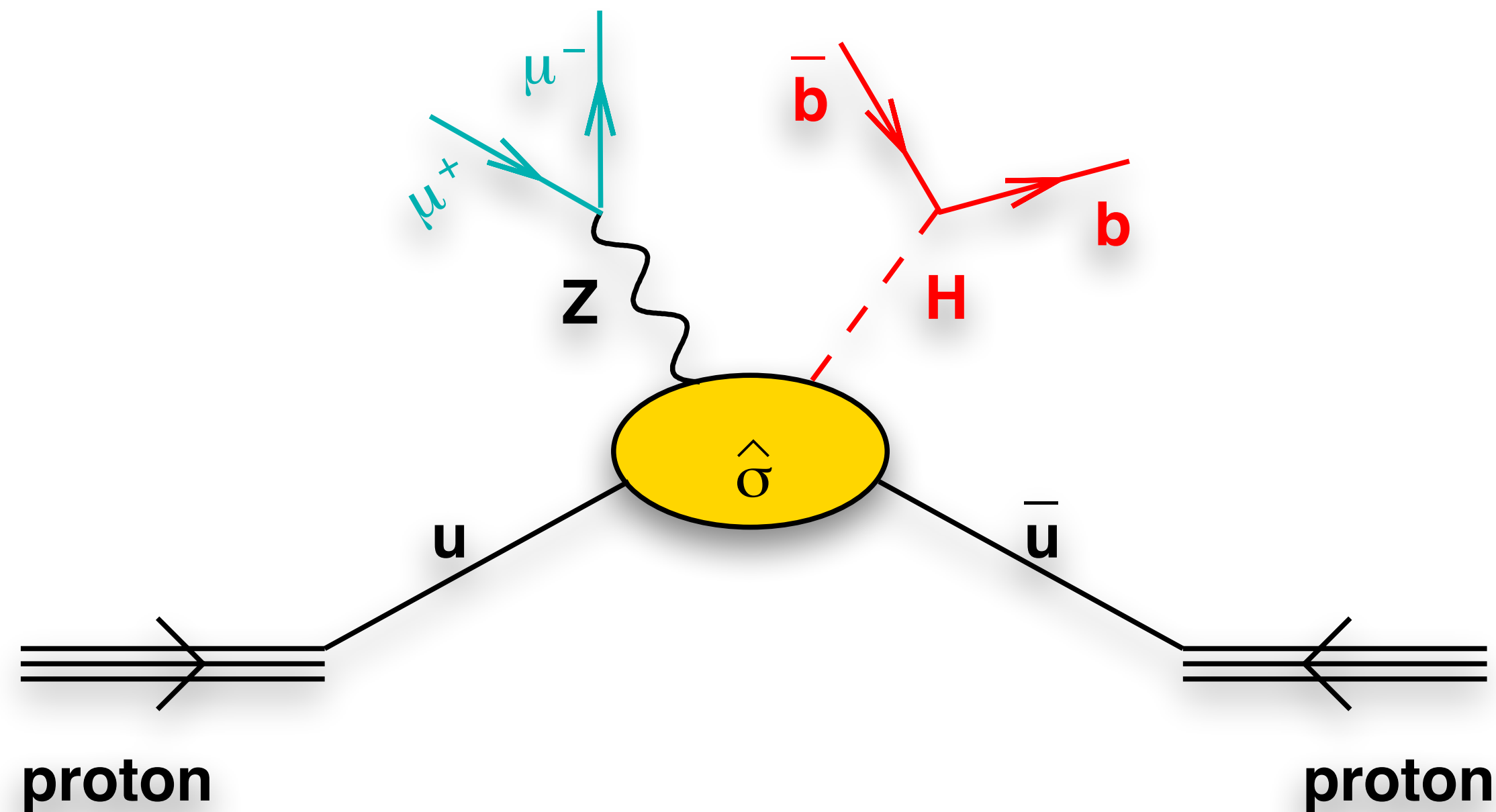
*Parton distribution function (PDF): e.g. number of up quarks carrying fraction  $x_1$  of proton's momentum*



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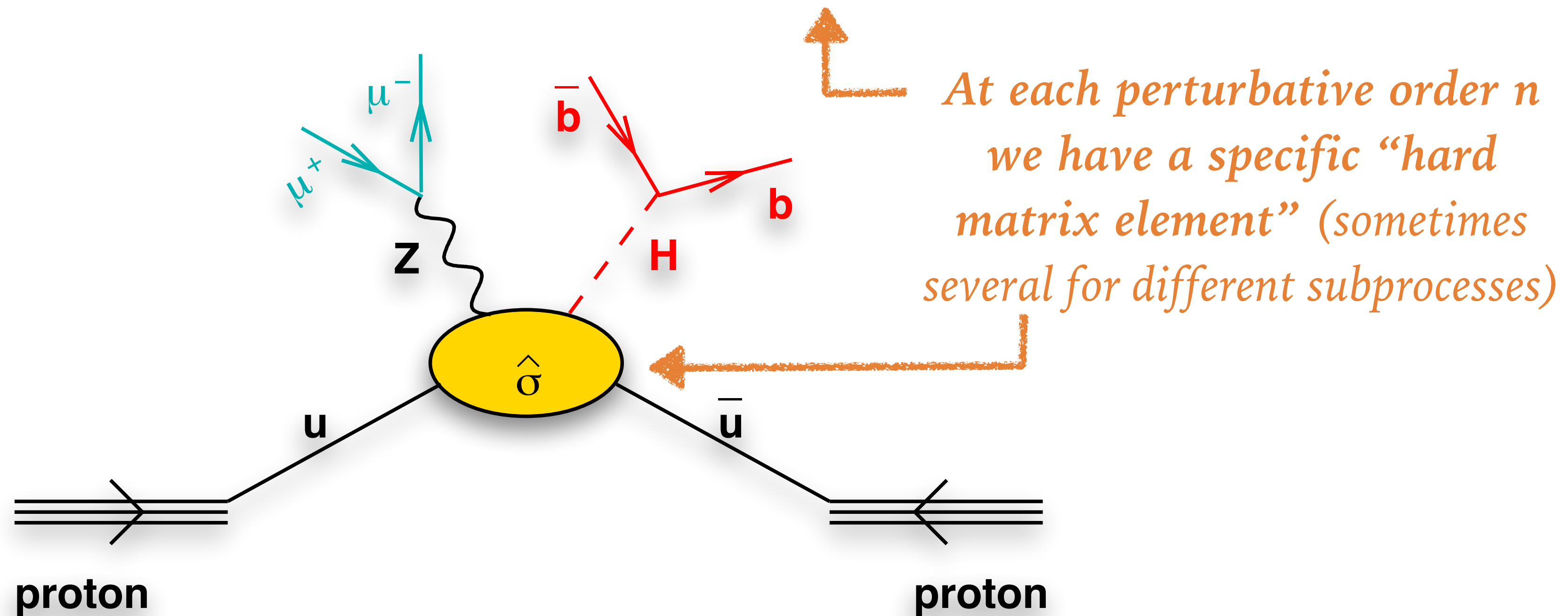
*Perturbative sum over powers of the strong coupling: typically we use first 2-4 orders*

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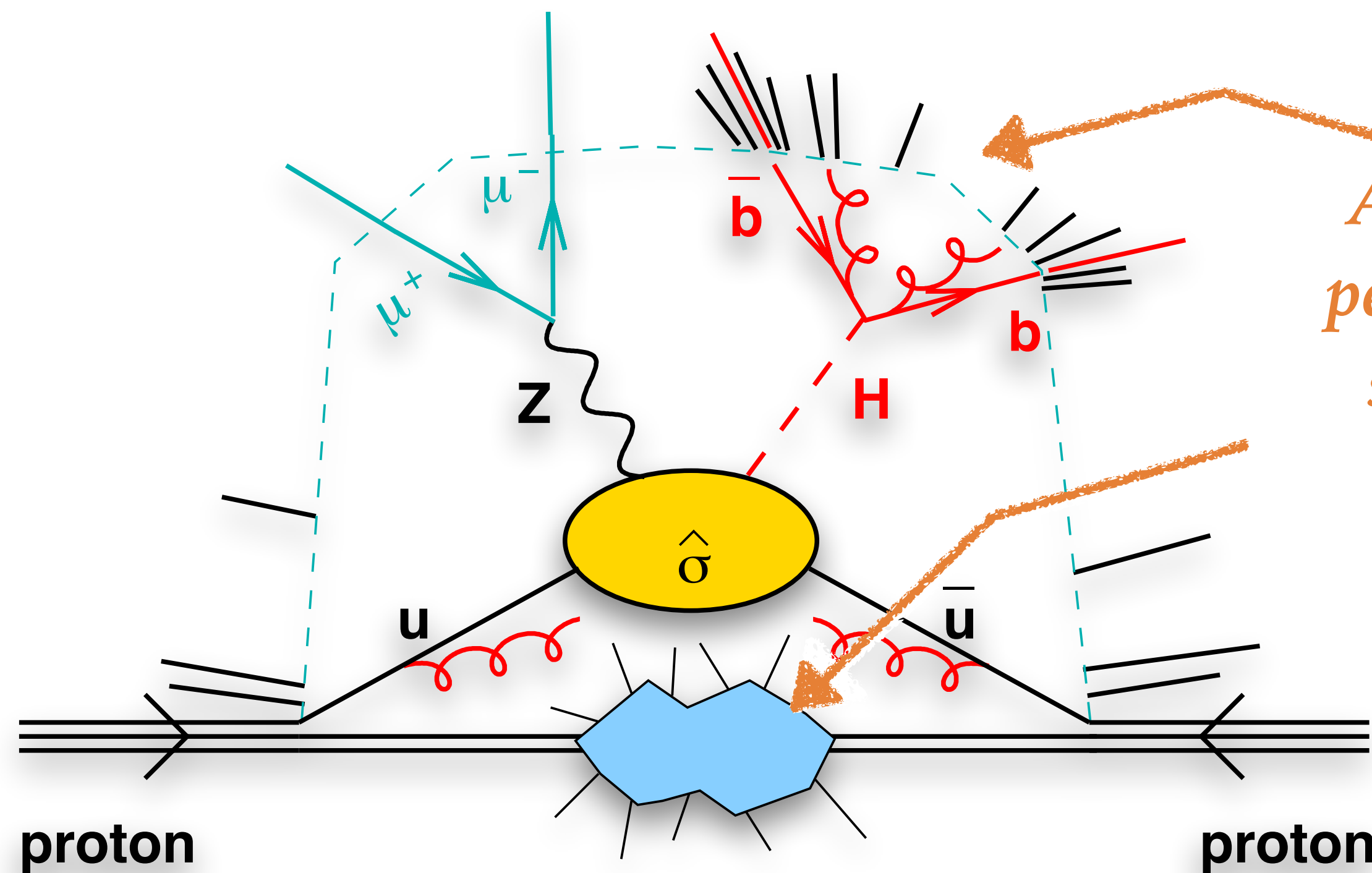
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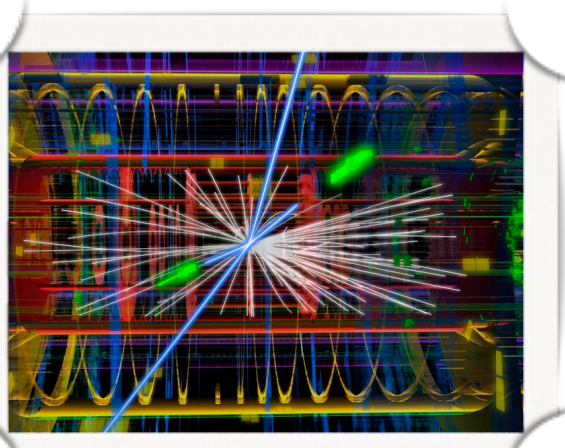
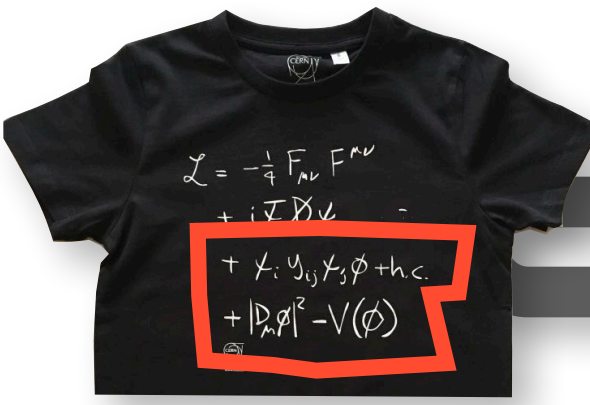


*Additional corrections from non-perturbative effects: higher “twist”, suppressed by powers of QCD scale ( $\Lambda$ ) / hard scale*

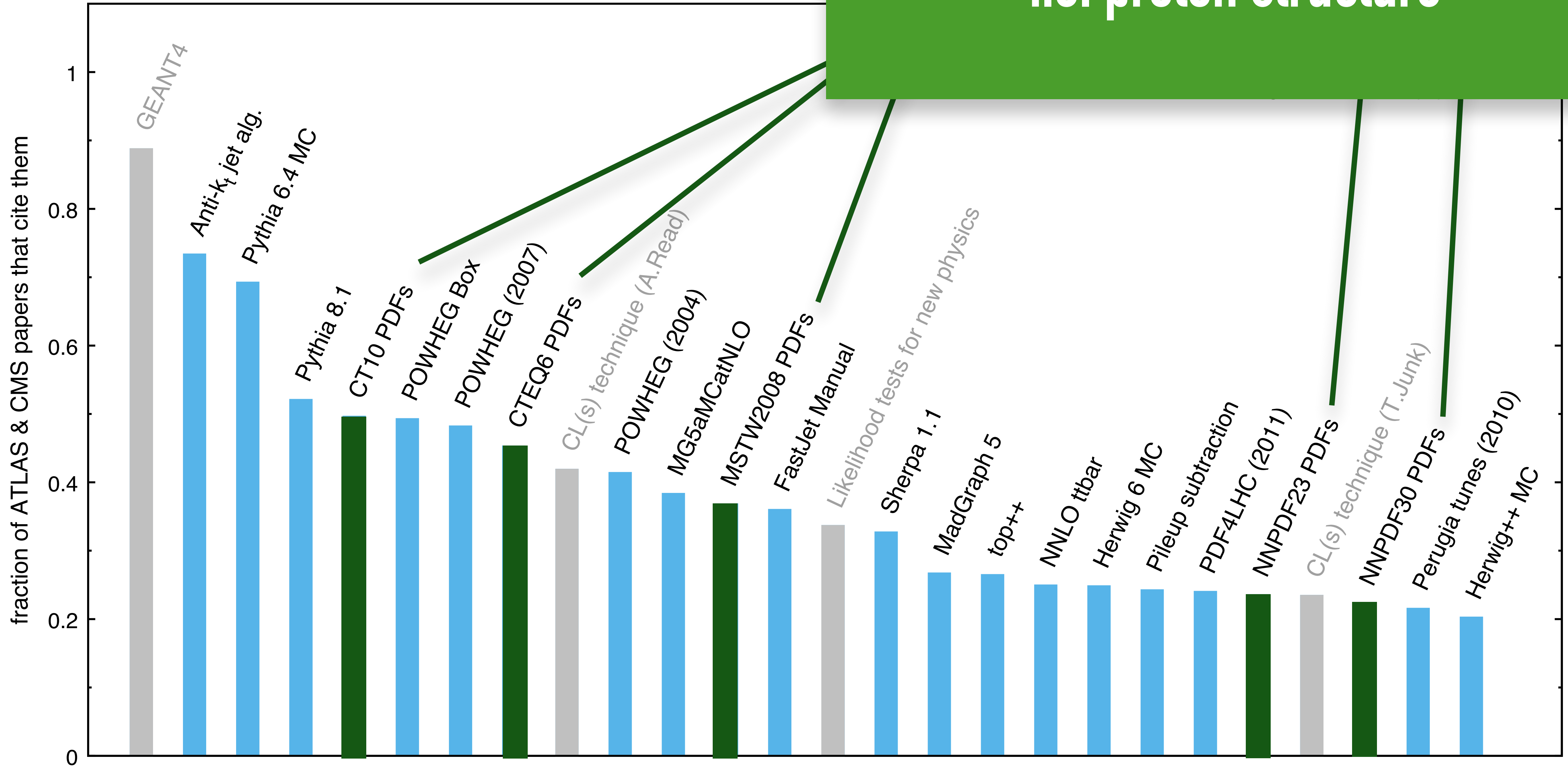
# parton distribution functions (PDFs)

For visualisations of PDFs and related quantities,  
a good place to start is

<http://apfel.mi.infn.it/> (ApfelWeb)



# knowing what goes into a collision i.e. proton structure

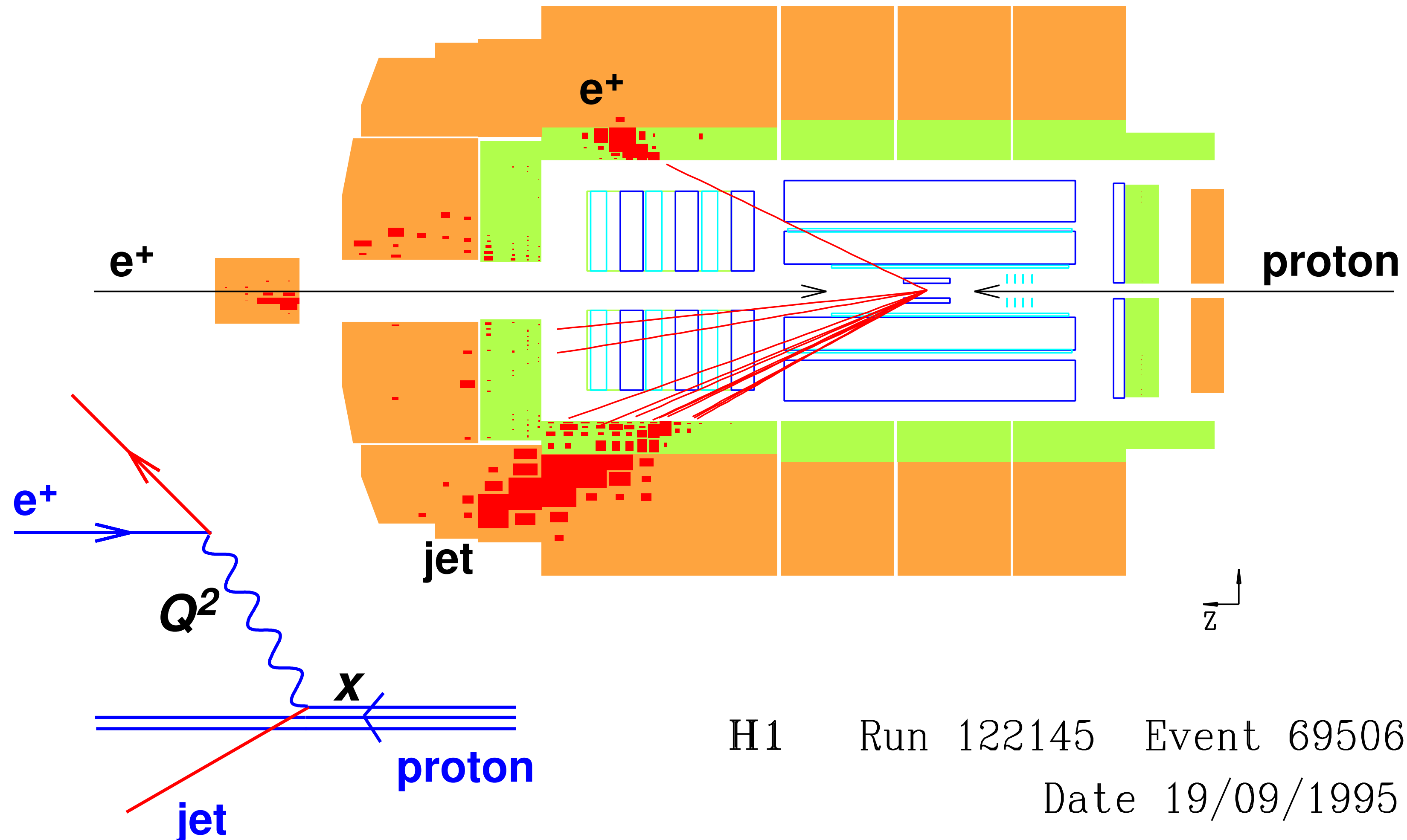


Plot by GP Salam based on data from InspireHEP

# Deep Inelastic Scattering — the simpler context to determine PDFs



$$Q^2 = 25030 \text{ GeV}^2; \quad y = 0.56; \quad \mathbf{x=0.50}$$



two major kinematic variables:

$x \approx$  longitudinal momentum fraction of struck quark

$Q^2 \approx$  photon virtuality  $\rightarrow$  transverse resolution at which is probes proton structure

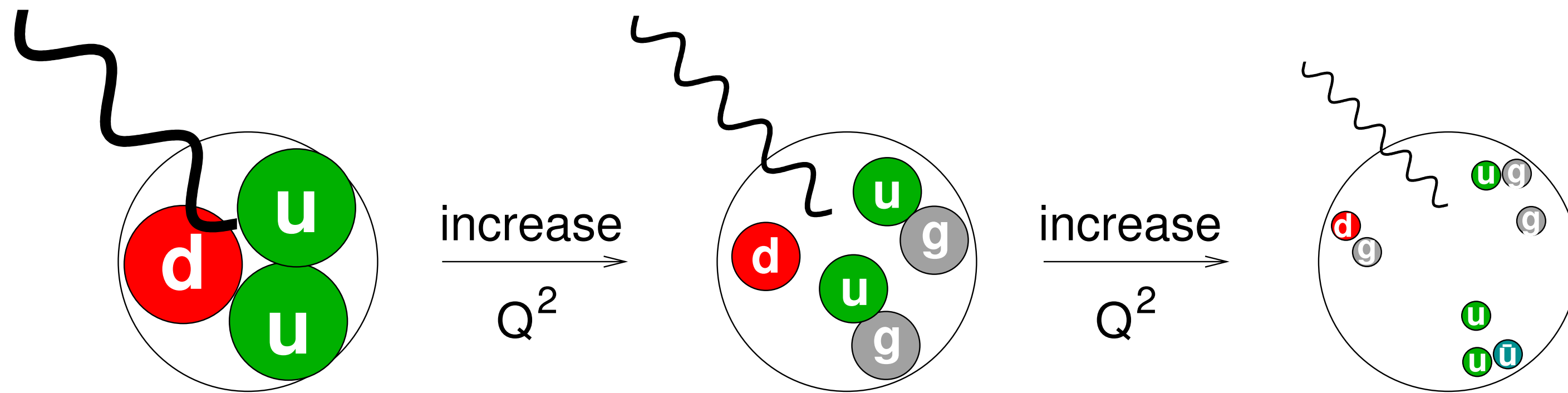
# Parton distribution and DGLAP

---

- Write up-quark distribution in proton as

$$f_{u/p}(x, \mu_F^2)$$

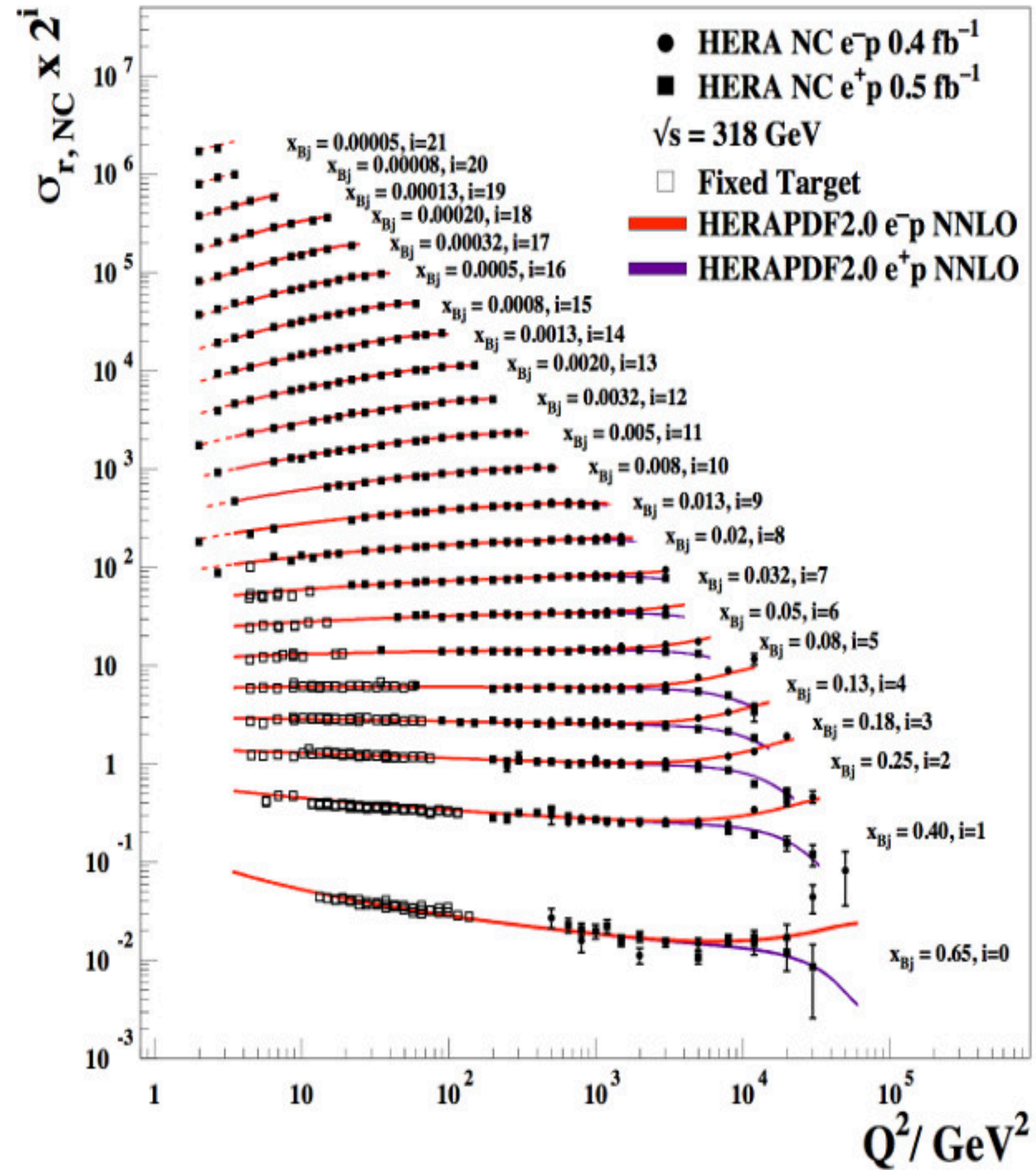
- $\mu_F$  is the **factorisation scale** — a bit like the renormalisation scale ( $\mu_R$ ) for the running coupling.
- As you vary the factorisation scale, the parton distributions evolve with a renormalisation-group type equation



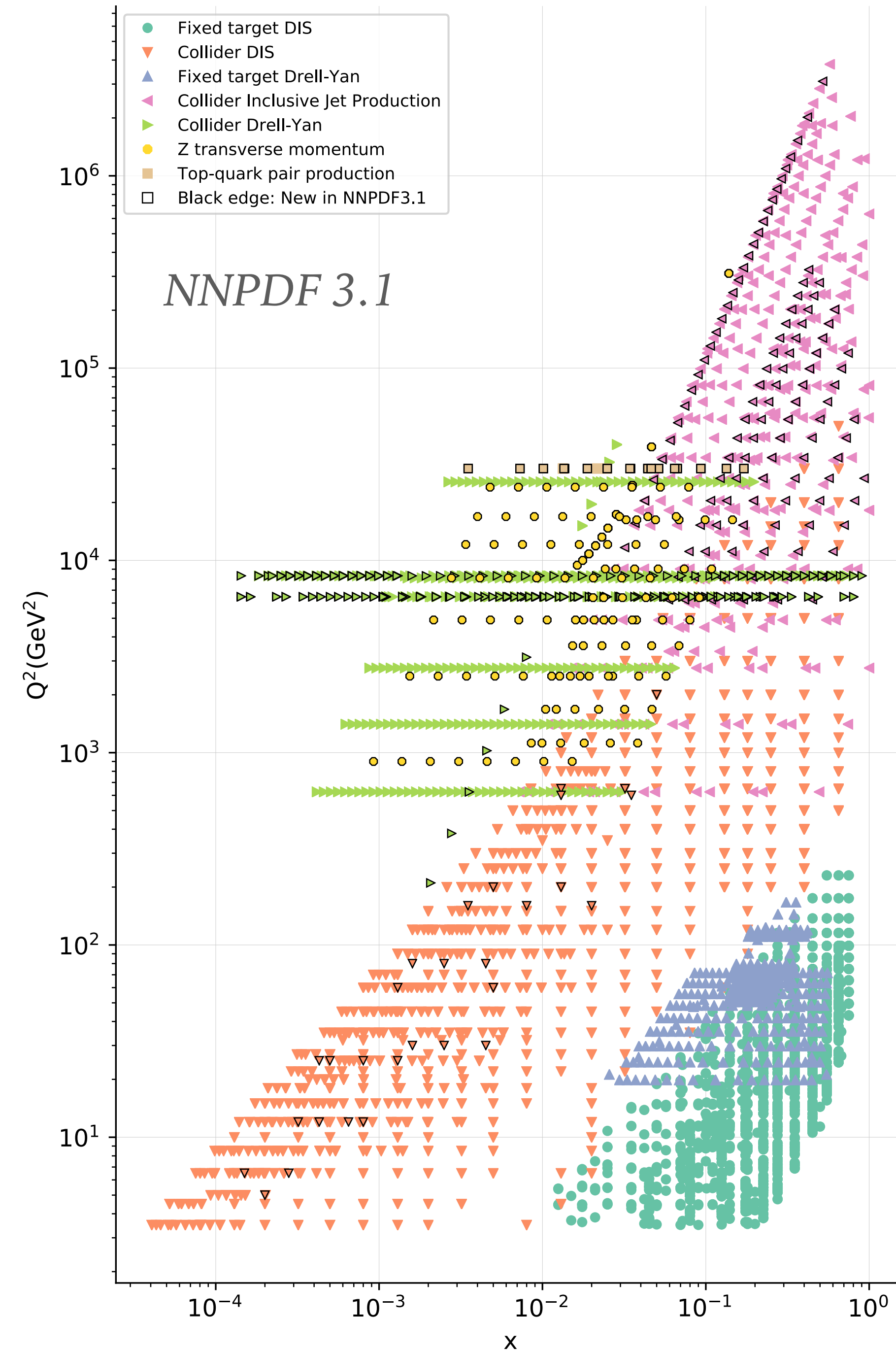
Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations

# Today's PDF fits

## H1 and ZEUS



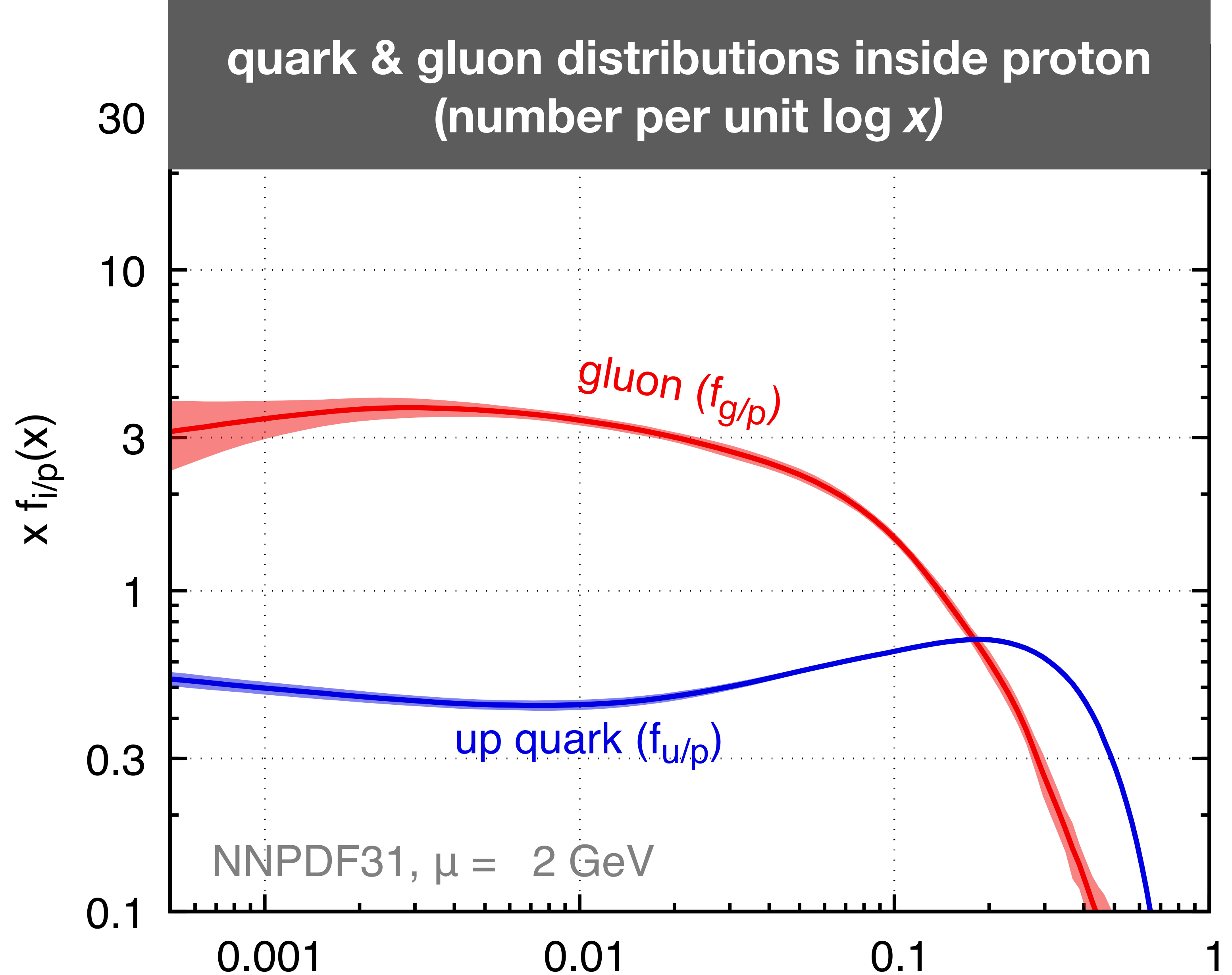
## Kinematic coverage





# Today's PDF fits

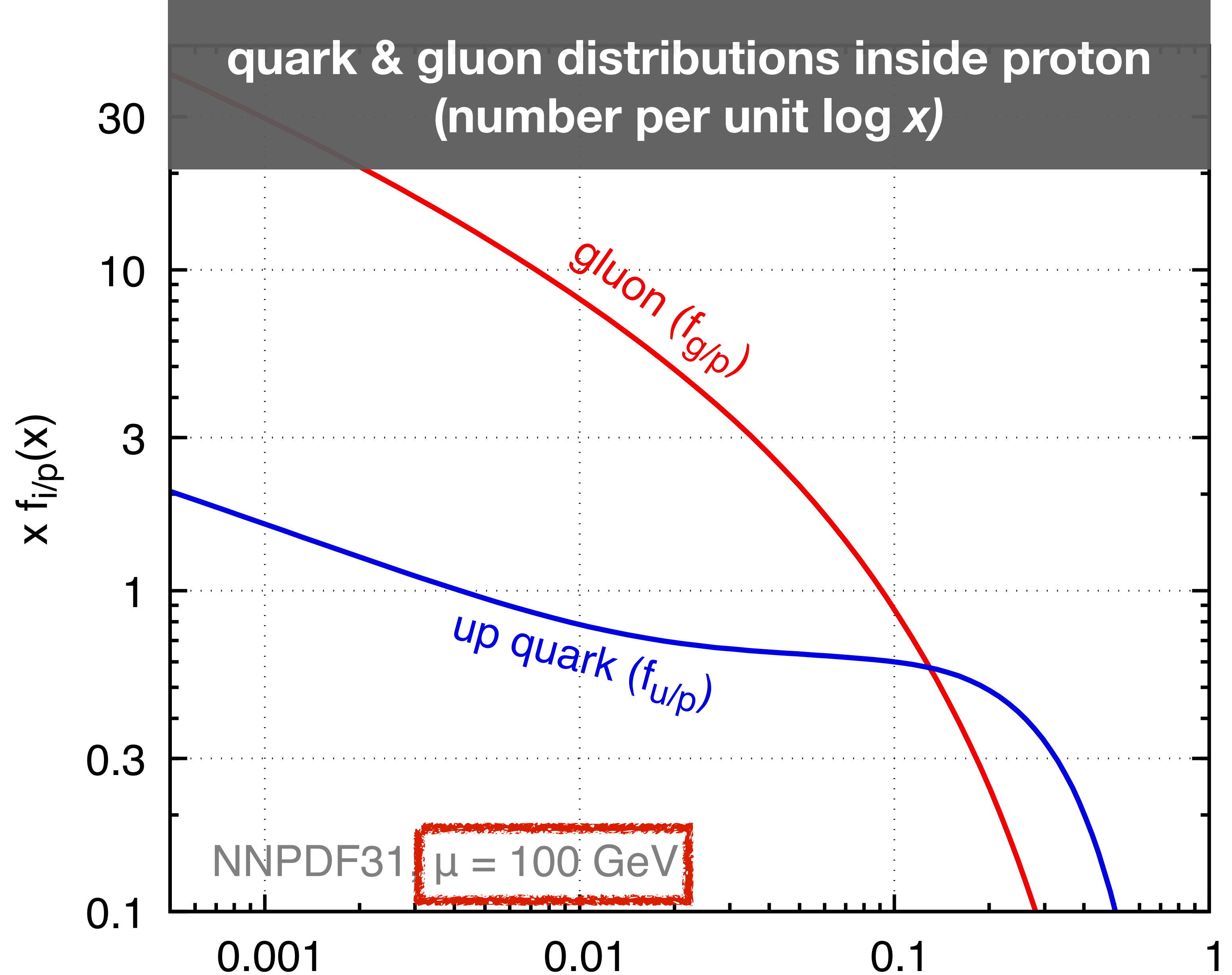
- ▶ LHC EW physics probes  $x \sim m_H/\sqrt{s} \sim 0.01$
- ▶ gluon distribution is  $\sim 10\times$  larger than (up) quark distribution



$x$  = fraction of proton momentum  
carried by quark/gluon

# Today's PDF fits

- ▶ LHC EW physics probes  $x \sim m_H/\sqrt{s} \sim 0.01$
- ▶ gluon distribution is  $\sim 10\times$  larger than (up) quark distribution
- ▶ viewing proton at scales from 2 GeV to 100 GeV, DGLAP evolution modifies PDFs by  $\sim \times 2-10$



$x$  = fraction of proton momentum  
carried by quark/gluon

# fixed-order calculations

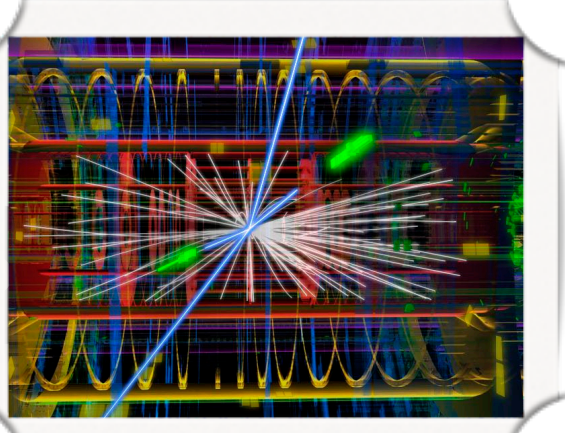
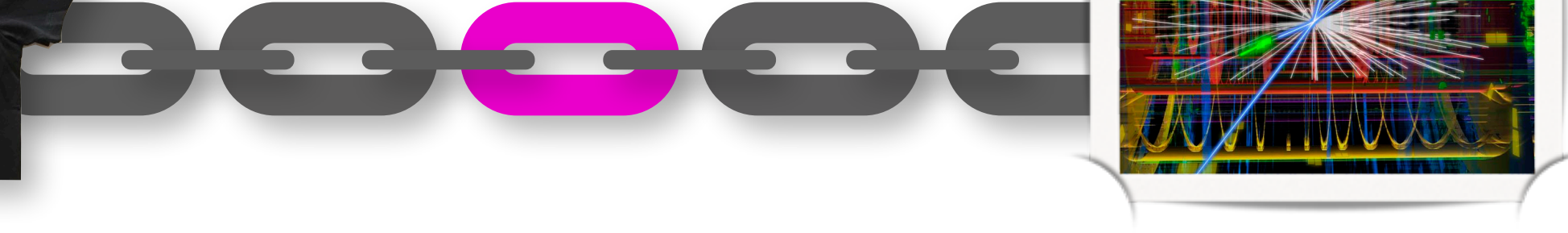
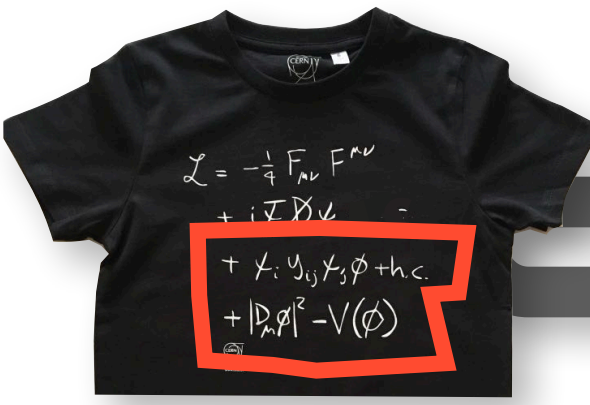
$$\sigma \sim \sigma_2 \alpha_s^2 + \sigma_3 \alpha_s^3 + \sigma_4 \alpha_s^4 + \sigma_5 \alpha_s^5 + \dots$$

**LO**

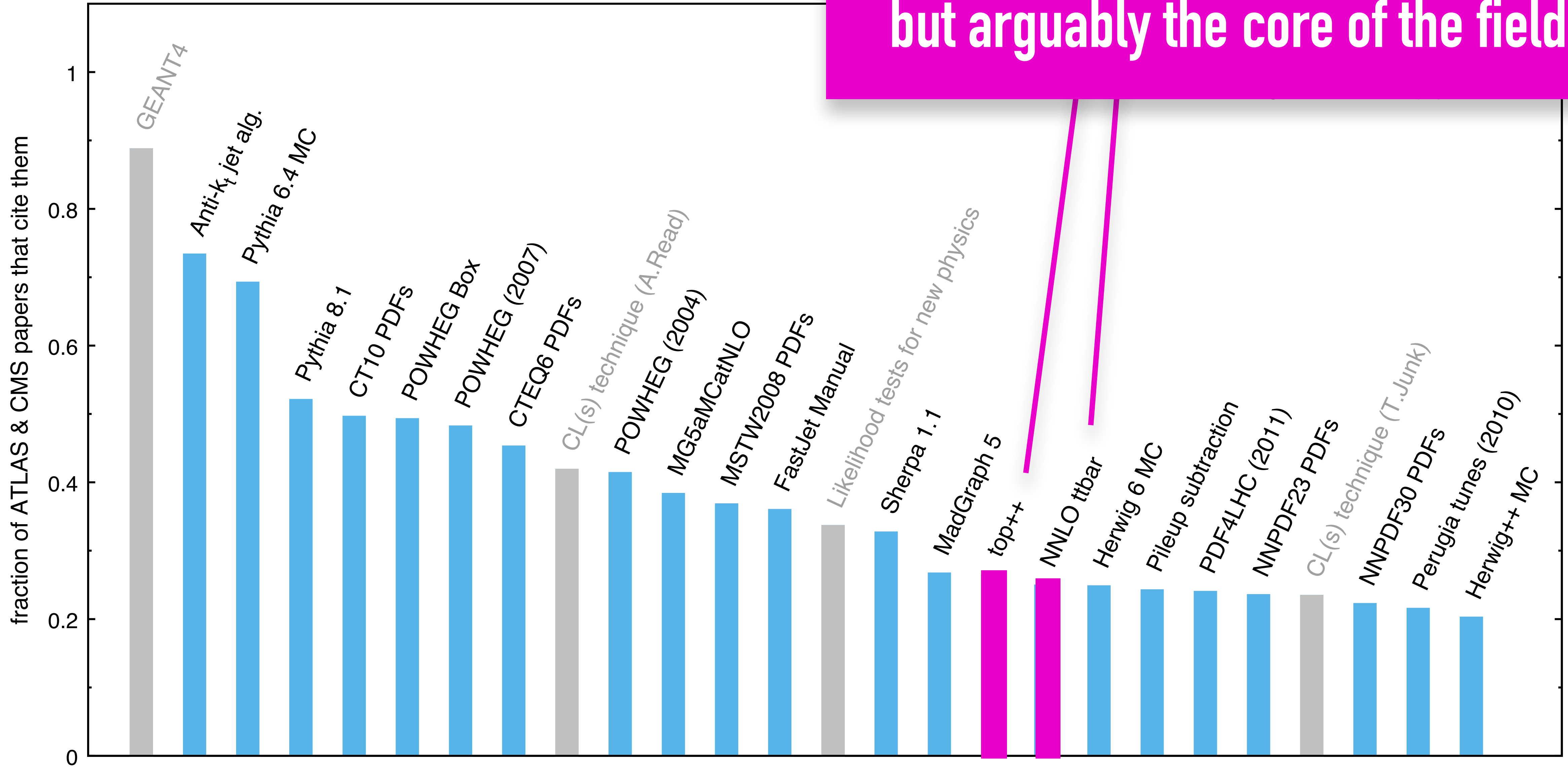
**NLO**

**NNLO**

**N3LO**



fixed order calculations  
(only modestly represented in plot,  
but arguably the core of the field)

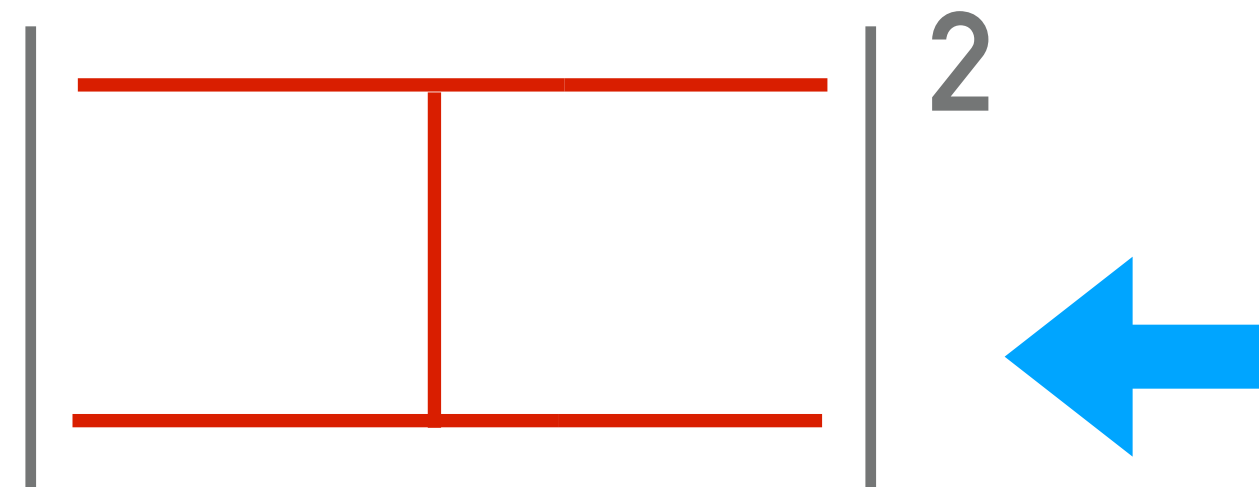


Plot by GP Salam based on data from InspireHEP

# Ingredients for a calculation (generic $2 \rightarrow 2$ process)

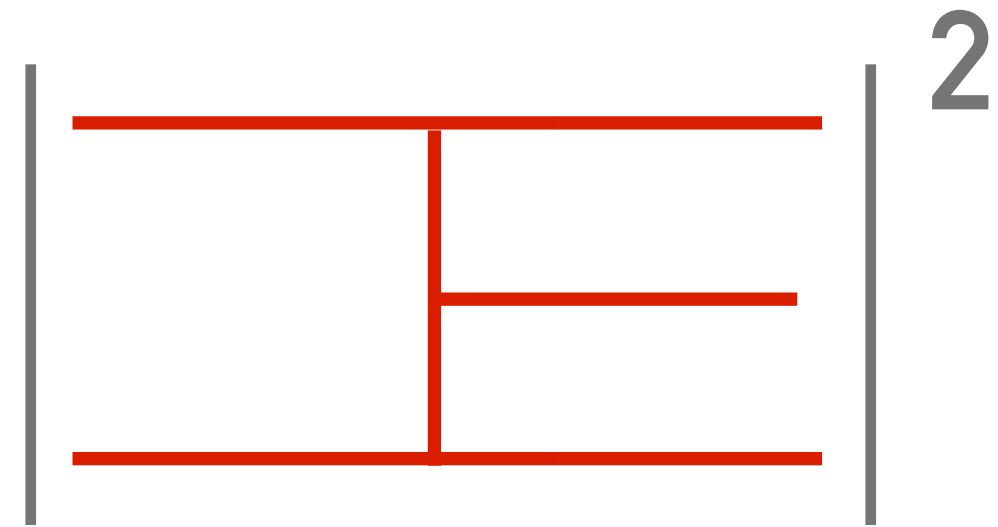
LO

Tree  
 $2 \rightarrow 2$



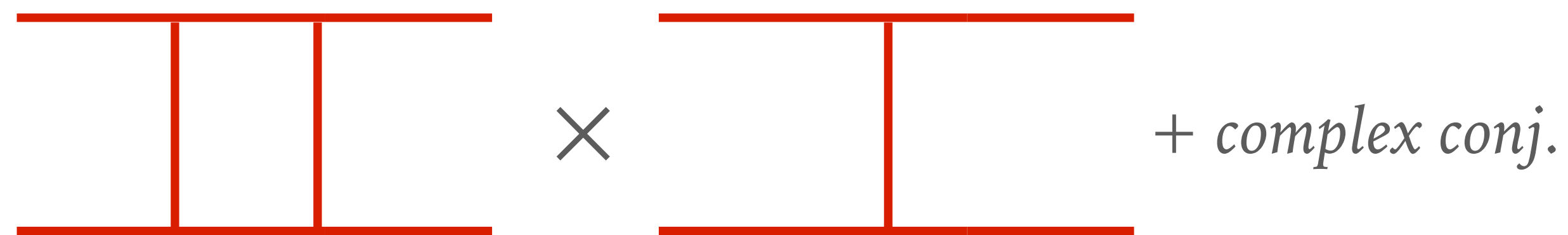
to illustrate the concepts, we don't care what the particles are — just draw lines

Tree  
 $2 \rightarrow 3$

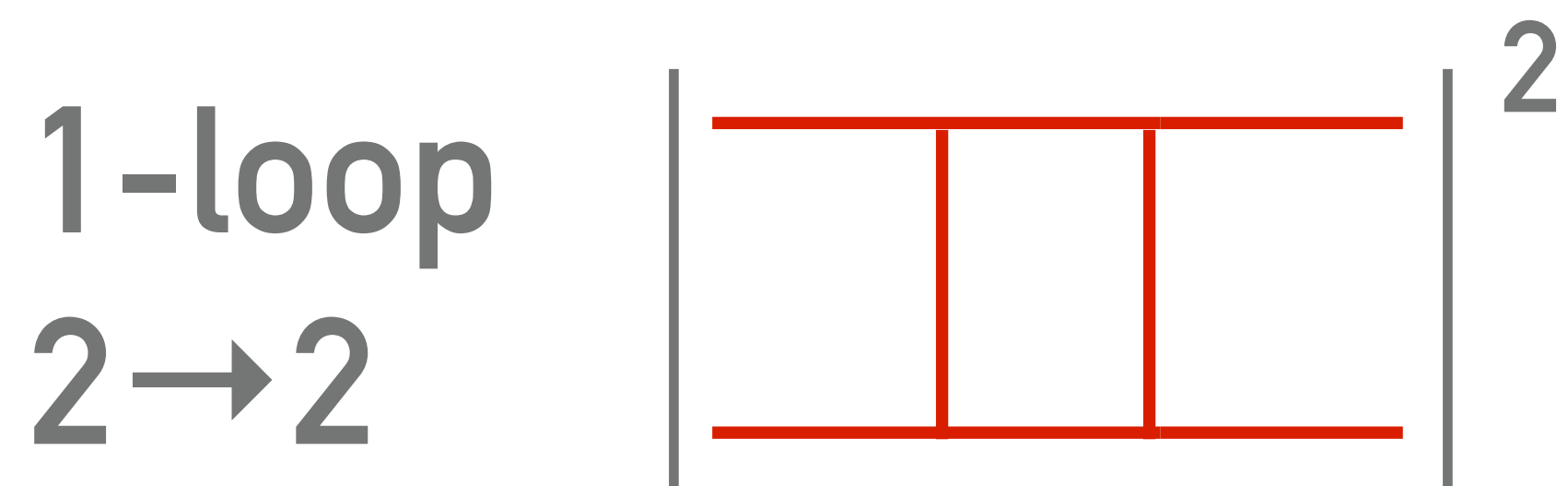
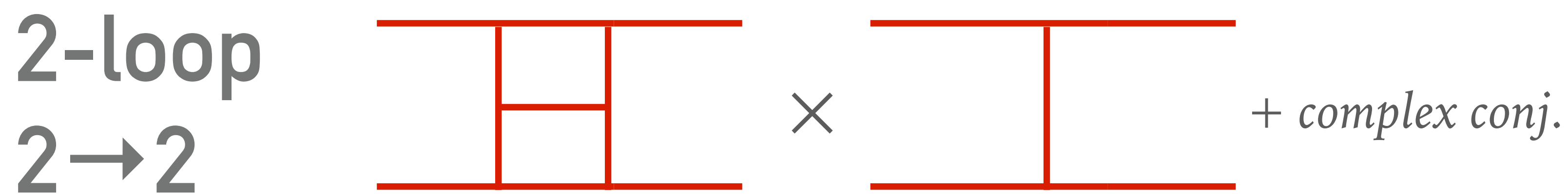
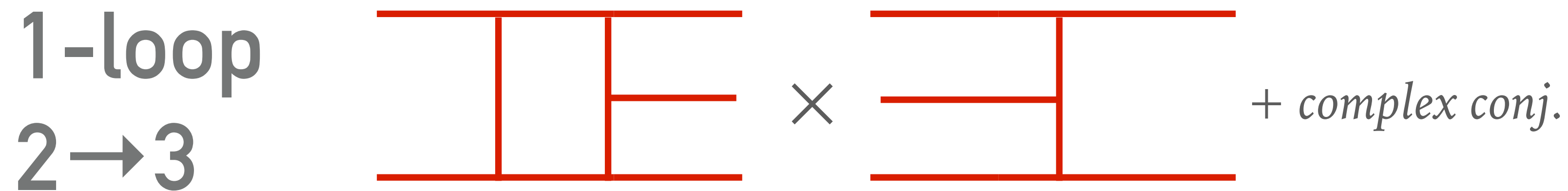
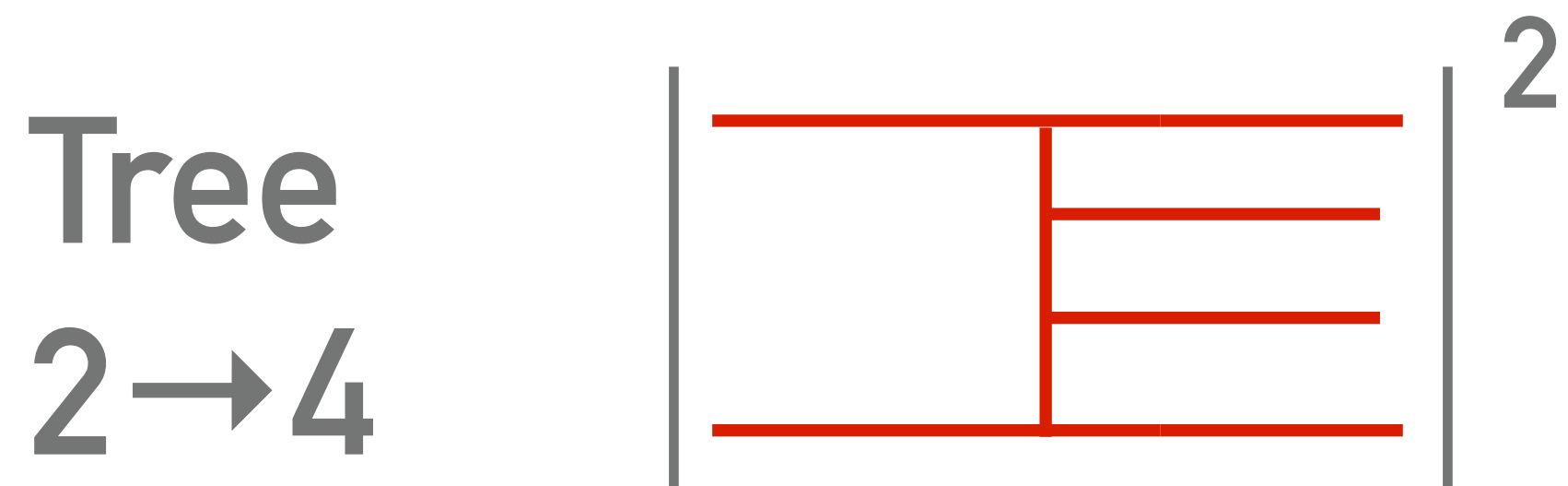


NLO

1-loop  
 $2 \rightarrow 2$

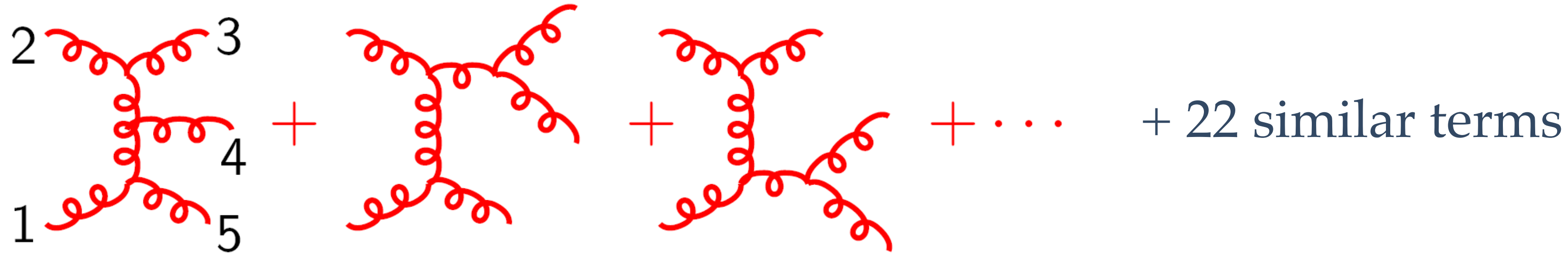


# Ingredients for a calculation (generic 2→2 process)



# doing better than Feynman diagrams to calculate individual terms

slide adapted from Fabrizio Caola



amp =

$$\begin{aligned}
 &+ F(\text{CiE1}, \text{Ci2}, \text{CiE4}) * F(\text{CiE2}, \text{Ci2}, \text{CiXX}) * F(\text{CiE3}, \text{CiE5}, \text{CiXX}) * ( \\
 &\quad - e1.e3*e2.e5*e4.p1*s14^-1 \\
 &\quad - 1/2*e1.e3*e2.e5*e4.p4*s14^-1 \\
 &\quad - 1/2*e1.e4*e2.e3*e5.p1*s14^-1 \\
 &\quad + 1/2*e1.e4*e2.e3*e5.p4*s14^-1 \\
 &\quad + 1/2*e1.e4*e2.e5*e3.p1*s14^-1 \\
 &\quad - 1/2*e1.e4*e2.e5*e3.p4*s14^-1 \\
 &\quad + e1.e5*e2.e3*e4.p1*s14^-1 \\
 &\quad + 1/2*e1.e5*e2.e3*e4.p4*s14^-1 \\
 &\quad - 1/2*e1.p1*e2.e3*e4.e5*s14^-1 \\
 &\quad + 1/2*e1.p1*e2.e5*e3.e4*s14^-1 \\
 &\quad - e1.p4*e2.e3*e4.e5*s14^-1 \\
 &\quad + e1.p4*e2.e5*e3.e4*s14^-1 \\
 &\quad )
 \end{aligned}$$

$$\begin{aligned}
 &+ F(\text{CiE1}, \text{Ci2}, \text{CiE4}) * F(\text{CiE2}, \text{Ci2}, \text{Ci4}) * F(\text{CiE3}, \text{CiE5}, \text{Ci4}) * ( \\
 &\quad - 2*e1.e2*e3.e5*e4.p1*p1.p3*s14^-1*s124^-1 \\
 &\quad - 2*e1.e2*e3.e5*e4.p1*p1.p4*s14^-1*s124^-1 \\
 &\quad + 2*e1.e2*e3.e5*e4.p1*p2.p3*s14^-1*s124^-1 \\
 &\quad - 2*e1.e2*e3.e5*e4.p1*p3.p4*s14^-1*s124^-1 \\
 &\quad - e1.e2*e3.e5*e4.p4*p1.p3*s14^-1*s124^-1 \\
 &\quad - e1.e2*e3.e5*e4.p4*p1.p4*s14^-1*s124^-1 \\
 &\quad + e1.e2*e3.e5*e4.p4*p2.p3*s14^-1*s124^-1 \\
 &\quad - e1.e2*e3.e5*e4.p4*p3.p4*s14^-1*s124^-1 \\
 &\quad + e1.e2*e3.p1*e4.p1*e5.p1*s14^-1*s124^-1 \\
 &\quad - 3*e1.e2*e3.p1*e4.p1*e5.p2*s14^-1*s124^-1 \\
 &\quad + e1.e2*e3.p1*e4.p1*e5.p3*s14^-1*s124^-1 \\
 &\quad + e1.e2*e3.p1*e4.p1*e5.p4*s14^-1*s124^-1 \\
 &\quad + 1/2*e1.e2*e3.p1*e4.p4*e5.p1*s14^-1*s124^-1 \\
 &\quad - 3/2*e1.e2*e3.p1*e4.p4*e5.p2*s14^-1*s124^-1 \\
 &\quad + 1/2*e1.e2*e3.p1*e4.p4*e5.p3*s14^-1*s124^-1 \\
 &\quad + 1/2*e1.e2*e3.p1*e4.p4*e5.p4*s14^-1*s124^-1 \\
 &\quad + 3*e1.e2*e3.p2*e4.p1*e5.p1*s14^-1*s124^-1 \\
 &\quad )
 \end{aligned}$$

=

Massive simplification!

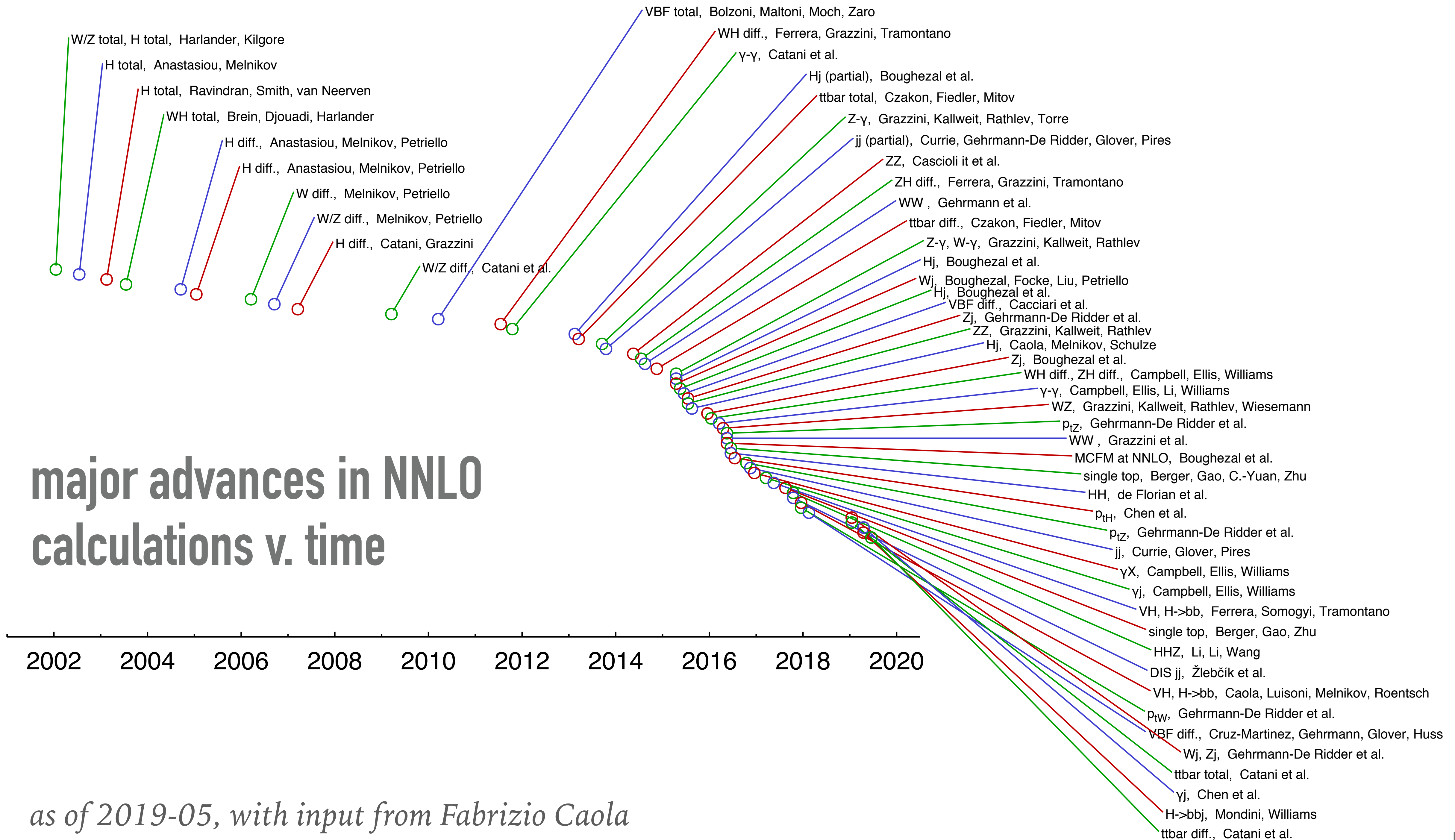
$$A_5^{\text{tree}}(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0$$

$$A_5^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

$$A_5^{\text{tree}}(1^-, 2^+, 3^-, 4^+, 5^+) = i \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

mathematically equivalent

# major advances in NNLO calculations v. time



as of 2019-05, with input from Fabrizio Caola

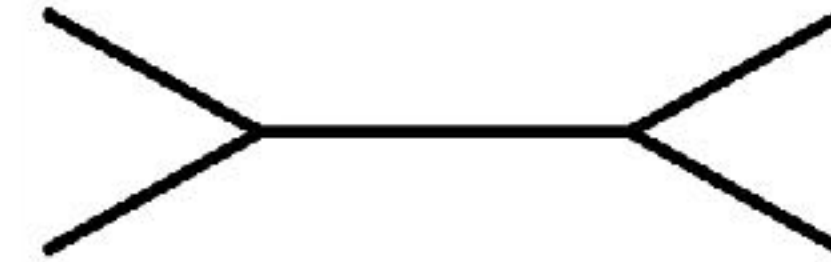


# Higher precision needs more legs & more loops

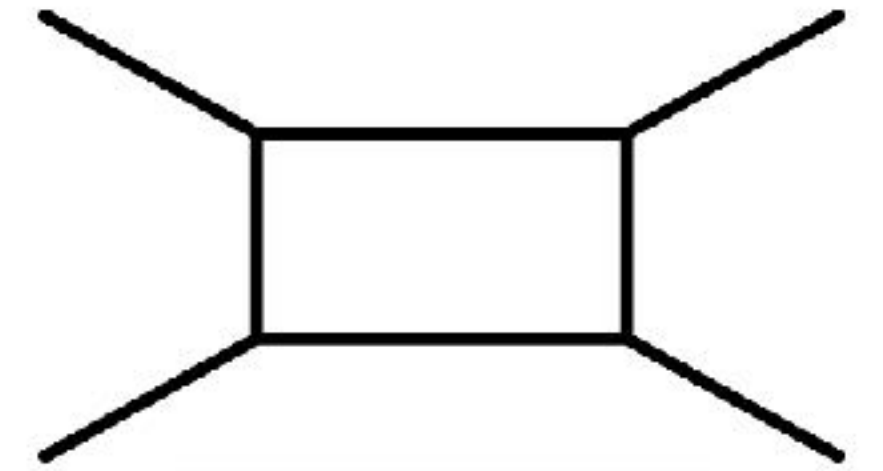
## Analytic Form of the Planar Two-Loop Five-Parton Scattering Amplitudes in QCD

S. Abreu,<sup>a</sup> J. Dormans,<sup>b</sup> F. Febres Cordero,<sup>b,c</sup> H. Ita,<sup>b</sup> B. Page,<sup>d</sup> and V. Sotnikov<sup>b</sup>

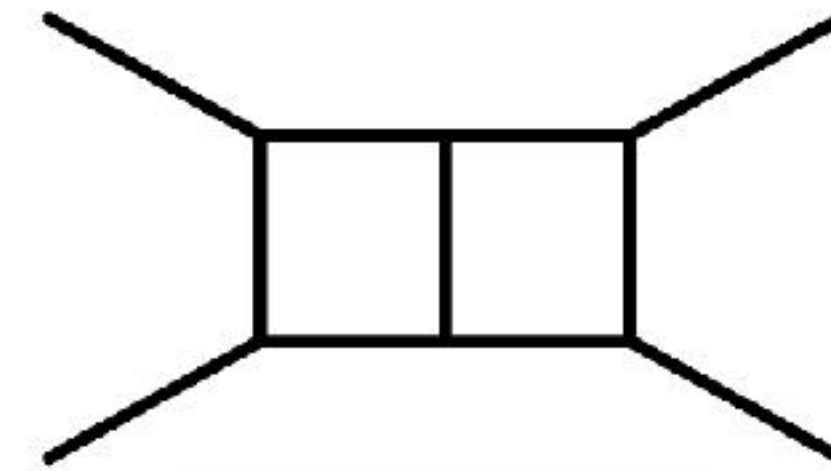
**ABSTRACT:** We present the analytic form of all leading-color two-loop five-parton helicity amplitudes in QCD. The results are analytically reconstructed from exact numerical evaluations over finite fields. Combining a judicious choice of variables with a new approach to the treatment of particle states in  $D$  dimensions for the numerical evaluation of amplitudes, we obtain the analytic expressions with a modest computational effort. Their systematic simplification using multivariate partial-fraction decomposition leads to a particularly compact form. Our results provide all two-loop amplitudes required for the calculation of next-to-next-to-leading order QCD corrections to the production of three jets at hadron colliders in the leading-color approximation.



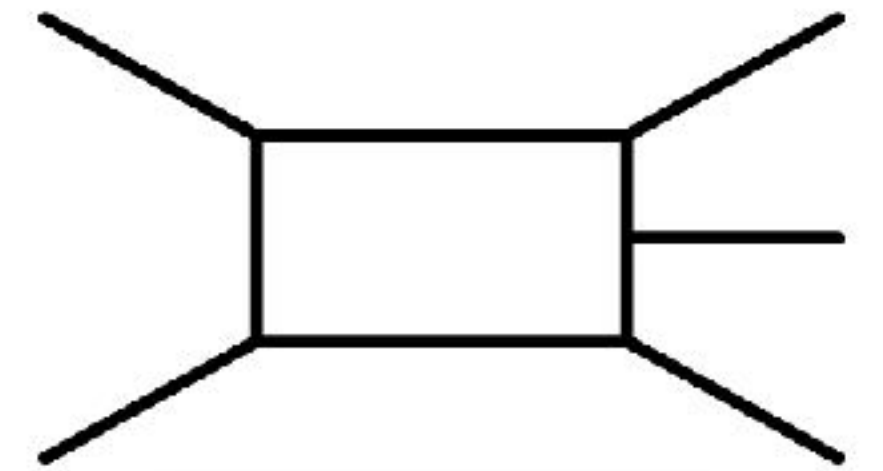
no-loop 4-legs



1-loop 4-legs



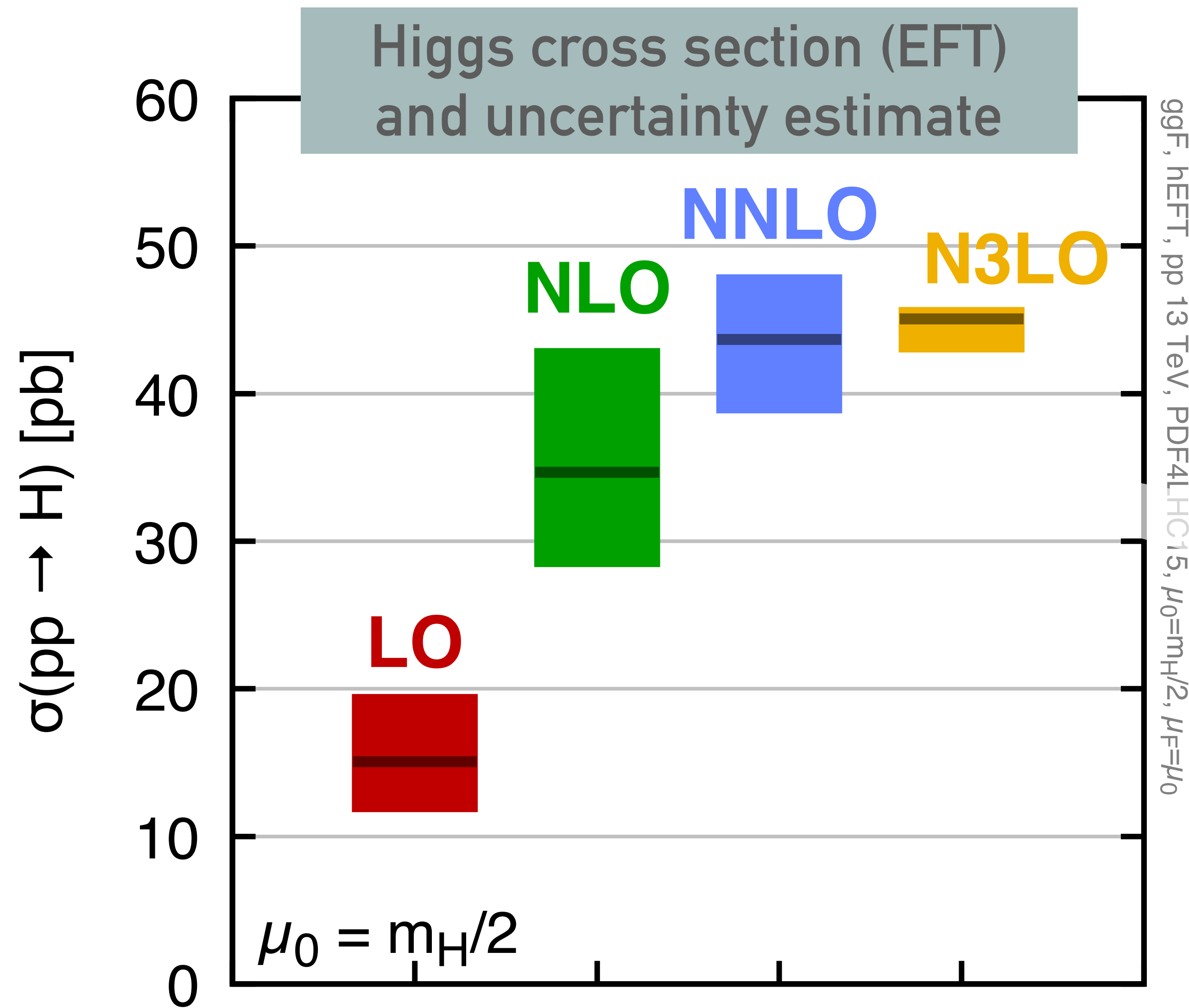
2-loop 4-legs



1-loop 5-legs

Order in perturbation theory:  $\alpha_s^{n_{\text{legs}} + n_{\text{loops}} - 2}$

# perturbative series for Higgs production ( $gg \rightarrow H$ )



results from [arXiv:1503.06056](https://arxiv.org/abs/1503.06056)

even though  $\alpha_s(m_H) \approx 0.11$ ,  
 perturbative series requires a  
 number of orders in order to start  
 converging

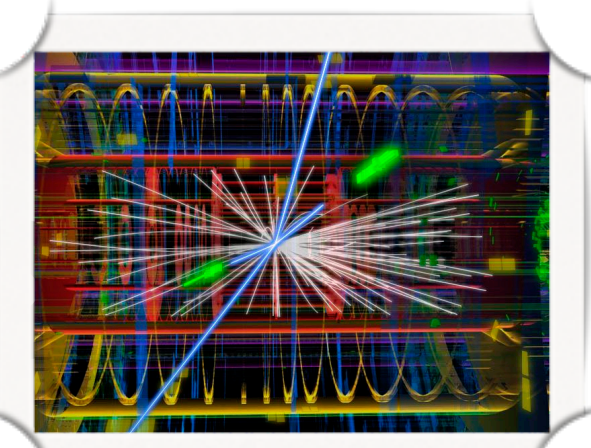
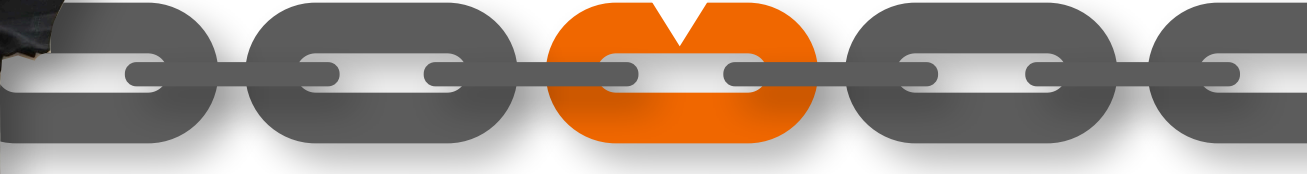
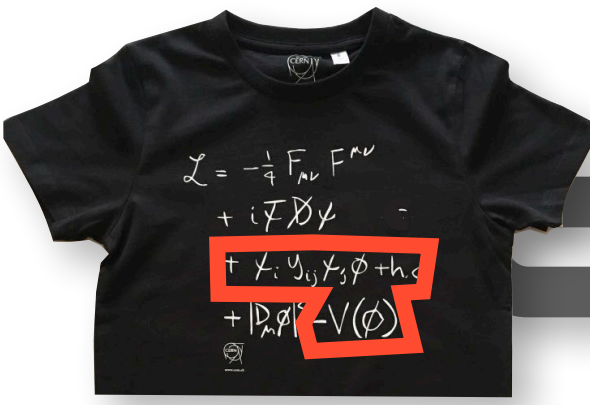
a similar phenomenon holds for  
 almost all hadron collider cross  
 sections (though not usually quite  
 this bad)

NB: here, only the renorm. scale  $\mu$  ( $=\mu_R$ ) has been varied to  
 estimate uncertainty. In real life you need to change renorm.  
 and factorisation ( $\mu_F$ ) scales.

# Monte Carlo event generators

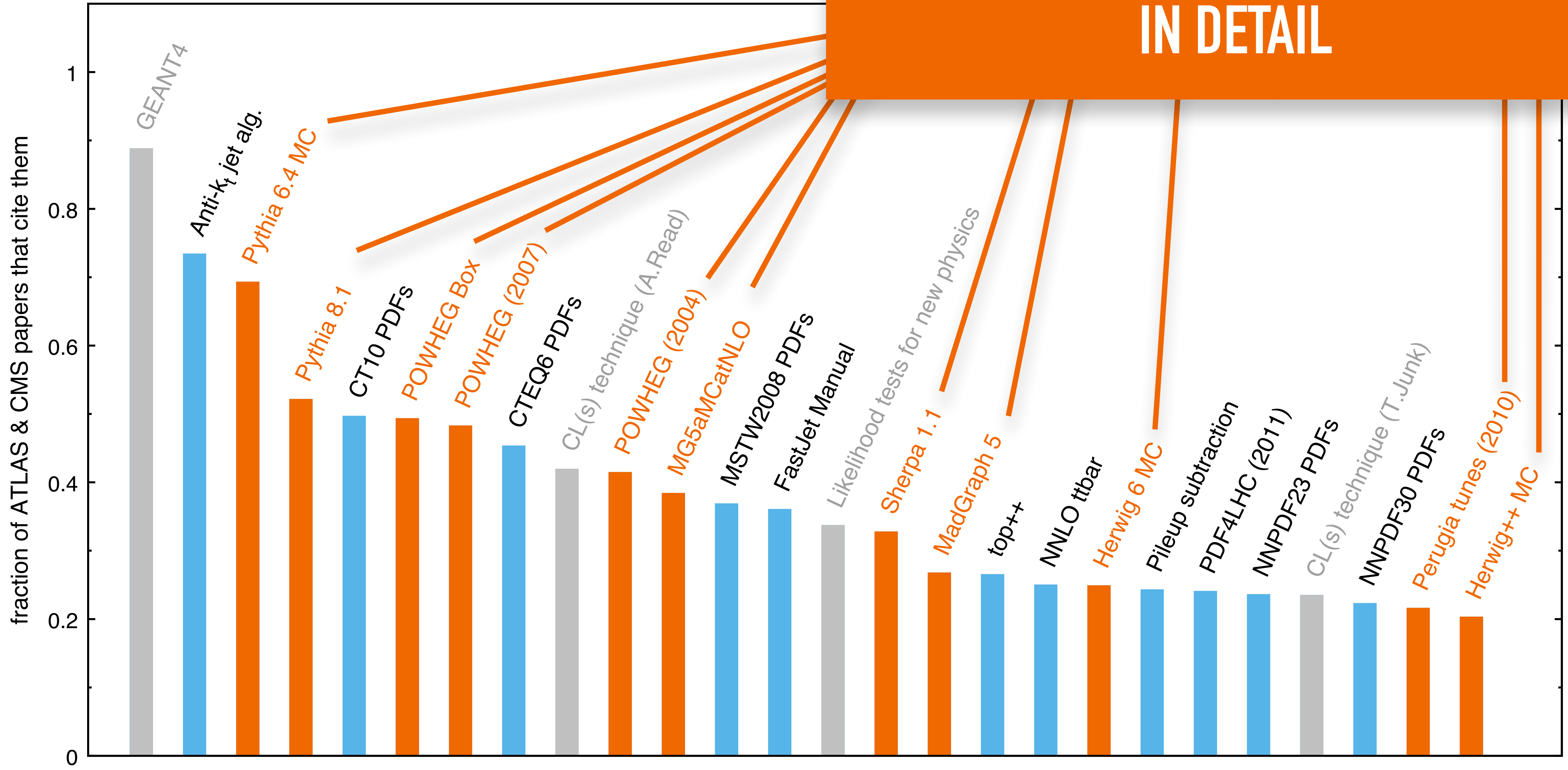
---

*see e.g. [arXiv:1202.1251](#), [PDG review](#)*



# predicting what collider events look like

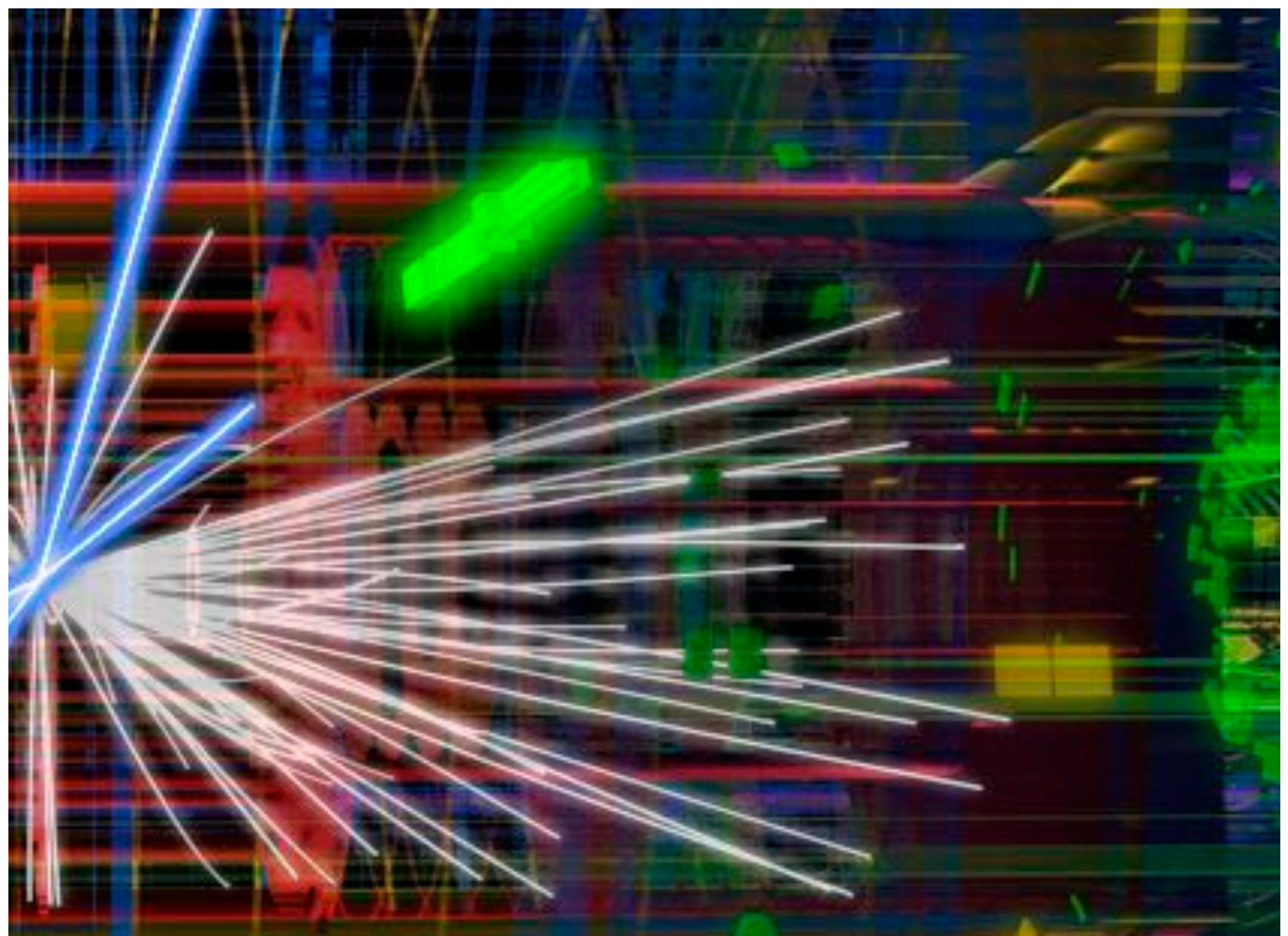
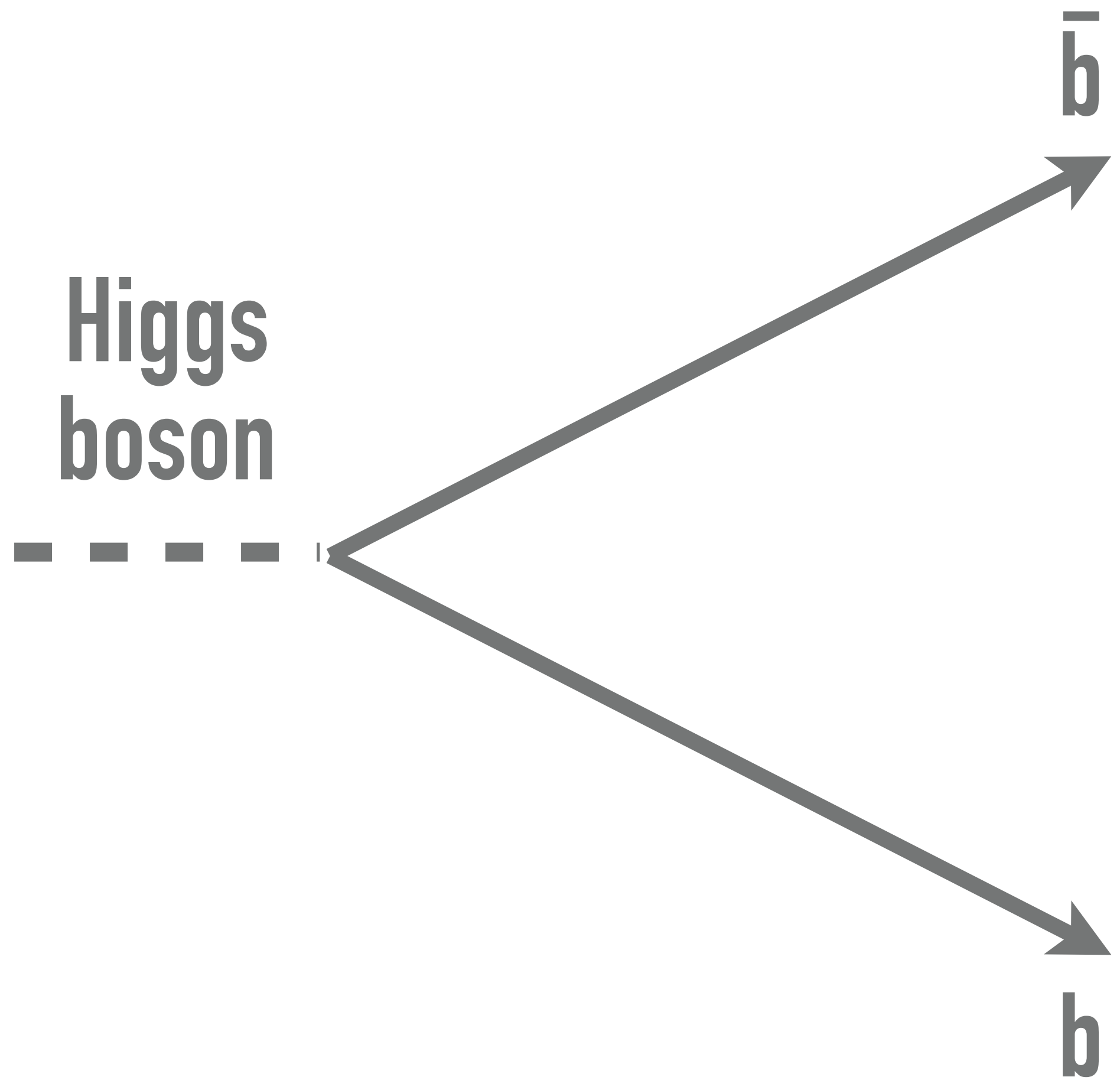
## IN DETAIL



Plot by GP Salam based on data from InspireHEP

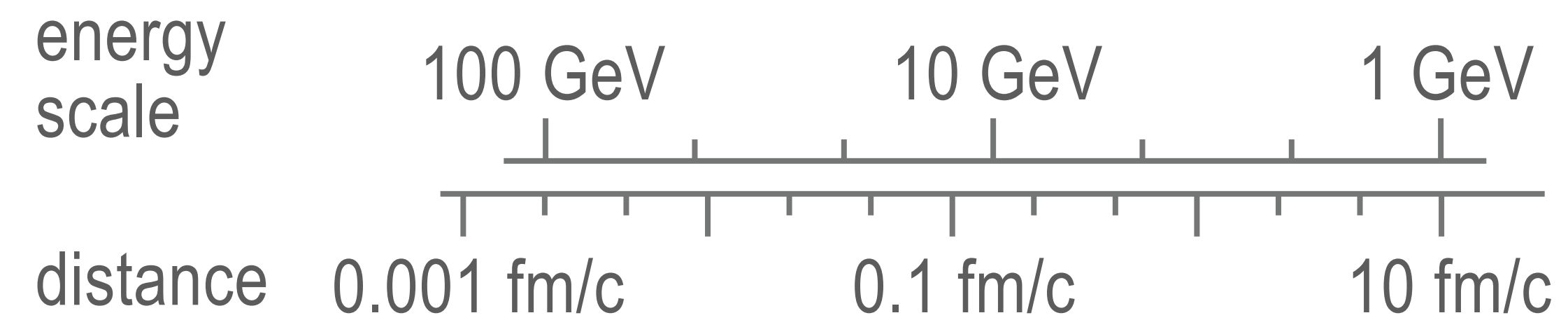
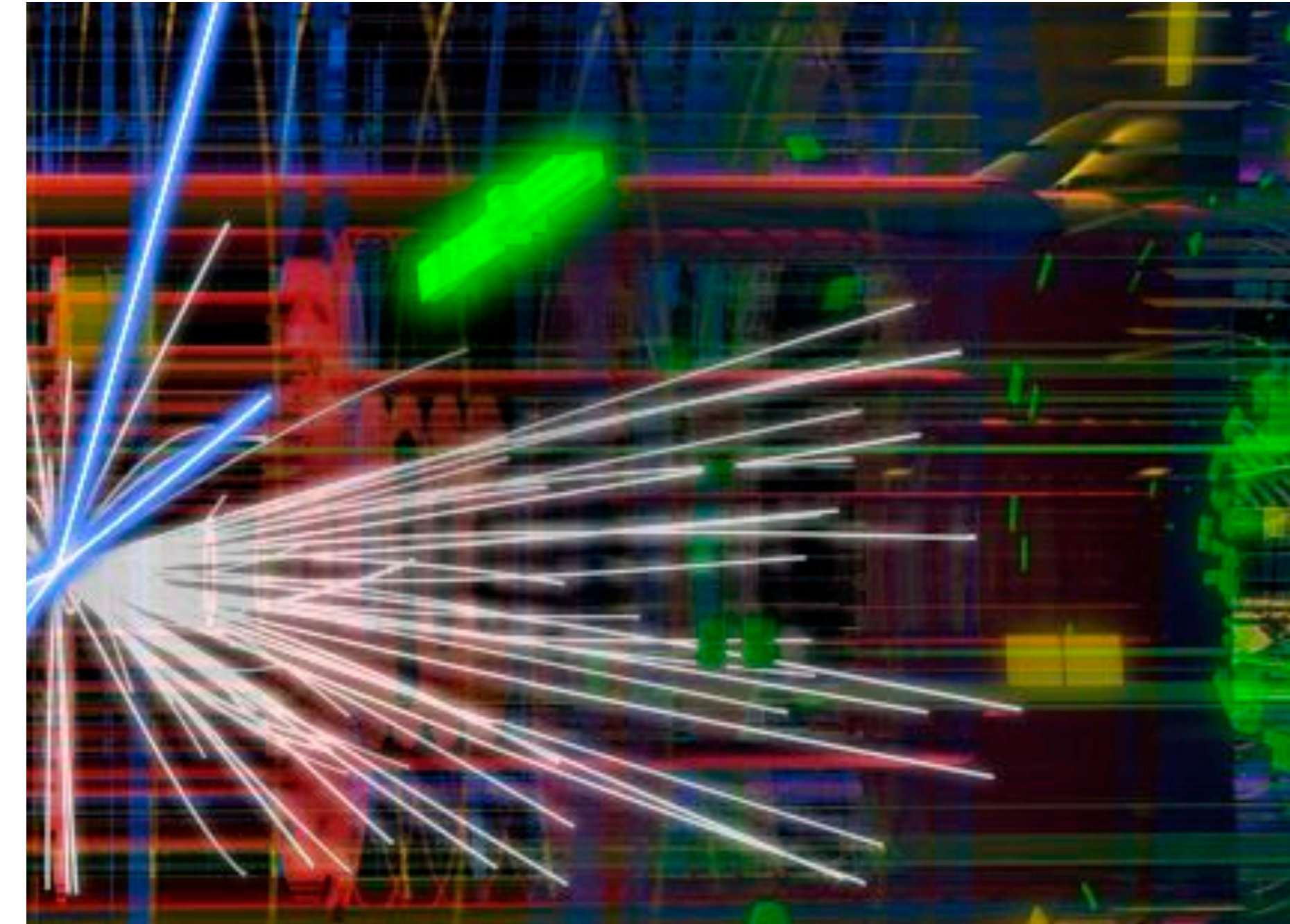
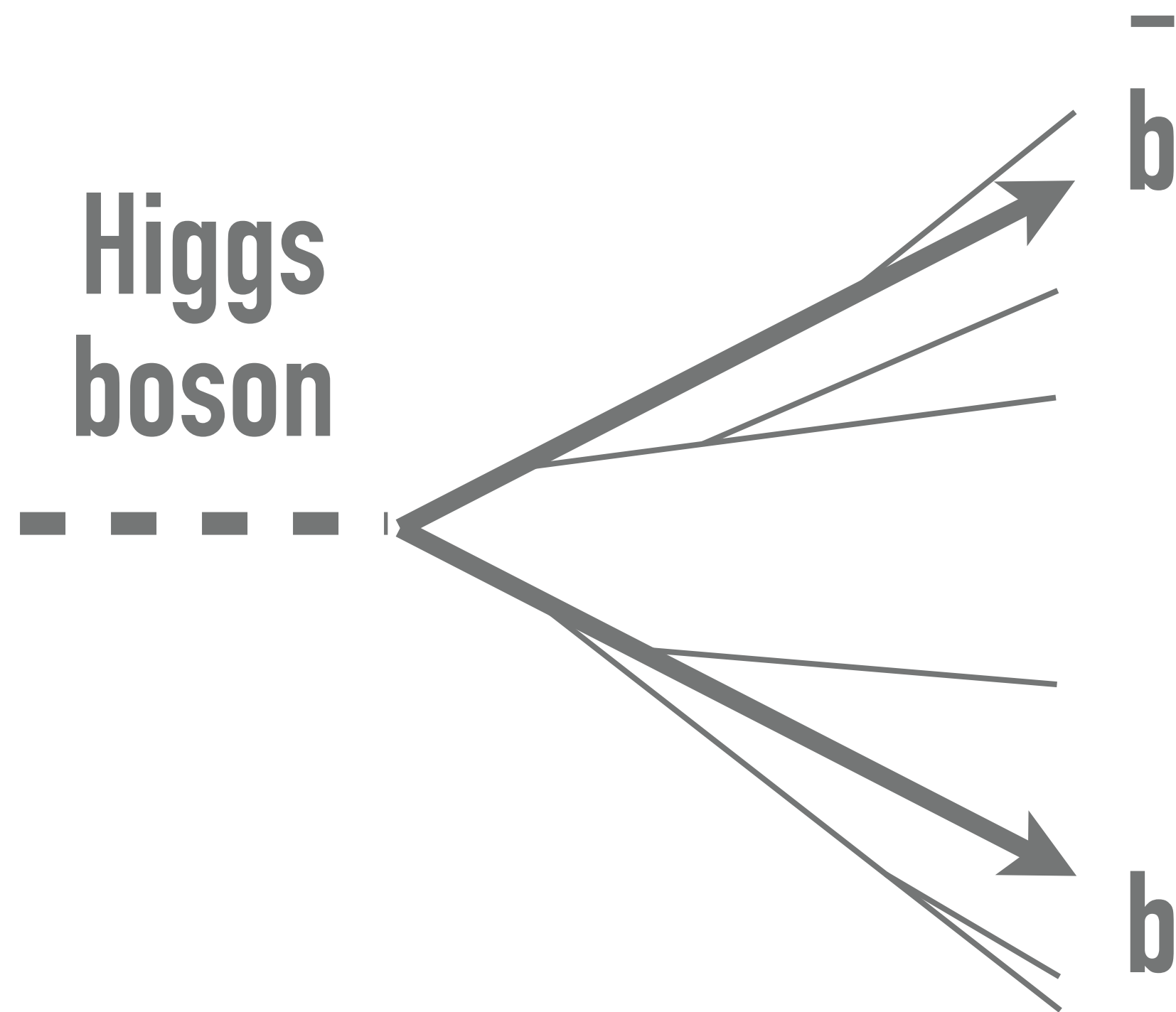
# QCD Parton Shower

[parton = quark or gluon]

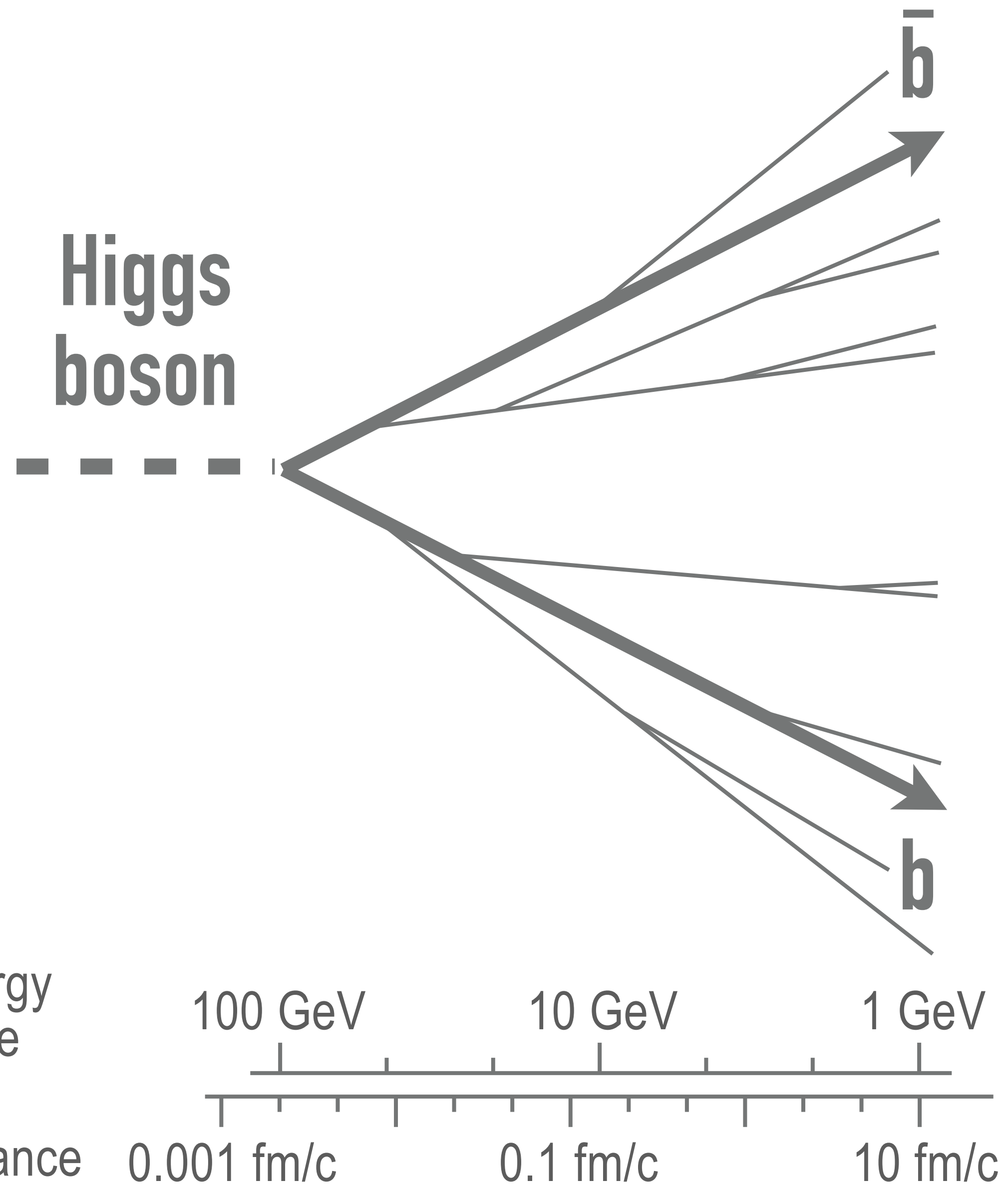


# QCD Parton Shower

[parton = quark or gluon]



# QCD Parton Shower [parton = quark or gluon]



Pattern of branching usually simulated with a **Monte Carlo Parton Shower algorithm**

Experiments **always compare data to Monte Carlo simulations** to establish fundamental hypotheses

Robustness & accuracy of multi-scale properties of these simulations is one of the open questions of the field

# At its simplest: the perturbative part of event generators

---

$$\sum_{n=0}^{\infty} \prod_{i=1}^n \left( \begin{array}{c} \diagup \\ \rightarrow \\ \diagdown \end{array} \right) = \dots \begin{array}{c} \diagup \\ \rightarrow \\ \diagdown \end{array}$$

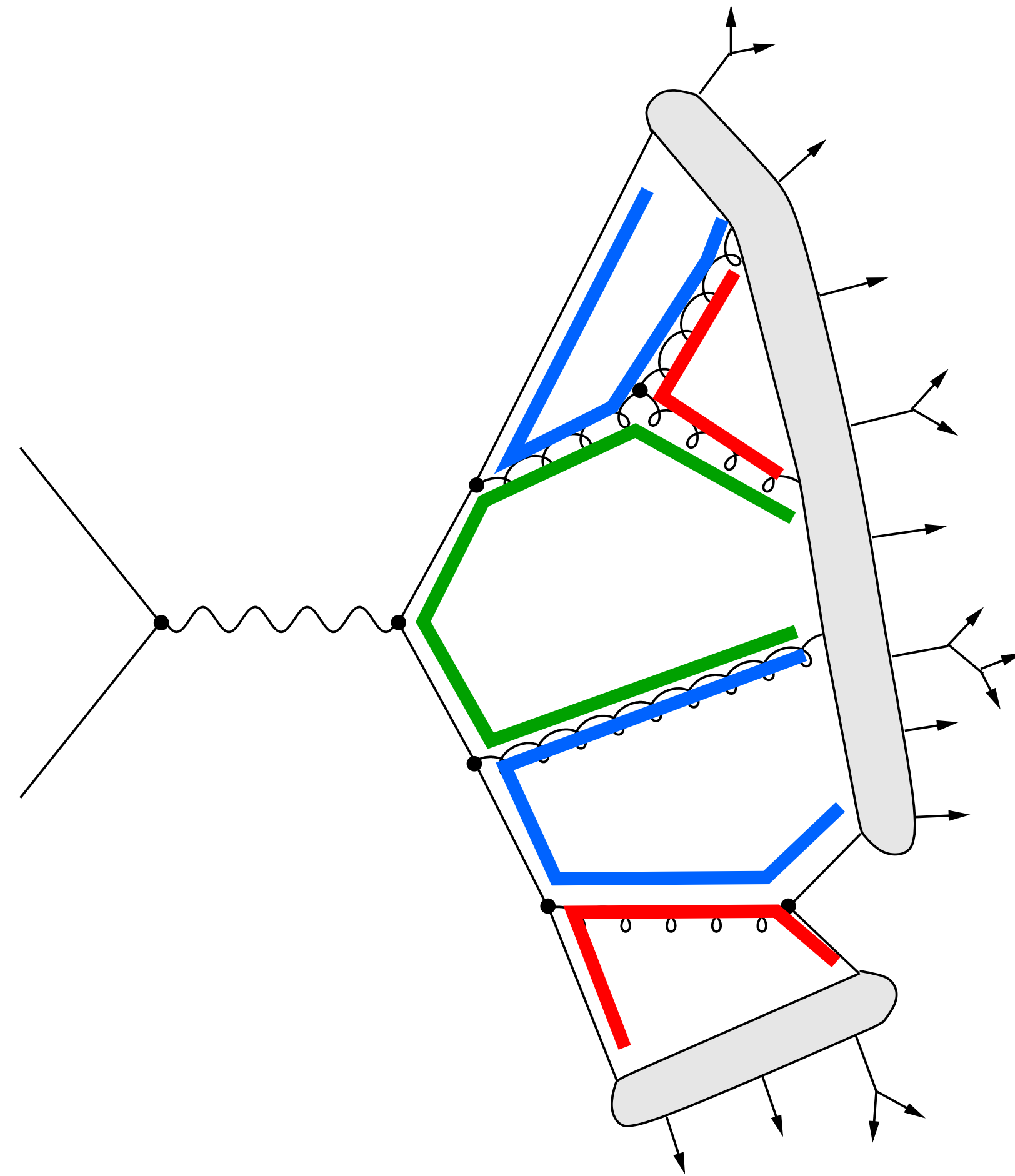
iteration of  $2 \rightarrow 3$  (or  $1 \rightarrow 2$ ) splitting kernel

in what sense does it give the right answer when you ask arbitrary questions about the final state?  
cf. [arXiv:1805.09327](https://arxiv.org/abs/1805.09327)

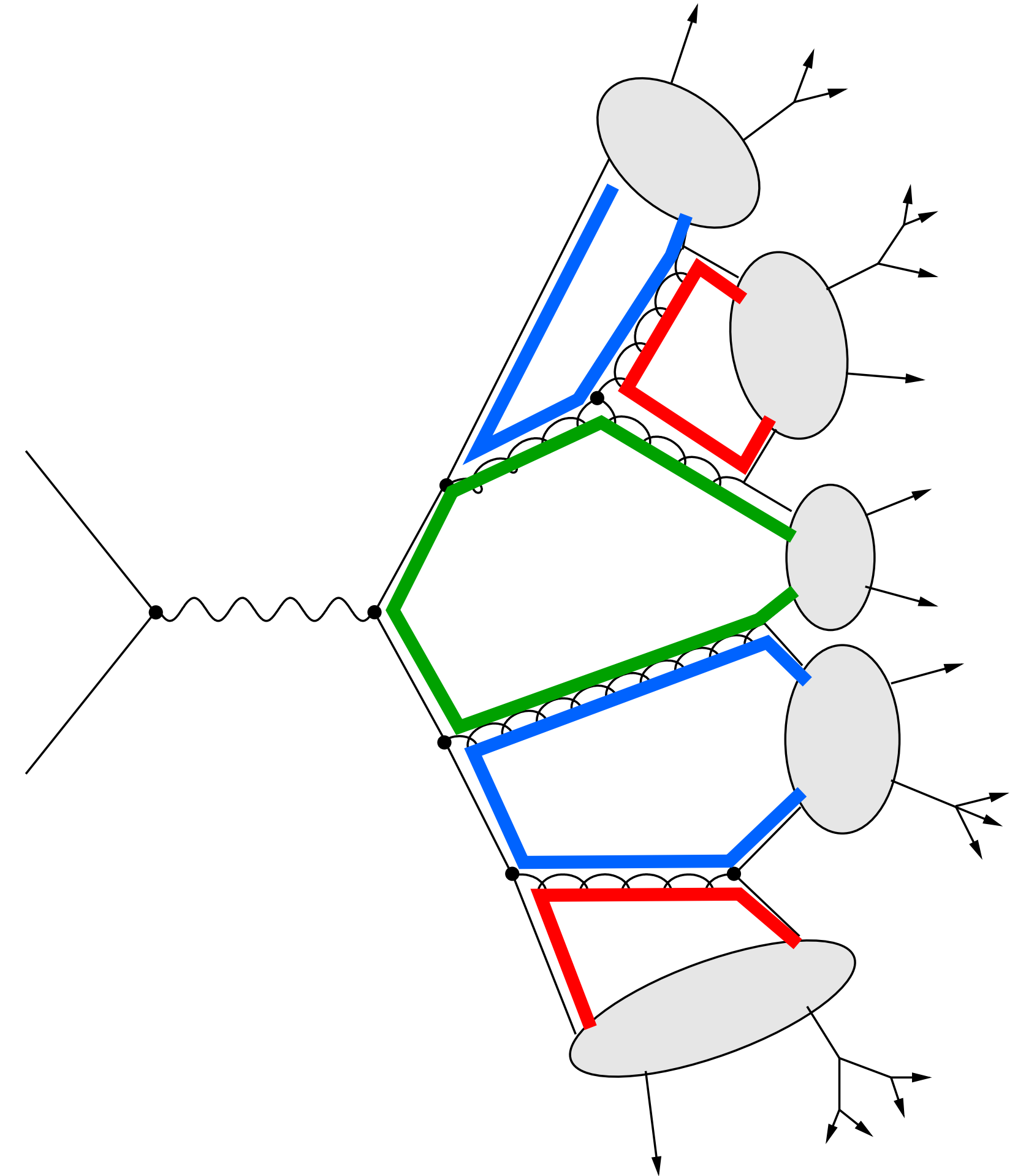


# parton–hadron transition (“**hadronisation**”) can, today, only be modelled

reorganise  
coloured  
partons  
into  
colour-  
singlet  
hadrons



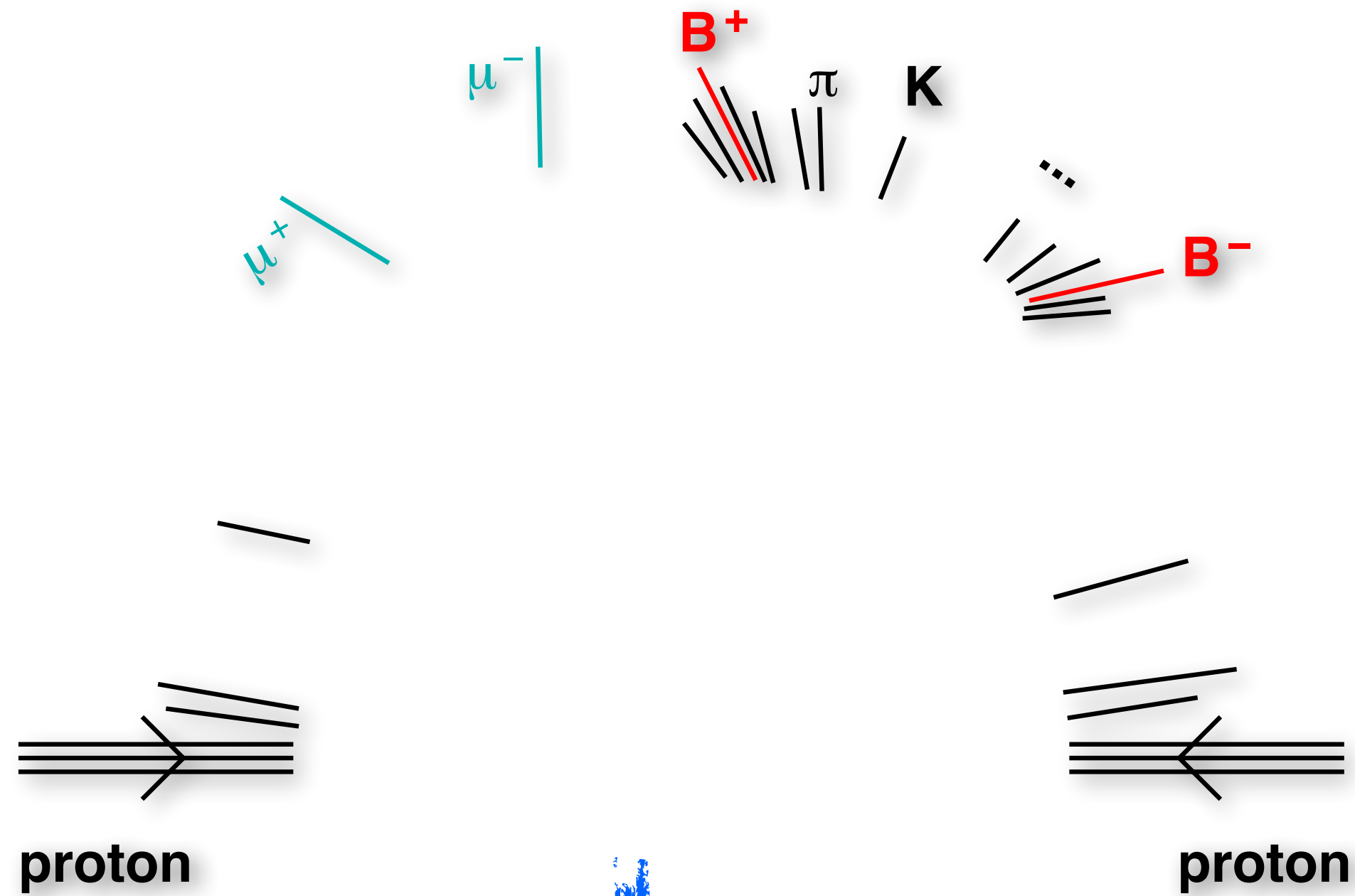
String Fragmentation  
(Pythia and friends)



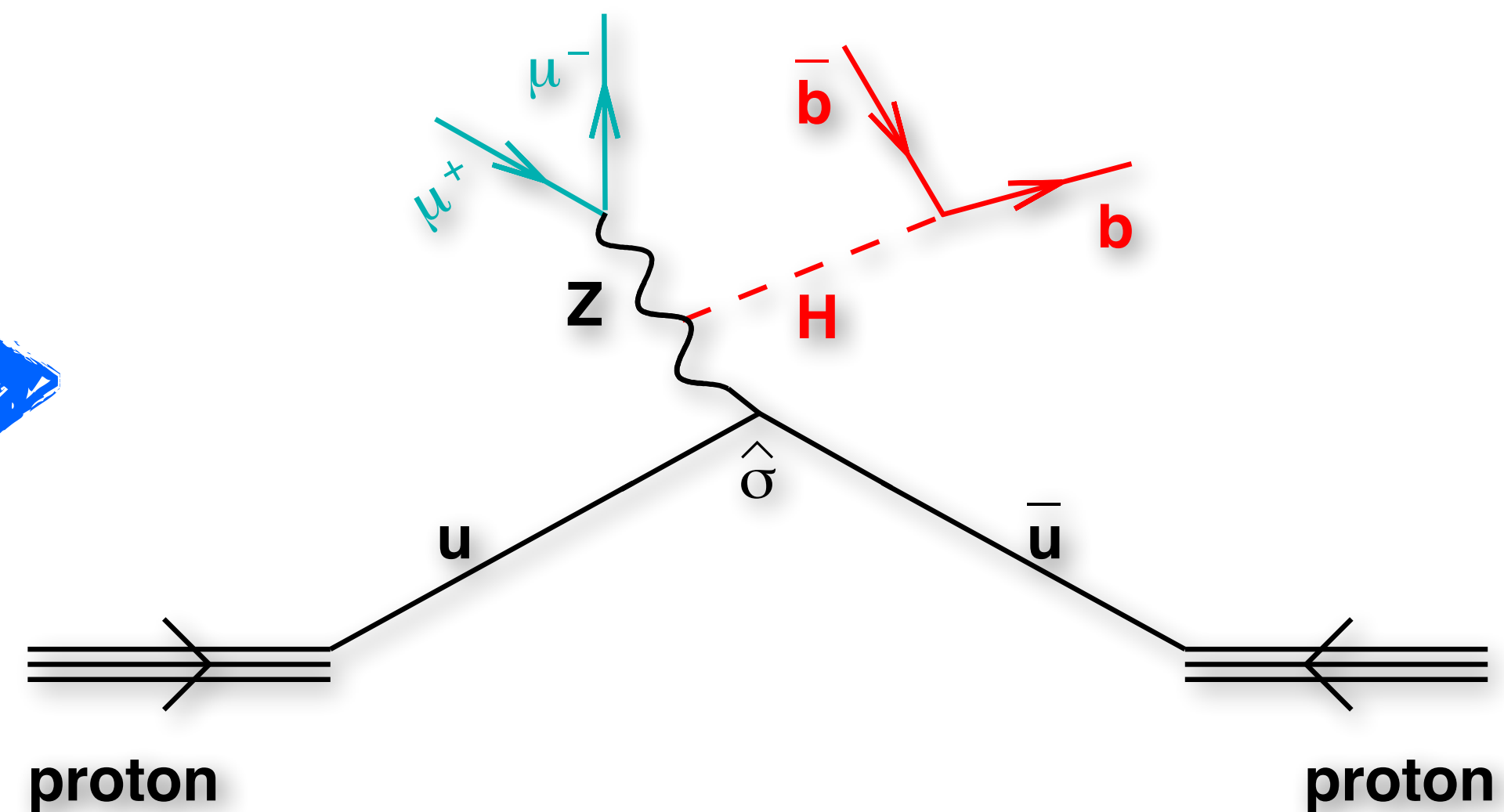
Cluster Fragmentation  
(Herwig) (& Sherpa)

# jets

*i.e. how we make sense of the hadronic part of events*



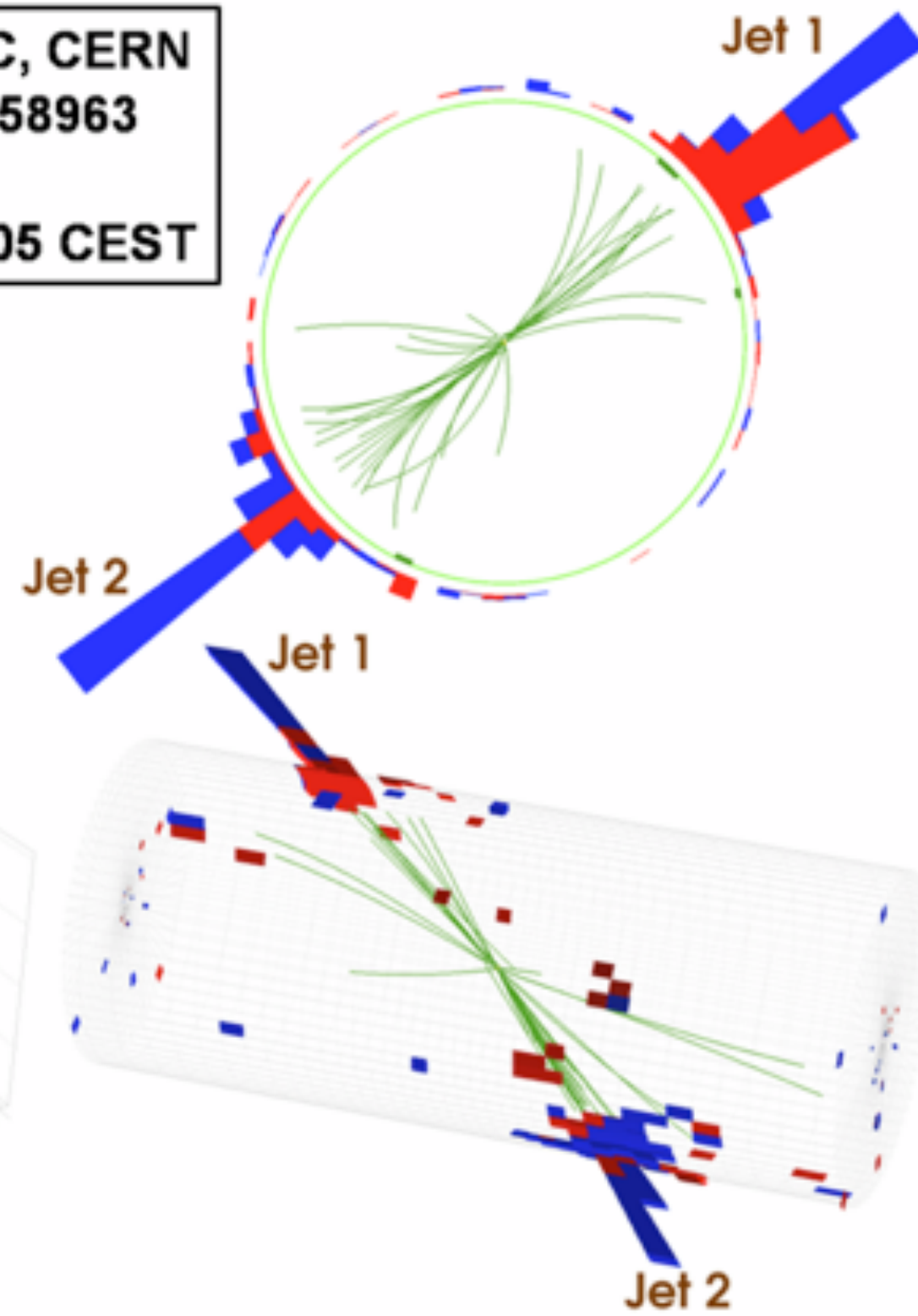
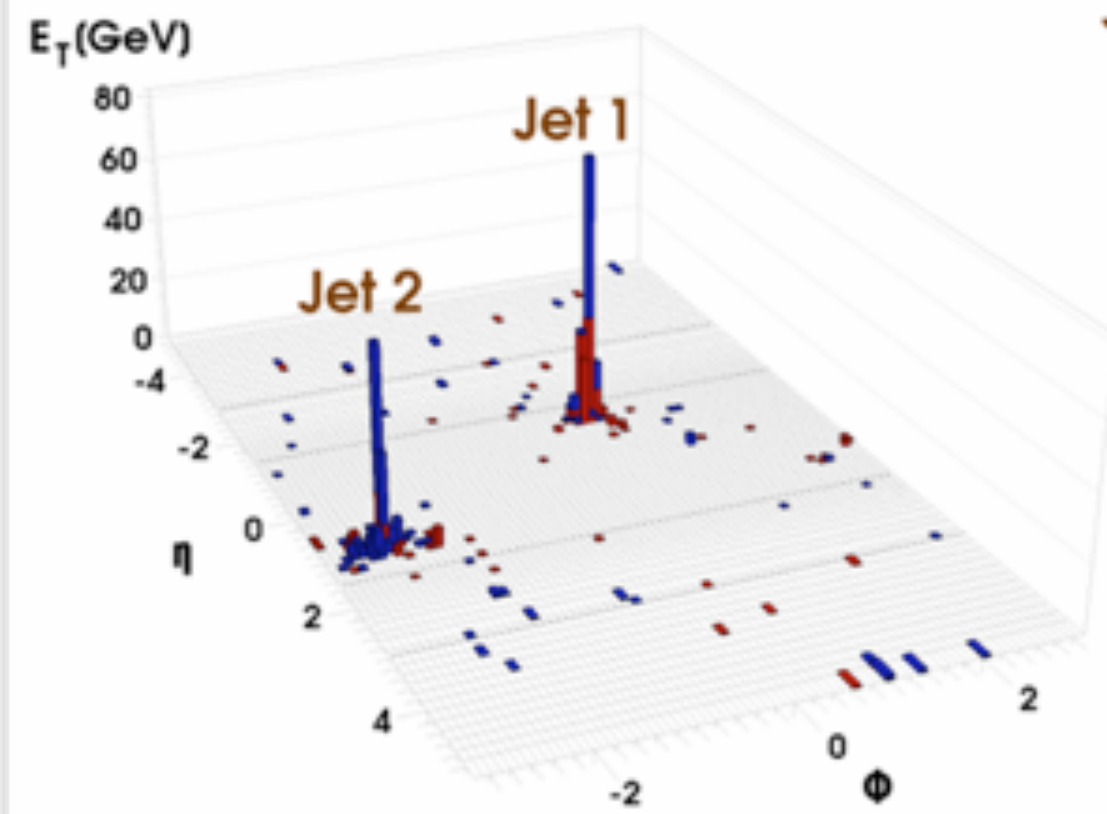
Interpretation



see e.g.  
[arXiv:0906.1833](https://arxiv.org/abs/0906.1833)  
[arXiv:1901.10342](https://arxiv.org/abs/1901.10342)

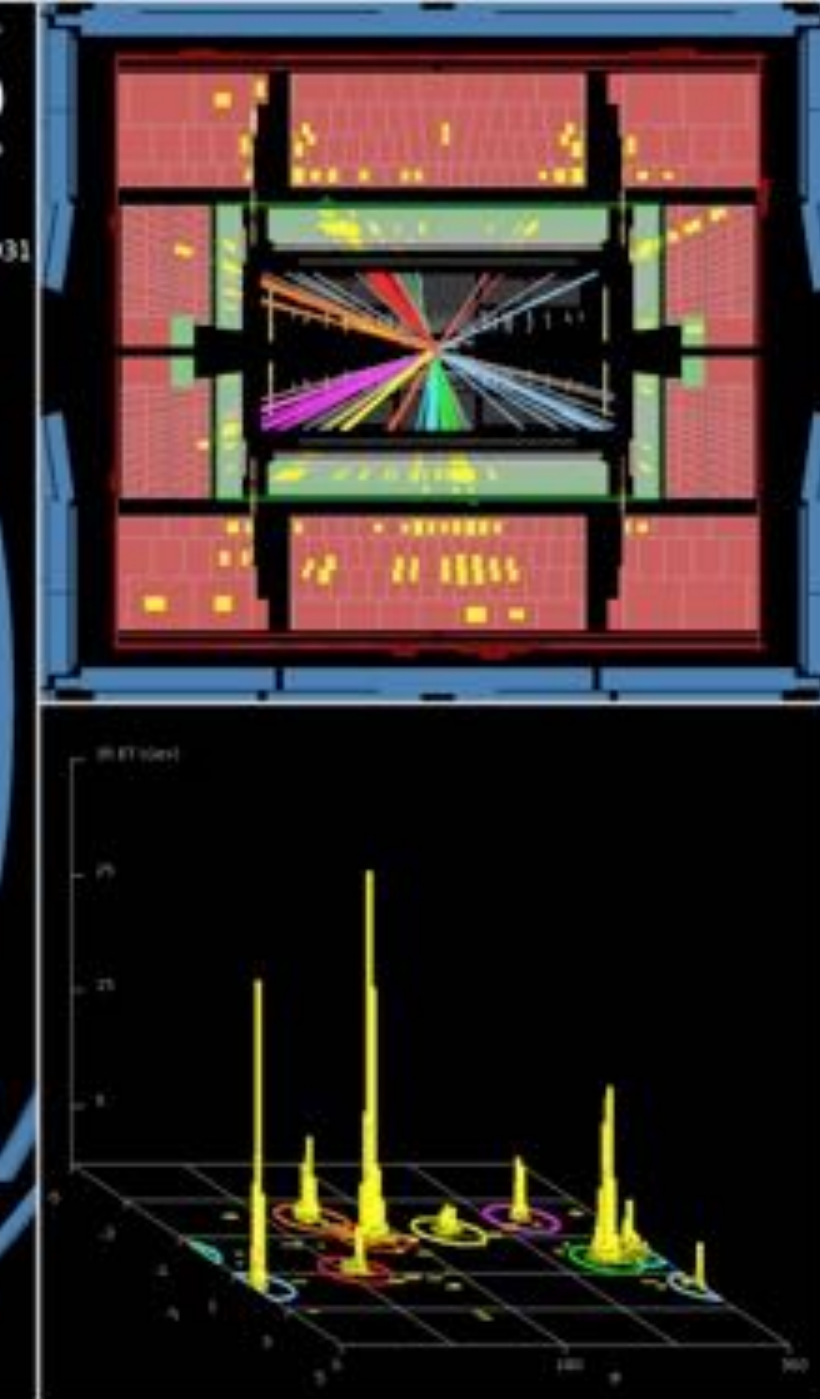
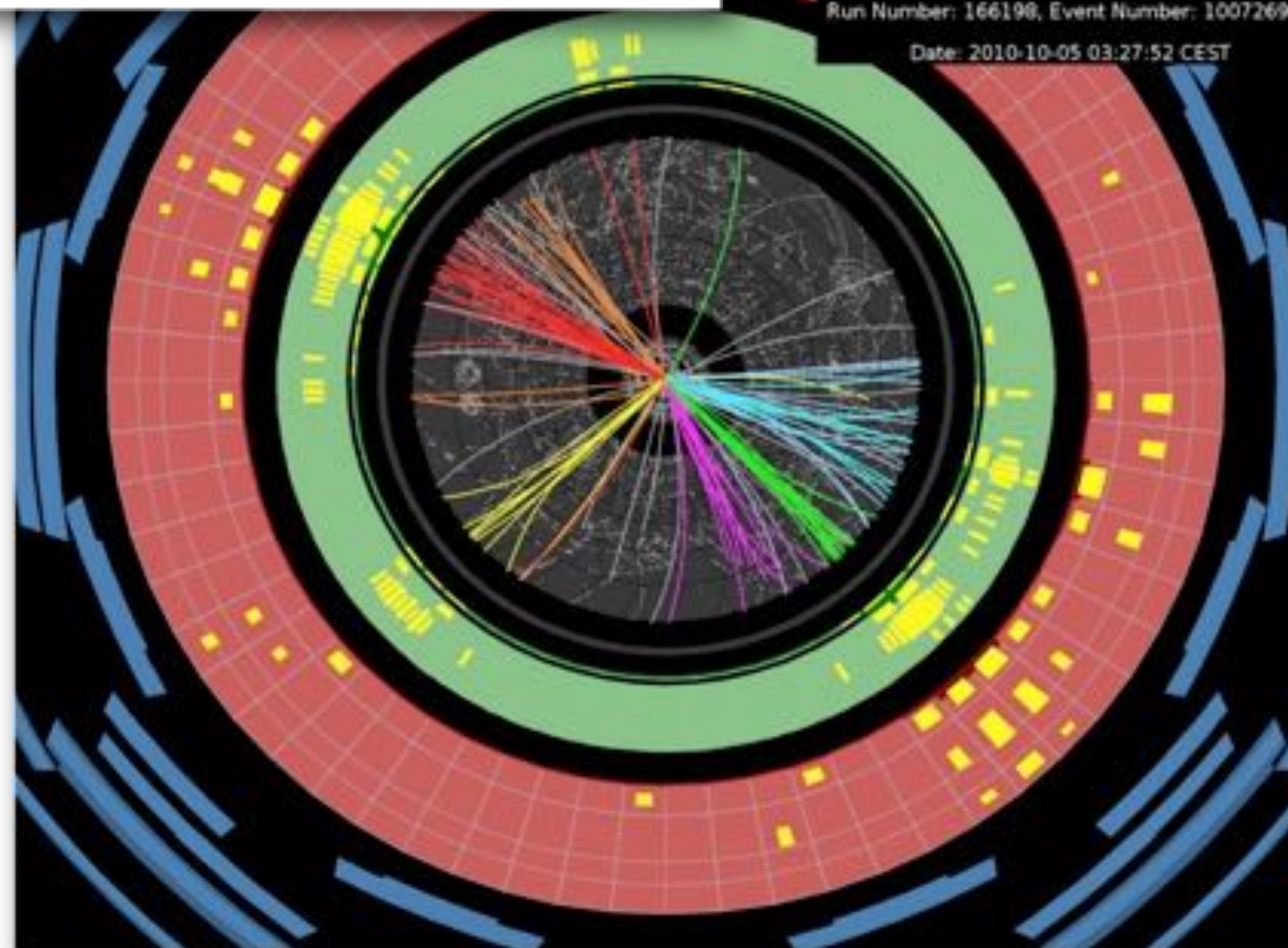


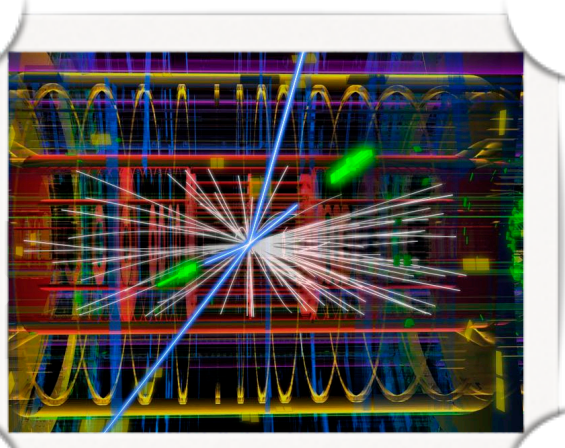
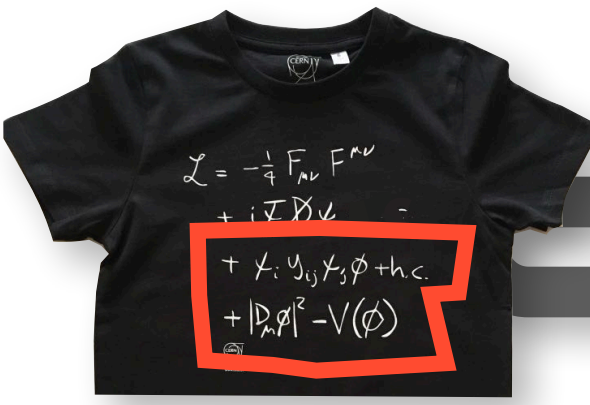
CMS Experiment at LHC, CERN  
Run 133450 Event 16358963  
Lumi section: 285  
Sat Apr 17 2010, 12:25:05 CEST



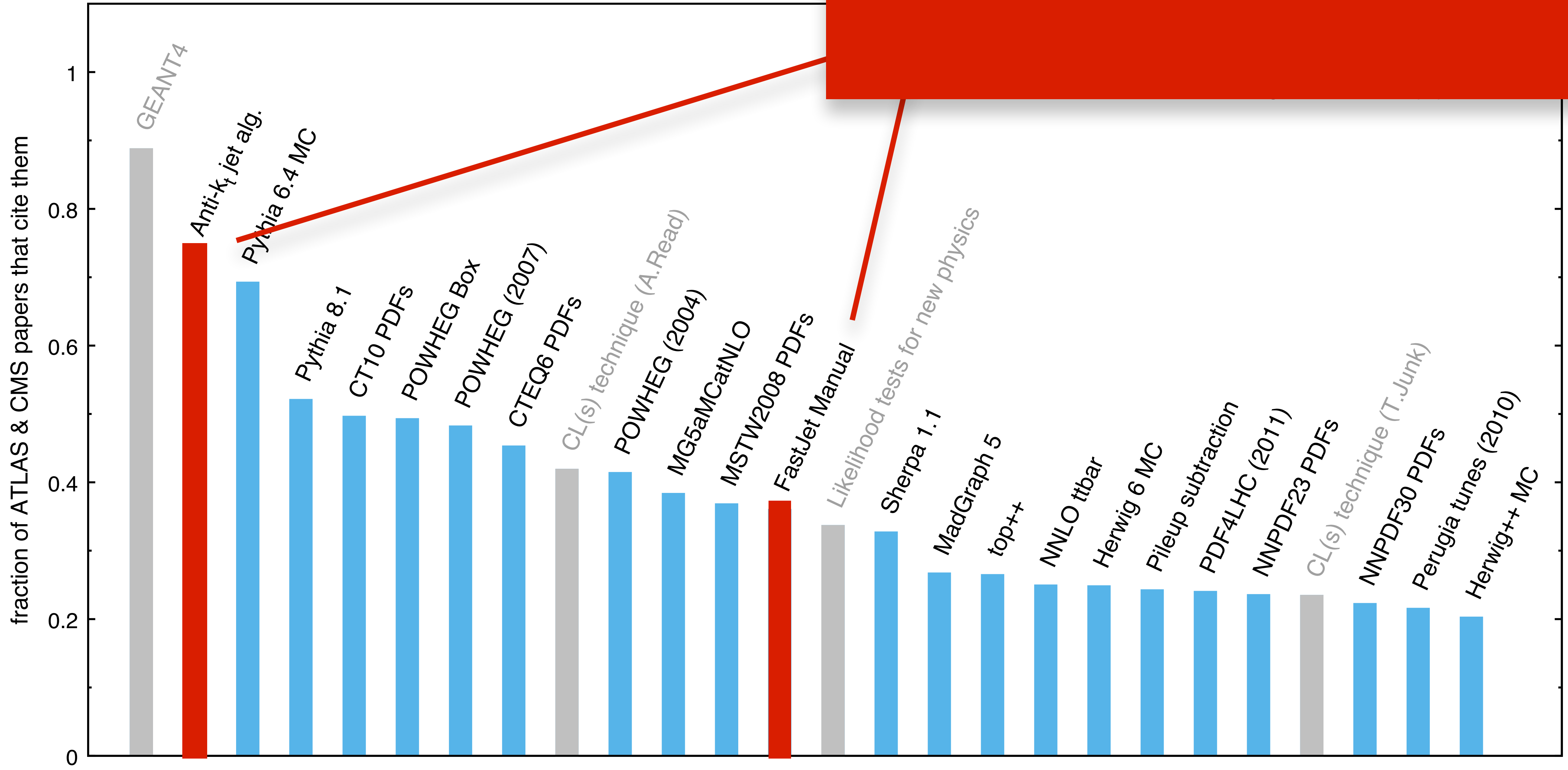
# jets

*i.e. how we make sense of the hadronic part of events*





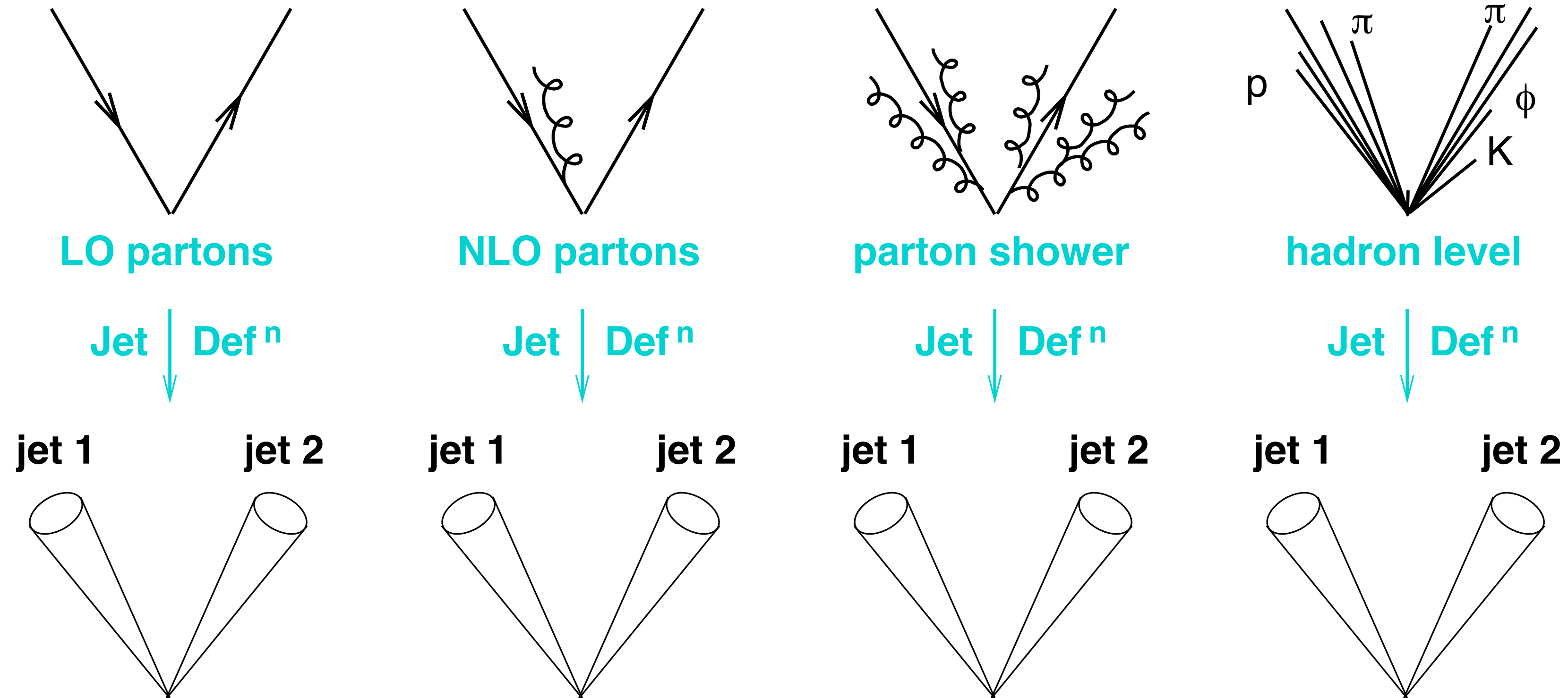
# jets: organising event information



Plot by GP Salam based on data from InspireHEP

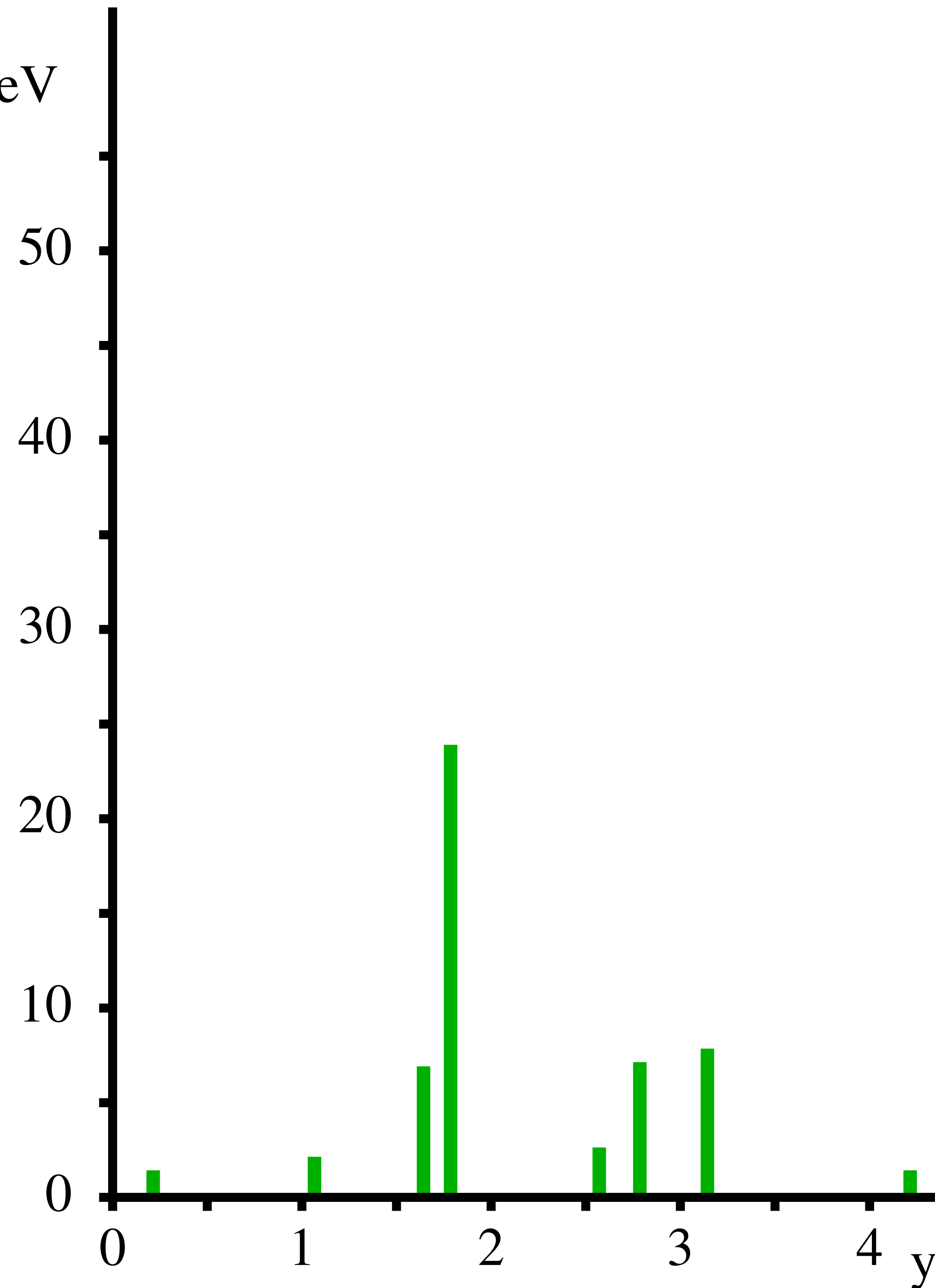
# what should a jet definition achieve? A projection to a simple picture of energy flow

---



projection to jets should be resilient to QCD effects

$p_t/\text{GeV}$

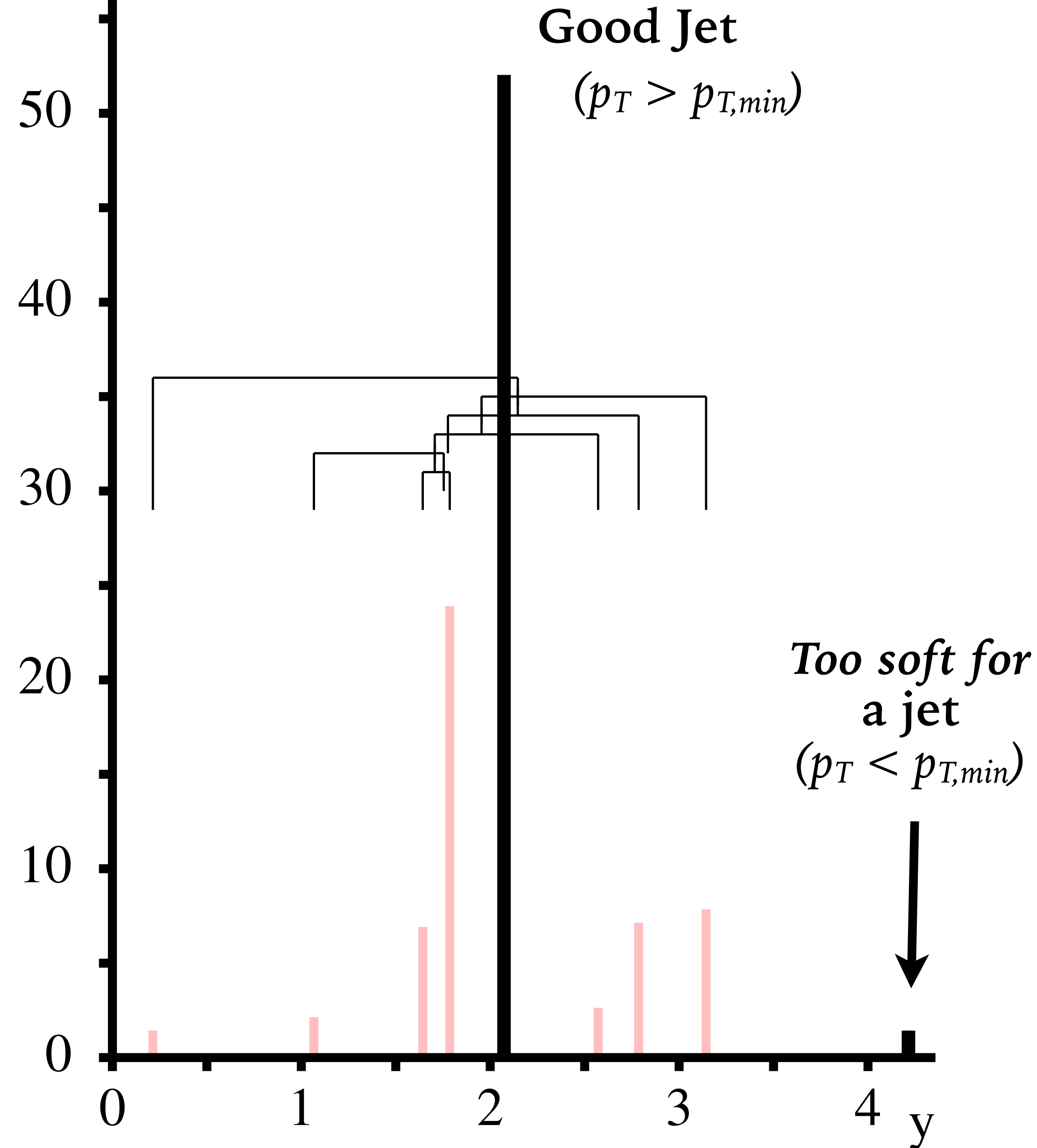


## anti- $k_t$ jet algorithm

---

- ▶ successive recombination of closest pair of particles (with some distance measure)
- ▶ parameter for reach in angle ( $R$ )
- ▶ parameter for minimum energy of jet ( $p_{t,\text{min}}$ )

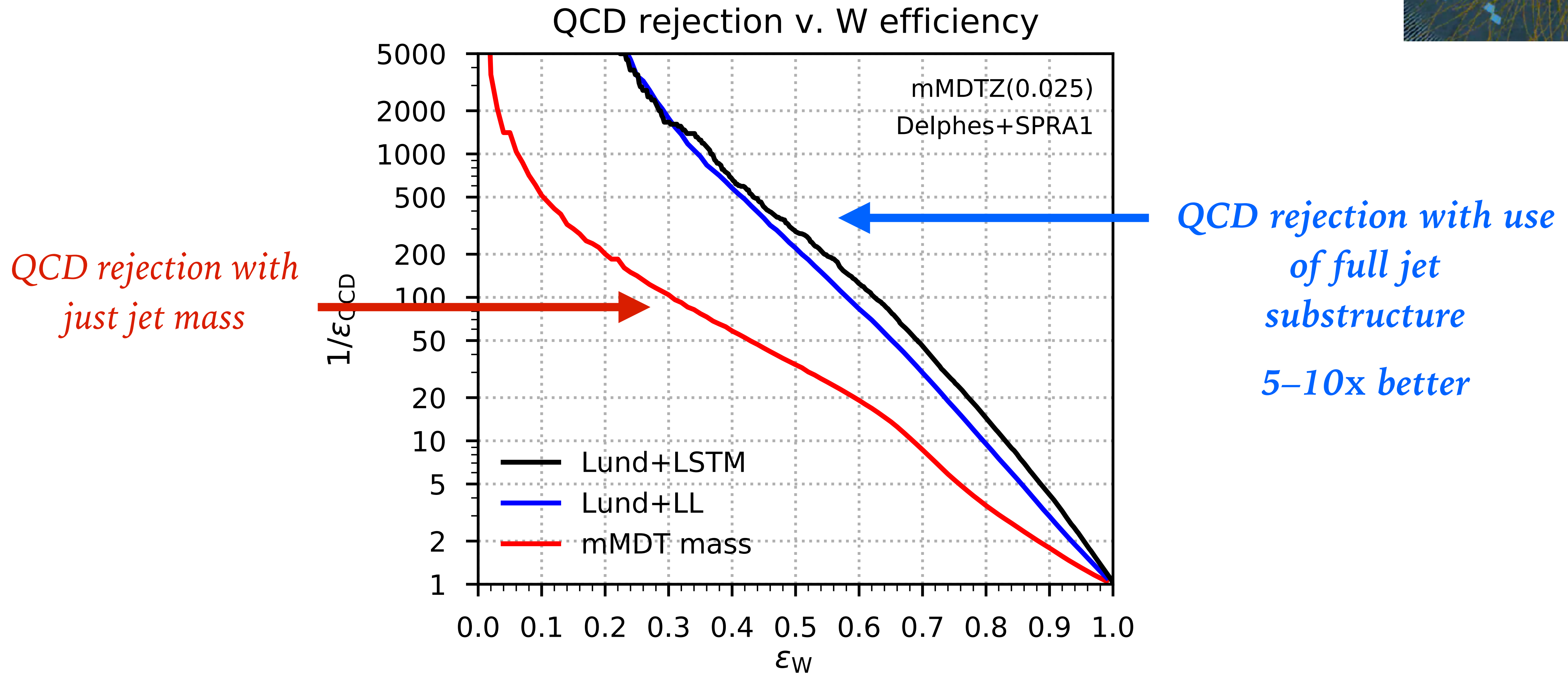
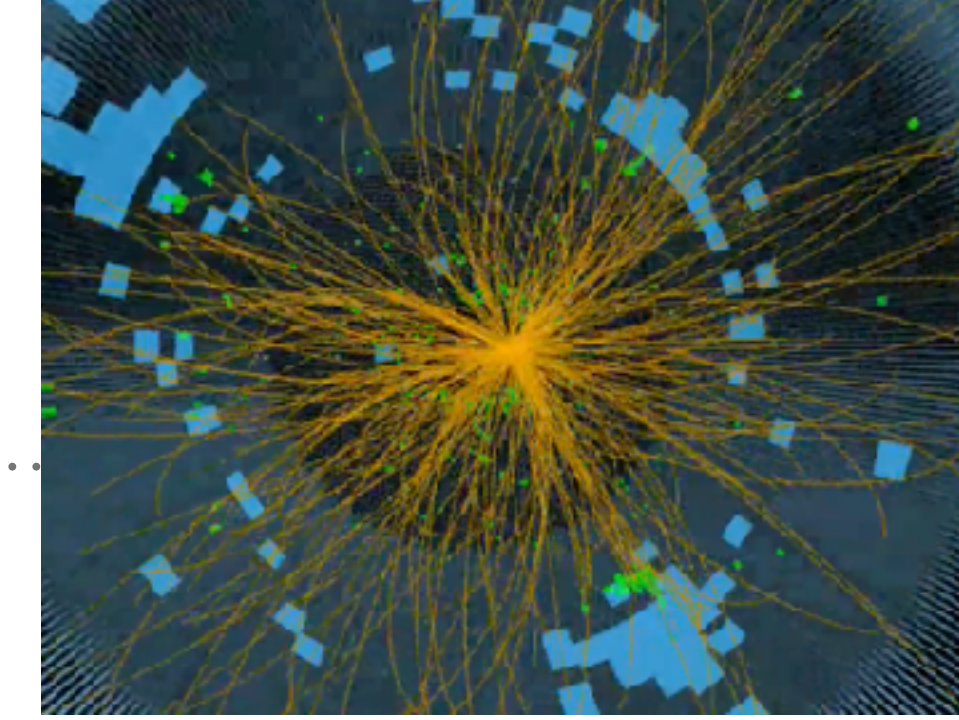
$p_t/\text{GeV}$



## anti- $k_t$ jet algorithm

- ▶ successive recombination of closest pair of particles (with some distance measure)
- ▶ parameter for reach in angle ( $R$ )
- ▶ parameter for minimum energy of jet ( $p_{t,min}$ )

# using full event information: jet substructure for W tagging



taken from Dreyer, GPS & Soyez '18



For identifying spatial clusters, we have implemented both centroid-linkage hierarchical clustering using **FastJet** [...]

Via the qSR software, FastJet can analyze a typical super-resolution dataset within a few seconds. By storing the full tree structure, the user can quickly re-cluster data and compare the resulting clusters at varying characteristic sizes.

New Results

## qSR: A software for quantitative analysis of single molecule and super-resolution data

J. Owen Andrews, Arjun Narayanan, Jan-Hendrik Spille, Won-Ki Cho, Jesse D. Thaler, Ibrahim I. Cisse

doi: <https://doi.org/10.1101/146241>

[Abstract](#) [Info/History](#) [Metrics](#) [Data Supplements](#) [Preview PDF](#)

### Abstract

We present a software for quantitative analysis of single molecule based super-resolution data. The software serves as an open-source platform integrating multiple algorithms for rigorous spatial and temporal characterizations of protein clusters in super-resolution data of living, or fixed cells.

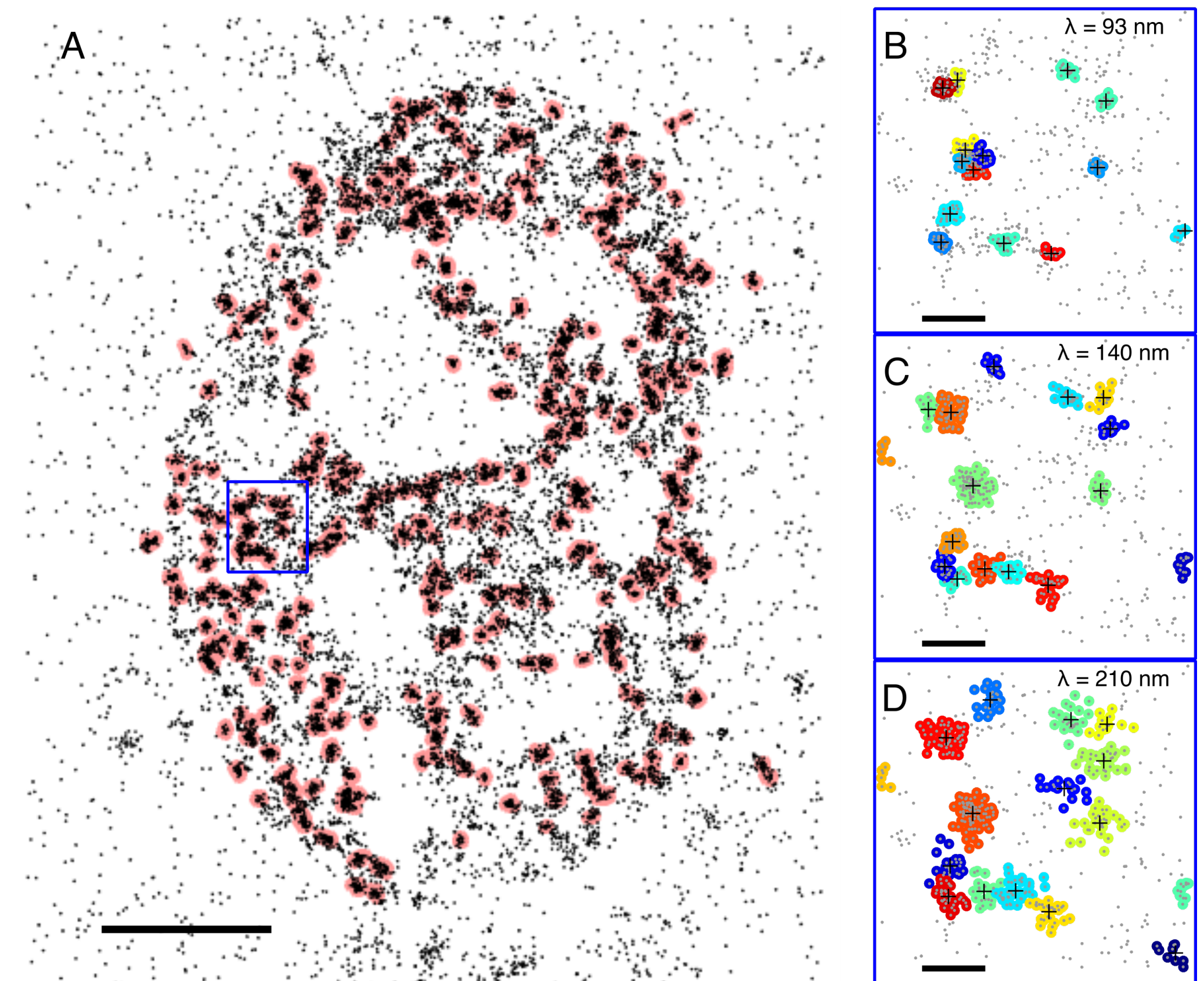


Figure S6: FastJet hierarchical clustering. (A) FastJet clusters found with a length scale of 140nm. (B-D) Zoomed in view of the region in the blue box from A. The clusters were generated by cutting the tree with a length scale of 93 nm, 140 nm, and 210 nm respectively. The black + signs mark the centroids of each cluster. Scale Bars – A: 5  $\mu$ m B - D: 500 nm

# closing

---

*does it work?*

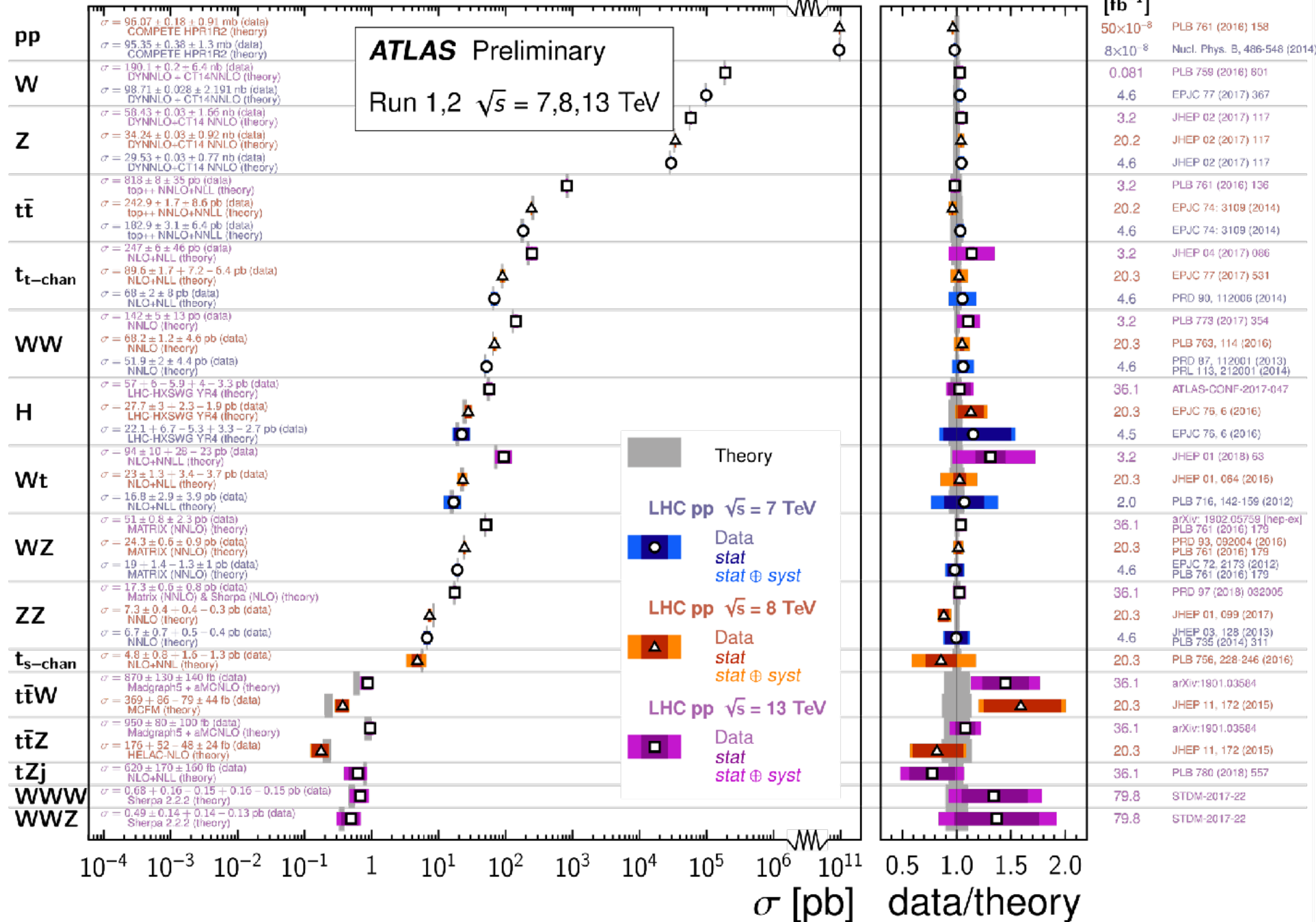
*does it work sufficiently well?*

# Standard Model Total Production Cross Section Measurements

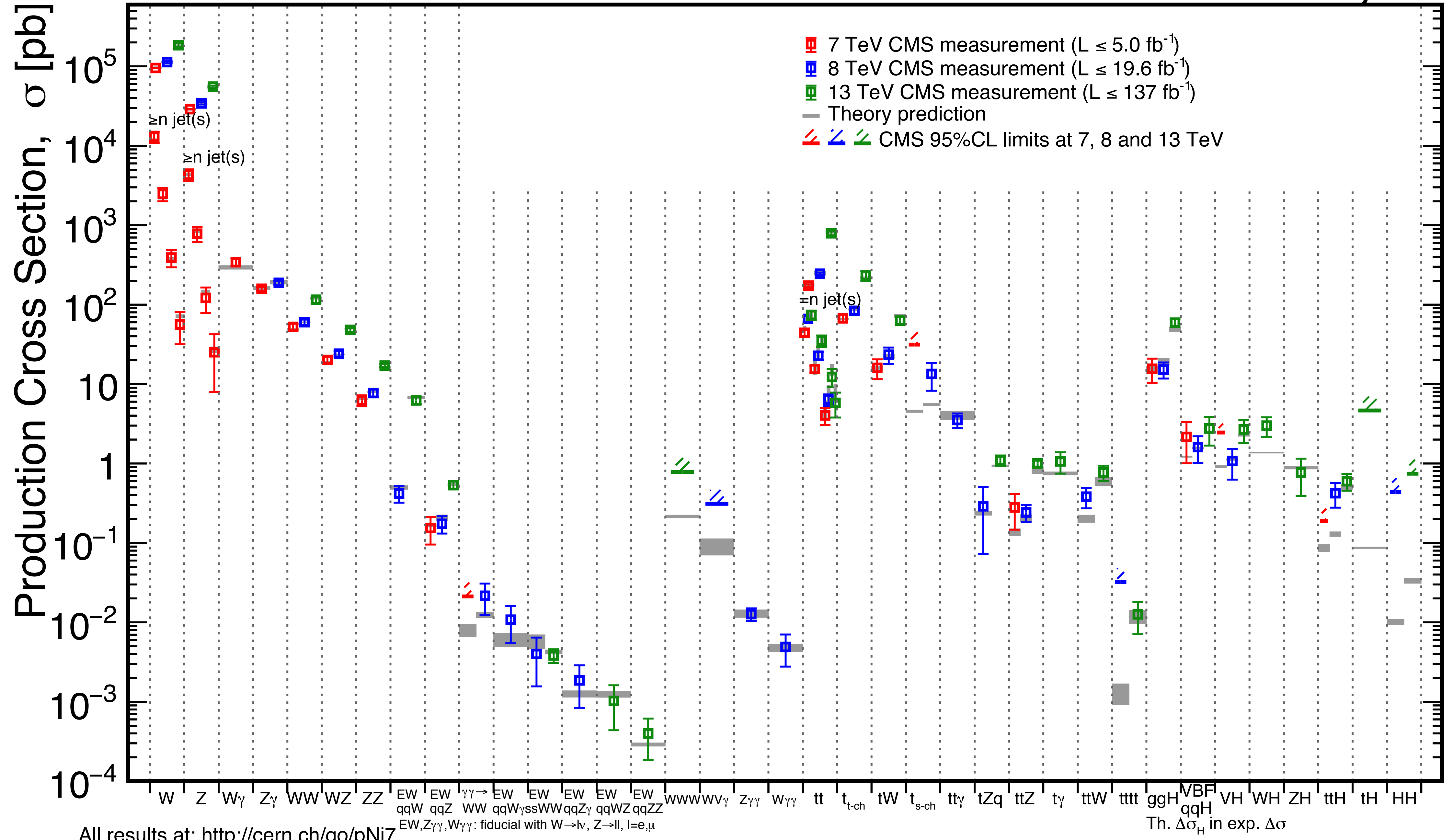
Status:  
March 2019

$\int \mathcal{L} dt$   
[fb<sup>-1</sup>]

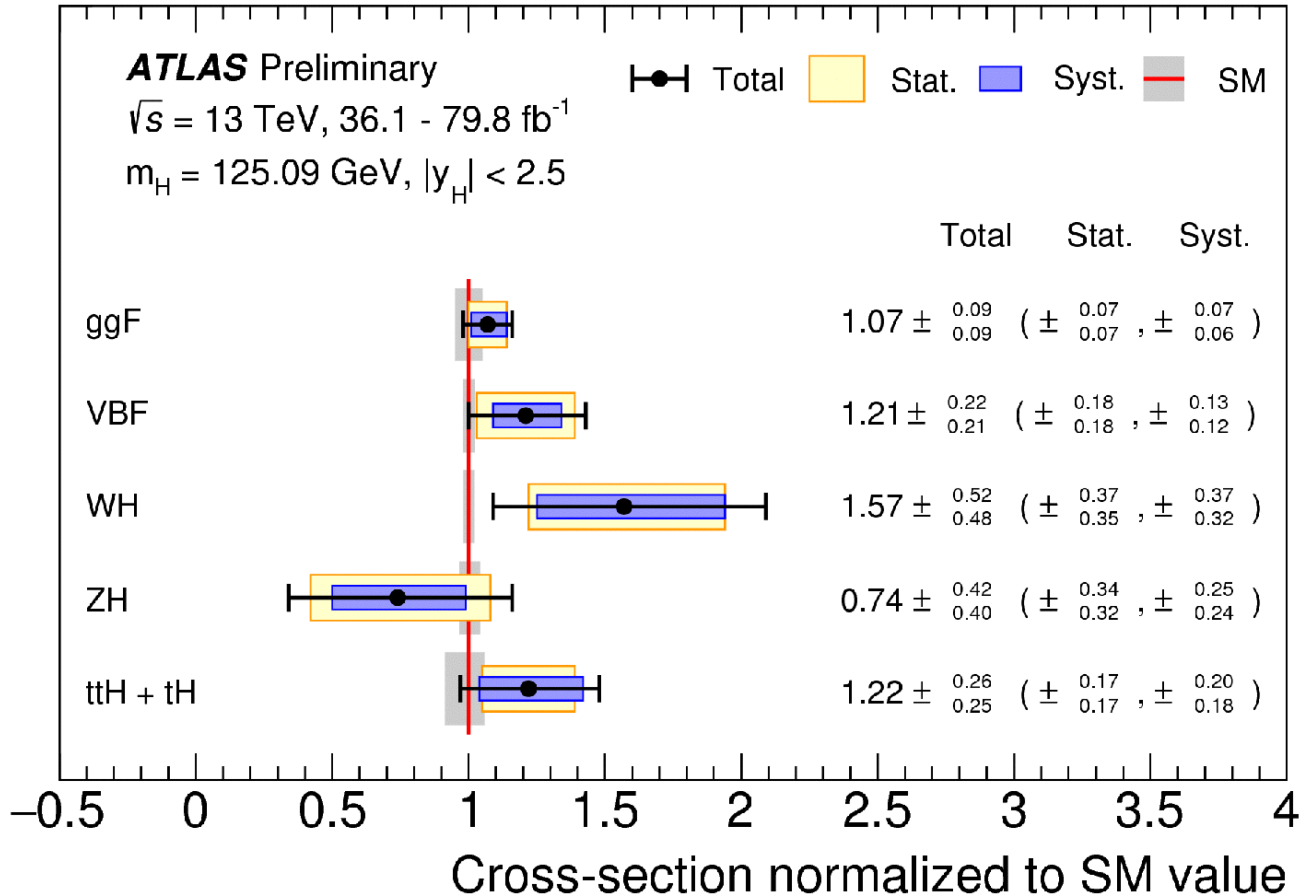
Reference



vast array of  
LHC data  
agrees with  
QCD predictions

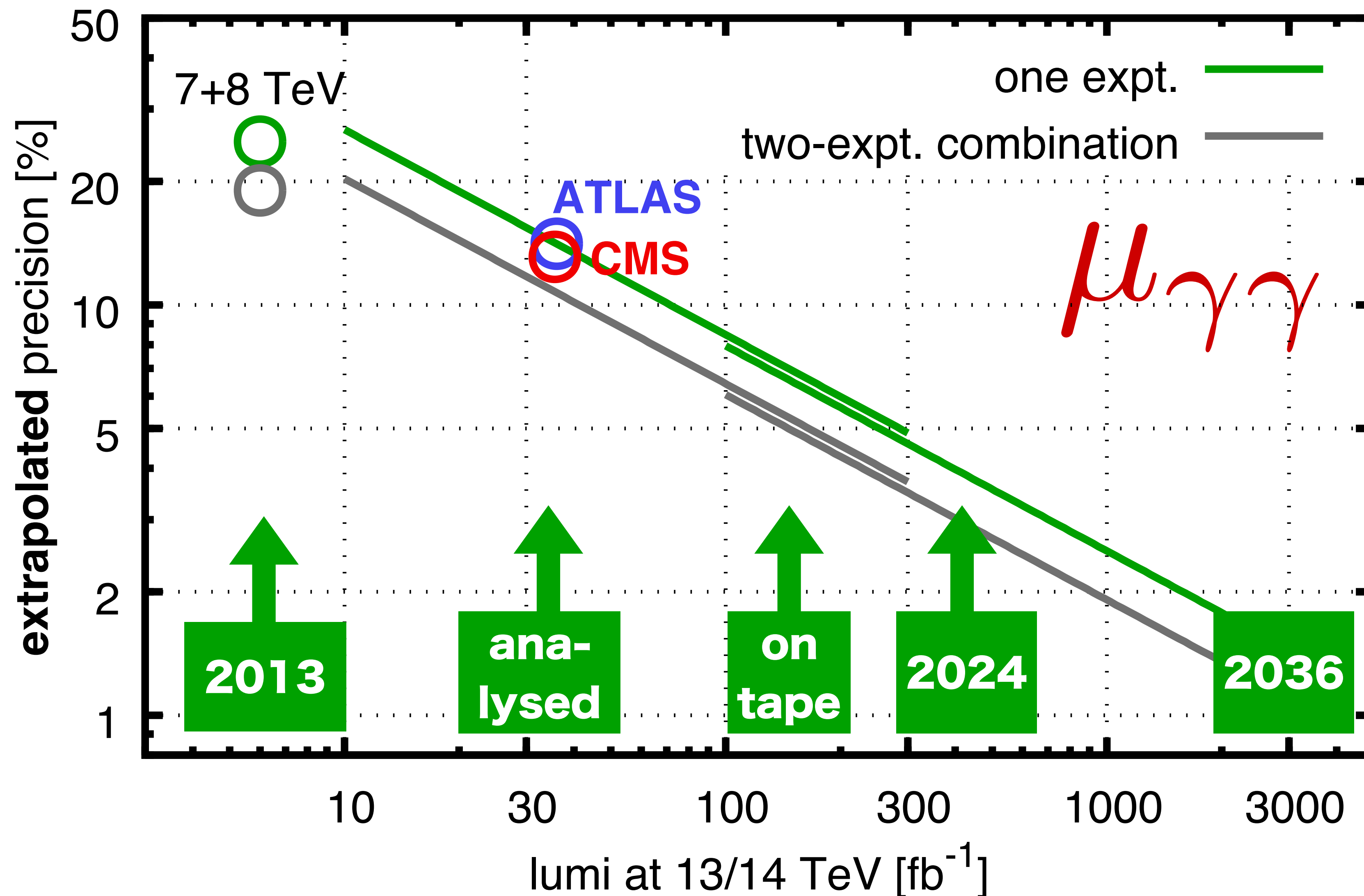


# Higgs cross-sections v. QCD theory



# Higgs precision ( $H \rightarrow \gamma\gamma$ ) : optimistic estimate v. luminosity & time

extrapolation of  $\mu_{\gamma\gamma}$  precision from 7+8 TeV results

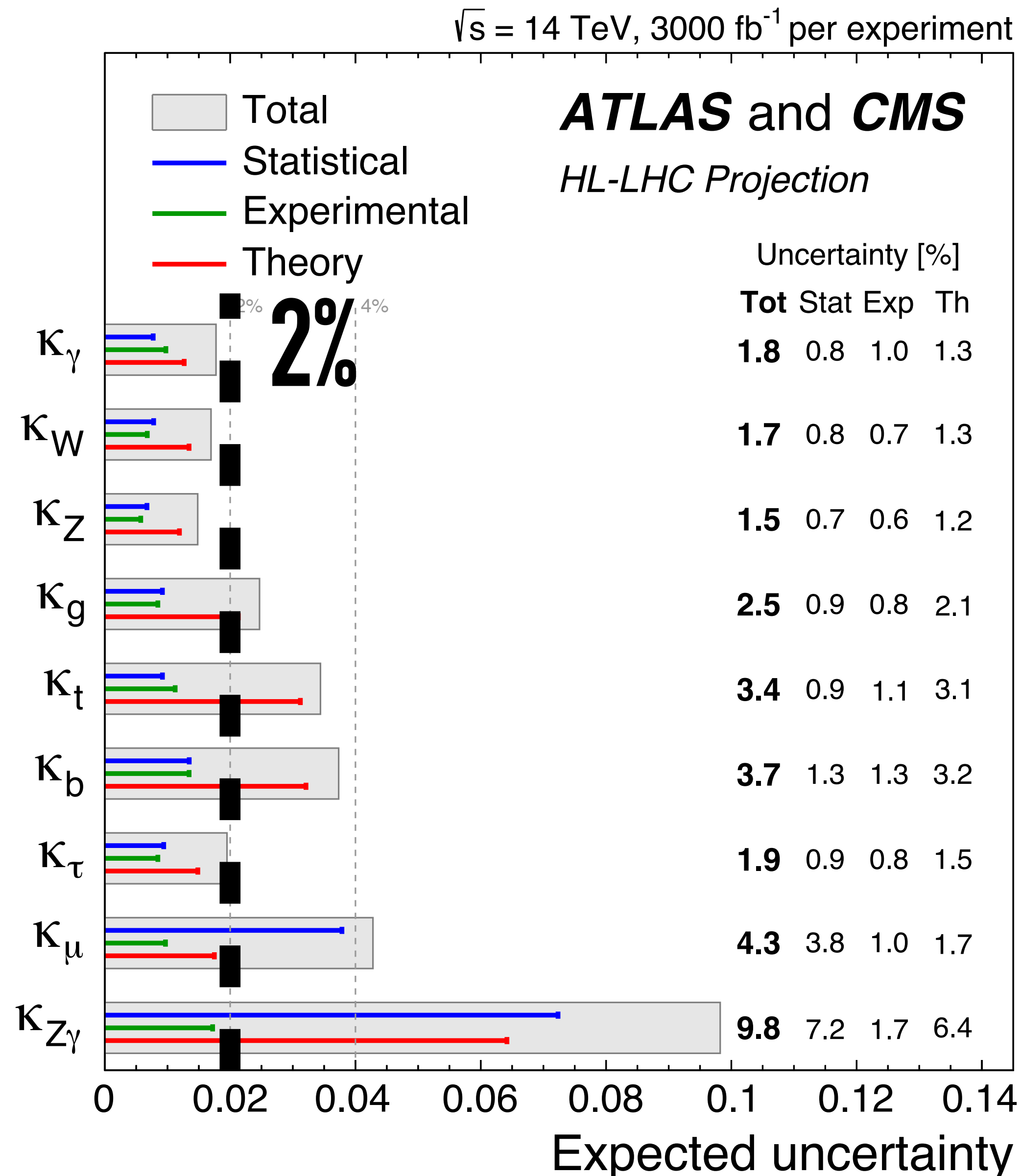


Today, Higgs coupling precisions are in the 10-20% range.

The LHC has the statistical potential to take Higgs physics from “observation” to 1–2% precision

1  $\text{fb}^{-1}$  =  $10^{14}$  collisions

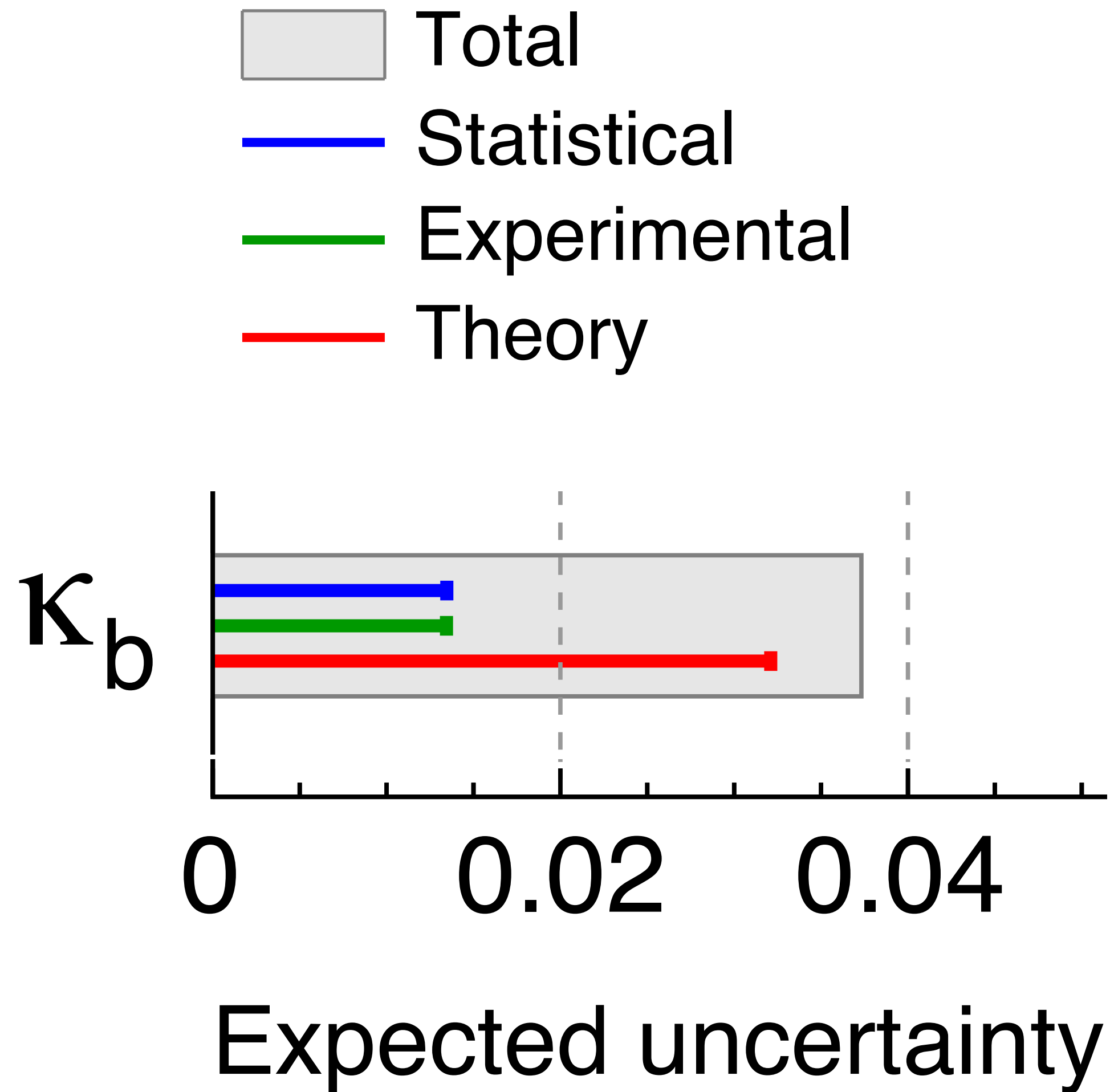
# HL-LHC official Higgs coupling projections (by ~2036)



We wouldn't consider electromagnetism established (textbook level) if we only knew it to 10%

HL-LHC can deliver 1–2% for a range of couplings

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**if theoretical interpretations can be made sufficiently accurate**

**theory (QCD) uncertainty dominates, even with an assumption of  $\times 2$  improvement by 2030s**

**can we ensure that QCD is up to the task?**