Small-x

(a micro review)

Gavin Salam
LPTHE, Université Paris VI et Paris VII

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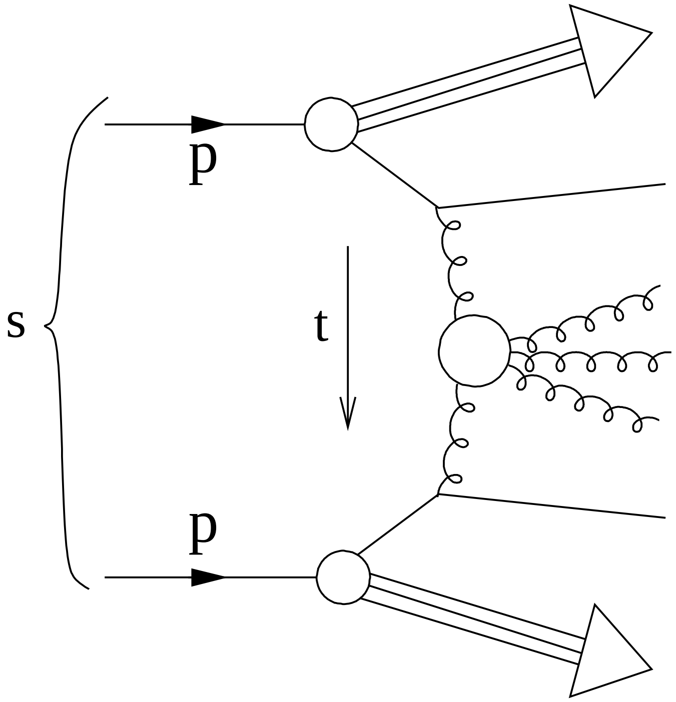
Outline

- ✗ Introduction
- ✗ Experimental searches for BFKL
- ✗ NLL theory status
- ✗ Saturation/BKP

What's it (supposed to be) good for?

Small-x physics is implicated whenever longitudinal energy exchange is much larger than transverse ($s \gg t, u$):

big rapidity intervals



<u>Situations crying out for understanding</u>

- ✗ pp, γp total cross sections
  (Is it really $\sim s^{0.08}$? Unitarity?)
- ✗ properties of minimum bias events
  (*e.g.* NP corrections to PT, radiation)
- ✗ small-x structure functions
  (*e.g.* input for collider predictions, νN)
- ✗ Diffraction
  (*e.g.* diffractive Higgs production)

(Prediction of growth of pp cross section was aim of original BFKL calculation.)

BUT: most of these problems are non-perturbative.

What's it (actually) good for?

All calculations are based on *resummation of logarithms:* $(x \sim 1/s)$

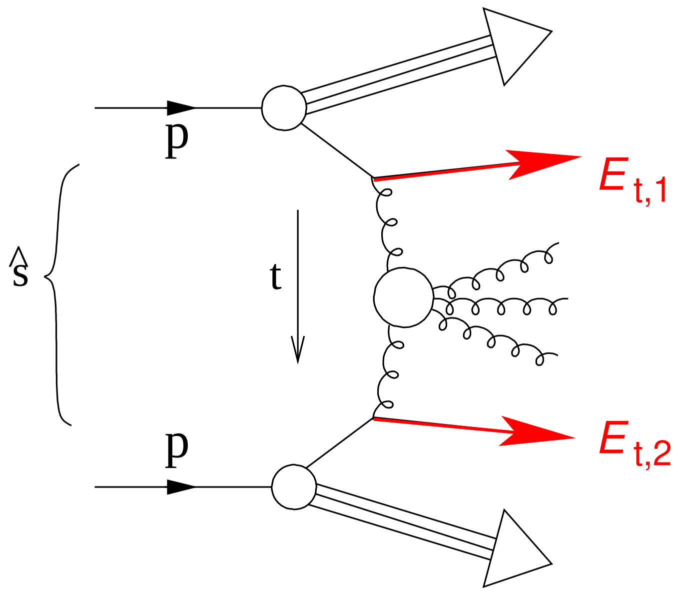
$(\alpha_s \ln s)^n$ (LL), $\alpha_s(\alpha_s \ln s)^n$ (NLL) NB: $\alpha_s \sim \alpha_s(t)$

For this to work, *coupling has to be perturbative.* $\Rightarrow$ largish transverse momentum.

Reactions where it can be used:

Take two virtual probes (jet, γ^*) separated by a large rapidity interval.

- ✗ Mueller-Navelet jets
- ✗ DIS forward jets
- ✗ $\gamma^*\gamma^*$ cross section



$$\hat{s} \gg E_{t,1}^2 \sim E_{t,1}^2 \gg \Lambda_{QCD}^2$$

Eliminate a non-perturbative scale by factorization

- ✗ small-x *splitting functions*

Main predictions?

Original leading log (LL) calculation (BFKL), summation of $(\alpha_s \ln s)^n$, predicted

$$\sigma \sim s^{4\ln 2 \frac{\alpha_s N_C}{\pi}} \sim s^{0.5}$$

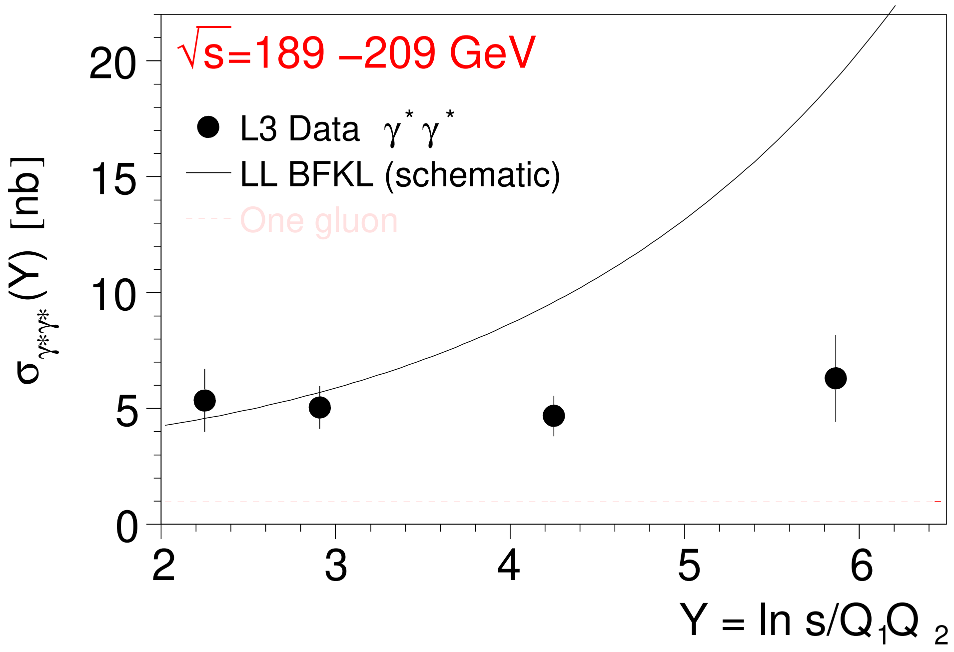
Experimentally strong signal

- ✗ rapidly increasing cross section
- ✗ larger emitted transverse momenta (ordering in rapidity; but not in k_t)

Theoretically exciting

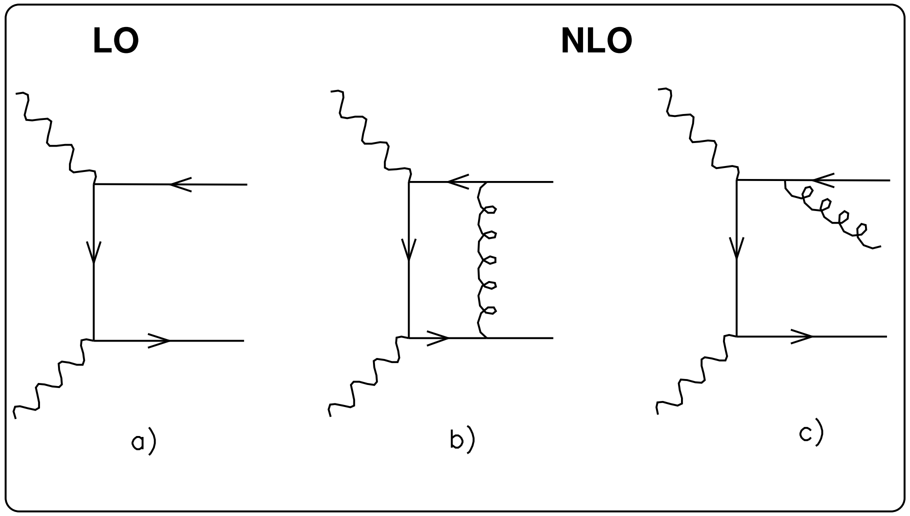
- ✗ leads to 'violation' of unitarity ($\ln^2 s$)
- ✗ implies *dense* systems of *weakly interacting* partons
- ✗ 'colour-glass' condensate

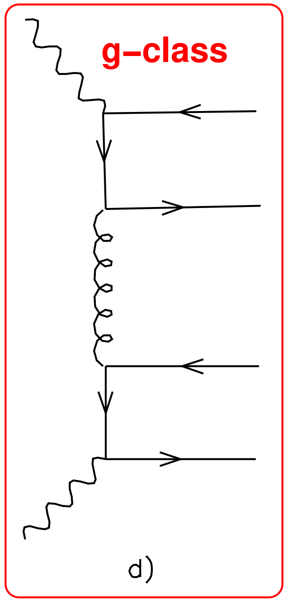
cleanest test should be $\gamma^*\gamma^*$:

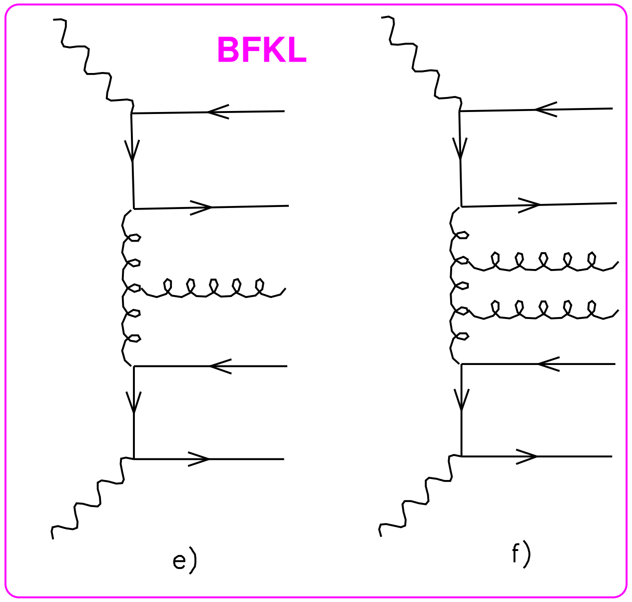


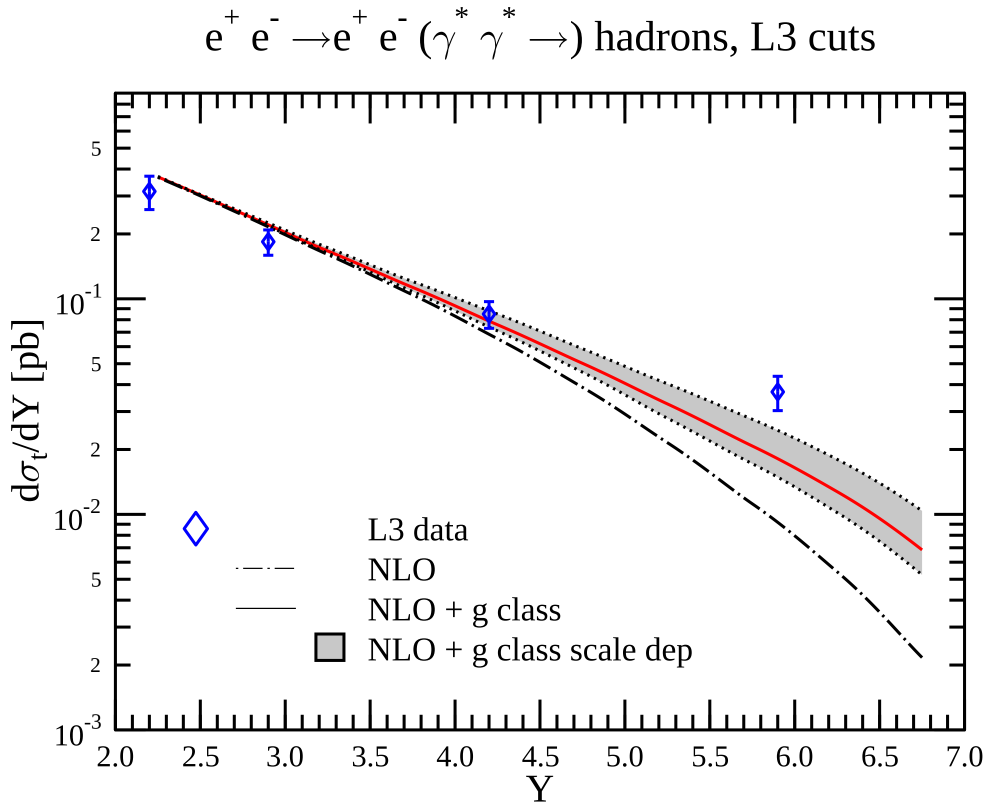
Data clearly incompatible with LL BFKL.

Do we need BFKL at all?









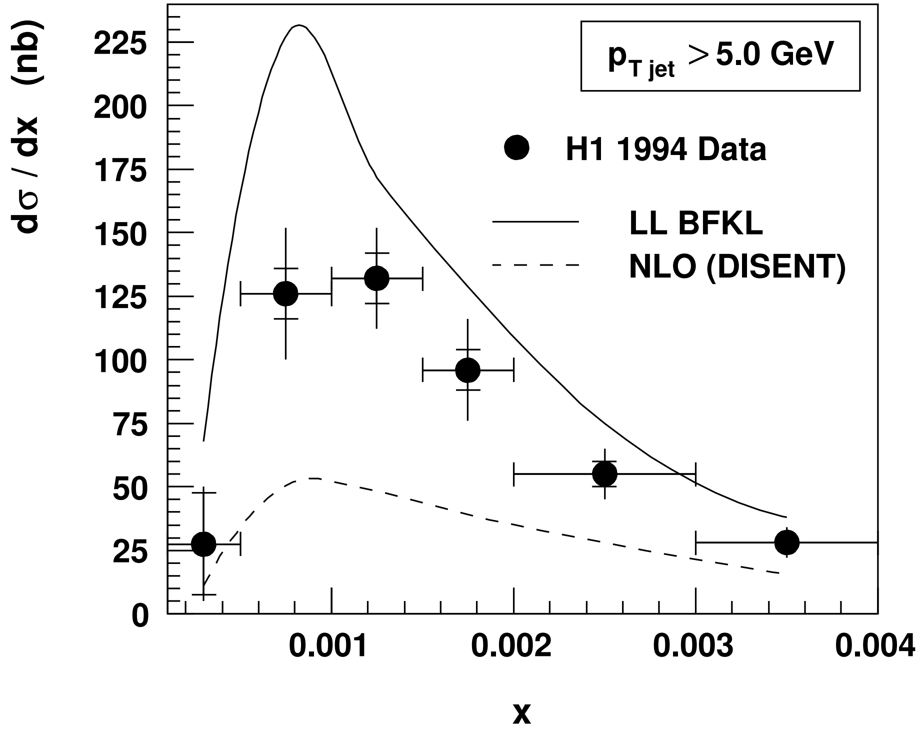
NLO calculations by Del Duca et al. of non-BFKL contributions (quark exchange)

+ first BFKL contribution: bare gluon exchange, $\sigma \sim s^0$. Not enough.

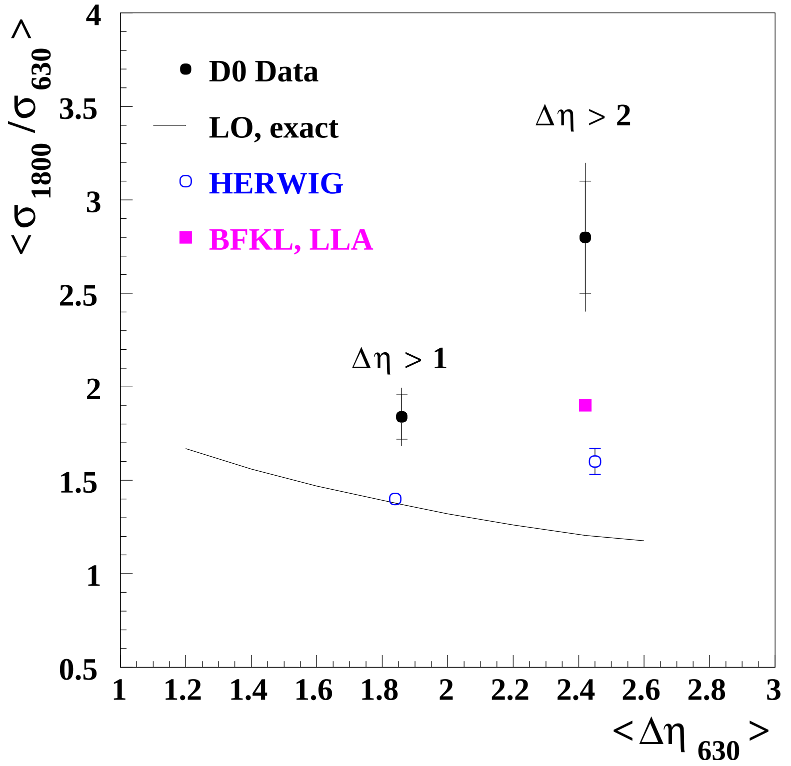
need something BFKL-like(?)

(NB: not all data in agreement)

Do we need BFKL at all (cont.)?



HERA *forward-jet* data also show LL BFKL too large, but 1-gluon exchange (NLO) too small.

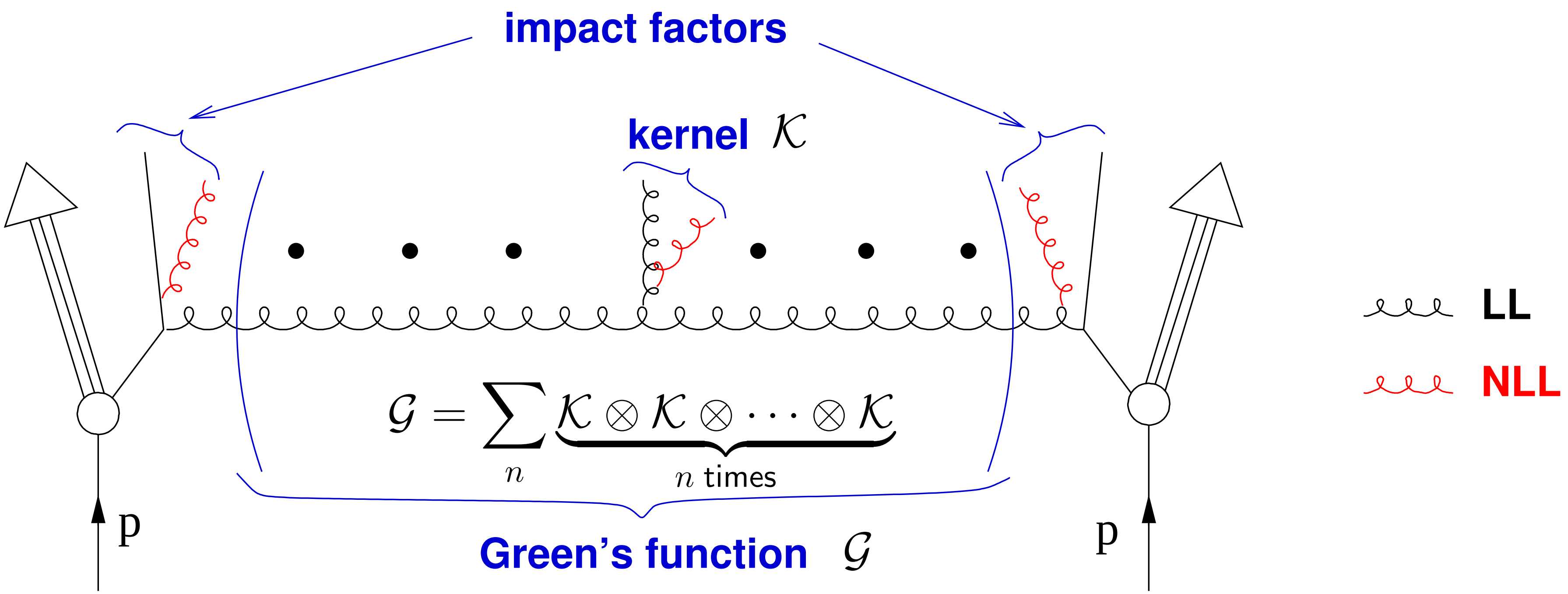


Only D0 data *(Mueller-Navelet jets)* show a rise which is larger than BFKL (but possibly large sub-asymptotic effects).

NB: for moderate $\ln s$ Herwig can simulate BFKL; asymptotically $\sigma \sim e^{\sqrt{\ln s}}$

Overhauling the theory? Going beyond LL...

Break the problem into constituent parts.



Approximate higher orders estimates:
(semi-analytical calcn's & MC's)

- ✗ running coupling
- ✗ full splitting function ($1/z + \ldots$)
- ✗ 'kinematic constraint'
- ✗ energy conservation

Exact higher orders:

- ✗ NLL kernel (1998)
- ✗ NLL impact factors (2002...)
- ✗ can be supplemented with approx. higher orders

NLL kernel — 10 years' work

L.N. Lipatov and V.S. Fadin, *Sov. J. Nucl. Phys.* **50** (1989) 712; V.S. Fadin, R. Fiore and M.I. Kotsky, *Phys. Lett.* **B 539** (1995) 181; V.S. Fadin, R. Fiore and M.I. Kotsky, *Phys. Lett.* **B 387** (1996) 593 [hep-ph/9605357]; V.S. Fadin, and L.N. Lipatov, *Nucl. Phys.* **B 406** (1993) 259; V.S. Fadin, R. Fiore and A. Quartarolo, *Phys. Rev.* **D 50** (1994) 5893 [hep-th/9405127]; V.S. Fadin, R. Fiore, and M.I. Kotsky, *Phys. Lett.* **B 389** (1996) 737 [hep-ph/9608229]; V.S. Fadin and L.N. Lipatov, *Nucl. Phys.* **B 477** (1996) 767 [hep-ph/9602287]; V.S. Fadin, M.I. Kotsky and L.N. Lipatov, *Phys. Lett.* **B 415** (1997) 97; I.A. Korchemskaya and G.P. Korchemsky, *Phys. Lett.* **B 387** (1996) 346 [hep-ph/9607229]; J. Blumlein, V. Ravindran and W.L. van Neerven, *Phys. Rev.* **D 58** (1998) 091502 [hep-ph/9806357]; S. Catani, M. Ciafaloni and F.Hautman, *Phys. Lett.* **B 242** (1990) 97; S. Catani, M. Ciafaloni and F.Hautman, *Nucl. Phys.* **B 366** (1991) 135; V.S. Fadin, R. Fiore, A. Flashi, and M.I. Kotsky, *Phys. Lett.* **B 422** (1998) 287 [hep-ph/9711427]; V. Del Duca, *Phys. Rev.* **D 54** (1996) 989; V. Del Duca, *Phys. Rev.* **D 54** (1996) 4474; V. Del Duca and C.R. Schmidt, *Phys. Rev.* **D 57** (1998) 4069 [hep-ph/9711309]; V. Del Duca and C.R. Schmidt, hep-ph/9810215.

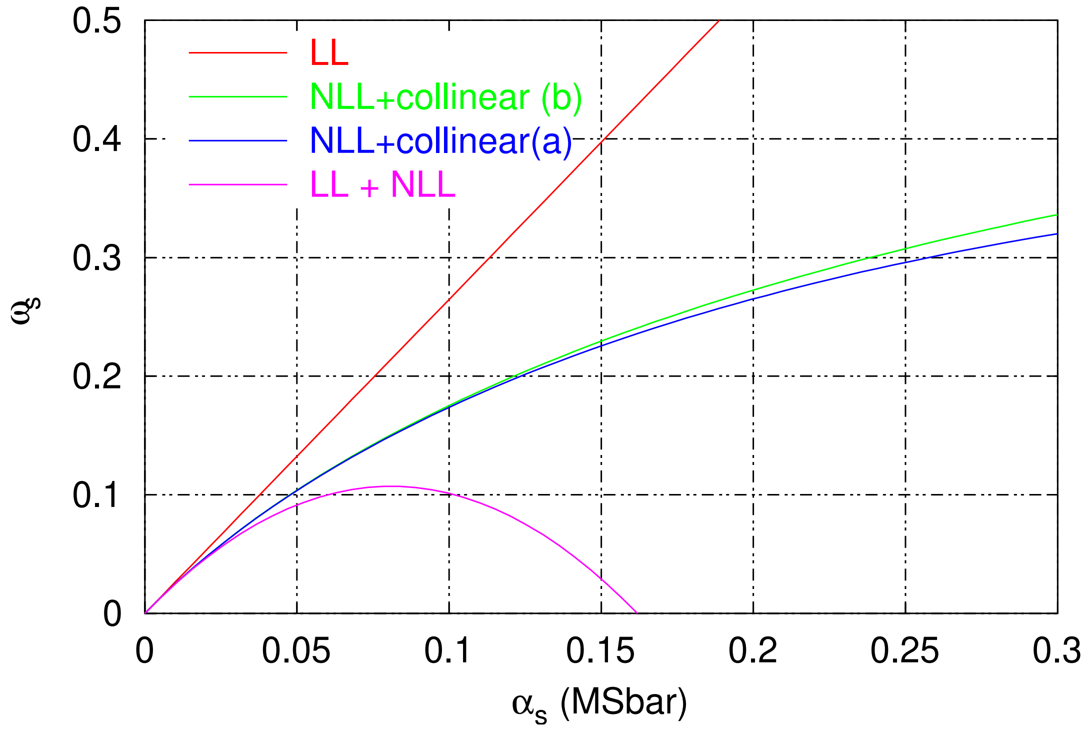
Put together in 1998:

M. Ciafaloni and G. Camici, *Phys. Lett.* **B 430** (1998) 127 [hep-ph/9803389];
M. Ciafaloni, *Phys. Lett.* **B 429** (1998) 349 [hep-ph/9801322];
G. Camici and M. Ciafaloni, *Phys. Lett.* **B 412** (1997) 396 [hep-ph/9707390];
V.S. Fadin and L.N. Lipatov, *Phys. Lett.* **B 429** (1998) 127 [hep-ph/9802290].

NLL kernel results

Power ω associated with s^ω growth gets $\mathcal{O}(\alpha_s)$ correction: $(n_f = 0)$

$$\omega \simeq 2.65\alpha_s(1 - 6.18\alpha_s)$$



Very poor convergence!

Is that the end of BFKL?

Yet higher orders may sort out the problem. . .

Several proposals were made for all-orders improvement (Schmidt, Brodsky et al) — but sensitive to a 'free-parameter' or a scheme choice.

One proposal (Ciafaloni, Colferai, GPS), *predicts NLL corrections to within better than 7%.*

- ✗ Large NLLx corrections related to collinear logs $(\alpha_s \ln^{(2)} Q^2)^n$
- ✗ Can be used to estimate collinear pieces of N^{n}LL
- ✗ Full NLL+collinear is *stable to within 10-15%.*

e.g. Forshaw et al. '99

Beware: Kernel $\leftrightarrow$ Green's function non trivial

Each iteration of the kernel will have a different scale for the coupling

The typical scale in the evolution may $\neq$ scale of 'hard probes'

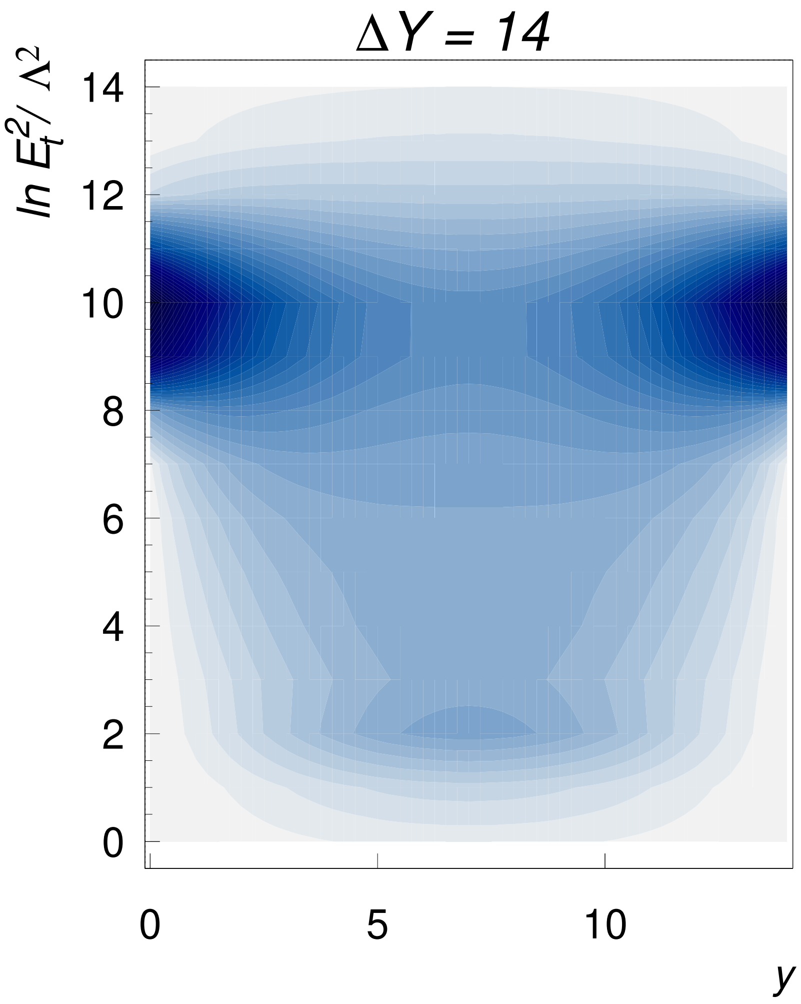
Much formal analysis is still limited to LL BFKL case (but expect extensions to NLL soon).

Fig. illustrates relevant scales in LL evolution for two probes with

- ✗ $\ln E_t^2/\Lambda^2 = 10 \Leftrightarrow E_t \sim 35$ GeV
- ✗ $\Delta Y = 14 \Leftrightarrow x \simeq 10^{-6}$

General issue for reliable predictions:

vital to identify & separate out non-perturbative contribution

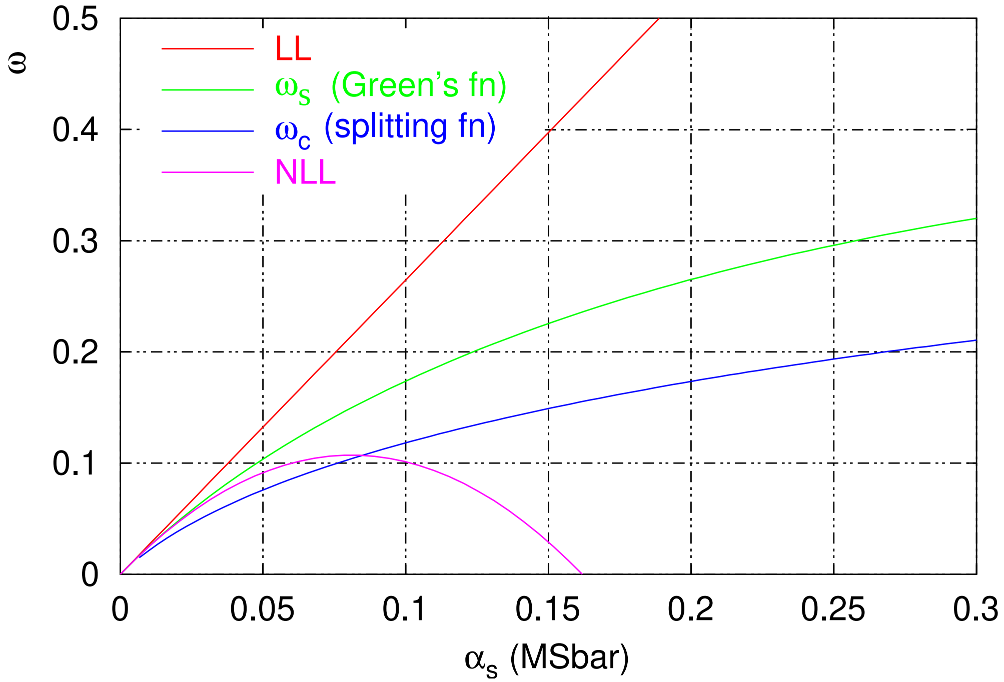


Kernel $\leftrightarrow$ splitting function

Iteration of kernel is subtle

- ✗ must factorize out all iterations below Q^2.
- ✗ asymmetry introduces big corrections of order $\alpha_s(b\alpha_s)^{2n/3}$

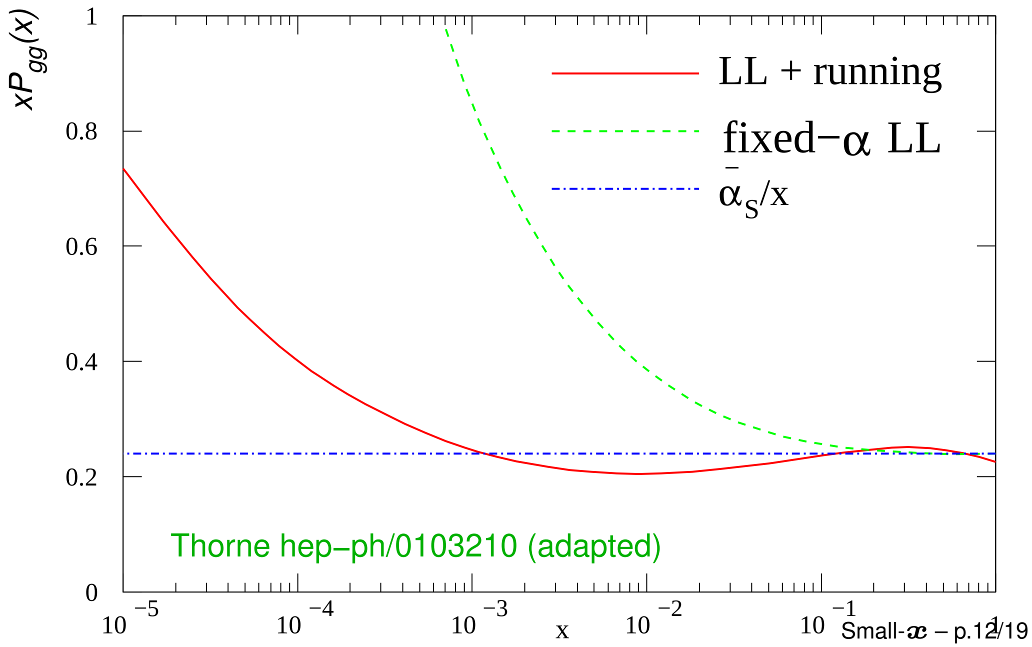
Hancock & Ross '92
Ciafaloni, Colferai & GPS '99



Not only power that is reduced

- ✗ Analytic structure of splitting function is changed (branch point $\longrightarrow$ pole)
- ✗ growth sets in at much smaller-x
- ✗ Related formulations compatible with HERA data

Thorne '99-'01
Altarelli Ball & Forte '01



Impact factors — almost complete

Photon impact factor

Bartels, Gieseke, Kyrieleis & Qiao '00, '01
Fadin, Ivanov & Kotsky '00 '01

- ✗ Real and virtual pieces calculated separately.
- ✗ Cancellations between them and subtraction of LL pieces are almost ready (being written up).
- ✗ Phase space integrals (all now finite) remain to be done.

Jet impact factor

Ciafaloni & Colferai '98, '99
Bartels, Colferai & Vacca '00, '01

- ✗ Real and virtual pieces known.
- ✗ Cancellations between them and subtraction of LL pieces are done.
- ✗ Phase space integrals (all now finite) remain to be done (inserting the jet algorithm).

Qu.: *What will the convergence be like?* It may be necessary to do a collinear improvement here too.

Saturation and all that

Rising cross section $\sigma \sim s^\omega$ is not possible for ever

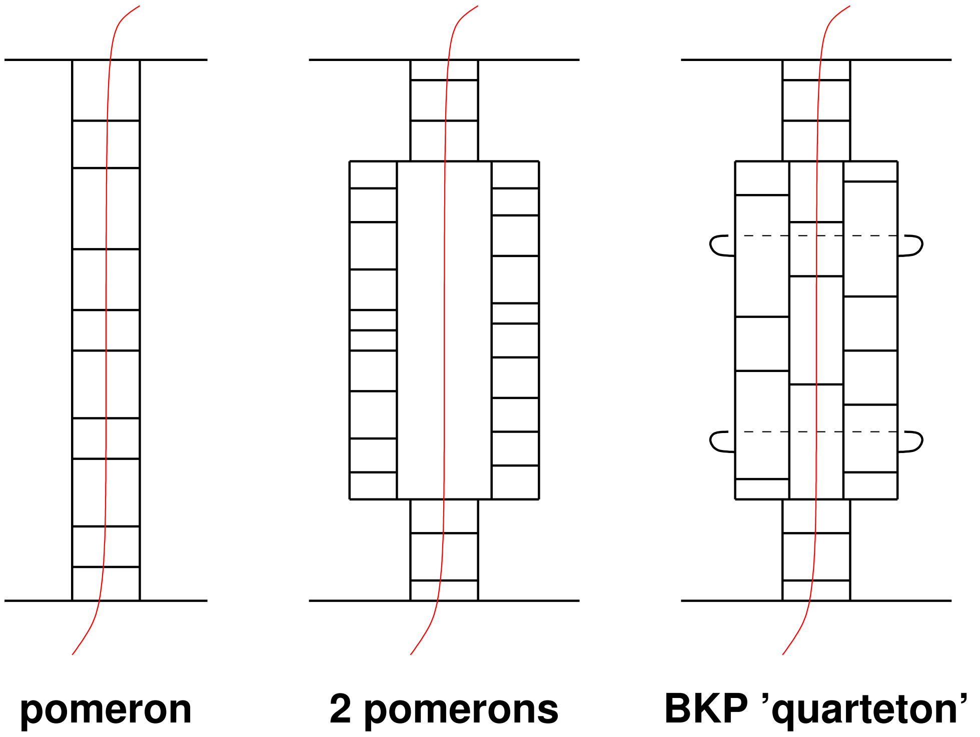
(OK, so far we can barely confirm that it rises at all. . .)

✗ Unitarity (probability of anything happening ≤ 1)

✗ Saturation (you can't fit too many gluons into one place)

✗ Other diagrams start contribution

multi-(reggeized)-gluon states



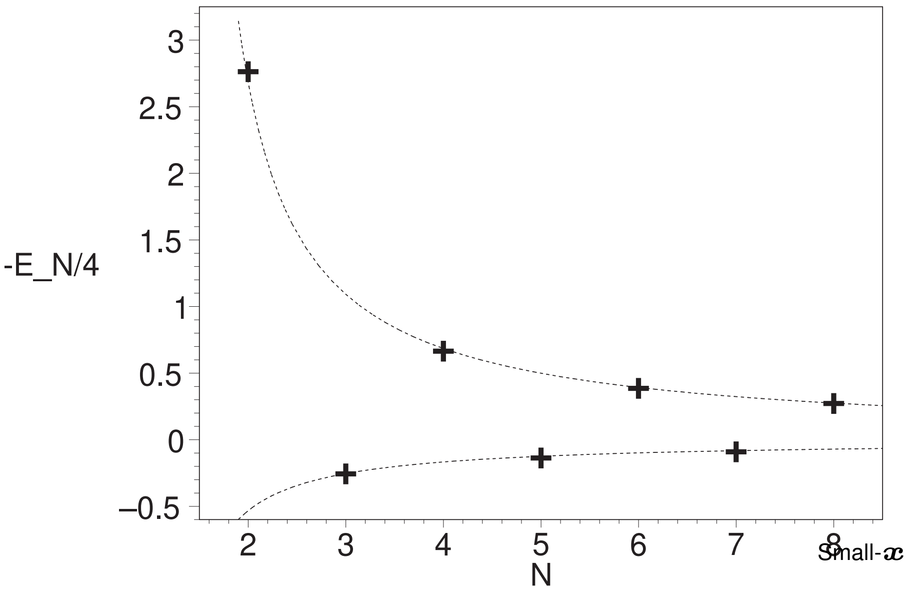
- ✗ Pomeron is two-gluon state: $s^{4\ln 2\bar{\alpha}_s}$
- ✗ 2-pomerons give: $s^{8\ln 2\bar{\alpha}_s}$
- ✗ Since 1981 ∃ Bartels, Kwiecinski Praszalowicz equation (BKP) for four-gluon state. But no solution: $\boxed{s^{???}}$ . . .

What dominates? 2-pomeron or BKP?

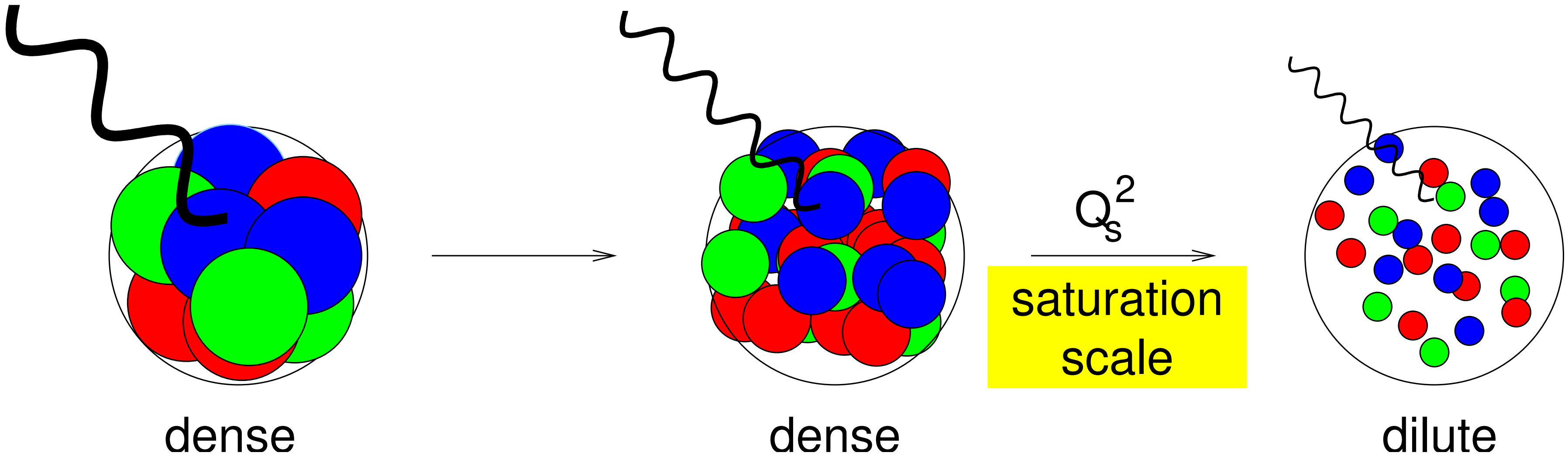
Methods based on integrable models have solved the problem

Derkachov, Korchemsky, Kotanski & Manashov hep-ph/0204124
de Vega & Lipatov hep-ph/0204245

Two-pomeron dominates. . . (But still issues to be resolved — couplings)



Saturation scales



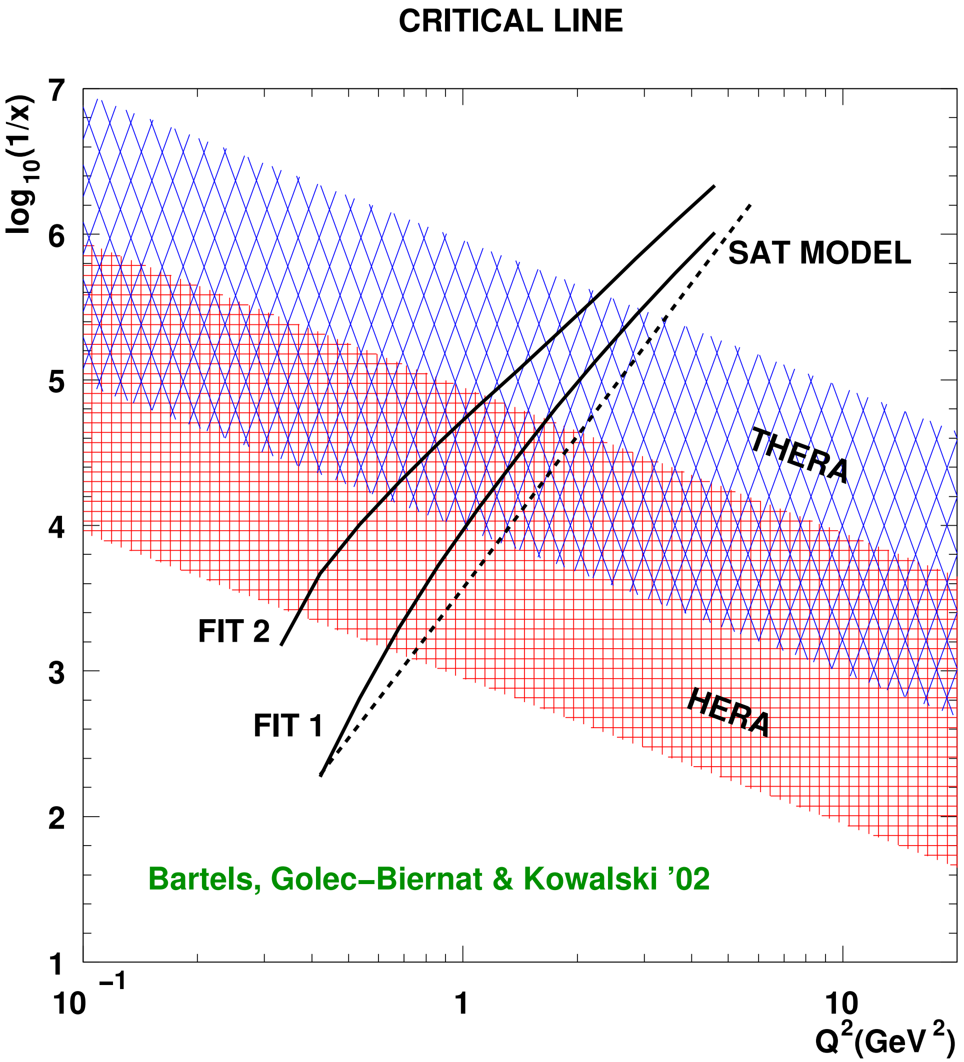
Below saturation scale: *dense system of gluons* $(\rho \sim 1/\alpha_s)$

Above saturation scale: *dilute system of gluons* $(\rho \ll 1/\alpha_s)$

Saturation scales (cont.)

<u>Big business at HERA</u>

- ✗ Models including saturation are fitted to HERA data
- ✗ Saturation sets in (perhaps?) just at limit of perturbative region
- ✗ Rises with decreasing x



Saturation scales (cont.)

Big business at HERA

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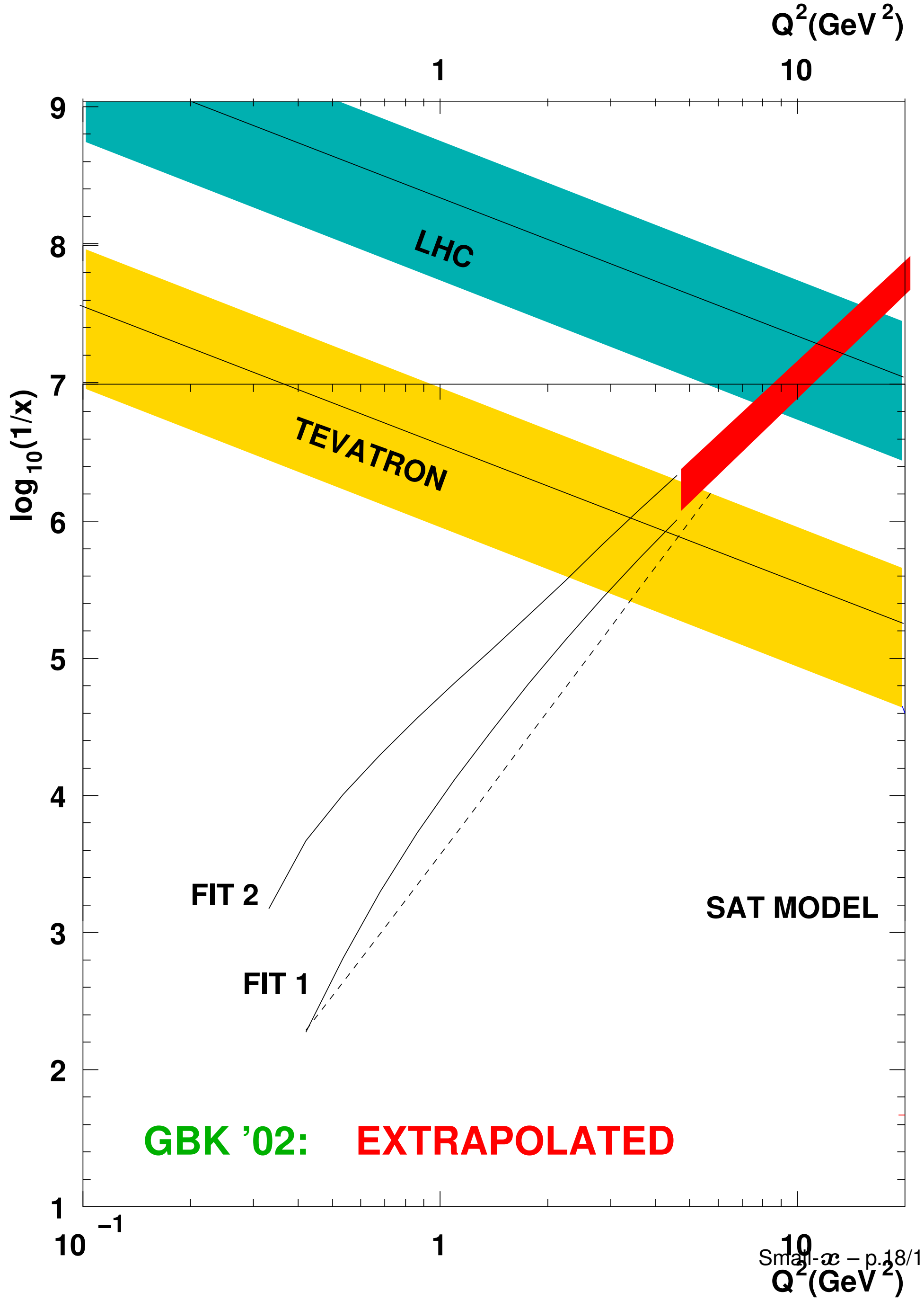
Back of the envelope: Tevatron? LHC?

- ✗ Typical transverse momentum in minimum bias is Q_s^2
- ✗ Convert from DIS using $x \sim \dfrac{Q_s^2(x)}{s}$
- ✗ LHC minimum bias $k_T \simeq 2\times$ Tevatron minimum bias?
- ✗ Very rough? *But beware: transverse momentum/collision could rise much faster than the cross section*



Conclusions

- ✗ LL BFKL excluded by the data. But evidence for something beyond pure gluon exchange
- ✗ The *NLL ingredients are almost all there* — expect full NLL predictions soon!
- ✗ NLL corrections must be supplemented by all-orders collinear effects (otherwise bad convergence)
- ✗ Interesting developments on the multi-gluon ladders and saturation fronts (see also talk by Kwiecinski).
  - ☞ New insights on minimum bias?

- ✗ Many things not discussed
  - ☞ Event generators (LDC, Small-x, Cascade)
  - ☞ Related approaches, CCFM
  - ☞ Modifications of BFKL (energy conservation, kinematic constraints)