# Progressi recenti in QCD ad alte energie

Gavin Salam

LPTHE — Univ. Paris VI & VII and CNRS

Convegno informale di fisica teorica

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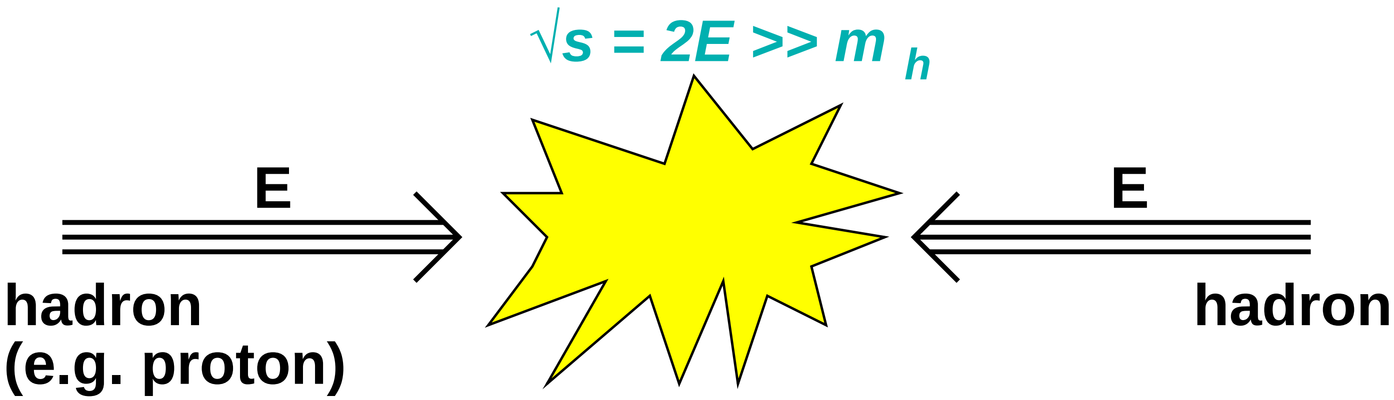
# High-energy limit

One of the major unsolved problems of QCD (and Yang-Mills theory in general) is the understanding of its *high-energy limit.*

I.e. the limit in which C.O.M. energy ($\sqrt{s}$) is much larger than *all other scales* in the problem.




Want to understand:

- asymptotic behaviour of cross section, $\sigma_{hh}(s) \sim ??$
- properties of final states for large $s$.

# Active field

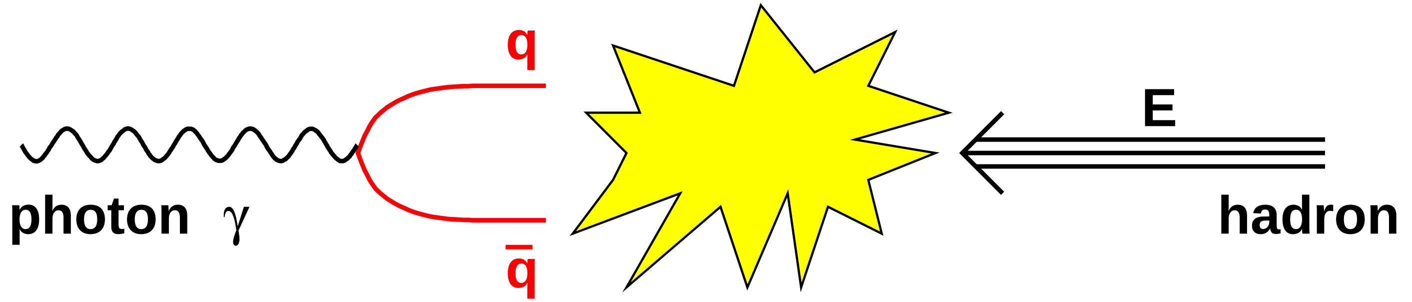
- $\sim 100$ articles / year.
- Difficult to give both introduction to field and discussion of major important recent developments.

Therefore:

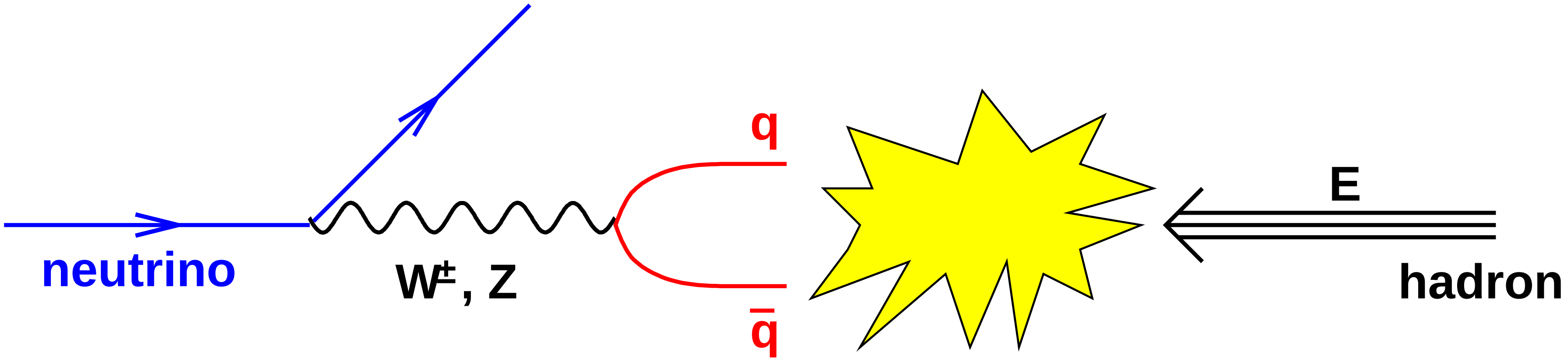
- Brief introduction:
  - Motivations
  - Basic approach
- Concentrate on a small (personal) selection of developments.
- Give pointers to some other major recent results.

# Not just for hadrons

Problem is must more general than just for hadrons. E.g. photon can *fluctuate* into a quark-antiquark (hadronic!) state:

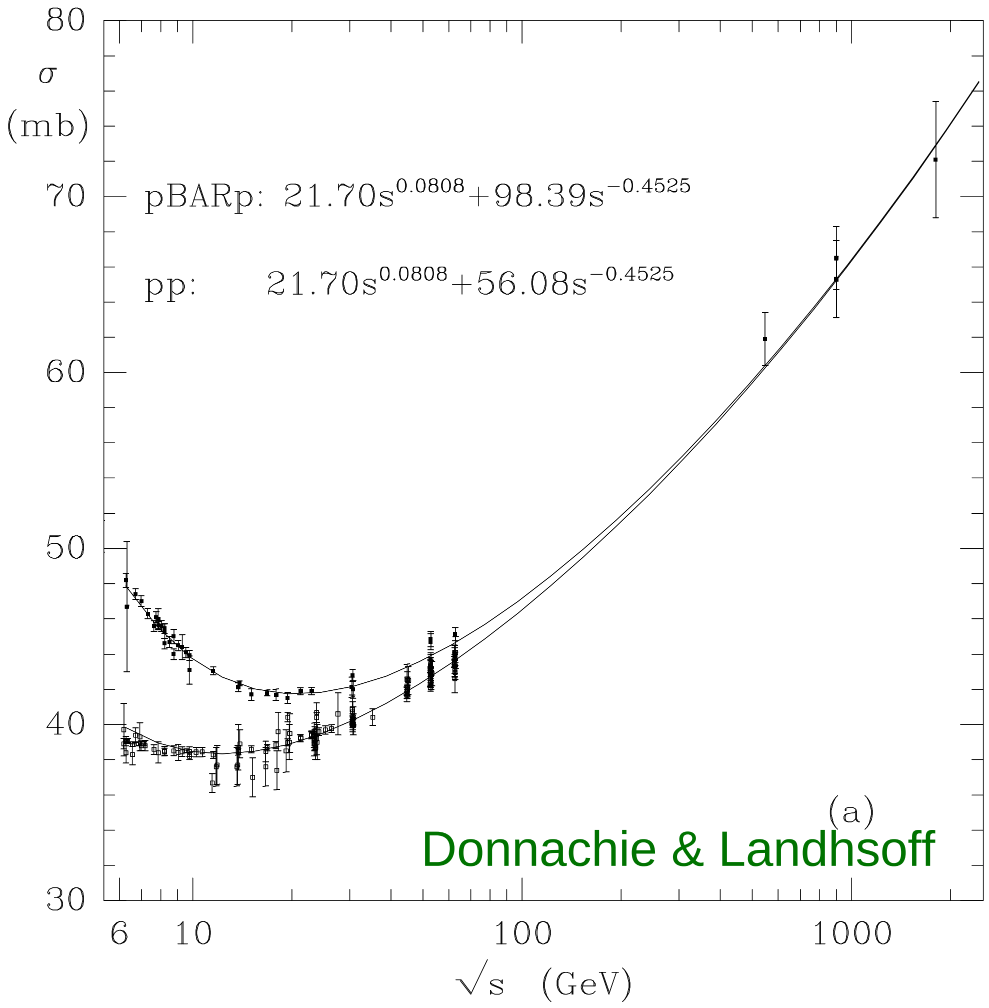



Even a neutrino can behave like a hadron



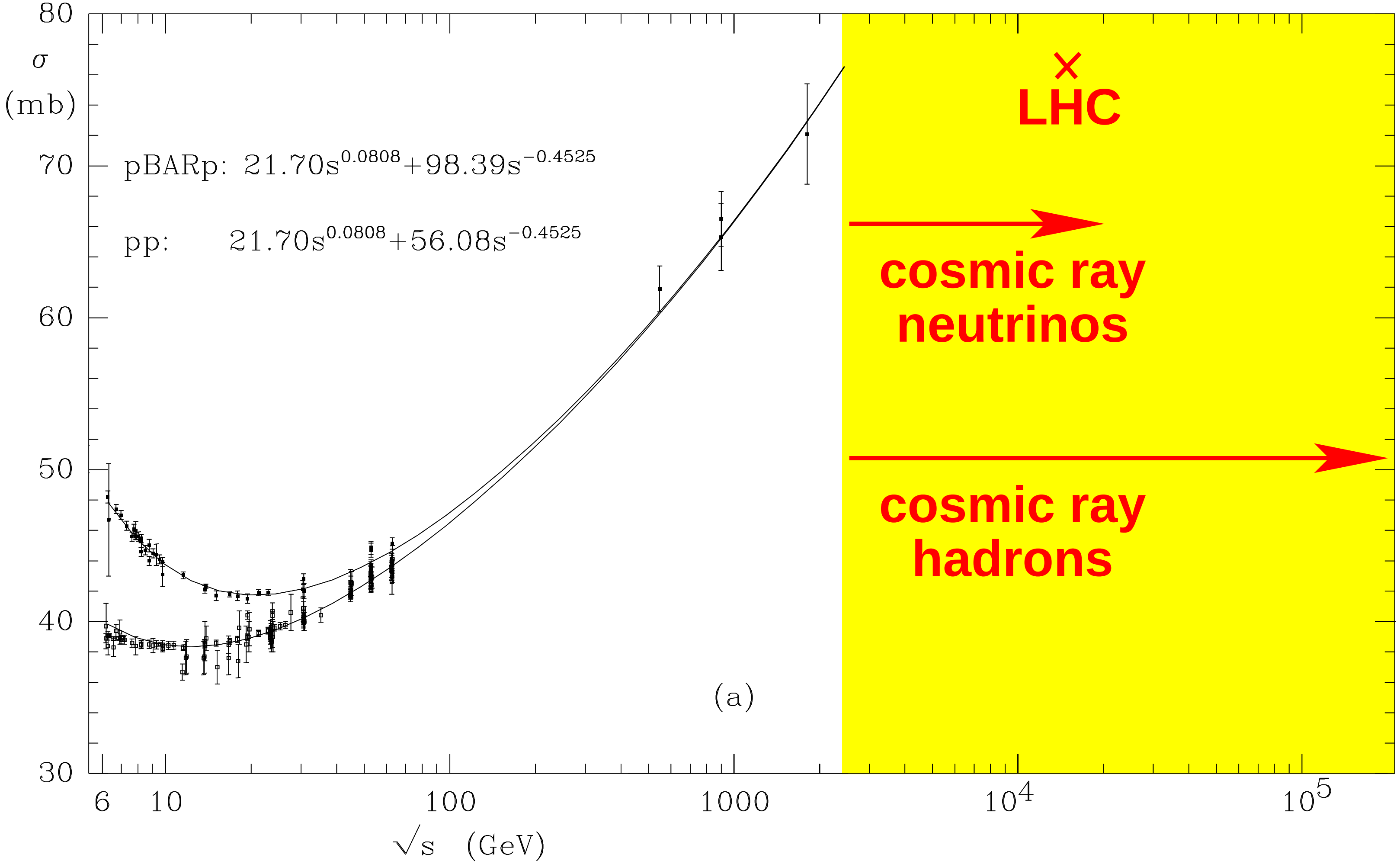

Hadronic component dominates high-energy cross section

# Experimental knowledge



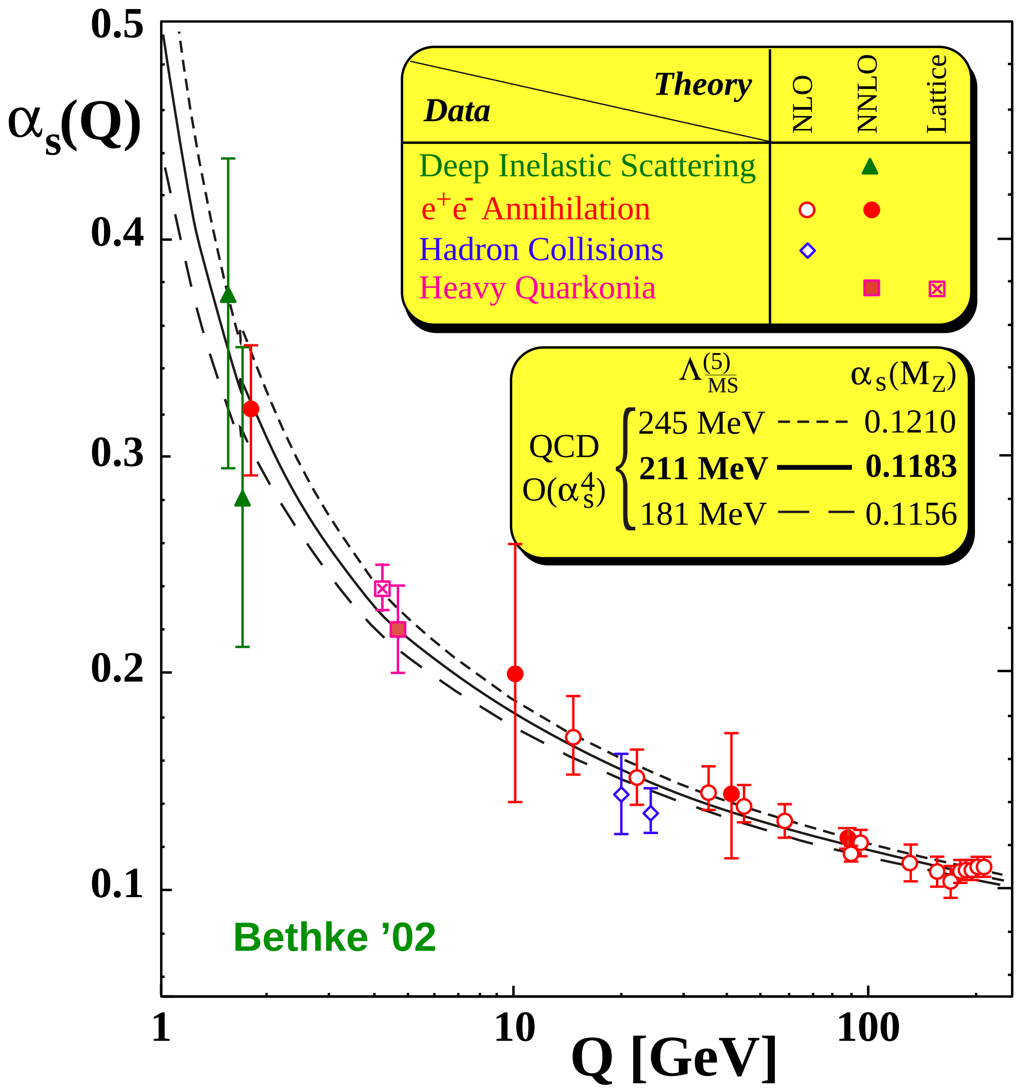

- Some knowledge exists about behaviour of cross section experimentally
- Slow rise as energy increases
- Data insufficient to make reliable statements about functional form
  - $\sigma \sim s^{0.08}$?
  - $\sigma \sim \ln^2 s$?
- Would like theoretical knowledge…

# Experimental knowledge




*Future experiments go to much higher energies.*

# What can perturbative QCD say?




Recall: QCD is *asymptotically free* — coupling ($\alpha_s$) decreases at large scales:

$$\frac{d\alpha_s(Q^2)}{d\ln Q^2} \simeq -\beta_0\alpha_s^2$$

*Corollary:* at low scales (e.g. proton mass, $1$ GeV) perturbation theory breaks down:

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- Calculation of proton-proton scattering is *beyond* perturbative QCD.

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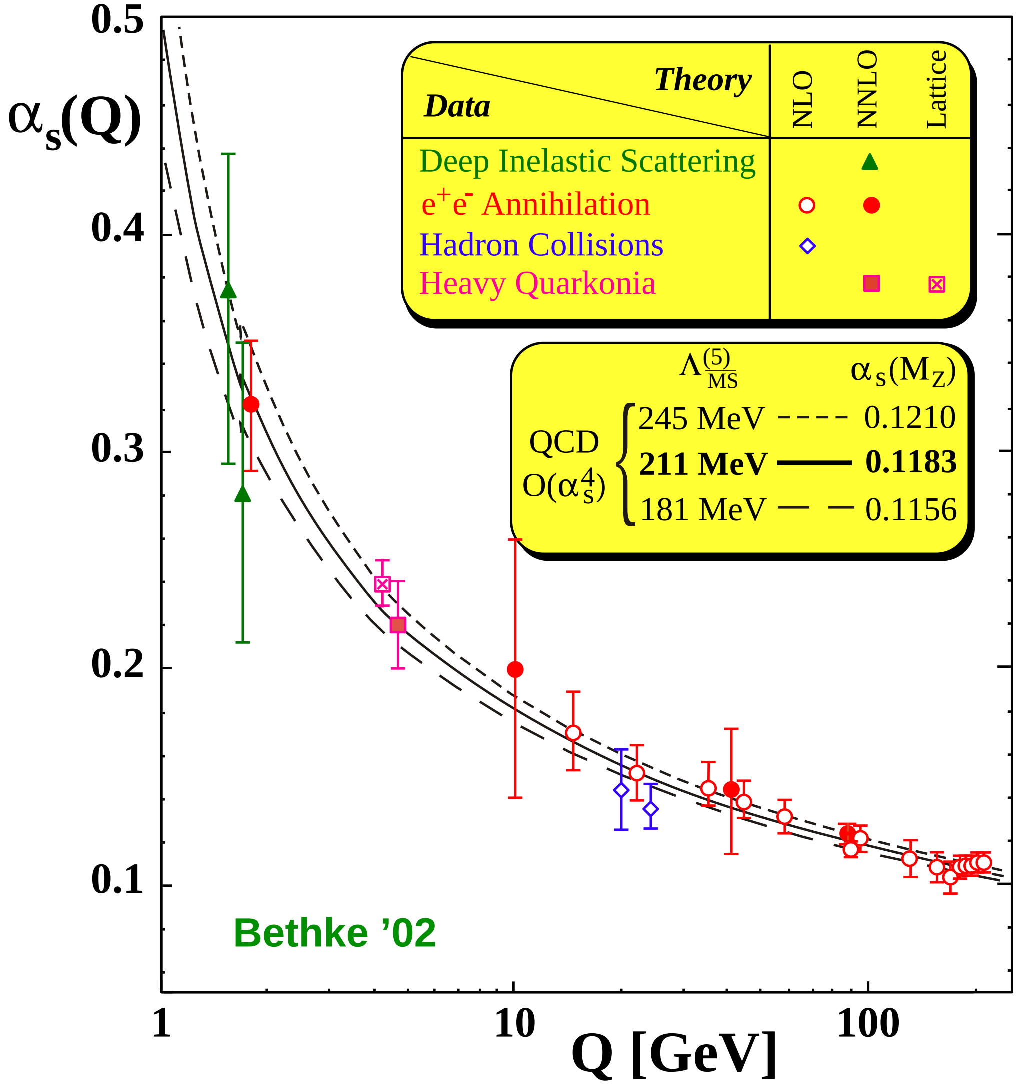


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Forget applicability for now — just examine field-theory behaviour

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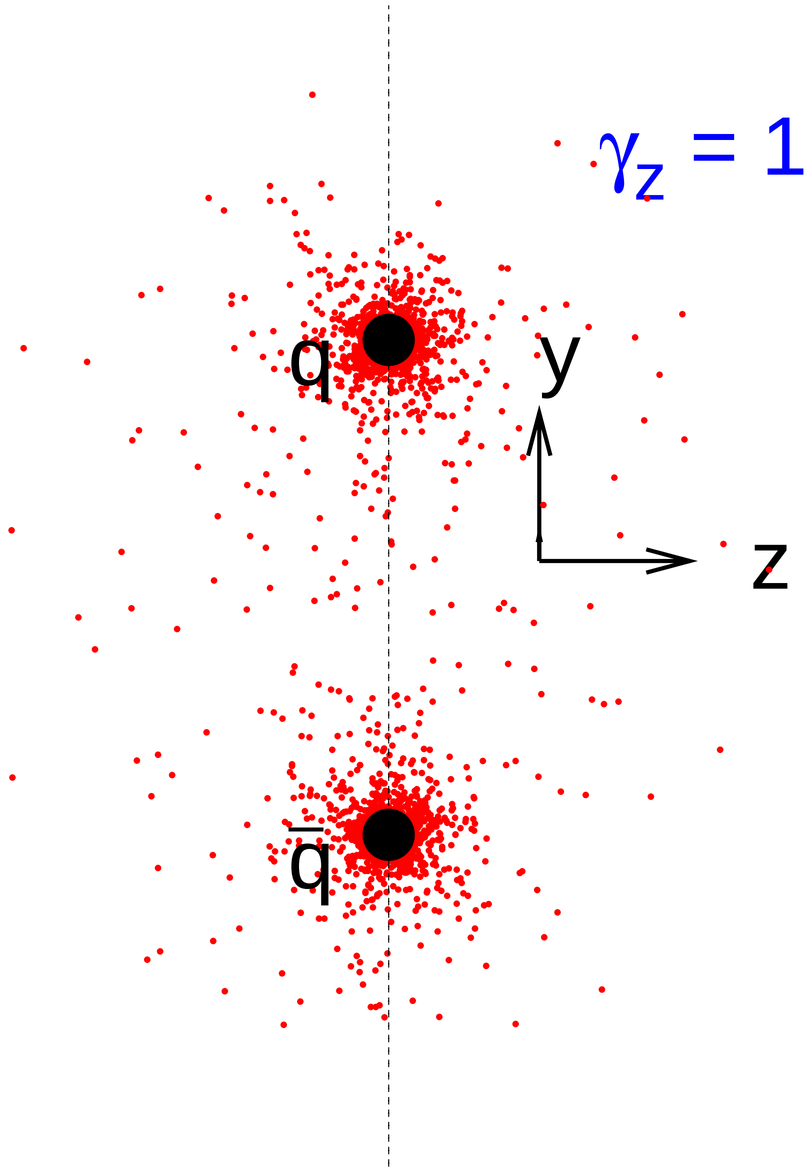


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QCD $\simeq$ QED

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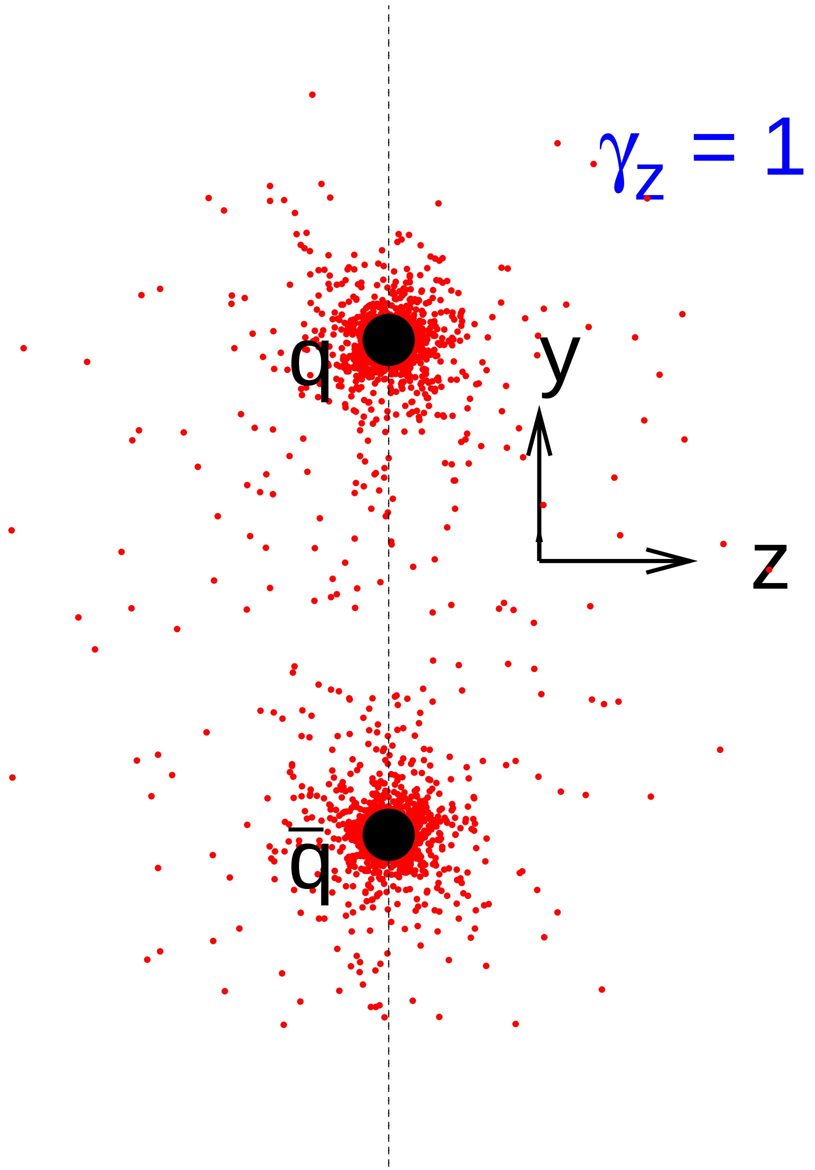


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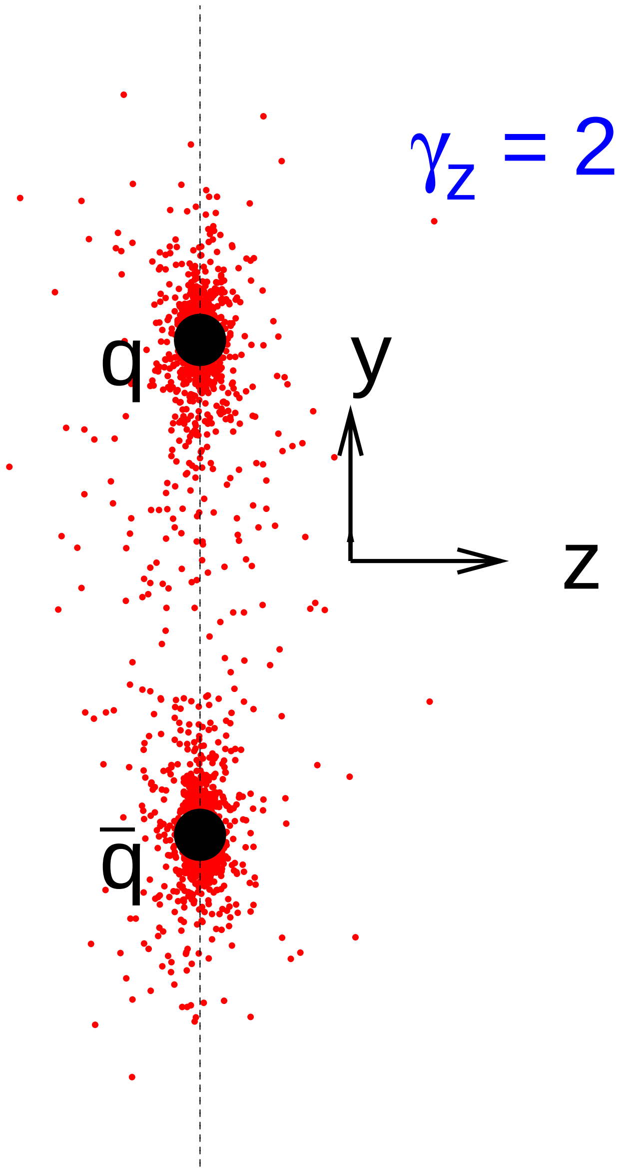


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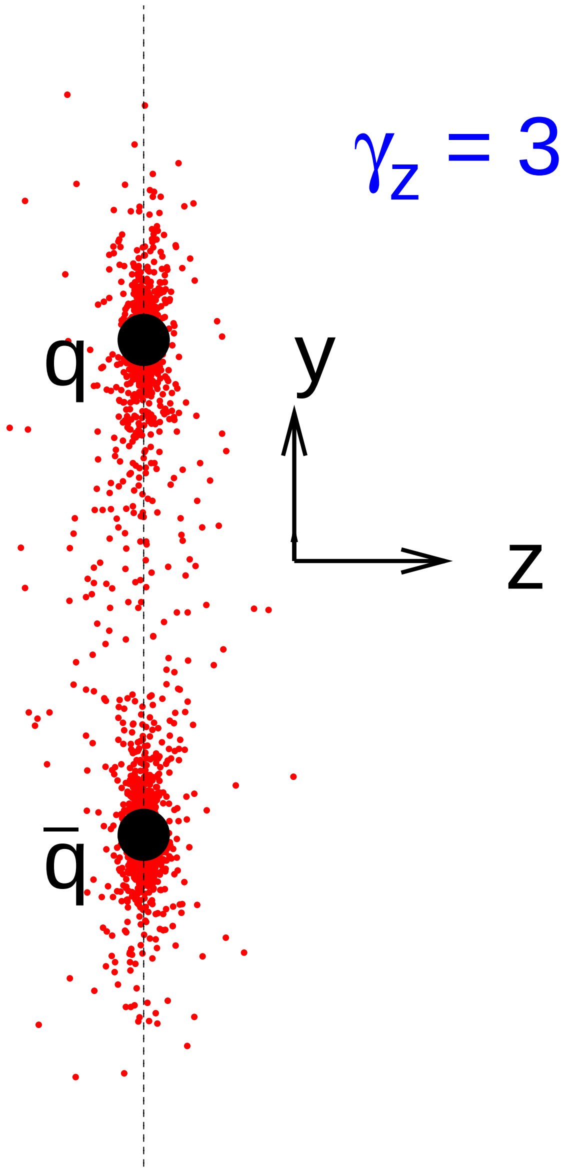


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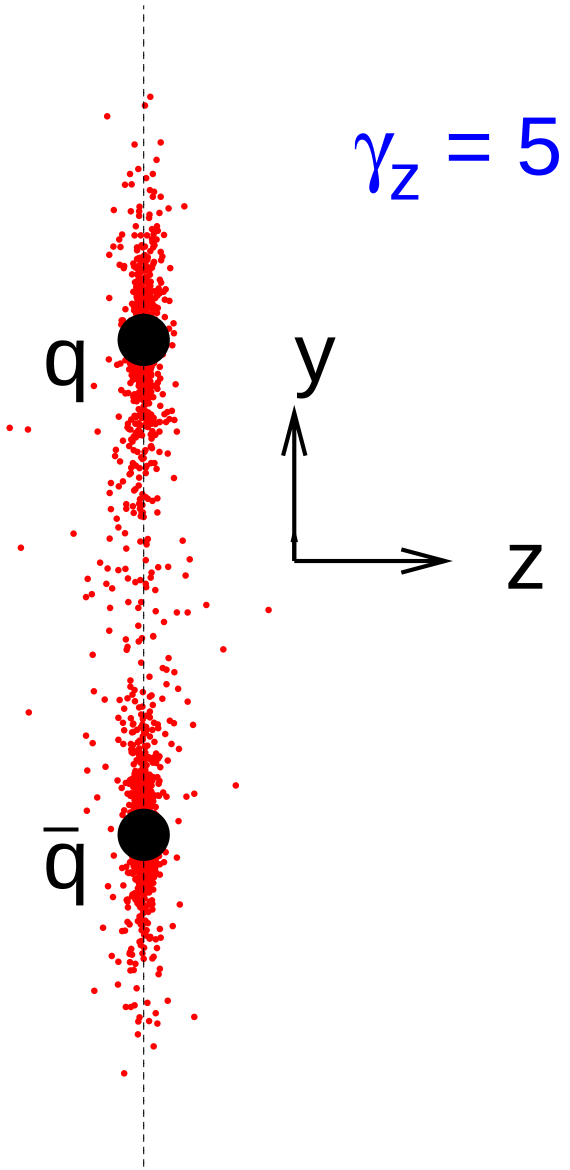


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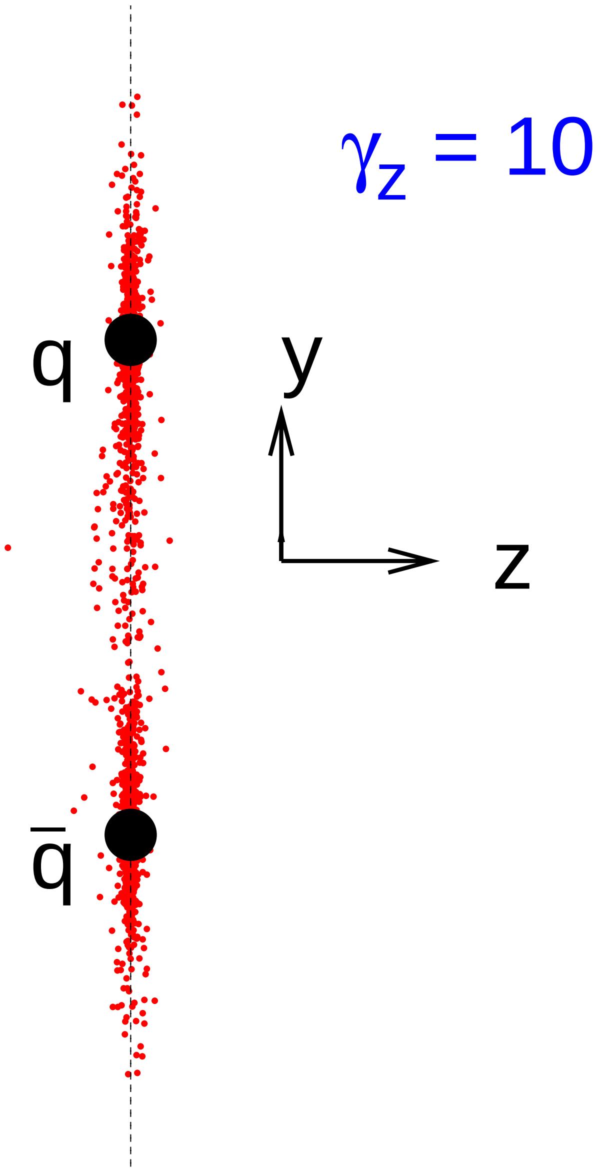


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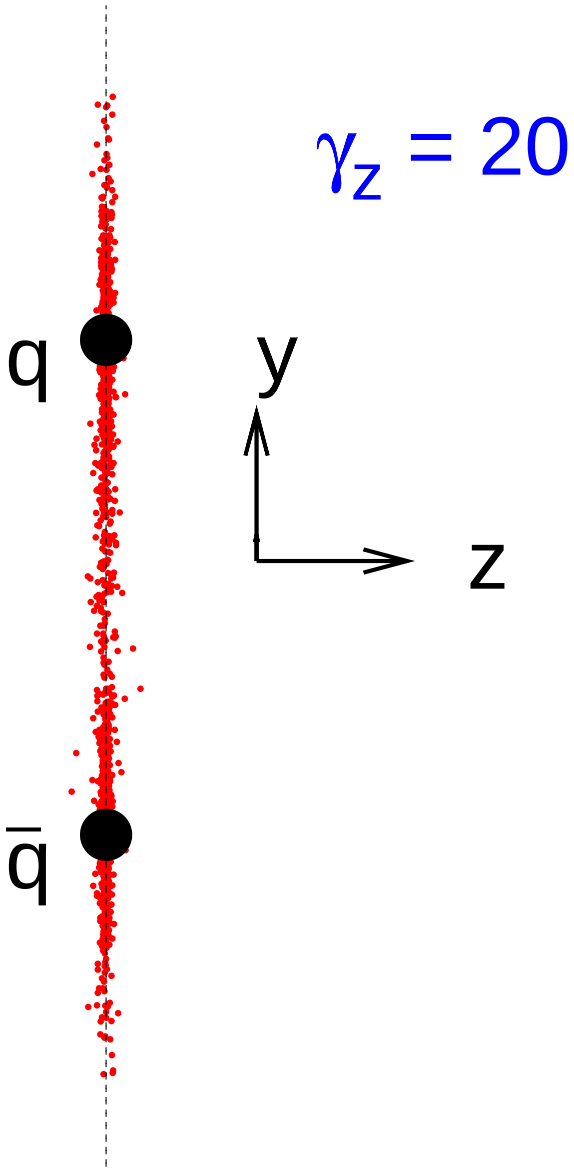


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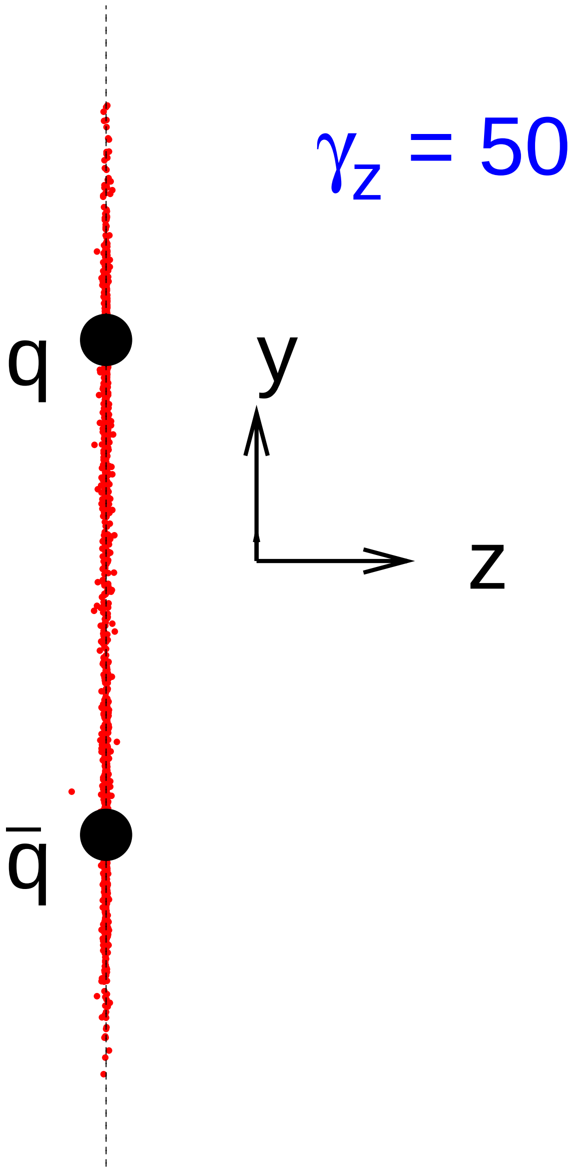


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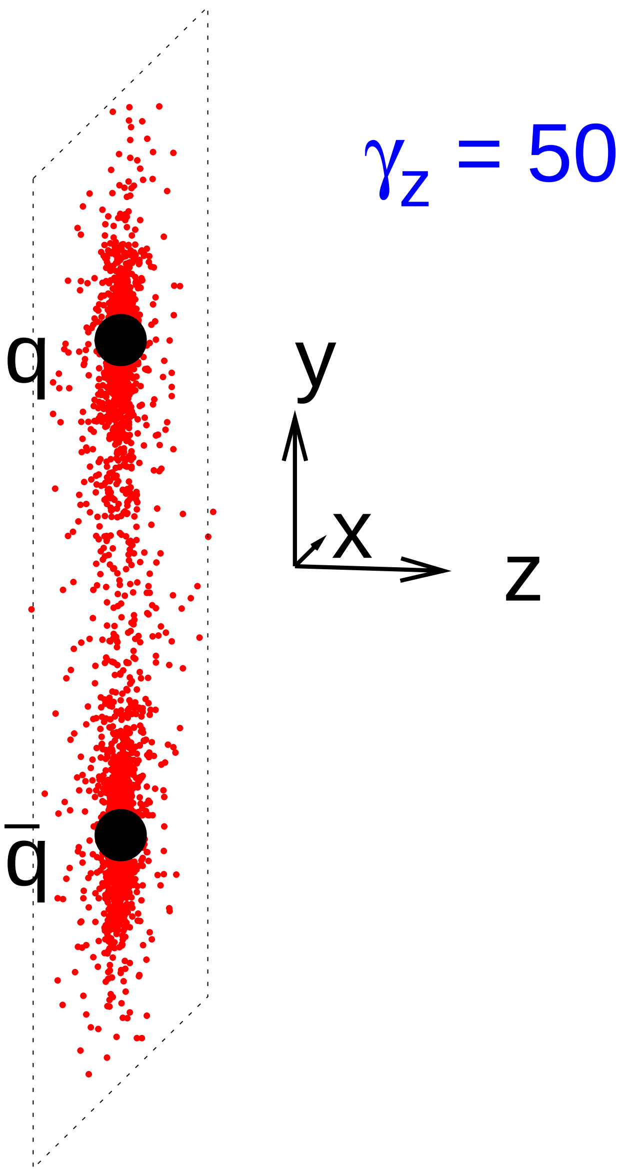


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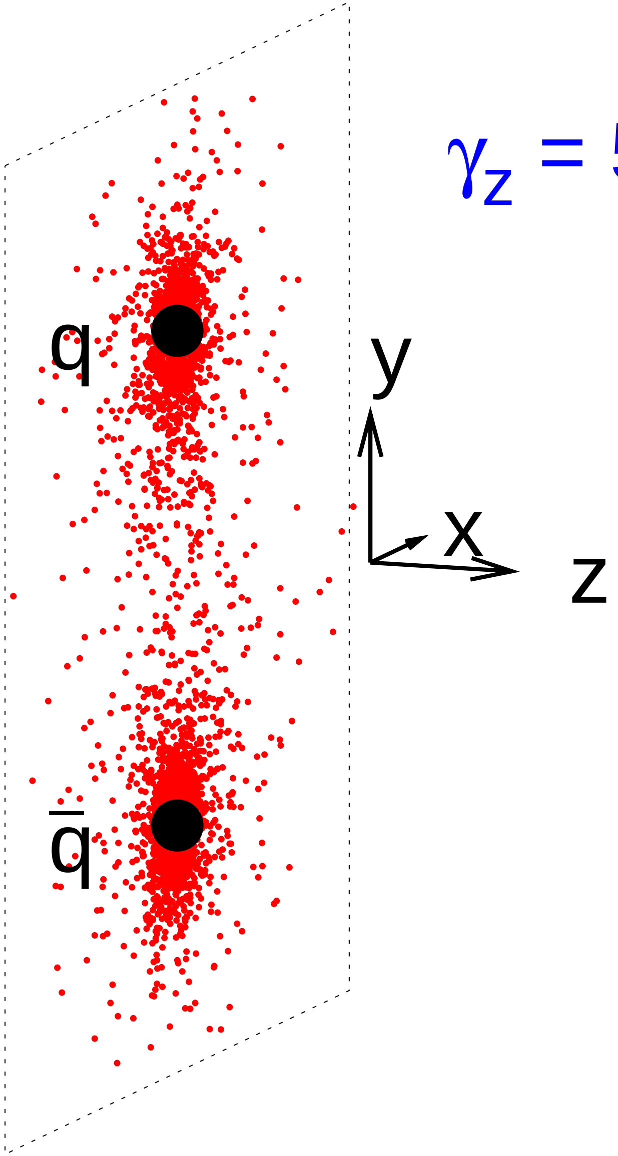


$\gamma_z = 50$

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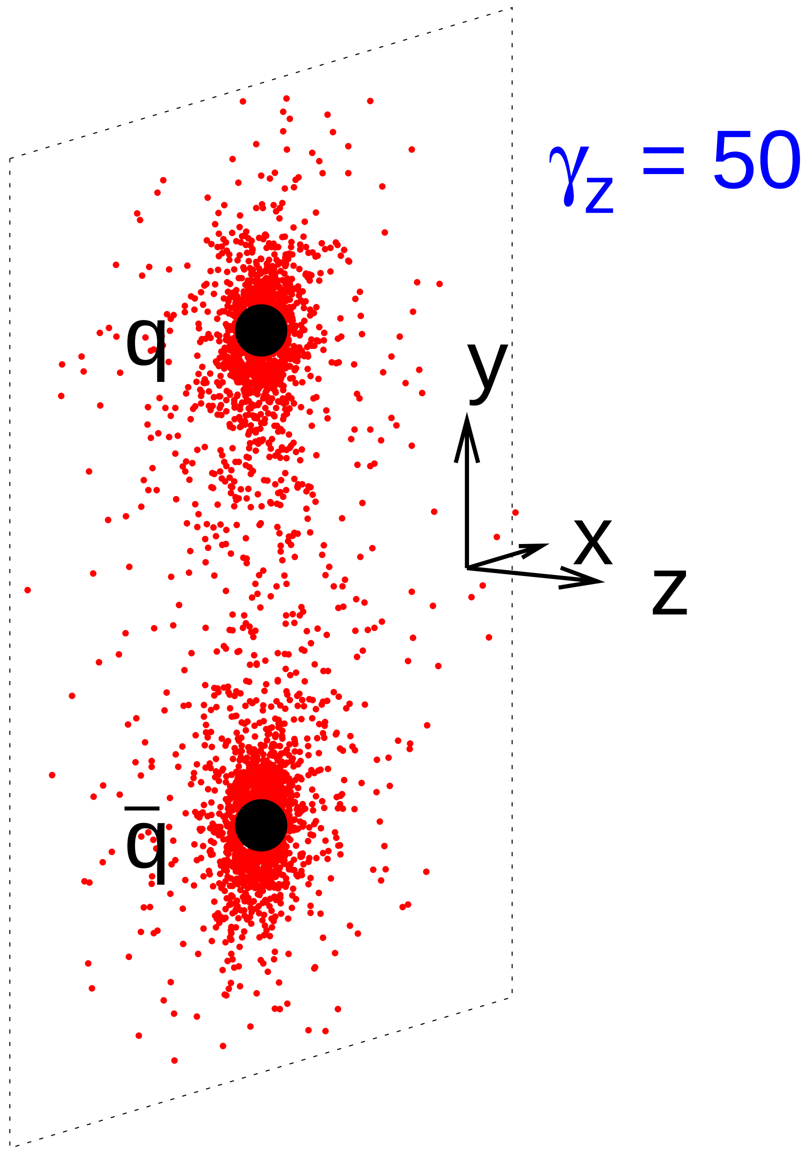


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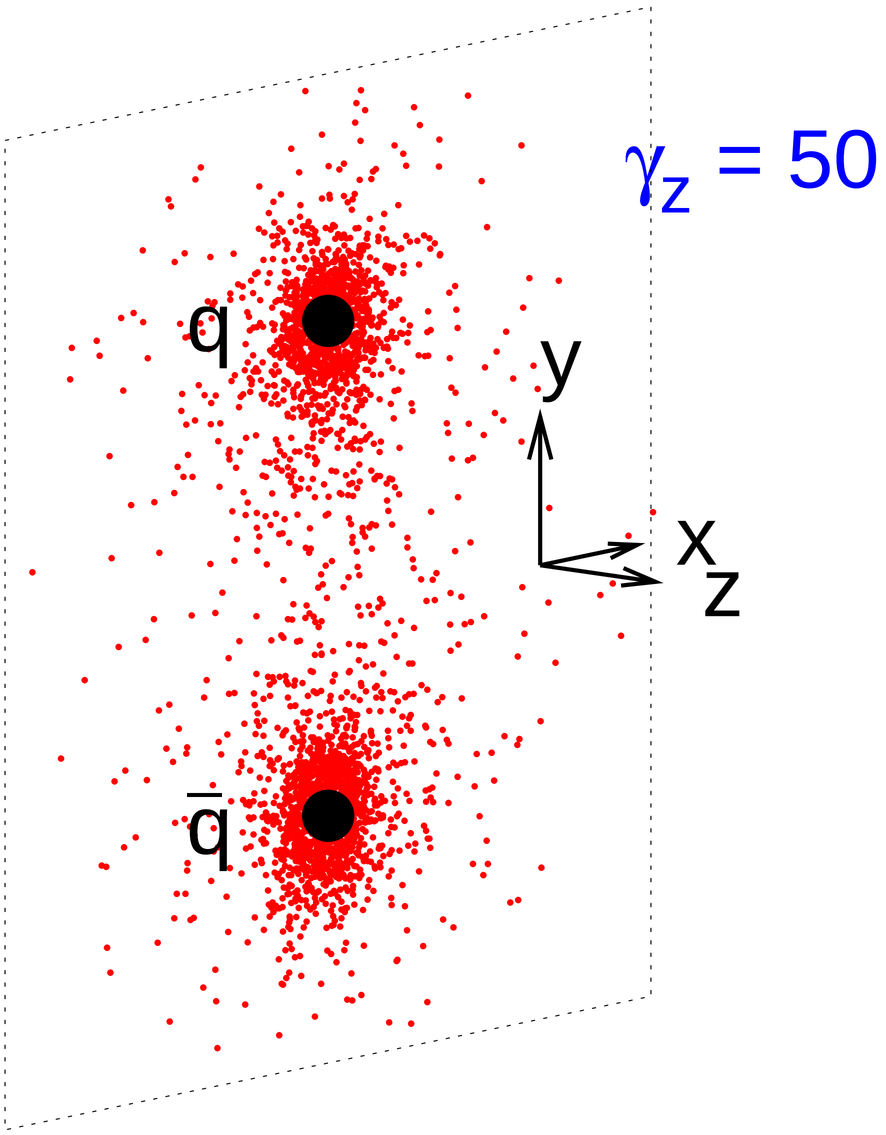


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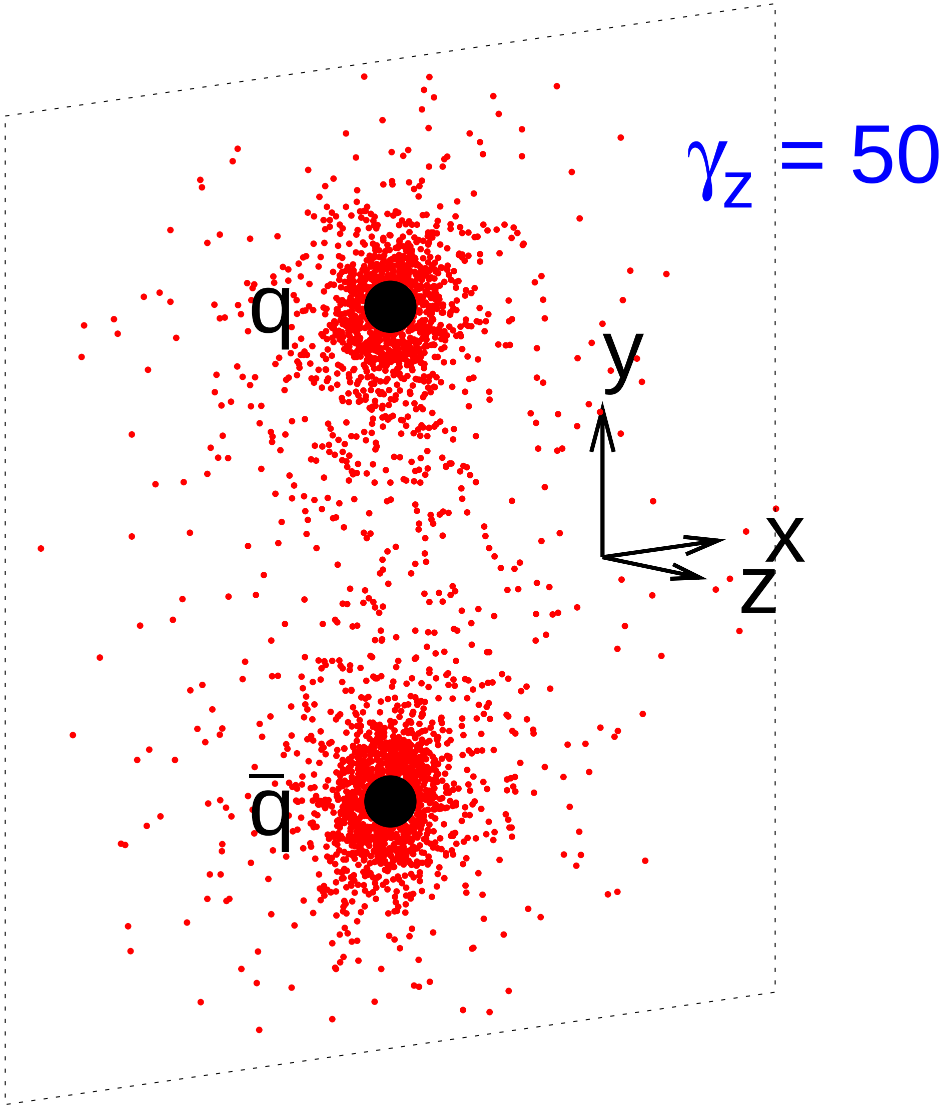


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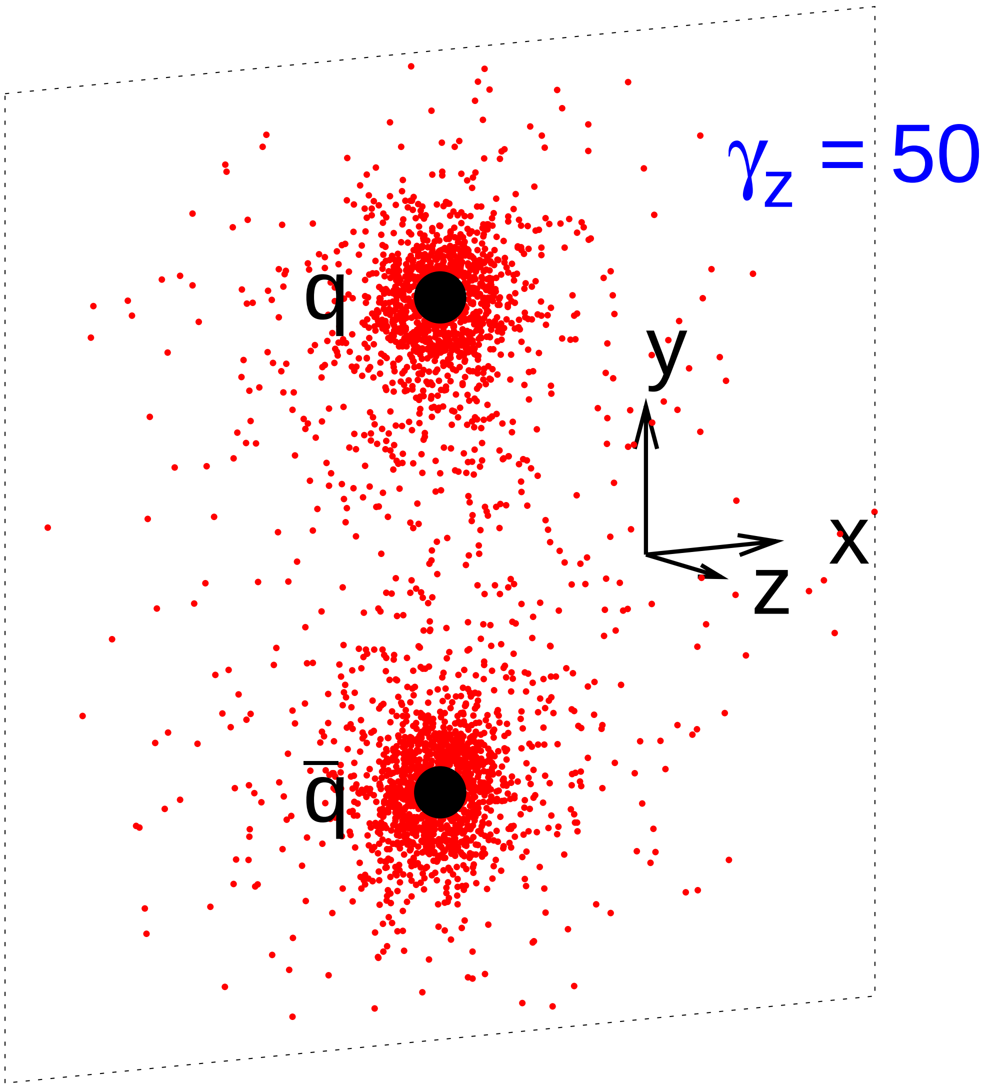


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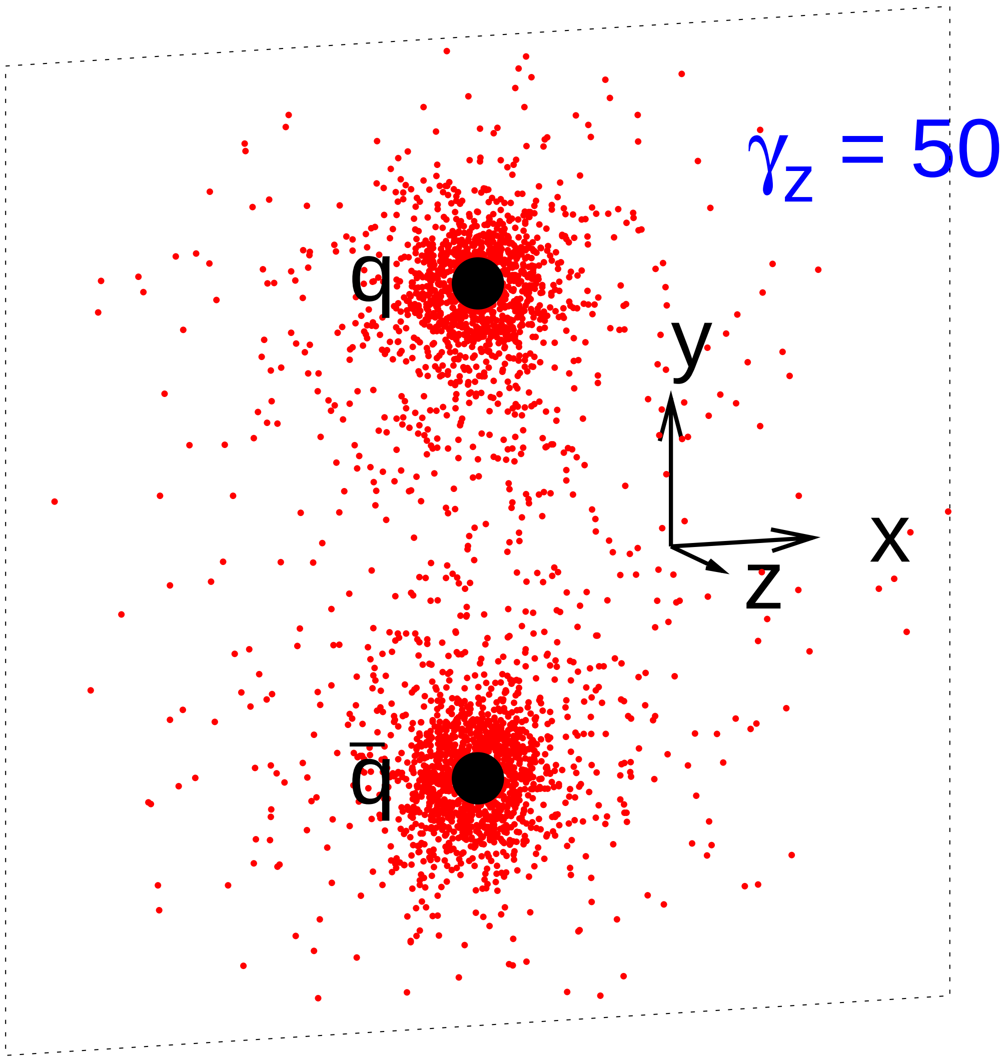


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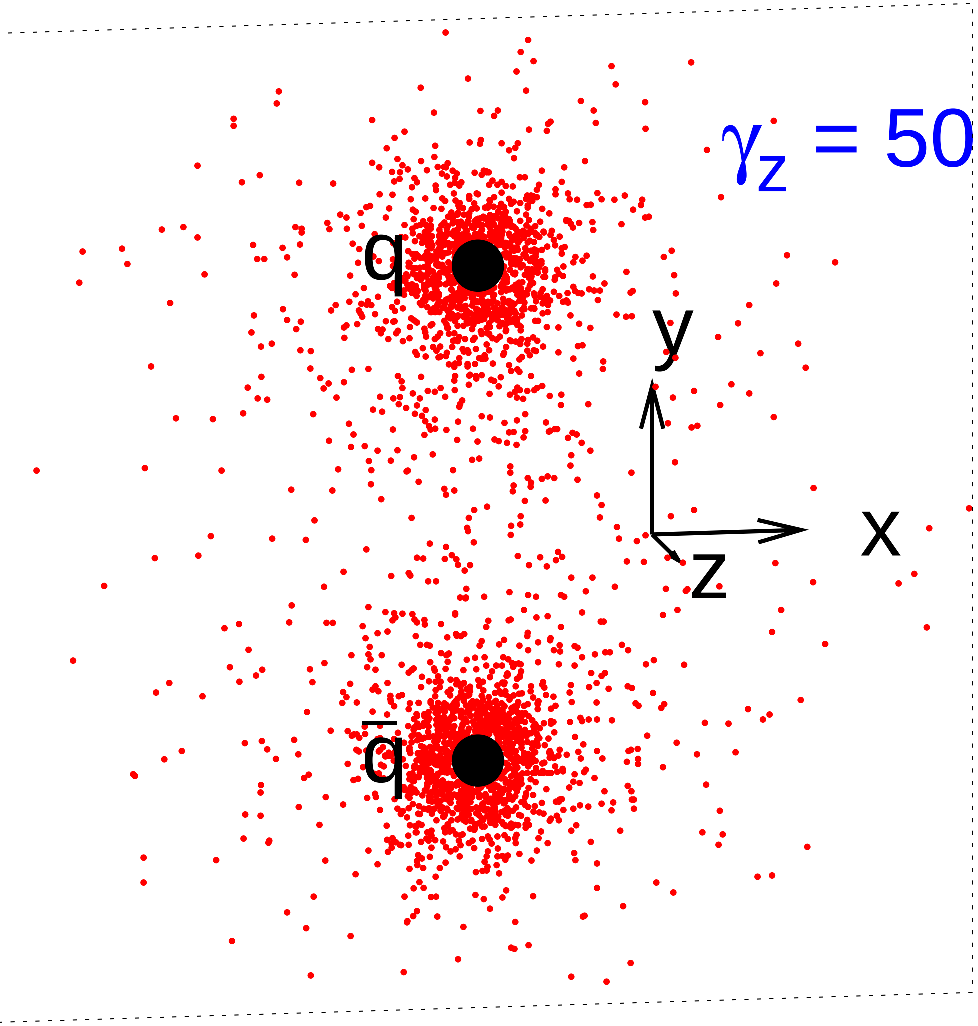


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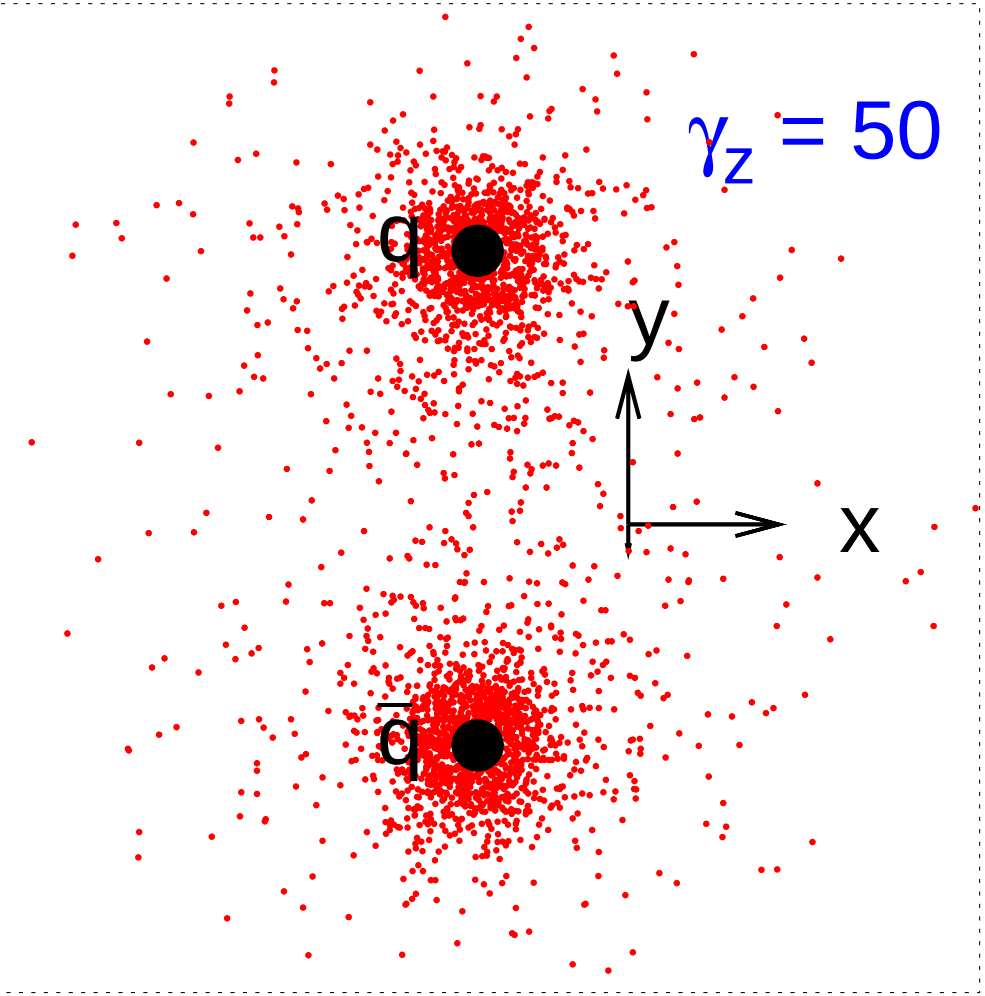


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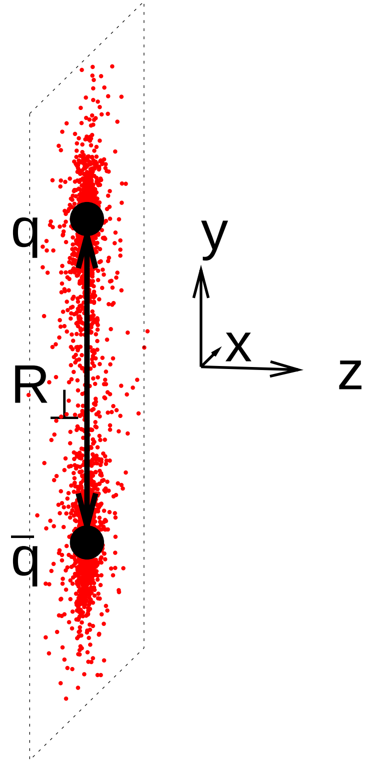
- There remains *non-trivial transverse structure.*
  - Fields are those of a dipole in 2+1 dimensions

# Total number of gluons

Longitudinal structure of energy density ($N_c$ = # of colours):

$$\frac{d\epsilon}{dz} \sim \frac{\alpha_s N_c}{\pi} \times \frac{\gamma_z \delta(z)}{R_\perp} \times \text{transverse}$$



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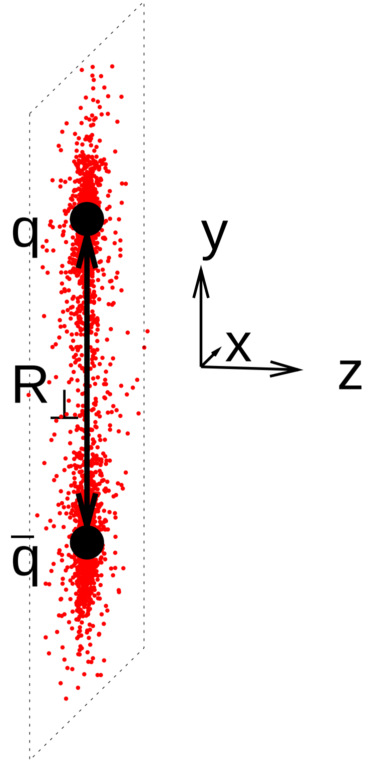


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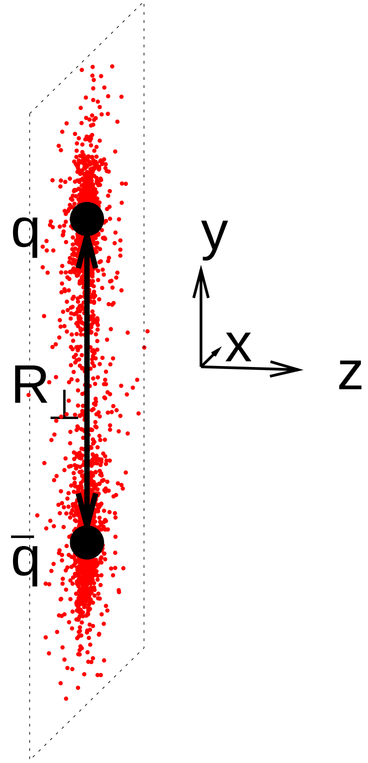


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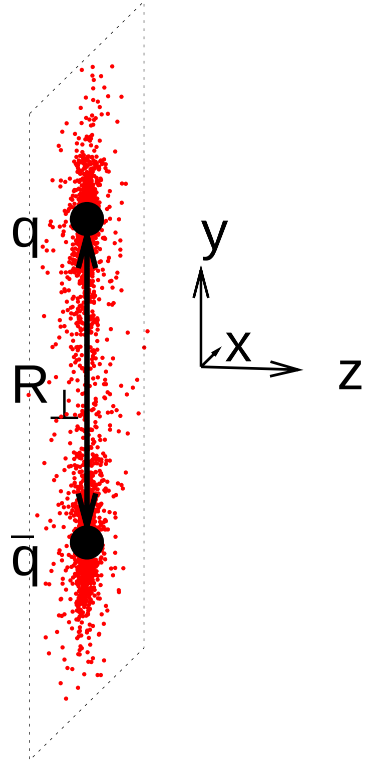


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Total number of gluons:

$$n \sim \frac{\alpha_s N_c}{\pi} \ln \gamma_z \times \text{transverse}$$

# High-energy limit $\sqrt{s} \sim \gamma_z \longrightarrow \infty$

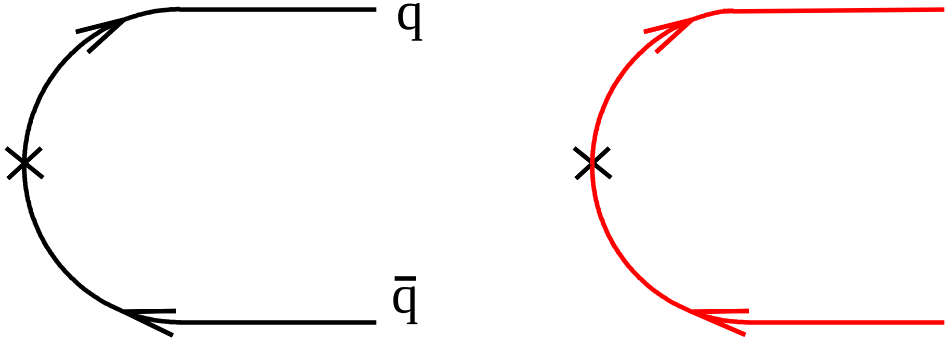
- Calculation so far is first-order perturbation theory.
- Fixed order perturbation theory is reliable if series converges quickly.
- At high energies, $n \sim \alpha_s \ln \gamma_z \gg 1$.
- What happens with higher orders?

$$(\alpha_s \ln \gamma_z)^n ?$$

Leading Logarithms. Any fixed order potentially non-convergent...

# Multiple gluon emission

Start with bare quark-antiquark dipole:



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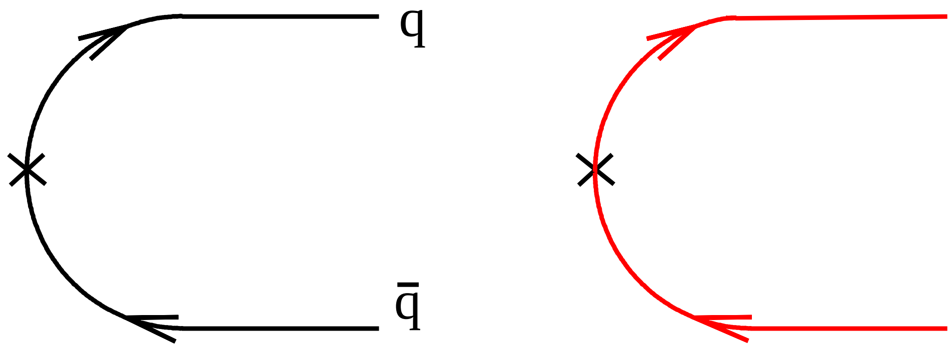


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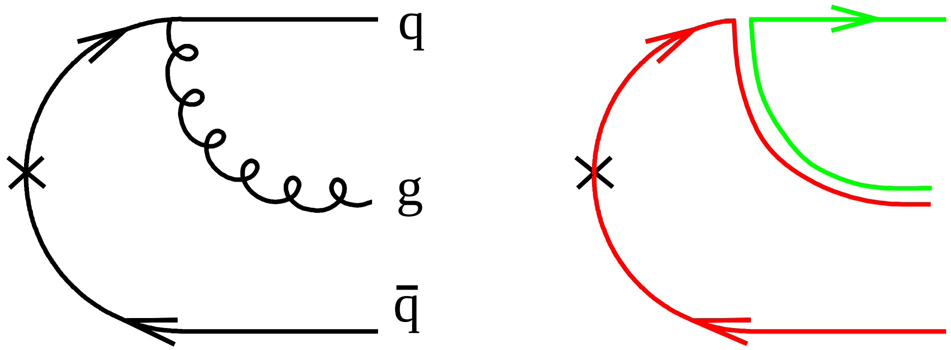

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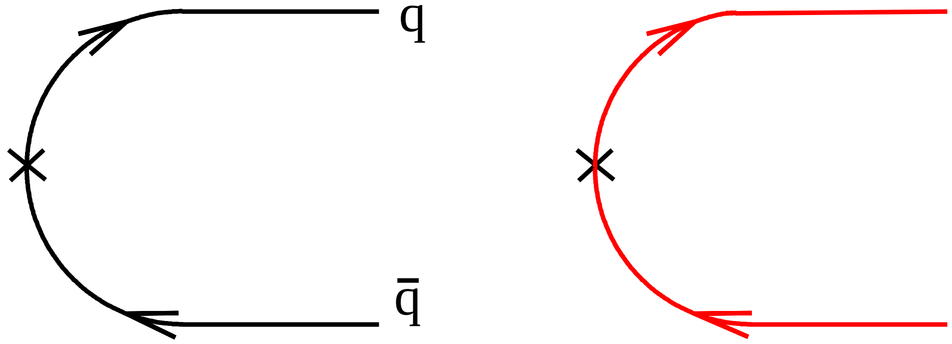


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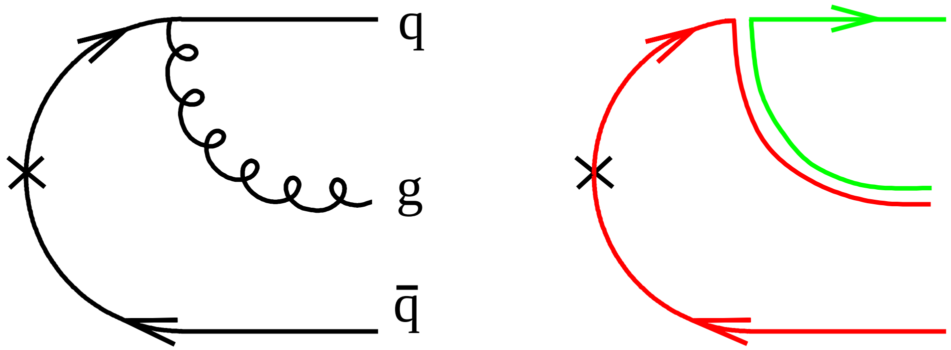


*Emission of 1 gluon is like QED case — modulo additional colour factor*
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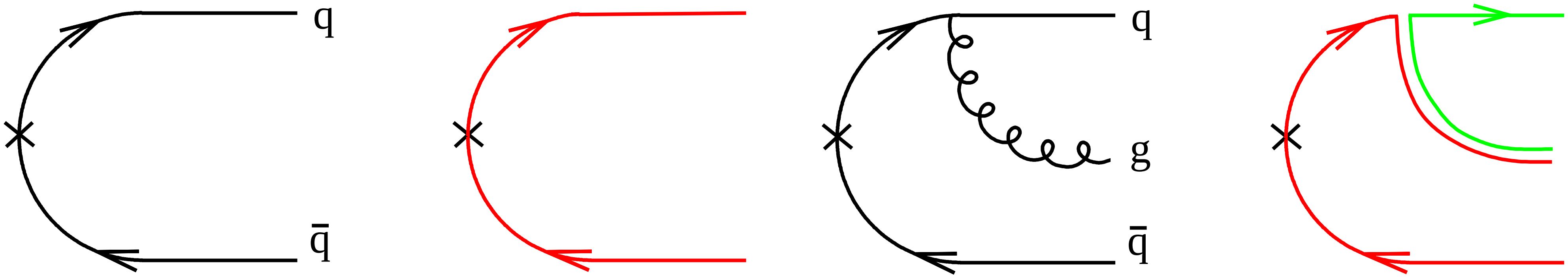


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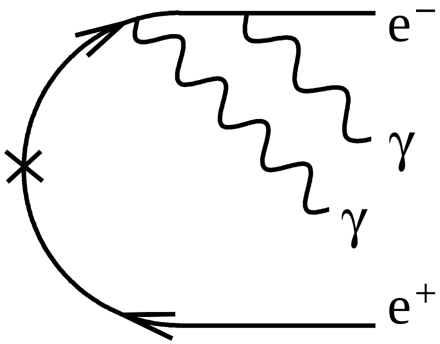

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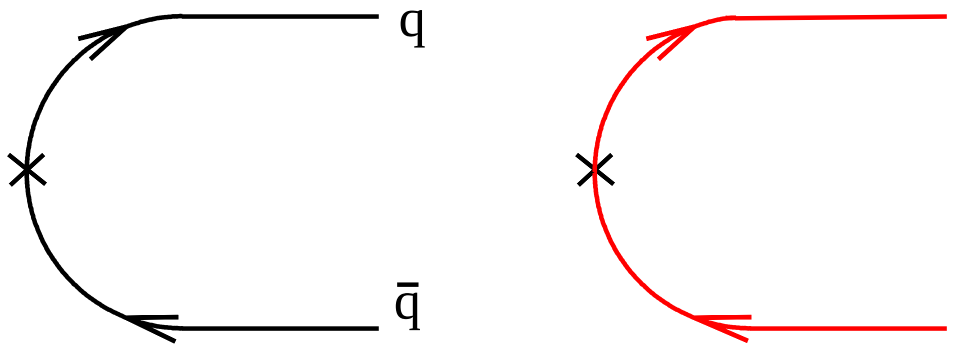


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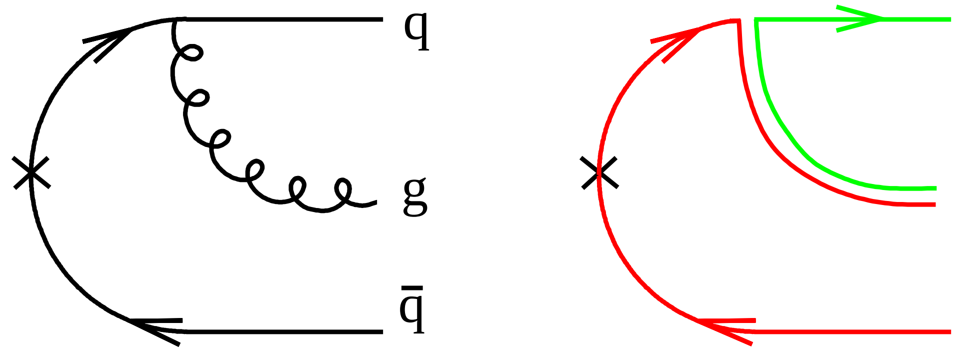


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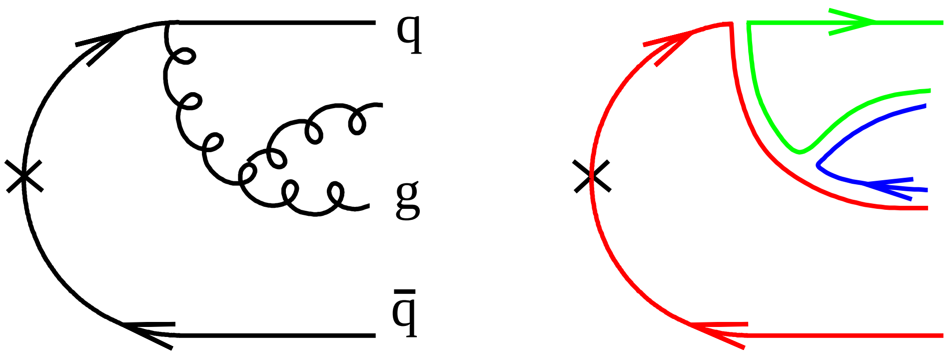
- In QCD original dipole is converted into two new dipoles, which *emit independently*.



# Iterating gluon emission

Problem is self-similar: dipole $\longrightarrow$ 2 dipoles $\longrightarrow$ 4 dipoles $\longrightarrow$ . . .

Number of dipoles (or gluons) grows *exponentially:*

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Tranverse part $\longrightarrow$ many complications/interest

- transverse part is *conformally invariant* ➥Extensive mathematical studies
- In high-energy limit it reduces to a pure number: $4 \ln 2$

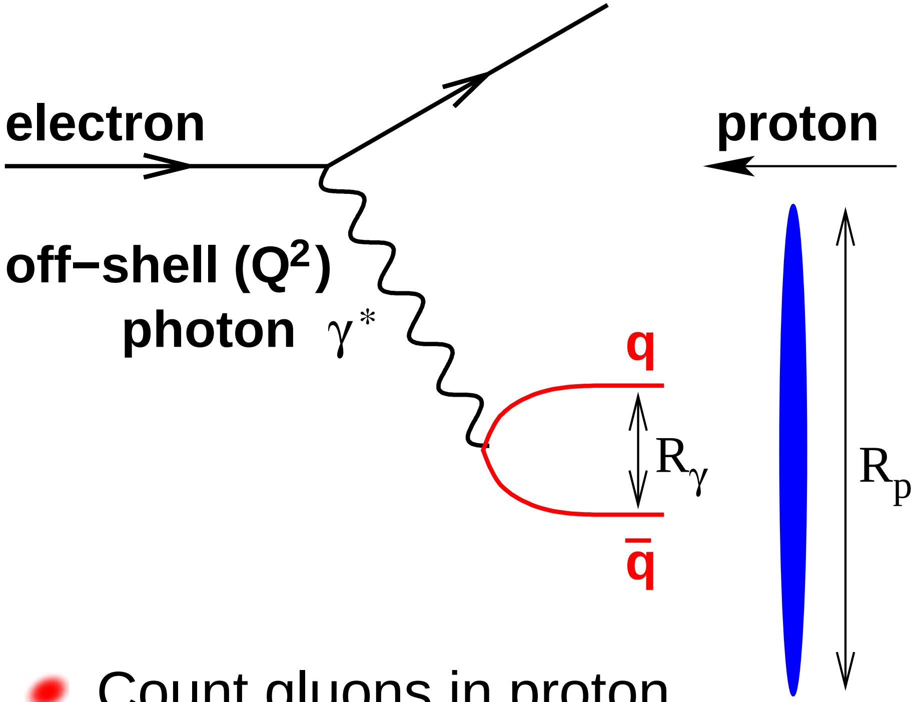
$$n \sim \gamma_z^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} \sim \gamma_z^{0.5}$$

BFKL Pomeron (1976)

- Strong signal: *rapid growth of number of gluons at high energy*
  $\Rightarrow$ similar rapid growth of scattering cross sections

# Experimental tests: HERA (since mid 90's)

Make problem semi-perturbative




- Count gluons in proton at given transverse scale

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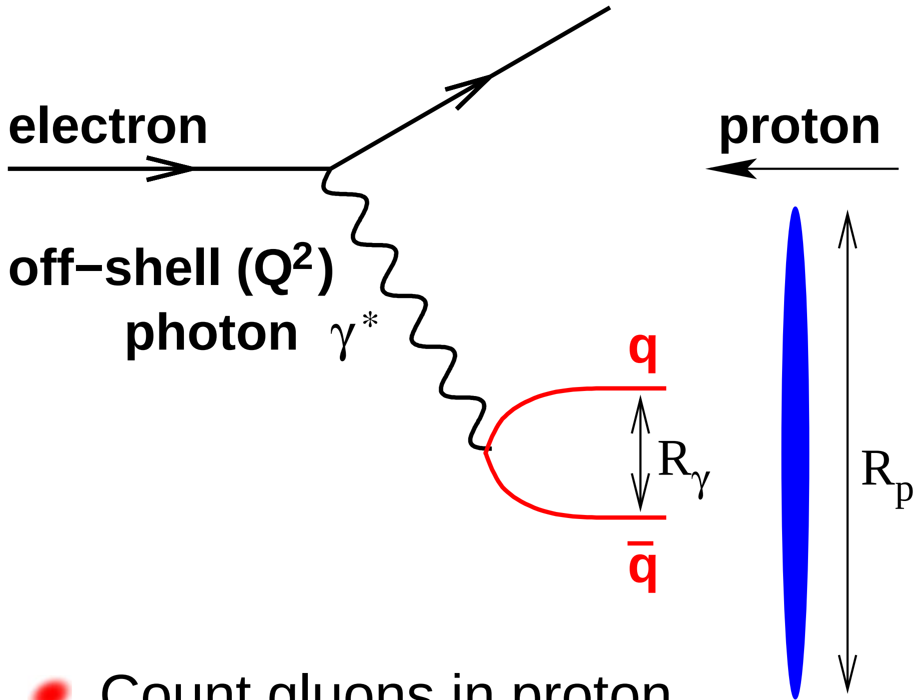
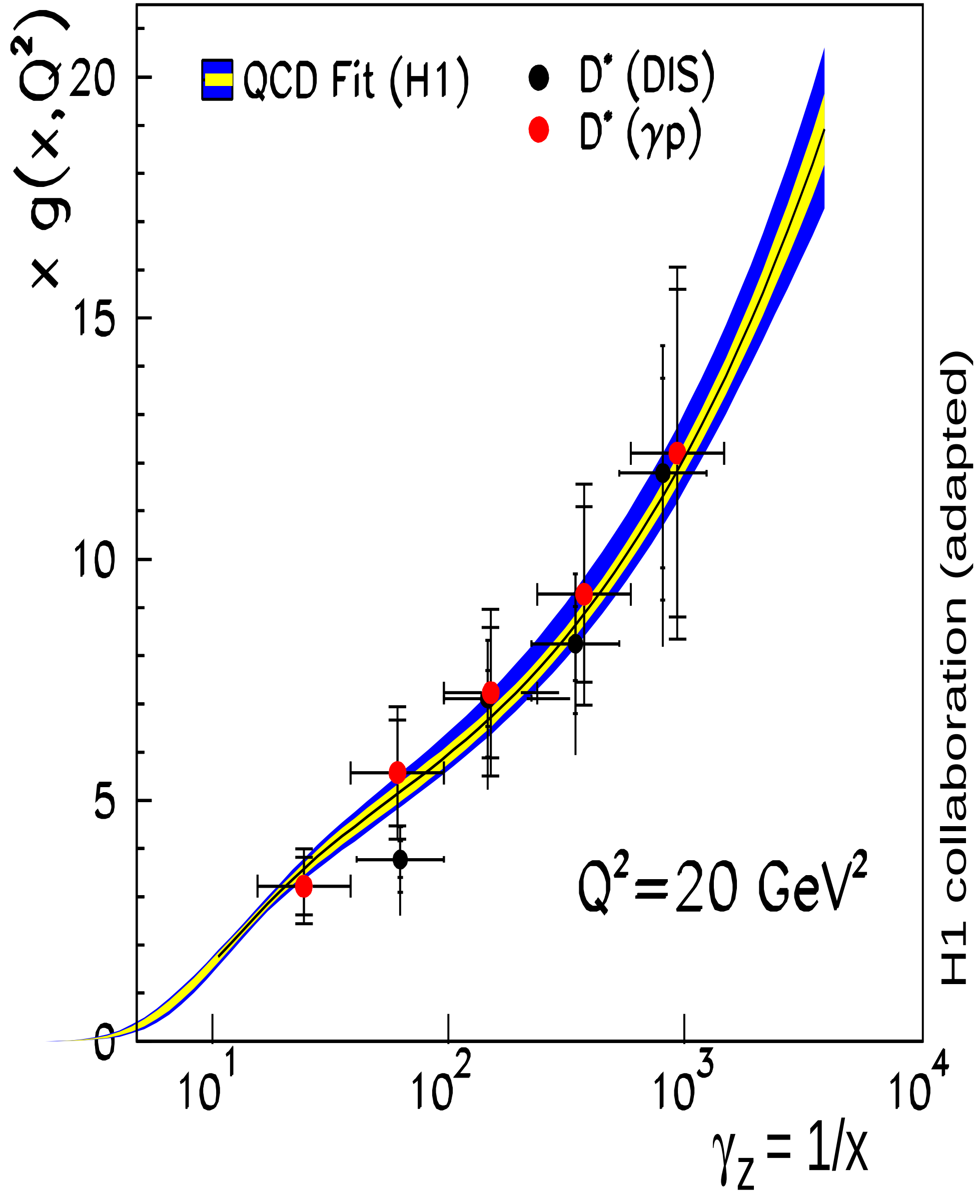
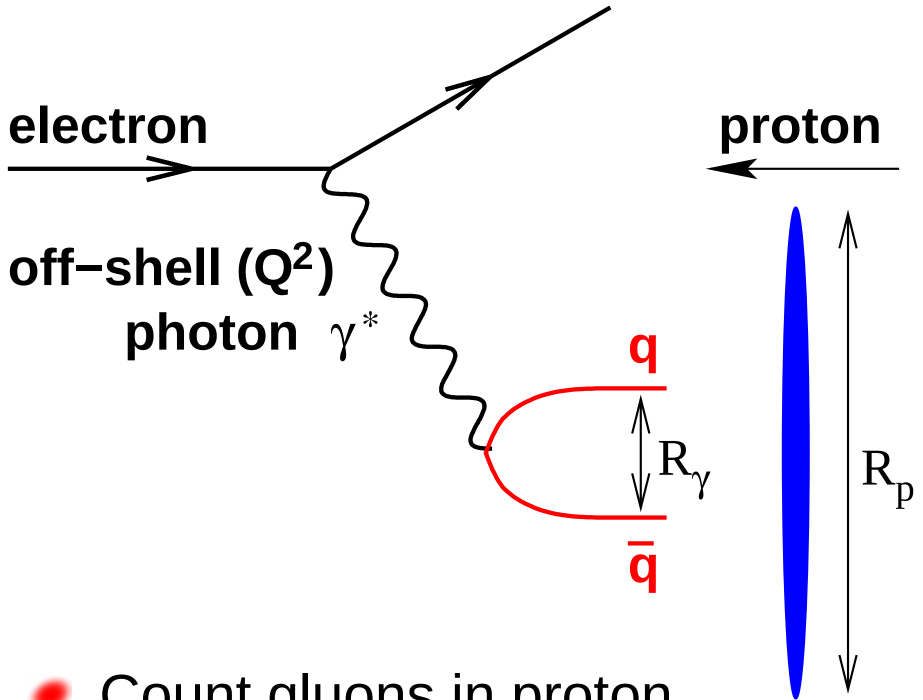


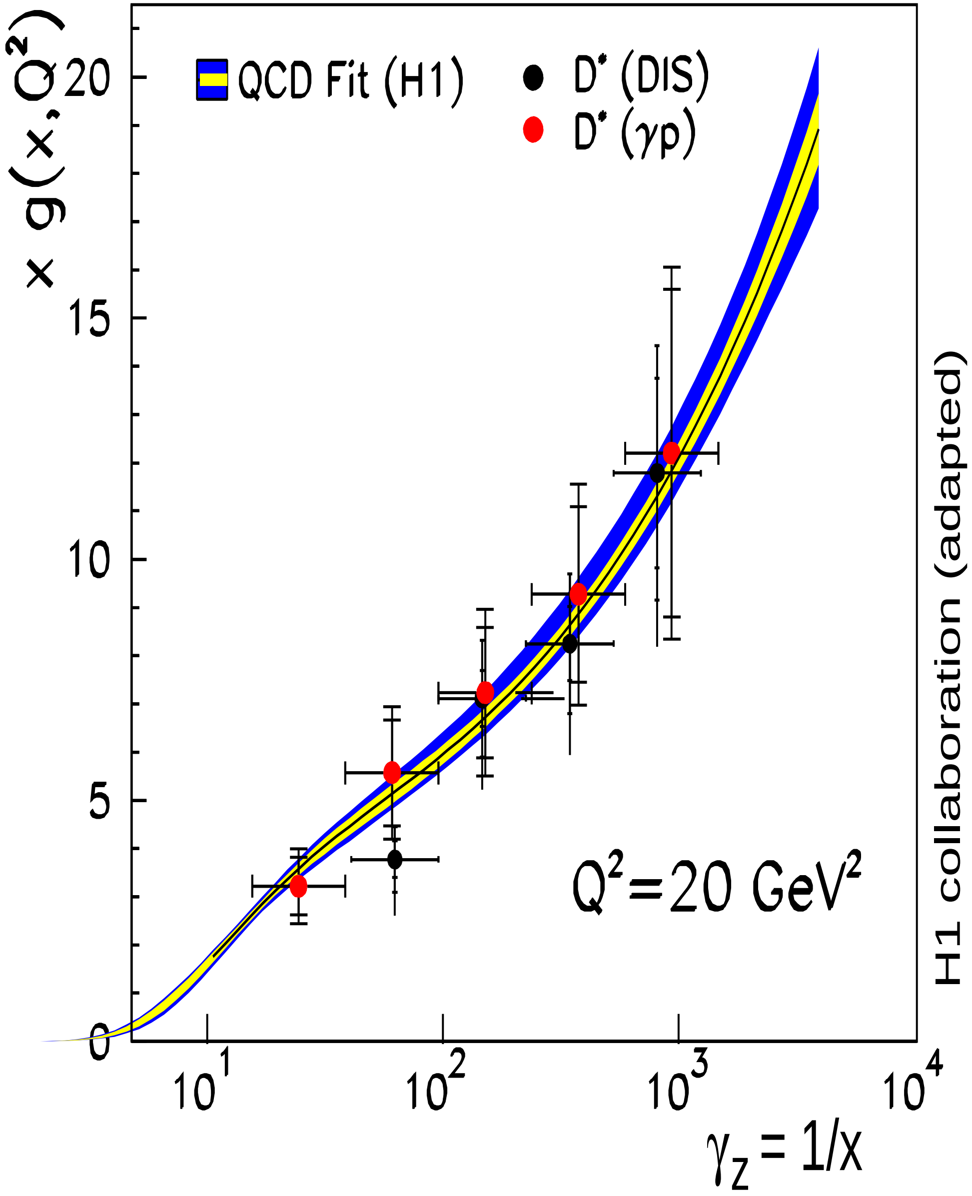

- Count gluons in proton at given transverse scale
- *Clear rise* in gluon density at high energies (low momentum fractions $x$).

# Experimental tests: HERA (since mid 90's)

Make problem semi-perturbative




- Count gluons in proton at given transverse scale
- *Clear rise* in gluon density at high energies (low momentum fractions $x$).
- But power $\sim 0.25$ — *about half the predicted value of $0.5$.*



# Next-to-leading logarithmic corrections

- Leading-log calculations are rarely sufficient in QCD.
- Next-to-leading log (NLL) 'BFKL': one of the most 'epic' NLL calculations in QCD
  $\sim$ 15 papers for separate pieces
  Put together 1998: Fadin & Lipatov, Camici & Ciafaloni

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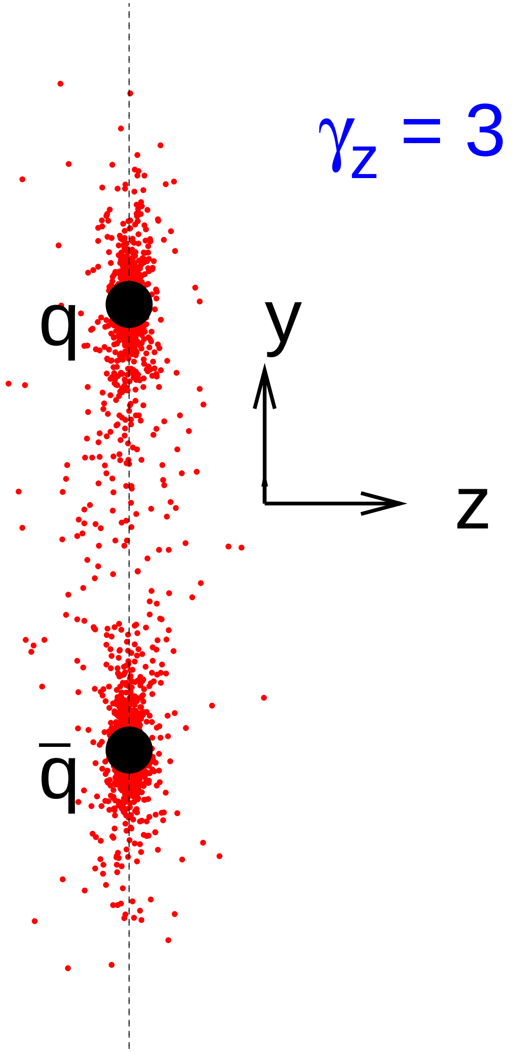
- Unexpected result:

$$\text{power} = \frac{\alpha_s N_c}{\pi} 4 \ln 2 (1 - 6.2\alpha_s) \simeq -0.1$$

- Why such a large effect? What does it mean? Just as inconsistent with data as LL result!

# Integrate over $\gamma_z$: small $\gamma_z$ not flattened




- First branching occurs for $\ln \gamma_z \sim \frac{c}{\alpha_s}$
- In practice $c$ is small: $\gamma_z \sim 2 - 5$
- Energy-distribution $\neq$ perfect $\delta(z)$

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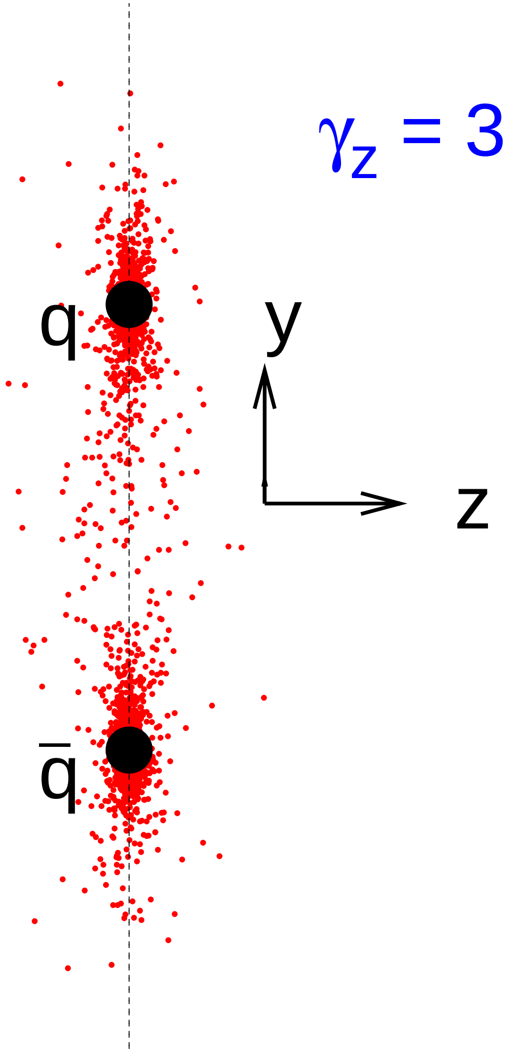


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Ciafaloni '88

Andersson et al; Kwiecinski et al '96

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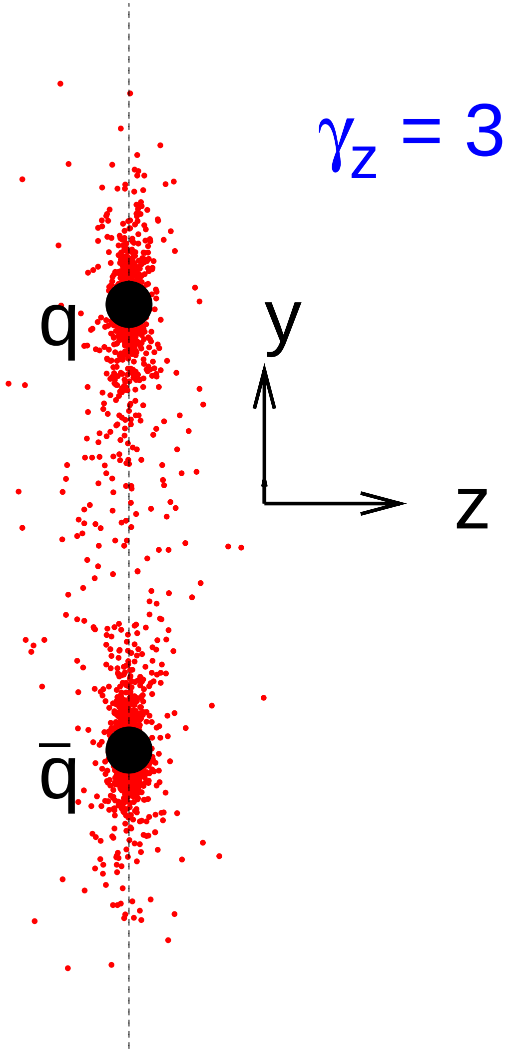


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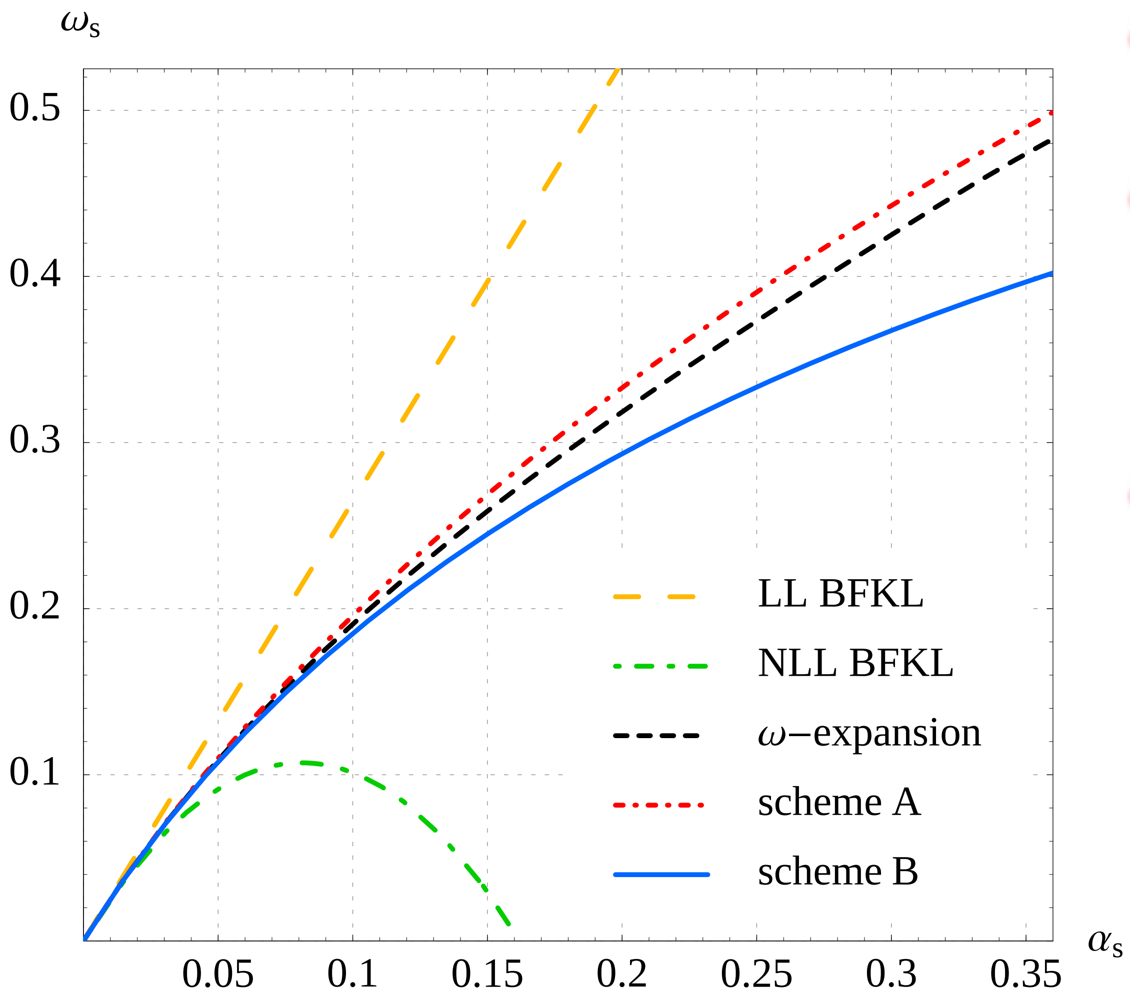
Andersson et al; Kwiecinski et al '96

- Dominant part $\equiv$ double & single $\perp$ logs
  - Responsible for $\sim 90\%$ of NLL corrections
  - Can be used to supplement NLL at all orders

GPS; Ciafaloni & Colferai, '98–99

# Inclusion of all-orders transverse-longitudinal mixing




Ciafaloni, Colferai, GPS, Staśto, '03

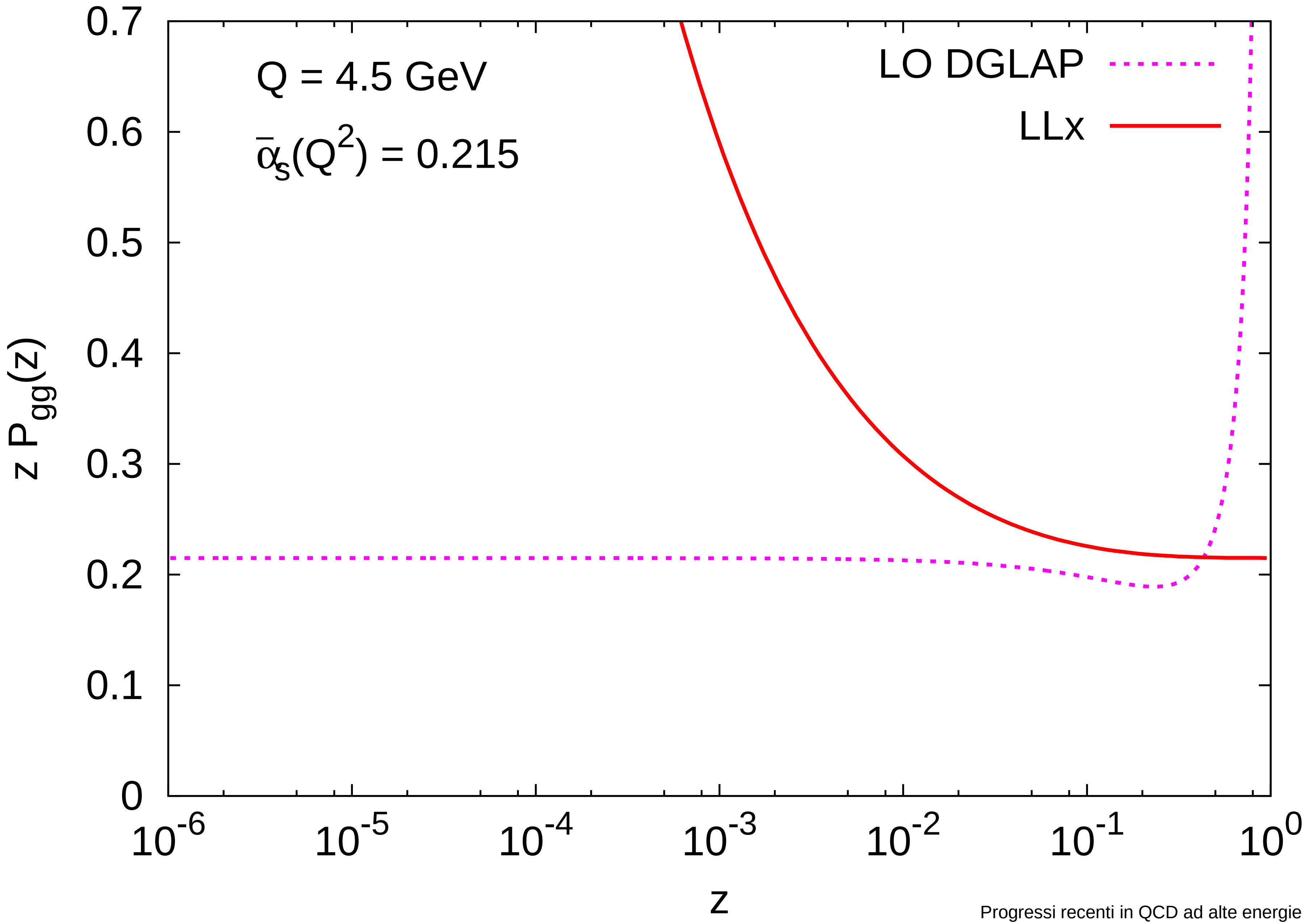
- Significant stabilisation of power.
- Power is consistent with experiments

And other theorists!

Altarelli, Ball, Forte, '04 *prelim.*

- Good starting point for phenomenology

# E.g.: small-$x$ resummed $P_{gg}(x)$ splitting function



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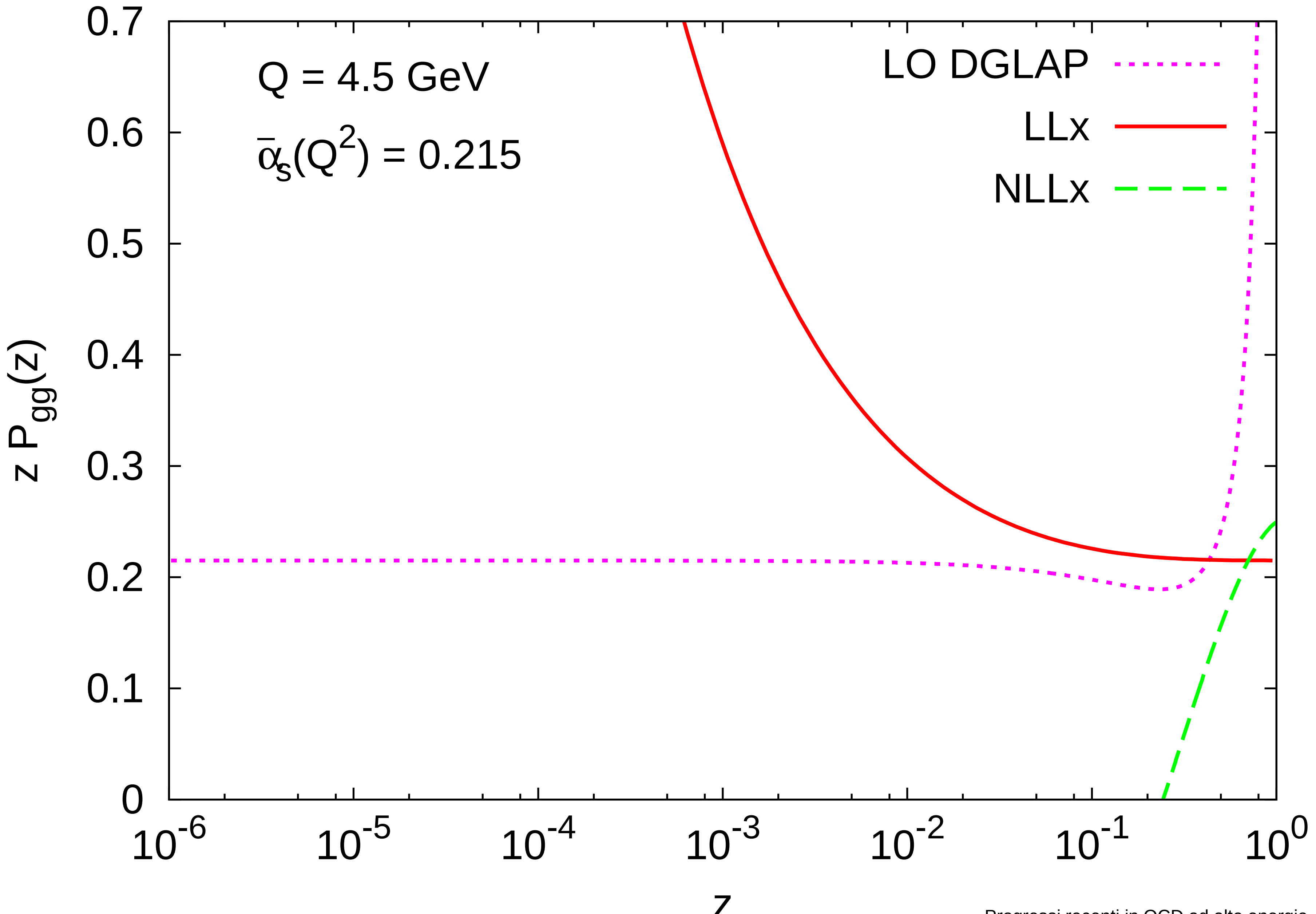

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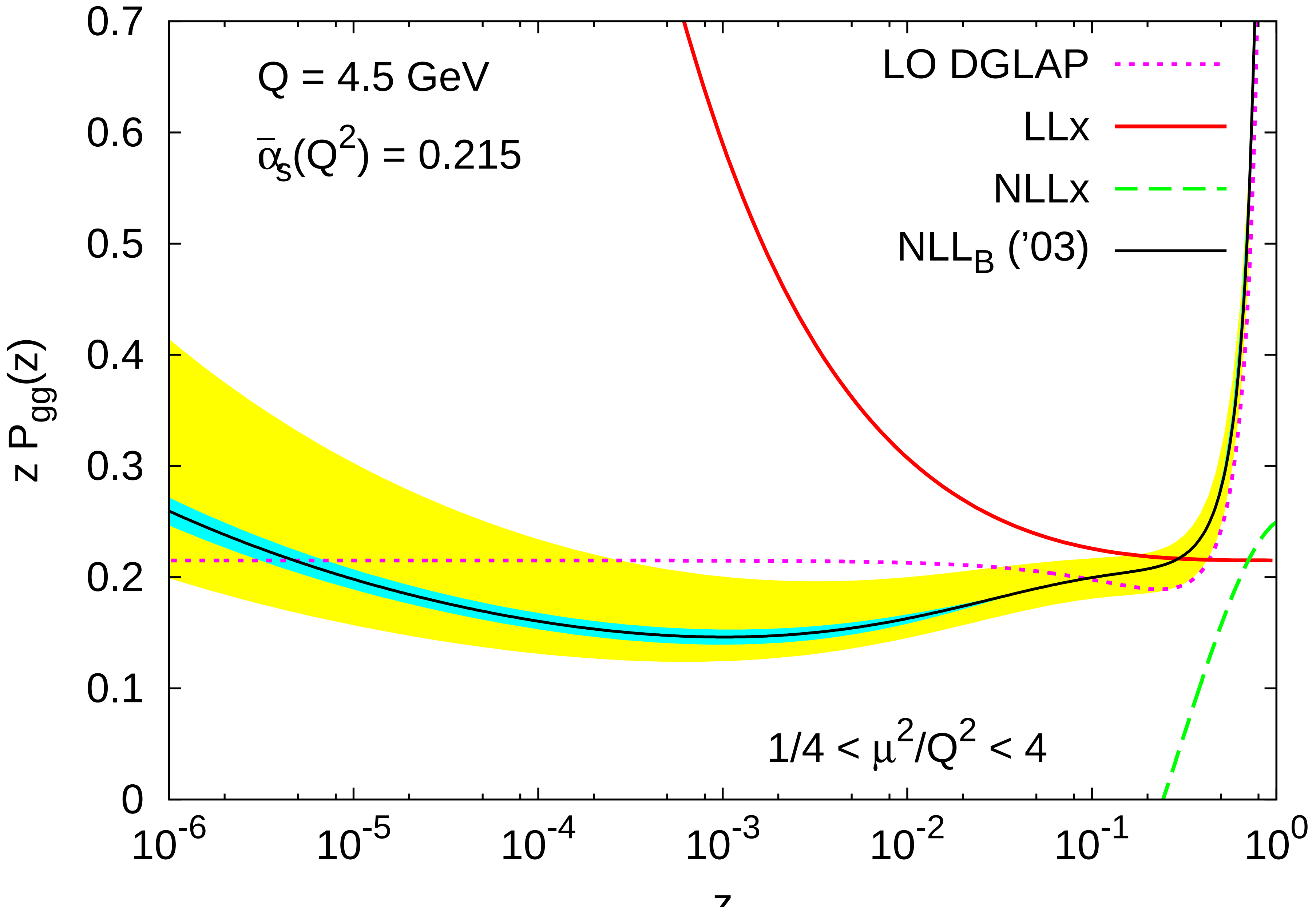

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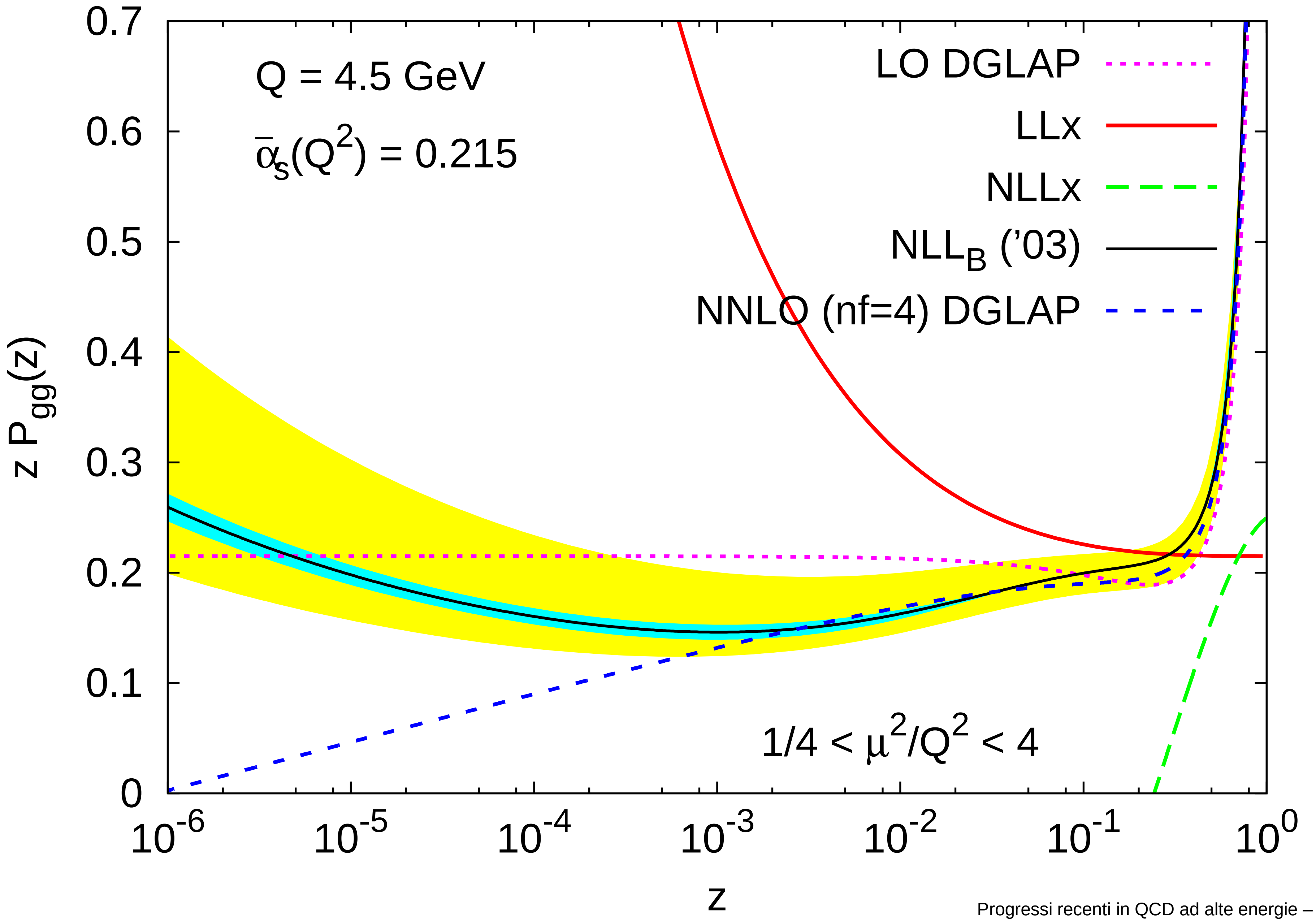


# Detailed phenomenology?

Big job! Work in progress. . .

- Understand coupling to external states
- Conformal-invariance broken by running-coupling effects & must regularise coupling in infrared
- NLL equation more difficult to solve
- Extract splitting & coefficient function for structure function analyses

Bartels, Colferai, Gieseke, Kyrieleis, Qiao ’00–04

Mueller & Kovchegov; Ciafaloni, Mueller & Taiuti, ’98–00

Thorne ’99–01

Altarelli, Ball & Forte ’99–04

Andersen & Sabio-Vera ’03–04

Ciafaloni, Colferai, GPS & Staśto ’99–04

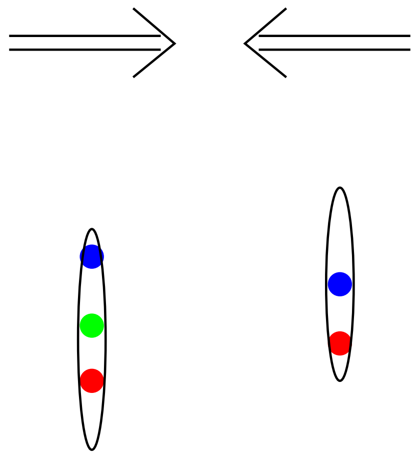
- Putting it all together

??? ’04–0x?

# Does cross section grow for ever?

Cross section growth occurs because of:

- increase in *area* of projectile



$\mathbf{P_{interaction} \ll 1}$

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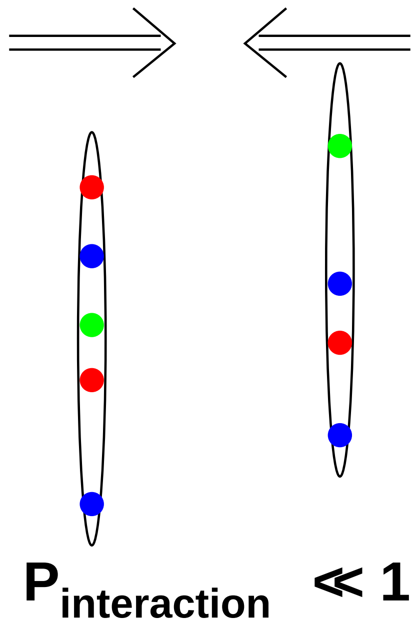

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Cross section growth occurs because of:

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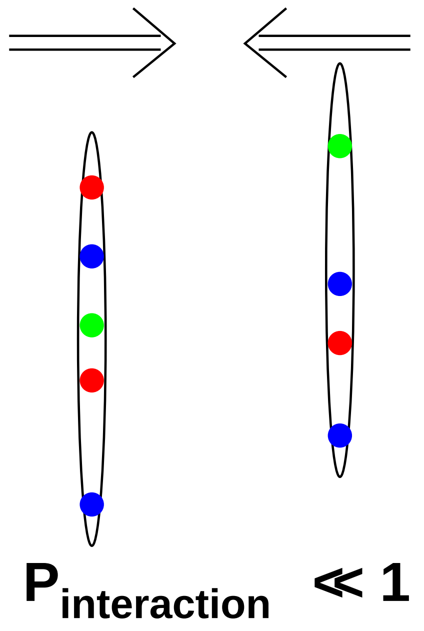

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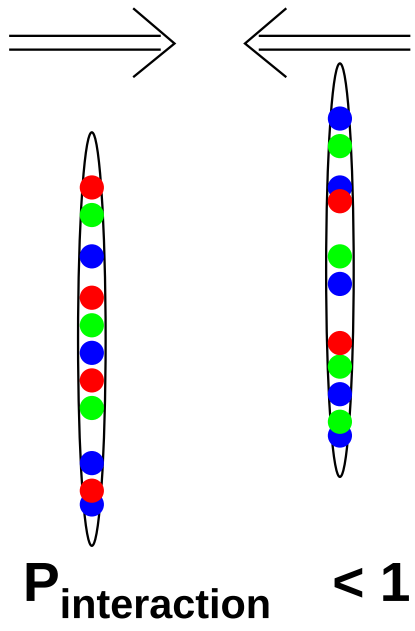

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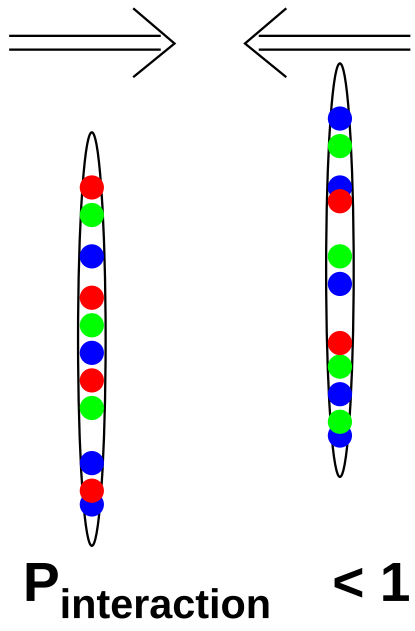
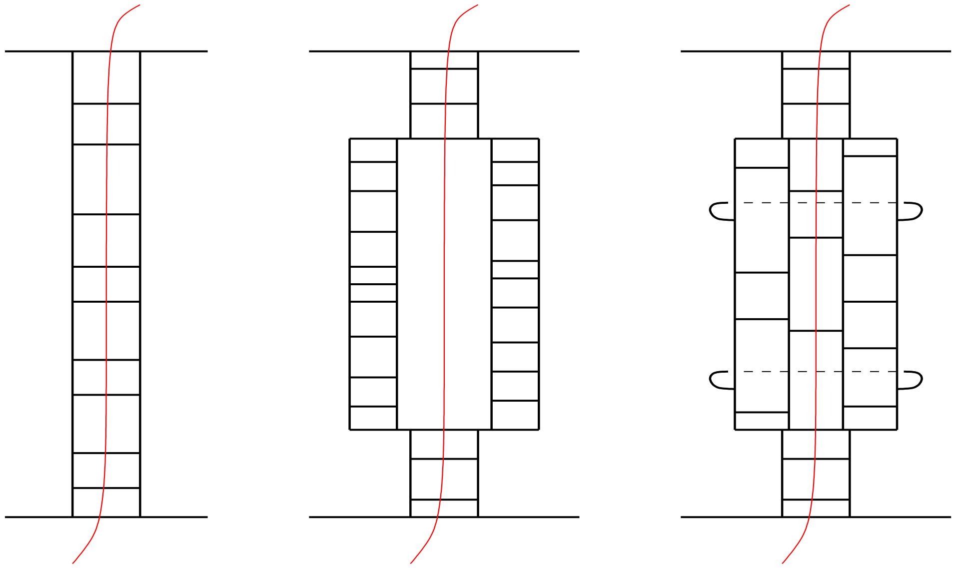



Unitarity requires *non-linearity* (not present so far):

- multiple interactions
- saturation of gluon density
- higher-order correlators between gluon fields

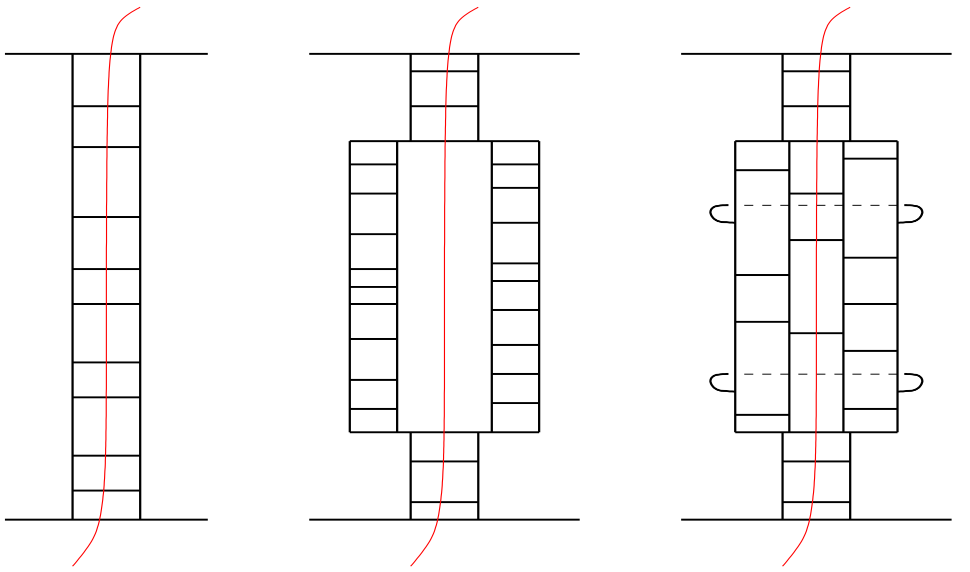
# Non-linearity $\Leftrightarrow$ correlations




- Pomeron is two-gluon state: $s^{4 \ln 2 \bar{\alpha}_s}$
- 2-pomerons give: $s^{8 \ln 2 \bar{\alpha}_s}$
- Since 1981 $\exists$ Bartels, Kwiecinski Praszalowicz equation (BKP) for four-gluon state. But no solution: $s^{???}$ ...

What dominates? 2-pomeron or BKP?

# Non-linearity ⇔ correlations



**pomeron (dipole)** — **2 pomerons (dipole–dipole)** — **BKP 'quarteton' (quadrupole)**

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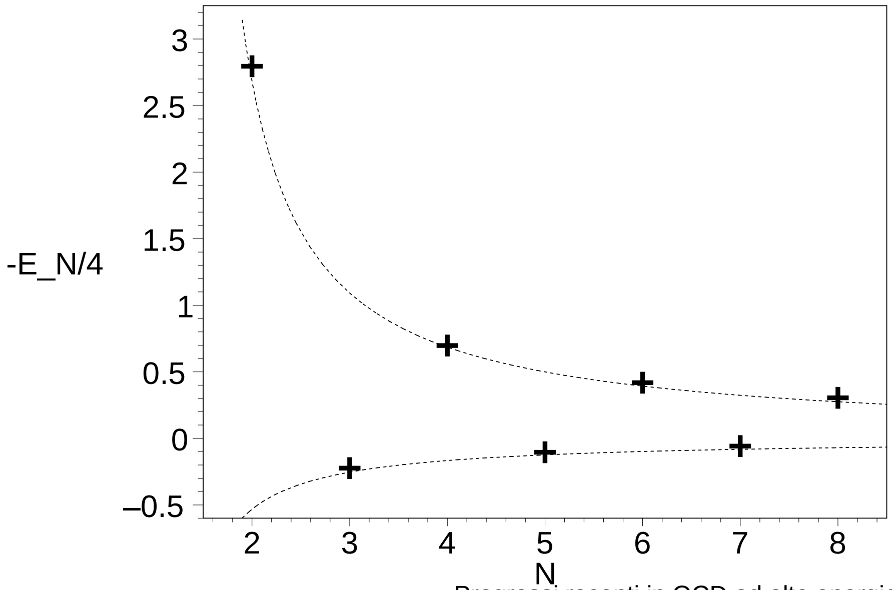
*Conformal invariance ⇔ methods based on integrable models*

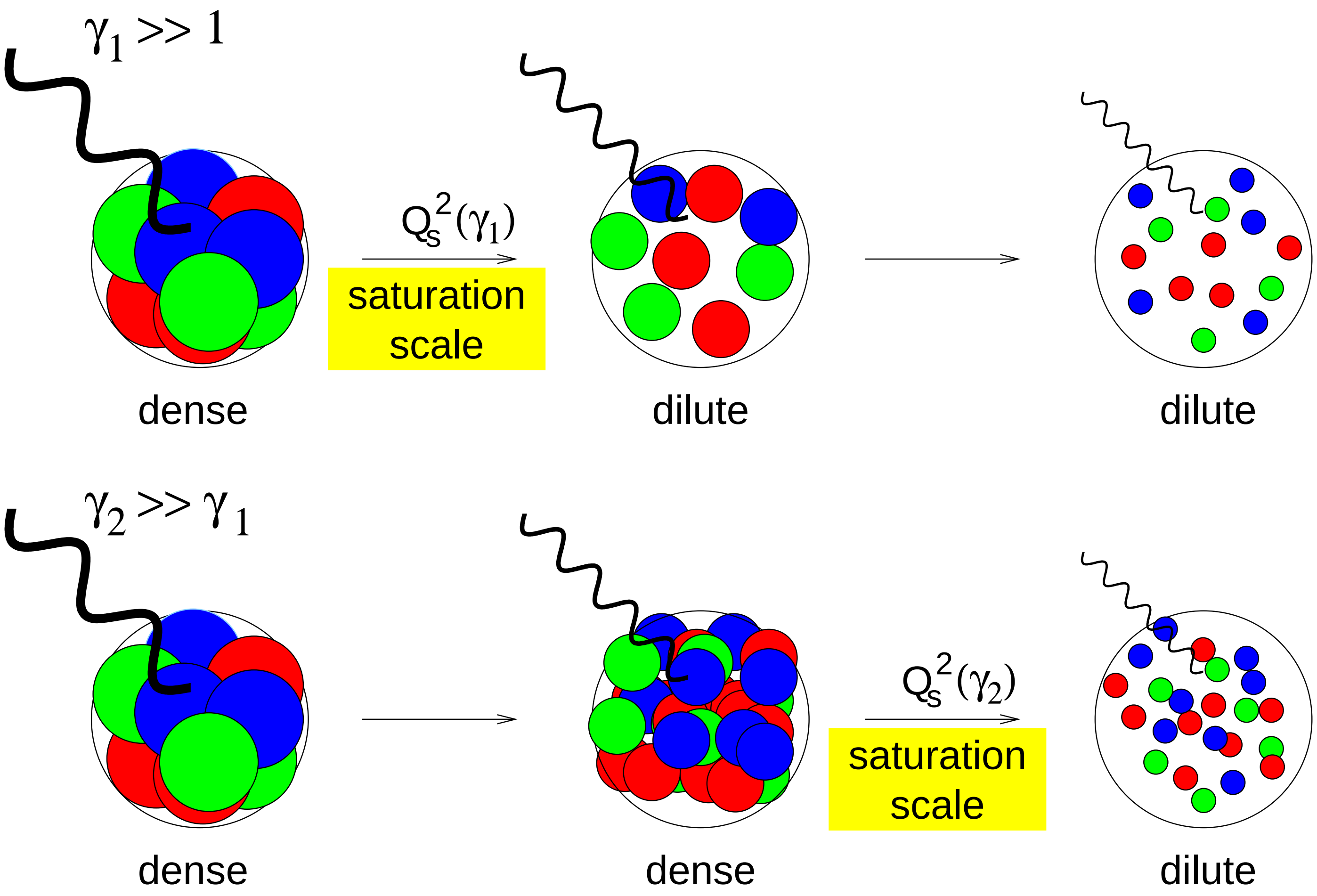
Derkachov, Korchemsky, Kotanski & Manashov '02

de Vega & Lipatov '02

Two-pomeron dominates. . . (But still issues to be resolved — couplings)



# Saturation scales (applies mostly to proton)




Idea of a *maximum (saturated) density* of gluons ($\rho \sim 1/\alpha_s$)

[Colour Glass Condensate]

Saturation scale — resolution param. *separating* saturated and non-saturated.

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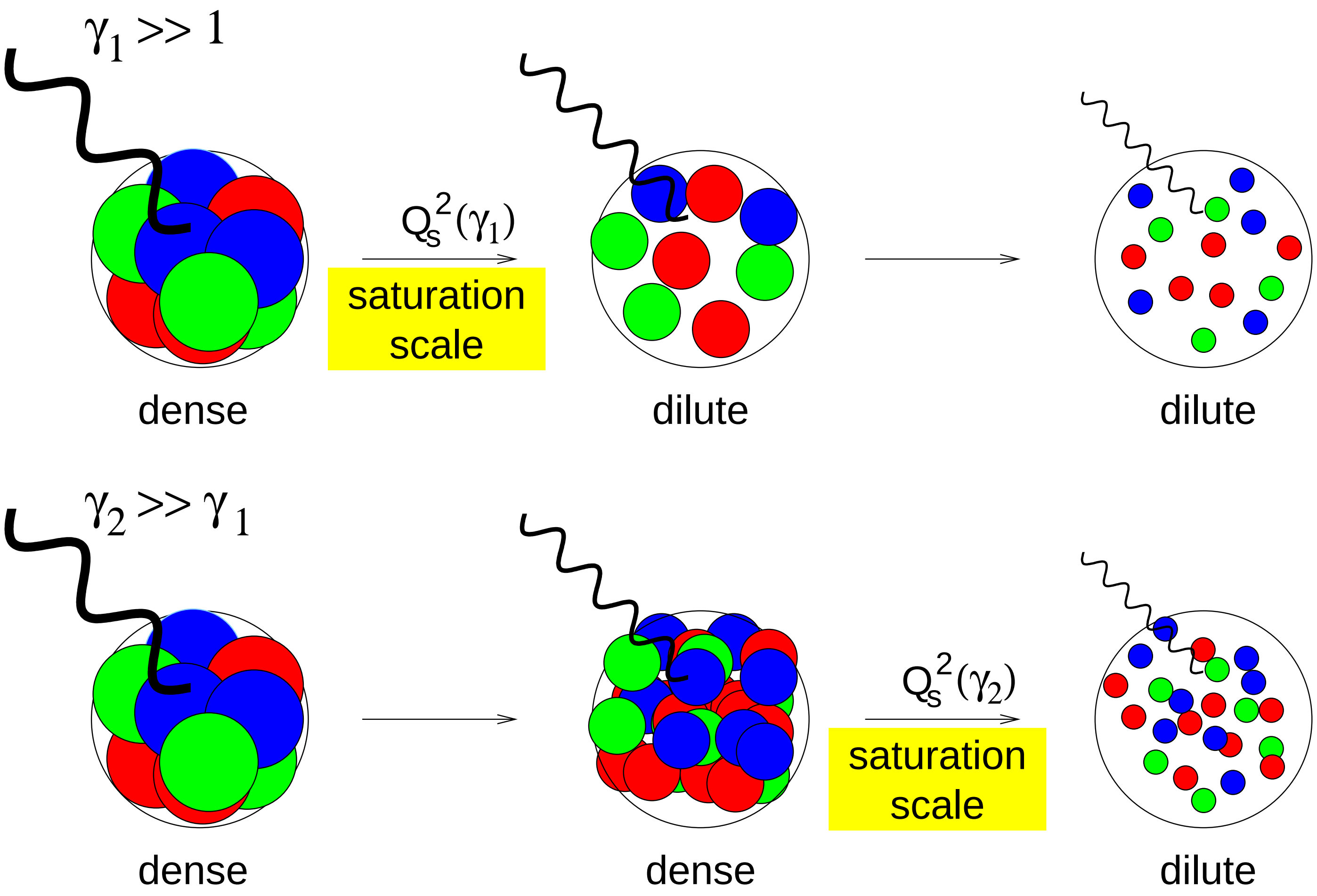


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Can be perturbative even for $p\bar{p}$ collisions

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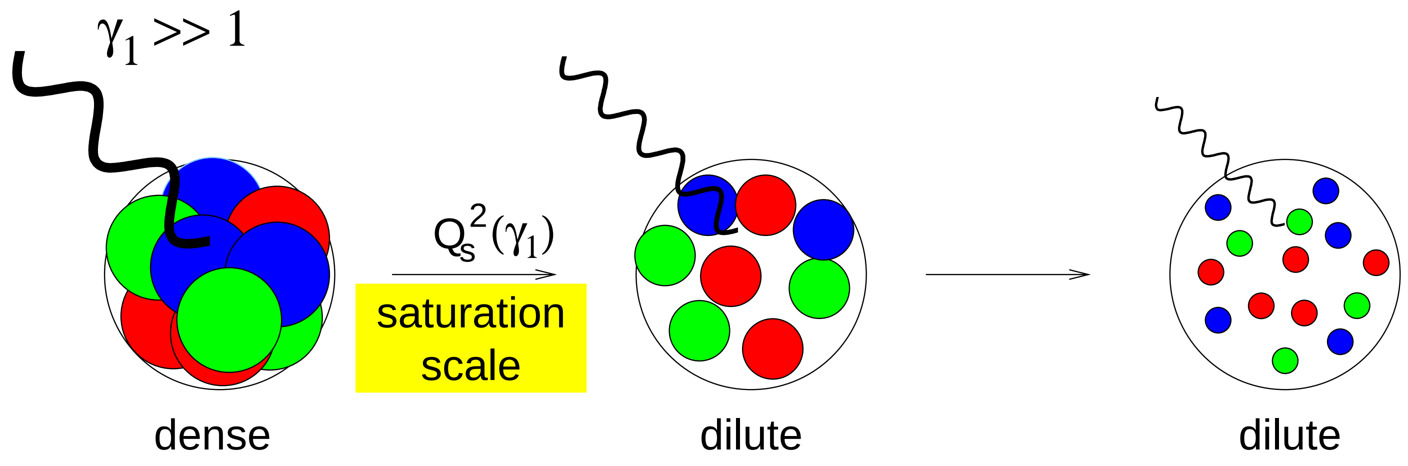
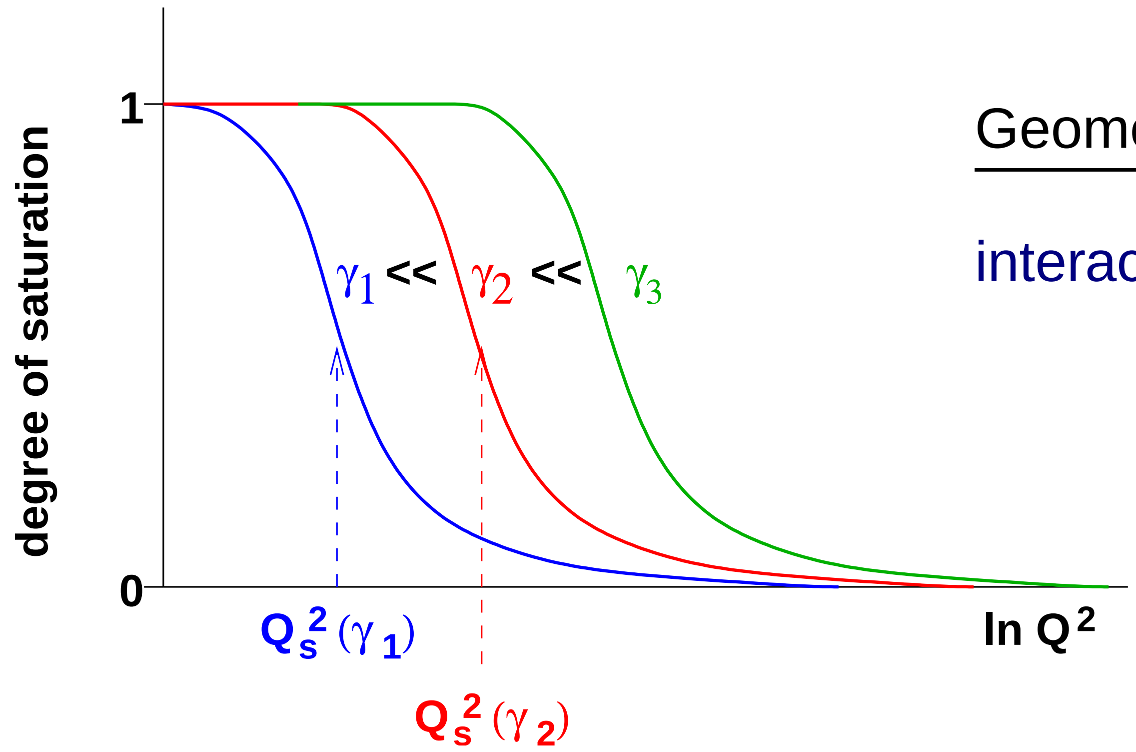


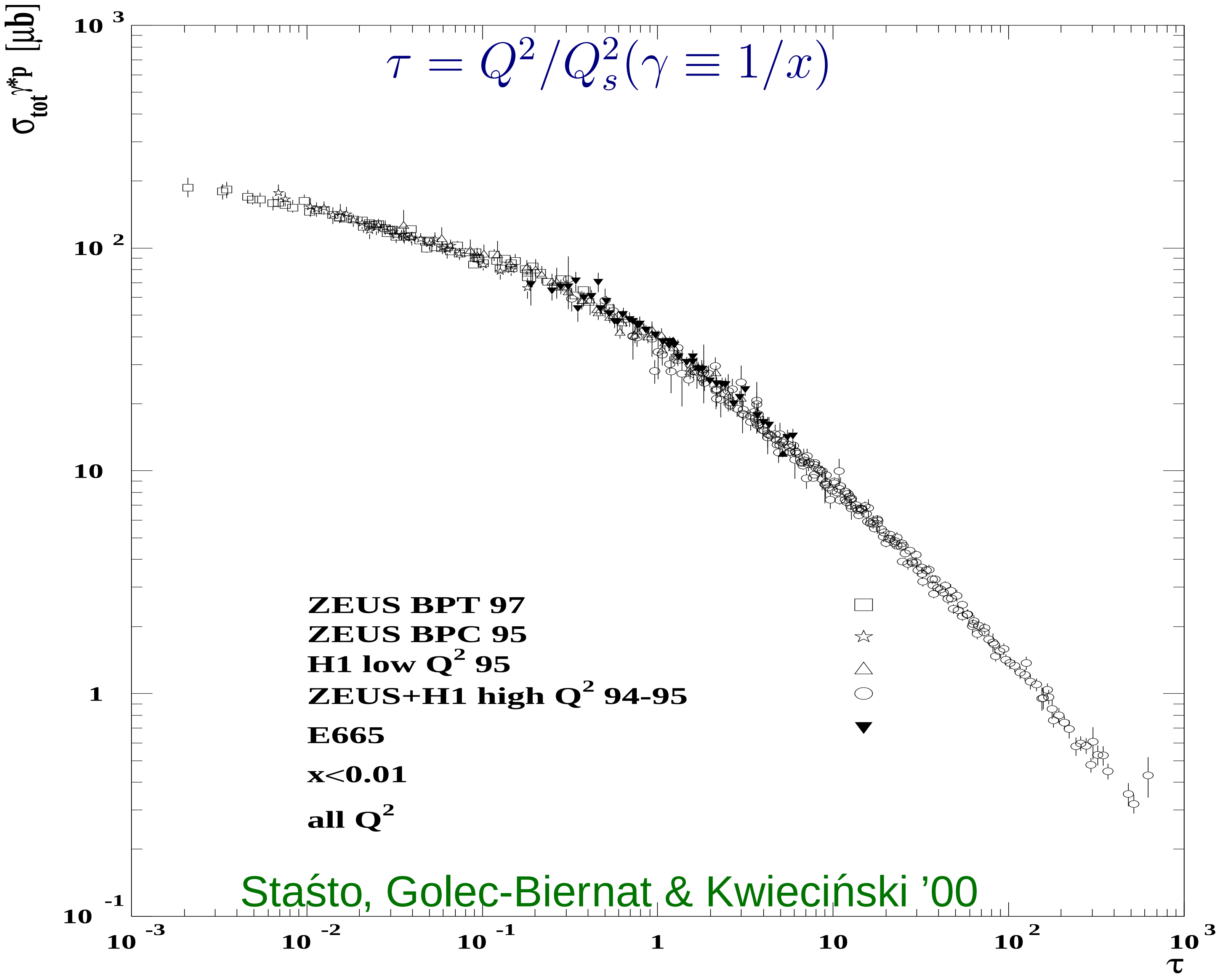




## Geometric scaling

$$\text{interaction}(\gamma_z, Q^2) \simeq f\left(\frac{Q^2}{Q_s^2}\right)$$

Golec-Biernat & Wüsthoff '99

# Geometric scaling in data



# Origins of geometric scaling

Balitsky-Kovchegov equation for saturation

$$\frac{d\langle n\rangle}{d\ln\gamma_z} = \frac{\alpha_{\mathsf{s}} N_c}{\pi}\mathcal{K}\otimes\langle n\rangle - \alpha_{\mathsf{s}}^2\langle nn\rangle$$

Usually replaced with (mean-field approx.)

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Widely studied. Recently pointed out to be in same class as Fisher-Kolmogorov-Petrovsky-Piscounov (KPP) equation

Munier & Peschanski, ’03

$$\partial_t u(t,x) = \partial_x^2 u(t,x) + u(t,x)(1-u(t,x))$$

(Review: Ebert, Van Saarloos ’00)

➥ Geometric scaling is a rigorous property in mean-field approximation

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*OPEN QUESTION: what happens beyond mean-field approximation?*

# Conclusions. . .

Main recent developments

- Transition from qualitative (LL) to quantitative tools (NLL + improvements):
  - BFKL growth of cross sections is theoretically robust & calculable
- Vast investigation of saturation
  - development of a number of approaches (MNZ; BK; JIMWLK)
  - study of solutions mostly in mean-field approx.

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## Not discussed, but important

- Going beyond ‘inclusive quantities’ (total cross sections)
  - diffraction (valuable for understanding saturation)
  - other final-state properties

# EXTRA SLIDES

# Separate longitudinal and transverse

$$\vec{E} = \frac{e\gamma}{4\pi} \frac{\vec{r}}{(r_x^2 + r_y^2 + \gamma^2 r_z^2)^{3/2}}$$

Relativistic contraction $\Rightarrow$ natural *separation between longitudinal and transverse* ($\perp$) directions.

Do *fourier transform in $z$-direction.*, *i.e.*

- Describe field in terms of transverse position (frozen) and longitudinal momentum (dominant component)

Result is just field for a *$2$-dimensional system*

$$\vec{E}_{\perp, p_z} = \int dr_z e^{ip_z . r_z} \vec{E} \Rightarrow \frac{e}{2\pi} \frac{\vec{r}_\perp}{r_\perp^2}$$

NB:

- Separate treatment of $z$ and $\perp$ directions only makes sense for $p_z \gg 1/r_\perp$
- Ignore $e^{ip_z . r_z}$ factor by considering $p_z \ll \gamma / r_\perp$

# Dipole; Energy density in field $\Rightarrow$ # of quanta

Now consider a neutral system (QCD systems always neutral) — a dipole with charges at origin and $\vec{R}_\perp$:

$$\vec{E}_{\perp,p_z} = \frac{e}{2\pi}\left(\frac{\vec{r}_\perp}{r_\perp^2} - \frac{\vec{R}_\perp - \vec{r}_\perp}{|\vec{R} - \vec{r}_\perp|^2}\right)$$

Find *energy density*: $\epsilon = \frac{1}{4\pi}(E^2 + B^2)$ and use $|\vec{E}| = |\vec{B}|$ (& fudge $\times 2$):

$$\frac{d\epsilon}{dp_z\, d^2\vec{r}_\perp} = \frac{e^2}{4\pi^3}\,\frac{R_\perp^2}{r_\perp^2|\vec{R}_\perp - \vec{r}_\perp|^2}$$

Interpret as *number (n) of photons* (divide by photon energy $p_z$):

$$\frac{dn}{dp_z d^2\vec{r}_\perp} = \frac{1}{p_z}\frac{e^2}{4\pi^3}\,\frac{R_\perp^2}{r_\perp^2|\vec{R}_\perp - \vec{r}_\perp|^2}$$