Event-shape power corrections: putting the hadrons into hadronisation

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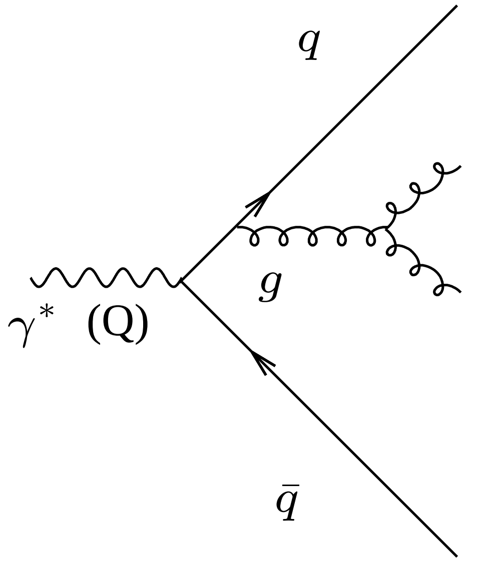
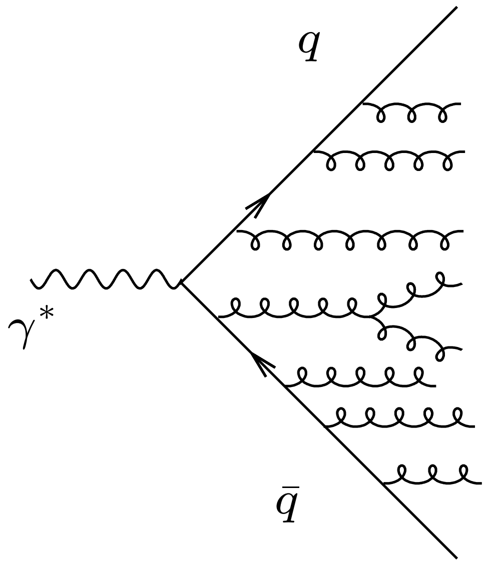
Setting the scene

Calculations of high-energy processes in perturbative QCD always involve *quarks and gluons*. One can do fixed order perturbative (PT) calculations, *e.g.* $O(\alpha_s)$, $O(\alpha_s^2)$ involving a limited number of particles

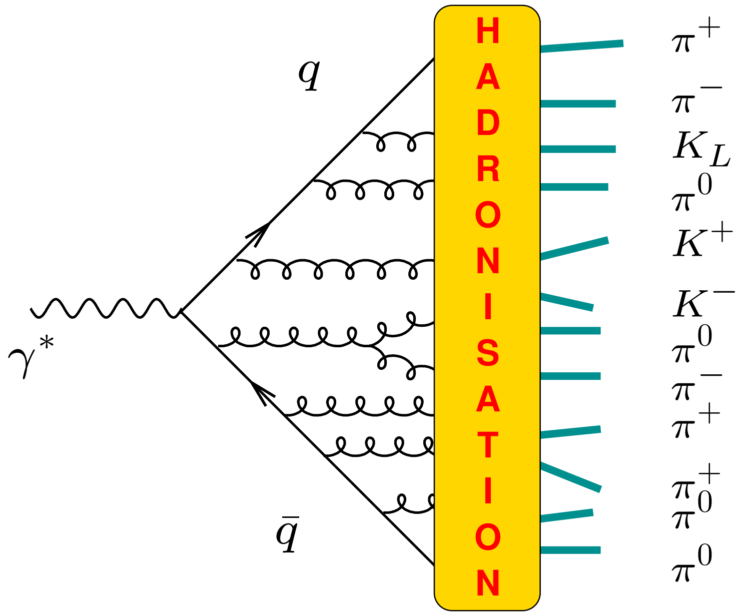


It is also possible to calculate properties of *ensembles of many quarks and gluons* if most of them are soft or at small angles (collinear).



This is the limit of perturbative calculations.

But in the lab we never see quarks and gluons (partons), only colourless *HADRONS*.



Suppose we have an observable which measures properties of the final state: then its value calculated within PT QCD (even at arbitrarily high order) will differ from the experimentally measured value. This difference can be ascribed to *hadronisation*.

We would like to understand these hadronisation corrections:

✗ Practical importance in making predictions for high-energy colliders.

✗ Extracting information about α_s and structure functions, from final-state observables.

✗ Just for the sake of it: *i.e.* to obtain an understanding of the dynamics of QCD in the strong coupling regime.

What methods are available?

For many aspects of QCD ($e.g.$ spectrum of hadrons, hadron decay rates, moments of structure functions) there exist powerful, well-defined non-perturbative methods: *lattice calculations of appropriate operators* (sometimes used in combination with methods such as chiral perturbation theory).

But for hadronisation the lattice currently offers *little promise*:

✗ hadronisation cannot (as yet) be expressed in terms of any well-defined operators;

✗ it involves a wide range of scales (fast-moving particles, slow-moving particles);

✗ it requires the treatment of many particles in a large volume;

✗ it is a very ‘Minkowskian’ process (time-dependence).

The above points make it very difficult, both technically and conceptually, to see how to use lattice calculations to make predictions about hadronisation.

What about Monte Carlo event generators?

These contain very sophisticated models of hadronisation:

✗ PYTHIA, ARIADNE: Lund string fragmentation model

✗ HERWIG: cluster fragmentation

These models are *extremely valuable tools*, both for experimenters and theorists. But:

✗ They depend on unknown parameters (sometimes many), which have to be tuned with existing data.

✗ We know that they work at today's energies: what happens when we go to higher energies?

✗ It's difficult to use them to develop a more fundamental understanding of hadronisation.

Alternative approach(es)

Over the past few years a search has gotten underway for a more basic understanding of hadronisation corrections. Many theorists have been active in recent years:

Akhoury, Ball, Banfi, Beneke, Braun, Dasgupta, Dokshitzer, Gardi, Grunberg, Korchemsky, Lucenti, Manohar, Marchesini, Magnea, Nason, Rathsman, GPS, Seymour, Smye, Sterman, Tafat, Trócsanyi, Webber, Wise, Zakharov, Zanderighi.

It's a learning process. A significant part of the theoretical work has been triggered by interaction (sometimes ≡ theorists being told that their approaches fail!) with experimenters from:

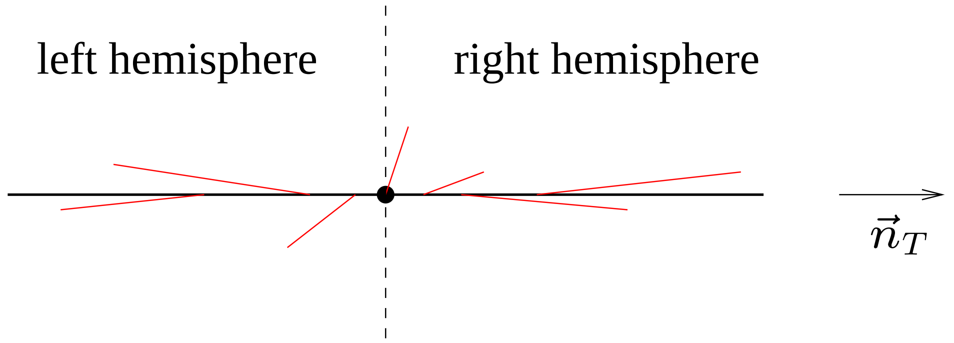
ALEPH, DELPHI, H1, JADE, L3, OPAL, ZEUS

The second part of the seminar is about one such case:

- ✗ First we'll look at a standard approach for studying hadronisation corrections.
- ✗ See some inconsistencies in very specific situations.
- ✗ We'll identify their origin → find that these inconsistencies were evidence of a more generalised problem...

Where do we start?

We're going to study some observables which are relatively inclusive ($i.e.$ we are not interested in the fate of individual particles, but rather in the general flow of energy): *Event Shapes in e^+e^-*.



$$\text{thrust} \quad T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i.\vec{n}}{\sum_i |\vec{p}_i|}\,, \qquad \tau = 1 - T\,,$$

$$C\text{-parameter} \quad C = \frac{3}{2}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}\,,$$

$$\text{jet-mass} \quad \rho = \frac{\left(\sum_{i\in\text{hemisphere}} p_i\right)^2}{\left(\sum_i E_i\right)^2}\,,$$

$$\text{broadening} \quad B_T = \frac{\sum_i p_{ti}}{\sum_i |\vec{p}_i|}\,.$$

Define also:

$$\text{heavy-jet mass} \quad \rho_h = \max\{\rho_{\text{left}}, \rho_{\text{right}}\}\,,$$

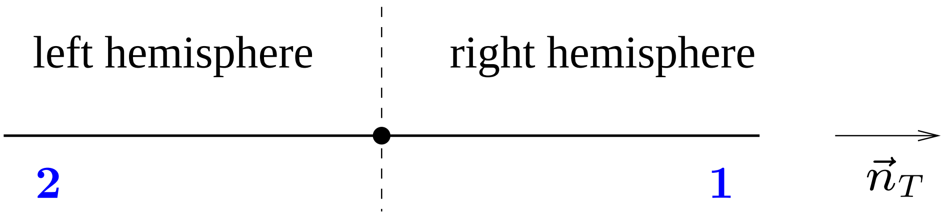
$$\text{wide-jet broadening} \quad B_W = \max\{B_{\text{left}}, B_{\text{right}}\}\,.$$

Why all these variables? We're interested in a range of variables because each one measures different properties of the event.

Let's take the *thrust* and study it in more detail.

$$T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i.\vec{n}}{\sum_i |\vec{p}_i|}\,, \qquad \tau = 1 - T\,.$$

For an event with two back-to-back particles:



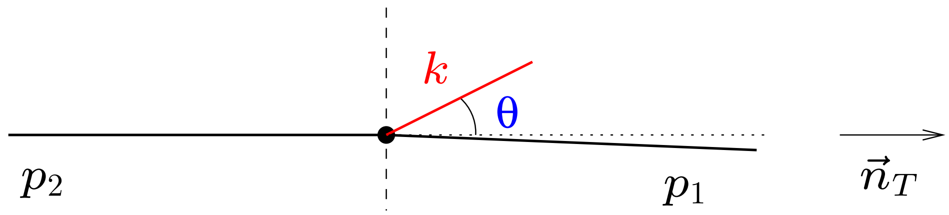
$\vec{p}_1 = -\vec{p}_2$ and thrust axis ($\vec{n}_T$) is parallel to $\vec{p}_1$

➥ $$T = 1\,, \qquad \boxed{\tau \equiv 1 - T = 0}$$

All other variables also *go to zero in the two-jet limit*.

$$T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i.\vec{n}}{\sum_i |\vec{p}_i|}\,, \qquad \tau = 1 - T\,.$$

Observable becomes non-trivial when we have three or more particles



If extra particle is *soft* (low energy) then recoil is negligible:

$$\tau \simeq \frac{|\vec{k}| - |\vec{k}.\vec{n}_T|}{Q} = \frac{k_t}{Q}\tan\frac{\theta}{2}, \qquad (\theta < \pi/2)$$
$$= \frac{k_t}{Q}\, e^{-|\eta|}$$

where Q is centre of mass energy, k_t is the transverse momentum and the (pseudo)rapidity $\eta = -\ln\tan\theta/2$.

So thrust (τ) is a measure of *flow of transverse momentum at large angles*.

When we talk about hadronisation corrections to event shapes what we really refer to is *how hadronisation affects the mean transverse momentum* per unit rapidity

Renormalons

A ‘non-perturbative’ approach widely used in recent years, which tries to push perturbation theory to its limit.

Schematically, to get the mean value of τ (soft limit) we just integrate over the perturbative particle-production probability, multiplied by the value of the observable

$$\langle \tau \rangle \simeq \underbrace{\frac{2C_F}{\pi} \int d\eta \int^Q \frac{dk_t}{k_t} \alpha_s(k_t)}_{\text{emission probability}} \underbrace{\frac{k_t}{Q} e^{-|\eta|}}_{\text{observable}}$$

The integral over η gives a factor of 2:

$$\langle \tau \rangle \simeq \frac{4C_F}{\pi} \int \frac{dk_t}{Q} \alpha_s(k_t)$$

But α_s has a Landau pole:

$$\alpha_s(k_t) = \frac{1}{b_0 \ln k_t^2/\Lambda^2}, \qquad b_0 = \frac{11C_A - 2n_f}{12\pi}$$

Integrating over it gives an ill-defined answer: *estimate ambiguity* from residue, which is of order

$$\frac{4C_F}{\pi} \frac{\Lambda}{Q}$$

The same ambiguity arises if one examines the perturbation series order by order (the smallest term is of this size). This (roughly) is the problem of *infrared renormalons.*

Interpretation?

Infrared renormalons indicate a *fundamental limit of perturbation theory*. To go beyond one needs to understand the non-perturbative behaviour of the theory.

We can hypothesise

- ✗ that non-perturbative effects will be of the same order of magnitude as the ambiguity (reasonable).
- ✗ that non-perturbative effects will be proportional to the ambiguity (pushing our luck?).

To illustrate what we mean we need to consider more than just a single observable. For a system consisting of a hard quark, anti-quark and a soft gluon, all variables have the same dependence on k_t but different dependences on the rapidity:

τ	C	ρ	$\mathcal{V}$
$\frac{k_t}{Q} e^{-\lvert\eta\rvert}$	$\frac{k_t}{Q} \frac{3}{\cosh \eta}$	$\frac{k_t}{Q} e^{-\eta}\, \Theta(\eta)$	$\frac{k_t}{Q} f_{\mathcal{V}}(\eta)$

$$\langle \mathcal{V} \rangle = \underbrace{\frac{2C_F}{\pi} \int \frac{dk_t}{k_t} \alpha_s(k_t) \frac{k_t}{Q}}_{\text{universal}} \underbrace{\int d\eta f_{\mathcal{V}}(\eta)}_{\mathcal{V}-\text{dependent}}$$

The rapidity and transverse momentum integrals *factorise*.

Rapidity integral:

$$c_{\mathcal{V}} = \int d\eta f_{\mathcal{V}}(\eta)$$

$\mathcal{V}$	τ	C	ρ
$c_{\mathcal{V}}$	2	3π	1

One way of doing the k_t integral is to hypothesise a *universal, non-perturbative, infrared finite coupling* α_s^{NP}. Attribute non-perturbative corrections to the difference between 'universal' coupling and perturbative coupling:

Dokshitzer & Webber 1995

$$\langle \delta\mathcal{V} \rangle = \frac{2C_F}{\pi} \int^{\mu_I} \frac{dk_t}{k_t} \frac{k_t}{Q} \left(\alpha_s^{\mathrm{NP}}(k_t) - \alpha_s^{\mathrm{PT}}(k_t) \right) \cdot c_{\mathcal{V}}$$

Schematically, this gives

$$\boxed{\langle \delta\mathcal{V} \rangle = \frac{2C_F}{\pi} \frac{\mu_I}{Q} \left(\alpha_0(\mu_I) - \langle \alpha_s^{\mathrm{PT}} \rangle_{\mu_I} \rangle \right) \cdot c_{\mathcal{V}}}$$

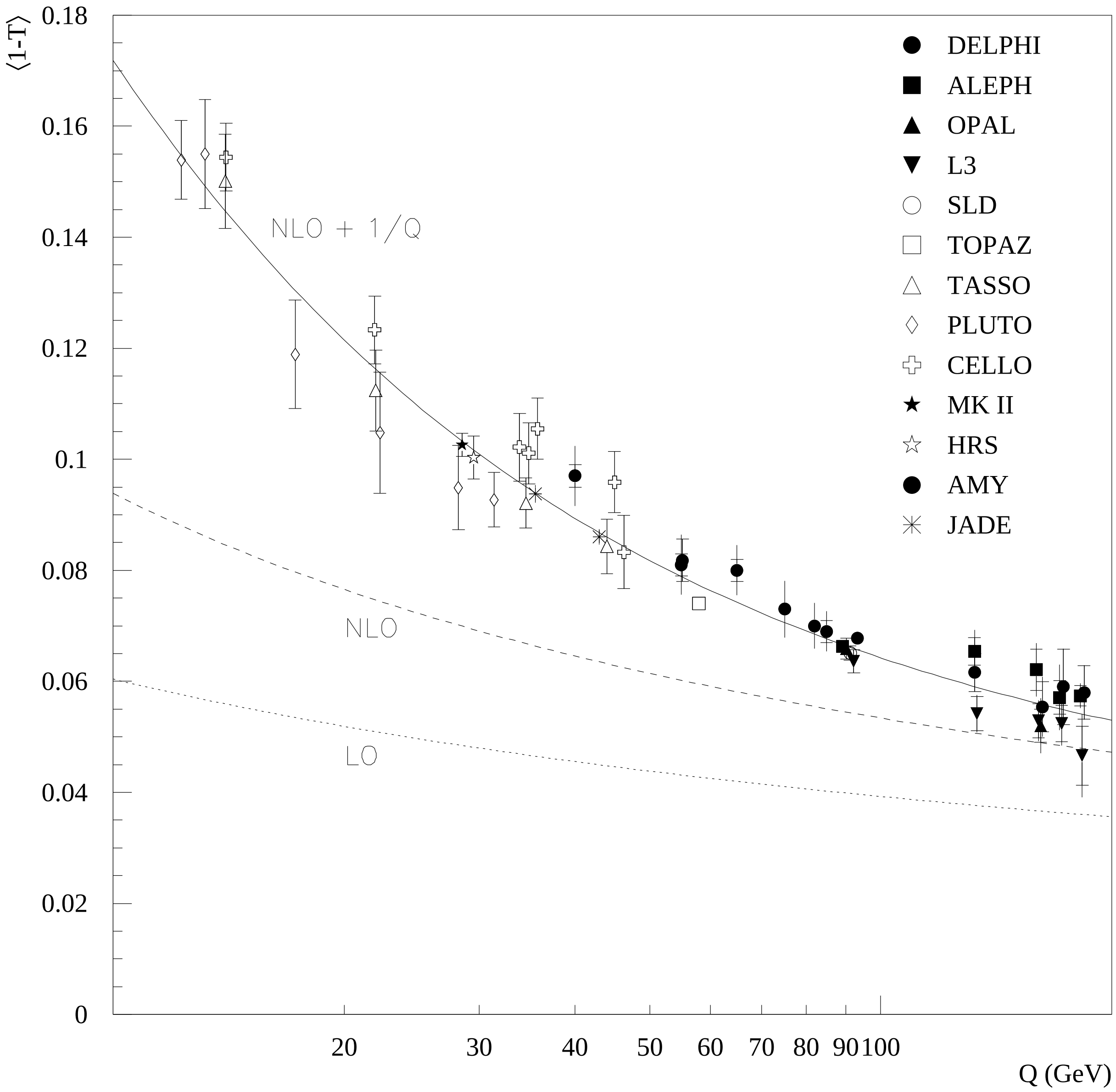
where we've introduced the (a priori unknown) first moment of the coupling:

$$\alpha_0 = \frac{1}{\mu_I} \int_0^{\mu_I} dk_t \, \alpha_s^{\mathrm{NP}}(k_t)$$

NB: interpretation of answer in terms of moment of coupling is specific to the renormalon analysis.

Form of answer $\propto c_{\mathcal{V}}/Q$ is a more general consequence of independence of particle production on rapidity. (*c.f.* Feynman 1972).

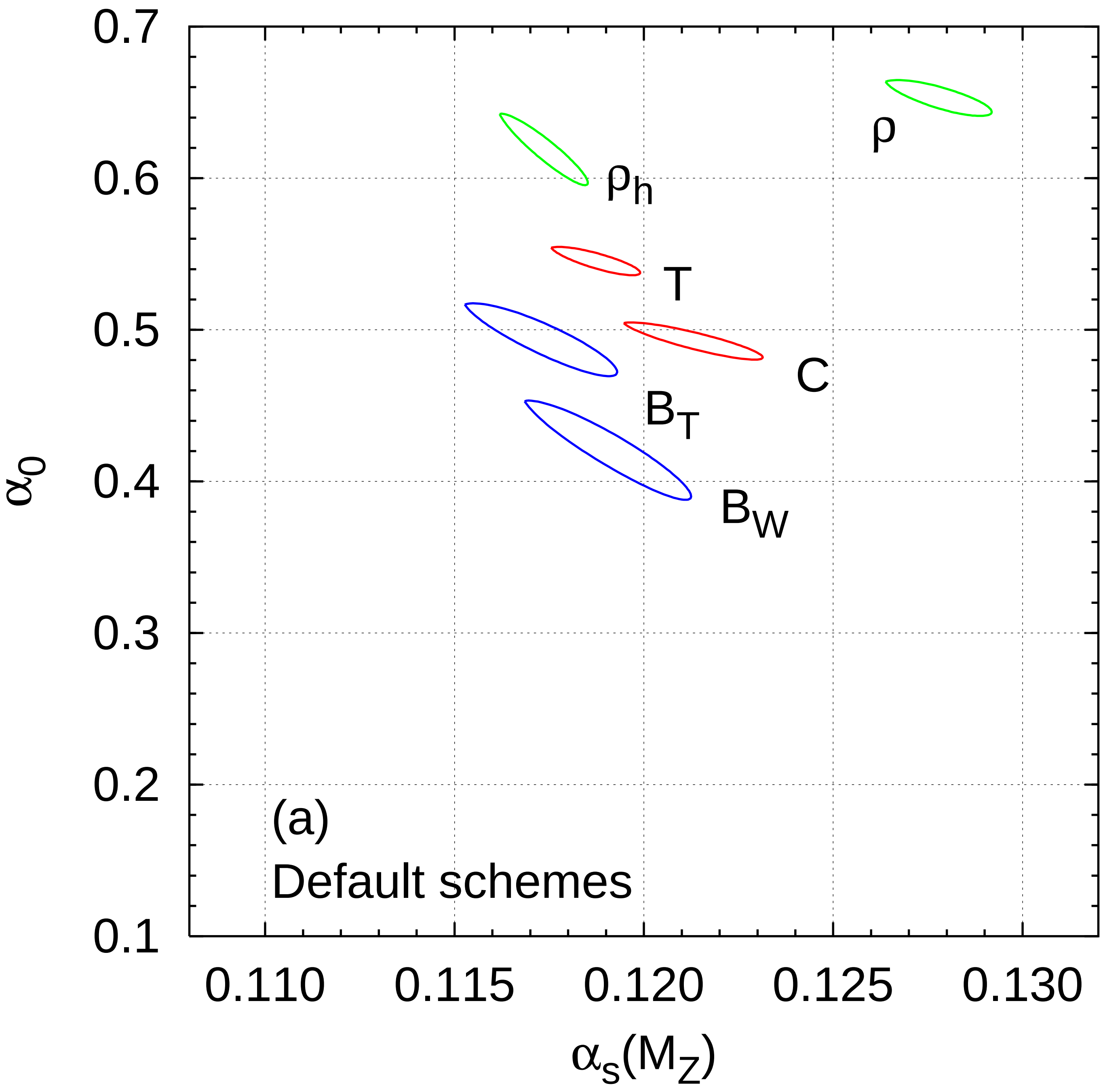
PT and NP contributions to τ



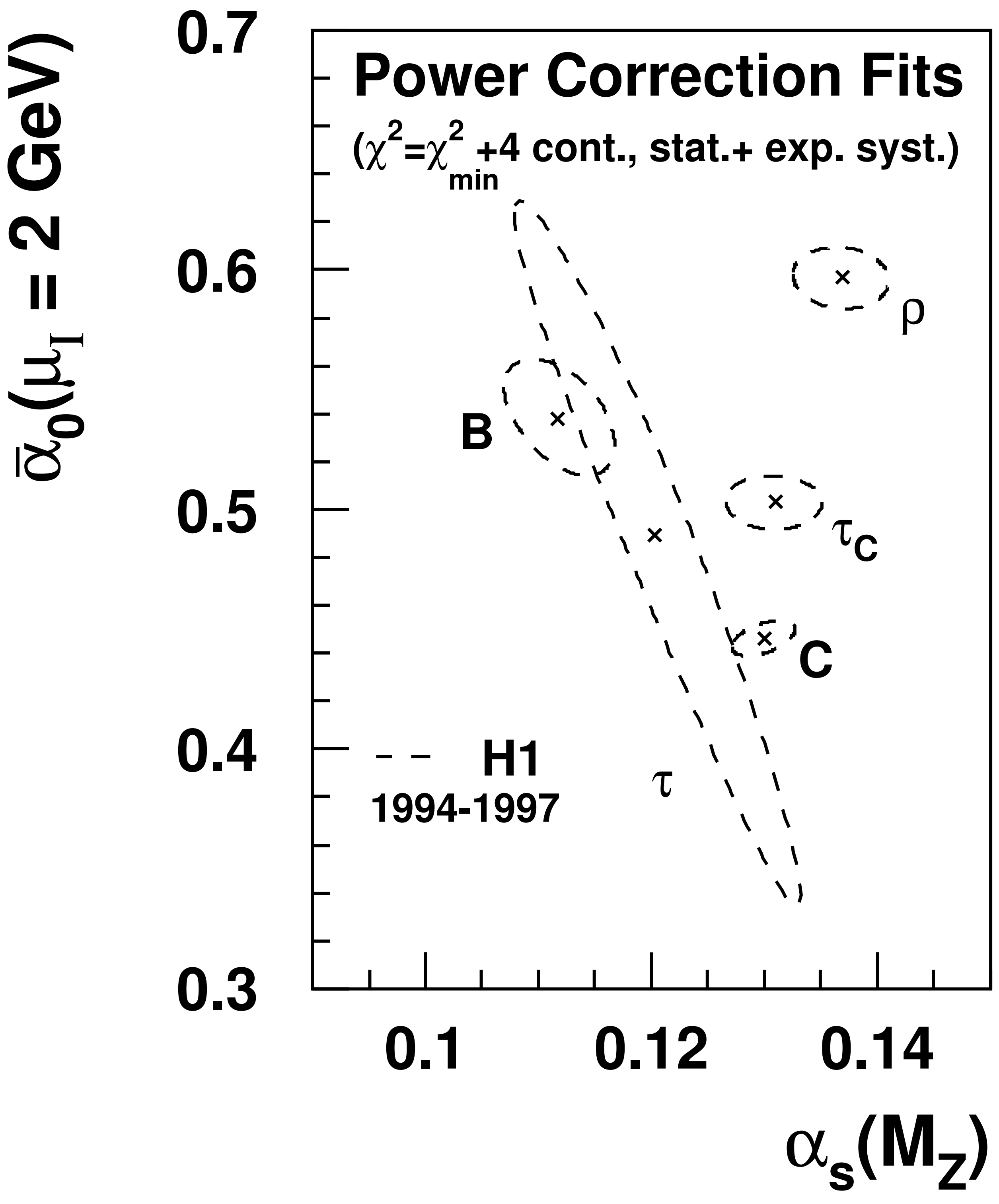
Testing universality

We can compare next-to-leading PT predictions plus NP contribution to data, fitting for α_s and α_0. If universality holds then both α_s and α_0 should be *independent of the observable.*

If we do it in e^+e^- we obtain (1-σ contours):



One can do the same thing in DIS



Clear problem with jet masses (ρ, ρ_h).

This problem has been known about for a couple of years. What is it that we're missing?

With Bryan Webber, we noticed another puzzle. For an arbitrary ensemble of partons one can show that

$$\rho_{\text{left}} + \rho_{\text{right}} \leq \tau$$

Yet if we look at the e^+e^- data we find

Q [GeV]		$\langle \rho_{\text{left}} + \rho_{\text{right}} \rangle$	$\langle \tau \rangle$
22	(TASSO)	0.1366	0.1124
44	(TASSO)	0.0989	0.0843
91	(L3)	0.0742	0.0636
91	(DELPHI)	0.0739	0.0678
133	(DELPHI)	0.0661	0.0616
183	(DELPHI)	0.0594	0.0580

What's more, the violation of the inequality seems to be by a piece of order $\sim 1/Q$. *What's going on?*

A clue was provided by Hasko Stenzel (on ALEPH) who showed that with JETSET, at parton level the inequality is satisfied — *only after hadronisation is it violated.*

Why the difference between hadron and parton levels?

The only 'mathematical' difference between parton and hadron levels is that *partons are massless*, while *hadrons have masses*. The inequality (and everything else so far) was derived assuming all particles are massless...

$$\text{thrust} \quad T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i.\vec{n}}{\sum_i |\vec{p}_i|}\,, \qquad \tau = 1 - T\,,$$

$$\text{jet-mass} \quad \rho = \frac{\left(\sum_{i\in\text{hemisphere}} p_i\right)^2}{\left(\sum_i E_i\right)^2}\,,$$

Thrust is defined just in terms of 3-momenta (insensitive to masses), whereas jet-mass involves both energies and 3-momenta.

Simplest case of violation of inequality is for an ensemble made of two massive back-to-back particles (mass m):

$$\tau = 0$$

$$\rho_{\text{left}} + \rho_{\text{right}} = \frac{2m^2}{Q^2}$$

(we'll see in a while that we can also get $1/Q$ terms).

Can we eliminate mass effects?

The mass effects seem to come directly from explicit mass-dependence in the definition of the observable (sensitivity to difference between energy and 3-momentum).

Simplest way of eliminating mass-dependence is to *redefine the observables!* There are two obvious possibilities:

p-scheme: redefine observable purely in terms of particle 3-momenta ($i.e.$ replace energies with $|\vec{p}|$). [Default definitions of τ, C, B are already in the p-scheme.]

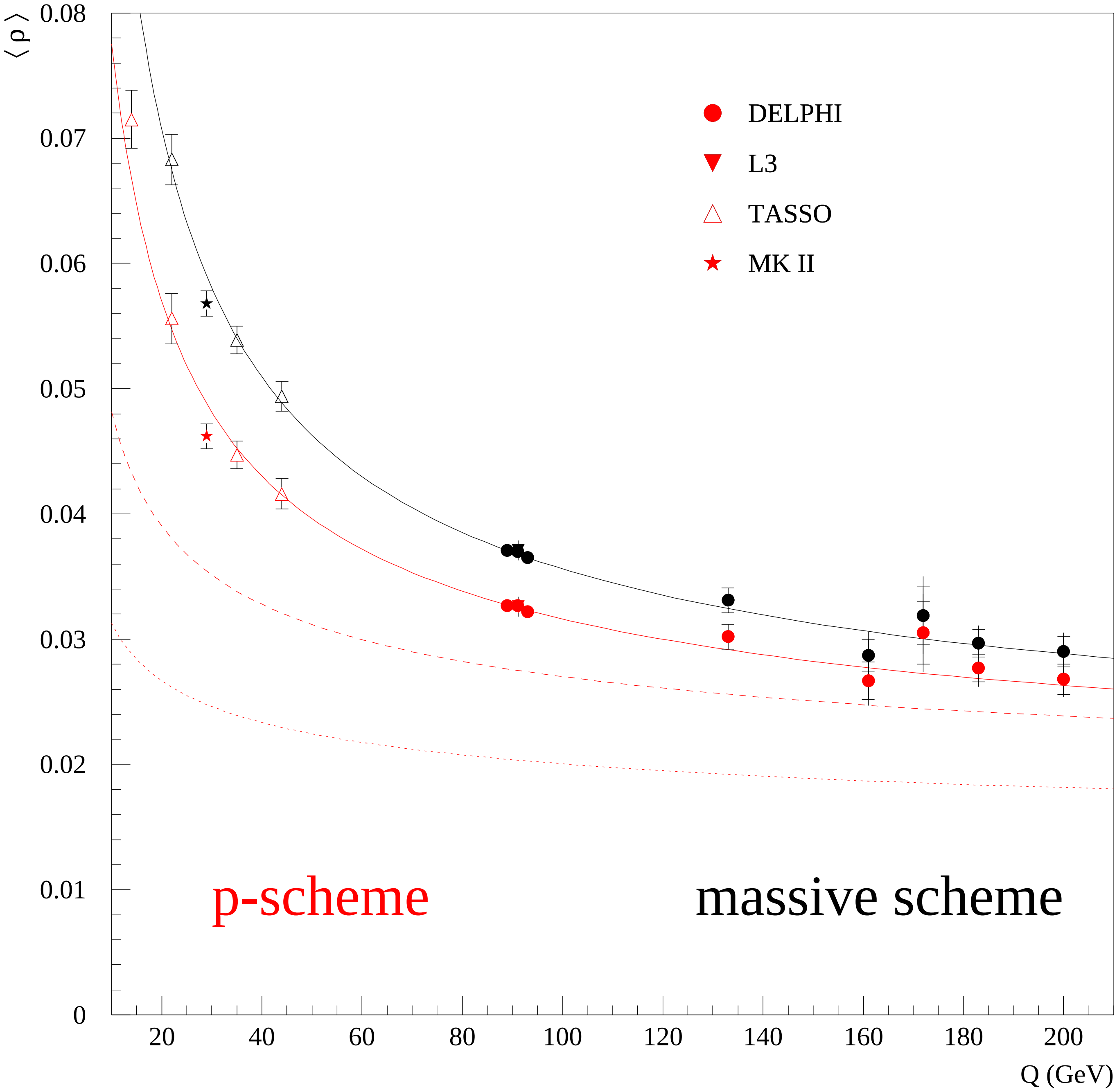
E-scheme: redefine observable purely in terms of particle energies and angles ($i.e.$ replace $\vec{p}$ with $\frac{E}{|\vec{p}|}\vec{p}$).

When this was proposed there was no clear indication of which was better.

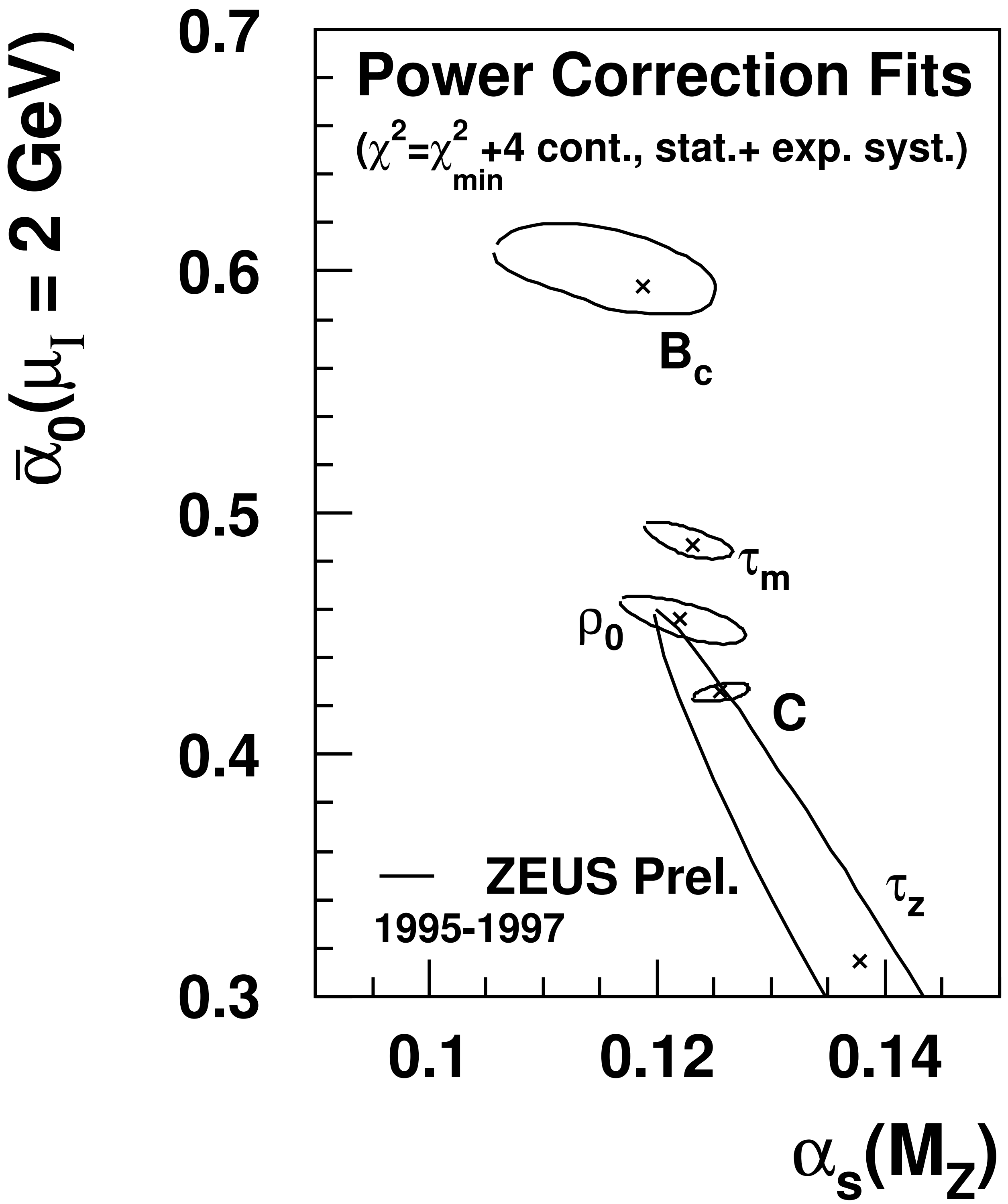
Since then DELPHI, ZEUS and H1 have measured the jet mass with these new definitions and first results came out about a year ago.

Results in default and p-schemes in e^+e^-

In e^+e^- p and E-scheme data are available only from DELPHI. For other experiments we 'correct' the data to the new schemes using Ariadne:



DIS results, ρ in p-scheme



In earlier H1 plots ρ (measured in default scheme) was quite inconsistent with other variables. In the ZEUS (and H1) p-scheme measurements the fit result changes enormously and now seems to be consistent with other variables. *Mass effects make a big difference.*

Can we understand mass effects?

Since mass effects are large we'd like to have a theoretical handle on them. Problem investigated in collaboration with with Daniel Wicke (DELPHI).

Particle masses modify the soft limit of an observable. In the case of the jet mass we have (soft particle of mass m):

$$\rho = \frac{k_t}{Q} e^{-\eta}\, \Theta(\eta) \Rightarrow \frac{\sqrt{k_t^2 + m^2}}{Q} e^{-\eta}\, \Theta(\eta)$$

Let's consider a case where $k_t^2 > m^2$. We can expand the square root to get

$$\frac{k_t}{Q} + \frac{m^2}{2k_t Q} + \cdots$$

For a slowish particle $k_t \sim m \sim \Lambda_{QCD}$ the extra piece is of order Λ/Q.

This shows that *mass effects are formally of the same order as 'traditional' non-perturbative corrections*. But they differ in that they cannot be given a perturbative interpretation ($i.e.$ they are associated with $\pi, K, p, \ldots$ masses, not just an extension of the coupling into the infrared).

Generalise previous treatment by including mass effects

$$\langle \delta \mathcal{V} \rangle = \underbrace{\sum_{\gamma,\pi,\ldots} \int dk_t \frac{dn}{dk_t d\eta}}_{\text{was } \frac{2C_F}{\pi}\int \frac{dk_t}{k_t}\alpha_s(k_t)} \left(c_{\mathcal{V}} \frac{k_t}{Q} + \gamma_{\mathcal{V}} \frac{m^2}{k_t Q} + \cdots \right)$$

$\gamma_{\mathcal{V}}$ is a new observable-dependent coefficient which comes out after integrating over angles.

$\mathcal{V}$	τ	C	ρ	$\rho_{p\text{-scheme}}$
$\gamma_{\mathcal{V}}$	$-\pi/2+1$	$-3\pi/4$	$1/2$	$-\pi/4+1/2$
$\gamma_{\mathcal{V}}/c_{\mathcal{V}}$	-0.285	-0.25	0.5	-0.285

Mass corrections are not proportional to $c_{\mathcal{V}}$, so they *break universality*. Pattern is consistent with data:

✗ Jet mass has a *positive* mass correction.

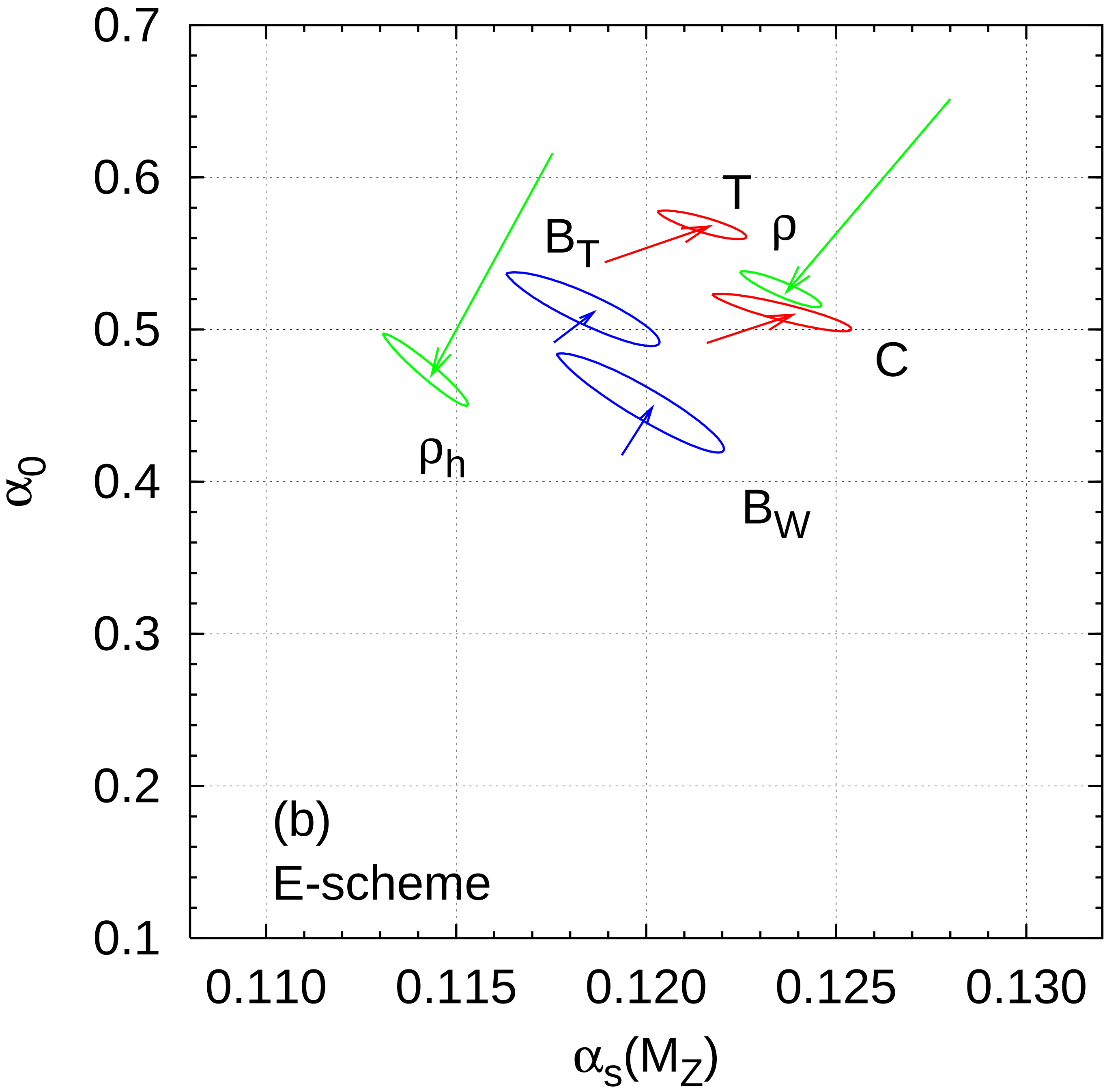
✗ Other variables have a *negative* mass correction, (even though the observable definitions appear not to be directly sensitive to particle masses.)

✗ Redefined jet-mass in p-scheme also has a negative mass correction: we expect *approximate universality* across variables defined *in the p-scheme*.

What about the E-scheme?

This scheme is *privileged*, because $\gamma_{\mathcal{V}} \equiv 0$ for all variables.

Therefore in the E-scheme, *universality-breaking mass effects disappear* altogether.



[Used ARIADNE to correct data to E-scheme, except for DELPHI data.]

End of the story?

Actually only the start... (But it's nearly the end of the seminar, so time only for a brief overview!)

Two major issues arise from a detailed study of mass effects

✗ Energy-dependence of mass effects.

✗ Issue of different hadronic levels.

Energy-dependence

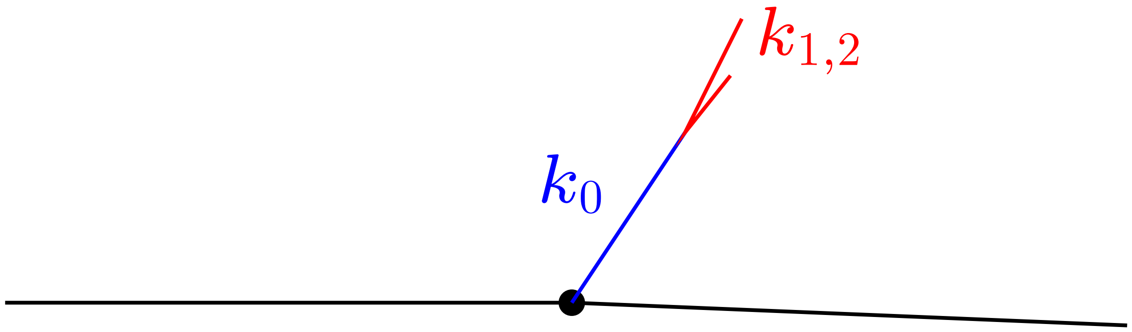
'Traditional' hadronisation corrections come from a piece of event shape going as

$$c_{\mathcal{V}} \sum_i \frac{k_{ti}}{Q}$$

Mass corrections come from a piece going as

$$\gamma_{\mathcal{V}} \sum_i \frac{m_i^2}{k_{ti} Q}$$

Both scale as $1/Q$. But *different sensitivity to soft/collinear branching.*



Traditional piece is unchanged by soft/collinear branching:

$$\frac{k_{t0}}{Q} \simeq \frac{k_{t1}}{Q} + \frac{k_{t2}}{Q}$$

Mass-dependent piece changed completely by such branching:

$$\frac{m_0^2}{k_{t0}Q} \neq \frac{m_1^2}{k_{t1}Q} + \frac{m_2^2}{k_{t2}Q}$$

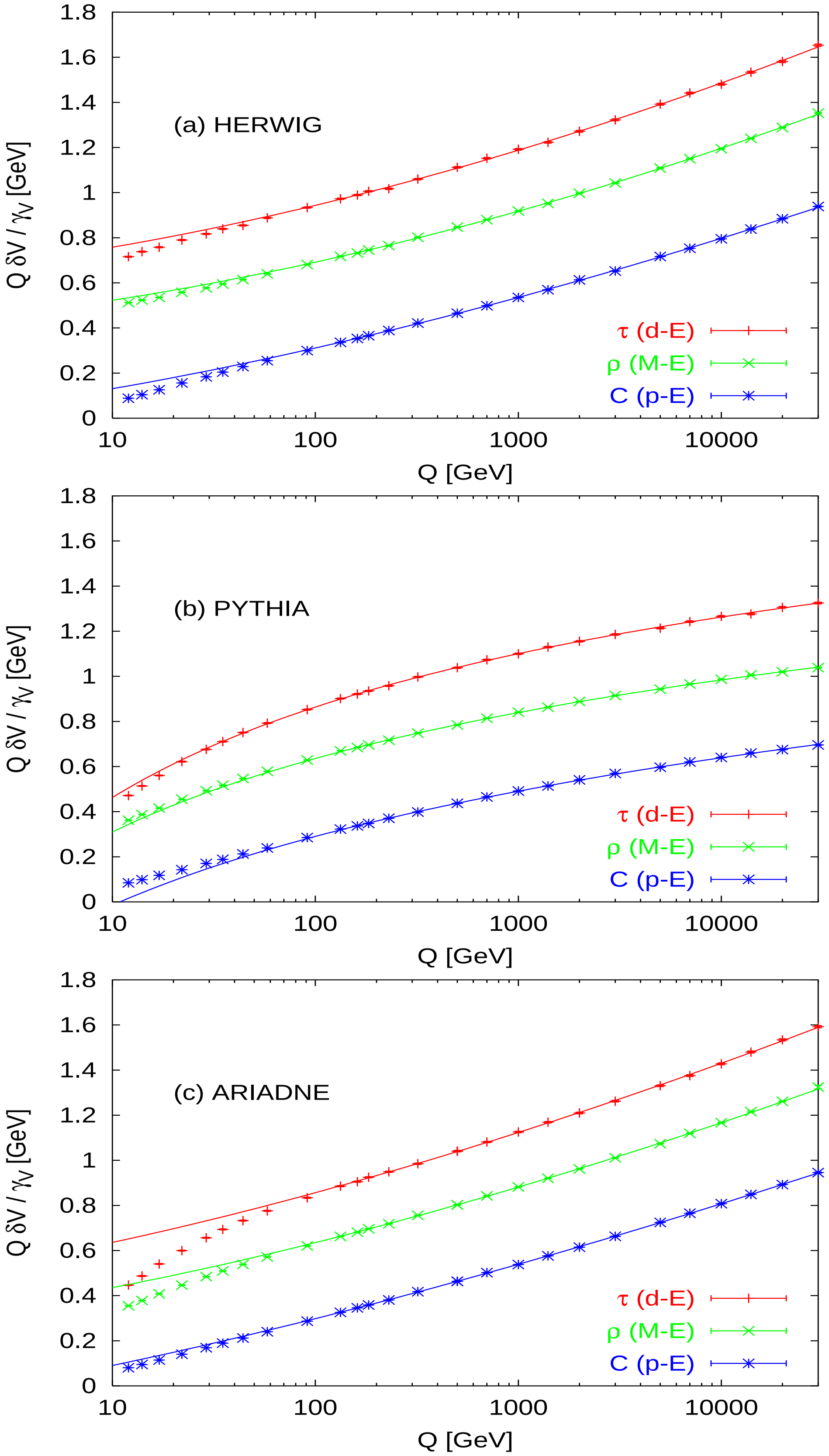
A proper treatment of mass-dependence requires that one take into account collinear and soft branching of gluons $\Rightarrow$ *as for multiplicity of hadrons.*

Quantities like multiplicities are *infrared and collinear unsafe.* But, accounting for coherence of gluon radiation, their Q-dependence (*anomalous dimension*) is perturbatively calculable:

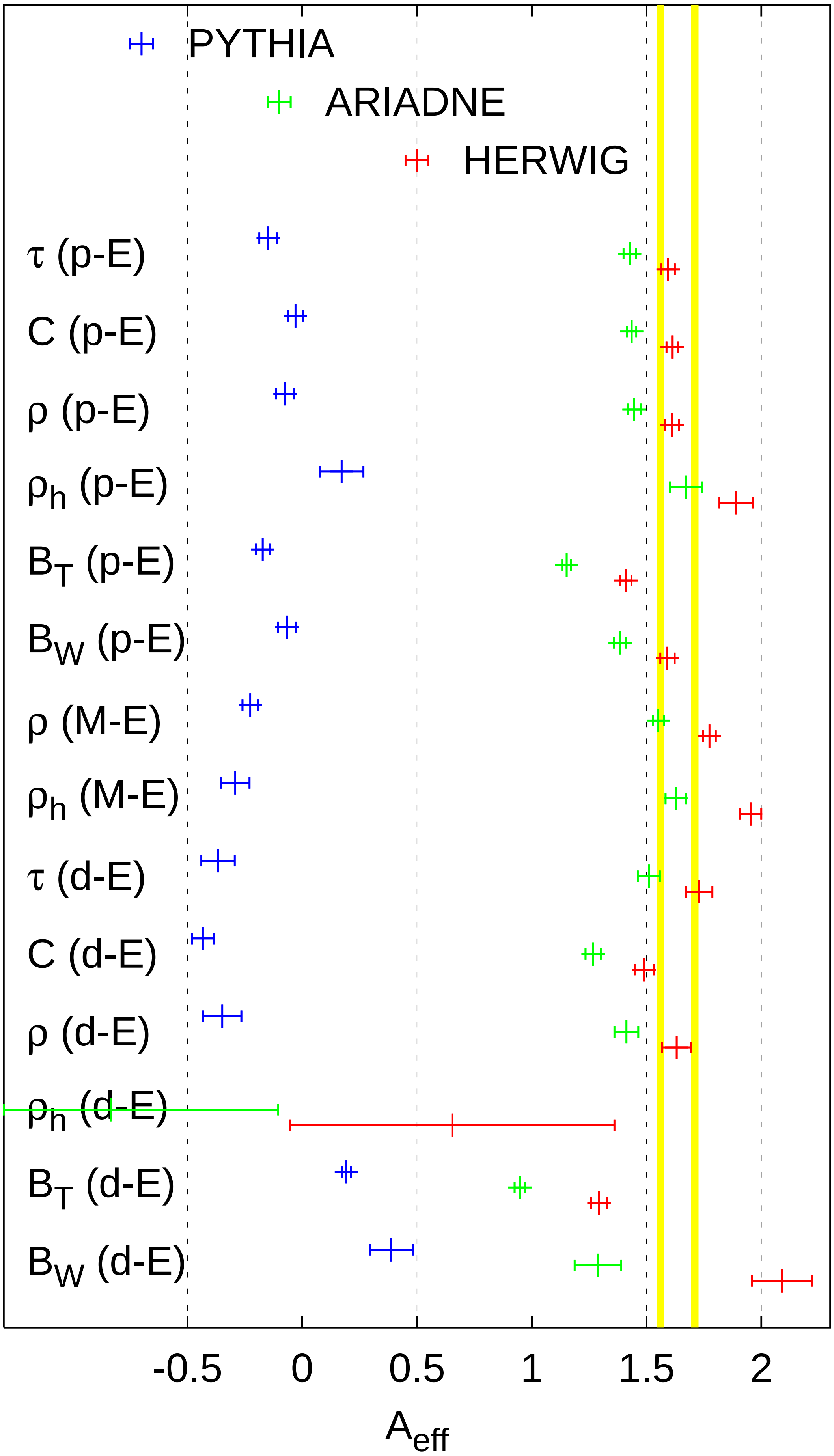
$$\sum_i \frac{m_i^2}{k_{ti}Q} \sim \frac{\ln^A Q/\Lambda}{Q}, \qquad A = \frac{C_A}{\pi b_0} \simeq 1.6$$

Conclusion: we expect mass-dependent effects to go as

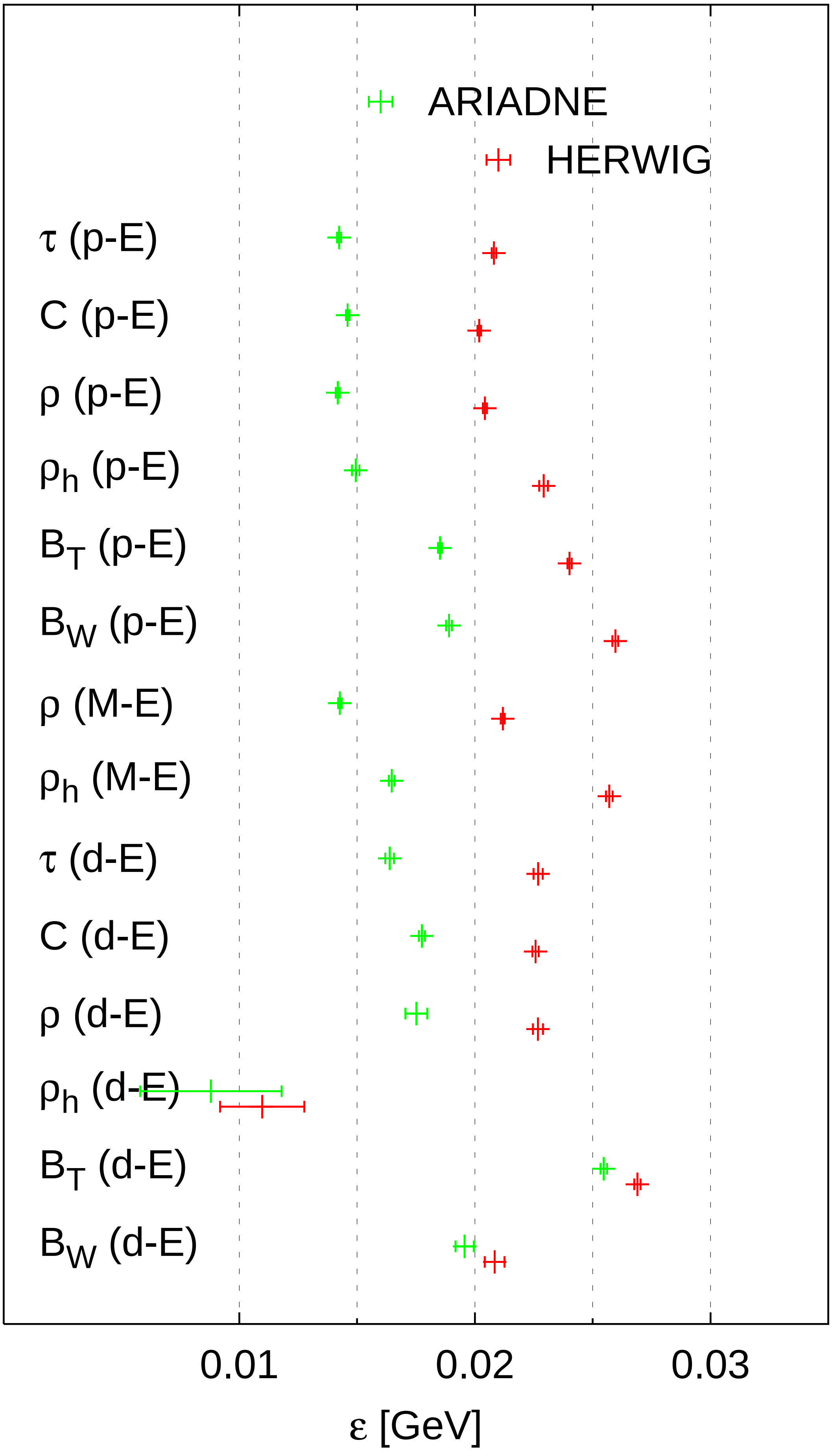
$$\boxed{\langle \delta_m \mathcal{V} \rangle = \gamma_{\mathcal{V}} \frac{\ln^A Q/\Lambda}{Q} \cdot \epsilon} + O\left(\frac{\ln^{A/2} Q/\Lambda}{Q}\right)$$



Test power of $\ln Q$



Test proportionality to γ_ν



Hadronic level is non-unique

Who cares about these non-universal mass effects? After all we can measure in the E-scheme and they should be zero...

Experiments measure hadrons a few metres after their creation. But this is an *arbitrary choice* dictated by 'technological limitations of detectors'!

We could in principle define hadron level to be

- ✗ Resonance level: stage where first hadronic resonances are formed.
- ✗ Decay level: after all hadrons have decayed (long lifetime of proton is a minor practical inconvenience).

Hadronic decay liberates an extra amount of transverse momentum $\sim m^2/k_t$. *Very similar to usual mass effects.*

So for event shapes, difference between various any two hadronic levels (ℓ) will scale as

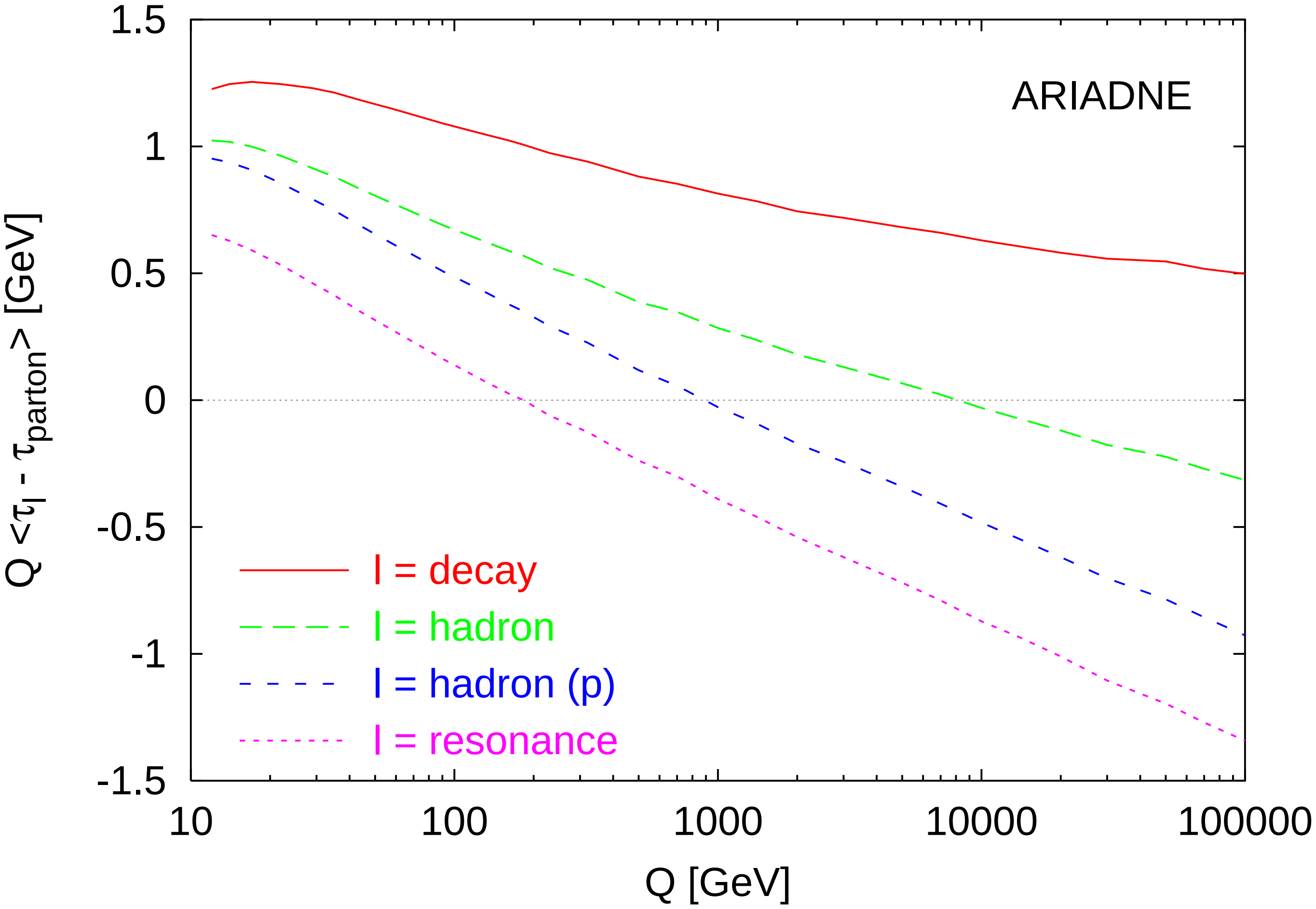
$$\langle \delta_\ell \mathcal{V} \rangle \sim \frac{\ln^A Q/\Lambda}{Q}$$

It can be shown that these 'decay' corrections are *always proportional to coefficient of universal power corrections,* $c_{\mathcal{V}}$.

Corollary

Even in the E-scheme, *hadronisation corrections cannot be pure $1/Q$;* for an arbitrary hadronic level they can always be contaminated (even dominated) by $(\ln^A Q)/Q$ contributions.

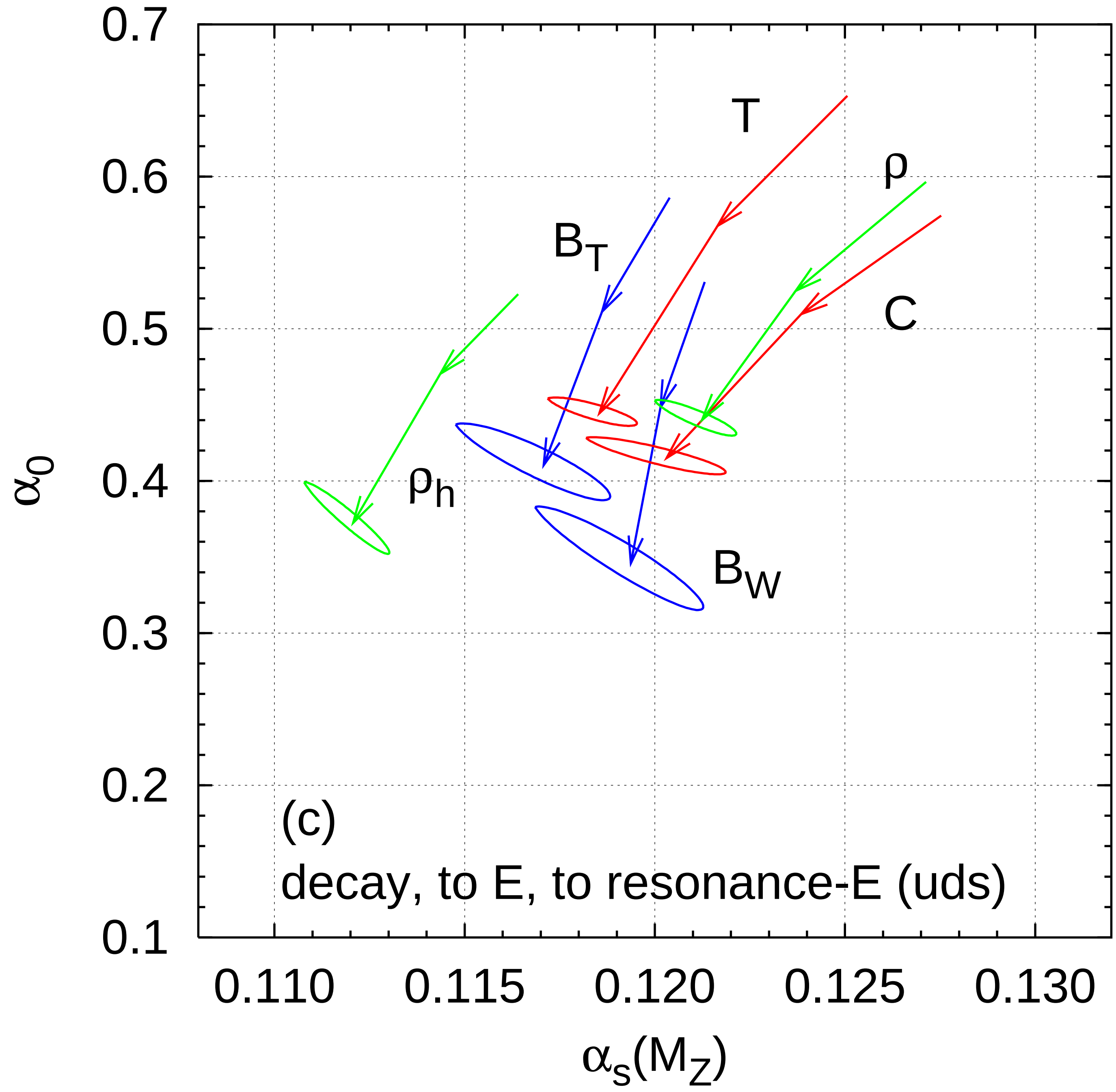
Hadronisation corrections in ARIADNE:



Fits for different hadronic levels

At $Q = M_Z$ observables can differ by over 15% between resonance level ('first set of hadrons after Lund fragmentation') and decay level ('all hadrons decay to pairs of massless particles').

This has a significant effect on fits.



Conclusions

In general:

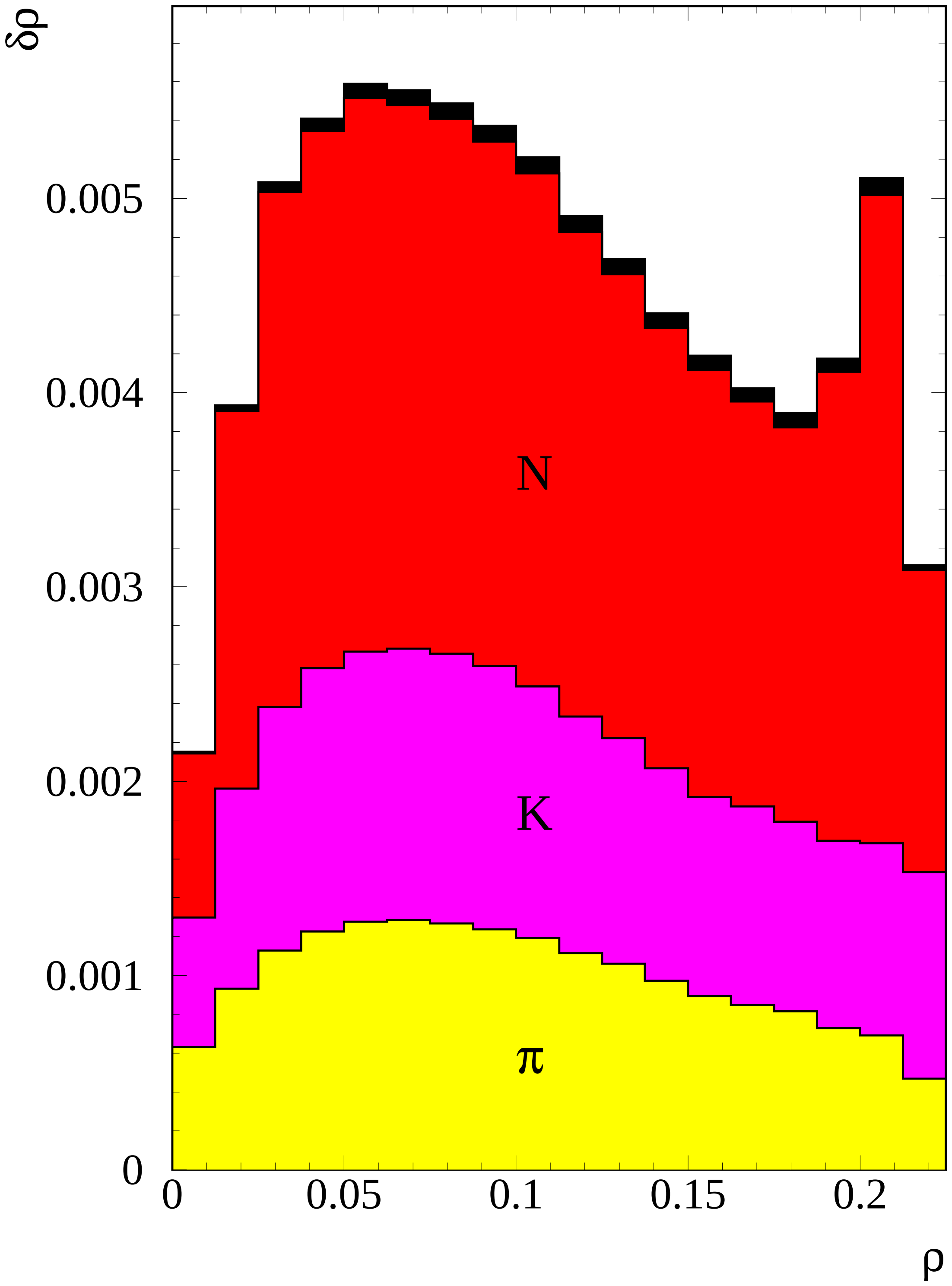
- ✗ There's a long way to go before we fully understand hadronisation corrections.
- ✗ Active discussion between theory and experiment is a vital part of the game.

More specifically:

- ✗ Mass effects scale as $(\ln^A Q)/Q$ with $A \simeq 1.6$.
- ✗ To test universality we need suitable definitions of observables (free of non-universal mass effects) — ideally E-scheme.
- ✗ Even in the E-scheme hadronisation is contaminated by $(\ln^A Q)/Q$ contributions, associated with the decay of hadrons, (and possibly also the creation of massive hadrons from the massless partons).

From a practical point of view:

- ✗ Experimentally universality seems to be doing well.
- ✗ Extraction (and interpretation) of α_s and α_0 is subject to systematics associated with choice of hadronic level.



δρ
0.005
0.004
0.003
0.002
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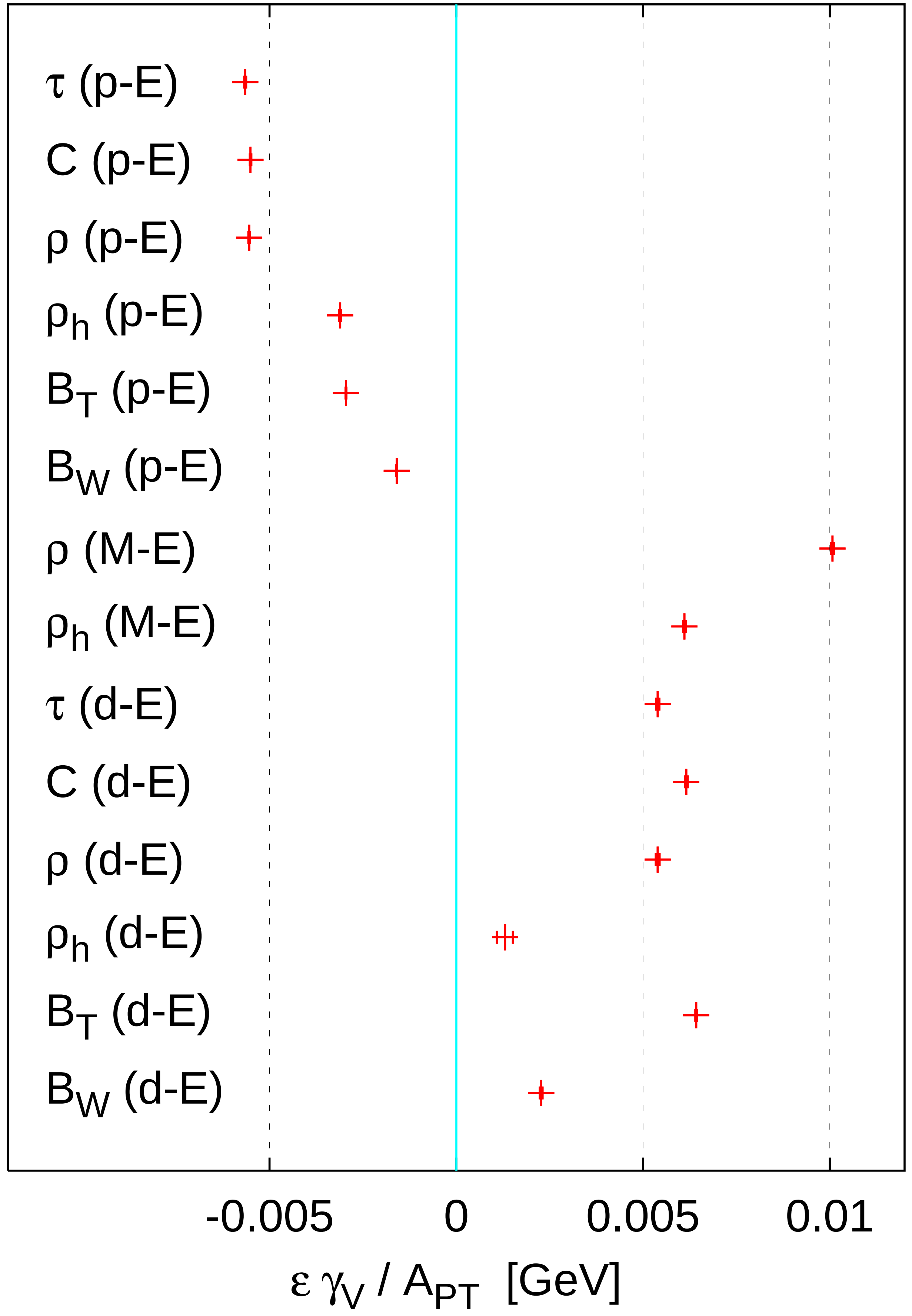
ρ
N
K
π



τ (p-E)
C (p-E)
ρ (p-E)
ρh (p-E)
BT (p-E)
BW (p-E)
ρ (M-E)
ρh (M-E)
τ (d-E)
C (d-E)
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ρh (d-E)
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BW (d-E)
-0.005
0
0.005
0.01
ε γV / APT [GeV]