CAESAR: Computer automated resummations

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Provide a wealth of information, e.g.:

- Measurements of the coupling α_s and its renormalization group running
- Measurements/cross checks of the values of the colour factors of QCD
- Studies of connection between parton-level (QCD calculations of quarks and gluons) and (the real) hadron-level



At the base of all these studies lie perturbative predictions for the distribution of the event shape.

Leading order (LO) $\equiv \mathcal{O}(\alpha_{s})$:

By hand

$$\frac{1}{\sigma}\frac{d\sigma}{d(1-T)} = \frac{\alpha_{\mathsf{s}}C_F}{2\pi} \left[\frac{2(3T^2 - 3T + 2)}{T(1-T)}\ln\frac{2T - 1}{1-T} - \frac{3(3T - 2)(2-T)}{1-T}\right]$$

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Imbalance leads to large logarithms in distribution of event shape:

$$\operatorname{Prob}(1 - T < \tau) \simeq 1 - \frac{\alpha_{\mathsf{s}} C_F}{2\pi} \ln^2 \tau + \dots \qquad (\tau \ll 1)$$

Large Logarithms at all orders

There is a soft and a collinear divergence (\mapsto logs) for each emitted gluon.

At all orders, probability of event being two-jet like has *poorly convergent perturbation series:*

$$P(1 - T < \tau) \equiv \Sigma(\tau) = 1 + \sum_{n=1}^{\infty} R_{n,2n} \alpha_{s}^{n} \ln^{2n} \tau + \dots$$

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Today's state of the art involves *exponentiation* and resummation of *Leading Logs* (LL) and *Next-to-Leading Logs* (NLL):

$$\Sigma(\tau) \simeq \exp\left[\sum_{n=1}^{\infty} \left(\underbrace{G_{n,n+1} \alpha_{s}^{n} \ln^{n+1} \tau}_{\mathsf{LL}} + \underbrace{G_{n,n} \alpha_{s}^{n} \ln^{n} \tau}_{\mathsf{NLL}} + \cdots\right)\right]$$

NB: $\alpha_s^n \ln^{2n} \tau$ in Σ , but only $\alpha_s^n \ln^{n+1} \tau$ in exponent.

The analytical resummation industry...

$e^+e^- ightarrow$ 2 jets

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DIS 1+1 jet

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e^+e^- , DY, DIS 3 jets

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Average: 1 observable per article

Automating it?

- Symbolic manipulation programs (Mathematica, etc.)? E.g. like Feyncalc.
 - Observables have complex definitions (jet algorithms, maximisations)
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 - Only know if program is suitable for observable if you've already done most of the resummation...
 - Matching with fixed order (LO, NLO, NNLO) is complex

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Computer Automated Expert Semi-Analytical Resummation (CAESAR)

Observable must have standard functional form for soft & collinear gluon emission

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- Determine coefficients a_{ℓ} , b_{ℓ} , d_{ℓ} and $g_{\ell}(\phi)$ for emissions close to each hard Born parton (leg) ℓ .
- Require *continuous globalness*, *i.e.* uniform dependence on k_t independently of emission direction ($a_1 = a_2 = \cdots = a$)

Multiple emission properties

Require recursive infrared & collinear safety (schematic)

$$\left[\lim_{\epsilon \to 0}, \lim_{\epsilon' \to 0}\right] \frac{1}{\epsilon} V(\{p\}, \epsilon k_1, \epsilon' \epsilon k_2, \ldots) = 0$$

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- For NLL accuracy, determine function \mathcal{F} (by MC)

$$\mathcal{F}(C_1 r'_1, \dots, C_n r'_n) = \left\langle \exp\left\{-R' \ln \frac{V(k_1, \dots, k_m)}{\max\{V(k_1), \dots, V(k_m)\}}\right\} \right\rangle$$

 ${\mathcal F}$ contains all *relevant* info about observable's dependence on multiple emissions.

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$$\mathcal{F} = \lim_{\epsilon' \to 0} \frac{\epsilon'^{R'}}{R'} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m+1} \sum_{\ell_i=1}^{n} C_{\ell} r'_{\ell_i} \int_{\epsilon'}^{1} \frac{d\zeta_i}{\zeta_i} \int_{0}^{2\pi} \frac{d\phi_i}{2\pi} \right) \delta(\ln \zeta_1) \times \\ \times \exp\left(-R' \ln \lim_{\epsilon \to 0} \frac{V(\{\tilde{p}\}, \kappa_1(\zeta_1 \epsilon), \dots, \kappa_{m+1}(\zeta_{m+1} \epsilon))}{\epsilon} \right).$$

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$$\ln \Sigma(v) = -\sum_{\ell=1}^{n} C_{\ell} \left[r_{\ell}(v) + r'_{\ell}(v) \left(\ln \bar{d}_{\ell} - b_{\ell} \ln \frac{2E_{\ell}}{Q} \right) \right. \\ \left. + B_{\ell} T \left(\frac{\ln 1/v}{a + b_{\ell}} \right) \right] + \sum_{\ell=1}^{n_{i}} \ln \frac{f_{\ell}(x_{\ell}, v^{\frac{2}{a + b_{\ell}}} \mu_{f}^{2})}{f_{\ell}(x_{\ell}, \mu_{f}^{2})} \\ \left. + \ln S \left(T \left(\frac{\ln 1/v}{a} \right) \right) + \ln \mathcal{F}(C_{1}r'_{1}, \dots, C_{n}r'_{n}) , \right]$$

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$$\begin{split} C_{\ell} &= \text{colour factor } (C_F \text{ or } C_A), \qquad f_{\ell}(x_{\ell}, \mu_f^2) = \text{parton distributions} \\ r_{\ell}(L) &= \int_{v^{\frac{2}{a+b_{\ell}}}Q^2}^{v^{\frac{2}{a+b_{\ell}}}Q^2} \frac{dk_t^2}{k_t^2} \frac{\alpha_{\mathsf{s}}(k_t)}{\pi} \ln\left(\frac{k_t}{v^{1/a}Q}\right)^{a/b_{\ell}} + \int_{v^{\frac{2}{a+b_{\ell}}}Q^2}^{Q^2} \frac{dk_t^2}{k_t^2} \frac{\alpha_{\mathsf{s}}(k_t)}{\pi} \ln\frac{Q}{k_t} , \\ S(T(\frac{1}{a}\ln 1/v)) &= \text{large-angle logarithms (process dependence)} \end{split}$$

. . .

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leg ℓ	a_ℓ	b_ℓ	$g_\ell(\phi)$	d_ℓ	$\langle \ln g_\ell(\phi) \rangle$
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summations – p.12/14



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Resummed thrust for Tevatron

• run II regime $\sqrt{s} = 1.96\,{
m TeV}$

- cut on rapidity $|\eta| < 1$
- cut on jet transverse energy $E_T > 50 GeV$



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Longer-term Outlook

- Matching with fixed order (in progress) \rightarrow Phenomenology
- Extending scope (*e.g.* non-global observables?)