High-energy QCD and the search for large perturbative field strengths

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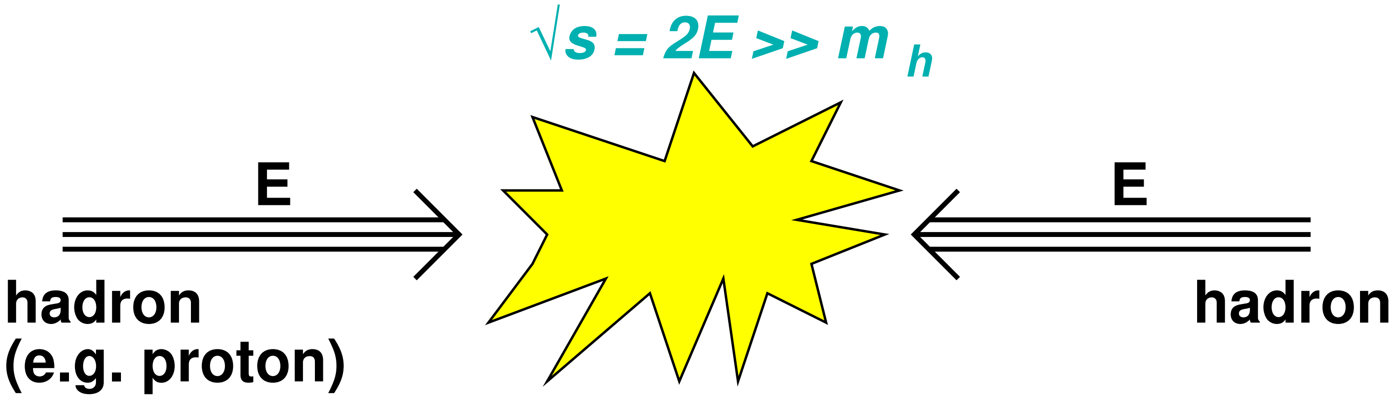
High-energy limit

One of the major unsolved problems of QCD (and Yang-Mills theory in general) is the understanding of its *high-energy limit.*

I.e. the limit in which C.O.M. energy ($\sqrt{s}$) is much larger than *all other scales* in the problem.

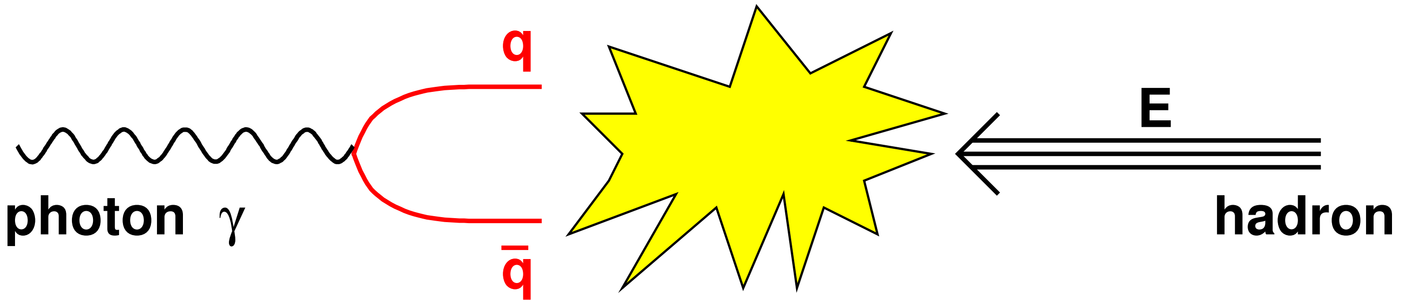


Want to understand:

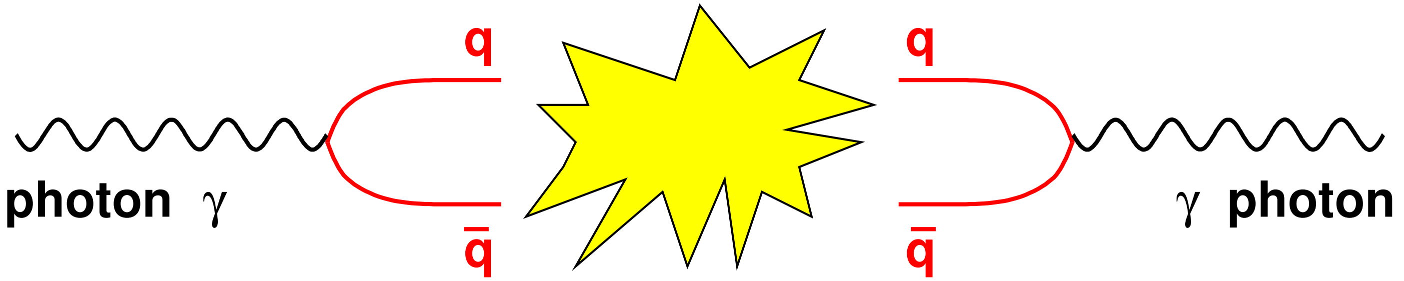
- ✗ asymptotic behaviour of cross section, $\sigma_{hh}(s) \sim ??$
- ✗ properties of final states for large s.

Not just for hadrons

Problem is must more general than just for hadrons. E.g. photon can *fluctuate* into a quark-antiquark (hadronic!) state:



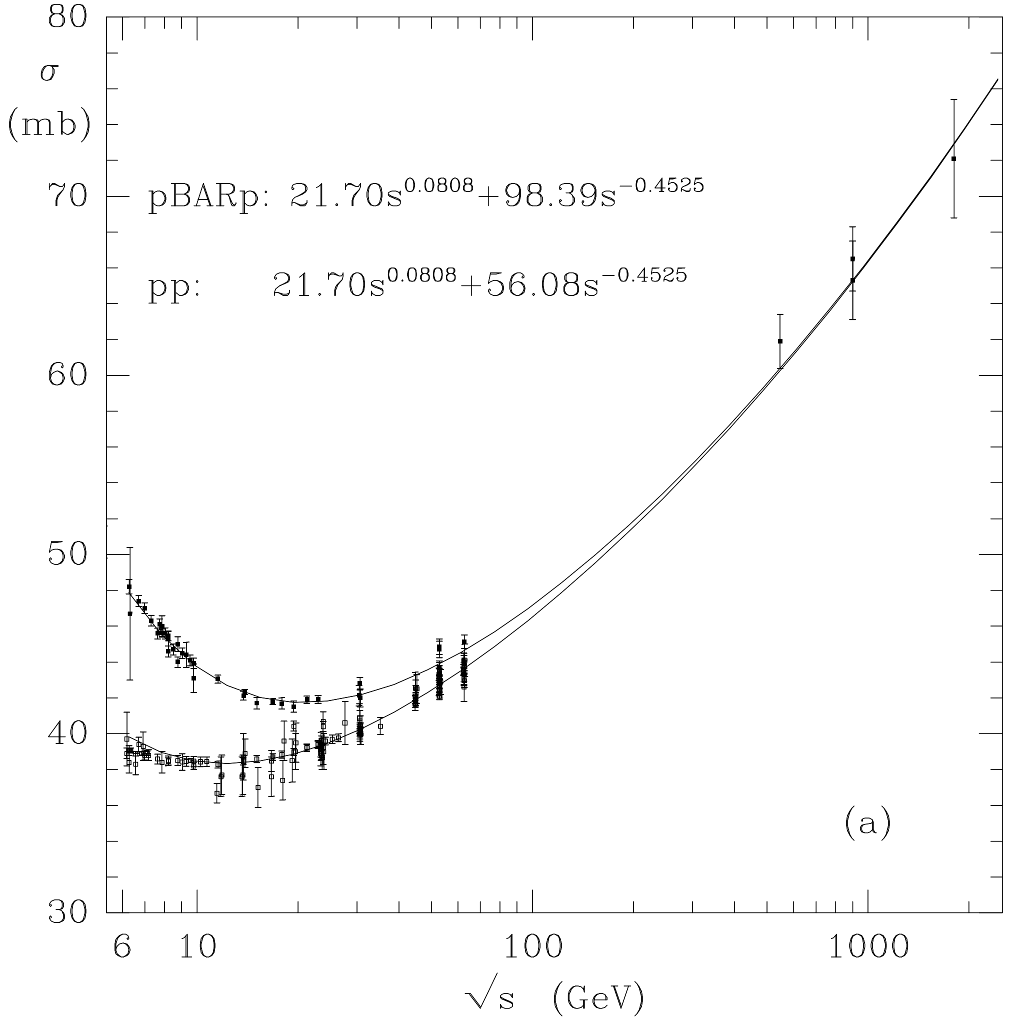
Or even



These *hadronic fluctuations dominate* the high-energy cross section.

(Analogous considerations can be applied to electron (/muon/tau) or neutrino interactions with hadrons.)

Experimental knowledge



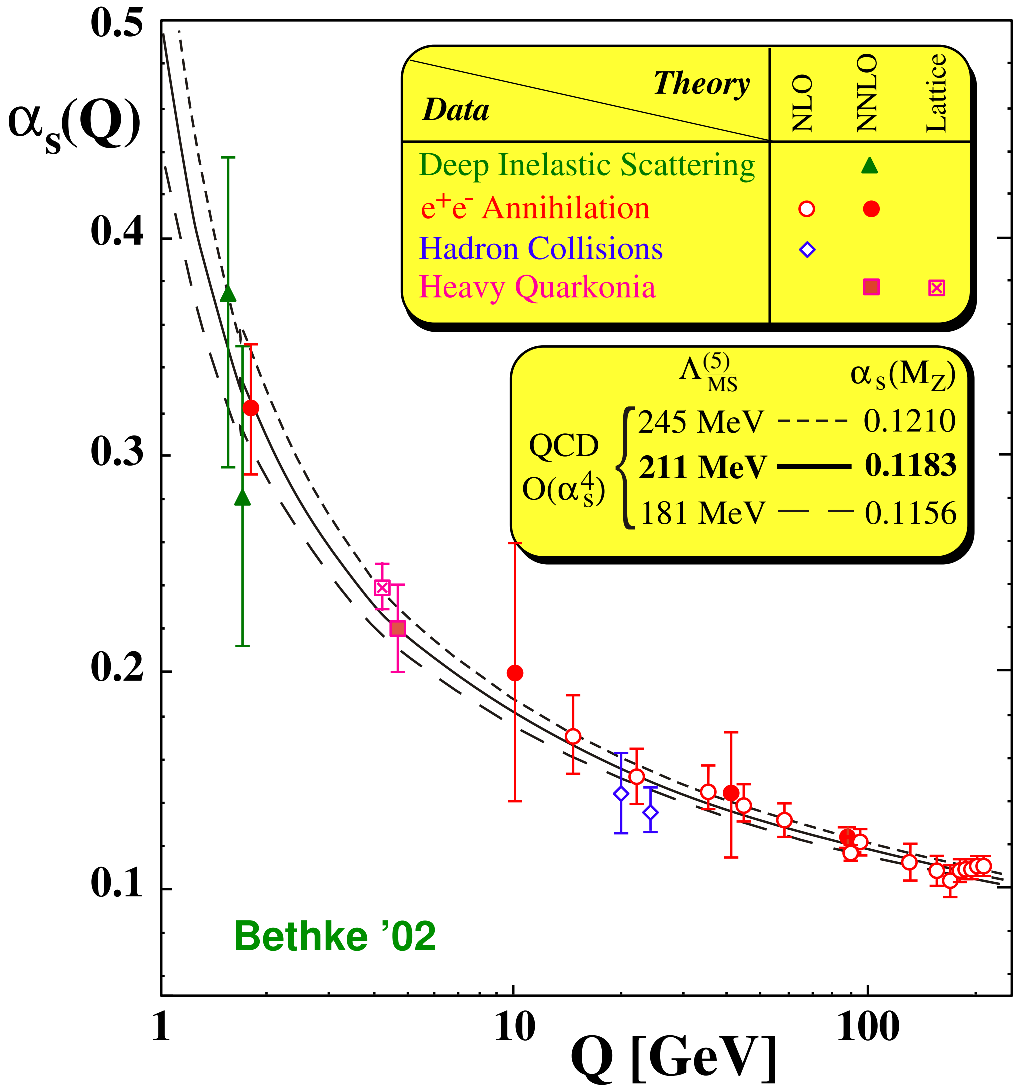
- ✗ Some knowledge exists about behaviour of cross section experimentally
- ✗ Slow rise as energy increases
- ✗ Data insufficient to make reliable statements about functional form
  - ☛ $\sigma \sim s^{0.08}$?
  - ☛ $\sigma \sim \ln^2 s$?
- ✗ Would like theoretical knowledge...

Applications

Many experiments involve very high-energy interactions (beyond currently measured energies).

- ✗ Understanding needed to make sense of experimental data
  - ☛ e.g.detection of neutrino cosmic-rays: want knowledge of X-section to *interpret number of events* (νp interaction in detector) *as a neutrino flux*. [$\sqrt{s} \sim 10^4$ GeV]
- ✗ Nuisance value(!):
  - ☛ High-energy proton-proton collisions (LHC – $\sqrt{s} = 14000$ GeV) will be used to search for new particles, e.g. Higgs boson (rare events). *Huge radiation background* from 'plain' pp collisions:
    - Complicates extraction of any 'new physics' signals
    - damages the detector — 100's of (wo)man years are being invested to make the detector radiation hard!
  - ☛ But the amount radiation can currently only be *estimated based on lower-energy* experiments (uncertainty $\sim$ factor of two).

What can perturbative QCD say?



Recall: QCD is *asymptotically free* — coupling (α_s) decreases at large scales:

$$\frac{d\alpha_s(Q^2)}{d\ln Q^2} \simeq -\beta_0 \alpha_s^2$$

Corollary: at low scales (e.g. proton mass, 1 GeV) perturbation theory breaks down:

- ✗ Cannot use language of quarks and gluons
- ✗ Calculation of proton-proton scattering is *beyond* perturbative QCD.

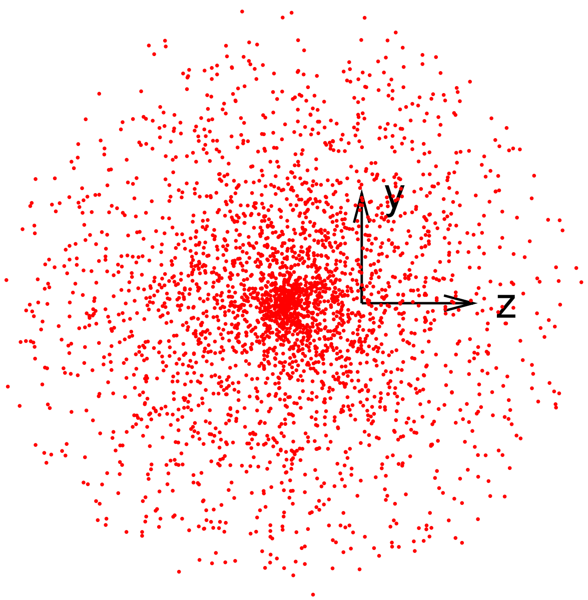
Forget applicability for now — examine field-theory behaviour anyway!

How to approach the problem?

Basic issue: distribution of quanta (photons/gluons) in a fast moving object

Many ways of looking at the problem. Simplest (ignoring rigour) is to start by considering *electric field of a fast-moving charge,* in QED:

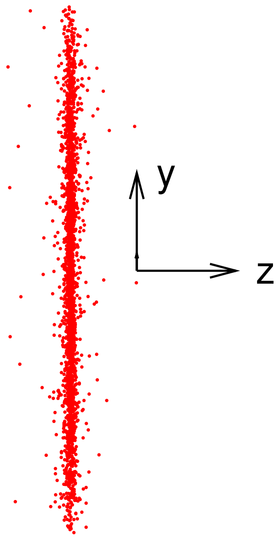
Stationary



$$\vec{E} = \frac{e}{4\pi}\frac{\vec{r}}{r^3}$$

Fast moving

$\Longrightarrow \; v_z$



$$\beta = v_z/c \to 1$$

$$\gamma \;=\; \frac{1}{\sqrt{1-\beta^2}} \;\gg\; 1$$

Large $\gamma \equiv$ large $\sqrt{s}$

$$\vec{E} = \frac{e\gamma}{4\pi}\frac{\vec{r}}{(r_x^2 + r_y^2 + \gamma^2 r_z^2)^{3/2}}$$

Separate longitudinal and transverse

$$\vec{E} = \frac{e\gamma}{4\pi} \frac{\vec{r}}{(r_x^2 + r_y^2 + \gamma^2 r_z^2)^{3/2}}$$

Relativistic contraction $\Rightarrow$ natural *separation between longitudinal and transverse* ($\perp$) directions.

Do *fourier transform in z-direction.*, *i.e.*

- ✗ Describe field in terms of transverse position (frozen) and longitudinal momentum (dominant component)

Result is just field for a 2*-dimensional system*

$$\vec{E}_{\perp,p_z} = \int dr_z e^{ip_z.r_z} \vec{E} \Rightarrow \boxed{\frac{e}{2\pi} \frac{\vec{r}_\perp}{r_\perp^2}}$$

NB:

- ✗ Separate treatment of z and $\perp$ directions only makes sense for $p_z \gg 1/r_\perp$
- ✗ Ignore $e^{ip_z.r_z}$ factor by considering $p_z \ll \gamma/r_\perp$

Dipole; Energy density in field $\Rightarrow$ # of quanta

Now consider a neutral system (QCD systems always neutral) — a dipole with charges at origin and $\vec{R}_\perp$:

$$\vec{E}_{\perp,p_z} = \frac{e}{2\pi}\left(\frac{\vec{r}_\perp}{r_\perp^2} - \frac{\vec{R}_\perp - \vec{r}_\perp}{|\vec{R} - \vec{r}_\perp|^2}\right)$$

Find *energy density*: $\epsilon = \frac{1}{4\pi}(E^2 + B^2)$ and use $|\vec{E}| = |\vec{B}|$ (& fudge $\times 2$):

$$\frac{d\epsilon}{dp_z\, d^2\vec{r}_\perp} = \frac{e^2}{4\pi^3}\,\frac{R_\perp^2}{r_\perp^2|\vec{R}_\perp - \vec{r}_\perp|^2}$$

Interpret as *number (n) of photons* (divide by photon energy p_z):

$$\boxed{\frac{dn}{dp_z d^2\vec{r}_\perp} = \frac{1}{p_z}\frac{e^2}{4\pi^3}\,\frac{R_\perp^2}{r_\perp^2|\vec{R}_\perp - \vec{r}_\perp|^2}}$$

Total number of photons

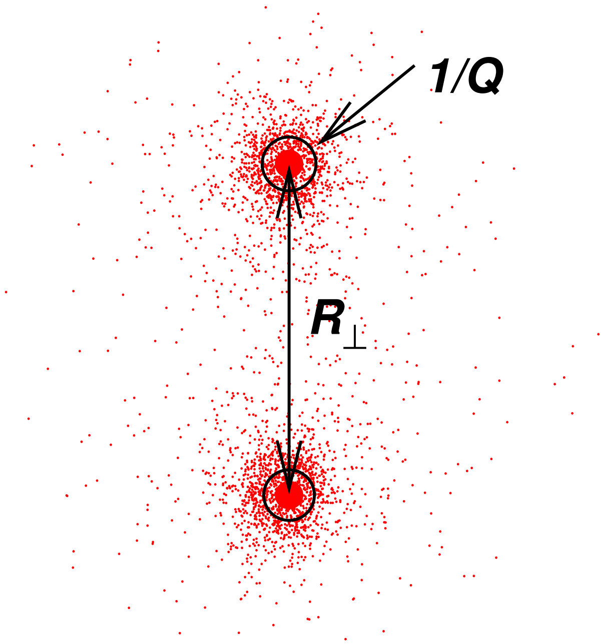
$$\frac{dn}{dp_z d^2\vec{r}_\perp} = \frac{1}{p_z}\frac{\alpha}{\pi^2}\frac{R_\perp^2}{r_\perp^2|\vec{R}_\perp - \vec{r}_\perp|^2}$$

Valid for $\frac{1}{R_\perp} \ll p_z \ll \frac{\gamma}{R_\perp}$

Integrate over p_z to get total number of photons at given $\vec{r}_\perp$:

$$\frac{dn}{d^2\vec{r}_\perp} = \frac{\alpha \ln\gamma}{\pi^2}\frac{R_\perp^2}{r_\perp^2(\vec{R}_\perp - \vec{r}_\perp)^2}$$

Enhancement at high energies (γ)



Can also integrate over transverse coordinates (impact parameter) — up to some large effective transverse momentum scale $Q \gg 1/R_\perp$, *i.e.* $r_\perp \gtrsim 1/Q$, $|\vec{R}_\perp - \vec{r}_\perp| \gtrsim 1/Q$:

$$n(Q^2) \simeq \frac{2\alpha \ln\gamma \ln(Q^2 R_\perp^2)}{\pi}$$

Enhancement at high transverse momenta (Q^2)

Logarithmic enhancements

- ✗ Logarithmic enhancements — $\alpha \ln \gamma$, $\alpha \ln \gamma \ln Q^2$ are *typical of gauge theories.*
- ✗ *At high energies* $\alpha \ln \gamma \gtrsim 1$, *convergence* of expansion in powers of α can be *ruined* by large coefficients, *e.g.* $(\alpha \ln \gamma)^n$.
- ✗ For *QED, problem is not too serious* — α is tiny & analysis so far gives most of picture.
- ✗ For *QCD* α_s *is only moderately small* & analysis so far is just beginning of story.

Fundamental difference of QCD is that it is non-abelian (SU(3))

gluon is charged

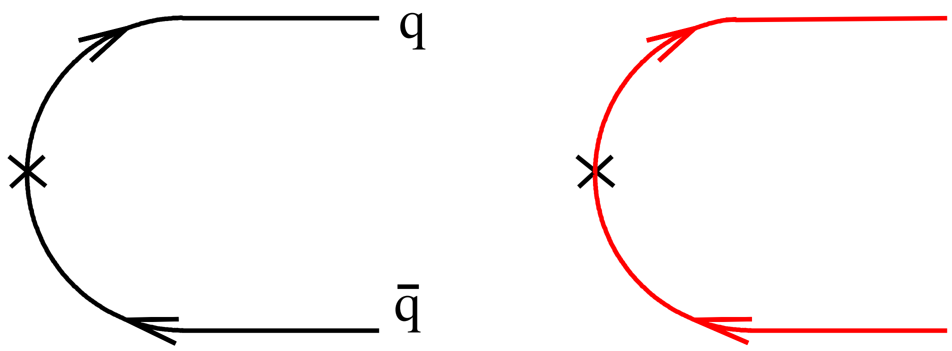
QCD basics (large-N_c approx.)

Feynman diagram **Colour flow**

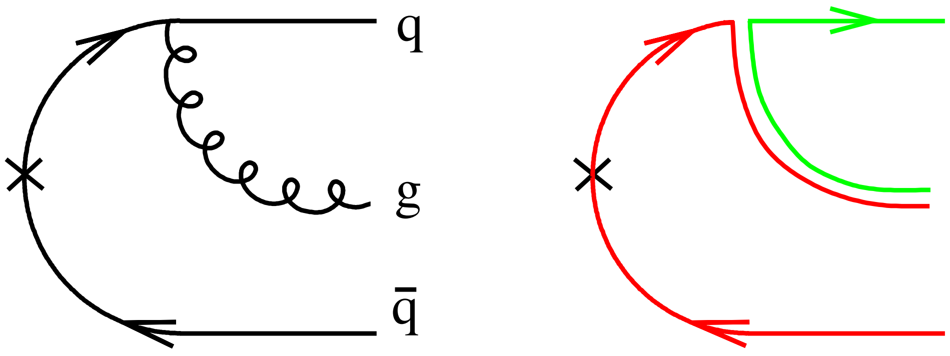
- ✗ Quarks come in 3 ‘colours’ ($N_c = 3$). Gluons emission ‘repaints’ the colour of the quark.
- ✗ *i.e.* gluon carries away one colour and brings in a different one [this simple picture $\equiv$ approx of many colours].
- ✗ gluon itself is charged with both colour and anti-colour [*c.f.* two lines with different directions].

Gluon emission — QCD

Start with bare quark-antiquark dipole:



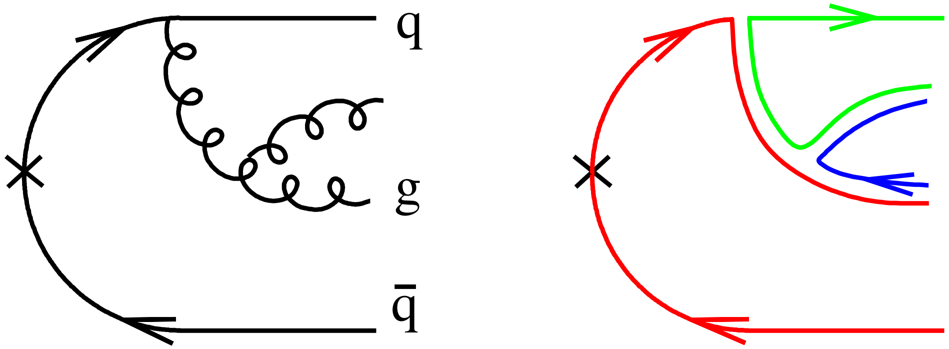
Emit a gluon:



Emission of 1 gluon is like QED case — modulo additional colour factor (number of different ways to repaint quark):

$$\boxed{\alpha \to \alpha_s N_c/2} \text{ (approx)}$$

- ✗ In QED subsequent photons are emitted by *original dipole*
- ✗ In QCD original dipole is converted into two new dipoles, which *emit independently*.



BFKL Equation

Emission from dipole (splitting of dipoles) *iterates:*

$$\frac{dn}{dp_z}(p_z, R_\perp) = \frac{1}{p_z}\frac{\alpha_s N_c}{2\pi}\mathcal{K} \otimes n(p_z, \ldots) \qquad \mathcal{K} \equiv \int \frac{R_\perp^2\, d^2\vec{r}_\perp}{\pi r_\perp^2 |\vec{R}_\perp - \vec{r}_\perp|^2}$$

Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation ['76]

BFKL accounts for all *Leading Logarithmic (LL) terms* (new formal expansion):

$$(\alpha_s \ln p_z)^n$$

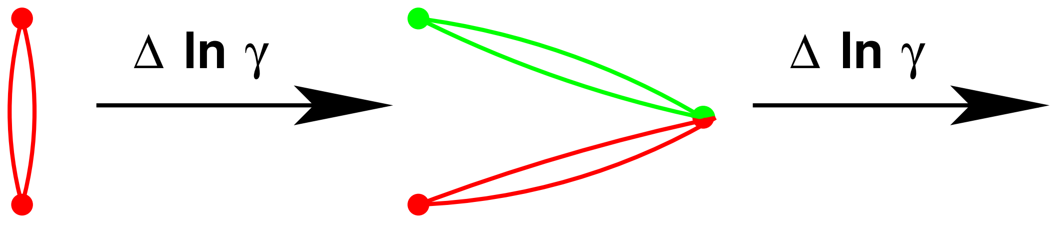
Solution of equation (number of dipoles or number of gluons):

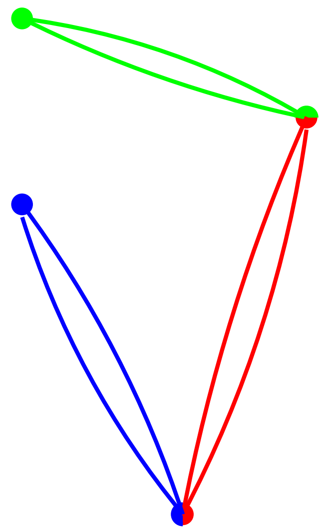
$$n(p_z, R) \sim \exp\left(\frac{\alpha_s N_c}{\pi} 4 \ln 2 \cdot \ln \frac{p_z^{max}}{p_z} \ln\ \gamma \qquad \right) = \gamma^{\frac{\alpha_s N_c}{\pi} 4 \ln 2}$$

✗ $\alpha_s \simeq 0.2$, $N_c/\pi \simeq 1$, $4\ln 2 \simeq 2.8$; *Power* $\simeq 0.5$.

Conformal invariance

Kernel $\dfrac{R_\perp^2\, d^2\vec{r}_\perp}{r_\perp^2 |\vec{R}_\perp - \vec{r}_\perp|^2}$ is *conformally invariant* — various consequences. . .





e.g. : Growth in area

BFKL growth is not just just increase in number of gluons/dipoles.

Gluons can be produced *far* from original dipole — because of conformal (scale) invariance each step in $\ln\gamma$ translates to a constant factor of increase in area.

Area covered by some given (minimum) density scales as

$$\text{Area} \sim \gamma^{\frac{\alpha_s N_c}{\pi}\cdot 2.441695\ldots}$$

BFKL: rising cross sections

$$n_{\mathsf{dipoles}}(\gamma, R) \sim \gamma^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} \qquad \Leftrightarrow \qquad \sigma_{\mathsf{hh}} \sim s^{\frac{\alpha_s N_c}{\pi} 4 \ln 2}, \qquad \frac{\alpha_s N_c}{\pi} 4 \ln 2 \simeq 0.5$$

- ✗ Completely incompatible with rise of $p\bar{p}$ cross section ($\sim s^{0.08}$)
  - ☞ $p\bar{p}$ is simply beyond perturbation theory
- ✗ experimentally spectactular — if observable in some process. . .
- ✗ Raises many theoretical issues — high gluon densities should lead non-linear effects (high fields, but still perturbative)

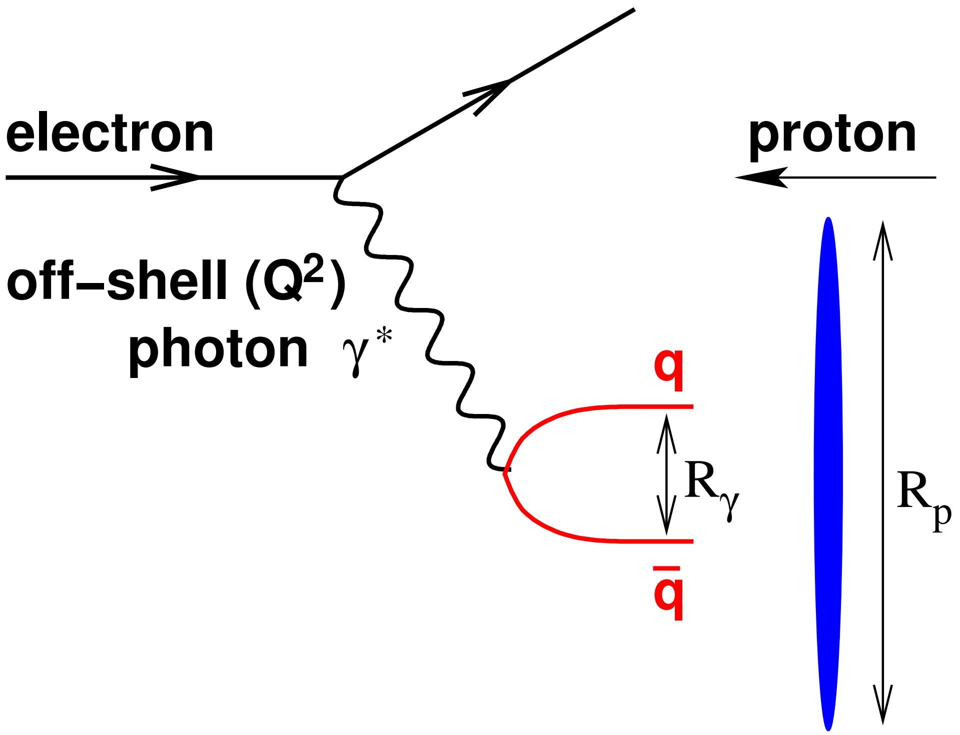
How can we search for it experimentally?

- ✗ Need to ensure we are in regime where perturbation theory can be applied
  - ☞ Choose appropriate hadronic scales (small R)

Getting small transverse sizes (needed for $\alpha_s \ll 1$) *and* asymptotically large collission energies is experimentally difficult.

In general collide two hadronic probes — try a compromise: *make one of them small*

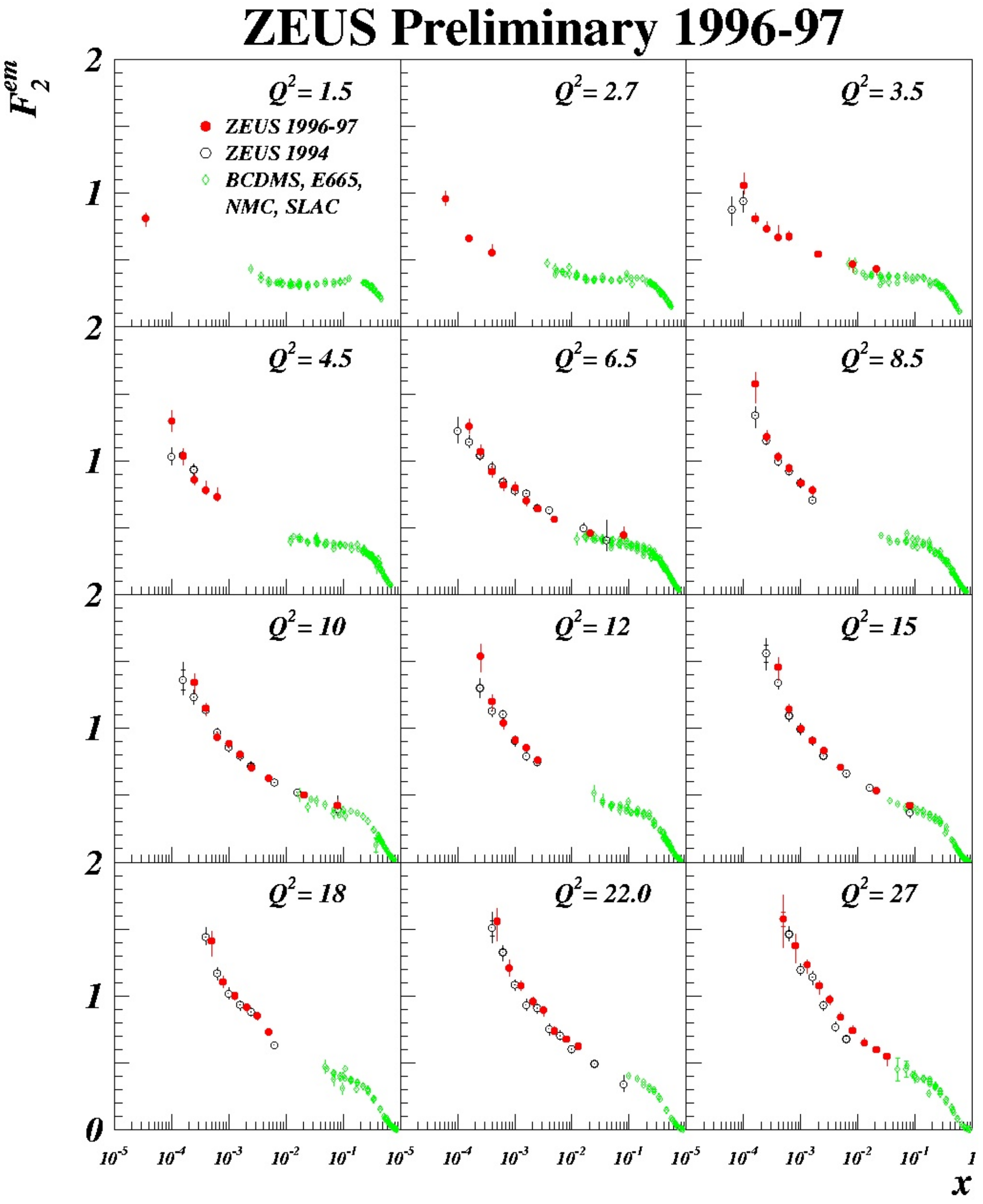
$$R_\gamma \sim \frac{1}{Q} \ll R_p \sim \frac{1}{m_p}$$



- ✗ $q\bar{q}$ probe measures (roughly) number of gluons in proton up to scale Q
- ✗ Process is called *Deep Inelastic Scattering (DIS)* [more traditionally viewed as photon hitting quarks in proton]
- ✗ Some of physics perturbative ($m_p \ll p_t \lesssim Q$)
- ✗ But if $\ln Q^2 \gtrsim \ln s$ we have competition between

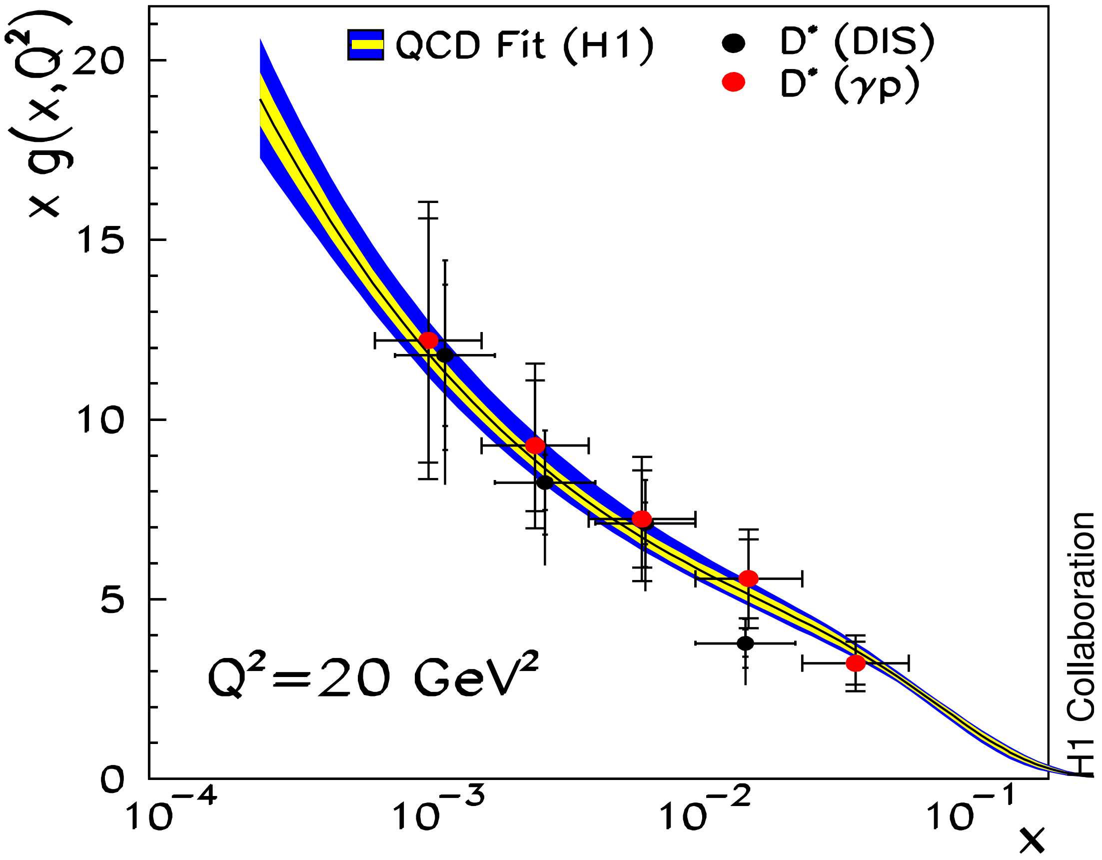
$$(\alpha_s \ln s)^n \quad \text{v.} \quad (\alpha_s \ln s \ln Q^2)^n$$

HERA ep collider — rich source of data



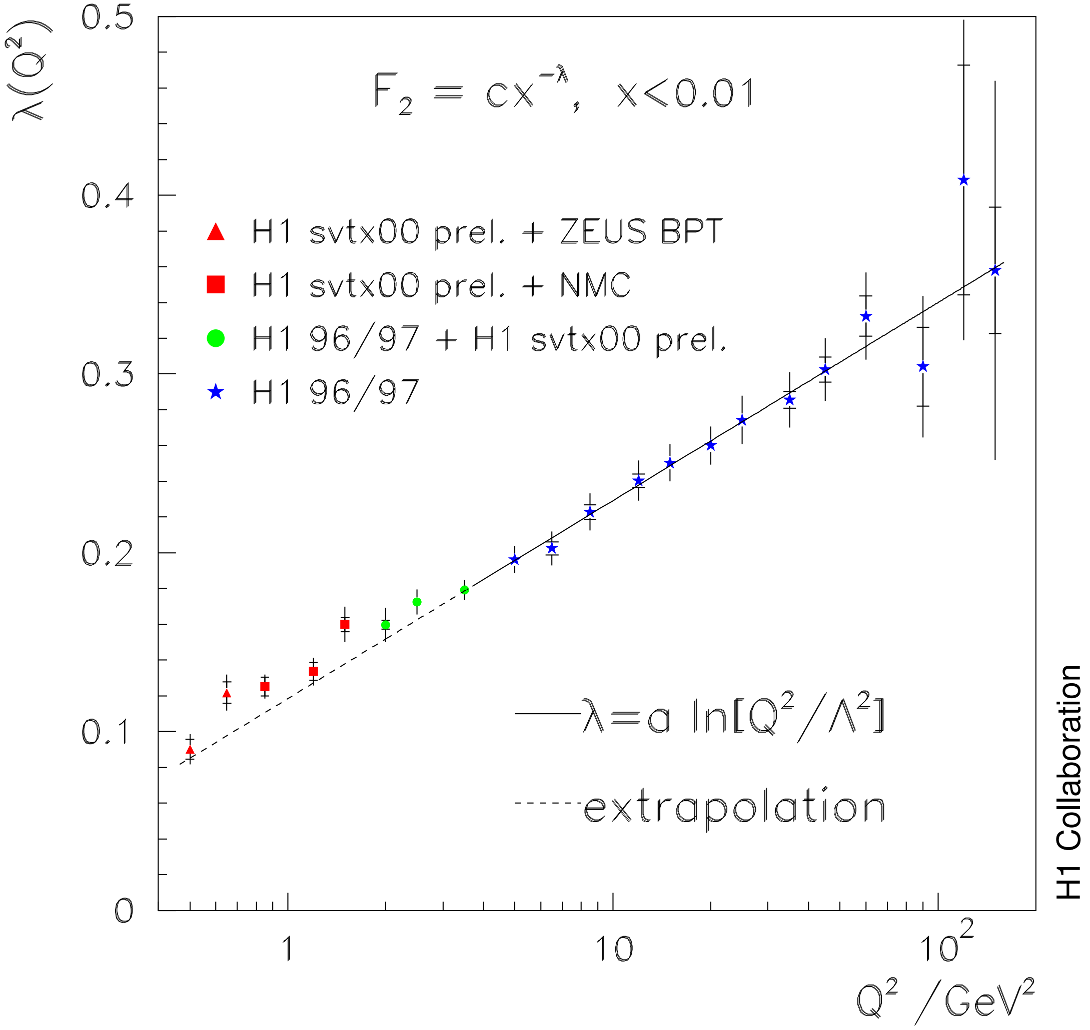
- ✗ F_2 is rescaled cross section
- ✗ $x = \dfrac{p_z}{p_{z,\text{proton}}} \sim \dfrac{1}{s}$
- ✗ Clear rise of cross section at high energies (low x).

Large number of gluons



- ✗ Convert cross sections into estimate of number of gluons
- ✗ Various independent extractions
- ✗ Up to 20 gluons per unit $\ln x$ (or unit $\ln p_z$)!

Is it BFKL?



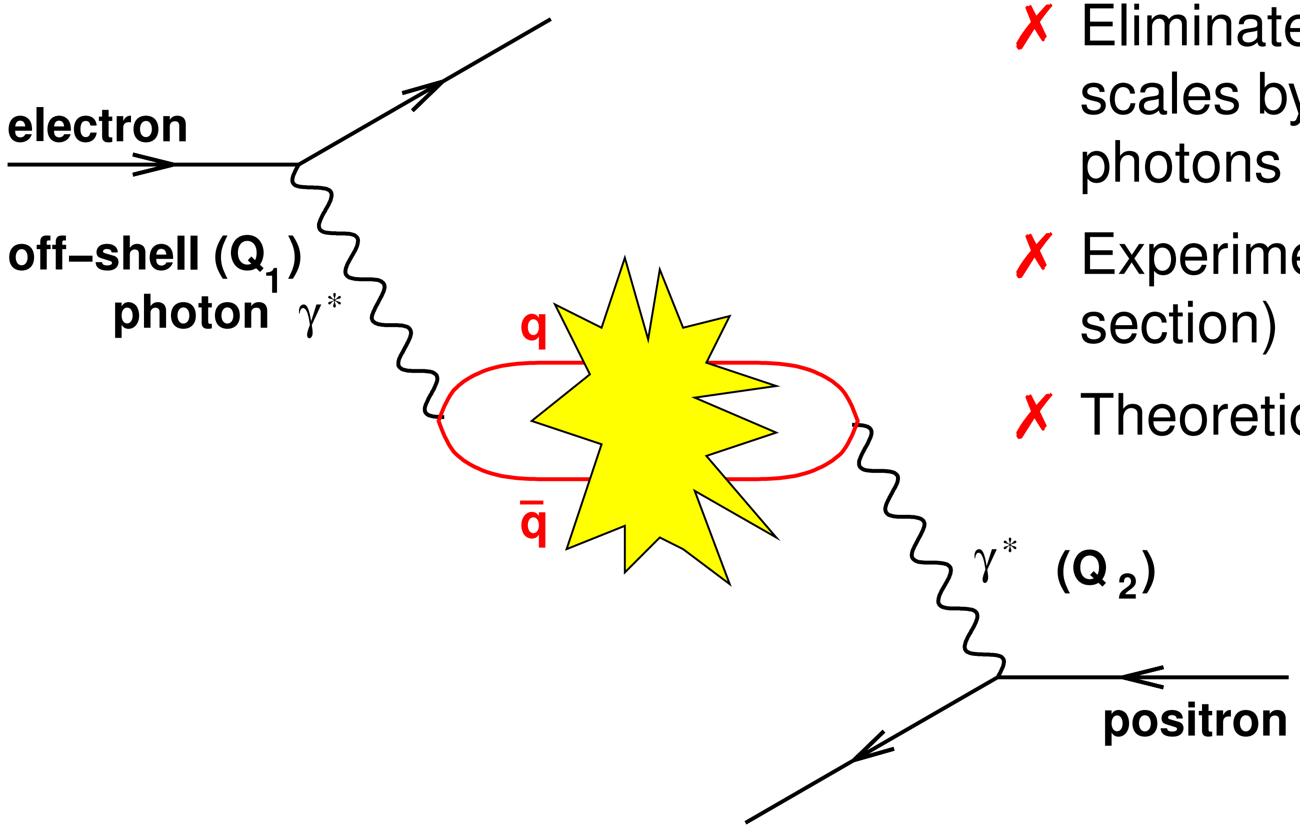
- ✗ Check if BFKL by looking at power (λ) of x
- ✗ For BFKL, expect $\lambda \simeq 0.5$
- ✗ *Definitely not BFKL!*

There is some growth — where does it come from?

It seems to be due to combination of $x \ll 1$ and $Q^2 \gg m_p^2$ — resummation of terms $(\alpha_s \ln \frac{1}{x} \ln Q^2)^n$:

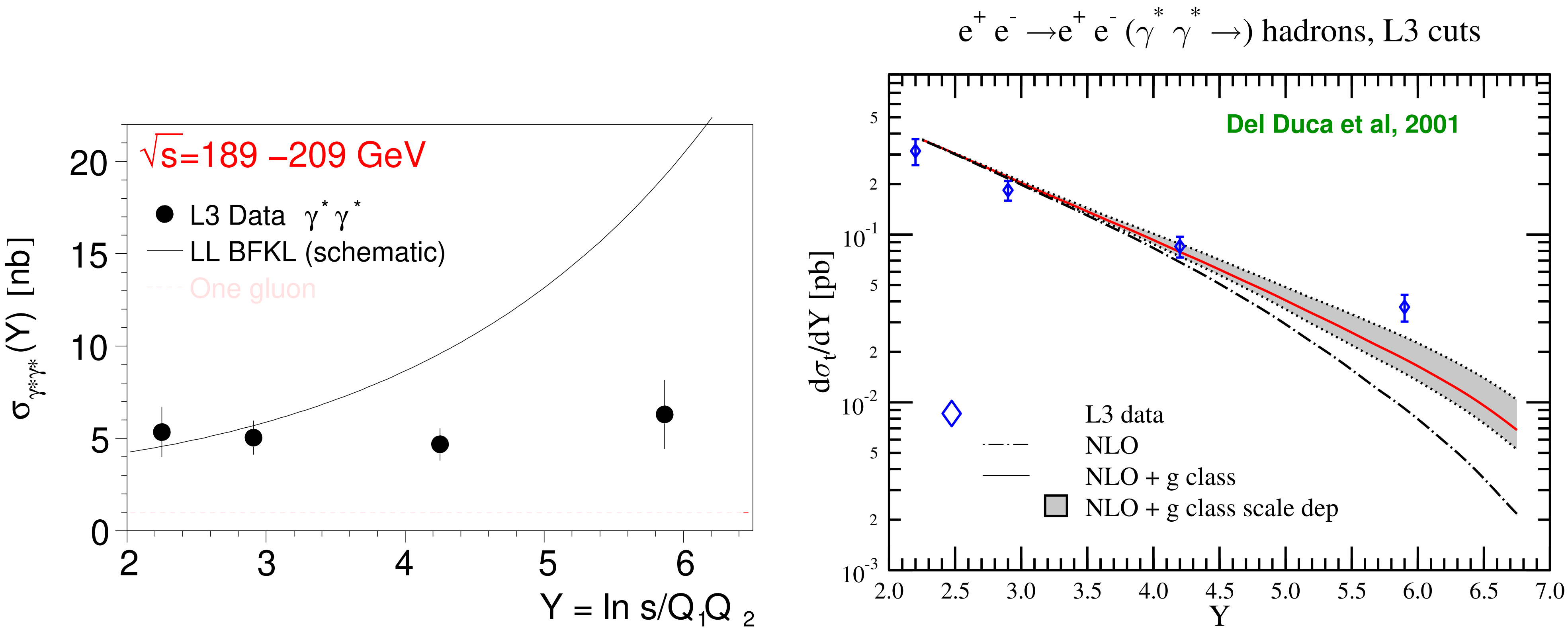
$$\sigma \sim \exp\left[c\sqrt{\alpha_s \ln Q^2 \ln x}\right]$$

$\gamma^*\gamma^*$ collisions



- ✗ Eliminate ratios of transverse scales by colliding two virtual photons $Q_1 \sim Q_2$
- ✗ Experimentally difficult (small cross section)
- ✗ Theoretically clean

Results from LEP collider



- ✗ Here too, data clearly incompatible with LL BFKL.
- ✗ But perhaps some evidence for a weaker growth

Where has BFKL disappeared to?!

BFKL behaviour is a rigorous prediction of field theory — why do we not see it?

Recall procedure

$$\vec{E} = \frac{e\gamma}{4\pi} \frac{\vec{r}}{(r_x^2 + r_y^2 + \gamma^2 r_z^2)^{3/2}}$$

$$\vec{E}_{\perp,p_z} = \int dr_z e^{ir_z \cdot p_z} \vec{E} \Rightarrow \frac{e}{2\pi} \frac{\vec{r}_\perp}{r_\perp^2}$$

Valid for $\quad \frac{1}{r_\perp} \ll p_z \ll \frac{\gamma}{r_\perp}$

Integrated $\frac{dp_z}{p_z}$ in this range to get $\ln \gamma$

Problem is that when we talk of re-summing $(\alpha_s \ln \gamma)^n$ we have made a mistake:

$$\ln \gamma \Rightarrow \ln \gamma - \mathcal{O}(1)$$
$$\ln^n \gamma \Rightarrow \ln^n \gamma - \mathcal{O}(n)$$

The extra $\mathcal{O}(1)$ pieces give contributions $\alpha_s(\alpha_s \ln \gamma)^n$:

Next-to-Leading Logarithms (NLL)

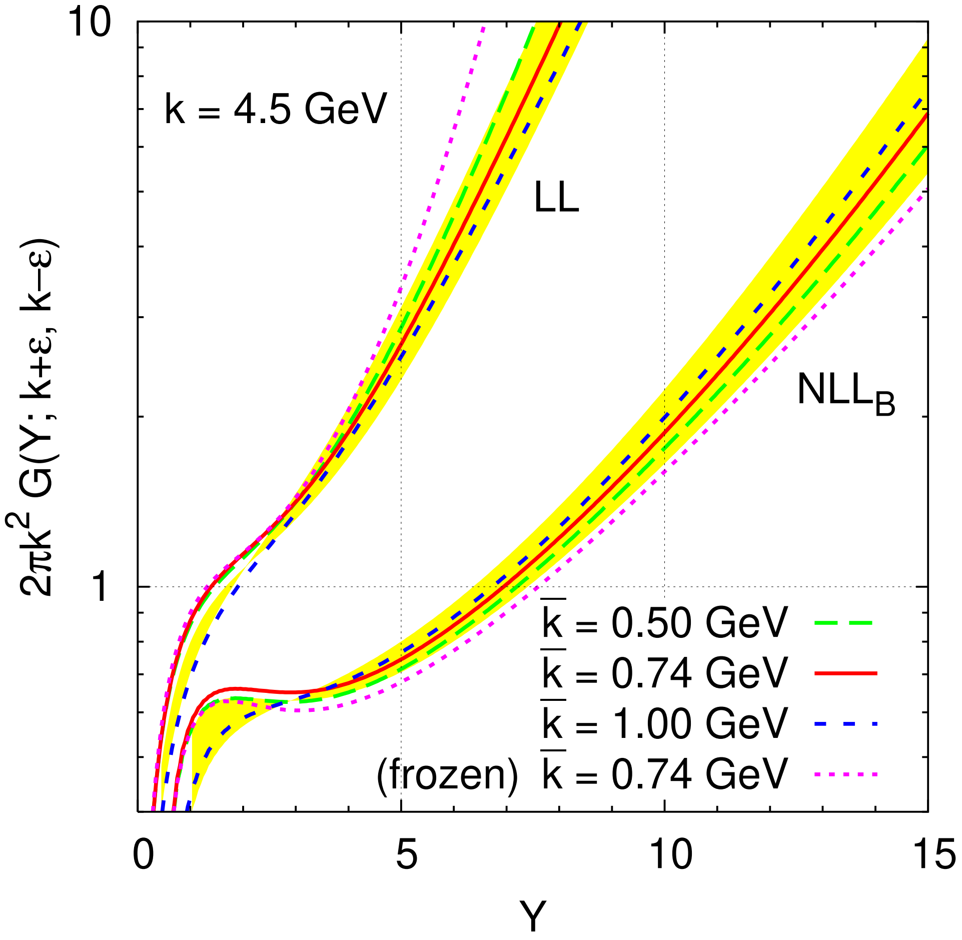
Calculated 1989–1998.

NLL Result for power:

$$\lambda = \frac{\alpha_s N_c}{\pi} 4 \ln 2 \, (1 - 6.2\alpha_s) \quad \Rightarrow \quad \lambda \simeq -0.13 \text{ (for } \alpha_s = 0.2)$$

Is negative power end of BFKL?

- ✗ Formally, no — just due to bad truncation of a poorly convergent series: $\lambda = 2.6\alpha_s(1 - 6.2\alpha_s + ??\alpha_s^2 - \ldots)$
- ✗ And in practice?
  - Perhaps not — negative NLL correction has clear physical origin (internal iteration of $(\alpha_s \ln Q^2)^n$ terms)
  - Can use same ideas to estimate N^{n}LL terms
  - Find a stable result at end



- ✗ power is weaker than LL, but positive (range: 0.20–0.30)
- ✗ At moderately high energies ($Y = \ln s \lesssim 5$) preasymptotics kill growth
- ✗ Just beyond today's experimental energies(!) BFKL growth should set in

Consistency

Experimental

- ✗ Experimentally, BFKL is elusive
- ✗ That elusiveness is not inconsistent with theory expectations
- ✗ Full NLL predictions for comparison are in progress — but presence of high-energy growth seems theoretically solid.

What about internal consistency?

- ✗ Can we have s^λ growth of total cross section for ever?
- ✗ What happens as density of gluons increases?

Unitarity

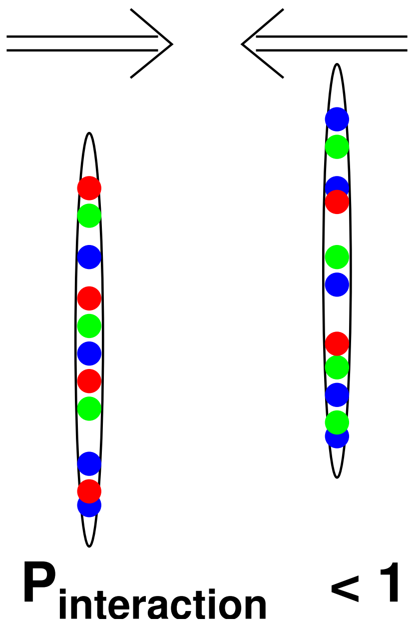
Cross section growth occurs because of:

✗ increase in *area* of projectile

- ☛ not possible for ever, because eventually reach confinement sizes $(1/m_\pi)$

✗ increase in *density* of projectile

- ☛ but for fixed collision impact-parameter probability of interaction is bounded ≤ 1 (*unitarity condition*).



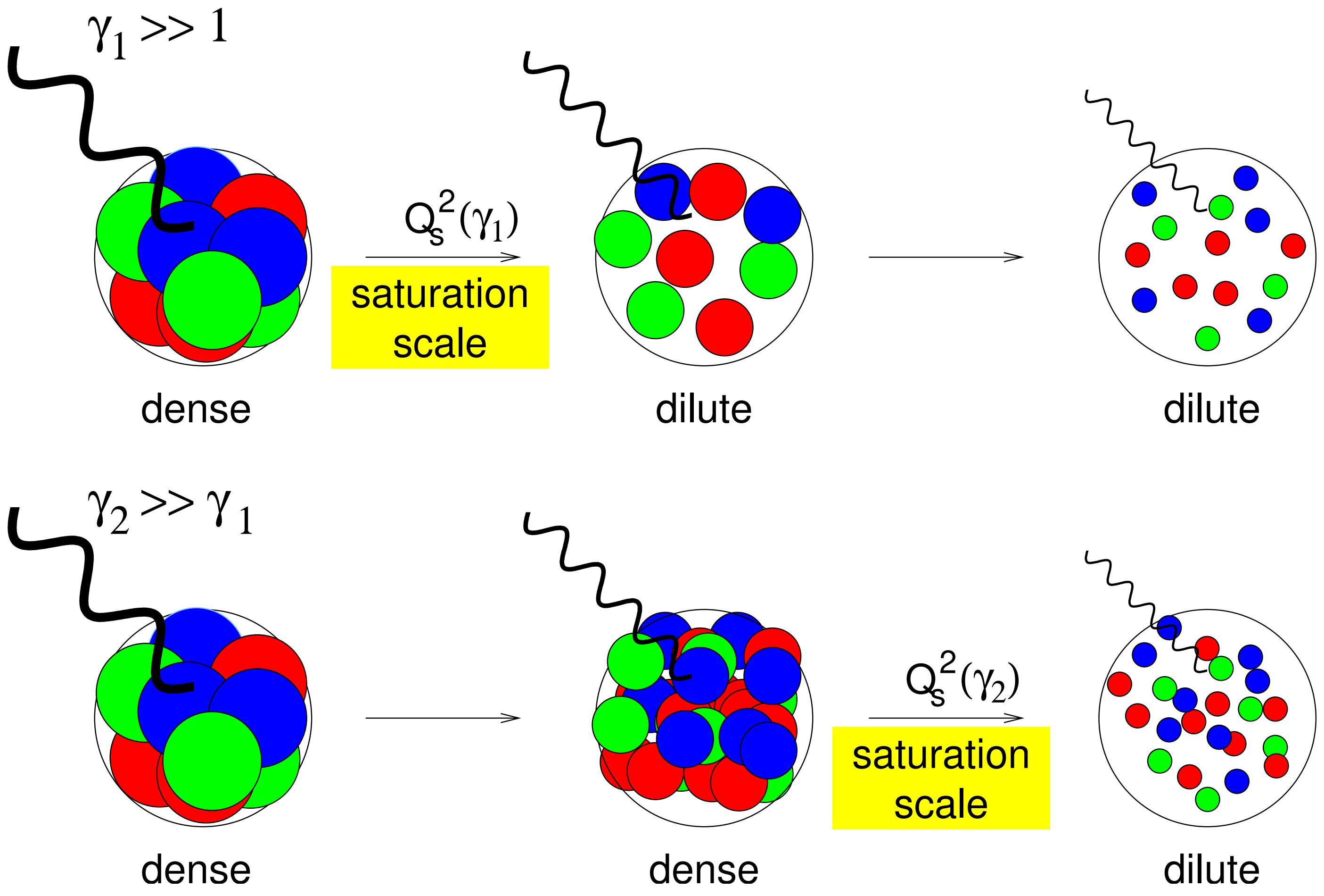
Raw BFKL accounts for neither of these issues:

✗ Confinement can at best be (poorly) modeled

✗ Violation of unitarity is consequence of linearity equations — if densities are so high that there are typically n scatterings per collision, BFKL counts contribution n to interaction probability — right answer is ~ 1.

Can put in multiple scattering (need care + not whole story)

Saturation scales (applies mostly to proton)



Idea of a *maximum (saturated) density* of gluons $(\rho \sim 1/\alpha_s)$

[Colour Glass Condensate]

Saturation scale — resolution parameter that *separates* saturated and non-saturated.

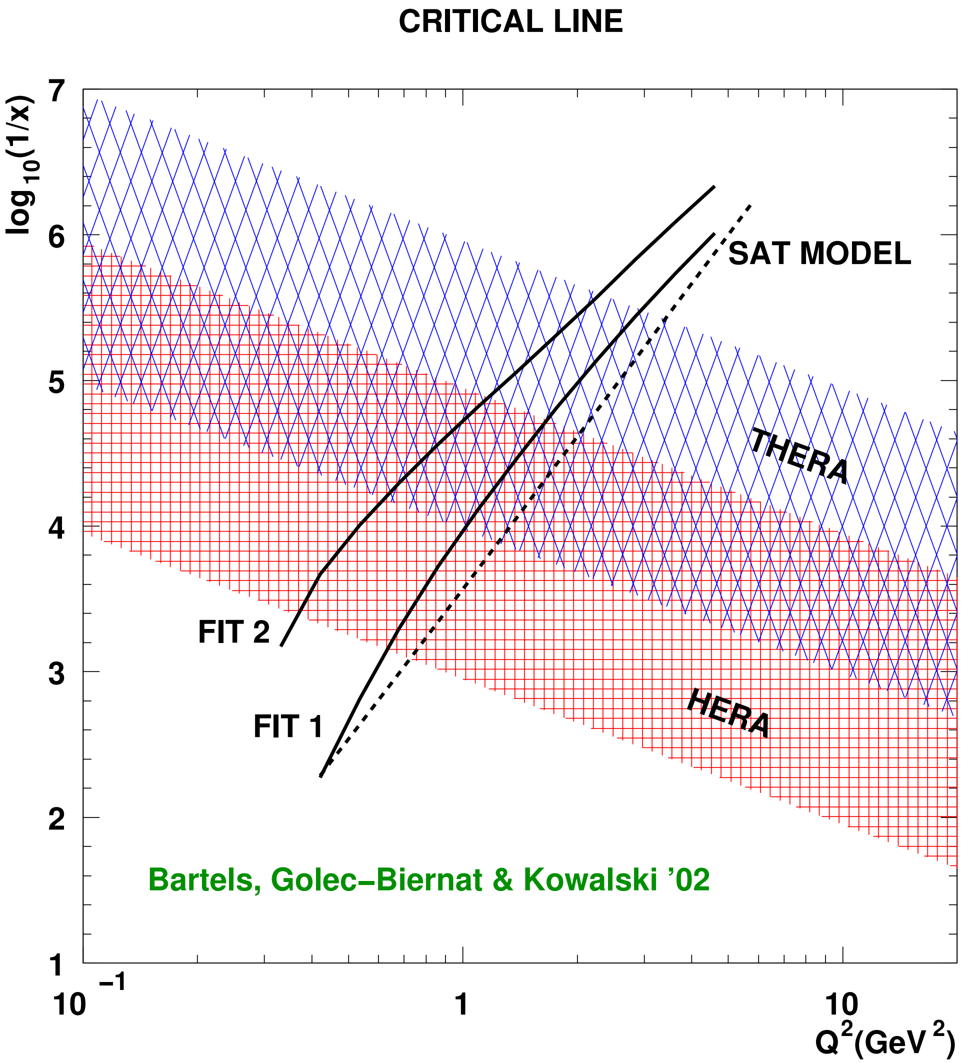
Saturation scales (cont.)

Big business at HERA collider

- ✗ Models including saturation are fitted to HERA data
- ✗ Saturation sets in (perhaps?) just at limit of perturbative region
- ✗ Rises with decreasing x

Interesting possibility:

- ✗ Since saturation scale > 1 GeV, perhaps even properties (radiation) of plain proton-proton scattering can be partially described perturbatively.



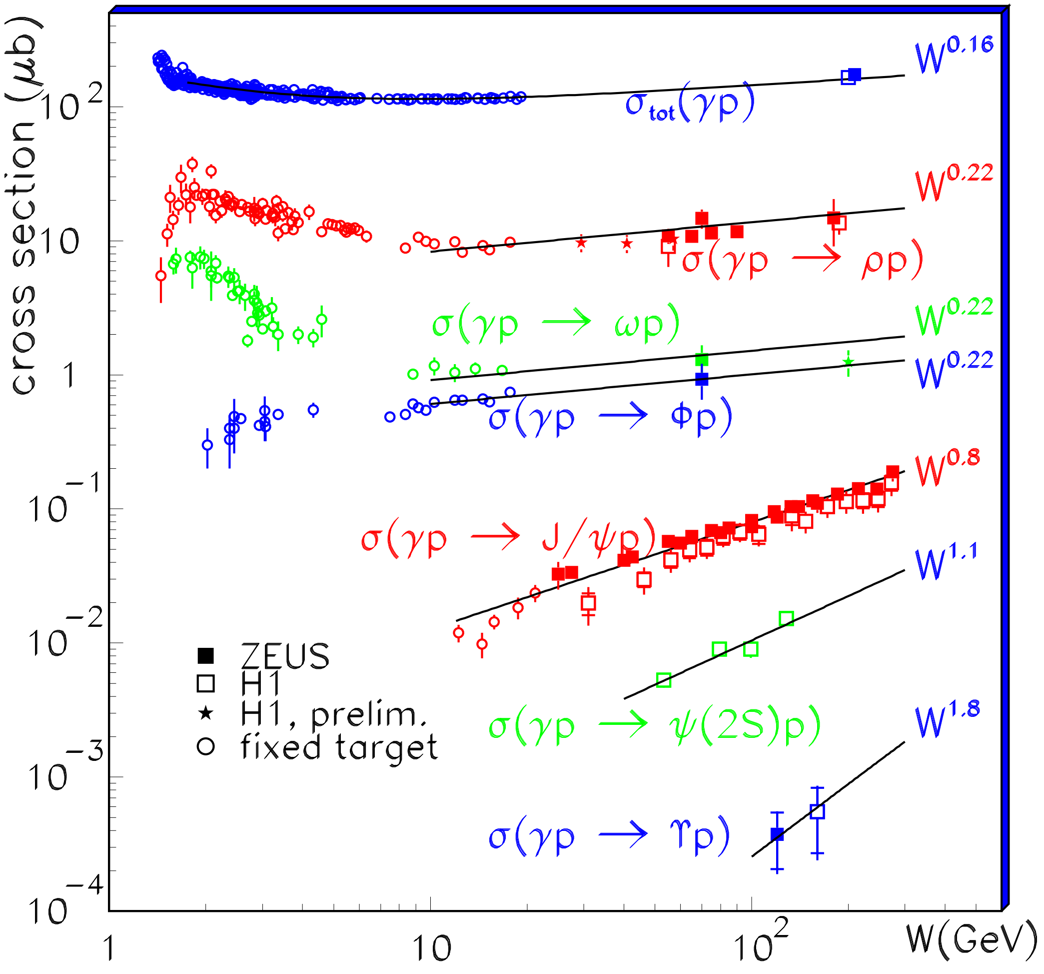
Conclusions

Search for full understanding of high-energy limit of QCD continues. . .

Much not discussed:

- ✗ All of the details!
- ✗ Diffraction (source of much of our understanding on saturation)
- ✗ Nuclear collisions (overlap of nuclear wavefunctions gives high gluon densities)
- ✗ Iteration of n-gluon states (*e.g.* quadrupole, hexapole)
  - ☛ Conformal invariance & relation to integrable models
- ✗ Final states — what comes out of a high-energy collision.

Exclusive vector mesons



These are processes which essentially measure the *square of the gluon distribution* — form a small component of total cross section that grows twice as fast:

$$\sigma_{\text{excl}} \sim s^{2\lambda} = W^{4\lambda}$$

Transverse scale is roughly particle mass (m)

	m^2 [GeV^2]	$\lambda_{tot}(m^2)$
ρ, ω, ϕ	< 1	0.08
J/ψ	10	0.22
Υ	100	0.35