Fall and rise of the gluon splitting function (at small x)

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In collaboration with M. Ciafaloni, D. Colferai and A. Staśto

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Introduction

- At high energies ($\sqrt{s}$) $\equiv$ small x, cross sections are supposed to rise rapidly — domain of BFKL physics $\equiv$ *resummation of logarithms of s* (or x):

$$\sigma \sim \sum_{n=0} \alpha_s^n \ln^n s \sim s^{4 \ln 2 \frac{\alpha_s N_C}{\pi}} \sim s^{0.5}$$

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Hard to measure experimentally
&
of limited wider relevance

Intro: semi-perturbative studies

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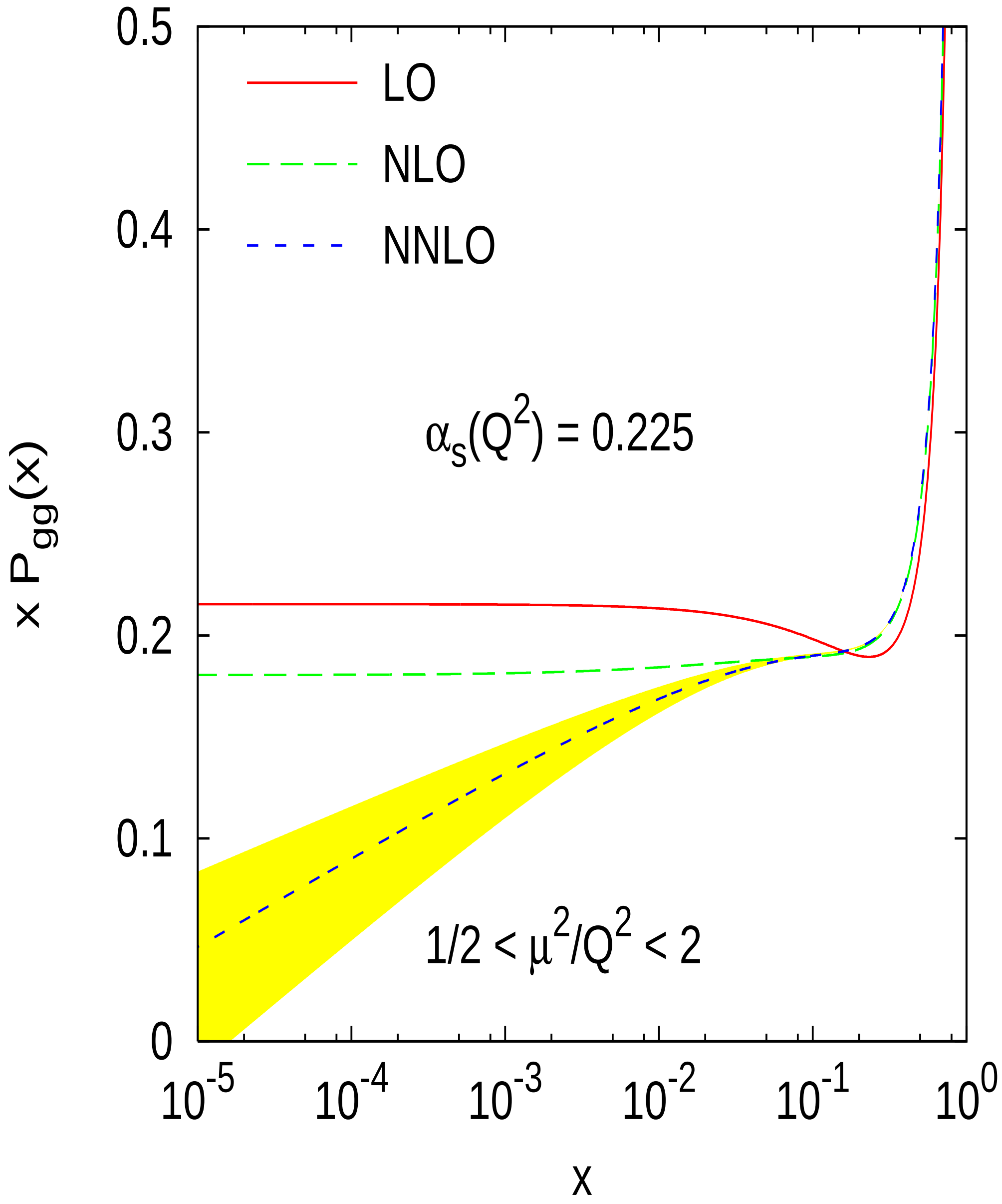
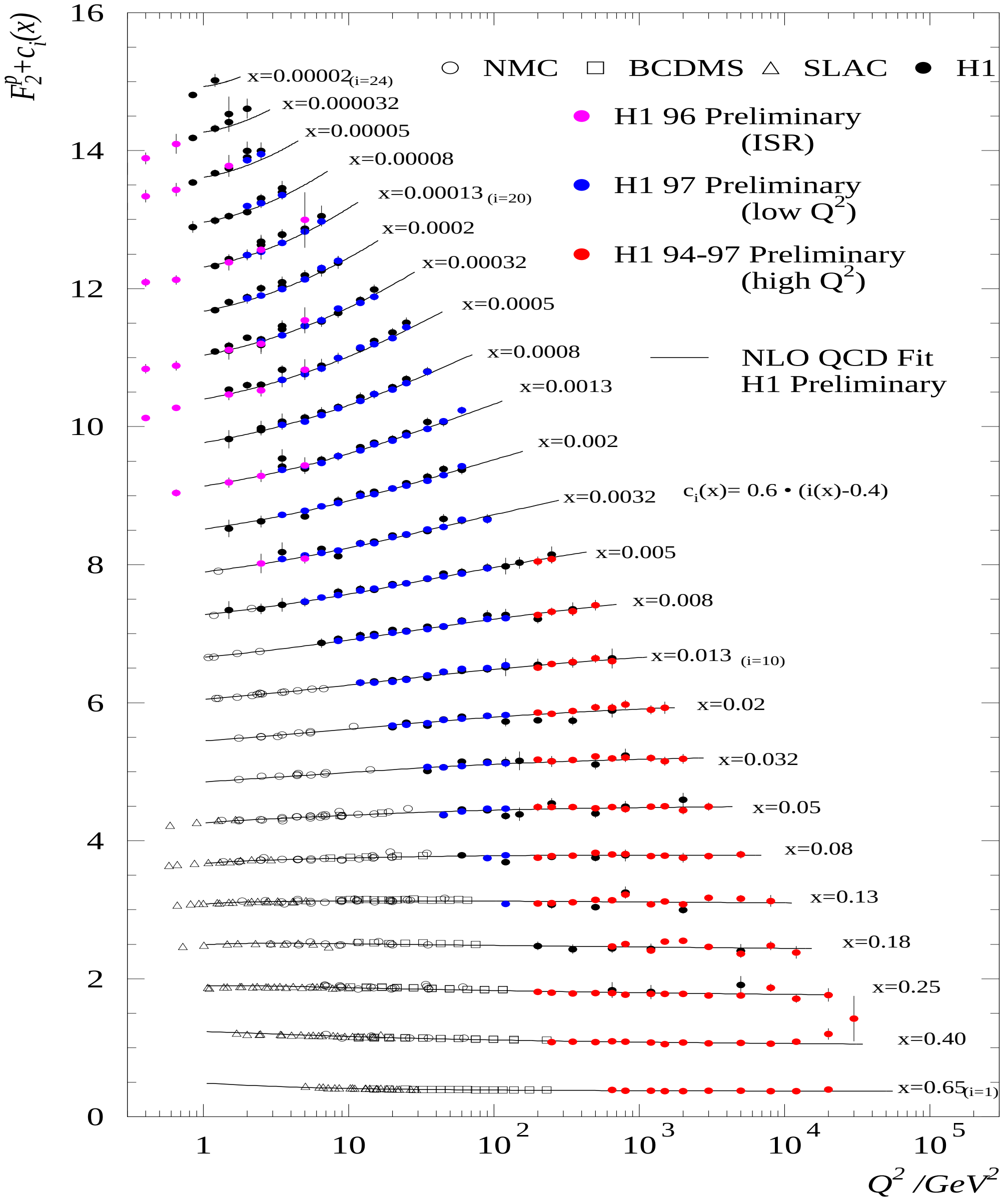


NLO DGLAP versus data



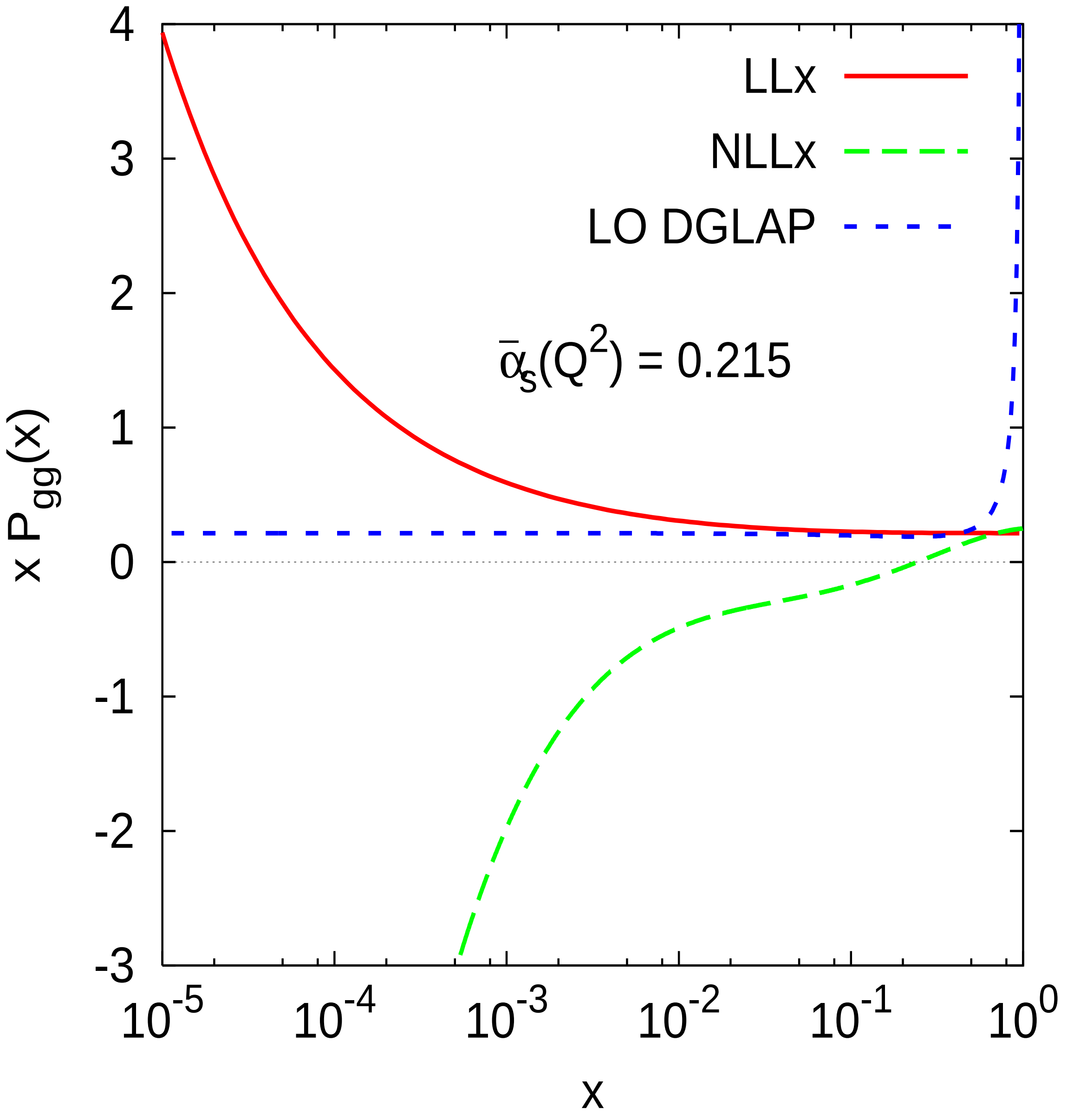
- NLO DGLAP fits give good description of data
- So do preliminary NNLO DGLAP fits
- Evidence of some problems for very small $x \lesssim 10^{-3}$
  - instabilities from NLO to NNLO
  - negative gluons

LLx, NLLx?

<u>Resummation status</u>

- LLx terms rise very fast, $xP_{gg}(x) \sim x^{-0.5}$.
  Incompatible with data.
  Ball & Forte '95
- NLLx terms go negative very fast.
  No one's even tried fitting the data!

[NB: Taking NLLx terms of P_{gg} is almost the worst possible expansion]



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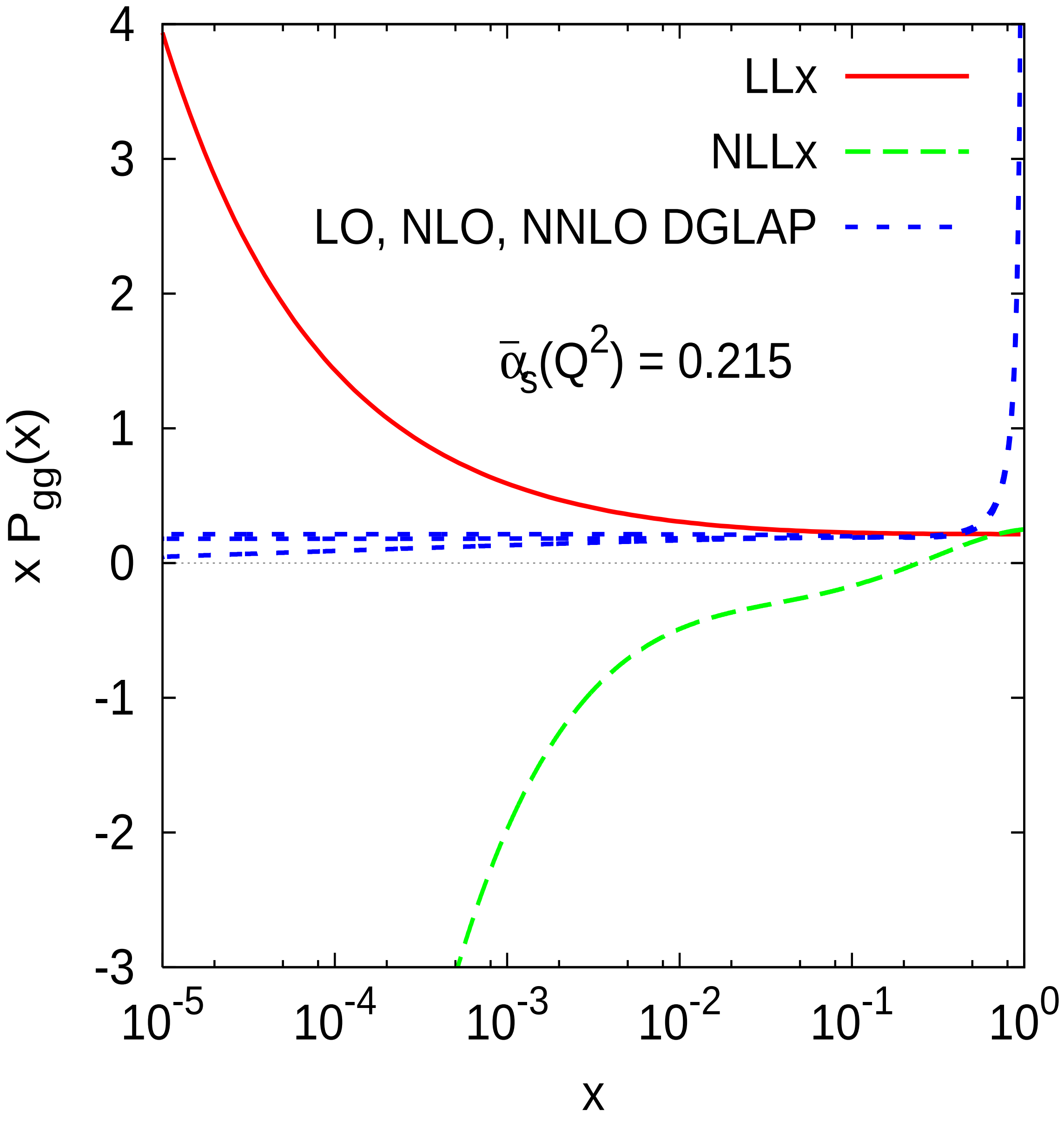
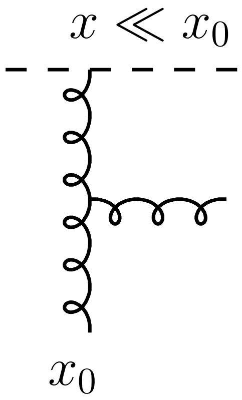


‘Improving’ on NLLx? Start with kernel...

BFKL

α_s


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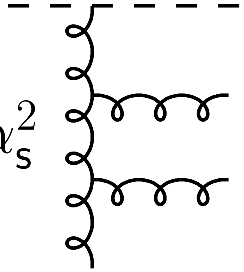
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DGLAP

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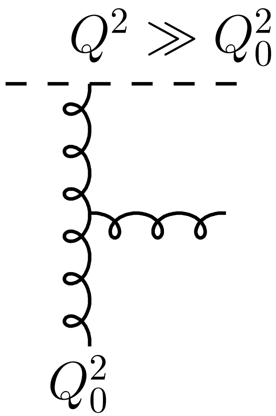

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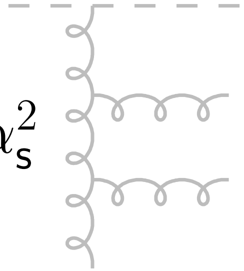
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anti-DGLAP

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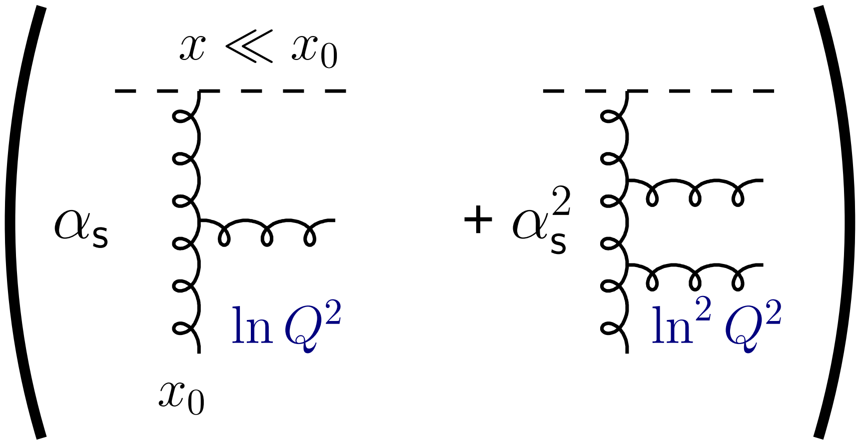

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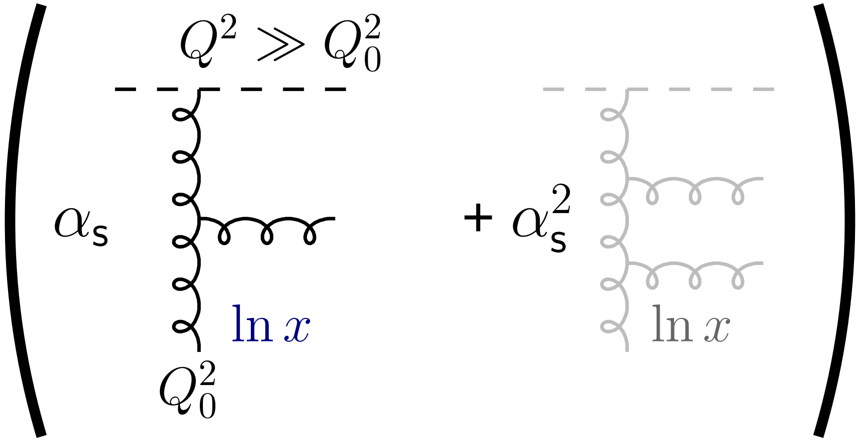

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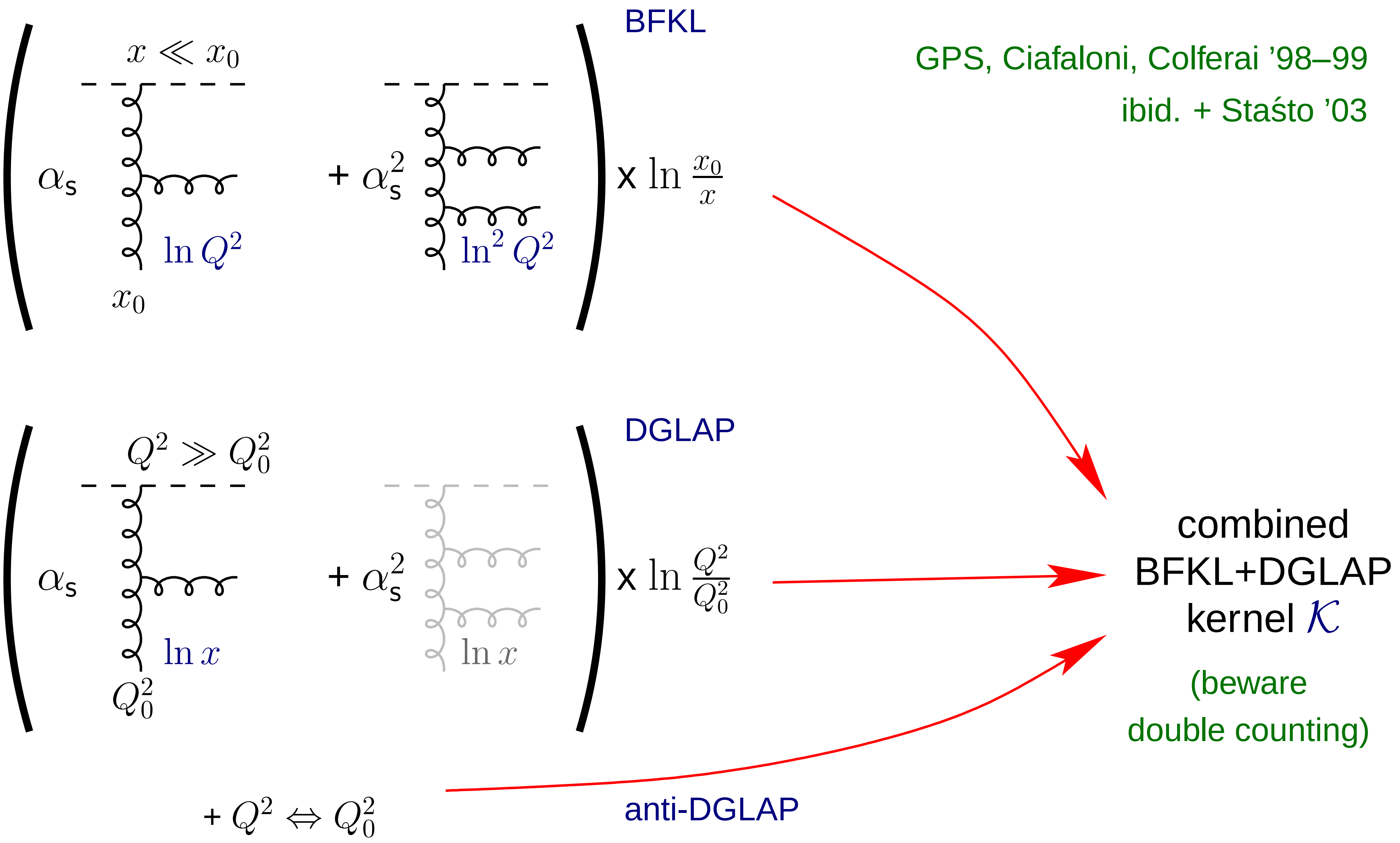
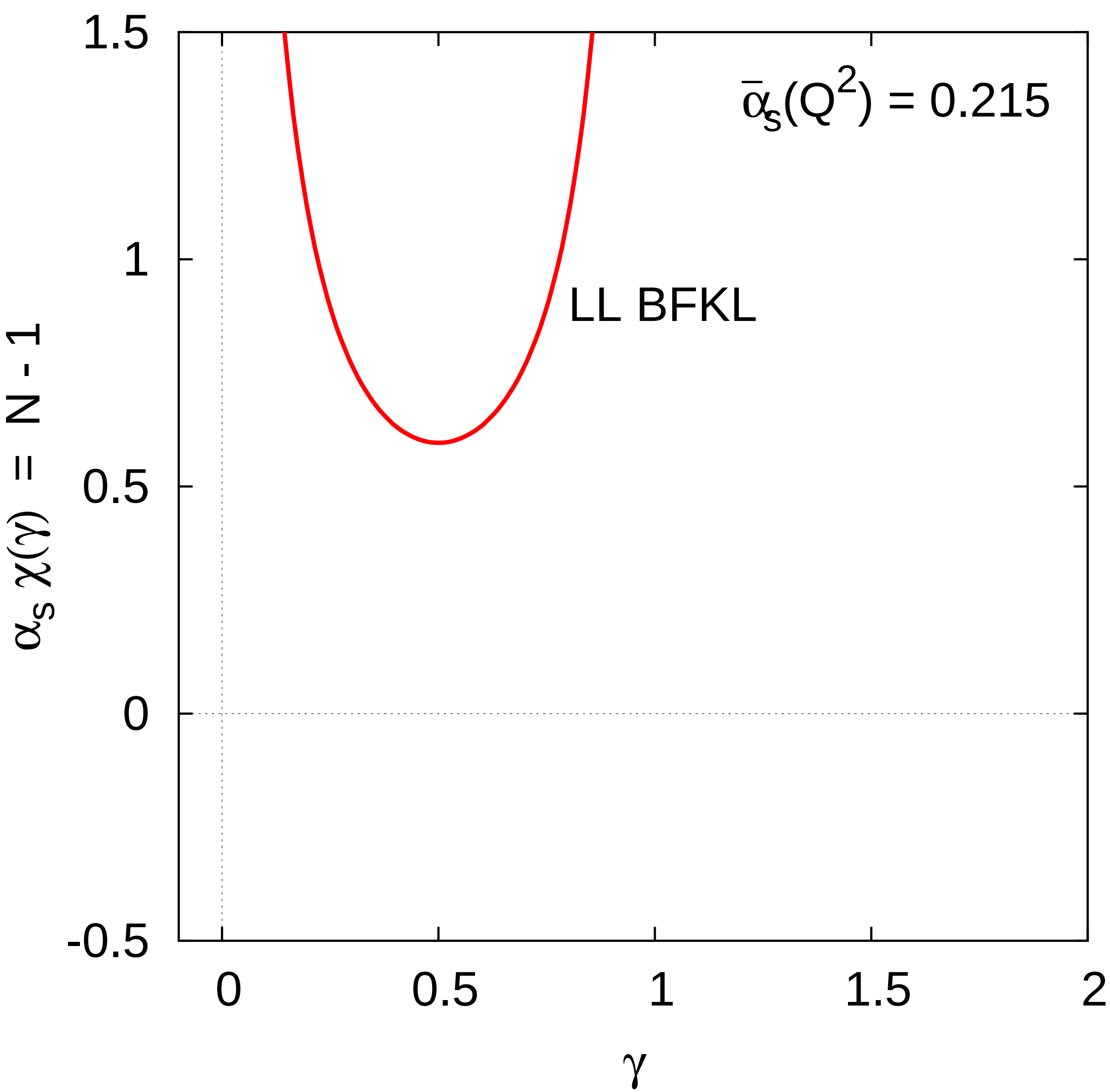


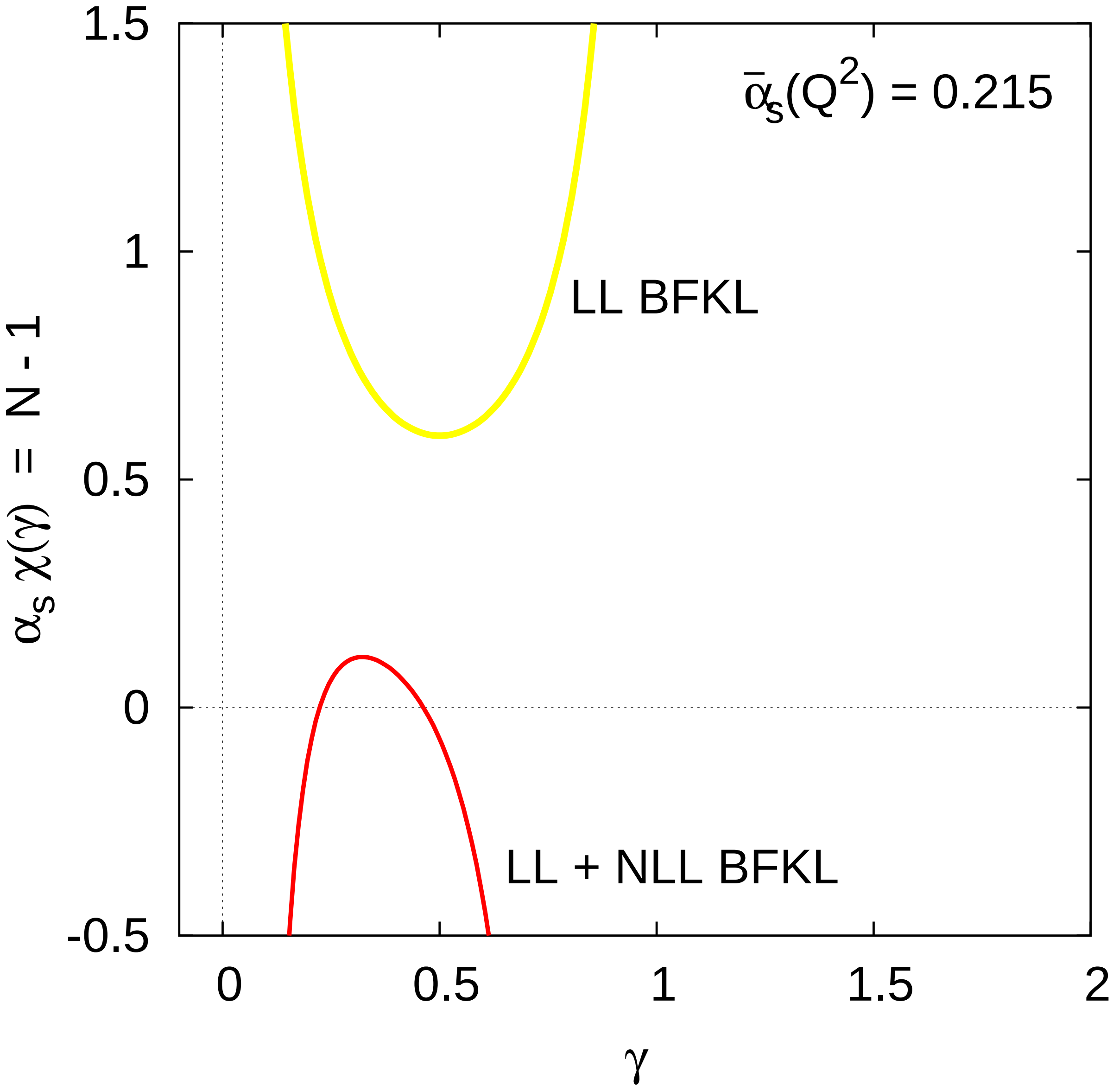
Building up the kernel. . .



Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)

$$\bar{\alpha}_s \chi(\gamma) = = \int \frac{dk^2}{k^2} \left(\frac{k^2}{k_0^2}\right)^\gamma \mathcal{K}(k, k_0)$$

Height of minimum is 'BFKL power'



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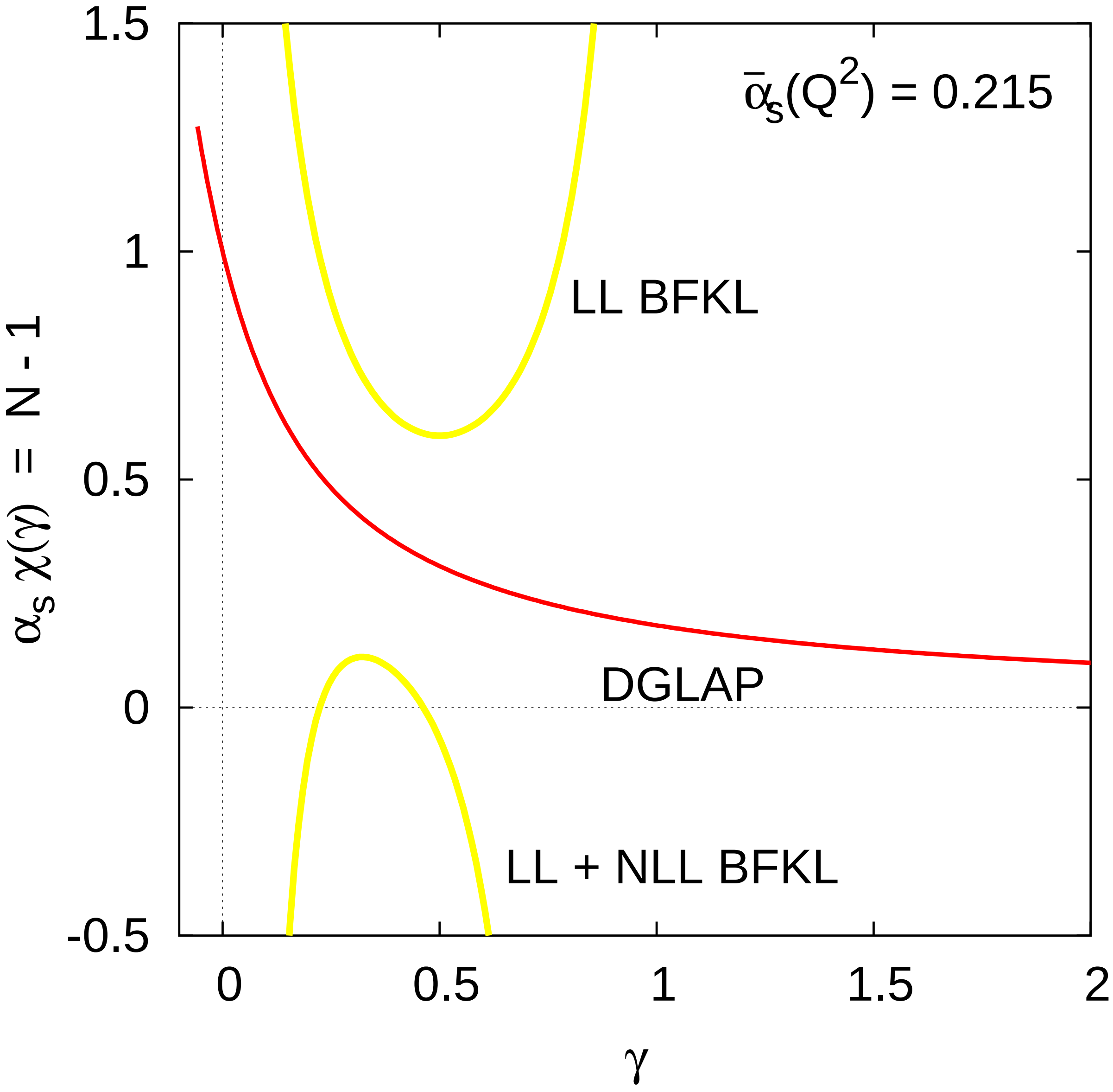

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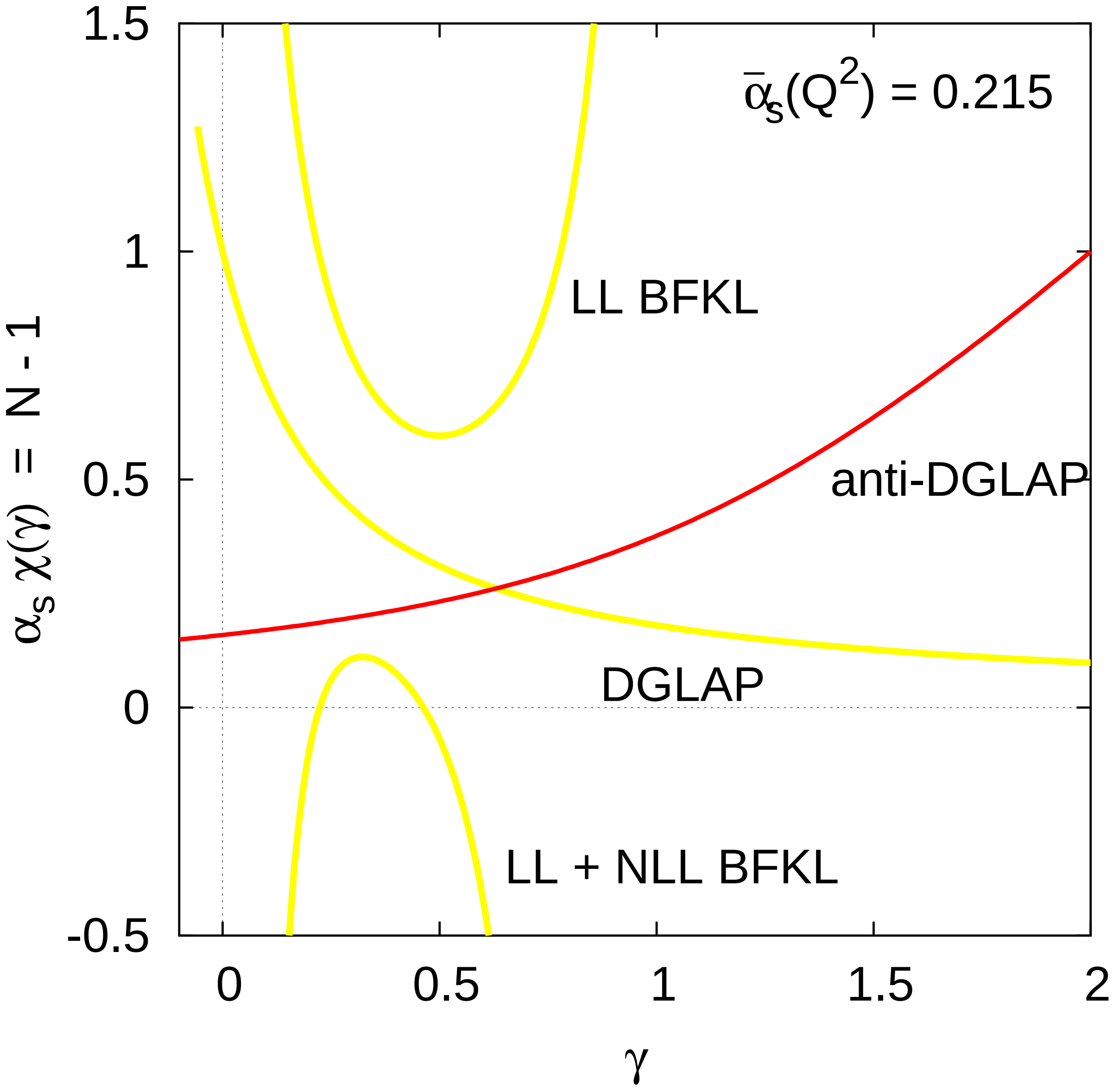

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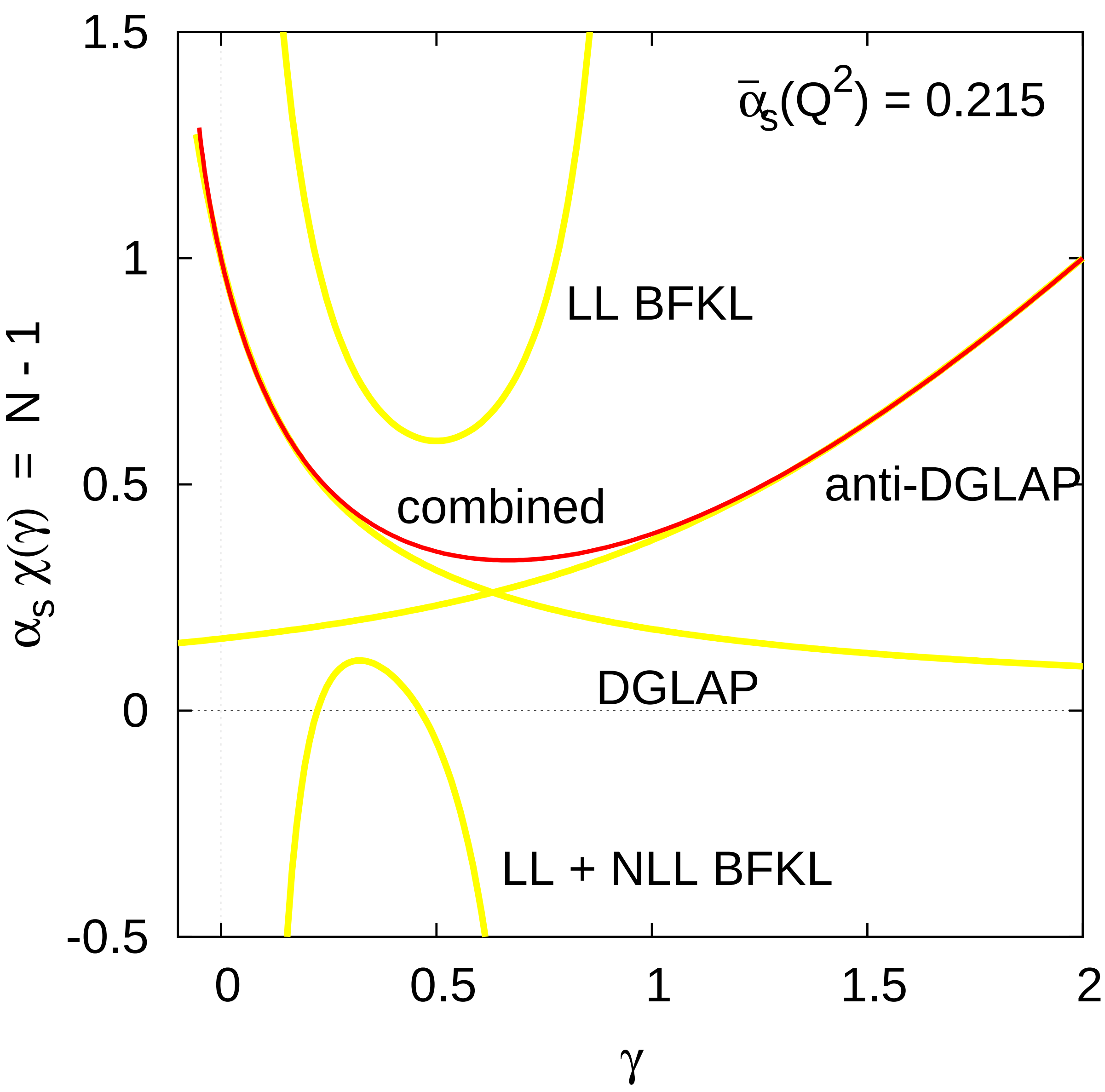

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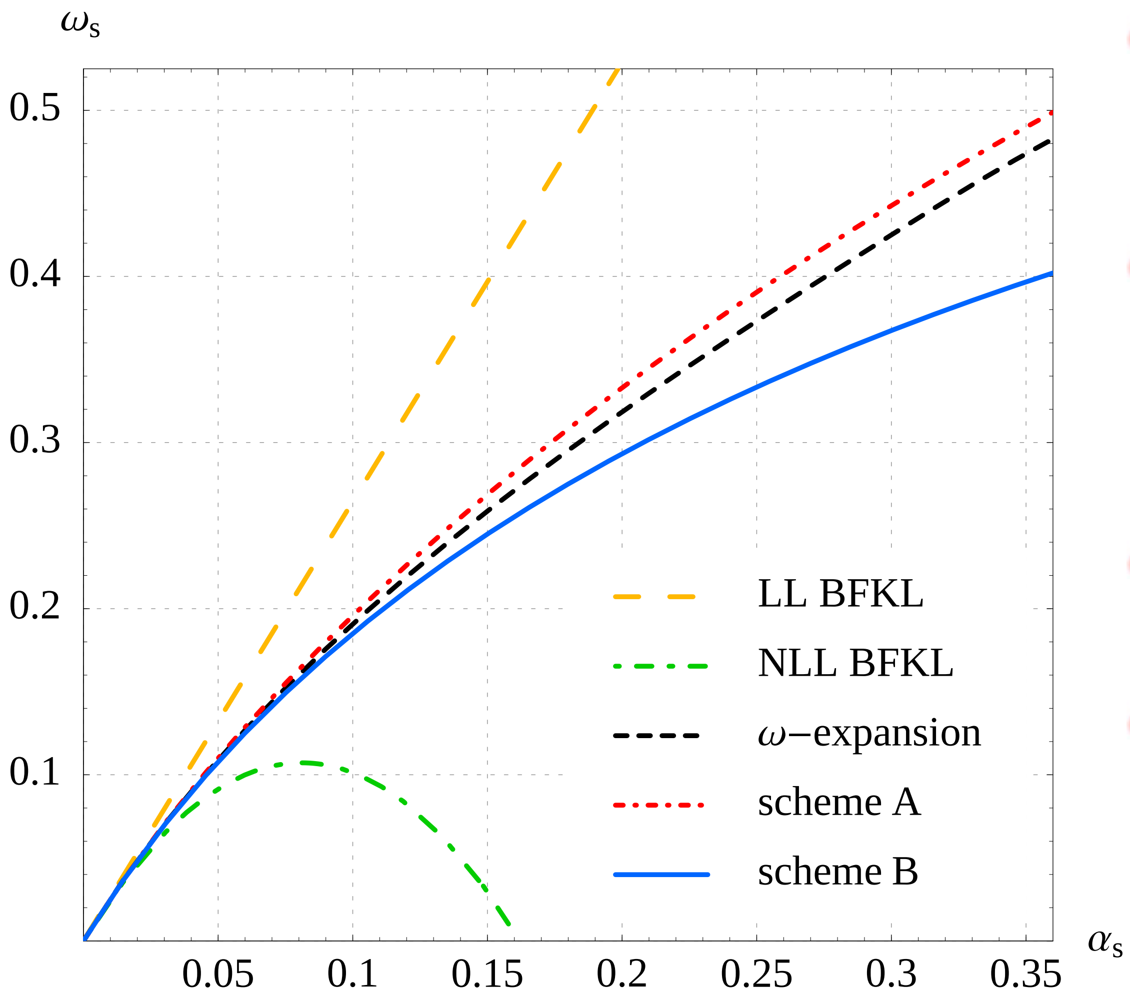

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- With same logic, other theorists find similar results!
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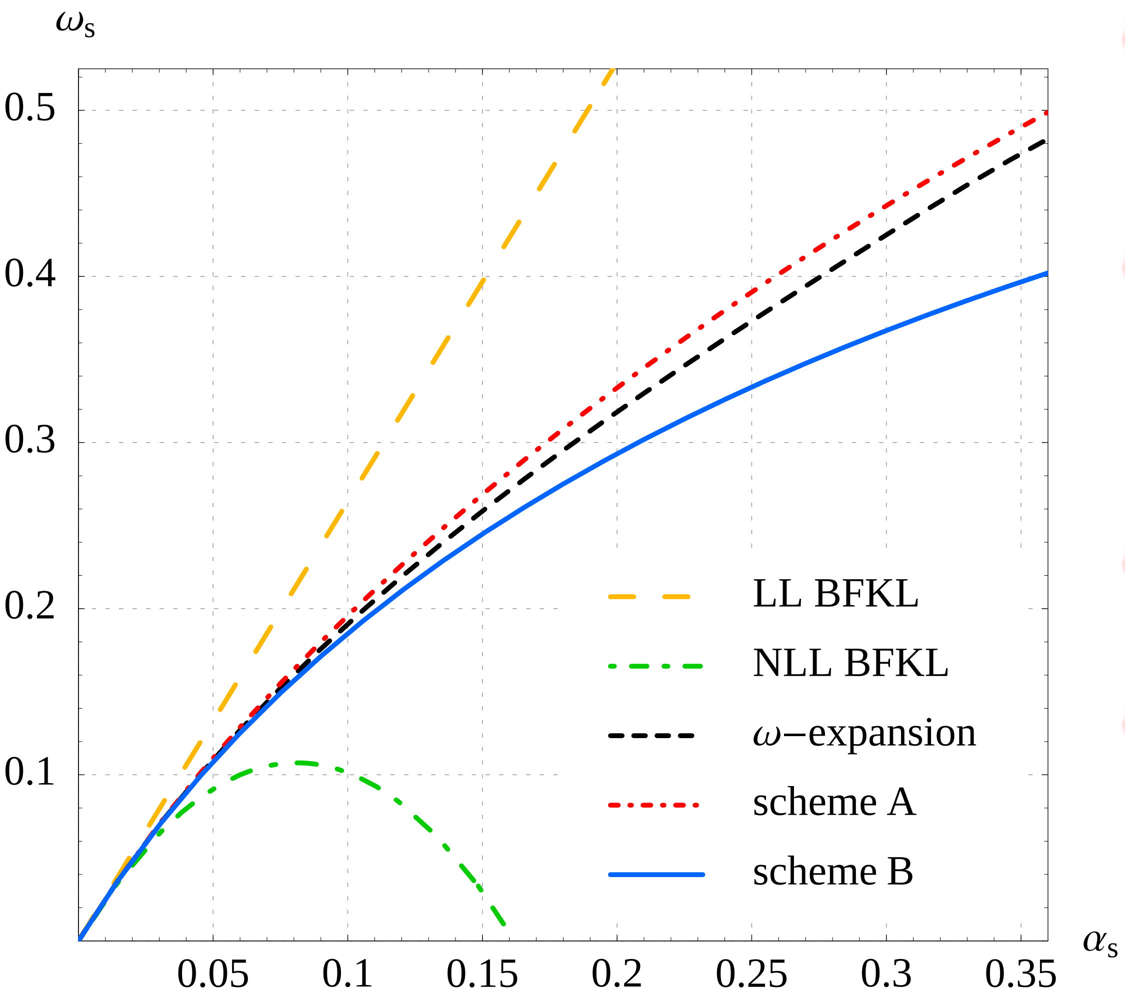

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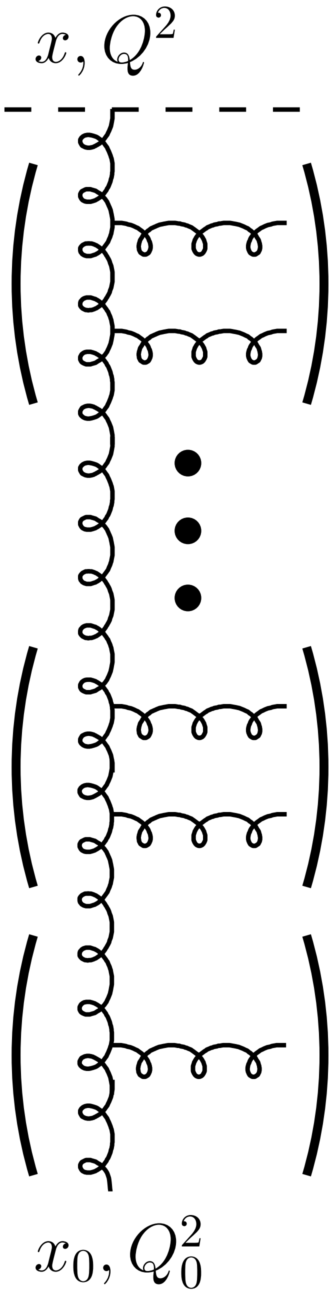
NB: power shown here is property of *kernel*, not of cross sections...

Iteration of kernel $\Rightarrow$ Green function



Green function: $G\left(\ln \frac{x}{x_0}; Q_0, Q\right)$

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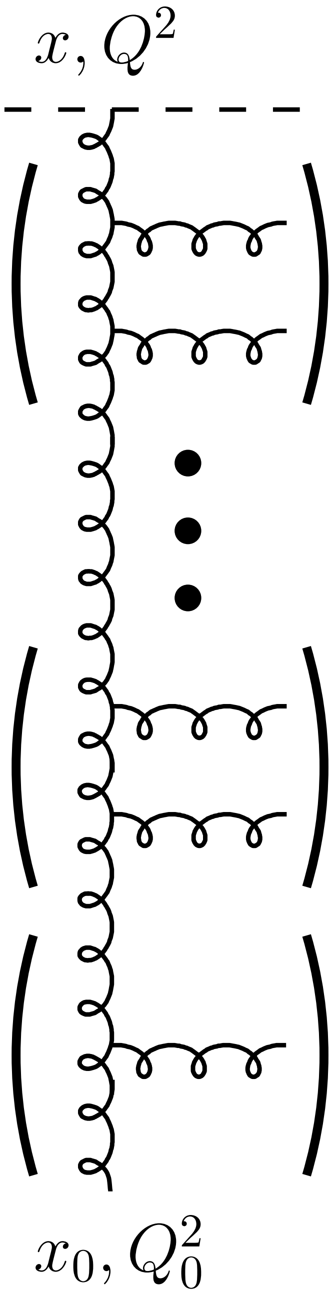

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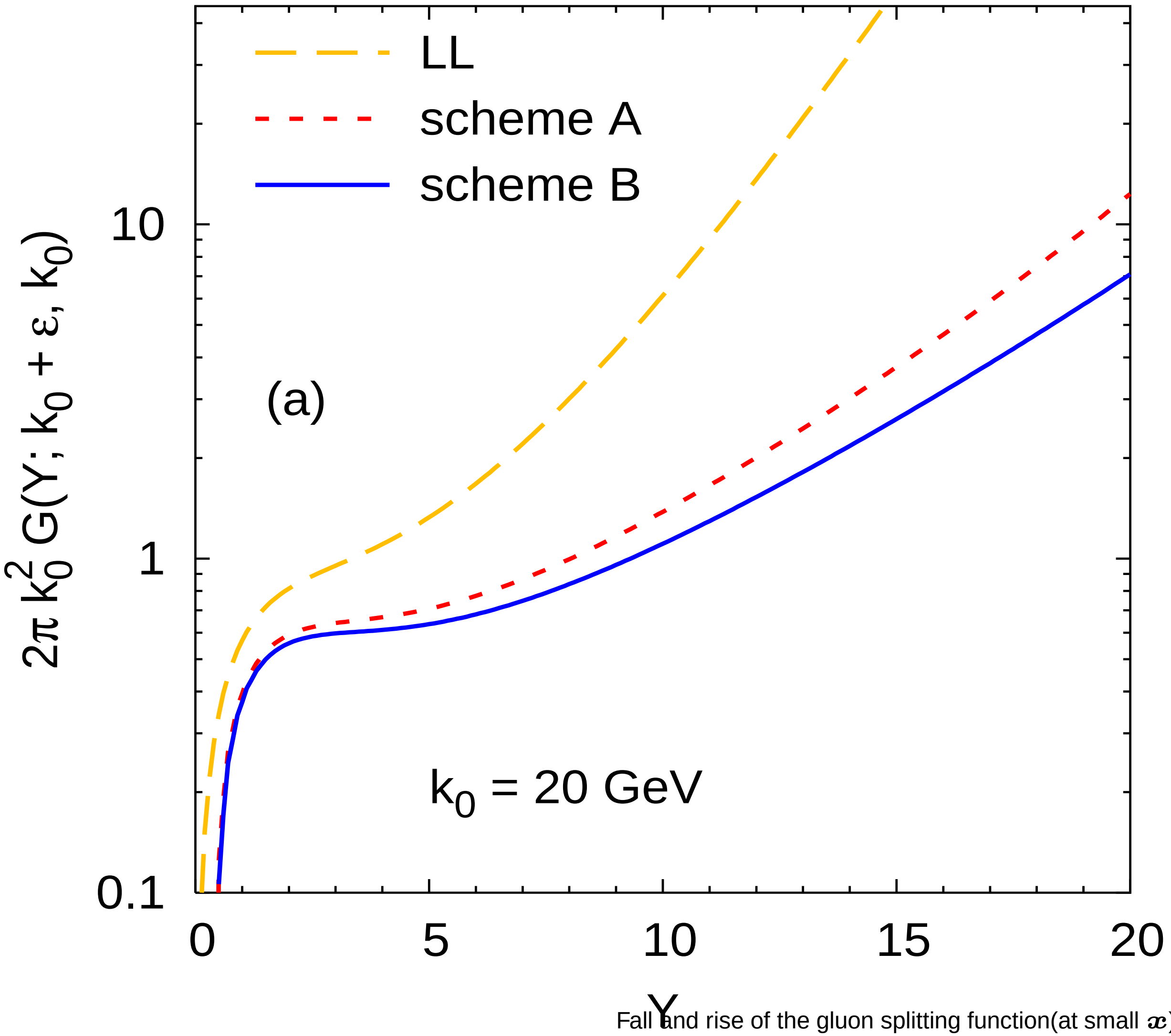

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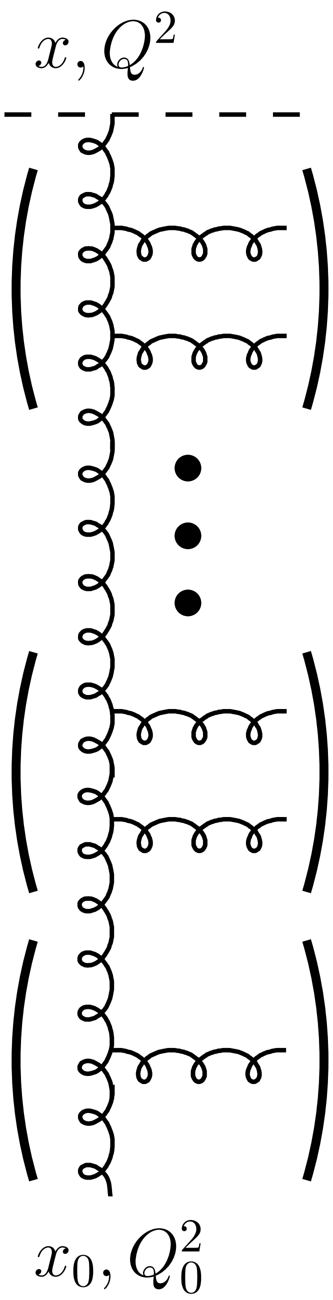

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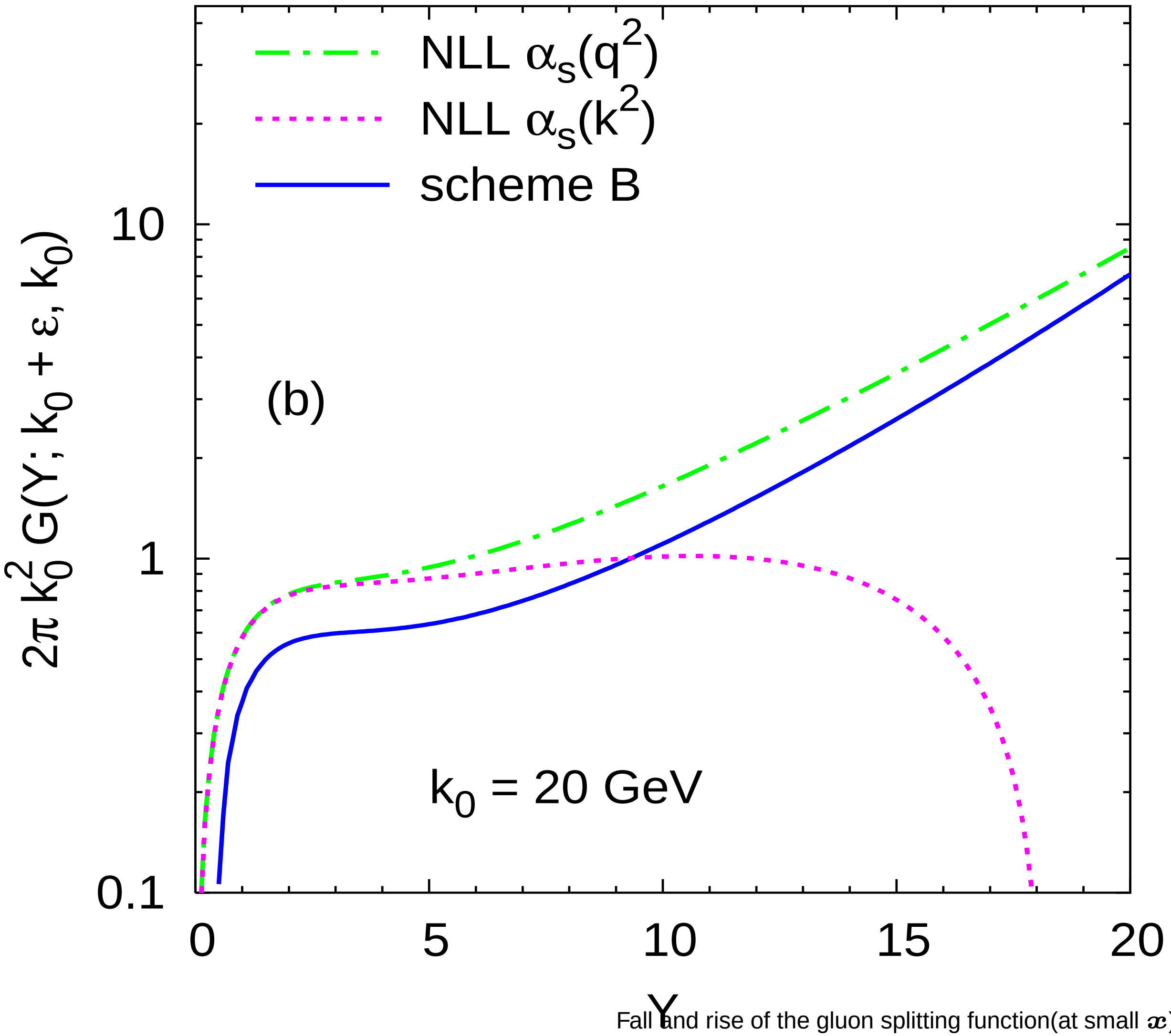

Green function $\Rightarrow$ effective DGLAP splitting function

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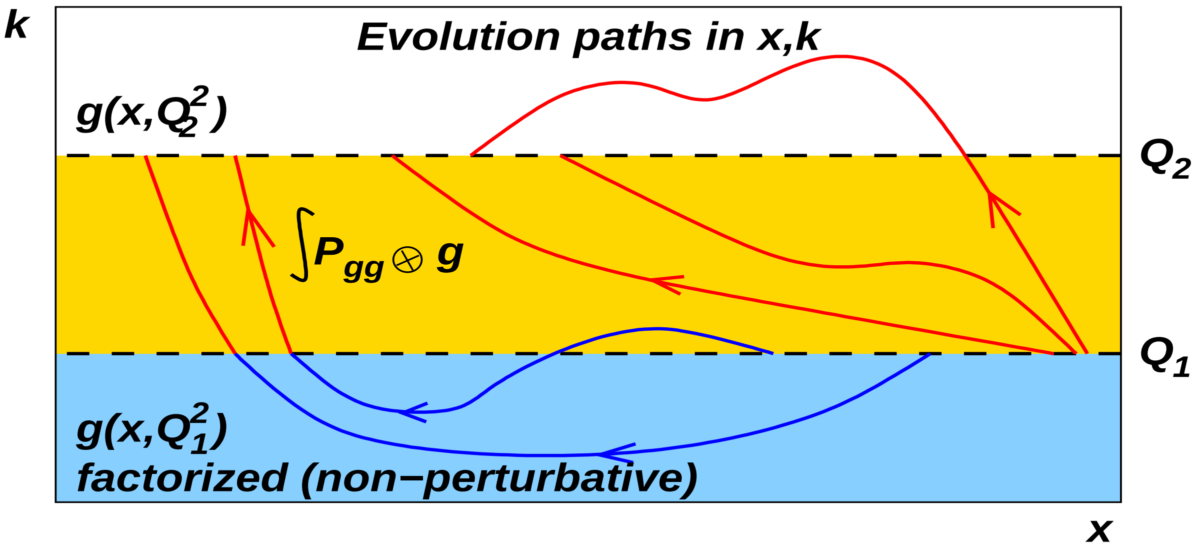
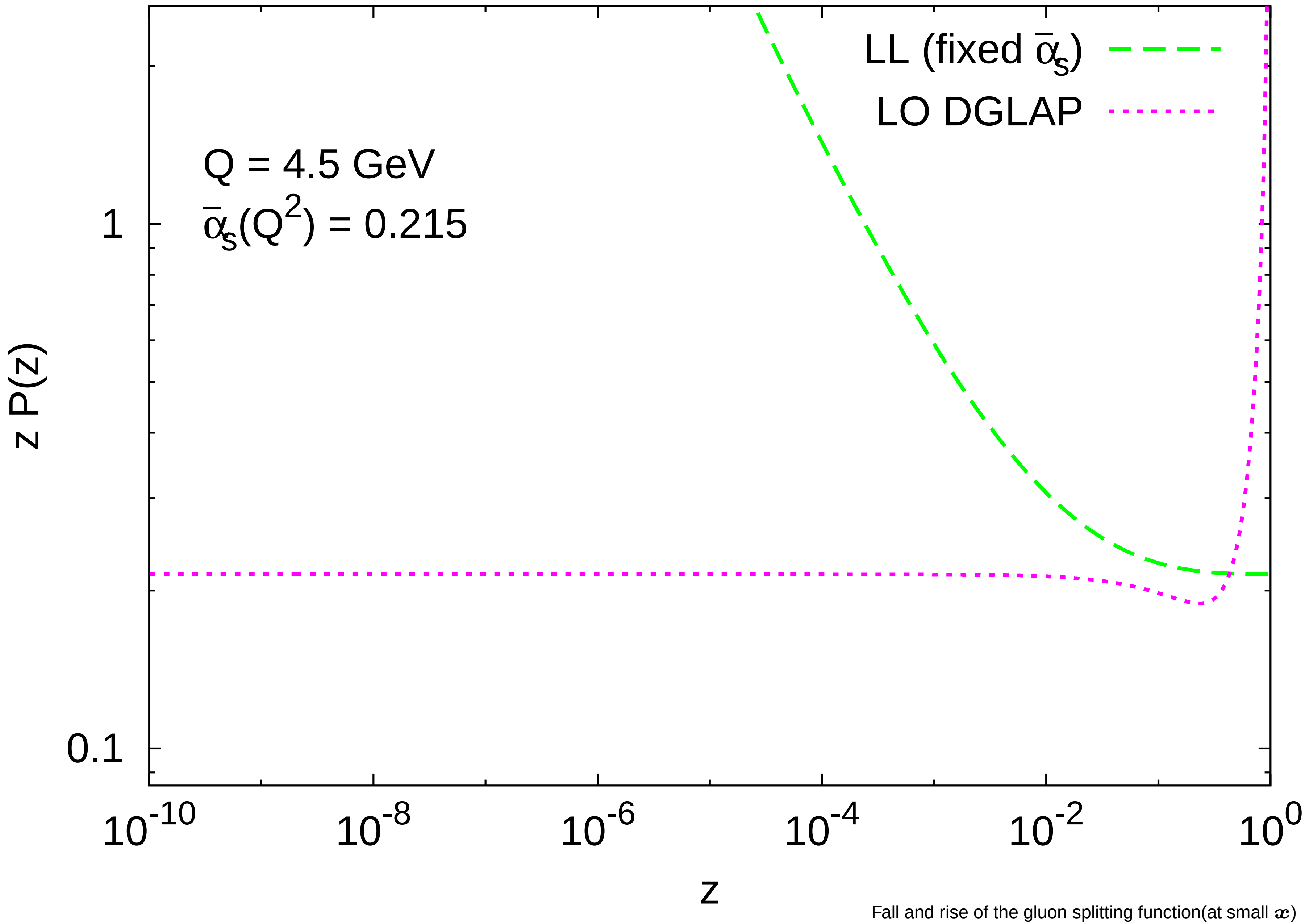
<u>Factorisation</u>

- Splitting function: red paths
- Green function: all paths



$P_{gg}(z)$ splitting function results



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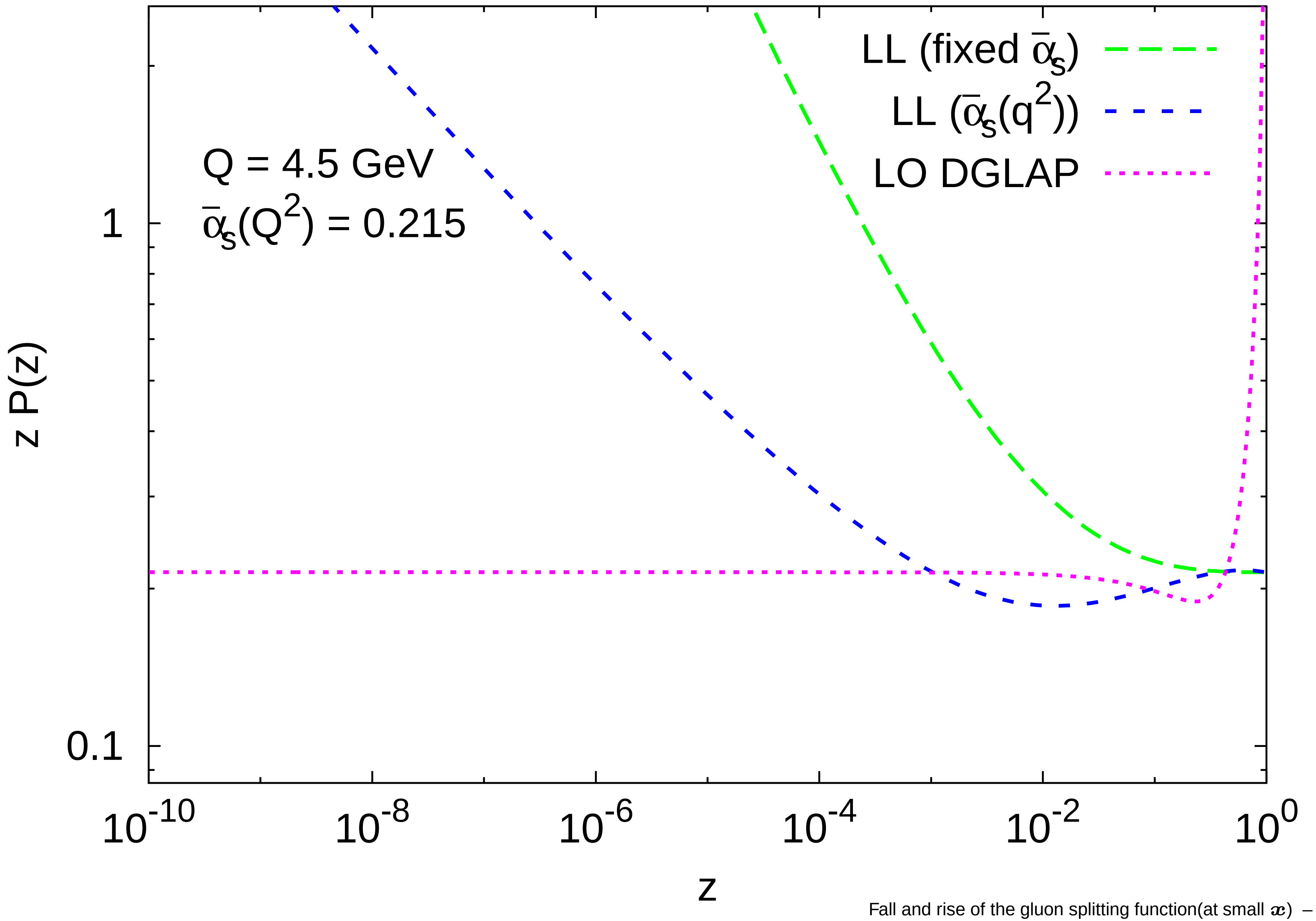

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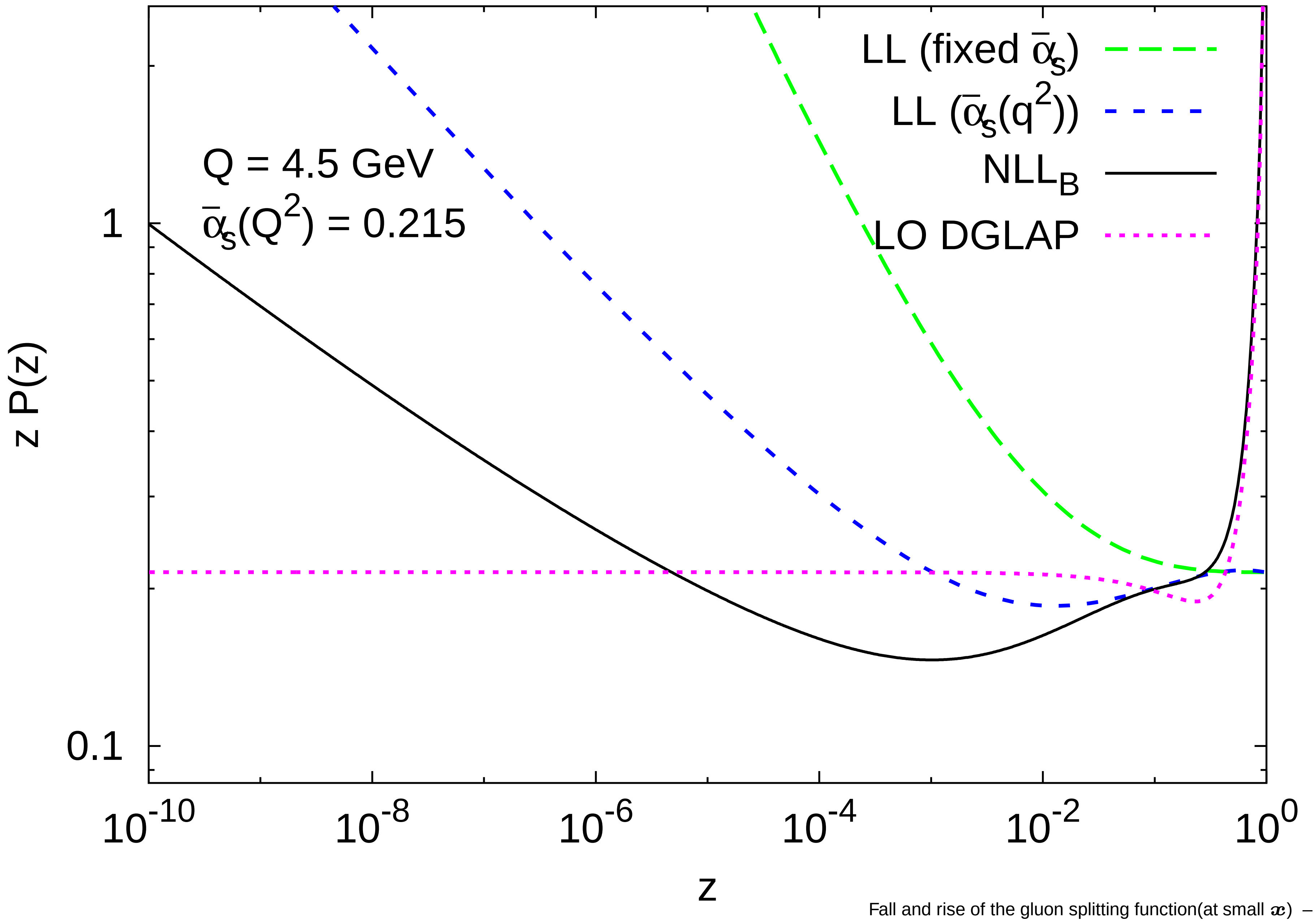

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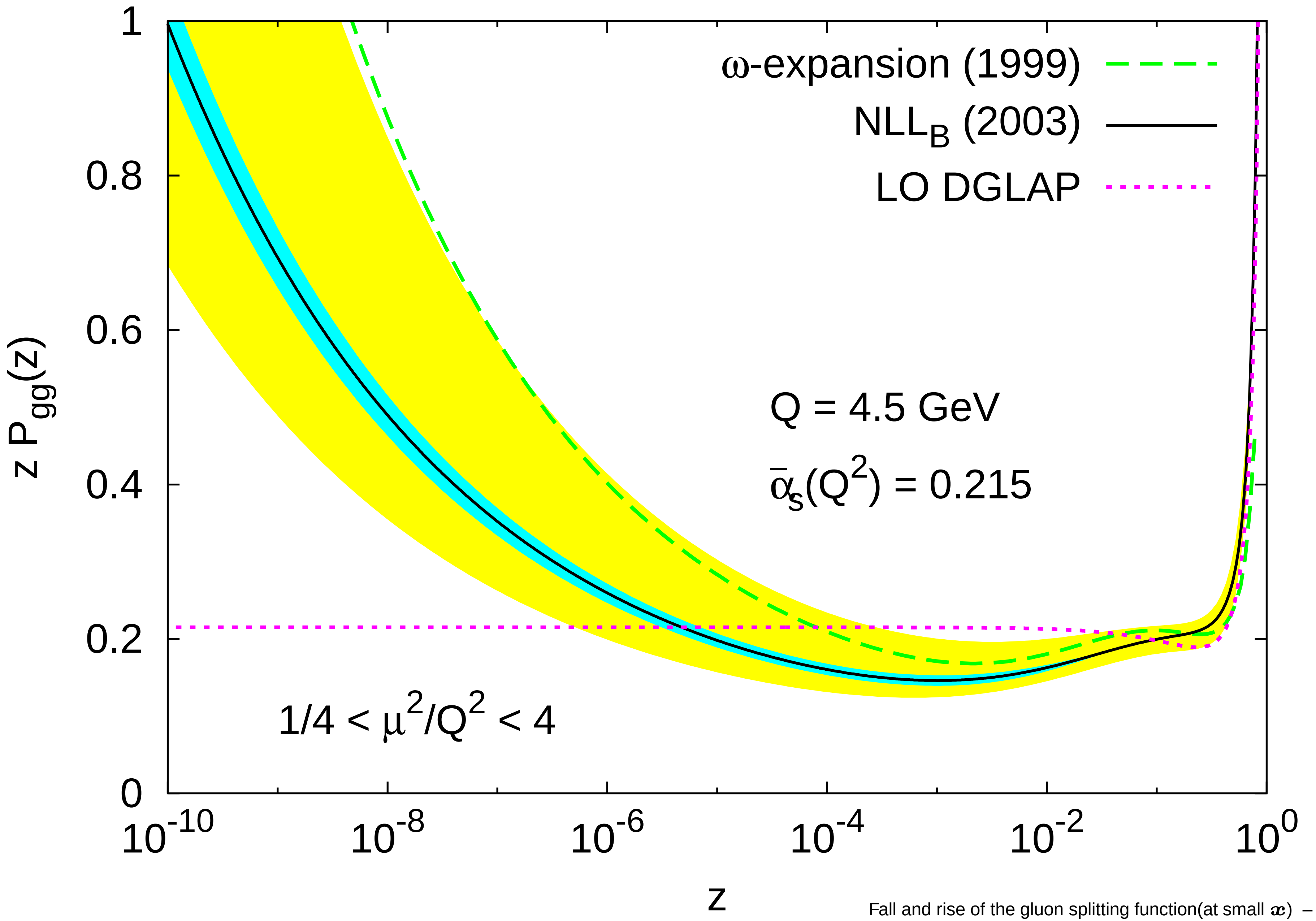

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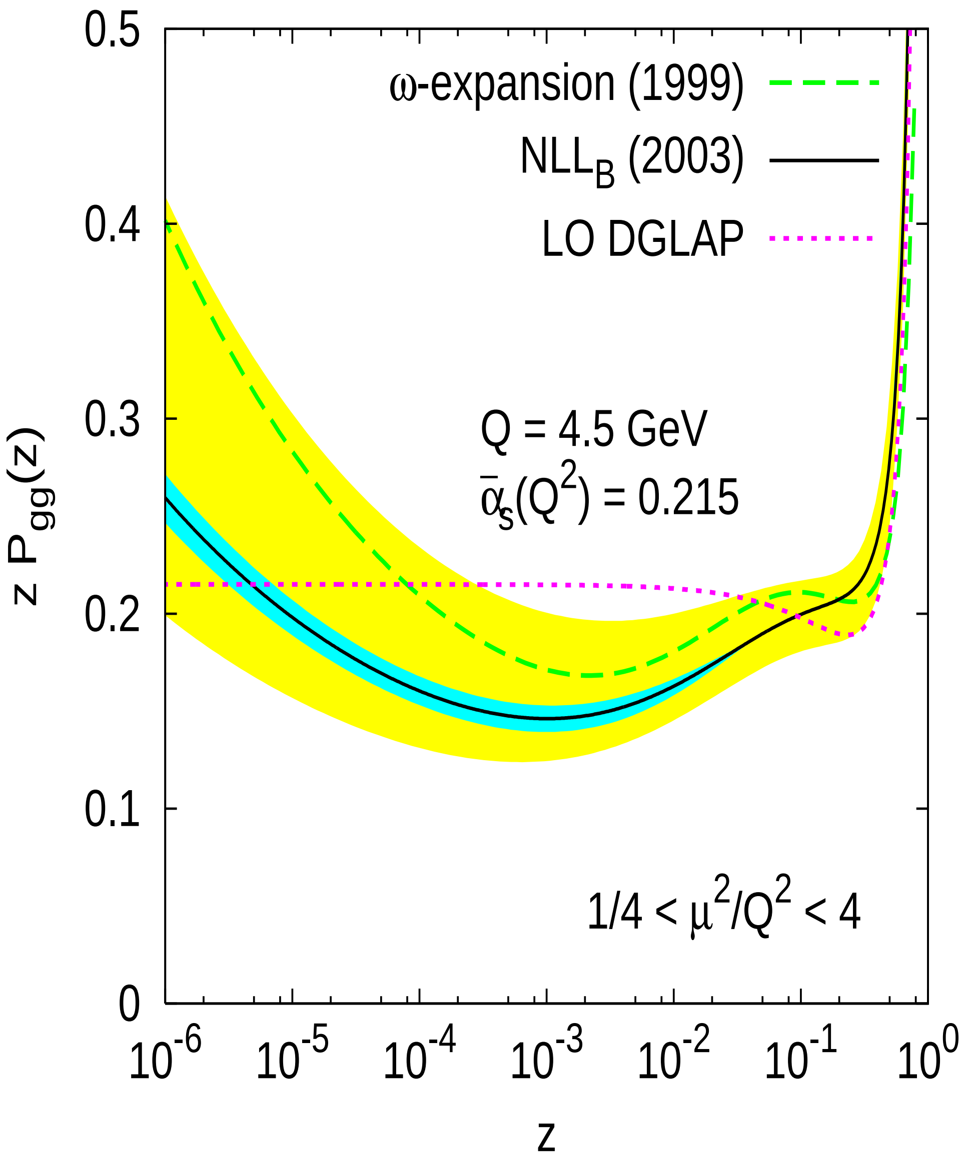
- Main feature is a *dip at* $x \sim 10^{-3}$



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Questions:

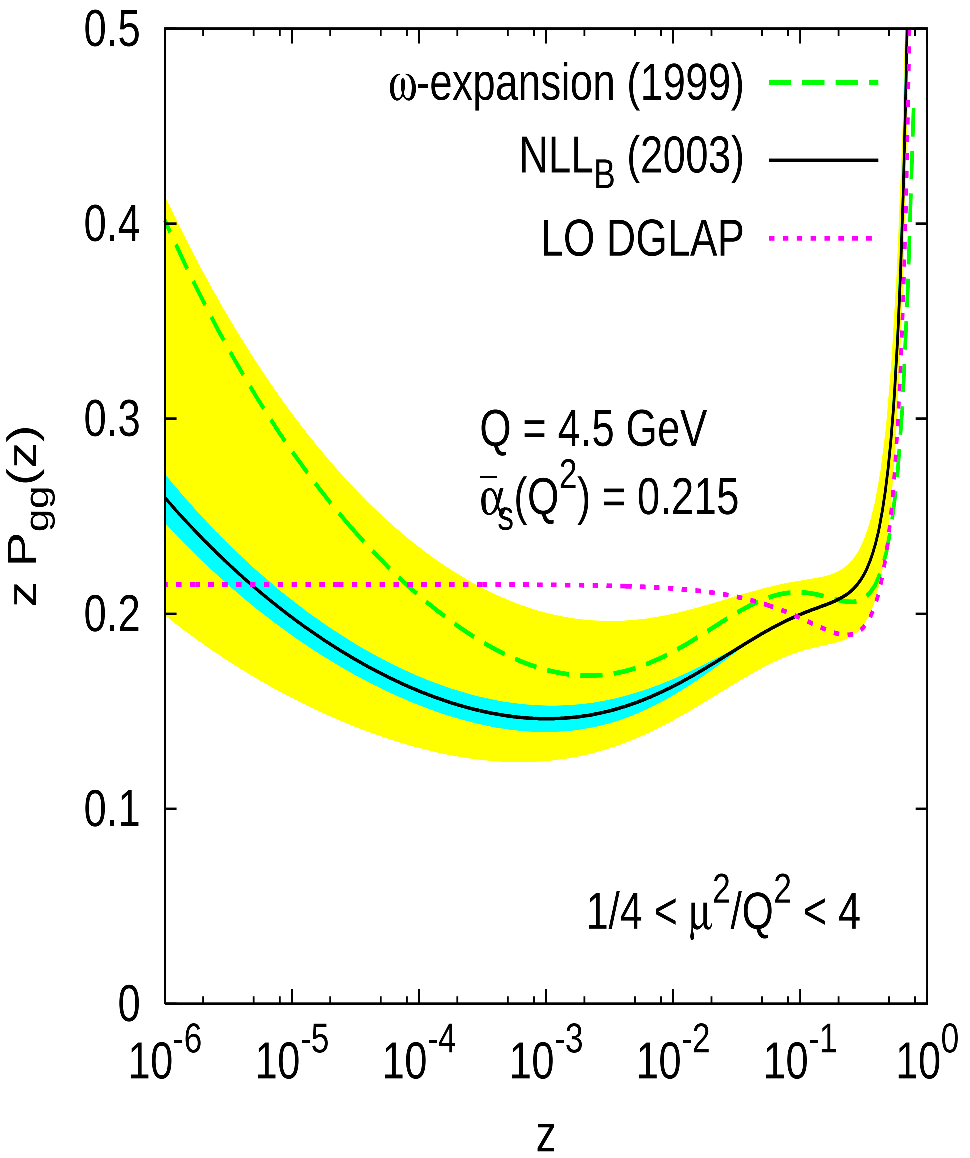
- Various 'dips' have been seen

  Thorne '99, '01 (running α_s, NLLx)

  ABF '99–'03 (fits, running α_s)

  CCSS '01,'03 (running α_s, NLL$_B$)

  Is it always the same dip?



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- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at* $x \sim 10^{-3}$

Questions:

- Various 'dips' have been seen

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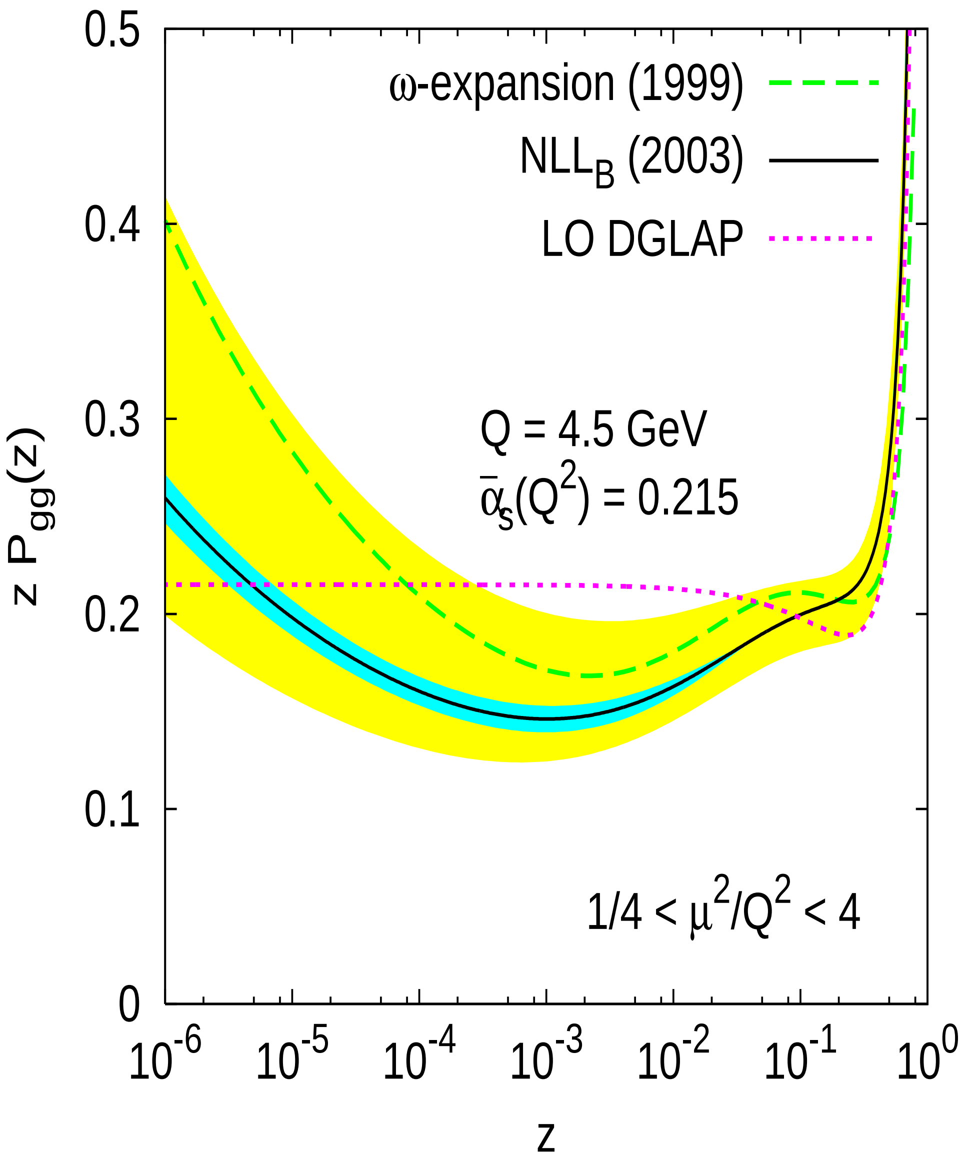

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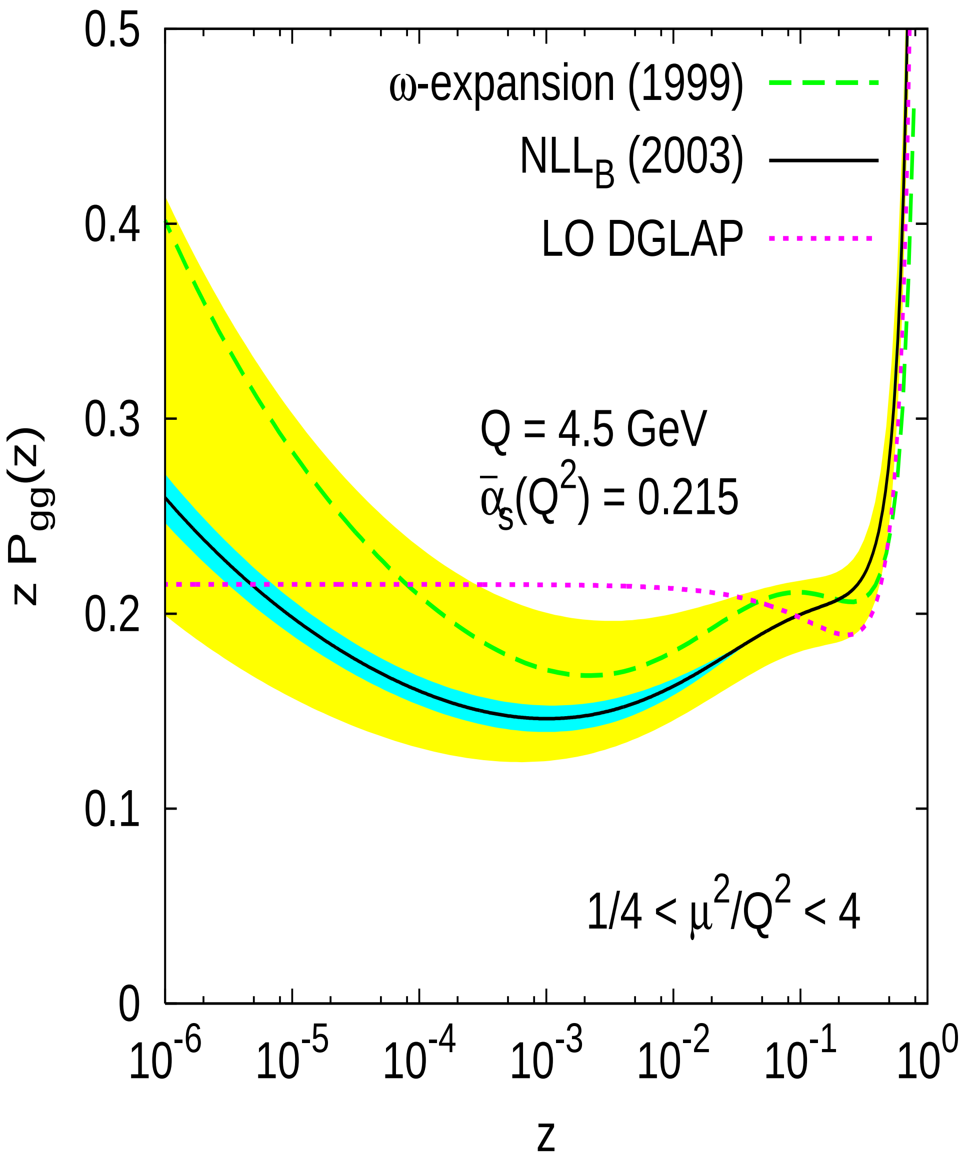

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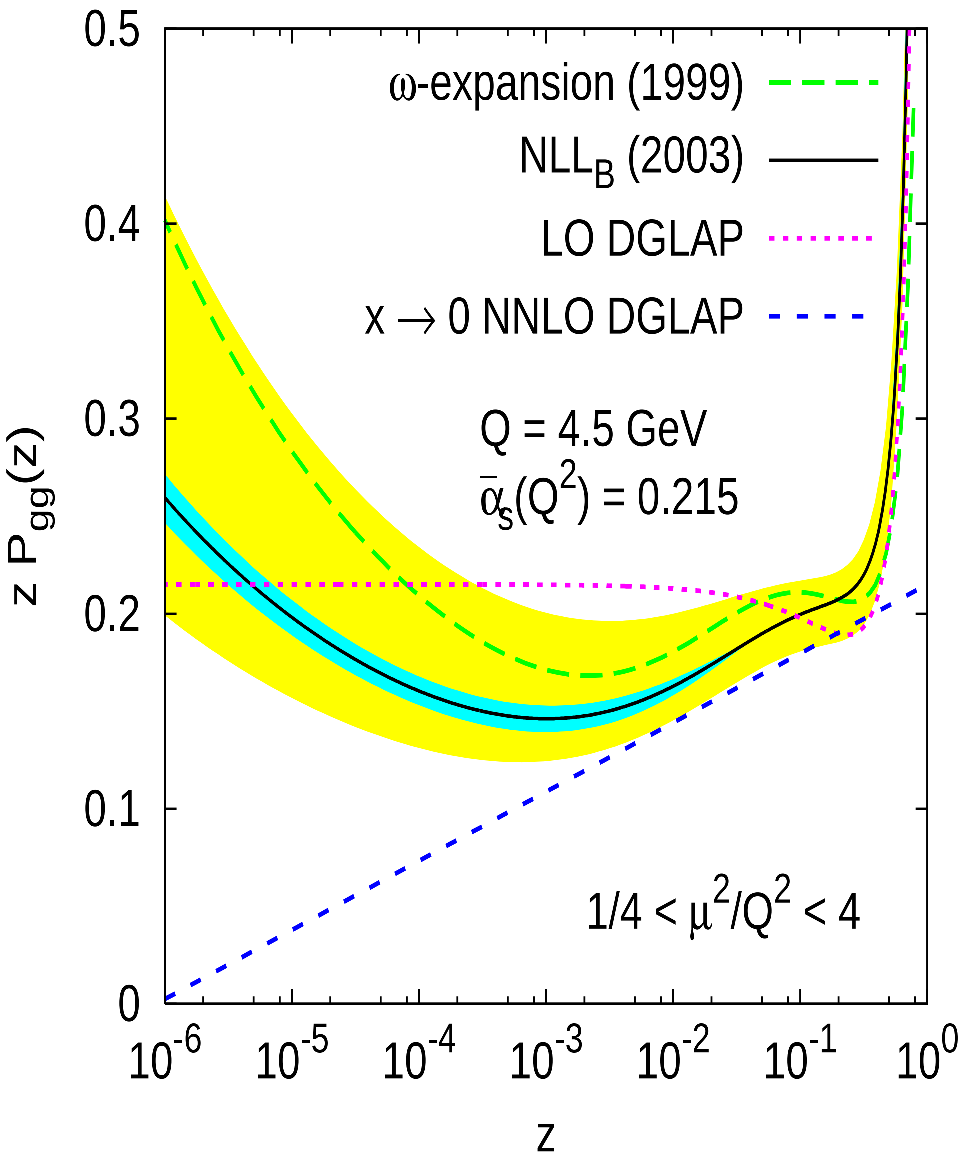

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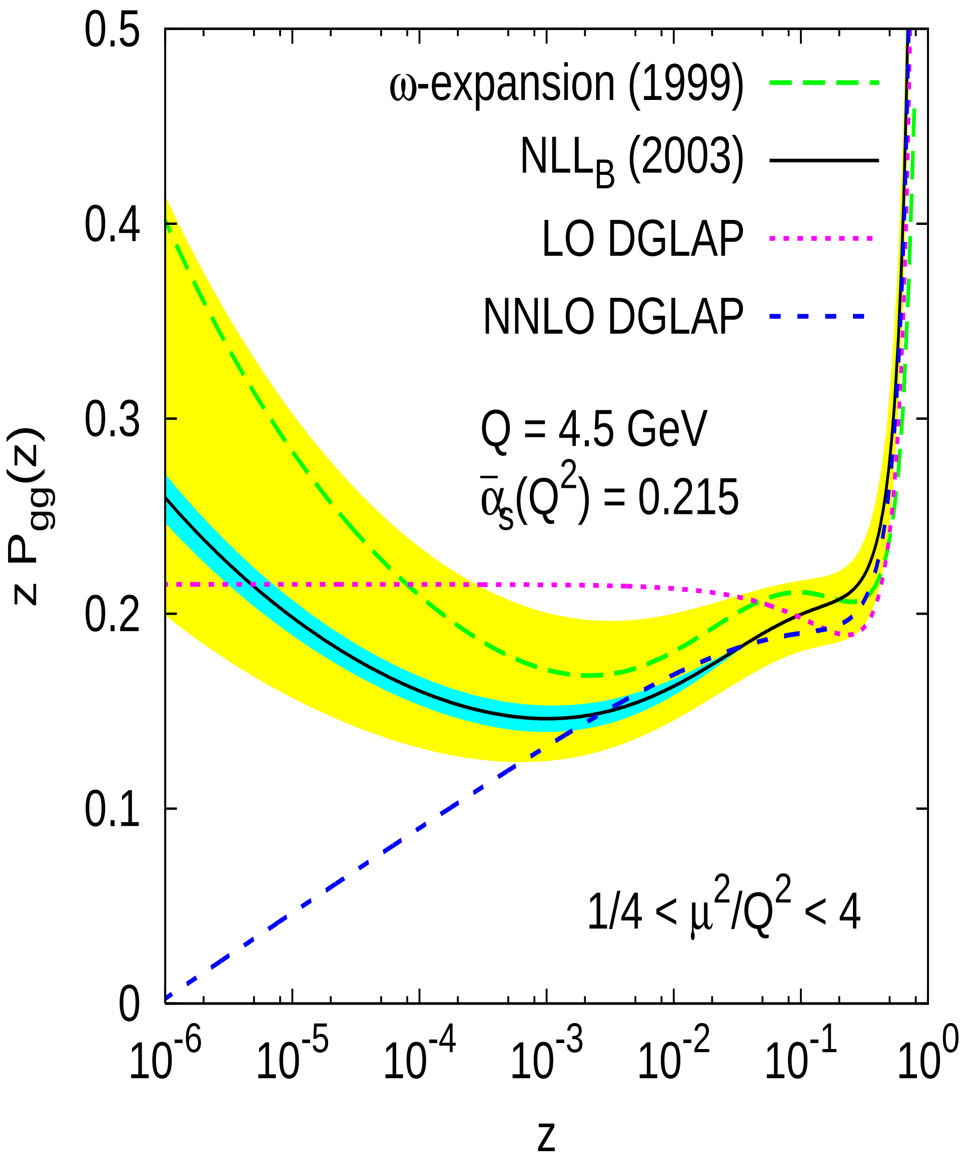

Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
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$\vdots$				

const.

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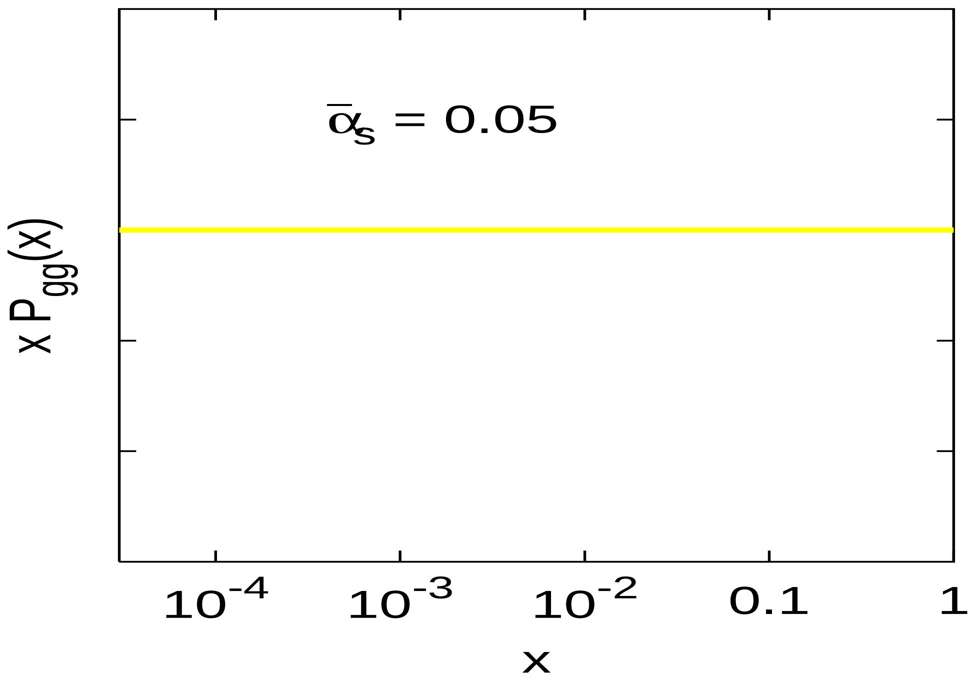

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At moderately small x, first terms with x-dependence are

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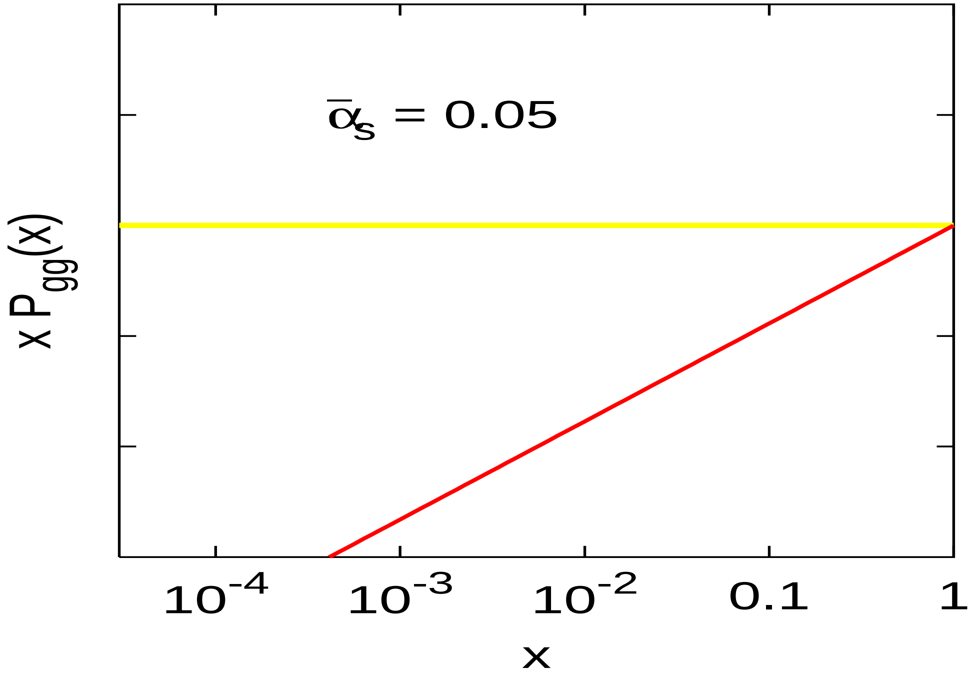

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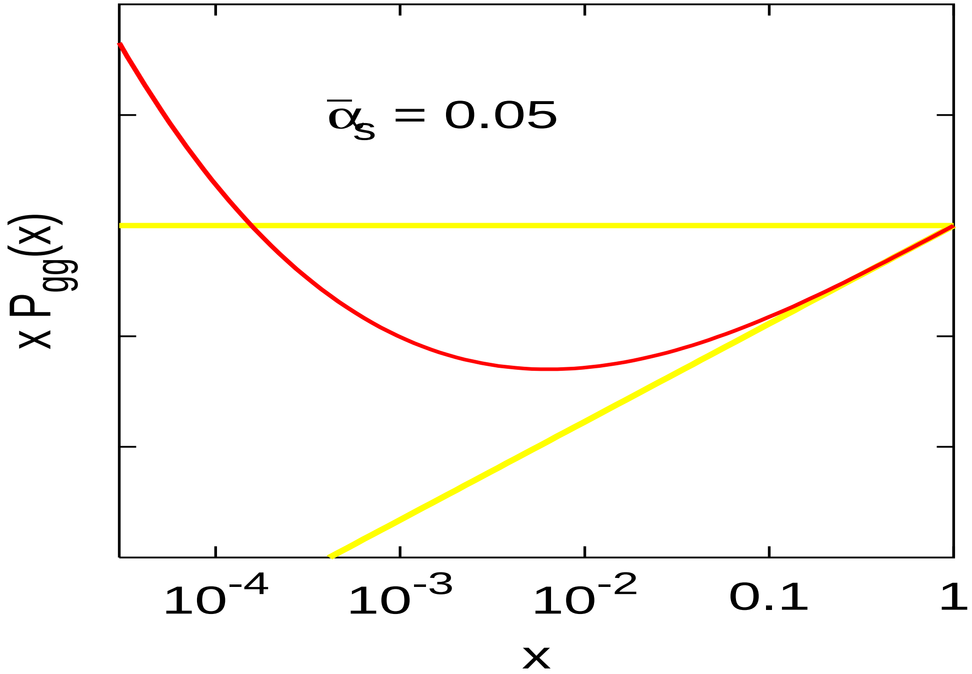

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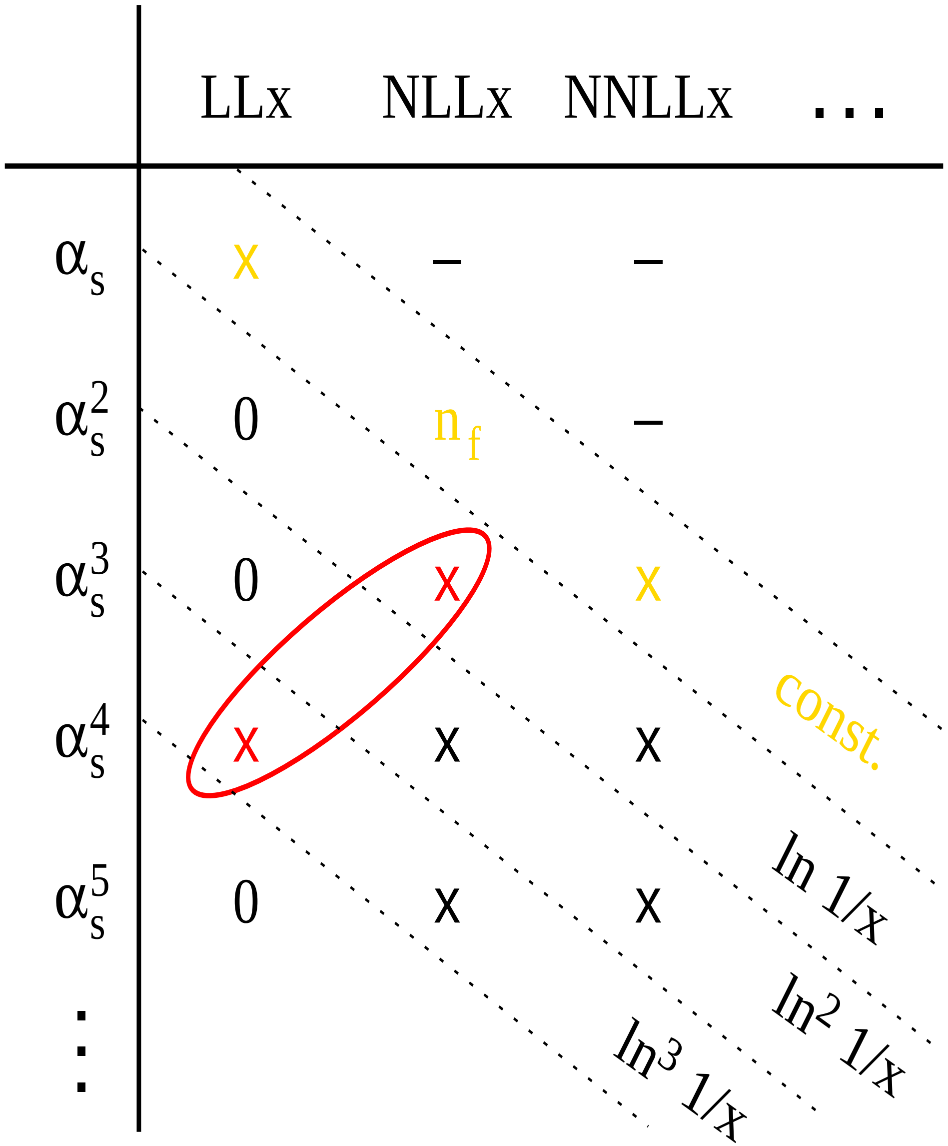

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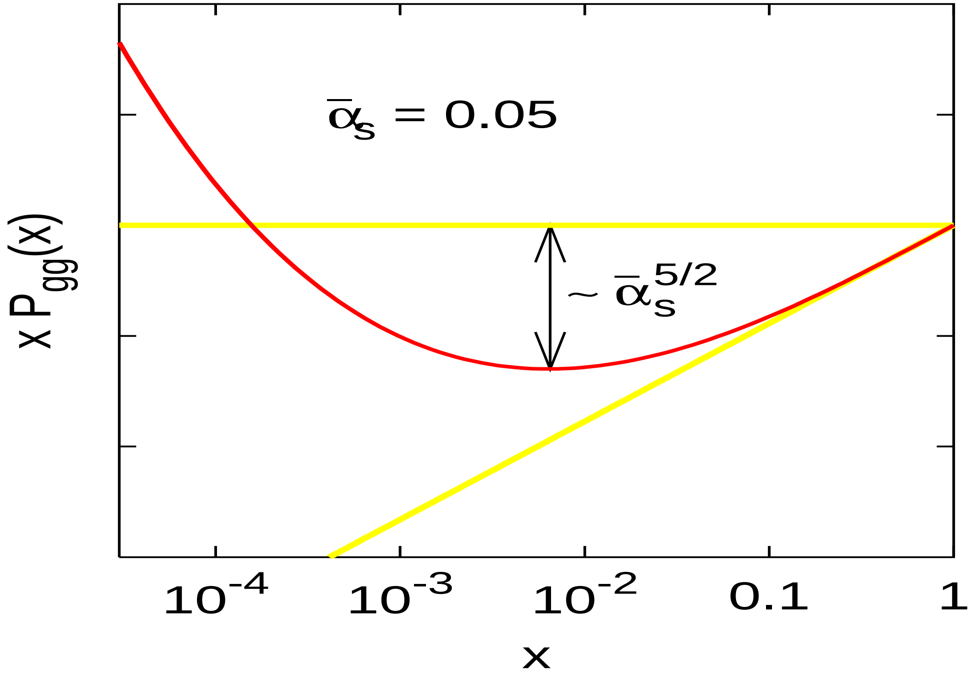
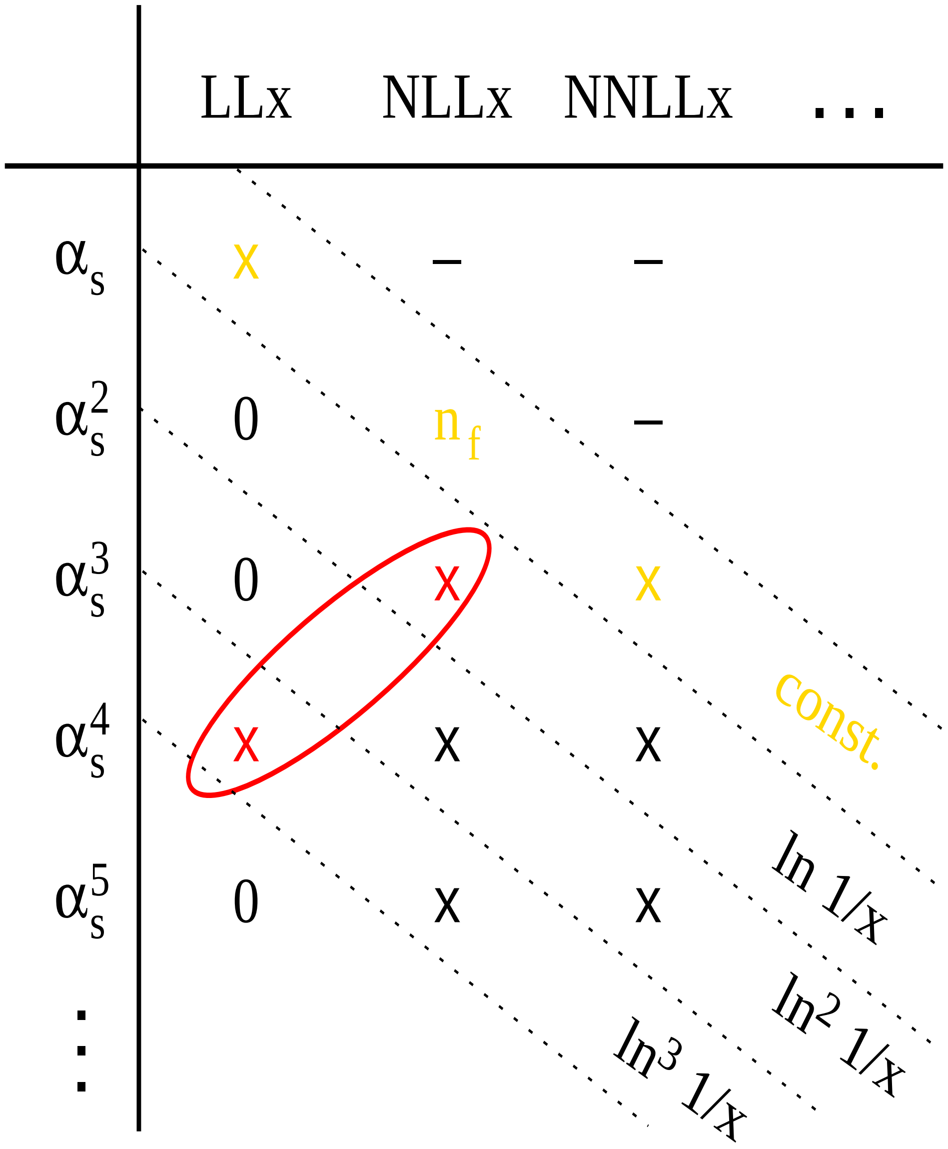
Minimum when

$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln\frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$



Systematic expansion in $\sqrt{\alpha_s}$



Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}}$$

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$$-d \simeq -1.237\bar{\alpha}_s^{5/2}$$

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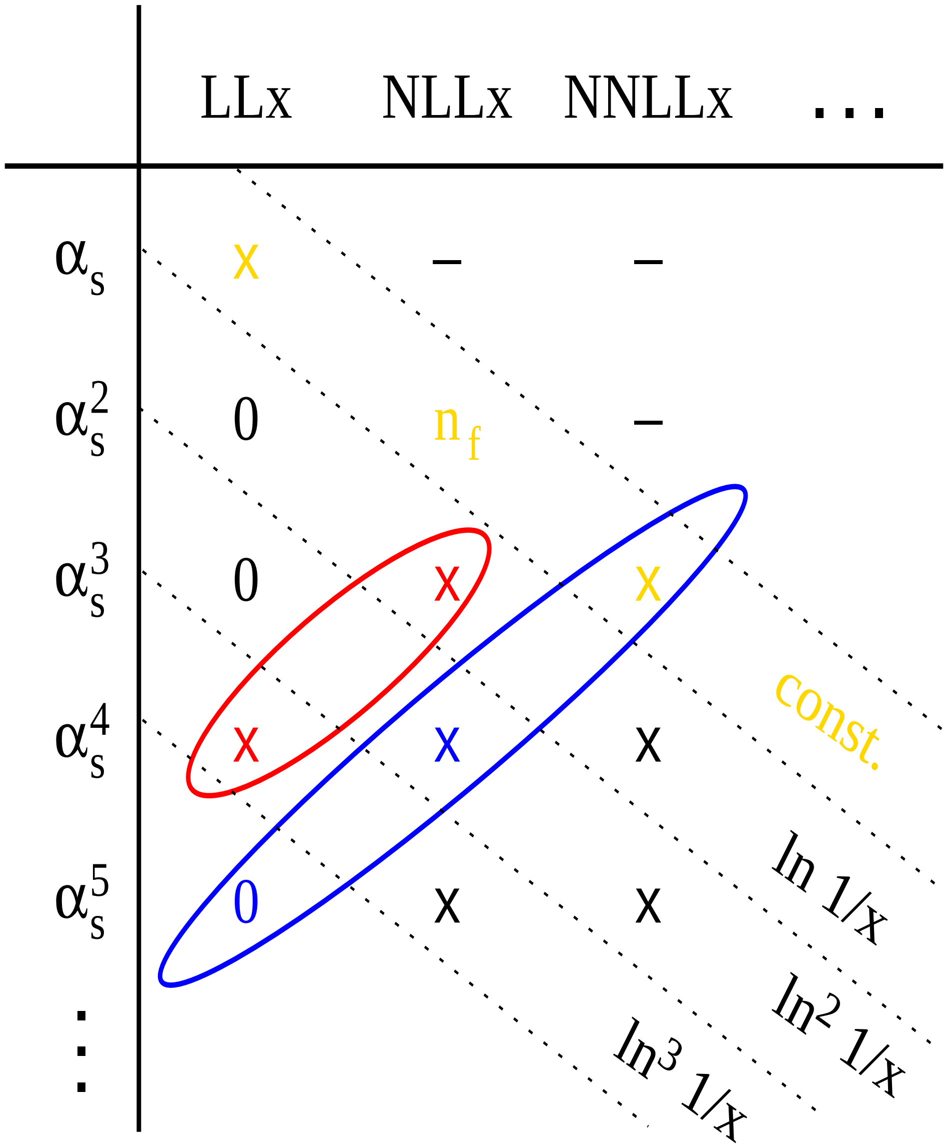

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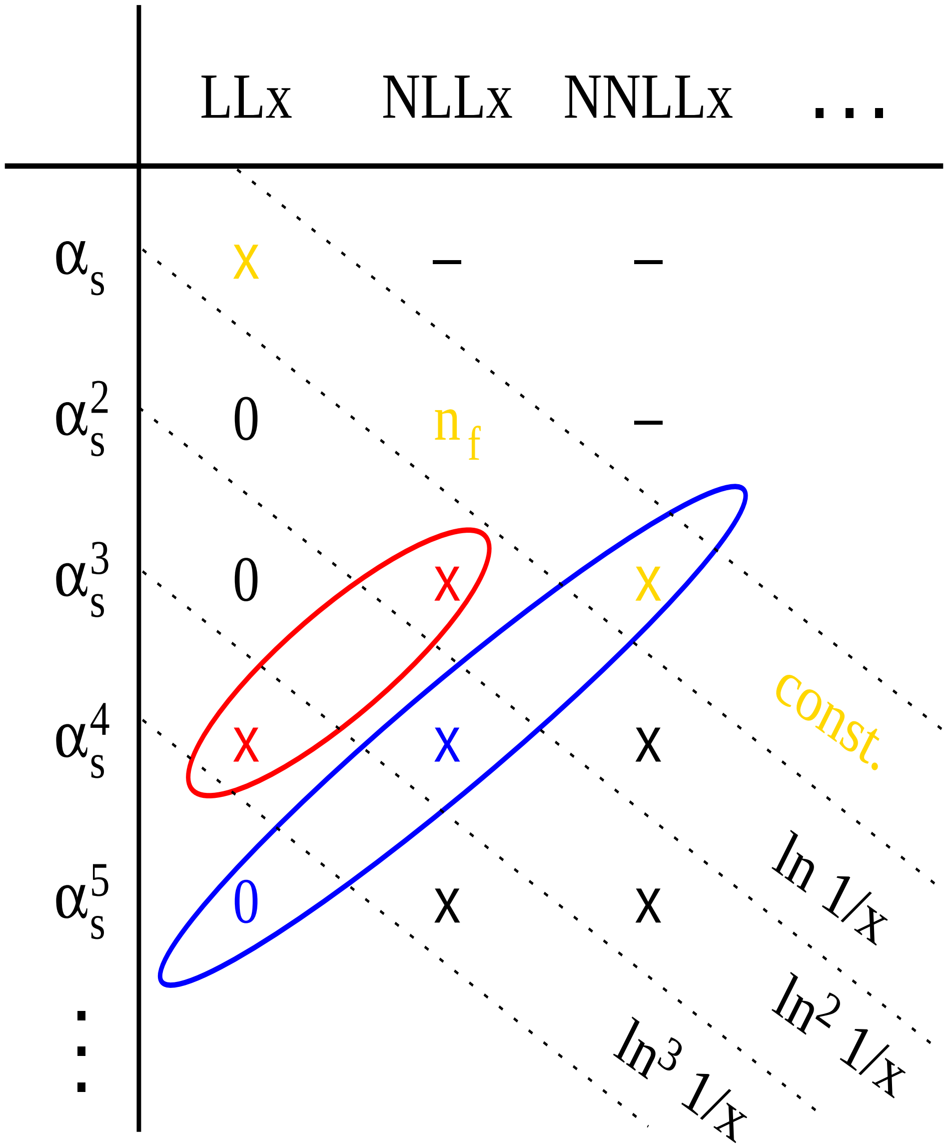

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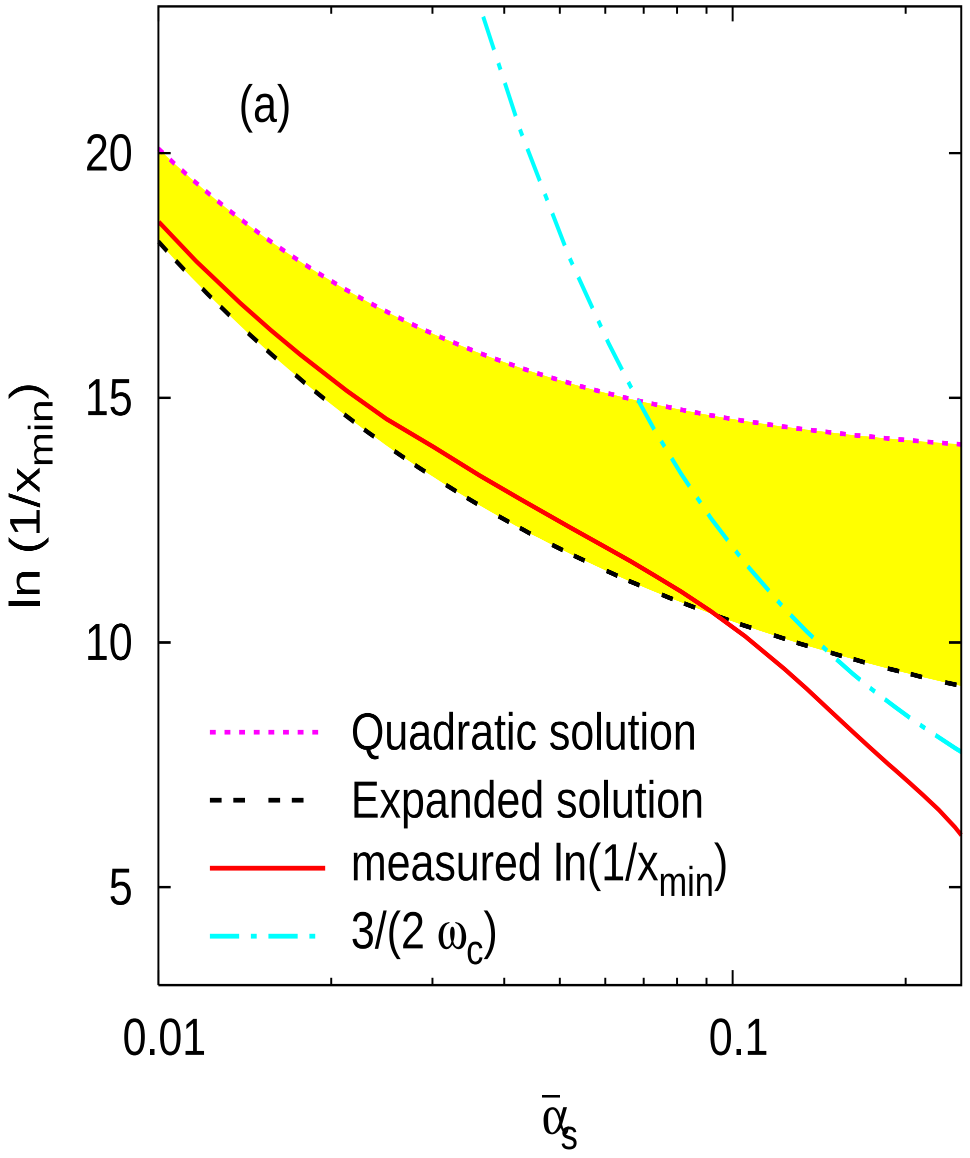
NB:

- convergence is very poor

  As ever at small x!

- higher-order terms in expansion need NNLLx info

Test dip properties v. BFKL+DGLAP resummation



Test position of dip v. α_s

- Band is uncertainty due to higher orders in $\sqrt{\alpha_s}$
- At small α_s, good agreement $\longrightarrow$ confirmation of 'dip mechanism'
- At moderate α_s, normal small-x resummation effects 'collide' with dip

$$\ln \frac{1}{x_{\min}} \lesssim \frac{3}{2\omega_c}$$

Dip then comes from interplay between $\alpha_s^3 \ln x$ (NNLO) term and full resummation.

[Actually, story more complex]

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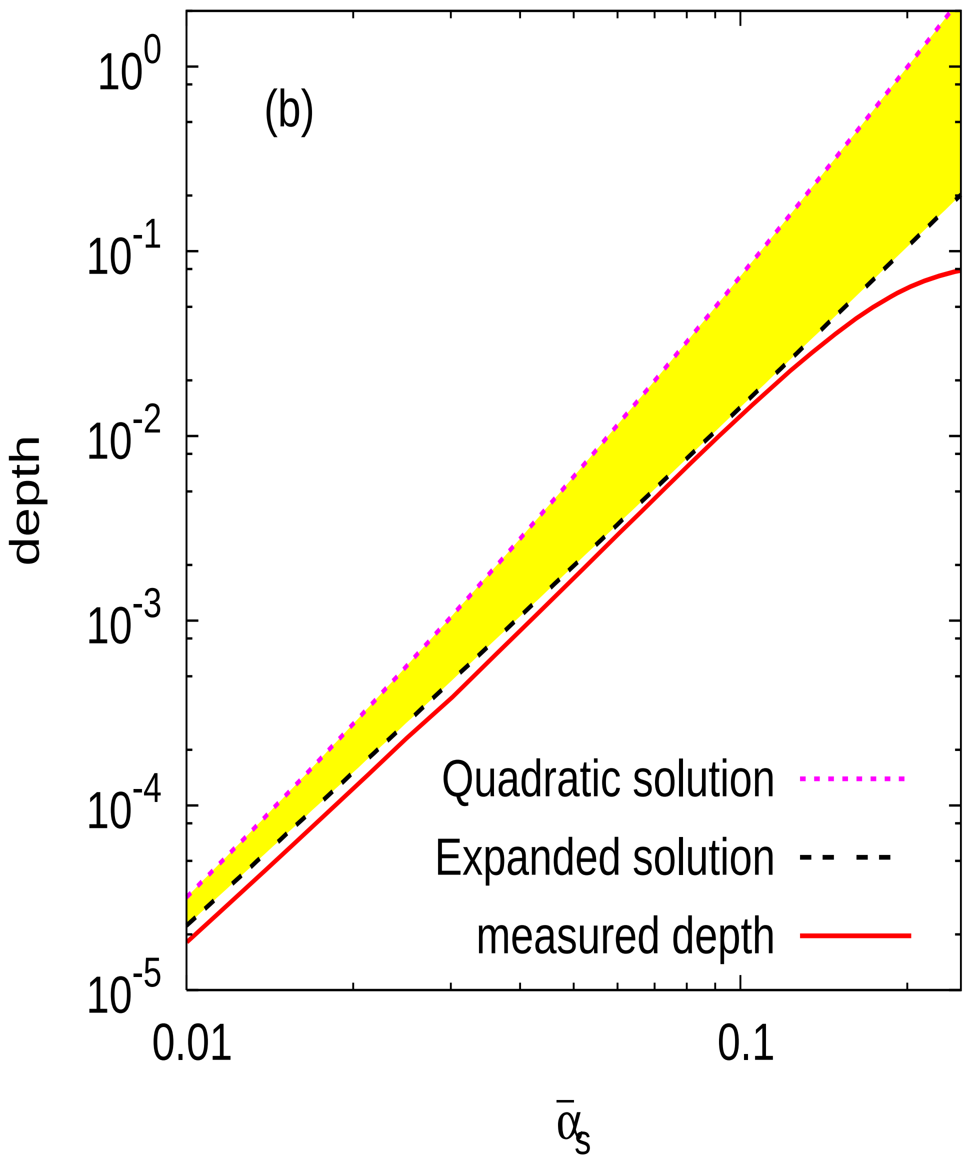

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- similar conclusions!

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- Work still needed for phenomenology. . .