Factorisation and diffusion at small $x^\dagger$

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How far can we take BFKL before being affected by the non-perturbative region?

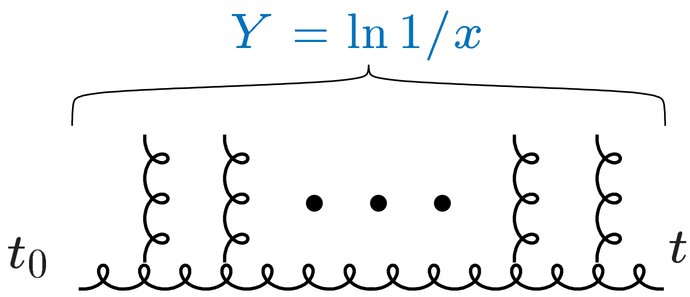
Look in two contexts:

- DIS: Factorisation .
- Mueller-Navelet, forward jets, $\gamma^*\gamma^*$: Diffusion & tunneling.

† Talk given at the Florence QCDNET meeting, September 1999. This file is a slightly revised version of January 2000.

Look at Green's function ($t = \ln Q^2$):

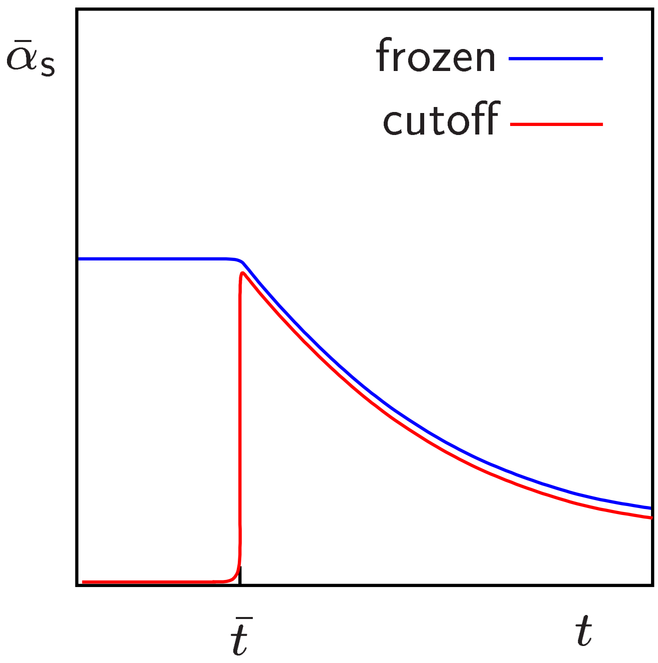
$G(Y, t_0, t)$



Use running $\bar{\alpha}_s$:

$$\bar{\alpha}_s(t) = \frac{\alpha_s N_c}{\pi} = \frac{1}{bt}$$

regularised at scale $\bar{t} \sim 1$.



High-energy behaviour is generally a power: $G(Y) \sim e^{\omega Y}$. Different powers arise:

- Perturbative (PT) powers (cf. Dimitri's talk): $\omega_c(t)$, $\omega_s(t)$, both of $\mathcal{O}(\bar{\alpha}_s(t))$.
- Non-perturbative (NP) power: $\omega_{\mathbb{P}} \sim \mathcal{O}(\bar{\alpha}_s(\bar{t}))$, exact value depends also on regularisation.

Since $t \gg \bar{t}$, we have $\omega_{\mathbb{P}} \gg \omega_s(t) \sim \omega_c(t)$. At very high energies, terms with NP power dominate.

DIS: $G(Y, t_0, t)$ with $t \gg t_0 \simeq \bar{t}$

Since $t_0 \simeq \bar{t}$, evolution starts off NP: Green's function cannot be calculated; as usual in DIS... What about anomalous dimensions?

Normal DIS: Mellin transform

$$G_\omega(t_0, t) = \int \mathrm{d}Y \, \mathrm{e}^{-\omega Y} G(Y, t_0, t)$$

satisfies

$$\gamma_{\text{eff}}(\omega, t) \equiv \frac{\mathrm{d}}{\mathrm{d}t} \ln G_\omega(t_0, t)$$

where the anomalous dimension, $\gamma_{\text{eff}}(\omega, t)$, depends only on perturbative QCD + higher twist pieces. This is factorisation.

Mueller (1996) suggested out that factorisation might not hold at small x, because of diffusion: since BFKL branching is not ordered in transverse momentum, gluon chains can come in and out of NP region.

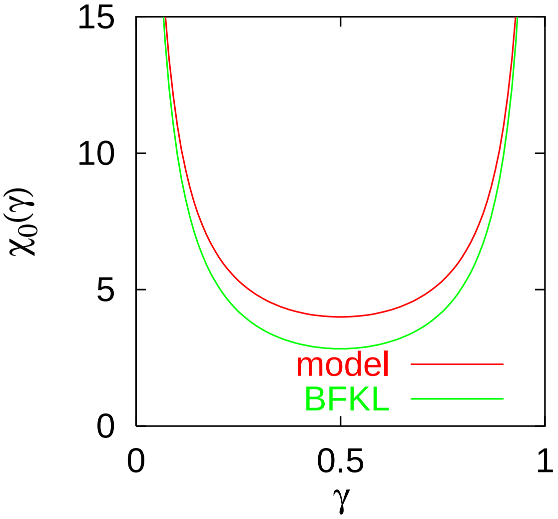
Collinear model (simple version)

Contains running coupling, diffusion and correct collinear behaviours:

$$\omega G_\omega(t,t_0) = \delta(t-t_0) +$$
$$+ \underbrace{\int_{-\infty}^{t} dt'\, \bar{\alpha}_s(t)\, G_\omega(t_0,t')}_{A} + \underbrace{\int_{t}^{\infty} dt'\, \bar{\alpha}_s(t')\, G_\omega(t_0,t')\, \mathrm{e}^{t-t'}}_{B}$$

Same polar structure as BFKL:

$$\chi_0(\gamma) = \frac{1}{\gamma} + \frac{1}{1-\gamma}$$
$$\chi_1(\gamma) = -\frac{b}{(1-\gamma)^2}$$



Easy to solve, as a coupled linear differential equation:

$$\begin{aligned}
\frac{\mathrm{d}A}{\mathrm{d}t} &= \bar{\alpha}_s G + \frac{\bar{\alpha}_s'}{\bar{\alpha}_s} A \\
\frac{\mathrm{d}B}{\mathrm{d}t} &= -\bar{\alpha}_s G + B \\
G &= A + B
\end{aligned}$$

Factorisation

Since problem is equivalent to a linear 2nd order DE, there are two independent solutions:

- F_R which is regular to the right ($t \to \infty$) — for $t > \bar{t}$ it is independent of the NP regularisation.
- F_L which is regular to the left ($t \to -\infty$) — contains all the dependence on the NP regularisation, including $\omega_{\mathbb{P}}$ singularity.

Green's function is

$$G_\omega(t, t_0) = F_R(t) F_L(t_0) \Theta(t - t_0) + (t_0 \leftrightarrow t)$$

And the anomalous dimension is

$$\gamma_{\text{eff}} = \frac{\mathrm{d}}{\mathrm{d}t} \ln F_R(t)$$

which has no dependence on $t \Rightarrow$ factorisation is exact in this model

Factorisation in BFKL — PRELIMINARY

Essential differences: many regular solutions, going as e^{-nt}, one for each pole, $1/(\gamma + n)$ of the characteristic function. Numerically, find coefficients such that

$$\gamma_{\text{eff}} = \gamma_{\text{eff,PT}} + \mathcal{O}\left(\frac{e^{-t}}{\omega - \omega'_{\mathbb{P}}}\right)$$

Translated to x, Q^2 space, splitting function has a higher-twist piece which grows rapidly at small x:

$$xP_{gg}(x, Q^2) \sim \underbrace{\bar{\alpha}_s^2\, x^{-\omega_c(Q^2)}}_{\text{PT}} + \mathcal{O}\left(\frac{x^{-\omega'_{\mathbb{P}}}}{Q^2}\right)$$

$(\omega_c(Q^2) \sim \bar{\alpha}_s(Q^2) \ll \omega'_{\mathbb{P}} < \omega_{\mathbb{P}})$

But after convolution with Green's function (which grows as $x^{-\omega_{\mathbb{P}}}$), relative correction to scaling violations are small

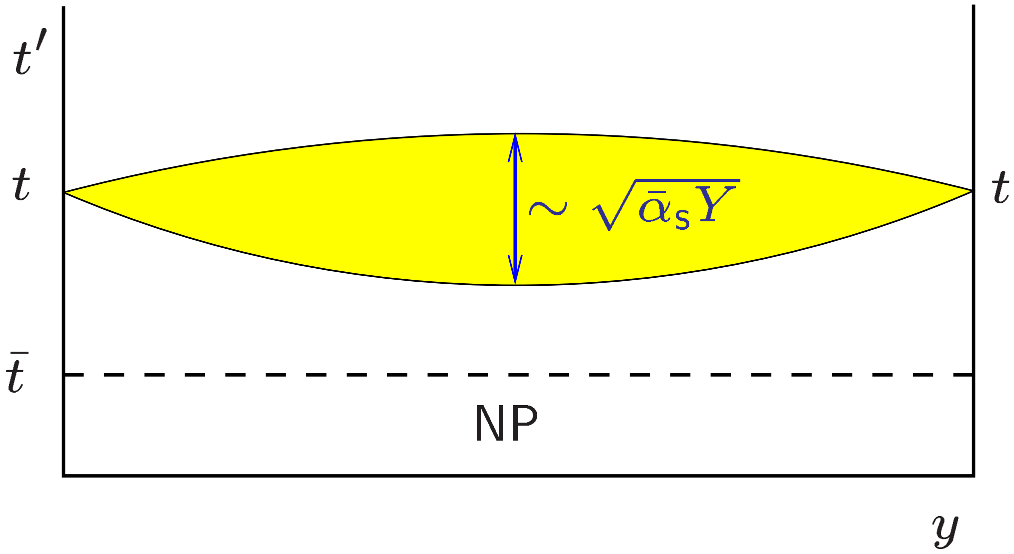
$$\mathcal{O}\left(\frac{1}{Q^2}\right)$$

$\gamma^*\gamma^*$, forward jets: processes with $t_0 \simeq t \gg \bar{t}$

Green's function is perturbatively calculable up to a certain Y. To see what Y, look at Bartels cigar: contour plot of

$$\frac{G(y, t_0, t')G(Y-y, t', t)}{G(Y, t_0, t)}$$

Coloured region represents domain of dominant paths in transverse momentum–rapidity space, reached through diffusion:

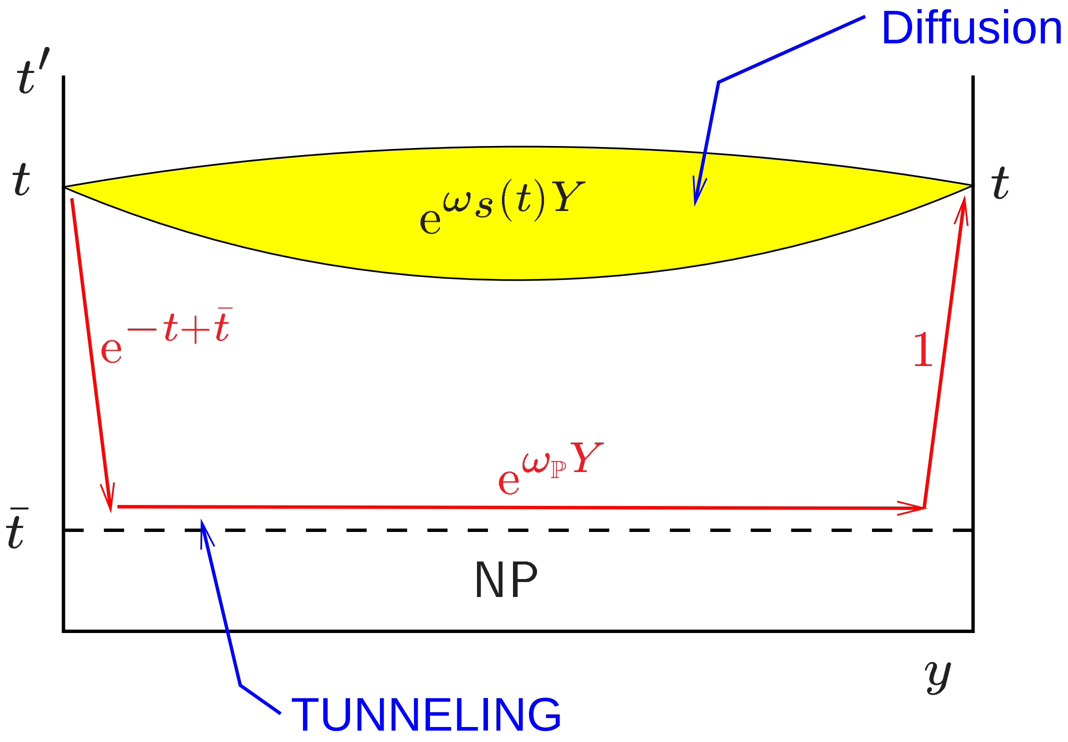


Make simple estimate of when cigar reaches NP region. Width must be $\simeq t \sim 1/\bar{\alpha}_s$ (Mueller):

$$\sqrt{\bar{\alpha}_s Y_{NP}} \sim \frac{1}{\bar{\alpha}_s} \Rightarrow \boxed{Y_{NP} \sim \frac{1}{\bar{\alpha}_s^3} \sim t^3}$$

Tunneling

BFKL has more than diffusion. Diffusion is exact when all steps in t are small. In QCD, large changes in t are possible for a moderate price:



Normal evolution and tunneling compete ($\omega_{\mathbb{P}} \gg \omega_s(t)$):

$$G(Y, t, t) \sim e^{\omega_s(t)Y} + e^{-t+\omega_{\mathbb{P}}Y}$$

Tunneling takes over when

$$\boxed{Y \simeq \frac{t}{\omega_{\mathbb{P}} - \omega_s(t)}}$$

NP region enters through tunneling ($\frac{t}{\omega_{\mathbb{P}}-\omega_s}$) long before it does through diffusion (t^3).

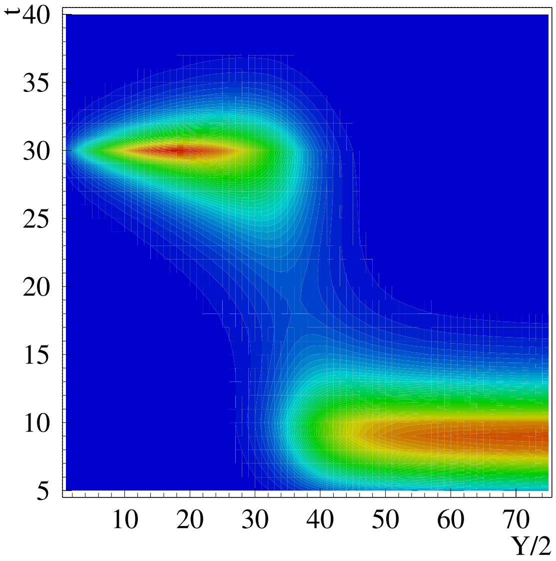
Animation

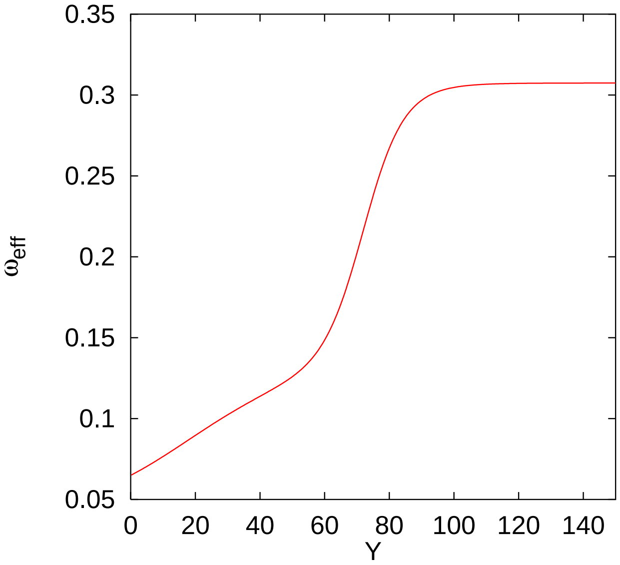
Illustrate process with an animation: show 'Bartels cigar' for parameters: $t_0 = t = 30$, $\bar{t} = 10$, as a function of Y. Extreme parameters have been chosen in order to illustrate clearly the relevant processes.

Shown for collinear model with b=1 and no higher order effects other than running coupling (quick to implement).

Changing collinear model $\rightarrow$ BFKL does not change the basic characteristics.

Bartels duck





Take line through middle of cigars, as a function of Y

$$\frac{[G(Y/2,t,t')]^2}{G(Y,t,t)}$$

Effective exponent:

$$\omega_{\text{eff}} \equiv \frac{d \ln G(Y,t,t)}{dY}$$

At point where NP takes over, see a step in ω_{eff}.

Conclusions

Deep Inelastic Scattering:

- Diffusion is not incompatible with exact factorisation: e.g. collinear model.
- BFKL does have $1/Q^2$ corrections: while enhanced ($\sim x^{-\omega_{\mathbb{P}}}$) for small-$x$ splitting functions, they are small for the scaling violations ($\sim \ln x$).

Processes with two (similar) hard scales, $\gamma^*\gamma^*$, etc.:

- In addition to diffusion, there is tunneling.
- Tunneling puts the limit on the applicability of perturbative QCD:

$$Y \simeq \frac{t}{\omega_{\mathbb{P}} - \omega_s(t)}(1 + \omega_{\mathbb{P}})$$

- At moderate Q^2 values tunneling and diffusion are probably equally relevant.
- Big question is: what is $\omega_{\mathbb{P}}$? Almost certainly it will be a unitarised $\omega_{\mathbb{P}}$ . . .