CAESAR: Computer automated resummations

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9eme Recontre Itzykson
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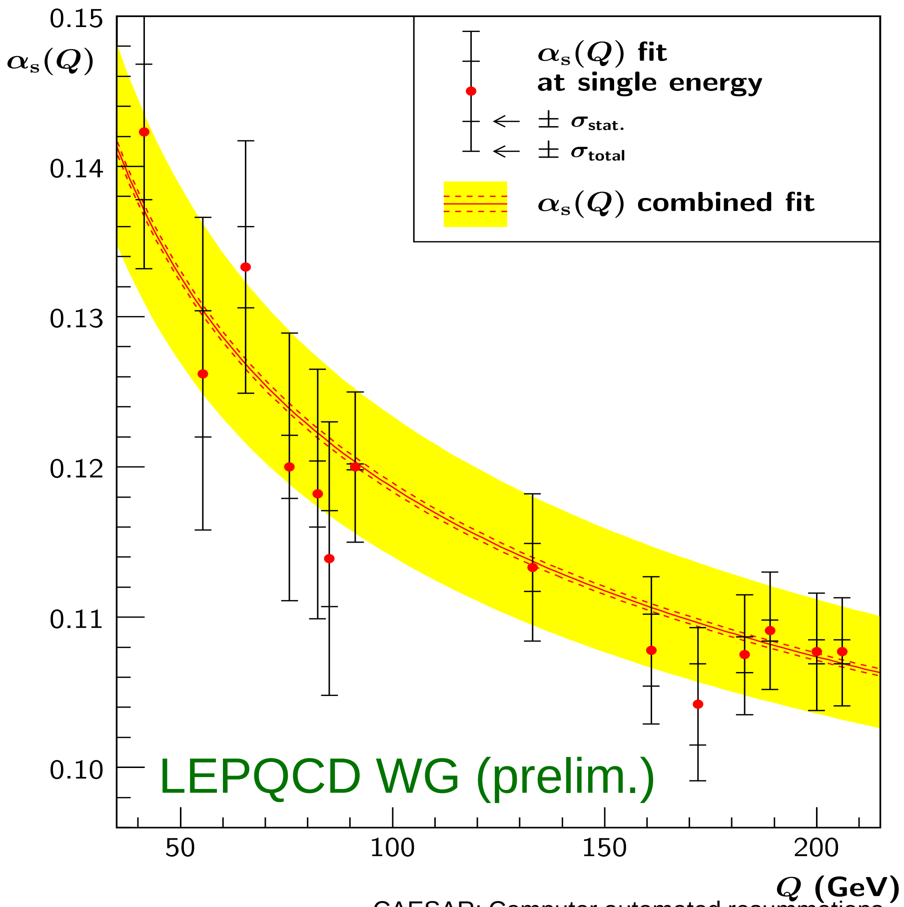

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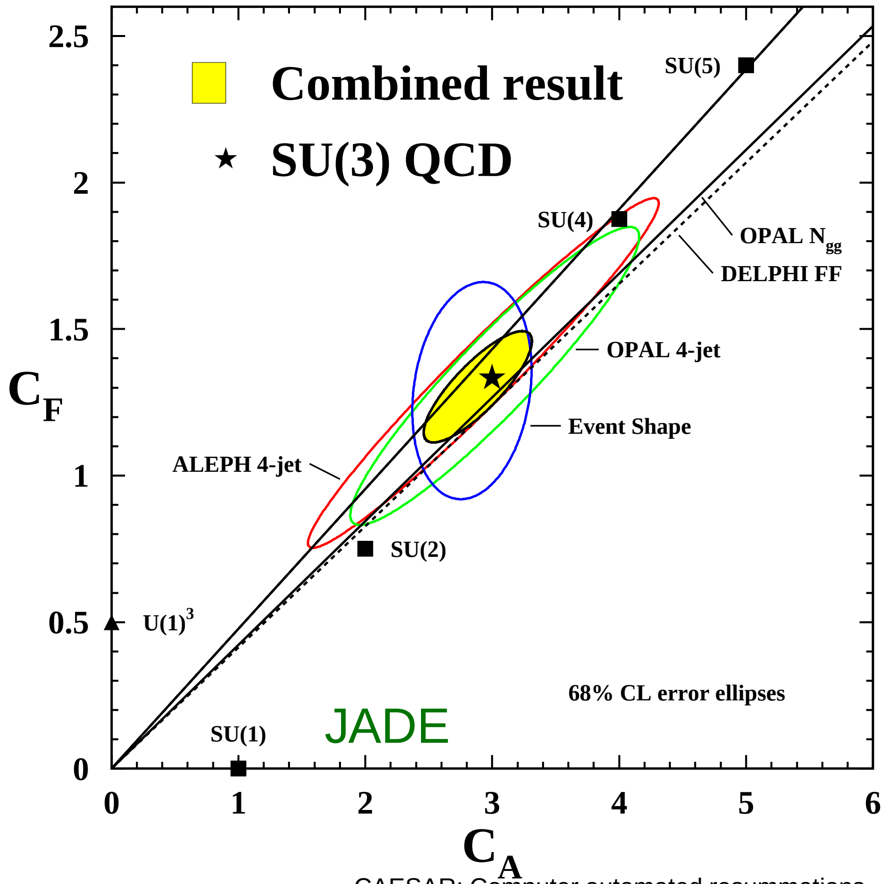

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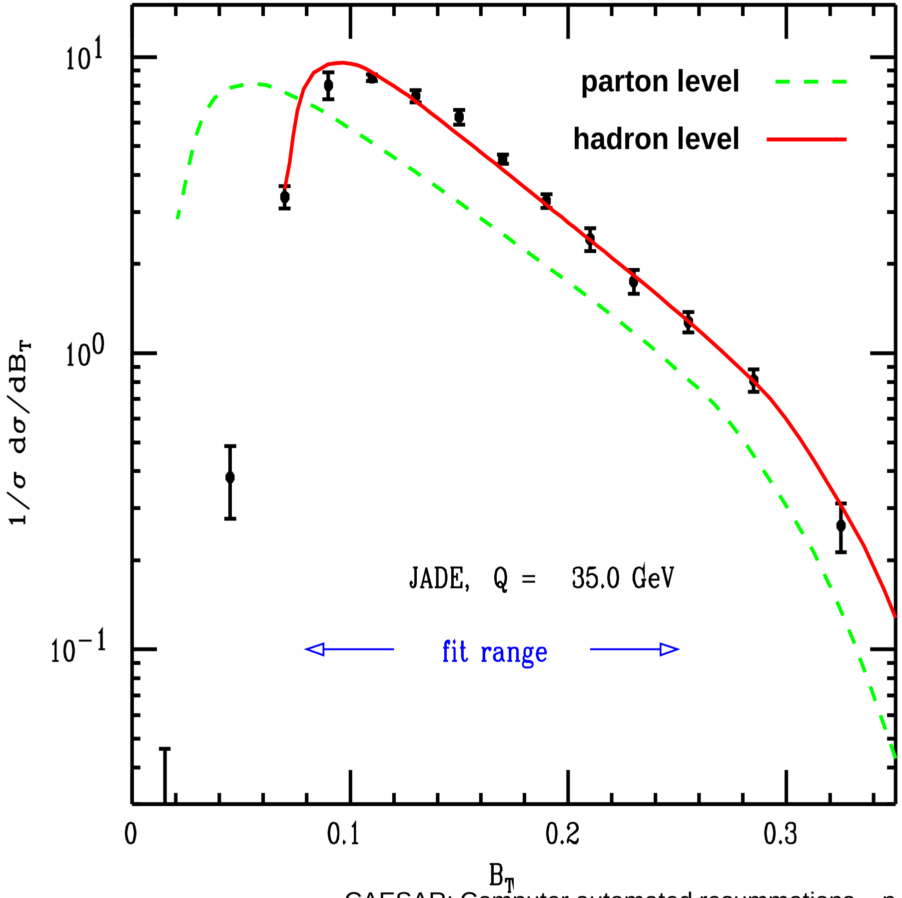
- Measurements/cross checks of the values of the colour factors of QCD
- Studies of connection between parton-level (QCD calculations of quarks and gluons) and (the real) hadron-level



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- Studies of event-shapes for three (2+1) jet events is vital for testing non-perturbative (NP) models
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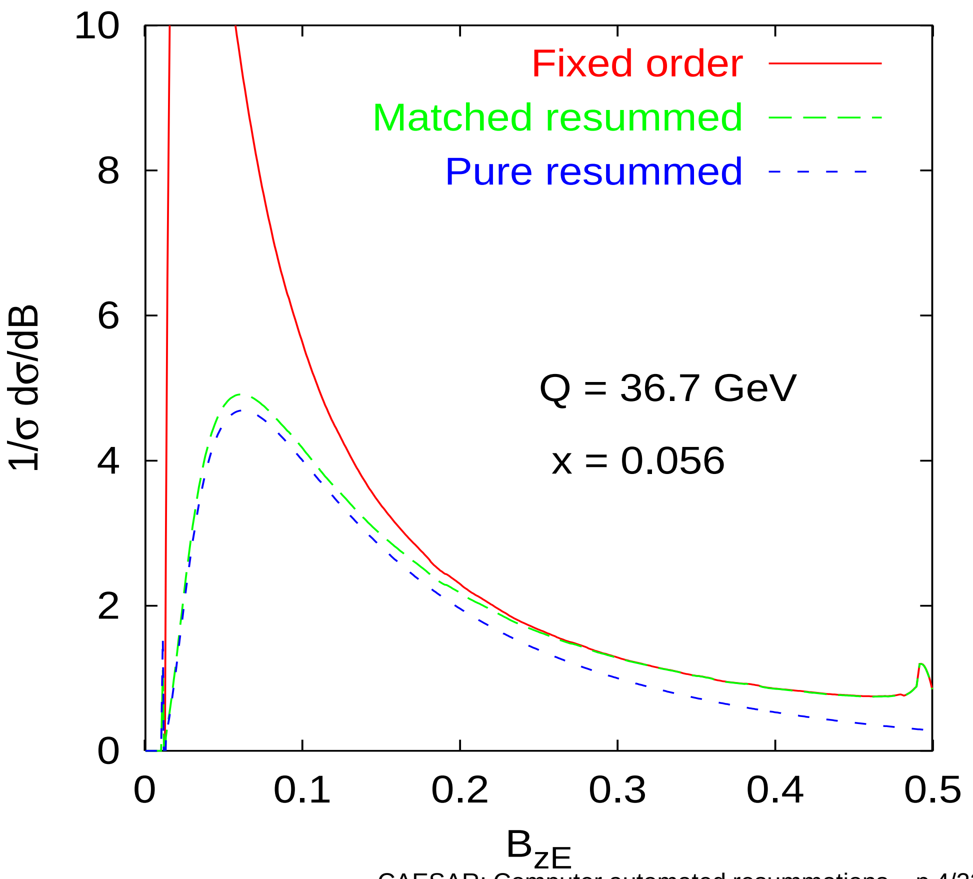
- A wide range of studies are possible in hadron-hadron colliders (dijet events, W, Z + jet)
  - analogous studies to those in e^+e^- and DIS
  - purely analytical studies of underlying event and minium bias properties

Need the theoretical tools

- Account for hard gluon/quark radiation (fixed order)
  - ✓ Programs calculate NLO predictions given subroutine for observable
  - ✓ Many processes: Event2, Disent, Disaster++, NLOJET++, MCFM, . . .
  - ✗ distributions diverge in soft/collinear region
- Account for multiple soft/collinear radiation (NLL resummation)
  - ✓ Ensures sensible distribution
  - ✗ Has to be done by hand for each observable & process



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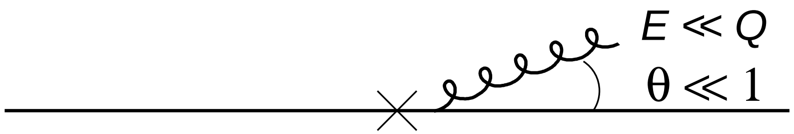

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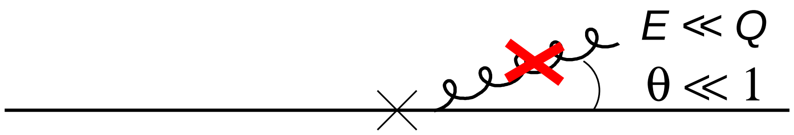

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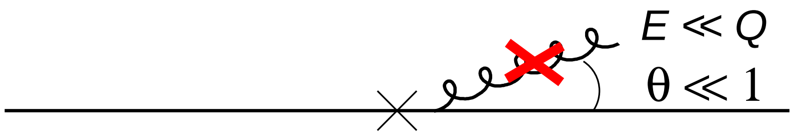
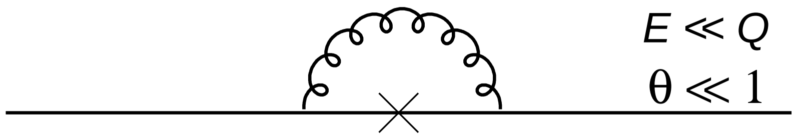


- virtual corrections are unaffected

$$-\frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(\theta E)$$



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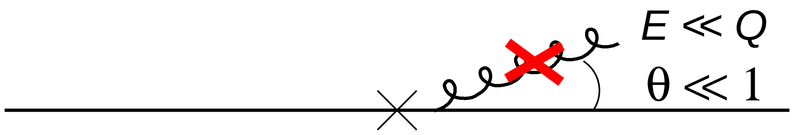

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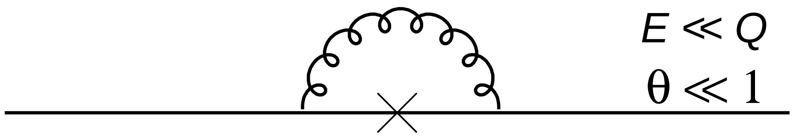

Imbalance leads to large logarithms in distribution of event shape:

$$\mathrm{Prob}(1-T<\tau) \simeq 1 - \frac{\alpha_s C_F}{2\pi}\ln^2\tau + \ldots \qquad (\tau \ll 1)$$

Resummation: Large Logarithms at all orders

There is a soft and a collinear divergence ($\mapsto$ logs) for each emitted gluon.

At all orders, probability of event being two-jet like has *poorly convergent perturbation series:*

$$P(1 - T < \tau) \equiv \Sigma(\tau) = 1 + \sum_{n=1} R_{n,2n}\, \alpha_s^n \ln^{2n} \tau + \dots$$

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Today's state of the art involves *exponentiation* and resummation of *Leading Logs* (LL) and *Next-to-Leading Logs* (NLL):

$$\Sigma(\tau) \simeq \exp\left[\sum_{n=1} \Big(\underbrace{G_{n,n+1}\, \alpha_s^n \ln^{n+1} \tau}_{\text{LL}} + \underbrace{G_{n,n}\, \alpha_s^n \ln^{n} \tau}_{\text{NLL}} + \cdots \Big)\right]$$

NB: $\alpha_s^n \ln^{2n} \tau$ in Σ, but only $\alpha_s^n \ln^{n+1} \tau$ in exponent.

The analytical resummation industry. . .

$e^+e^- \to$ 2 jets

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F. Krauss and G. Rodrigo, *Resummed jet rates for e^+e^- annihilation into massive quarks,* Phys. Lett. B **576** (2003) 135

DIS 1+1 jet

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M. Dasgupta and GPS, *Resummed event-shape variables in DIS,* JHEP **0208** (2002) 032

e^+e^-, DY, DIS 3 jets

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A. Banfi , Y. L. Dokshitzer, G. Marchesini and G. Zanderighi, *Near-to-planar 3-jet events in and beyond QCD perturbation theory,* Phys. Lett. B **508** (2001) 269

A. Banfi , Y. L. Dokshitzer, G. Marchesini and G. Zanderighi, *QCD analysis of D-parameter in near-to-planar three-jet events,* JHEP **0105** (2001) 040

A. Banfi , G. Marchesini, G. Smye and G. Zanderighi, *Out-of-plane QCD radiation in hadronic Z0 production,* JHEP **0108** (2001) 047

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Average: 1 observable per article

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- Symbolic manipulation programs (Mathematica, etc.)? E.g. like Feyncalc.
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  - Matching with fixed order (LO, NLO, NNLO) is complex

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Hybrid analytical & numerical

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Computer Automated Expert Semi-Analytical Resummation (CAESAR)

In practice (1)?

Single emission properties

- Observable must have standard functional form for soft & collinear gluon emission

$$V(\{p\},k) = d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi) \,.$$

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- Require *continuous globalness*, *i.e.* uniform dependence on k_t independently of emission direction ($a_1 = a_2 = \cdots = a$)

Example: thrust in e^+e^-

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A limit on thrust,

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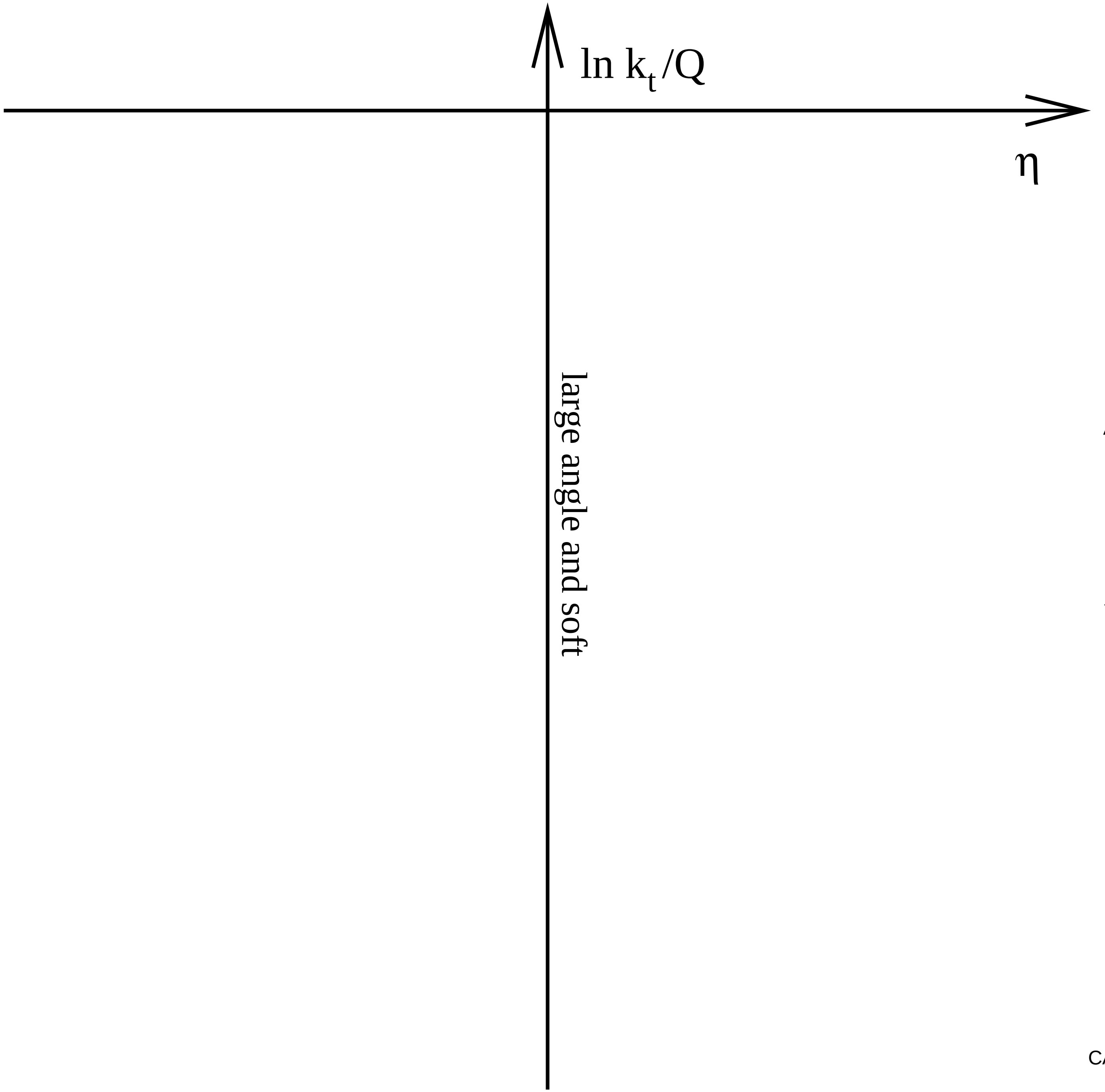

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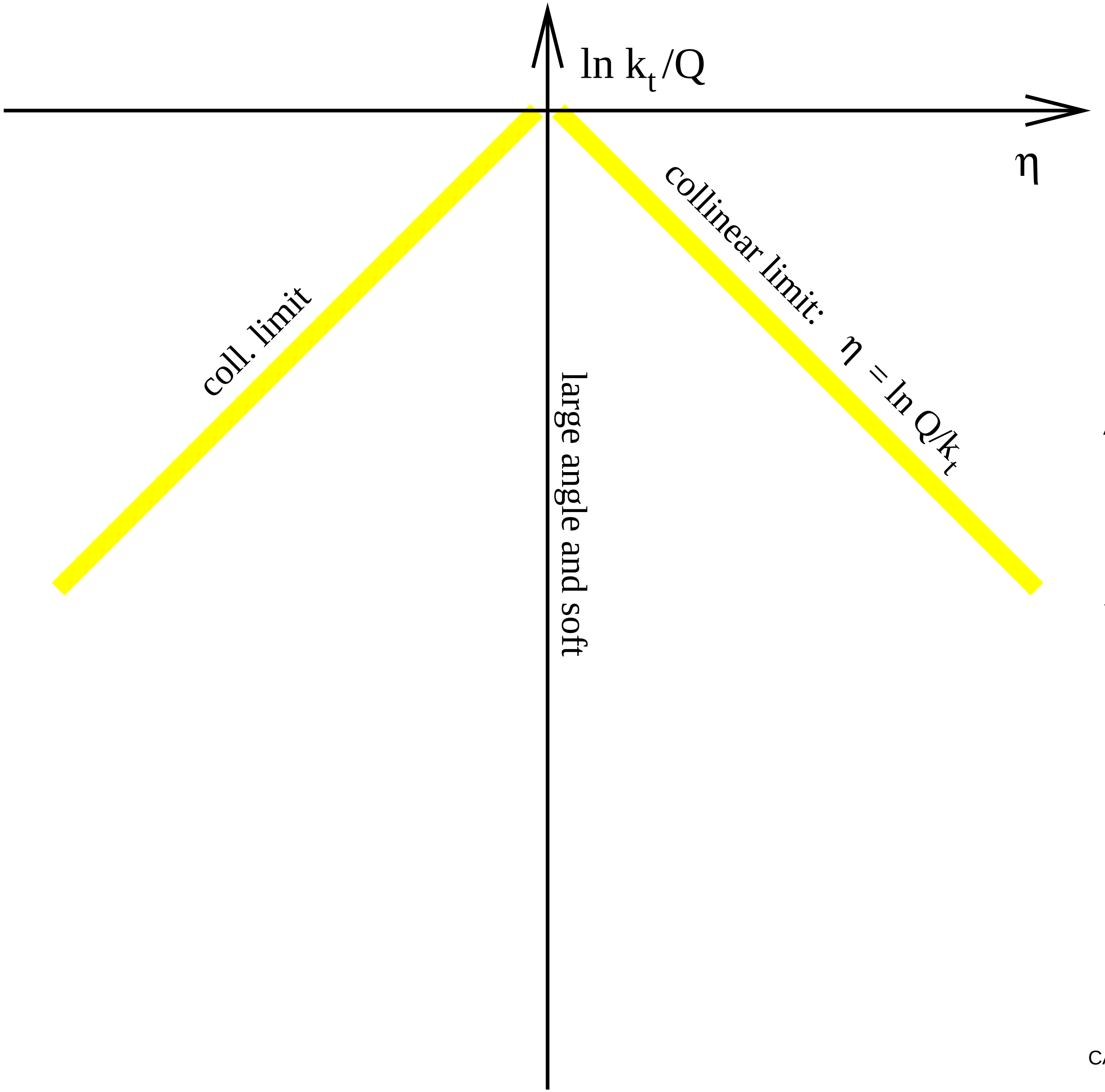

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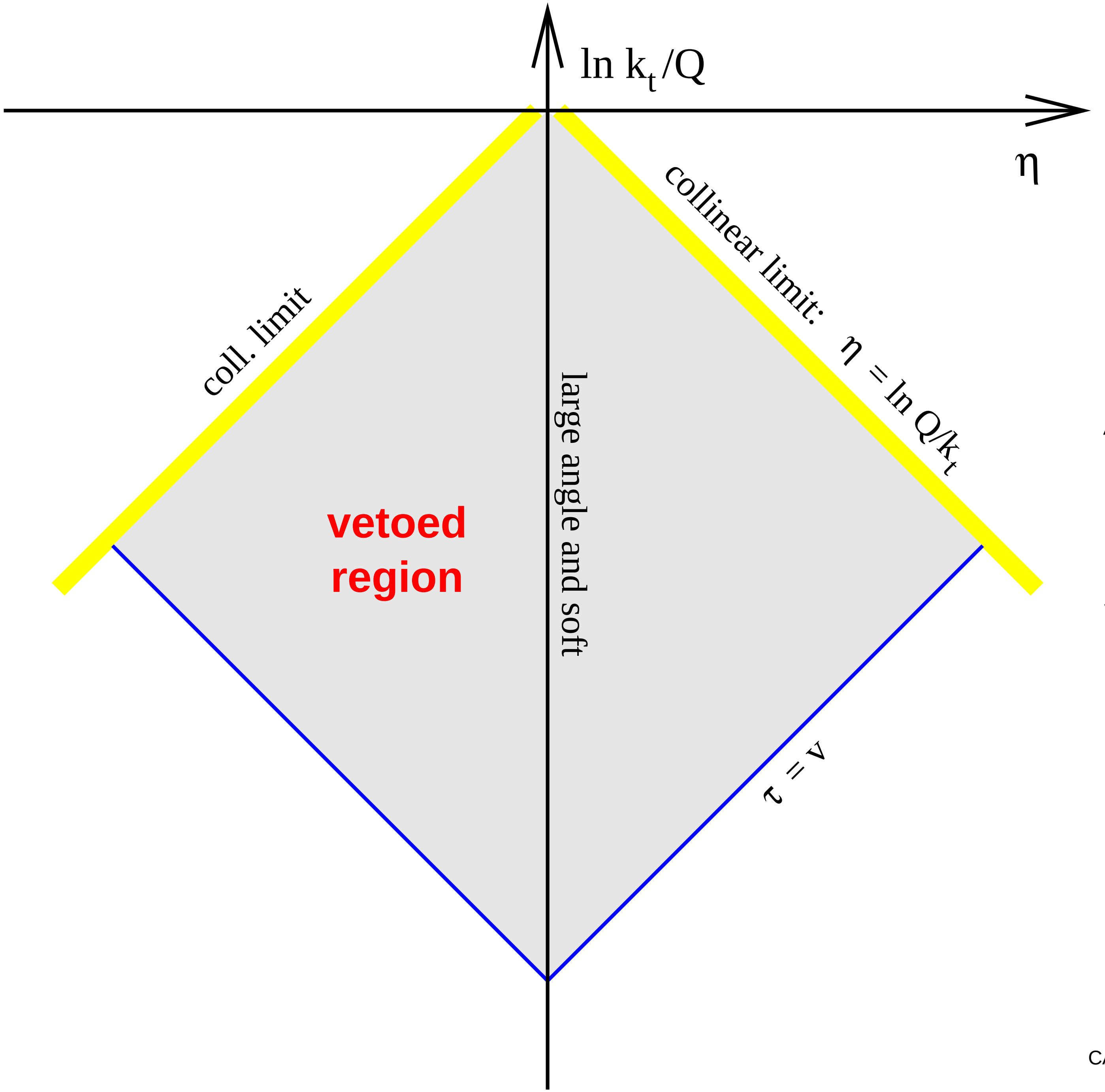

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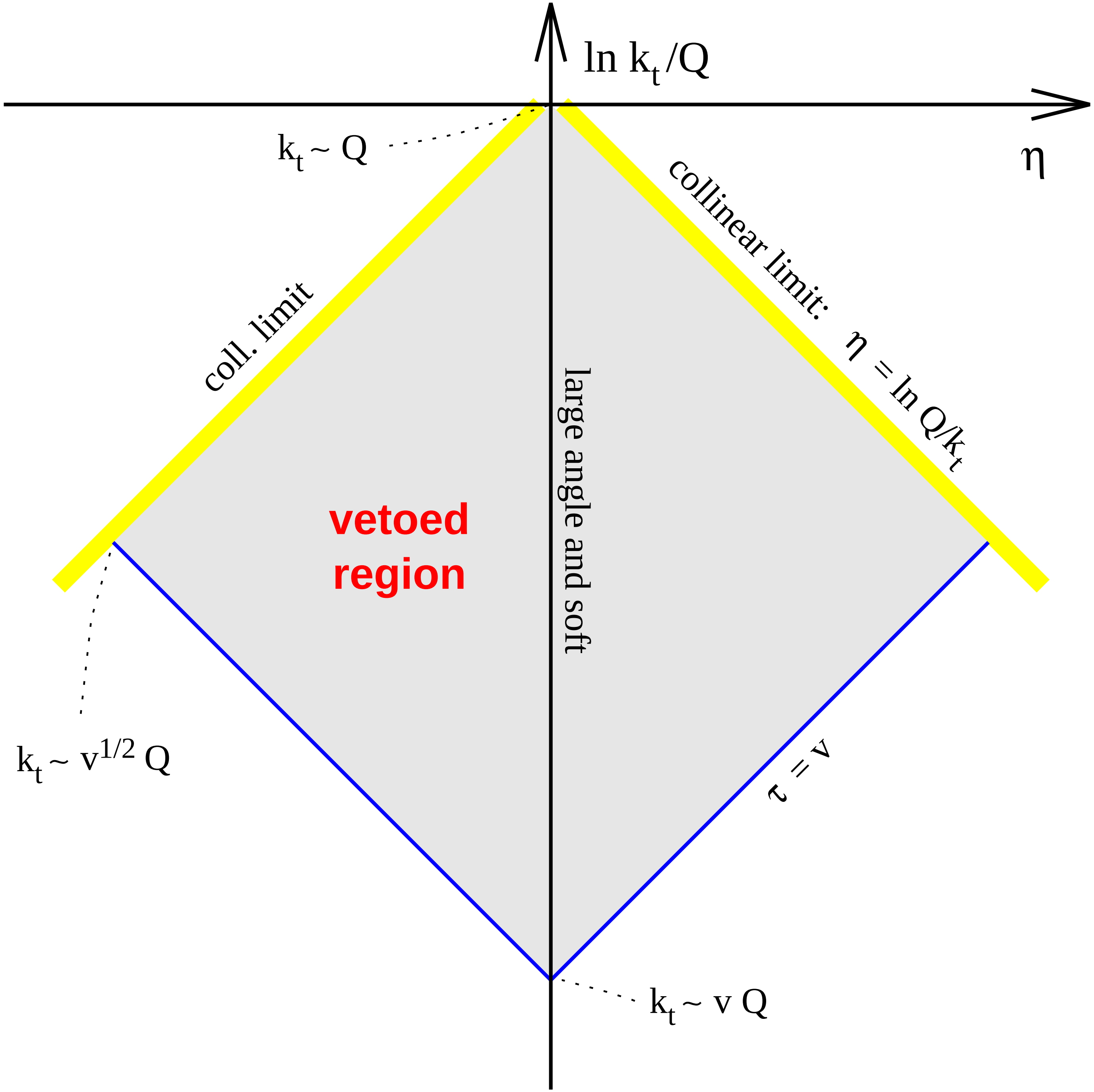

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determine vetoed region (LL & NLL).

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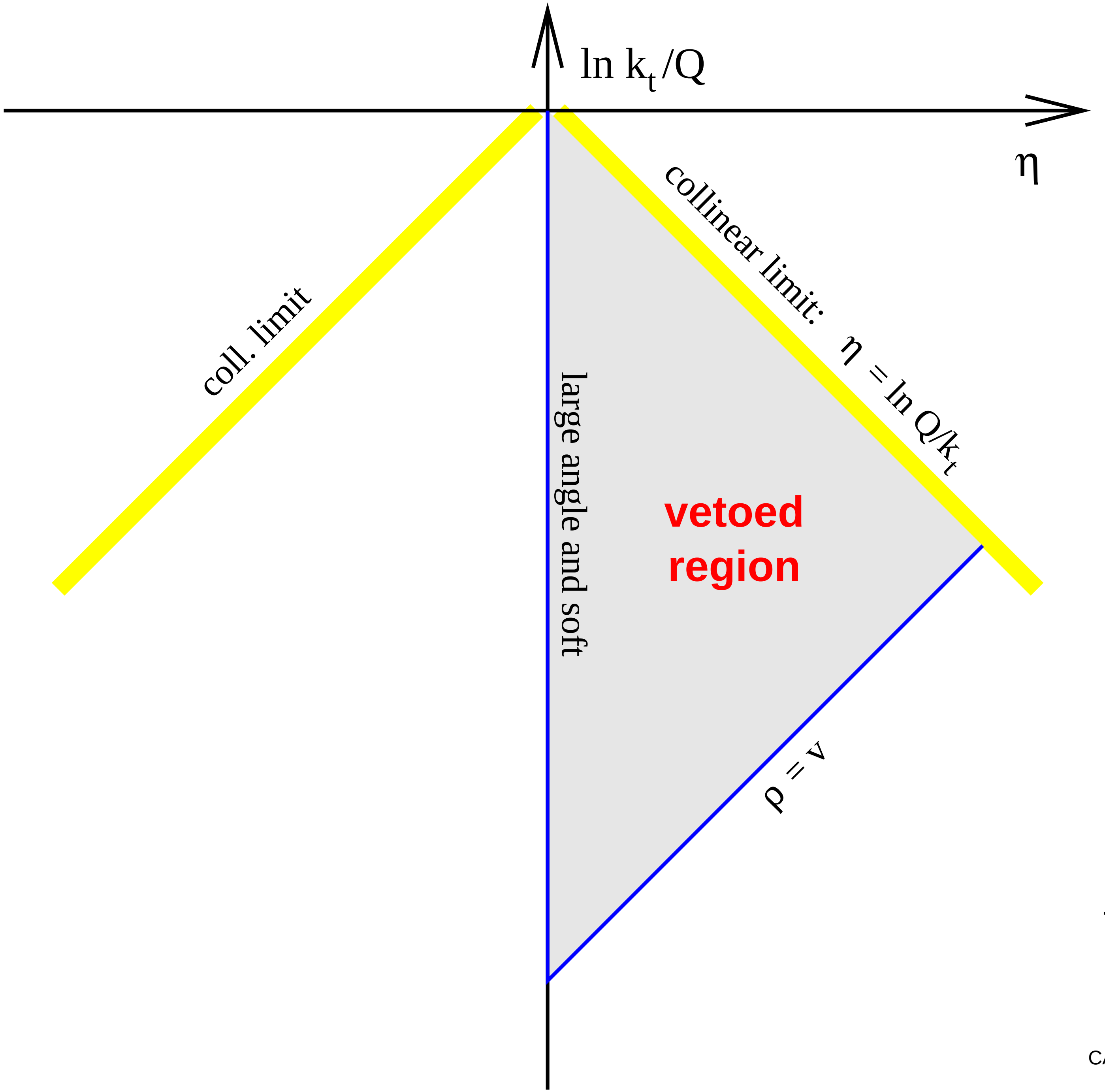

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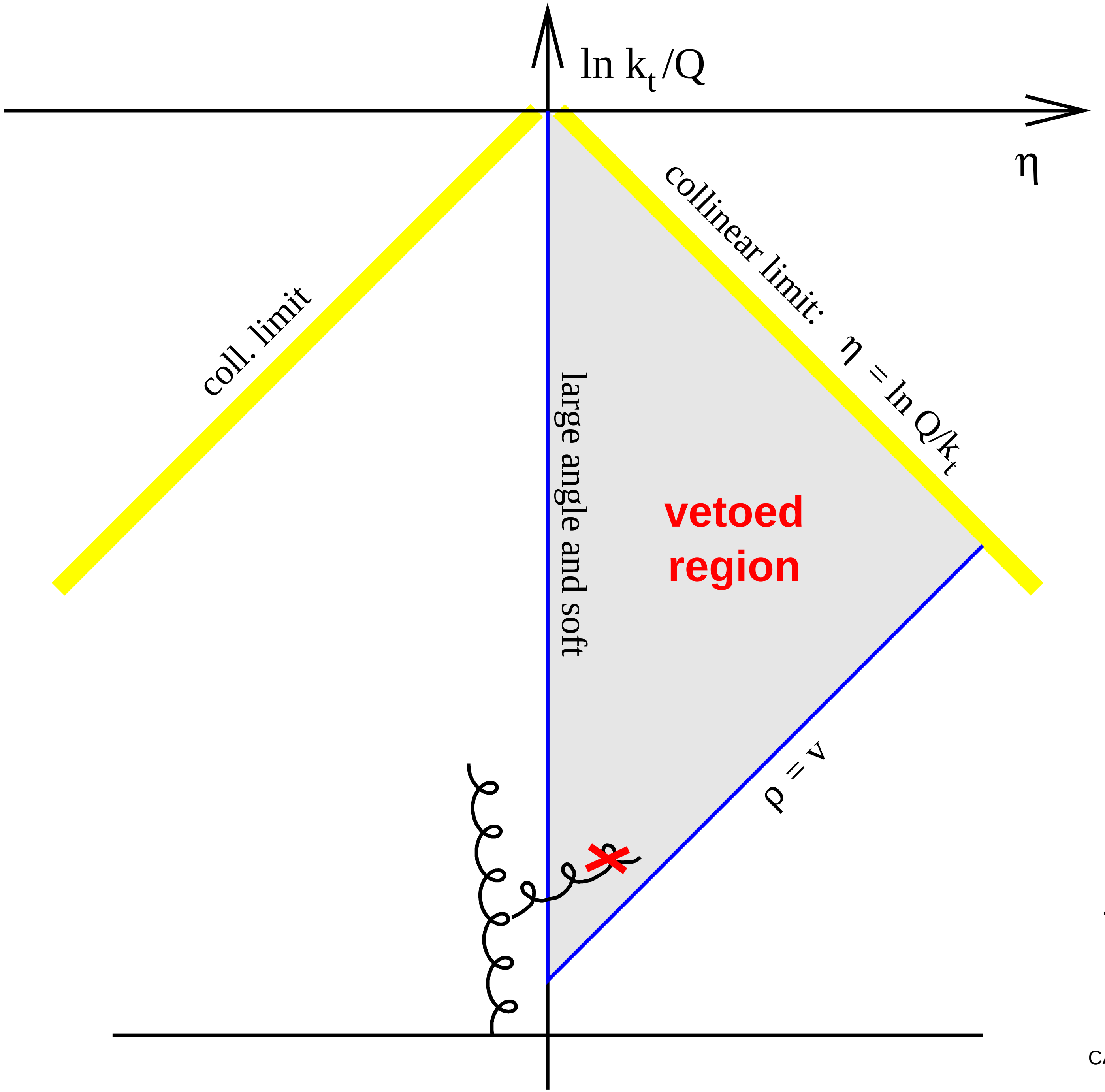

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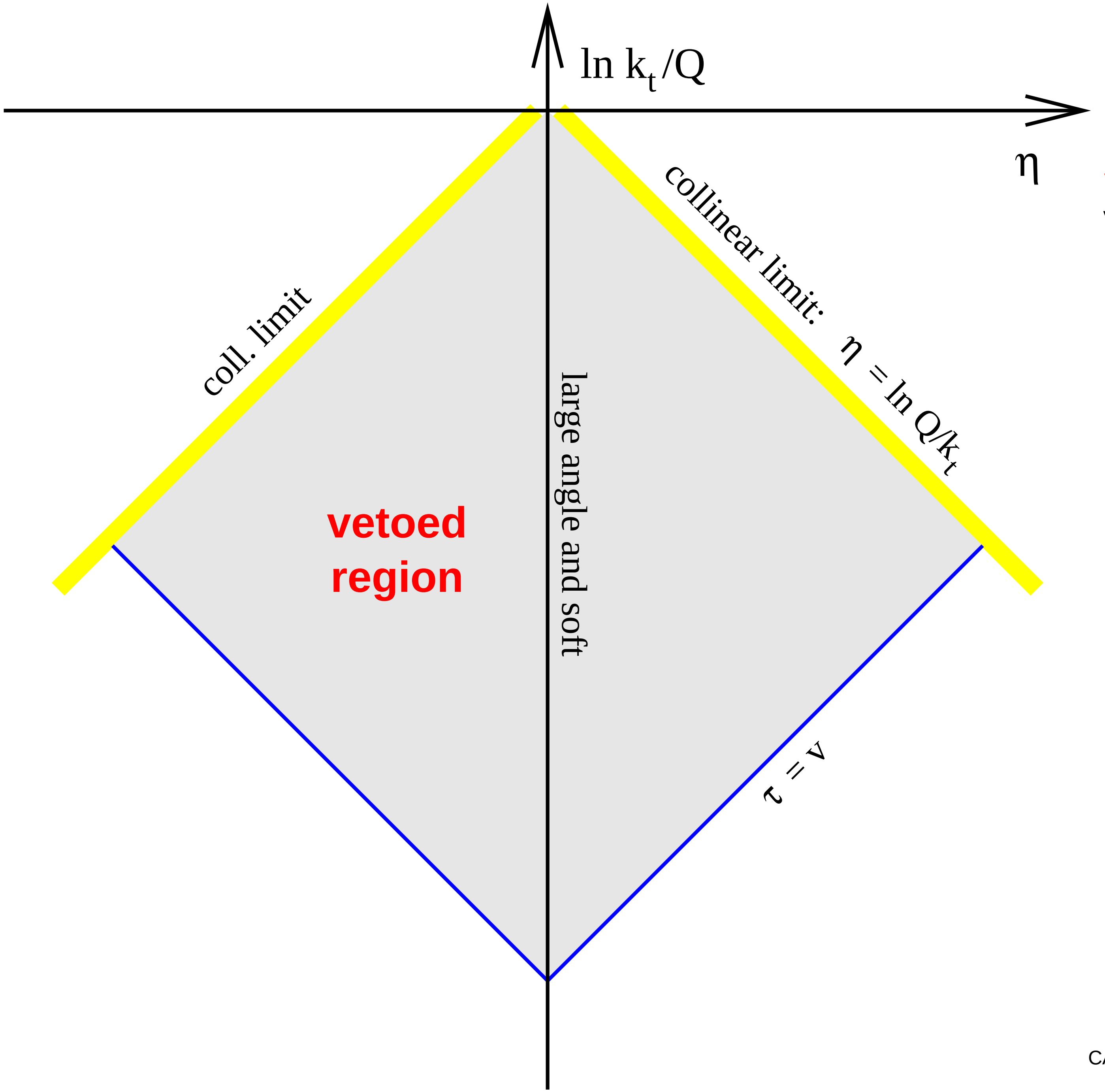
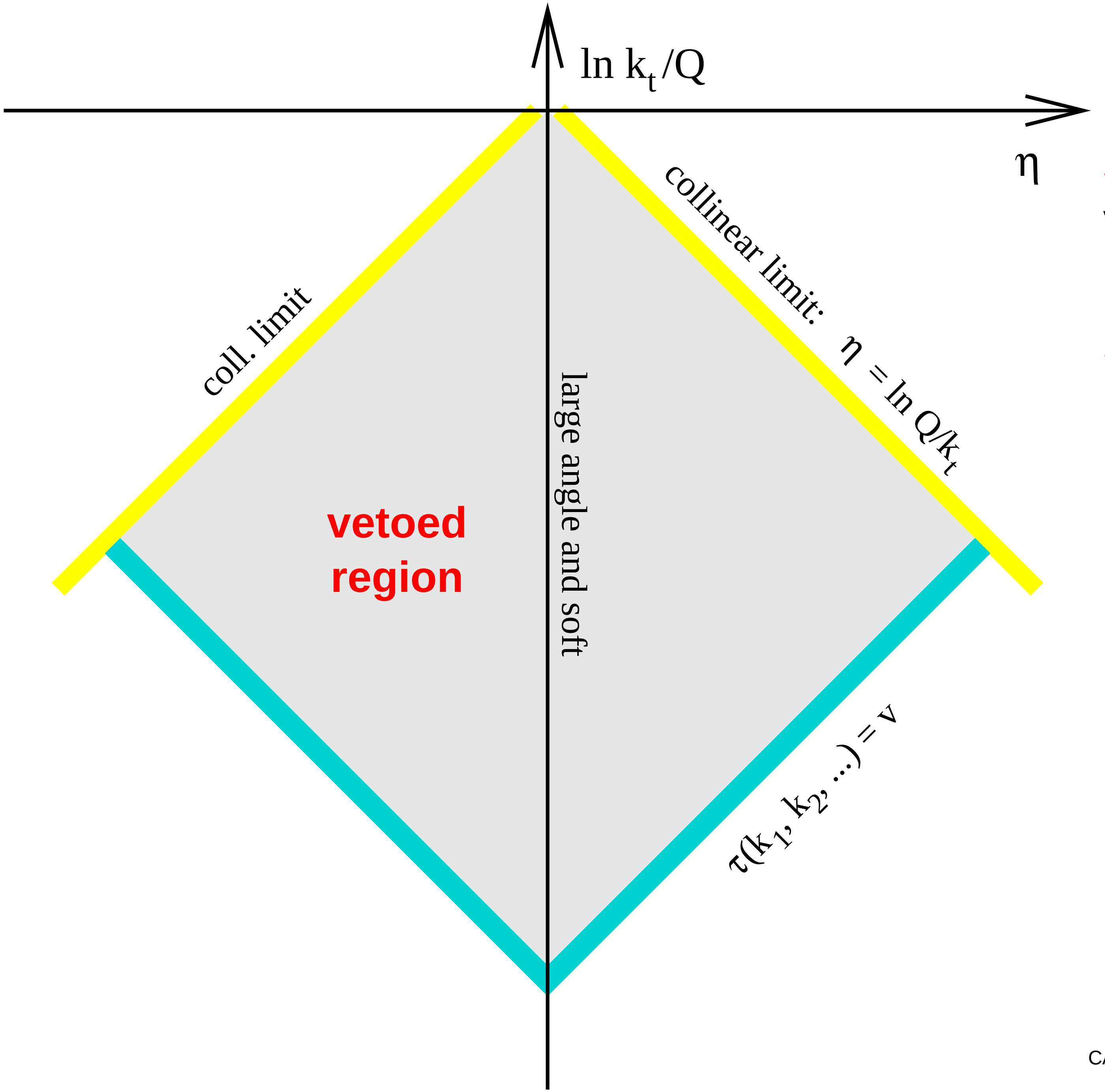
This is *recursive infrared & collinear (rIRC) safety*

Meaning of rIRC safety



Sharp single-emission veto boundary

Meaning of rIRC safety

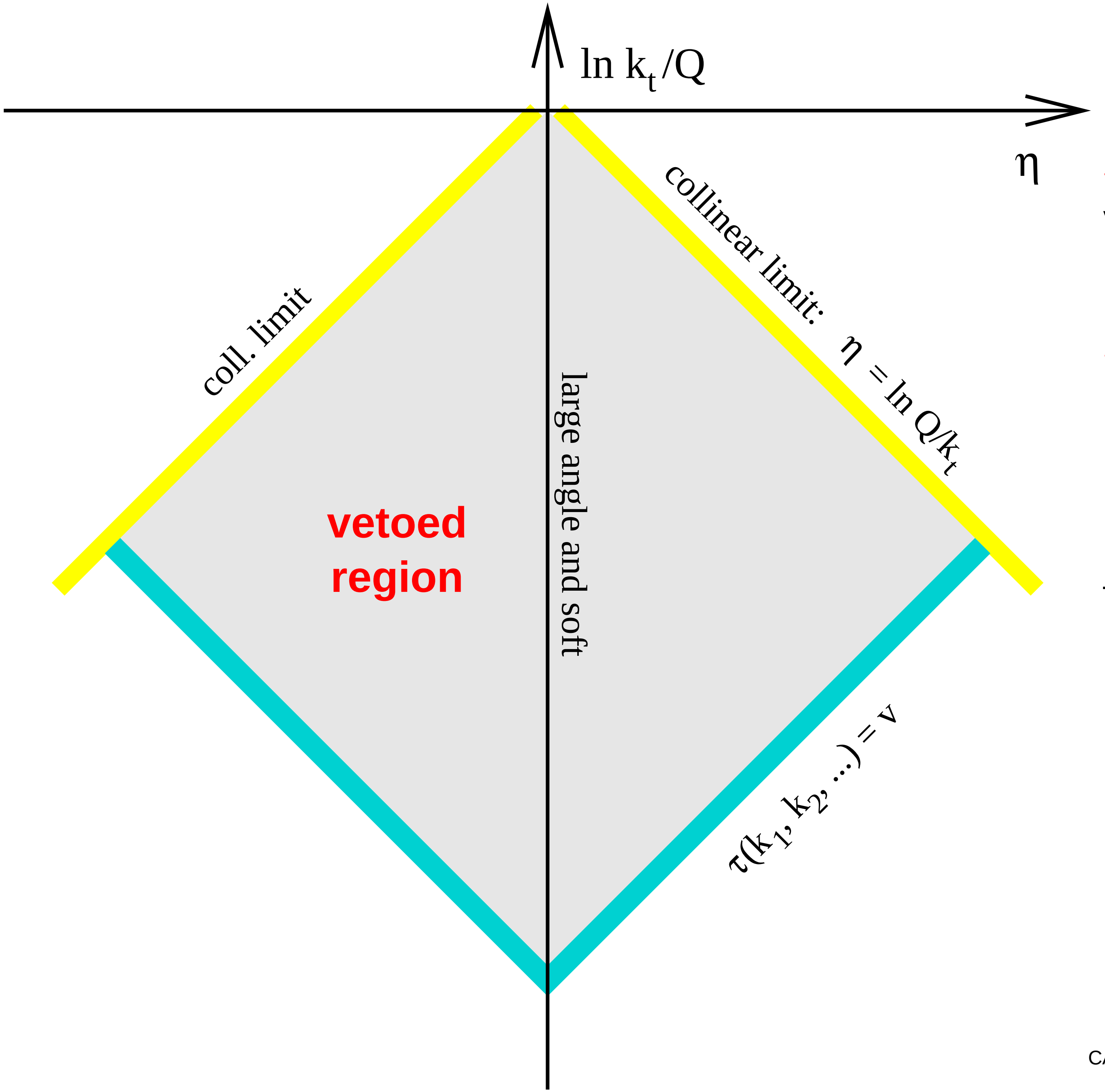


Sharp single-emission veto boundary

↓

fuzzy, but still *local* multi-emission veto boundary

Meaning of rIRC safety



Sharp single-emission veto boundary

↓

fuzzy, but still *local* multi-emission veto boundary

It guarantees:

- double logs (LL) *exponentiate*
- ‘multiple-emission’ corrections are *single-logarithmic* (NLL)

Multi-emission single logs

Determine function $\mathcal{F}$ that accounts for *average extra veto* to be applied to exponentiated double logs.

Given by:

$$\mathcal{F} = \lim_{\epsilon\to 0} \frac{\epsilon^{R'}}{R'} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m+1} \sum_{\ell_i=1}^{n} C_\ell r'_{\ell_i} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \right) \delta(\ln \zeta_1) \times$$
$$\times \exp\left(-R' \ln \lim_{\bar{v}\to 0} \frac{V(\{\tilde{p}\}, \kappa_1(\zeta_1 \bar{v}), \ldots, \kappa_{m+1}(\zeta_{m+1}\bar{v}))}{\bar{v}} \right).$$

- Result explicitly depends on single-logarithmic quantity $r'_\ell \sim \alpha_s \ln 1/v$ (roughly: length of boundaries of vetoed region, for each leg).
- *Evaluated by Monte Carlo* integration.
- Potentially *divergent for non-rIRC* safe observables.

General Result (e^+e^-, DIS, hh; 2, 3, 4 jets)

Given info from previous pages, *final answer is analytical:*

$$\ln \Sigma(v) = -\sum_{\ell=1}^{n} C_\ell \left[r_\ell(v) + r'_\ell(v) \left(\ln \bar{d}_\ell - b_\ell \ln \frac{2E_\ell}{Q} \right) \right.$$
$$\left. + B_\ell T\left(\frac{\ln 1/v}{a+b_\ell} \right) \right] + \sum_{\ell=1}^{n_i} \ln \frac{f_\ell(x_\ell, v^{\frac{2}{a+b_\ell}} \mu_f^2)}{f_\ell(x_\ell, \mu_f^2)}$$
$$+ \ln S\left(T\left(\frac{\ln 1/v}{a} \right) \right) + \ln \mathcal{F}(C_1 r'_1, \dots, C_n r'_n) \,,$$

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C_ℓ = colour factor (C_F or C_A), $\qquad f_\ell(x_\ell,\mu_f^2)$ = parton distributions

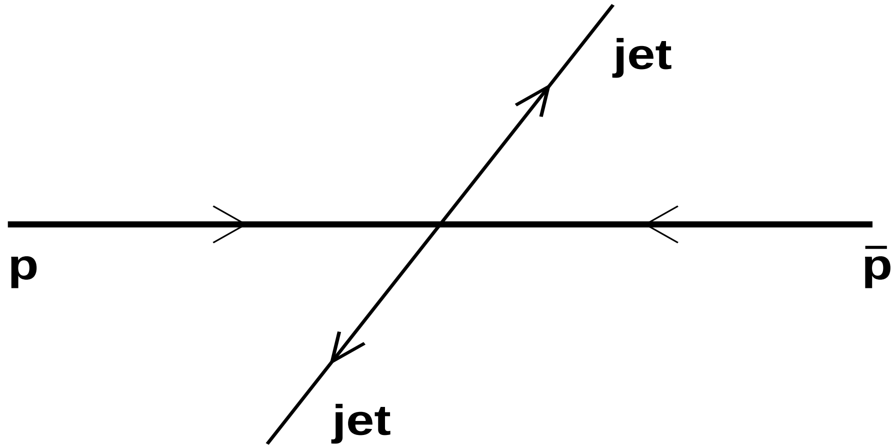
$$r_\ell(L) = \int_{v^{\frac{2}{a}}Q^2}^{v^{\frac{2}{a+b_\ell}}Q^2} \frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t)}{\pi}\ln\left(\frac{k_t}{v^{1/a}Q}\right)^{a/b_\ell} + \int_{v^{\frac{2}{a+b_\ell}}Q^2}^{Q^2} \frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t)}{\pi}\ln\frac{Q}{k_t}\,,$$

$S(T(\frac{1}{a}\ln 1/v))$ = large-angle logarithms (process dependence)

...

Example: global thrust in hadronic dijet production

In hadronic-dijet scattering, invent observables to measure *deviation* from Born event.



E.g.: global transverse thrust

$$1 - T_{t,g} \equiv 1 - \max_{\vec{n}_t} \frac{\sum_i |\vec{p}_{ti} \cdot \vec{n}_t|}{\sum_i p_{ti}}\,,$$

Properties of global transverse thrust

leg ℓ	a_ℓ	b_ℓ	$g_\ell(\phi)$	d_ℓ	$\langle \ln g_\ell(\phi) \rangle$
1	1.000	0.000	tabulated	1.02062	-1.859
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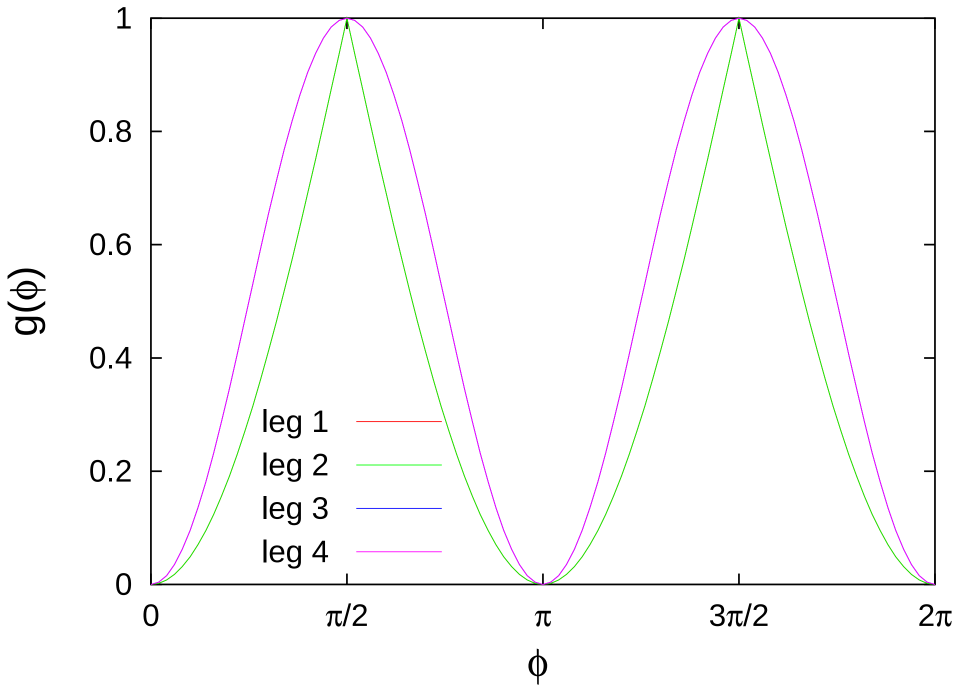

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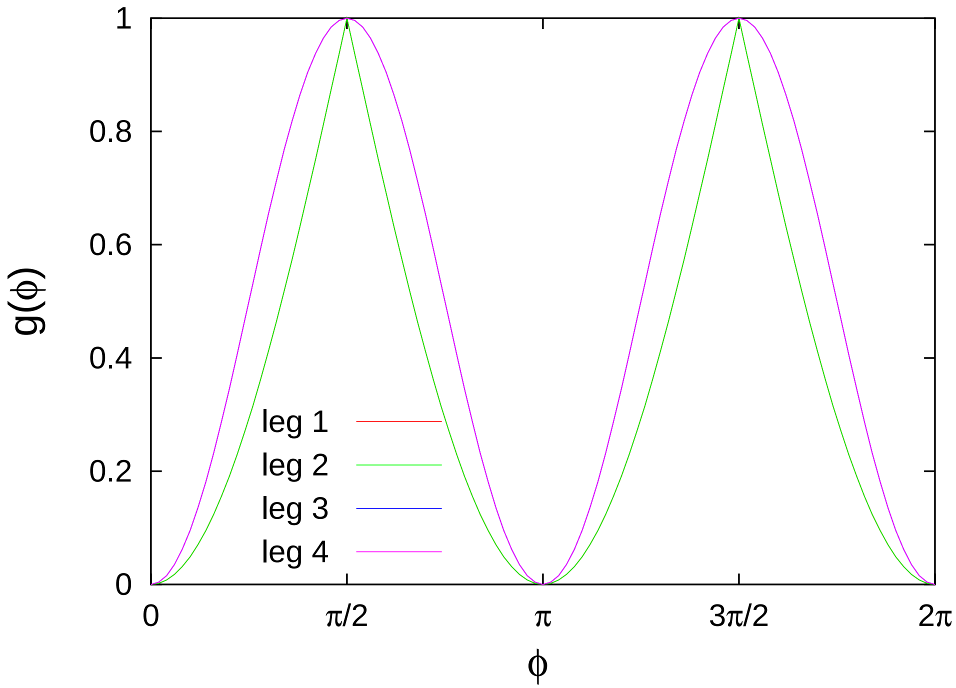

Test	result
continuously global	T
rec. IRC safe (cond. 1)	T
rec. IRC safe (cond. 2a)	T
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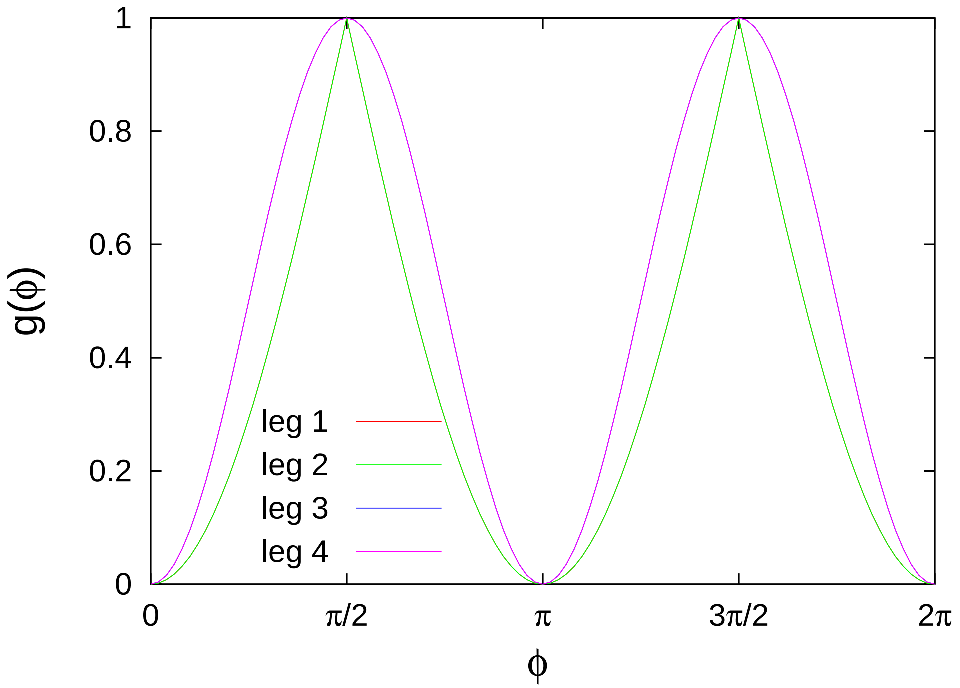

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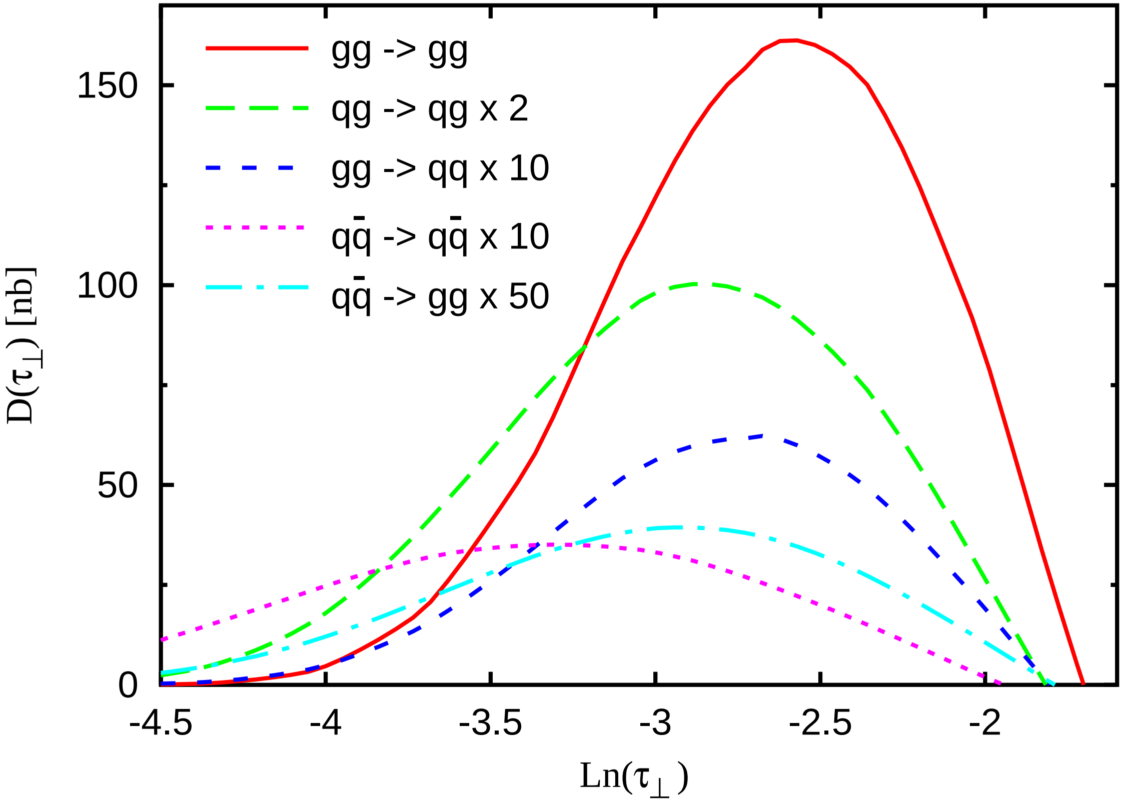
$$\mathcal{F} = \frac{e^{-\gamma_E R'}}{\Gamma(1+R')}$$

Resummed thrust for Tevatron

- run II regime $\sqrt{s} = 1.96$ TeV
- cut on jet-rapidity $|\eta| < 1$
- cut on jet transverse energy $E_T > 50$GeV



PRELIMINARY!

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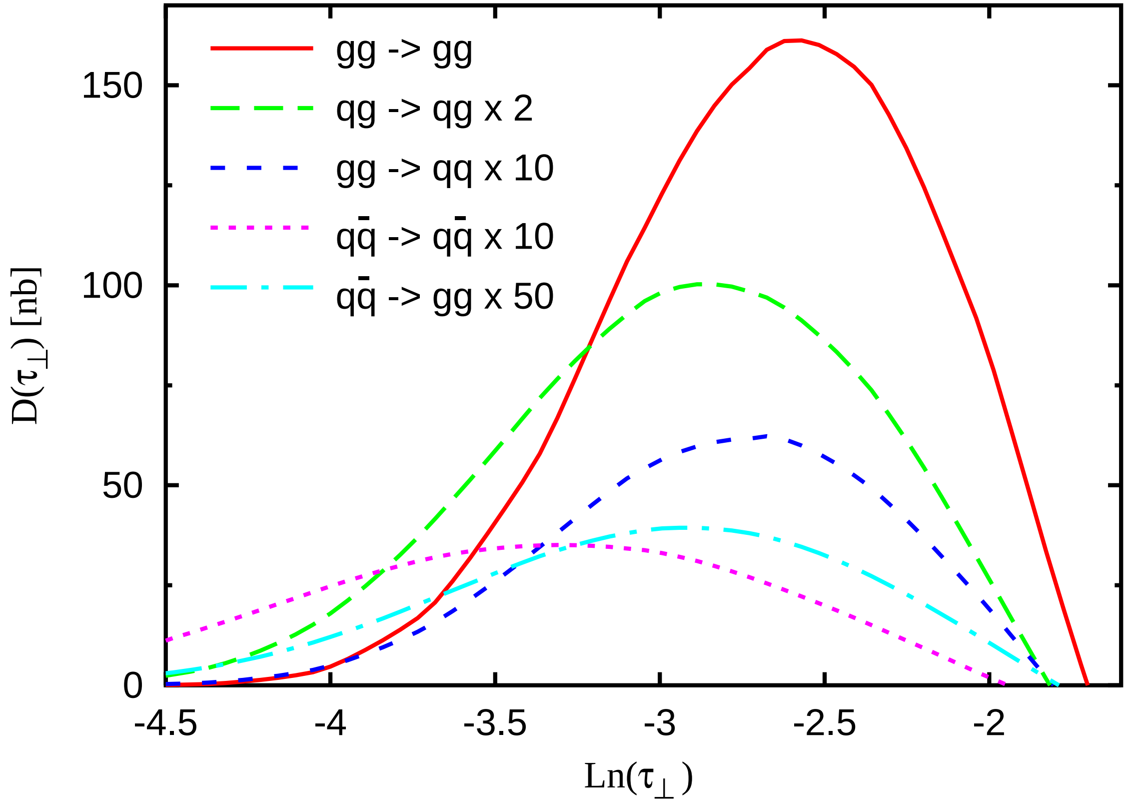

PRELIMINARY!

Many other observables are being studied

Conclusions/Outlook

Status

- Powerful new tool
- In the past year it has moved from ‘proof of concept’ to near completion.
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- Completion of some details & writeup
- Making program available

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Longer-term Outlook

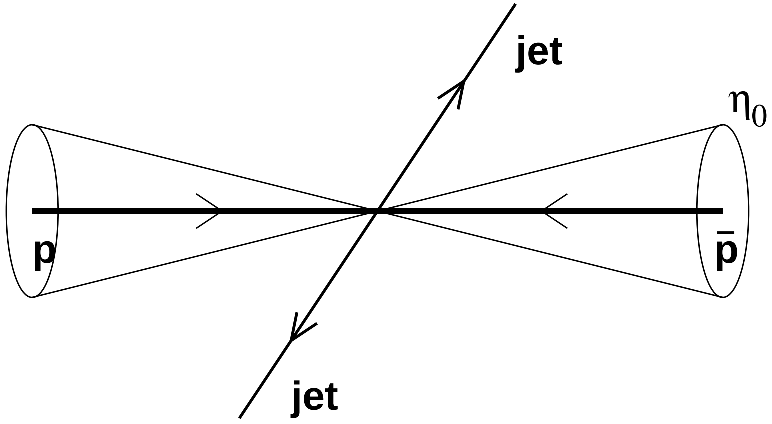
- Matching with fixed order (in progress) $\longrightarrow$ Phenomenology
- Extending scope (*e.g.* non-global observables?)

EXTRA SLIDES

Observables in hadronic dijet production

Cut around the beam $|\eta| < \eta_0$

➥ Problems with globalness



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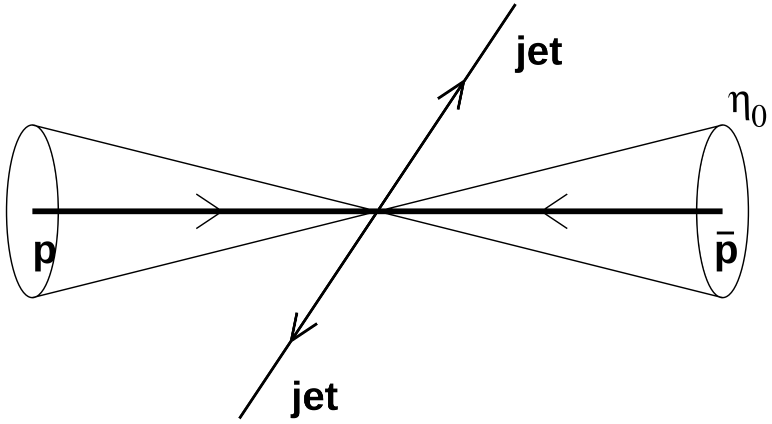

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Directly global observables: $\eta_0 \gg 1$

Transverse thrust

$$T_T = \frac{1}{E_T} \max_{\vec{n}_T} \sum_i |\vec{p}_{ti} \cdot \vec{n}_T|$$

Thrust minor

$$T_m = \frac{1}{E_T} \sum_i |p_i^{out}|$$

Predictions valid as long as
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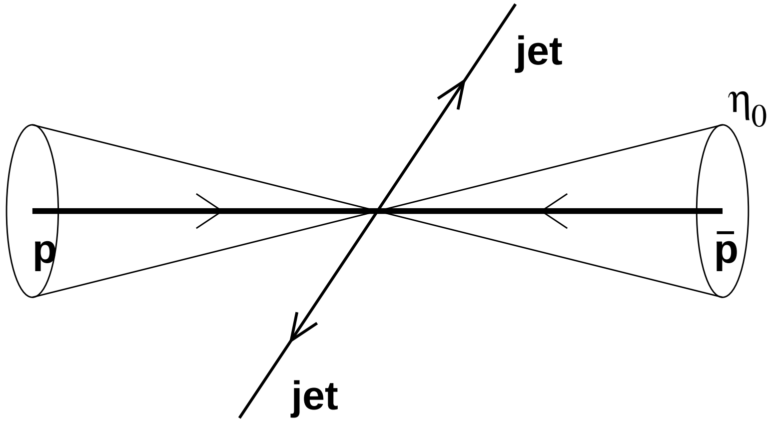

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Indirectly global observables: $\eta_0 = \mathcal{O}(1)$

Transverse thrust

$$T_T = \frac{1}{E_{T,\eta_0}} \left(\max_{\vec{n}_T} \sum_{|\eta_i|<\eta_0} |\vec{p}_{ti} \cdot \vec{n}_T| - \Big|\sum_{|\eta_i|<\eta_0} \vec{p}_{ti}\Big| \right)$$

Thrust minor

$$T_m = \frac{1}{E_{T,\eta_0}} \left(\sum_{|\eta_i|<\eta_0} |p_i^{out}| + \Big|\sum_{|\eta_i|<\eta_0} \vec{p}_{ti}\Big| \right)$$

Predictions valid as usual

but $\mathcal{F}$ diverges for $R' = R'_c$