Resummation

Gavin Salam

CERN, Princeton & LPTHE/CNRS (Paris)

QCD in the LHC Era
A Meeting in Honour of Bryan Webber
Cambridge, UK, 22 September 2010

Resummation implies accounting for a (logarithmically) enhanced subset of terms at each and every order of the perturbative series, e.g.

$$V \ll 1: \qquad \sigma(V) \simeq \sigma_0 \sum_{n=0}^{\infty} \quad \alpha_s^n \ln^{2n} V \quad + \mathcal{O}\left(\alpha_s^n \ln^{2n-1} V\right)$$

There are many ways in which Bryan has been involved in this. Among them:

- Herwig & parton-shower development
- MC@NLO
- CKKW

But usually, by "resummation," we mean **analytically** extracting the functions corresponding to Leading Logarithms (LL), NLL, etc.

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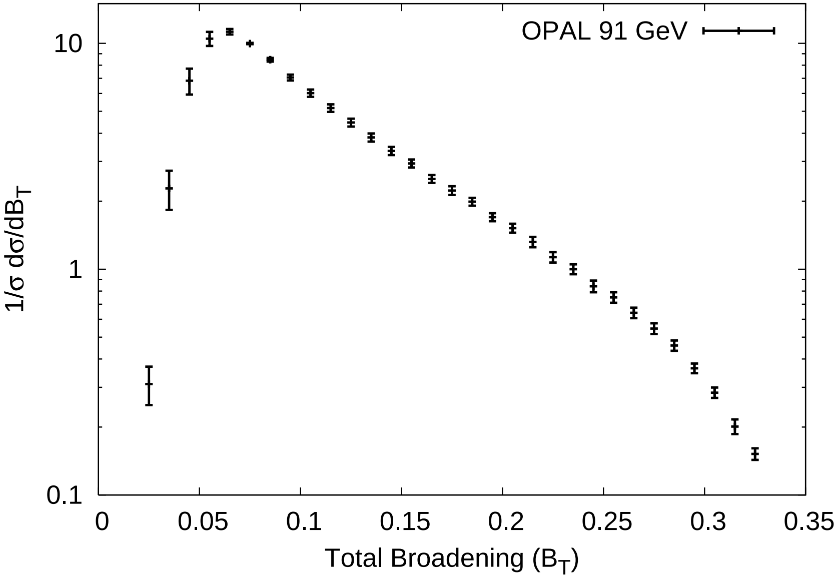
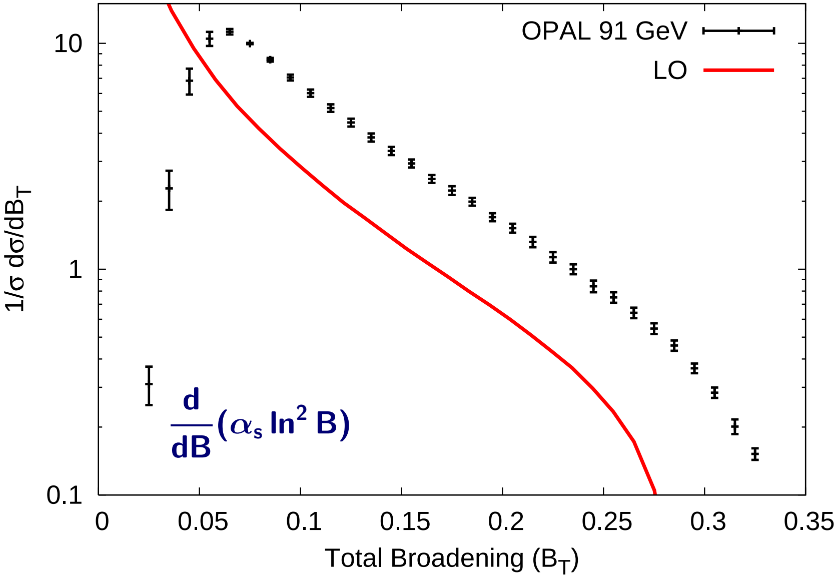

OPAL 91 GeV
10
1
0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Total Broadening (B_T)
$1/\sigma\ d\sigma/dB_T$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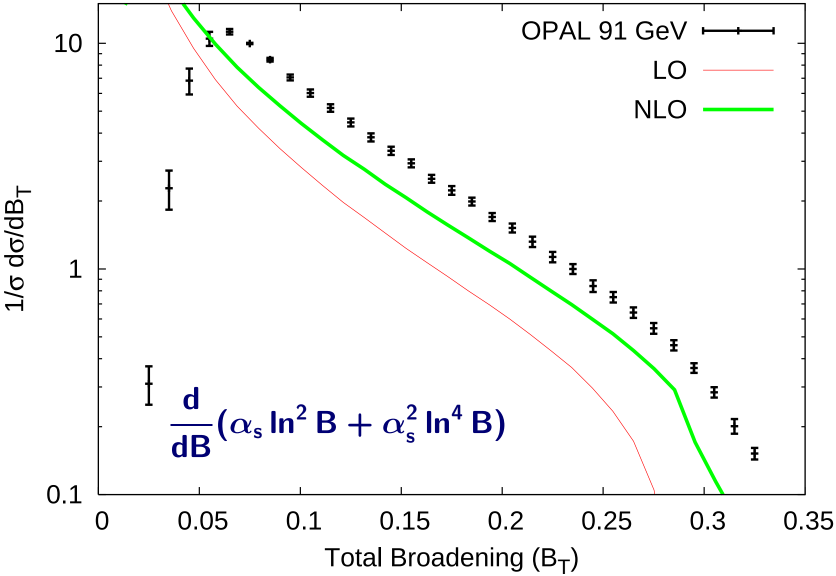
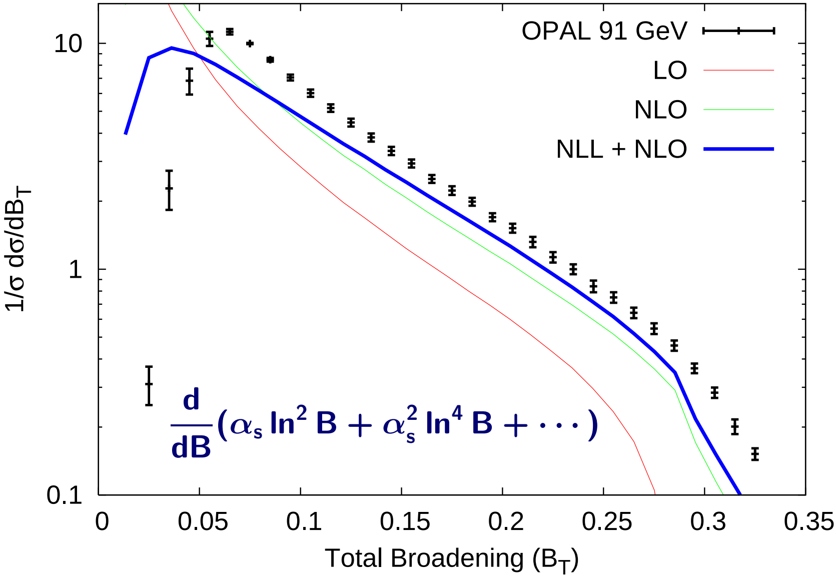
$1/\sigma\, d\sigma/dB_T$
Total Broadening (B_T)
$\frac{d}{dB}(\alpha_s \ln^2 B)$

Why resummation is needed




OPAL 91 GeV
LO
NLO
NLL + NLO
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0.2
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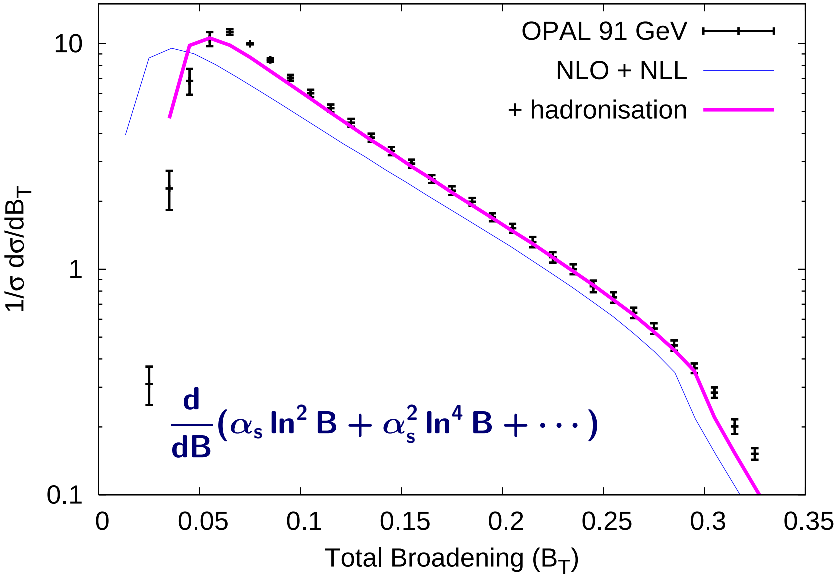
0.3
0.35
1/σ dσ/dB_T
Total Broadening (B_T)
$\frac{d}{dB}(\alpha_s \ln^2 B + \alpha_s^2 \ln^4 B + \cdots)$


OPAL 91 GeV
NLO + NLL
+ hadronisation
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0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
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$1/\sigma\ d\sigma/dB_T$
Total Broadening (B_T)
$\frac{d}{dB}\left(\alpha_s \ln^2 B + \alpha_s^2 \ln^4 B + \cdots\right)$

To talk about enhanced terms at all orders, you need to pick out a physical variable that is large or small and whose logs you sum.

There are quite a few options

- $v \ll 1$ where $v =$ 1-Thrust, Broadening, etc.
  deviation of events from perfect 2-jet nature
- $y \ll 1$, where y is jet resolution parameter
  jet rates, jet multiplicities, etc.
- $q_T \ll m_{DY/H/...}$ where q_T is Drell-Yan/Higgs transverse momentum
  for helping discover the Higgs at LHC
- $x \ll 1$, where x is fraction of parton's momentum carried by a hadron
  Understanding hadron multiplicites, testing ideas like LPHD, etc.
- $x \ll 1$, where x is fraction of proton's momentum carried by a parton
  HERA physics, LHC moderate p_t, heavy-ion collisions
- $1 - x \ll 1$, threshold resummation
  Approaching the edge of Tevatron/LHC kinematic reach

Bryan's resummation-related work. . .

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5) C. Anastasiou, G. Dissertori, F. Stockli *et al.*, "QCD radiation effects on the H $\to$ WW $\to \ell\nu\ell\nu$ signal at the LHC," JHEP **0803** (2008) 017. [arXiv:0801.2682 [hep-ph]].
6) C. Ewerz, B. R. Webber, "Multiplicity of (mini)jets at small x," JHEP **9908** (1999) 019. [hep-ph/9907430].
7) Y. L. Dokshitzer, G. Marchesini, B. R. Webber, "Nonperturbative effects in the energy energy correlation," JHEP **9907** (1999) 012. [hep-ph/9905339].
8) C. Ewerz, B. R. Webber, "Jet rates at small x to single logarithmic accuracy," JHEP **9904** (1999) 022. [hep-ph/9904244].
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11) S. Catani, B. R. Webber, "Infrared safe but infinite: Soft gluon divergences inside the physical region," JHEP **9710** (1997) 005. [hep-ph/9710333].
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13) S. Catani, Y. L. Dokshitzer, B. R. Webber, "Average number of jets in deep inelastic scattering," Phys. Lett. **B322** (1994) 263-269.
14) P. Nason, B. R. Webber, "Scaling violation in e+ e- fragmentation functions: QCD evolution, hadronization and heavy quark mass effects," Nucl. Phys. **B421** (1994) 473-517.
15) S. Catani, L. Trentadue, G. Turnock *et al.*, "Resummation of large logarithms in e+ e- event shape distributions," Nucl. Phys. **B407** (1993) 3-42.
16) S. Catani, G. Turnock, B. R. Webber, "Jet broadening measures in e+ e- annihilation," Phys. Lett. **B295** (1992) 269-276.
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18) S. Catani, Y. L. Dokshitzer, F. Fiorani *et al.*, "Average number of jets in e+ e- annihilation," Nucl. Phys. **B377** (1992) 445-460.
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27) C. P. Fong, B. R. Webber, "Higher Order Qcd Corrections To Hadron Energy Distributions In Jets," Phys. Lett. **B229** (1989) 289.
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Bryan's first resummation paper

VOLUME 43, NUMBER 23 PHYSICAL REVIEW LETTERS 3 DECEMBER 1979

Noncollinearity of Jets in Quantum Chromodynamics

P. E. L. Rakow and B. R. Webber
Cavendish Laboratory, Cambridge, England
(Received 5 September 1979)

Quantum chromodynamics predicts significant noncollinearity of two-jet processes, resulting from recoil against gluons outside the jets. Jet angular radii measured in collinear experiments should therefore be much larger than those predicted by Sterman and Weinberg. Exact calculations of this effect in first-order perturbation theory are presented and compared with numerical estimates of nonperturbative contributions. The result of resumming large logarithms to all orders is also presented.

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But 1979 wasn't just the year in which Bryan started doing resummations.

The same article also included his first work on new hypothetical particles, specifically the fluon

> angle Δ around the quark's initial momentum. To logarithmic accuracy, the transverse momenta of emitted fluons are strongly ordered so that only the first one can knock the quark out of the cone. In the notation of Ref. 10, the improved formula is therefore

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Two seminal final-state resummation papers

Physics Letters B 269 (1991) 432–438
North-Holland

PHYSICS LETTERS B

New clustering algorithm
for multijet cross sections in e^+e^- annihilation☆

S. Catani [a,b,1], Yu.L. Dokshitzer [c,d], M. Olsson [d], G. Turnock [a] and B.R. Webber [a]

[a] *Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, UK*
[b] *INFN, Sezione di Firenze, Largo Fermi 2, I-50125 Florence, Italy*
[c] *Leningrad Nuclear Physics Institute, Gatchina, SU-188 350 Leningrad, USSR*
[d] *Department of Theoretical Physics, University of Lund, Sölvegatan 14A, S-22362 Lund, Sweden*

Received 2 August 1991

Cross sections for $e^+e^- \to n$-jets, as functions of the jet resolution parameter y_{cut}, are computed according to a new clustering algorithm. The jet multiplicity n is defined in such a way that jets i and j with energies E_i and E_j at relative angle θ_{ij} are resolved if $y_{ij} = 2(1-\cos\theta_{ij})\min(E_i^2, E_j^2)/s > y_{cut}$, where s is the centre-of-mass energy squared. Using this algorithm, large higher-order corrections at small values of y_{cut} can easily be evaluated. Our calculations include resummation of leading and next-to-leading logarithms of y_{cut} to all orders in QCD perturbation theory. This enables us to predict the jet cross sections at small y_{cut} for arbitrary n. Simple analytical results for $n \leqslant 5$ are presented.

Nuclear Physics B407 (1993) 3–42
North-Holland

NUCLEAR
PHYSICS B

Resummation of large logarithms in e^+e^- event shape distributions *

S. Catani [1]

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

L. Trentadue

Dipartimento di Fisica, Università di Roma II, Tor Vergata, I-00173 Rome, Italy

G. Turnock

Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, UK

B.R. Webber [2]

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Received 29 January 1993
Accepted for publication 27 May 1993

The k_t algorithm
$\sim$ 680 citations
[see Yuri's talk]

"CTTW"
$\sim$ 300 citations
[this talk]

For example, thrust, heavy-jet mass, jet broadening, etc.

Calculation of $R(v)$, probability that event shape has value smaller than v:

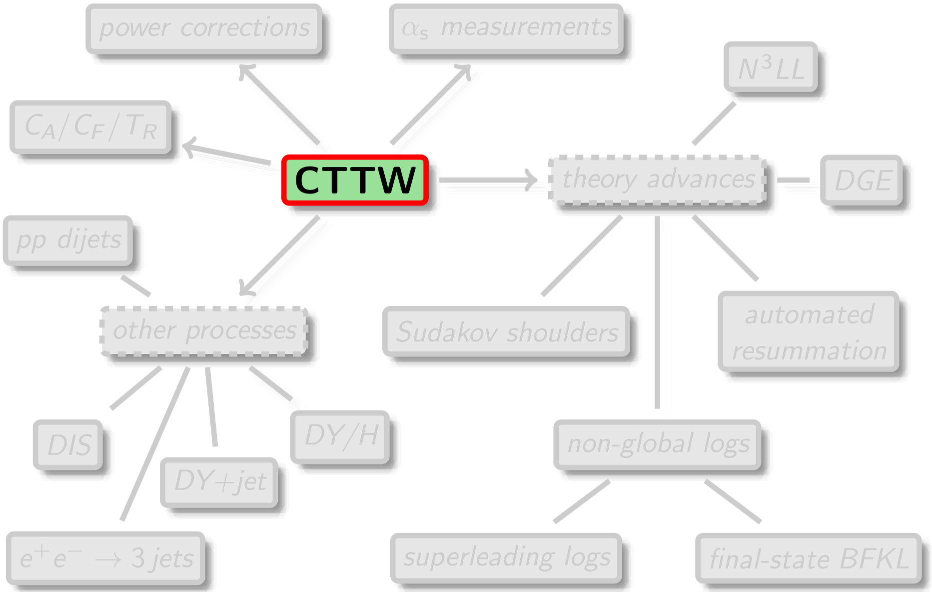
$$R(v) \simeq (1 + C_1\alpha_s) \exp\left[\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \mathcal{O}\left(\alpha_s^n L^{n-1}\right)\right], \qquad \begin{array}{l} L \equiv \ln \frac{1}{v}, \\ L \gg 1 \end{array}$$

Catani, Trentadue, Turnock & Webber '93

Their calculations of LL function $g_1(\alpha_s L)$ and NLL function $g_2(\alpha_s L)$ held as state of the art for 15 years.

Until N^3LL thrust (except cusp) in Becher & Schwartz '08
& heavy-jet mass: Chien & Schwartz '10

Some of the legacy of CTTW



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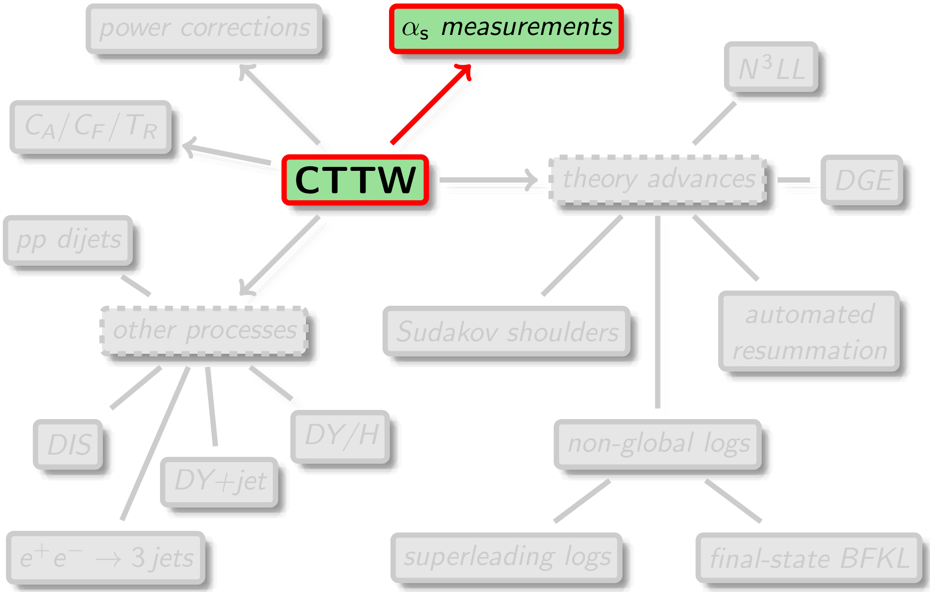
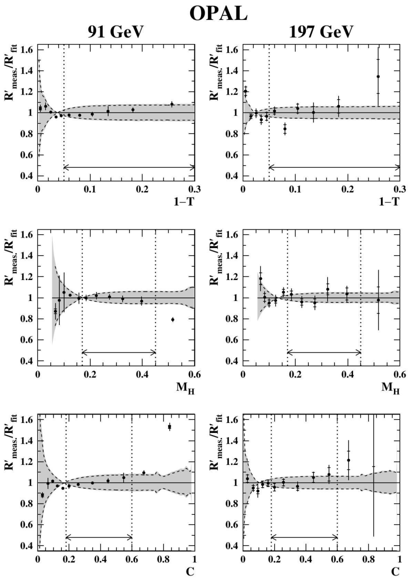
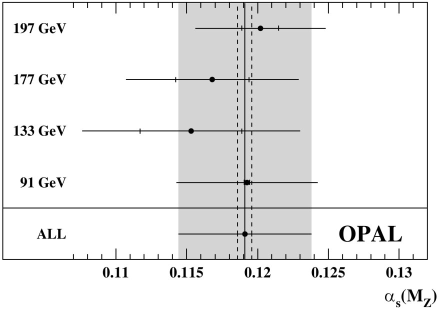


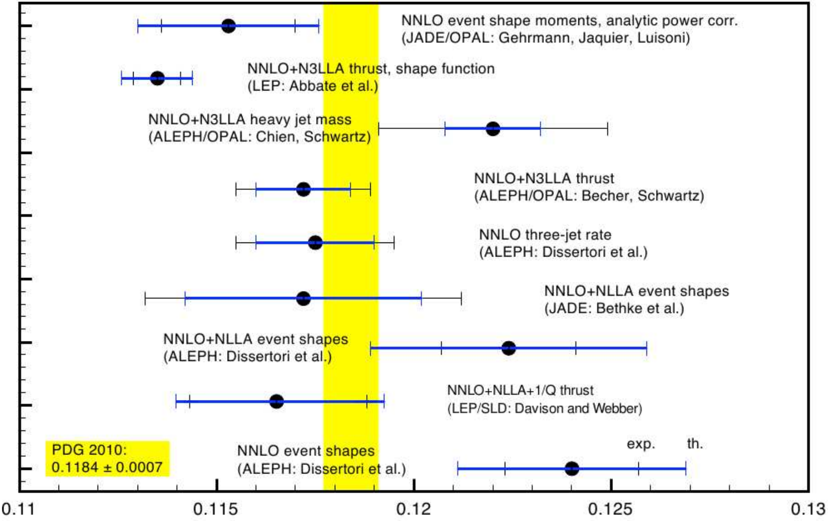




[OPAL '05]

And similar results from ALPEH, DELPHI, JADE, L3 & SLD!

Recent event-shape α_s determinations



adapted from Gehrmann '10

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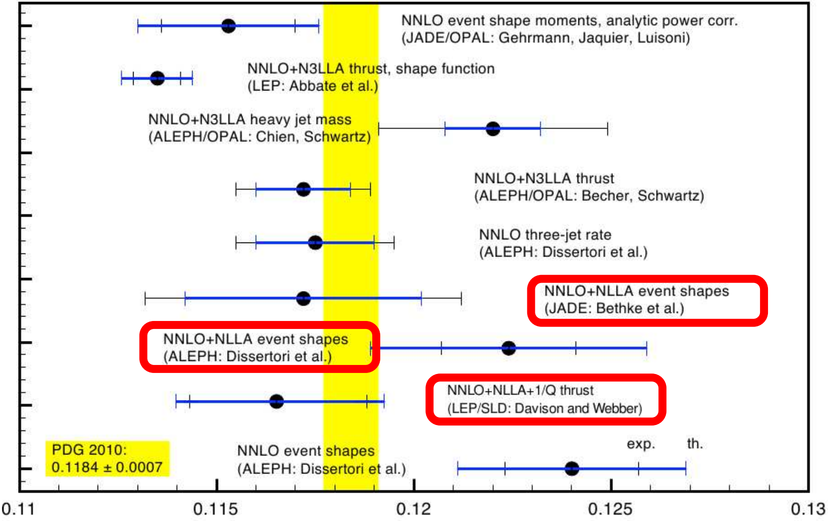

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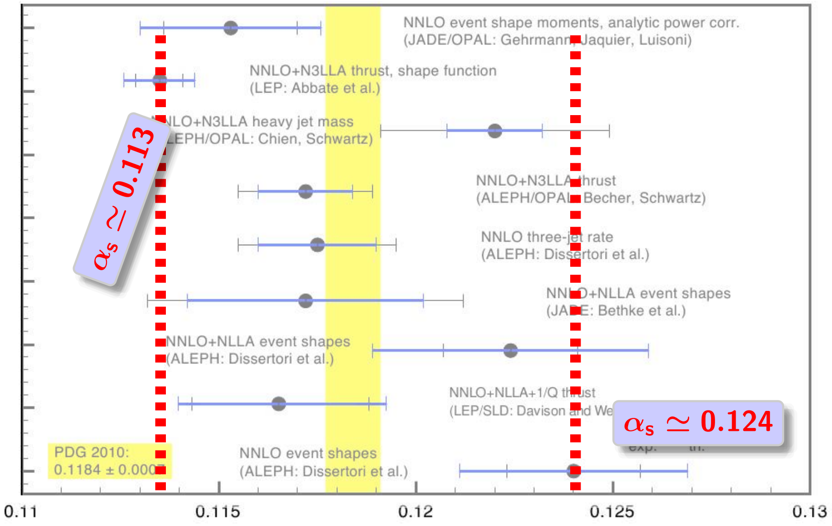

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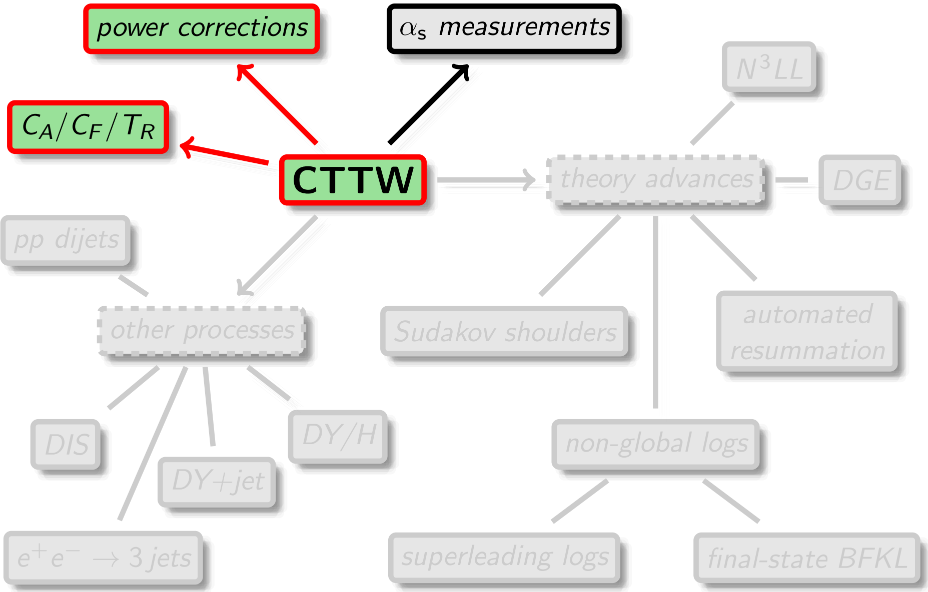
power corrections
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N^3LL
$C_A/C_F/T_R$
CTTW
theory advances
DGE
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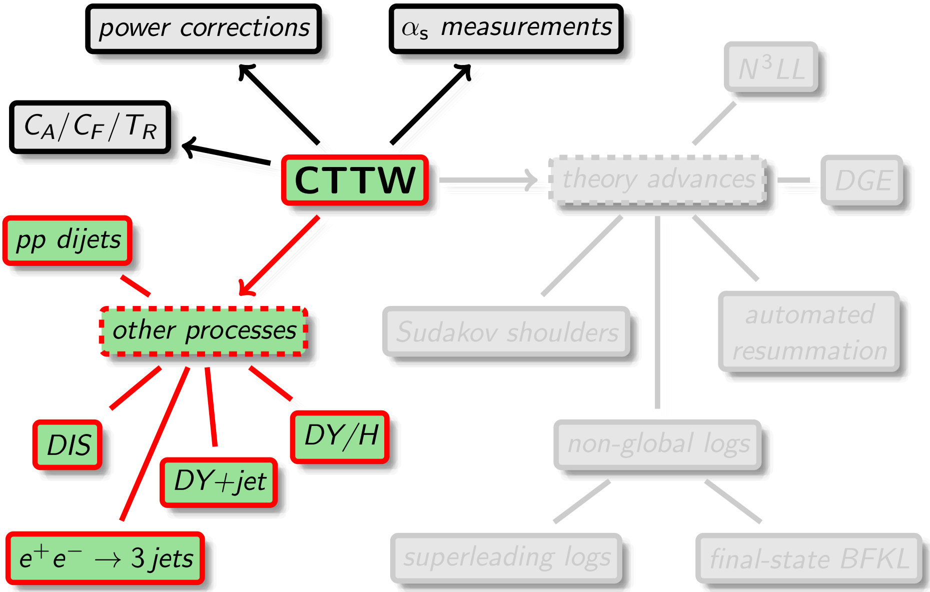
other processes
Sudakov shoulders
automated resummation
DIS
DY/H
DY+jet
non-global logs
$e^+e^- \to 3$ jets
superleading logs
final-state BFKL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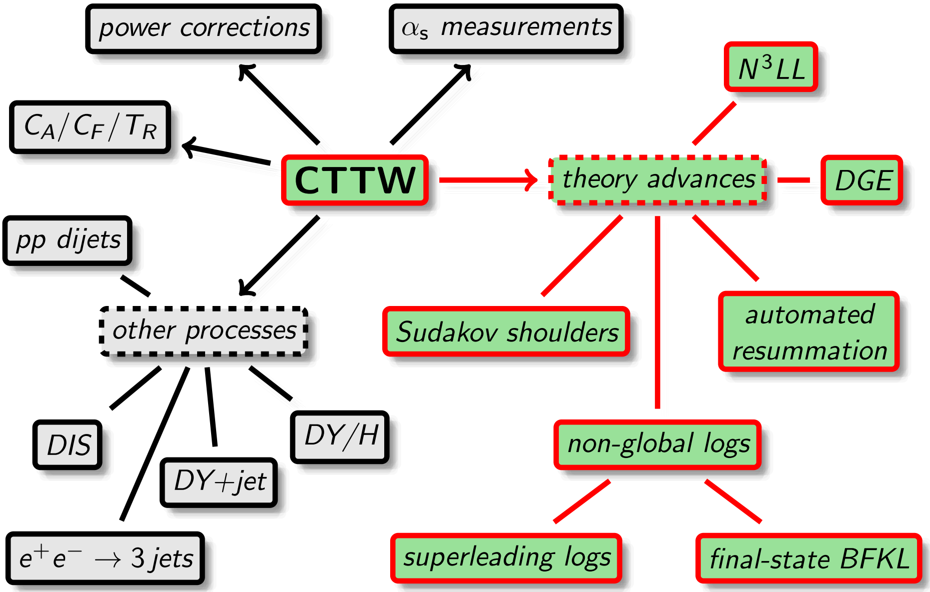
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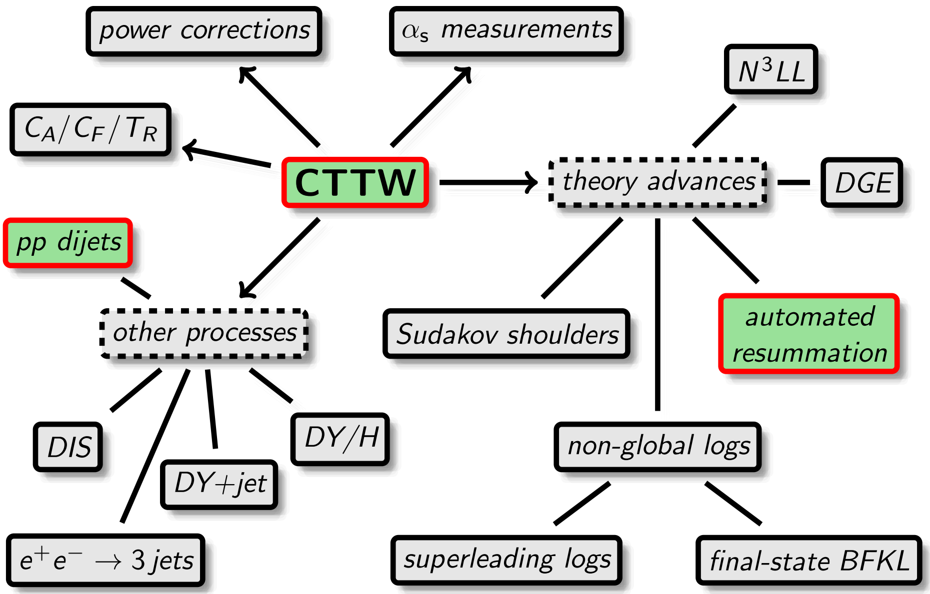
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Bryan and collaborators originally resummed T, m_H, B_W, B_T, C (twice), y_3 all for $e^+e^- \to 2\,$jets

About one paper per observable

After the first few observables it becomes technical rather than challenging, especially for more complex observables (broadenings) and/or processes (e.g. multijet)...

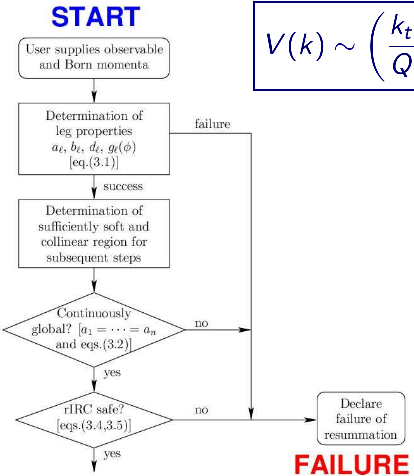
cf. the $e^+e^- \to 3\,$jet series of papers by Andrea, Giulia, Pino & Yuri
or the DIS series by Mrinal & GPS

For LHC, can we get an expert system to do the resummation for us?

Computer **A**utomated **E**xpert **S**emi-**A**nalytical **R**esummer

Banfi, GPS, Zanderighi '03–'05

$$V(k) \sim \left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta_\ell}$$



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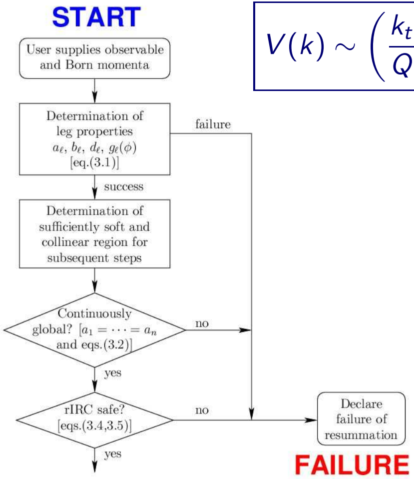
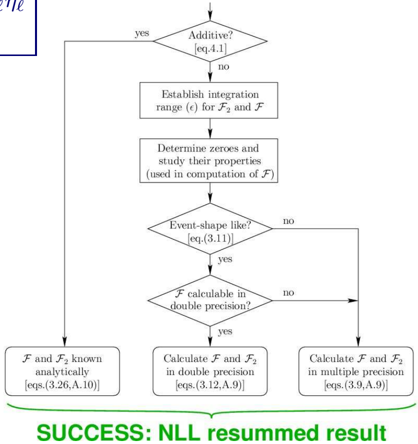
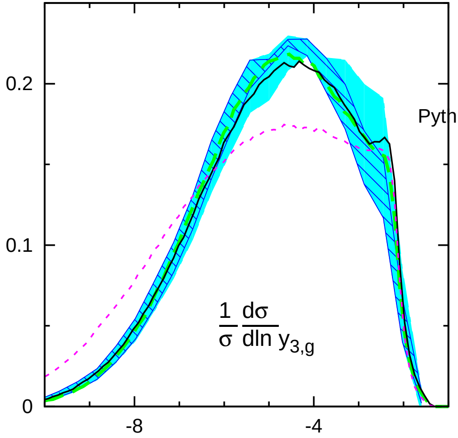




[Banfi, GPS & Zanderighi '10]



TEVATRON

Tevatron, 1.96 TeV

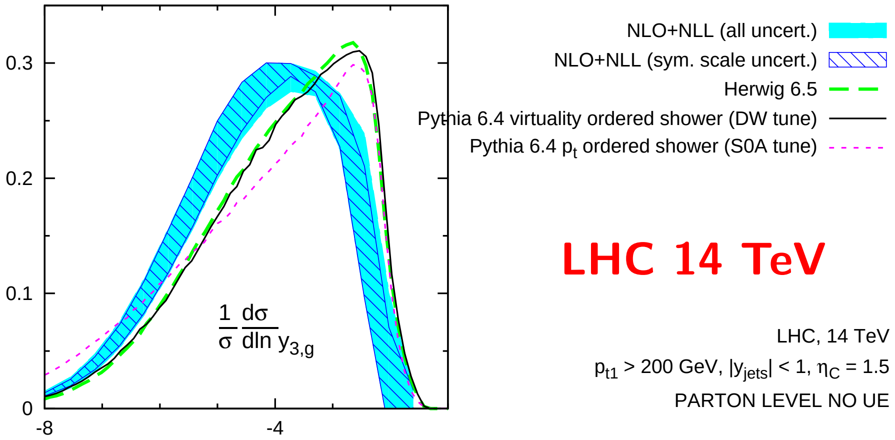
p_{t1} > 200 GeV, $|y_{jets}|$ < 0.7, η_C = 1

PARTON LEVEL NO UE

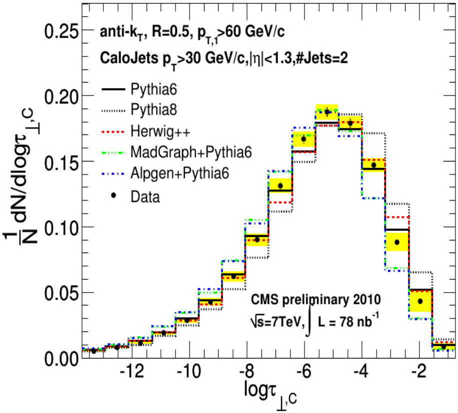
Tevatron at $p_t \sim 200$ GeV is dominated by quark scattering
Monte Carlos and (Caesar NLL + NLOJet) agree well

Hadron-collider event shapes: what do we learn?

[Banfi, GPS & Zanderighi '10]



LHC(14) at $p_t \sim 200$ GeV is dominated by gluon scattering
Monte Carlos seem significantly harder than NLL+NLO



Resummations are for ev. shapes defined in terms of particles:

$$T_\perp \propto \max_{\vec{n}_\perp} \sum_{i \in \text{particles}} \vec{p}_{\perp,i}$$

First LHC measurements, from CMS, are defined in terms of jets

$$T_\perp \propto \max_{\vec{n}_\perp} \sum_{i \in \text{jets}} \vec{p}_{\perp,i}$$

Changes resummation dramatically (often → non-global).

cf Banfi, Dasgupta et al jet azimuthal decorrelations

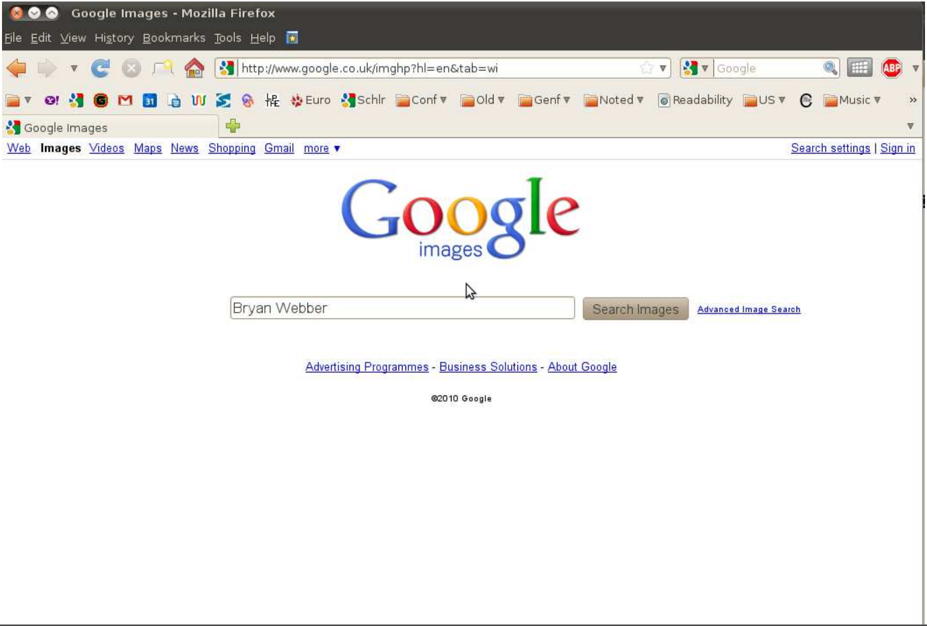
But even with jet-based definition, & no resummation, first LHC results show clear discriminatory power of event shape data.

Conclusions

To keep them short:

scope of resummation is not about to be exhausted!

Pictures of Bryan



Clearly much happier 15 years later!

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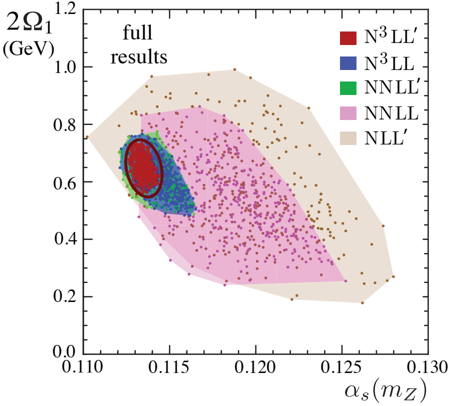
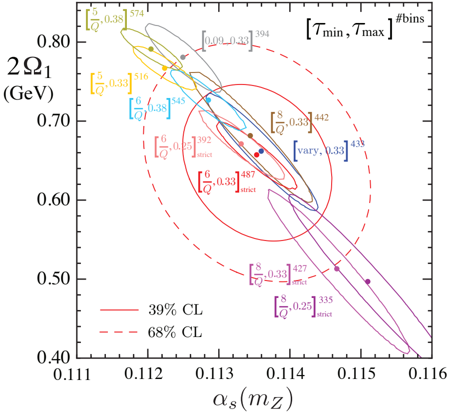
Wishing you even more happiness (and fun papers) over the years to come!

EXTRAS


$2\Omega_1$ (GeV)
full results
N^3LL'
N^3LL
$NNLL'$
NNLL
NLL'
$\alpha_s(m_Z)$


$2\Omega_1$ (GeV)
$[\tau_{min}, \tau_{max}]^{\#bins}$
39% CL
68% CL
$\alpha_s(m_Z)$