Giant K factors

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Work performed with Mathieu Rubin and Sebastian Sapeta, arXiv:1006.2144

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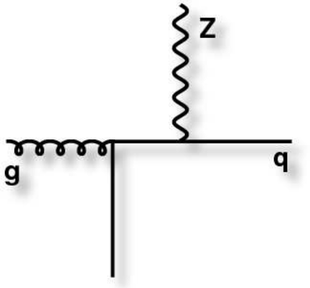
Many searches for New Physics (eg. SUSY) rely on Leading Order predictions for backgrounds.

eg. Z+4jet background to gluino pair production
with NLO technology rapidly becoming mature for such cases

LO often considered good to within a factor of 2
NLO to within 10-20%

This talk is about cases where such "rules of thumb" fail (spectacularly)

The Z+jet process



Use MCFM to examine various properties of such events at LO and NLO.

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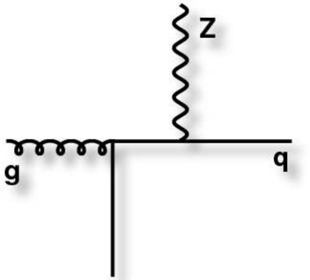

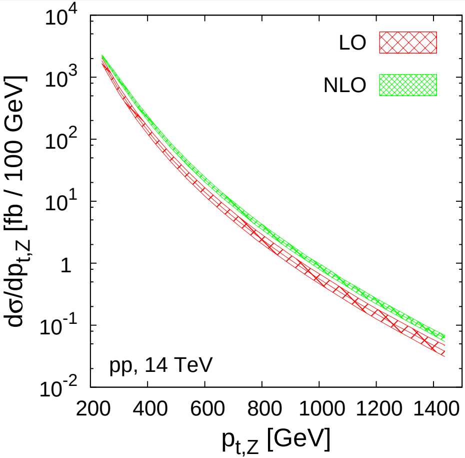

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p_t spectrum of **Z boson** gets K-factor of 1.5

Fairly standard kind of occurrence

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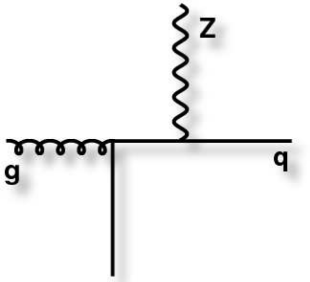

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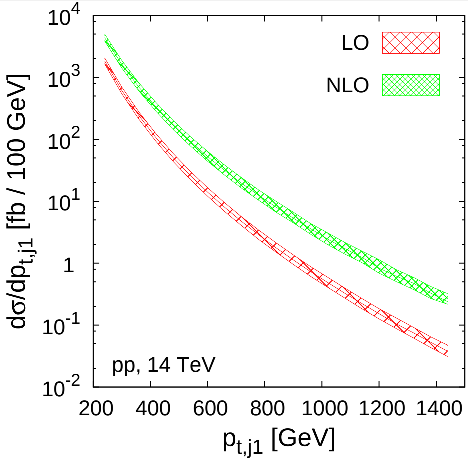
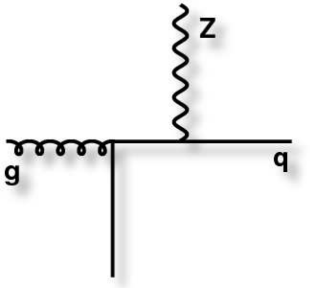


p_t spectrum of **leading jet** gets K-factor of 5–10

related issues in Butterworth, Davison, Rubin & GPS '08

Bauer & Lange '09; Denner, Dittmaier, Kasprzik & Much '09

The Z+jet process



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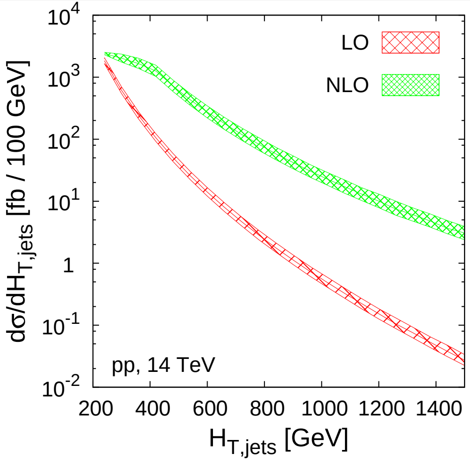
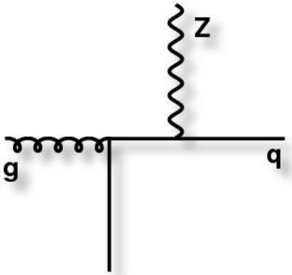


$H_{T,jets} \equiv \sum_{i \in jets} p_{t,i}$ gets K-factor of up to 100

Such things are not supposed to happen with $\alpha_s = 0.1$!

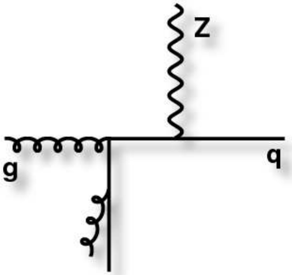
Why the large K-factors?

Leading Order



$\alpha_s \alpha_{EW}$

Next-to-Leading Order



$\alpha_s^2 \alpha_{EW}$

LHC probes scales well above EW scale, $\sqrt{s} \gg M_Z$.
EW bosons are **light**. New log-enhanced topologies appear.

$H_{T,jets}$ is extreme, because at LO $H_{T,jets} \simeq p_{t,jet\,1}$; NLO: $H_{T,jets} \simeq 2p_{t,jet\,1}$

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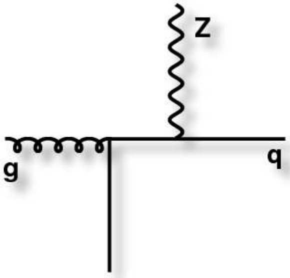

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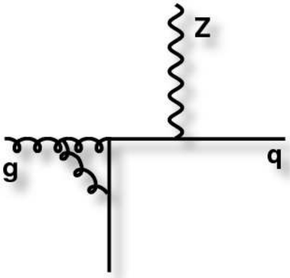

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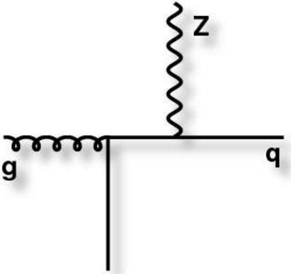

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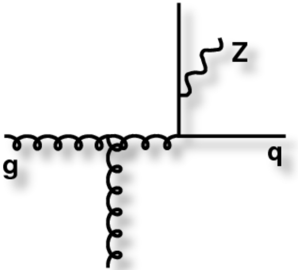

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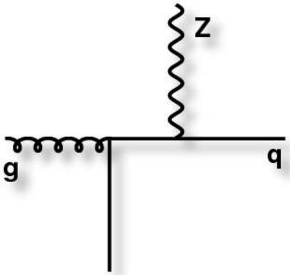

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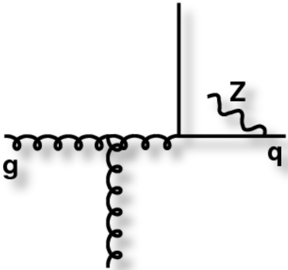

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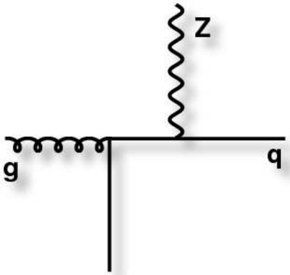

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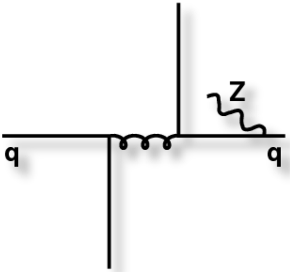

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We calculated Z+jet@NLO.

But giant K-factors are dominated by **"LO"** Z+2-parton piece of Z+jet@NLO.

We know LO calculations aren't reliable.

We'd ideally want to combine Z+jet@NLO with Z+2-jet@NLO

without double counting

without having to do full Z+jet@NNLO calculation

First try something simpler:

Take the "leading" process

[Z + jet @ LO]

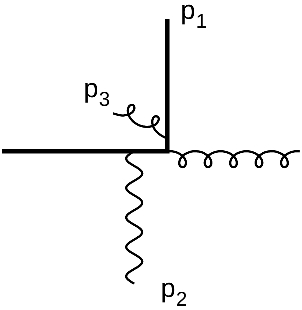
and add in process with one extra jet.

[i.e. include Z + 2 jets @ LO]

approximate the 1-loop Z+jet term, by requiring cancellation of all divergences

[those from singly unresolved limit of Z + 2 jets]

The LoopSim idea: $\bar{n}$LO

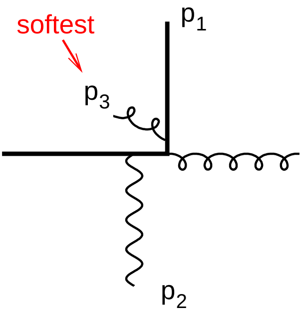


Z + 2 partons

$$|M^2(p_1, p_2, p_3)|$$

- Identify softest or most collinear parton [with help of a jet algorithm]
- "Loop" it ≡ remove it from event, reshuffle other momenta; weight of looped event is $(-1)\times$ weight of tree-level event

The LoopSim idea: $\bar{n}$LO



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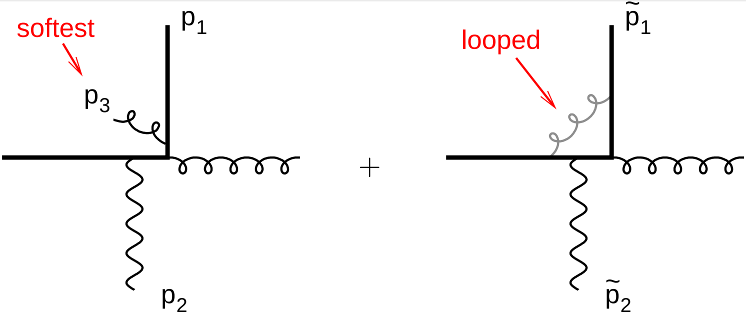

Z + 2 partons — $|M^2(p_1, p_2, p_3)|$

Z + 1 parton + 1 sim. loop — $-|M^2(p_1, p_2, p_3)|$

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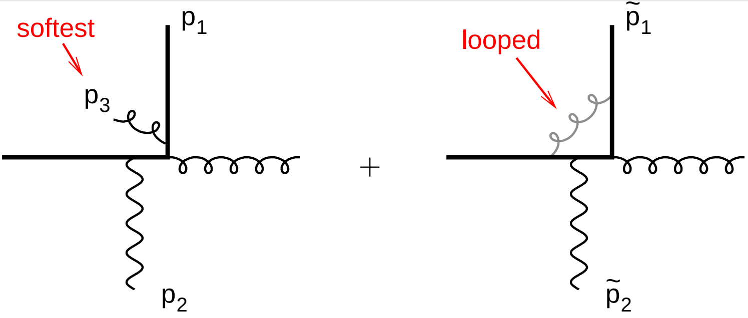
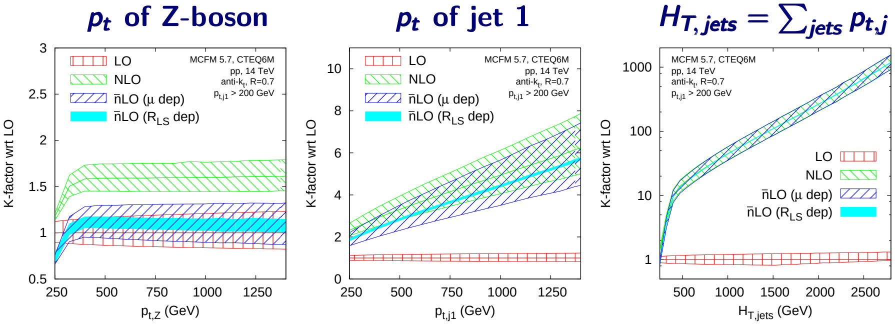


Z + 2 partons	+	Z + 1 parton + 1 sim. loop
$\|M^2(p_1, p_2, p_3)\|$		$-\|M^2(p_1, p_2, p_3)\|$

- Identify softest or most collinear parton [with help of a jet algorithm]
- "Loop" it $\equiv$ remove it from event, reshuffle other momenta; weight of looped event is $(-1)\times$ weight of tree-level event

This cancels all the "single-unresolved" divergences in the Z+2 events

$\bar{n}$LO results (K-factors, normalised to LO)



When the K-factors are large, $\bar{n}$LO agrees well with NLO

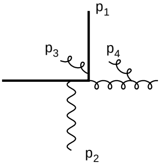
MLM matching also does a similar job
cf. de Aquino, Hagiwara, Li & Maltoni '11

Differences between and LoopSim and MLM/CKKW matching:

1. Does not rely on shower (✓: simplicity; ✗: not easily integrated with shower MCs)
2. Does not need arbitrary separation of Z+1/Z+2/etc. samples with (hard-to-choose) momentum cutoff
3. Can easily be extended beyond LO matching

$\bar{n}\bar{n}$LO and $\bar{n}$NLO

add tree-level Z+3,
cancel divergences in single + doubly unresolved limits: **$\bar{n}\bar{n}$LO**



$|M^2(p_1, p_2, p_3, p_4)|$

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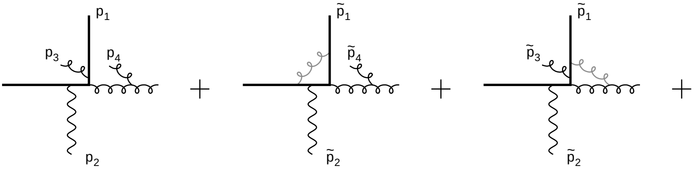
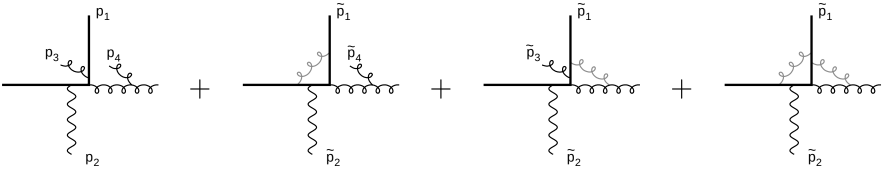


$|M^2(p_1, p_2, p_3, p_4)| \quad -|M^2(p_1, p_2, p_3, p_4)| \quad -|M^2(p_1, p_2, p_3, p_4)|$

provides approximation of 1-loop

$\bar{n}\bar{n}$LO and $\bar{n}$NLO

add tree-level Z+3,
cancel divergences in single + doubly unresolved limits: **$\bar{n}\bar{n}$LO**



$$|M^2(p_1,p_2,p_3,p_4)| \quad -|M^2(p_1,p_2,p_3,p_4)| \quad -|M^2(p_1,p_2,p_3,p_4)| \quad +|M^2(p_1,p_2,p_3,p_4)|$$

provides approximation of 1-loop and 2-loop contributions; total sums to zero

$\bar{n}\bar{n}$LO and $\bar{n}$NLO

add tree-level Z+3,
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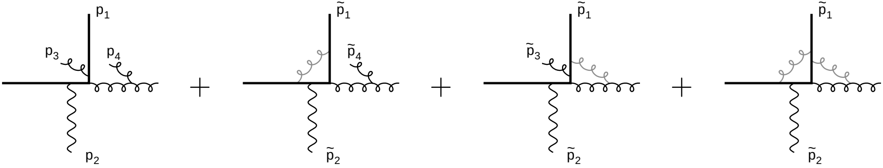

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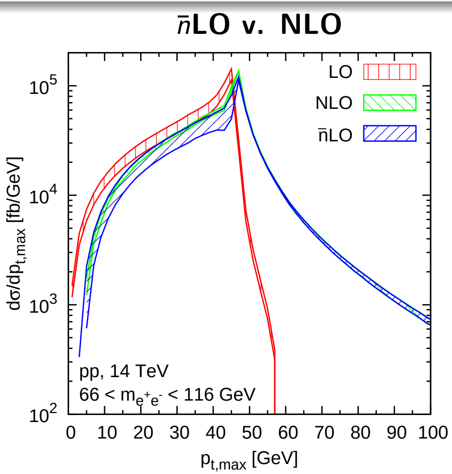
add in (exact Z+2 @ 1-loop) − (approximate Z+2 @ 1-loop)
+ extra simulated 2-loop piece to cancel new Z+2@1-loop divergences
This is $\bar{n}$NLO

allows combination of Z+1@NLO with Z+2@NLO

Testing NLO Merging, in 3 processes

1. Z@NLO with Z+j@NLO
2. Z+j@NLO with Z+2j@NLO
3. 2j@NLO with 3j@NLO

Validation: Drell-Yan lepton p_t, $\bar{n}$NLO v. NNLO



Z (i.e. DY) with Z+j from MCFM & LoopSim

For $p_{t\,ell} \gtrsim \frac{1}{2}M_Z + \Gamma_Z$ (giant K-factor!) it had to work
For $p_{t,\ell} \lesssim \frac{1}{2}M_Z + \Gamma_Z$ it's remarkable that it still works

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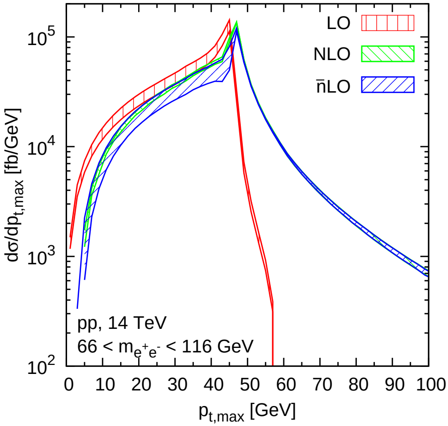
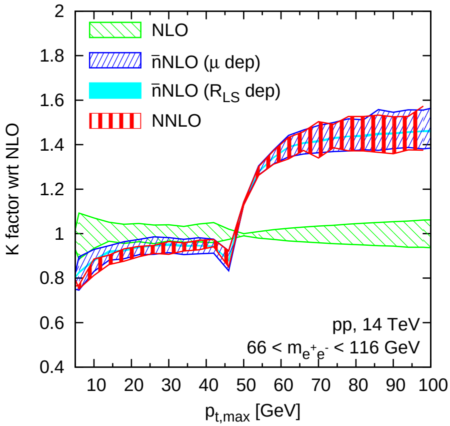
$\bar{n}$LO v. NLO



$\bar{n}$NLO v. NNLO



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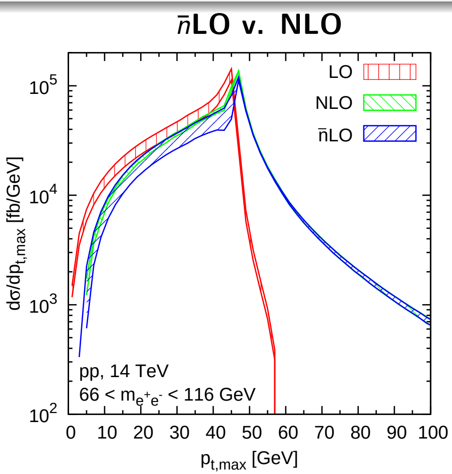

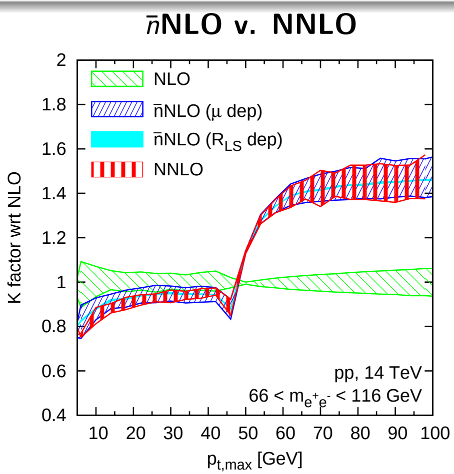

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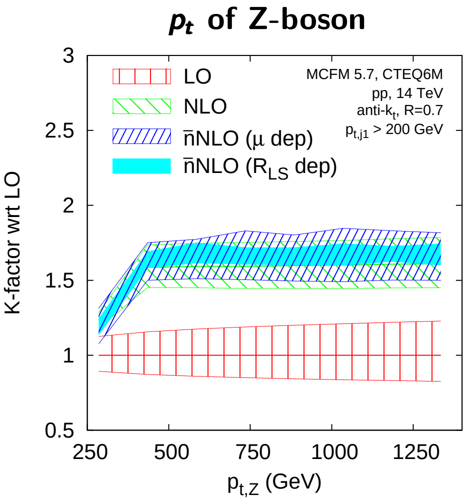
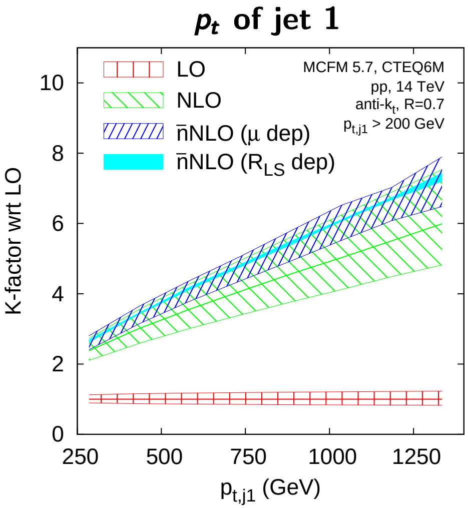
n̄NLO for Z+j observables



- p_{tZ} distribution didn't have giant K-factors.
- $\bar{n}$NLO brings no benefit

  To get improvement you would need exact 2-loop terms

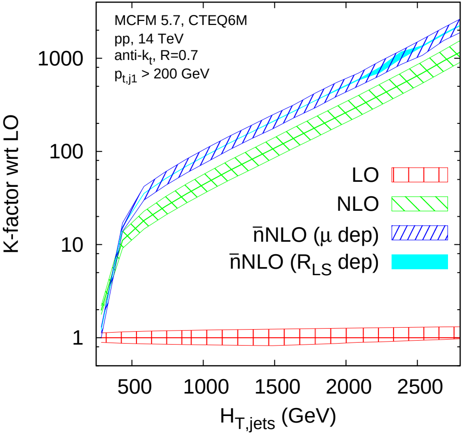
n̄NLO for Z+j observables



- p_{tj} distribution seems to converge at $\bar{n}$NLO
- scale uncertainties reduced by $\sim$ factor 2

n̄NLO for Z+j observables

$H_{T,jets} = \sum_{jets} p_{t,j}$



- Signficiant further enhancement for $H_{T,jets}$
- n̄NLO brings clear message:

$H_{T,jets}$ is not a good observable!

What's the problem with H_T?

H_T (effective mass) type observables are widely used in searches

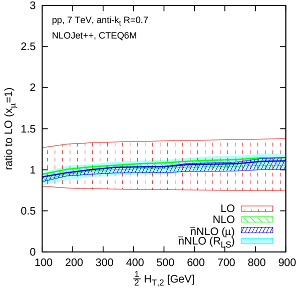
- H_T has a steeply falling distribution (like p_{tj}, p_{tZ})
- At each order (NLO, NNLO), an extra (soft) jet contributes to the H_T sum — e.g. from ISR
- That shifts H_T up, which translates to a substantial increase in the cross section

We can test this hypothesis for plain jet events, using a truncated sum,

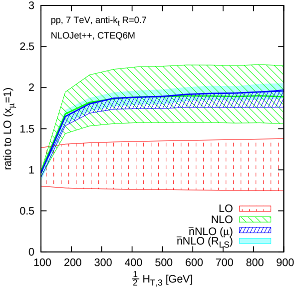
$$H_{T,n} = \sum_{i=1}^{n} p_{t,\text{jet } i}$$

$H_{T,n}$ in (di)jet events

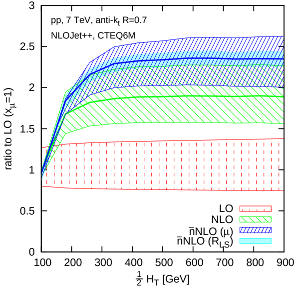
$H_{T,2}$



$H_{T,3}$



$H_{T,\infty}$



A clear message:
for a process with n objects at lowest order, use $H_{T,n}$

Do you know what gets used in your experiment's searches?
Many writers of LHC SUSY proceedings didn't...

Some take-home messages

Be aware that giant K-factors exist

Always look one order beyond the leading order, for example with MLM/CKKW matching

New tool to get good predictions in such cases: **LoopSim**

Basically an "operator" to generate approximations to unknown loops

Combine Z+j@NLO, Z+2j@NLO to get "$\bar{n}$NLO" Z+jet

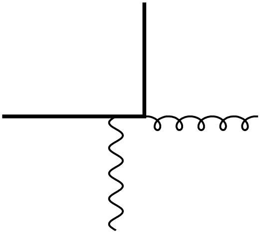
It sometimes works even beyond "giant-K-factor" regions

Watch out for H_T

Even for simple processes, it converges very poorly unless you define it carefully (limit number of objects in sum)

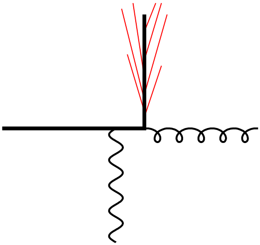
EXTRAS

Add Z+1jet, Z+2jet + shower



Z+parton

Add Z+1jet, Z+2jet + shower



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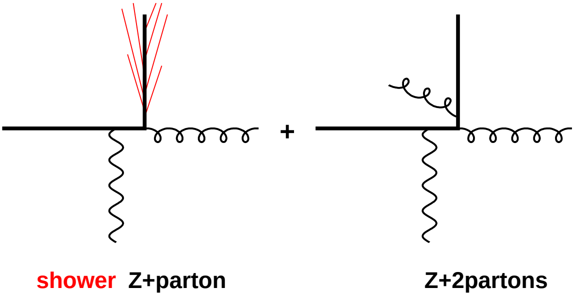

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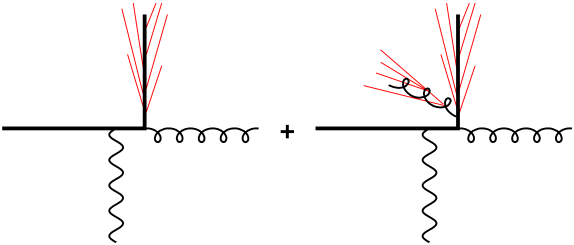
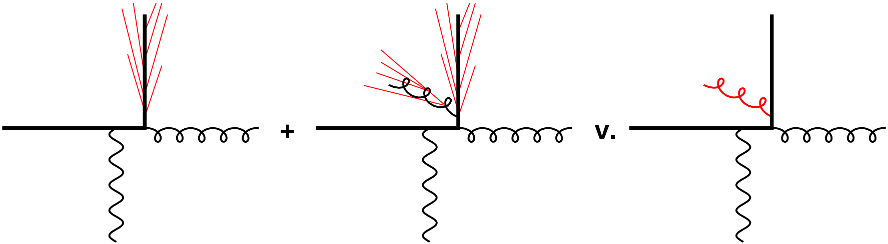


shower Z+parton **shower Z+2partons**

Add Z+1jet, Z+2jet + shower

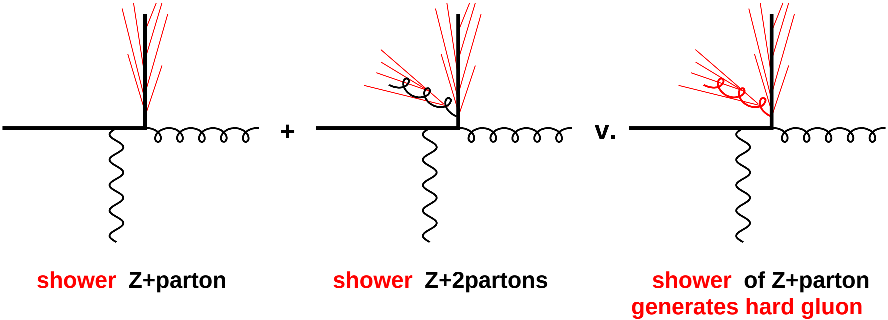


shower Z+parton

shower Z+2partons

shower of Z+parton generates hard gluon

Add Z+1jet, Z+2jet + shower



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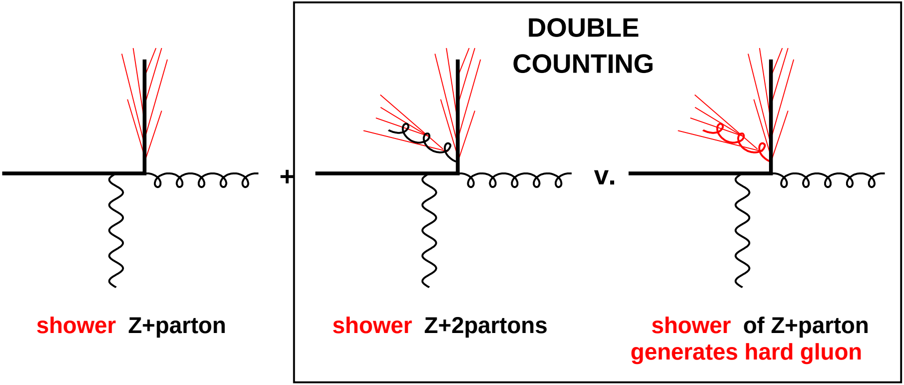
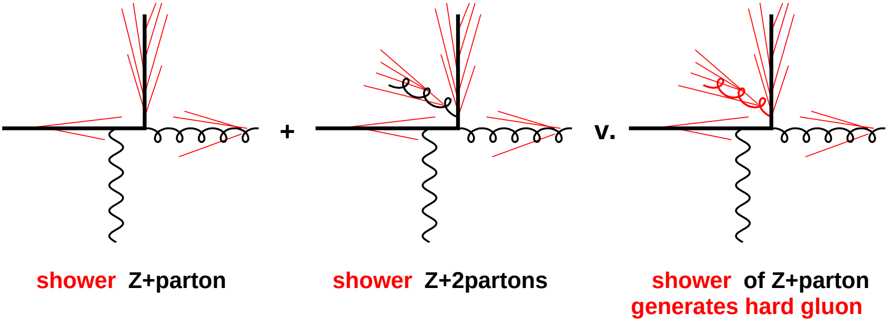


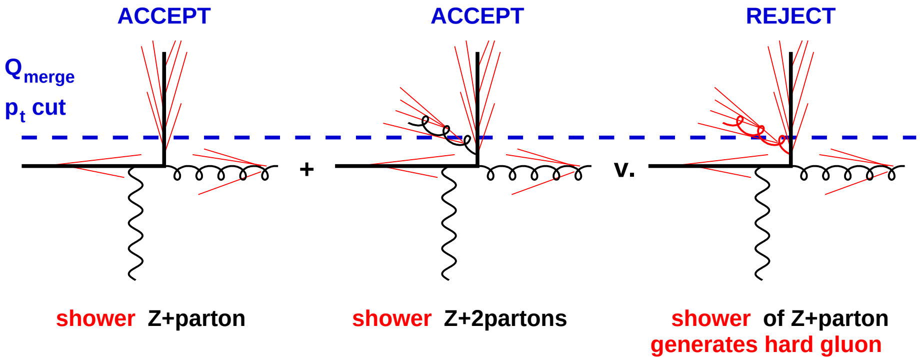
Z + parton implicitly includes part of Z + 2 partons
It's just that the 2nd parton isn't always explicitly "visible"

cartoon of MLM merging of Z+j and Z+2j



- MLM merging relies on parton shower to help figure out what fraction of Z + parton is really Z + 2 partons.
- Our aim is to do that without the parton shower

cartoon of MLM merging of Z+j and Z+2j



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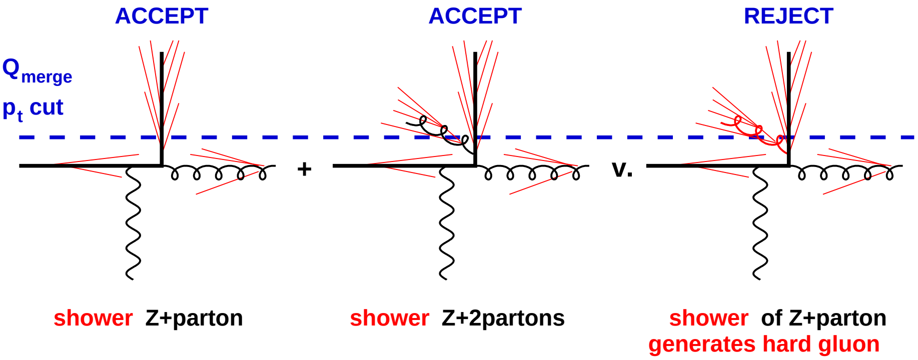

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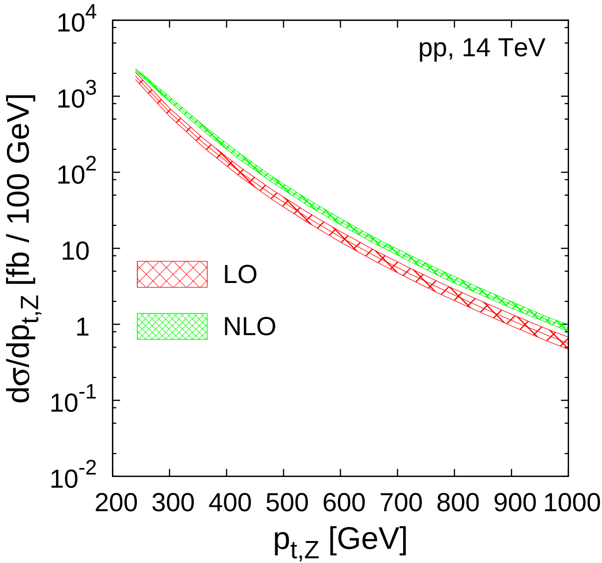
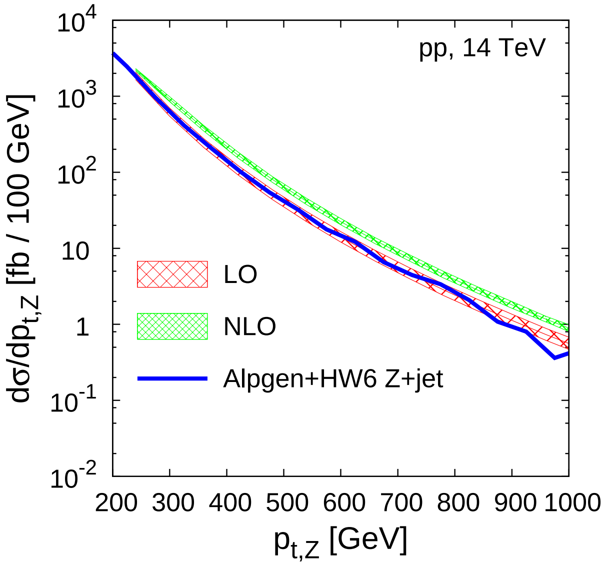
Testing Alpgen + Herwig + MLM Matching

p_t of Z-boson



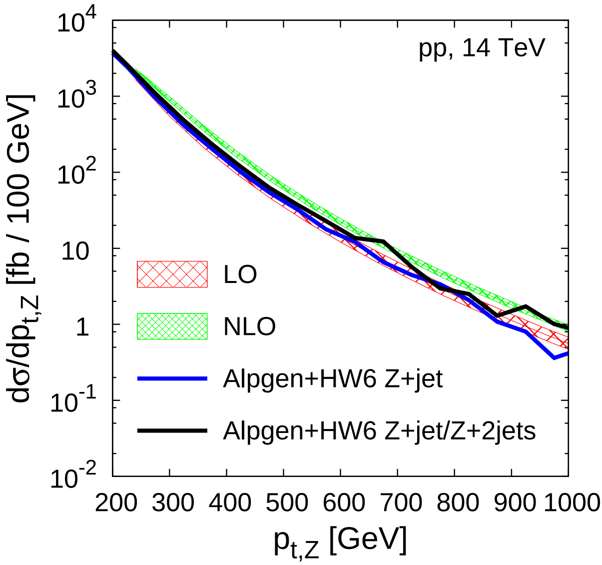
Testing Alpgen + Herwig + MLM Matching

p_t of Z-boson



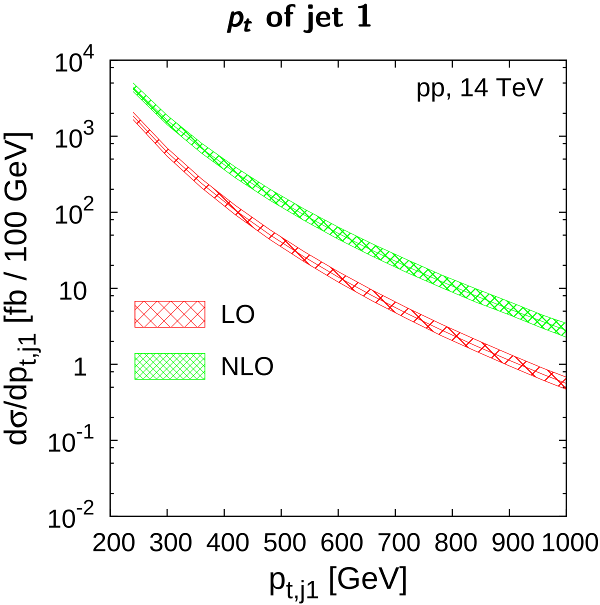
Testing Alpgen + Herwig + MLM Matching

p_t of Z-boson

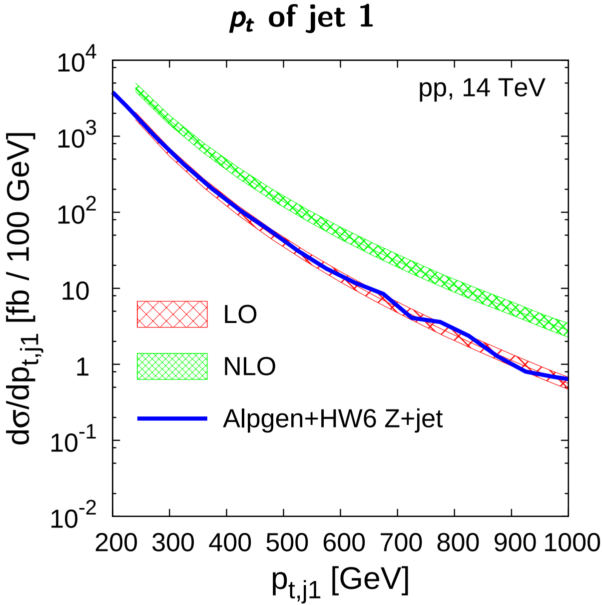


All predictions similar and stable

Testing Alpgen + Herwig + MLM Matching

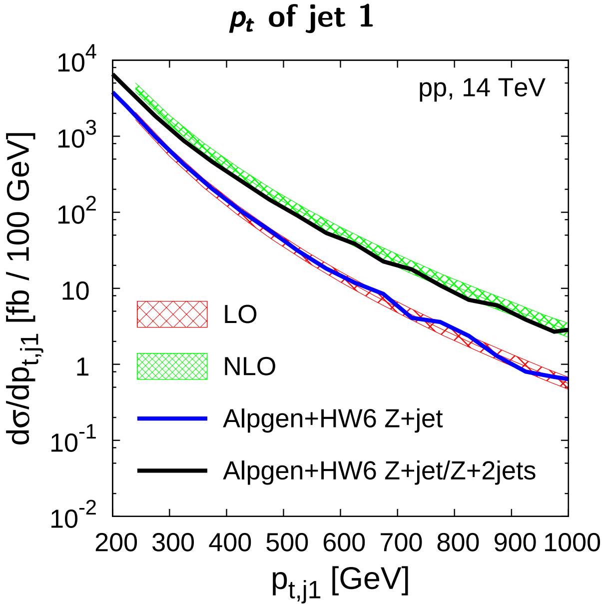


Testing Alpgen + Herwig + MLM Matching



Showered Z+j $\simeq$ LO

Testing Alpgen + Herwig + MLM Matching

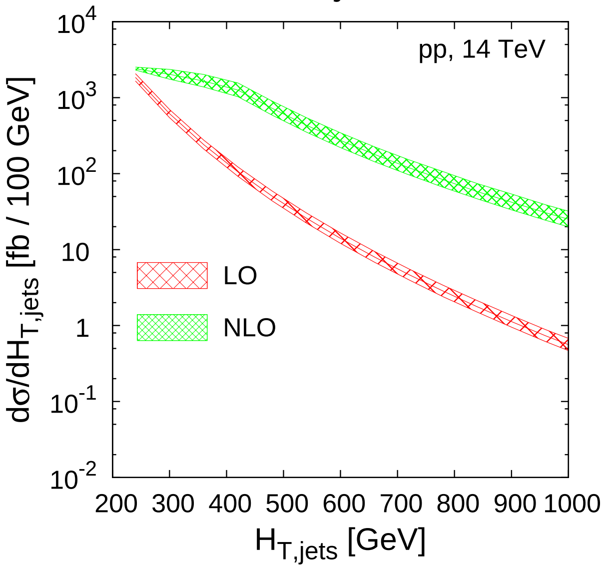


Showered Z+j $\simeq$ LO

Showered Z+j/Z+2j $\simeq$ NLO

Testing Alpgen + Herwig + MLM Matching

$H_{T,jets} = \sum_{jets} p_{t,j}$



Testing Alpgen + Herwig + MLM Matching

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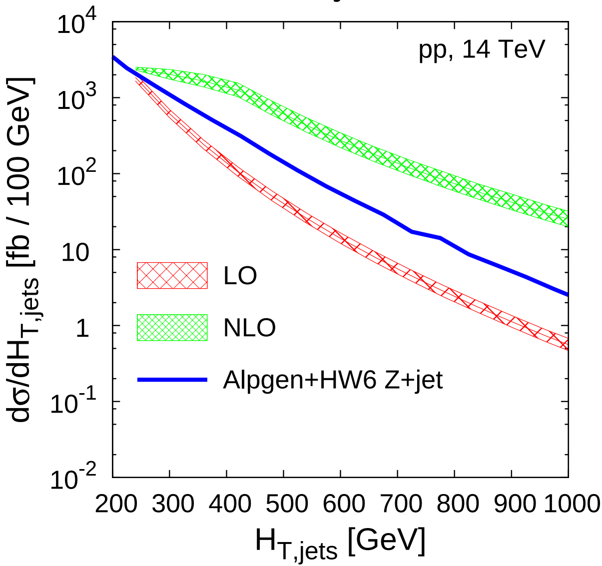

Showered Z+j $\neq$ LO

Testing Alpgen + Herwig + MLM Matching

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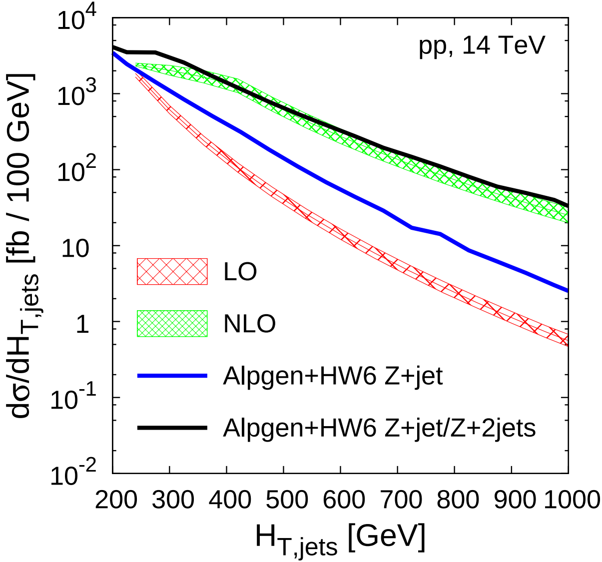

Showered Z+j $\neq$ LO

Showered Z+j/Z+2j $\simeq$ NLO