Remarks on theory uncertainty shapes and on scale variation

Gavin Salam (CERN)

HXSWG – THUTF
CERN, 20 May 2013

There's been a lot of – informal – discussion of what probability distribution (shape) to attribute to theory uncertainties (THU)

[Gaussian, top-hat, log-normal, something in between]

while THU magnitude is taken from scale variation.

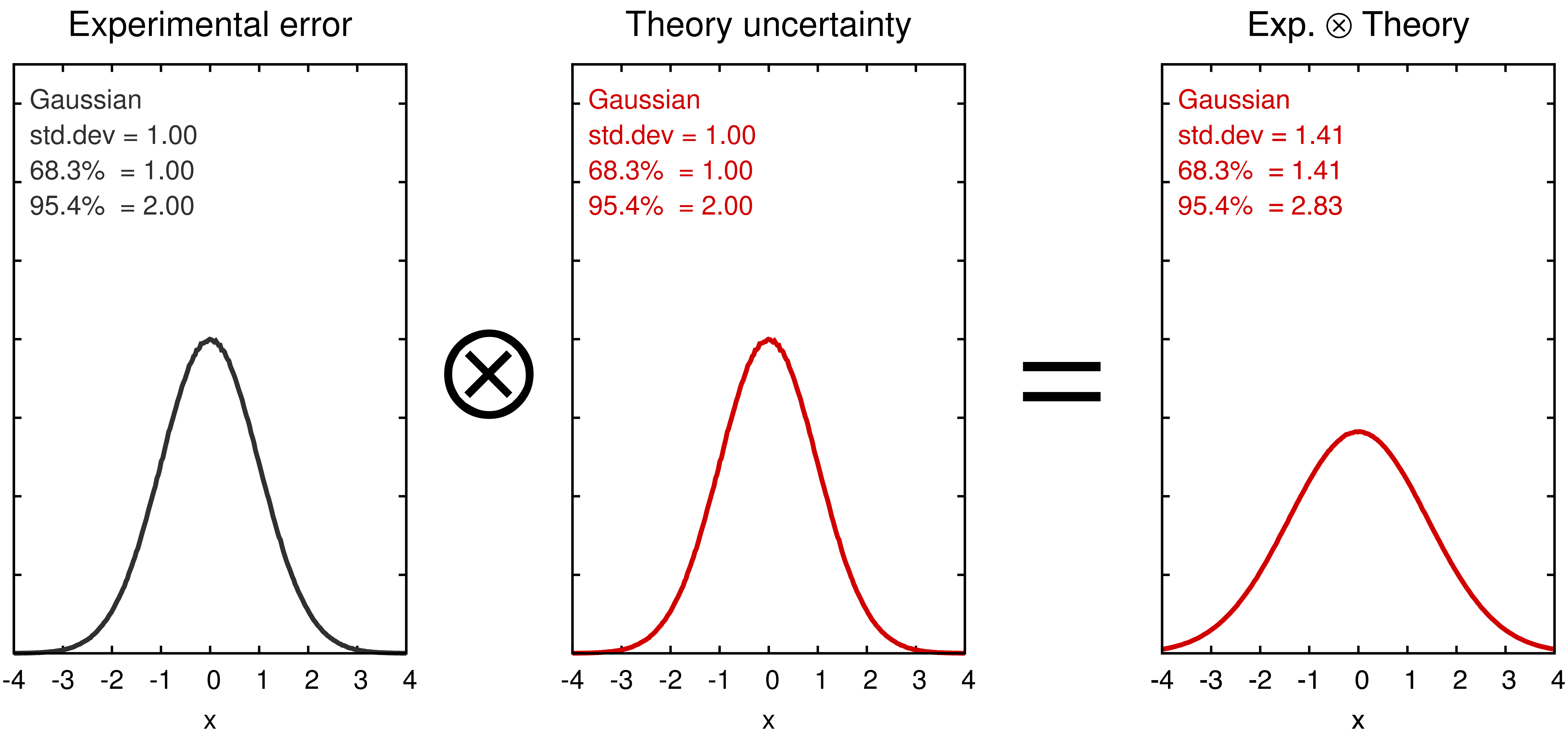
Aim of this talk:

Add simple material towards that discussion, including some ongoing partially-baked thinking

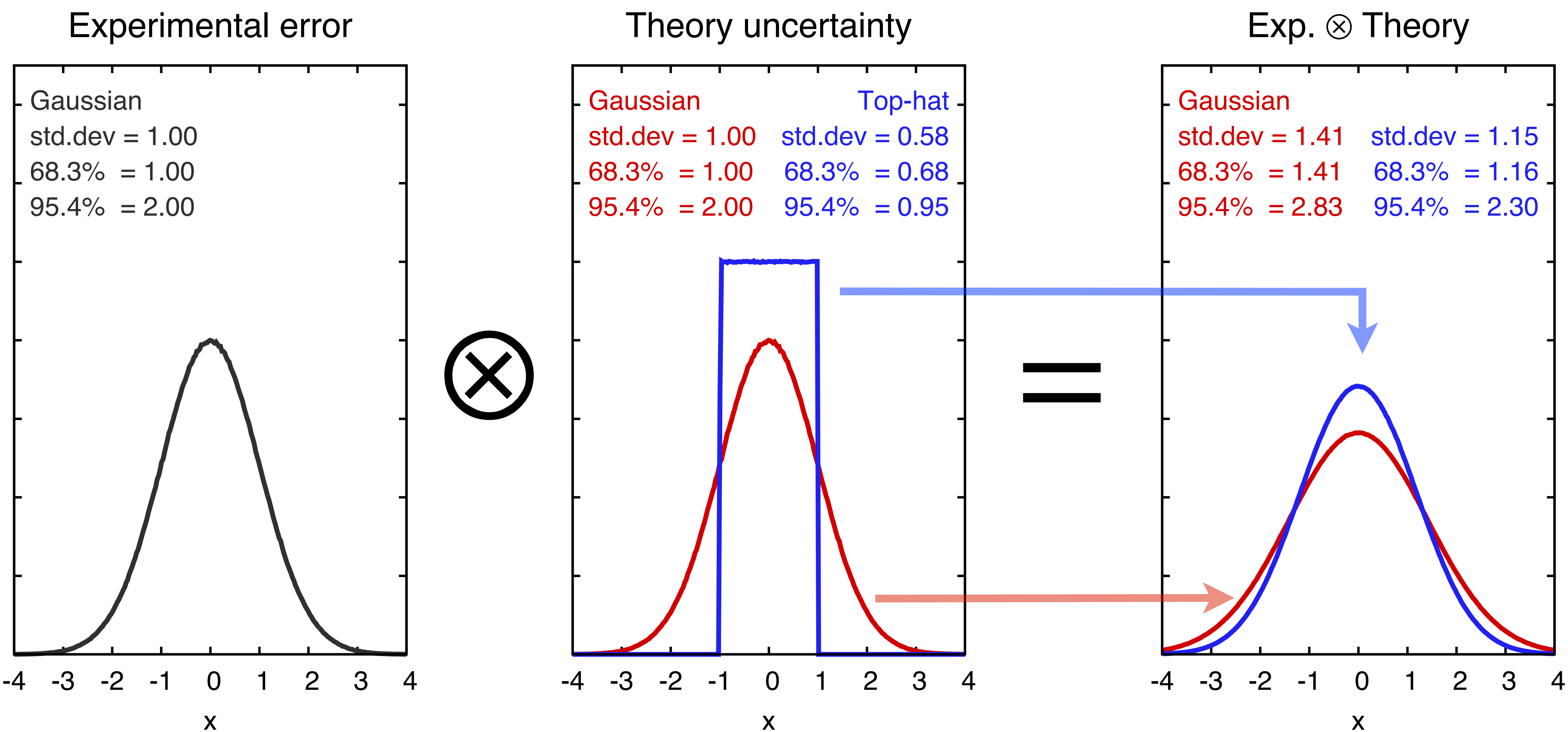
1) about impact of shape of THU

2) about scale variation for estimating magnitude of missing higher orders

As a baseline, consider similar Exp & TH uncertainties. Let's start with scenario of Gaussian TH unc.



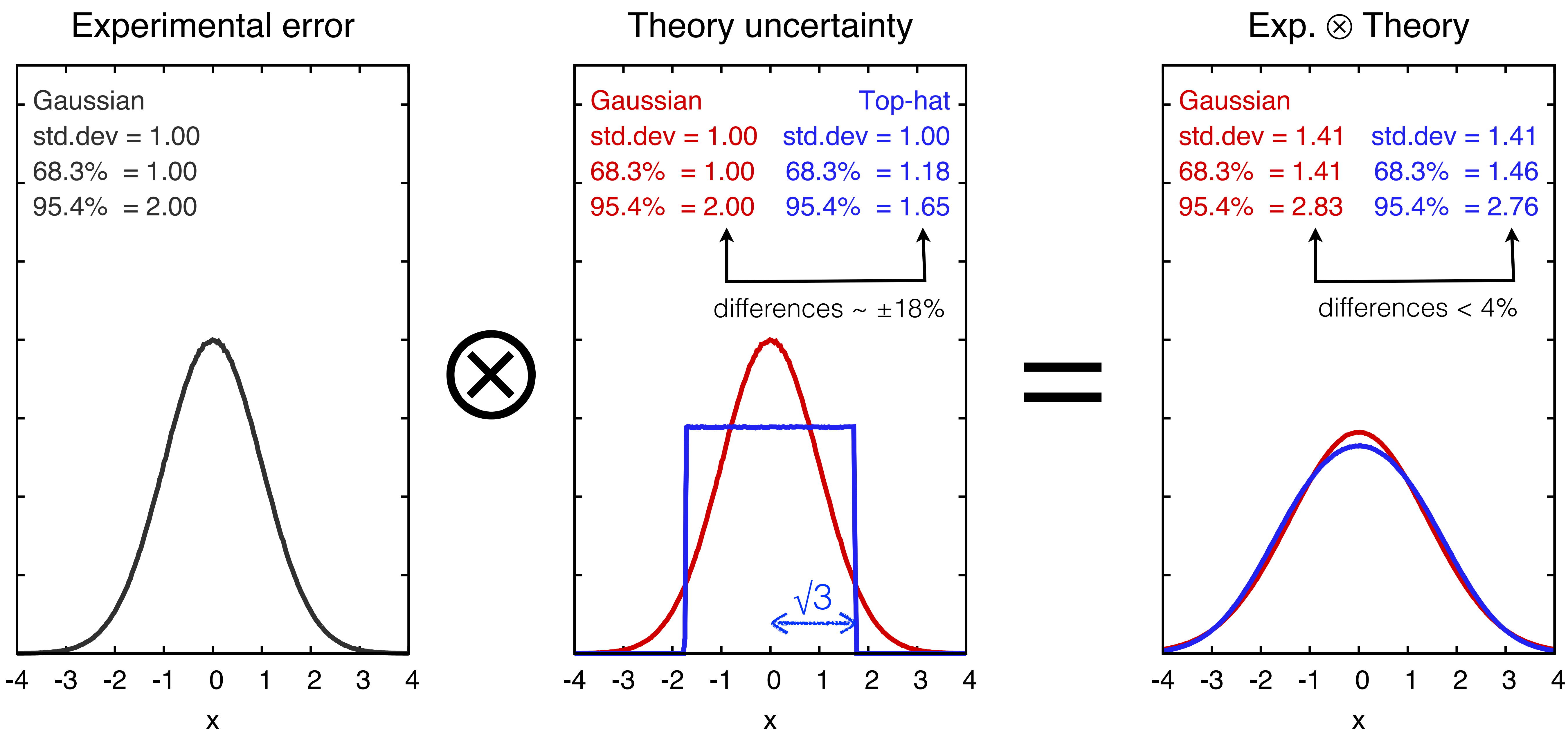
Next: compare with a top-hat TH uncertainty
Choose its half-width = Gaussian std.dev.



People often say top-hat is more conservative than Gaussian
But, here, top-hat actually gives a **smaller** final uncertainty.

Because choice for width → smaller std.dev. ($1/\sqrt{3} \simeq 0.58$) than for Gaussian (1)

To compare "properly", choose top-hat std.dev = Gaussian std.dev.



(Gaussian ⊗ Top-hat) almost identical to (Gaussian ⊗ Gaussian)

This is the central-limit theorem in action

Key feature of THU is its standard deviation

In that context, my (minority?) view is to go with Gaussian THU shape, because

(a) it has just one parameter & is simple for the statistics

(b) it reflects the fact that uncalculated higher orders can take us beyond some given scale variation band

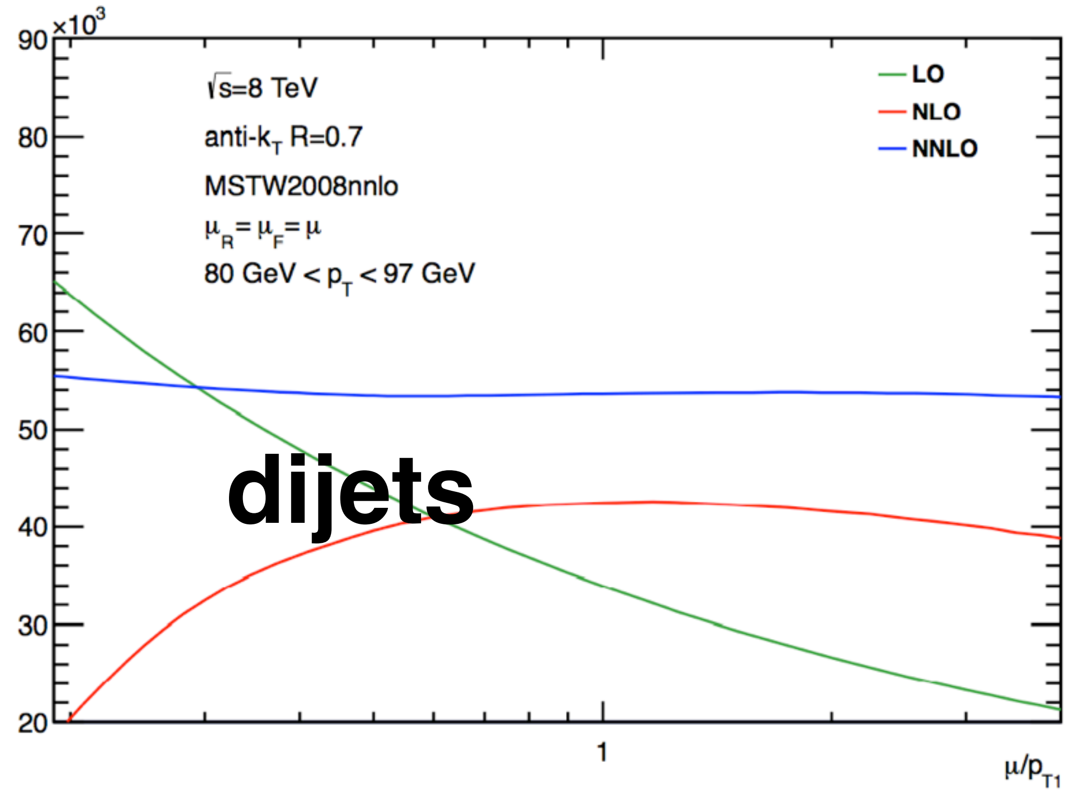
Still need to make choice for std.dev. →
Convention: the size of the (max?) scale variation?

This looks pretty conservative.
But we should remember that HXSWG is seen as an “authority” and there will be pressure for the rest of the community to adopt its choices (cf ST for W+jet ratios).

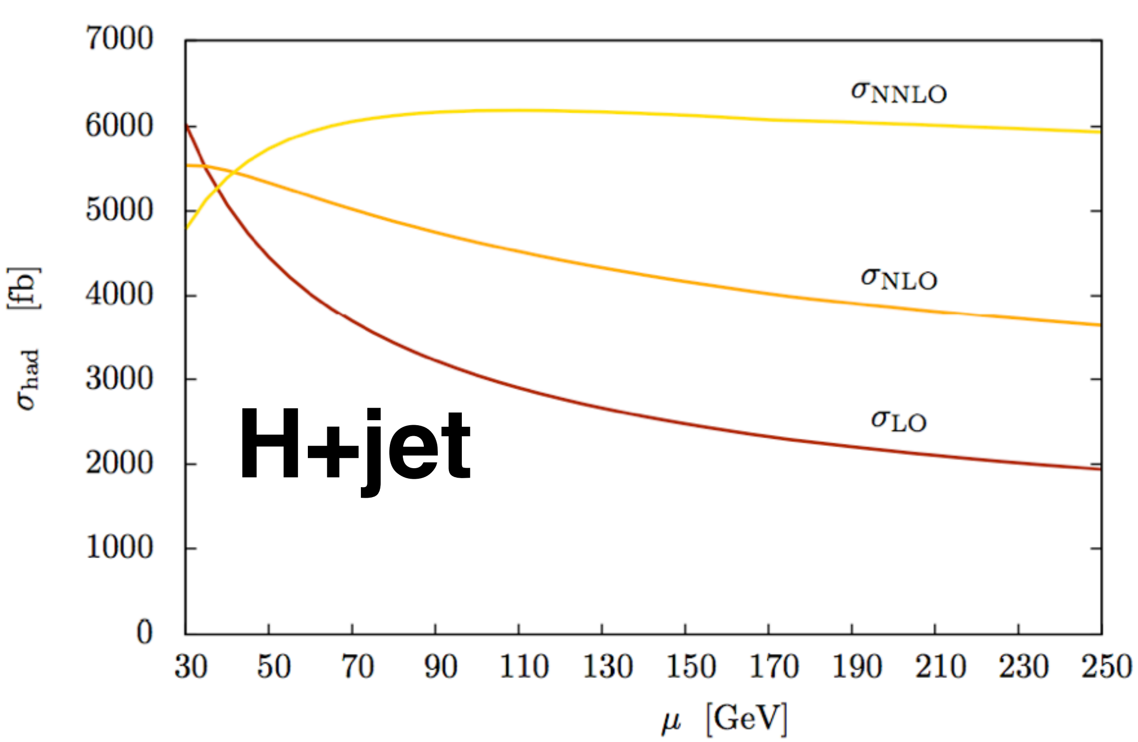
If we go for a more sophisticated procedure I think it the bar should be quite high for demonstrating it brings a substantial advantage.

How much should we rely on scale variation?

[Gehrmann-De Ridder, Gehrmann, Glover, Pires, 1301.7310]



[Boughezal, Caola, Melnikov, Petriello, Schulze, 1302.6216]



There's no shortage of cases where (sometimes partial) NNLO is at or beyond edge of NLO scale variation

[Czakon, Fiedler & Mitov 1303.6254]

$t\bar{t}$ @ LHC8

LO: 145^{+49}_{-34} pb

NLO: 213^{+25}_{-27} pb

NNLO: 239^{+9}_{-15} pb

top++, MSTW2008NNLO, $\mu = m_t$

Scale variation gives an uncertainty
But to what extent is it a measure of ***the*** uncertainty?

Toy model:

(1) Take a running coupling where

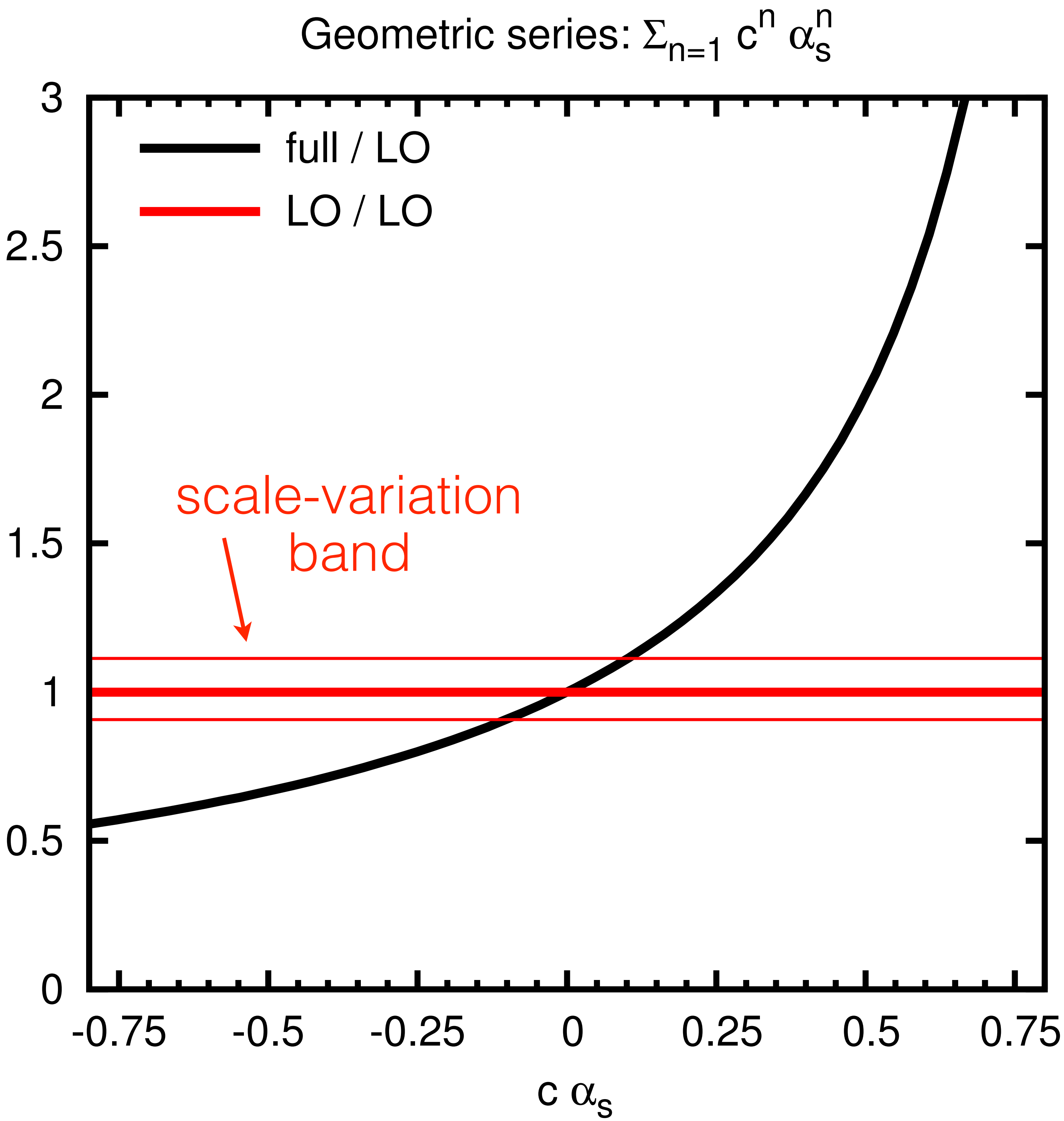
$\beta_0 = \beta_{0,QCD}$
$\beta_1 = \beta_2 = \ldots = 0$

(2) Consider a simple perturbative series that you can sum to all orders. E.g.

$$\sigma = \frac{c\,\alpha_s(M)}{1 - c\alpha_s(M)} = c\,\alpha_s + c^2\alpha_s^2 + c^3\alpha_s^3 + \cdots$$

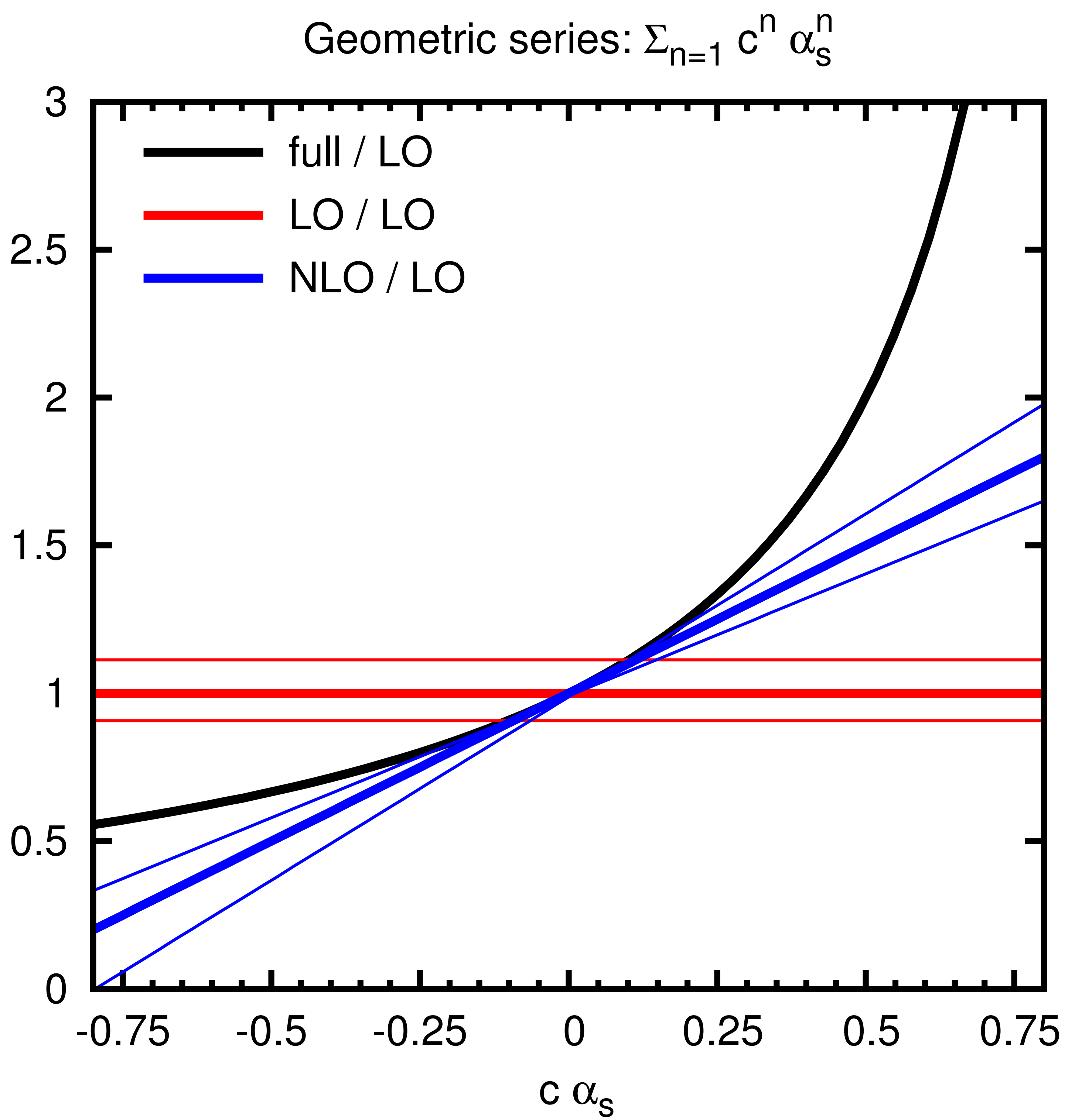
simplest possible series in QCD: corresponds to coupling at one scale expressed in terms of coupling at another (reference) scale M

Now examine truncations of series, as a function of c for $\alpha_s = 0.12$



LO: scale variation mostly useless.

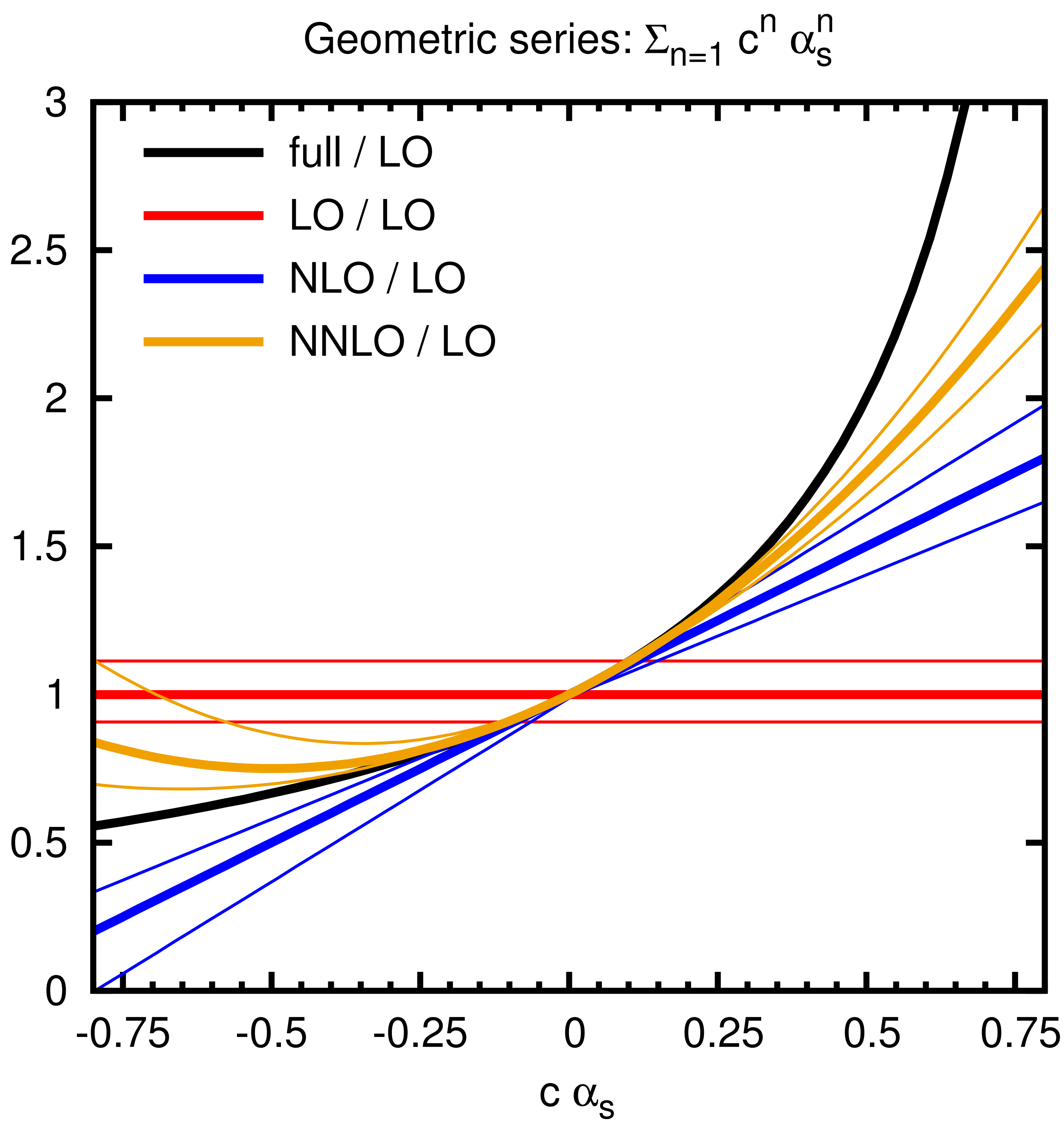
Now examine truncations of series, as a function of c for $\alpha_s = 0.12$



LO: scale variation mostly useless.

NLO: its usefulness extends further, but at some point breaks down.

Now examine truncations of series, as a function of c for $\alpha_s = 0.12$

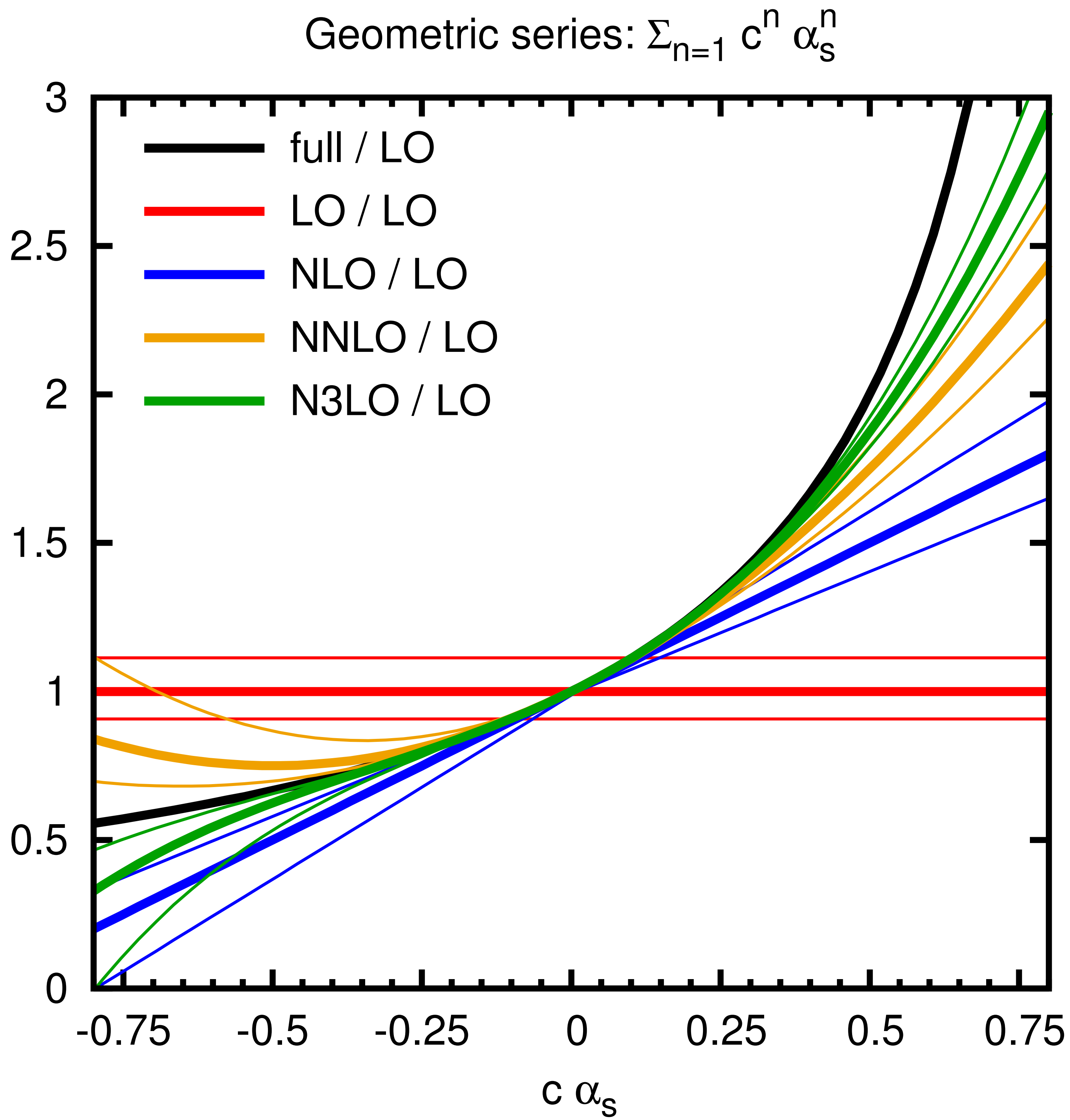


LO: scale variation mostly useless.

NLO: it's usefulness extends further, but at some point breaks down.

NNLO: ditto

Now examine truncations of series, as a function of c for $\alpha_s = 0.12$

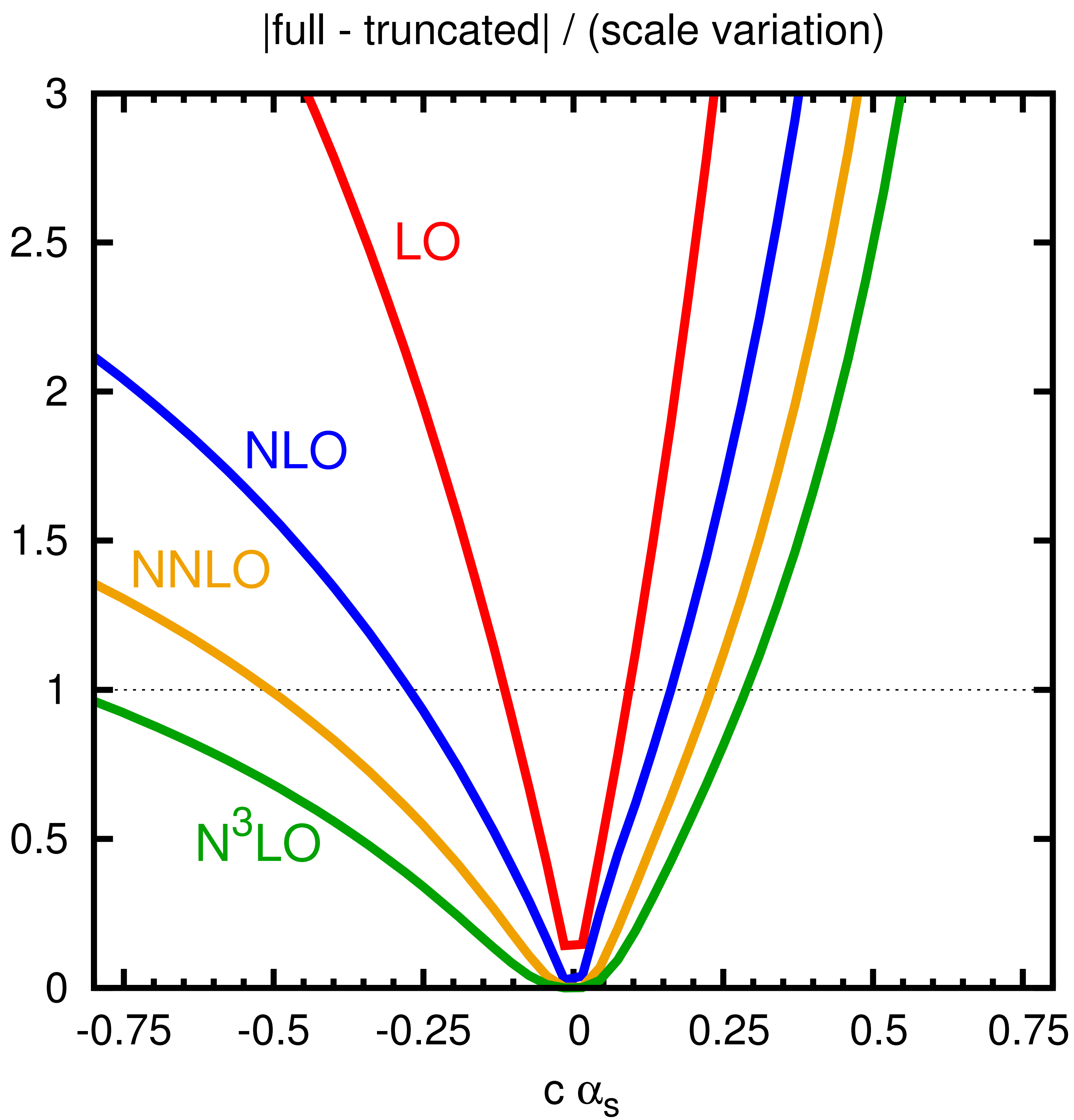


LO: scale variation mostly useless.

NLO: it's usefulness extends further, but at some point breaks down.

NNLO: ditto
NNNLO: ditto

Now examine truncations of series, as a function of c for $\alpha_s = 0.12$



LO: scale variation mostly useless.

NLO: it's usefulness extends further, but at some point breaks down.

NNLO: ditto
NNNLO: ditto

$$\sigma = \sum_{n=1} (c\,\alpha_s)^n$$

Higgs $(\mu = m_H)$
$\mathcal{N} \times (\alpha_s^2 + 11\alpha_s^3 + 62\alpha_s^4)$

Normalised to LO, what's missing from N^{p}LO is:

$$\sim c^{p+1}\alpha_s^{p+1}$$

Scale var^n (c » 1) gives:

$$\sim (p+1) \cdot c^p \alpha_s^{p+1} \quad *$$

Ratio scale uncertainty/ true missing higher orders:

$$\sim \frac{p+1}{c}$$

For poorly converging series (c » 1), scale variation **parametrically** underestimates the uncertainty.

At higher orders (≡ for larger p) scale variation works further, but for large enough c inevitably breaks down

*coefficient is $\frac{23}{6\pi}\ln 2 \simeq 0.85$

Conclusions?

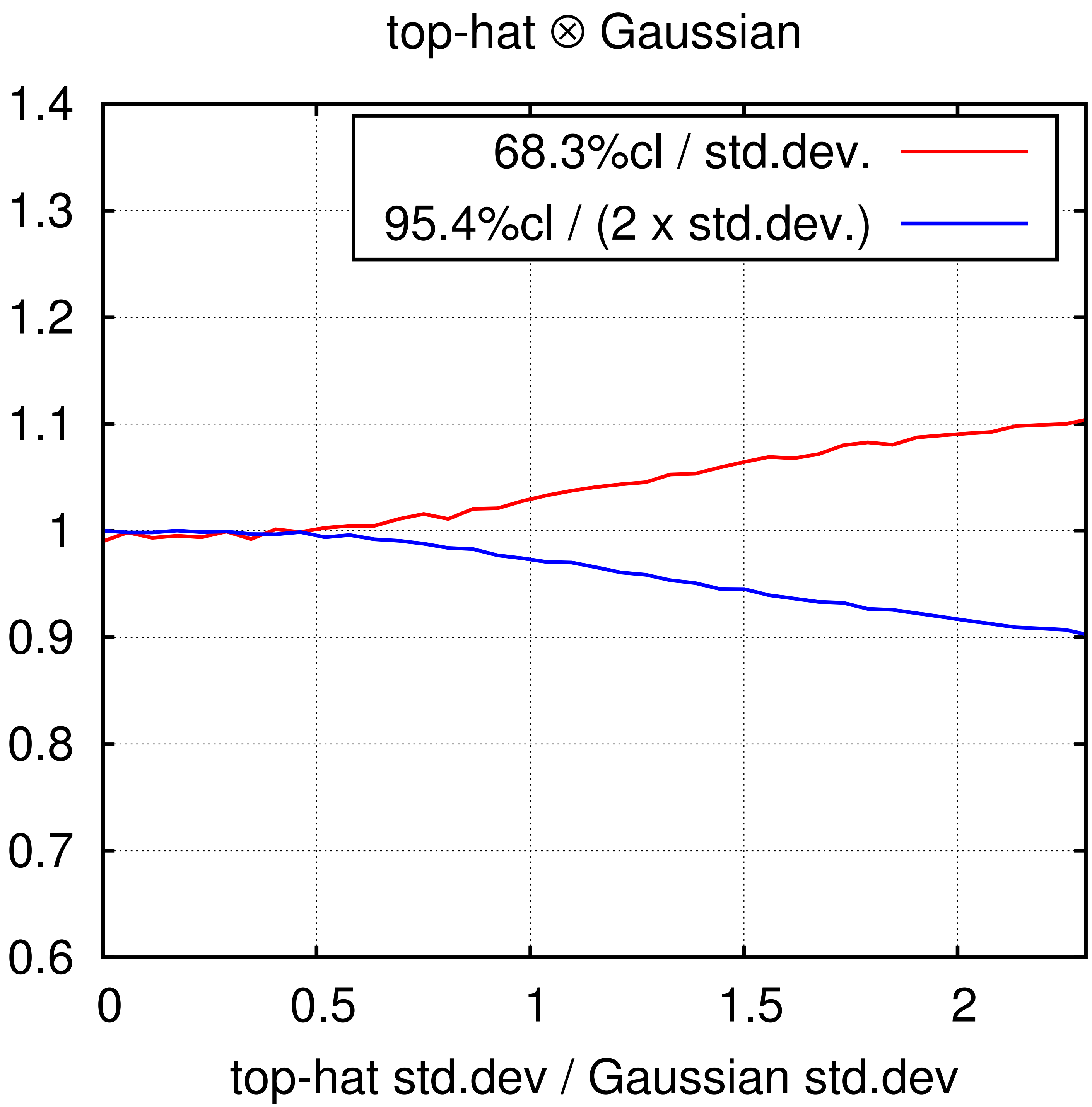
Two messages

- The key aspect of THU shape is its standard deviation
- Scale variation estimates **one** source of uncertainty, but can parametrically underestimate **total** uncertainty.

Open questions

- Should THU shape be anything other than Gaussian? With what std. dev.?
- Does scale variation fail for Higgs production?
- Is there a good alternative/complement to scale var^{n}?

EXTRAS


top-hat ⊗ Gaussian
68.3%cl / std.dev.
95.4%cl / (2 x std.dev.)
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
0.5
1
1.5
2
top-hat std.dev / Gaussian std.dev

Another (view of the same) issue with scale variation: consider two series, with two different α_s values:

$$4\alpha_s + 16\alpha_s^2 + 64\alpha_s^3 = 0.875 \pm 0.039\,, \qquad \text{for } \alpha_s = 0.125$$

$$2\alpha_s + 4\alpha_s^2 + 8\alpha_s^3 = 0.875 \pm 0.077\,, \qquad \text{for } \alpha_s = 0.25$$

But the two series are actually identical:

$$0.5 + 0.25 + 0.125$$

Why should the uncertainty on their sum then depend on the underlying factorisation between α_s and coefficients?

This is an intrinsic property of scale variation, which says that uncertainty ~ α_s x (last term) without taking into account overall structure of series