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LHC PHYSICS @ 1% PRECISION?

Gavin Salam, CERN

EPP Theory Seminar, SLAC, April 1st 2016 Talk in part inspired by discussions at KITP Santa Barbara



The LHC and its Experiments



 ~16.5 mi circumference, ~300 feet underground • 1232 superconducting twin-bore Dipoles (49 ft, 35 t each) • Dipole Field Strength 8.4 T (13 kA current), Operating Temperature 1.9K • Beam intensity 0.5 A (2.2 10⁻⁶ loss causes quench), 362 MJ stored energy

ATLAS: general purpose



ALICE: heavy-ion physics



CMS: general purpose



LHCb: B-physics



+ TOTEM, LHCf

LHC – TWO ROLES – A DISCOVERY MACHINE **AND A PRECISION MACHINE**



Increase in luminosity brings discovery reach and precision



PRECISION LHC PHYSICS NEEDS PRECISION THEORY

Progress on calculations has been stunning in the past years

- ► N3LO Higgs
- ► Many processes at NNLO
- ► NLO + PS automation
- ► First NNLO + PS
- NNLL Resumations
- \succ EW + QCD, etc.

The intention with this talk?

Start asking questions about what precision goals we might set ourselves, what obstacles we will meet, what techniques and measurements might help us progress

This progress is essential for LHC precision physics, but also only part of the story.



PRECISION LHC PHYSICS NEEDS PRECISION THE N³LO Higgs production Anastasiou, Duhr, Dulat, Herzog & Mistlberger '15

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The intent

Only collider process known to this accuracy: O(10⁷) phase space integrals, O(10⁵) interference diagrams, O(10³) three-loop master integrals. A truly amazing technical achievement

And a result that *really* matters for future Higgs physics



Generic reaction: WOW! How did they do it?

G. Zanderighi - CERN & Oxford University

Start asking questions about With at precision goals we might set ourselves, what obstacles we will meet, what techniques and measurements might help us progress



PRECISION LHC PHYSICS NEEDS PRECISION THEORY

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What precision should we have as a target?



HIGGS TODAY & TOMORROW

Production process	ATLAS+CMS
μ_{ggF}	$1.03^{+0.17}_{-0.15}$
$\mu_{ m VBF}$	$1.18^{+0.25}_{-0.23}$
μ_{WH}	$0.88^{+0.40}_{-0.38}$
μ_{ZH}	$0.80^{+0.39}_{-0.36}$
μ_{ttH}	$2.3^{+0.7}_{-0.6}$

ATLAS-CMS Run I combination

In most cases, stat. errors are largest single source

Best channels $\sim \pm 20\%$

Decay channel	ATLAS+CMS
$\mu^{\gamma\gamma}$	$1.16^{+0.20}_{-0.18}$
μ^{ZZ}	$1.31^{+0.27}_{-0.24}$
μ^{WW}	$1.11_{-0.17}^{+0.18}$
$\mu^{ au au}$	$1.12^{+0.25}_{-0.23}$
μ^{bb}	$0.69^{+0.29}_{-0.27}$

HL-LHC prospects?

x2.5 in cross section x150 in luminosity (→ 3000 fb⁻¹) ~ 400 times more events

⇒ stat. errors in 1-2% range



DI-HIGGS PRODUCTION AT HL-LHC (HH \rightarrow 4b, 3ab⁻¹)

Category		signal	background		$S/\sqrt{B_{\rm tot}}$	$S/\sqrt{B_{4b}}$	$S/B_{ m tot}$	S/B_{4b}
		$N_{ m ev}$	$N_{ m ev}^{ m tot}$	$N_{ m ev}^{ m 4b}$				
Doostod	no PU	290	$1.2 \cdot 10^4$	$8.0 \cdot 10^{3}$	2.7	3.2	0.03	0.04
DOOSted	PU80+SK+Trim	290	$3.7\cdot 10^4$	$1.2 \cdot 10^4$	1.5	2.7	0.01	0.02
Intermediate	no PU	130	$3.1 \cdot 10^{3}$	$1.5 \cdot 10^{3}$	2.3	3.3	0.04	0.08
Intermediate	PU80+SK+Trim	140	$5.6\cdot 10^3$	$2.4 \cdot 10^{3}$	1.9	2.9	0.03	0.06
Docolurad	no PU	630	$1.1 \cdot 10^{5}$	$5.8 \cdot 10^4$	1.9	2.7	0.01	0.01
nesorved	PU80+SK	640	$1.0\cdot 10^5$	$7.0 \cdot 10^4$	2.0	2.6	0.01	0.01
Combined	no PU				4.0	5.3	7	
Complified	PU80+SK+Trim				3.1	4.7		

Behr, Bortoletto, Frost, Hartland, Issever & Rojo, 1512.08928

Key signal channels will need ~1% control of complex bkgds





DATA-DRIVEN BKGD ESTIMATES: NON-SMOOTHNESS AT 1% LEVEL



Standard experimental techniques, like data-driven bkgd estimates, can be skewed by O(1%) theoretical subtleties.







p_⊤ [GeV]



systematics.

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Top quark pair, CMC-PDFs, LHC 14 TeV



At HL-LHC, Statistical errors on ttbar production will be < 1% up to Mtt ~ 2 TeV

IN THE FUTURE?

- high-pt W, Z
- high-mass Drell-Yan
- high-mass ttbar

Will all be at ~1% statistical level up to and even beyond the TeV scale.

With leptonic final states, there's a chance systematic errors may also be < 1%.



A YY RESONANCE?



95% C.L. limit σ(pp→ S→γγ) (fb)



1000x more lumi → 1% ?

OVERALL, 1% SEEMS AN INTERESTING FIGURE TO HAVE IN MIND

- To start thinking about getting there, let's work through the "inputs":
- ► the strong coupling
- \blacktriangleright PDFs

- And the types of process:
- inclusive / purely leptonic
- processes with jets



Input parameters? **Concentrate on CKs**

(almost) all theory predictions for LHC are based on perturbation theory, e.g.

 $\sigma = a_s \sigma_1 + a_s^2 \sigma_2 + \dots$



PDG World Average: $\alpha_s(M_Z) = 0.1181 \pm 0.0013 (1.1\%)$

Bethke, Dissertori & GPS in PDG '16







PDG World Average: $\alpha_s(M_Z) = 0.1181 \pm 0.0013 (1.1\%)$

- 1004.4285, 1408.4169)
- suggest
- $a_s(M_Z) = 0.1184 \pm 0.0012(1\%)$
- ► Worries include missing perturbative effects in 3–4

Most consistent set of independent determinations is from lattice

► Two best determinations are from same group (HPQCD,

 $a_s(M_Z) = 0.1183 \pm 0.0007$ (0.6%) [heavy-quark correlators] $a_s(M_Z) = 0.1183 \pm 0.0007 (0.6\%)$ [Wilson loops]

Error criticised by FLAG, who

perturbative contributions, nonflavour transition at charm mass

[addressed in some work], etc.

E+E-EVENT SHAPES AND JET RATES

Two "best" determinations are from same group (Hoang et al, 1006.3080,1501.04111) $a_s(M_Z) = 0.1135 \pm 0.0010 (0.9\%)$ [thrust] $a_s(M_Z) = 0.1123 \pm 0.0015 (1.3\%)$ [C-parameter]

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- thrust & C-parameter are highly correlated observables
- Analysis valid far from 3-jet region, but not too deep into 2-jet region — at LEP, not clear how much of distribution satisfies this requirement
- thrust fit shows noticeable sensitivity to fit region (Cparameter doesn't)

WHAT WAY FORWARDS FOR α_s ?

- ➤ We need to settle question of whether "small" (0.113) a_s is possible. LHC data already weighing in on this (top data), further info in near future (Z p_T, *cf*. later slides)
 ATLAS Preliminary
- To go beyond 1%, best hope is probably lattice QCD — on a 10-year timescale, there will likely be enough progress that multiple groups will have high-precision determinations

NB: top-quark mass choice affects this plot

PDFS

UNCERTAINTIES ON PARTONIC LUMINOSITIES — V. RAPIDITY(Y) AND MASS

gluon-gluon luminosity uncertainty

WHAT ROUTE FOR PROGRESS?

- Current status is 2–3% for core "precision" region
- Path to 1% is not clear e.g. Z p_T's strongest constraint is on qg lumi, which is already best known (why?)
- It'll be interesting to revisit the question once ttbar, incl. jets, Z p_T, etc. have all been incorporated at NNLO
- Can expts. get better lumi determination?
- [is it time for PDFs to include theory uncertainties?]

quark-gluon luminosity uncertainty

Z P_T: the "ideal" hard process?

One obvious thing to talk about is N3LO Higgs

But in terms of precision, both for data and theory, $Z p_T$ is a more immediate testing ground for 1% effects.

(& unlike $Z \& W prod^n$ it's sensitive to a_s)

Z p_T: uncertainties somewhat smaller for ATLAS than CMS

REMARKS

- ➤ Looks like scale uncertainties are ±1-2% (but how well does series converge?) ___
- In key 50–100 GeV region, data seem
 ~4% higher than NNLO theory
- This could have important implications for a_s and PDFs (smaller a_s will not help!)
- What about non-perturbative effects?

Non-perturbative effects in Z p_T

- ► Inclusive Z cross section should have $\sim \Lambda^2/M^2$ corrections ($\sim 10^{-4}$?)
- \succ Z p_T is **not inclusive** so corrections can be $\sim \Lambda/M$.
- ► It seems size of effect can't be probed by turning MC hadronisation on/off [maybe by modifying underlying MC] parameters?]

Non-perturbative effects in Z p_T

- ► Inclusive Z cross section should have $\sim \Lambda^2/M^2$ corrections ($\sim 10^{-4}$?)
- Z p_T is not inclusive so corrections can be $\sim \Lambda/M$.
- Size of effect can't be probed by turning MC hadronisation on/off [maybe by modifying underlying MC] parameters?]
- Shifting Z p_T by a finite amount illustrates what could happen

Multi-Parton Interactions?

Naively, you'd expect these are not correlated with Z p_T — but in at least one MC (Pythia 6) switching them on/ off changes distribution by O(1%)

(with MPI) / (without MPI)

PROCESSES WITH (MEASURED) JETS

much less inclusive wrt QCD radiation subject to larger hadronisation effects

THE JET IN Z+JET @ NNLO

	1- j	et cross	section	5	
	$\sigma_{ m LO}~(m pb)$	$\sigma_{\rm NLO}~({\rm pb})$	$\sigma_{\rm NNLO} \ ({\rm pb})$	$K_{\rm NLO}$	$K_{\rm NNLO}$
8 TeV	$4.17\substack{+0.55 \\ -0.47}$	$6.59_{-0.53}^{+0.62}$	$6.86^{+0.01}_{-0.13}$	1.58	1.04
$13 { m TeV}$	$9.12^{+0.88}_{-0.79}$	$14.90^{+1.29}_{-1.06}$	$15.54_{-0.24}^{+0.01}$	1.63	1.04

- ► NNLO K-factor is 4%
- ► Residual scale uncertainty <2%

HIGGS JET VETO @ N3LO + NNLL

Anastasiou, Duhr, Dulat, Herzog & Mistlberger 1503.06056 Boughezal, Caola, Melnikov, Petriello & Schulze 1504.07922 Banfi, Caola, Dreyer, Monni, GPS, Zanderighi & Dulat 1511.02886

- ► N3LO effects at 2–4%
- Residual uncertainty up to 4% (fairly conservative estimate)

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- ► N3LO effects at 2–4%
- Residual uncertainty up to 4% (fairly conservative)
- rather stable (~2%) wrt jet-p_T resummation effects

2 KINDS OF EFFECT IN SUCH PROCESSES ?

- "Inclusive" correction to process as a whole (insofar as this is meaningful)
- corrections related to jet fragmentation

Can we make such a distinction more meaningful?

VECTOR-BOSON FUSION \rightarrow **HIGGS**

double DIS approximation is powerful tool for VBF, using structure functions for the W/Z production (Han, Valencia & Willenbrock 1992, NNLO by Bolzoni et al 1003.4451)

- Now being extended to N3LO, shows scale uncertainties $\ll 1\%$ for observables inclusive wrt the jets
- good stability from NNLO to N3LO

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N3LO VBF (no cuts)

VBF with cuts on jets: Projection to Born method

(a) Born VBF process

using VBF 3-jet @ NLO from Jäger, Schissler & Zeppenfeld, 1405.6950

Cacciari, Dreyer, Karlberg, GPS & Zanderighi, 1506.02660

Inclusive part only (with VBF cuts)

NNLO is 1% effect

Full calculation (with VBF cuts)

NNLO is up to 8% effect

Almost all of which comes from jet fragmentation

Can we examine same idea in other contexts? E.g. inclusive jet spectrum

- > There is no way of defining the "inclusive" part in most cases
- > But there are arguments that for a jet radius $R_m \approx 1$, ISR and FSR effects mostly cancel each other [Soyez, 1006.3634]
- > So try looking at effect of NNLO corrections relative $R_m = 1$ [can be done with NLO 3-jet calcⁿ from NLOJET++]

$$\sigma^{\mathrm{NNLO}_R}(R, R_m) \equiv \sigma_0 +$$

Dasgupta, Dreyer, GPS & Soyez, 1602.01110

 $\sigma_1(R) + [\sigma_2(R) - \sigma_2(R_m)]$

R-dependent piece of NLO NNLO, relative to R_m

► Full NNLO will have an additional NNLO term associated with the effective K-factor for the "inclusive" piece — we miss that part (and unlike VBF, it may not be small)

$\textbf{NNLO}_{R} \ \textbf{\& small-R resummation}$

to explore full R-range, need resummation as well

 $\sigma(R) = \sigma(R_0 = 1) \times \operatorname{ratio}(R, R_0)_{\text{fixed-order} + \operatorname{LL}_R}$

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 $\sigma(R) = \sigma(R_0 = 1) \times \operatorname{ratio}(R, R_0)_{\text{fixed-order} + \operatorname{LL}_R}$

NON-PERTURBATIVE EFFECTS & JETS

Often discussed for inclusive jet spectrum But relevant for any process involving jets

INCLUSIVE JETS

Two effects: Hadronisation & "Underlying Event" (UE/MPI) 5 – 15% effects, often of opposite signs

Dasgupta, Dreyer, GPS & Soyez, 1602.01110

REMARKS

- Non-pert. effects are always relevant at accuracies we're interested in
- Watch out for cancellation between "hadronisation" and MPI/UE (separate physical effects)
- Definition of perturbative / nonperturbative is ambiguous
- Alternative to MC: analytical estimates.
 MC's have strong pT dependence, missing in analytical estimates

non-perturbative effects may become a key limitation at 1%

POWERFUL HANDLE: EXPLORE A RANGE OF JET RADI

3 effects:

- > perturbative ($\sim \ln R$)
- ► hadronisation (~ 1/R)
- > MPI/UE ($\sim R^2$)
- To disentangle them, need $\geq 3 \text{ R}$ values:
- ► 0.6–0.7: large MPI/UE
- ► 0.4: non-pert. effects cancel?
- ► 0.2–0.3: large hadronisation

ratio of inclusive jet spectra at R=0.4 and 0.6

Dasgupta, Dreyer, GPS **POWERFUL HANDLE: EXPLORE A RANGE OF JET RADI** & Soyez, 1602.01110 ratio of inclusive jets at R=0.2 and 0.4 3 effects: pp, 2.76 TeV, CT10 > perturbative ($\sim \ln R$) |y| < 0.5, anti-k_t alg. =0.4) \blacktriangleright hadronisation (~ 1/R) **0.8** – $0.5\mu_0 < \mu_R, \mu_F < 2\mu_0, \underline{R}_0 = 1$ atio $\sigma(p_t; R=0.2)/\sigma(p_t; R$ > MPI/UE ($\sim R^2$) 0.6 To disentangle them, need $\geq 3 \text{ R}$ values: 0.4 ALICE data +++ $NLO \times (NP \text{ corr.})$ 0.2 $NNLO_R \times (NP \text{ corr.})$ ► 0.2–0.3: large hadronisation

- ► 0.6–0.7: large MPI/UE
- ► 0.4: non-pert. effects cancel?

This one usually missing (except ALICE); needs small-R resummation

 $(NNLO+LL_R) \times (NP \text{ corr.})$ 0 80 40 60 120 20 100 pt [GeV]

COMMENTS / CONCLUSIONS

- ► We're entering the precision era today, notably with 1% Z p_T distribution (first hadron-collider process $\propto \alpha_s$ known with this precision)
- these remains to be developed...
- Processes with jets need a dedicated effort to improve the precision

> 1% precision is something that we will want to reach for a range of processes to get full value out of the "precision" part of LHC's programme (Higgs, top, dilepton, ...)

> Even a Z can have non-perturbative corrections — framework for understanding

BACKUP

EXPERIMENTAL PERSPECTIVES

Generation of pseudo-data: the Z pt

Generate pseudo-data for the transverse momentum distribution of Z bosons decaying into leptons Statistical uncertainties determined from **number of events per bin**, after a binning optimisation Added a **2% systematic uncertainty** to the statistical uncertainty

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Generation of pseudo-data: high-mass Drell-Yan

Generate pseudo-data for the invariant mass distribution of di-electrons and di-muons

Statistical uncertainties determined from **number of events per bin**, after a binning optimisation

Added a **2% systematic uncertainty** to the statistical uncertainty

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Generation of pseudo-data: top quark pair

Generate pseudo-data for the **invariant mass distribution** in the **leptonic final state** Statistical uncertainties determined from number of events per bin, after a binning optimisation Added a **3% systematic uncertainty** to the statistical uncertainty

Juan Rojo

ABSOLUTE CROSS-SECTIONS MEASURED TO $\sim 1\%$?

Beam Imaging and Luminosity Calibration arXiv:1603.03566v1 [hep-ex]

March 14, 2016

Markus Klute, Catherine Medlock, Jakob Salfeld-Nebgen Massachusettes Institute of Technology

We discuss a method to reconstruct two-dimensional proton bunch densities using vertex distributions accumulated during LHC beam-beam scans. The x-y correlations in the beam shapes are studied and an alternative luminosity calibration technique is introduced. We demonstrate the method on simulated beam-beam scans and estimate the uncertainty on the luminosity calibration associated to the beam-shape reconstruction to be below 1%.

Z PT

CMS Z p_T uncertainties (normalised to total fiducial)

Figure 1: Relative uncertainties in percent of the normalised fiducial cross section measurement. Each plot shows the q_T dependence in the indicated ranges of |y|.

Uncertainties seem significantly larger for CMS.

Where are the differences wrt ATLAS?

1504.03511

Figure 4: Uncertainty from various sources on $(1/\sigma) d\sigma/d\phi_{\eta}^*$ (top) and $(1/\sigma) d\sigma/dp_{T}^{\ell\ell}$ (bottom) for events with 66 GeV < $m_{\ell\ell}$ < 116 GeV and $|y_{\ell\ell}|$ < 2.4. Left: electron-pair channel at dressed level. Right: muon-pair channel at bare level.

Uncertainties seem significantly larger for CMS.

Where are the differences wrt ATLAS?

1512.02912

VBF HIGGS PRODUCTION

Structure Function Approach

upper and lower sectors.

• this picture is accurate to more than 1% [Bolzoni et al. (2012)], [Ciccolini, Denner, Dittmaier (2008)], [Andersen et al. (2008)]

One can think of VBF Higgs production as a double Deep Inelastic Scattering (DIS×DIS) with no cross-talk between the

[Han, Valencia, Willenbrock (1992)]

- the factorisation of the two sectors is exact if one imagines two copies of QCD, QCD₁ and QCD₂, respectively for the upper and lower sectors.
- all DIS coefficients are known to NNLO and almost all to $N^{3}LO$.
- as the DIS coefficients are inclusive over the hadronic final state, the calculation cannot provide differential results.

Beyond the Structure Function Approach

The calculation is based on two ingredients:

- 1. An "inclusive" contribution
 - use the Structure Function Approach and use four-vectors q_1, q_2 to assign Born-like kinematics using the equations below
 - use the projected Born-like momenta to compute differential distributions

Beyond the Structure Function Approach

The calculation is based on two ingredients: 2. An "exclusive" contribution

approximation

The counter-events cancel identically with the projected terms from the "inclusive" contribution.

• use the electroweak H + jjj NLO calculation in the factorized

[Figy et al. (2007)], [Jäger et al. (2014)]

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• for each parton, keep track of whether it belongs to the upper or lower sector, and compute vector-boson momenta q_1, q_2

• for each event add counter-event with projected Born kinematics and opposite weight

Beyond the Structure Function Approach

Schematically we express the "projection-to-Born" (P2B) method as

$$d\sigma = \int d\Phi_B(B+V) + \int d\Phi_B(B+$$

$d\Phi_R R$ $\mathrm{d}\Phi_R R_{P2B} + \int \mathrm{d}\Phi_R R - \int \mathrm{d}\Phi_R R_{P2B}$ "exclusive" contribution oution (c) NNLO "exclusive" part (from VBF H+3j@NLO) double real double-real counterevent +one-loop single real one-loop single-real counterevent

UNIVERSITY OF

Phenomenology

We study 13 TeV LHC collisions with $M_H = 125$ GeV and NNPDF3.0_nnlo_as118. We use the following VBF cuts:

- Jets defined with anti- k_t , R = 0.4 and $p_t > 25$ GeV
- Two hardest jets within |y| < 4.5
- High dijet invariant mass, $M_{j_1j_2} > 600$ GeV, and separation, $\Delta y_{j_1 j_2} > 4.5$
- Hardest jets in opposite hemispheres, $y_{i_1}y_{i_2} < 0$

We choose a central scale which approximates well $\sqrt{Q_1Q_2}$ and symmetrically vary by a factor 2 up and down

$$\mu_0^2(p_{t,H}) = \frac{M_H}{2} \sqrt{\left(\frac{M_H}{2}\right)^2 + p_{t,H}^2}$$

NON-PERTURBATIVE EFFECTS

Jet v. Z in Z+jet process

hadron / parton

(with MPI) / (without MPI)

2 – 5% effects for jets

Higgs jet veto

1 – 3% effects for jets

Banfi, GPS, Zanderighi 1203.5773

VBF (leading jet)

Cacciari, Dreyer, Karlberg, GPS & Zanderighi, 1506.02660 [unpublished backup plots]

Non-perturbative corrections

SMALL-R

NLL SMALL-R TERMS

R

subleading terms

R

