## Predicting the Fine Structure of Collider Events

based on 1805.09327 with Dasgupta, Dreyer, Hamilton, Monni \& 2002.11114 (idem + Soyez)

## LBNL via Zoom,

 May 2020Gavin Salam*<br>Rudolf Peierls Centre for Theoretical Physics \& All Souls College, Oxford




## The context of this talk: LHC physics (colour-coded by directly-probed energy scales)

| Standard-model |
| :---: |
| physics |
| (QCD \& electroweak) |
| $100 \mathrm{MeV}-4 \mathrm{TeV}$ |

top-quark physics

170 GeV - O(TeV)
Higgs physics
$125 \mathrm{GeV}-500 \mathrm{GeV}$
direct new-particle searches
flavour physics (bottom \& some charm)
heavy-ion physics
$100 \mathrm{GeV}-8 \mathrm{TeV}$
$1-5 \mathrm{GeV}$

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## Broadband searches (here an example with 704 event classes)



ATLAS, arXiv:1807.07447
$13 \mathrm{TeV}, 3.2 \mathrm{fb}^{-1}$
General search

Just one illustration out of many searches at the LHC

## high $\mathrm{p}_{\mathrm{T}}$ Higgs \& [SD] jet mass

We wouldn't trust electromagnetism if we'd only tested at one length/ momentum scale.

New Higgs interactions need testing at both low and (here) high momenta.


## high-pT <br> Z $\rightarrow$ bb

high-pT
$\mathrm{H} \rightarrow \mathrm{bb}$
(2.5 б)


## LHC luminosity v. time



## UNDERLYING THEORY

$$
\begin{aligned}
\mathcal{L} & =-\frac{1}{4} F_{\mu \nu} F^{\mu \nu} \\
& +i F^{\prime} D \psi \\
& +x_{i} y_{i} \psi_{s} \phi+h \phi \\
& +\left|D_{n}, \gamma\right|^{2}-V(\phi)
\end{aligned}
$$

how do you make quantitative connection?

## EXPERIMENTAL DATA

| $\begin{aligned} \mathcal{L} & =-\frac{1}{4} F_{N \nu} F^{\mu \nu} \\ & +i F D \psi \end{aligned}$ | how do you make quantitative connection? |
| :---: | :---: |
| $\begin{aligned} & +x_{i} y_{0} y_{s} \phi+h_{c} \mid \\ & +\left\|D_{p},\right\|^{2}-V(\phi) \end{aligned}$ |  |



## UNDERLYING THEORY

$$
\begin{aligned}
\mathcal{L} & =-\frac{1}{4} F_{\mu \nu} F^{\mu \nu} \\
& +i \neq D \psi \\
& +x_{i} y_{i j} \psi_{s} \phi+h_{L} \\
& +\left|D_{\mu} \phi\right|^{2}-V(\phi)
\end{aligned}
$$

how do you make quantitative connection?

through a chain

of experimental
and theoretical links
[in particular Quantum Chromodynamics (QCD)]

## What are the links?

ATLAS and CMS (big LHC expts.) have written 715 articles since 2017
links $\equiv$ papers they cite
quantum chromodynamics (QCD) theory papers
experimental \& statistics papers

## predicting full particle structure that comes out of a collision



## general purpose Monte Carlo event generators:

 THE BIG 3

Herwig 7


Pythia 8


Sherpa 2
used in ~95\% of ATLAS/CMS publications they do an amazing job of simulation vast swathes of data; collider physics would be unrecognisable without them


# schematic view of key components of QCD predictions and Monte Carlo event simulation 



## schematic view of key components of QCD predictions and Monte Carlo event simulation


schematic view of key components of QCD predictions and Monte Carlo event simulation pattern of particles in MC can be directly compared to pattern in experiment


# using full event information 

how much information is hidden among the hundreds of particles produced in a collisions?
pure QCD event

event with Higgs \& Z boson decays


## Machine learning and jet/event structure

- Project a jet onto a fixed $n \times n$ pixel image in rapidity-azimuth, where each pixel intensity corresponds to the momentum of particles in that cell.
- Can be used as input for classification methods used in computer vision, such as deep convolutional neural networks.

(a) ParticleNet

Qu E Guskos, arXiv:1902.08570

## using full jet/event information for H/W/Z-boson tagging



## using full jet/event information for H/W/Z-boson tagging

Dreyer 2020 (work in progress) QCD rejection with just jet mass (SD/mMDT) i.e. 2008 tools $\mathcal{E}$ their 2013/14 descendants


## QCD rejection

 with use of full jet substructure (2019 tools) 100x betterFirst started to be exploited by Thaler $\mathcal{E}$ Van Tilburg with "N-subjettiness" (2010/11)
can we trust machine learning? A question of confidence in the training...

Unless you are highly confident in the information you have about the markets, you may be better off ignoring it altogether

- Harry Markowitz (1990 Nobel Prize in Economics) [via S Gukov]


## Concrete example: azimuthal structure in jets



## Concrete example: azimuthal structure in jets



## Concrete example: azimuthal structure in jets



## Concrete example: azimuthal structure in jets



## Concrete example: azimuthal structure in jets



(machine-learning) quark/gluon discrimination trained on this simulation will learn to exploit a feature that doesn't exist in real events

# what is a (Monte Carlo) parton shower? 

illustrate with dipole / antenna showers

Gustafson \& Pettersson 1988, Ariadne 1992, main Sherpa \& Pythia8 showers, option in Herwig7,
Vincia $\mathcal{E}$ Dire showers $\mathcal{E}$ (partially) Deductor shower

## Example of radioactive decay (limit of long half-life)

Constant decay rate $\mu$ per unit time, total time $t_{\max }$. Find distribution of emissions.

1. write as coupled evolution equations for probability $P_{0}, P_{1}, P_{2}$, etc., of having $0,1,2, \ldots$ emissions

$$
\frac{d P_{n}}{d t}=-\mu P_{n}(t)+\mu P_{n-1}(t)
$$

[easy to implement in Monte Carlo approach]

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$$
\frac{d P_{n}}{d t}=\frac{-\mu P_{n}(t)}{n \rightarrow n+1}+\mu P_{n-1}(t)
$$

[easy to implement in Monte Carlo approach]

Monte Carlo solution (repeat following procedure many times to get distribution of $n,\left\{t_{i}\right\}$ )
a. start with $n=0, t_{0}=0$
b. Choose random number $r(0<r<1)$ and find $t_{n+1}$ that satisfies

$$
r=e^{-\mu\left(t_{n+1}-t_{n}\right)}
$$

c. If $t_{n+1}<t_{\text {max }}$, increment $n$, go to step b

## Monte Carlo worked example

E.g. for decay rate $\mu=1$, total time $t_{\max }=2$

- start with $n=0, t_{0}=0$
$\rightarrow$ random number $r=0.6 \rightarrow t_{1}=t_{0}+\log (1 / r)=0.51$ [emission 1]
> random number $r=0.3 \rightarrow t_{2}=t_{1}+\log (1 / r)=1.71$ [emission 2]
$>$ random number $r=0.4 \rightarrow t_{3}=t_{2}+\log (1 / r)=2.63\left[>t_{\max }\right.$, so stop]
- This event has two emissions at times $\left\{t_{1}=0.51, t_{2}=1.71\right\}$

Monte Carlo solution (repeat following procedure many times to get distribution of $n$, $\left\{t_{i}\right\}$ )
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## QCD shower: an evolution equation (in evolution scale v, e.g. 1/trans.mom.)



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Start with q-qbar state.
Throw a random number to determine down to what scale state persists unchanged

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\frac{d P_{2}(v)}{d v}=-f_{2 \rightarrow 3}^{q \bar{q}}(v) P_{2}(v)
$$

## QCD shower: an evolution equation (in evolution scale v, e.g. 1/trans.mom.)



Start with q-qbar state.
Throw a random number to determine down to what scale state persists unchanged

At some point, state splits ( $2 \rightarrow 3$, i.e. emits gluon). Evolution equation changes

$$
\frac{d P_{3}(v)}{d v}=-\left[f_{2 \rightarrow 3}^{q g}(v)+f_{2 \rightarrow 3}^{g \bar{q}}(v)\right] P_{3}(v)
$$

gluon is part of two dipoles $(q g),(g \bar{q})$, each treated as independent (many showers use a large $\mathbf{N}_{\mathrm{C}}$ limit)

## QCD shower: an evolution equation (in evolution scale v, e.g. 1/trans.mom.)



## recent directions of parton-shower work?

1. including $2 \rightarrow 4$ (or $1 \rightarrow 3$ ) splittings
2. subleading colour corrections (dipole picture is large $\mathrm{N}_{\mathrm{C}}$ )
3. EW showers

## Including $1 \rightarrow 3$ splittings $(\equiv 2 \rightarrow 4)$

> Jadach et al, e.g. 1504.06849, 1606.01238 > Höche, Krauss \& Prestel, 1705.00982,
> Li \& Skands, 1611.00013 Höche \& Prestel, 1705.00742, Dulat, Höche \& Prestel, 1805.03757

$$
\begin{aligned}
& D_{j i}^{(0)}(z, \mu)=\delta_{i j} \delta(1-z) \quad \leftrightarrow \quad \bigodot_{j} / \bigodot_{i} \\
& D_{j i}^{(1)}(z, \mu)=-\frac{1}{\varepsilon} P_{j i}^{(0)}(z) \quad \bigodot_{i} \underbrace{6^{60}}_{j} / \bigodot_{i} \\
& D_{j i}^{(2)}(z, \mu)=-\frac{1}{2 \varepsilon} P_{j i}^{(1)}(z)+\frac{\beta_{0}}{4 \varepsilon^{2}} P_{j i}^{(0)}(z)+\frac{1}{2 \varepsilon^{2}} \int_{z}^{1} \frac{\mathrm{~d} x}{x} P_{j k}^{(0)}(x) P_{k i}^{(0)}(z / x)
\end{aligned}
$$

Equations from slides by Höche

## Hierarchy of subleading colour corrections


cf. also work by Hatta \& Ueda, 1304.6930; Nagy \& Soper papers; Hoche \& Reichelt, 2001.11492

EW showers (esp. beyond LHC)
Bauer, Ferland \& Webber, 1703.08562, 1808.08831
Bauer, De Jong, Nachman, Provasoli, 1904.03196 see also Chen, Han E Tweedie, 1611.00788 $\mathcal{E}$ Sjostrand $\mathcal{E}$ collab


## W emission affects only left-handed quarks

$\rightarrow$ strong polarisation of quarks in unpolarised proton (at high enough energies)

# what does a parton shower achieve? 

not just a question of ingredients,
but also the final result of assembling them together

# what should a parton shower achieve? 

not just a question of ingredients,
but also the final result of assembling them together

## it's a complicated issue...

> For a total cross section, e.g. for Higgs production, it's easy to talk about systematic improvements (LO, NLO, NNLO, ...). But they're restricted to that one observable

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> With a parton shower (+hadronisation) you produce a "realistic" full set of particles. You can ask questions of arbitrary complexity:
> the multiplicity of particles
> the total transverse momentum with respect to some axis (broadening)

- the angle of 3rd most energetic particle relative to the most energetic one [machine learning might "learn" many such features]


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> the multiplicity of particles
> the total transverse momentum with respect to some axis (broadening)

- the angle of 3rd most energetic particle relative to the most energetic one [machine learning might "learn" many such features]
how can you prescribe correctness \& accuracy of the answer, when the questions you ask can be arbitrary?


## The standard answer so far

It's common to hear that showers are Leading Logarithmic (LL) accurate.
That language, widespread for multiscale problems, comes from analytical resummations. E.g. for (famous) "Thrust"

$$
T=\max _{\vec{n}_{T}} \frac{\sum_{i}\left|\vec{p}_{i} \cdot \vec{n}_{T}\right|}{\sum_{i}\left|\vec{p}_{i}\right|}
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$$



2-jet event: $\quad T \simeq 1$

$$
\begin{aligned}
& \sigma\left(1-T<e^{-L}\right)=\sigma_{\text {tot }} \exp \left[\frac{\left[g_{1}\left(\alpha_{s} L\right)\right.}{L L}+\frac{g_{2}\left(\alpha_{s} L\right)}{\mathrm{NLL}}+\frac{\alpha_{s} g_{3}\left(\alpha_{s} L\right)}{\mathrm{NNLL}}+\frac{\left.\alpha_{s}^{2} g_{4}\left(\alpha_{s} L\right)+\cdots\right]}{\left.\alpha_{s} \ll 1, L \gg 1\right]}\right. \\
& \quad \text { Catani, Trentadue, Turnock } \& \text { Webber '93 }
\end{aligned}
$$

## The standard answer so far

Sometimes you may see statements like "Following standard practice to improve the logarithmic accuracy of the parton shower, the soft enhanced term of the splitting functions is rescaled by $1+a_{s}(t) /(2 \pi) K "\left[K \sim A_{2}\right.$ in cusp anomalous dimension]

## Questions:

1) Which is it? LL or better?
2) For what known observables does this statement hold?
3) What good is it to know that some handful of observables is LL (or whatever) when you want to calculate arbitrary observables?
4) Does LL even mean anything when you do machine learning?
5) Why only "LL" when analytic resummation can do so much better?
6) Do better ingredients (e.g. higher-order splitting functions) make better showers?

## Back to radioactive decay example: two ways of writing result

Constant decay rate $\mu$ per unit time, total time $t$. Find distribution of emissions.

1. write as coupled evolution equations for probability $P_{0}, P_{1}, P_{2}$, etc., of having $0,1,2, \ldots$ emissions

$$
\frac{d P_{n}}{d t}=-\mu P_{n}(t)+\mu P_{n-1}(t)
$$

[easy to implement in
Monte Carlo approach]
2. or as explicit formula

[here Poisson distribution; in QCD: effective matrix element]

## Our proposal for baseline shower accuracy ("NLL")

## Resummation

Require single logarithmic accuracy (control of terms $\alpha_{s}^{n} L^{n}$ for all observables where this makes sense)
> global event shapes (thrust, broadening, angularities, jet rates, energy-energy correlations, ...)
> non-global observables

- fragmentation / parton-distribution functions
> [multiplicity, get NLL $\alpha_{s}^{n} L^{2 n-1}$, cf. original Herwig angular-ordered shower from 1980's)


## Matrix elements

Effective tree-level matrix elements generated by the shower should be correct for any multiplicity $N$ if all emissions are well separated in a Lund diagram.

## Phase space: two key variables (+ azimuth)

$$
\begin{aligned}
& \bar{E} \\
& \theta\left(\text { or } \eta=-\ln \tan \frac{\theta}{2}\right) \quad \eta \text { is called (pseudo) rapidity } \\
& p_{t}=E \theta \quad p_{t}\left(\text { or } p_{\perp}\right) \text { is a transverse momentum }
\end{aligned}
$$

jet with $R=0.4, p_{t}=200 \mathrm{GeV}$

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## When do we require effective shower $\left|M^{2}\right|$ to be correct?

$\ln p_{t}$


- a shower with simple $1 \rightarrow 2$ or $2 \rightarrow 3$ splittings can't reproduce full matrix element
> but QCD has amazing factorisation properties - simplifications in presence of energy or angular ordering
> we should be able to reproduce $\left|M^{2}\right|$ when all emissions well separated in Lund diagram $d_{12} \gg 1, d_{23} \gg 1, d_{15} \gg 1$, etc.


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- a shower with simple $1 \rightarrow 2$ or $2 \rightarrow 3$ splittings can't reproduce full matrix element
- but QCD has amazing factorisation properties - simplifications in presence of energy or angular ordering
- we are allowed to make a mistake (by $\mathcal{O}(1)$ factor) when a pair is close by, e.g. $d_{23} \sim 1$


## key shower elements and their consequences

## Key element \#1 in a shower: evolution/ordering variable

- Radioactivity example had just a time variable $\rightarrow$ only choice for evolution variable
- A shower has two (logarithmic) variables, e.g. angle and $p_{t}$ : what do you choose for the evolution variable?


Option 1: $\theta$ as the evolution/ordering variable (default in Herwig; Marchesini \& Webber '84)

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- (1) and (2) have similar angles; large- $N_{C}$ matrix-element is simple if composed with ordered energies ( 2 , then 1 ); but shower generates (1) first (larger angle, but smaller energy), i.e. disordered energies $\rightarrow$ gets wrong matrix element


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Option 2: $p_{t}$ as the evolution/ordering variable, together with use of colour dipoles (default in most showers, introduced in Ariadne; Gustafson, Pettersson \& Lonnblad c. 1988)

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$\checkmark$ emissions at commensurate angles are produced in "right" order (red after green), and so with correct large- $N_{C}$ dipole matrix element (Bassetto, Ciafaloni Marchesini, 1983)

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- emissions at commensurate $p_{t}$ affect each other, e.g. if (2) emitted after (1), it modifies kinematics of (1)

Andersson, Gustafson, Sjogren '92,
Nagy \& Soper 0912.4534, 1401.6366
Dasgupta, Dreyer, Hamilton, Monni \& GPS $\underline{1805.09327}$

## Key element \#2 in a shower: kinematic map (local within dipole for many showers)

Start with dipole $(\tilde{i}-\tilde{j}) \rightarrow$ emit gluon $k$ to get two dipoles $(i-k)$ and $(k-j)$

$$
\left.\begin{array}{rl}
p_{k} & =a_{k} \tilde{p}_{i}+b_{k} \tilde{p}_{j}+k_{\perp} \\
p_{i} & =a_{i} \tilde{p}_{i}+b_{i} \tilde{p}_{j}-k_{\perp} \\
p_{j} & =a_{j} \tilde{p}_{i}
\end{array}\right\} \quad \text { kinematic map }
$$


transverse recoil assigned to end that is closer in angle in dipole c.o.m. frame

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## designing new showers

in the large- $N_{C}$ limit, without spin correlations (spin correlations solved by Collins 1986; beyond leading $N_{C}$ more subtle)

## core principles

1. for a new emission $k$, when it is generated far in the Lund diagram from any other emission ( $\left|d_{k i}^{\text {Lund }}\right| \gg 1$ ), it should not modify the kinematics (Lund coordinates) of any preceding emission by more than an amount $\exp \left(-p\left|d_{k i}^{\text {Lund }}\right|\right)$, where $p=\mathcal{O}(1)$
2. when $k$ is distant from other emissions, generate it with matrix element and phasespace (and associated Sudakov)

$$
\frac{d \Phi_{k}}{d \Phi_{k-1}} \frac{\left|M_{1 \ldots k}\right|^{2}}{\left|M_{1 \ldots(k-1)}\right|^{2}}
$$

[simple forms known from factorisation properties of matrix-elements]
3. emission $k$ should not impact $d \Phi \times|M|^{2}$ ratio for subsequent distant emissions unless
a. they are at commensurate angle (or on $k$ 's Lund "leaf"), or
b. $k$ was a hard collinear splitting, which can affect other hard collinear splittings (cross-talk on same leaf $\equiv$ DGLAP, cross-talk on other leaves $\equiv$ spin correlations)

## Handle \#1: choice of recoil scheme for emission from ij dipole

> Dipole-local scheme (PanLocal)

$$
\left.\begin{array}{rl}
p_{k} & =a_{k} \tilde{p}_{i}+b_{k} \tilde{p}_{j}+k_{\perp}, \\
p_{i} & =a_{i} \tilde{p}_{i}+b_{i} \tilde{p}_{j}-f k_{\perp}, \\
p_{j} & =a_{j} \tilde{p}_{i}+b_{j} \tilde{p}_{j}-(1-f) k_{\perp}
\end{array}\right\} \quad \text { kinematic map }
$$

$f=1$ (0) when $k$ collinear to $i(j)$;
transition when $k$ bisects ( $i j$ ) opening angle in event c.o.m. frame
(normal dipole/antenna showers have transition in the dipole c.o.m. frame)


## Comparing PanLocal and standard Dipole recoil

## PanLocal

$\bar{q} g_{1}$ dipole
$\perp$ recoil from $g_{1}$


## Standard Dipole



## Handle \#2: ordering variable

Use an ordering variable intermediate between transverse momentum and angle


Use an ordering variable intermediate between transverse momentum and angle


Use an ordering variable intermediate between transverse momentum and angle


Use an ordering variable intermediate between transverse momentum and angle

> $v$ is ordering variable
> maps to contour in Lund diagram at an angle $\tan ^{-1} \beta$

$$
v=p_{t} e^{-\beta|\eta|} / \rho
$$

> require $0<\beta<1$ (in practice use $\beta=0.5$ )
> Ensures that commensurate- $p_{t}$ emissions are produced at successively smaller angles (avoids major recoil in gg dipole)

## Alternative approach: PanGlobal

Dipole-local map doesn't handle transverse recoil

$$
\begin{aligned}
\bar{p}_{k} & =a_{k} \tilde{p}_{i}+b_{k} \tilde{p}_{j}+k_{\perp}, \\
\bar{p}_{i} & =\left(1-a_{k}\right) \tilde{p}_{i}, \\
\bar{p}_{j} & =\left(1-b_{k}\right) \tilde{p}_{j} .
\end{aligned}
$$


kinematic map

Whole event scaled and boosted after each emission to restore 4-momentum conservation.
Works with $0 \leq \beta<1$ (i.e. can be used with $p_{t}$ ordering)


$$
\overline{\mathrm{i} \equiv q}
$$

## Personal comment

> I don't like either of these approaches
> they involve "hacks" in order to satisfy the core principles, but are physically unappealing in some respects (recoil assigned to particles that shouldn't "physically" get it)

- I believe one could find other solutions that are better
> But for now, they serve as a proof of principle that it is possible to construct a shower that satisfies our NLL conditions (some people believed this might not even be possible)

NB, Nagy E Soper 1401.6364 have elements related to PanLocal but with a global recoil

## validating new showers

## maybe one's concerned ? <br> how can we convince you (and ourselves) that we have achieved NLL?

## test matrix-element with subjet azimuthal difference



## test matrix-element with subjet azimuthal difference



- run full shower with specific value of $\alpha_{s}(Q)$
- ratio to NLL should be flat $\equiv 1$
> it isn't: have we got an NLL mistake? Or a residual subleading (NNLL) term?
> try halving $\alpha_{s}(Q)$, while keeping constant $\alpha_{s} L\left[L \equiv \ln k_{t 1} / Q\right]$
- NLL effects, $\left(\alpha_{S} L\right)^{n}$, should be unchanged, subleading ones, $\alpha_{s}\left(\alpha_{s} L\right)^{n}$, halved


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$\sqrt{ }$ extrapolation $\alpha_{s} \rightarrow 0$ agrees with NLL


## A side-note about running with small $\alpha_{s}$

> If you keep $\alpha_{s} L$ fixed, $\alpha_{s} \rightarrow 0$ implies $L \rightarrow \infty$
> In practice $\alpha_{s} L=-0.5$ with $\alpha_{s}=0.005$ gives $L=-100$, i.e. transverse momenta $\sim 10^{-44} Q$ (e.g. at LHC minimum $p_{t} \sim 10^{-5} Q$ )
> normal showers aren't designed to work over such a range of scales, quite a few problems needed to be solved ( $\sim 1.5$ years' thought \& work)
$>$ but if you want a numerical test of NLL (and, subsequently, higher orders), then you need a framework that can address such challenges

NB: to study Pythia8 / Dire-v1 dipole showers, we had to recode them (based on descriptions in papers \& inspection of their code)

## test matrix-element with subjet azimuthal difference



Now examine many showers
standard dipole showers (Pythia, Direvi) disagree with NLL by up to $60 \%$

PanLocal $\beta=0$ is also expected to disagree with NLL and does

All other "PanScales" showers (with valid $\beta$ values) agree with NLL

## Carry out similar shower/NLL ratio tests for many observables



## Carry out similar shower/NLL ratio tests for many observables



Relative deviation from NLL for $\alpha_{s} \rightarrow 0$

## Carry out similar shower/NLL ratio tests for many observables



Relative deviation from NLL for $\alpha_{s} \rightarrow 0$

## Carry out similar shower/NLL ratio tests for many observables



## Carry out similar shower/NLL ratio tests for many observables



## Carry out similar shower/NLL ratio tests for many observables



## Carry out similar shower/NLL ratio tests for many observables



All PanScales shower
that are expected to agree with NLL pass these tests
(Standard dipole showers don't)

## Conclusions

## conclusions

- Parton showers (and event generators in general), and their predictions of the fine structure of events, are an essential part of LHC's very broad physics programme
- Despite their central role, understanding of their accuracy has been elusive
> Minimal baseline for progress beyond 1980's technology is to achieve NLL accuracy $\equiv$ control of terms $\left(\alpha_{s} L\right)^{n}$
> We've demonstrated that is possible (with some caveats, final-state showers only, spin correlations still missing, leading- $N_{C}$ approx.)
- Essential elements that we hope can be of wider use:
> concrete criteria for specifying log accuracy
> core guiding principles that help achieve NLL accuracy
- powerful numerical approach to demonstrating shower accuracy


## BACKUP



FIG. 1. Left: distribution for the difference in azimuthal angle between the two highest- $k_{t}$ primary Lund declusterings in the Pythia8 dipole shower algorithm, normalised to the NLL result [53], [51]§4; successively smaller $\alpha_{s}$ values keep fixed $\alpha_{s} \ln k_{t 1}$. Middle: the same for the PanGlobal $(\beta=0)$ shower. Right: the $\alpha_{s} \rightarrow 0$ limit of the ratio for multiple showers. This observable directly tests part of our NLL (squared) matrix-element correctness condition. A unit value for the ratio signals success.



FIG. 2. Left: ratio of the cumulative $y_{23}$ distribution from several showers divided by the NLL answer, as a function of $\alpha_{s} \ln y_{23} / 2$, for $\alpha_{s} \rightarrow 0$. Right: summary of deviations from NLL for many shower/observable combinations (either $\Sigma_{\text {shower }}\left(\alpha_{s} \rightarrow\right.$ $\left.0, \alpha_{s} L=-0.5\right) / \Sigma_{\mathrm{NLL}}-1$ or $\left.\left(N_{\text {shower }}^{\text {subjet }}\left(\alpha_{s} \rightarrow 0, \alpha_{s} L^{2}=5\right) / N_{\mathrm{NLL}}^{\text {subjet }}-1\right) / \sqrt{\alpha_{s}}\right)$. Red squares indicate clear NLL failure; amber triangles indicate NLL fixed-order failure that is masked at all orders; green circles indicate that all NLL tests passed.


FIG. 3. Comparison of the ratio $\Sigma_{\text {shower }} / \Sigma_{\text {NLL }}$ between the toy shower and the full shower for three reference observables $\left(\sqrt{y_{23}}, B_{W}\right.$ and $\mathrm{FC}_{1}$ ), in the limit $\alpha_{s} \rightarrow 0$, as a function of $\alpha_{s} L$. For the full showers the figure shows the ratio of the shower prediction to the full NLL result, while for the toy shower it shows the ratio to the CAESAR-like toy shower. Three full showers are shown in each plot, each compared to the corresponding toy shower. The PanLocal full showers are shown in their dipole variants (identical conclusions hold for the antenna variant). Small $(0.5 \%)$ issues at $\lambda \gtrsim-0.1$ are a consequence of the fact that for the largest of the $\alpha_{s}$ values used in the extrapolation, the corresponding $L$ values do not quite satisfy $e^{L} \ll 1$.


FIG. 4. Fixed order results from the toy implementation of the standard dipole showers. The plots show the difference between the toy dipole shower and the (NLL-correct) CAESAR results for the $F_{n}$ coefficient of $\bar{\alpha}^{n}$ in the expansion of Eq. (33), divided by $L^{n}$. For an NLL-correct shower, the results should tend to zero for large negative $L$. The first row shows the result of $n=3$, the second row that of $n=4$. The columns correspond to different observables (thrust, slice transverse momentum and hemisphere $\sqrt{y_{23}}$ ). Observe how the results tend to constants (NLL discrepancy) or demonstrate a linear or even quadratic dependence on $L$ (super-leading logarithms). The coefficients have been fitted taking into account correlations between points, and we include powers down to $L^{-3}$ in the fit of $\delta F_{n} / L^{n}$. The fit range is from -100 to -5 and the quoted error includes both the (statistical) fit uncertainty and the difference in coefficients obtained with the range $[-100,-10]$ (added in quadrature).


FIG. 5. Analogue of Fig. 4, demonstrating the absence of NLL (or super-leading) issues at fixed order in the toy version of the PanLocal $\beta=0.5$ shower. At order $\bar{\alpha}^{4}$, we include fit terms down to $L^{-4}$.
toy dipole shower, Thrust

toy dipole shower, Thrust


FIG. 7. Toy-shower all-order result for the thrust ( $S_{\beta=1}$, Eq. (25)). Left: $\Sigma_{\text {dipole }} / \Sigma_{\text {NLL }}$, where the NLL result is given by running the CAESAR version of the shower. Four values of $\bar{\alpha}$ are shown, together with the extrapolation to $\bar{\alpha}=0$, showing that the all-order dipole-shower result (in our usual limit of fixed $\bar{\alpha} L$ and $\bar{\alpha} \rightarrow 0$ ) is consistent with the NLL result, despite the super-leading logarithmic terms that are visible in Fig. 4. Right: $\left(\Sigma_{\text {dipole }} / \Sigma_{\mathrm{NLL}}-1\right) / \bar{\alpha}$, again for three values of $\bar{\alpha}$ and the extrapolation to $\bar{\alpha}=0$. The fact that these curves converge is a sign that the all-order (toy) dipole-shower discrepancy with respect to NLL behaves as a term that vanishes proportionally to $\bar{\alpha}$, i.e. as an NNLL term. The results here involve fixed coupling, i.e. they do not include a correction of the form of Eq. (30).


FIG. 8. Checks of the $k_{t}$ algorithm subjet multiplicity. Left: the multiplicity as a function of $\frac{1}{2} \sqrt{\alpha_{s}(Q)} \ln y_{c u t}$, comparing the PanLocal $\beta=0.5$ shower (dipole variant) with the NLL prediction, for two choices of $\alpha_{s}$. Right: Eq. (50) for the same shower, for several $\alpha_{s}$ values, together with the $\alpha_{s} \rightarrow 0$ limit.

## Fundamental experimental calibrations (jets)



Jet energy scale, which feeds into hundreds of other measurements

Largest systematic errors (1-2\%) come from differences between MC generators
(here Sherpa v. Pythia)
$\rightarrow$ fundamental limit on LHC precision potential

