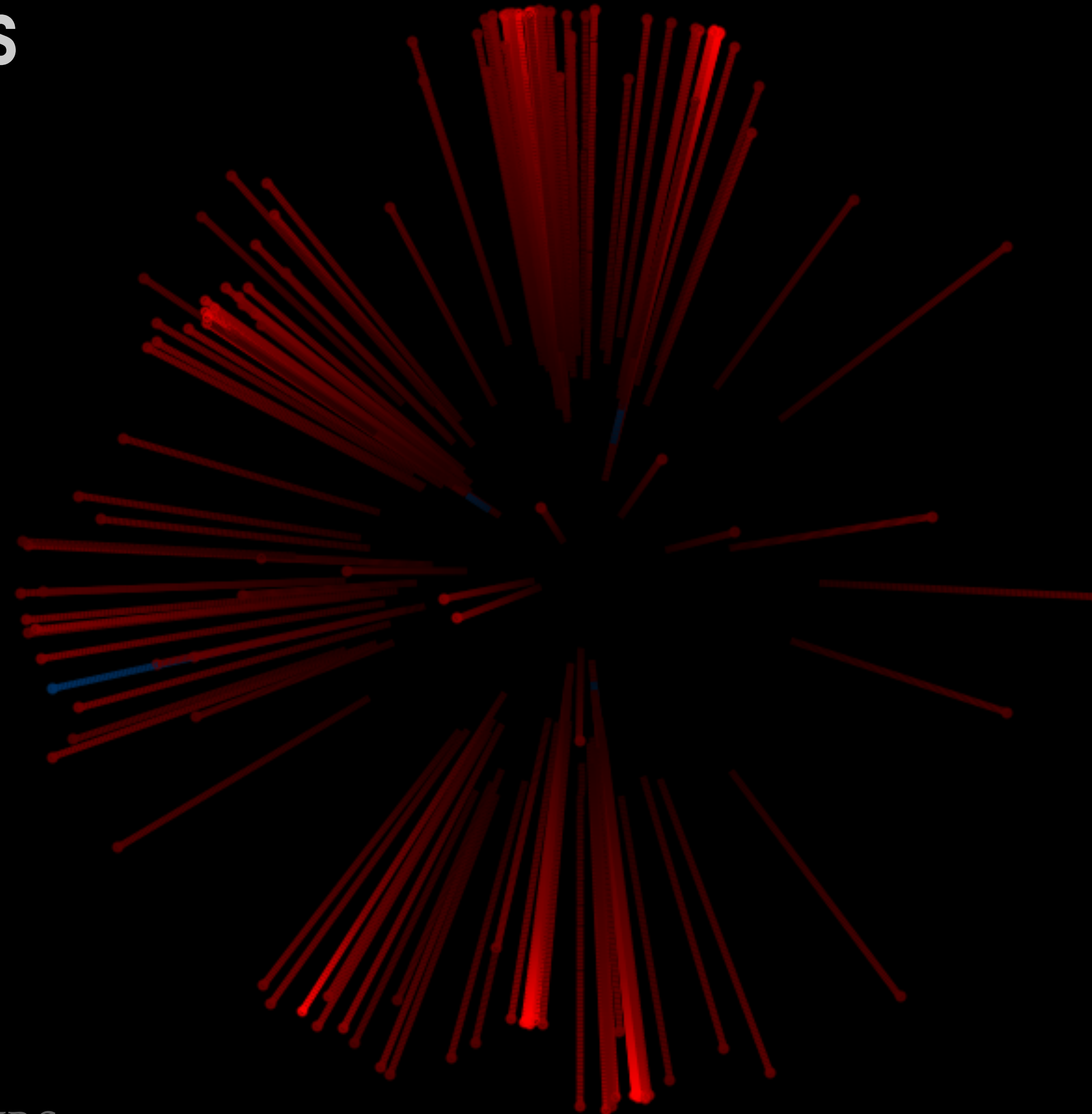


# Predicting the Fine Structure of Collider Events

based on 1805.09327  
with Dasgupta, Dreyer,  
Hamilton, Monni  
& 2002.11114  
(idem + Soyez)

RECONNECT 2020,  
IPPP Durham via zoom  
May 2020

**Gavin Salam\***  
Rudolf Peierls Centre for  
Theoretical Physics  
& All Souls College, Oxford



\* on leave from CERN and CNRS

# The context of this talk: LHC physics (colour-coded by directly-probed energy scales)

---

**Standard-model  
physics  
(QCD & electroweak)**

**100 MeV - 4 TeV**

**top-quark physics**

**170 GeV - 0(TeV)**

**Higgs physics**

**125 GeV - 500 GeV**

**direct new-particle  
searches**

**100 GeV - 8 TeV**

**flavour physics  
(bottom & some charm)**

**1 - 5 GeV**

**heavy-ion physics**

**100 MeV - 500 GeV**

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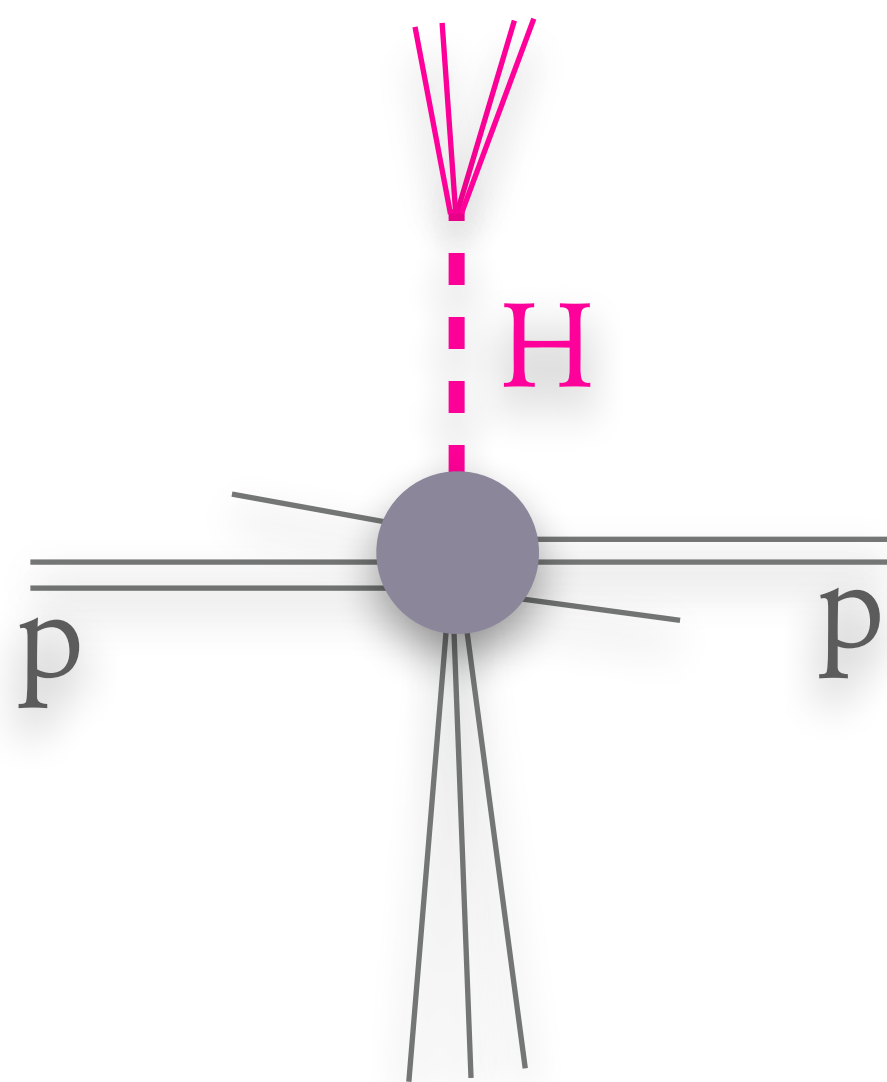
**100 MeV - 500 GeV**



# high $p_T$ Higgs & [SD] jet mass

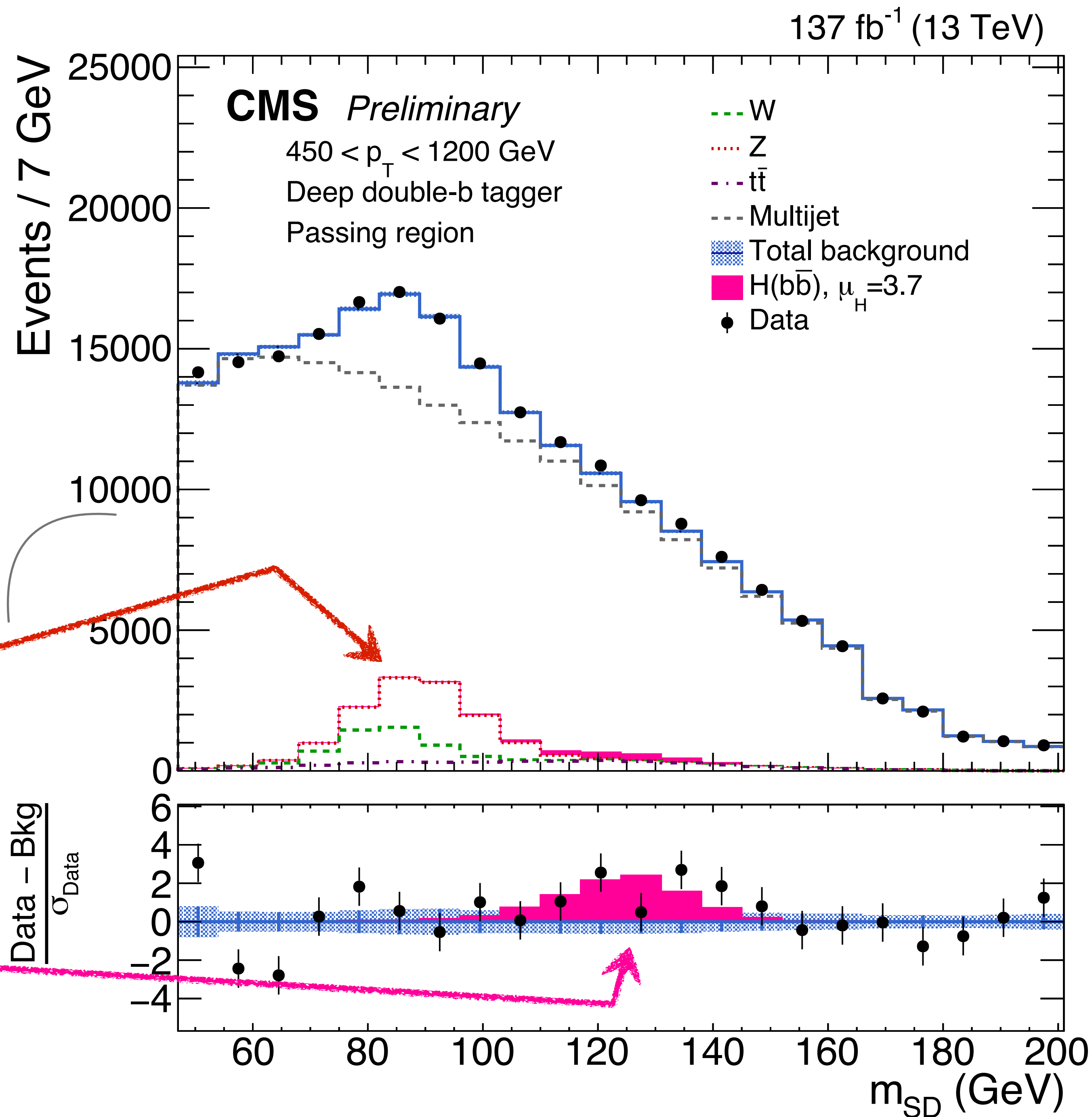
We wouldn't trust electromagnetism if we'd only tested at one length/momentum scale.

New Higgs interactions need testing at both low and (here) high momenta.

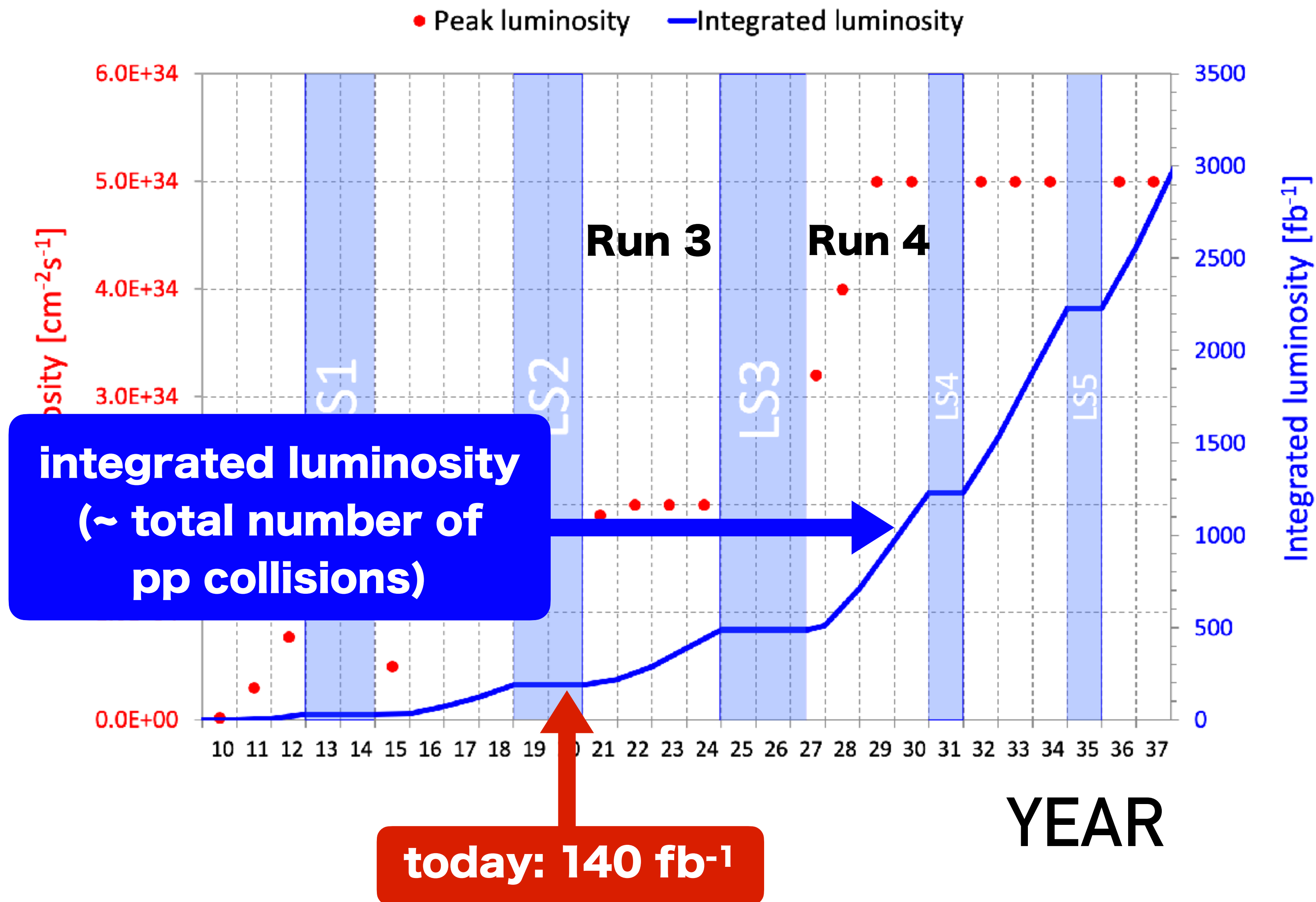


high- $p_T$   
 $Z \rightarrow b\bar{b}$

high- $p_T$   
 $H \rightarrow b\bar{b}$   
 (2.5  $\sigma$ )



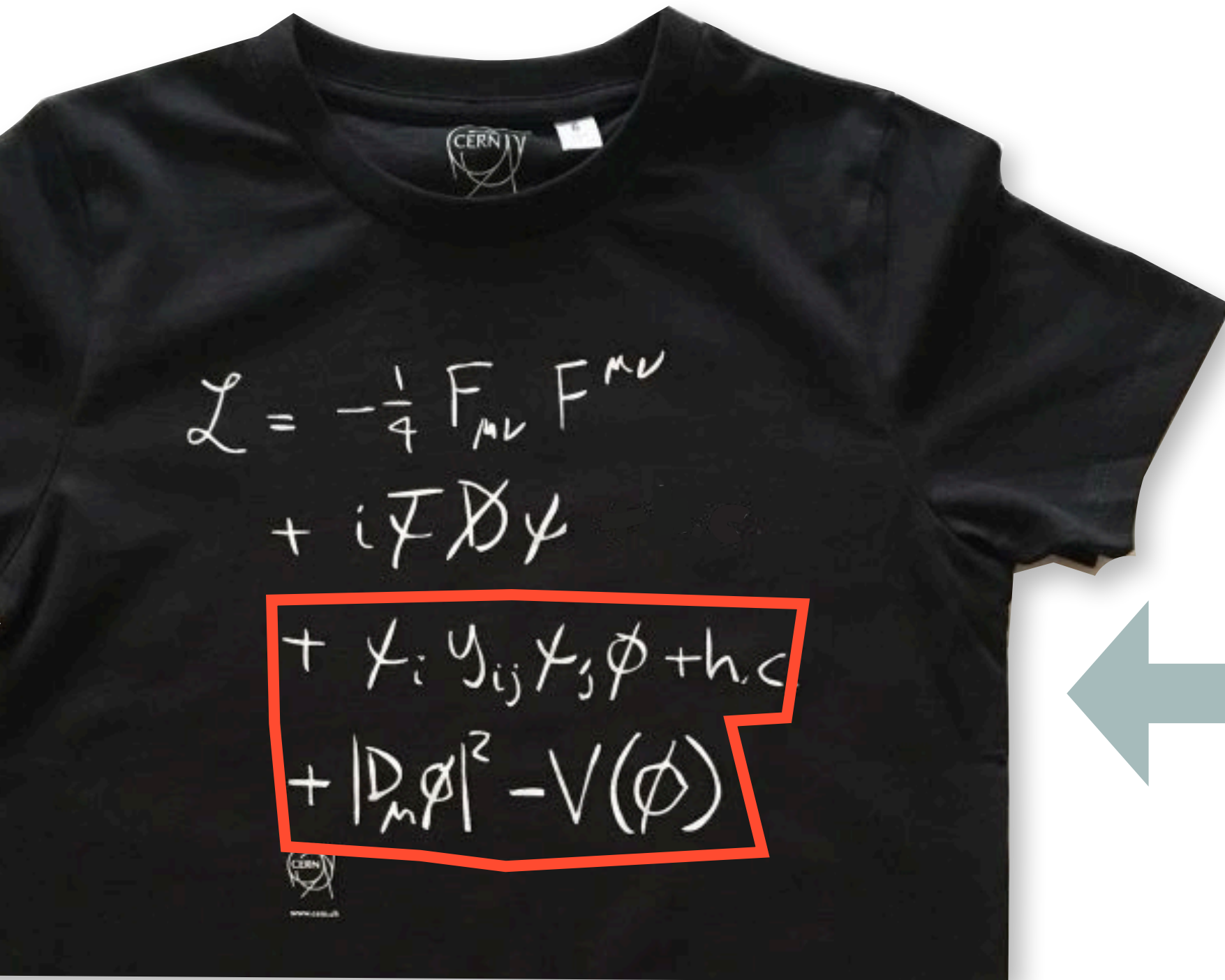
# LHC luminosity v. time



year	lumi (fb <sup>-1</sup> )	
2020	140	
2025	450	(× 3)
2030	1200	(× 8)
2037	3000	(× 20)

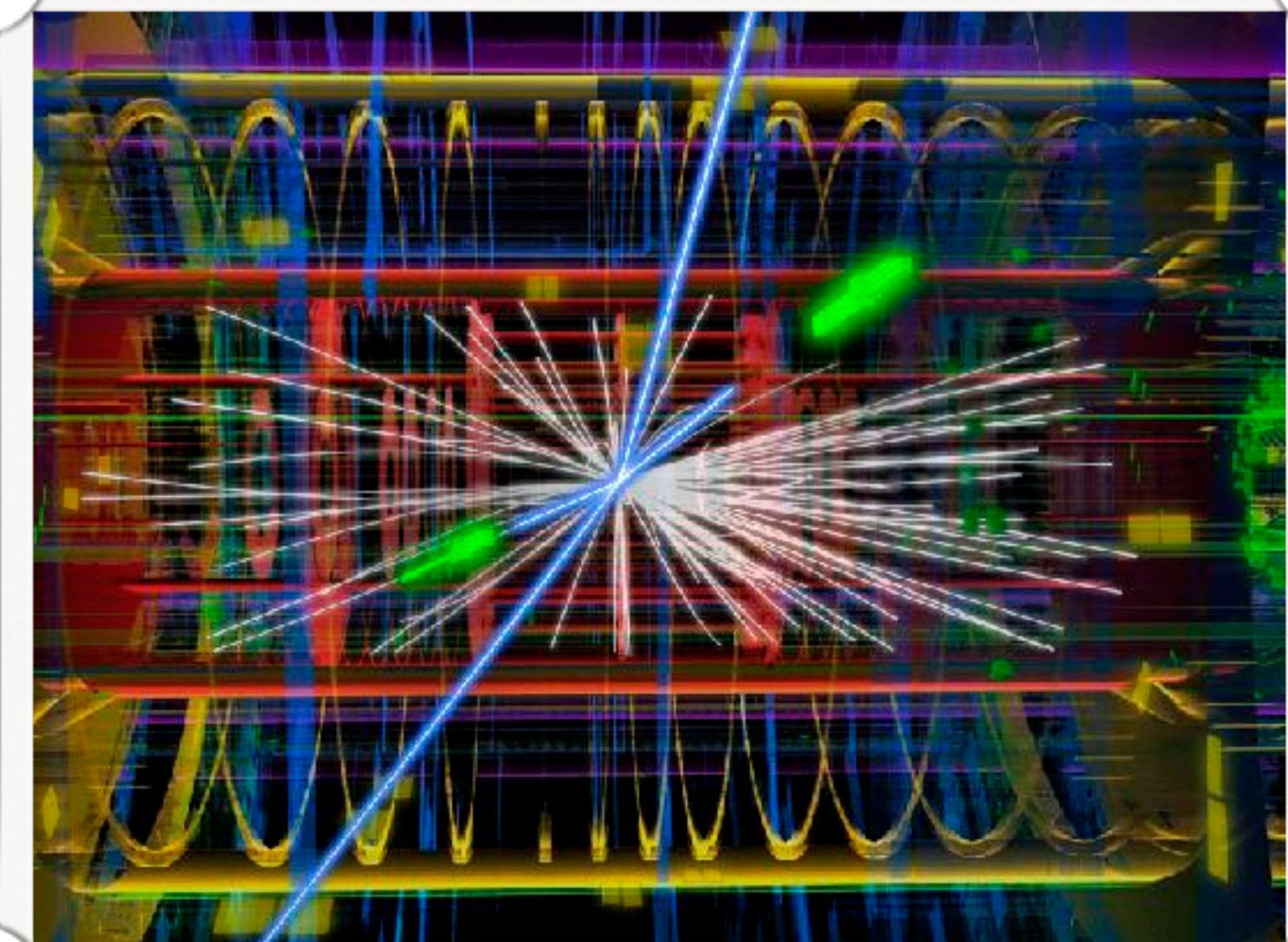
*95% of collisions still to be delivered*

# UNDERLYING THEORY

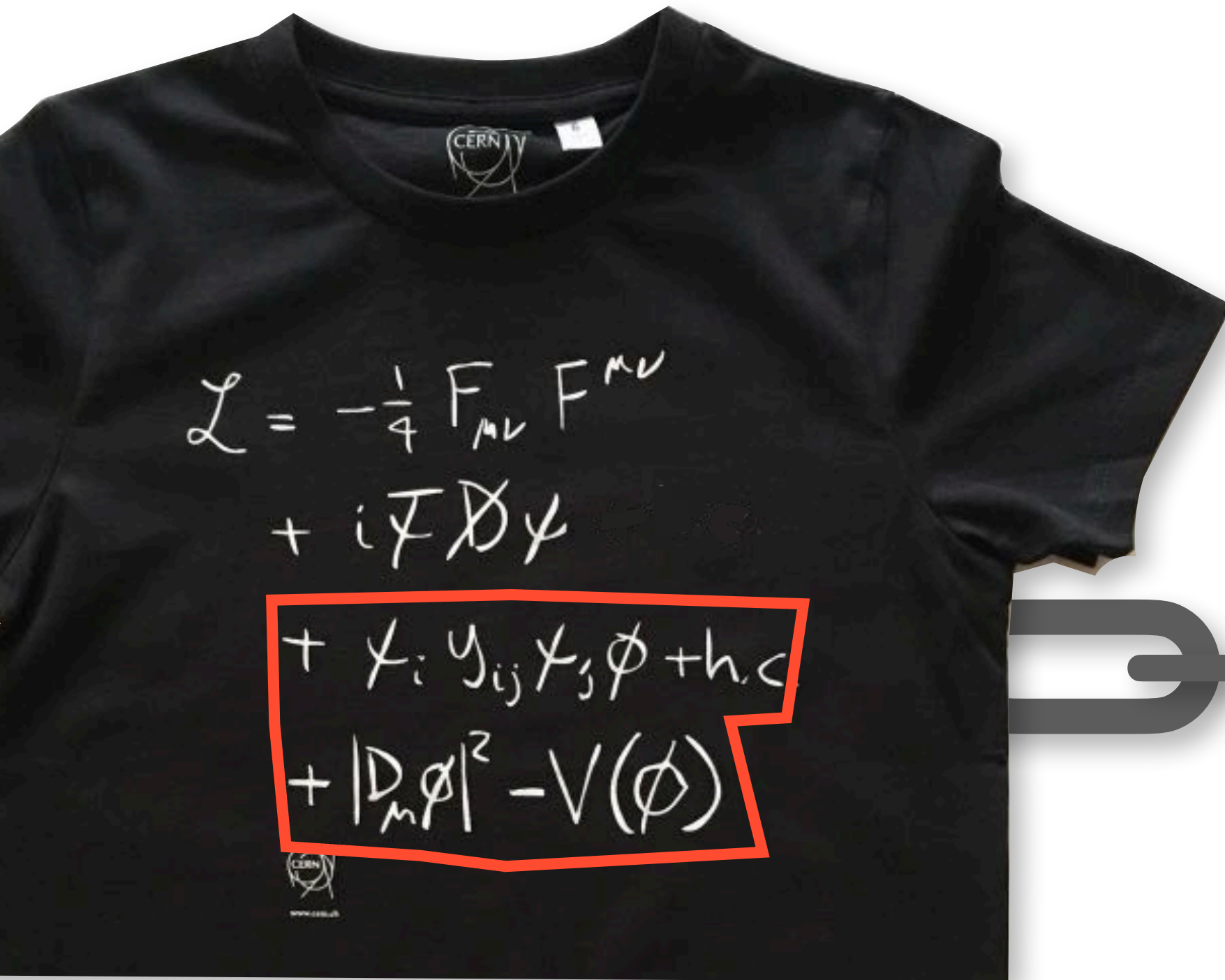


*how do you make  
quantitative  
connection?*

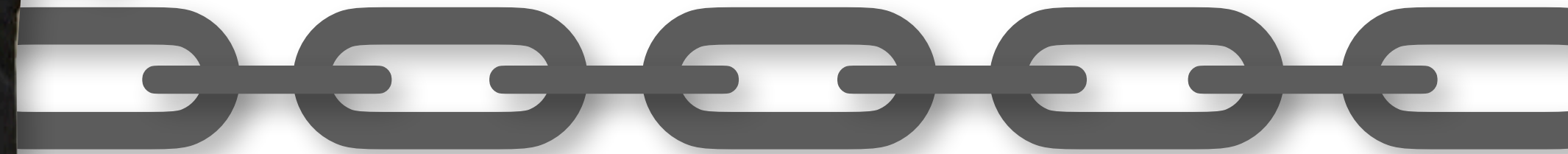
# EXPERIMENTAL DATA



# UNDERLYING THEORY

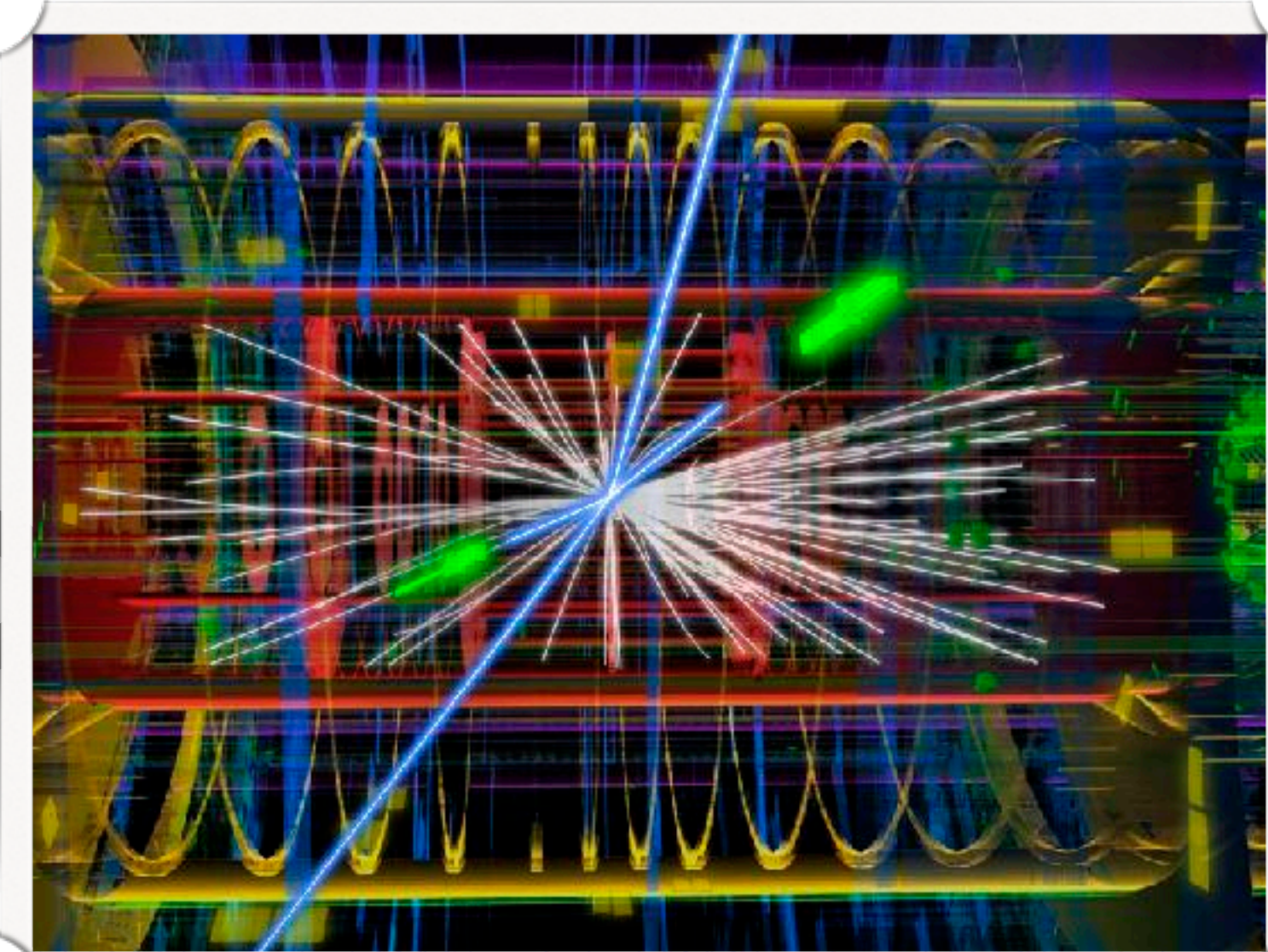


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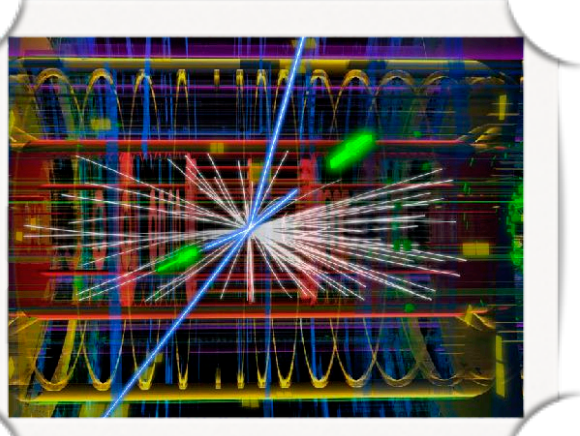
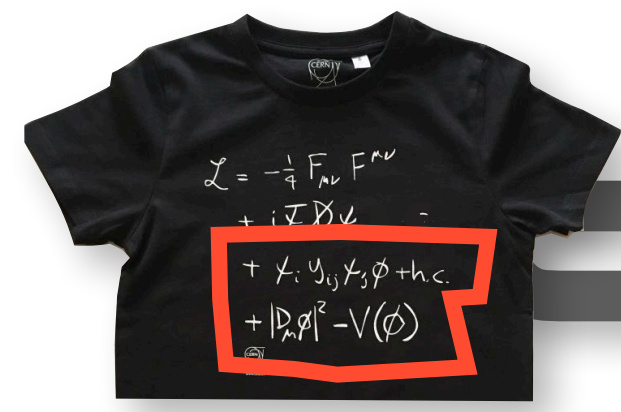
*through a chain  
of experimental  
and theoretical links*

# EXPERIMENTAL DATA



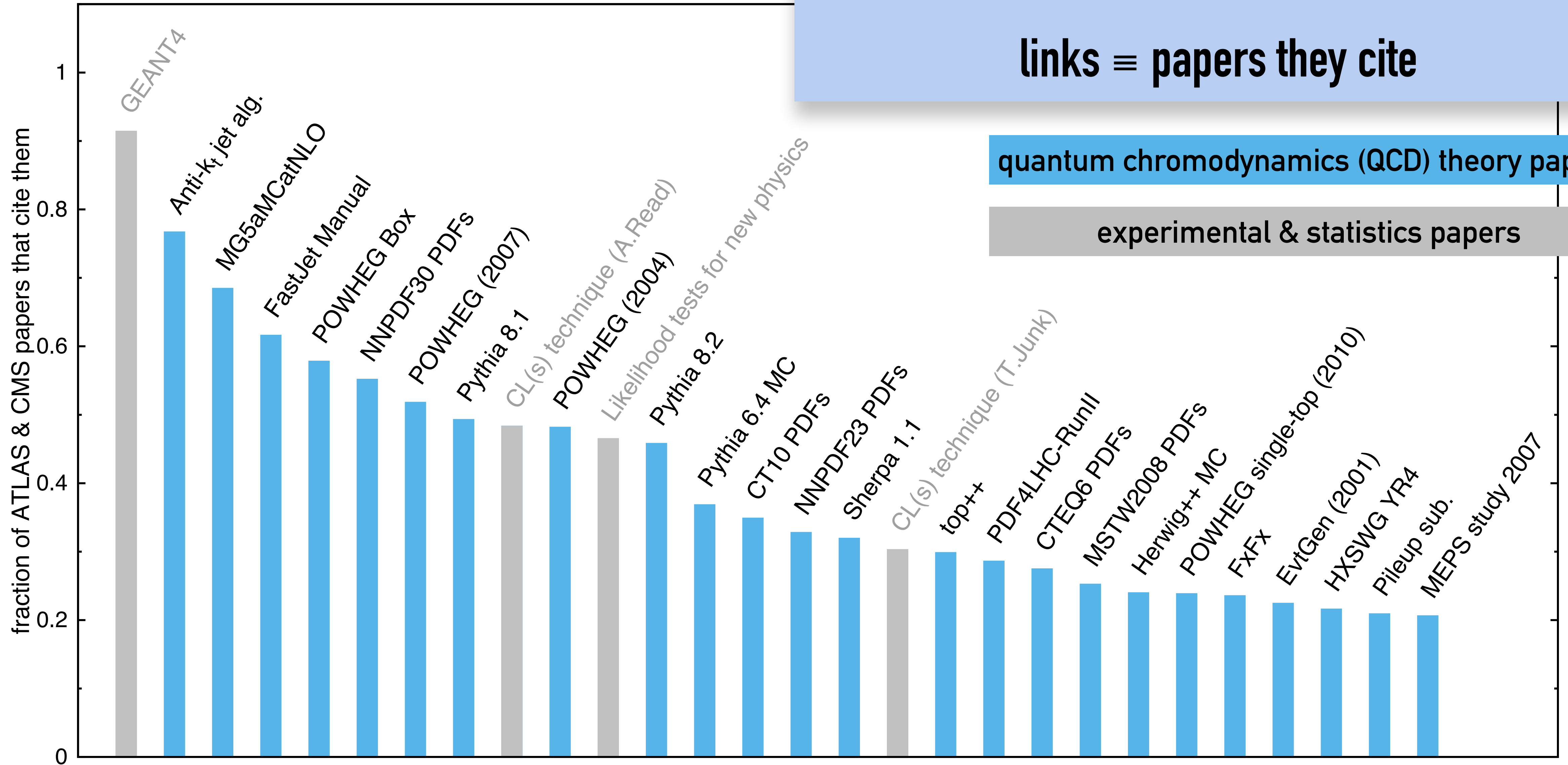
*[in particular Quantum Chromodynamics (QCD)]*



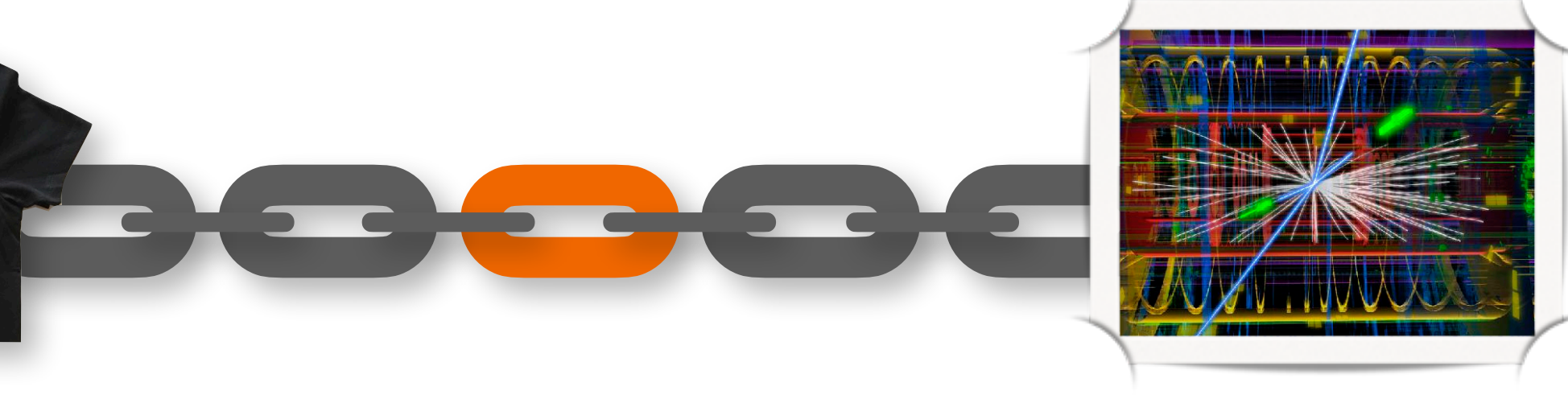
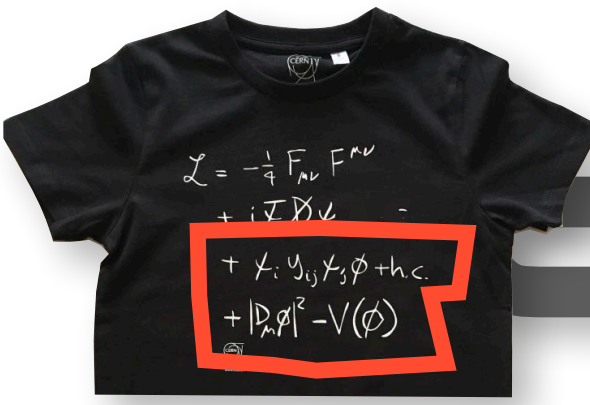


**What are the links?**  
**ATLAS and CMS (big LHC expts.) have written 715 articles since 2017**

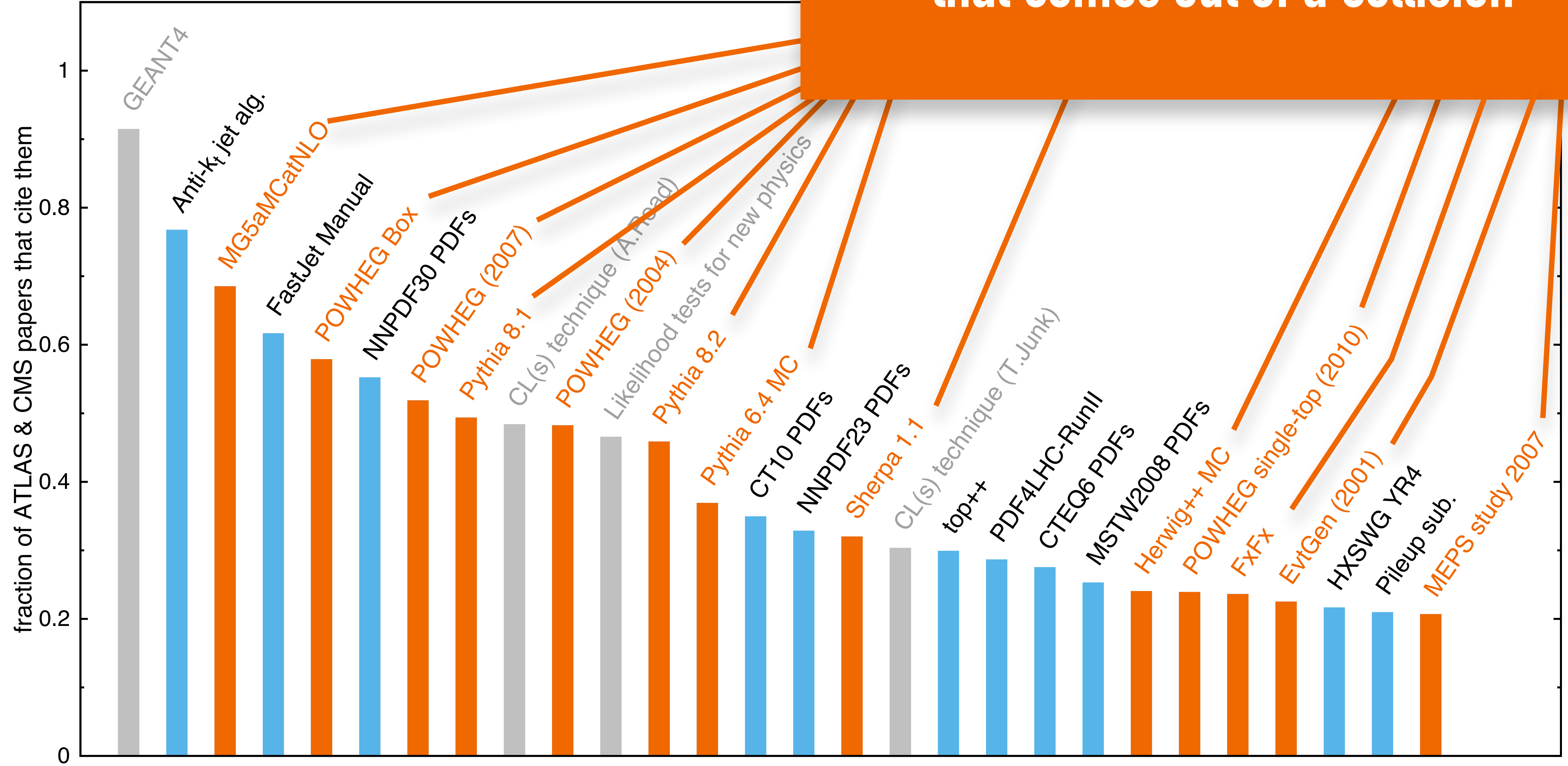
**links  $\equiv$  papers they cite**



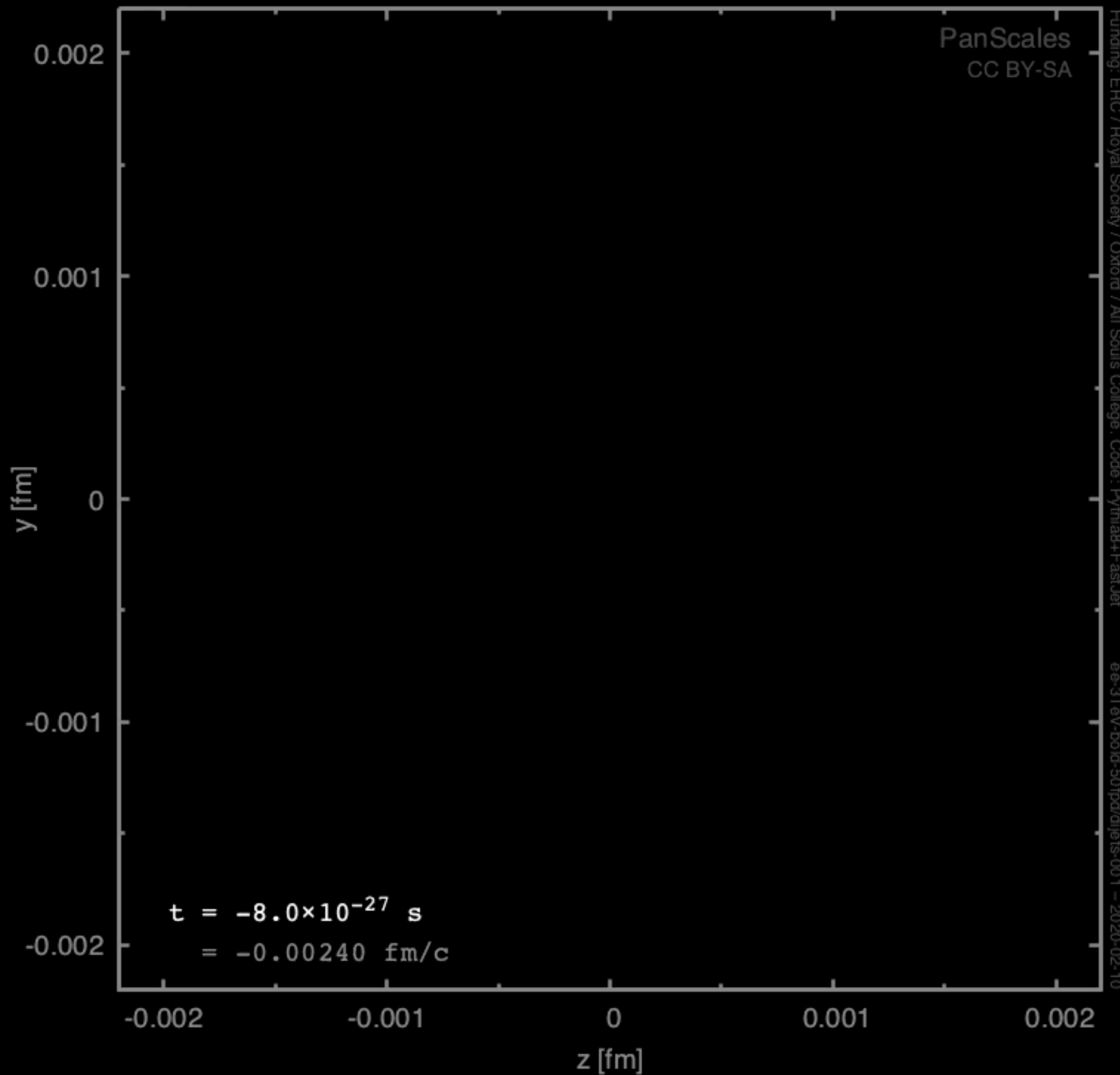
Plot by GP Salam based on data from InspireHEP



# predicting full particle structure that comes out of a collision

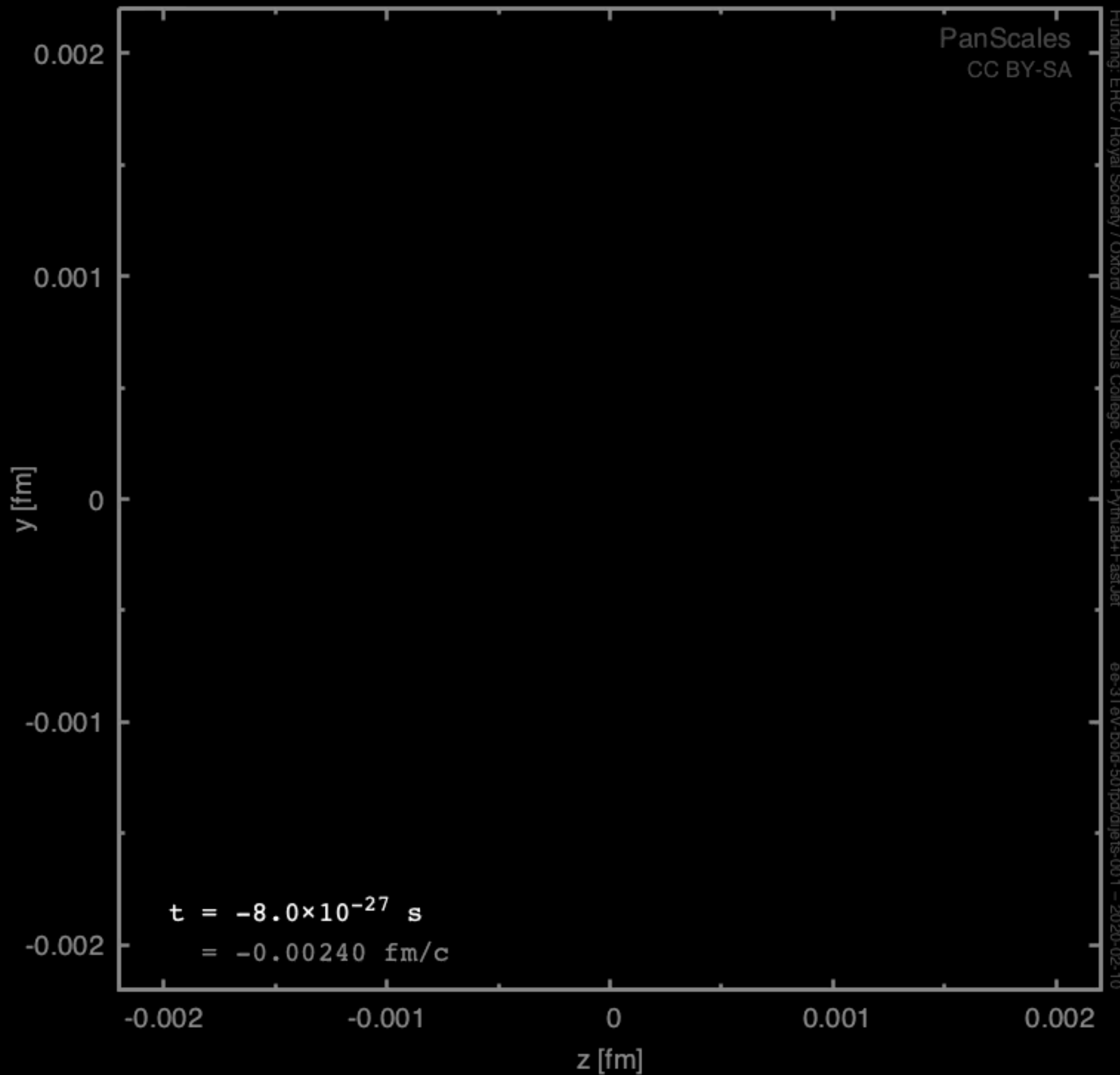


Plot by GP Salam based on data from InspireHEP



- incoming beam particle
- intermediate particle
- final particle

Event evolution spans 7 orders of magnitude in space-time



- incoming beam particle
- intermediate particle
- final particle

Event evolution spans 7 orders of magnitude in space-time

# general purpose Monte Carlo event generators:

## THE BIG 3



**Herwig 7**



**Pythia 8**

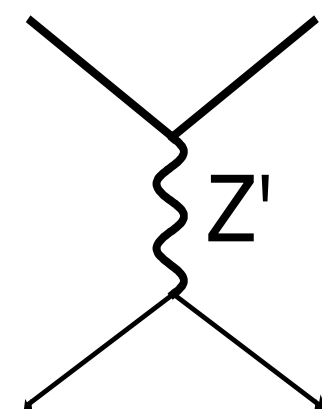


**Sherpa 2**

used in  $\sim 95\%$  of ATLAS/CMS publications  
they do an amazing job of simulation vast swathes of data;  
collider physics would be unrecognisable without them

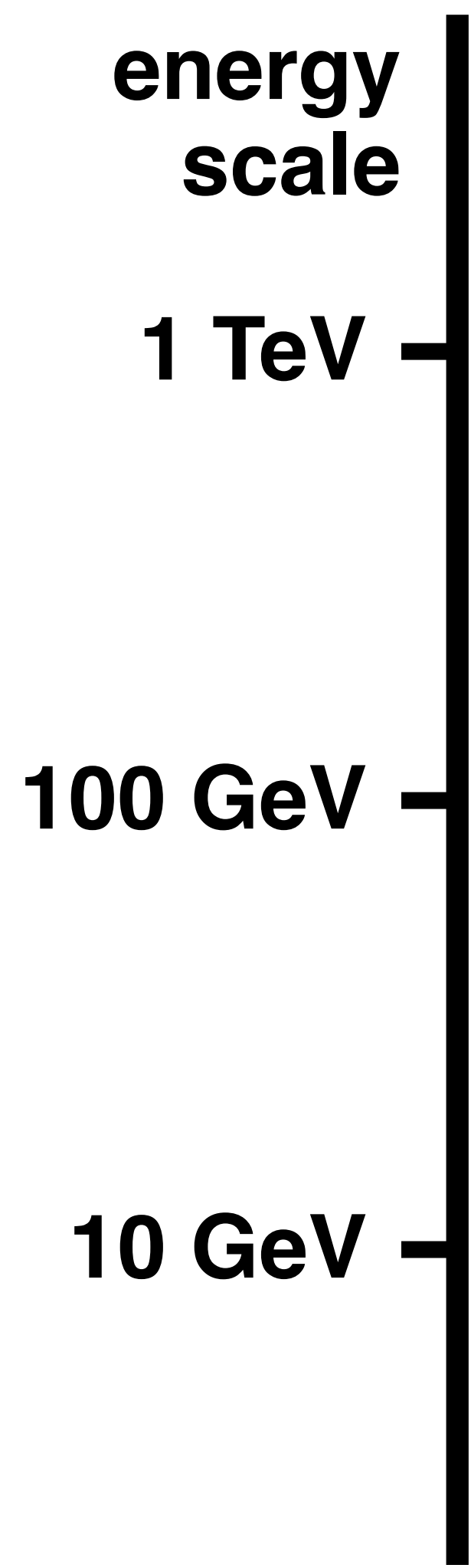
energy  
scale  
1 TeV

hard process



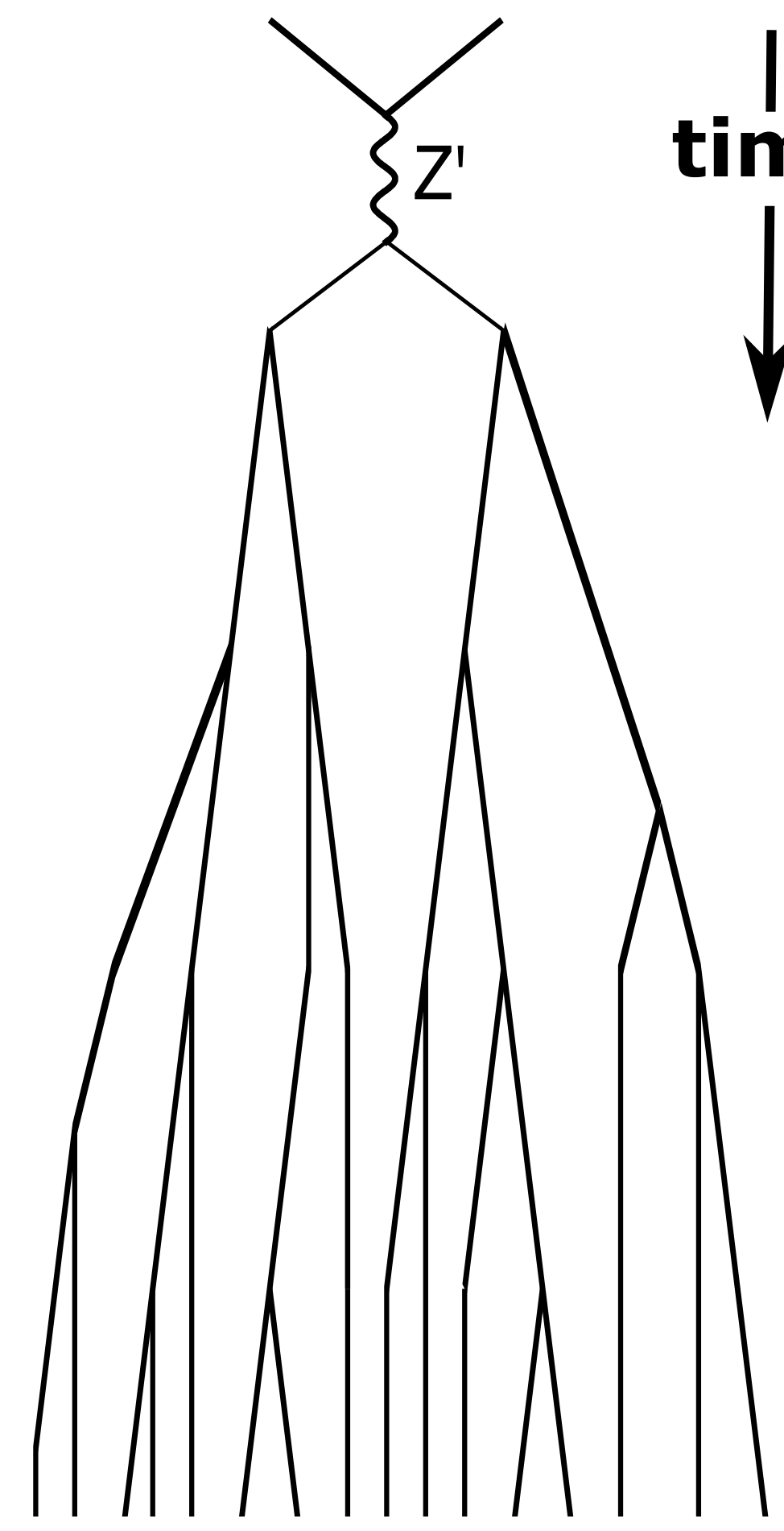
time

schematic view of key  
components of QCD  
predictions and Monte  
Carlo event simulation

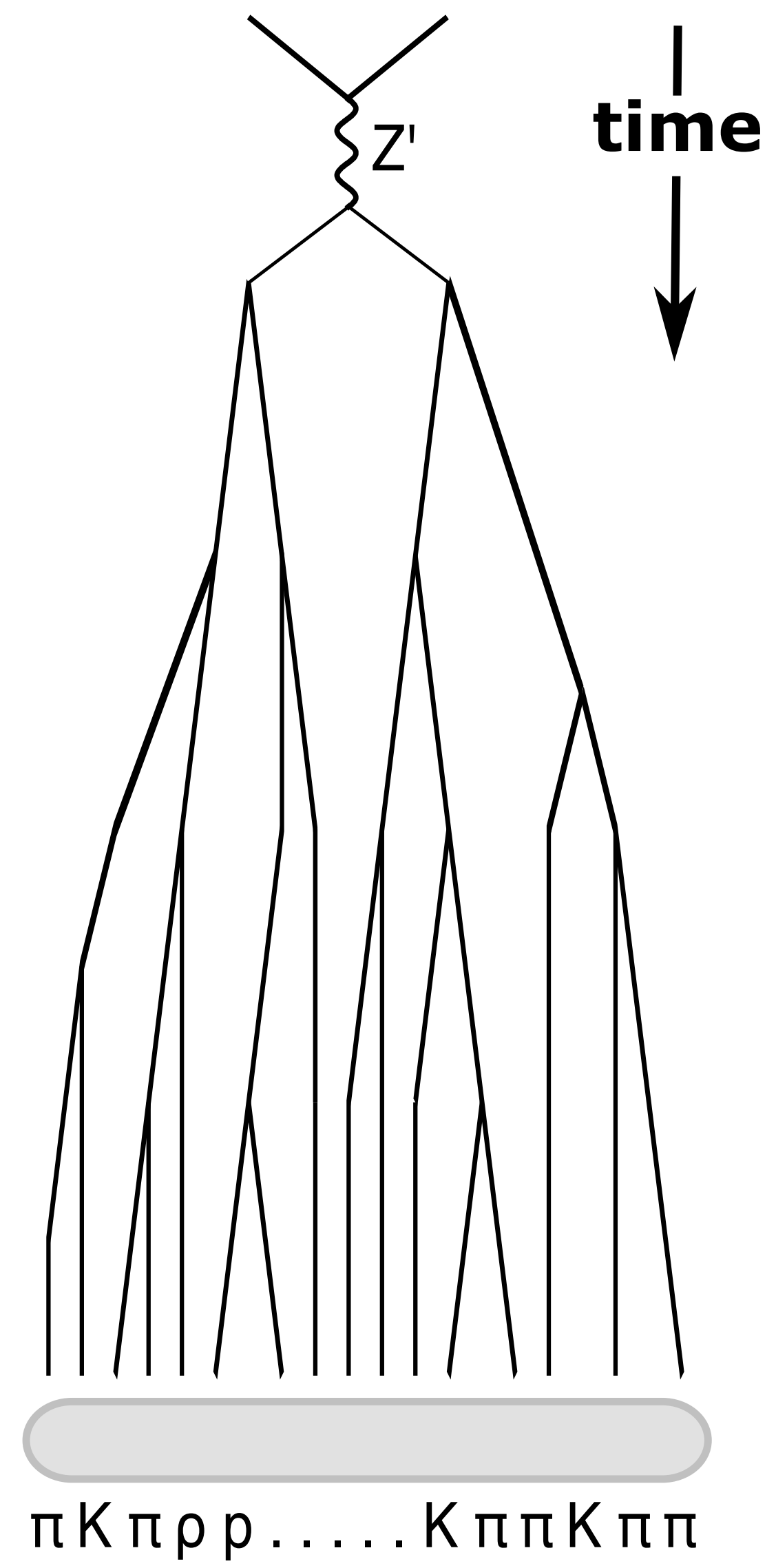
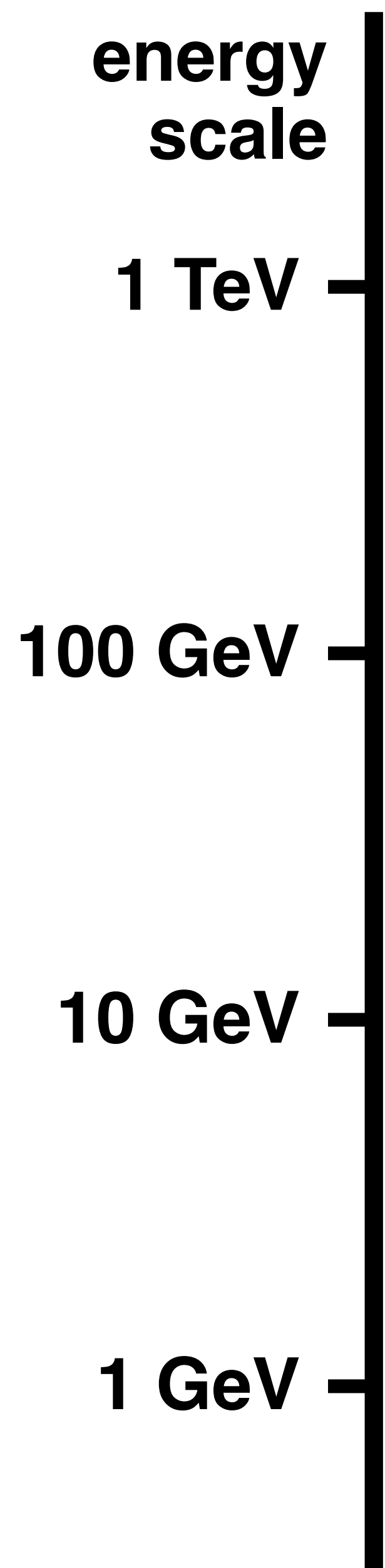


hard process

parton shower



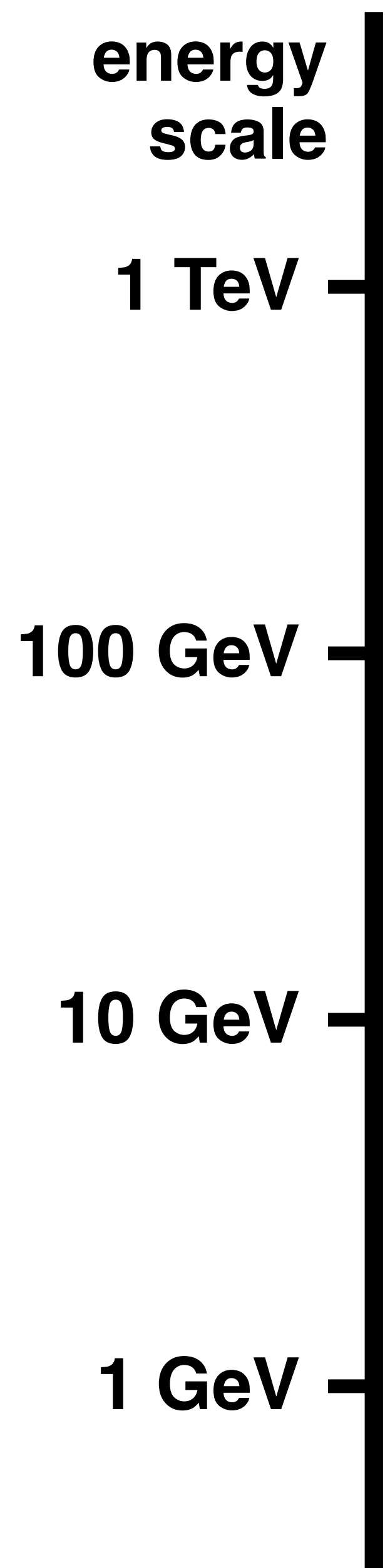
schematic view of key components of QCD predictions and Monte Carlo event simulation



schematic view of key components of QCD predictions and Monte Carlo event simulation

pattern of particles in MC can be directly compared to pattern in experiment

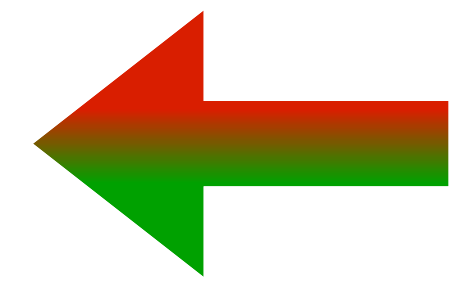
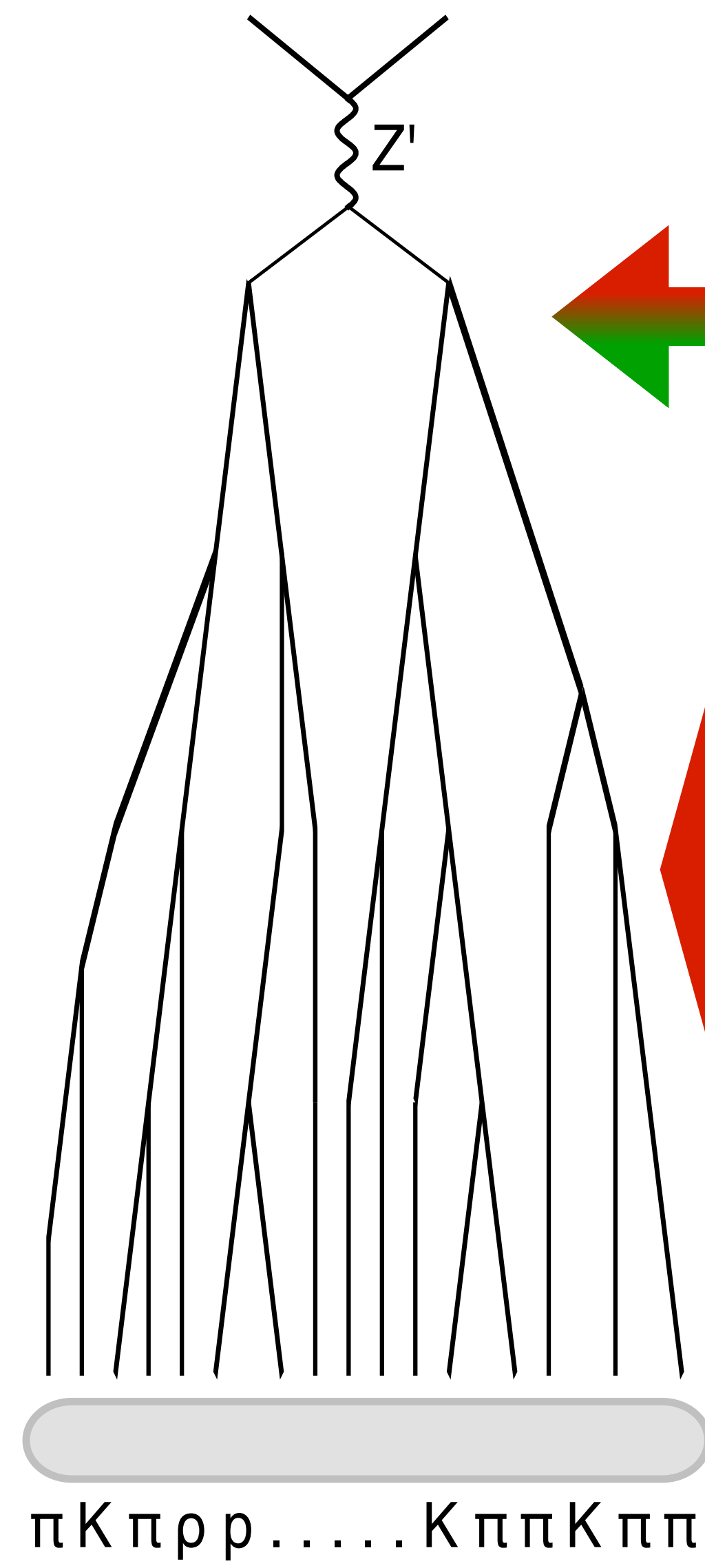




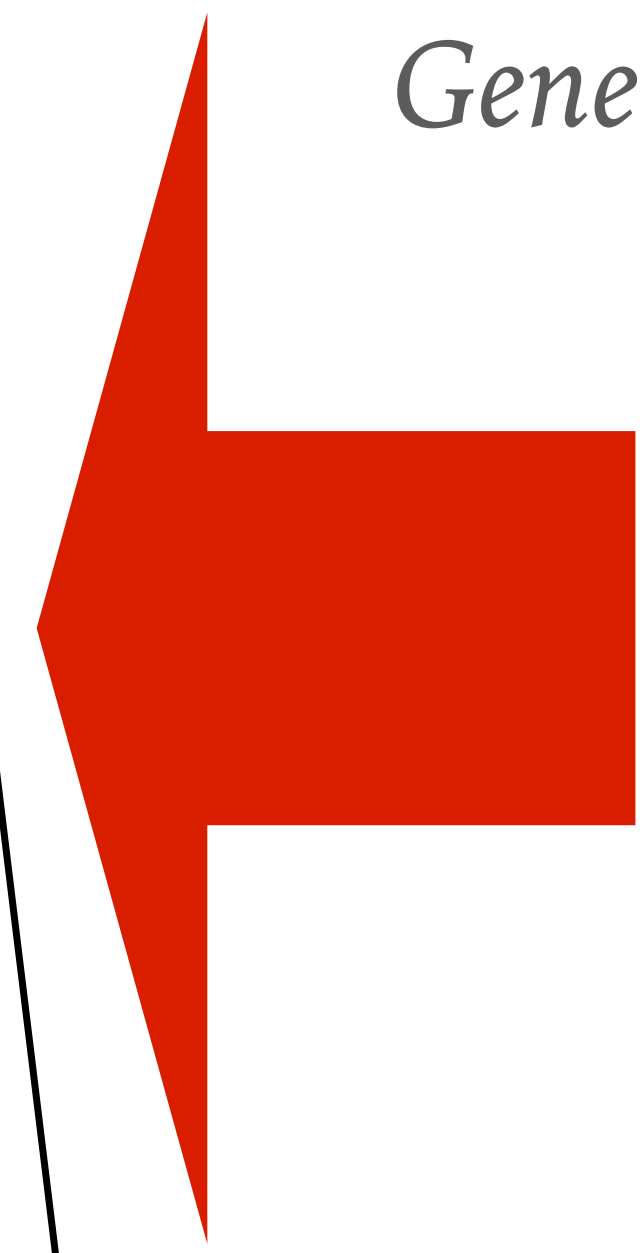
**hard process**

**parton shower**

**hadronisation**



*Much of past 20 years' work:  
MLM, CKKW, MC@NLO,  
POWHEG, MINLO, FxFx,  
Geneva, UNNLOPS, Vincia, etc.*



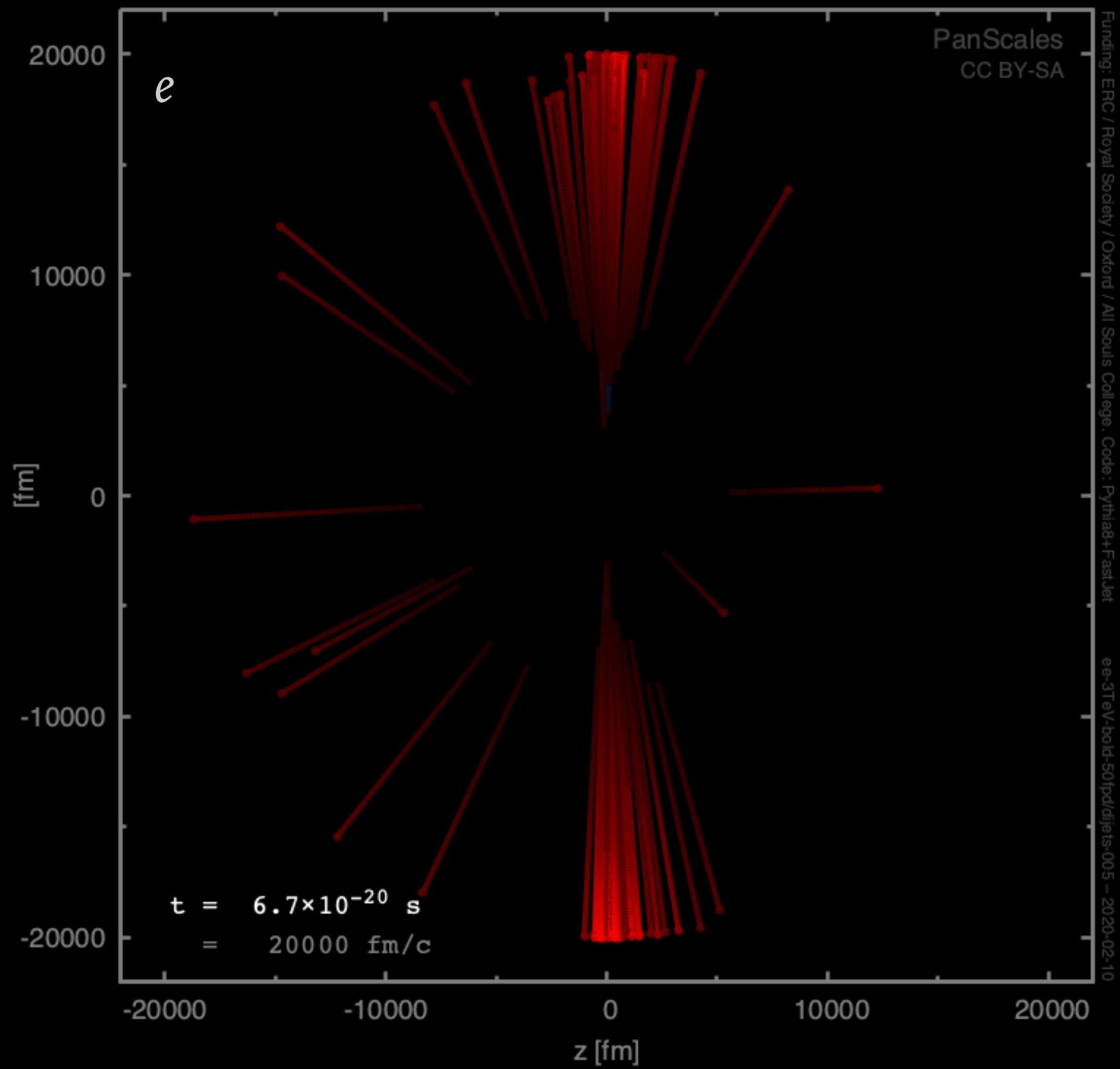
*This talk*

# using full event information

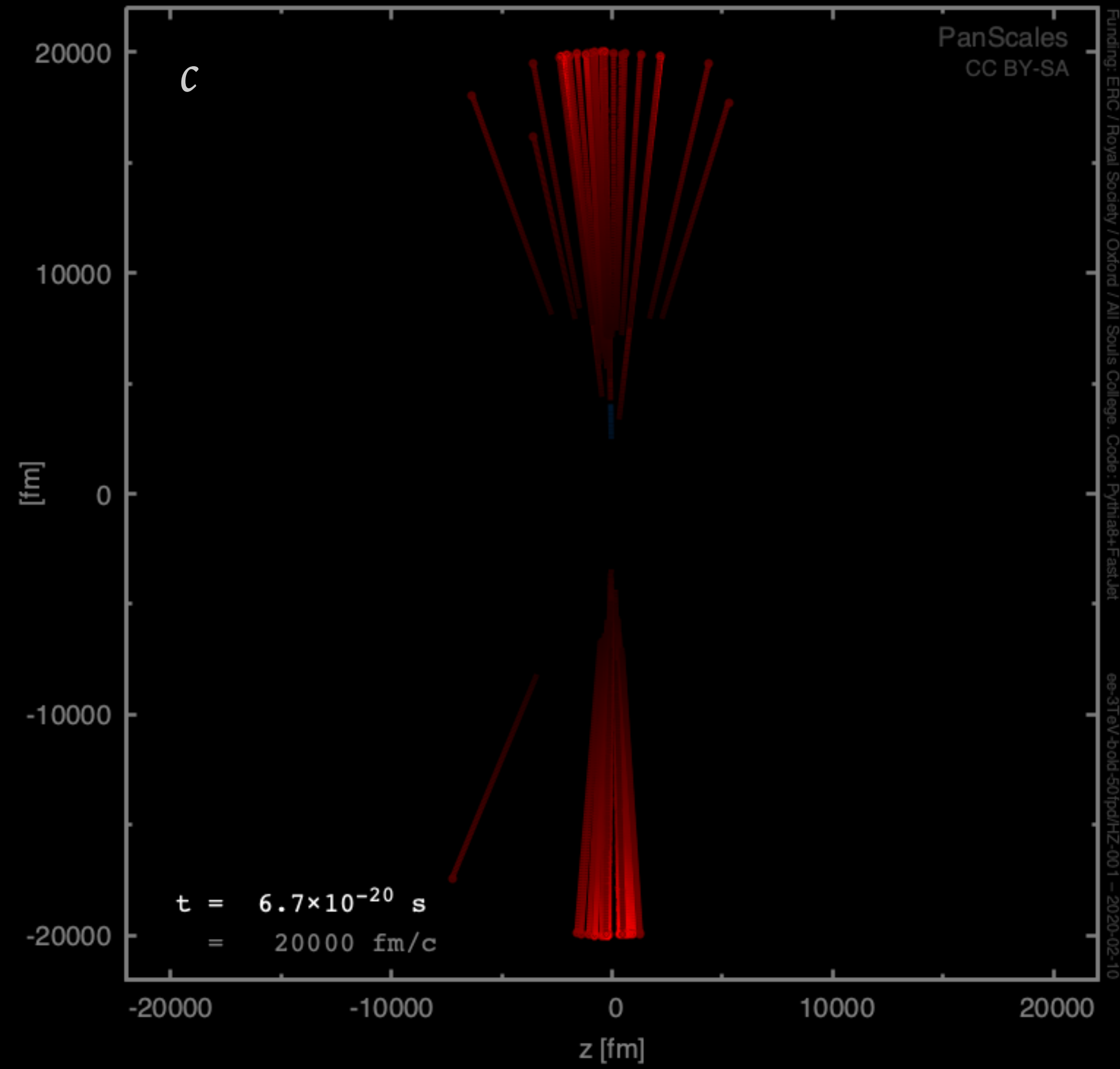
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*how much information is hidden among  
the hundreds of particles produced in a collisions?*

# pure QCD event

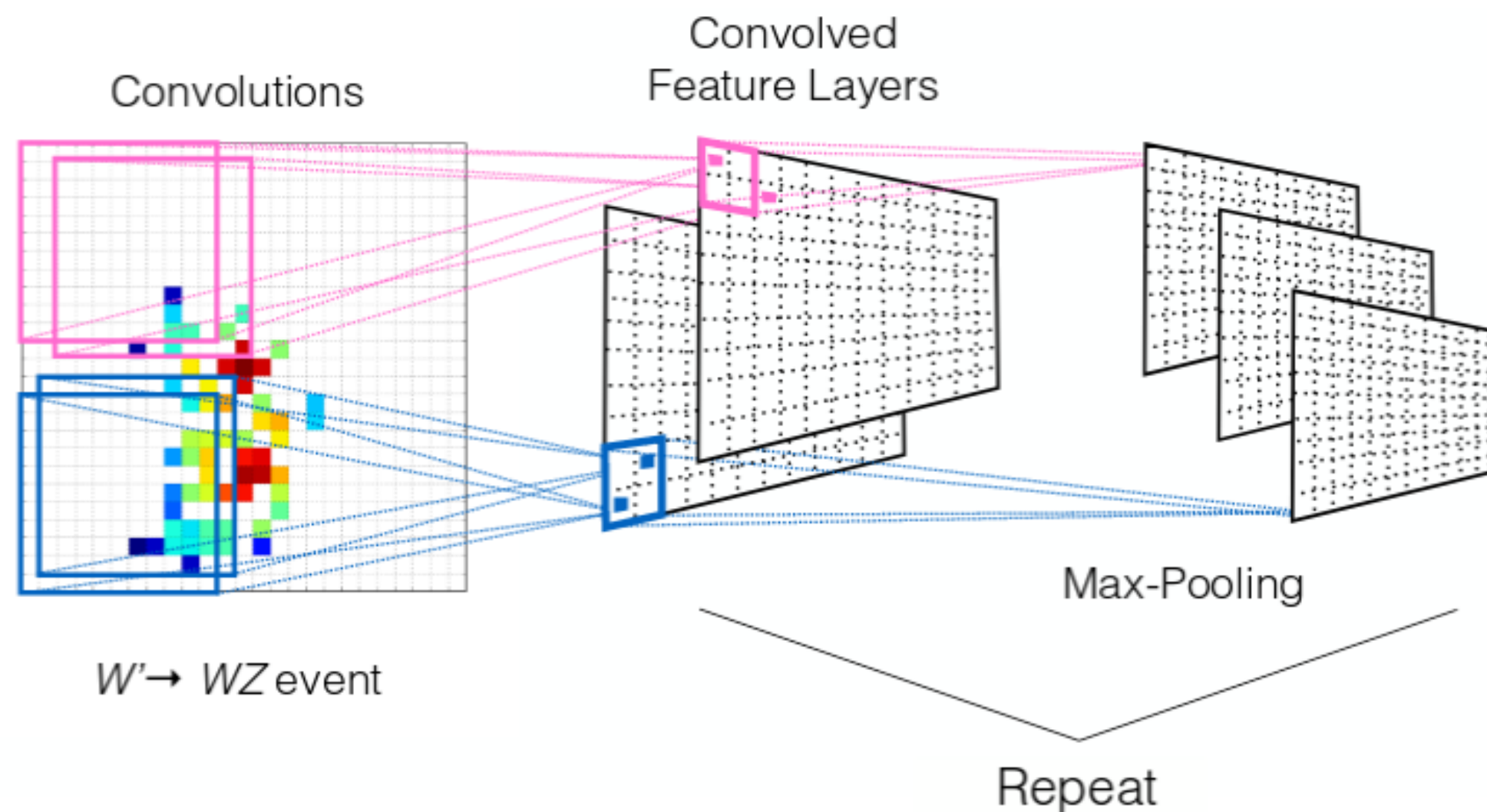
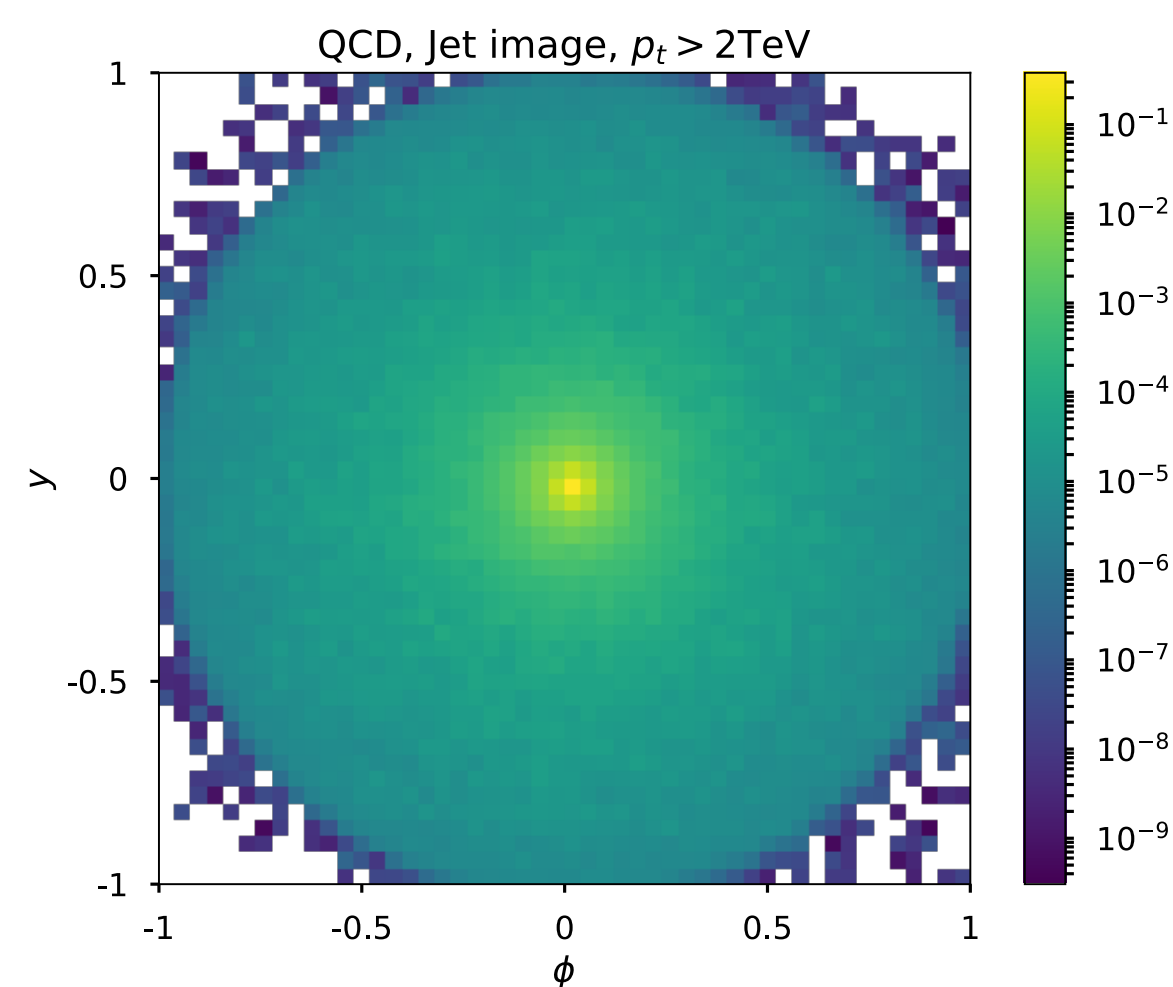


# event with Higgs & Z boson decays



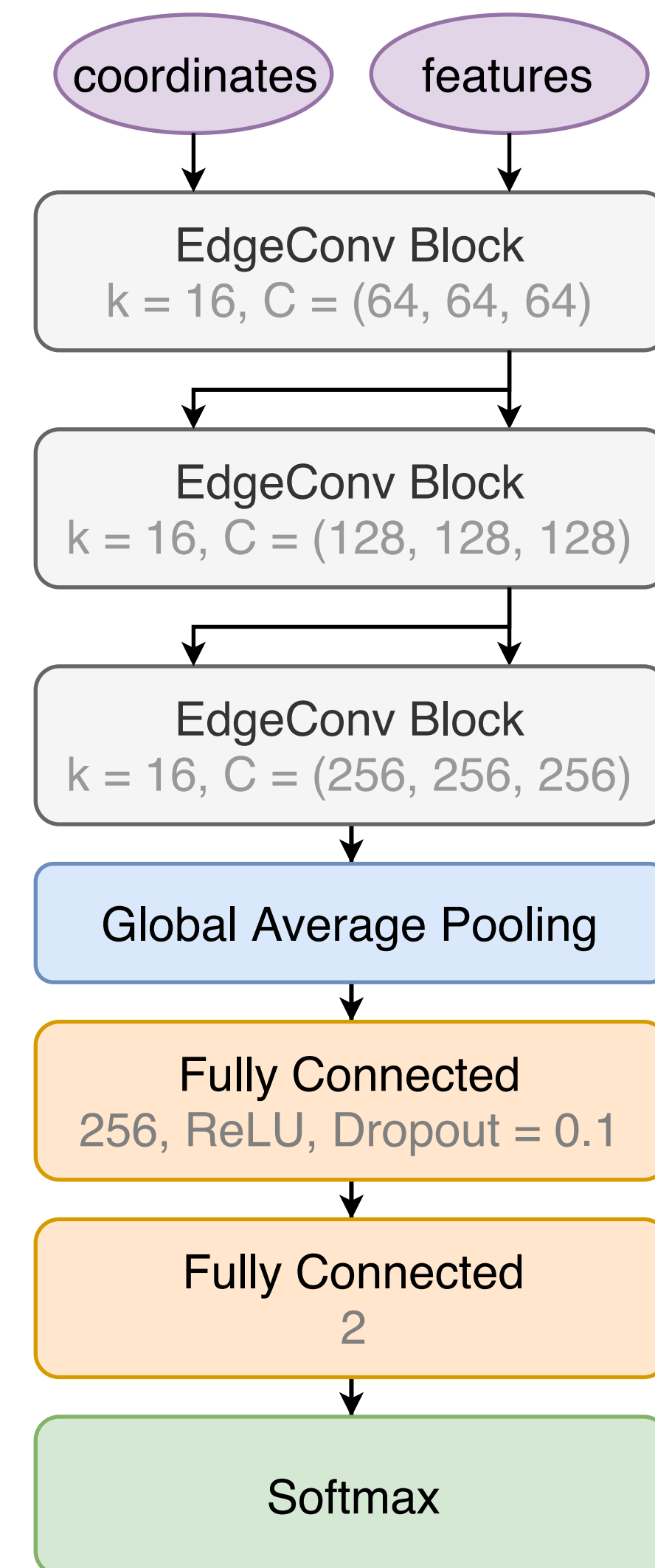
# Machine learning and jet/event structure

- ▶ Project a jet onto a fixed  $n \times n$  pixel image in rapidity-azimuth, where each pixel intensity corresponds to the momentum of particles in that cell.
- ▶ Can be used as input for classification methods used in computer vision, such as deep convolutional neural networks.



[Cogan, Kagan, Strauss, Schwartzman [JHEP 1502 \(2015\) 118](#)]

[de Oliveira, Kagan, Mackey, Nachman, Schwartzman [JHEP 1607 \(2016\) 069](#)]



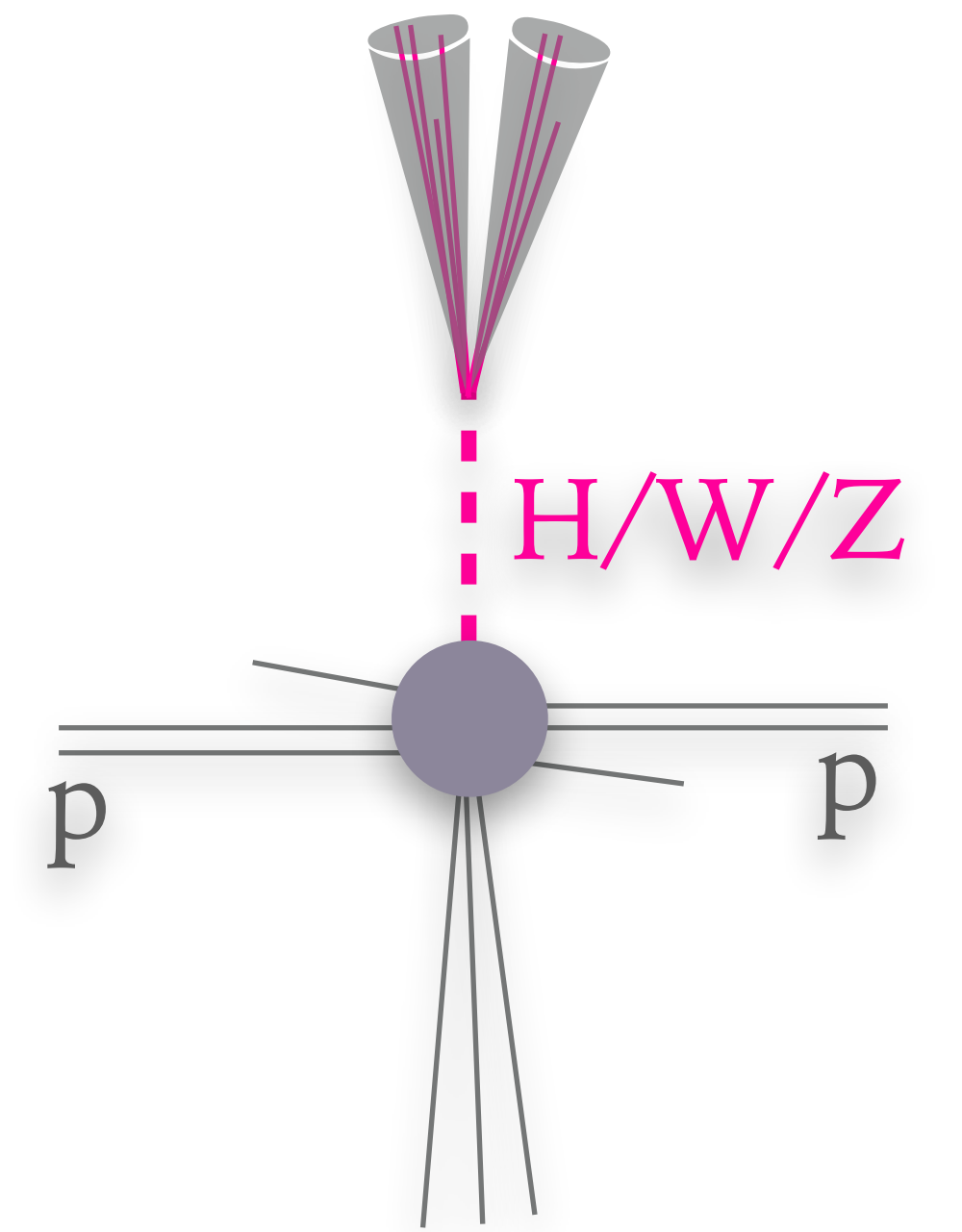
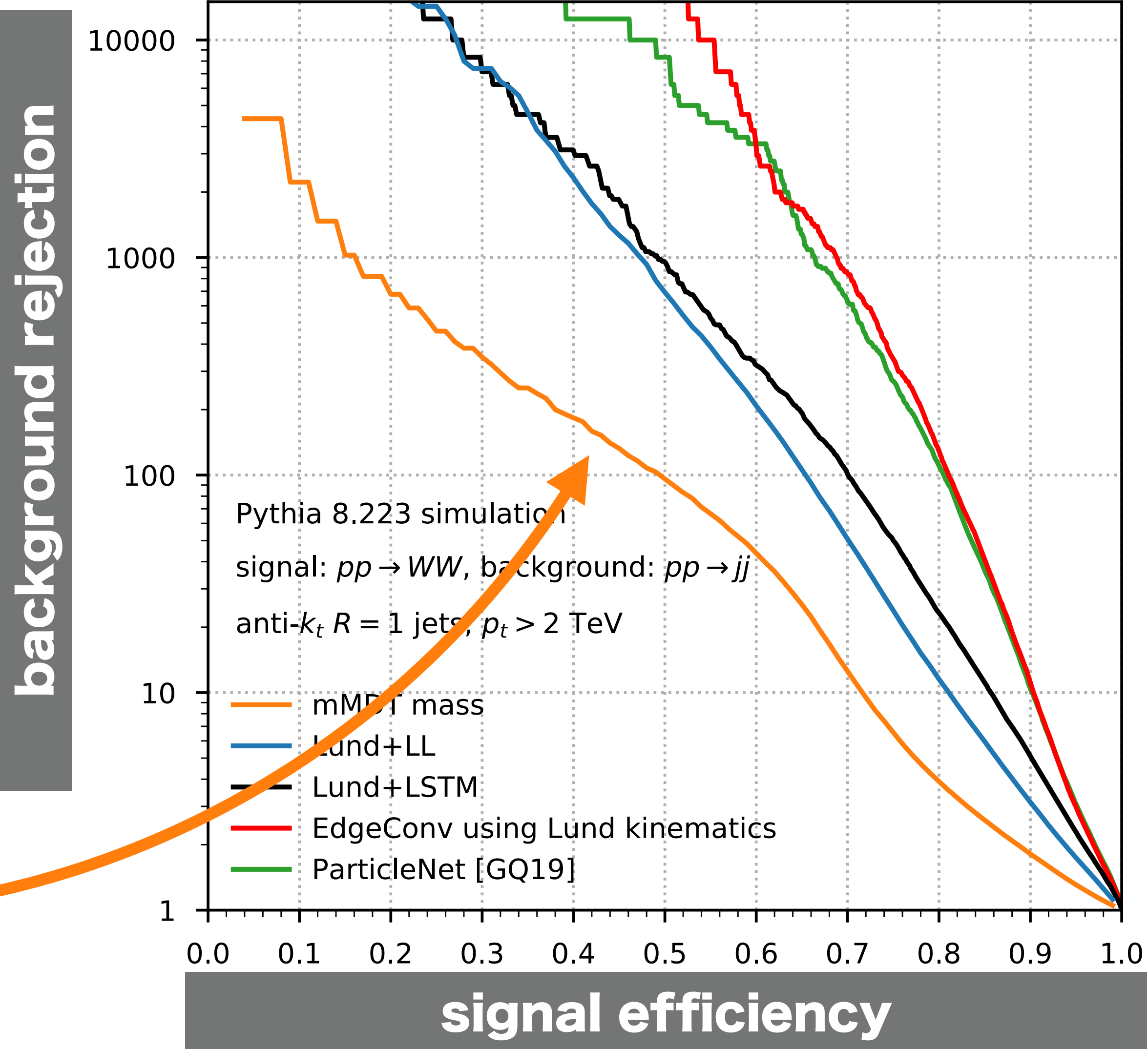
(a) ParticleNet

*Qu & Guskos,*  
*[arXiv:1902.08570](#)*

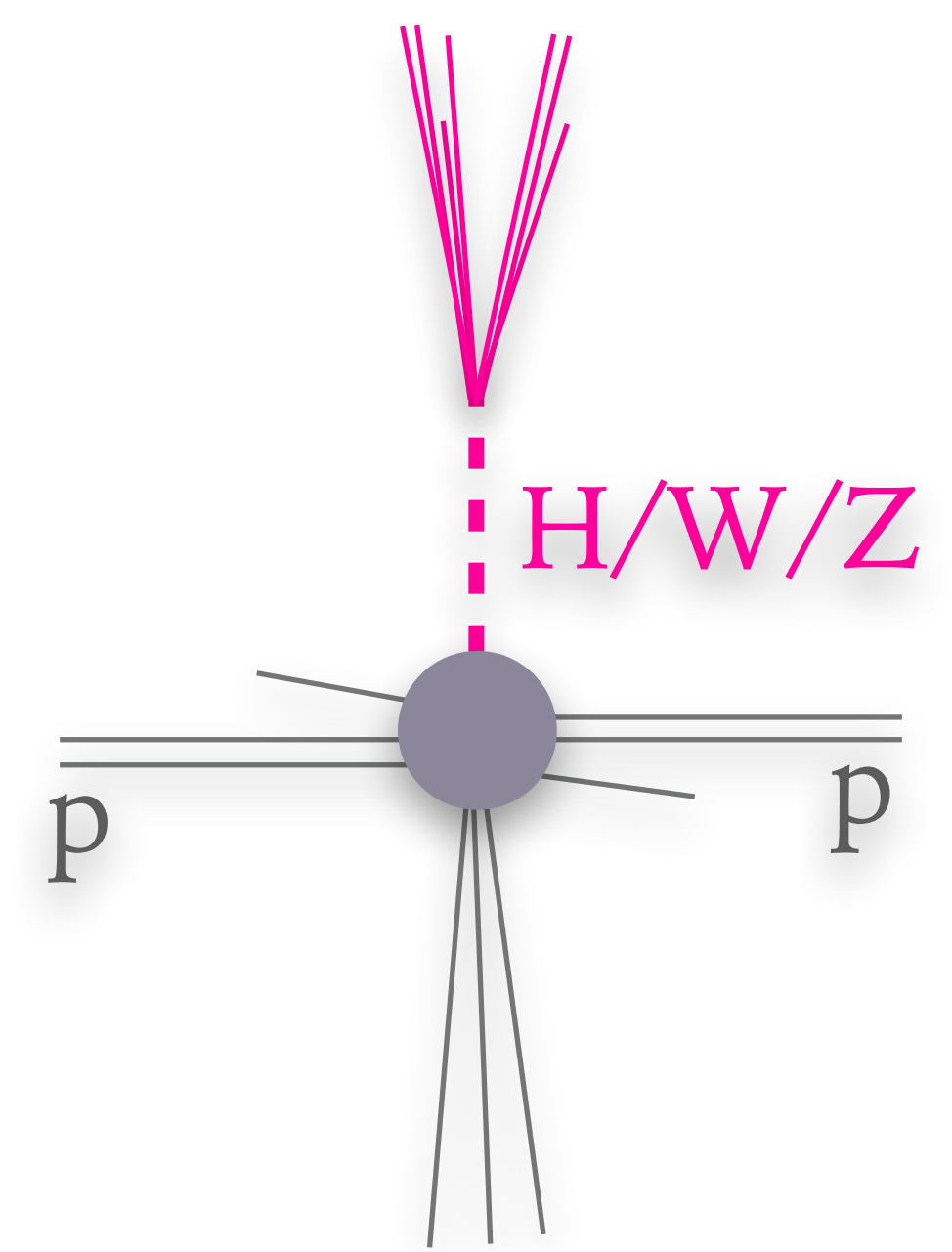
# using full jet/event information for H/W/Z-boson tagging

Dreyer 2020  
(work in progress)

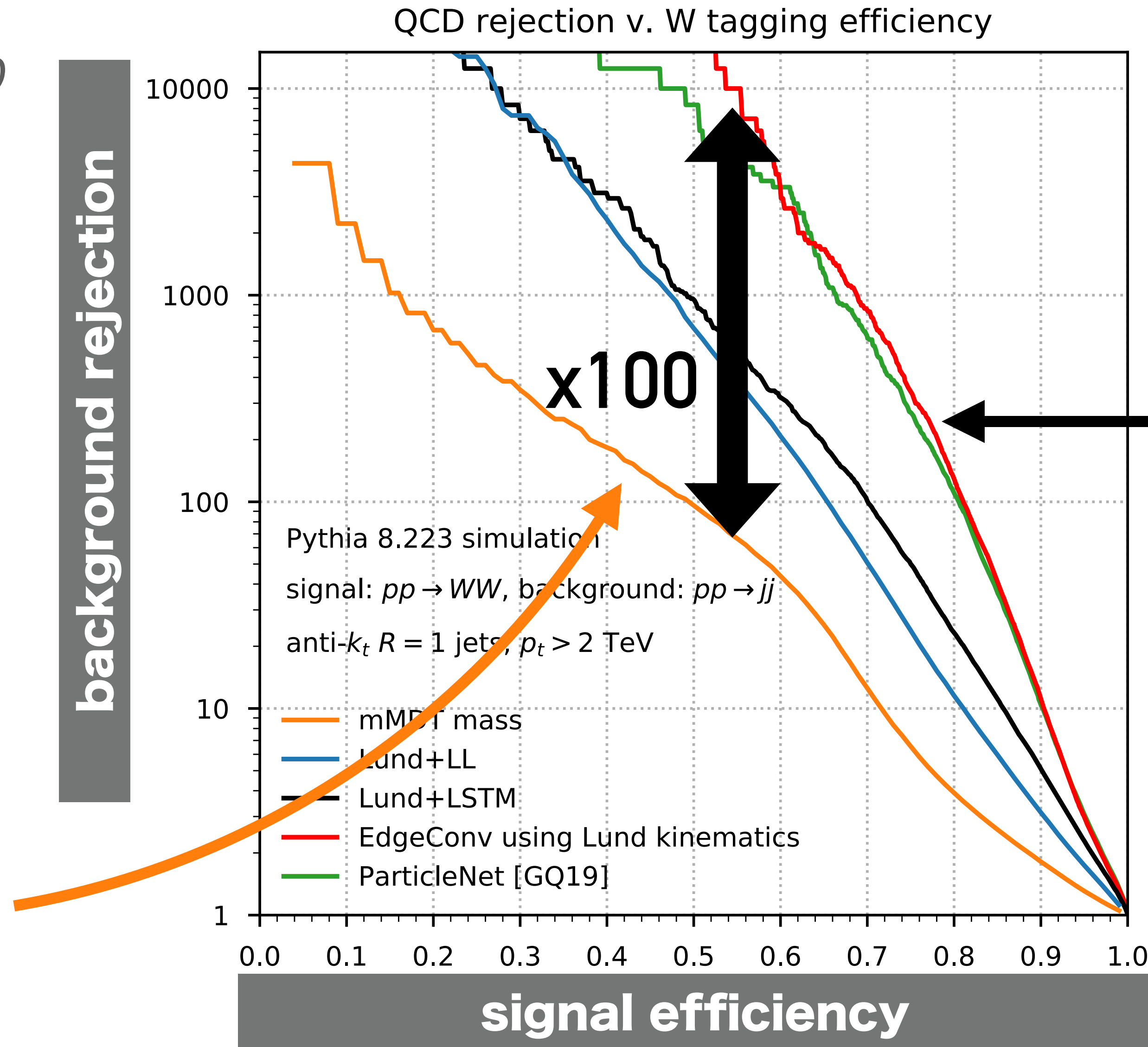
QCD rejection with just jet mass (SD/mMDT) i.e. 2008 tools & their 2013/14 descendants



# using full jet/event information for H/W/Z-boson tagging



Dreyer 2020  
(work in progress)



QCD rejection with just jet mass (SD/mMDT) i.e. 2008 tools & their 2013/14 descendants

QCD rejection with use of full jet substructure (2019 tools) 100x better

First started to be exploited by Thaler & Van Tilburg with "N-subjettiness" (2010/11)

# can we trust machine learning? A question of confidence in the training...

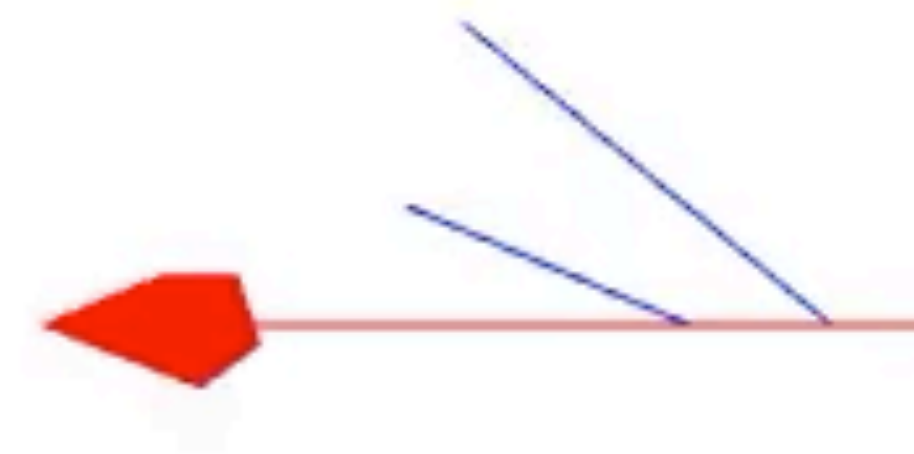


Unless you are highly confident in the information you have about the markets, you may be better off ignoring it altogether

*- Harry Markowitz (1990 Nobel Prize in Economics)  
[via S Gukov]*

# Concrete example: azimuthal structure in jets

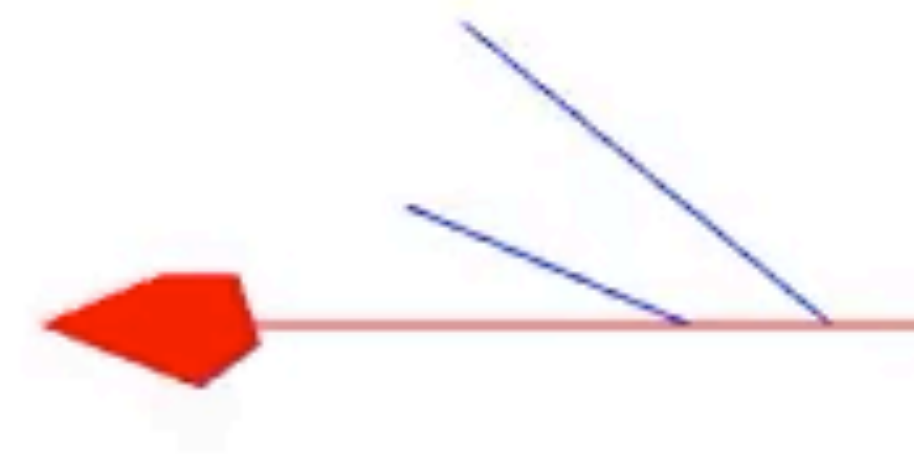
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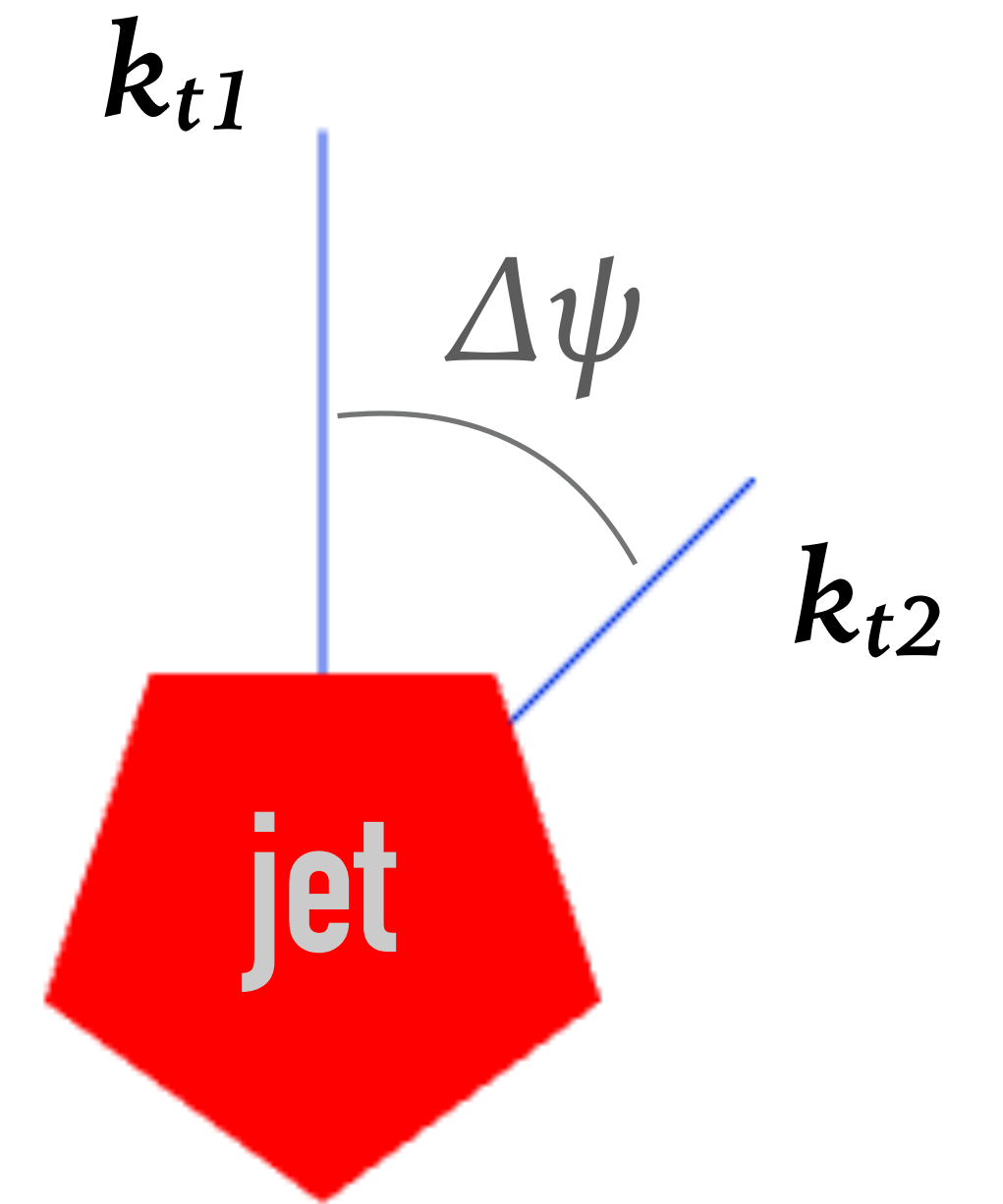
# Concrete example: azimuthal structure in jets

---

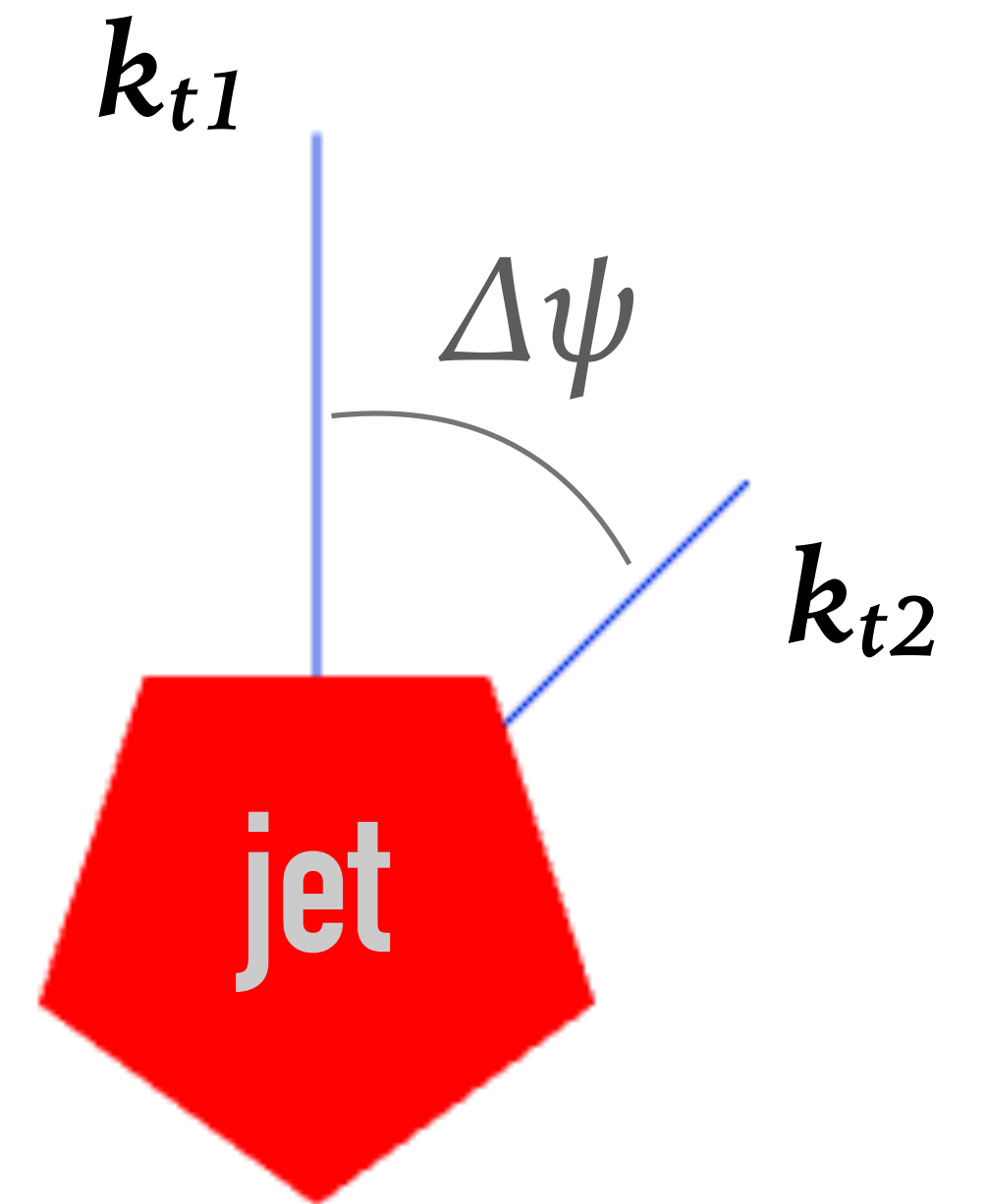
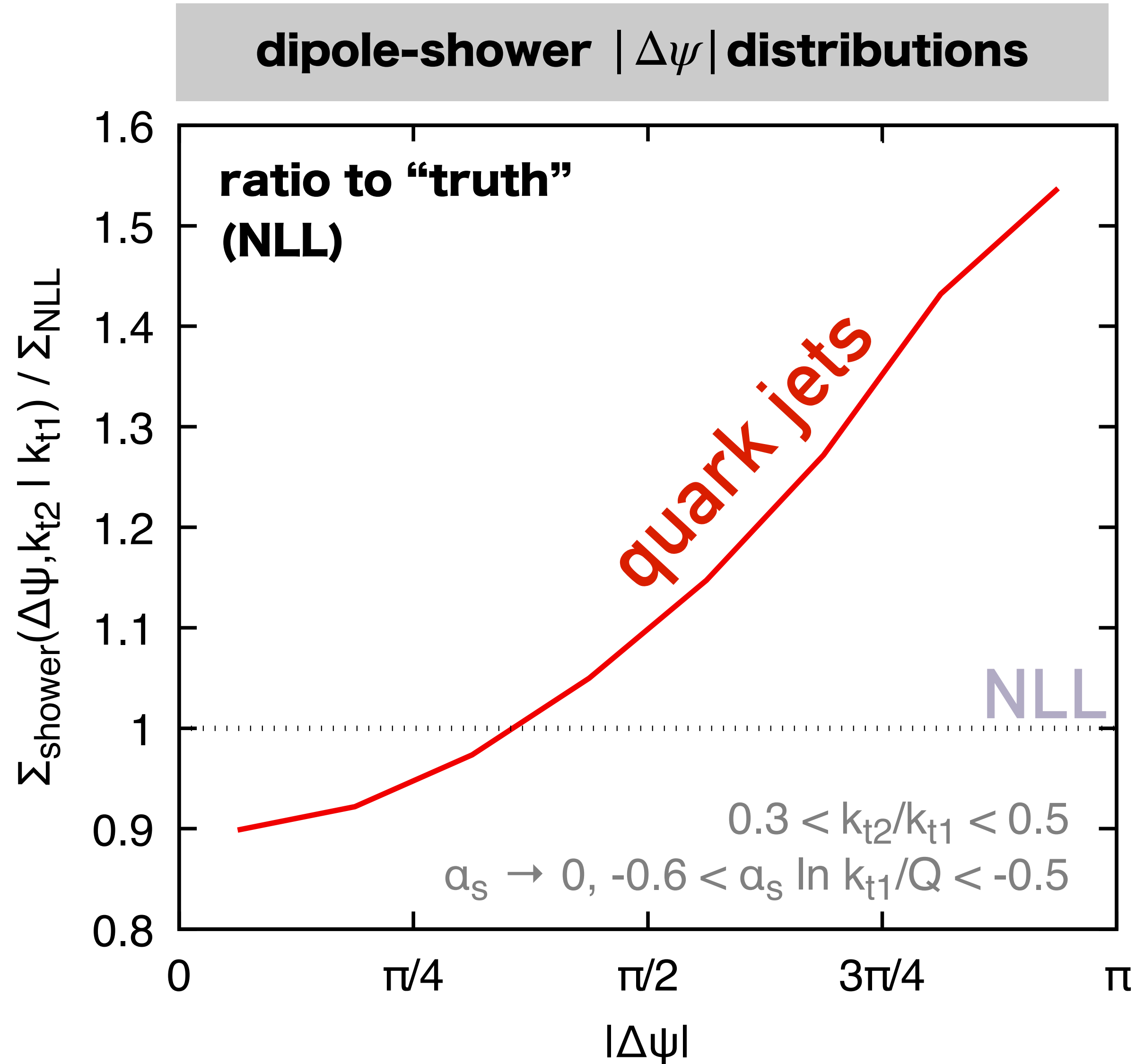


# Concrete example: azimuthal structure in jets

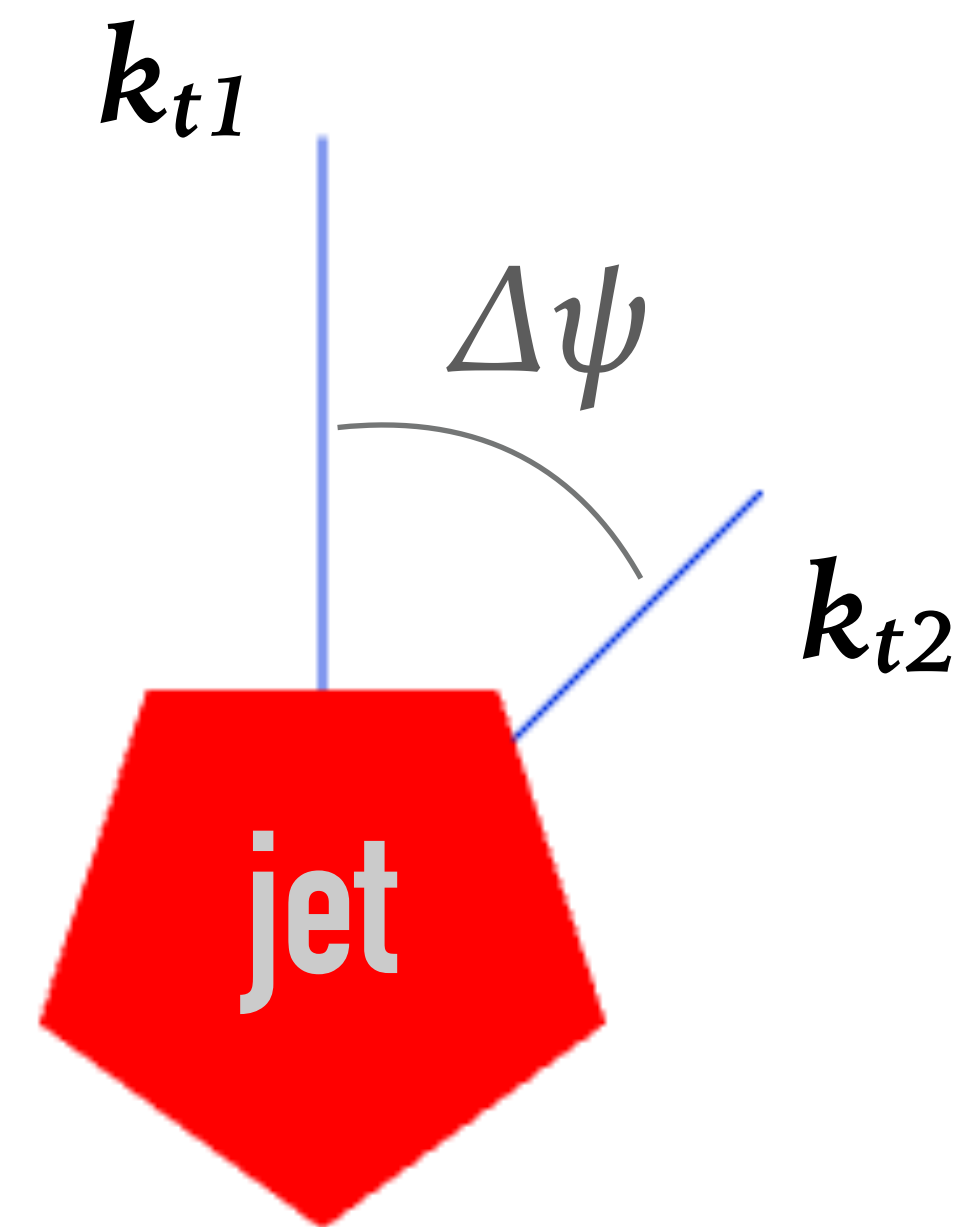
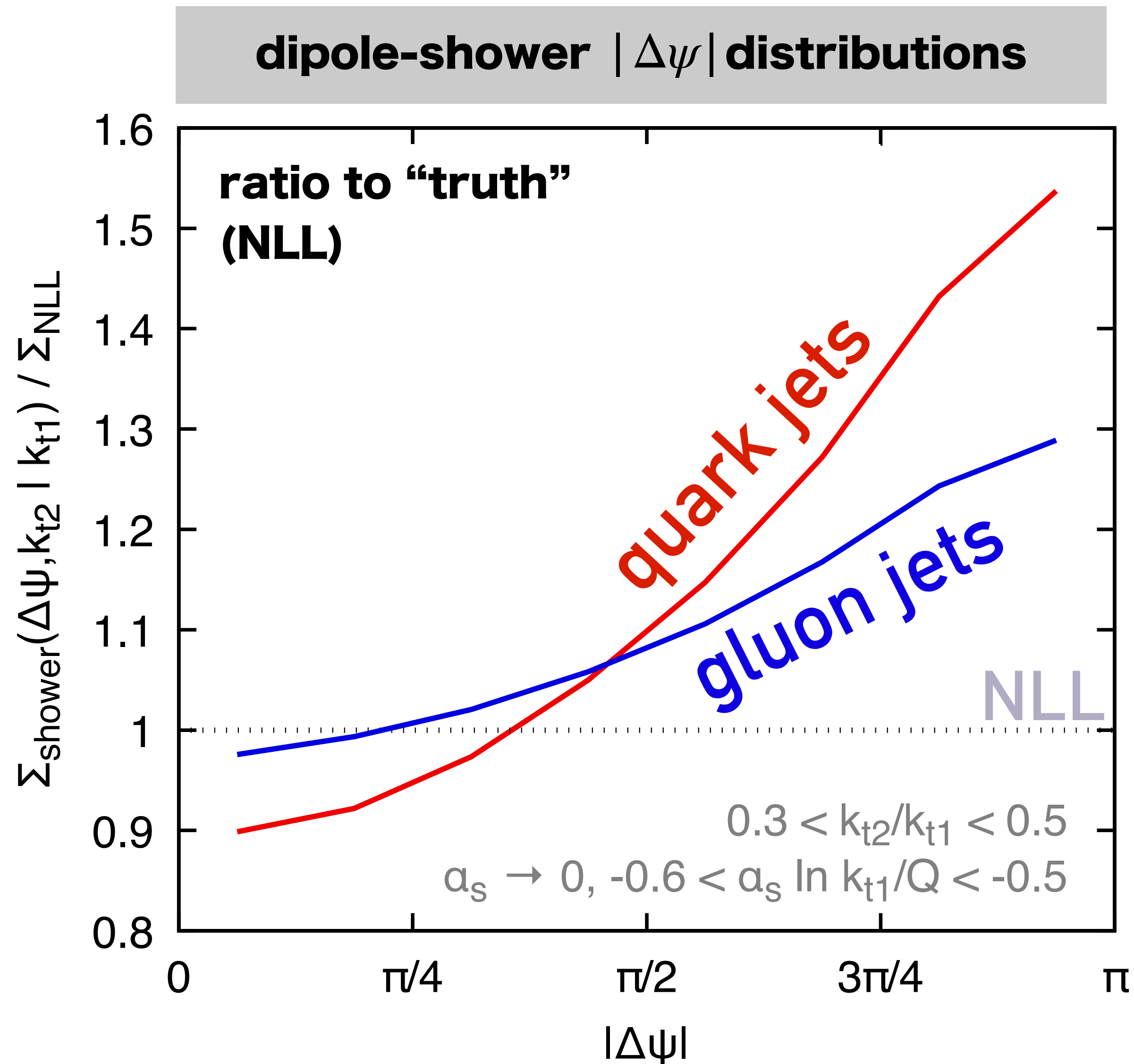
---



# Concrete example: azimuthal structure in jets



# Concrete example: azimuthal structure in jets



(machine-learning) quark/gluon discrimination trained on this simulation will **learn to exploit a feature that doesn't exist in real events**

# what is a (Monte Carlo) parton shower?

---

*illustrate with dipole / antenna showers*

*Gustafson & Pettersson 1988, Ariadne 1992, main Sherpa & Pythia8 showers, option in Herwig7,  
Vincia & Dire showers & (partially) Deductor shower*

# Example of radioactive decay (limit of long half-life)

---

Constant decay rate  $\mu$  per unit time, total time  $t_{\max}$ . Find distribution of emissions.

1. write as coupled evolution equations for probability  $P_0, P_1, P_2$ , etc., of having 0, 1, 2, ... emissions

$$\frac{dP_n}{dt} = -\mu P_n(t) + \mu P_{n-1}(t)$$

$n \rightarrow n+1$        $n-1 \rightarrow n$

[easy to implement in Monte Carlo approach]

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[easy to implement in Monte Carlo approach]

**Monte Carlo solution** (repeat following procedure many times to get distribution of  $n, \{t_i\}$ )

- a. start with  $n = 0, t_0 = 0$
- b. Choose random number  $r$  ( $0 < r < 1$ ) and find  $t_{n+1}$  that satisfies

$$r = e^{-\mu(t_{n+1} - t_n)}$$

[i.e. randomly sample exponential distribution]

- c. If  $t_{n+1} < t_{\max}$ , increment  $n$ , go to step b

# Monte Carlo worked example

E.g. for decay rate  $\mu = 1$ , total time  $t_{\max} = 2$

- ▶ start with  $n = 0, t_0 = 0$
- ▶ random number  $r = 0.6 \rightarrow t_1 = t_0 + \log(1/r) = 0.51$  [emission 1]
- ▶ random number  $r = 0.3 \rightarrow t_2 = t_1 + \log(1/r) = 1.71$  [emission 2]
- ▶ random number  $r = 0.4 \rightarrow t_3 = t_2 + \log(1/r) = 2.63$  [ $> t_{\max}$ , so stop]
- ▶ This event has two emissions at times  $\{t_1 = 0.51, t_2 = 1.71\}$

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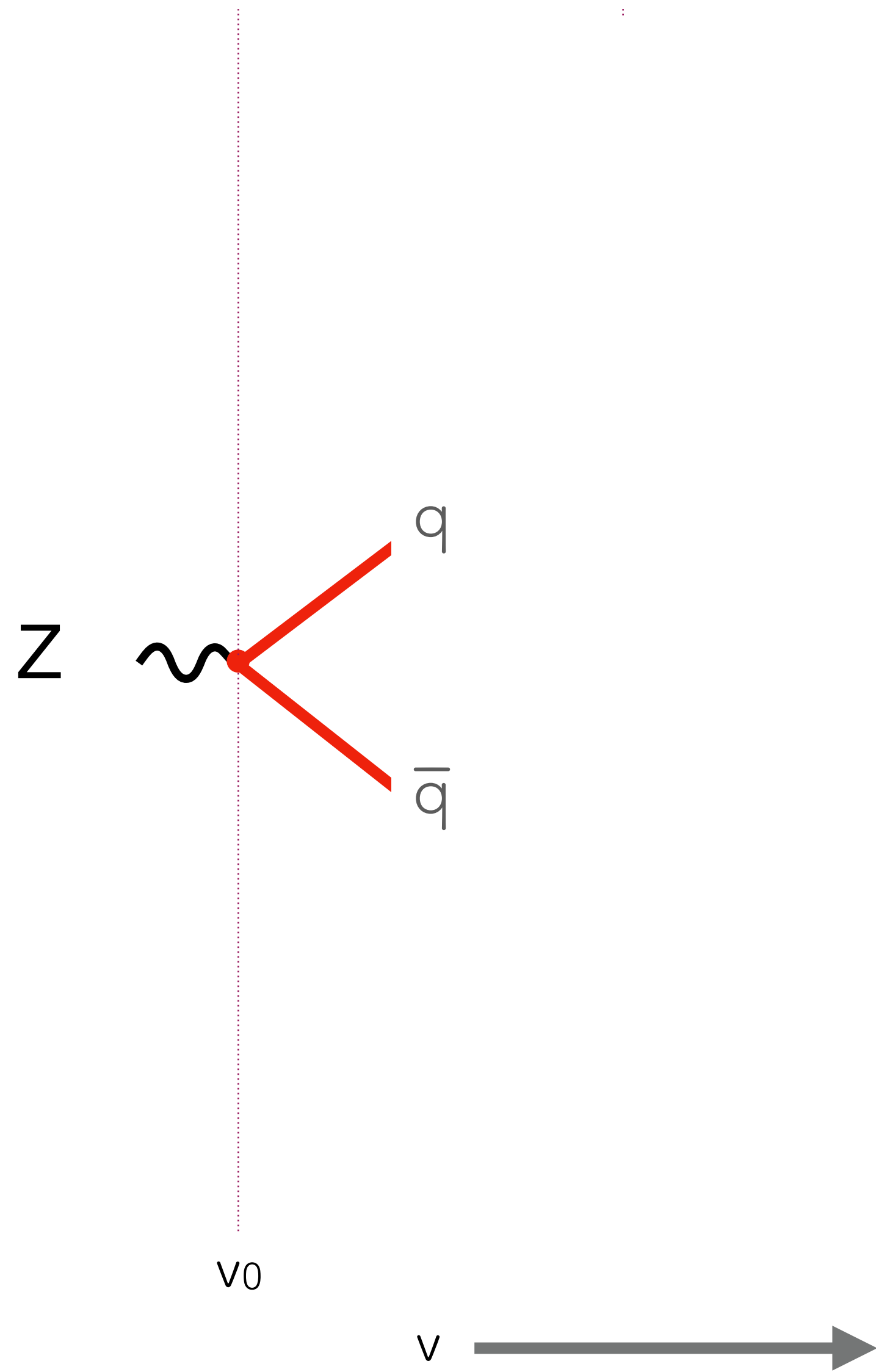


# QCD shower: an evolution equation (in **evolution scale $v$** , e.g. $1/\text{trans.mom.}$ )

Start with  $q$ - $q$ bar state.

Throw a random number to determine down to what scale state persists unchanged

$$\frac{dP_2(v)}{dv} = -f_{2 \rightarrow 3}^{q\bar{q}}(v) P_2(v)$$

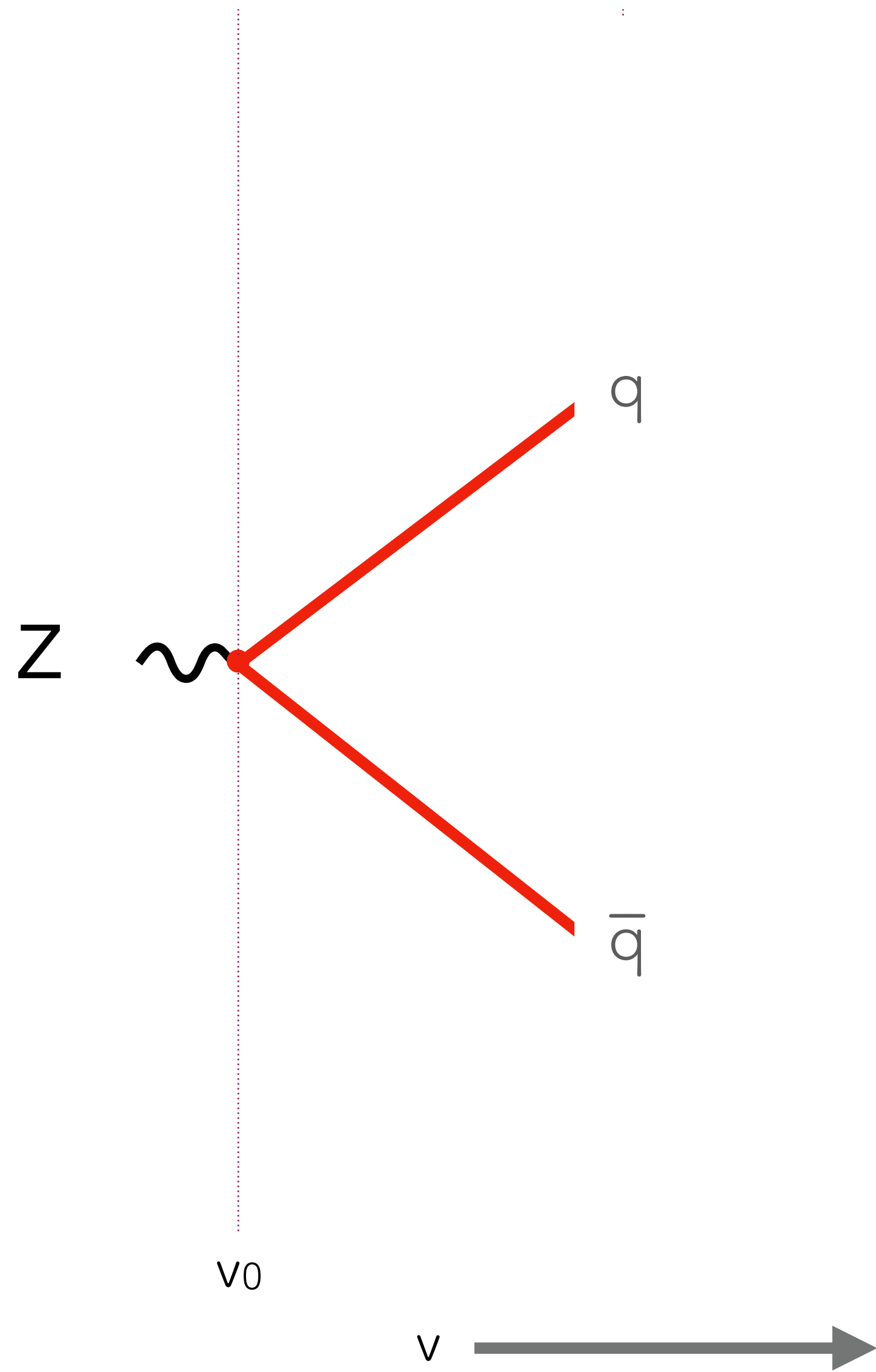


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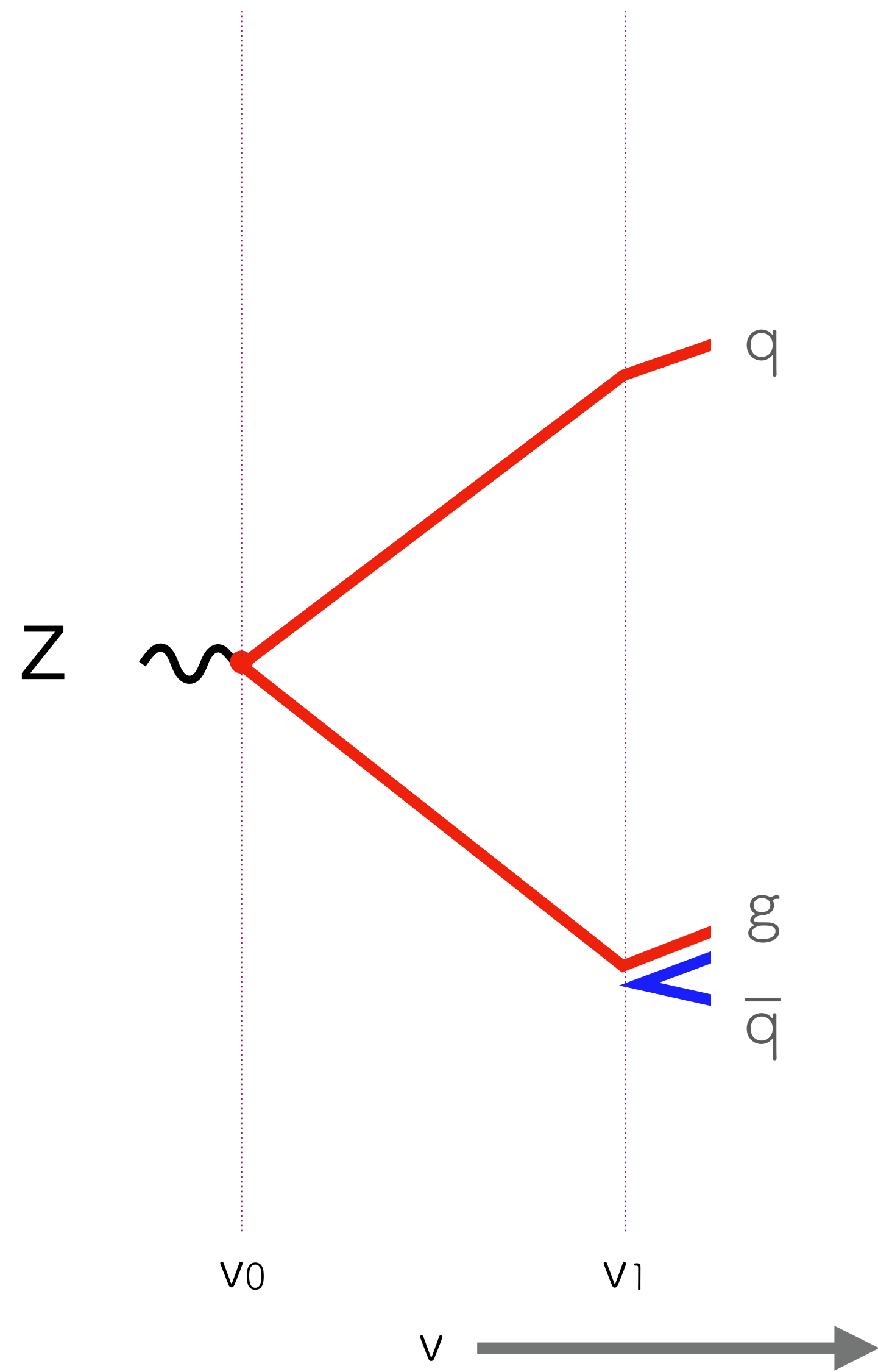
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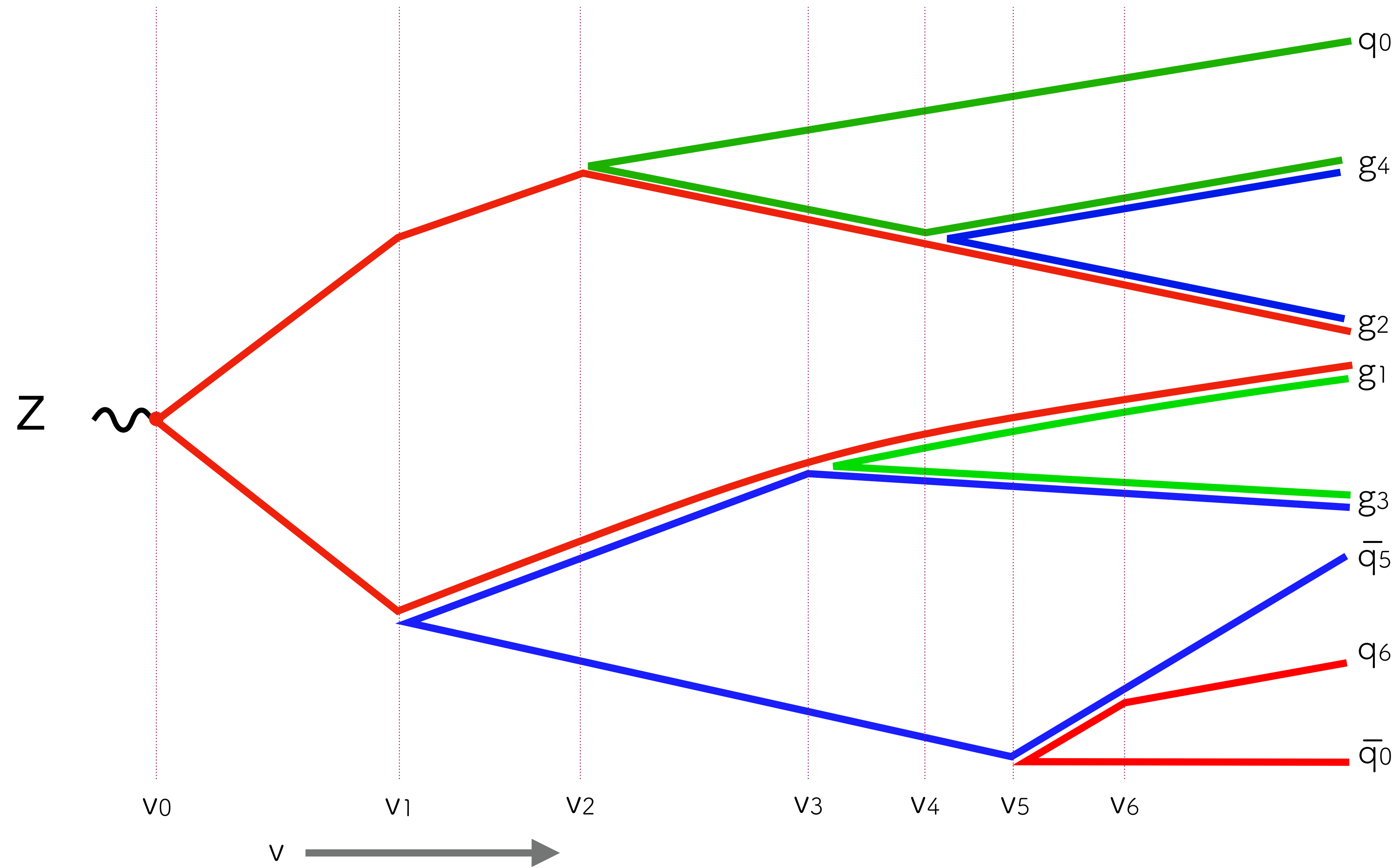
At some point, **state splits** ( $2 \rightarrow 3$ , i.e. emits gluon). Evolution equation changes

$$\frac{dP_3(v)}{dv} = - \left[ f_{2 \rightarrow 3}^{qg}(v) + f_{2 \rightarrow 3}^{g\bar{q}}(v) \right] P_3(v)$$

gluon is part of two dipoles  $(qg)$ ,  $(g\bar{q})$ , each treated as independent

**(many showers use a large  $N_C$  limit)**

# QCD shower: an evolution equation (in **evolution scale $v$** , e.g. $1/\text{trans.mom.}$ )



self-similar  
evolution  
continues until it  
reaches a non-  
perturbative  
scale

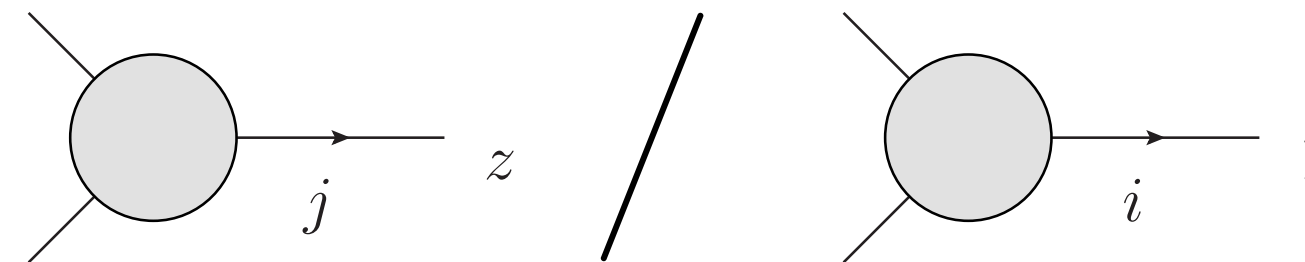
# recent directions of parton-shower work?

1. including  $2 \rightarrow 4$  (or  $1 \rightarrow 3$ ) splittings
2. subleading colour corrections (dipole picture is large  $N_C$ )
3. EW showers

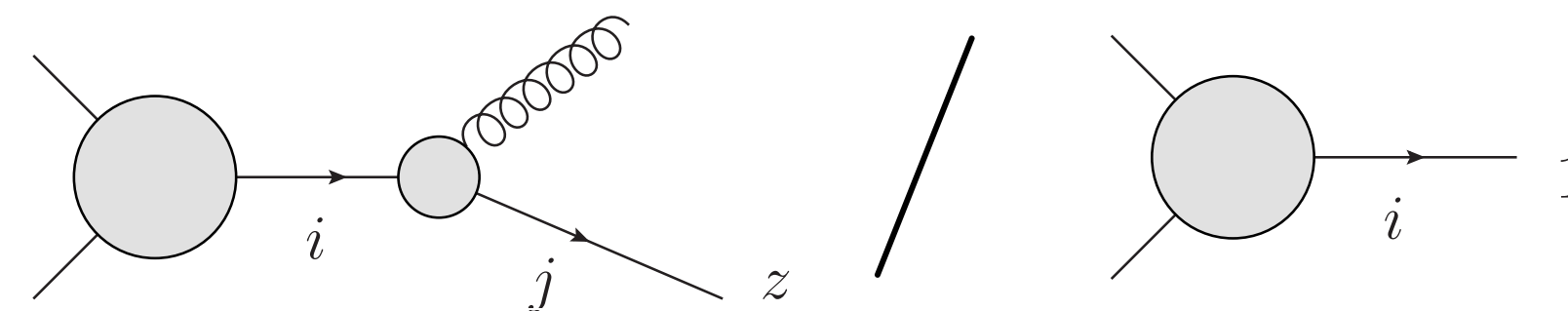
# Including $1 \rightarrow 3$ splittings ( $\equiv 2 \rightarrow 4$ )

- ▶ Jadach et al, e.g. 1504.06849, 1606.01238
- ▶ Li & Skands, 1611.00013
- ▶ Höche, Krauss & Prestel, 1705.00982,  
Höche & Prestel, 1705.00742,  
Dulat, Höche & Prestel, 1805.03757

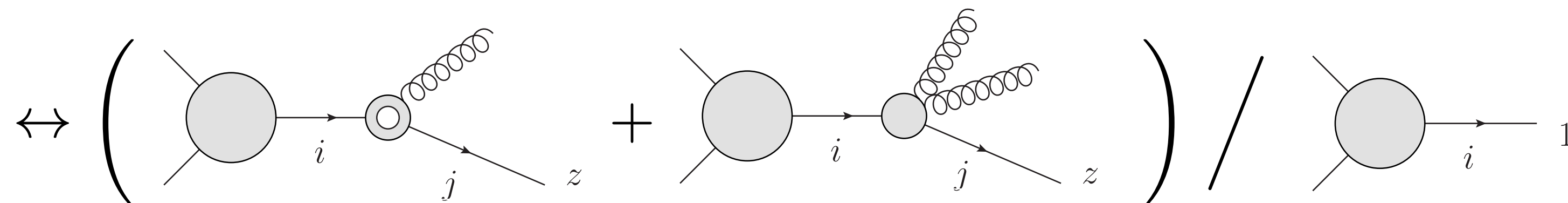
$$D_{ji}^{(0)}(z, \mu) = \delta_{ij} \delta(1-z)$$

 $\leftrightarrow$ 


$$D_{ji}^{(1)}(z, \mu) = -\frac{1}{\varepsilon} P_{ji}^{(0)}(z)$$

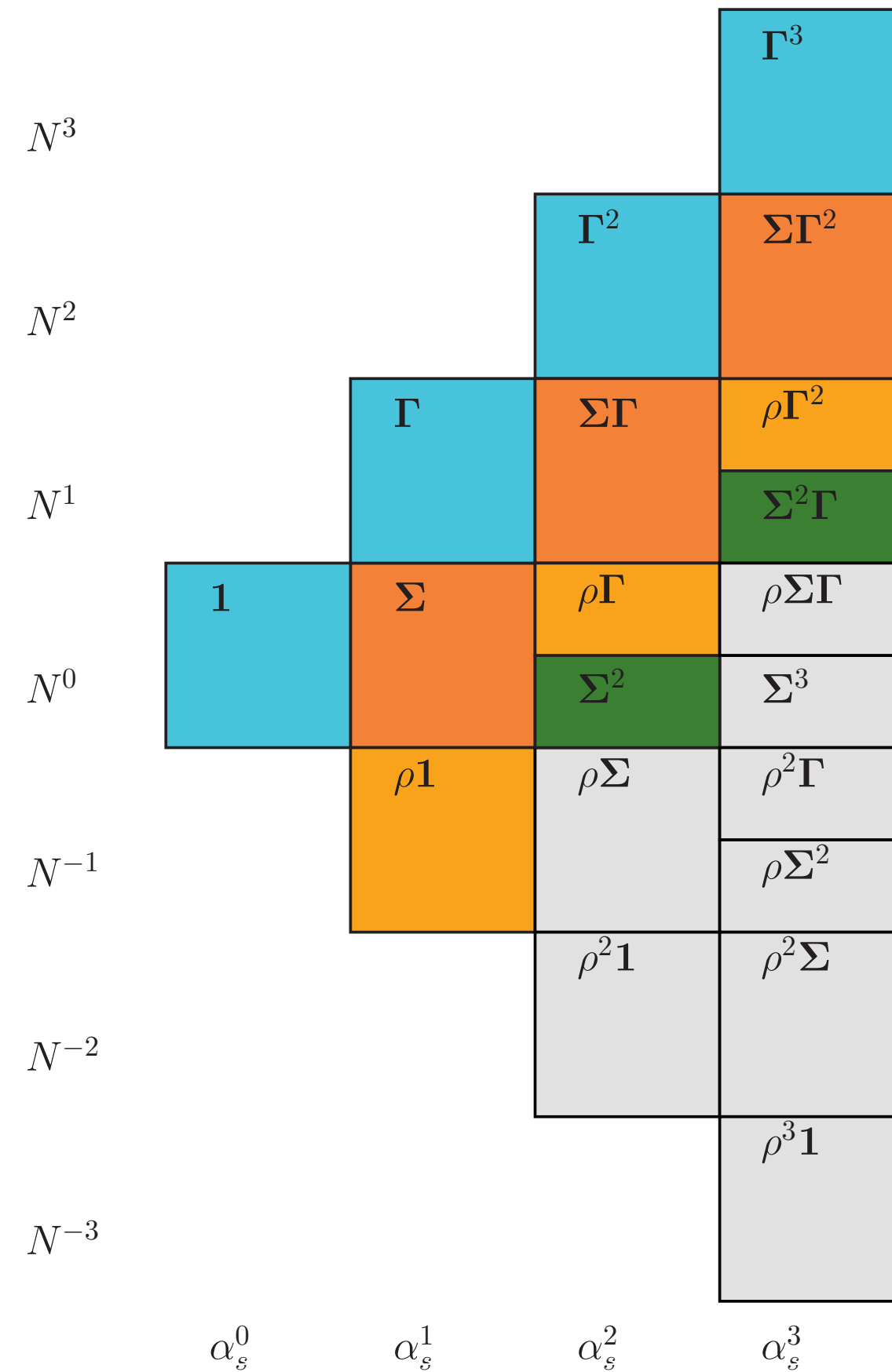
 $\leftrightarrow$ 


$$D_{ji}^{(2)}(z, \mu) = -\frac{1}{2\varepsilon} P_{ji}^{(1)}(z) + \frac{\beta_0}{4\varepsilon^2} P_{ji}^{(0)}(z) + \frac{1}{2\varepsilon^2} \int_z^1 \frac{dx}{x} P_{jk}^{(0)}(x) P_{ki}^{(0)}(z/x)$$

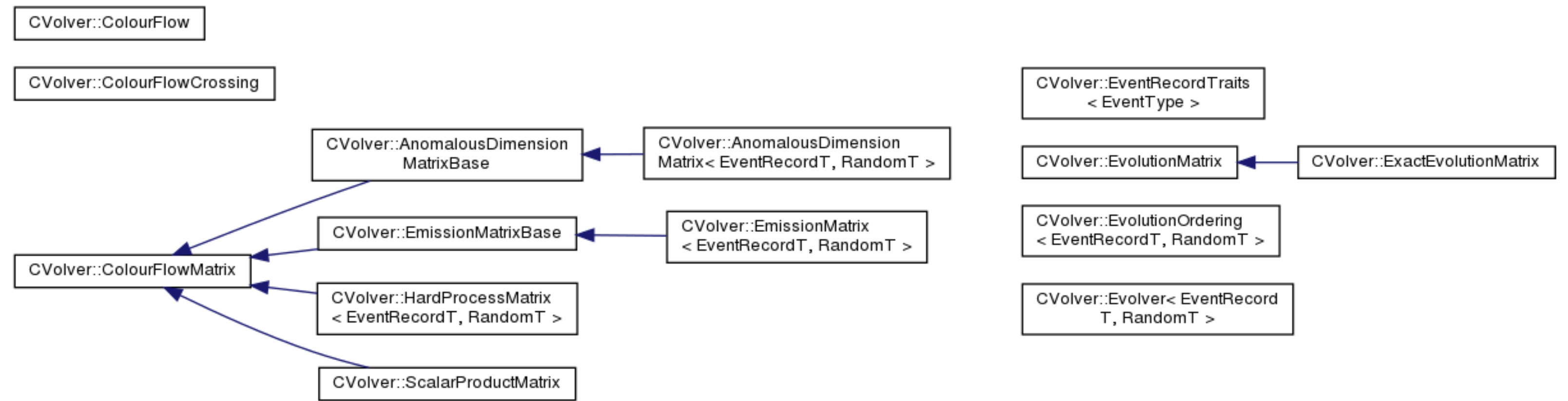


# Hierarchy of subleading colour corrections

Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044  
 Gieseke, Kirchgaesser, Plätzer, Siodmok – arXiv:1808.06770  
 Plätzer, Sjö Dahl, Thorén, arXiv:1808.00332  
 Forshaw, Holguin, Plätzer, arXiv:1905.08686, [2003.06399](#)



$$\mathbf{A}_n(E) = \mathbf{V}(E, E_n) \mathbf{D}_n \mathbf{A}_{n-1}(E_n) \mathbf{D}_n^\dagger \mathbf{V}^\dagger(E, E_n) \theta(E - E_n)$$

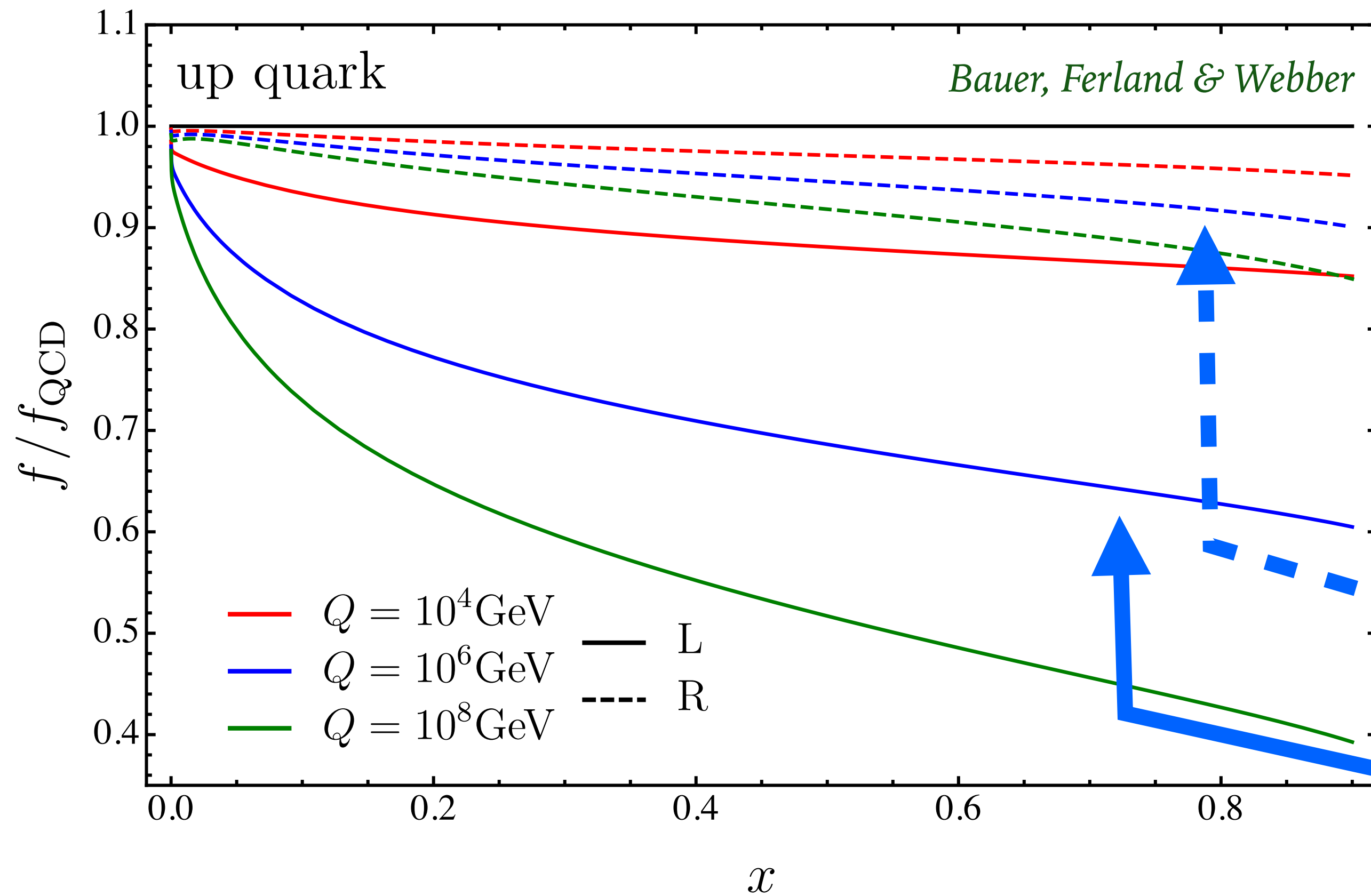


Plugin approach can accommodate anything from (N)GLs to full parton showers.

cf. also work by Hatta & Ueda, [1304.6930](#); Nagy & Soper papers; Hoche & Reichelt, [2001.11492](#)

# EW showers (esp. beyond LHC)

*Kleiss & Verheyen, [2002.09248](#)*  
*Bauer, Ferland & Webber, [1703.08562](#), [1808.08831](#)*  
*Bauer, De Jong, Nachman, Provasoli, [1904.03196](#)*  
*see also Chen, Han & Tweedie, [1611.00788](#)*  
*Sjostrand & Christiansen, [1401.5238](#)*  
 ...



W & Z emissions come with double logarithms

$$\alpha_{EW} \ln^2 \frac{Q}{m_W}$$

right-handed up quarks

left-handed up quarks

W emission affects only left-handed quarks

→ strong polarisation of quarks in unpolarised proton (at high enough energies)



# what does a parton shower achieve?

*not just a question of ingredients,  
but also the final result of assembling them together*

# what **should** a parton shower achieve?

*not just a question of ingredients,  
but also the final result of assembling them together*

## it's a complicated issue...

---

- For a total cross section, e.g. for Higgs production, it's easy to talk about systematic improvements (LO, NLO, NNLO, ...). But they're restricted to that one observable

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- With a parton shower (+hadronisation) you produce a “realistic” full set of particles. You can ask questions of arbitrary complexity:
  - **the multiplicity of particles**
  - **the total transverse momentum with respect to some axis (broadening)**
  - **the angle of 3rd most energetic particle relative to the most energetic one**  
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*[machine learning might “learn” many such features]*

**how can you prescribe correctness & accuracy of the answer,  
when the questions you ask can be arbitrary?**

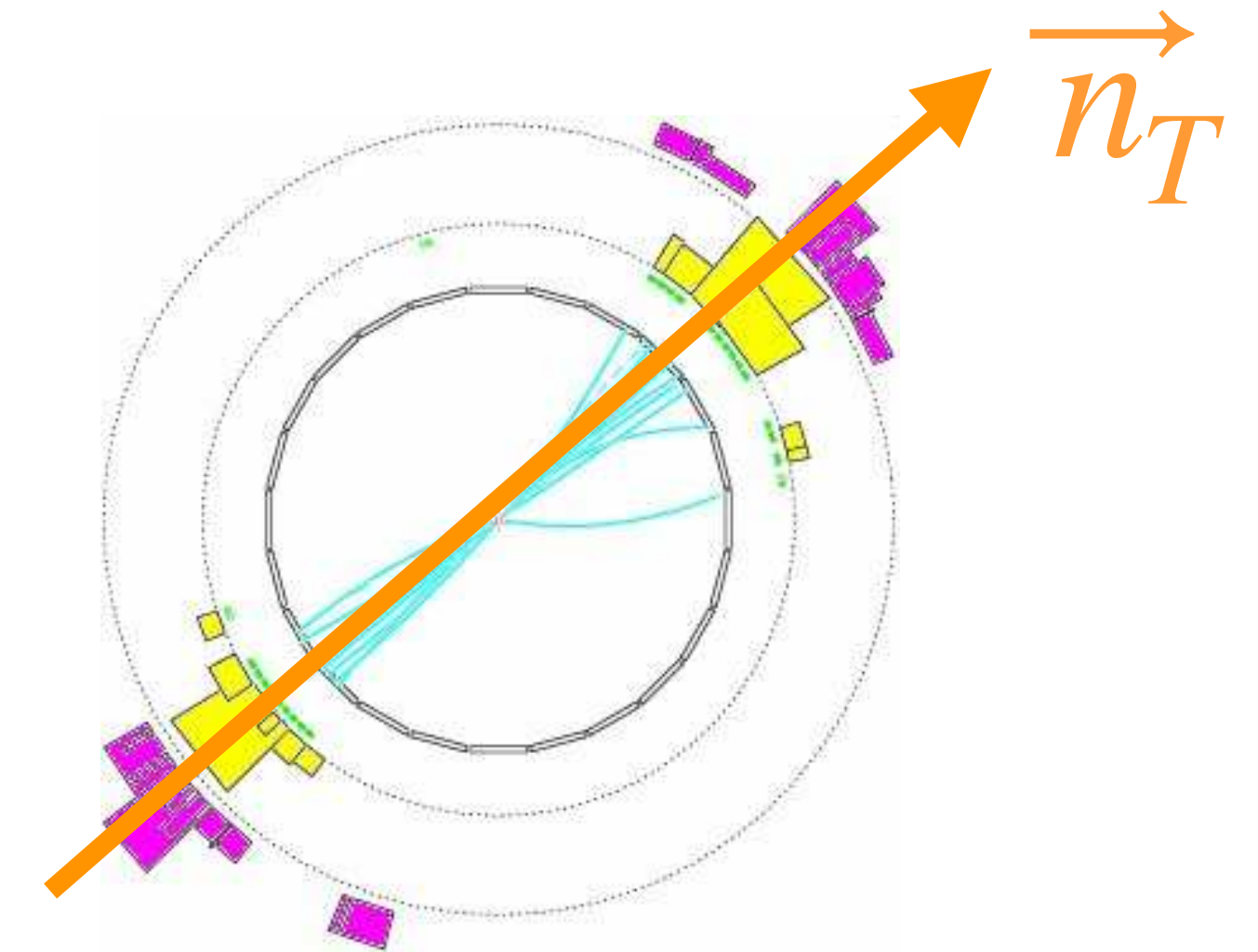
# The standard answer so far

---

It's common to hear that **showers are Leading Logarithmic (LL)** accurate.

That language, widespread for multiscale problems, comes from analytical resummations. E.g. for (famous) “Thrust”

$$T = \max_{\vec{n}_T} \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$



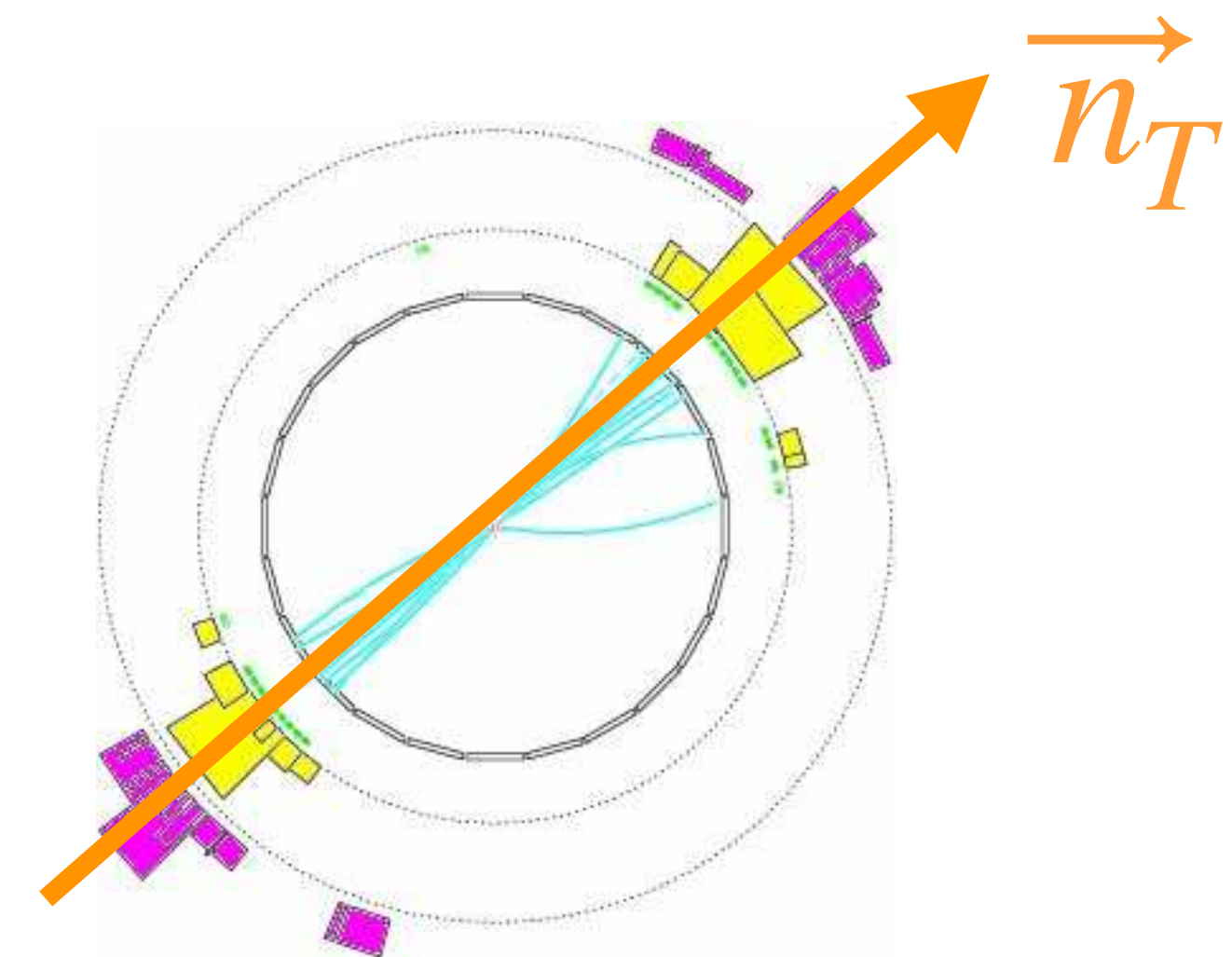
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$$\sigma(1 - T < e^{-L}) = \sigma_{tot} \exp \left[ \underbrace{Lg_1(\alpha_s L)}_{LL} + \underbrace{g_2(\alpha_s L)}_{NLL} + \underbrace{\alpha_s g_3(\alpha_s L)}_{NNLL} + \underbrace{\alpha_s^2 g_4(\alpha_s L)}_{N^3LL} + \dots \right]$$

$[\alpha_s \ll 1, L \gg 1]$

**LL**

**NLL**

**NNLL**

**N<sup>3</sup>LL**

Catani, Trentadue, Turnock & Webber '93

Becher & Schwartz '08

# The standard answer so far

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It's common to hear that **showers are Leading Logarithmic (LL)** accurate.

Showers extend to regime where  $\alpha_s L \sim 1$  (equivalently,  $L \sim 1/\alpha_s$ )

$$\sigma(1 - T < e^{-L}) = \sigma_{tot} \exp \left[ \underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \underbrace{\alpha_s^2 g_4(\alpha_s L)}_{\text{N}^3\text{LL}} + \dots \right]$$

$O(1/\alpha_s)$	$O(1)$	$O(\alpha_s)$	$O(\alpha_s^2)$
-----------------	--------	---------------	-----------------

At the very least, one wants control of  $O(1)$  terms, i.e. **NLL**.



# The standard answer so far

---

Sometimes you may see statements like “*Following standard practice to improve the logarithmic accuracy of the parton shower, the soft enhanced term of the splitting functions is rescaled by  $1 + a_s(t)/(2\pi)K$* ” [ $K \sim A_2$  in cusp anomalous dimension]

## Questions:

- 1) Which is it? LL or better?
- 2) For what known observables does this statement hold?
- 3) What good is it to know that some handful of observables is LL (or whatever) when you want to calculate arbitrary observables?
- 4) Does LL even mean anything when you do machine learning?
- 5) Why only “LL” when analytic resummation can do so much better?
- 6) Do better ingredients (e.g. higher-order splitting functions) make better showers?

# Back to radioactive decay example: **two ways of writing result**

Constant decay rate  $\mu$  per unit time, total time  $t$ . Find distribution of emissions.

1. write **as coupled evolution equations** for probability  $P_0, P_1, P_2$ , etc., of having  $0, 1, 2, \dots$  emissions

$$\frac{dP_n}{dt} = -\mu P_n(t) + \mu P_{n-1}(t)$$

$n \rightarrow n+1$        $n-1 \rightarrow n$

[easy to implement in Monte Carlo approach]

2. or **as explicit formula**

$$dP_n = \frac{1}{n!} e^{-\mu t_{\max}} (\mu dt_1) \dots (\mu dt_n)$$

*symmetry factor*      *virtual terms  
(summed to all orders)*       $|matrix-element|^2$   
 $\times$  *phasespace*

[here Poisson distribution;  
in QCD: effective matrix element]

# Our proposal for baseline shower accuracy (“NLL”)

---

## Resummation

Require single logarithmic accuracy (control of terms  $\alpha_s^n L^n$  for all observables where this makes sense)

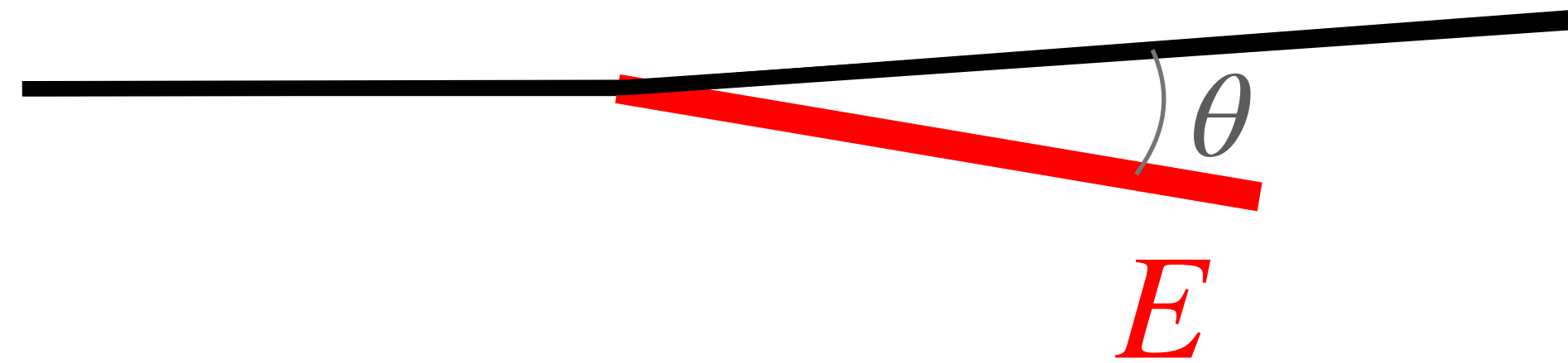
- global event shapes (thrust, broadening, angularities, jet rates, energy-energy correlations, ...)
- non-global observables
- fragmentation / parton-distribution functions
- [multiplicity, get NLL  $\alpha_s^n L^{2n-1}$ , cf. original Herwig angular-ordered shower from 1980's]

## Matrix elements

Effective tree-level matrix elements generated by the shower should be correct for any multiplicity  $N$  if all emissions are well separated in a *Lund diagram*.

# Phase space: two key variables (+ azimuth)

---



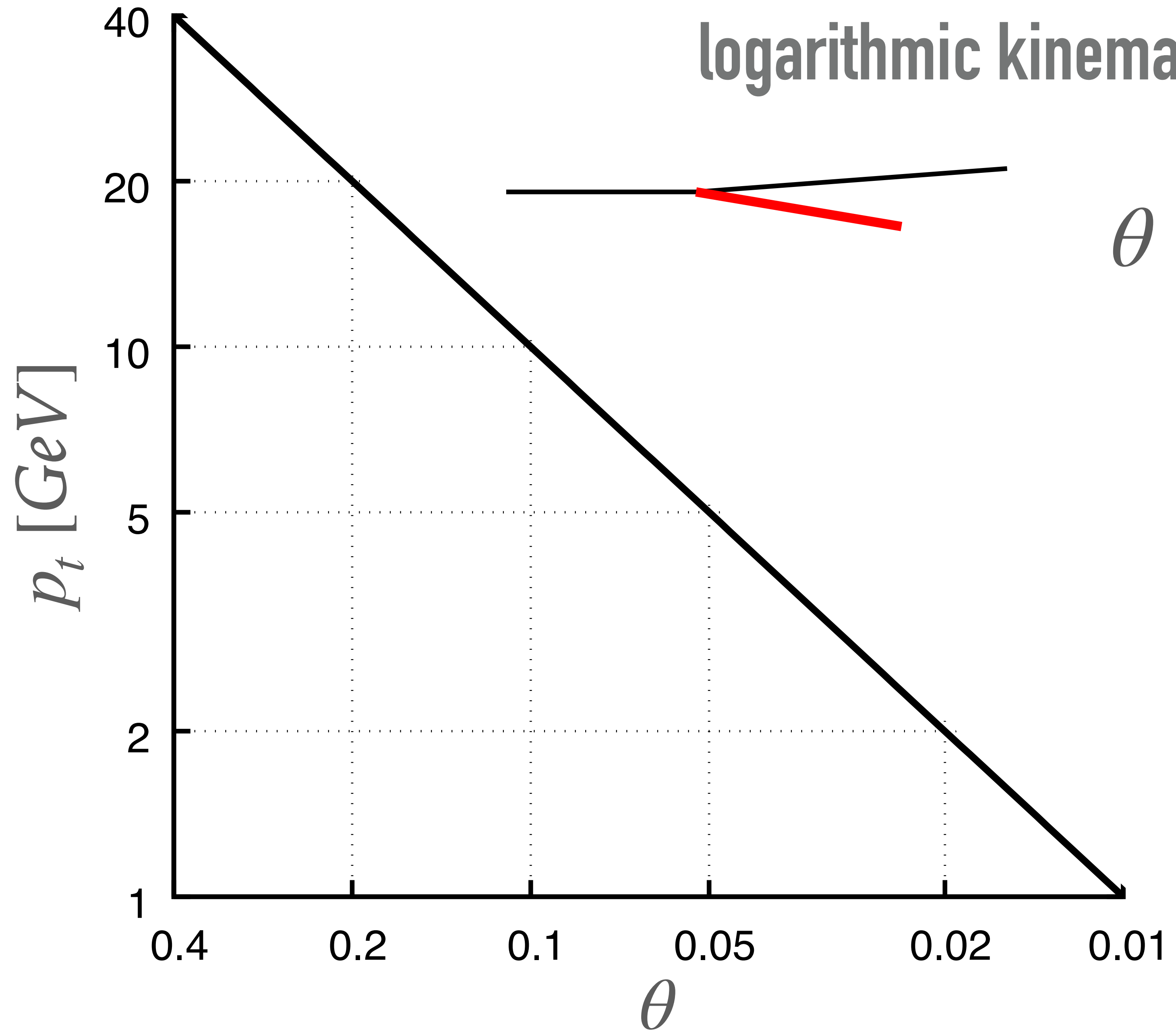
$$\theta \text{ (or } \eta = -\ln \tan \frac{\theta}{2} \text{)}$$

*$\eta$  is called (pseudo)rapidity*

$$p_t = E\theta$$

*$p_t$  (or  $p_{\perp}$ ) is a transverse momentum*

jet with  $R = 0.4$ ,  $p_t = 200 \text{ GeV}$



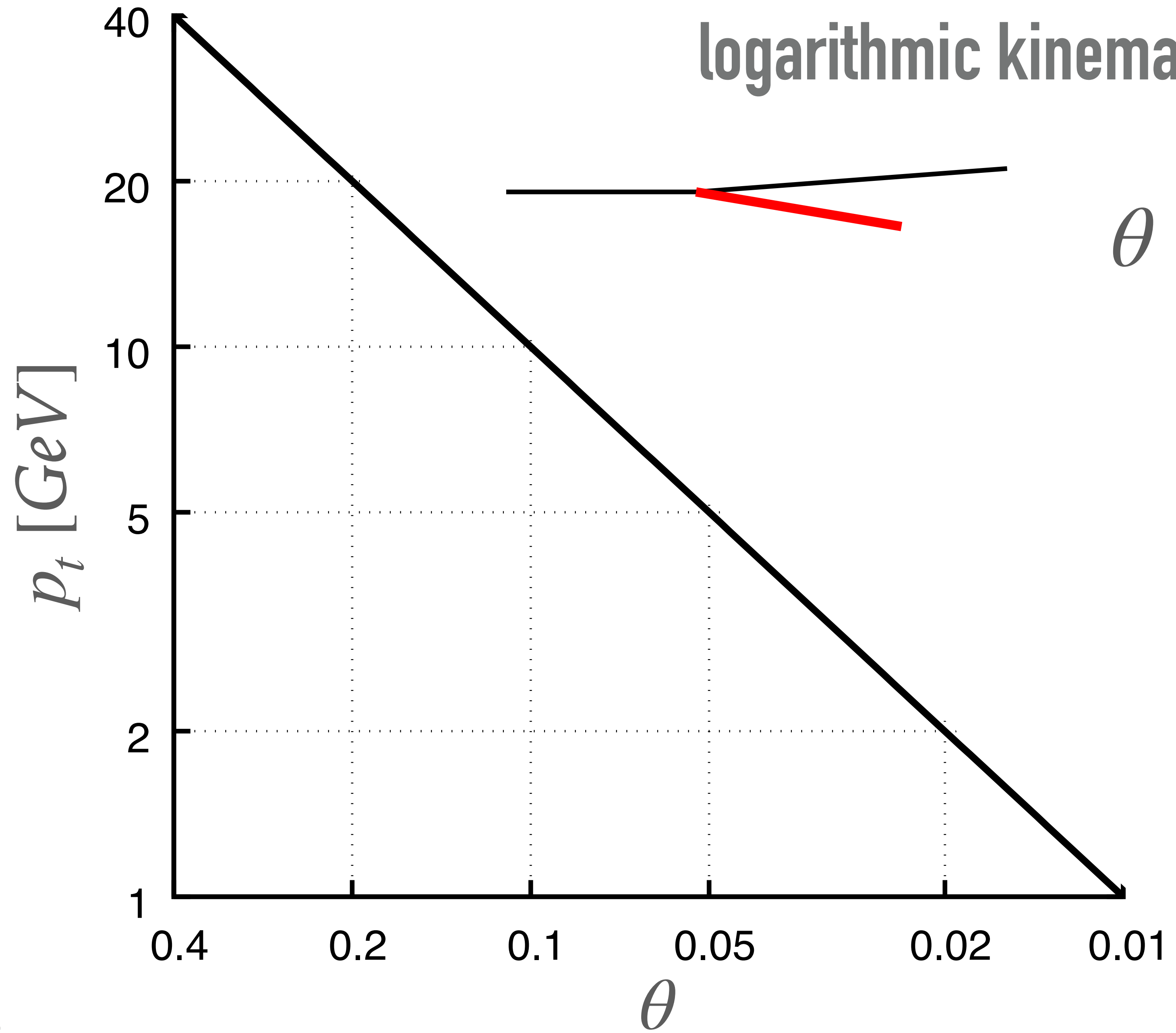
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Introduced for understanding Parton Shower Monte Carlos by B. Andersson, G. Gustafson L. Lonnblad and Pettersson 1989

# The Lund Plane

jet with  $R = 0.4$ ,  $p_t = 200$  GeV



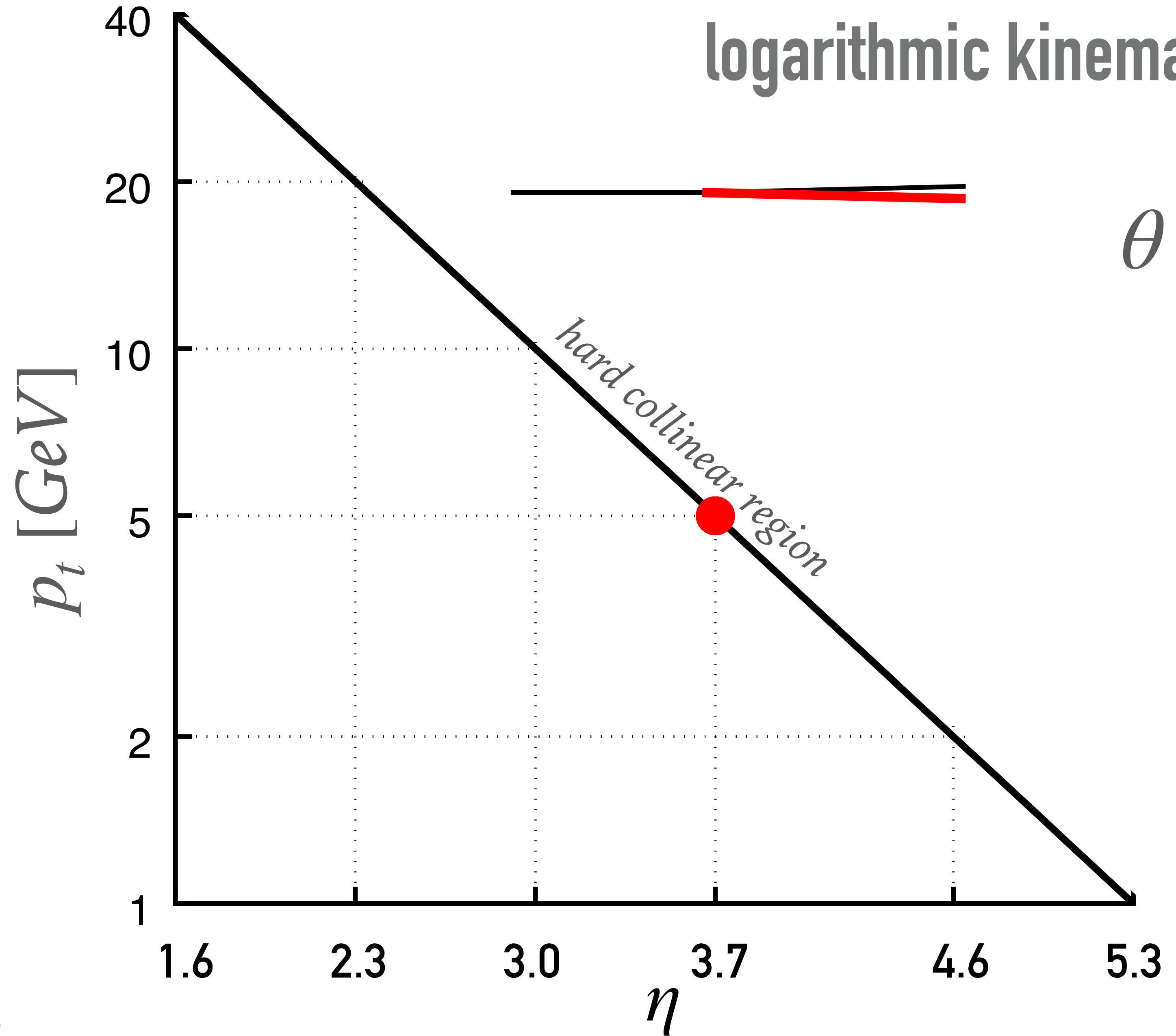
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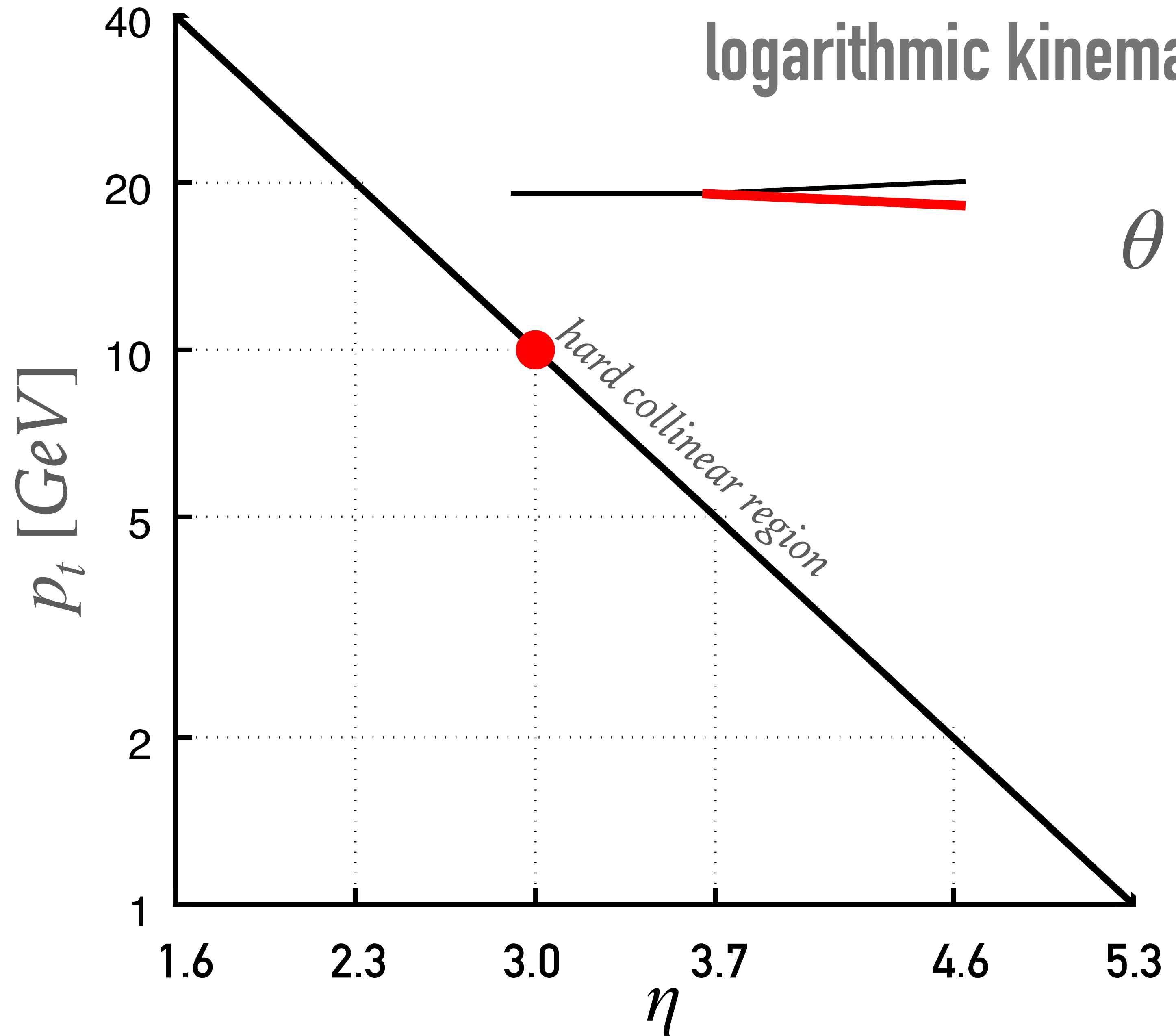
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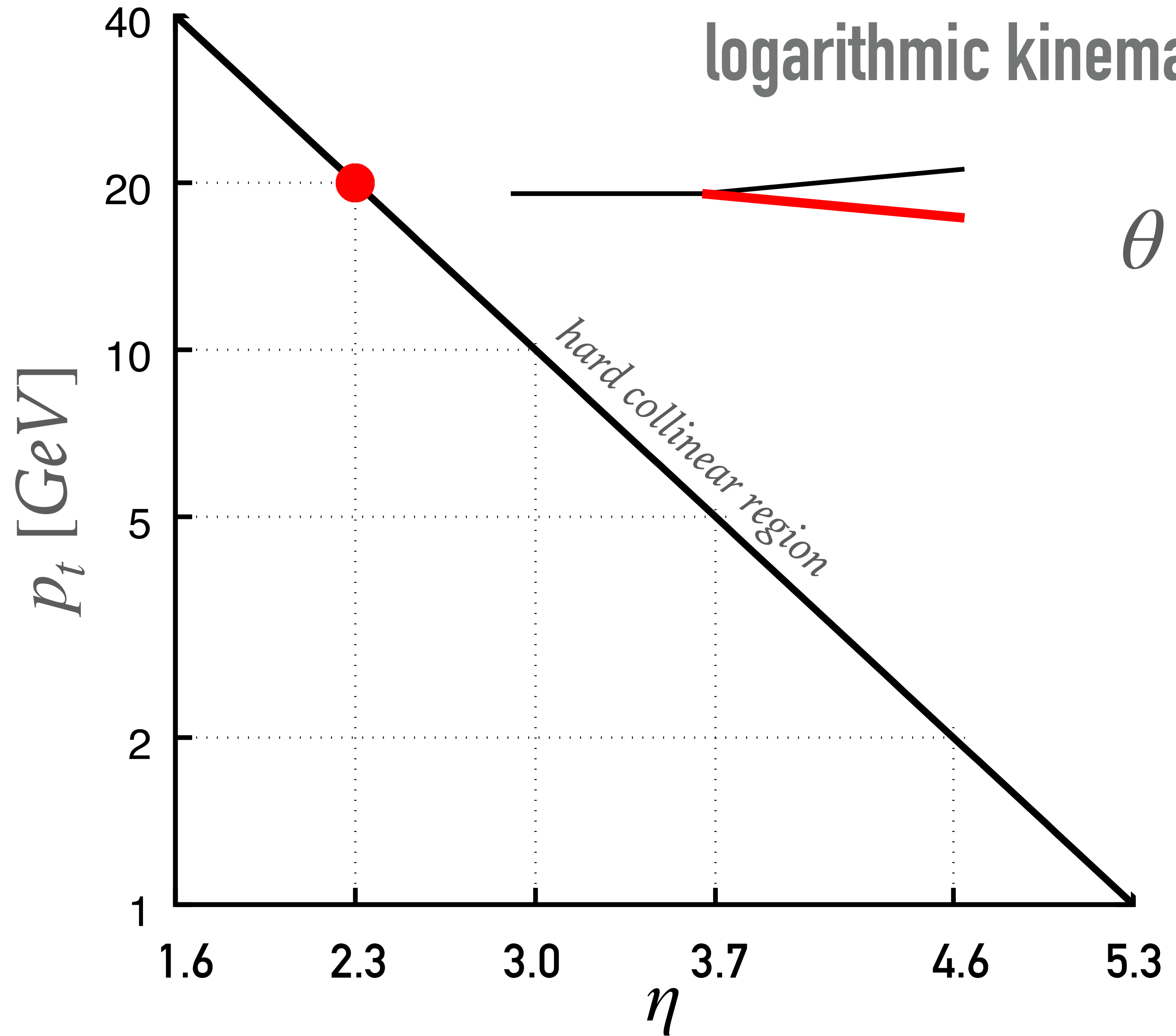
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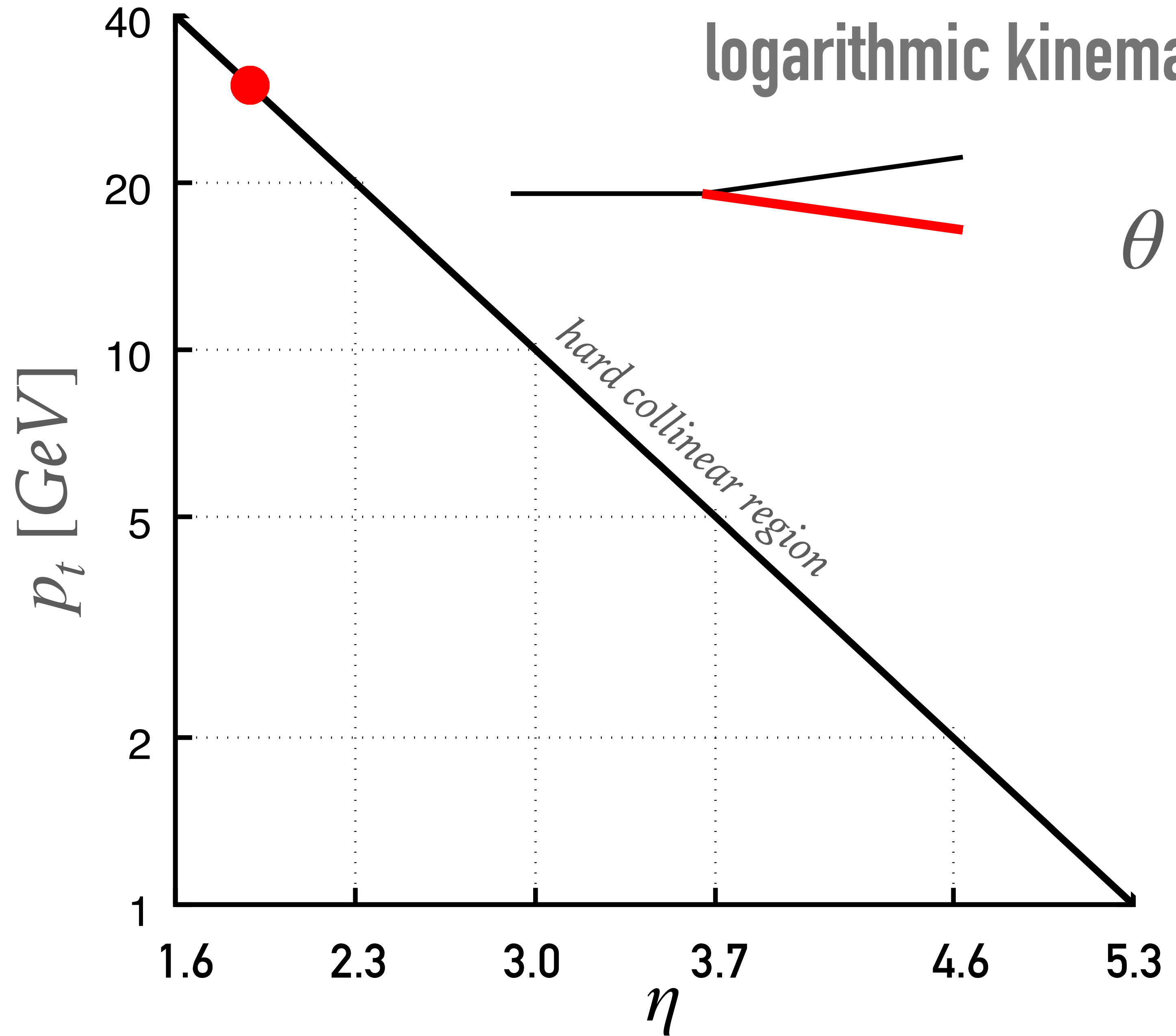
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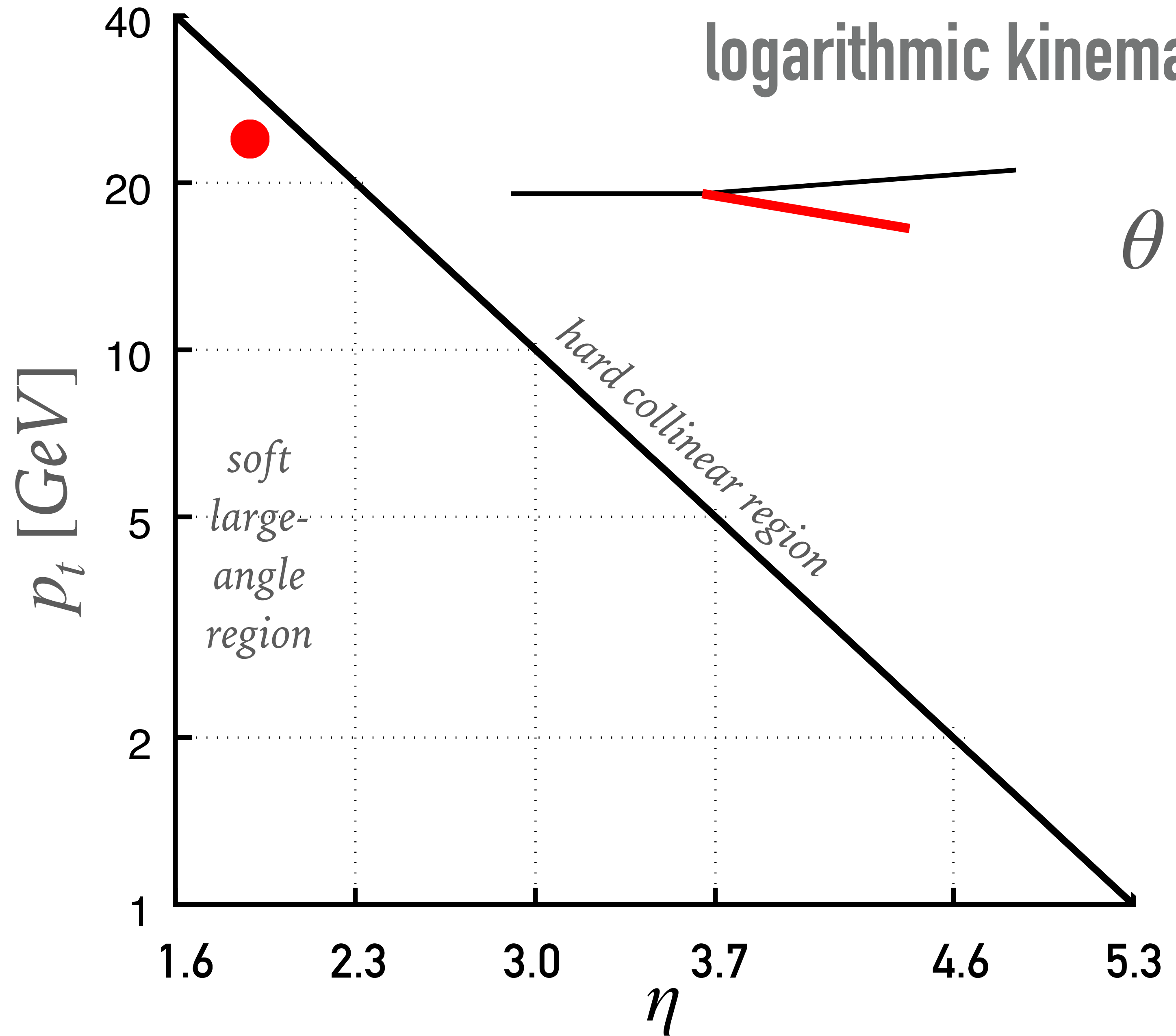
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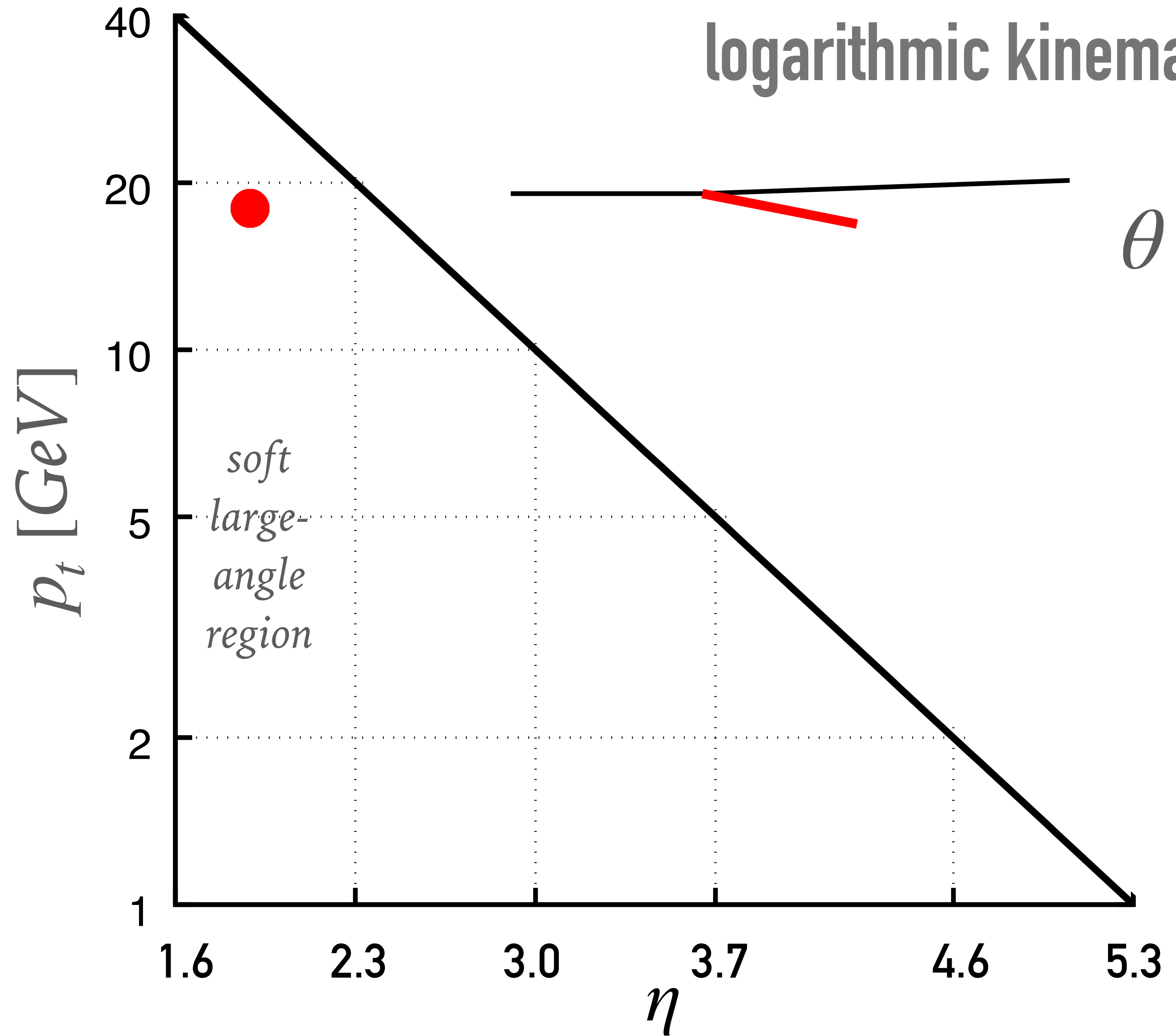
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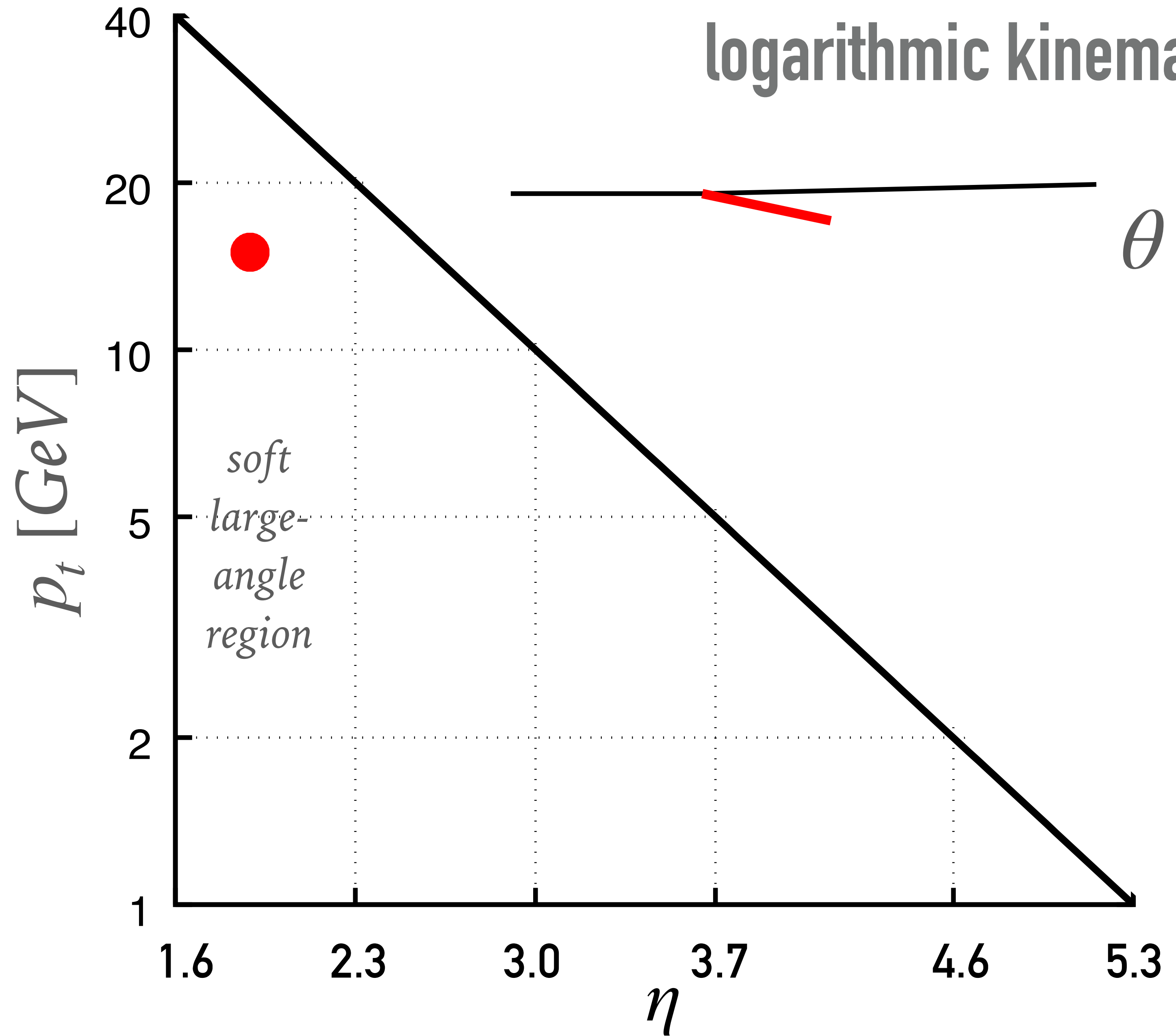
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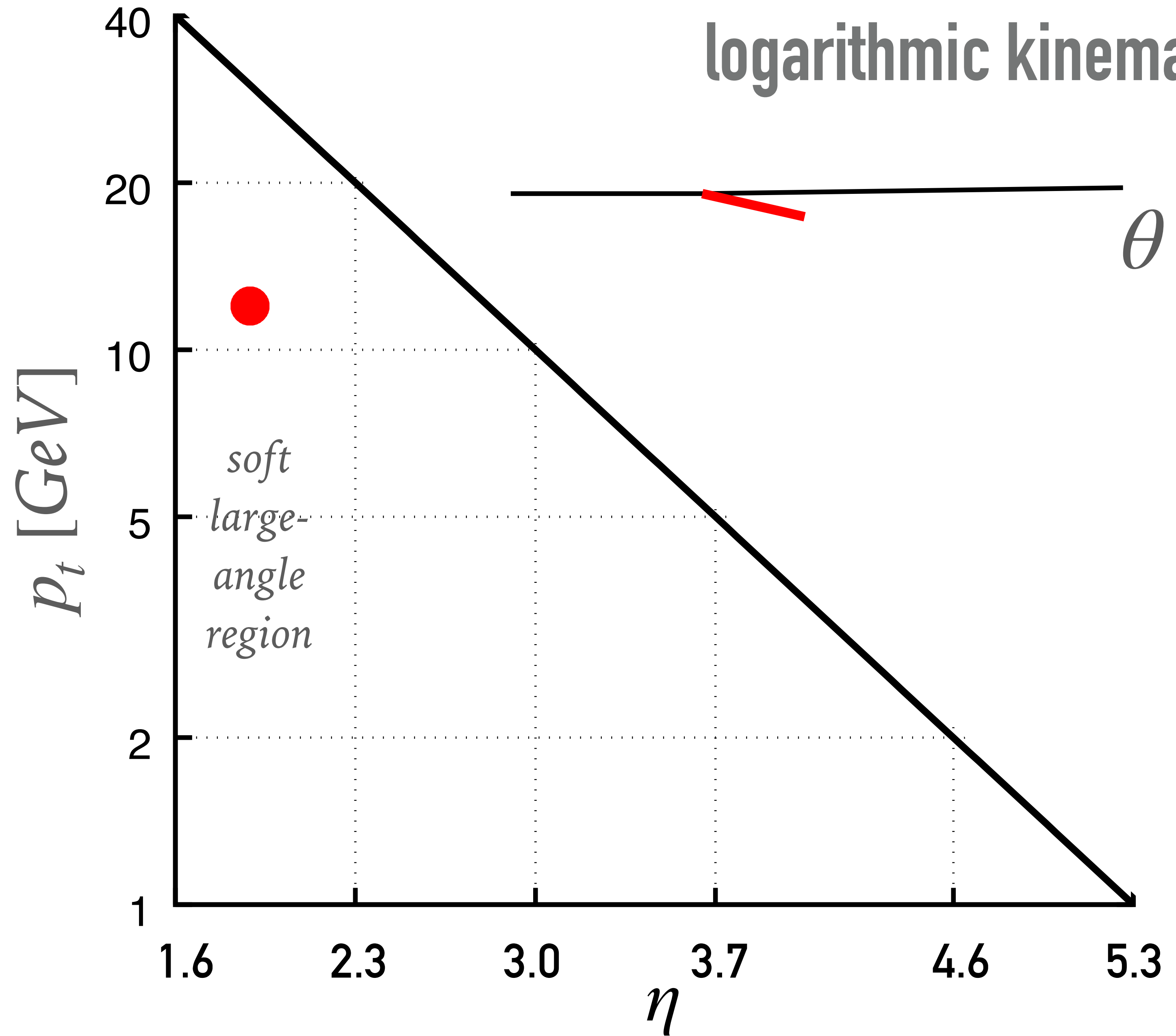
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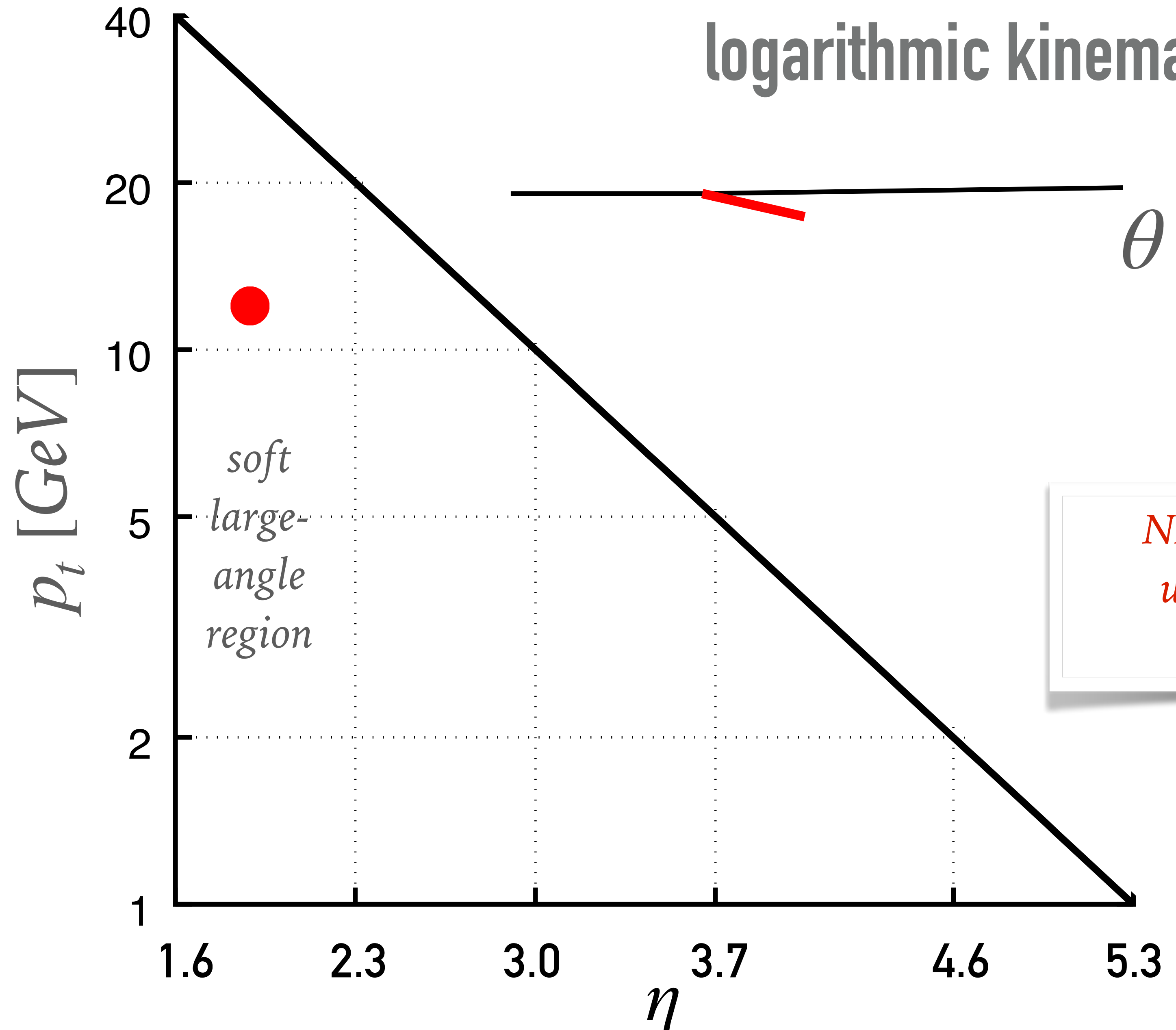
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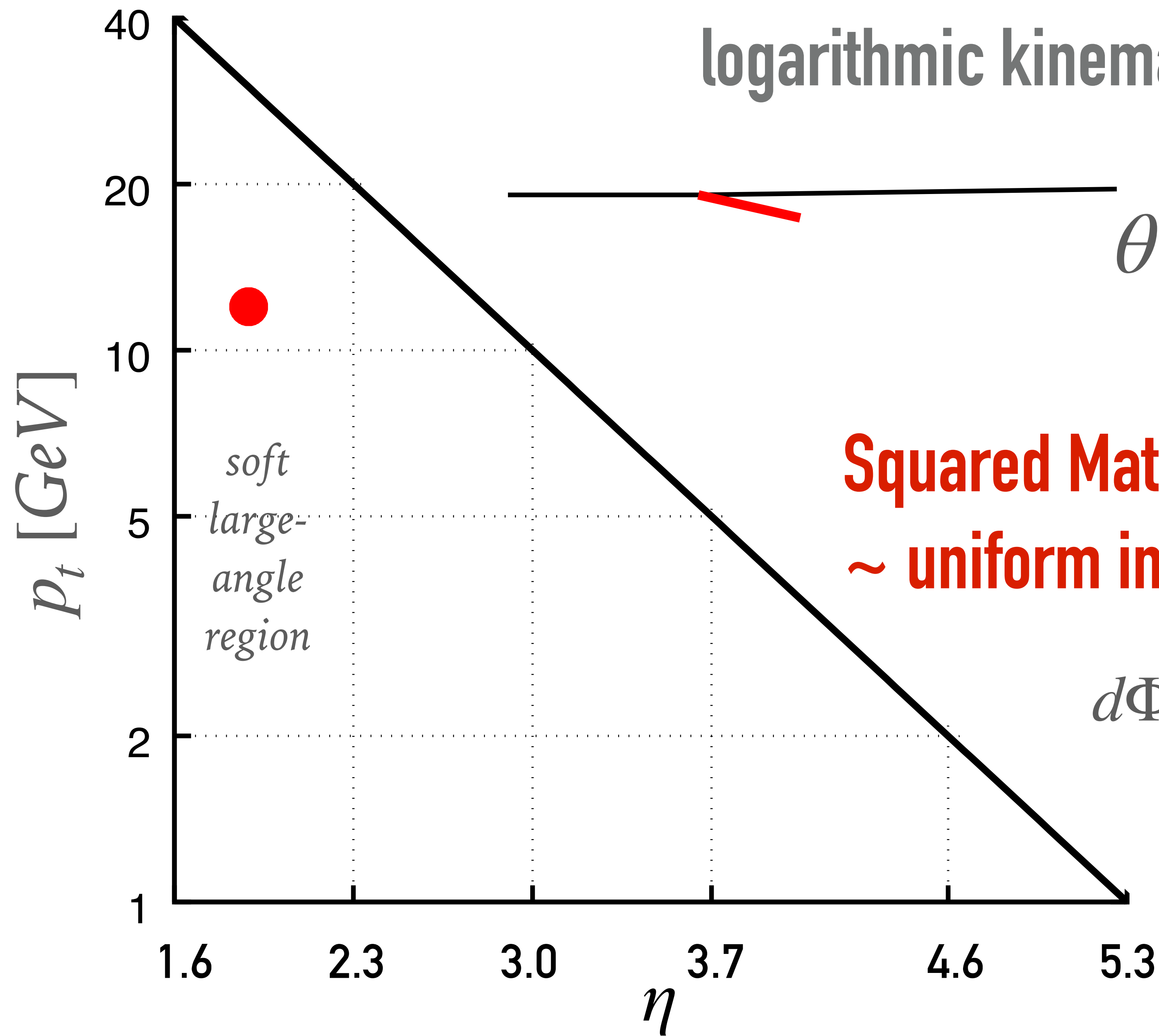
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*NB: Lund plane can be constructed event-by-event using Cambridge/Aachen jet clustering sequence, cf. Dreyer, GPS & Soyez '18*

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logarithmic kinematic plane whose two variables are

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**Squared Matrix Element  $\times$  phasespace**  
 **$\sim$  uniform in  $\ln p_t$  and  $\eta$**

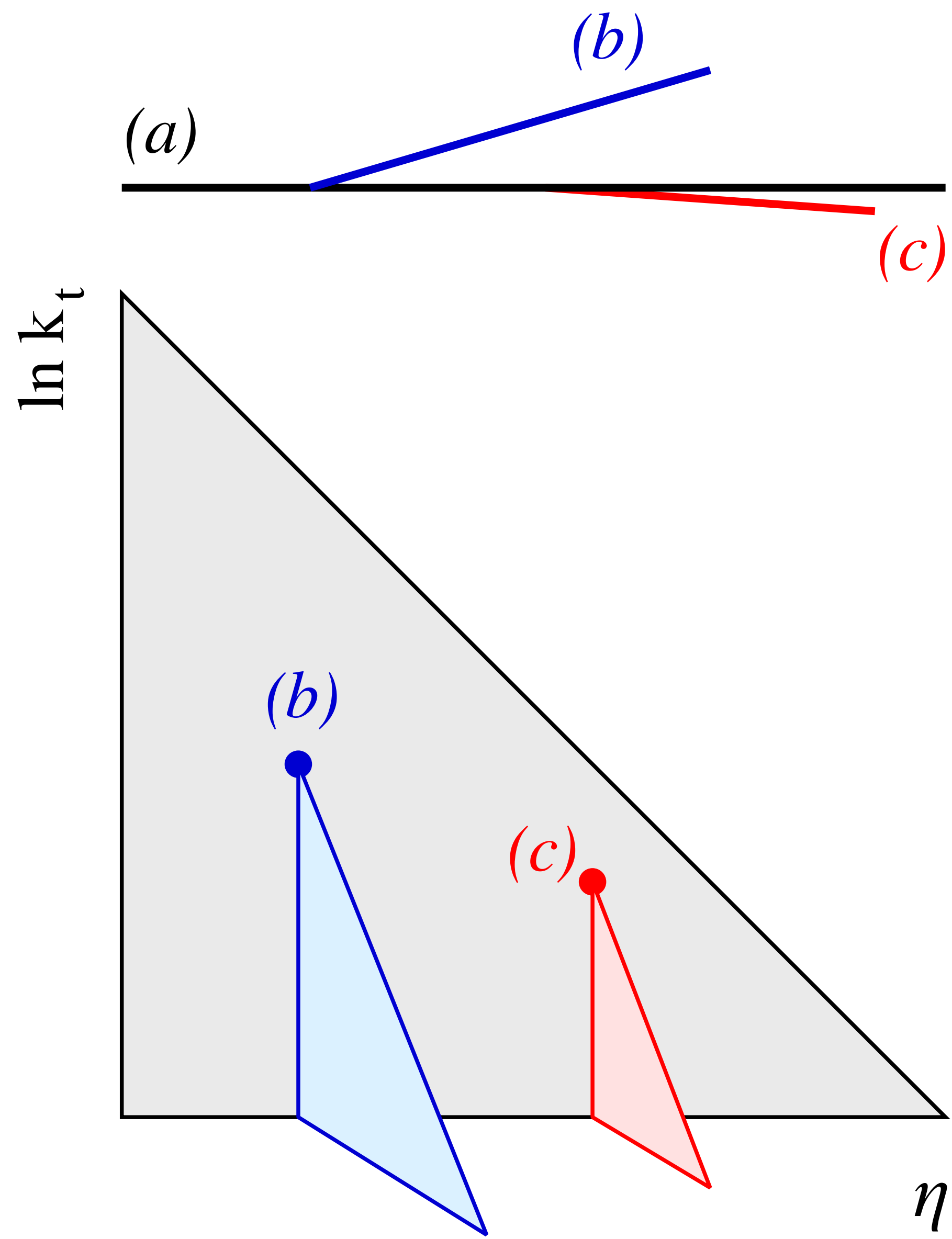
$$d\Phi |M^2| = \frac{2\alpha_s(p_t)C}{\pi} \frac{dp_t}{p_t} \frac{d\theta}{\theta} \frac{d\phi}{2\pi}$$

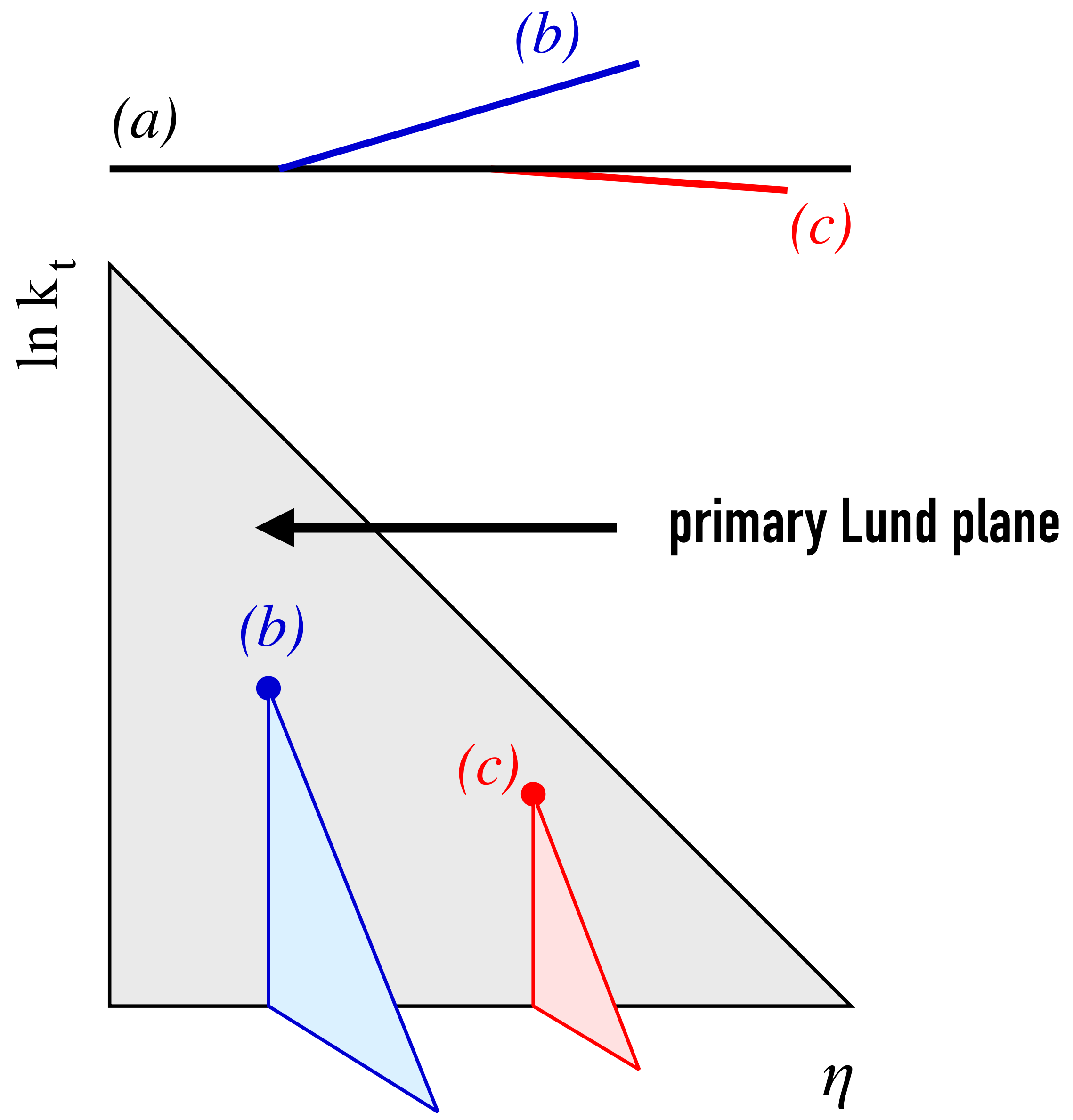
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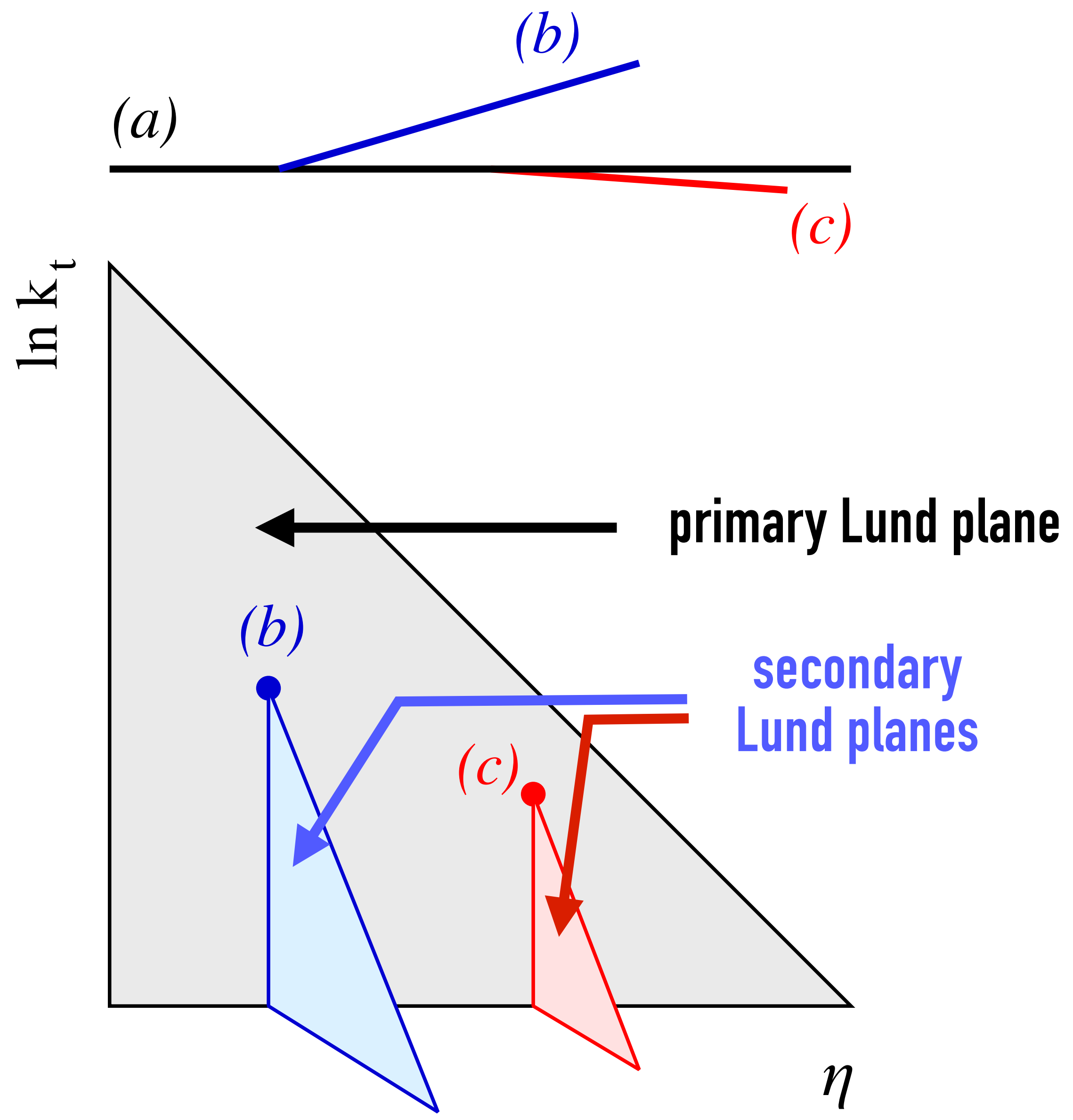
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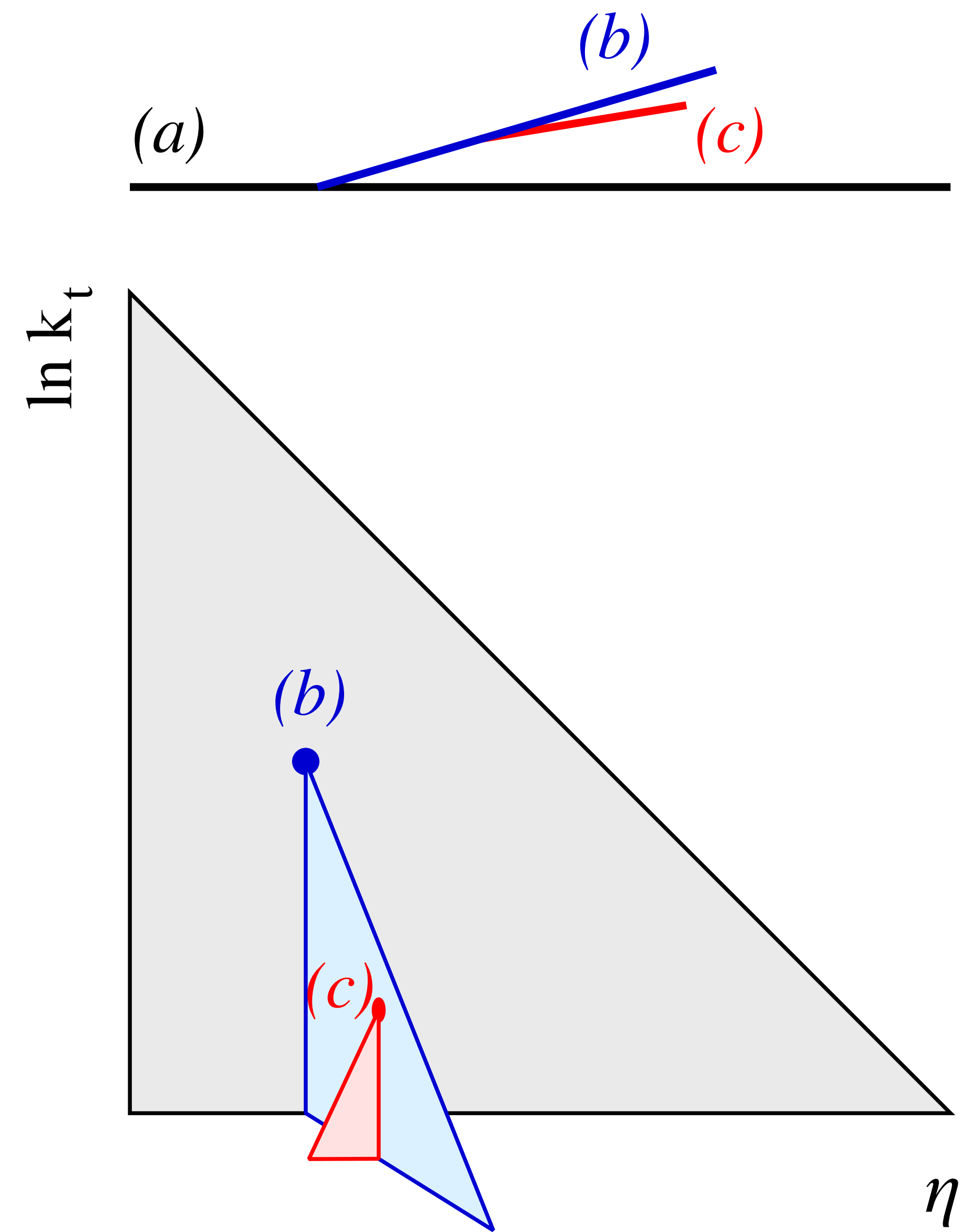
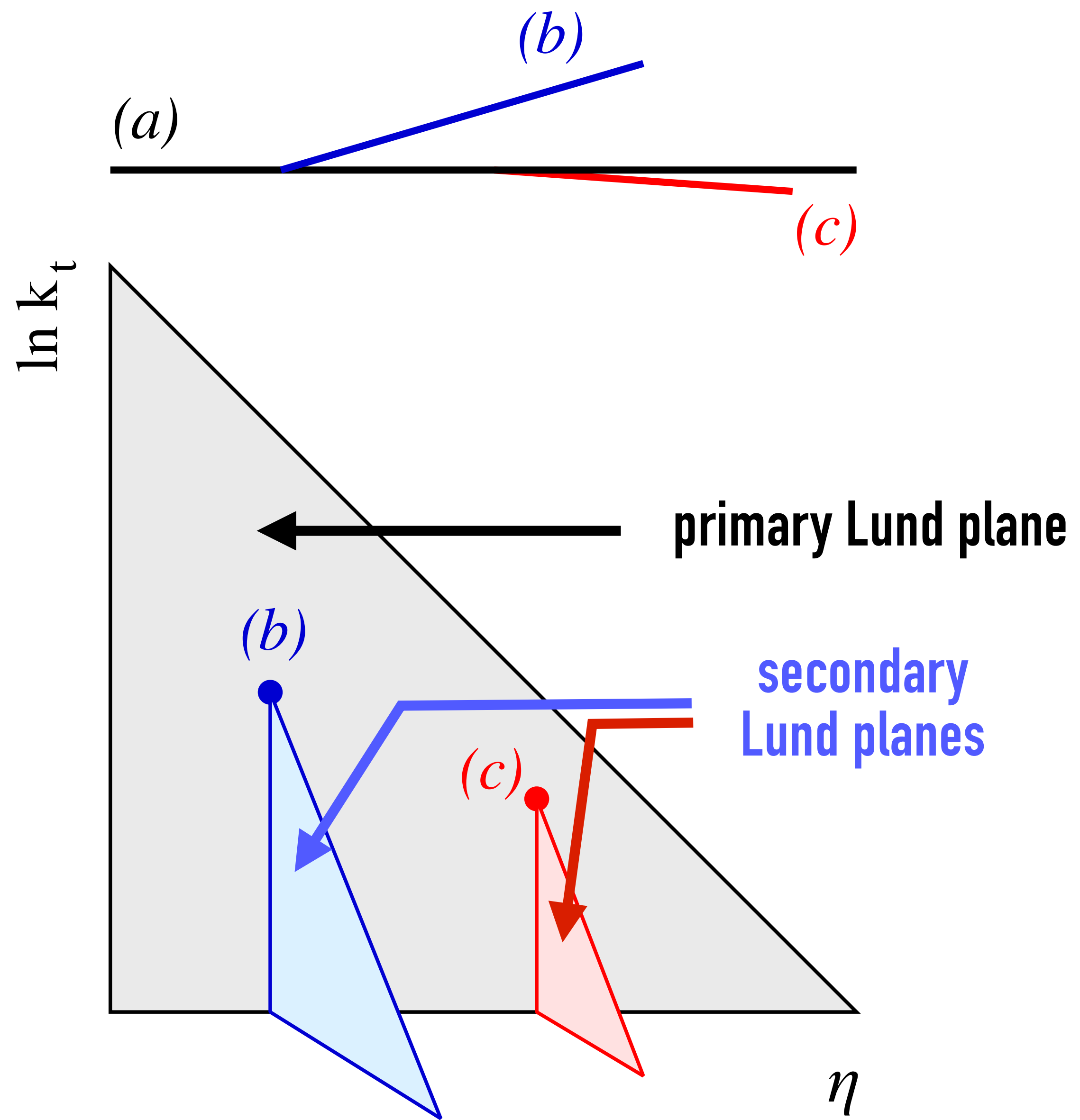


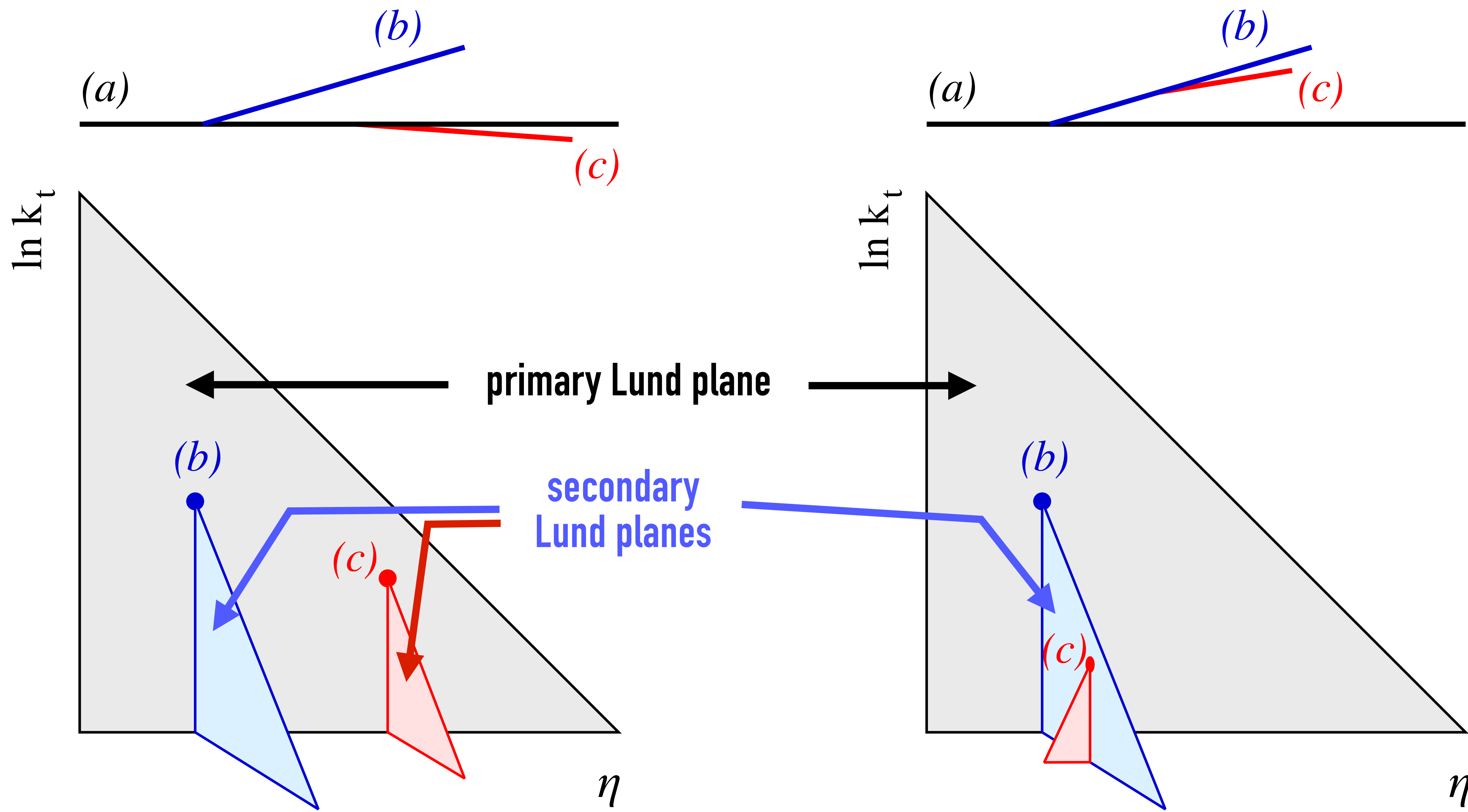
# LUND DIAGRAM

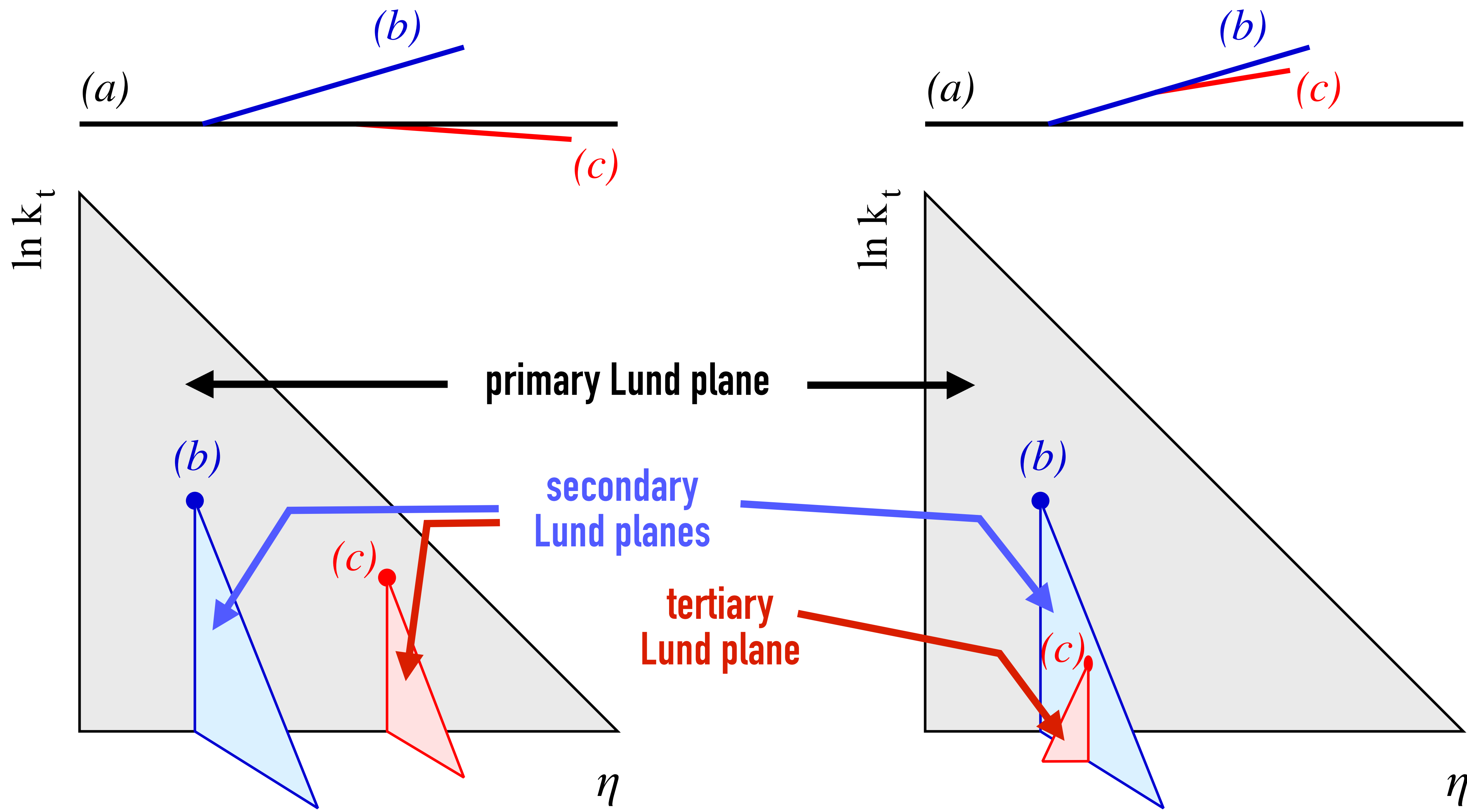








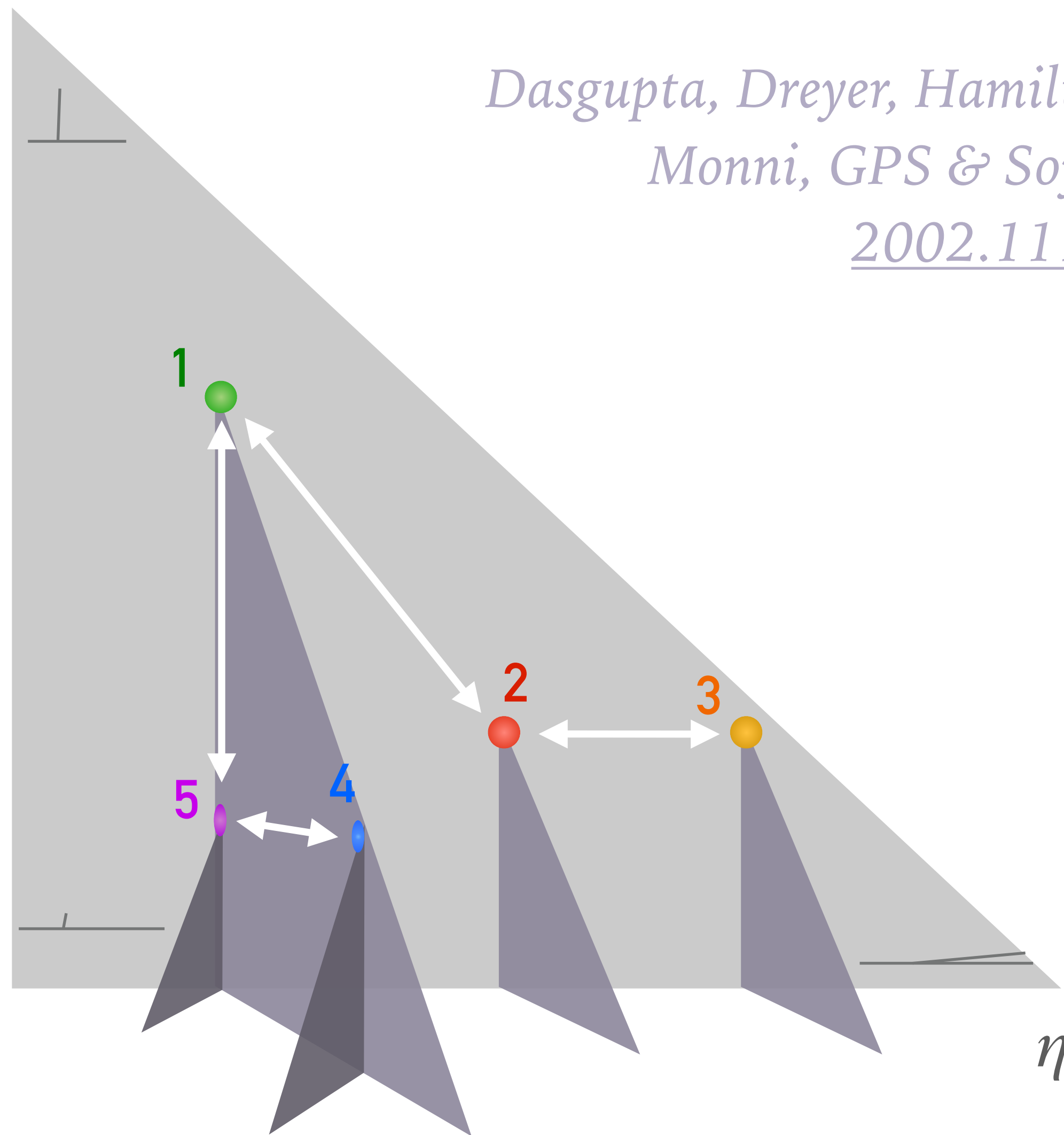




# When do we require effective shower $|M^2|$ to be correct?

$\ln p_t$

*Dasgupta, Dreyer, Hamilton,  
Monni, GPS & Soyez,  
[2002.11114](#)*

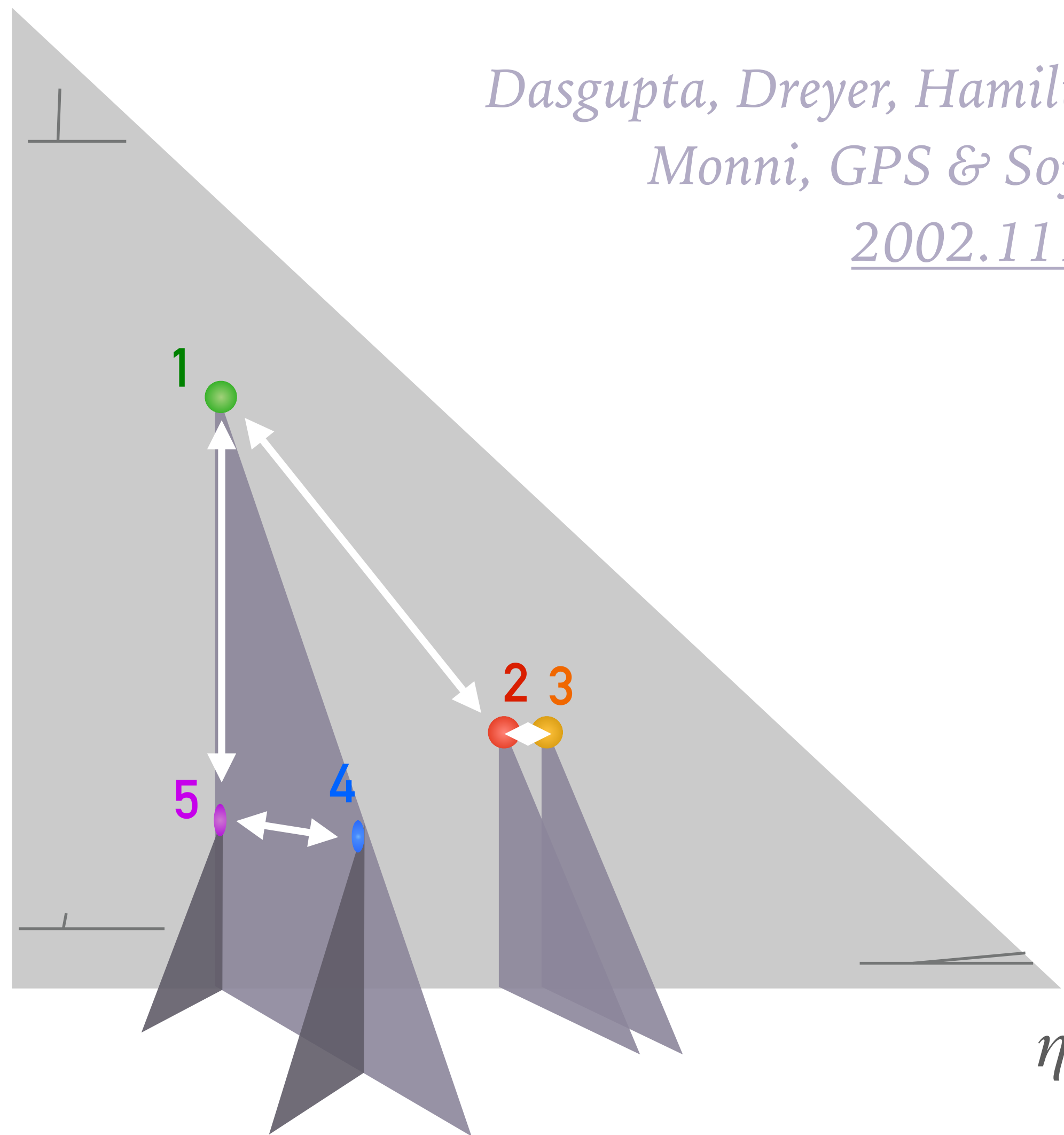


- a shower with simple  $1 \rightarrow 2$  or  $2 \rightarrow 3$  splittings can't reproduce full matrix element
- but QCD has amazing factorisation properties — simplifications in presence of energy or angular ordering
- **we should be able to reproduce  $|M^2|$  when all emissions well separated in Lund diagram**  
 $d_{12} \gg 1, d_{23} \gg 1, d_{15} \gg 1, \text{ etc.}$

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- a shower with simple  $1 \rightarrow 2$  or  $2 \rightarrow 3$  splittings can't reproduce full matrix element
- but QCD has amazing factorisation properties — simplifications in presence of energy or angular ordering
- **we are allowed to make a mistake (by  $\mathcal{O}(1)$  factor) when a pair is close by, e.g.  $d_{23} \sim 1$**



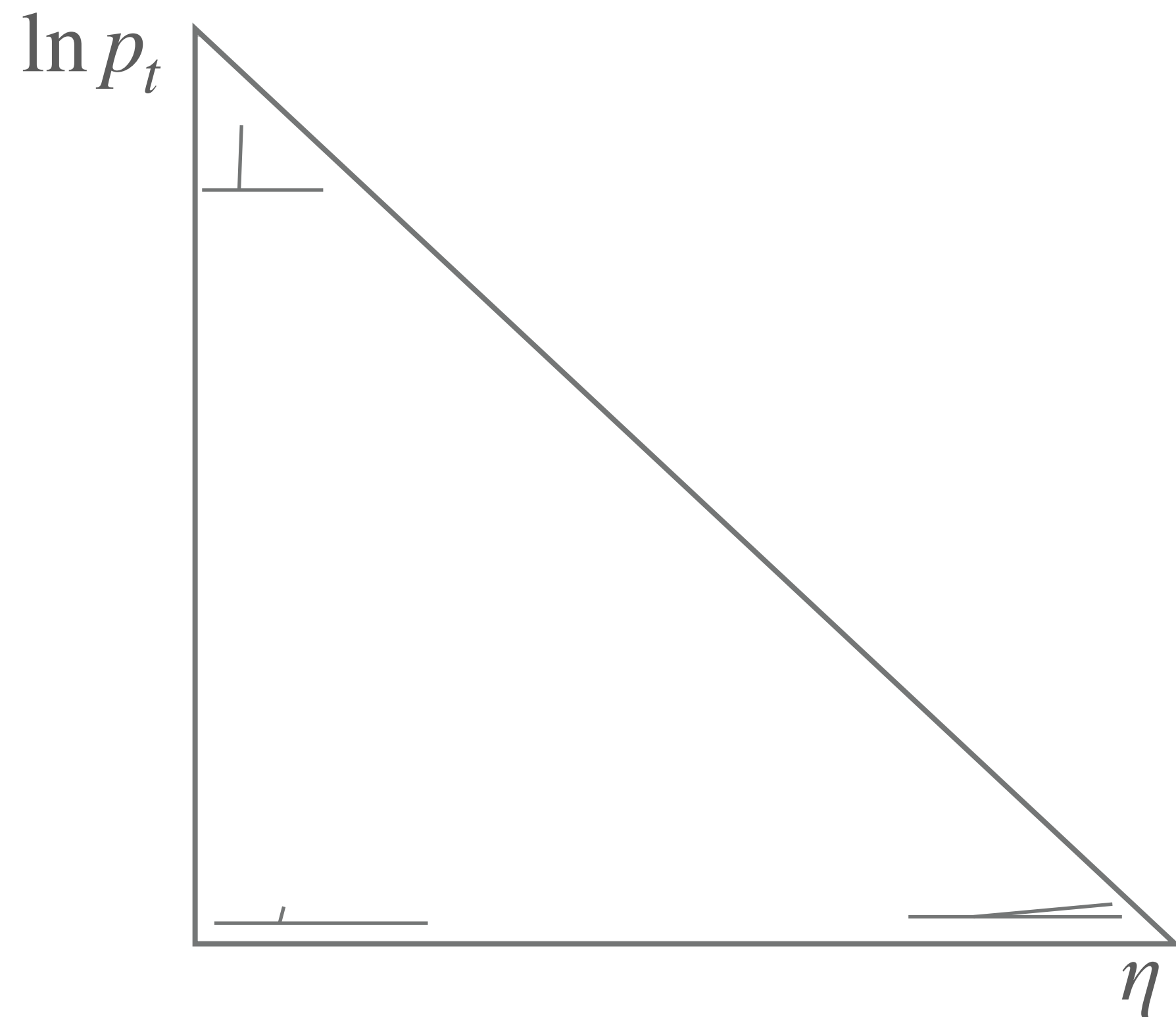
# key shower elements and their consequences

---

# Key element #1 in a shower: evolution/ordering variable

---

- Radioactivity example had just a time variable → only choice for evolution variable
- A shower has two (logarithmic) variables, e.g. angle and  $p_t$  : what do you choose for the evolution variable?

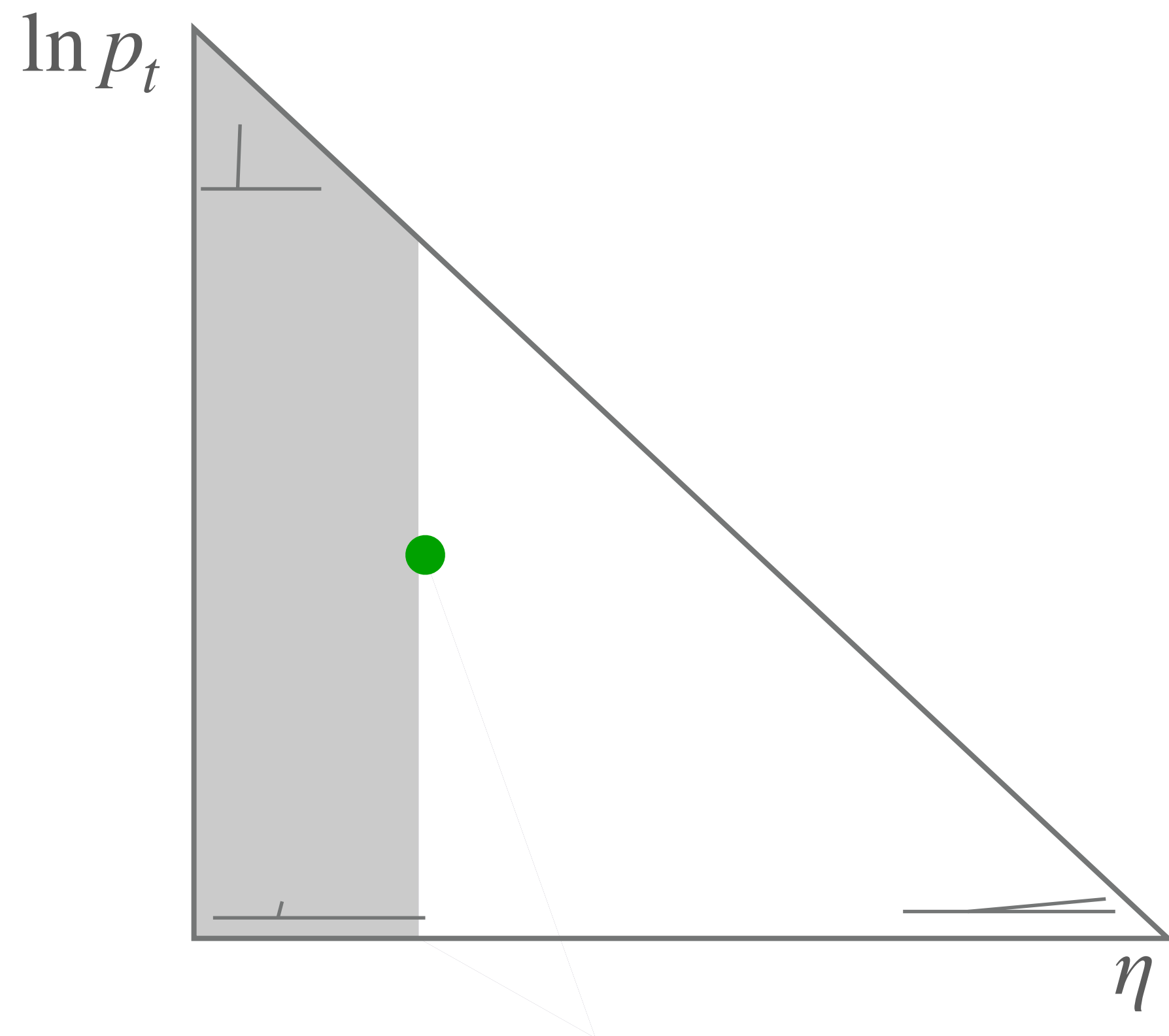


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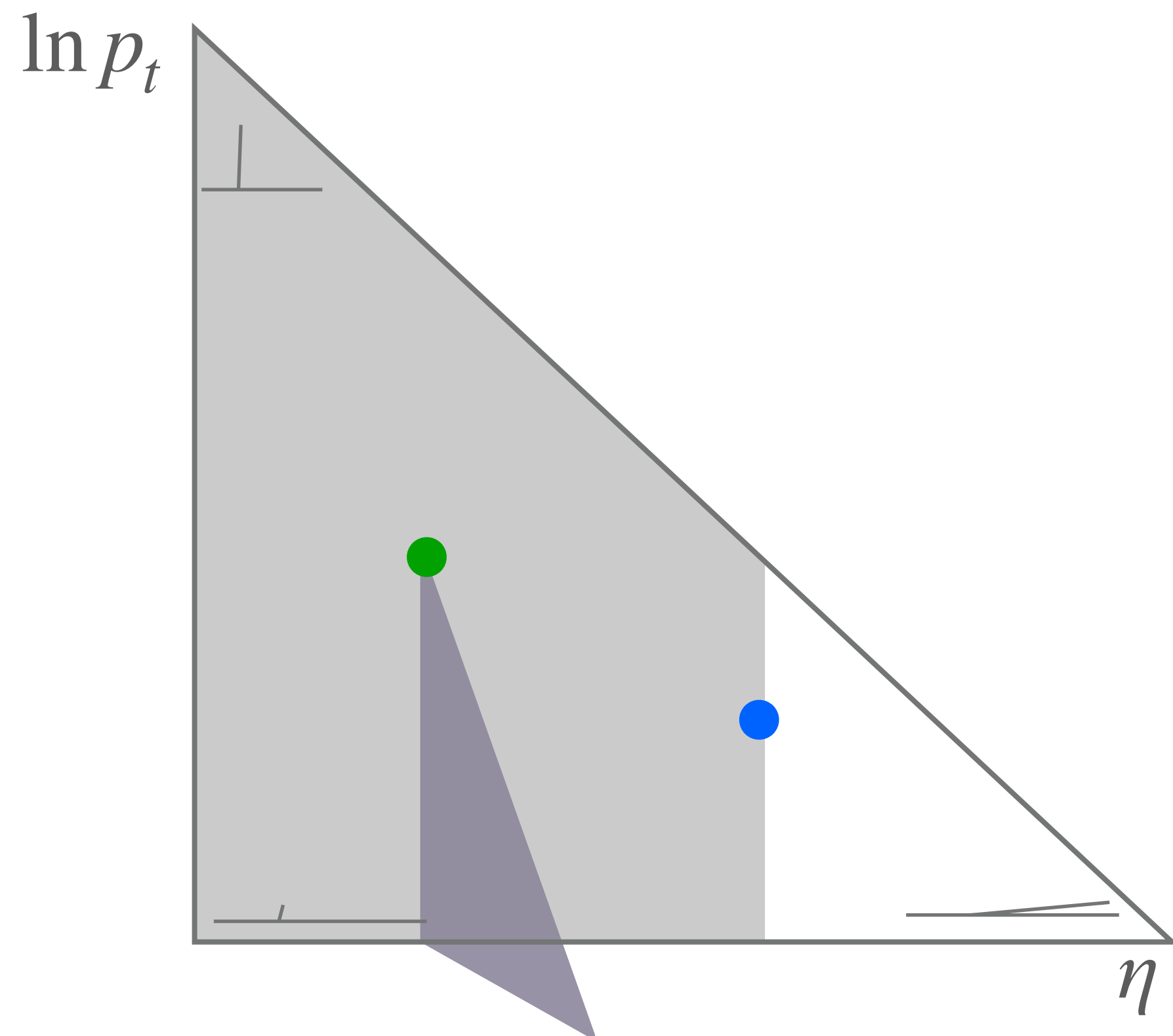


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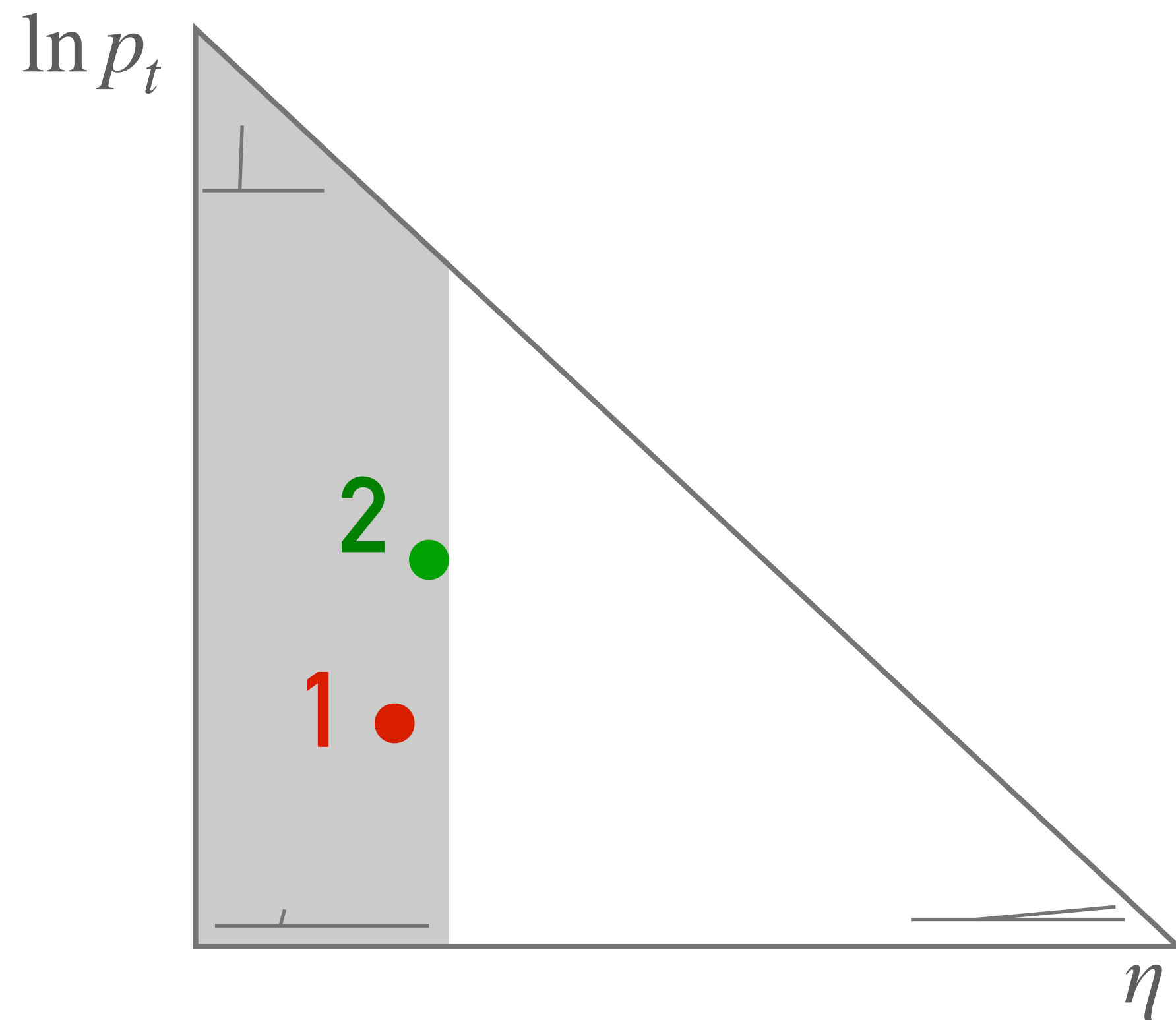
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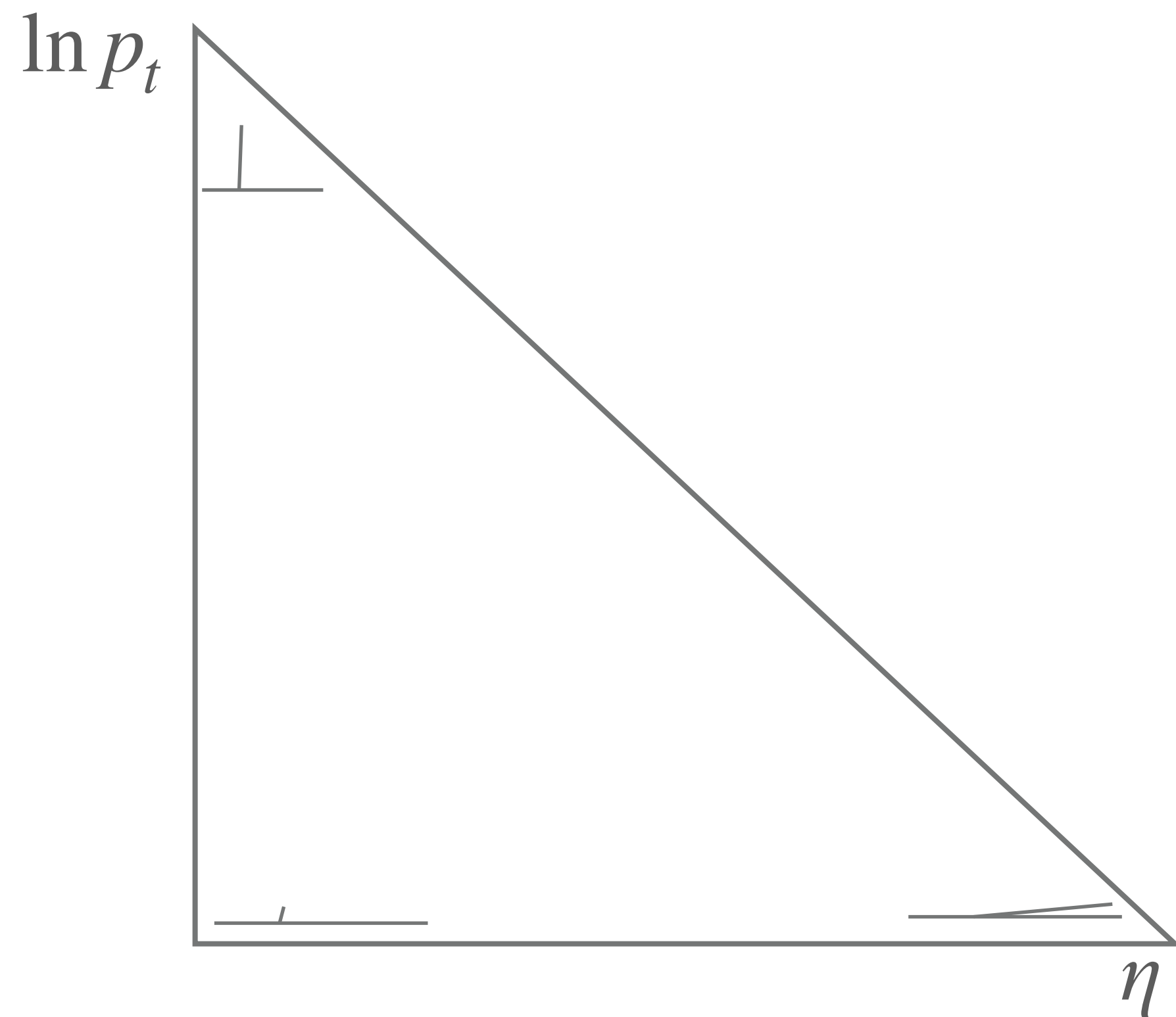
- ✓ very easy to respect (azimuthally averaged) colour coherence — **first shower to get correct multiplicity**
- ⊙ (1) and (2) have similar angles; large- $N_C$  matrix-element is simple if composed with ordered energies (2, then 1); but shower generates (1) **first** (larger angle, but smaller energy), i.e. disordered energies → **gets wrong matrix element**

*Banfi, Corcella & Dasgupta, [hep-ph/0612282](https://arxiv.org/abs/hep-ph/0612282)*

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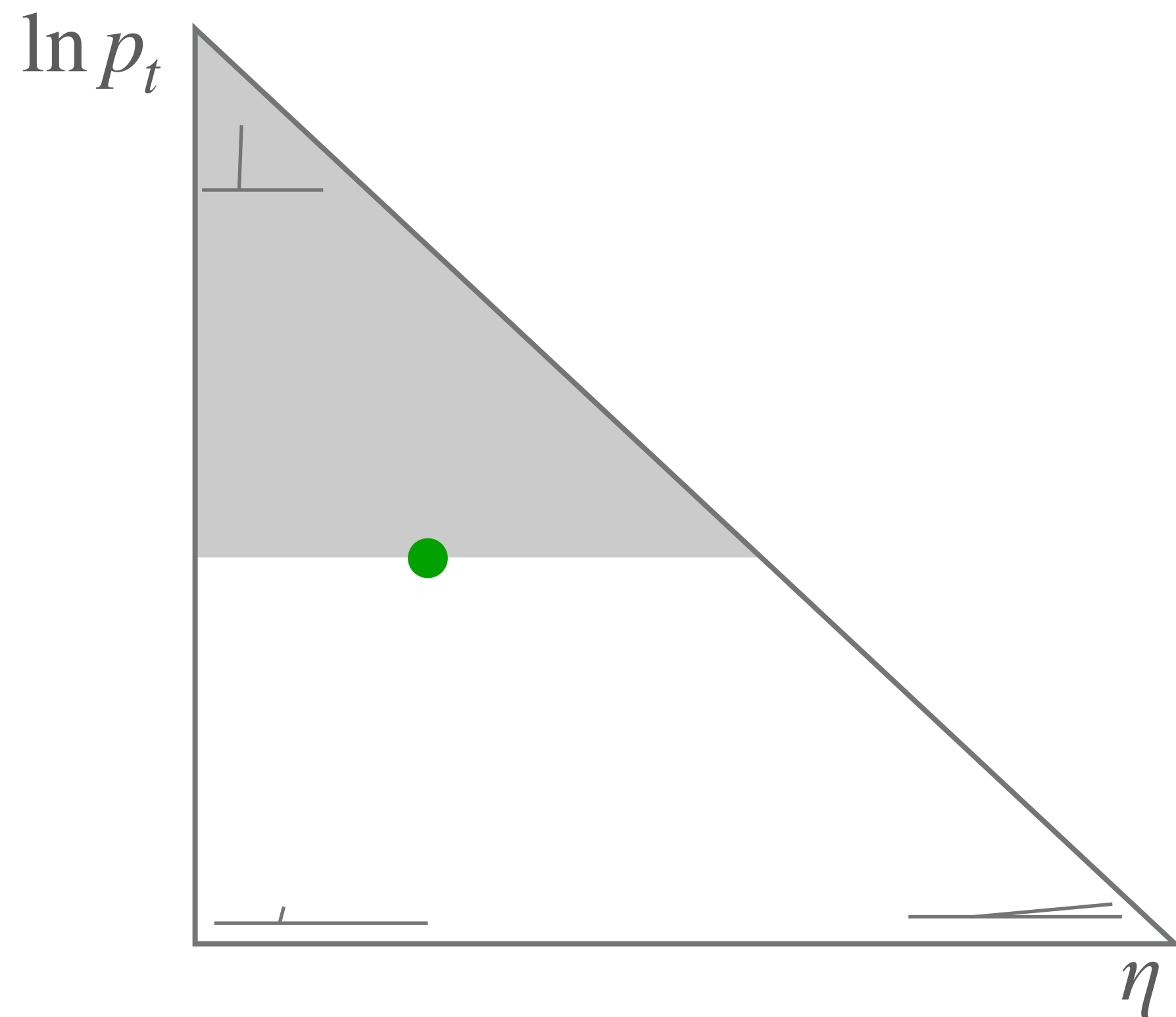


Option 2:  $p_t$  as the evolution/ordering variable, together with use of colour dipoles (default in most showers, introduced in Ariadne; Gustafson, Pettersson & Lonnblad c. 1988)

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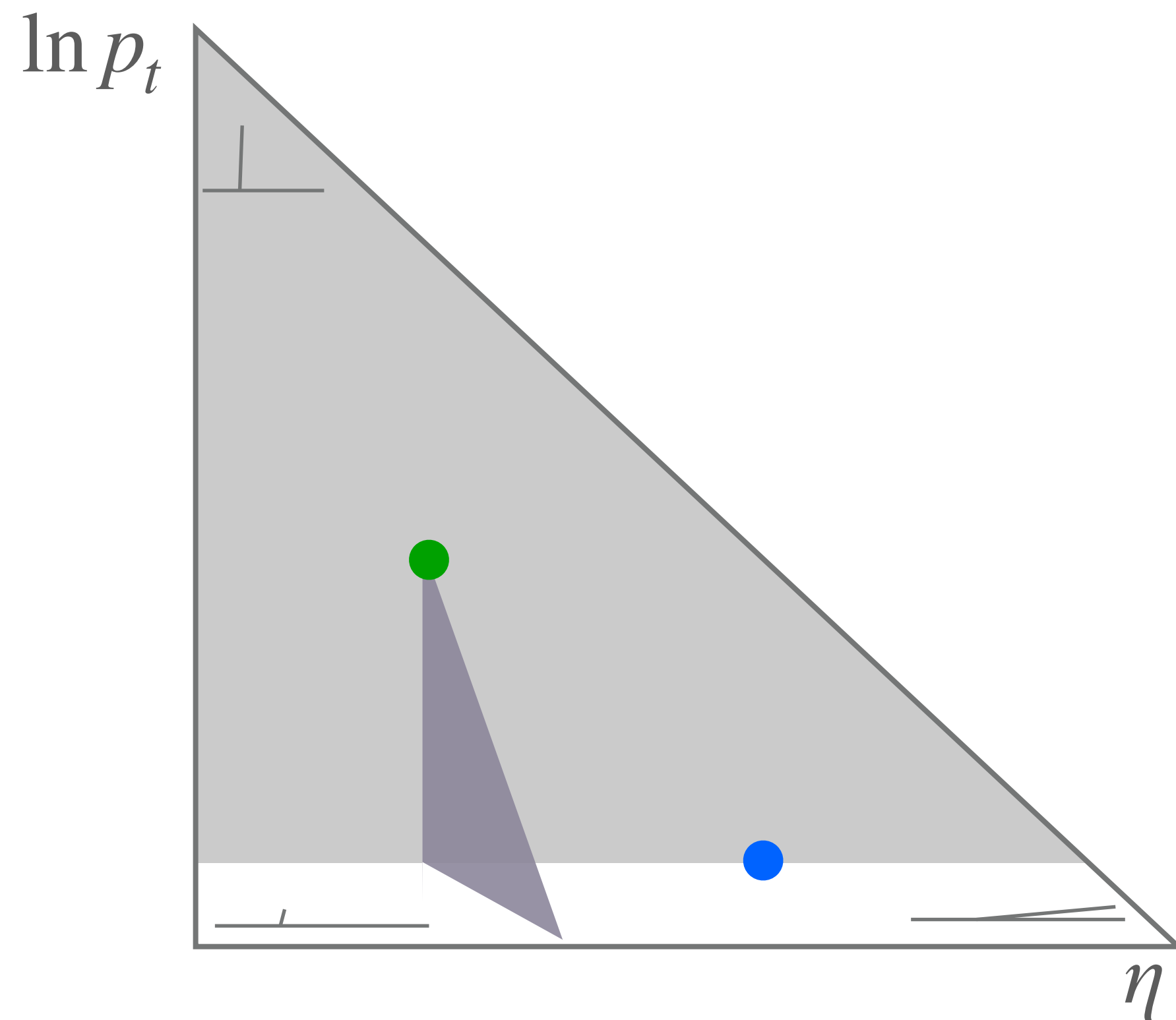


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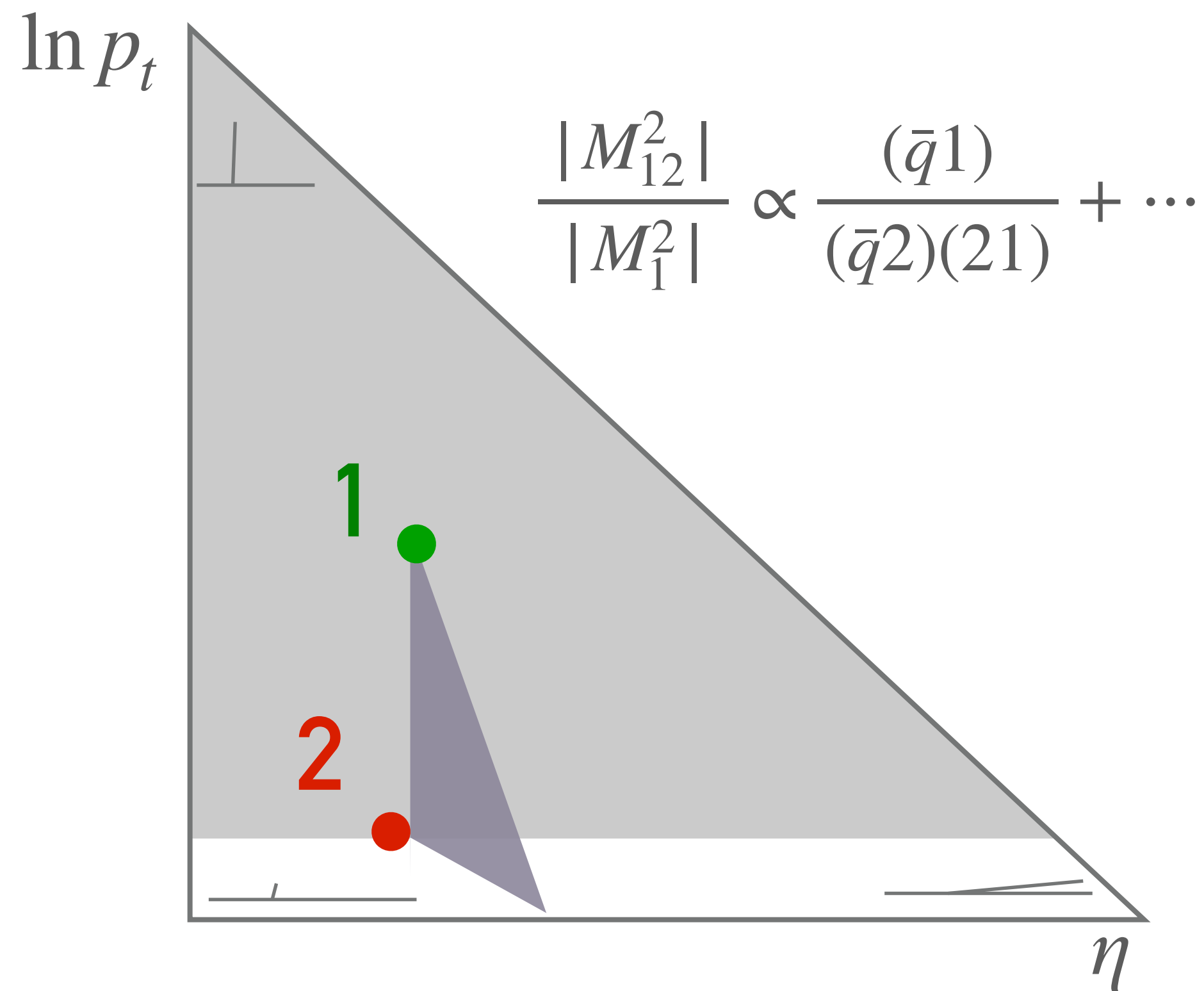


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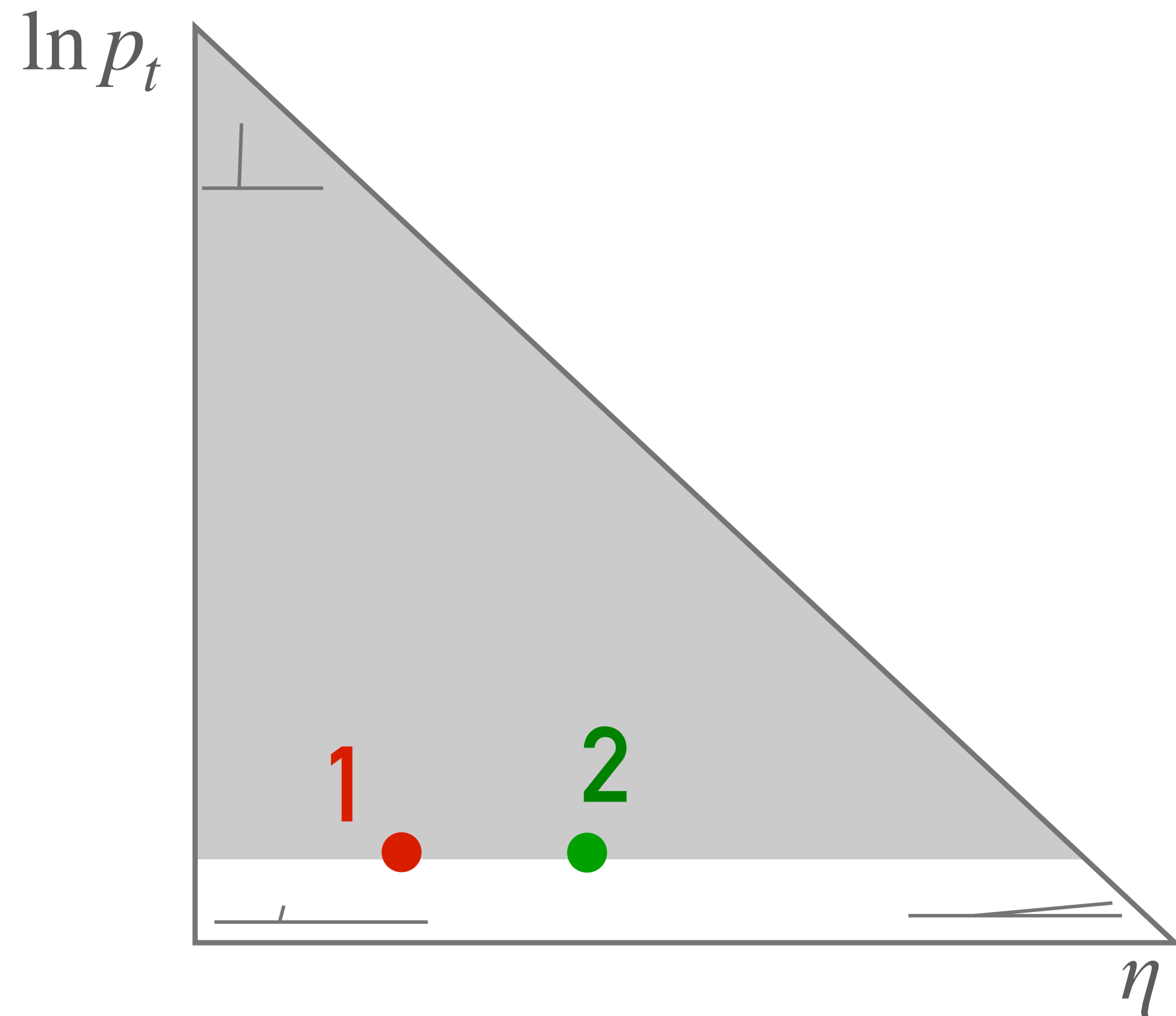


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- ✓ emissions at commensurate angles are produced in “right” order (red after green), and so with correct large- $N_C$  dipole matrix element (Bassetto, Ciafaloni Marchesini, 1983)

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- ⦿ emissions at commensurate  $p_t$  affect each other, e.g. if (2) emitted after (1), it modifies kinematics of (1)

*Andersson, Gustafson, Sjogren '92,*

*Nagy & Soper 0912.4534, 1401.6366*

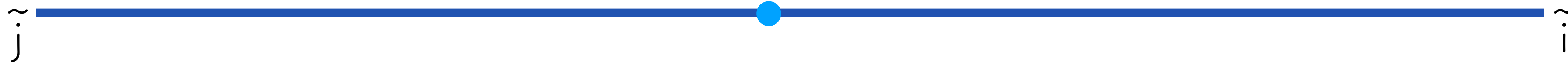
*Dasgupta, Dreyer, Hamilton, Monni & GPS 1805.09327*

## Key element #2 in a shower: **kinematic map** (local within dipole for many showers)

---

Start with dipole  $(\tilde{i} - \tilde{j}) \rightarrow$  emit gluon  $k$  to get two dipoles  $(i - k)$  and  $(k - j)$

$$\left. \begin{aligned} p_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_{\perp}, \\ p_i &= a_i \tilde{p}_i + b_i \tilde{p}_j - k_{\perp}, \\ p_j &= a_j \tilde{p}_i \end{aligned} \right\} \text{kinematic map}$$



transverse recoil assigned to end that is closer in angle in dipole c.o.m. frame

## Key element #2 in a shower: **kinematic map** (local within dipole for many showers)

---

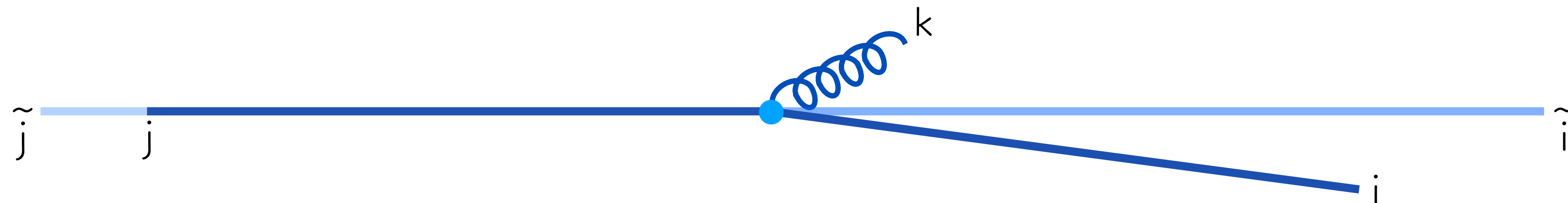
Start with dipole  $(\tilde{i} - \tilde{j}) \rightarrow$  emit gluon  $k$  to get two dipoles  $(i - k)$  and  $(k - j)$

$$p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_{\perp},$$

$$p_i = a_i \tilde{p}_i + b_i \tilde{p}_j - k_{\perp},$$

$$p_j = a_j \tilde{p}_i$$

} kinematic map



transverse recoil assigned to end that is closer in angle in dipole c.o.m. frame

# Key element #2 in a shower: **kinematic map** (local within dipole for many showers)

---

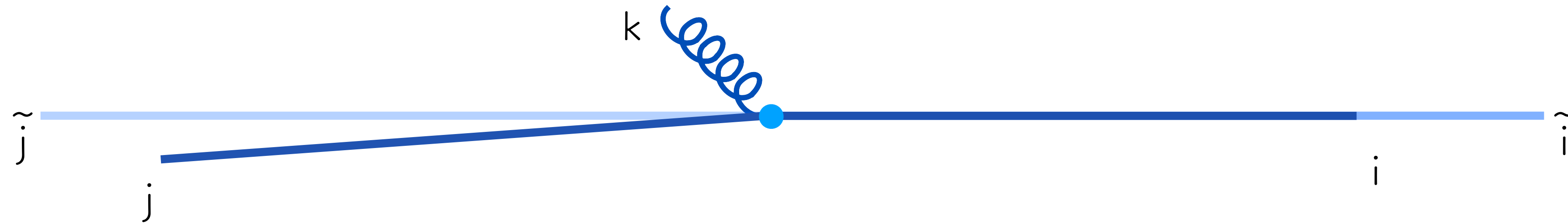
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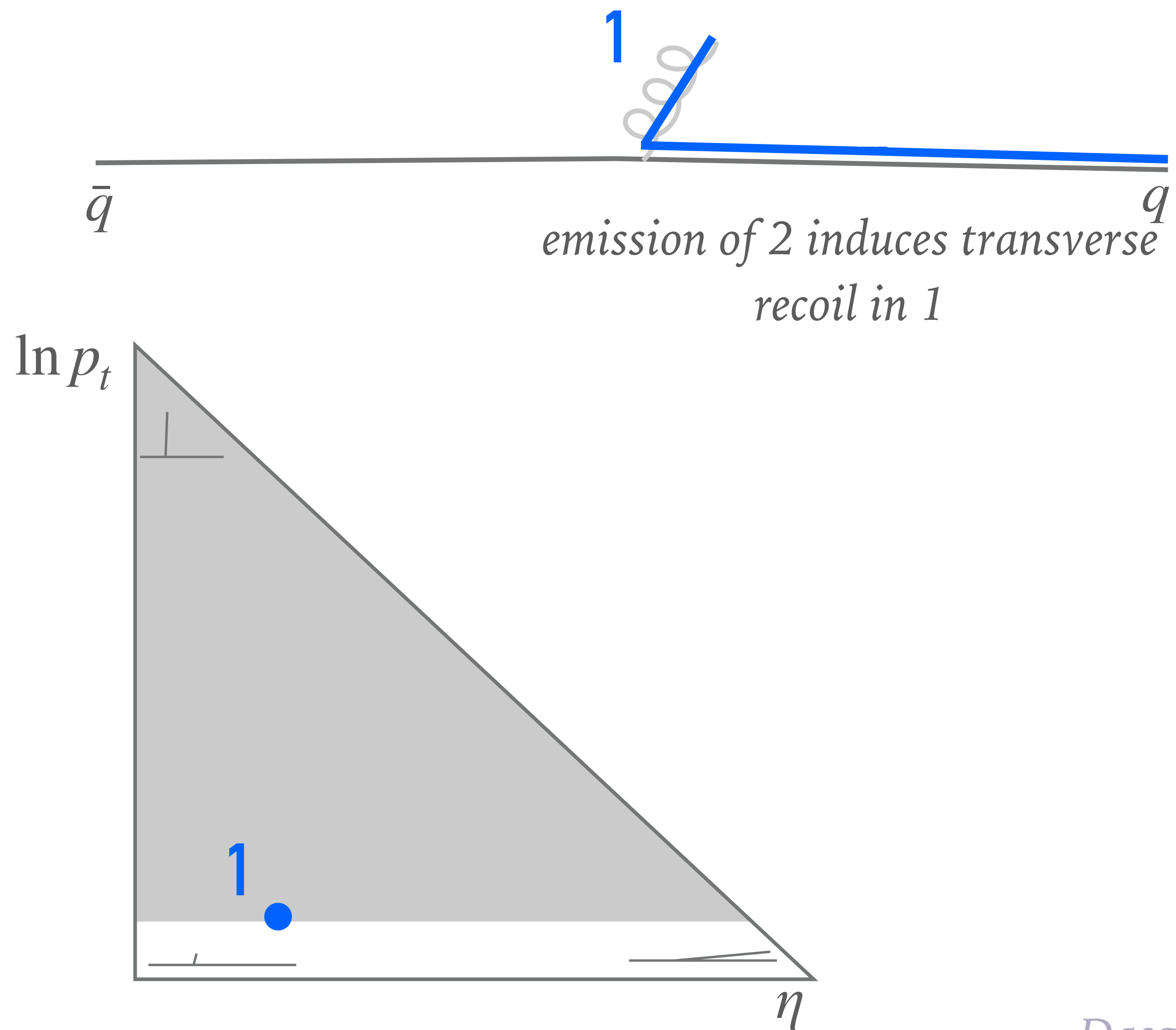
$$p_j = a_j \tilde{p}_i + b_j \tilde{p}_j - k_{\perp}$$

} kinematic map



transverse recoil assigned to end that is closer in angle in dipole c.o.m. frame

# Key element #2 in a shower: **kinematic map** (local within dipole for many showers)

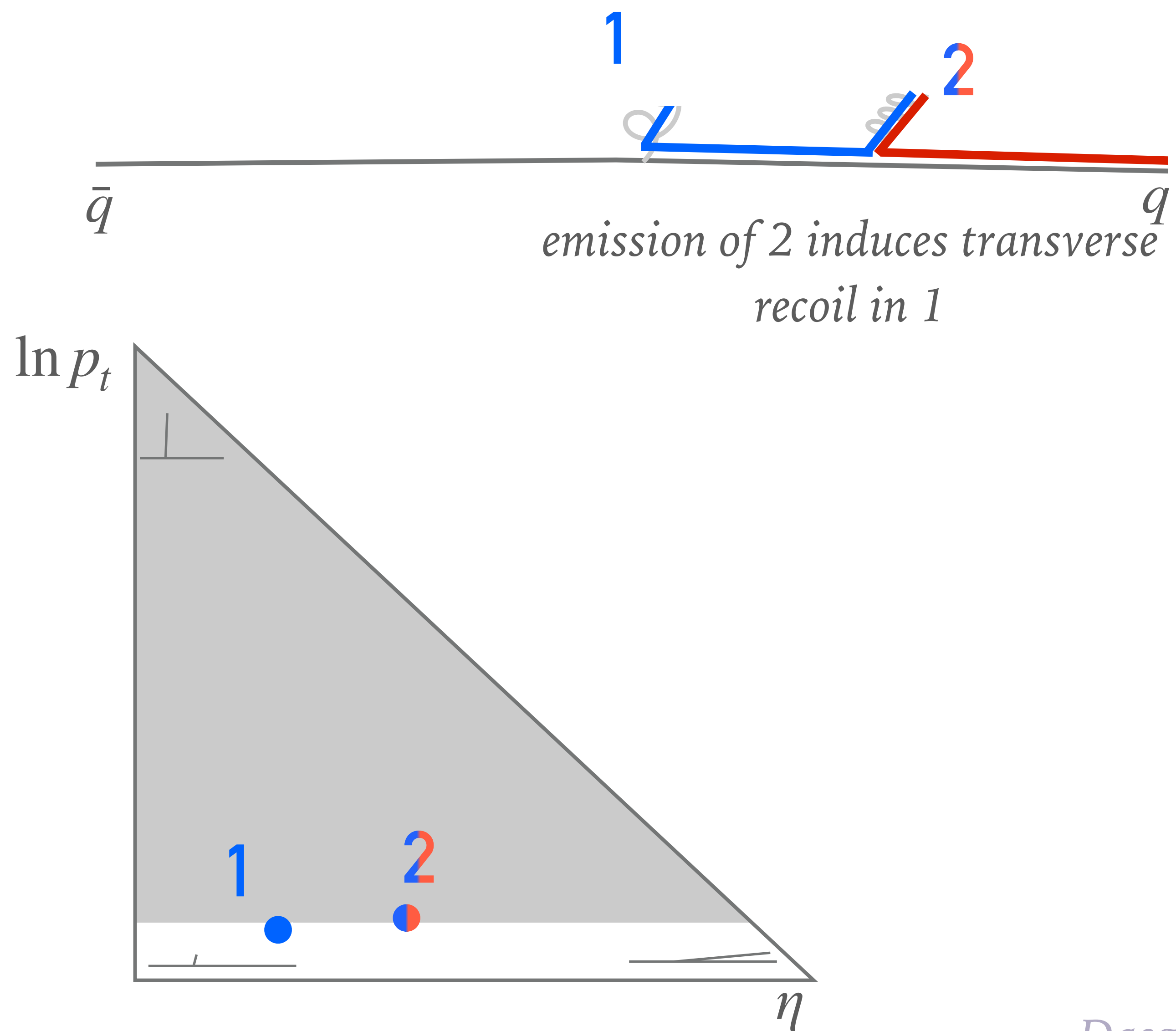


*Andersson, Gustafson, Sjogren '92,*

*Nagy & Soper 0912.4534, 1401.6366*

*Dasgupta, Dreyer, Hamilton, Monni & GPS 1805.09327*

# Key element #2 in a shower: **kinematic map** (local within dipole for many showers)

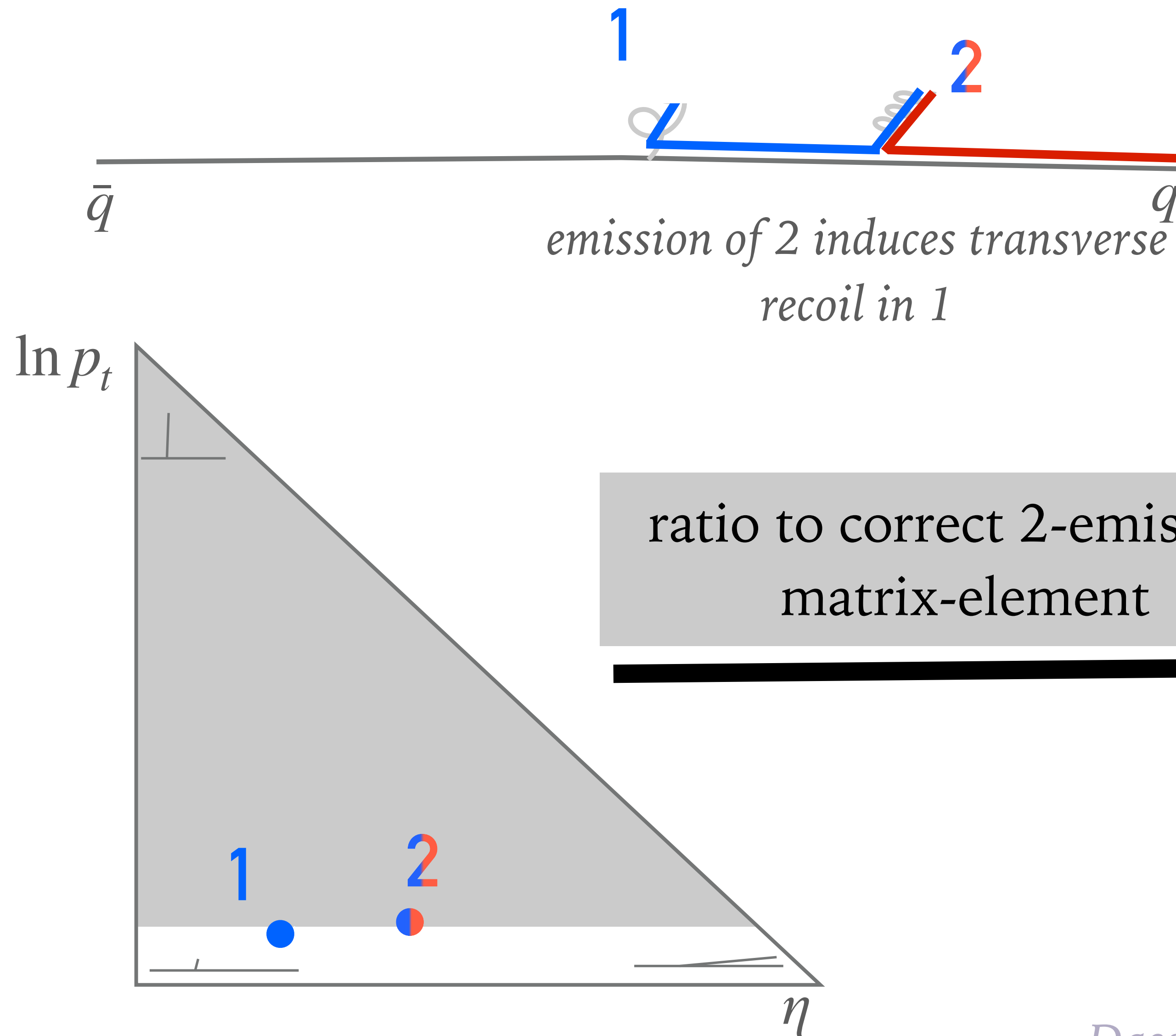


*Andersson, Gustafson, Sjogren '92,*

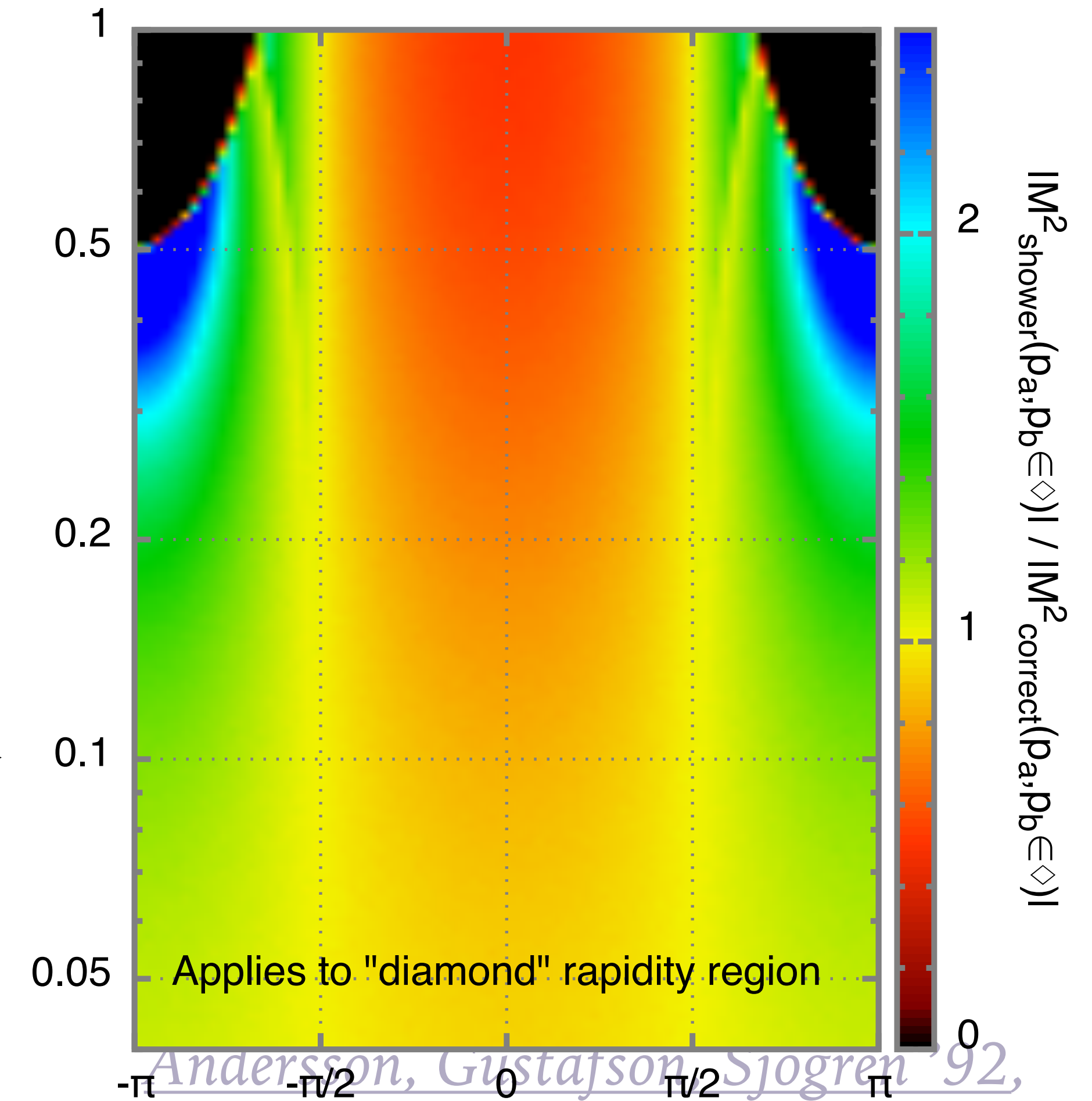
*Nagy & Soper 0912.4534, 1401.6366*

*Dasgupta, Dreyer, Hamilton, Monni & GPS 1805.09327*

# Key element #2 in a shower: **kinematic map** (local within dipole for many showers)



ratio of dipole-shower double-soft ME to correct result

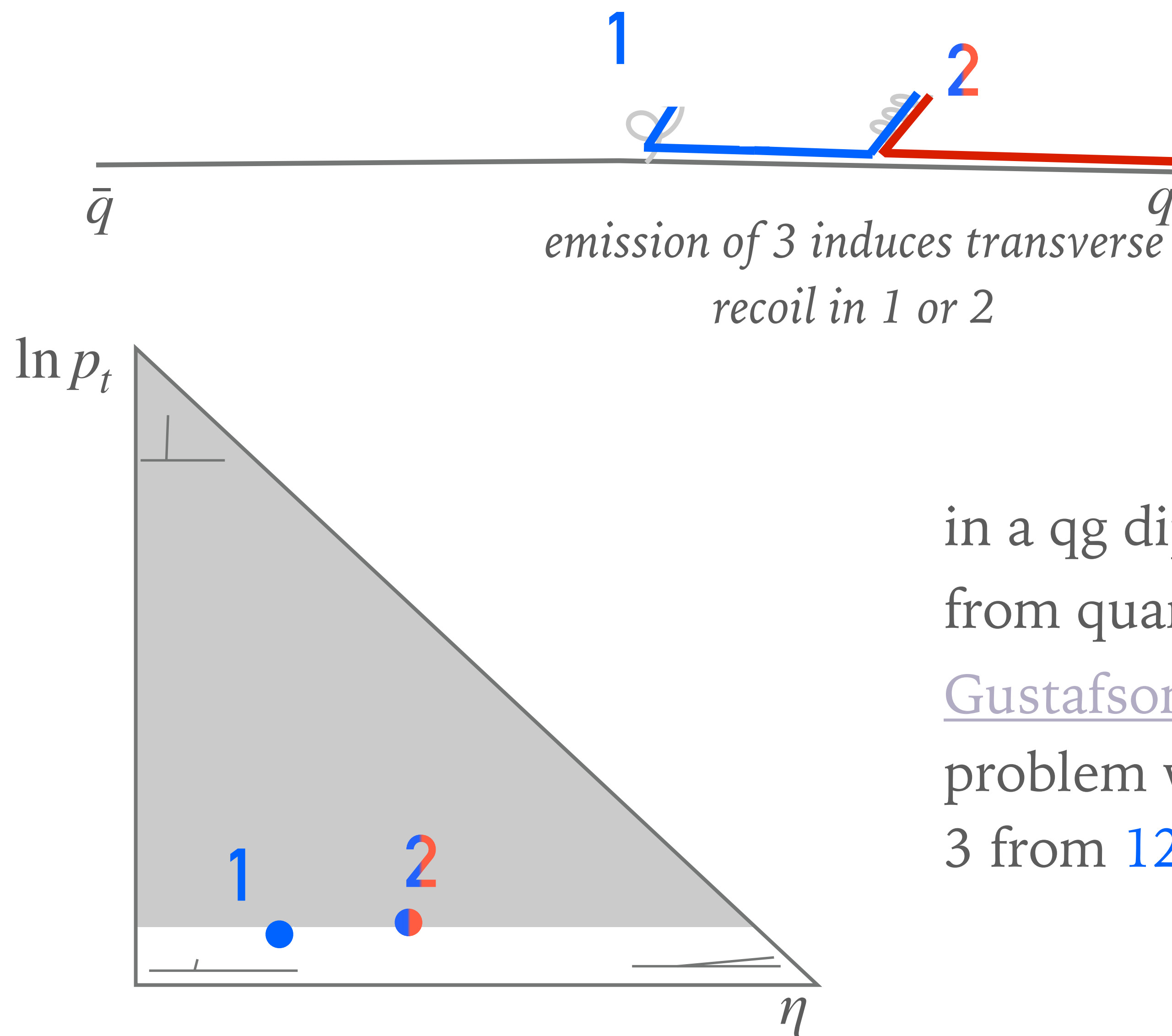


Andersson, Gustafson, Sjogren '92,  
Nagy & Soper 0912.4534, 1401.6366

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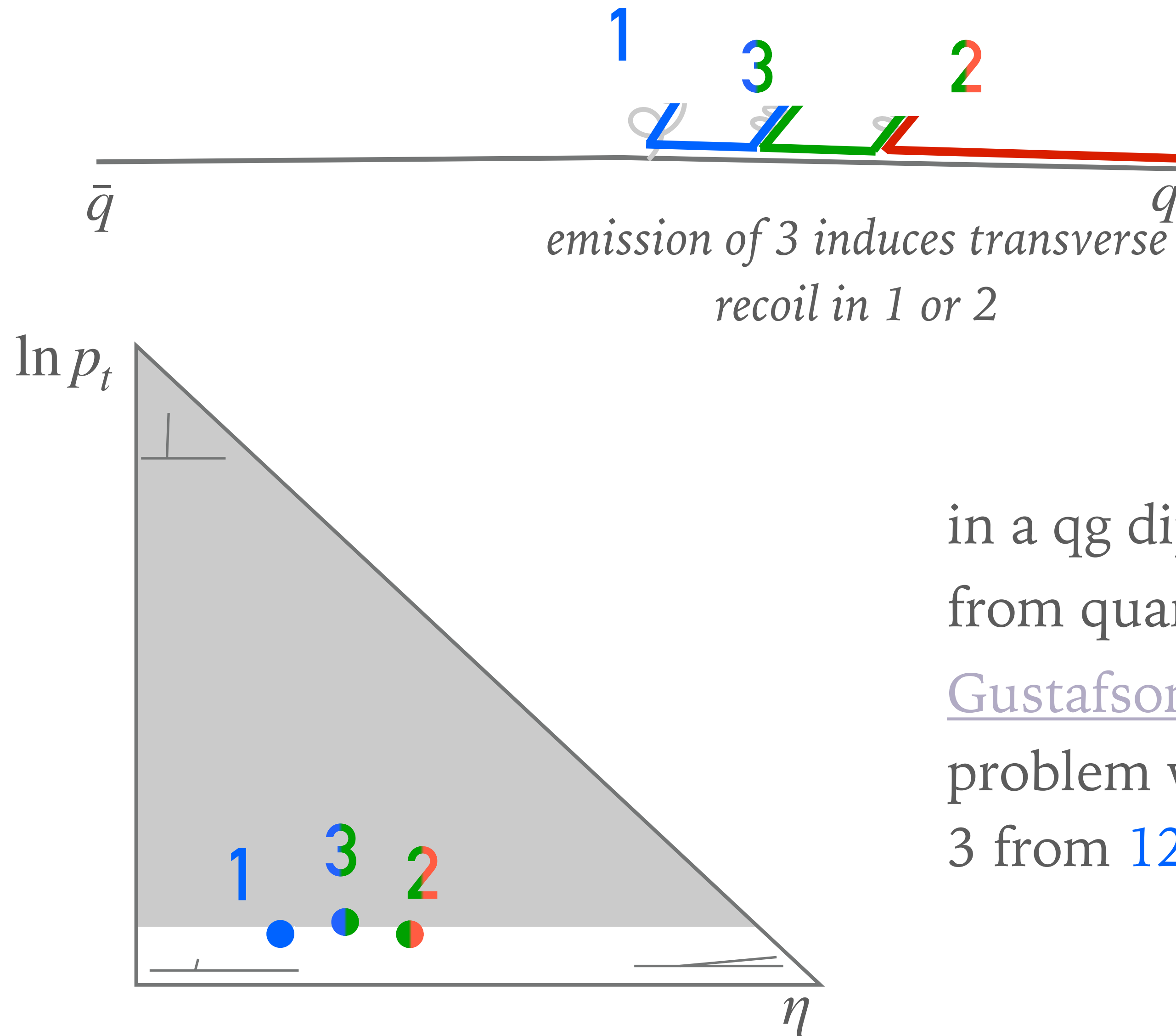


# Key element #2 in a shower: **kinematic map** (local within dipole for many showers)



in a  $qg$  dipole you can always force recoil to be taken from quark (solves problem at  $\alpha_s^2$ , [Andersson, Gustafson, Sjögren '92](#)), but with  $p_t$  ordering, problem will reappear for  $gg$  dipoles, e.g. emission of 3 from  $12$  dipole here

# Key element #2 in a shower: **kinematic map** (local within dipole for many showers)



in a  $qg$  dipole you can always force recoil to be taken from quark (solves problem at  $\alpha_s^2$ , [Andersson, Gustafson, Sjögren '92](#)), but with  $p_t$  ordering, problem will reappear for  $gg$  dipoles, e.g. emission of 3 from  $12$  dipole here

# designing new showers

---

in the large- $N_C$  limit, without spin correlations  
(spin correlations solved by Collins [1986](#);  
beyond leading  $N_C$  more subtle)

# core principles

---

1. for a new emission  $k$ , when it is generated far in the Lund diagram from any other emission ( $|d_{ki}^{Lund}| \gg 1$ ), **it should not modify the kinematics (Lund coordinates) of any preceding emission** by more than an amount  $\exp(-p |d_{ki}^{Lund}|)$ , where  $p = \mathcal{O}(1)$

2. when  $k$  is distant from other emissions, generate it with matrix element and phasespace (and associated Sudakov)

$$\frac{d\Phi_k}{d\Phi_{k-1}} \frac{|M_{1\dots k}|^2}{|M_{1\dots(k-1)}|^2}$$

[simple forms known from factorisation properties of matrix-elements]

3. emission  $k$  **should not impact  $d\Phi \times |M|^2$  ratio for subsequent distant emissions unless**

a. they are at commensurate angle (or on  $k$ 's Lund "leaf"), or

b.  $k$  was a hard collinear splitting, which can affect other hard collinear splittings (cross-talk on same leaf  $\equiv$  DGLAP, cross-talk on other leaves  $\equiv$  spin correlations)

# Handle #1: choice of recoil scheme for emission from ij dipole

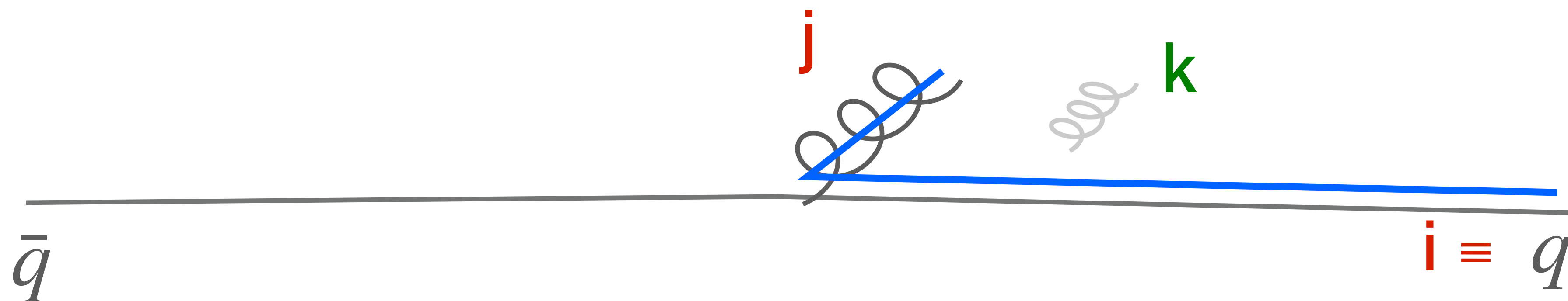
- Dipole-local scheme (**PanLocal**)

$$\left. \begin{aligned} p_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_{\perp}, \\ p_i &= a_i \tilde{p}_i + b_i \tilde{p}_j - f k_{\perp}, \\ p_j &= a_j \tilde{p}_i + b_j \tilde{p}_j - (1 - f) k_{\perp} \end{aligned} \right\} \text{kinematic map}$$

$f = 1$  (0) when  $k$  collinear to  $i$  ( $j$ );

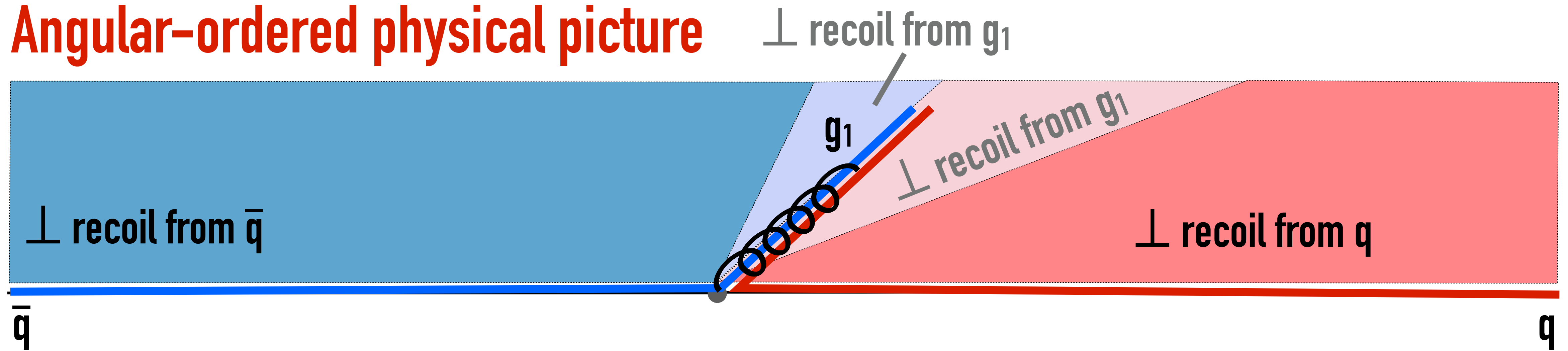
transition when  $k$  bisects ( $ij$ ) opening angle in **event** c.o.m. frame

(normal dipole/antenna showers have transition in the dipole c.o.m. frame)



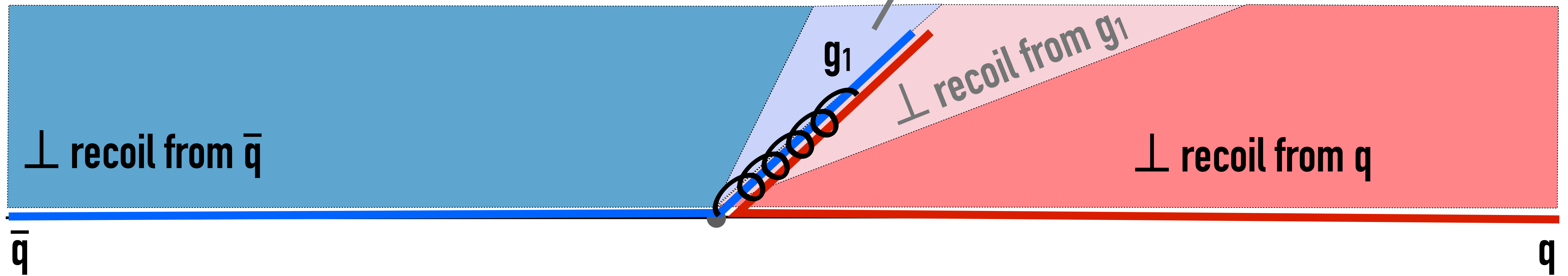
# “Physical” v standard dipole recoil

## Angular-ordered physical picture

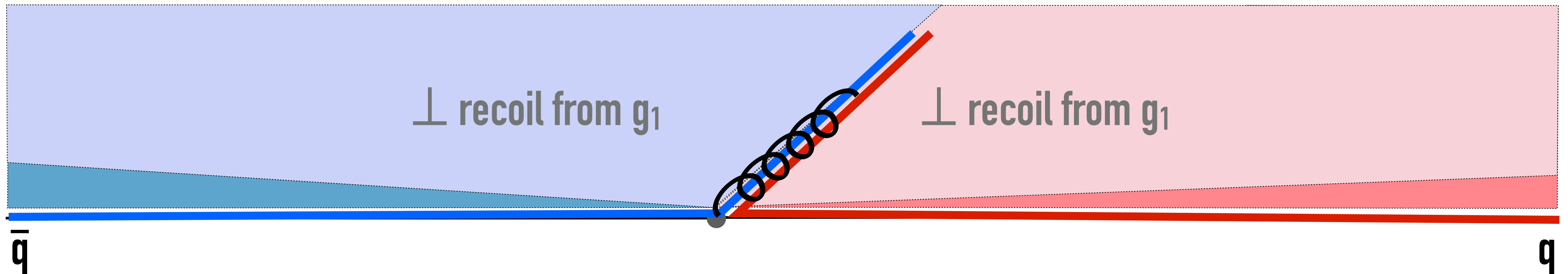


# “Physical” v standard dipole recoil

## Angular-ordered physical picture



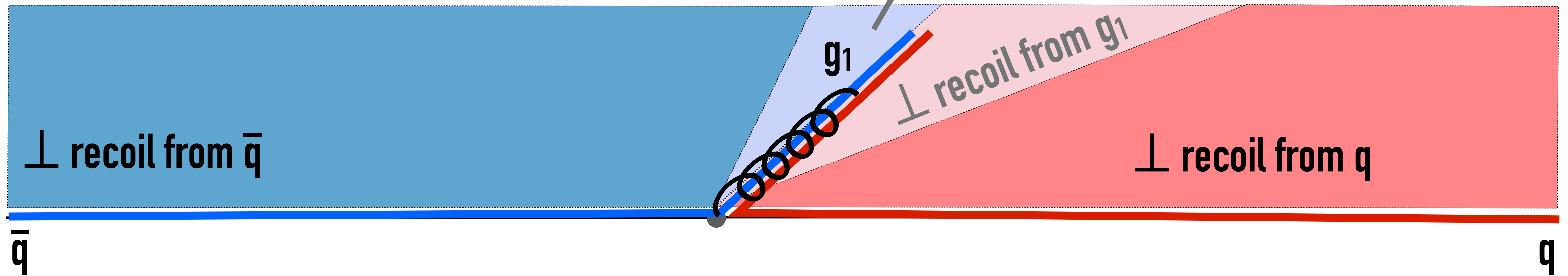
## Standard Dipole



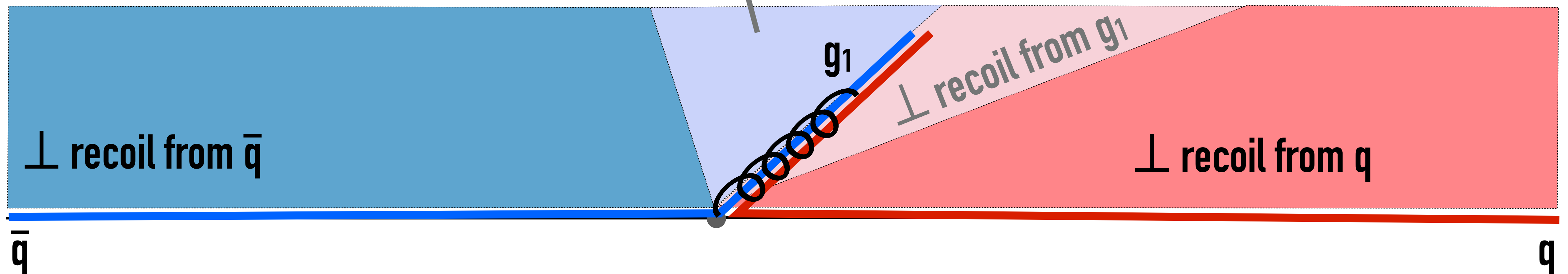
# “Physical” v. PanLocal recoil

*NB, Nagy & Soper 1401.6364  
have elements related to PanLocal  
but with a global recoil*

## Angular-ordered physical picture



## PanLocal

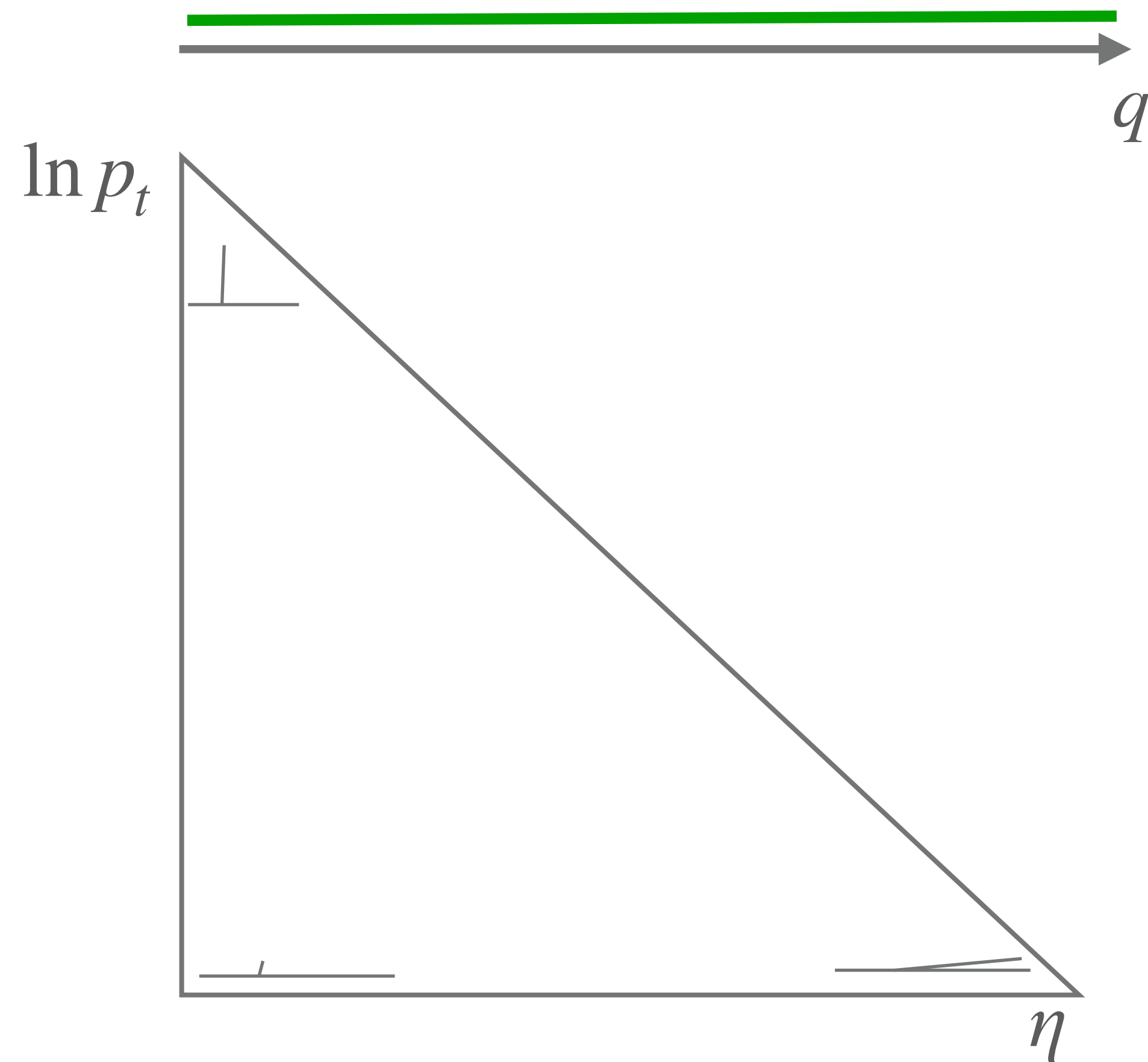




# Handle #2: ordering variable

---

Use an ordering variable intermediate between transverse momentum and angle



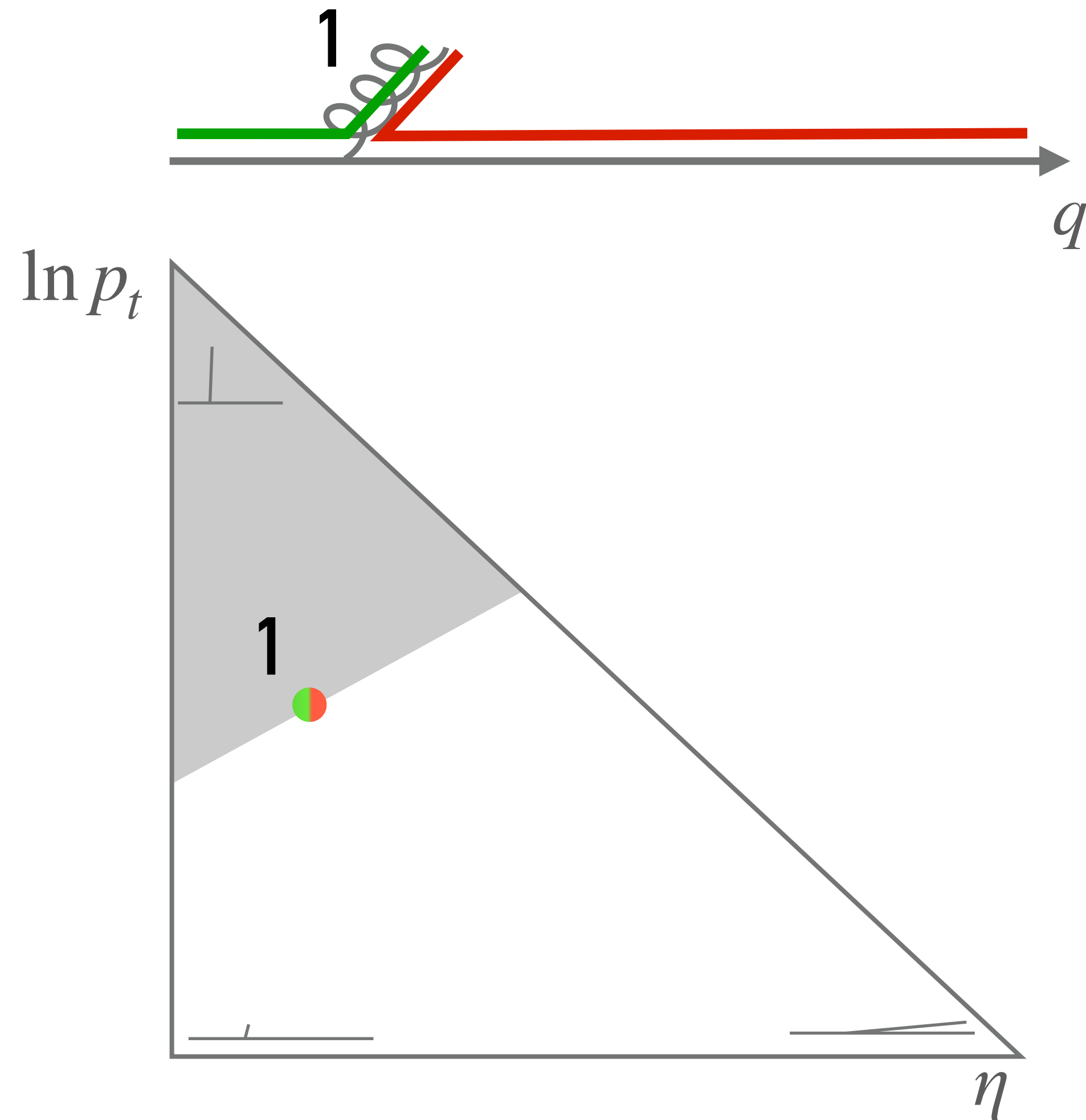
- ▶  $v$  is ordering variable
- ▶ maps to contour in Lund diagram at an angle  $\tan^{-1} \beta$

$$v = p_t e^{-\beta|\eta|} / \rho$$

- ▶ **require  $0 < \beta < 1$**   
(in practice use  $\beta = 0.5$ )

# Handle #2: ordering variable

Use an ordering variable intermediate between transverse momentum and angle



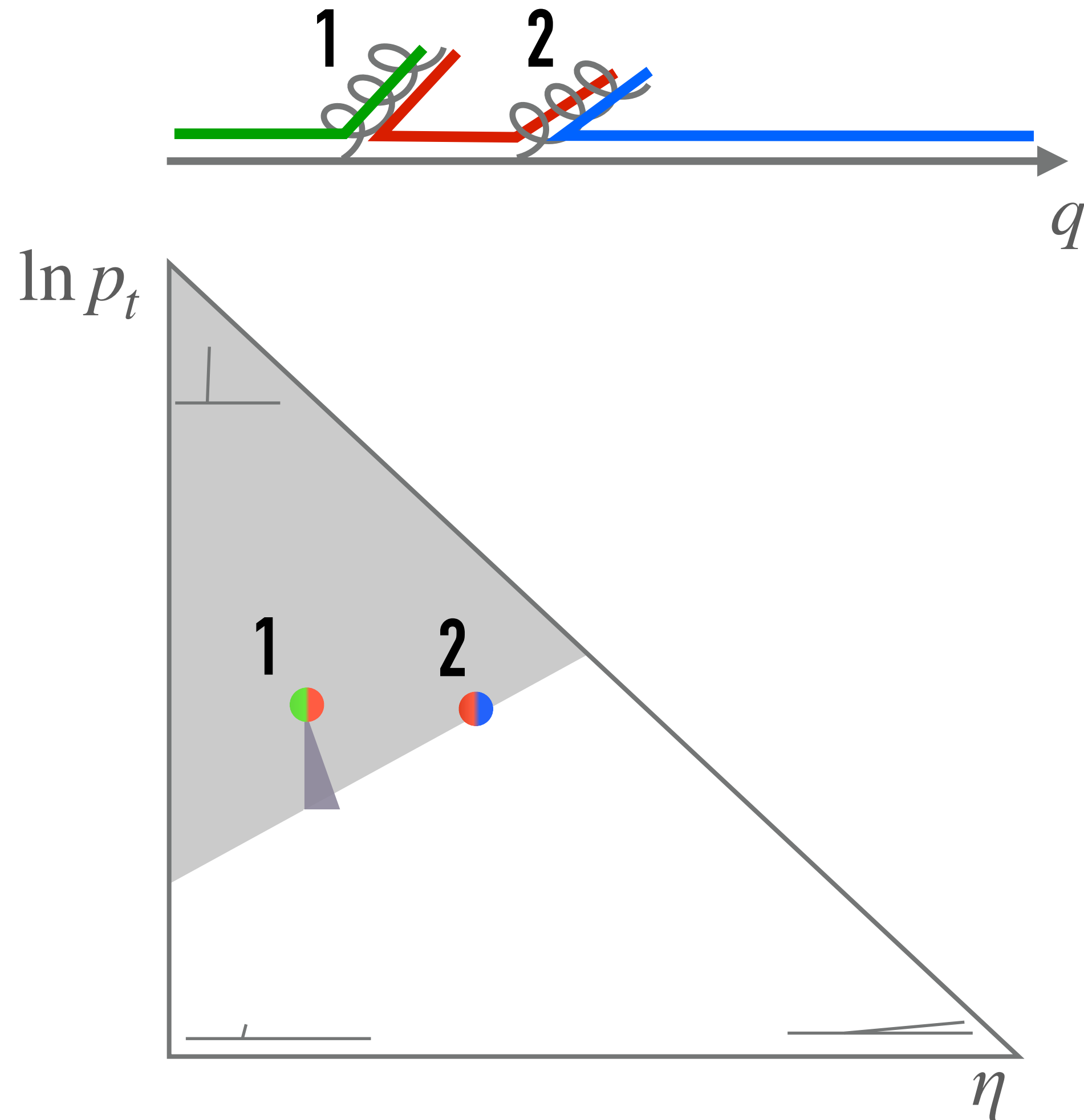
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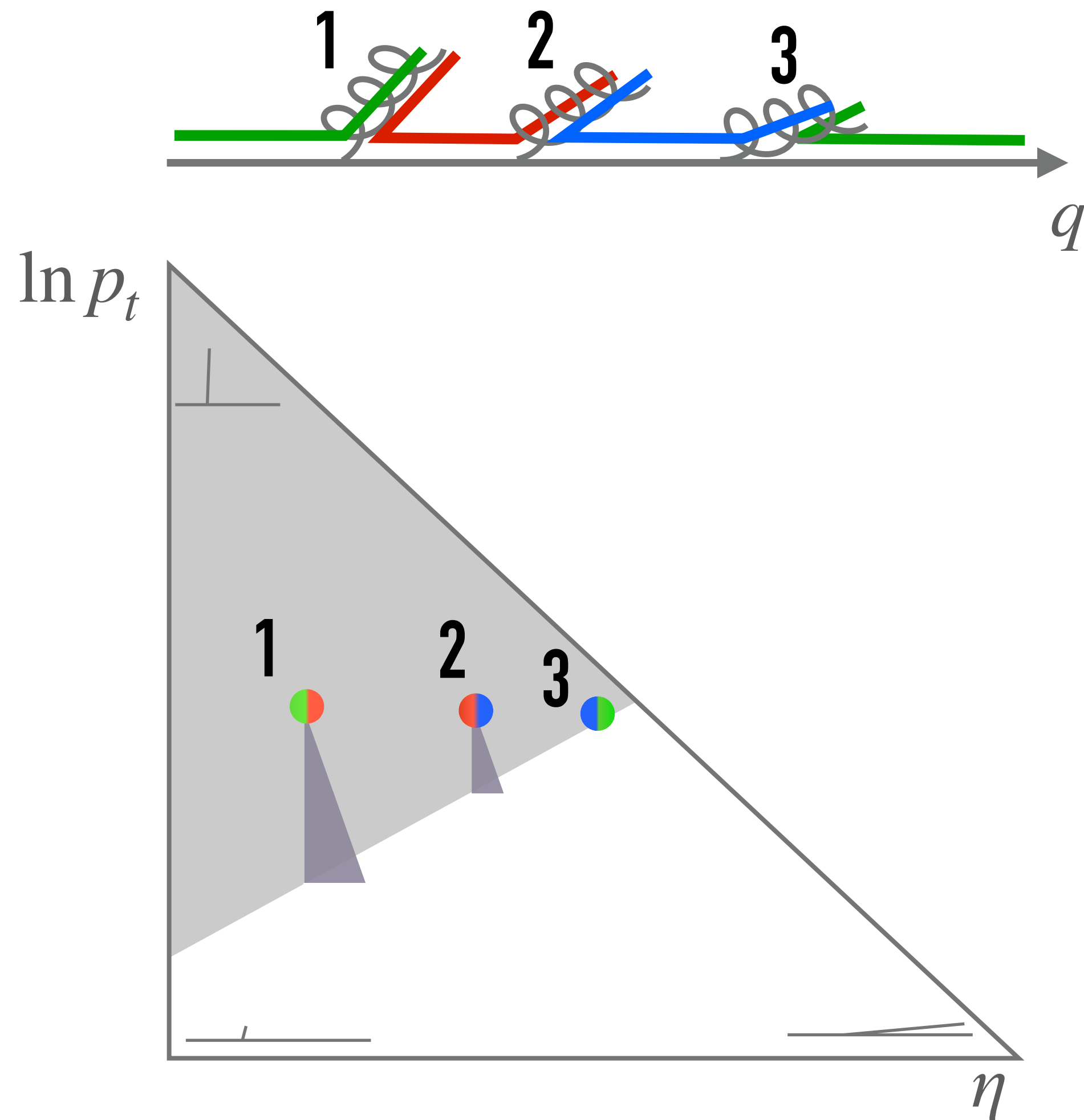
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Use an ordering variable intermediate between transverse momentum and angle



- ▶  $v$  is ordering variable
- ▶ maps to contour in Lund diagram at an angle  $\tan^{-1} \beta$

$$v = p_t e^{-\beta|\eta|} / \rho$$

- ▶ **require  $0 < \beta < 1$**   
(in practice use  $\beta = 0.5$ )
- ▶ Ensures that commensurate- $p_t$  emissions are produced at successively smaller angles (avoids major recoil in gg dipole)

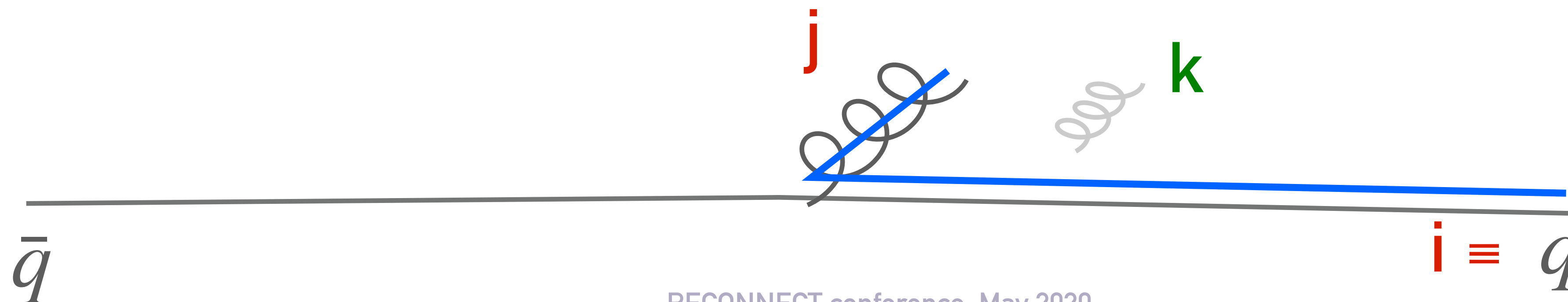
## Alternative approach: PanGlobal

Dipole-local map doesn't handle transverse recoil

$$\left. \begin{aligned} \bar{p}_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp, \\ \bar{p}_i &= (1 - a_k) \tilde{p}_i, \\ \bar{p}_j &= (1 - b_k) \tilde{p}_j. \end{aligned} \right\} \text{kinematic map}$$

Whole event scaled and boosted after each emission to restore 4-momentum conservation.

Works with  $0 \leq \beta < 1$  (i.e. can be used with  $p_t$  ordering)



# Personal comment

---

- I don't like either of these approaches
- they involve “engineering” in order to satisfy the core principles, but are physically unappealing in some respects (recoil assigned to particles that shouldn't “physically” get it)
- I believe one could find other solutions that are better
- But for now, they serve as a proof of principle that it is possible to construct a shower that satisfies our NLL conditions (some people believed this might not even be possible)
- *[having  $> 1$  solution is important, because differences between solutions can provide indication of size of uncontrolled NNLL terms]*

*NB, Nagy & Soper 1401.6364  
have elements related to PanLocal  
but with a global recoil*

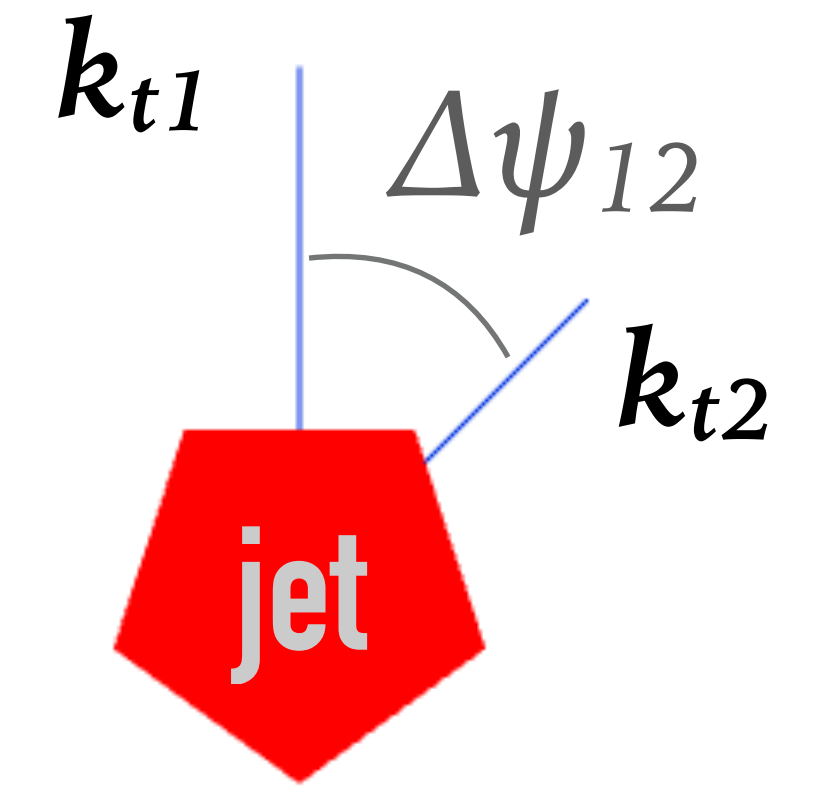
# validating new showers

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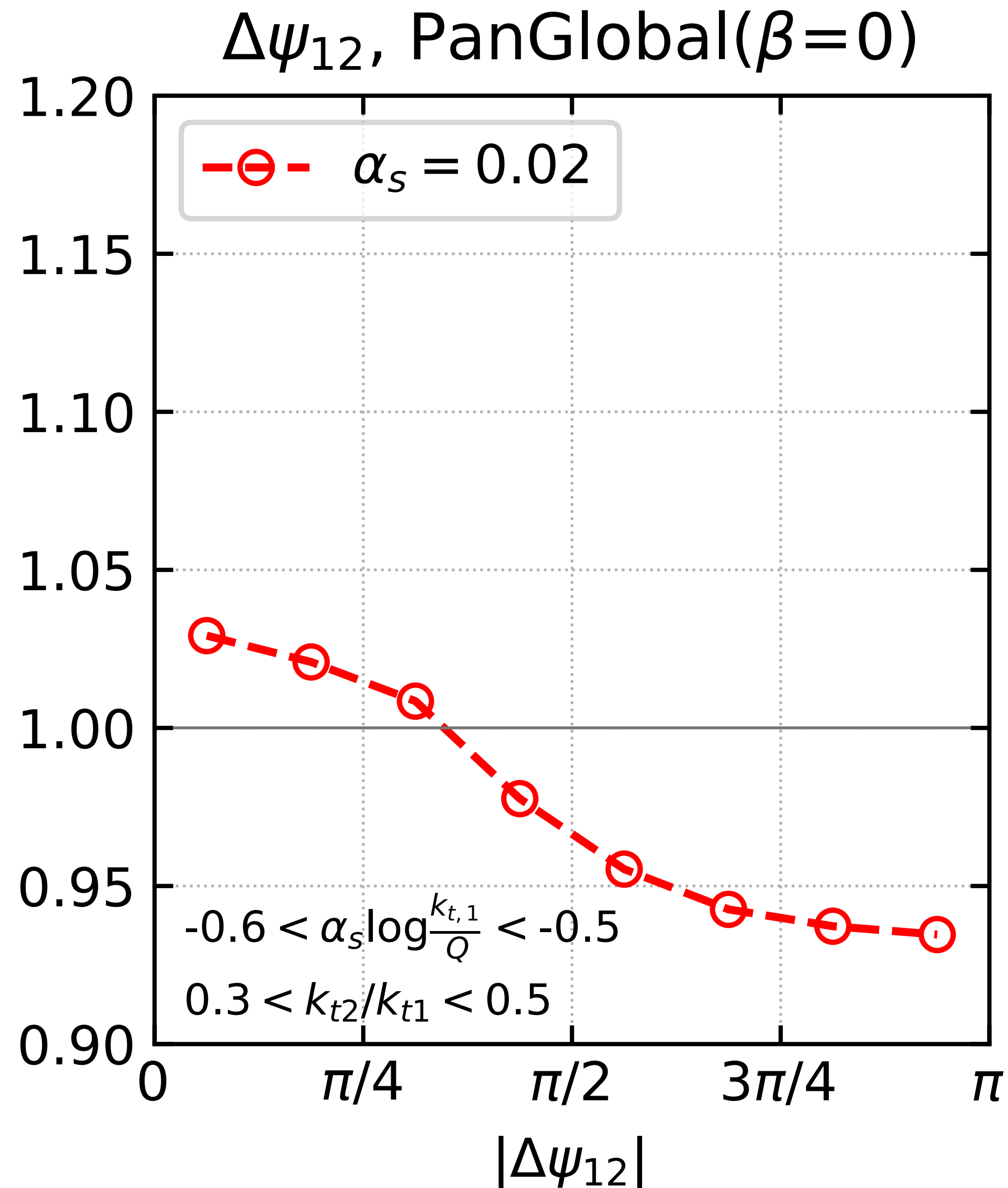
maybe one's concerned ?

how can we convince you (and ourselves) that we have  
achieved NLL?

# test matrix-element with subjet azimuthal difference



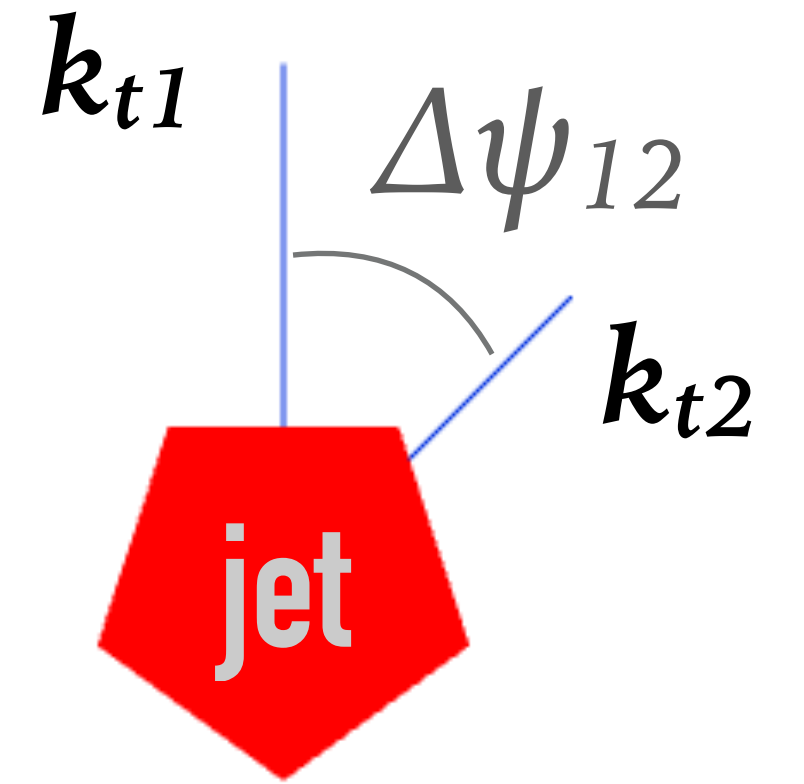
ratio  
to  
NLL



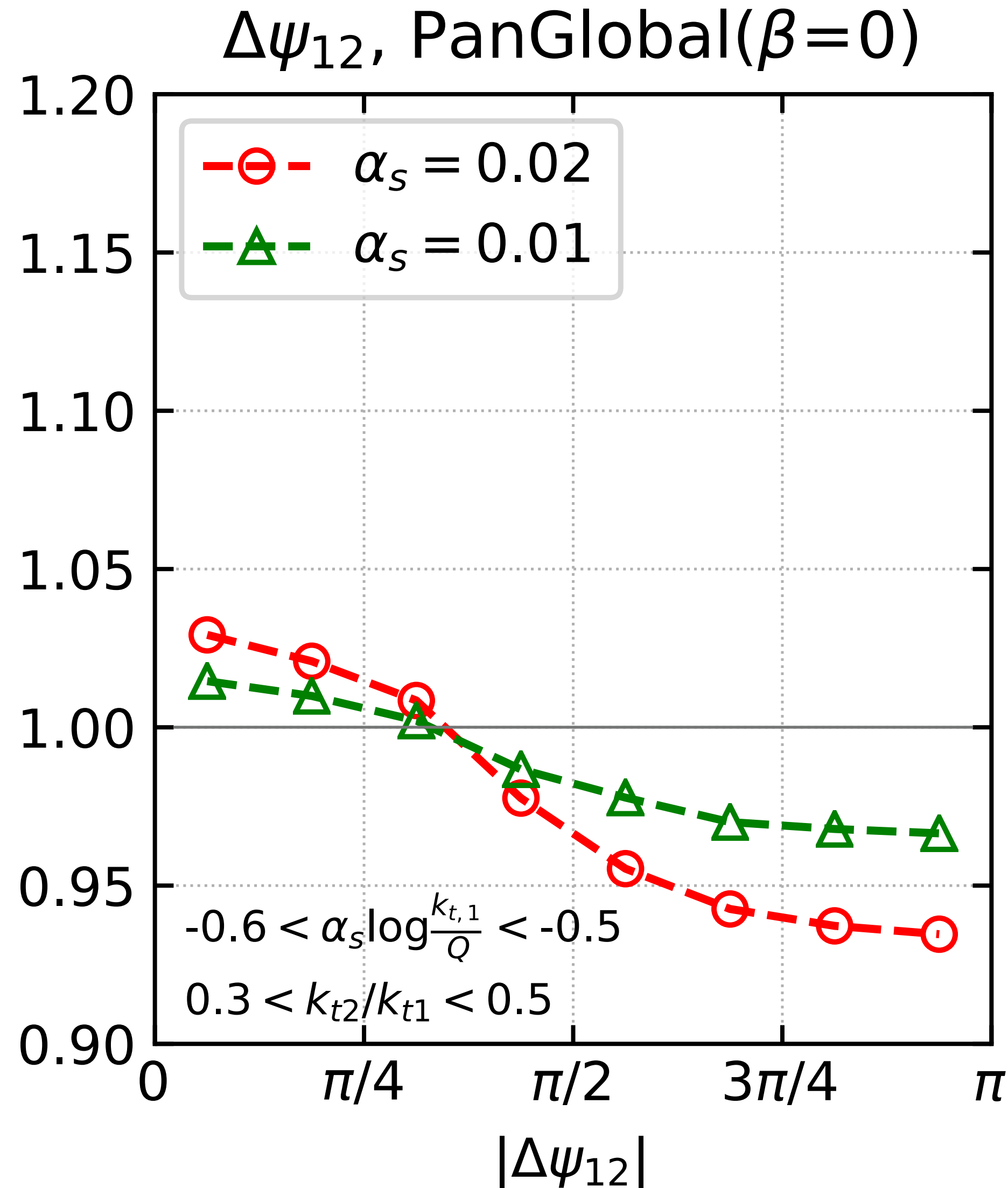
- ▶ run full shower with specific value of  $\alpha_s(Q)$
- ▶ ratio to NLL should be flat  $\equiv 1$
- ▶ it isn't: **have we got an NLL mistake? Or a residual subleading (NNLL) term?**



# test matrix-element with subjet azimuthal difference

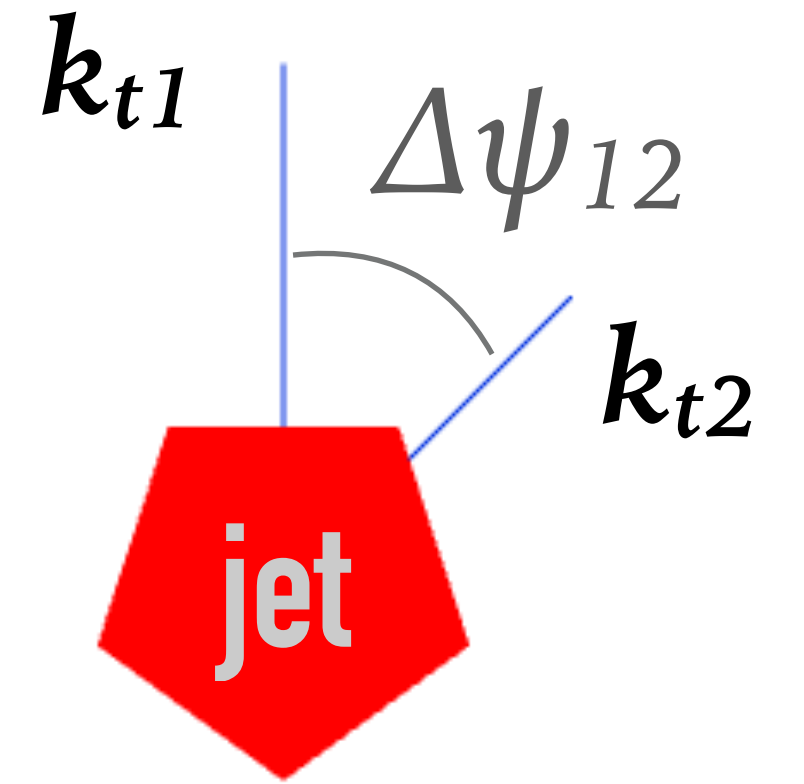


ratio to NLL

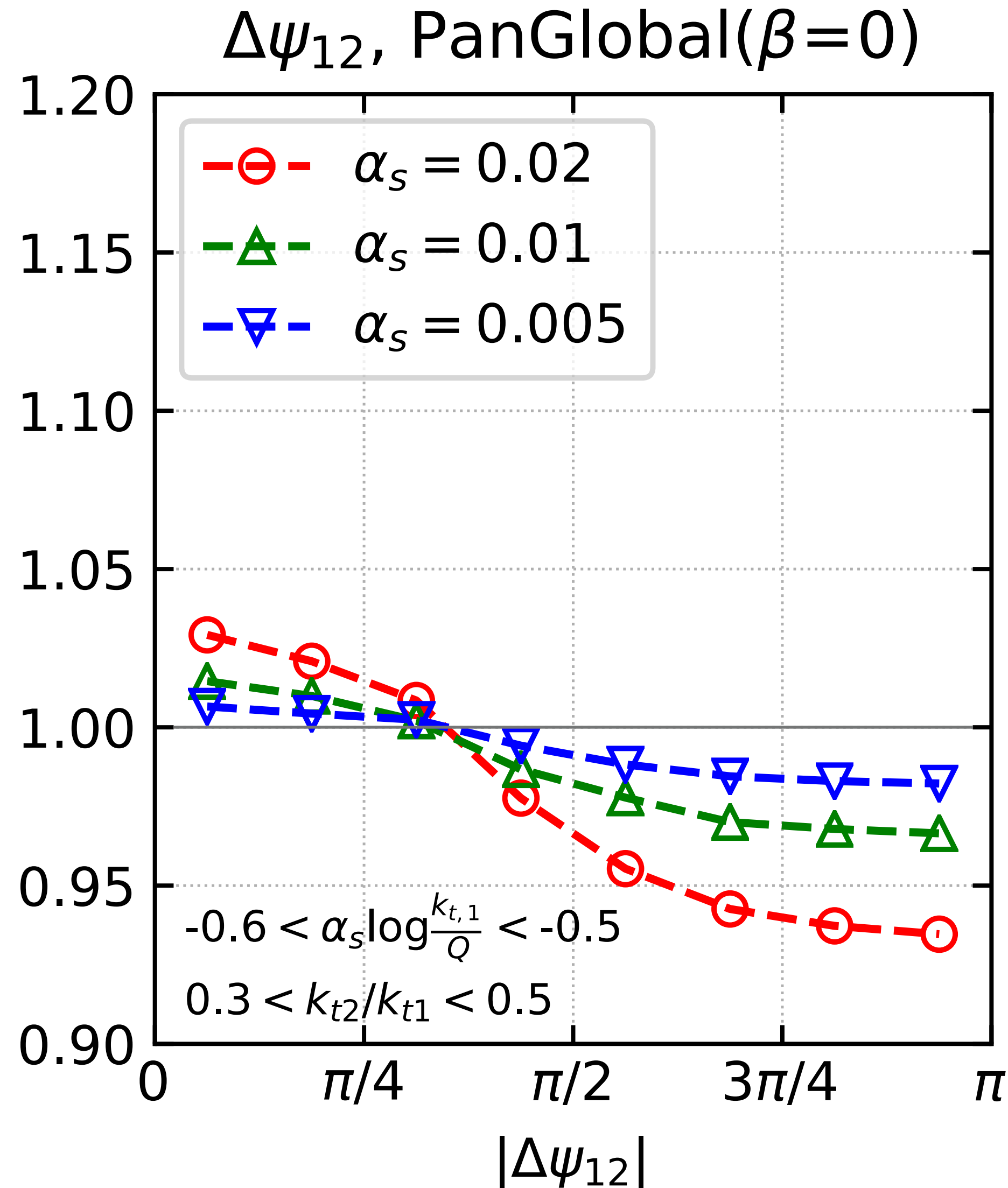


- ▶ run full shower with specific value of  $\alpha_s(Q)$
- ▶ ratio to NLL should be flat  $\equiv 1$
- ▶ it isn't: have we got an NLL mistake? Or a residual subleading (NNLL) term?
- ▶ **try halving  $\alpha_s(Q)$** , while keeping constant  $\alpha_s L$  [ $L \equiv \ln k_{t1}/Q$ ]
- ▶ **NLL effects,  $(\alpha_s L)^n$ , should be unchanged, subleading ones,  $\alpha_s(\alpha_s L)^n$ , halved**

# test matrix-element with subjet azimuthal difference

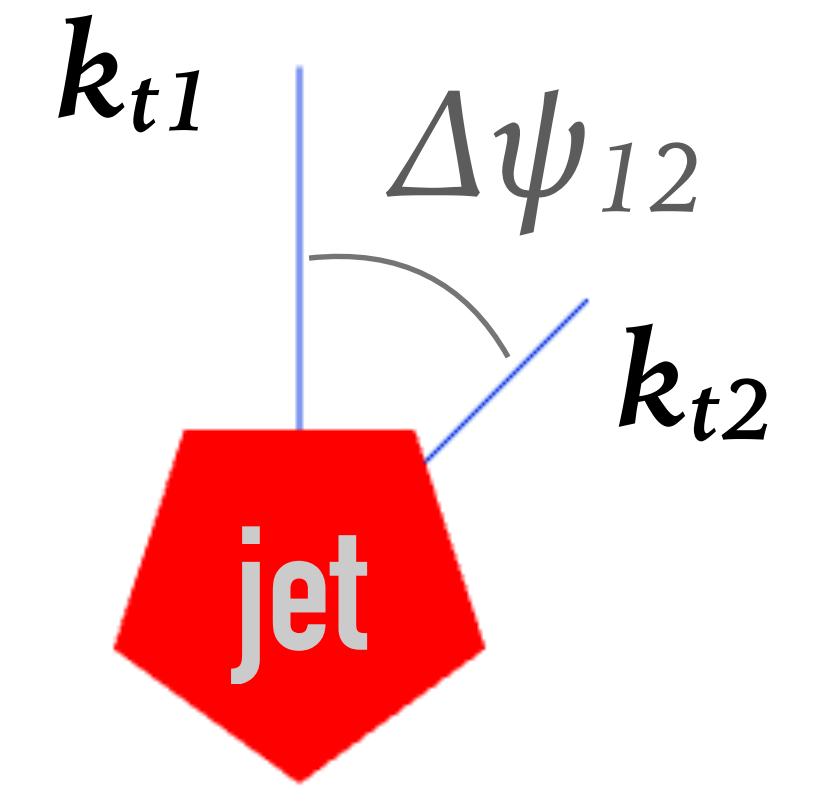


ratio to NLL

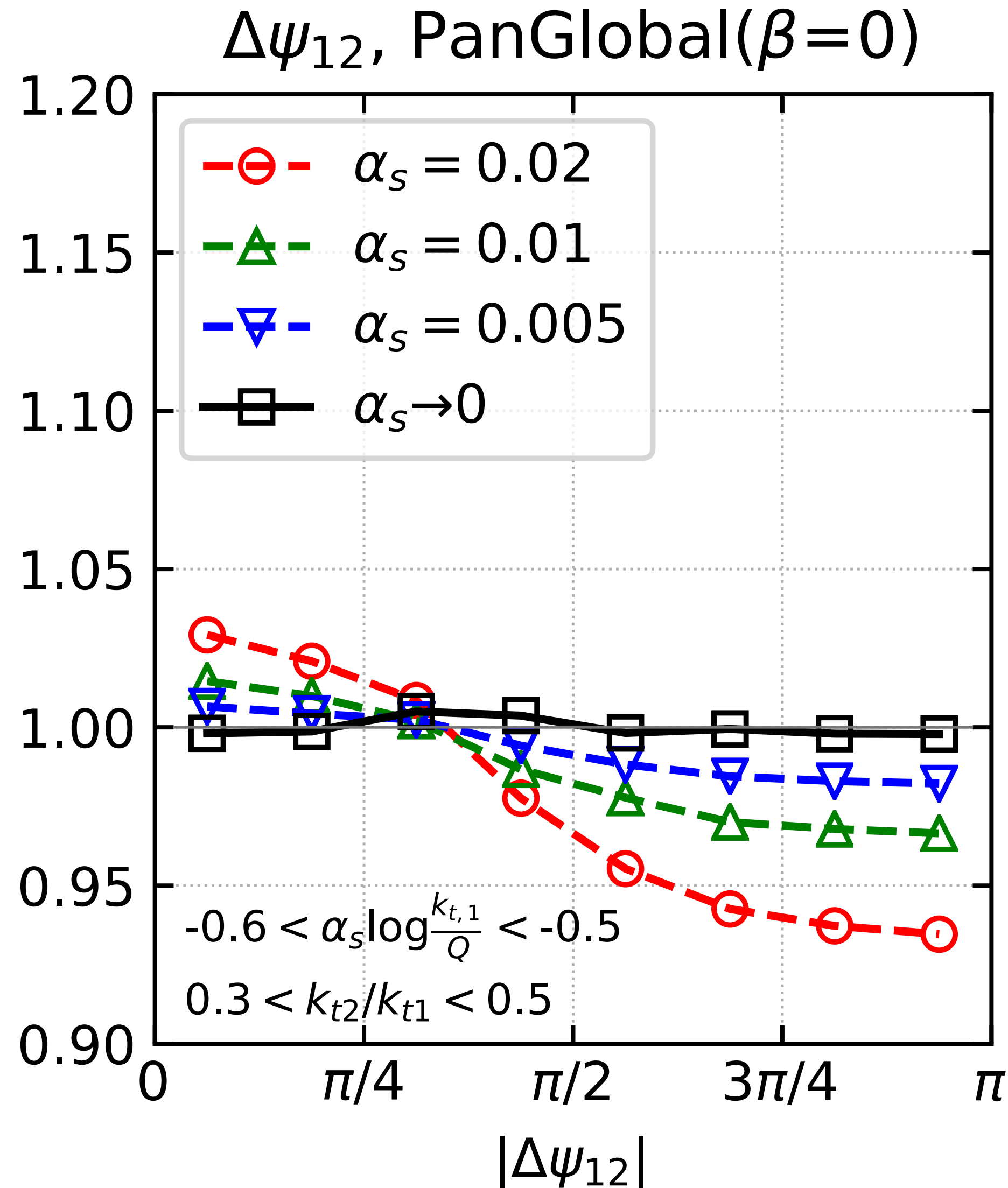


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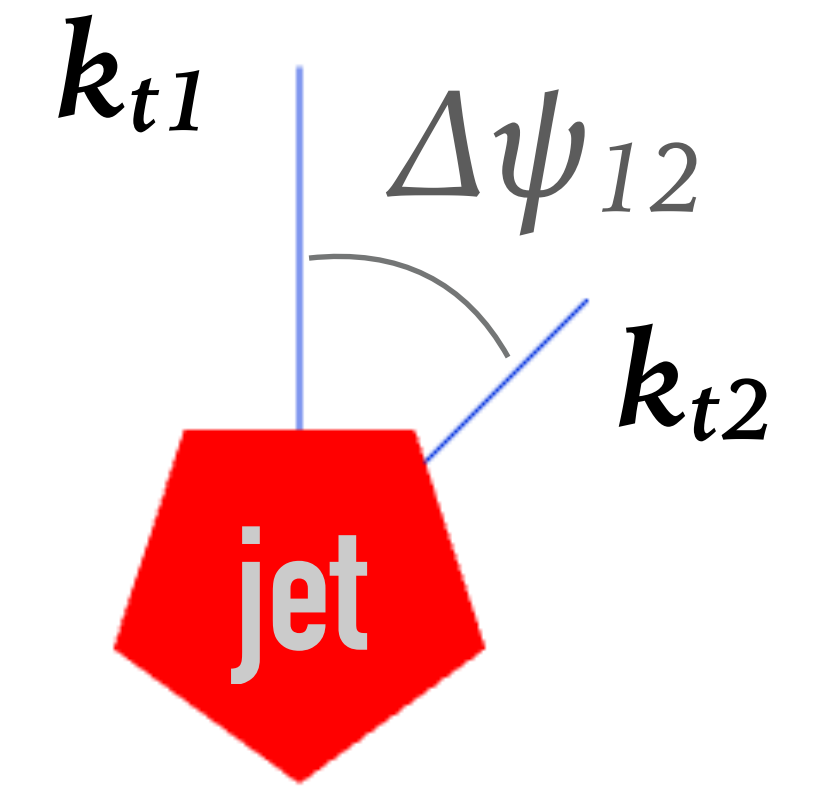


ratio  
to  
NLL

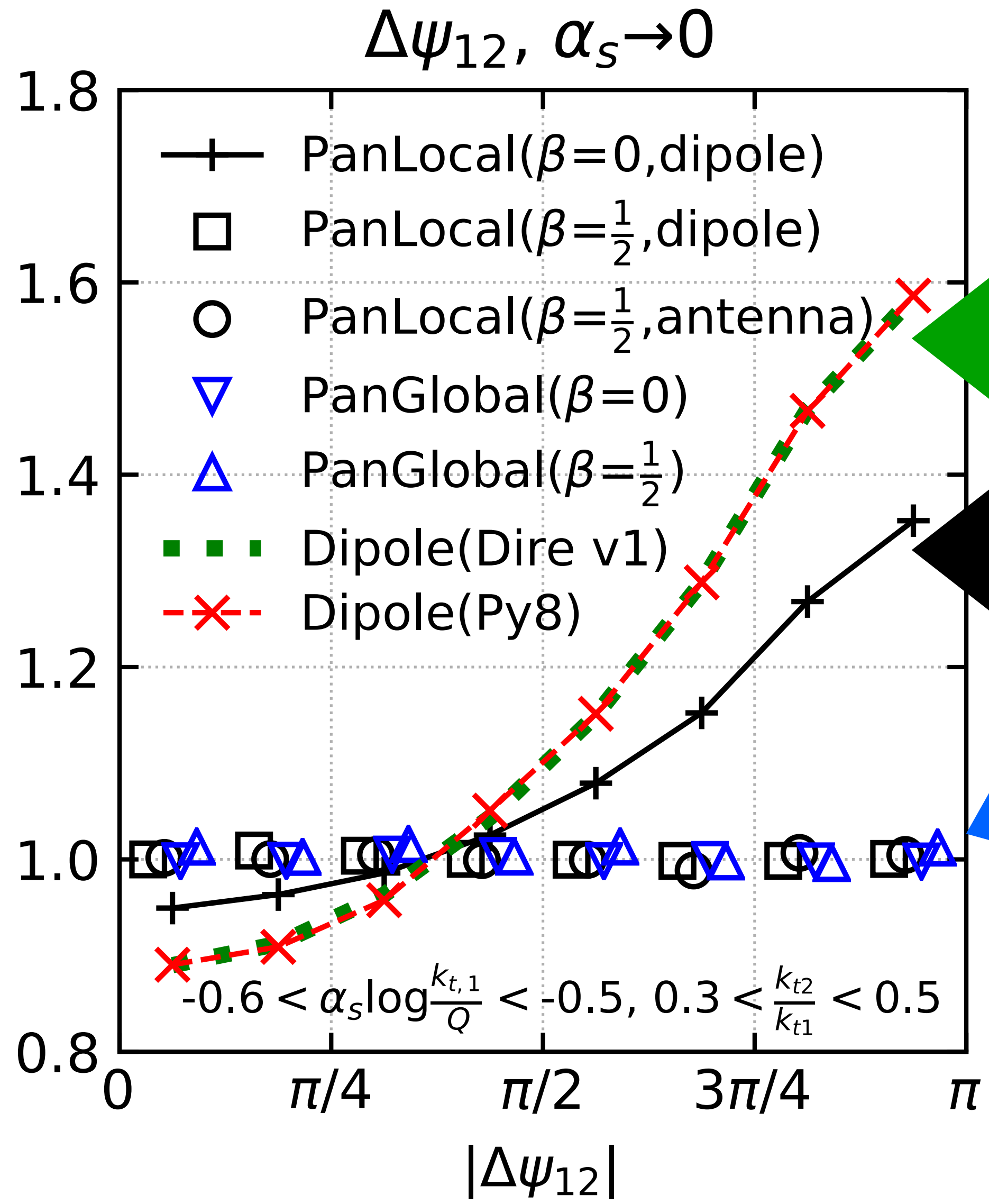


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- ▶ NLL effects,  $(\alpha_s L)^n$ , should be unchanged, subleading ones,  $\alpha_s(\alpha_s L)^n$ , halved
- ✓ **extrapolation  $\alpha_s \rightarrow 0$  agrees with NLL**

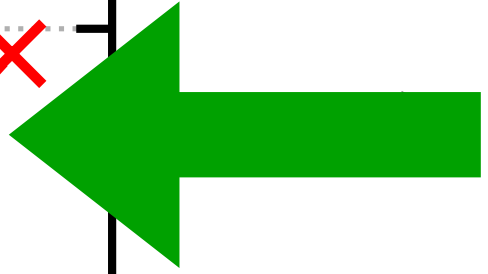
# test matrix-element with subjet azimuthal difference



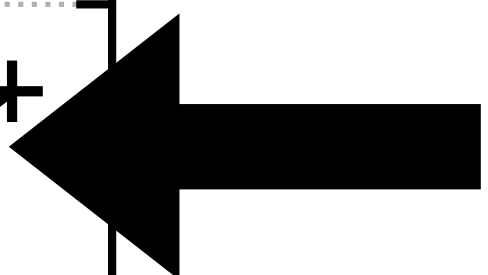
ratio to NLL



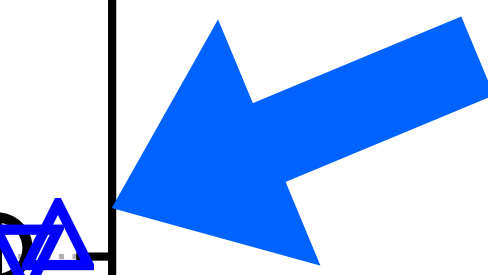
Now examine many showers



standard dipole showers (Pythia8, Dire-v1) disagree with NLL by up to 60%



PanLocal  $\beta = 0$  is also expected to disagree with NLL and does

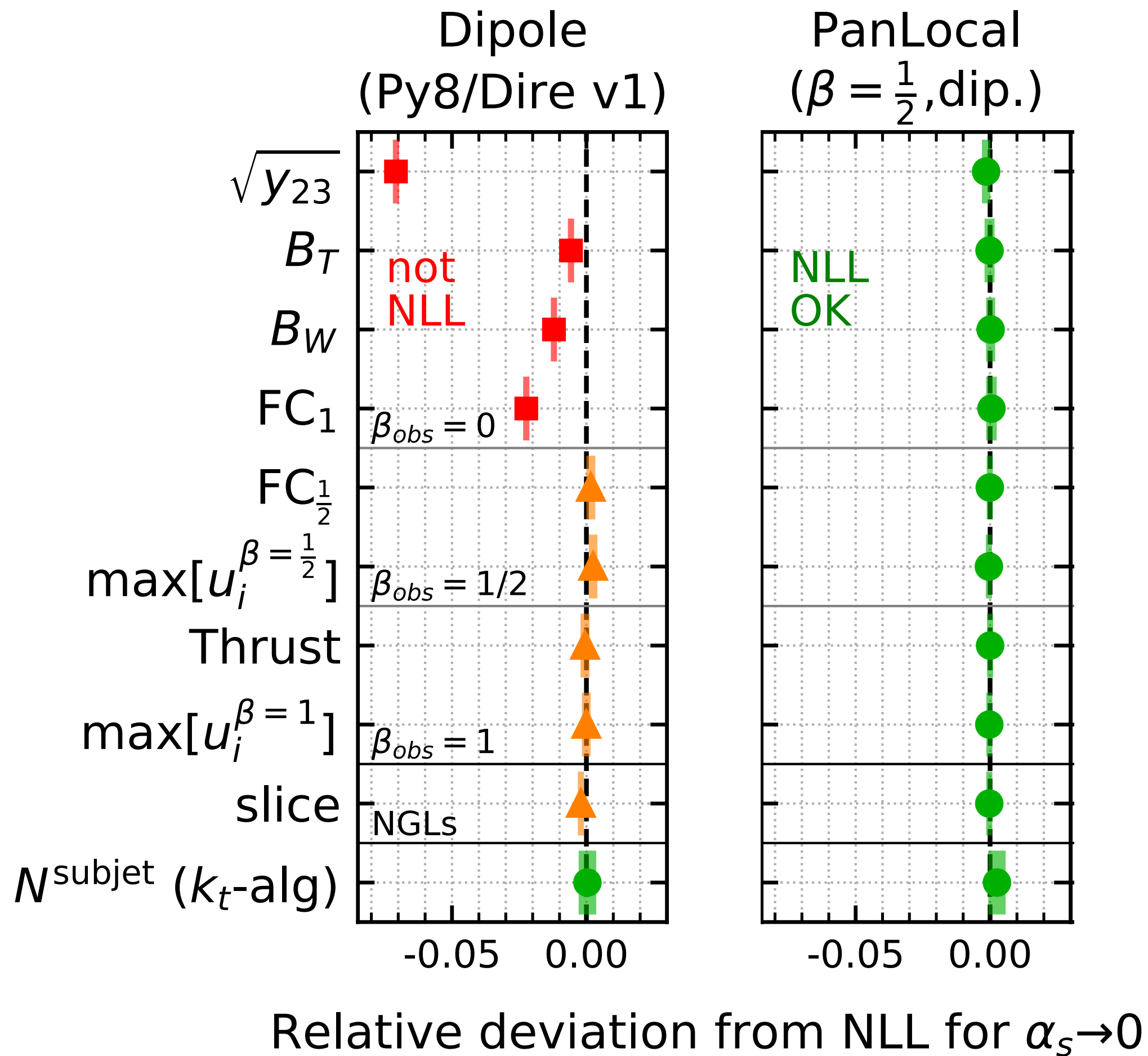


All other “PanScales” showers (with valid  $\beta$  values) agree with NLL

# Carry out similar shower/NLL ratio tests for many observables

**standard  
parton  
showers**

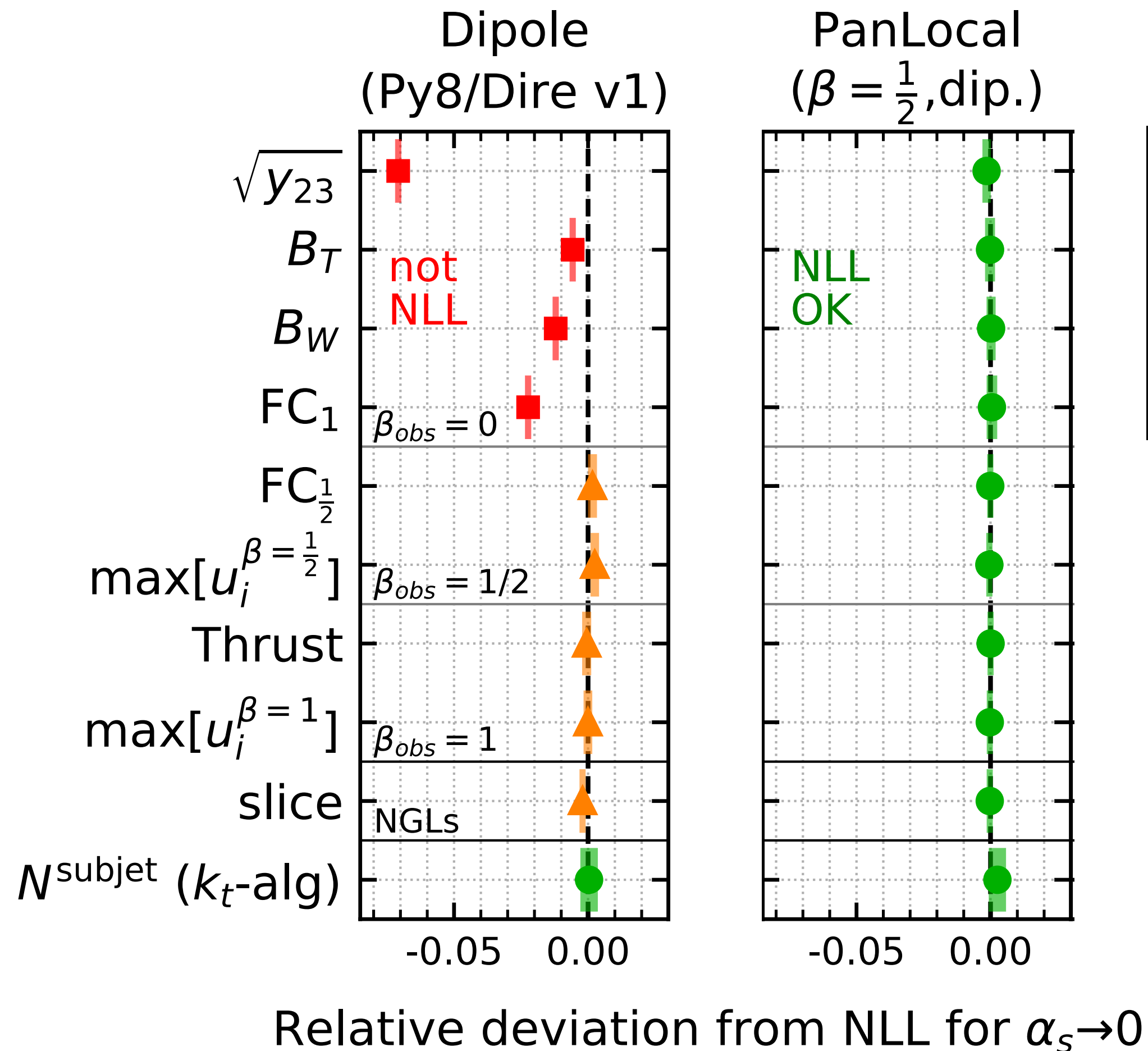
**new “PanScales” parton showers, designed  
specifically to achieve NLL accuracy**



# Carry out similar shower/NLL ratio tests for many observables

**standard  
parton  
showers**

**new “PanScales” parton showers, designed  
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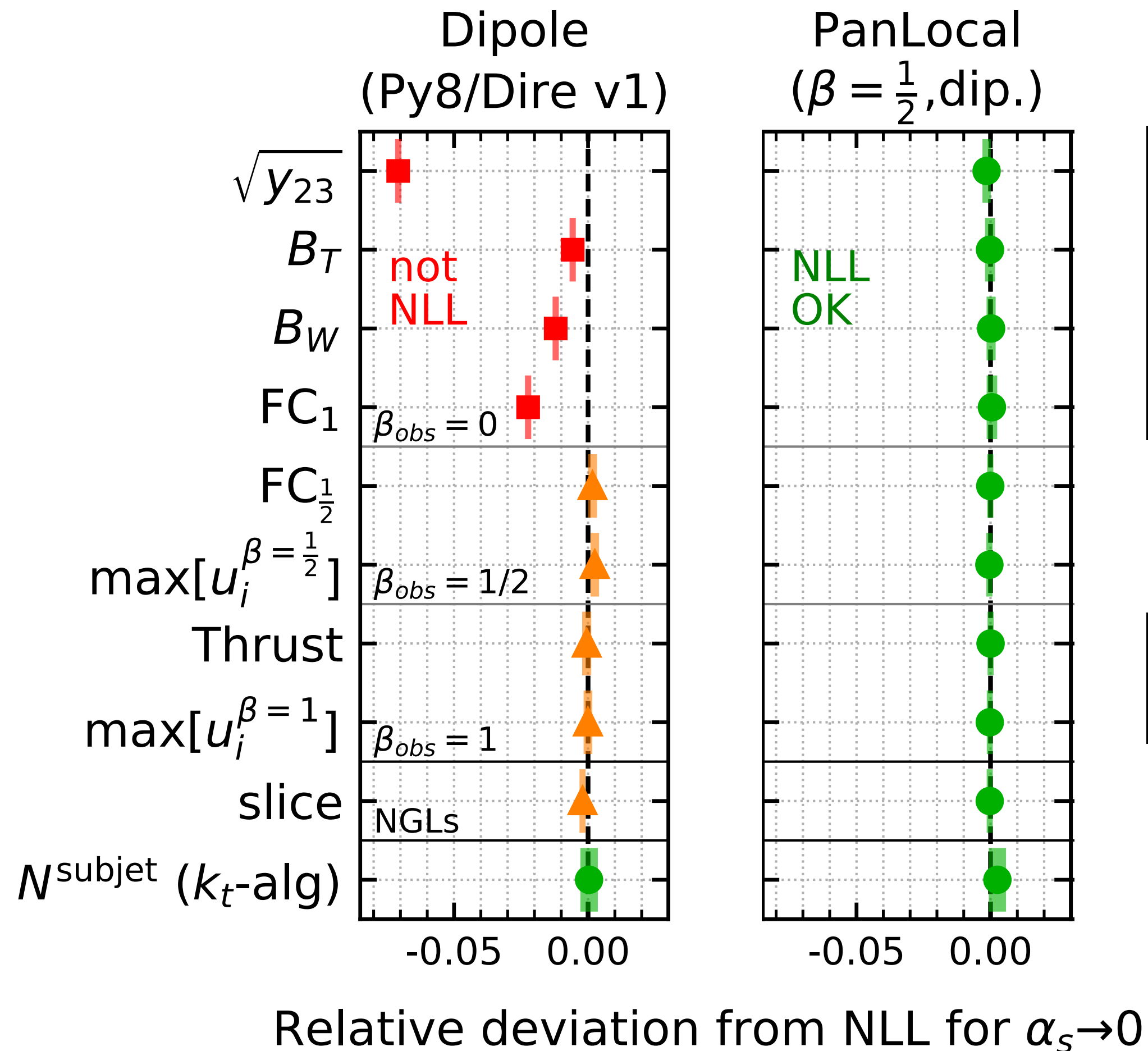


Event shapes sensitive to transverse momentum  
(jet broadenings, jet clustering transitions)

# Carry out similar shower/NLL ratio tests for many observables

**standard  
parton  
showers**

**new “PanScales” parton showers, designed  
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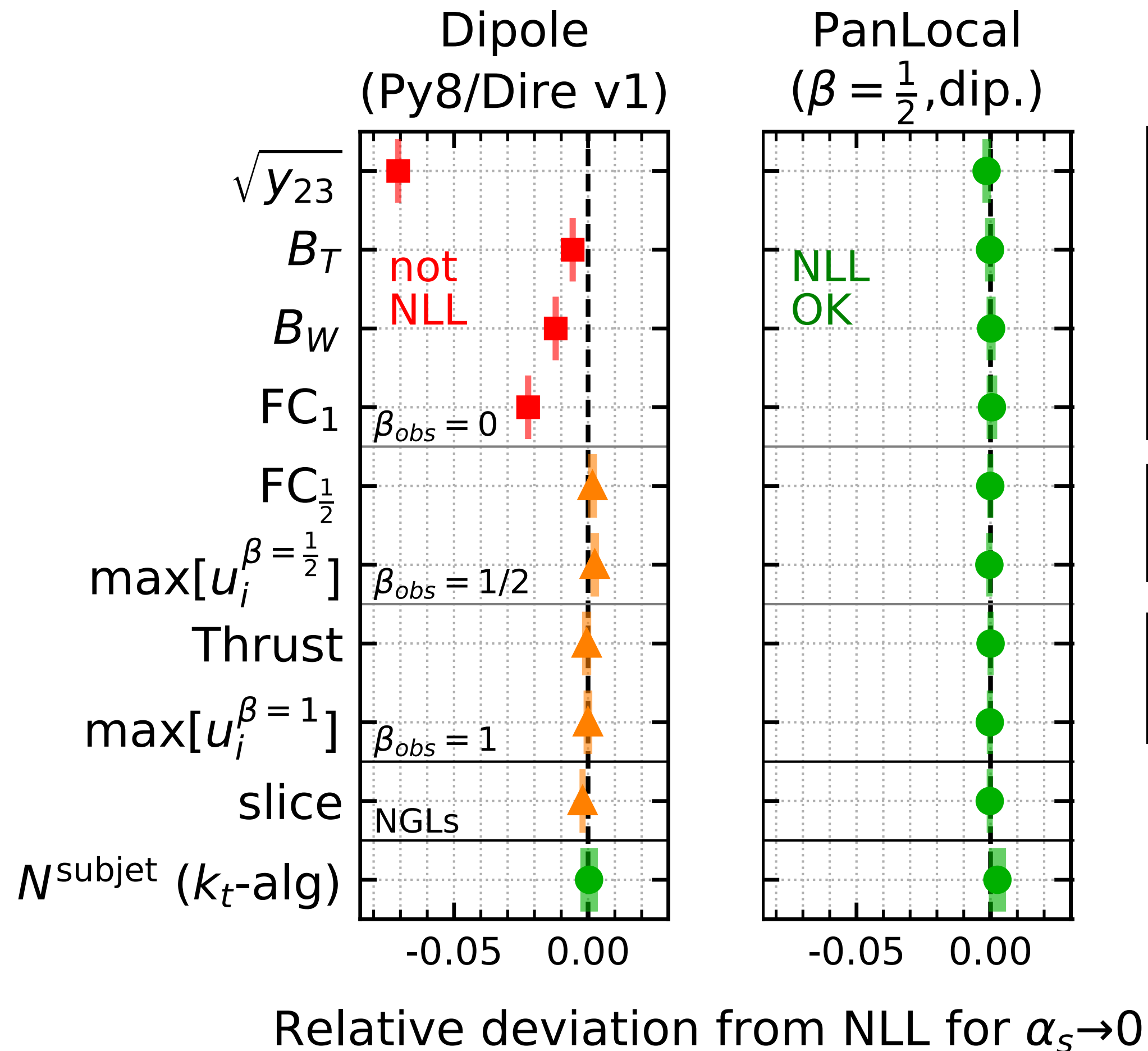
Event shapes sensitive to transverse momentum  
(jet broadenings, jet clustering transitions)

Event shapes like thrust

# Carry out similar shower/NLL ratio tests for many observables

standard  
parton  
showers

new “PanScales” parton showers, designed  
specifically to achieve NLL accuracy



Event shapes sensitive to transverse momentum  
(jet broadenings, jet clustering transitions)

Event shapes that probe  $p_t e^{-0.5|\eta|}$   
(like  $\beta = 0.5$  ordering variable)

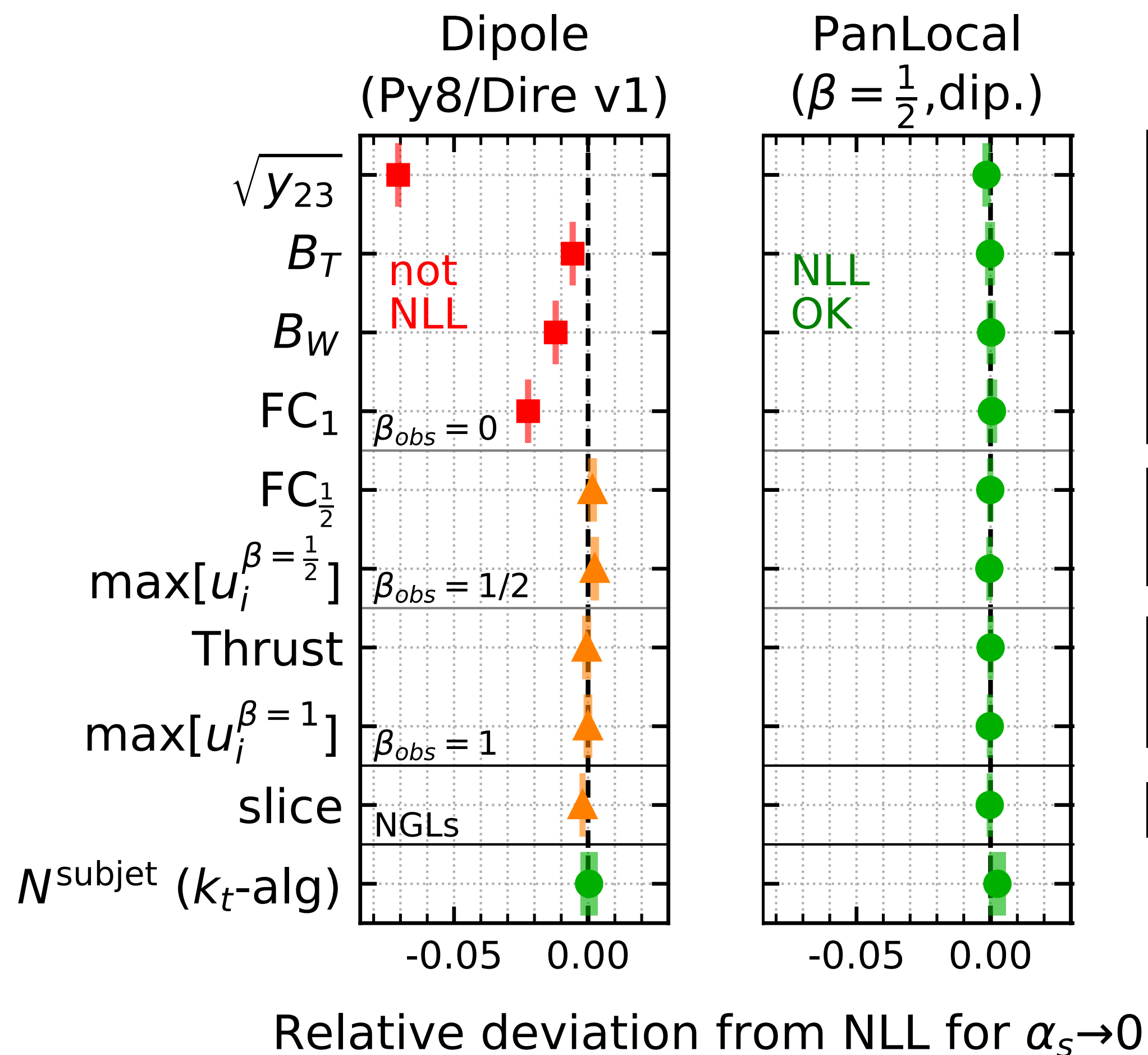
Event shapes like thrust



# Carry out similar shower/NLL ratio tests for many observables

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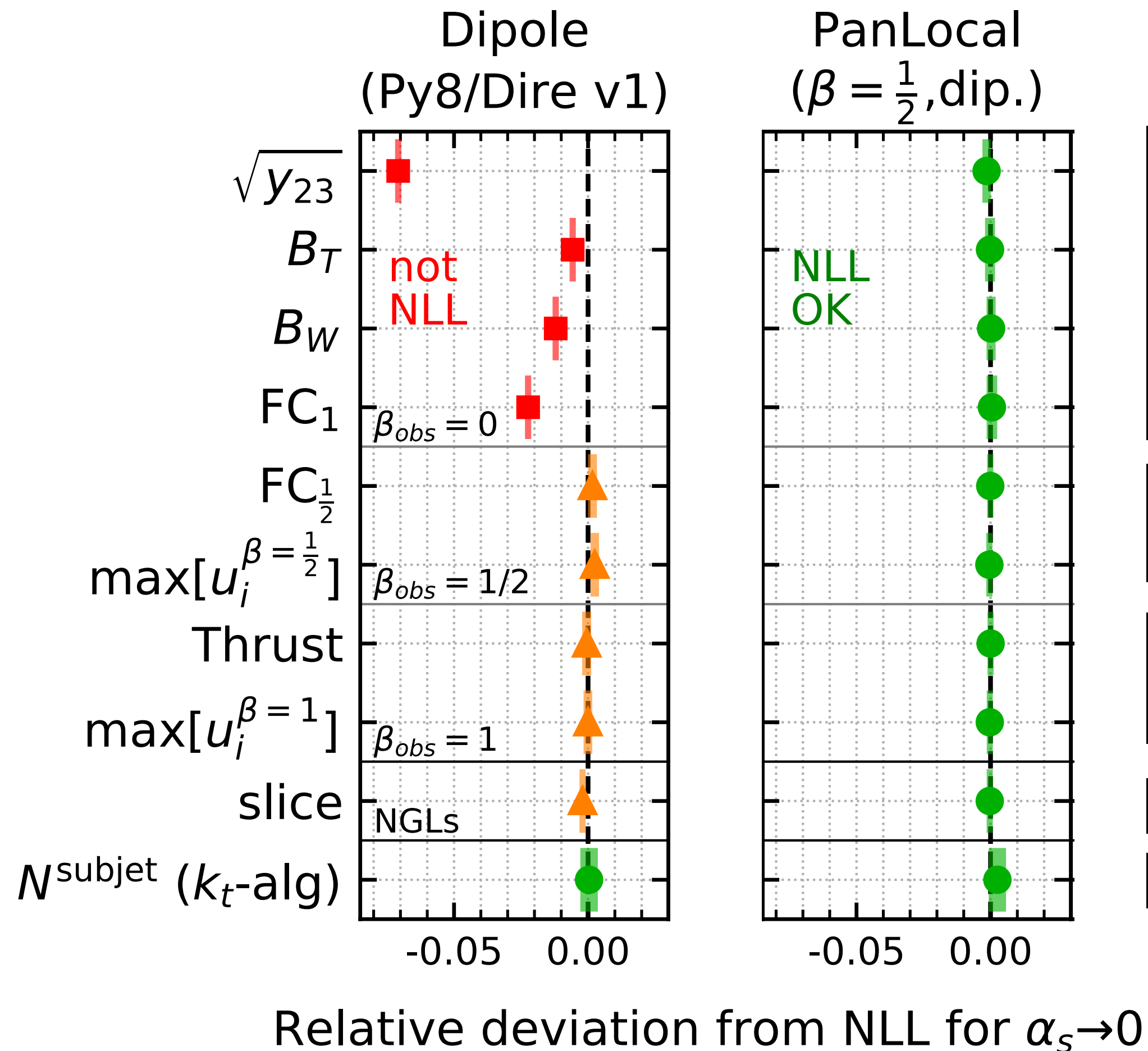
Event shapes like thrust

probe of non-global logarithms

# Carry out similar shower/NLL ratio tests for many observables

standard  
parton  
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new “PanScales” parton showers, designed  
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Event shapes sensitive to transverse momentum  
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Event shapes that probe  $p_t e^{-0.5|\eta|}$   
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Event shapes like thrust

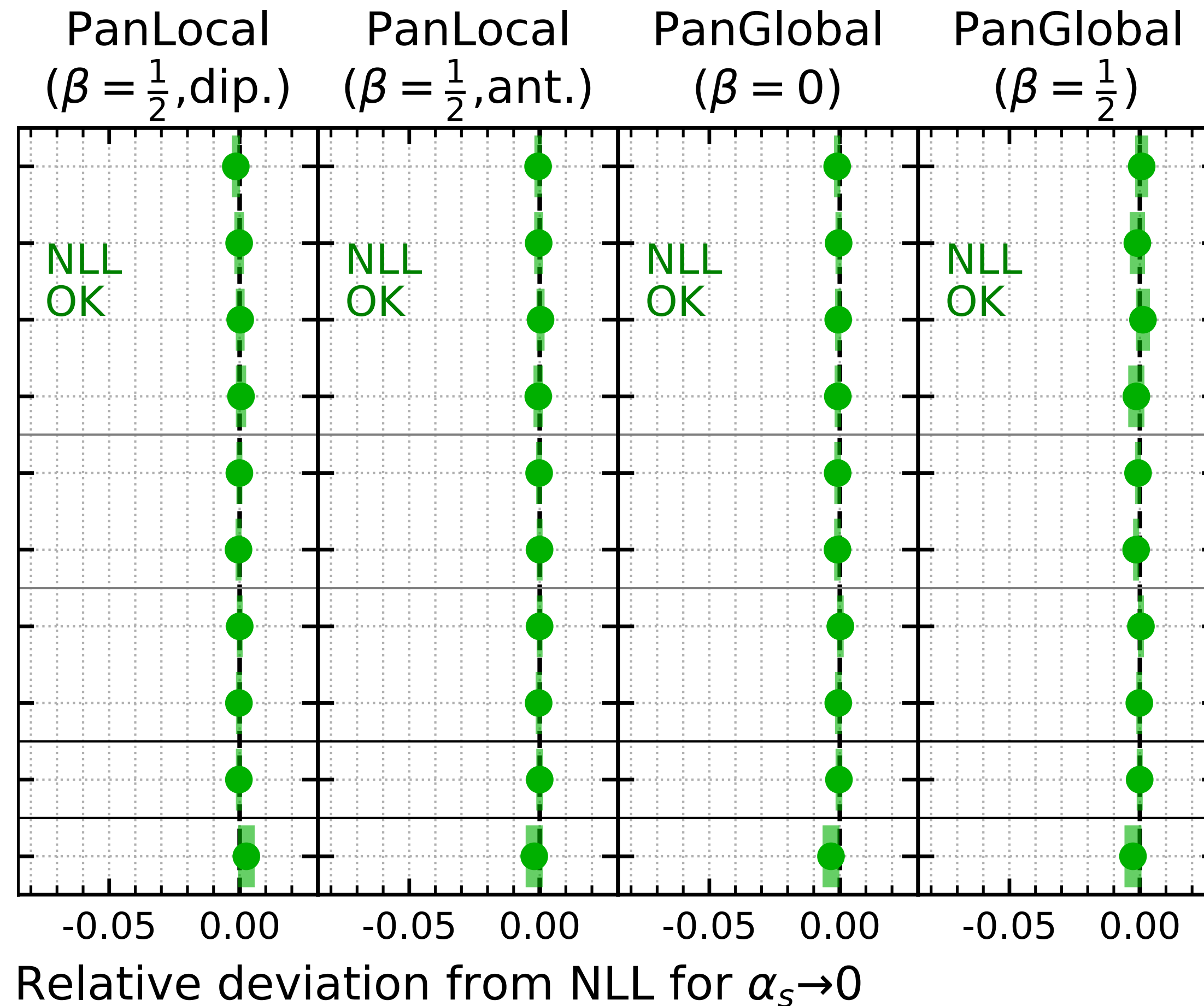
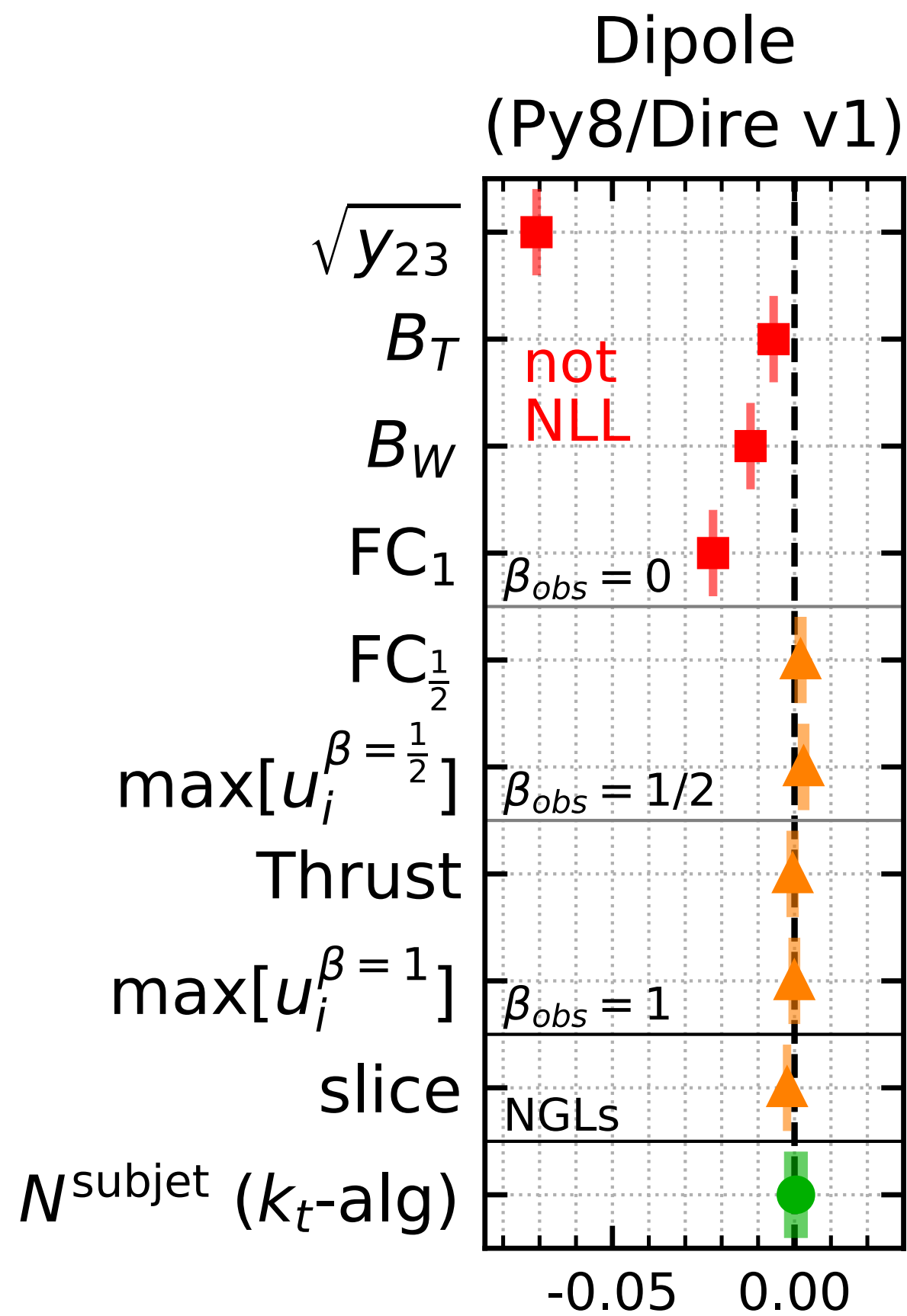
probe of non-global logarithms

standard jet multiplicity (probe of full recursive  
shower structure)

# Carry out similar shower/NLL ratio tests for many observables

**standard  
parton  
showers**

**new “PanScales” parton showers, designed  
specifically to achieve NLL accuracy**



*All PanScales shower  
that are expected to  
agree with NLL pass  
these tests*

*(Standard dipole  
showers don't)*

# Conclusions

# conclusions

---

- Parton showers (and event generators in general), and their predictions of the fine structure of events, are an essential part of LHC's very broad physics programme
- Pas two decades have seen much progress at the fixed-order / shower interface
- But understanding of shower logarithmic accuracy has been elusive
- Minimal baseline for progress beyond 1980's technology is to achieve NLL accuracy  $\equiv$  control of terms  $(\alpha_s L)^n$  — such terms are  $\mathcal{O}(1)$  for  $\alpha_s L \sim 1$ .
- We've demonstrated that is possible (with some caveats, final-state showers only, spin correlations still missing, leading- $N_C$  approx.)
- Essential elements that we hope can be of wider use:
  - concrete criteria for specifying log accuracy
  - core guiding principles that help achieve NLL accuracy
  - powerful numerical approach to demonstrating shower accuracy

**BACKUP**

# A side-note about running with small $\alpha_s$

---

- If you keep  $\alpha_s L$  fixed,  $\alpha_s \rightarrow 0$  implies  $L \rightarrow \infty$
- In practice  $\alpha_s L = -0.5$  with  $\alpha_s = 0.005$  gives  $L = -100$ ,  
**i.e. transverse momenta  $\sim 10^{-44} Q$**  (e.g. at LHC minimum  $p_t \sim 10^{-5} Q$ )
- normal showers aren't designed to work over such a range of scales,  
quite a few problems needed to be solved ( $\sim 1.5$  years' thought & work)
- but if you want a numerical test of NLL (and, subsequently, higher orders), then you need a framework that can address such challenges

NB: to study Pythia8 / Dire-v1 dipole showers, we had to recode them  
(based on descriptions in papers & inspection of their code)

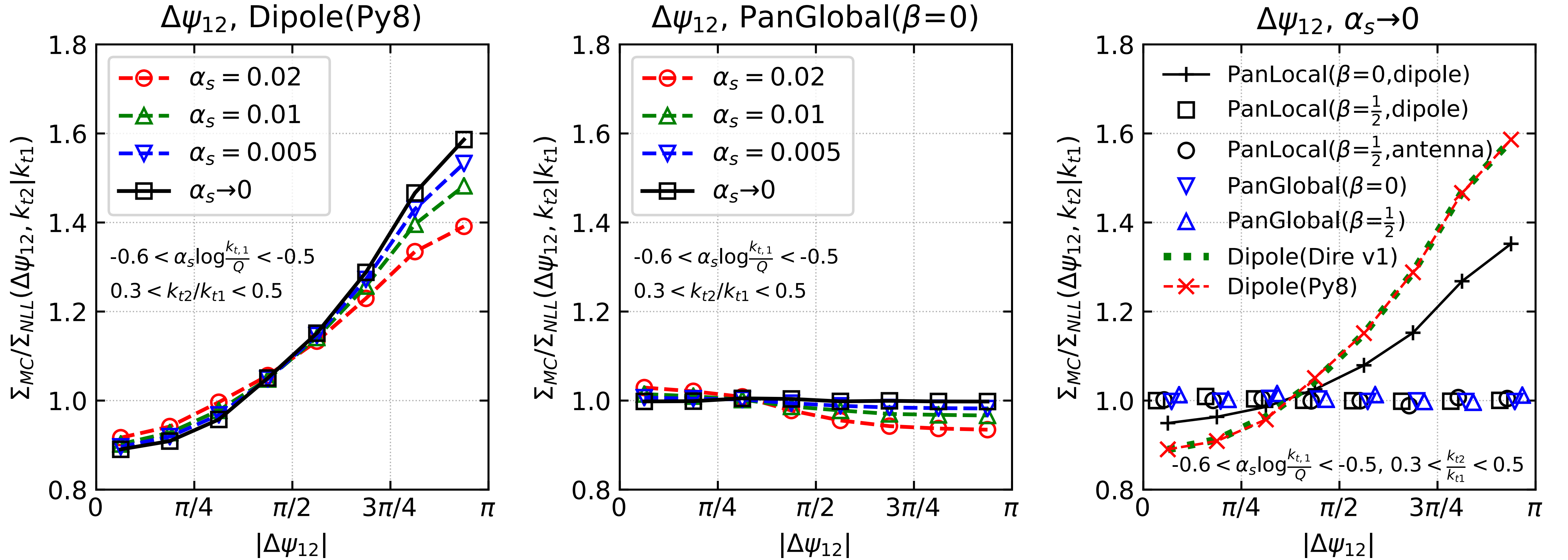


FIG. 1. Left: distribution for the difference in azimuthal angle between the two highest- $k_t$  primary Lund declusterings in the Pythia8 dipole shower algorithm, normalised to the NLL result [53], [51]§ 4; successively smaller  $\alpha_s$  values keep fixed  $\alpha_s \ln k_{t1}$ . Middle: the same for the PanGlobal( $\beta = 0$ ) shower. Right: the  $\alpha_s \rightarrow 0$  limit of the ratio for multiple showers. This observable directly tests part of our NLL (squared) matrix-element correctness condition. A unit value for the ratio signals success.



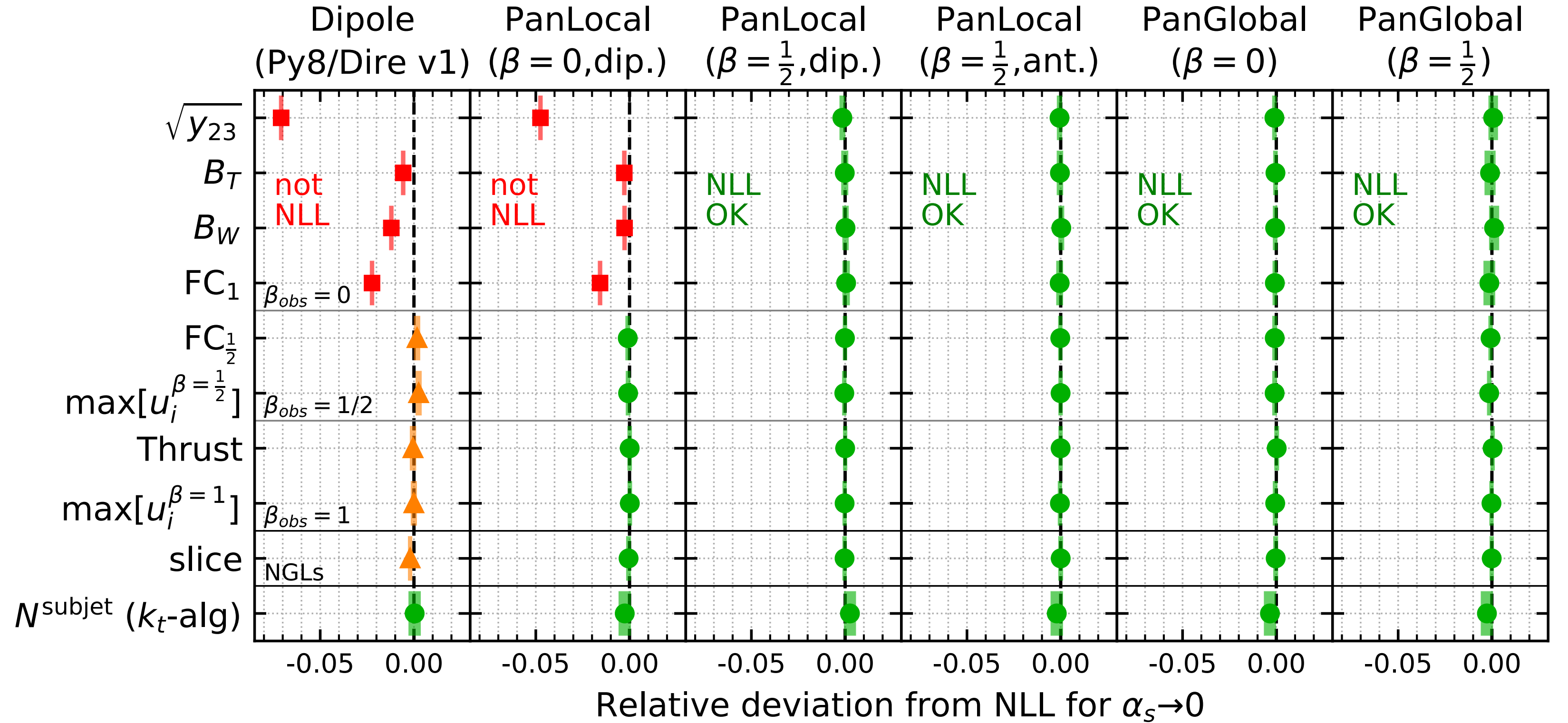
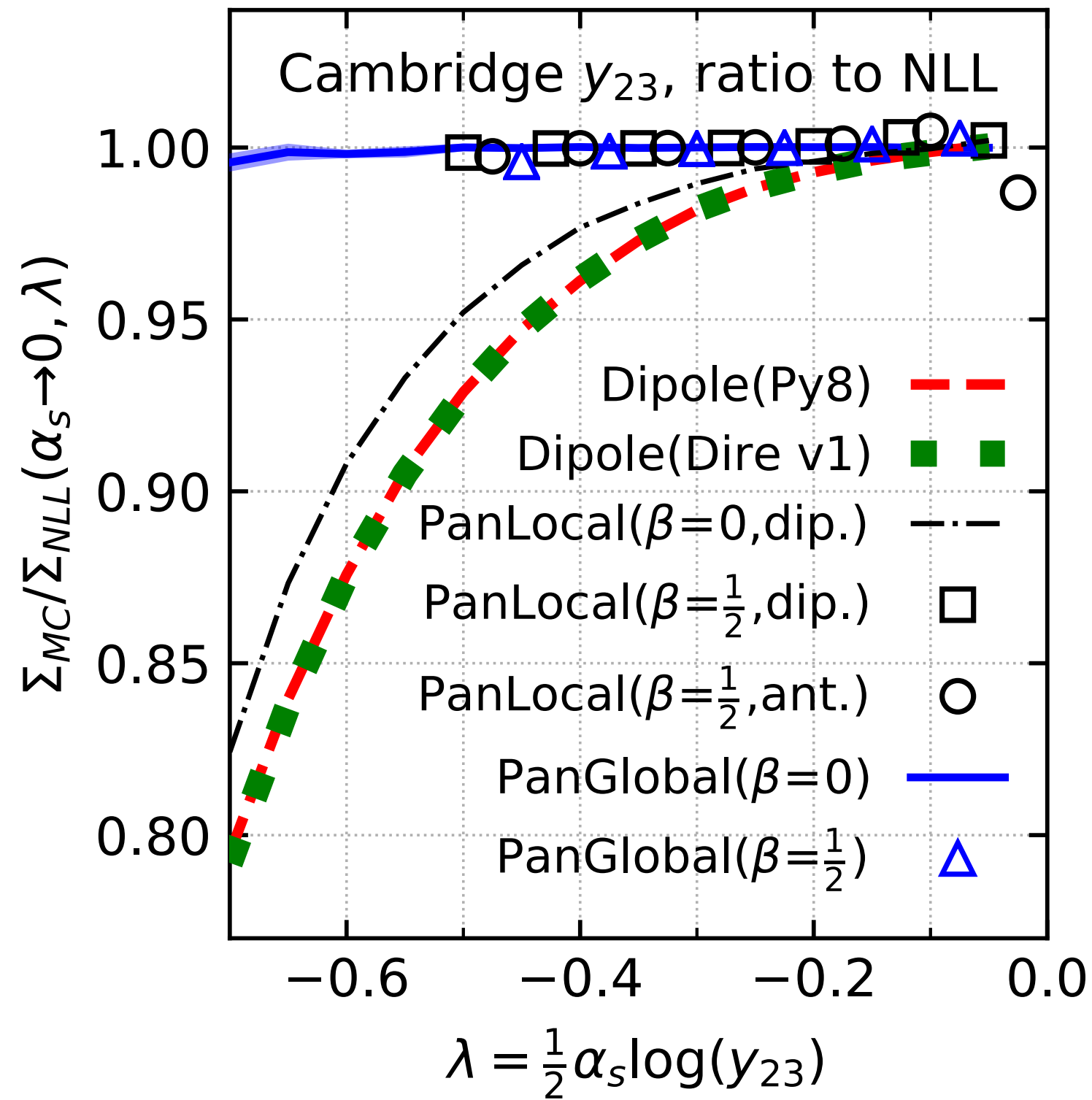


FIG. 2. Left: ratio of the cumulative  $y_{23}$  distribution from several showers divided by the NLL answer, as a function of  $\alpha_s \ln y_{23}/2$ , for  $\alpha_s \rightarrow 0$ . Right: summary of deviations from NLL for many shower/observable combinations (either  $\Sigma_{\text{shower}}(\alpha_s \rightarrow 0, \alpha_s L = -0.5)/\Sigma_{\text{NLL}} - 1$  or  $(N_{\text{shower}}^{\text{subject}}(\alpha_s \rightarrow 0, \alpha_s L^2 = 5)/N_{\text{NLL}}^{\text{subject}} - 1)/\sqrt{\alpha_s}$ ). Red squares indicate clear NLL failure; amber triangles indicate NLL fixed-order failure that is masked at all orders; green circles indicate that all NLL tests passed.

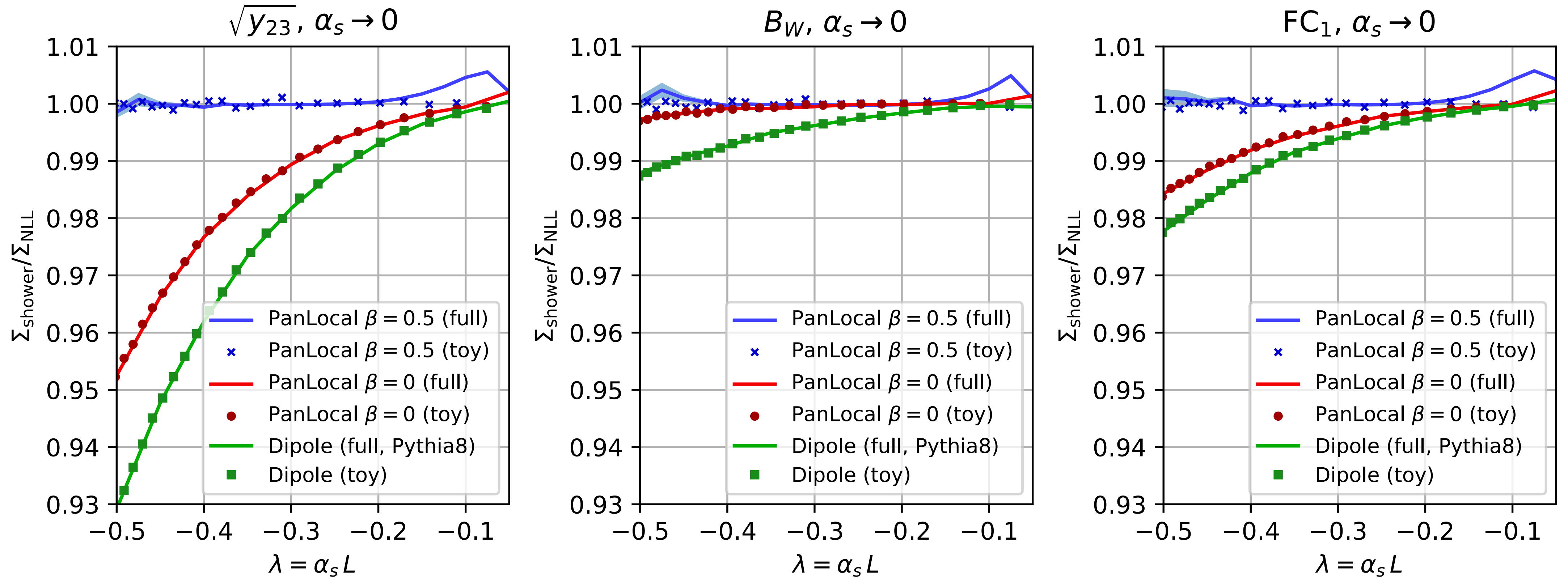


FIG. 3. Comparison of the ratio  $\Sigma_{\text{shower}}/\Sigma_{\text{NLL}}$  between the toy shower and the full shower for three reference observables ( $\sqrt{y_{23}}$ ,  $B_W$  and  $FC_1$ ), in the limit  $\alpha_s \rightarrow 0$ , as a function of  $\alpha_s L$ . For the full showers the figure shows the ratio of the shower prediction to the full NLL result, while for the toy shower it shows the ratio to the CAESAR-like toy shower. Three full showers are shown in each plot, each compared to the corresponding toy shower. The PanLocal full showers are shown in their dipole variants (identical conclusions hold for the antenna variant). Small (0.5%) issues at  $\lambda \gtrsim -0.1$  are a consequence of the fact that for the largest of the  $\alpha_s$  values used in the extrapolation, the corresponding  $L$  values do not quite satisfy  $e^L \ll 1$ .

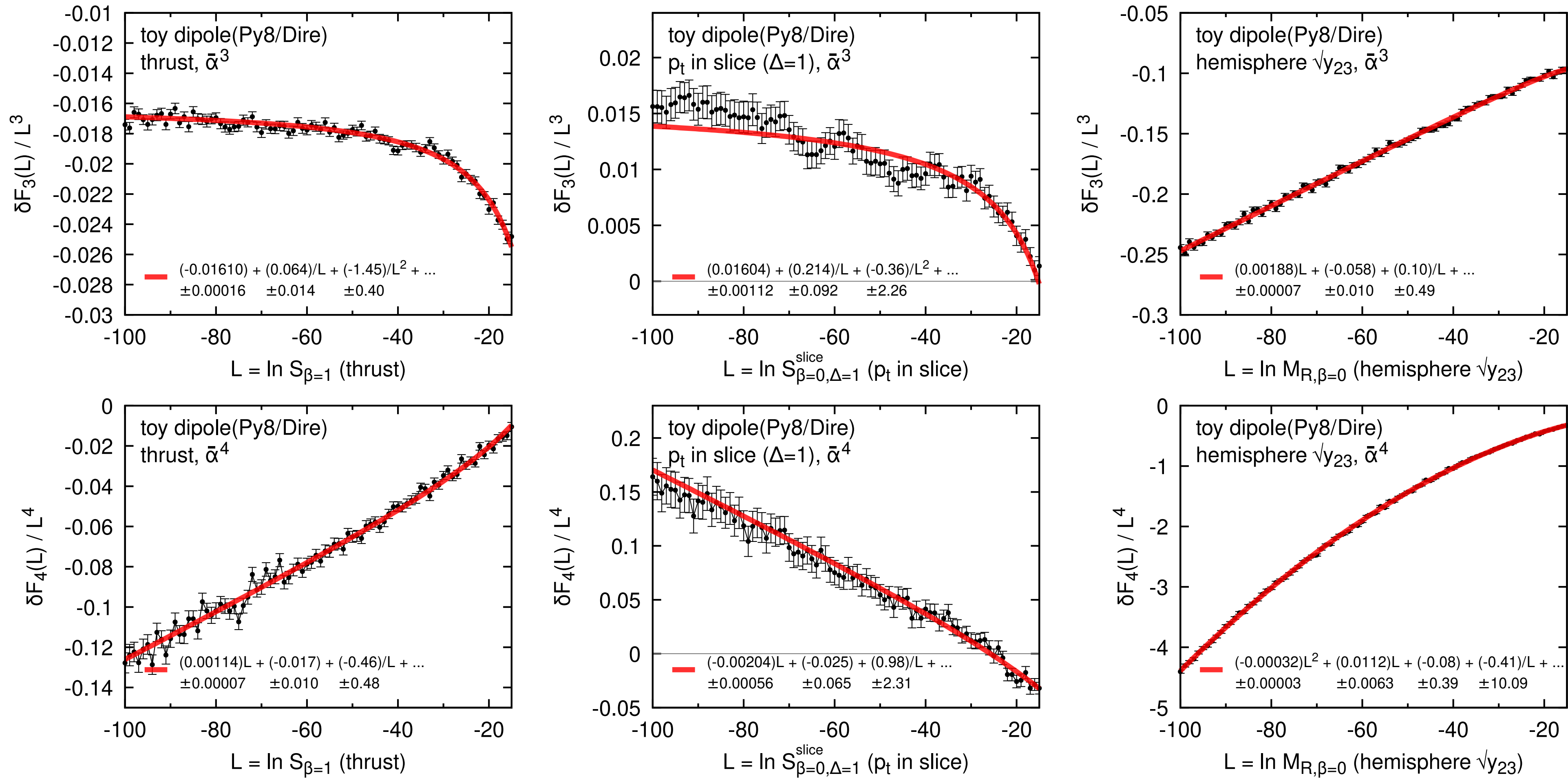


FIG. 4. Fixed order results from the toy implementation of the standard dipole showers. The plots show the difference between the toy dipole shower and the (NLL-correct) CAESAR results for the  $F_n$  coefficient of  $\bar{\alpha}^n$  in the expansion of Eq. (33), divided by  $L^n$ . For an NLL-correct shower, the results should tend to zero for large negative  $L$ . The first row shows the result of  $n = 3$ , the second row that of  $n = 4$ . The columns correspond to different observables (thrust, slice transverse momentum and hemisphere  $\sqrt{y_{23}}$ ). Observe how the results tend to constants (NLL discrepancy) or demonstrate a linear or even quadratic dependence on  $L$  (super-leading logarithms). The coefficients have been fitted taking into account correlations between points, and we include powers down to  $L^{-3}$  in the fit of  $\delta F_n / L^n$ . The fit range is from  $-100$  to  $-5$  and the quoted error includes both the (statistical) fit uncertainty and the difference in coefficients obtained with the range  $[-100, -10]$  (added in quadrature).

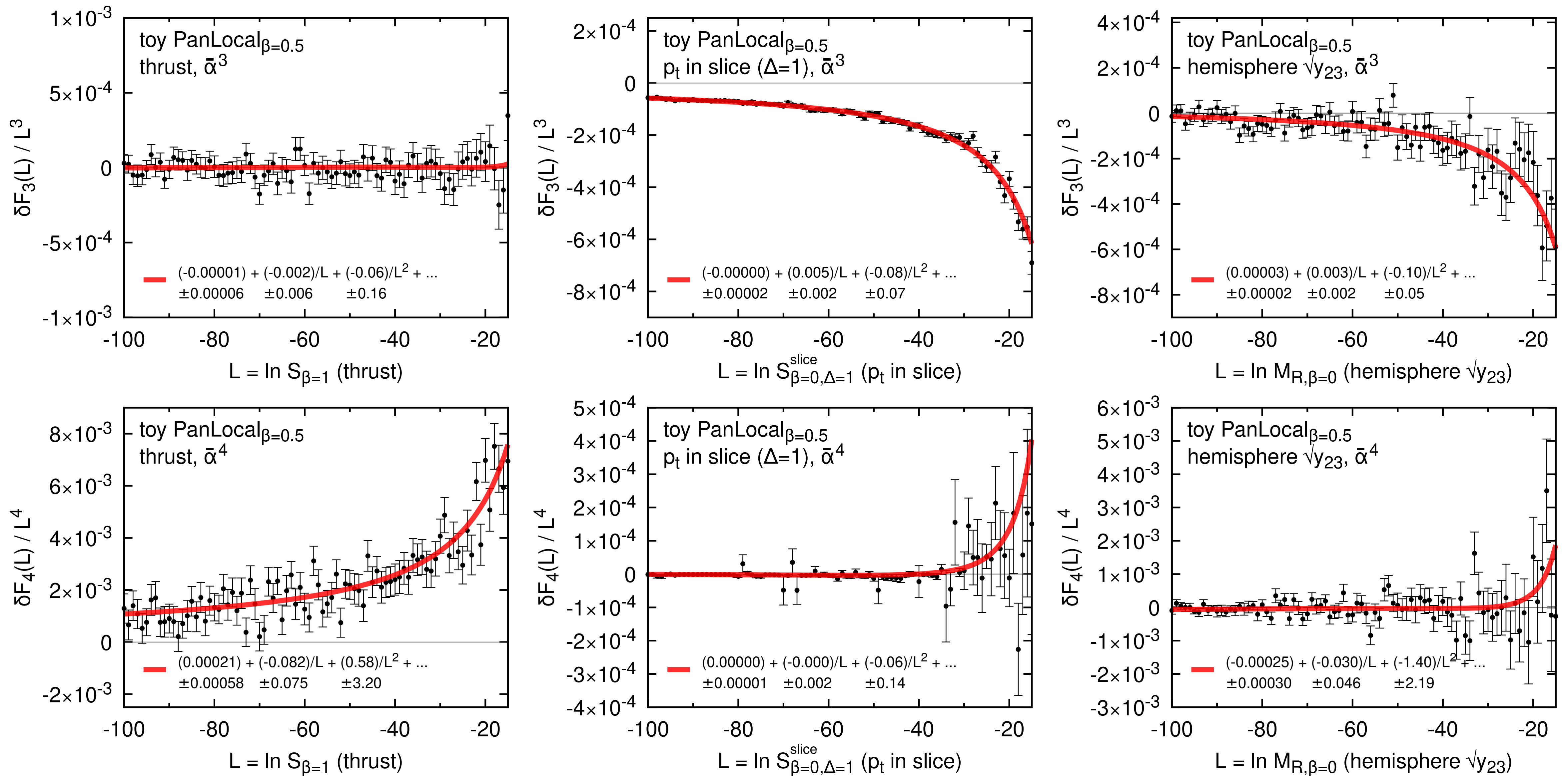


FIG. 5. Analogue of Fig. 4, demonstrating the absence of NLL (or super-leading) issues at fixed order in the toy version of the PanLocal  $\beta = 0.5$  shower. At order  $\bar{\alpha}^4$ , we include fit terms down to  $L^{-4}$ .

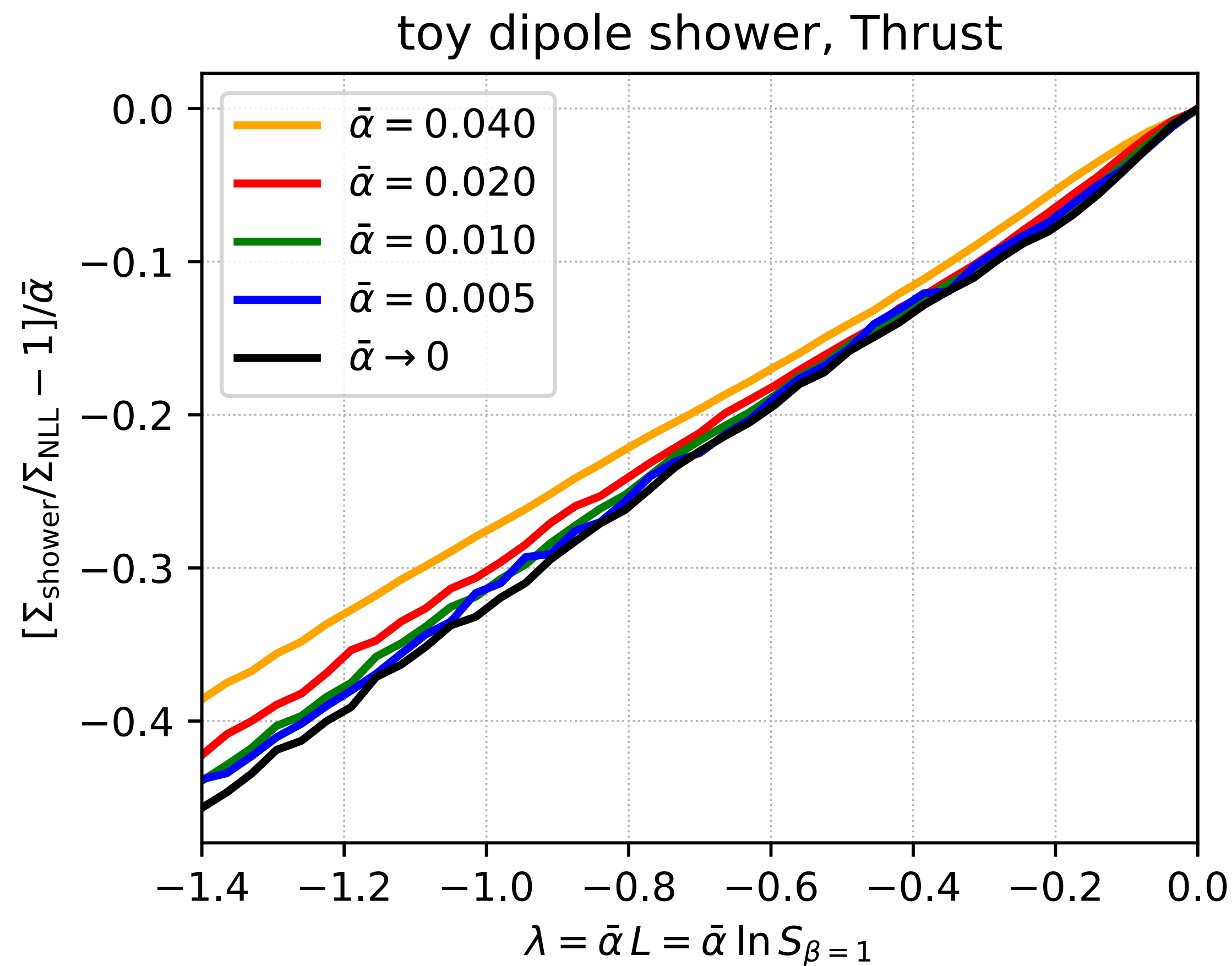
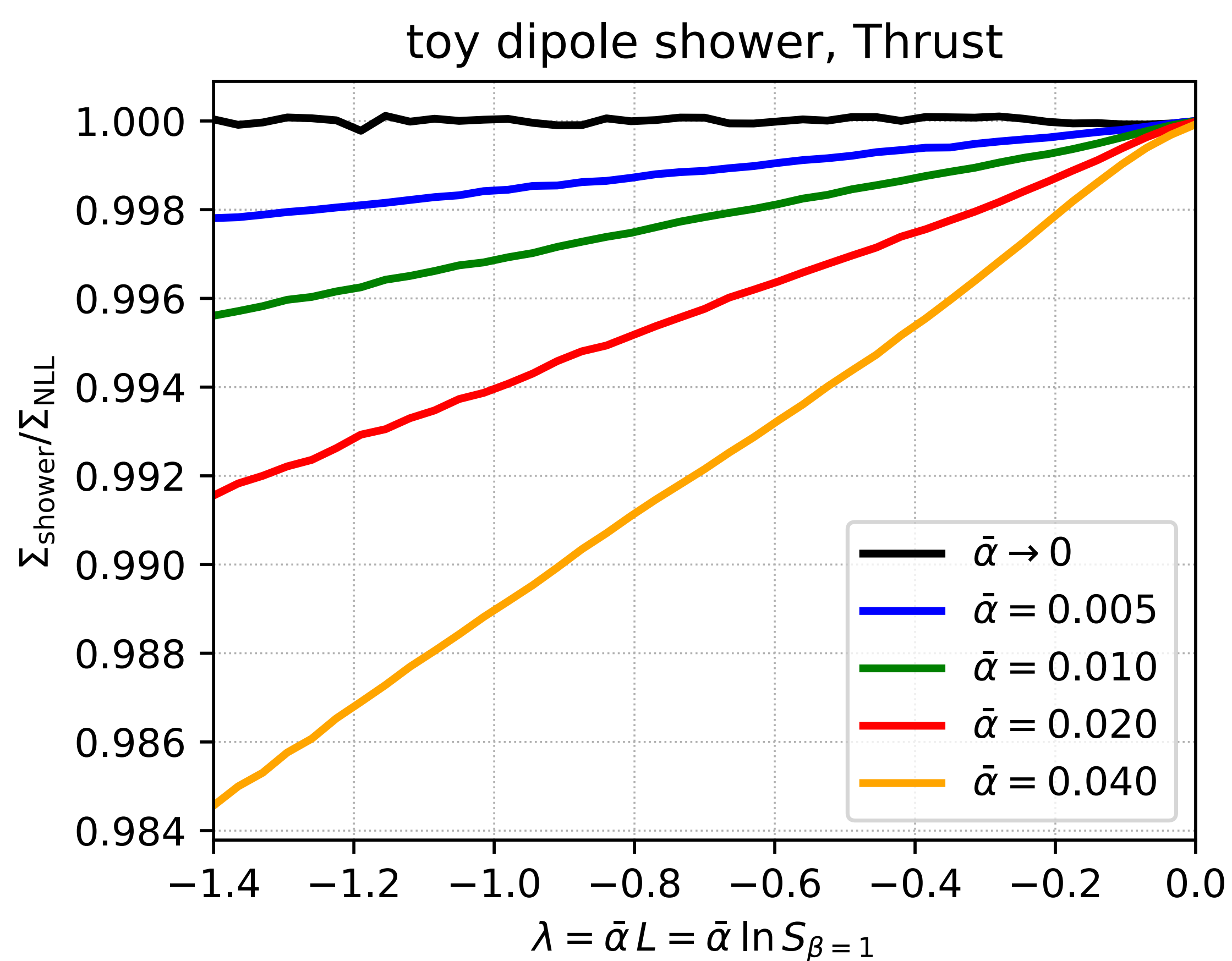


FIG. 7. Toy-shower all-order result for the thrust ( $S_{\beta=1}$ , Eq. (25)). Left:  $\Sigma_{\text{dipole}}/\Sigma_{\text{NLL}}$ , where the NLL result is given by running the CAESAR version of the shower. Four values of  $\bar{\alpha}$  are shown, together with the extrapolation to  $\bar{\alpha} = 0$ , showing that the all-order dipole-shower result (in our usual limit of fixed  $\bar{\alpha}L$  and  $\bar{\alpha} \rightarrow 0$ ) is consistent with the NLL result, despite the super-leading logarithmic terms that are visible in Fig. 4. Right:  $(\Sigma_{\text{dipole}}/\Sigma_{\text{NLL}} - 1)/\bar{\alpha}$ , again for three values of  $\bar{\alpha}$  and the extrapolation to  $\bar{\alpha} = 0$ . The fact that these curves converge is a sign that the all-order (toy) dipole-shower discrepancy with respect to NLL behaves as a term that vanishes proportionally to  $\bar{\alpha}$ , i.e. as an NNLL term. The results here involve fixed coupling, i.e. they do not include a correction of the form of Eq. (30).

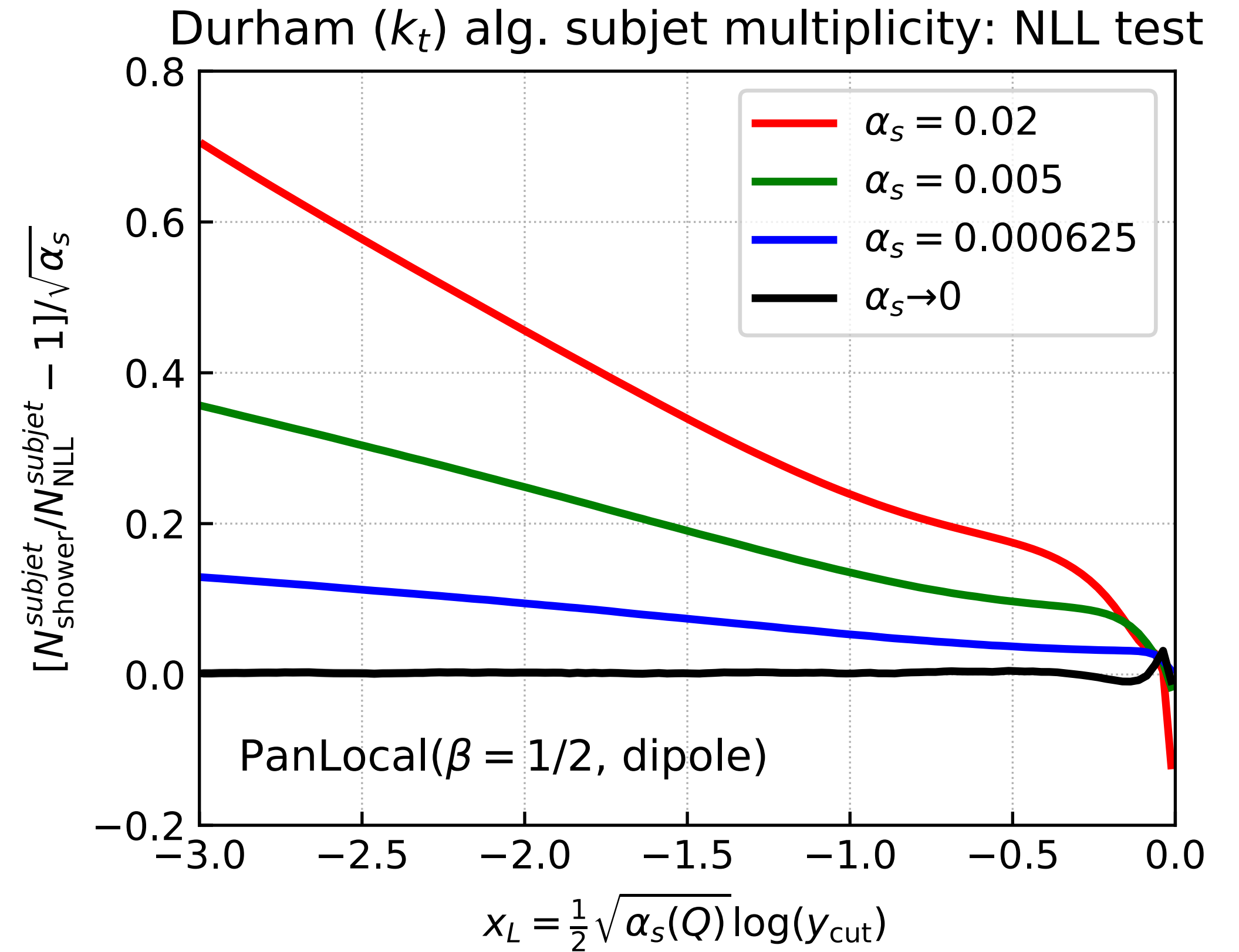
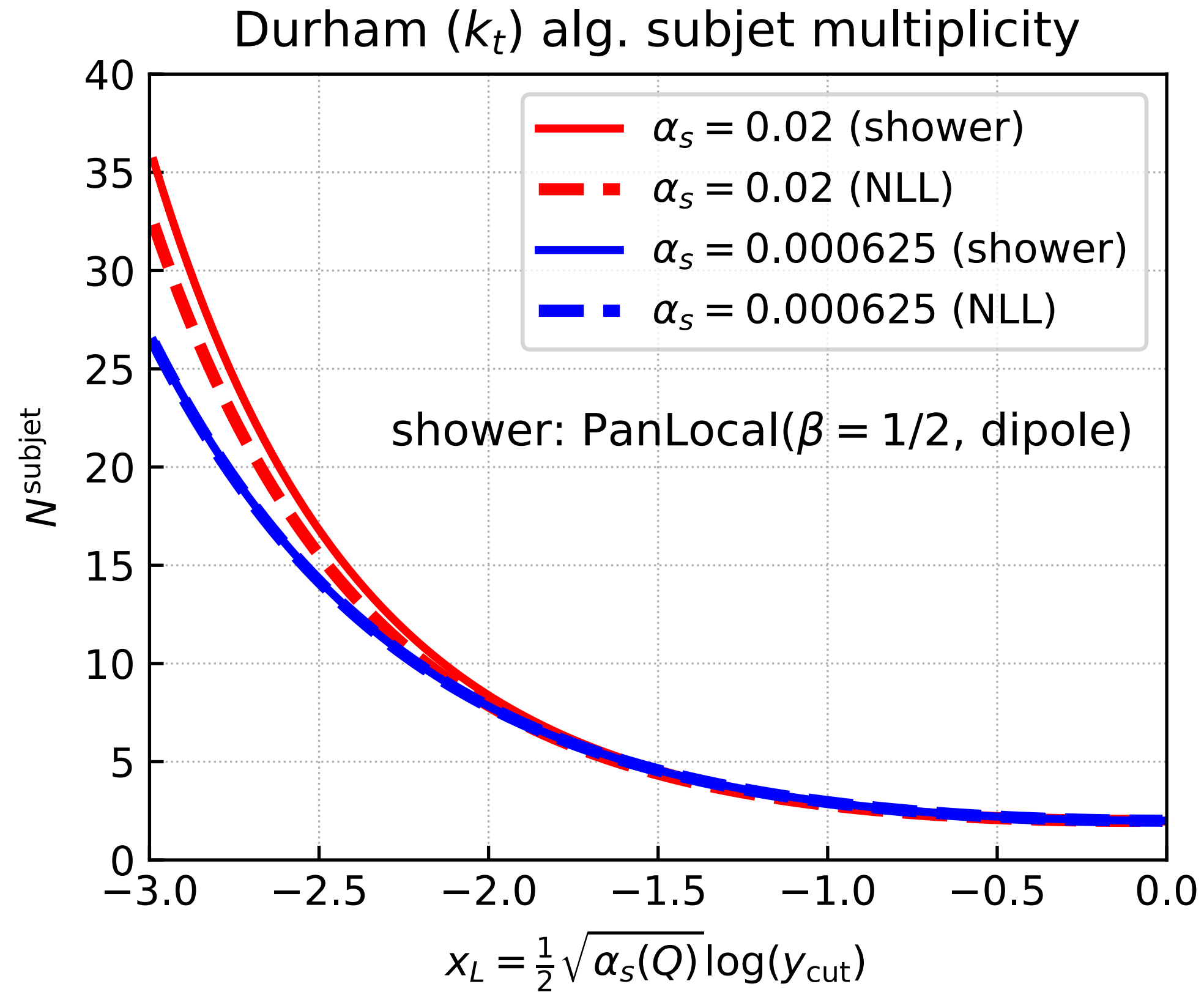
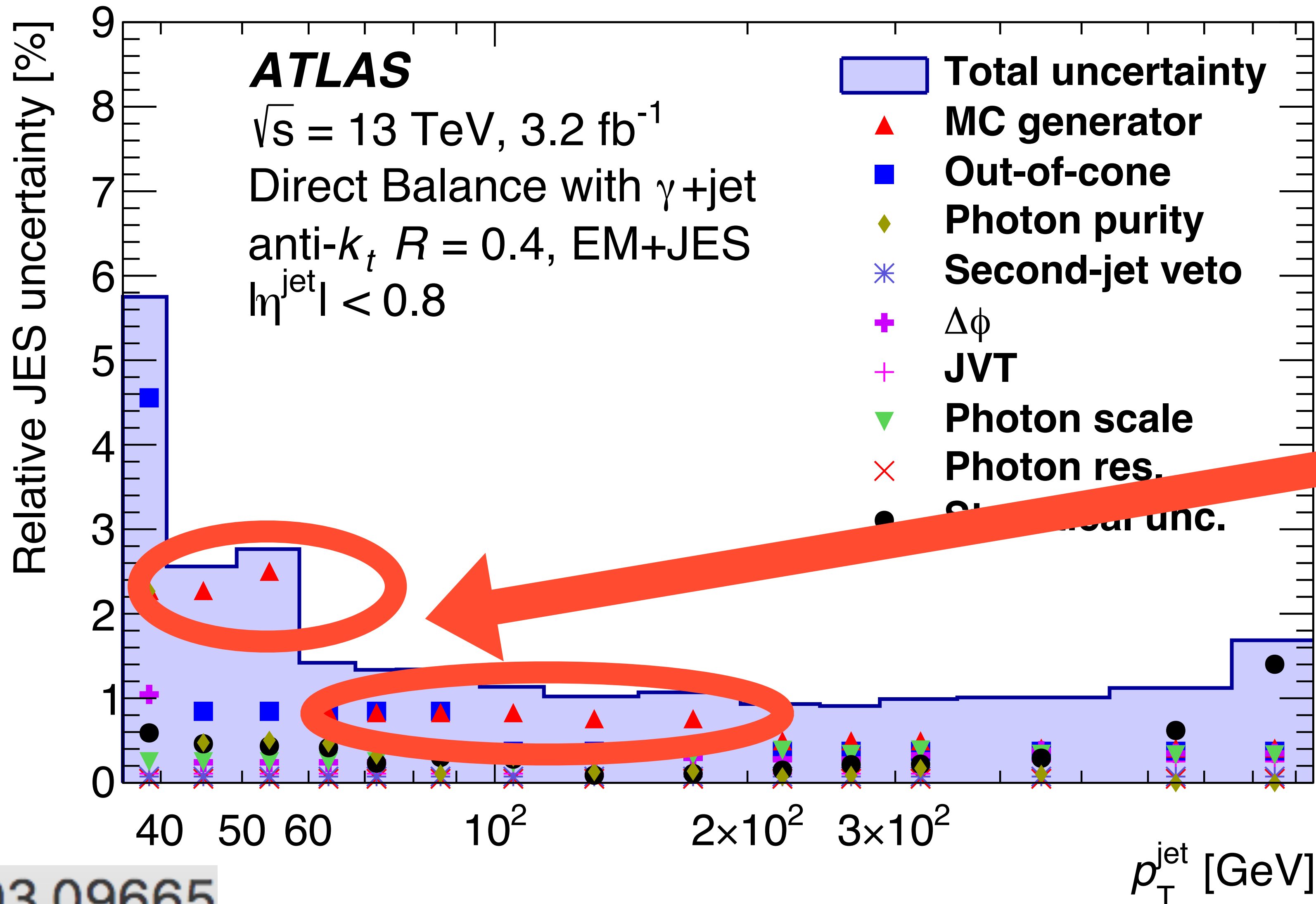
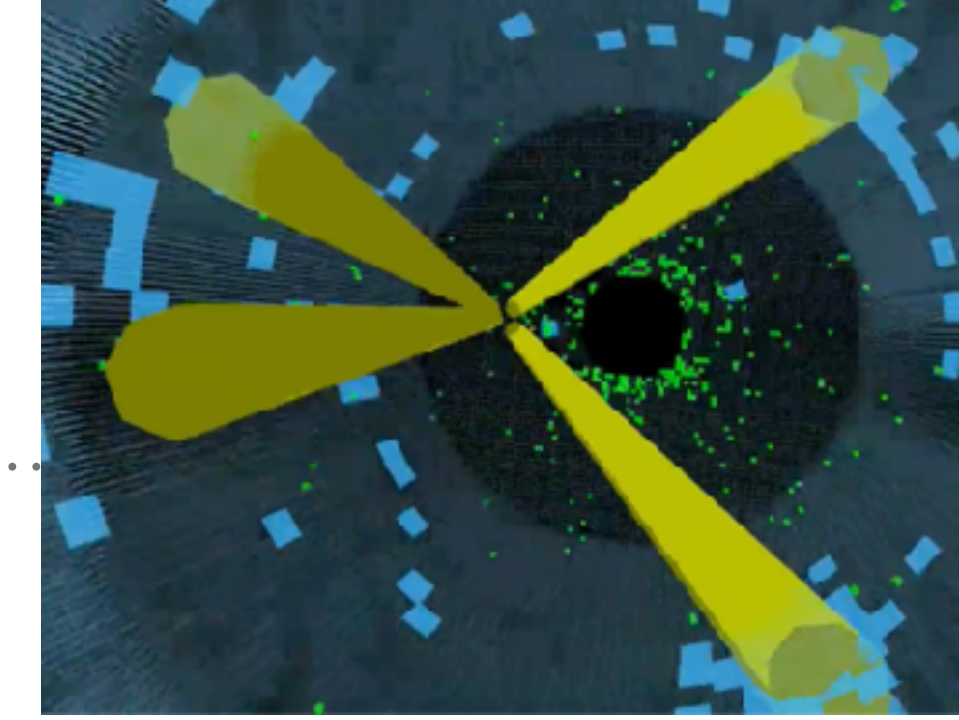


FIG. 8. Checks of the  $k_t$  algorithm subjet multiplicity. Left: the multiplicity as a function of  $\frac{1}{2} \sqrt{\alpha_s(Q)} \ln y_{\text{cut}}$ , comparing the PanLocal  $\beta = 0.5$  shower (dipole variant) with the NLL prediction, for two choices of  $\alpha_s$ . Right: Eq. (50) for the same shower, for several  $\alpha_s$  values, together with the  $\alpha_s \rightarrow 0$  limit.

# Fundamental experimental calibrations (jets)



Jet energy scale, which feeds into hundreds of other measurements

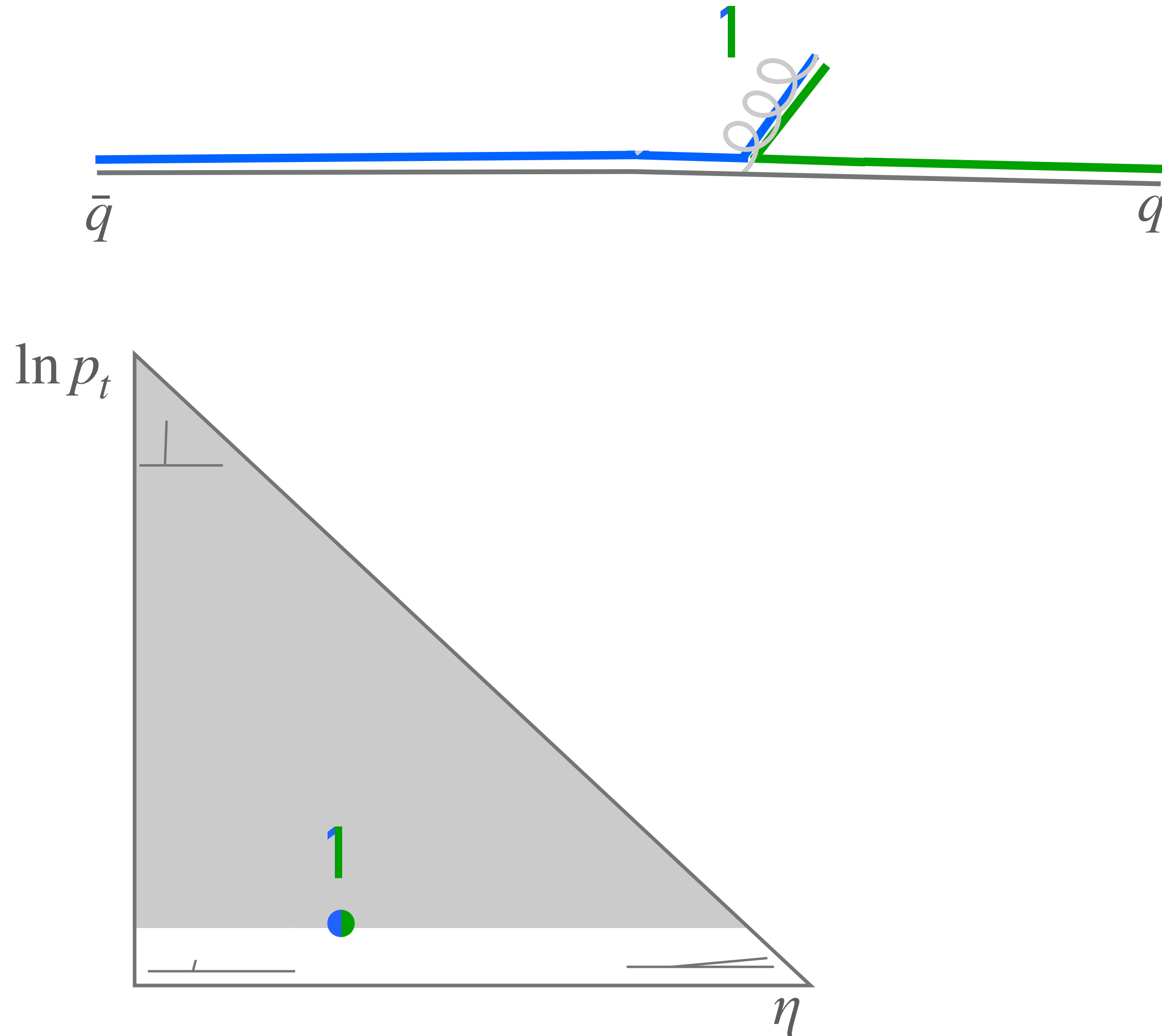
Largest systematic errors (1–2%) come from differences between MC generators

(here Sherpa v. Pythia)

→ fundamental limit on LHC precision potential

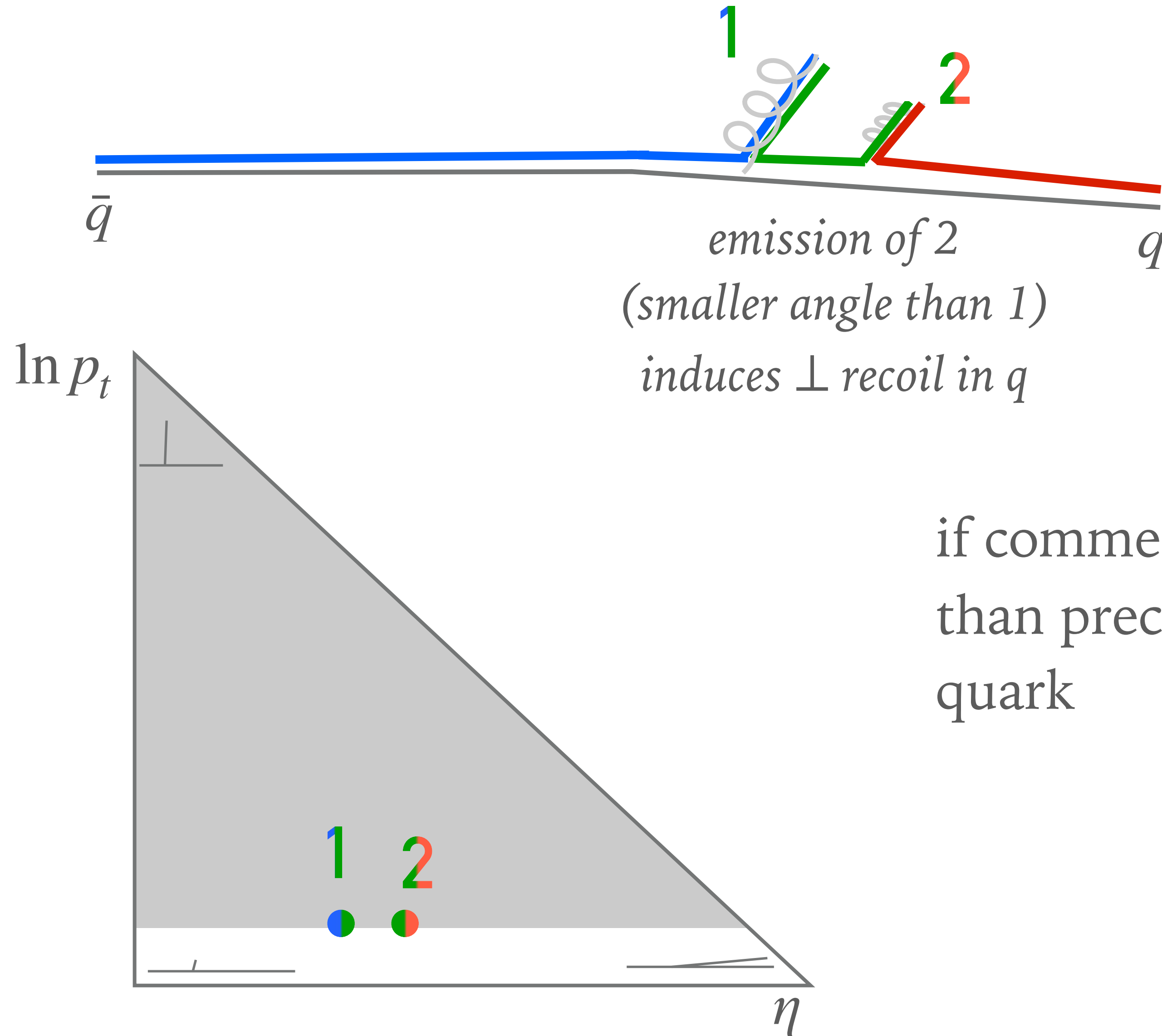
# What happens with PanLocal $\beta=0$ (i.e. $p_t$ ordered variant)

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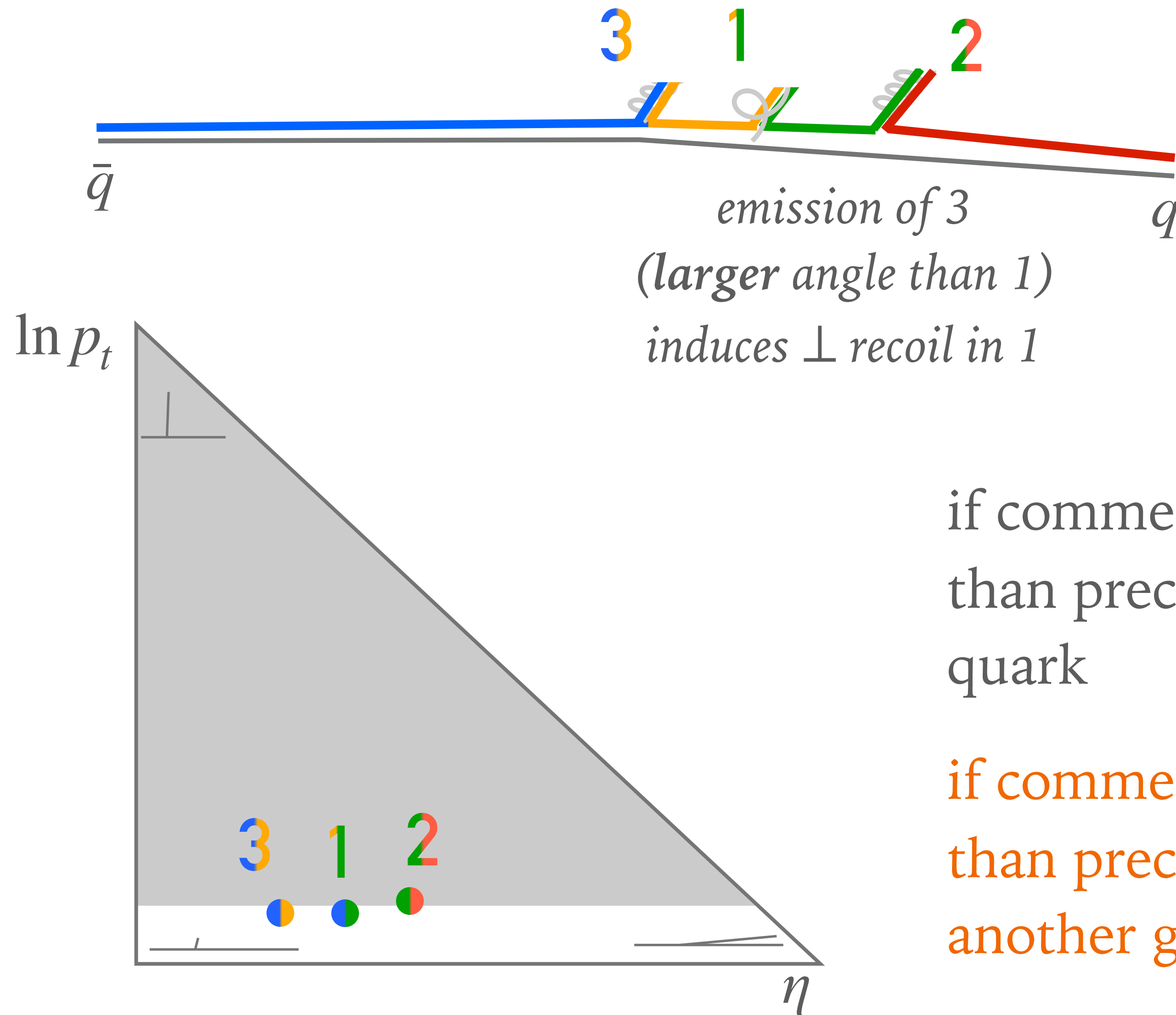


# What happens with PanLocal $\beta=0$ (i.e. $p_t$ ordered variant)



if commensurate  $p_t$  emission comes at smaller angle than preceding ones, recoil “correctly” assigned to quark

# What happens with PanLocal $\beta=0$ (i.e. $p_t$ ordered variant)



emission of 3  
(larger angle than 1)  
induces  $\perp$  recoil in 1

if commensurate  $p_t$  emission comes at smaller angle than preceding ones, recoil “correctly” assigned to quark

if commensurate  $p_t$  emission comes at larger angle than preceding ones, recoil “incorrectly” assigned to another gluon