# CUTS FOR 2-BODY DECAYS 

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## Linear $p_{\text {tH }}$ dependence of H acceptance, $\mathrm{f}\left(\mathrm{p}_{\text {th }}\right) \rightarrow$ impact on perturbative series



$$
f\left(p_{t, \mathrm{H}}\right)=f_{0}+f_{1} \cdot \frac{p_{t, \mathrm{H}}}{m_{\mathrm{H}}}+\mathcal{O}\left(\frac{p_{t, \mathrm{H}}^{2}}{m_{\mathrm{H}}^{2}}\right)
$$

See e.g. Frixione \& Ridolfi '97
Ebert \& Tackmann '19
idem + Michel \& Stewart ' 20
Alekhin et al ' 20

$$
\frac{d \sigma^{\mathrm{DL}}}{d p_{t, \mathrm{H}}}=\frac{\sigma_{\mathrm{tot}}}{p_{t, \mathrm{H}}} \sum_{n=1}^{\infty}(-1)^{n-1} \frac{2 \log ^{2 n-1} \frac{m_{\mathrm{H}}}{2 p_{t, \mathrm{H}}}}{(n-1)!}\left(\frac{2 C_{A} \alpha_{s}}{\pi}\right)^{n}
$$

$$
\sigma_{\mathrm{fid}}=\int \frac{d \sigma^{\mathrm{DL}}}{d p_{t, \mathrm{H}}} f\left(p_{t, \mathrm{H}}\right) d p_{t, \mathrm{H}}
$$

$$
=\sigma_{\text {tot }}\left[f_{0}+f_{1} \sum_{n=1}^{\infty}(-1)^{n+1} \frac{(2 n)!}{2(n!)}\left(\frac{2 C_{A} \alpha_{s}}{\pi}\right)^{n}+\cdots\right]
$$

## Behaviour of perturbative series in various log approximations

## Resummed

 results$$
\begin{aligned}
& \frac{\sigma_{\mathrm{asym}}-f_{0} \sigma_{\mathrm{inc}}}{\sigma_{0} f_{0}} \simeq 0.15_{\alpha_{s}}-0.29_{\alpha_{s}^{2}}+0.71_{\alpha_{s}^{3}}-2.39_{\alpha_{s}^{4}}+10.26_{\alpha_{s}^{5}}+\ldots \simeq 0.06 @ \mathrm{DL}, \\
& \simeq 0.15_{\alpha_{s}}-0.23_{\alpha_{s}^{2}}+0.44_{\alpha_{s}^{3}}-1.15_{\alpha_{s}^{4}}+3.83_{\alpha_{s}^{5}}+\ldots \simeq 0.06 @ L L, \\
& \simeq 0.18_{\alpha_{s}}-0.15_{\alpha_{s}^{2}}+0.29_{\alpha_{s}^{3}}+\ldots \quad \simeq 0.10 \text { @NNLL, } \\
& \simeq 0.18_{\alpha_{s}}-0.15_{\alpha_{s}^{2}}+0.31_{\alpha_{s}^{3}}+\ldots \quad \simeq 0.12 @ N 3 L L .
\end{aligned}
$$

Thanks to Pier Monni \& RadISH for supplying NN(N)LL distributions \& expansions, $\mu=m_{H} / 2$

- At DL \& LL (DL+running coupling) factorial divergence sets in from first orders
- Poor behaviour of N3LL is qualitatively similar to that seen by Billis et al ' 21
> At N3LO, there is extreme sensitivity to unphysically low $\mathrm{p}_{\mathrm{t}}$ (down to $\sim 1 \mathrm{MeV}$, see backup)
- Is the only solution to do resummation?


## Replace cut on leading photon $\rightarrow$ cut on product of photon pt's



$$
f\left(p_{t, \mathrm{H}}\right)=f_{0}+f_{2}\left(\frac{p_{t, \mathrm{H}}}{m_{\mathrm{H}}}\right)^{2}+\mathcal{O}\left(\frac{p_{t, \mathrm{H}}^{2}}{m_{\mathrm{H}}^{2}}\right) \quad \begin{aligned}
& \text { linear } \rightarrow \\
& \text { quadratic }
\end{aligned}
$$

$$
\frac{(2 n)!}{2(n!)}\left(\frac{2 C_{A} \alpha_{s}}{\pi}\right)^{n} \rightarrow \frac{1}{4^{n}} \frac{(2 n)!}{4(n!)}\left(\frac{2 C_{A} \alpha_{s}}{\pi}\right)^{n}
$$

Using product cuts dampens the factorial divergence

## Behaviour of perturbative series with product cuts

$$
\begin{aligned}
\frac{\sigma_{\mathrm{prod}}-f_{0} \sigma_{\mathrm{inc}}}{\sigma_{0} f_{0}} & \simeq 0.005_{\alpha_{s}}-0.002_{\alpha_{s}^{2}}+0.002_{\alpha_{s}^{3}}-0.001_{\alpha_{s}^{4}}+0.001_{\alpha_{s}^{5}}+\ldots \\
& \simeq 0.005_{\alpha_{s}}-0.002_{\alpha_{s}^{2}}+0.000_{\alpha_{s}^{3}}-0.000_{\alpha_{s}^{4}}+0.000_{\alpha_{s}^{5}}+\ldots \\
& \simeq 0.005_{\alpha_{s}}+0.002_{\alpha_{s}^{2}}-0.001_{\alpha_{s}^{3}}+\ldots \\
& \simeq 0.005_{\alpha_{s}}+0.002_{\alpha_{s}^{2}}-0.001_{\alpha_{s}^{3}}+\ldots
\end{aligned}
$$

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- Factorial growth of series strongly suppressed
> N3LO truncation agrees well with all-order result
> Per mil agreement between fixed-order and resummation gives confidence that all is under control


## There's lots more in the paper + code at https://github.com/gavinsalam/two-body-cuts

> interplay with rapidity cuts
> cuts where the acceptance is independent of $\mathrm{p}_{\mathrm{tH}}$ at small $\mathrm{p}_{\mathrm{tH}}$ (should have
perturbative properties of a high- $\mathrm{p}_{\mathrm{tH}}$ cross section)
> outline of extension to Drell-Yan
$\mathrm{p}_{\mathbb{H}}$ derivative of acceptance: white $=0$


Higgs rapidity
hardness and rapidity compensating ( $\mathrm{CBI}_{\mathrm{HR}}$ ) cuts

## Remarks

- Fixed-order perturbation theory can be badly compromised by existing (2-body) cuts
- Physics (e.g. $\mathrm{H} \rightarrow \mathrm{\gamma Y}$ ) will be more robust if we can reliably use both FO and resummed+FO results (and their central values and uncertainties are sunukar)
- FO problem can be solved by revisiting cuts:
> product and boost-invariant cuts give much better perturbative series
> Findings raise intriguing questions about our understanding of asymptotics of QCD perturbative series
- Cuts with little $\mathrm{p}_{\mathrm{tH}}$ dependence may be useful also, e.g., for extrapolating measurements to STXS or more inclusive cross sections, with limited dependence on BSM or non-pert effects.
- To discuss for LH: should experiments change cuts? Which ones should they adopt?Are further investigations needed to answer these questions?

Backup

## Asymmetric cuts and why the acceptance depends on the Higgs $p_{T}$



Numbers are for ATLAS $H \rightarrow \gamma \gamma p_{t}$ cuts, CMS cuts are similar

$$
p_{t, \pm}\left(p_{t, \mathrm{H}}, \theta, \phi\right)=\frac{m_{\mathrm{H}}}{2} \sin \theta \pm \frac{1}{2} p_{t, \mathrm{H}}|\cos \phi|+\frac{p_{t, \mathrm{H}}^{2}}{4 m_{\mathrm{H}}}\left(\sin \theta \cos ^{2} \phi+\csc \theta \sin ^{2} \phi\right)+\mathcal{O}_{3}
$$

$p_{t, \operatorname{prod}}\left(p_{t, \mathrm{H}}, \theta, \phi\right)=\sqrt{p_{t,+} p_{t,-}}=\frac{m_{\mathrm{H}}}{2} \sin \theta+\frac{p_{t, \mathrm{H}}^{2}}{4 m_{\mathrm{H}}} \frac{\sin ^{2} \phi-\cos ^{2} \theta \cos ^{2} \phi}{\sin \theta}+\mathcal{O}_{4}$

| Cut Type | cuts on | small- $p_{t, \mathrm{H}}$ dependence | $f_{n}$ coefficient | $p_{t, \mathrm{H}}$ transition |
| :---: | :---: | :---: | :---: | :---: |
| symmetric | $p_{t,-}$ | linear | $+2 s_{0} /\left(\pi f_{0}\right)$ | none |
| asymmetric | $p_{t,+}$ | linear | $-2 s_{0} /\left(\pi f_{0}\right)$ | $\Delta$ |
| sum | $\frac{1}{2}\left(p_{t,-}+p_{t,+}\right)$ | quadratic | $\left(1+s_{0}^{2}\right) /\left(4 f_{0}\right)$ | $2 \Delta$ |
| product | $\sqrt{p_{t,-}+p_{t,+}}$ | quadratic | $s_{0}^{2} /\left(4 f_{0}\right)$ | $2 \Delta$ |
| staggered | $p_{t, 1}$ | quadratic | $s_{0}^{4} /\left(4 f_{0}^{3}\right)$ | $\Delta$ |
| Collins-Soper | $p_{t, \mathrm{CS}}$ | none | - | $2 \Delta$ |
| $\mathrm{CBI}_{H}$ | $p_{t, \mathrm{CS}}$ | none | - | $2 \sqrt{2} \Delta$ |
| rapidity | $y_{\gamma}$ | quadratic | $f_{0} s_{0}^{2} / 2$ |  |

Table 1: Summary of the main hardness cuts, the variable they cut on at small $p_{t, \mathrm{H}}$, and the small- $p_{t, \mathrm{H}}$ dependence of the acceptance. For linear cuts $f_{n} \equiv f_{1}$ multiplies $p_{t, \mathrm{H}} / m_{\mathrm{H}}$, while for quadratic cuts $f_{n} \equiv f_{2}$ multiplies $\left(p_{t, \mathrm{H}} / m_{\mathrm{H}}\right)^{2}$ (in all cases there are additional higher order terms that are not shown). For a leading threshold of $p_{t, \text { cut }}, s_{0}=2 p_{t, \mathrm{cut}} / m_{\mathrm{H}}$ and $f_{0}=\sqrt{1-s_{0}^{2}}$, while for the rapidity cut $s_{0}=1 / \cosh \left(y_{\mathrm{H}}-y_{\mathrm{cut}}\right)$. For a cut on the softer lepton's transverse momentum of $p_{t,-}>p_{t, \text { cut }}-\Delta$, the right-most column indicates the $p_{t, \mathrm{H}}$ value at which the $p_{t,-}$ cut starts to modify the behaviour of the acceptance (additional $\mathcal{O}\left(\Delta^{2} / m_{\mathrm{H}}\right)$ corrections not shown). For the interplay between hardness and rapidity cuts, see sections 4.2, 4.3 and 5.2.

## Hardness [and rapidity] compensating boost invariant cuts ( CBI $_{\boldsymbol{H}}$ and $\mathrm{CBI}_{\boldsymbol{\mu}}$ )

Core idea 1: cut on decay $\mathrm{p}_{\mathrm{t}}$ in Collins-Soper frame

$$
\vec{p}_{t, \mathrm{CS}}=\frac{1}{2}\left[\vec{\delta}_{t}+\frac{\vec{p}_{t, 12} \cdot \vec{\delta}_{t}}{p_{t, 12}^{2}}\left(\frac{m_{12}}{\left.\left.\sqrt{m_{12}^{2}+p_{t, 12}^{2}}-1\right) \vec{p}_{t, 12}\right], \quad \vec{\delta}_{t}=\vec{p}_{t, 1}-\vec{p}_{t, 2}}\right.\right.
$$

Core idea 2: relax $\mathrm{p}_{\mathrm{t}, \mathrm{Cs}}$ cut at higher $\mathrm{p}_{\mathrm{t}, \mathrm{H}}$ values to maintain constant / maximal acceptance





## Sensitivity to low Higgs $\mathrm{p}_{\mathrm{t}}$ (and also scale bands): standard cuts



## Sensitivity to low Higgs $p_{t}$ (and also scale bands): sum \& product cuts




## Option of changing thresholds


$\mathrm{CBI}_{\mathrm{HR}}$ v. standard cuts


