

# Precision QCD at the LHC

NNPDF collaboration meeting  
Gargnano, September 2023

**Gavin Salam**

Rudolf Peierls Centre for  
Theoretical Physics  
& All Souls College, Oxford



Science and  
Technology  
Facilities Council



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UNIVERSITY OF  
OXFORD



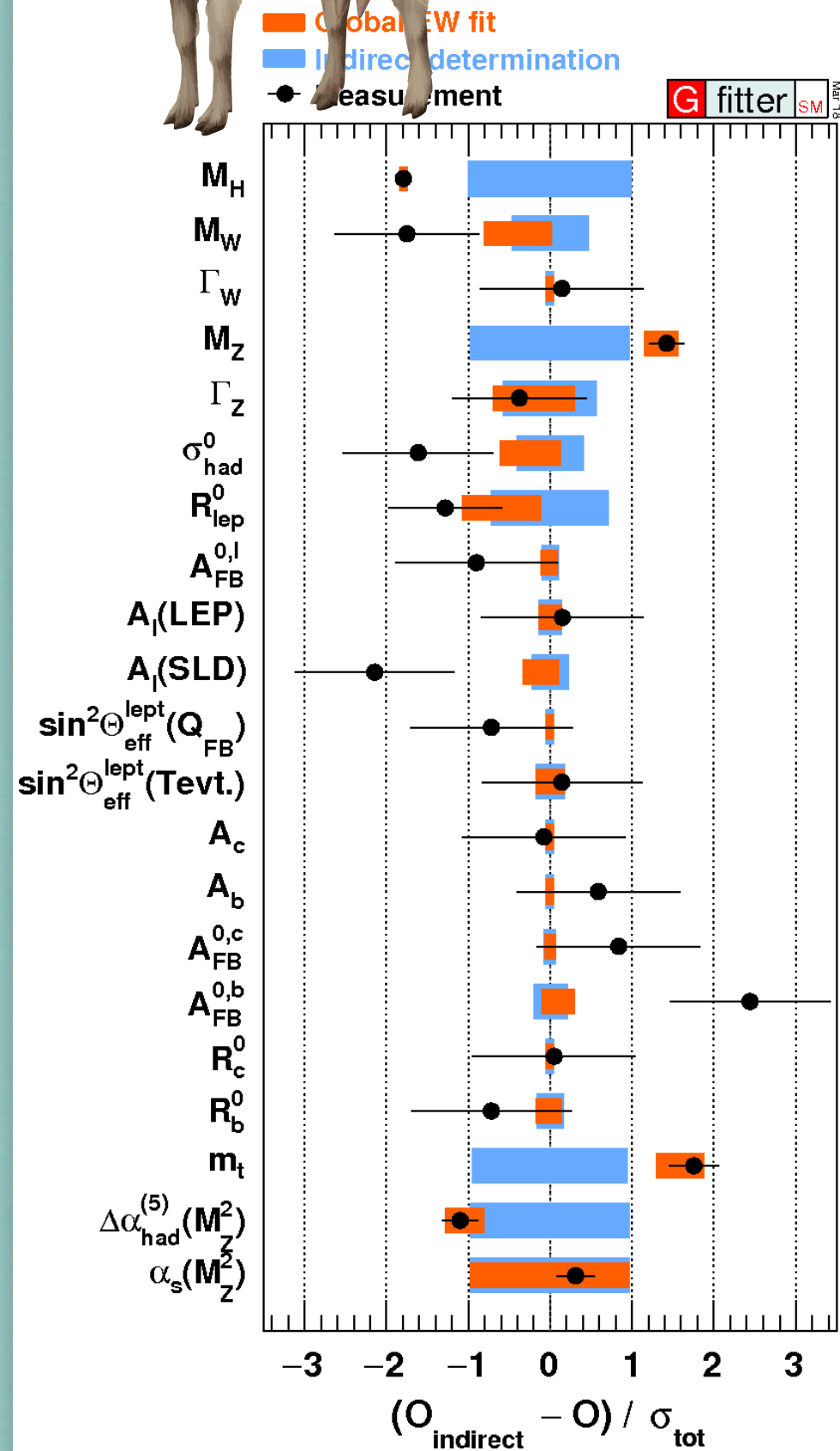
European Research Council  
Established by the European Commission

# Success of the SM [de Florian @ EPS-HEP 2023]

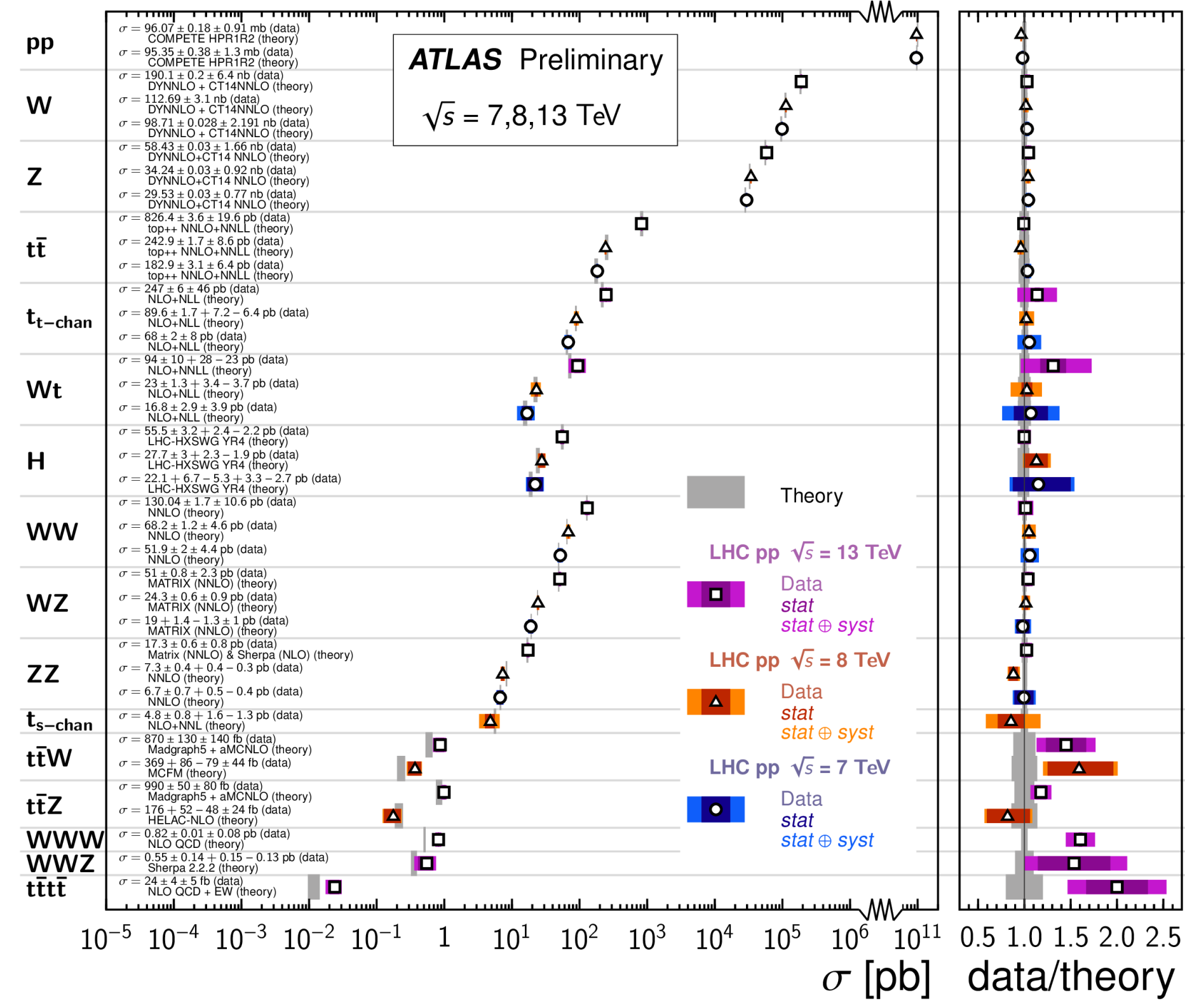


SM

## Everything looks SM-like at LHC Greatest Of All Theories



### Standard Model Total Production Cross Section Measurements



Standard Model and Higgs Theory

Daniel de Florian

2



# particle physics

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## “big unanswered questions”

about fundamental particles & their interactions  
(dark matter, matter-antimatter asymmetry,  
nature of dark energy, hierarchy of scales...)

v.

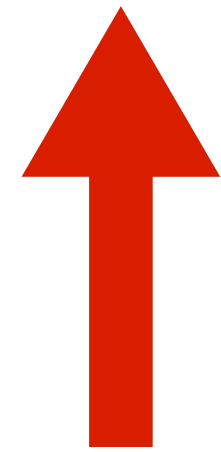
## “big answerable questions”

and how we go about answering them  
(nature of Higgs interactions, validity of SM up to higher scales,  
lepton flavour universality, pattern of neutrino mixing, ...)

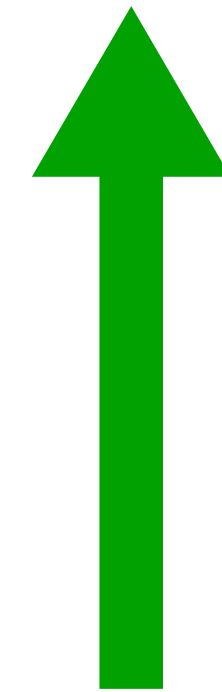
# The Lagrangian and Higgs interactions: two out of three qualitatively new!

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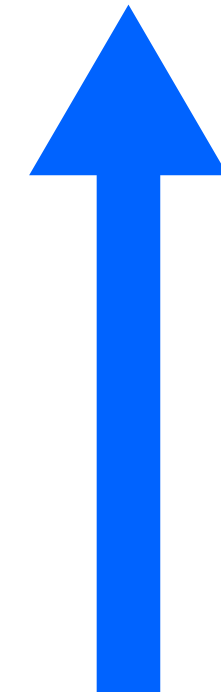
$$\mathcal{L}_{\text{SM}} = \dots + |D_\mu \phi|^2 + \psi_i y_{ij} \psi_j \phi - V(\phi)$$



Gauge interactions, structurally like those in QED, QCD, EW, **studied for many decades** (but now with a scalar)



Yukawa interactions. Responsible for fermion masses, and induces “fifth force” between fermions. **Direct study started only in 2018!**

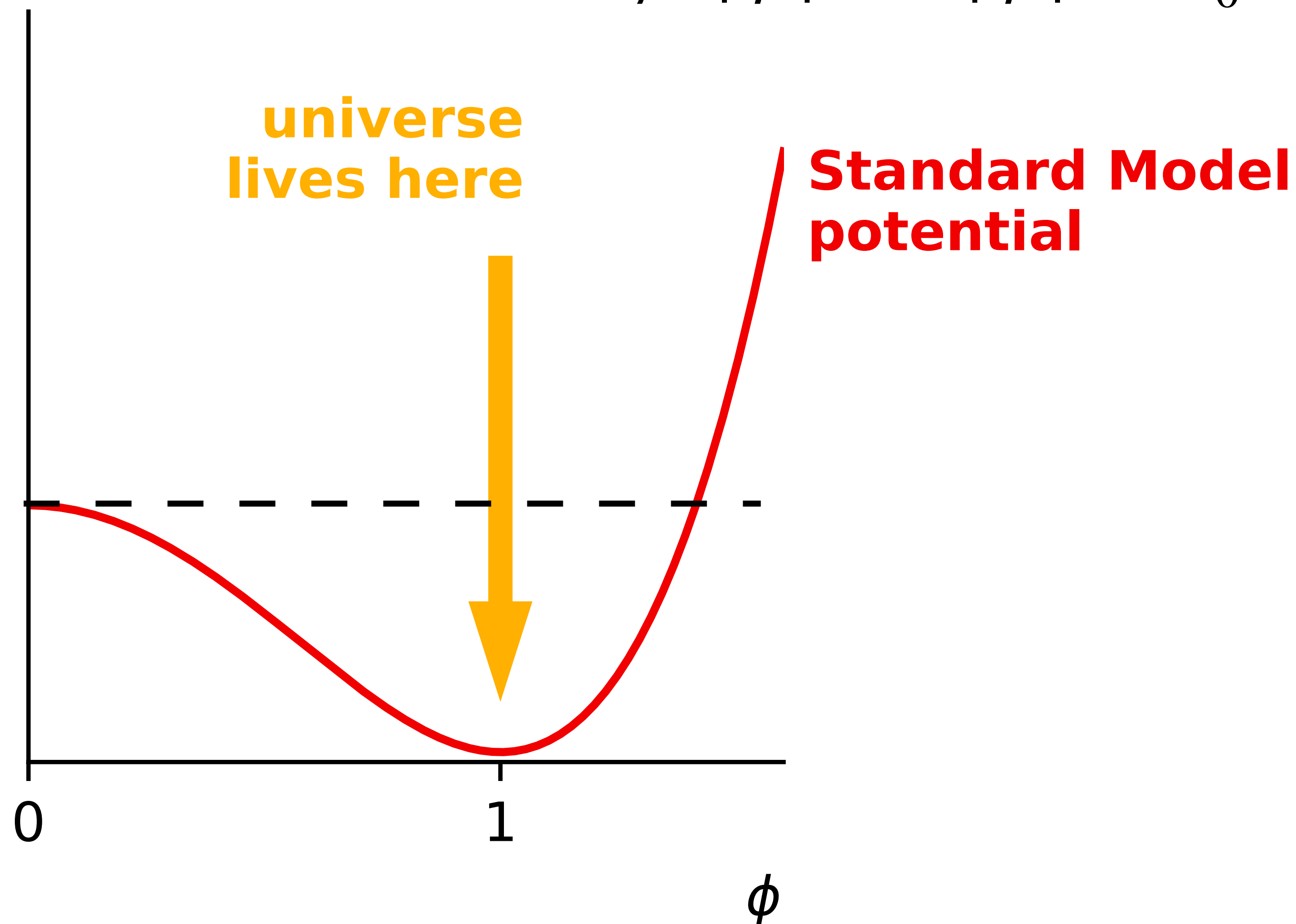


Higgs potential → self-interaction  
Holds the SM together.  
**Unobserved**

# Higgs potential

$V(\phi)$ , SM

$$V = -\mu^2 |\phi|^2 + \lambda |\phi|^4 + V_0$$

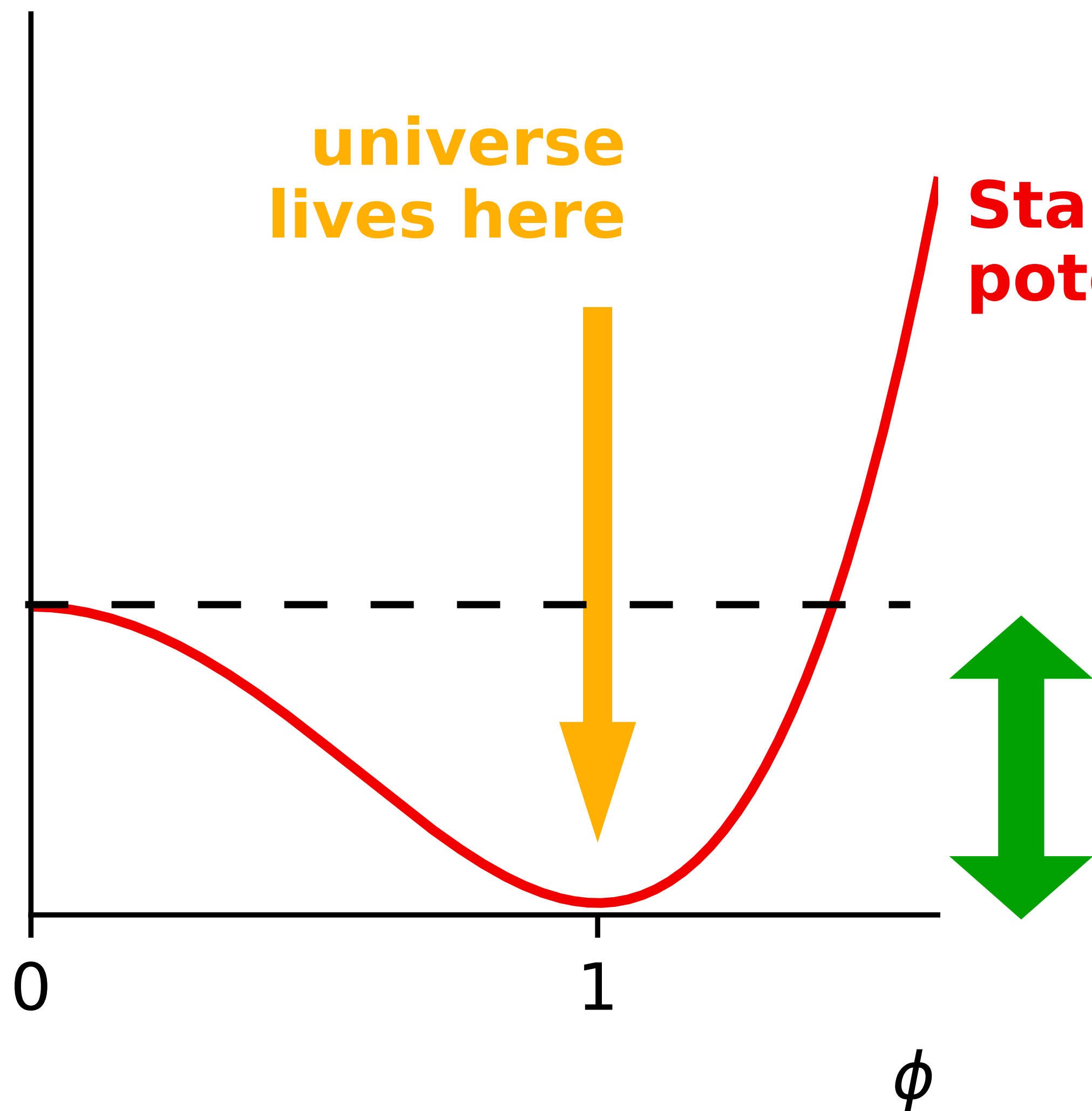


**Higgs mechanism  
gives mass to  
particles because  
Higgs field  $\phi$  is  
non-zero**

**& that's because the  
minimum of the SM  
potential is at  
non-zero  $\phi$**

# Higgs potential

$V(\phi)$ , SM

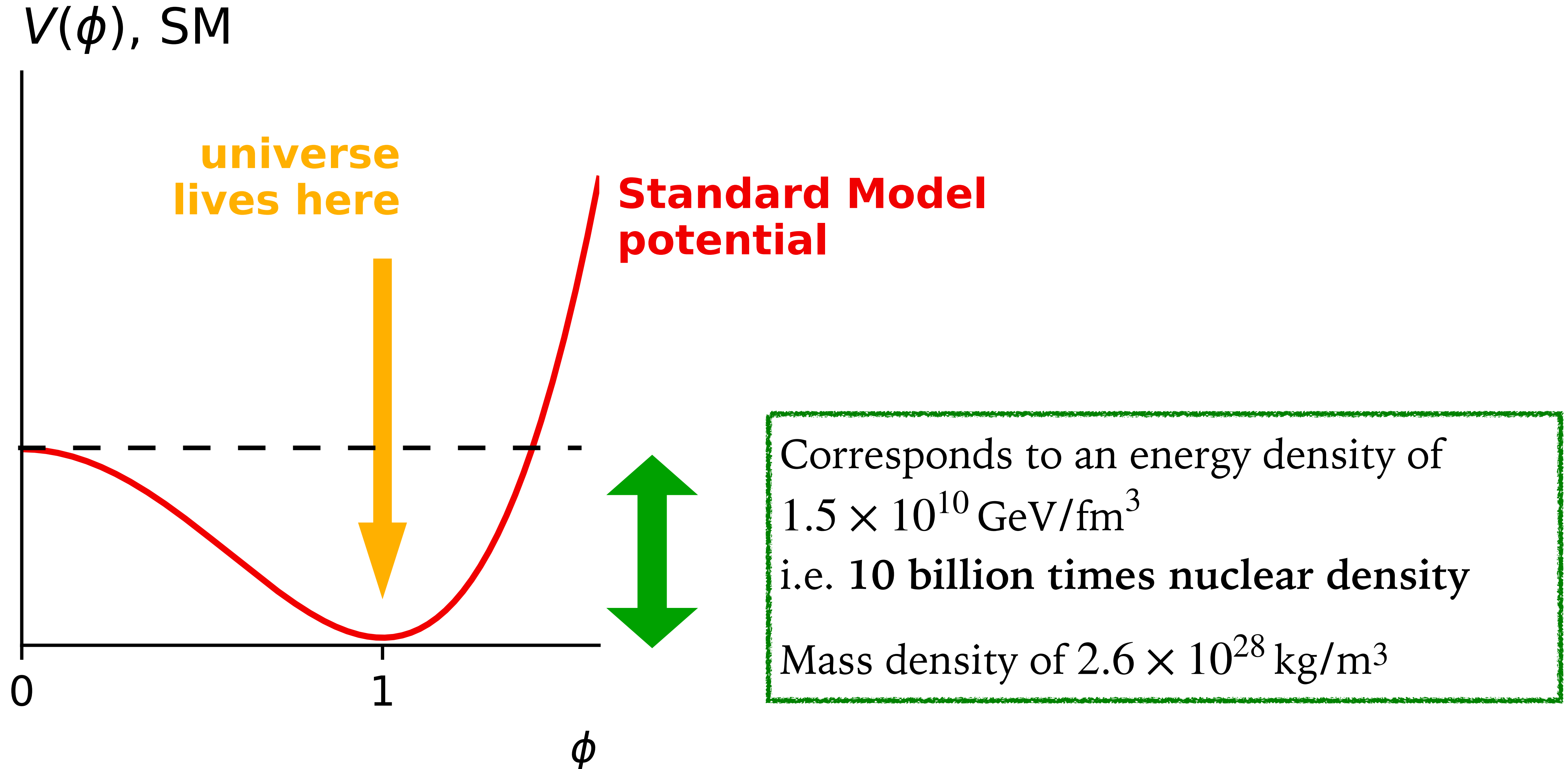


**Standard Model potential**

depth is  $\frac{m_H^2 v^2}{8}$  ( $m_H \simeq 125$  GeV,  $v \simeq 246$  GeV)

a fairly innocuous sounding  $(104 \text{ GeV})^4$

# Higgs potential – remember: it's an energy density



# What does $2.6 \times 10^{28} \text{ kg/m}^3$ mean?

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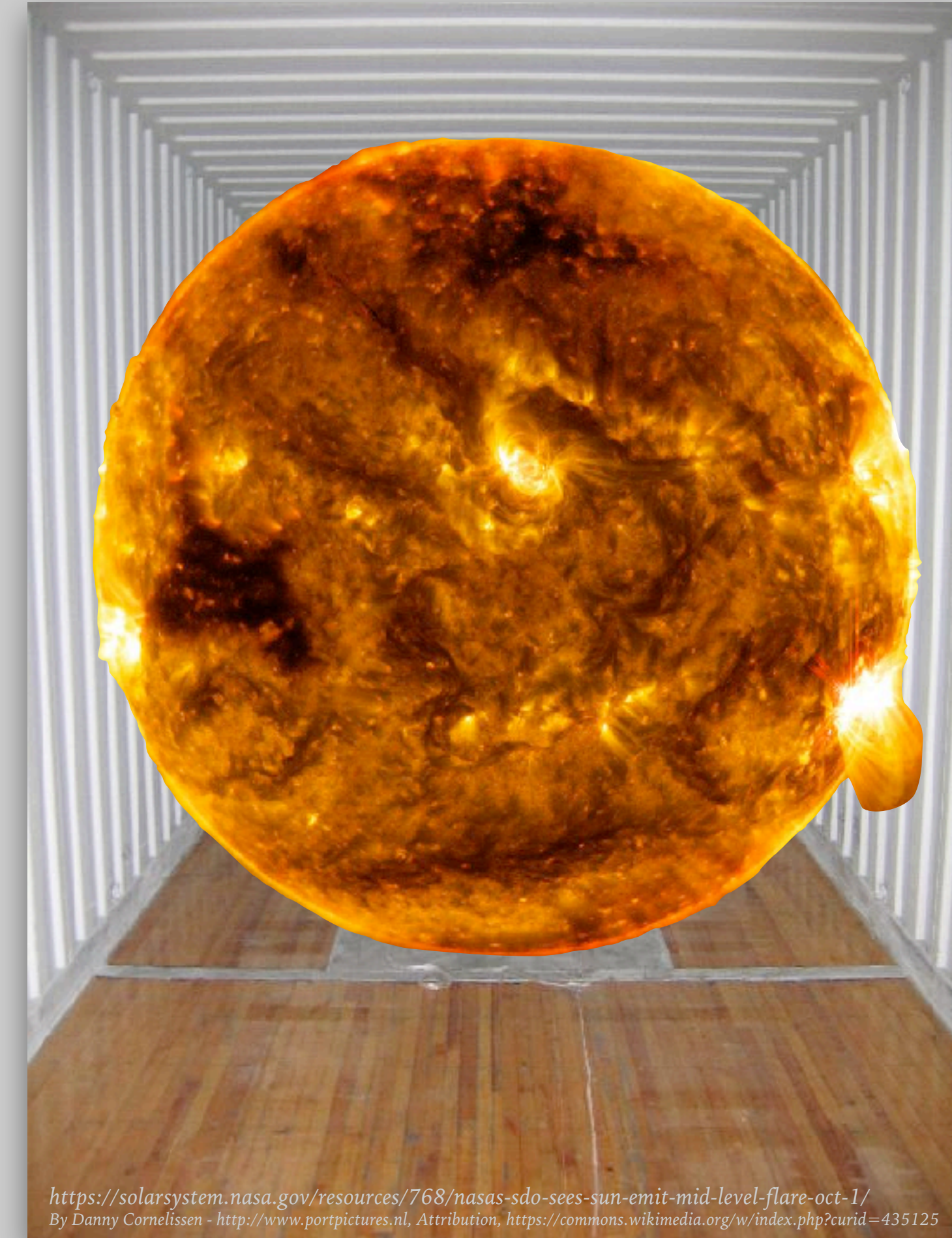
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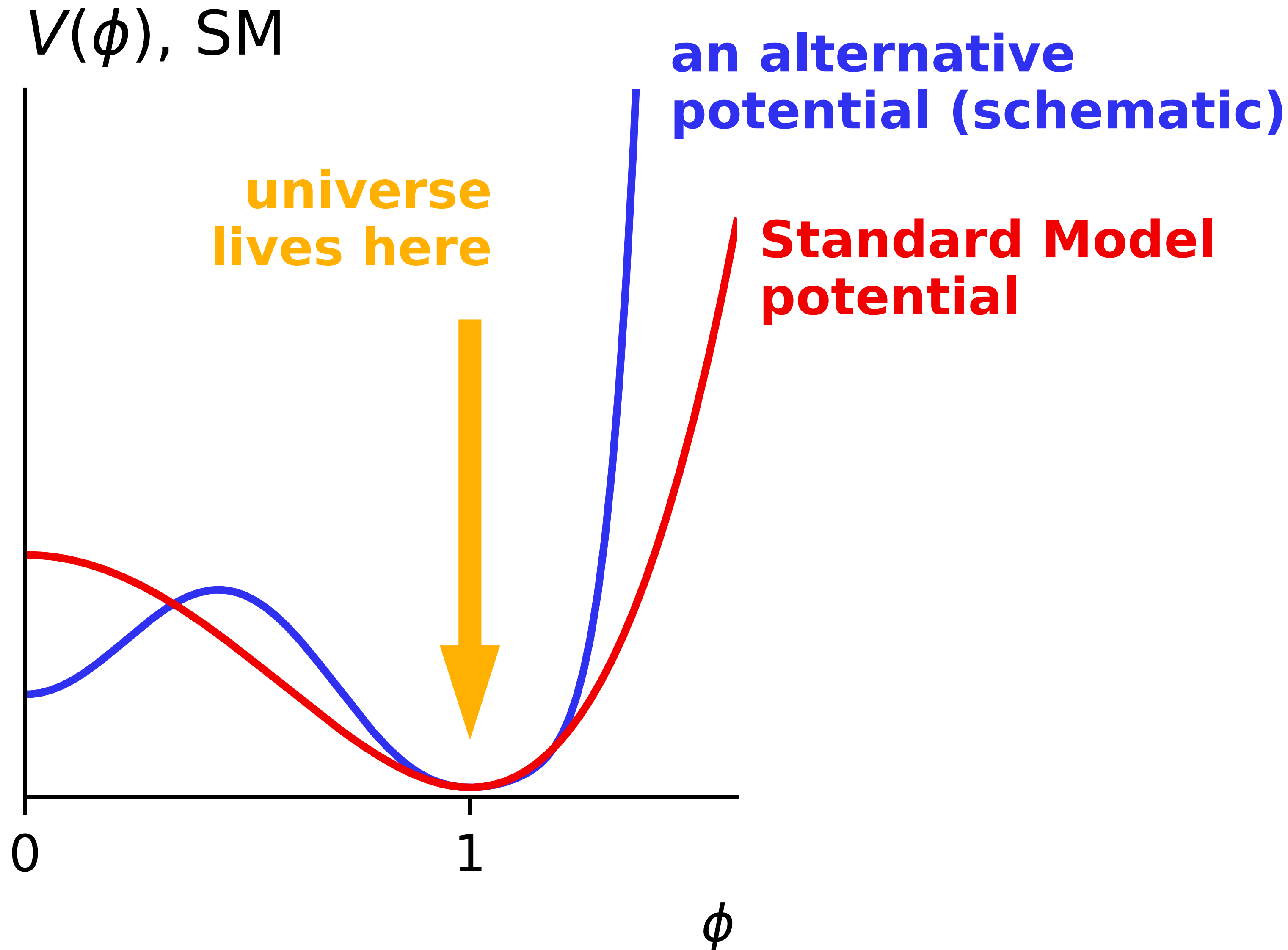
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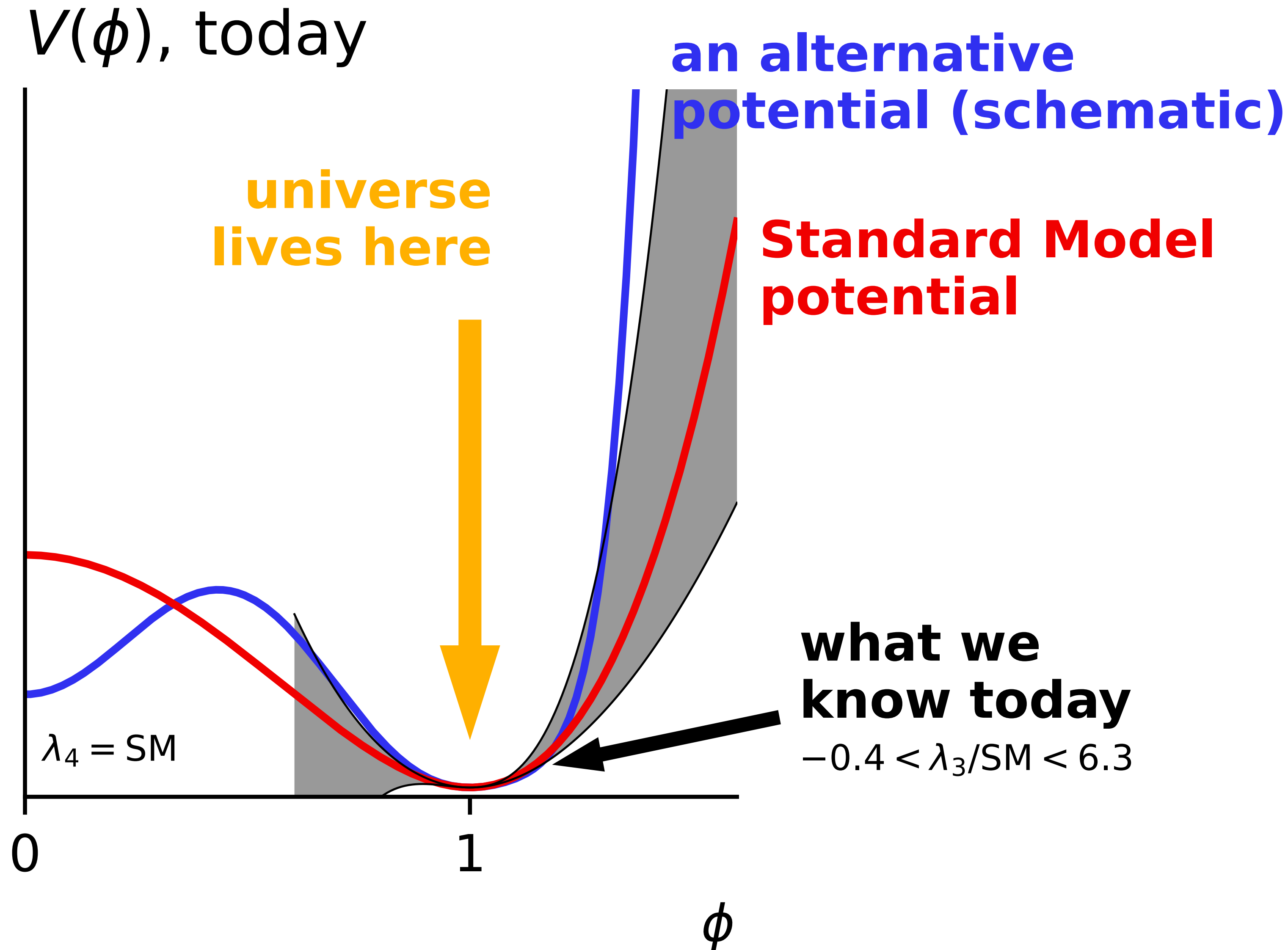
**fit the mass of the sun into a standard 40ft shipping container**

# Higgs potential — huge energy densities — yet to be experimentally confirmed

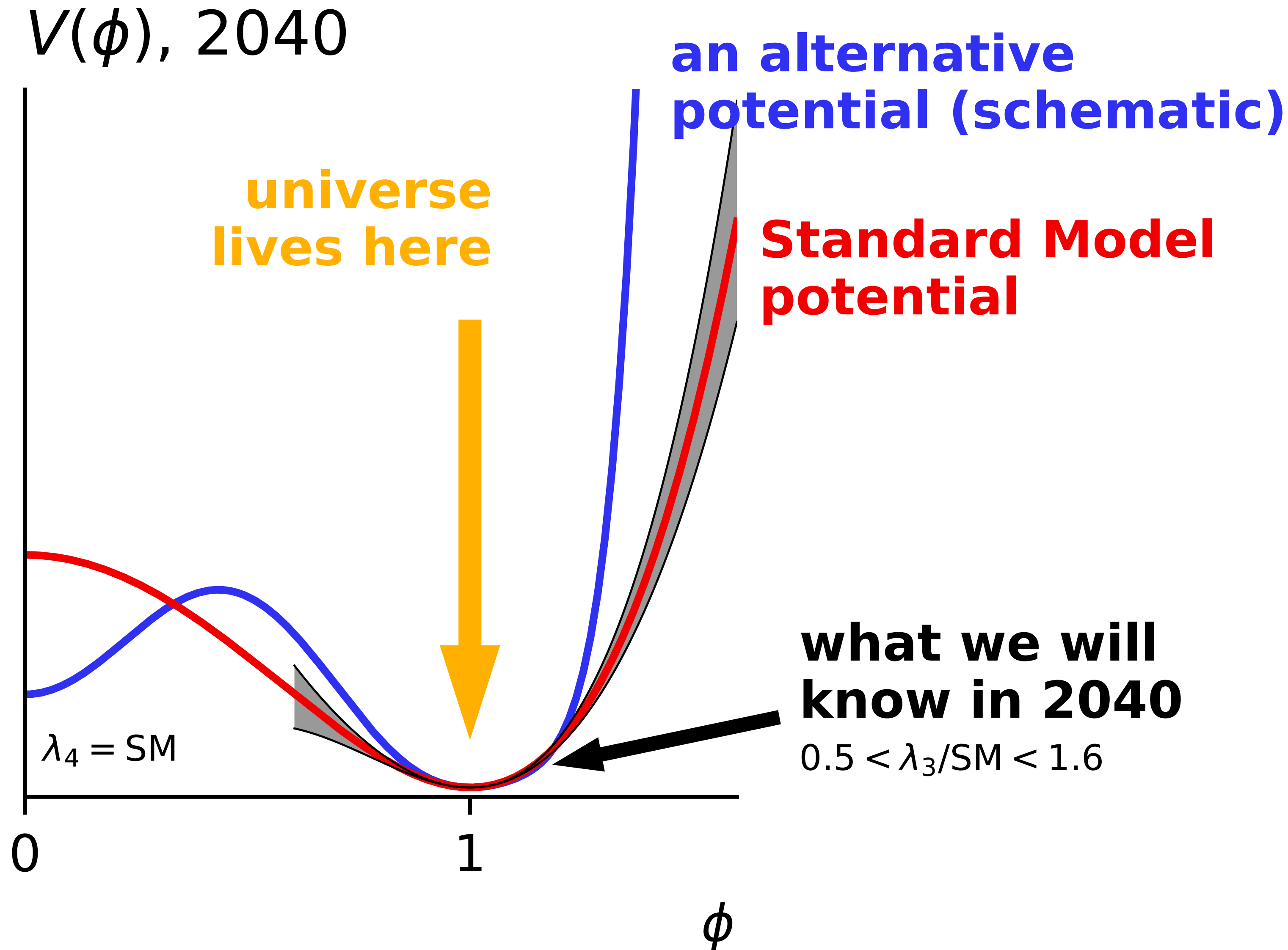
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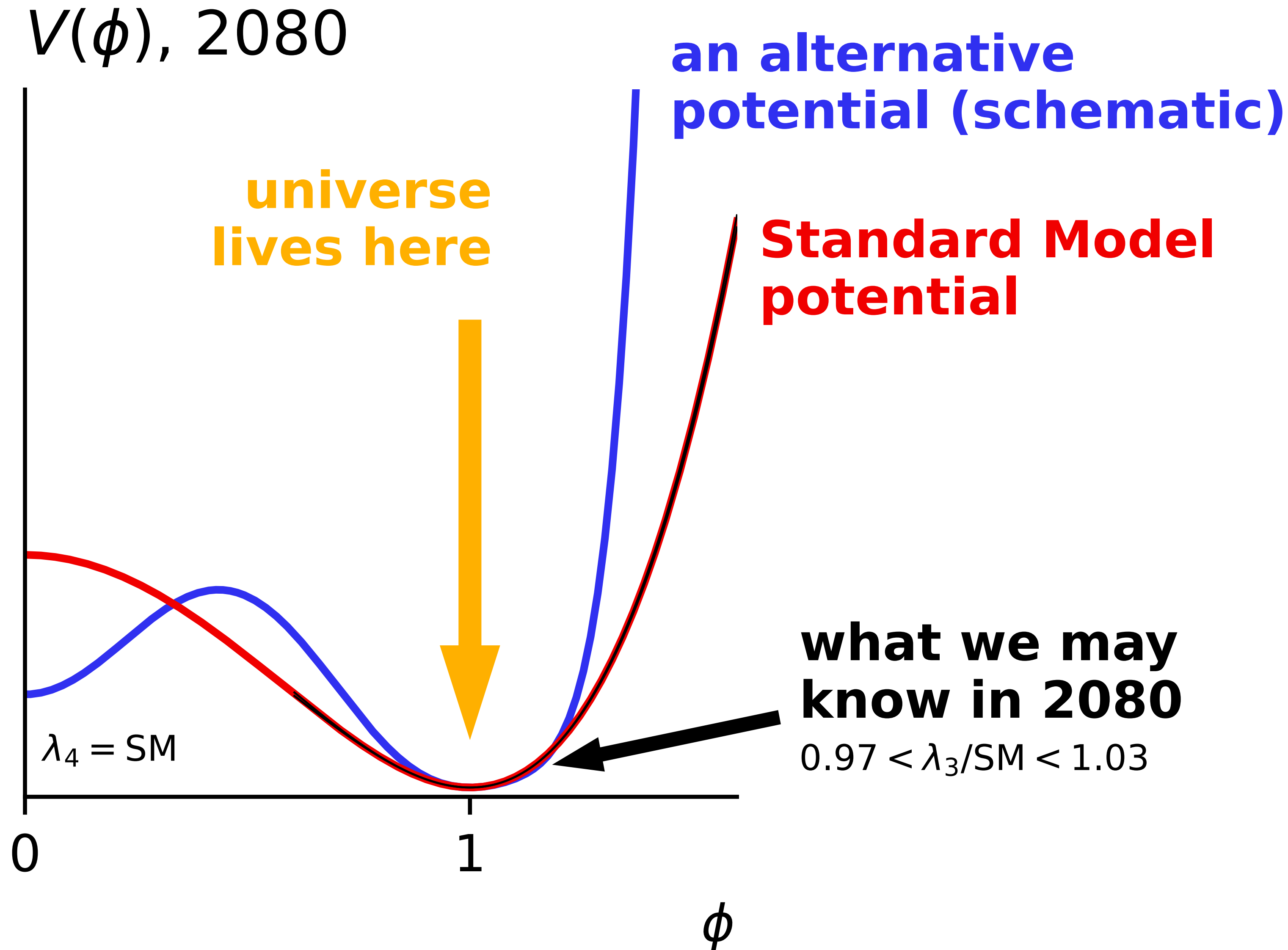
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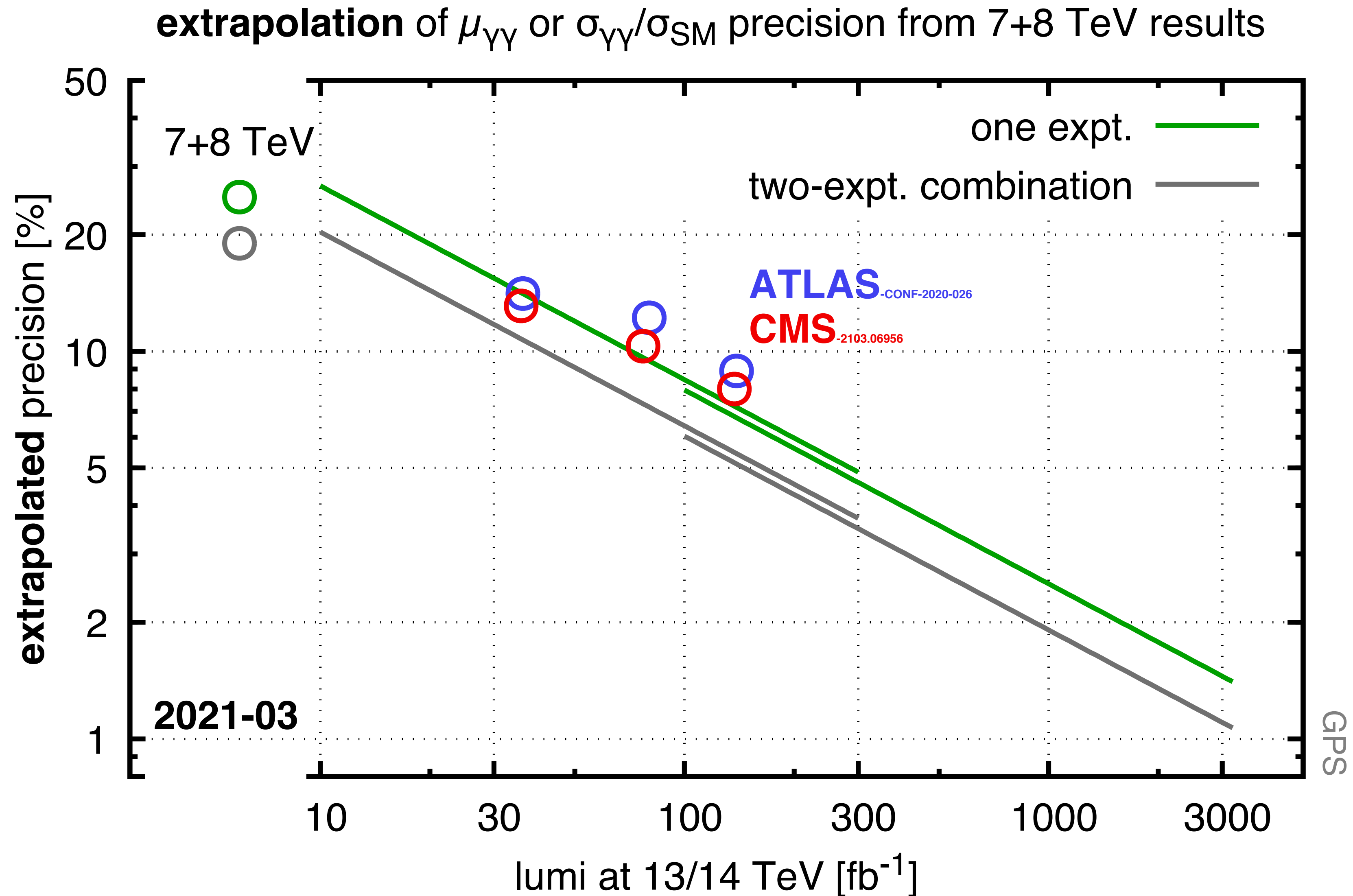
# Higgs potential — huge energy densities — yet to be experimentally confirmed



# Higgs potential — huge energy densities — yet to be experimentally confirmed



# The LHC is increasingly a precision machine, even for Higgs physics



1% uncertainty on  $\alpha_s$   
→ 2% uncertainty on  
Higgs cross section

# the master formula

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$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_{i/p}(x_1) f_{j/p}(x_2) \hat{\sigma}(x_1 x_2 s) \times [1 + \mathcal{O}(\Lambda/M)^p]$$

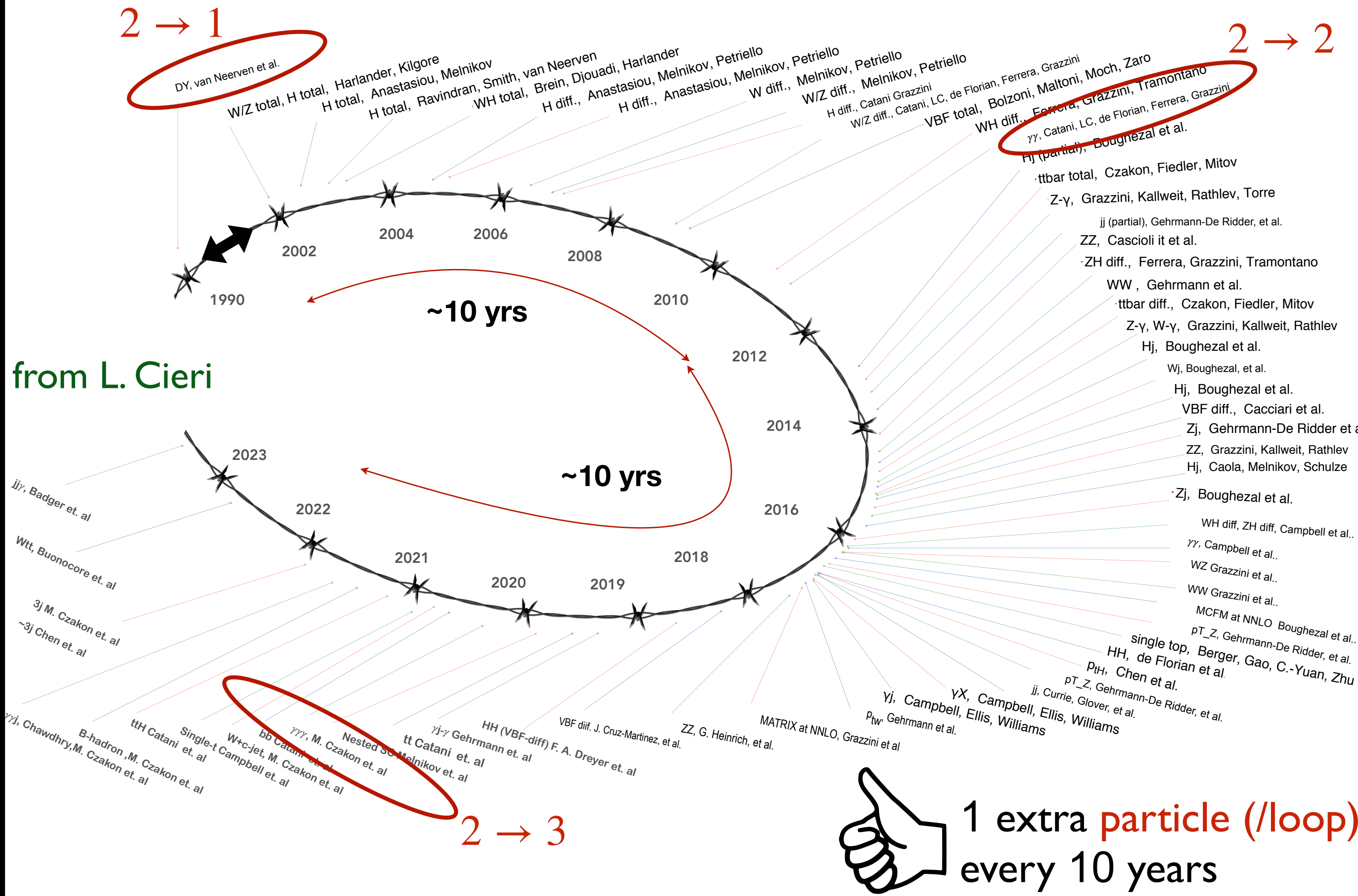


# the hard process

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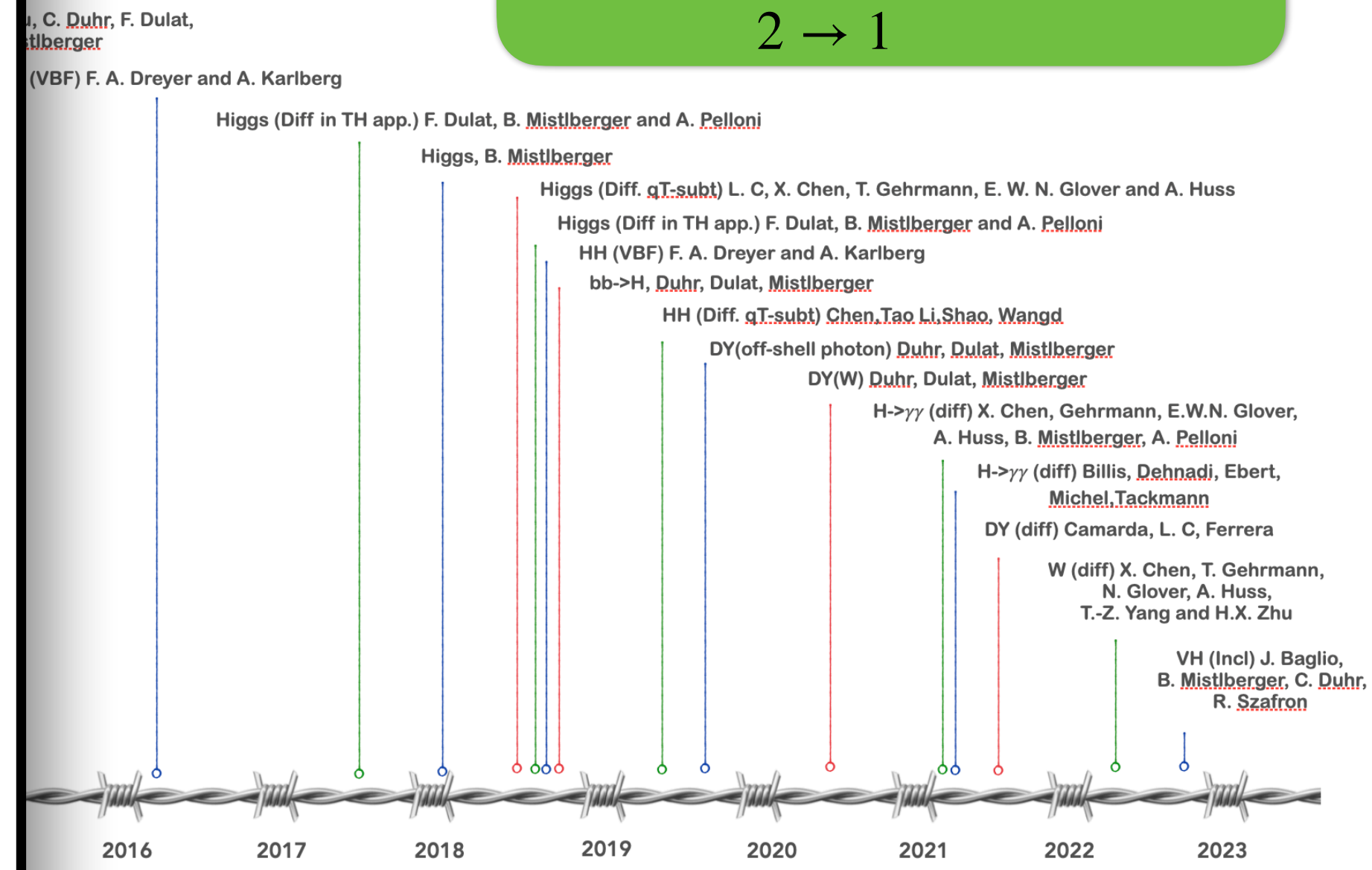
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# The NNLO revolution standard



# The N3LO revolution

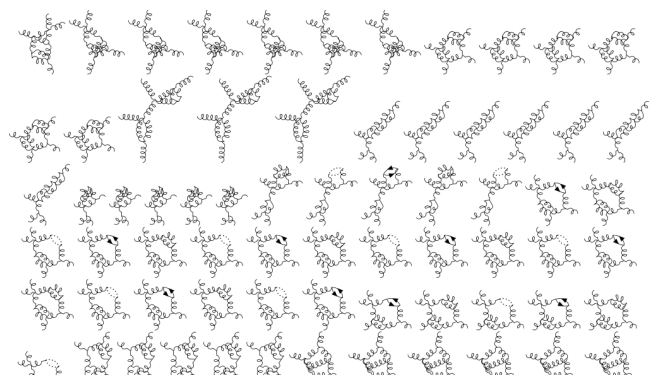
DY-like (no color in final state)  
2 → 1



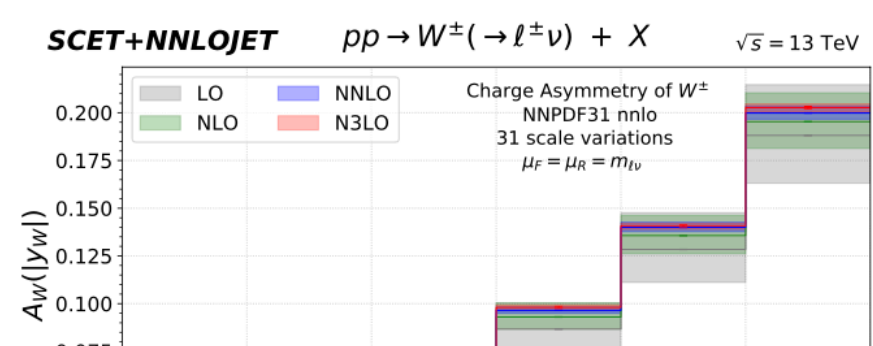
# N3LO for processes with "2 final state particles"?

3 loop amplitudes for  $pp \rightarrow jj$

Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi (2022)



# W transverse mass and charge asymmetry @N3LO



Chen, Gehrmann, Glover, Huss, Yang, Zhu (2022)  
qT subtraction

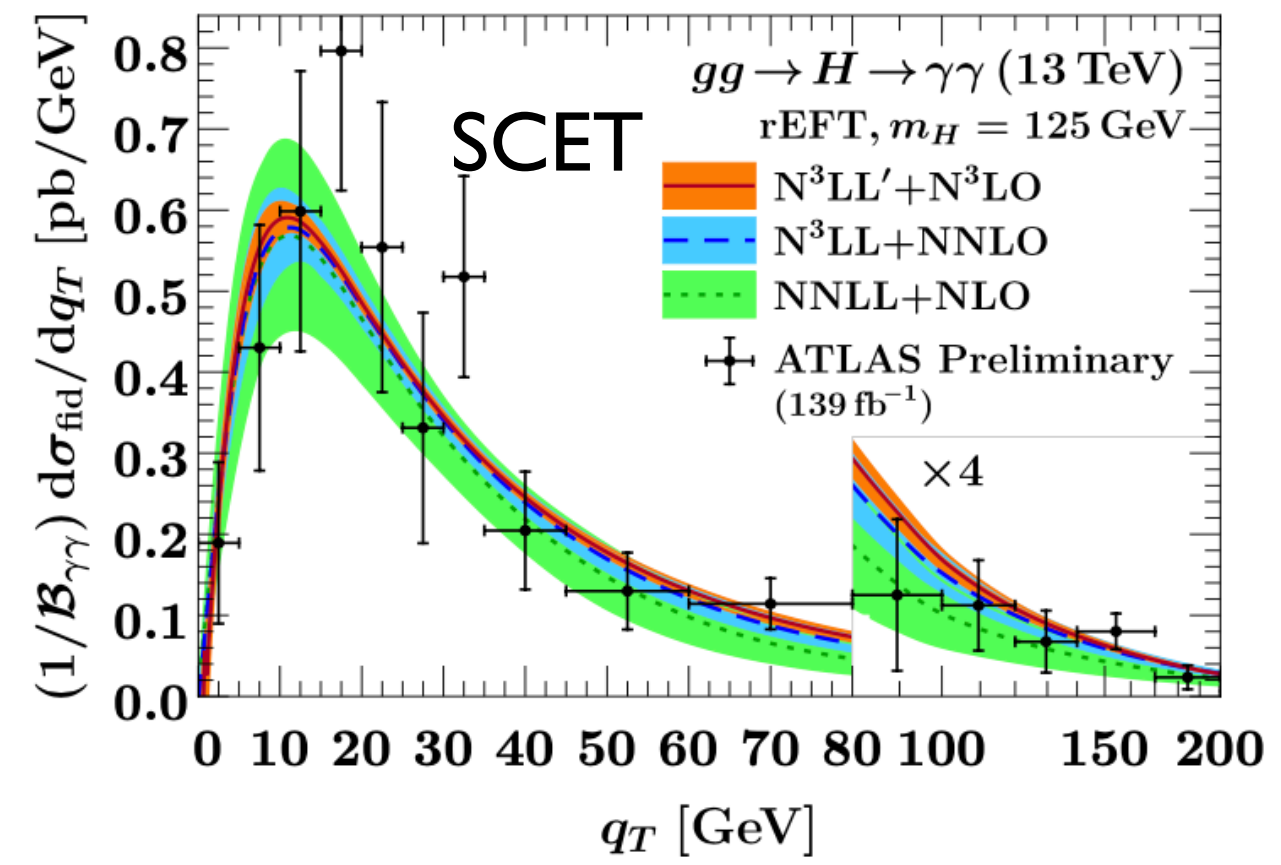
flat for rapidity distribution, about -2.5%  
Charge asymmetry relevant for pdfs



**Resummation:** state of the art for  $q_T$  resummation

- In the boundaries of phase space  $\longrightarrow$  soft and collinear emission

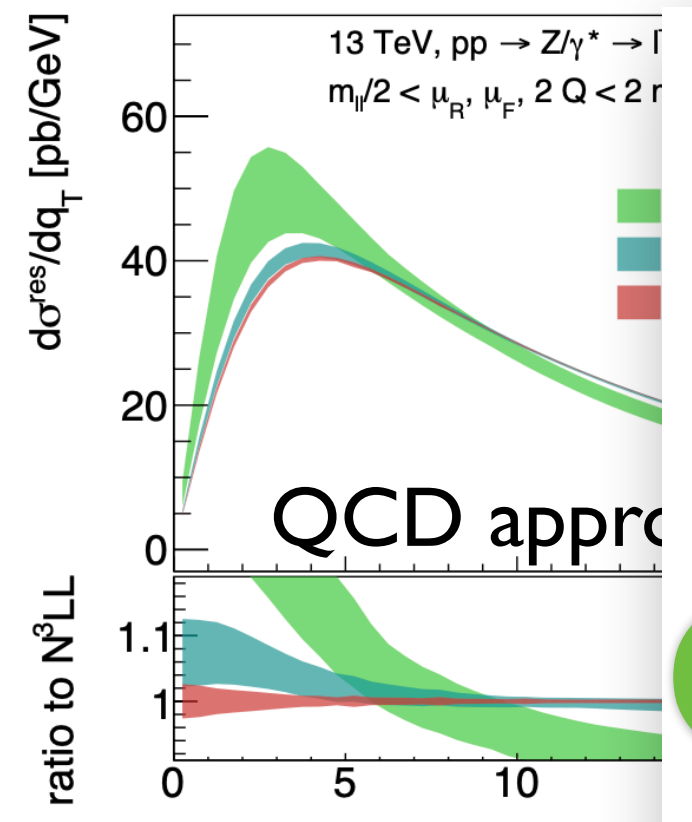
Large logs appear  $\longrightarrow$  spoil convergence of expansion in  $\alpha_s^n \log^{2n} \frac{q_t}{Q}$



Billis, Denhadi, Ebert, Michel, Tackmann(2021)

**N3LL' resummation + N3LO**

- ▶ Provide fiducial cross sections improved by  $q_T$  resummation
- ▶ Nice convergence and typically  $O(\%)$  corrections wrt pre



Camarda, Cier

**N3LL resum**

### Parton Shower accuracy

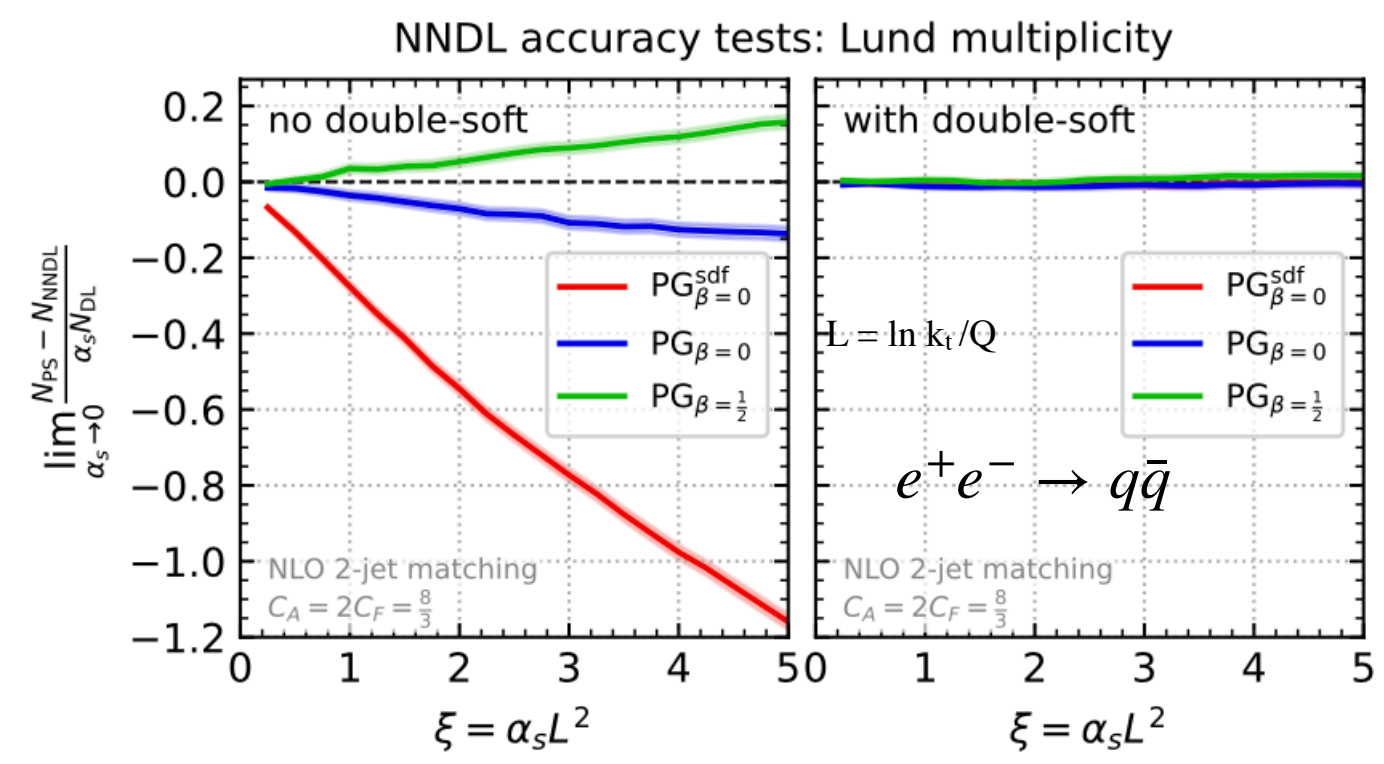
- ▶ PS is a core component of MC simulation: used in almost every analysis
- ▶ Standard parton showers are LL(+) accurate: limitation for precision

Dasgupta, Dreyer, Hamilton, Monni, Salam (2018)

▶ Several groups producing new generation of NLL PS for general observables

Nagy-Soper, Holguin-Forshaw-Platzer, PanScales, Herren-Höche-Krauss-Reichelt-Schönherr + ...

▶ **NEW:** various attempts towards even higher accuracy **NNLL**



PanScales: Ravasio, Hamilton, Karlberg, Salam, Scyboz, Soyez (2023)

- kinematics of the recoil
- color structure
- virtual contributions
- triple collinear
- **double soft**

Inclusion of double soft reproduces analytical results at order  $\alpha_s^n L^{2n-2}$

# PDFs

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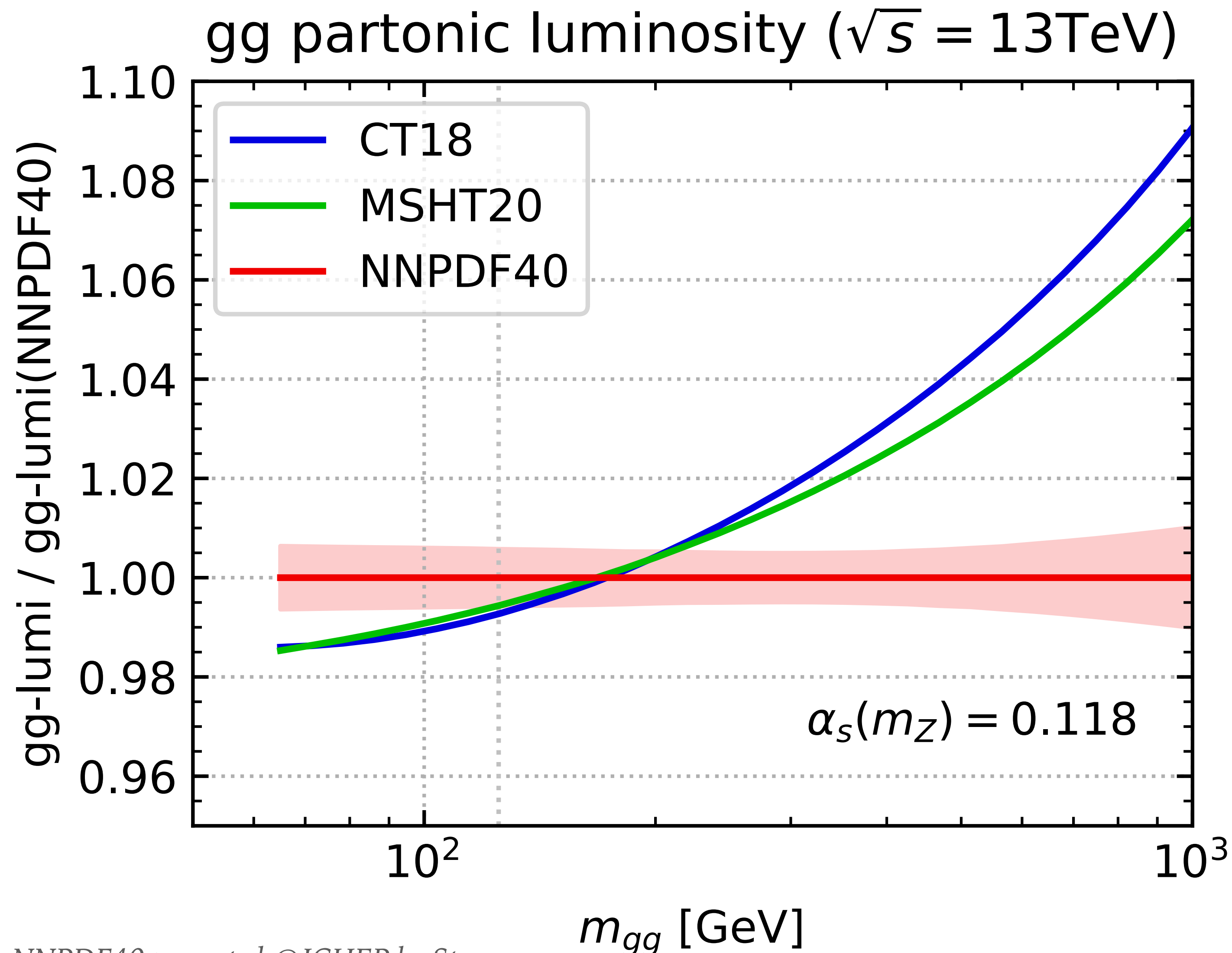
$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_{i/p}(x_1) f_{j/p}(x_2) \hat{\sigma}(x_1 x_2 s) \times [1 + \mathcal{O}(\Lambda/M)^p]$$

# PDFs

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# Comparing modern PDF sets



gg-lumi, ratio to PDF4LHC15 @  $m_H$

PDF4LHC15	1.0000	$\pm$	0.0184	↖
PDF4LHC21	0.9930	$\pm$	0.0155	
CT18	0.9914	$\pm$	0.0180	× 3
MSHT20	0.9930	$\pm$	0.0108	↙
NNPDF40	0.9986	$\pm$	0.0058	

Amazing that MSHT20 & NNPDF40 are reaching %-level precision

Differences include

- methodology (replicas & NN fits, tolerance factors, etc.)
- data inputs
- treatment of charm

At this level, QED effects probably no longer optional (MSHT20QED: 0.9870)

NNPDF40 presented @ICHEP by Stegeman

NB: PDF4LHC21 uses CT18/MSHT20/NNPDF31

# $\alpha_s$ from $Z p_T$

Table 1: Summary of the uncertainties for the determination of  $\alpha_s(m_Z)$ .

Experimental uncertainty	+0.00044	-0.00044
PDF uncertainty	+0.00051	-0.00051
Scale variations uncertainties	+0.00042	-0.00042
Matching to fixed order	0	-0.00008
Non-perturbative model	+0.00012	-0.00020
Flavour model	+0.00021	-0.00029
QED ISR	+0.00014	-0.00014
N4LL approximation	+0.00004	-0.00004
Total	+0.00084	-0.00088

Default PDF is MSHT20(aN3LO), gives **0.11828**

Table 2: Summary of N<sup>3</sup>LL fits with NNLO PDFs.

PDF set	$\alpha_s(m_Z)$	PDF uncertainty	$g$ [GeV <sup>2</sup> ]	$q$ [GeV <sup>4</sup> ]	$\chi^2/\text{dof}$
MSHT20 [32]	0.11839	0.00040	0.44	-0.07	96.0 /69
NNPDF40 [78]	0.11779	0.00024	0.50	0.08	116.0/69
CT18A [79]	0.11982	0.00050	0.36	-0.03	97.7 /69
HERAPDF20 [63]	0.11890	0.00027	0.40	-0.04	132.3/69

Difference of **0.00143**, significantly larger than quoted **~0.00086** error

# W mass

Table 2: Overview of fitted values of the  $W$  boson mass for different PDF sets. The reported uncertainties are the total uncertainties.

PDF-Set	$p_T^\ell$ [MeV ]	$m_T$ [MeV ]	combined [MeV ]
CT10	$80355.6^{+15.8}_{-15.7}$	$80378.1^{+24.4}_{-24.8}$	$80355.8^{+15.7}_{-15.7}$
CT14	$80358.0^{+16.3}_{-16.3}$	$80388.8^{+25.2}_{-25.5}$	$80358.4^{+16.3}_{-16.3}$
<b>CT18</b>	$80360.1^{+16.3}_{-16.3}$	$80382.2^{+25.3}_{-25.3}$	$80360.4^{+16.3}_{-16.3}$
MMHT2014	$80360.3^{+15.9}_{-15.9}$	$80386.2^{+23.9}_{-24.4}$	$80361.0^{+15.9}_{-15.9}$
MSHT20	$80358.9^{+13.0}_{-16.3}$	$80370.4^{+24.6}_{-25.1}$	$80356.3^{+14.6}_{-14.6}$
NNPDF3.1	$80344.7^{+15.6}_{-15.5}$	$80354.3^{+23.6}_{-23.7}$	$80345.0^{+15.5}_{-15.5}$
<b>NNPDF4.0</b>	$80342.2^{+15.3}_{-15.3}$	$80354.3^{+22.3}_{-22.4}$	$80342.9^{+15.3}_{-15.3}$

default

Difference of 17.9 MeV,  
greater than final quoted  
16.3 MeV error

Is there a single PDF that is the “right one”?

Choice  $\equiv$  systematic error...

Should it not be the same across analyses?

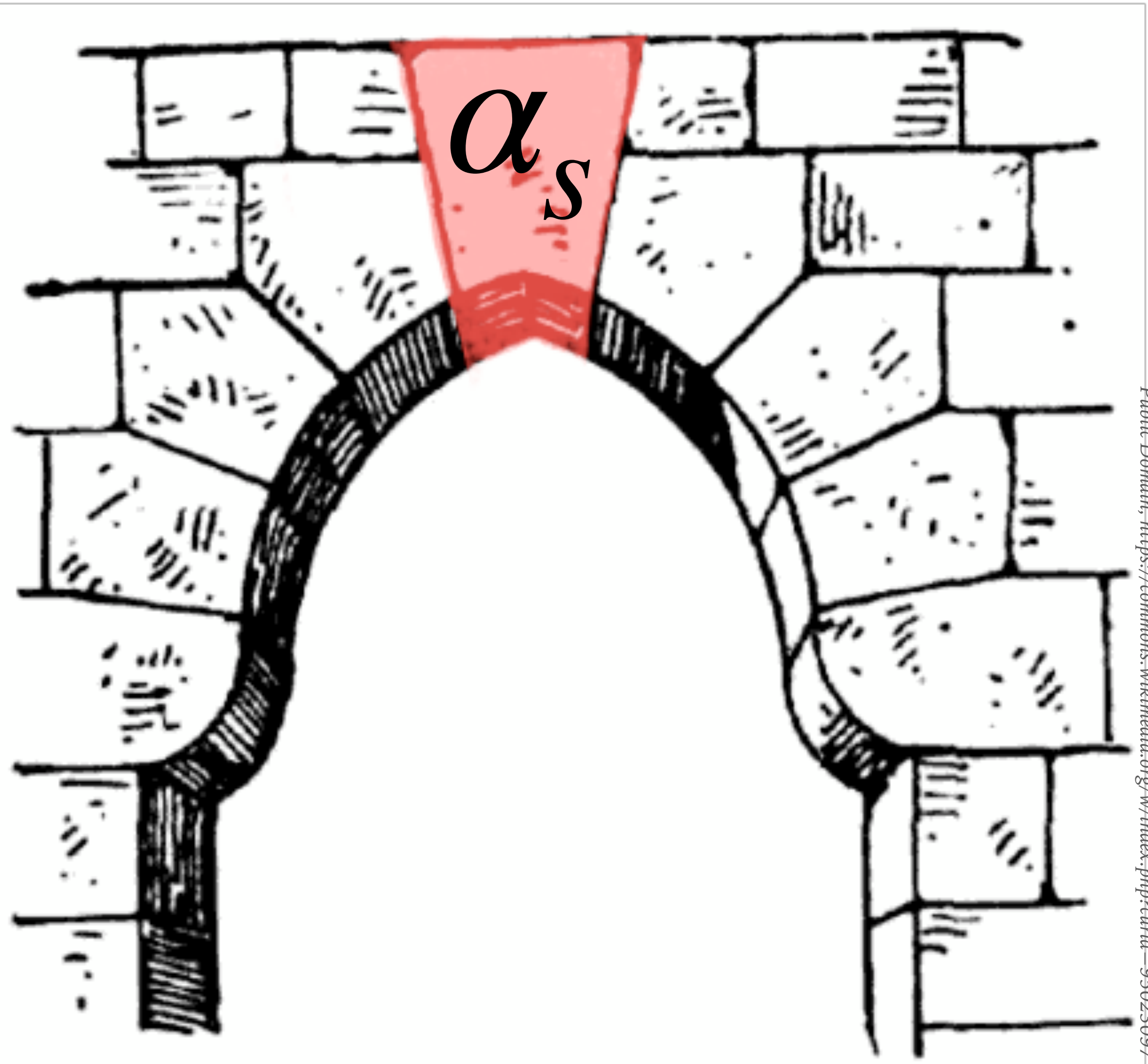
( $W$  mass relies profoundly on same  $Z$   $p_T$  distribution that goes into  $\alpha_s$  extraction)



# the non-perturbative part

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$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_{i/p}(x_1) f_{j/p}(x_2) \hat{\sigma}(x_1 x_2 s) \times [1 + \mathcal{O}(\Lambda/M)^p]$$



**the strong  
coupling**

# TESTS OF QCD \*

HD-PY 92/13

OPAL-CR093

October 23, 1992

Siegfried Bethke §

Physikalisches Institut, University of Heidelberg

Philosophenweg 12

D-6900 Heidelberg, Germany

DETERMINATIONS OF $\alpha_s$ .....	16
$\alpha_s$ from $e^+e^-$ Annihilations .....	18
$\alpha_s$ from Deep Inelastic Scattering .....	23
$\alpha_s$ from Hadron Collisions .....	24
$\alpha_s$ from Heavy Quarkonia Decays .....	24
$\alpha_s$ from Mass Splitting of Charmonium States .....	25
Summary of $\alpha_s$ Measurements .....	25

The final world average is thus quoted to be

$$\alpha_s(M_{Z^0}) = 0.118 \pm 0.007,$$

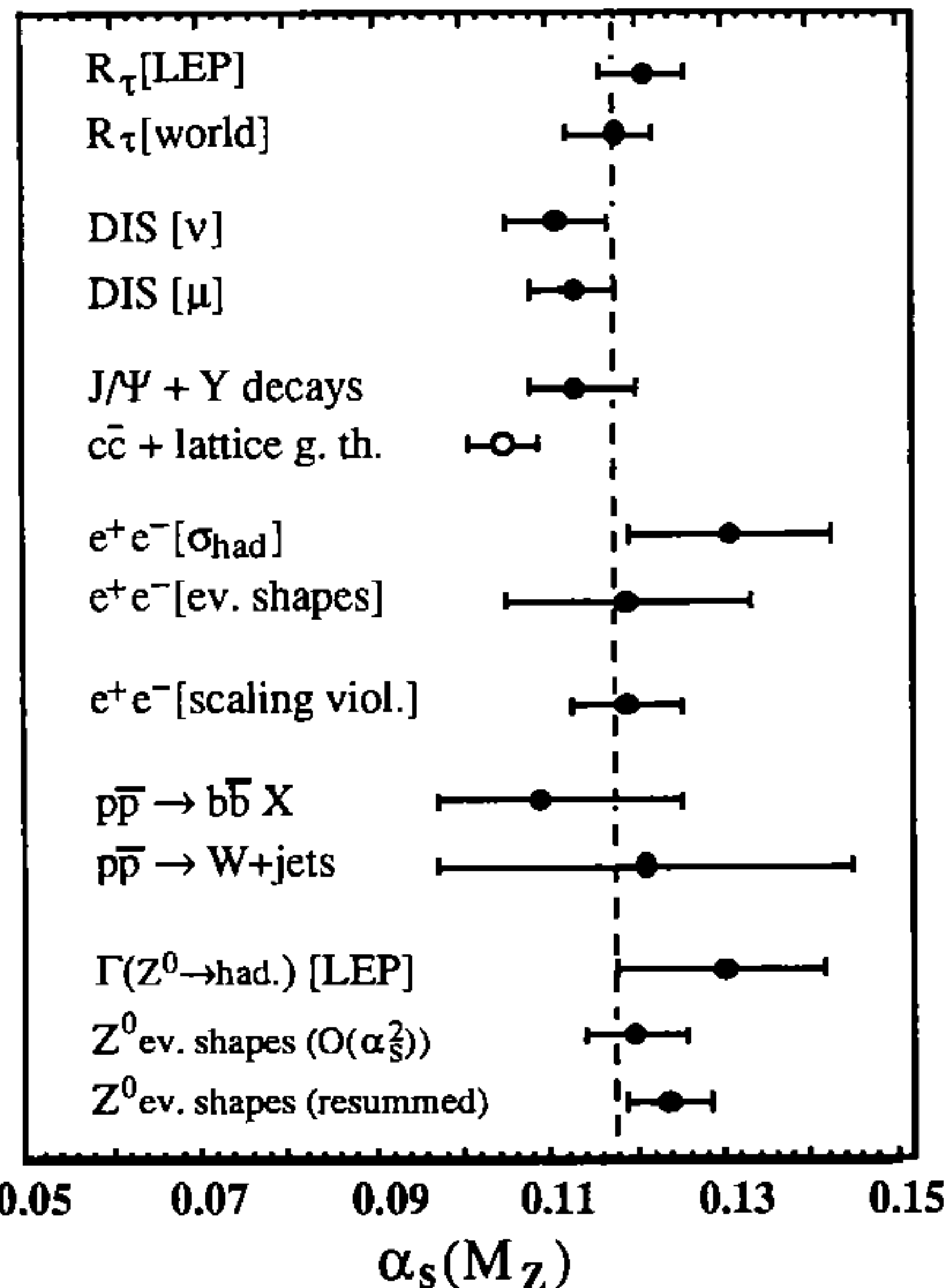


Fig. 31. Summary of measurements of  $\alpha_s(M_{Z^0})$ .

# Three decades of the strong coupling

Uncertainty has gone down by an order of magnitude to  $\sim 0.8\%$

central value has stayed stable, today

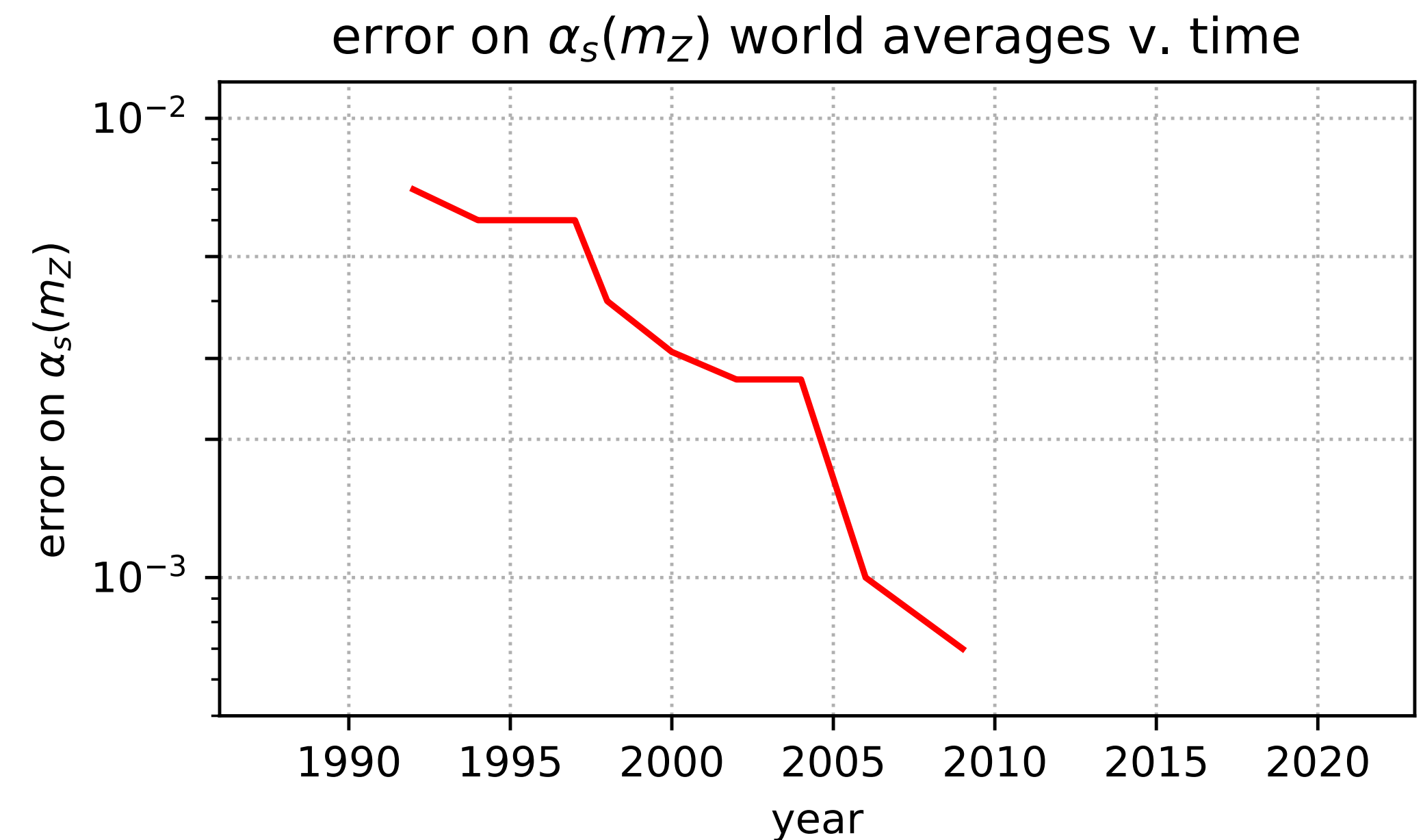
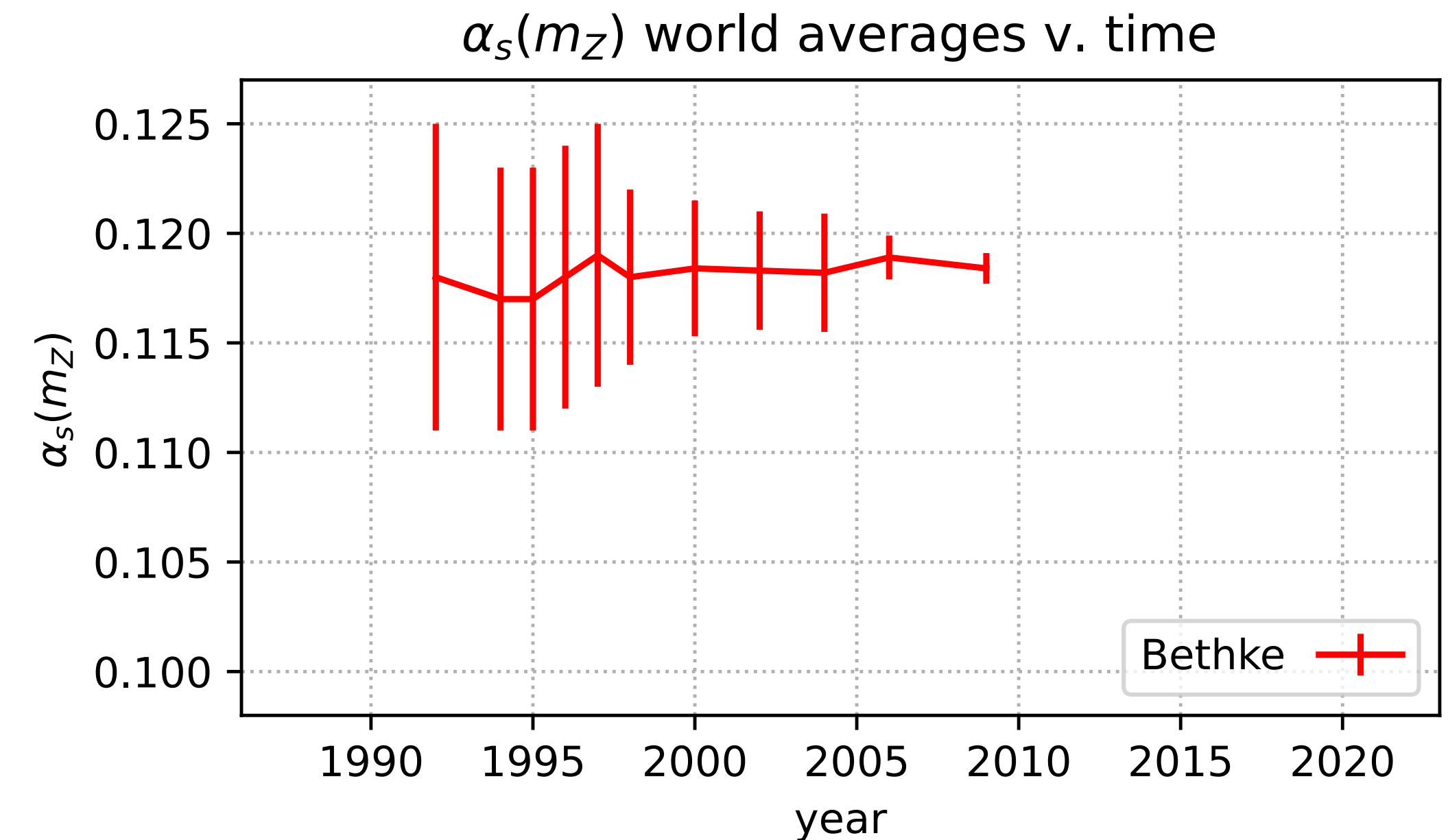
$$\alpha_s(m_Z) = 0.1179 \pm 0.0009$$

## Sources of improvement

- data (LEP, DIS,  $\sim$ LHC)
- better theory (e.g. NNLO, N3LL)
- better computers (e.g. for lattice)

## Challenges

- how to handle spread of error estimates (e.g. when systematic dominated)



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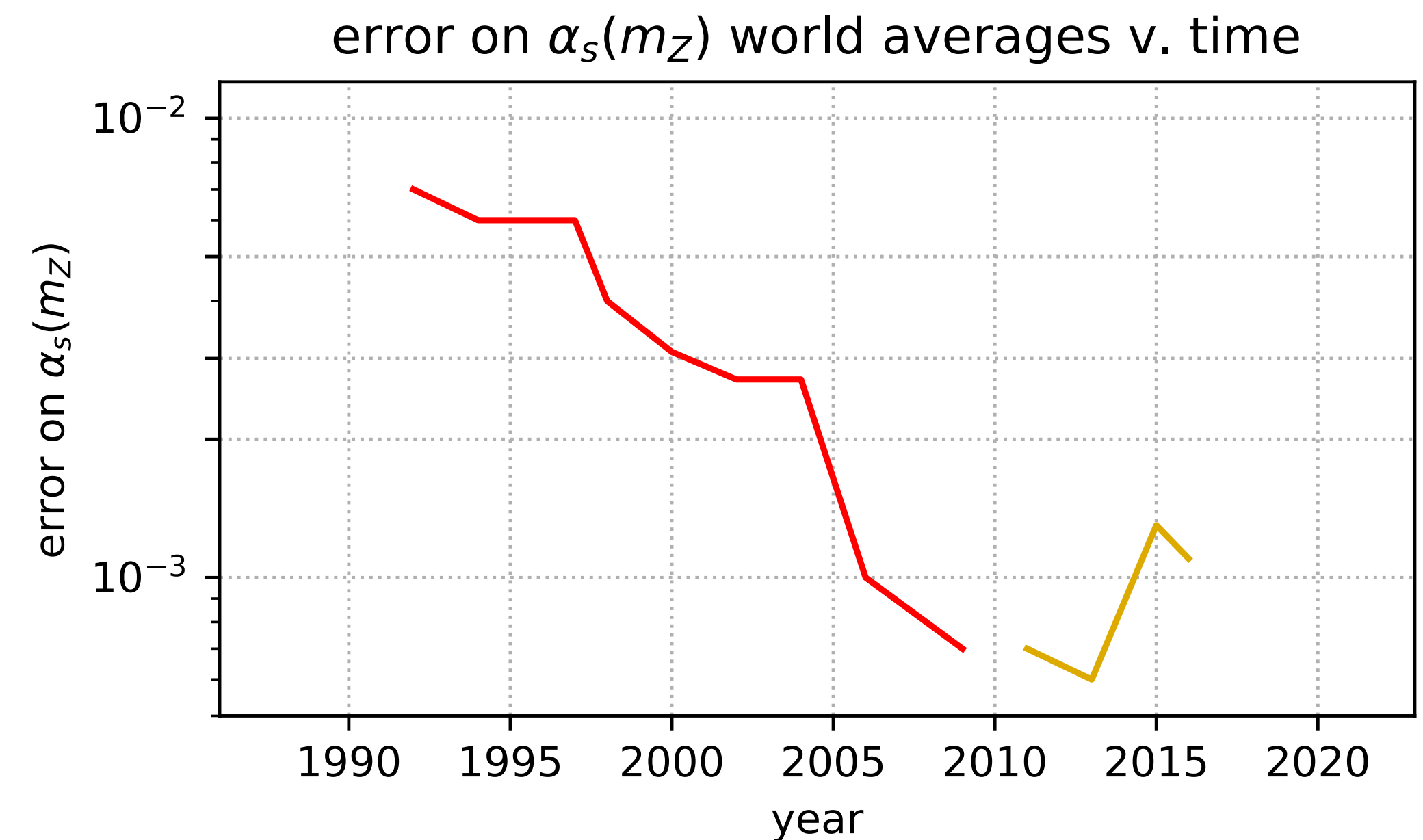
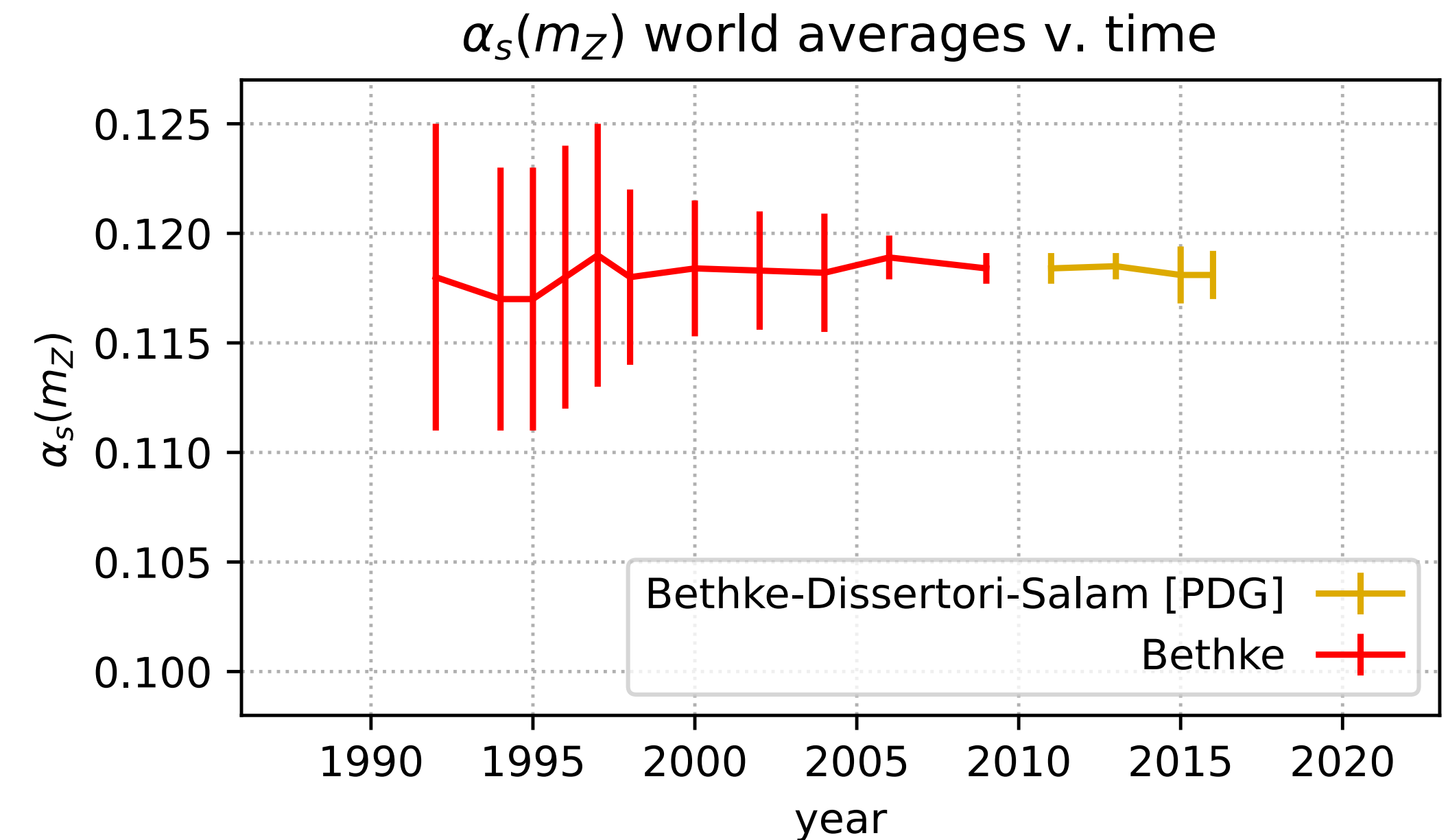
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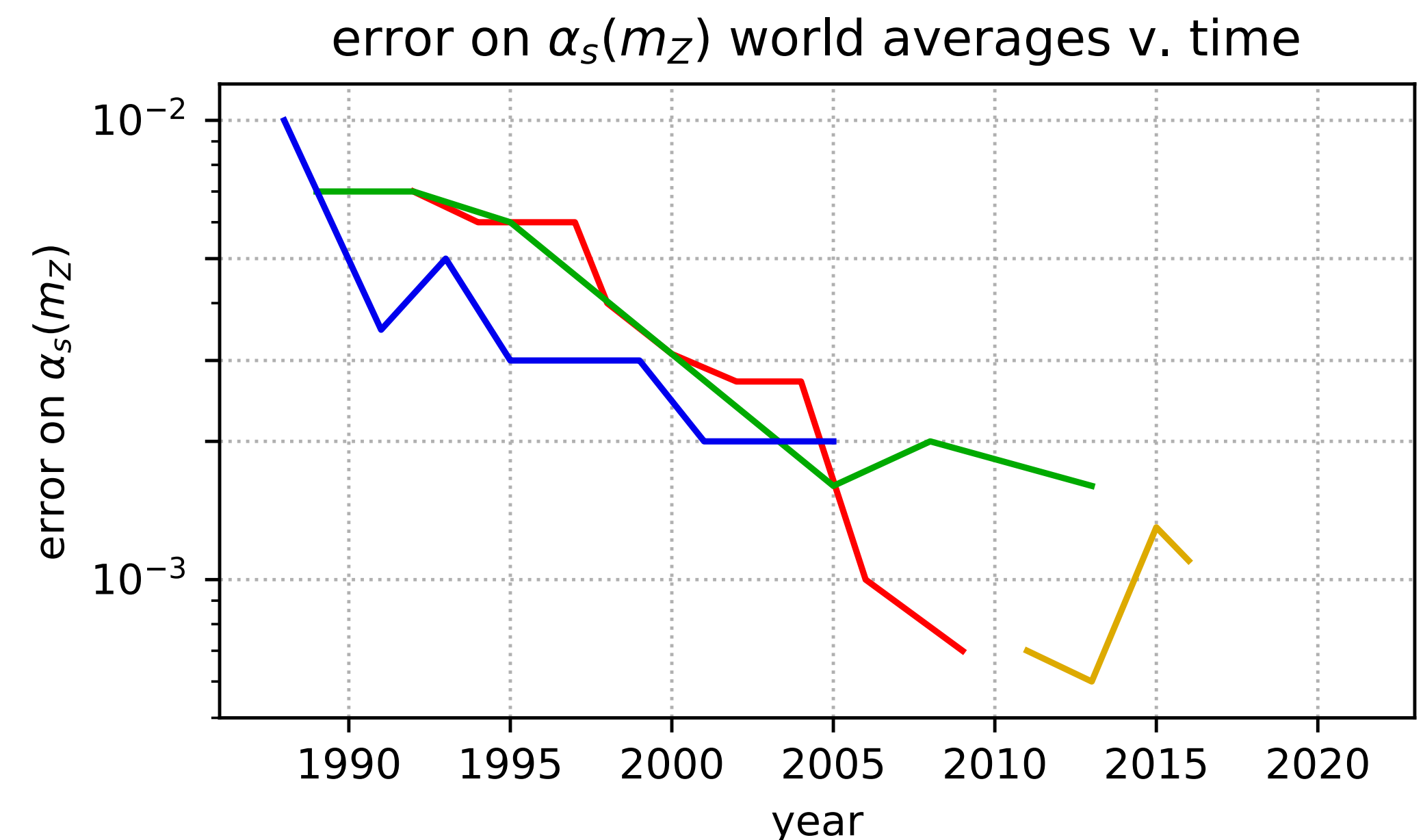
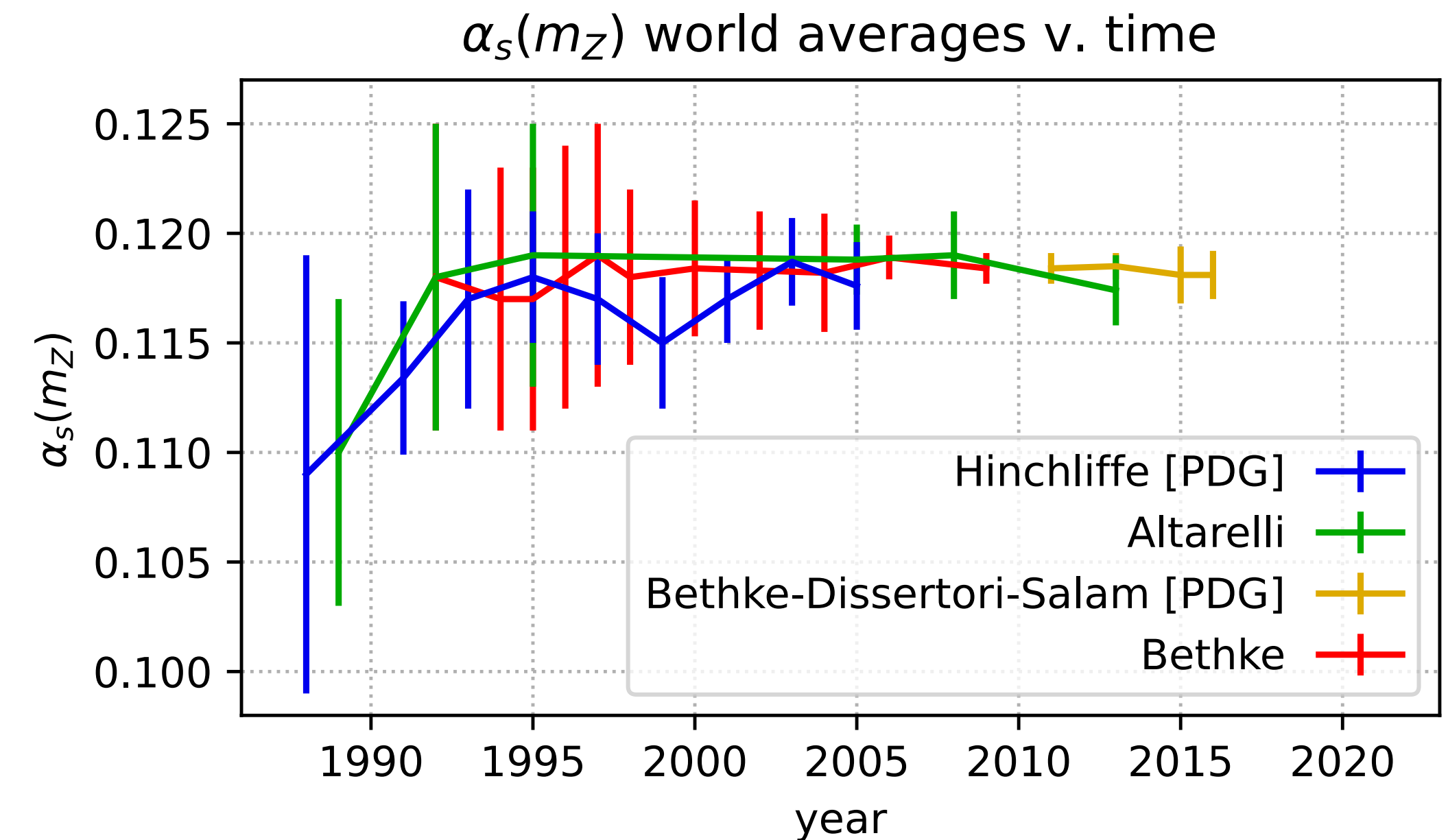
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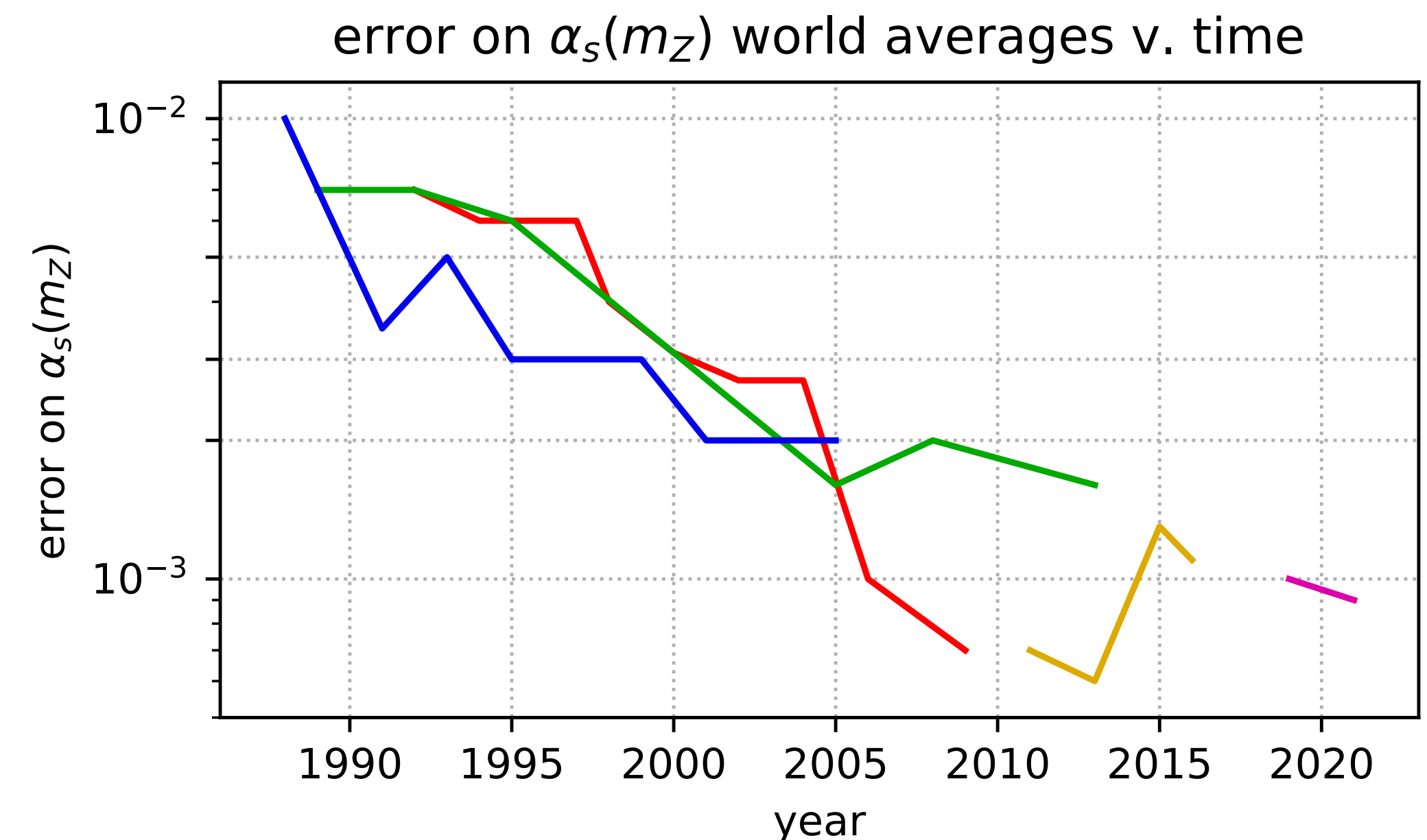
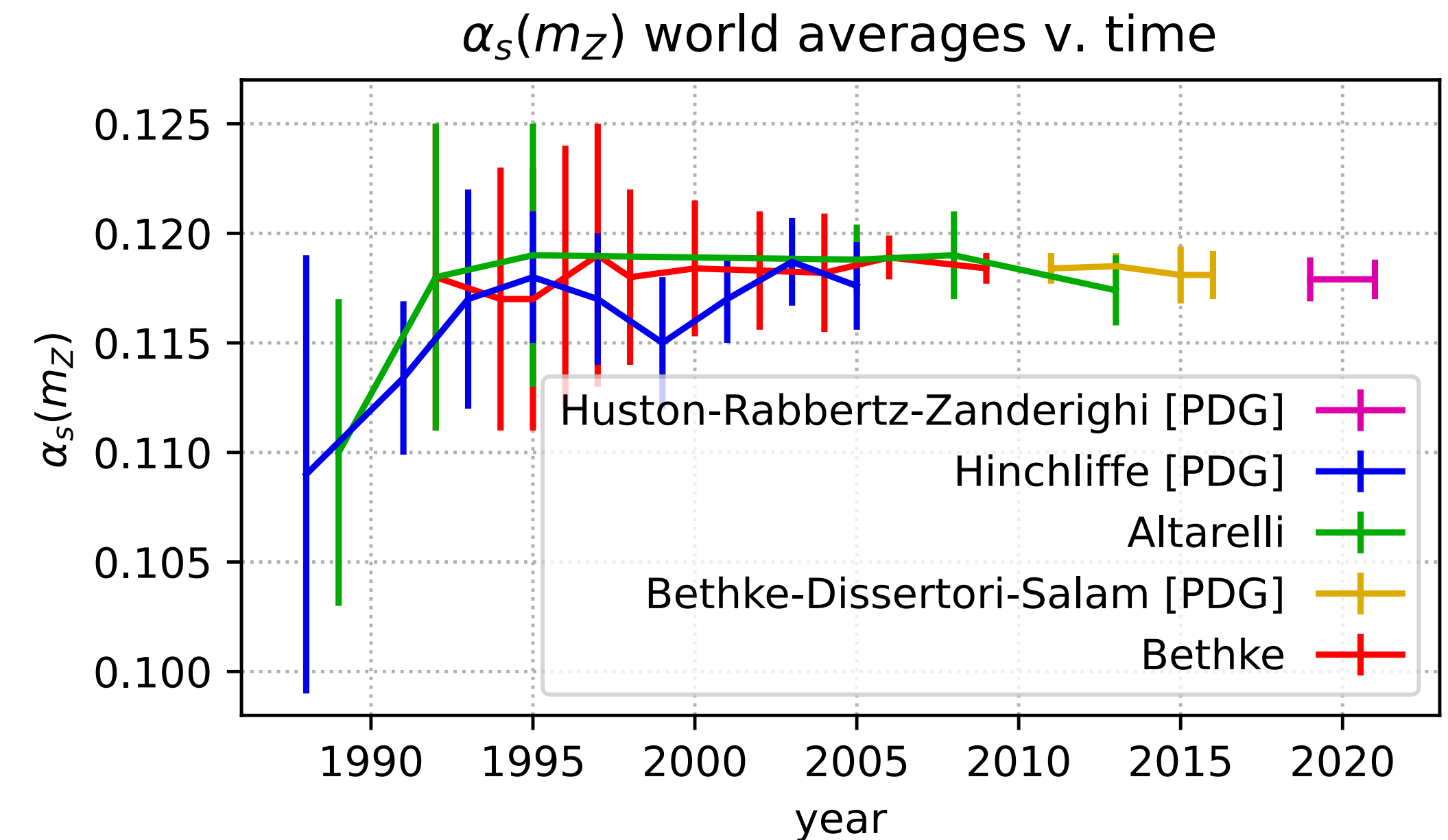
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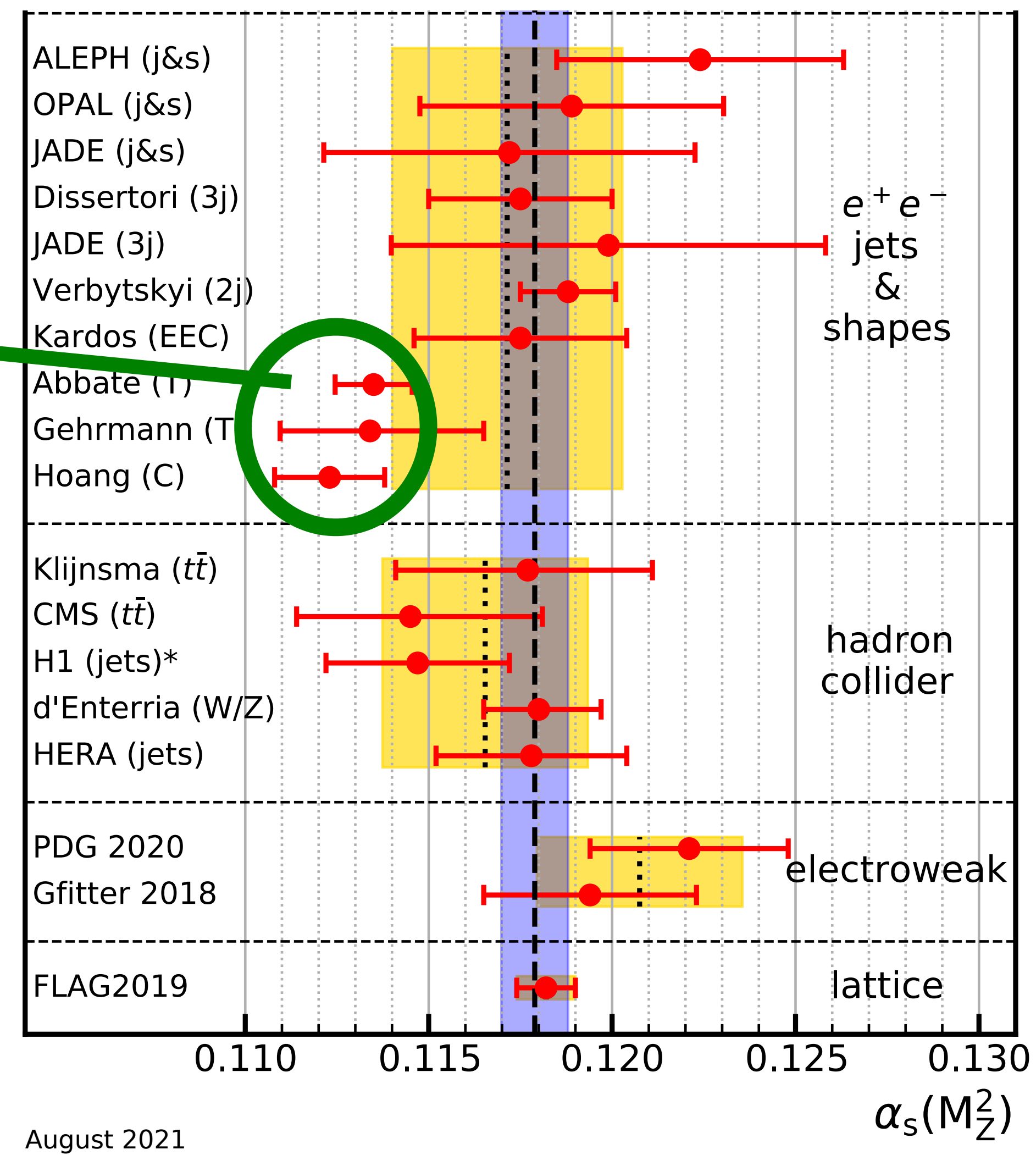
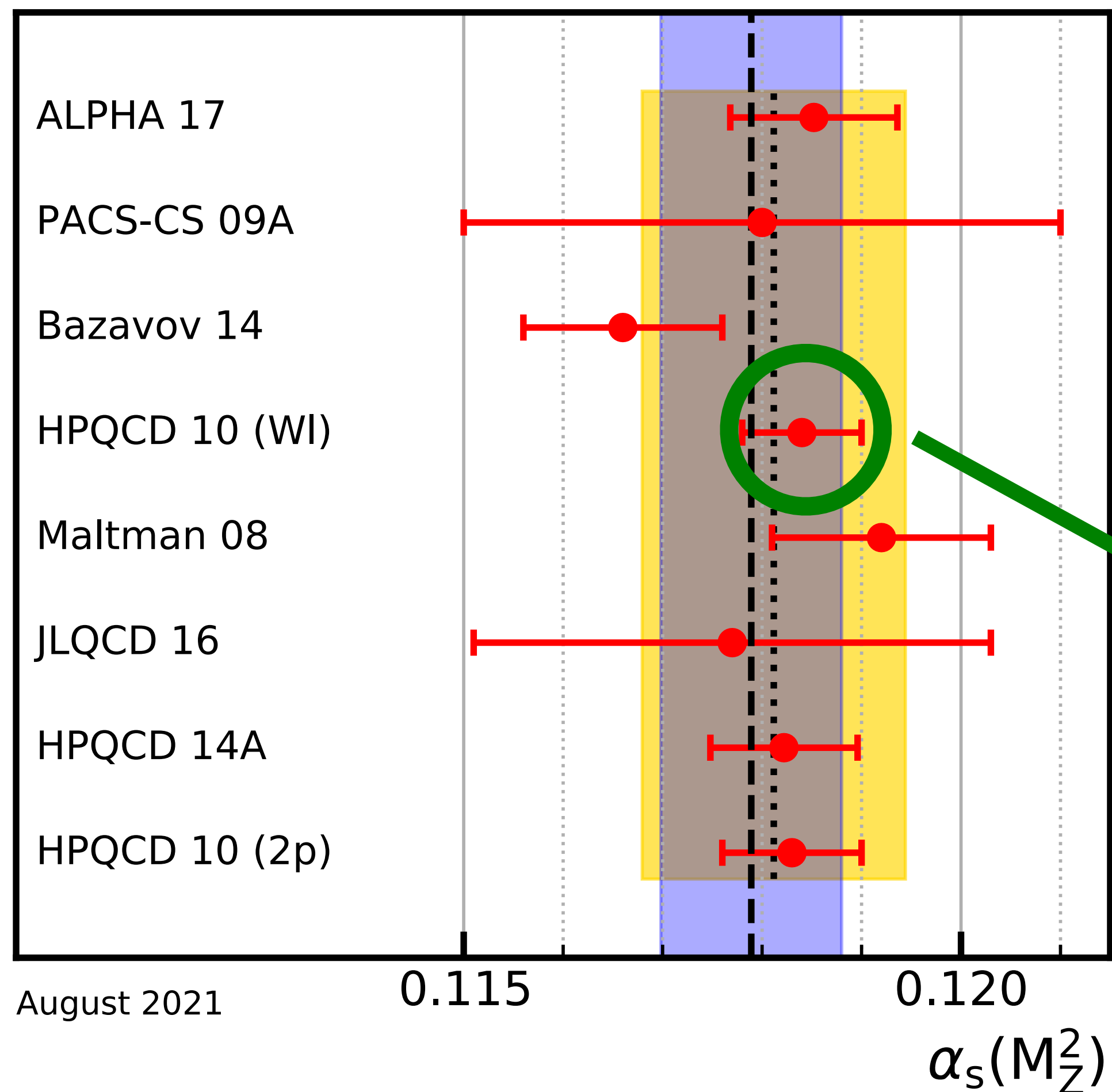
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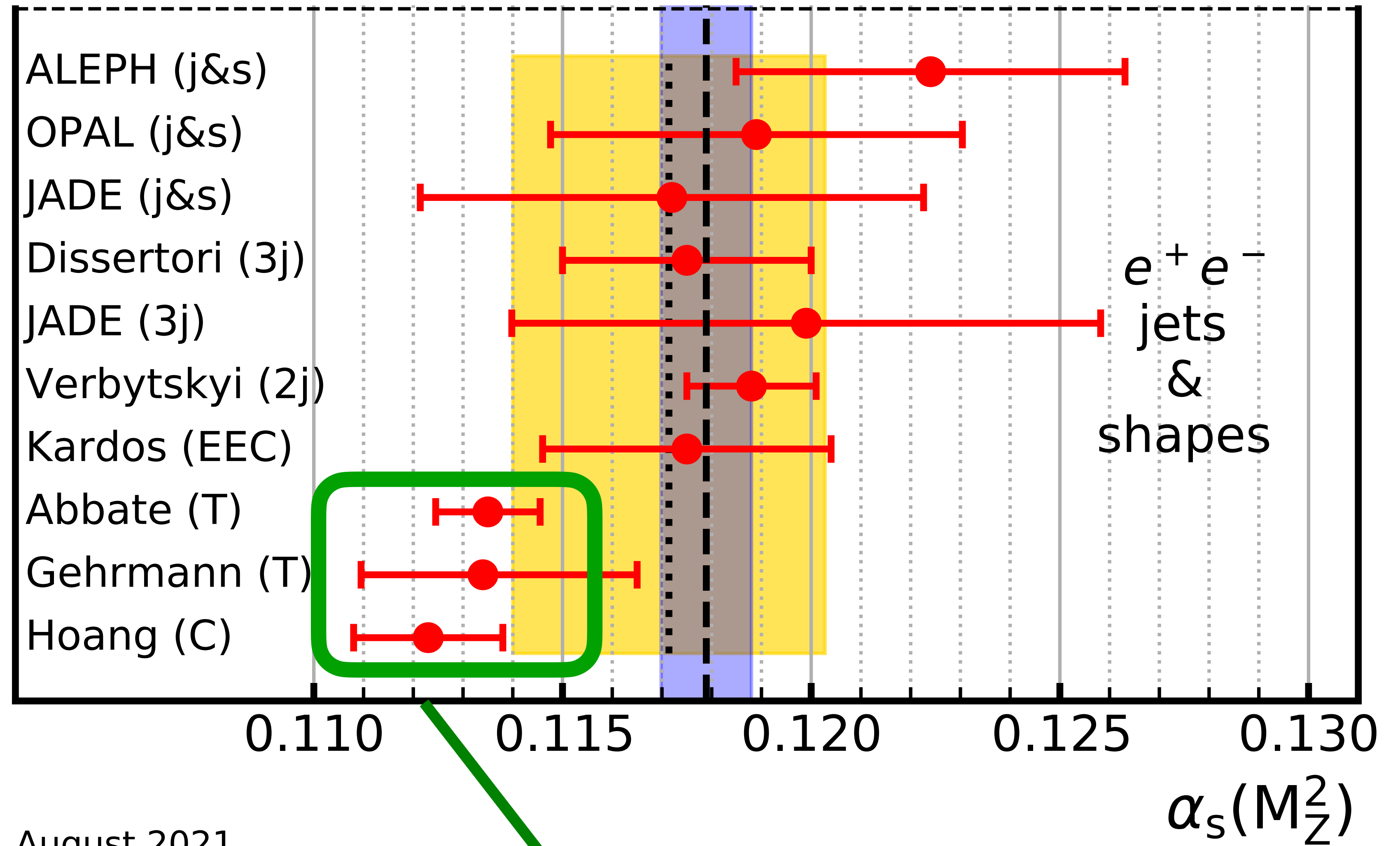




**Figure 9.5:** Lattice determinations that enter the FLAG2019 average. The yellow (light shaded) band and dotted line indicates the average value for this sub-field. The dashed line and blue (dark shaded) band represent the final world average value of  $\alpha_s(M_Z^2)$ .<sup>a</sup>

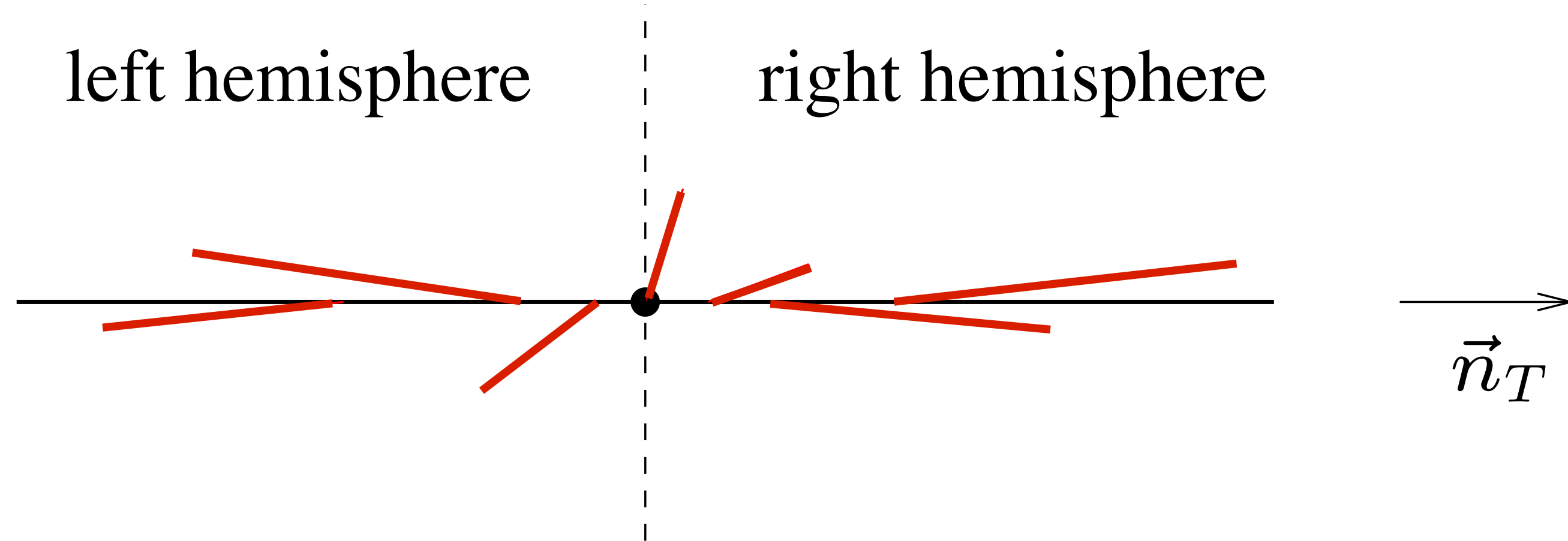


# Event shapes



August 2021

outliers and/or  
small errors



event shapes  
measure amount of  
radiation relative to  
simple



thrust  $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}, \quad \tau = 1 - T,$

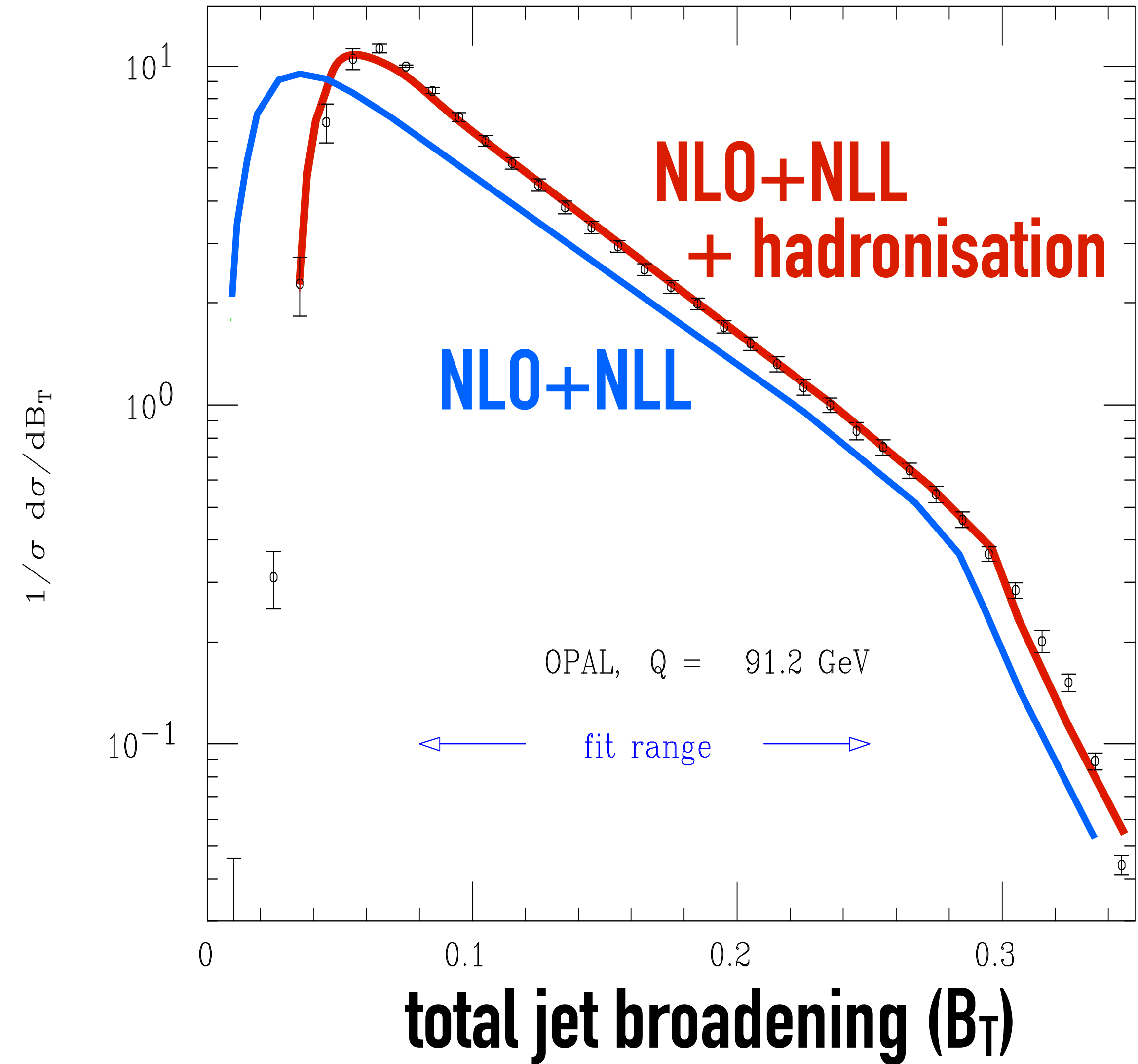
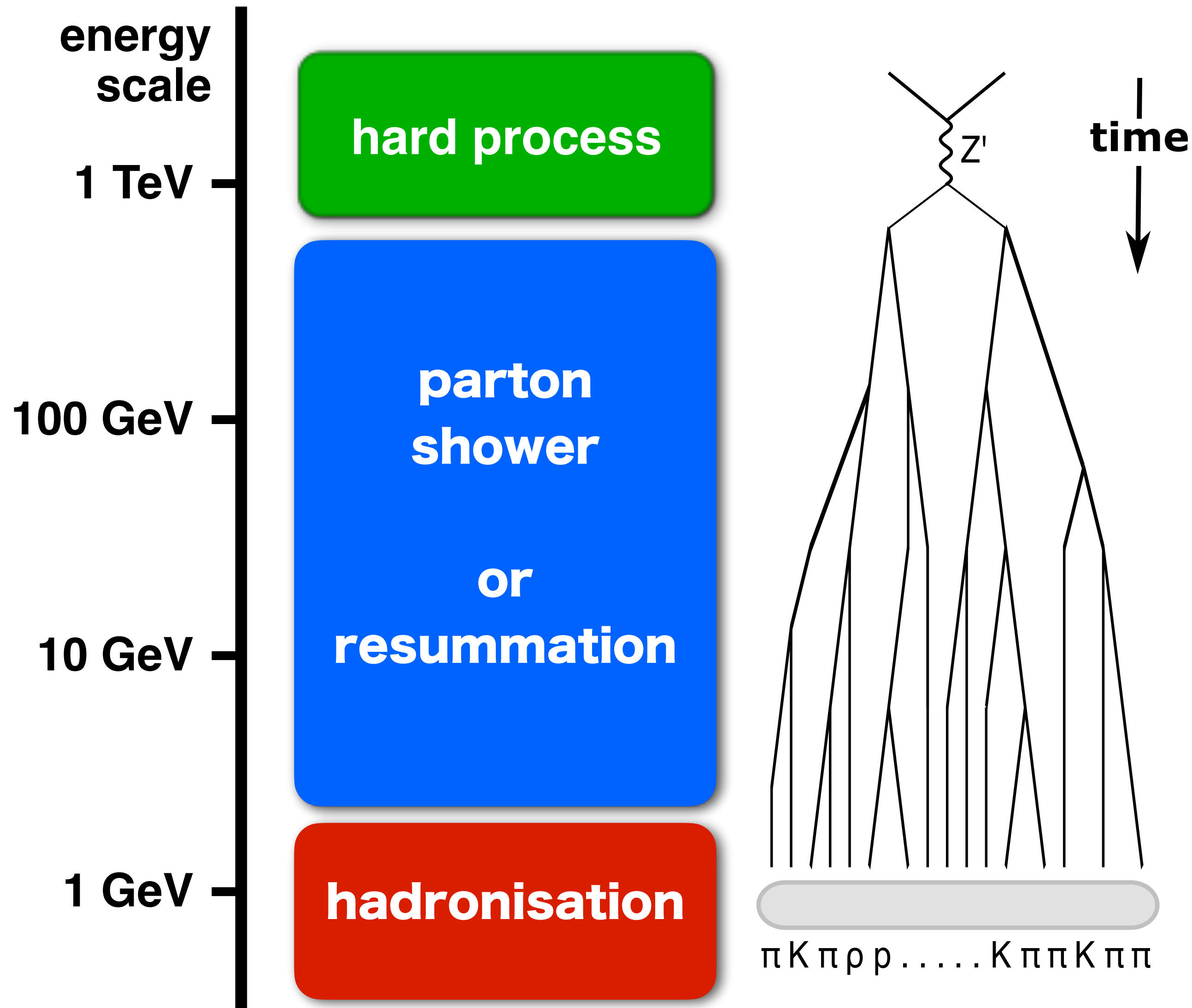
C-parameter  $C = \frac{3 \sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{2 (\sum_i |\vec{p}_i|)^2},$

jet-mass  $\rho = \frac{\left( \sum_{i \in \text{hemisphere}} p_i \right)^2}{(\sum_i E_i)^2},$

broadening  $B_T = \frac{\sum_i p_{Ti}}{\sum_i |\vec{p}_i|}.$

**NB: any issue that is present for event shapes is likely to be present also for pp & DIS jet measurements**

# non-perturbative physics & hadronisation: the bane of quantitative hadronic QCD



# The standard approach

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- Measure data at hadron (“particle”) level
- Run a general purpose Monte Carlo
- Determine the observable at parton level
- Determine the observable at hadron level
- Use the difference to correct a perturbative parton-level calculation to hadron-level, for comparison to data

**Fundamental & conceptual problem:**

**parton-level in a Monte Carlo  $\neq$  parton-level in a perturbative calculation**

# non-perturbative physics & hadronisation: the bane of quantitative collider QCD

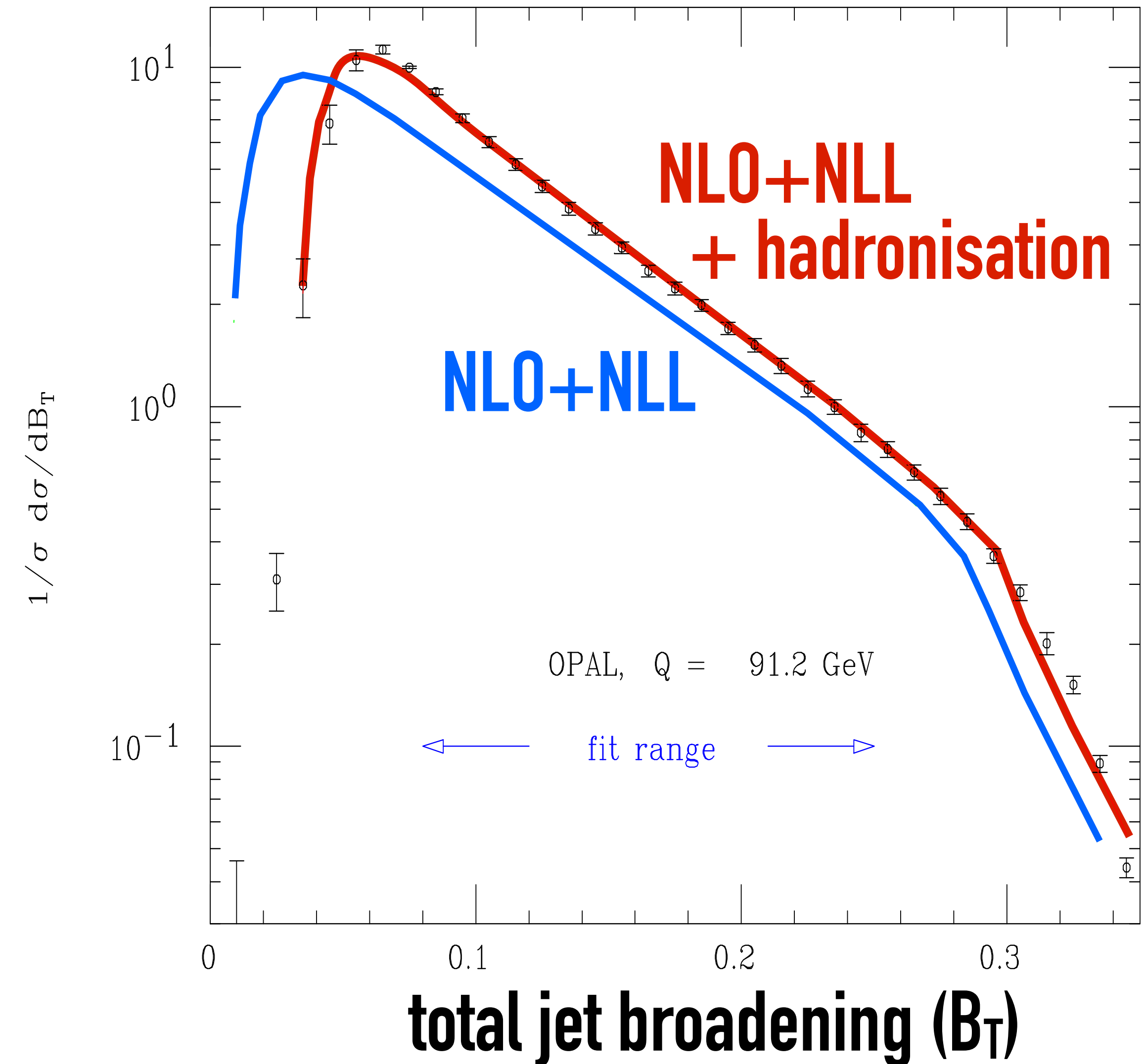
c. 1995, theorists proposed analytical approaches to quantifying hadronisation (Dokshitzer, Marchesini & Webber; Beneke & Braun; Manohar & Wise; Korchemsky & Sterman).

$$\delta V \sim \frac{c_V \alpha_0}{Q}$$

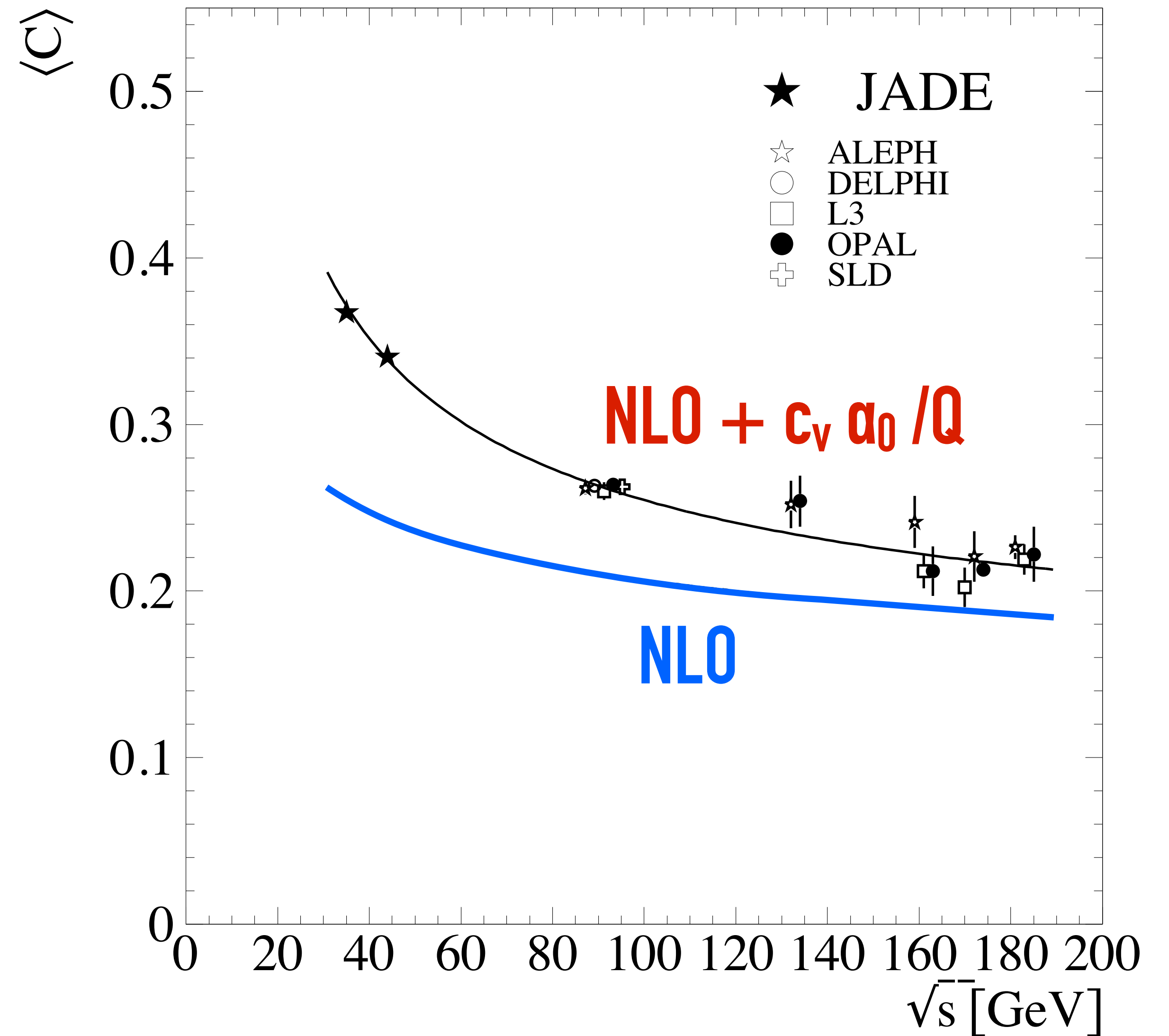
Did they match data?

Two key features to check:

- universality of  $\alpha_0$  across many shapes
- scaling with centre-of-mass energy  $Q$



**Studies of average values of event shapes v. Q (CoM energy) gave support to analytical approaches that tried to work around this problem**

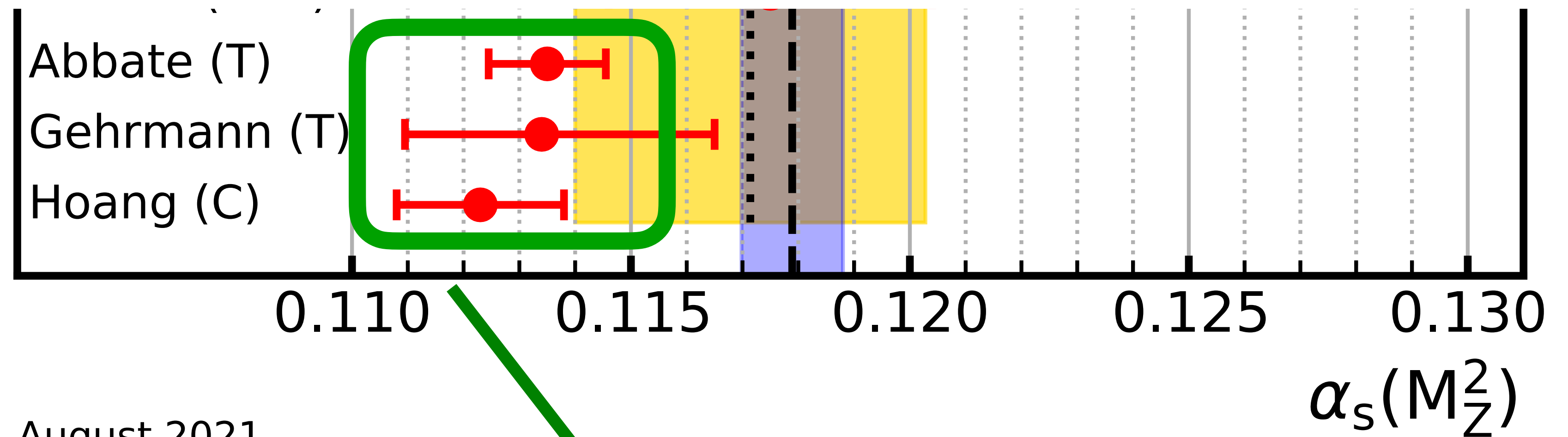


World average:  $\alpha_s(m_Z) = 0.1179 \pm 0.0009$

Thrust:  $\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{hadr}} \pm 0.0009_{\text{pert}}$  [1006.3080](#)

C-parameter:  $\alpha_s(m_Z) = 0.1119 \pm 0.0006_{\text{exp+had}} \pm 0.0013_{\text{pert}}$  [1501.04111](#)

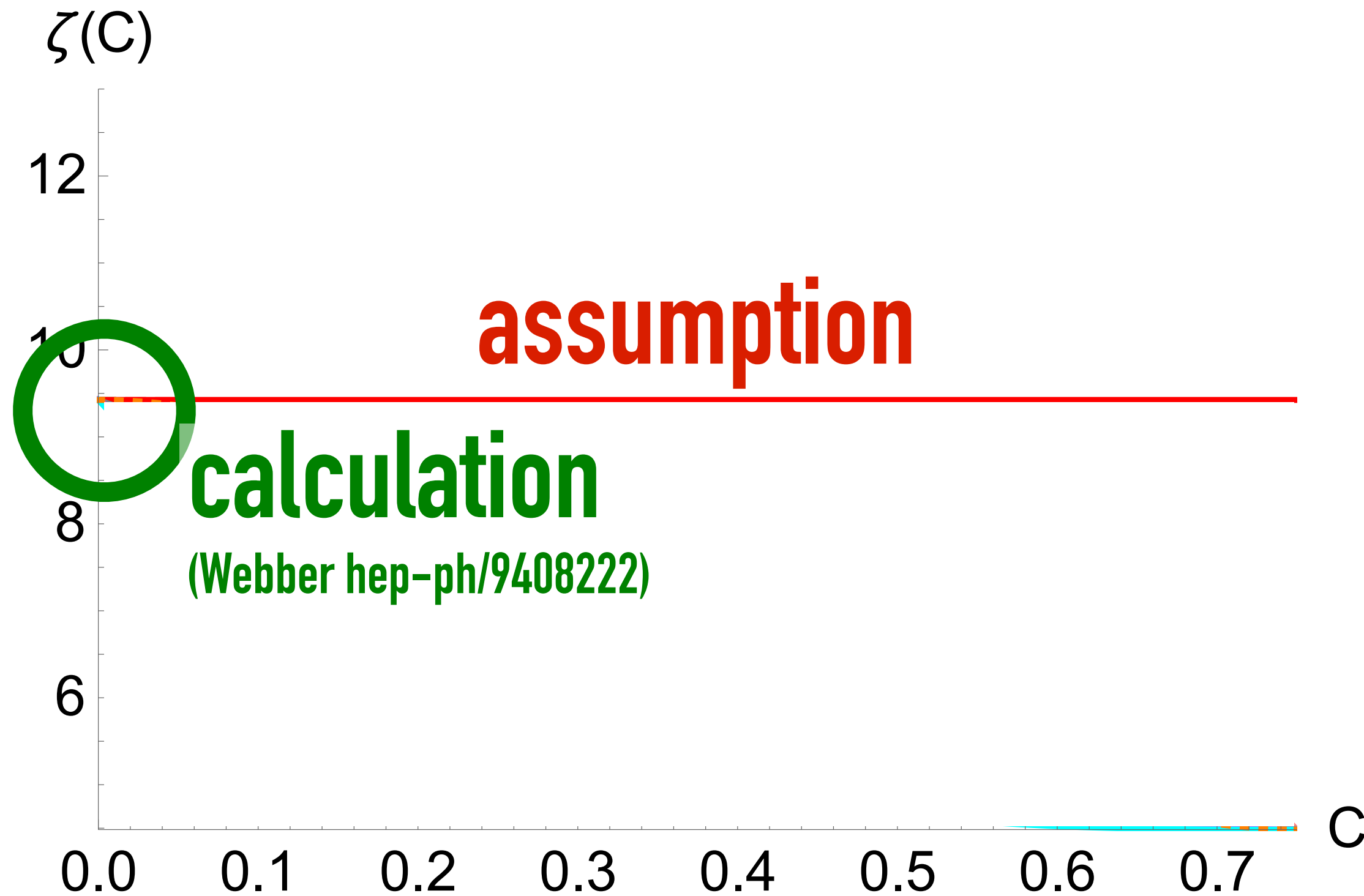
**NNLO + N3LL + 1/Q**



August 2021

**outliers and/or  
small errors**

# non-perturbative shift as $f^n$ of $C$



**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ .

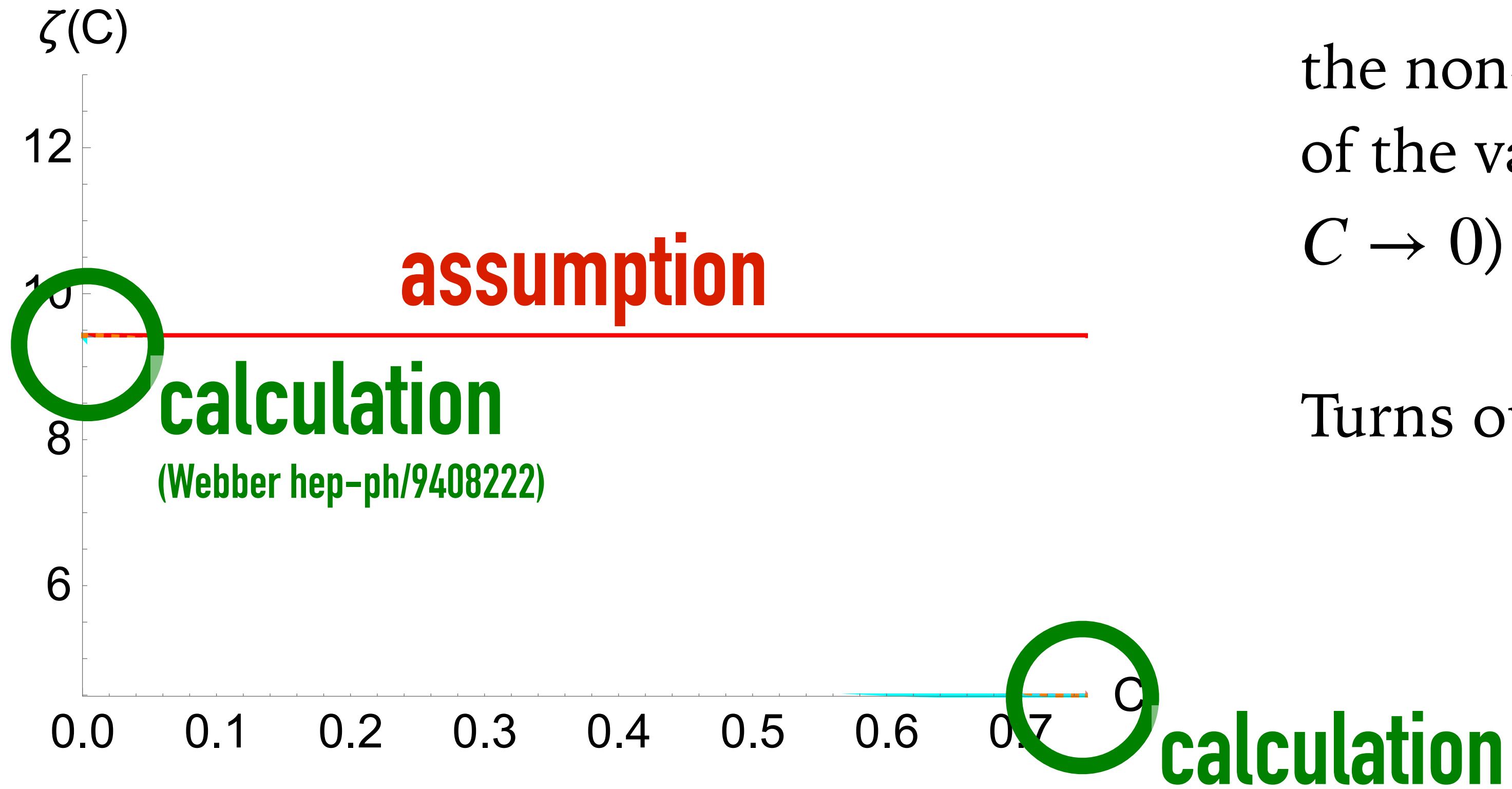
critical assumption in those high-precision fits:

the non-perturbative shift is independent of the value of the observable (valid when  $C \rightarrow 0$ )

Turns out not be true



# non-perturbative shift as $f^n$ of $C$



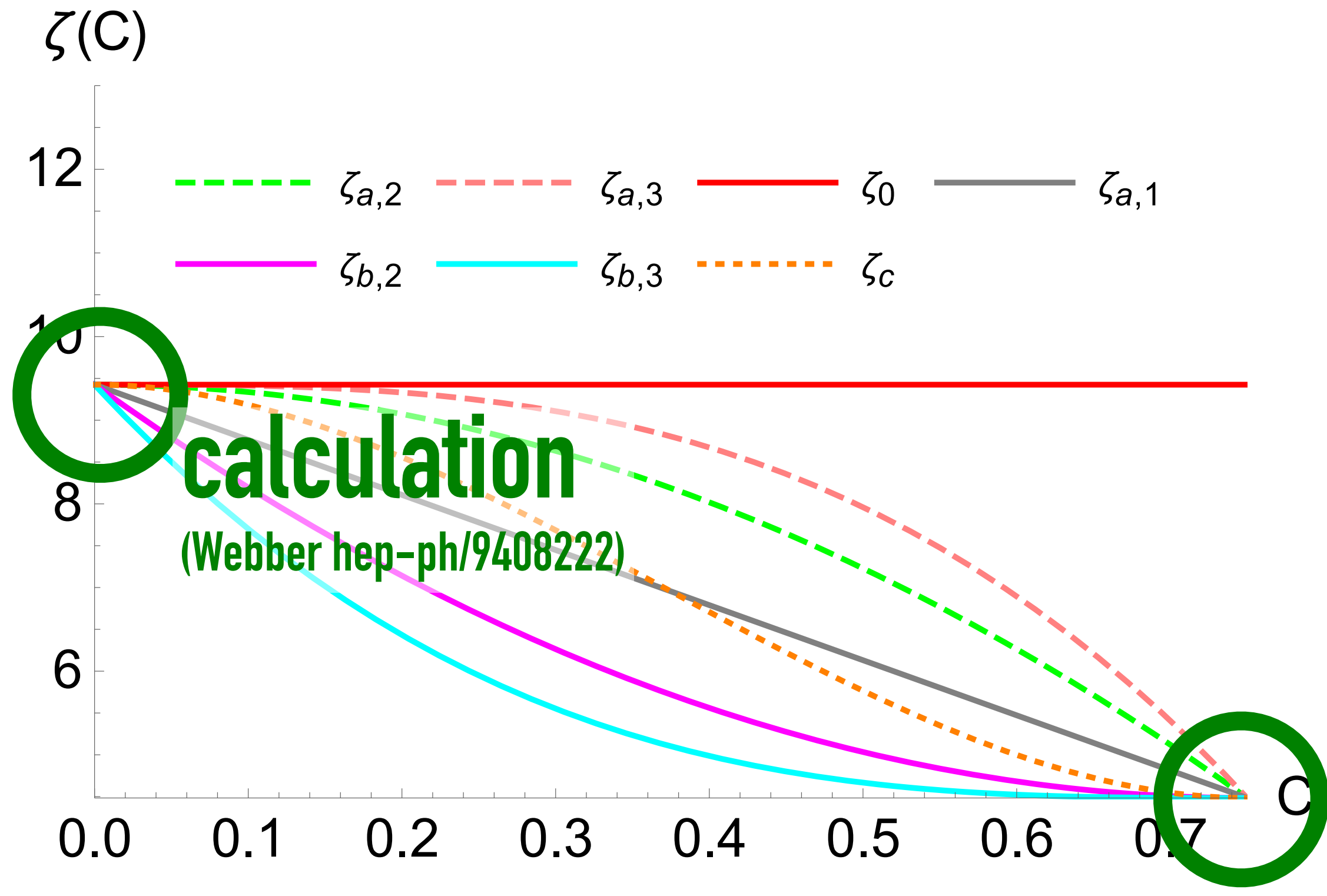
**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ . (Luisoni, Monni, GPS, 2012.00622)

critical assumption in those high-precision fits:

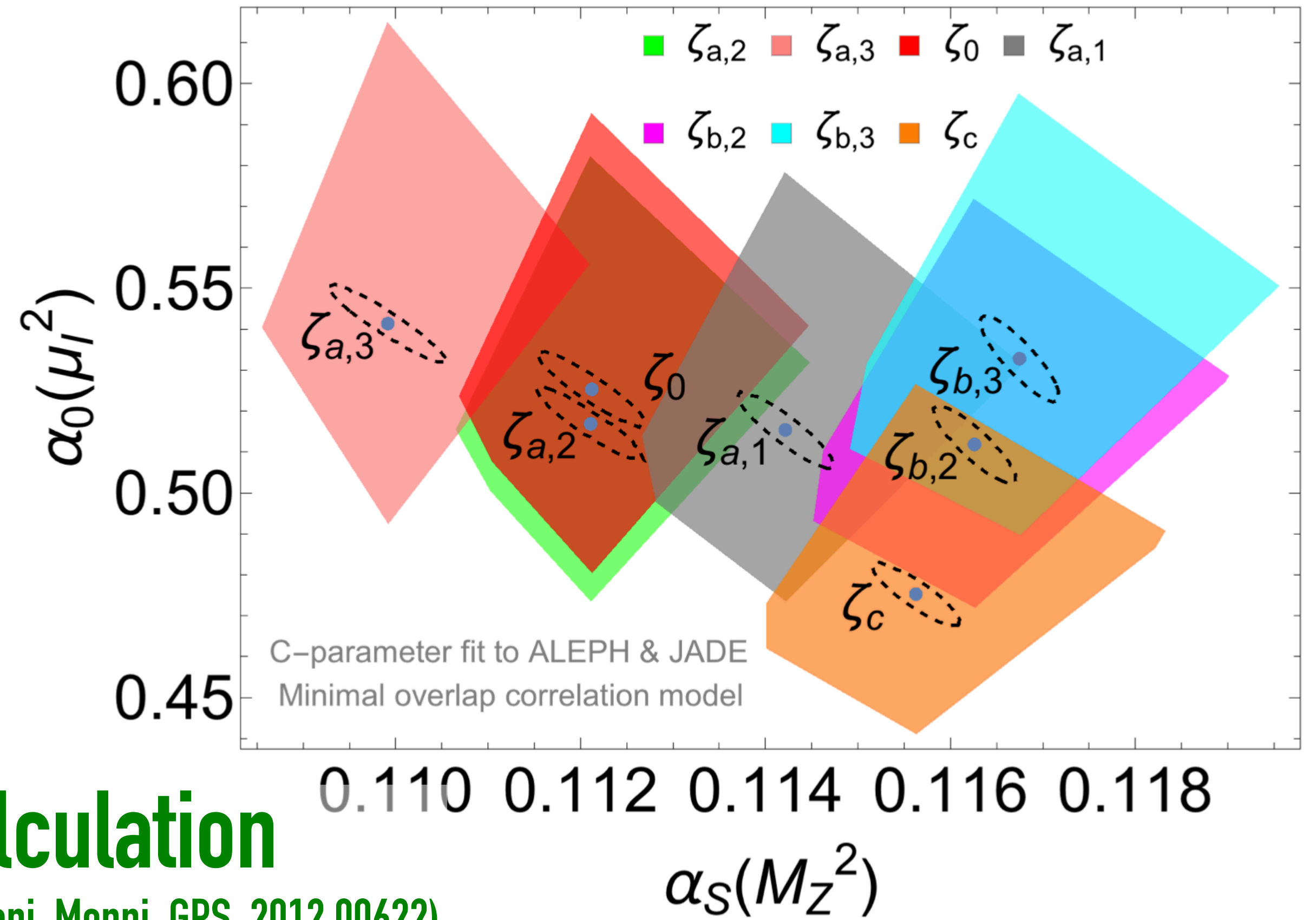
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Turns out not be true

# non-perturbative shift as $f^n$ of $C$

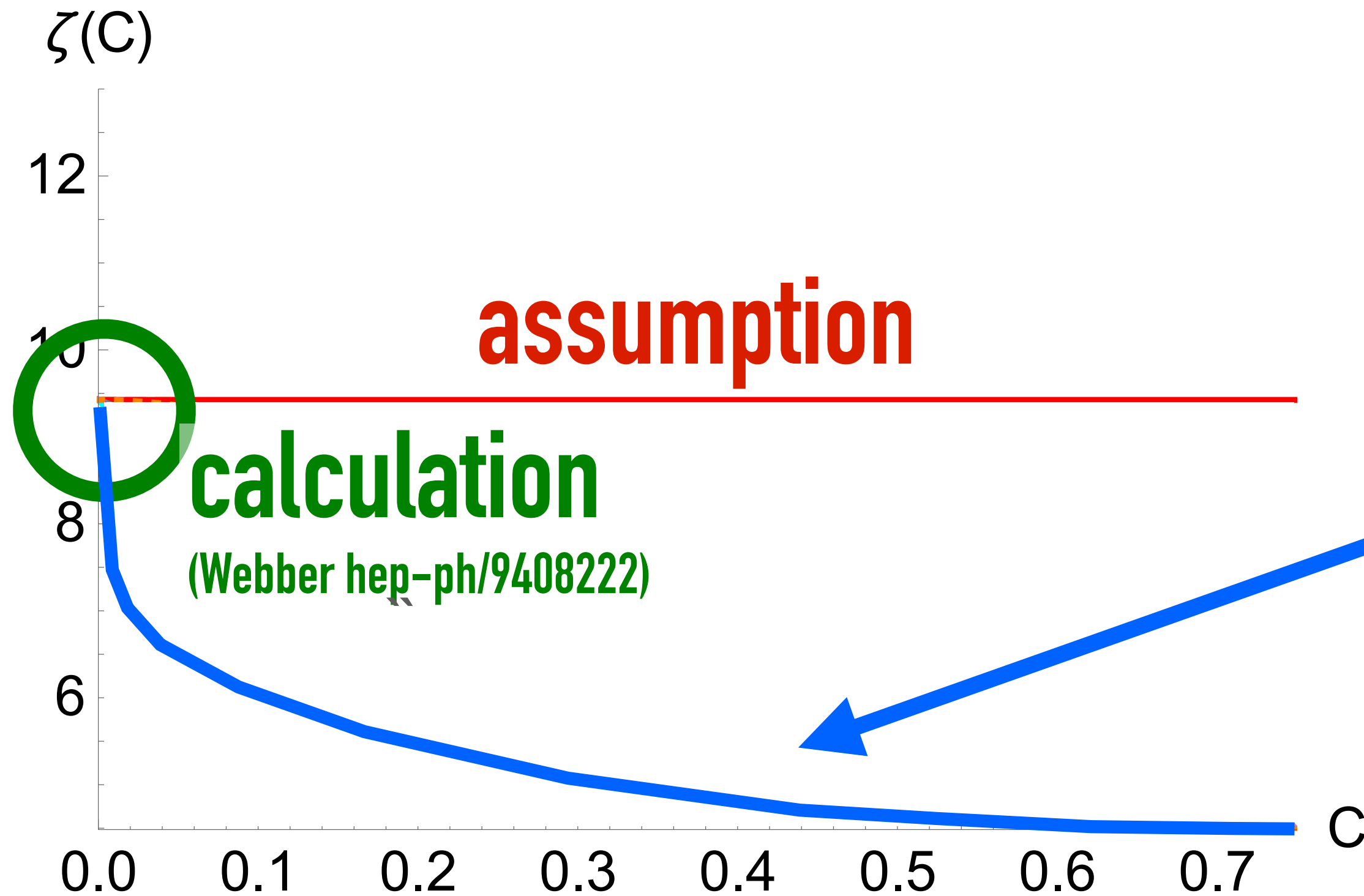


# fit results with different interpolations



**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ . (Luisoni, Monni, GPS, 2012.00622)

# non-perturbative shift as $f^n$ of $C$

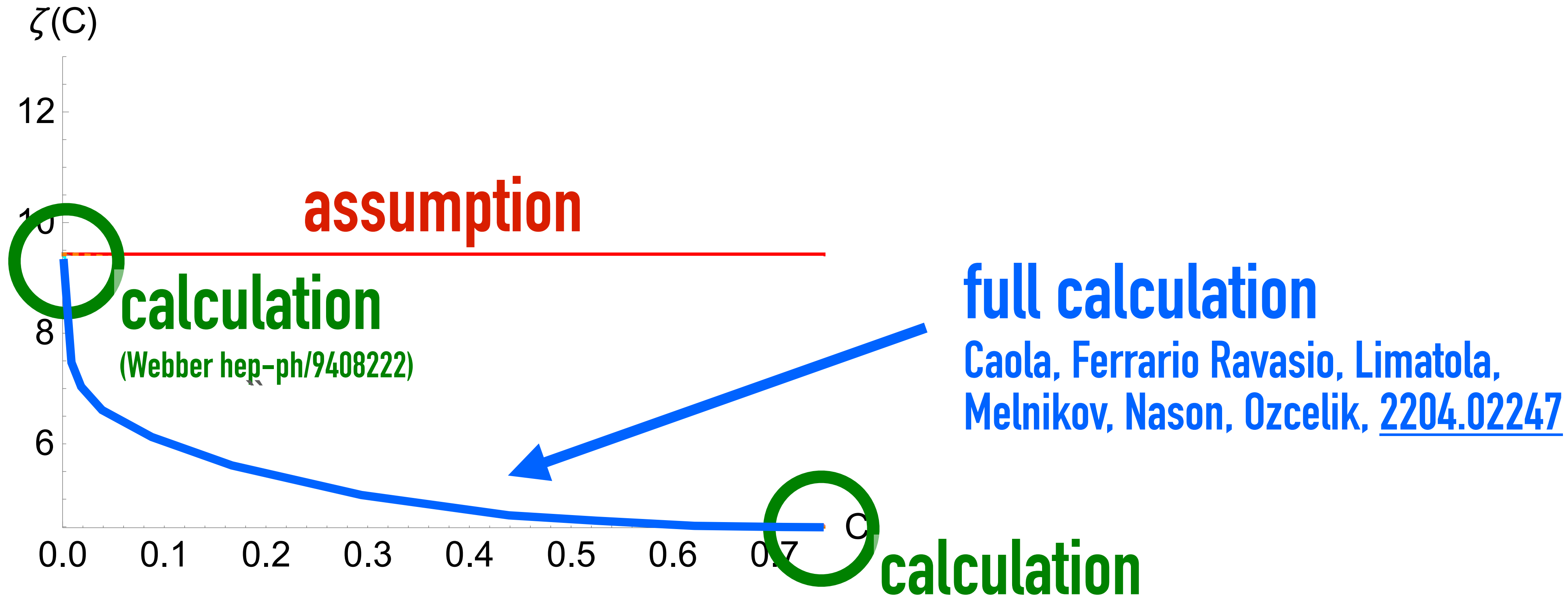


**full calculation**

Caola, Ferrario Ravasio, Limatola,  
Melnikov, Nason, Ozcelik, [2204.02247](https://arxiv.org/abs/2204.02247)

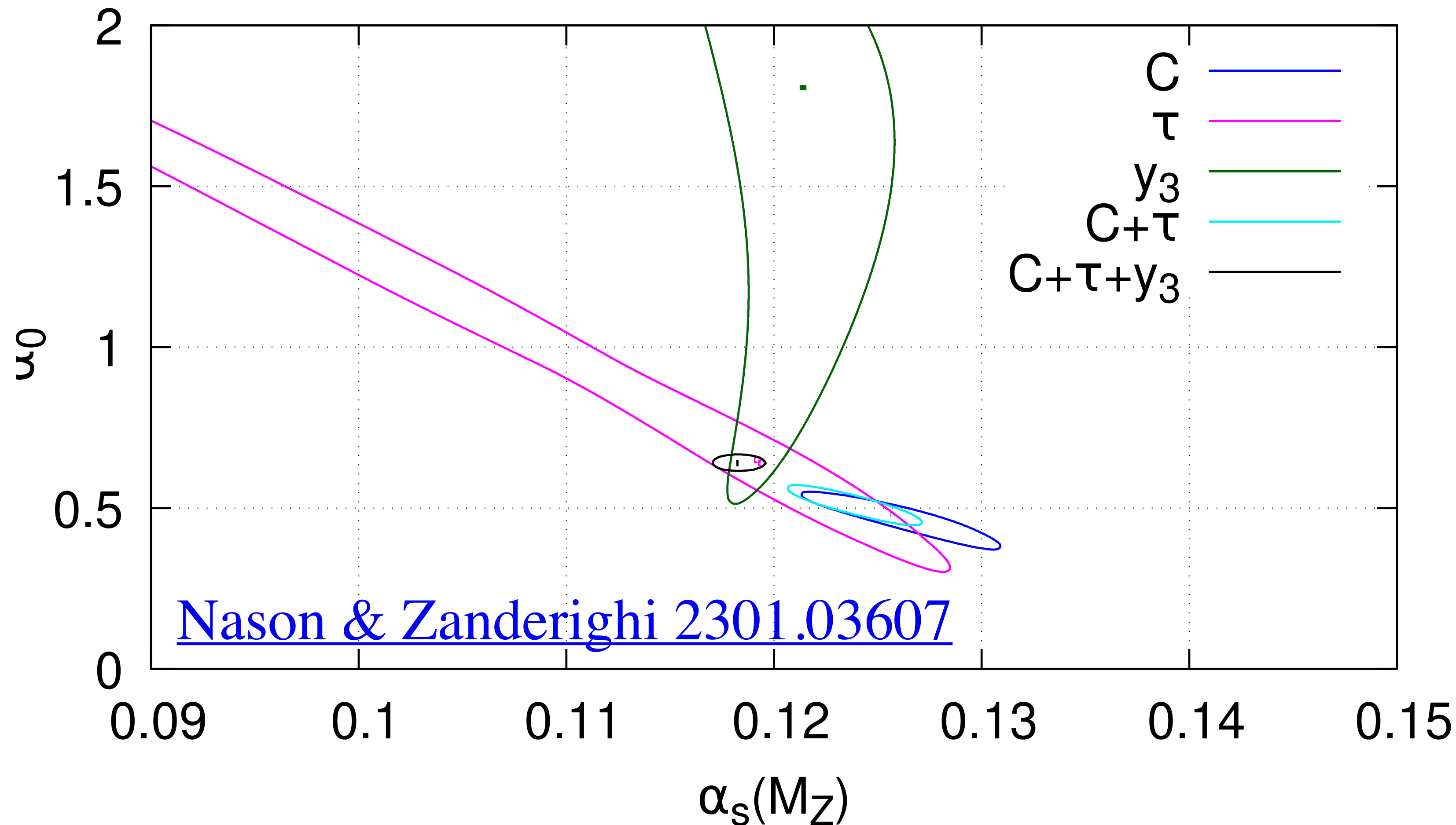
**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ .

# non-perturbative shift as $f^n$ of $C$



**Fig. 1.** Different functional forms for  $\zeta(C)$  function interpolating between the results at  $C = 0$  and  $C = 3/4$ .

# Fits with full (1st-order) non-perturbative correction



fits restricted to 3-jet  
region, NNLO + 1/Q

**fixed 1/Q:  $\alpha_s = 0.1132$**

**full 1/Q:  $\alpha_s = 0.1182$**

*“variations of our  
procedure can lead easily to  
differences of the order of a  
percent”*

**confirms strong sensitivity to our understanding of non-perturbative physics**

# the non-perturbative part at hadron colliders

---

$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_{i/p}(x_1) f_{j/p}(x_2) \hat{\sigma}(x_1 x_2 s) \times [1 + \mathcal{O}(\Lambda/M)^p]$$

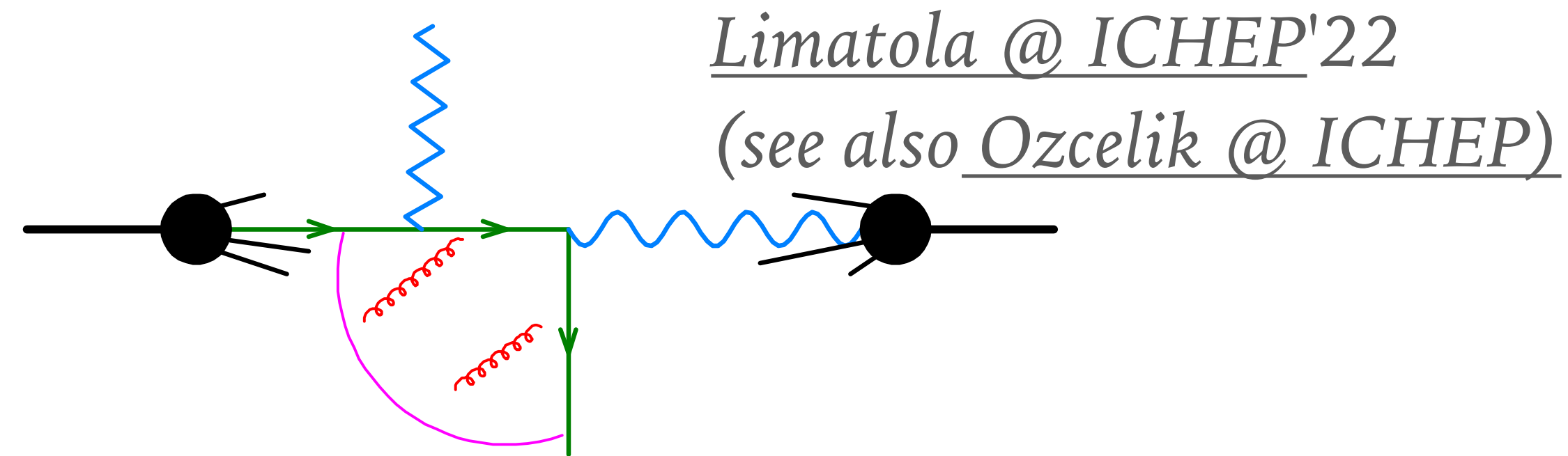
# What is value of $p$ in $(\Lambda/Q)^p$ ? [ $\Lambda \sim 1 \text{ GeV}$ ]

---

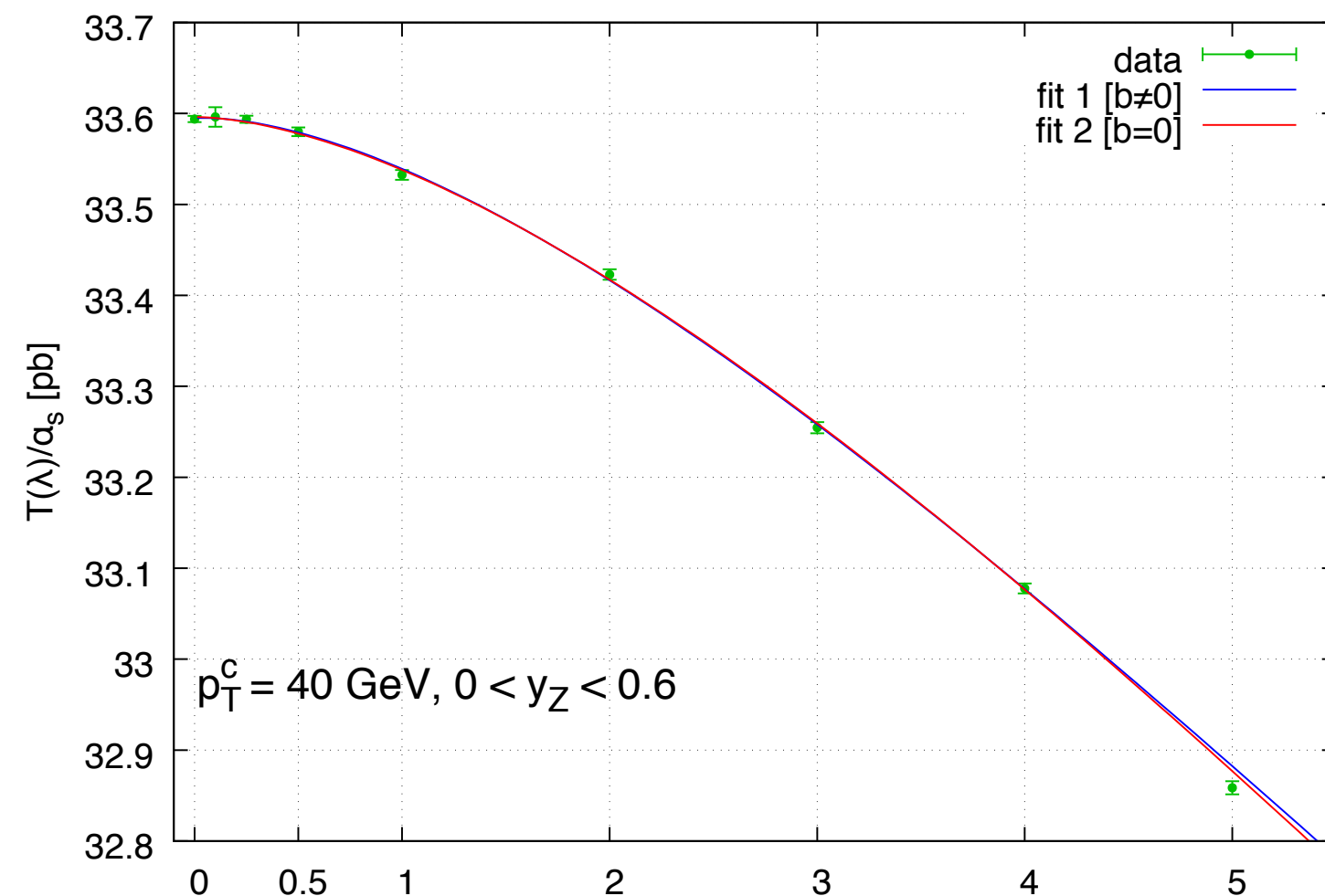
- ▶ Jet physics at LHC is dirty because  $p = 1$  (hadronisation & MPI)
- ▶ LEP event-shape (C-parameter, thrust)  $\alpha_s$  fit troubles are complex about because  $p = 1$ ,  $\Lambda \sim 0.5 \text{ GeV} \rightarrow (\Lambda/20\text{GeV}) \sim 2.5 \%$
- ▶ Hadron-collider inclusive and rapidity-differential Drell-Yan cross sections are believed to have  $p = 2$  (Higgs hopefully also), so leptonic / photonic decays should be clean, aside from isolation.  
 $\Lambda \sim 0.5 \text{ GeV} \rightarrow (\Lambda/125\text{GeV})^2 \sim 0.002 \%$   
[Beneke & Braun, hep-ph/9506452; Dasgupta, hep-ph/9911391]
- ▶ But at LHC, we're also interested in Z, W and Higgs production with non-zero  $p_T$   
Nobody knew if we have  $(\Lambda/p_T)^p$  with  $p = 1$  (a disaster) or  $p = 2$  (all is fine)

# What is value of $p$ in $(\Lambda/Q)^p$ for $Z p_T$ ?

- We consider the process  $d(p_1)\gamma(p_2) \rightarrow Z(p_3)d(p_4)$  to work in the *Large- $n_f$*  limit and to preserve the azimuthal color asymmetry ( $E_{CM} = 300$  GeV)



**absence of  $p=1$  term critical for viability of LHC precision programme**



We ([Ferrario Ravasio, GL, Nason \('20\)](#)) found

$$\langle O \rangle_\lambda^{(1)} \sim \left( \frac{\lambda}{p_T^c} \right)^2 \log \left( \frac{\lambda}{p_T^c} \right)$$

No numeric evidence of a IR linear renormalon for the transverse momentum of the  $Z$  boson!

**But note the log factor in  $p=2$  term — is this captured in intrinsic- $p_T$  models?**

*Ferrario Ravasio, Limatola & Nason, 2011.14114*

+ *analytic demonstration in Caola, Ferrario Ravasio, Limatola, Melnikov & Nason, 2108.08897, idem + Ozcelik 2204.02247*



# What is value of $p$ in $(\Lambda/Q)^p$ for top production?

Linear power corrections to top quark pair production in hadron collisions #1

Sergei Makarov (Karlsruhe U., TTP), Kirill Melnikov (Karlsruhe U., TTP), Paolo Nason (INFN, Milan Bicocca), Melih A. Ozelik (IJCLab, Orsay) (Aug 10, 2023)

e-Print: [2308.05526](#) [hep-ph]

[pdf](#) [cite](#) [claim](#) [reference search](#) [0 citations](#)

Linear power corrections to single top production processes at the LHC #2

Sergei Makarov (Karlsruhe U., TTP), Kirill Melnikov (Karlsruhe U., TTP), Paolo Nason (INFN, Milan Bicocca and Munich, Max Planck Inst.), Melih A. Ozelik (Karlsruhe U., TTP and IJCLab, Orsay) (Feb 6, 2023)

Published in: *JHEP* 05 (2023) 153 • e-Print: [2302.02729](#) [hep-ph]

[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [1 citation](#)

All-orders behaviour and renormalons in top-mass observables #5

Silvia Ferrario Ravasio (Milan Bicocca U. and INFN, Milan Bicocca), Paolo Nason (CERN and INFN, Milan Bicocca), Carlo Oleari (INFN, Milan Bicocca and Milan Bicocca U.) (Oct 25, 2018)

Published in: *JHEP* 01 (2019) 203 • e-Print: [1810.10931](#) [hep-ph]

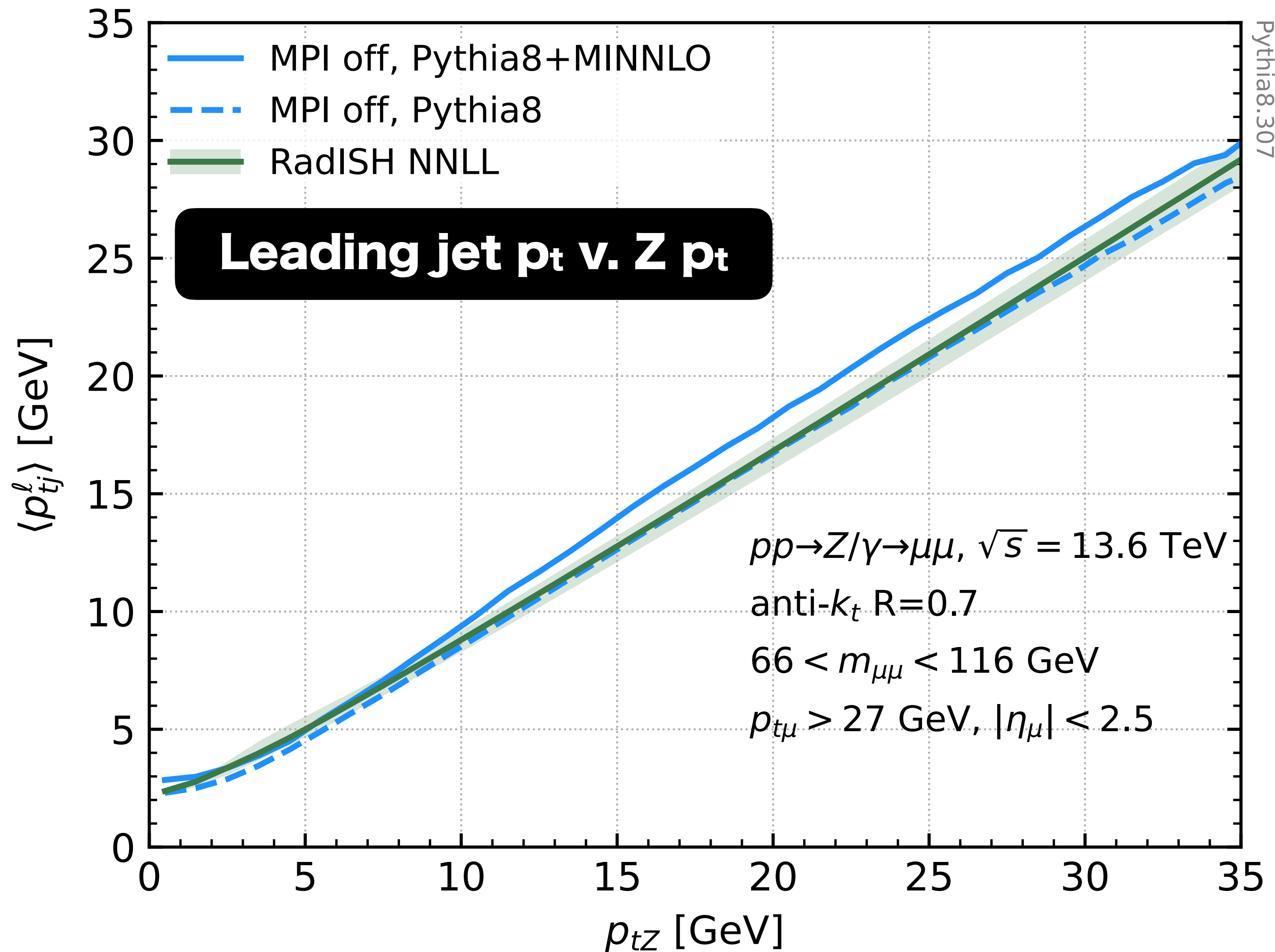
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [30 citations](#)

unless you  
choose a very  
special  
observable

$p=1$

# Underlying event & jets: $\Lambda/Q \rightarrow (\text{several GeV})/Q$

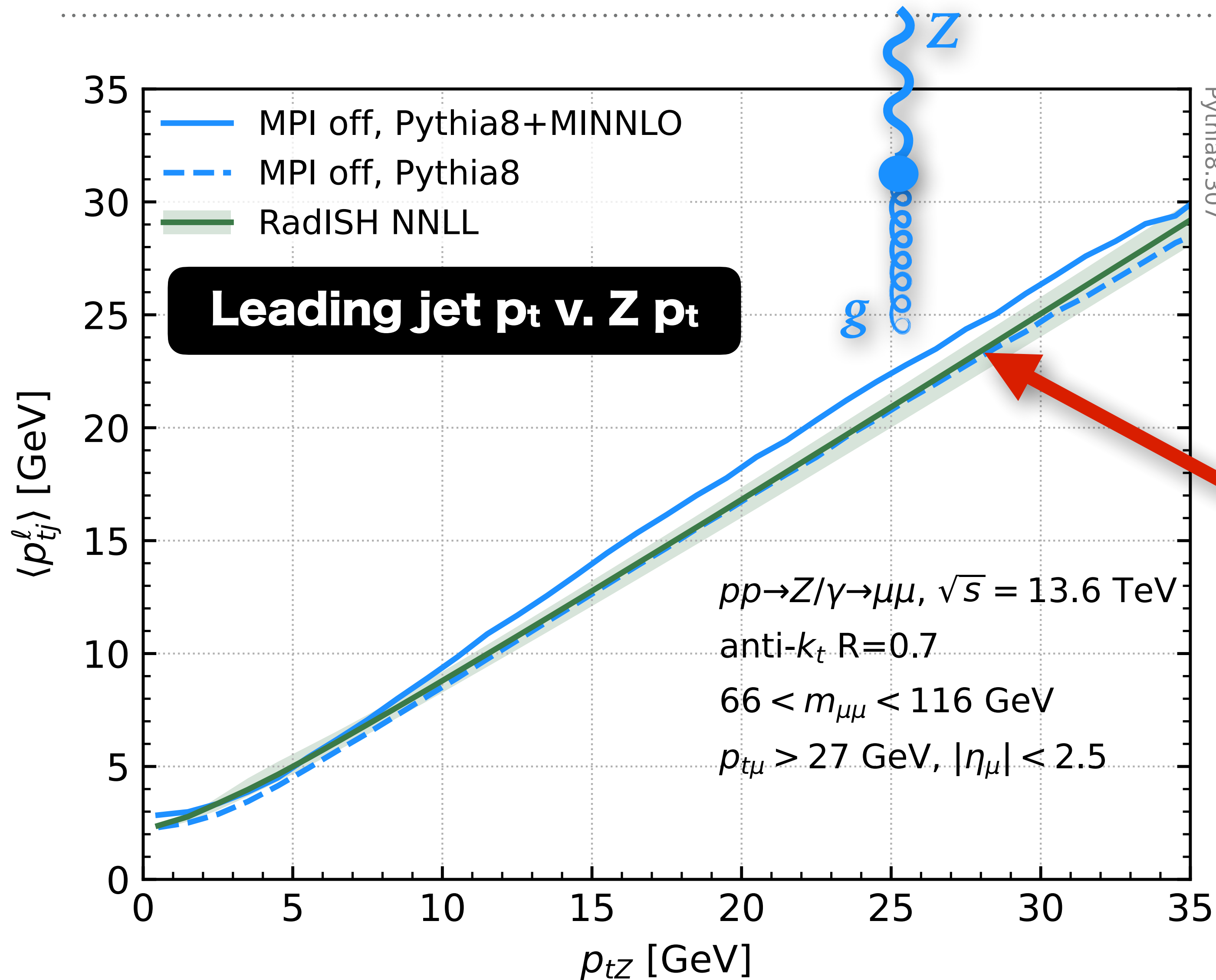
Andersen, Monni, Rottoli, GPS  
& Soto Ontoso, [2307.05693](#)



- Consider process with **MPI simulation turned off** (i.e. just 1HS)
- Look at avg.  $p_t$  of leading jet ( $p_{tj}^\ell$ ) as a function of  $Z p_t$  ( $p_{tZ}$ )

# Underlying event & jets: $\Lambda/Q \rightarrow (\text{several GeV})/Q$

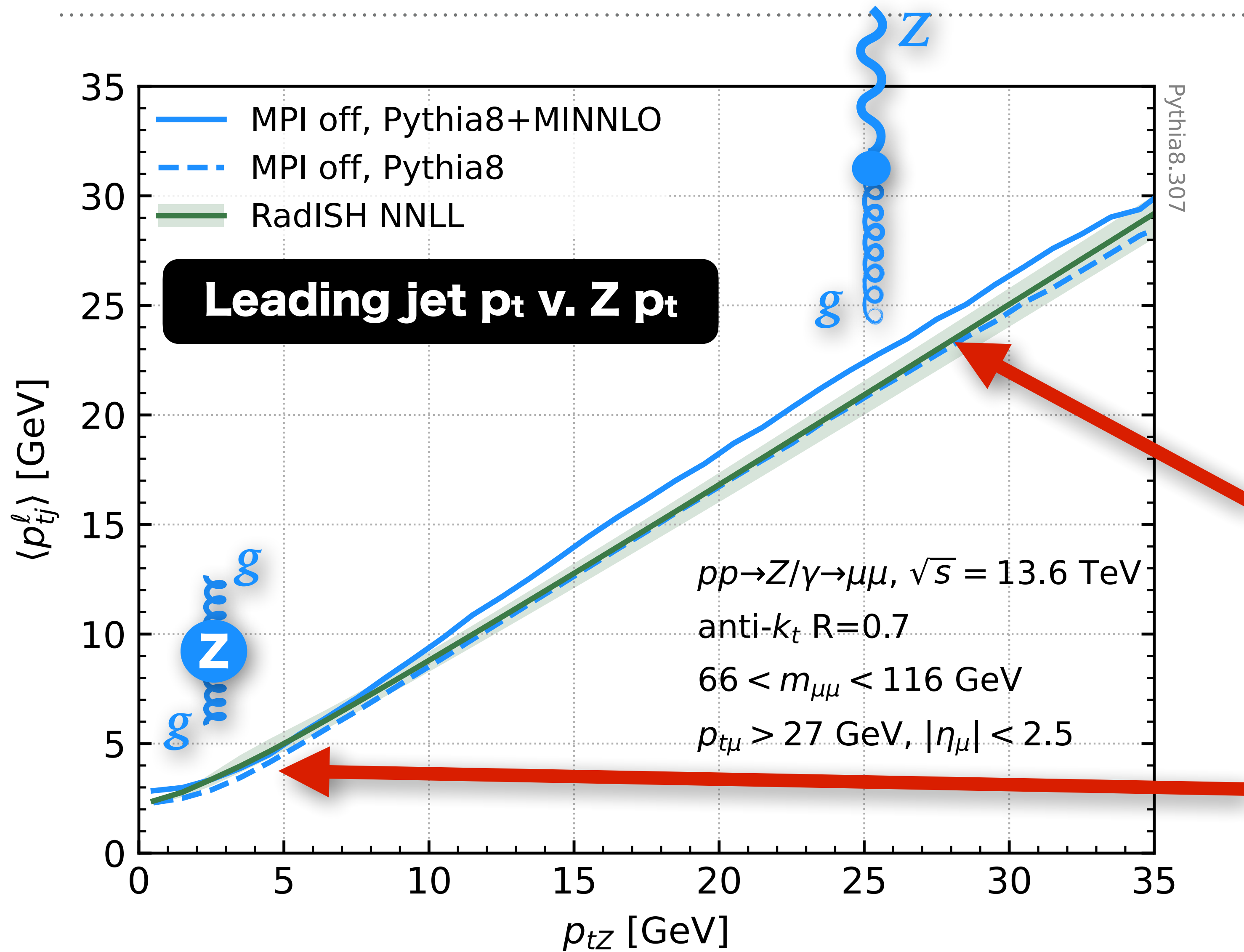
Andersen, Monni, Rottoli, GPS  
& Soto Ontoso, [2307.05693](#)



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- **Most of  $p_{tZ}$  range:** almost perfect linear correlation, since **leading jet balances  $p_{tZ}$**

# Underlying event & jets: $\Lambda/Q \rightarrow (\text{several GeV})/Q$

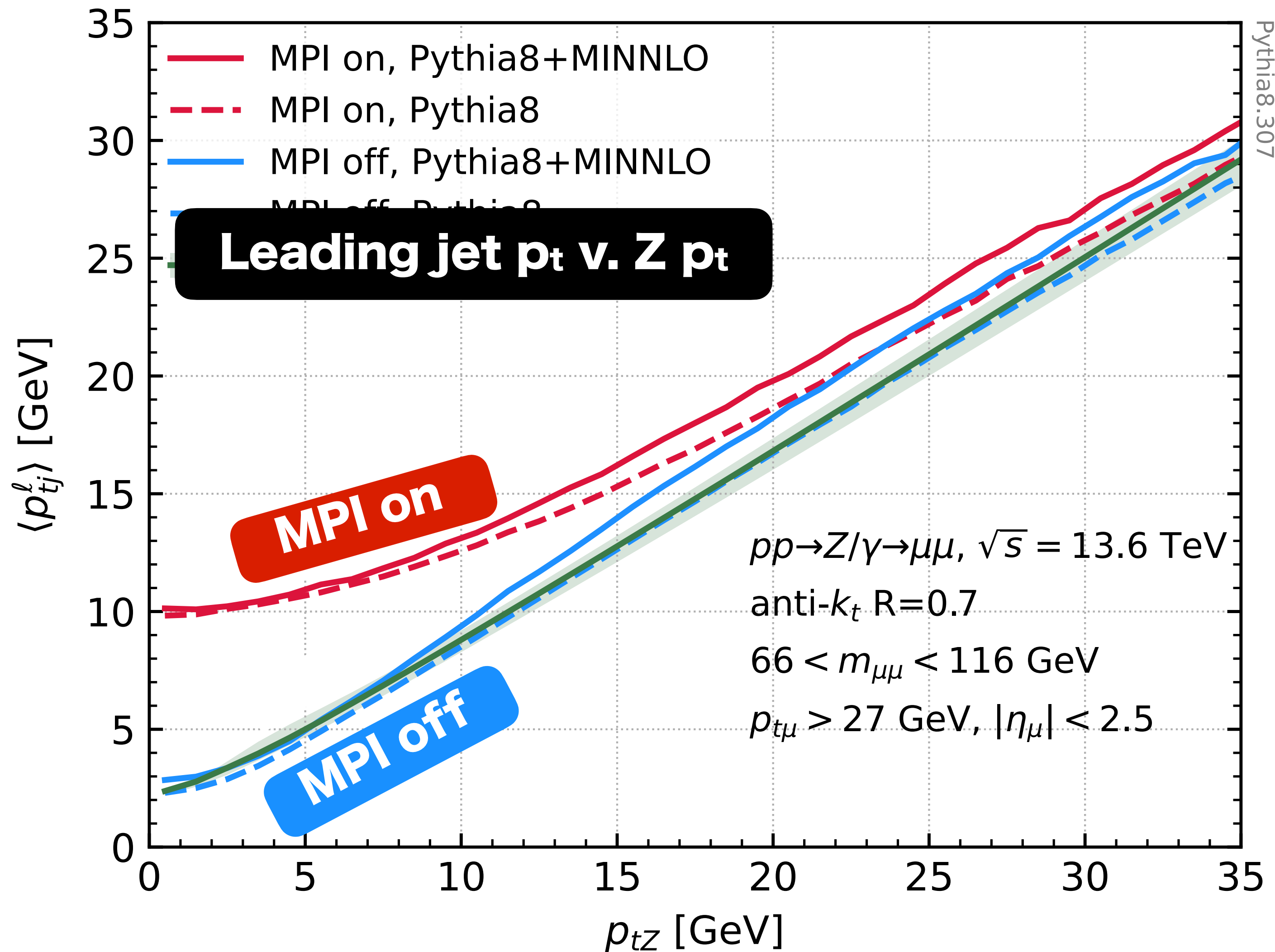
Andersen, Monni, Rottoli, GPS  
& Soto Ontoso, [2307.05693](#)



- Consider process with **MPI simulation turned off** (i.e. just 1HS)
- Look at avg.  $p_t$  of leading jet ( $p_{tj}^\ell$ ) as a function of Z  $p_t$  ( $p_{tZ}$ )
- **Most of  $p_{tZ}$  range:** almost perfect linear correlation, since **leading jet balances  $p_{tZ}$**
- **For  $p_{tZ} \rightarrow 0$ :**  $\langle p_{tj}^\ell \rangle$  saturates at about 2–3 GeV: **two soft jets balance each other**

# Underlying event & jets: $\Lambda/Q \rightarrow (\text{several GeV})/Q$

Andersen, Monni, Rottoli, GPS  
& Soto Ontoso, [2307.05693](#)



- next step: turn MPI on
- for  $p_{tZ} \rightarrow 0$ , leading jet  $p_t$  is now  $\sim 10$  GeV instead of 2–3 GeV [not so soft!]
- because there is almost always an MPI jet that is much harder than the soft jets from Z-process
- NB: jet studies take small radius of  $R=0.4$ , partly to mitigate MPI effects

# spurious perturbative behaviour

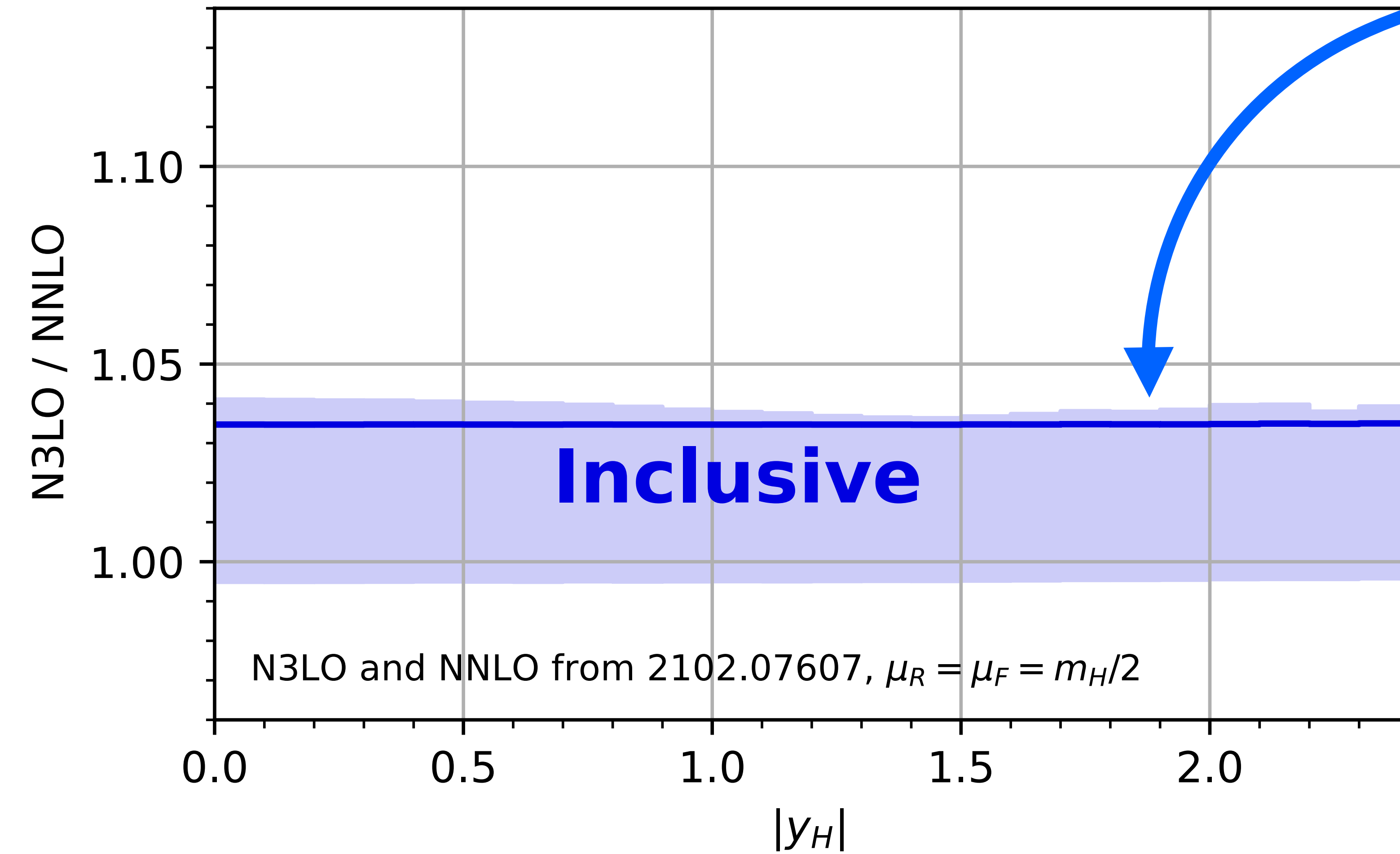
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$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_{i/p}(x_1) f_{j/p}(x_2) \hat{\sigma}(x_1 x_2 s) \times \left[ 1 + \mathcal{O}(\Lambda/M)^p \right]$$

Recent surprise:  $H \rightarrow \gamma\gamma$

inclusive N3LO  $\sigma$  uncertainties

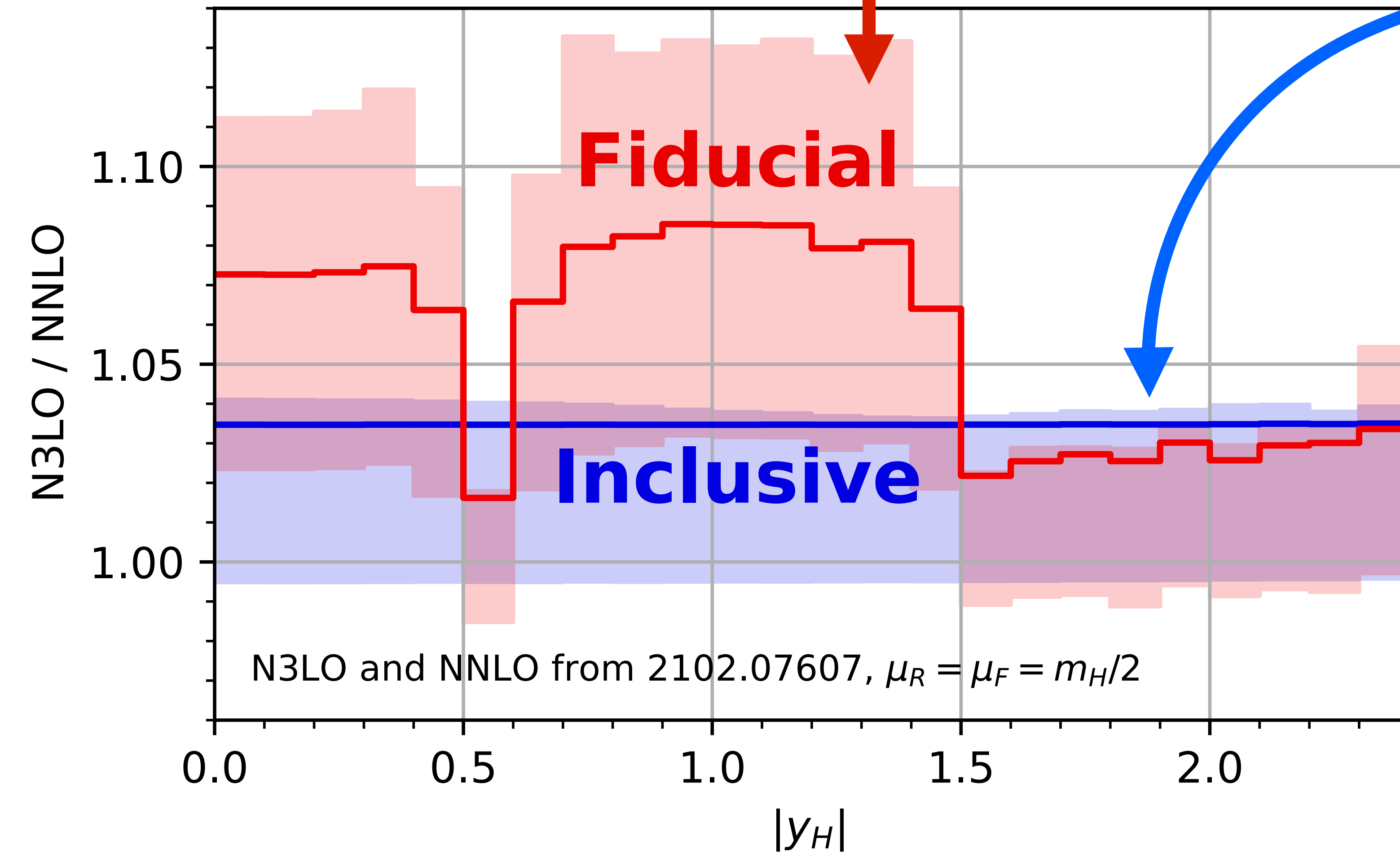
$H \rightarrow \gamma\gamma$ : N3LO K-factor



Chen, Gehrmann, Glover, Huss, Mistlberger & Pelloni, [2102.07607](#)

Recent surprise:  $H \rightarrow \gamma\gamma$  **fiducial N3LO**  $\sigma$  uncertainties  $\sim 2\times$  greater than **inclusive N3LO**  $\sigma$  uncertainties

$H \rightarrow \gamma\gamma$ : N3LO K-factor



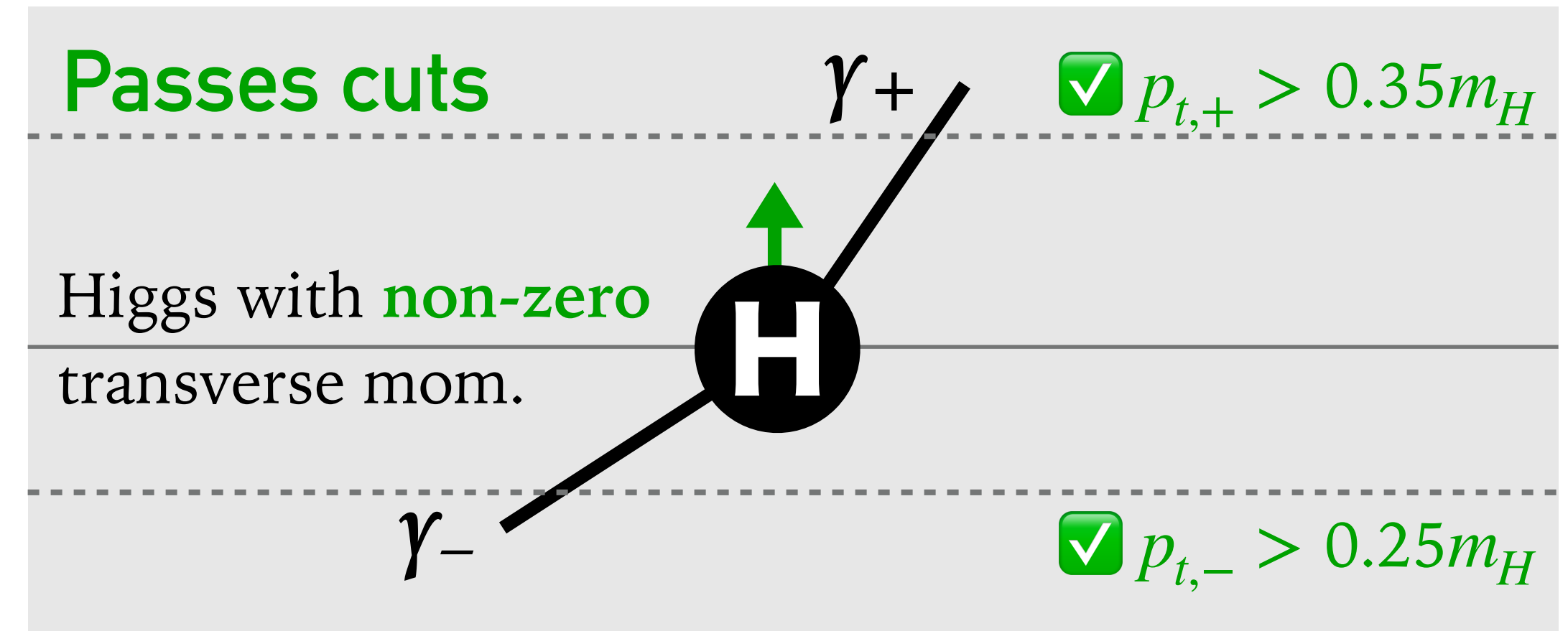
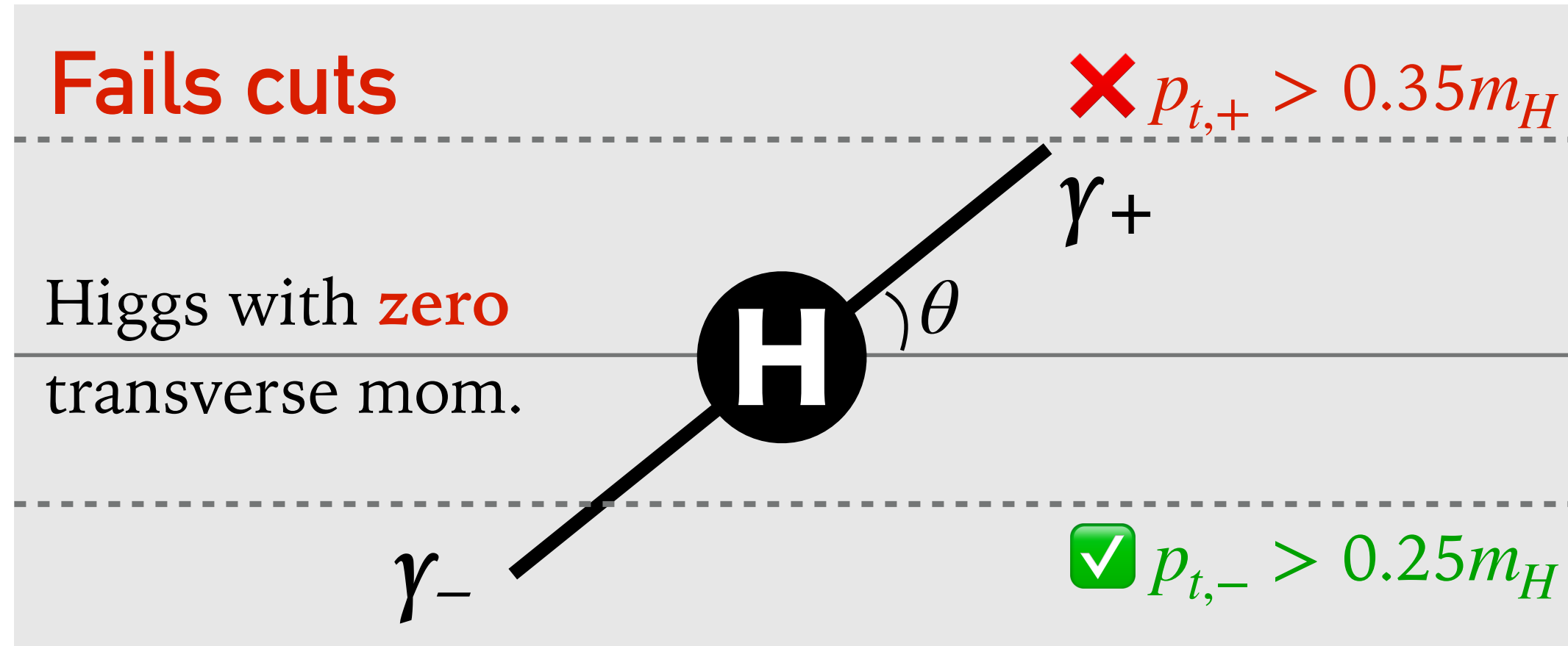
“Gold standard” fiducial cross section gives much worse prediction

Why?  
And can this be solved?

Chen, Gehrmann, Glover, Huss, Mistlberger & Pelloni, [2102.07607](#)



# Standard $p_{t,\gamma}$ cuts $\rightarrow$ Higgs $p_t$ dependence of acceptance



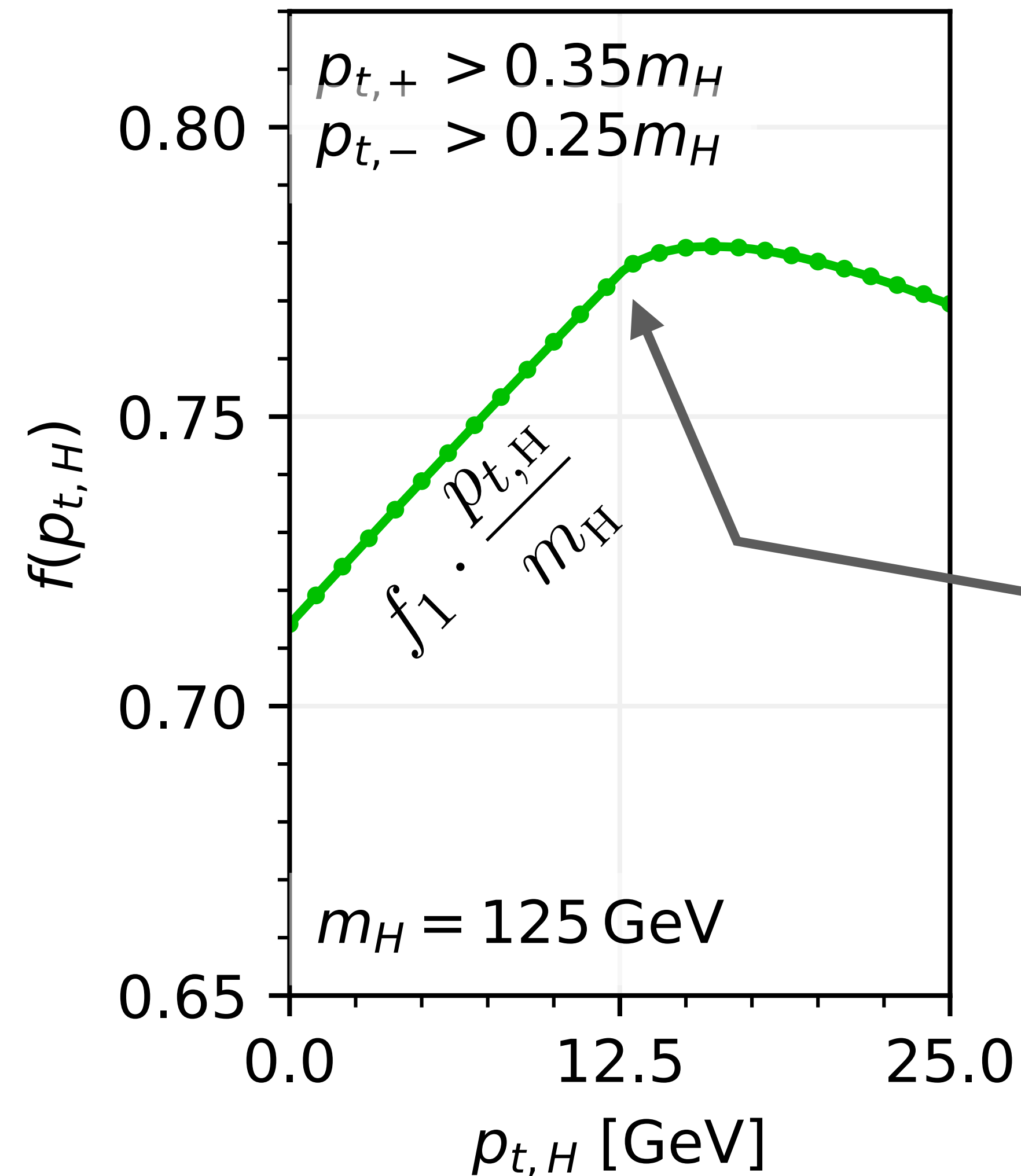
*Numbers are for ATLAS  $H \rightarrow \gamma\gamma$   $p_t$  cuts, CMS cuts are similar*

Expect acceptance to **increase with increasing  $p_{t,H}$**

$$p_{t,\pm}(p_{t,H}, \theta, \phi) = \frac{m_H}{2} \sin \theta \pm \frac{1}{2} p_{t,H} |\cos \phi| + \frac{p_{t,H}^2}{4m_H} (\sin \theta \cos^2 \phi + \csc \theta \sin^2 \phi) + \mathcal{O}_3,$$

# Linear $p_{t,H}$ dependence of H acceptance $\equiv f(p_{t,H})$

Acceptance for  $H \rightarrow \gamma\gamma$



$$f(p_{t,H}) = f_0 + f_1 \cdot \frac{p_{t,H}}{m_H} + \mathcal{O}\left(\frac{p_{t,H}^2}{m_H^2}\right)$$

See e.g. Frixione & Ridolfi '97  
 Ebert & Tackmann '19  
 idem + Michel & Stewart '20  
 Alekhin et al '20

$f_0$  and  $f_1$  are coefficients whose values depend on the cuts

effect of  $p_{t,-}$  cut sets in at  $0.1m_H$

define  $s_0 = \frac{2p_{t+,cut}}{m_H}$ :  $f_0 = \sqrt{1 - s_0^2} \simeq 0.71$ ,  $f_1 = \frac{2s_0}{\pi f_0} \simeq 0.62$

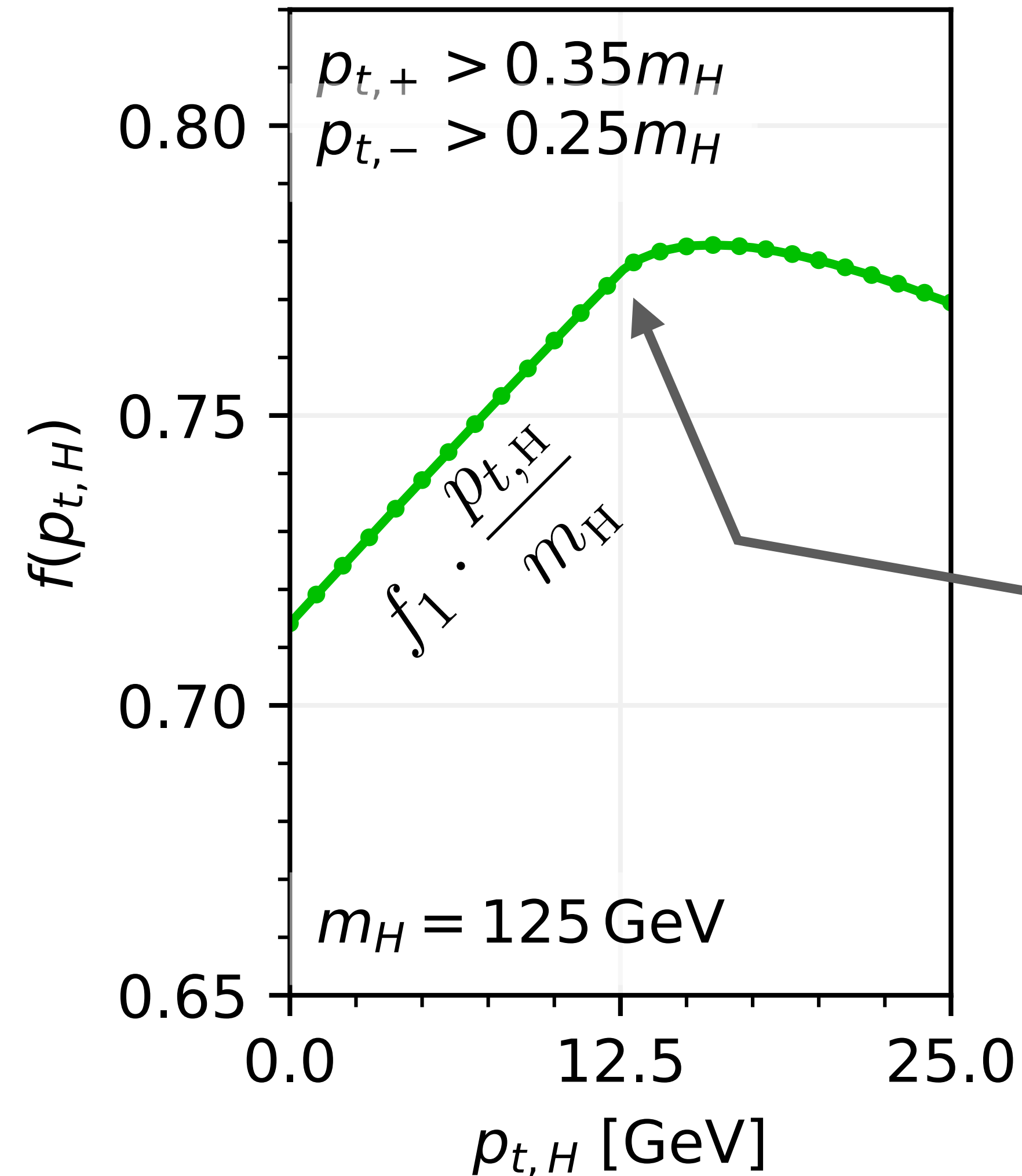
transition is at  $p_{t+,cut} - p_{t-,cut}$

# Linear $p_{t,H}$ dependence of H acceptance $\equiv f(p_{t,H})$

Acceptance for  $H \rightarrow \gamma\gamma$

$$f(p_{t,H}) = f_0 + f_1 \cdot \frac{p_{t,H}}{m_H} + \mathcal{O}\left(\frac{p_{t,H}^2}{m_H^2}\right)$$

See e.g. Frixione & Ridolfi '97  
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$f_0$  and  $f_1$  are coefficients whose values depend on the cuts

effect of  $p_{t,-}$  cut sets in at  $0.1 m_H$

$p_{t,H}$  dependence of acceptance (at 10% level)  $\rightarrow$  relating measured cross section and total cross section requires info about the  $p_{t,H}$  distribution.

# perturbative series for fiducial cross sections

$$f(p_{t,H}) = f_0 + f_1 \cdot \frac{p_{t,H}}{m_H} + \mathcal{O}\left(\frac{p_{t,H}^2}{m_H^2}\right)$$

Fiducial cross section depends on acceptance and Higgs  $p_t$  distribution

$$\sigma_{\text{fid}} = \int \frac{d\sigma}{dp_{t,H}} f(p_{t,H}) dp_{t,H}$$

To understand qualitative perturbative behaviour consider simple **(double-log)** approx for  $p_t$  distribution

$$\frac{d\sigma^{\text{DL}}}{dp_{t,H}} = \frac{\sigma_{\text{tot}}}{p_{t,H}} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2 \log^{2n-1} \frac{m_H}{2p_{t,H}}}{(n-1)!} \left(\frac{2C_A \alpha_s}{\pi}\right)^n$$

$$\int_0^{m_H} \frac{dp_{t,H}}{p_{t,H}} \frac{\alpha_s^n}{(n-1)!} \left(\log \frac{m_H}{p_{t,H}}\right)^{2n-1} \cdot \left(\frac{p_{t,H}}{m_H}\right) \sim \alpha_s^n \frac{(2n-1)!}{(n-1)!} \sim \alpha_s^n 2^{2n} n!$$

# Behaviour of perturbative series in various log approximations

$$\begin{aligned}
 \frac{\sigma_{\text{asym}} - f_0 \sigma_{\text{inc}}}{\sigma_0 f_0} &\simeq 0.15 \alpha_s - 0.29 \alpha_s^2 + 0.71 \alpha_s^3 - 2.39 \alpha_s^4 + 10.31 \alpha_s^5 + \dots &\simeq 0.06 \text{ @DL,} \\
 &\simeq 0.15 \alpha_s - 0.23 \alpha_s^2 + 0.44 \alpha_s^3 - 1.15 \alpha_s^4 + 3.86 \alpha_s^5 + \dots &\simeq 0.06 \text{ @LL,} \\
 &\simeq 0.18 \alpha_s - 0.15 \alpha_s^2 + 0.29 \alpha_s^3 + \dots &\simeq 0.10 \text{ @NNLL,} \\
 &\simeq 0.18 \alpha_s - 0.15 \alpha_s^2 + 0.31 \alpha_s^3 + \dots &\simeq 0.12 \text{ @N3LL.}
 \end{aligned}$$

**Resummed  
results**

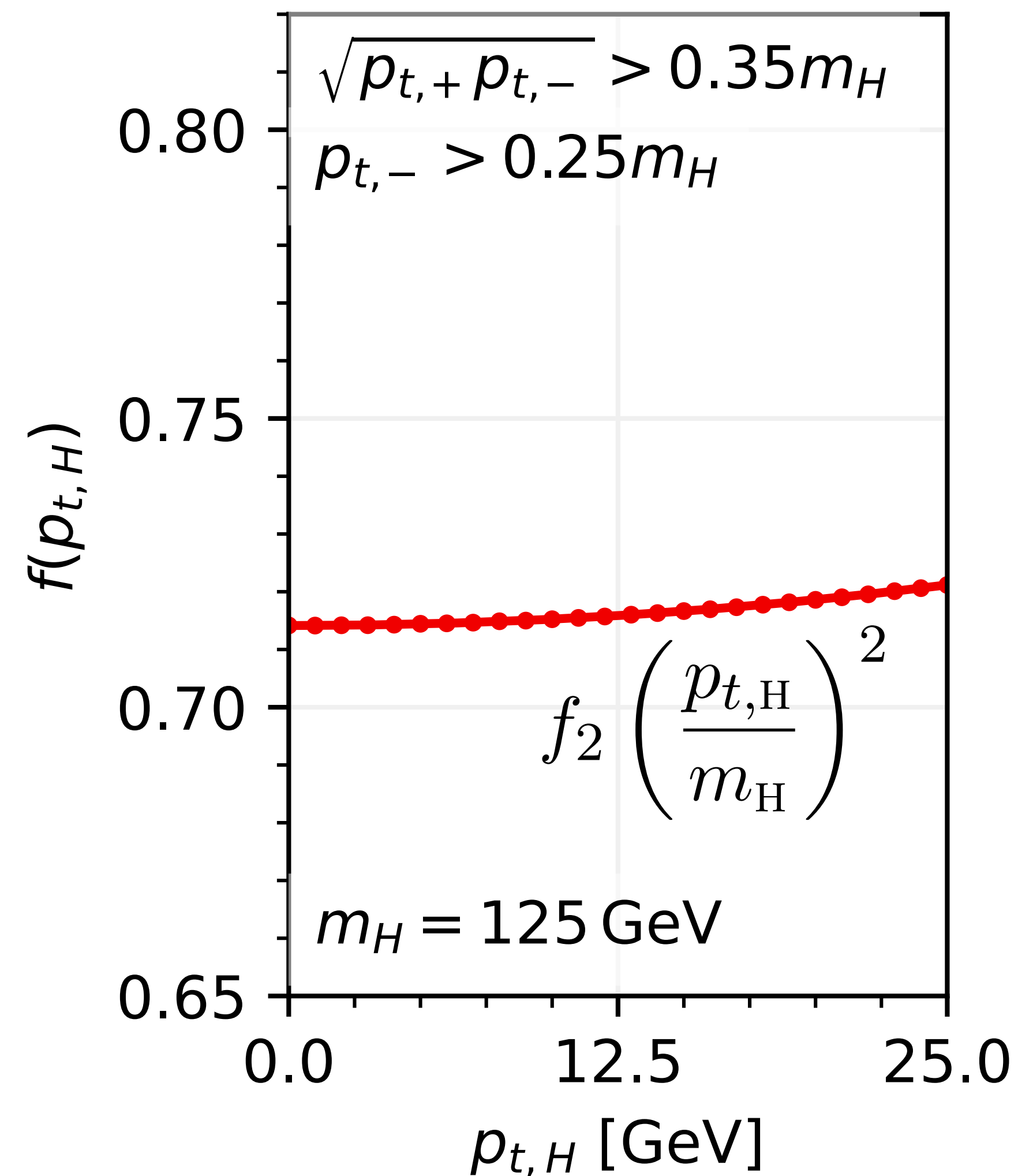
*Thanks to Pier Monni & RadISH for supplying NN(N)LL distributions & expansions,  $\mu = m_H/2$   
(relative to previous slide, this now has full expression for acceptance)*

- At DL & LL (DL+running coupling) **factorial divergence sets in from first orders**
- Poor behaviour of N3LL is qualitatively similar to that seen by Billis et al '21
- Theoretically similar to a power-suppressed ambiguity  $\sim (\Lambda_{\text{QCD}}/m_H)^{0.205}$   
[inclusive cross sections expected to have  $\Lambda^2/m^2$ ]

*GPS & Slade, 2106.08329*

# Replace cut on leading photon $\rightarrow$ cut on **product of photon $p_t$ 's**

Acceptance for  $H \rightarrow \gamma\gamma$



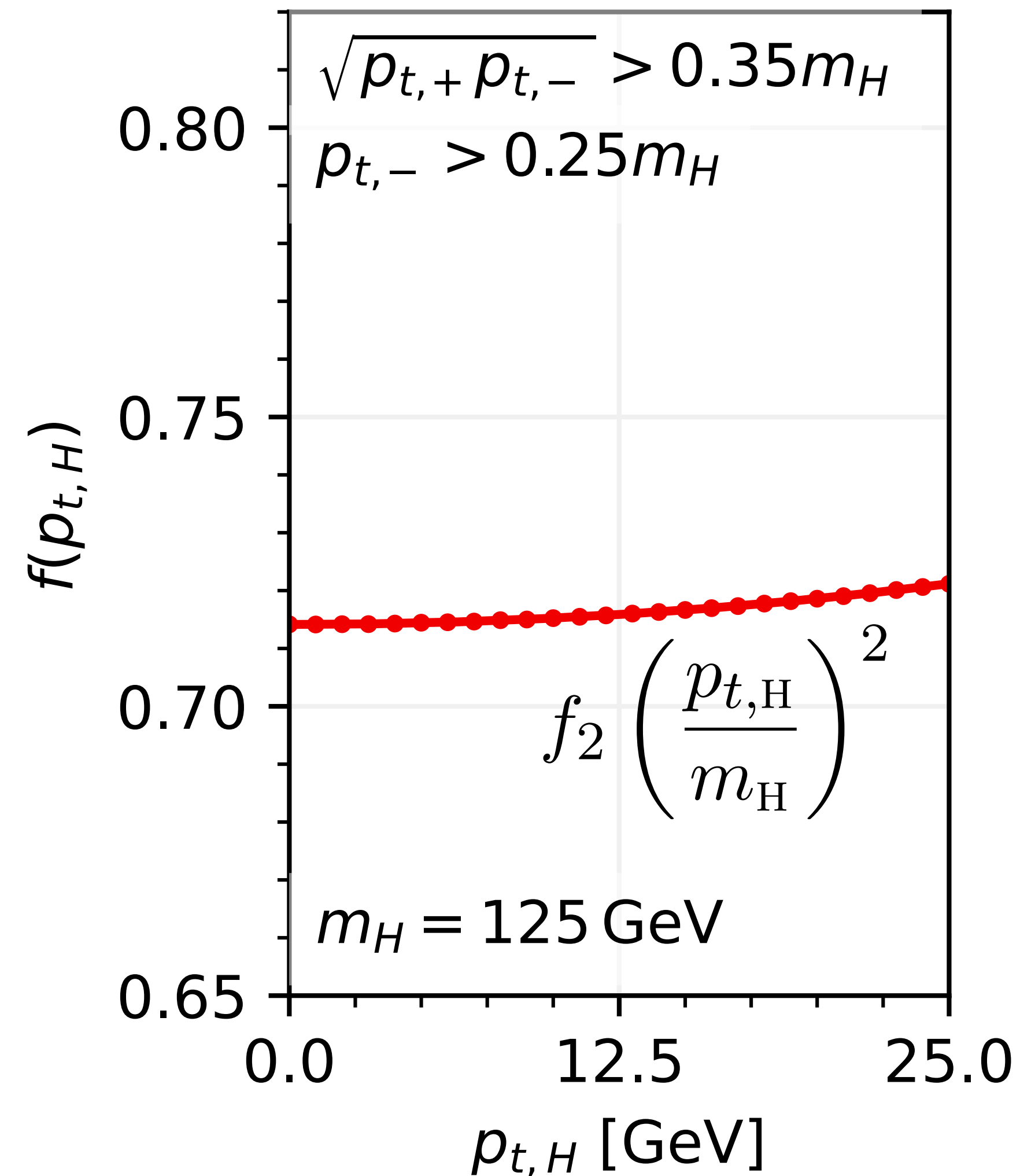
$$f(p_{t,H}) = f_0 + f_2 \left( \frac{p_{t,H}}{m_H} \right)^2 + \mathcal{O} \left( \frac{p_{t,H}^2}{m_H^2} \right)$$

**linear  $\rightarrow$   
quadratic**

NB: the cut on the softer photon is still maintained

# Replace cut on leading photon $\rightarrow$ cut on **product of photon $p_t$ 's**

Acceptance for  $H \rightarrow \gamma\gamma$



$$f(p_{t,H}) = f_0 + f_2 \left( \frac{p_{t,H}}{m_H} \right)^2 + \mathcal{O} \left( \frac{p_{t,H}^2}{m_H^2} \right) \quad \text{linear} \rightarrow \text{quadratic}$$

$$\frac{(2n)!}{2(n!)} \left( \frac{2C_A \alpha_s}{\pi} \right)^n \rightarrow \frac{1}{4^n} \frac{(2n)!}{4(n!)} \left( \frac{2C_A \alpha_s}{\pi} \right)^n$$

**Using product cuts dampens the factorial divergence**

NB: the cut on the softer photon is still maintained

# Behaviour of perturbative series with **product** cuts

$$\frac{\sigma_{\text{prod}} - f_0 \sigma_{\text{inc}}}{\sigma_0 f_0} \simeq 0.005 \alpha_s - 0.002 \alpha_s^2 + 0.002 \alpha_s^3 - 0.001 \alpha_s^4 + 0.001 \alpha_s^5 + \dots$$

$$\simeq 0.005 \alpha_s - 0.002 \alpha_s^2 + 0.000 \alpha_s^3 - 0.000 \alpha_s^4 + 0.000 \alpha_s^5 + \dots$$

$$\simeq 0.005 \alpha_s + 0.002 \alpha_s^2 - 0.001 \alpha_s^3 + \dots$$

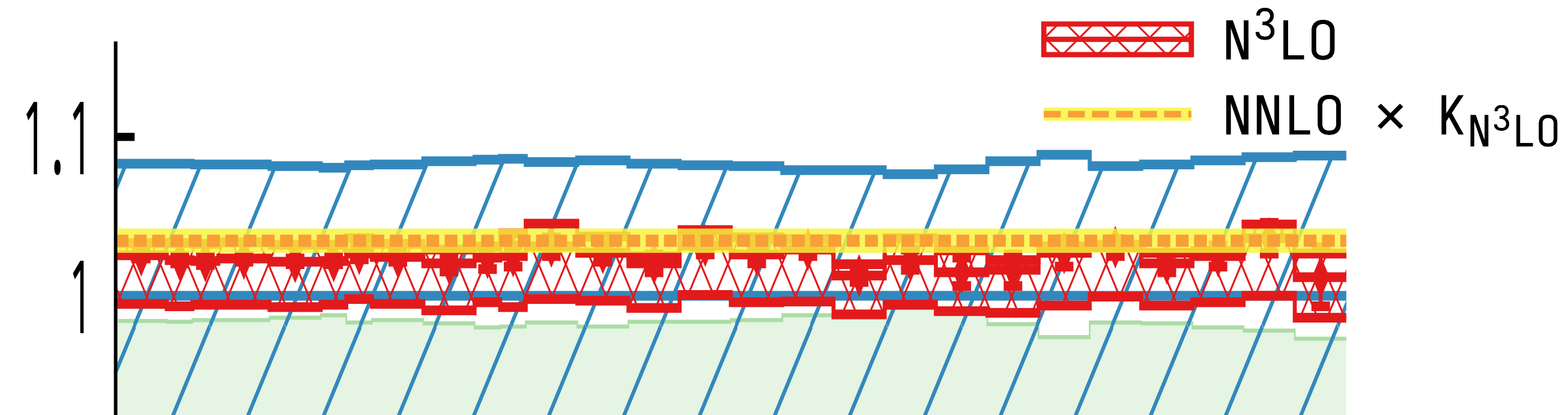
$$\simeq 0.005 \alpha_s + 0.002 \alpha_s^2 - 0.001 \alpha_s^3 + \dots$$

**Resummed results**

$\simeq 0.003$  @DL,  
 $\simeq 0.003$  @LL,  
 $\simeq 0.005$  @NNLL,  
 $\simeq 0.006$  @N3LL.

*Thanks to Pier Monni & RadISH for supplying NN(N)LL distributions & expansions,  $\mu = m_H/2$*

- Factorial growth of series strongly suppressed
- **N3LO truncation agrees well with all-order result**
- Per mil agreement between fixed-order and resummation **gives confidence that all is under control**



*Huss et al preliminary @ Higgs 2021*



# Cuts & N3LO fiducial DY cross sections

Same conceptual problem for DY, but reduced because of  $C_F$  instead of  $C_A$  colour factor

Magnitude of problem can be estimated from difference between fixed order and fixed-order + resummation (recall: resummation should not be needed for a fiducial cross section)

Order $k$	$\sigma$ [pb] Symmetric cuts		$\sigma$ [pb] Product cuts	
	$N^k$ LO	$N^k$ LO + $N^k$ LL	$N^k$ LO	$N^k$ LO + $N^k$ LL
0	$721.16^{+12.2\%}_{-13.2\%}$	—	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

┌ 0.5% difference ─┐

┌ no difference ─┐

Chen, Gehrmann, Glover, Huss & Monni, [2203.01565](#)

# non-pert corrections & PDFs?

---

$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_{i/p}(x_1) f_{j/p}(x_2) \hat{\sigma}(x_1 x_2 s) \times [1 + \mathcal{O}(\Lambda/M)^p]$$

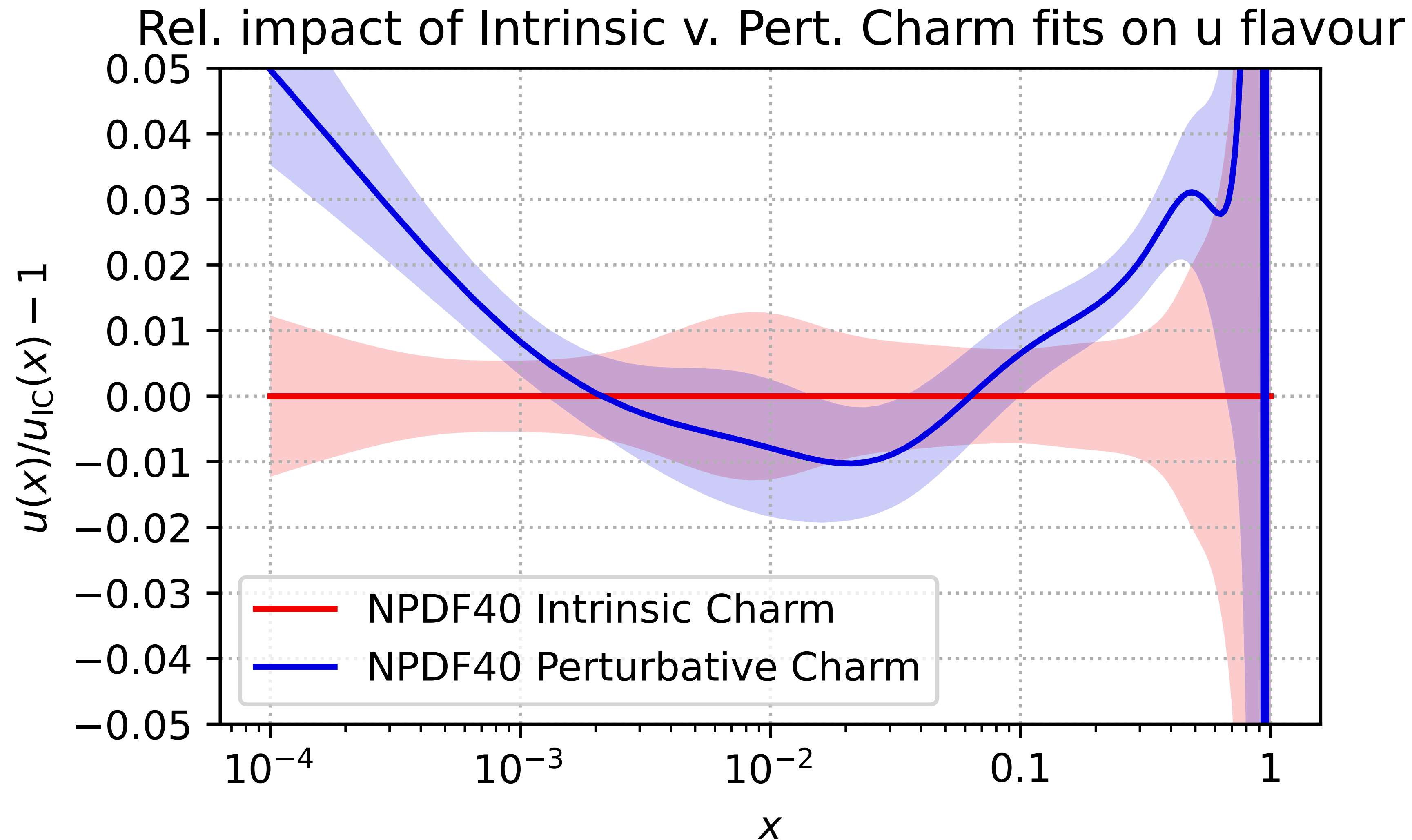
# NNPDF4.0 DIS struct. fn. datasets [2109.02653] — all involve $\Lambda^2/Q^2$

Dataset	Ref.	$N_{\text{dat}}$	$x$	$Q$ [GeV]	Theory
NMC $F_2^d/F_2^p$	[33]	260 (121/121)	[0.012, 0.680]	[2.1, 10.]	APFEL
NMC $\sigma^{\text{NC},p}$	[34]	292 (204/204)	[0.012, 0.500]	[1.8, 7.9]	APFEL
SLAC $F_2^p$	[35]	211 (33/33)	[0.140, 0.550]	[1.9, 4.4]	APFEL
SLAC $F_2^d$	[35]	211 (34/34)	[0.140, 0.550]	[1.9, 4.4]	APFEL
BCDMS $F_2^p$	[36]	351 (333/333)	[0.070, 0.750]	[2.7, 15.]	APFEL
BCDMS $F_2^d$	[36]	254 (248/248)	[0.070, 0.750]	[2.7, 15.]	APFEL
CHORUS $\sigma_{CC}^\nu$	[37]	607 (416/416)	[0.045, 0.650]	[1.9, 9.8]	APFEL
CHORUS $\sigma_{CC}^{\bar{\nu}}$	[37]	607 (416/416)	[0.045, 0.650]	[1.9, 9.8]	APFEL
NuTeV $\sigma_{CC}^\nu$ (dimuon)	[38, 39]	45 (39/39)	[0.020, 0.330]	[2.0, 11.]	APFEL+NNLO
NuTeV $\sigma_{CC}^{\bar{\nu}}$ (dimuon)	[38, 39]	45 (36/37)	[0.020, 0.210]	[1.9, 8.3]	APFEL+NNLO
[NOMAD $\mathcal{R}_{\mu\mu}(E_\nu)$ ] (*)	[111]	15 (—/15)	[0.030, 0.640]	[1.0, 28.]	APFEL+NNLO
[EMC $F_2^e$ ]	[44]	21 (—/16)	[0.014, 0.440]	[2.1, 8.8]	APFEL
HERA I+II $\sigma_{\text{NC},\text{CC}}^p$	[40]	1306 (1011/1145)	[ $4 \cdot 10^{-5}$ , 0.65]	[1.87, 223]	APFEL
HERA I+II $\sigma_{\text{NC}}^c$ (*)	[145]	52 (—/37)	[ $7 \cdot 10^{-5}$ , 0.05]	[2.2, 45]	APFEL
HERA I+II $\sigma_{\text{NC}}^b$ (*)	[145]	27 (26/26)	[ $2 \cdot 10^{-4}$ , 0.50]	[2.2, 45]	APFEL



**Table 2.1.** The DIS datasets analyzed in the NNPDF4.0 PDF determination. For each of them we indicate the name of the dataset used throughout this paper, the corresponding reference, the number of data points in the NLO/NNLO fits before (and after) kinematic cuts (see Sect. 4), the kinematic coverage in the relevant variables after cuts, and the codes used to compute the corresponding predictions. Datasets not previously considered in NNPDF3.1 are indicated with an asterisk. Datasets not included in the baseline determination are indicated in square brackets. The  $Q$  coverage indicated for NOMAD is to be interpreted as an integration range (see text).

# Intrinsic v. perturbative charm $\sim \Lambda^2/Q^2$ effect for $Q=m_c$



intrinsic charm v.  
perturbative charm fits  
are like including a  
 $\Lambda^2/Q^2$  effect in the fit

Doesn't just affect  
charm PDF, but, e.g.  
also up-quark PDF

Raises question of  
more general  $\Lambda^2/Q^2$   
effects in PDF fits at  
level of few-% accuracy

# Jet data has $\Lambda/Q$ corrections — a concern for $p_T$ cuts of 5 – 10 GeV

Dataset	Ref.	$N_{\text{dat}}$	$Q^2$ [GeV <sup>2</sup> ]	$p_T$ [GeV]	Theory
[ZEUS 820 (HQ) (1j)] (*)	[112]	30 (—/30)	[125,10000]	[8,100]	NNLOjet
[ZEUS 920 (HQ) (1j)] (*)	[113]	30 (—/30)	[125,10000]	[8,100]	NNLOjet
[H1 (LQ) (1j)] (*)	[115]	48 (—/48)	[5.5,80]	[4.5,50]	NNLOjet
[H1 (HQ) (1j)] (*)	[116]	24 (—/24)	[150,15000]	[5,50]	NNLOjet
[ZEUS 920 (HQ) (2j)] (*)	[114]	22 (—/22)	[125,20000]	[8,60]	NNLOjet
[H1 (LQ) (2j)] (*)	[115]	48 (—/48)	[5.5,80]	[5,50]	NNLOjet
[H1 (HQ) (2j)] (*)	[116]	24 (—/24)	[150,15000]	[7,50]	NNLOjet

**Table 2.2.** Same as Table 2.1 for DIS jet data.

# conclusions

# Concluding message

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For many parts of the field there is a clear path forward on precision

But we are reaching the point where we also need to reconsider the heuristics that get adopted for the  $(\Lambda/Q)^p$  terms

**backup**