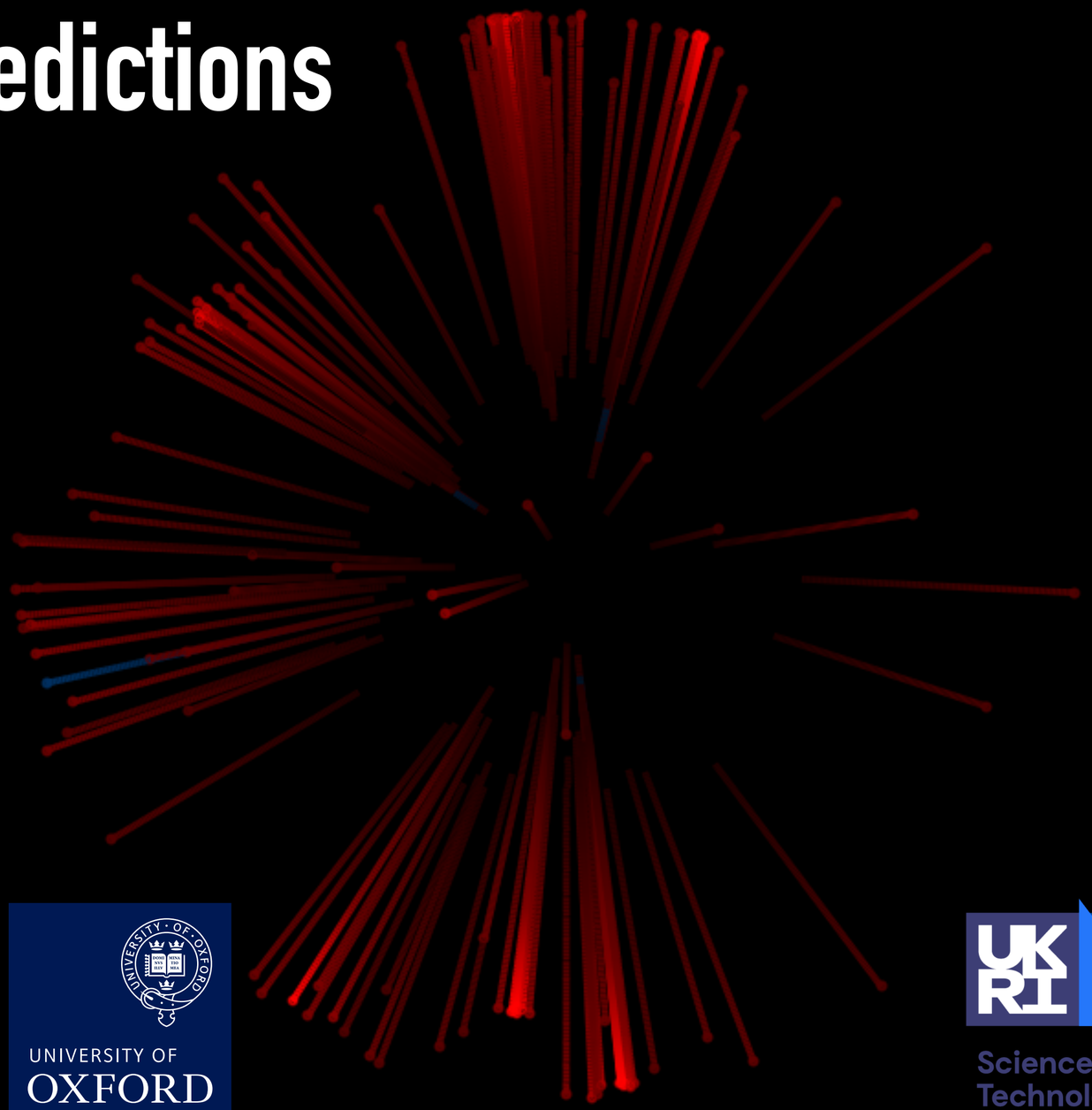
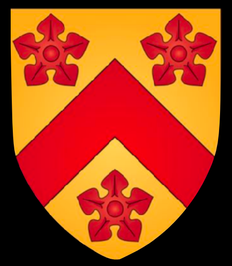
The challenge of accurate and physical QCD predictions

Theory Colloquium
CERN
30 July 2025



Gavin Salam
Rudolf Peierls Centre for Theoretical Physics
& All Souls College, Oxford

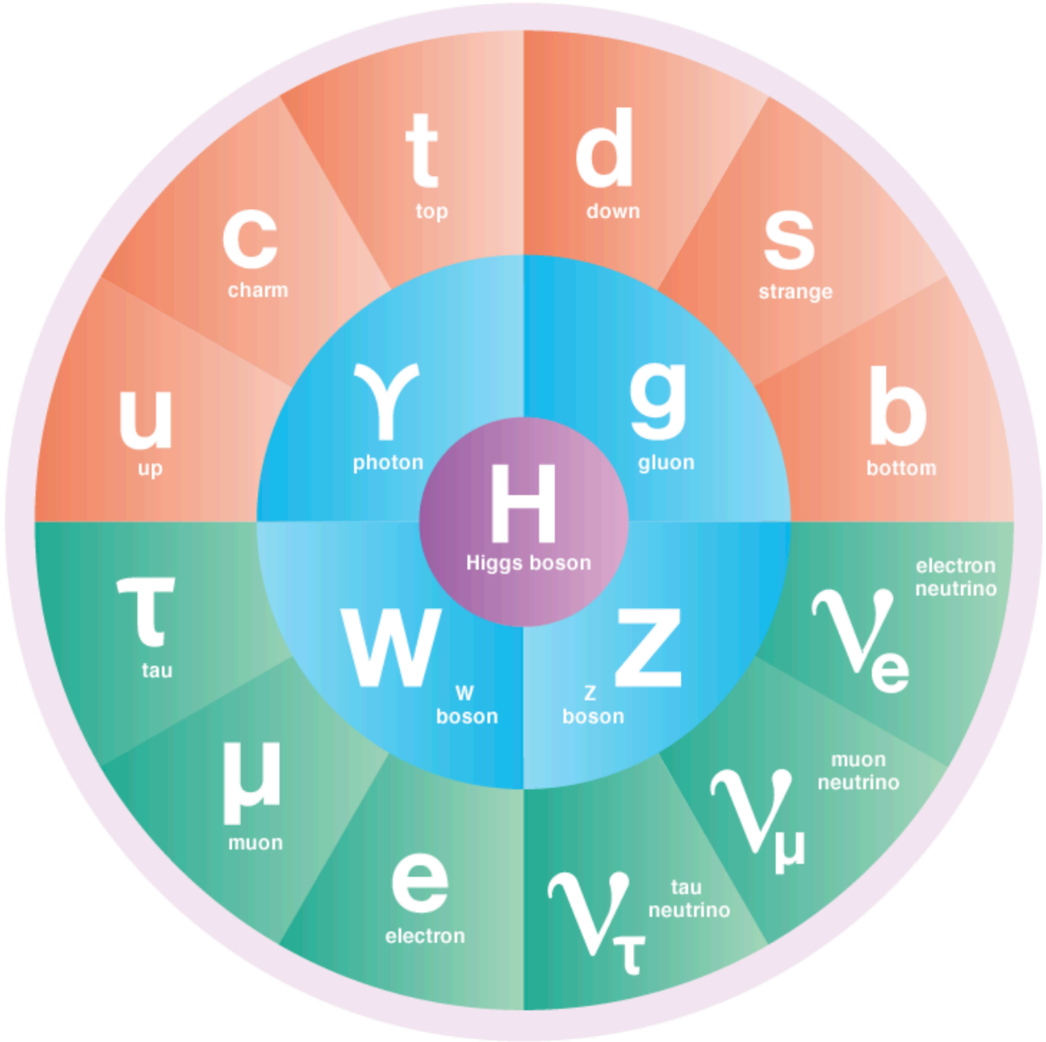




Science and Technology Facilities Council



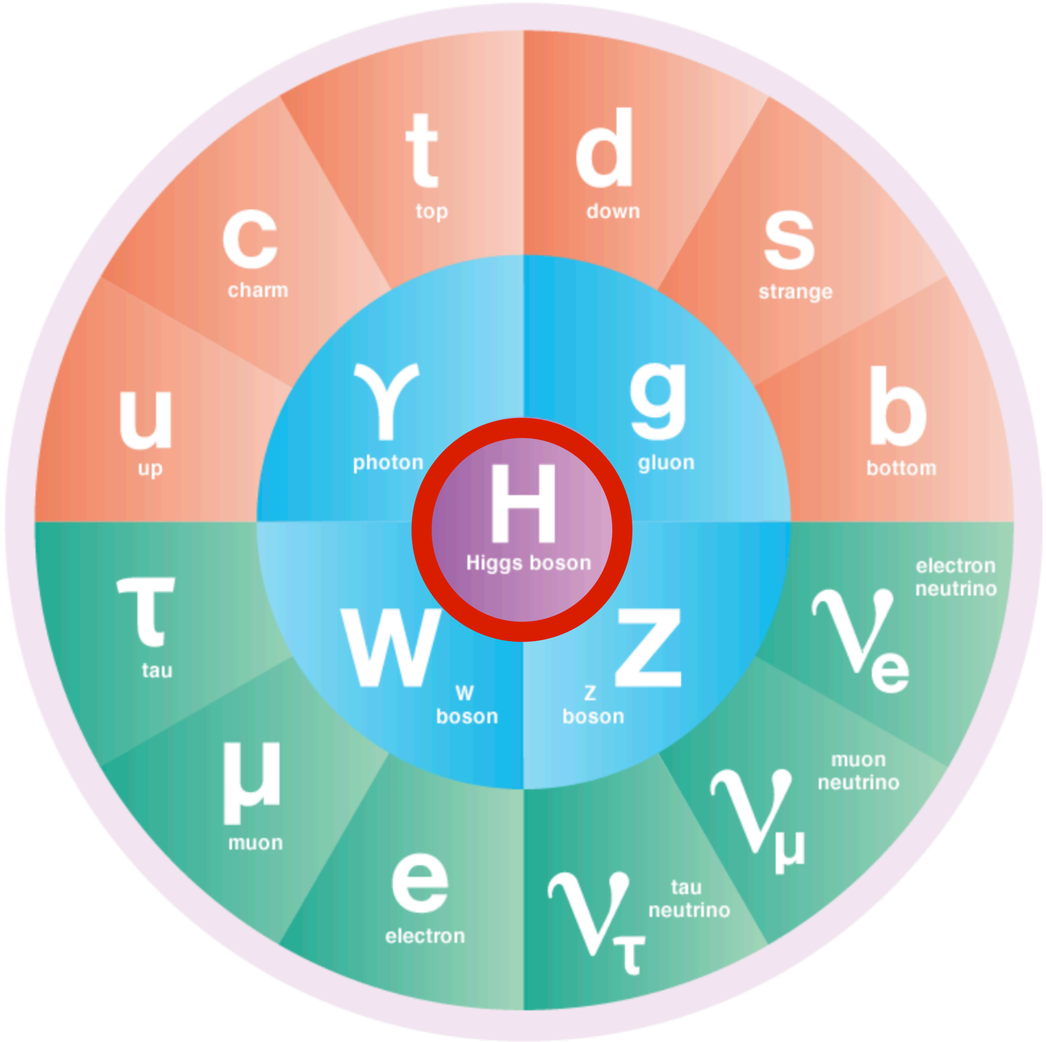
The Standard Model (SM)



https://www.symmetrymagazine.org/standard-model/

particles

The Standard Model (SM)

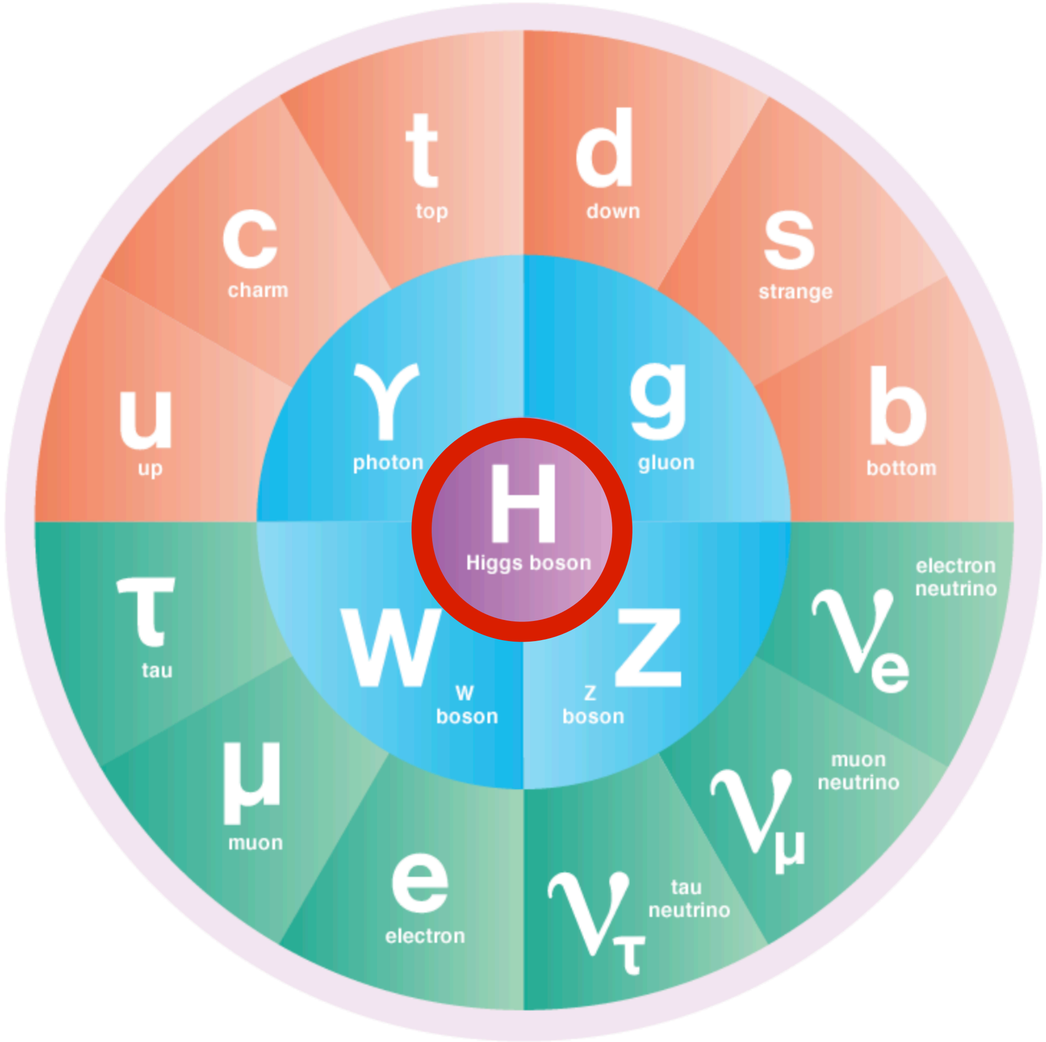


particles

https://www.symmetrymagazine.org/standard-model/

"the standard-model (SM) is complete"

The Standard Model (SM)



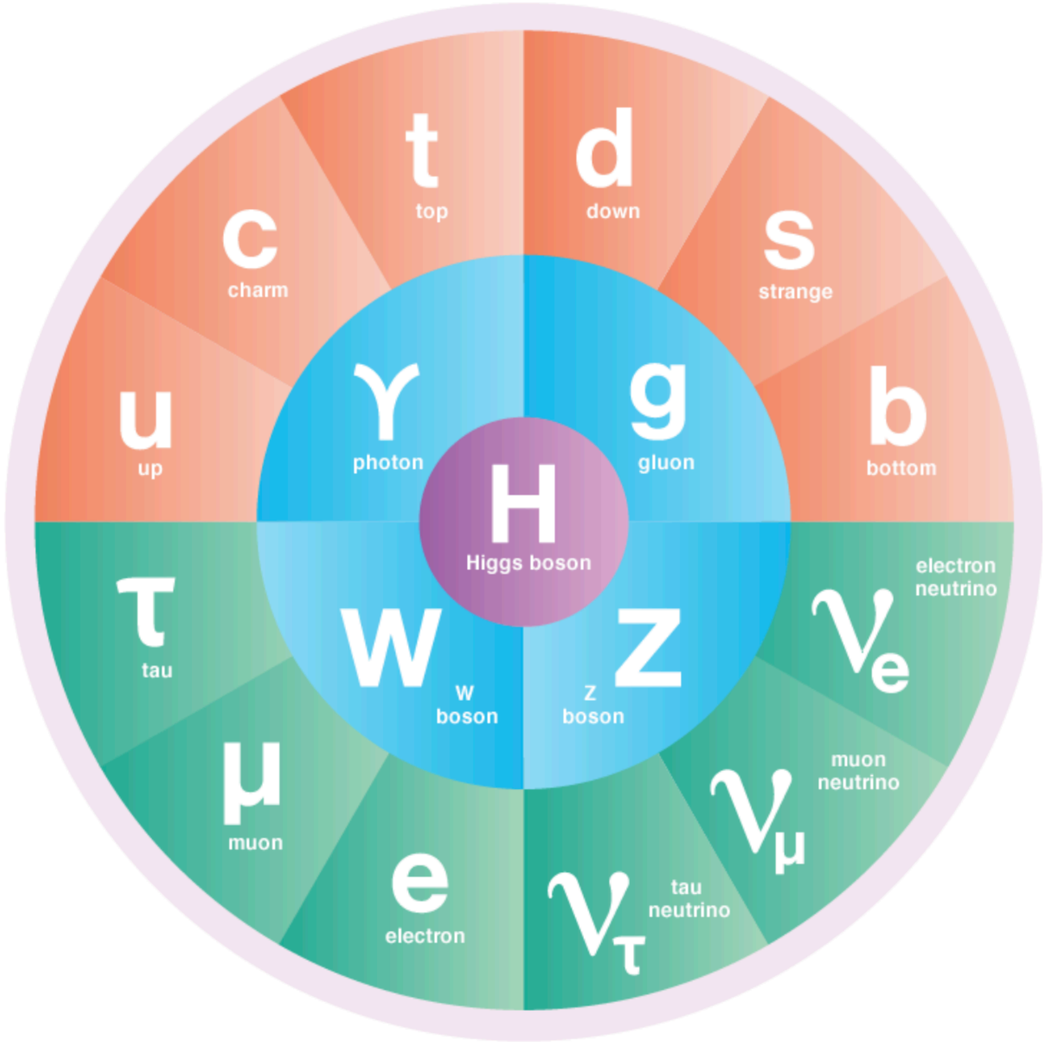
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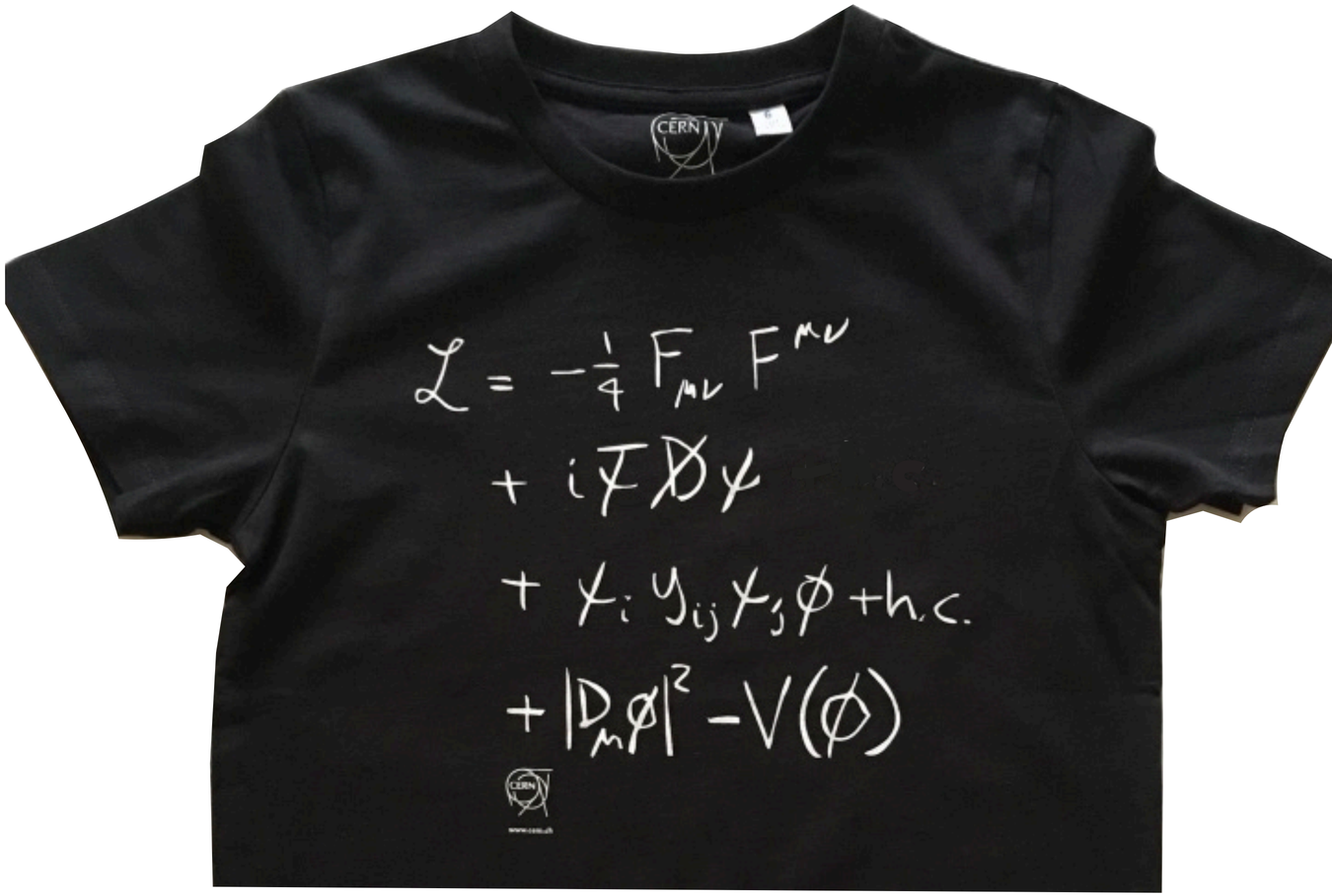
https://www.symmetrymagazine.org/standard-model/

"the standard-model (SM) is complete"

?

The Standard Model (SM)



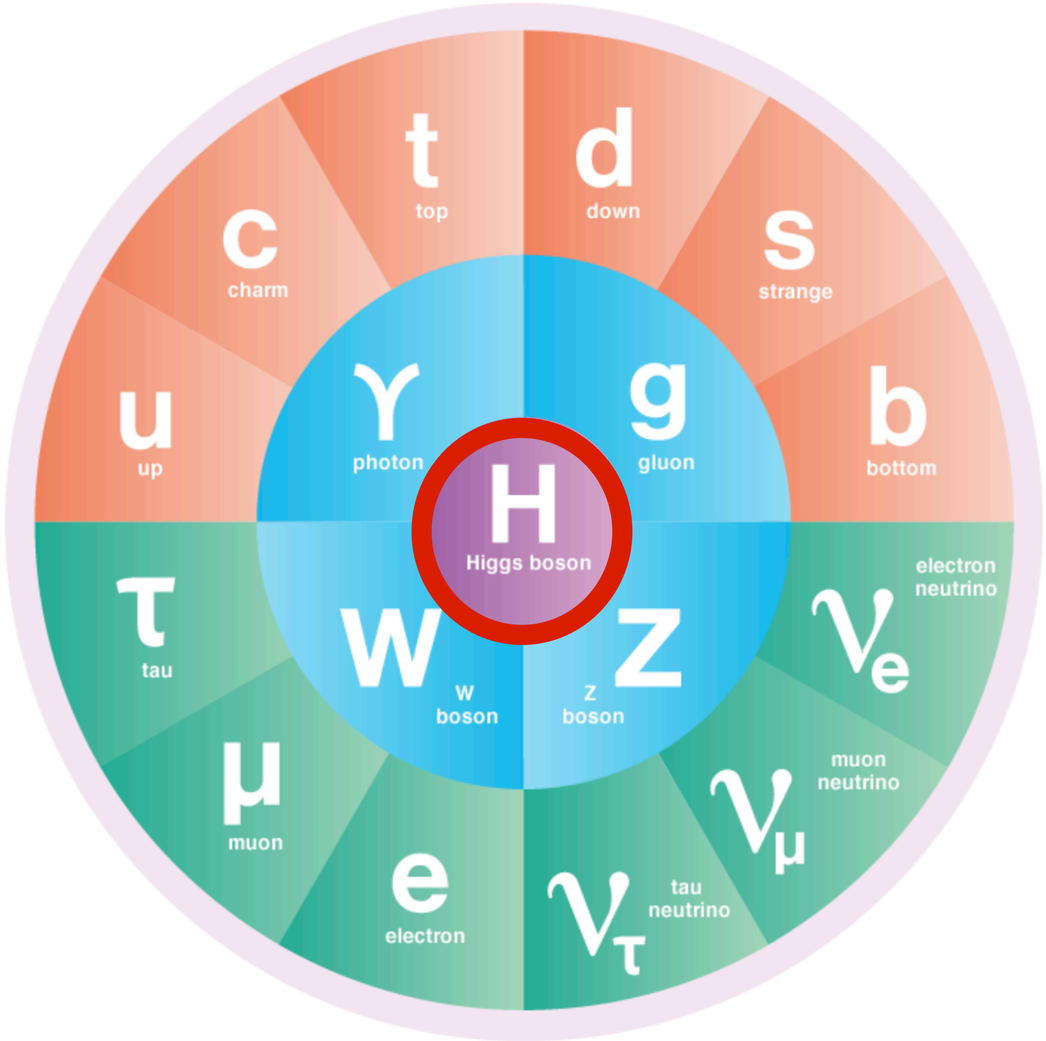


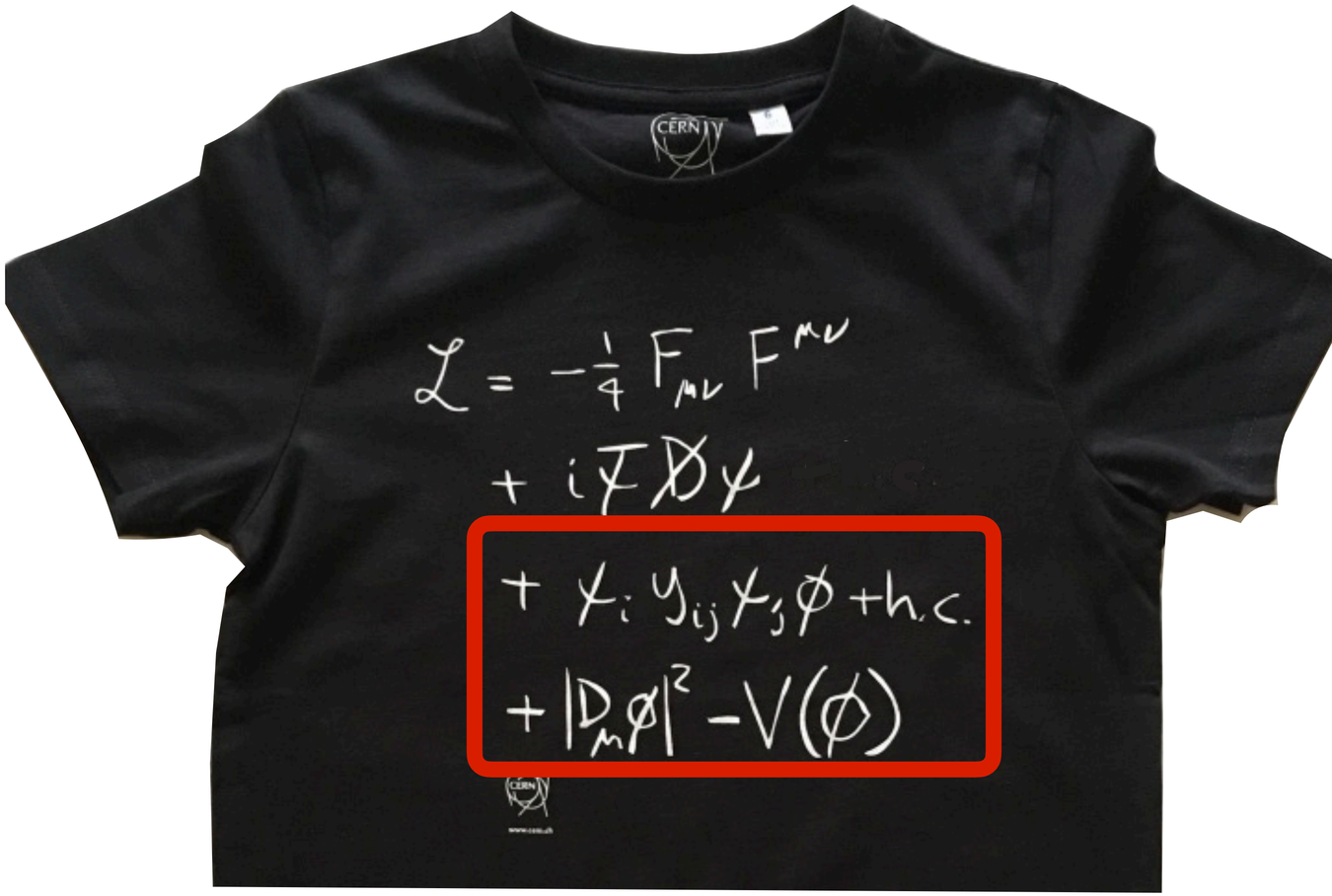
particles

interactions

https://www.symmetrymagazine.org/standard-model/

The Standard Model (SM)





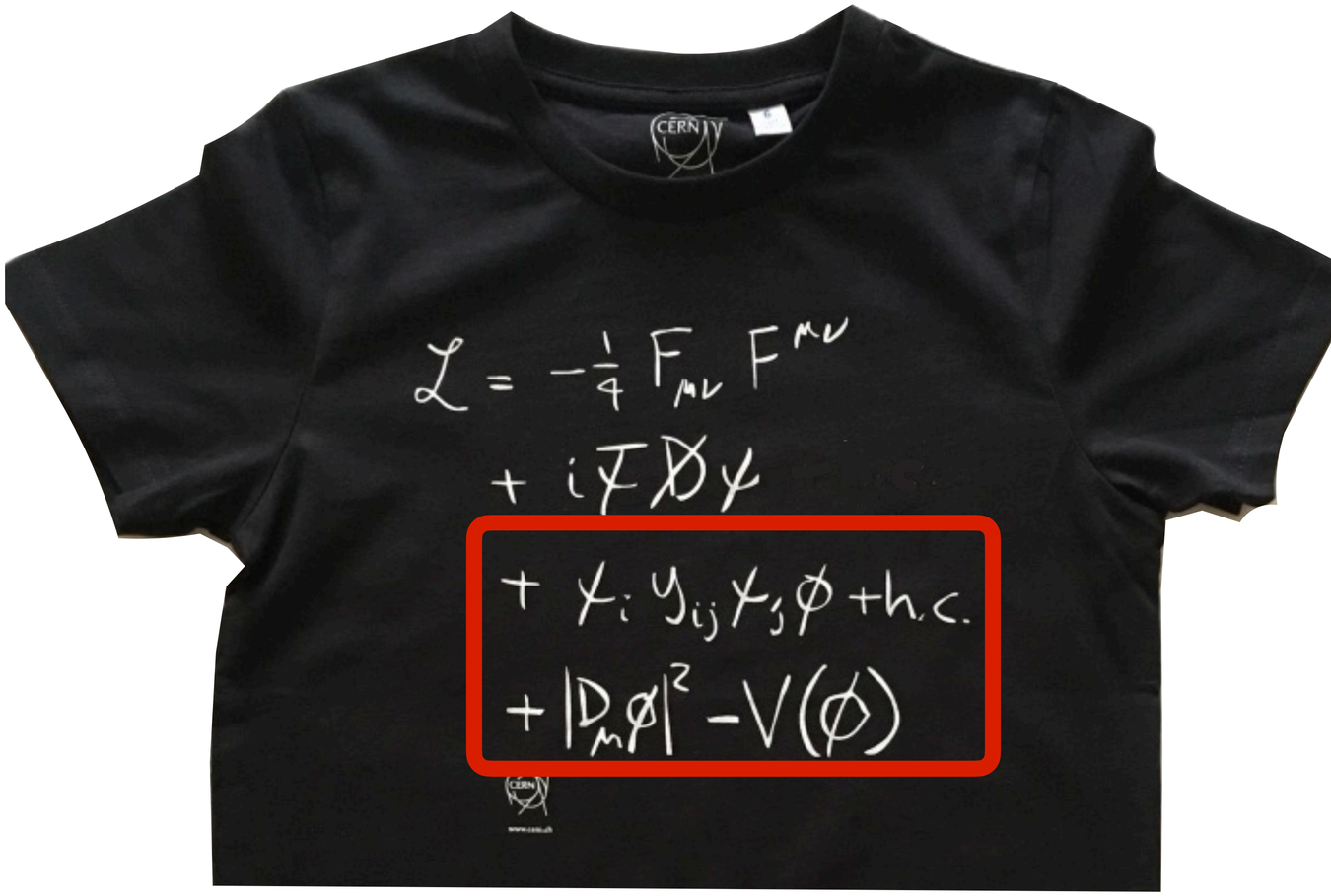
particles

interactions

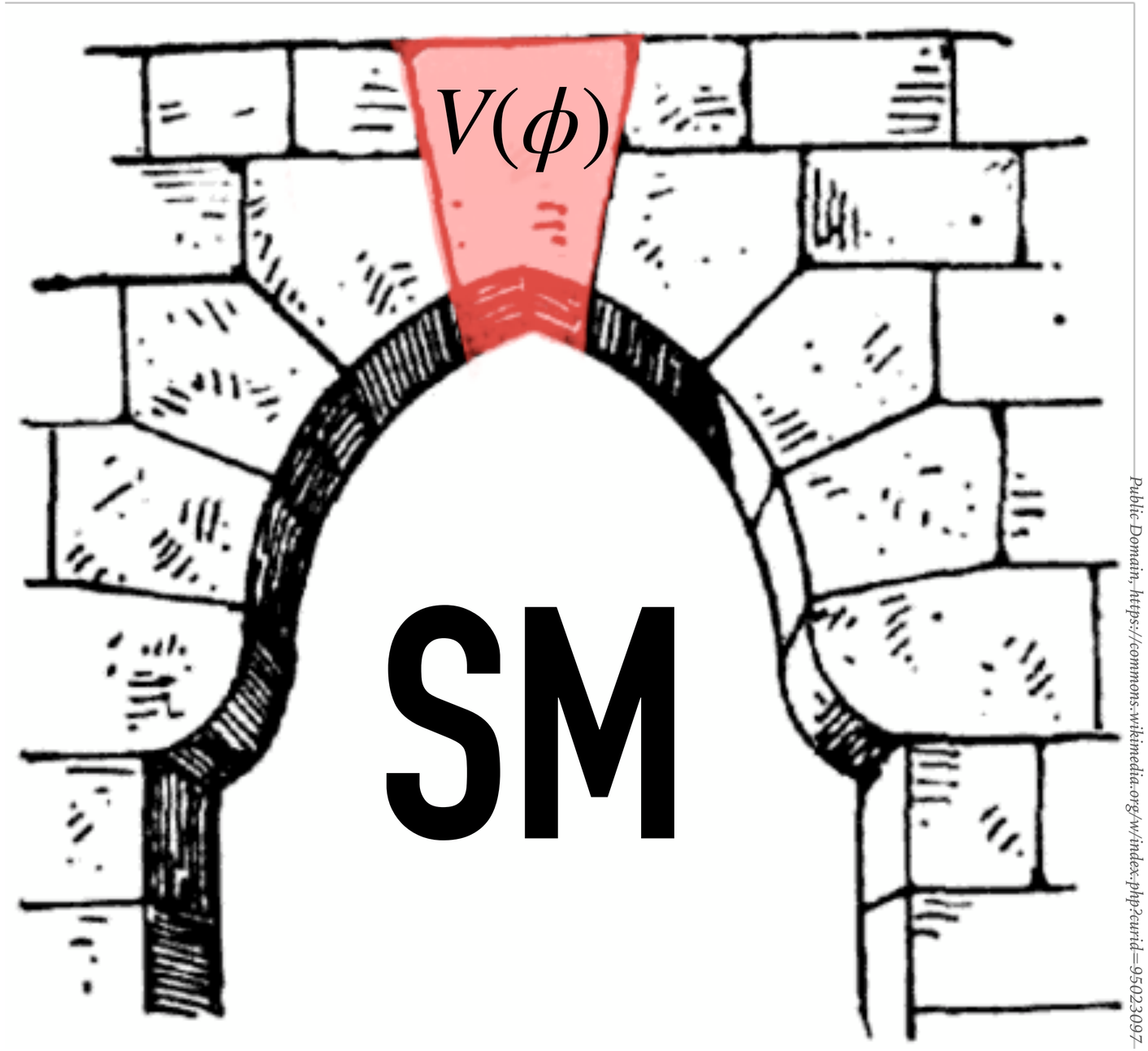
https://www.symmetrymagazine.org/standard-model/

The Standard Model (SM)

our experimental exploration of the Higgs–related SM interactions is only just starting



interactions



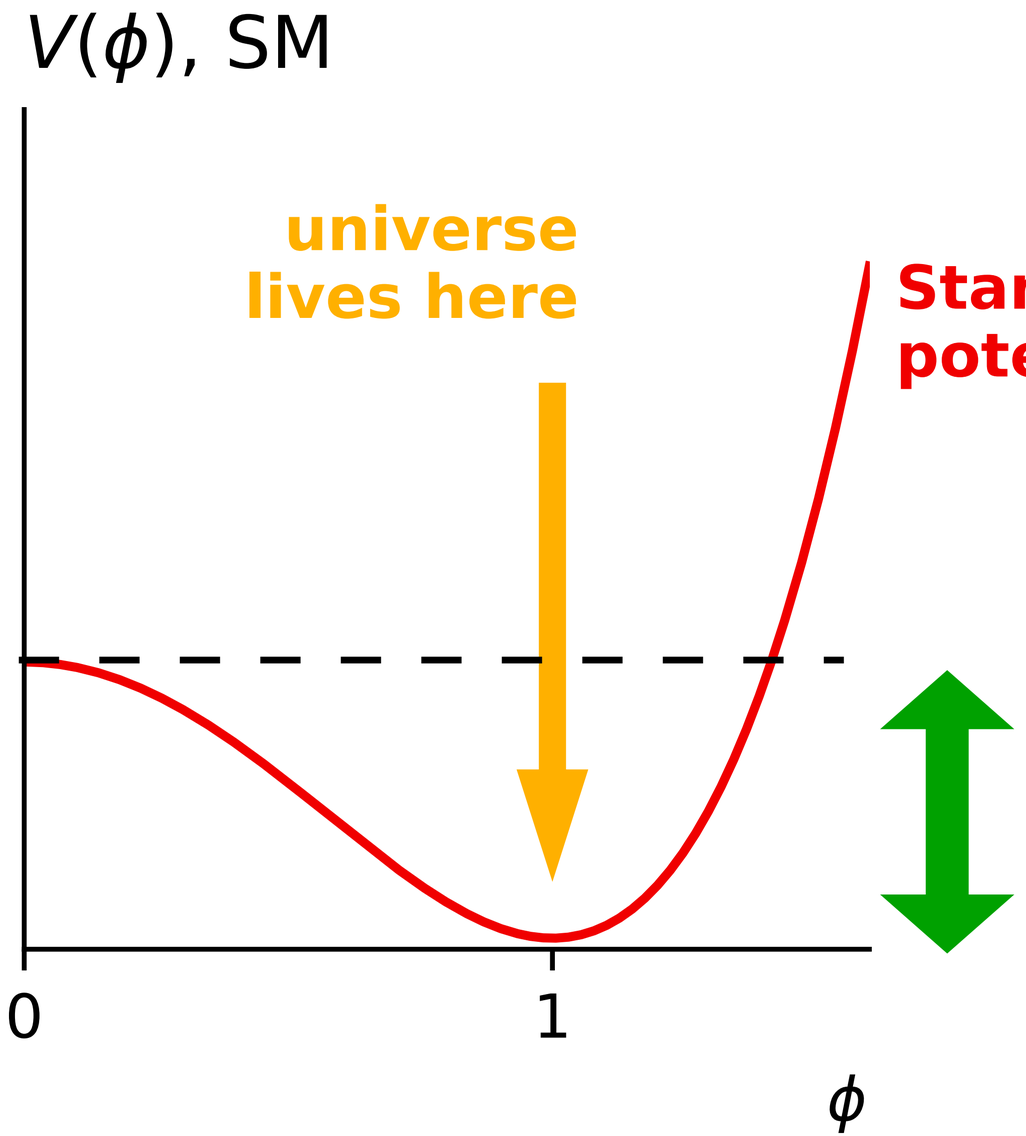
Public Domain, https://commons.wikimedia.org/w/index.php?curid=95023097

unique among all the fields we know, the Higgs field is the only one that is non-zero "classically"

Why?
Higgs potential?

Keystone of SM

Higgs potential – remember: it's an energy density



Corresponds to an energy density of
$1.5 \times 10^{10}\,\mathrm{GeV/fm^3}$

$E = mc^2 \rightarrow$ Mass density of $2.6 \times 10^{28}\,\mathrm{kg/m^3}$
i.e. **>40 billion times nuclear density**

https://en.wikipedia.org/wiki/Globe#/media/File:World_Globe_Map.jpg
https://en.wikipedia.org/wiki/Old_fashioned_glass#/media/File:Old_Fashioned_Glass.jpg
https://en.wikipedia.org/wiki/St._Jakob-Park#/media/File:StJakobParkB.JPG, by Sven Scharr, CC BY 3.0



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Earth at neutron star density

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https://en.wikipedia.org/wiki/Old_fashioned_glass#/media/File:Old_Fashioned_Glass.jpg
https://en.wikipedia.org/wiki/St._Jakob-Park#/media/File:StJakobParkB.JPG, by Sven Scharr, CC BY 3.0

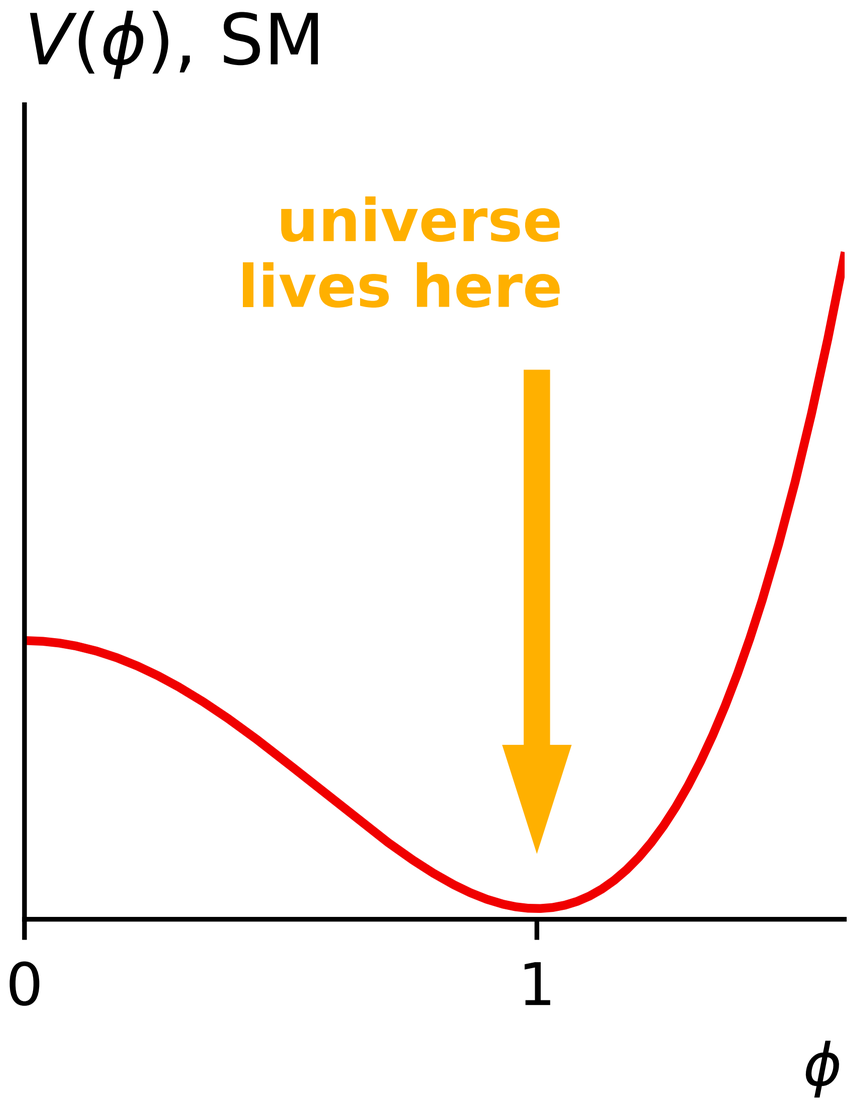


Earth at neutron star density



Earth at Higgs potential density

Higgs potential

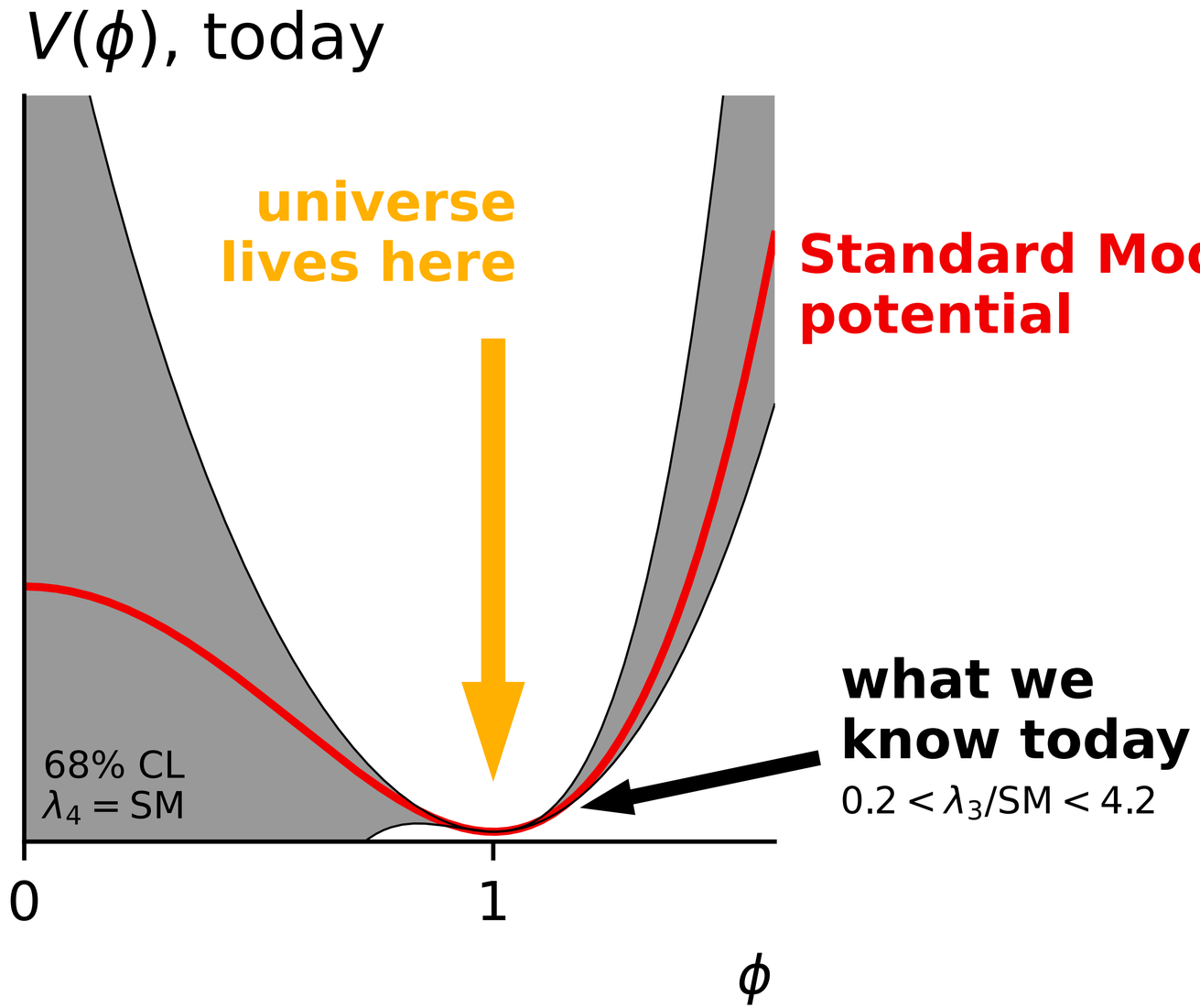


Studying H→HH probes specific mathematical property of the potential's shape:

its third derivative (λ_3), i.e. how asymmetric it is at the minimum

[reconstruction in plot assumes higher derivatives as in SM]

Higgs potential

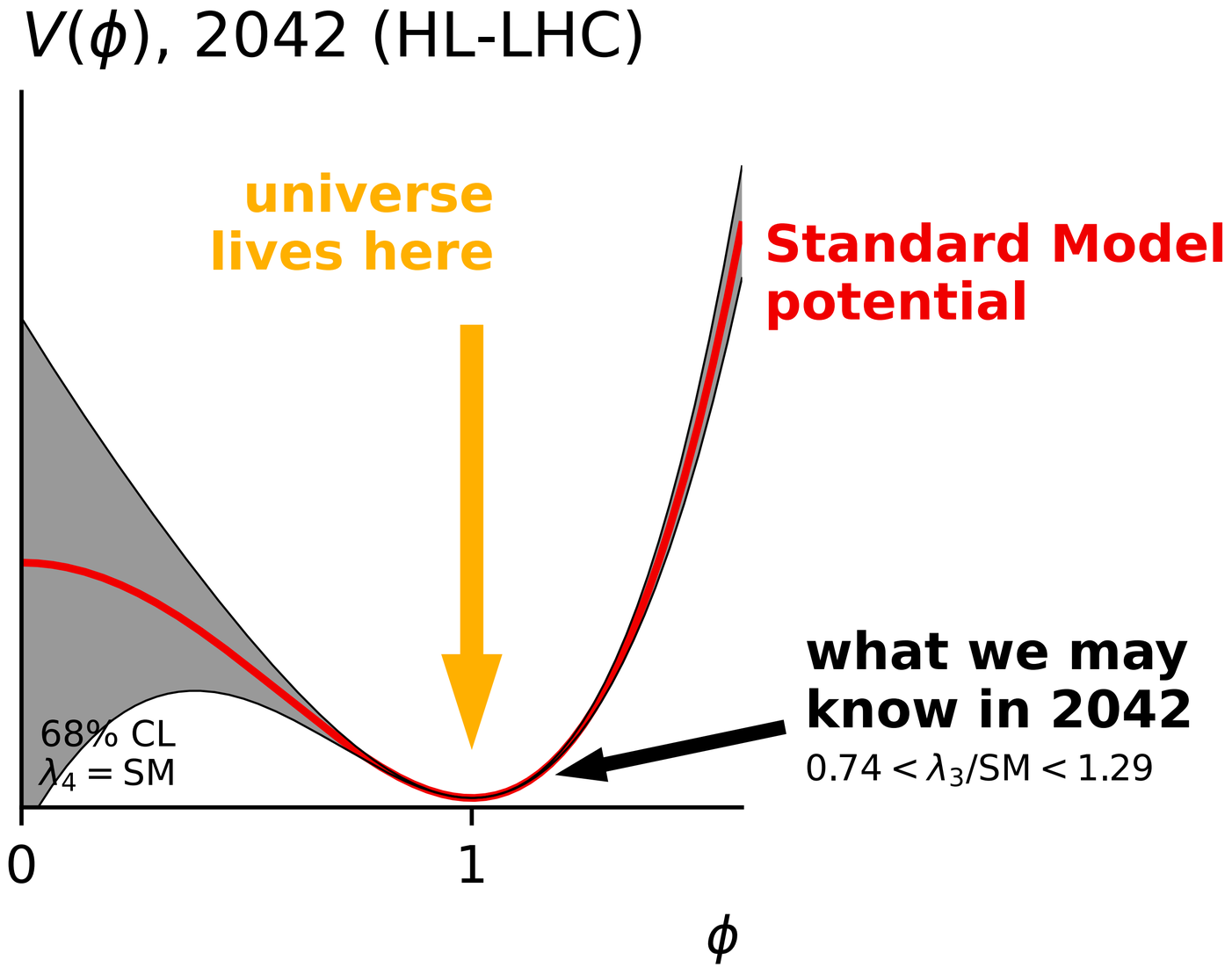


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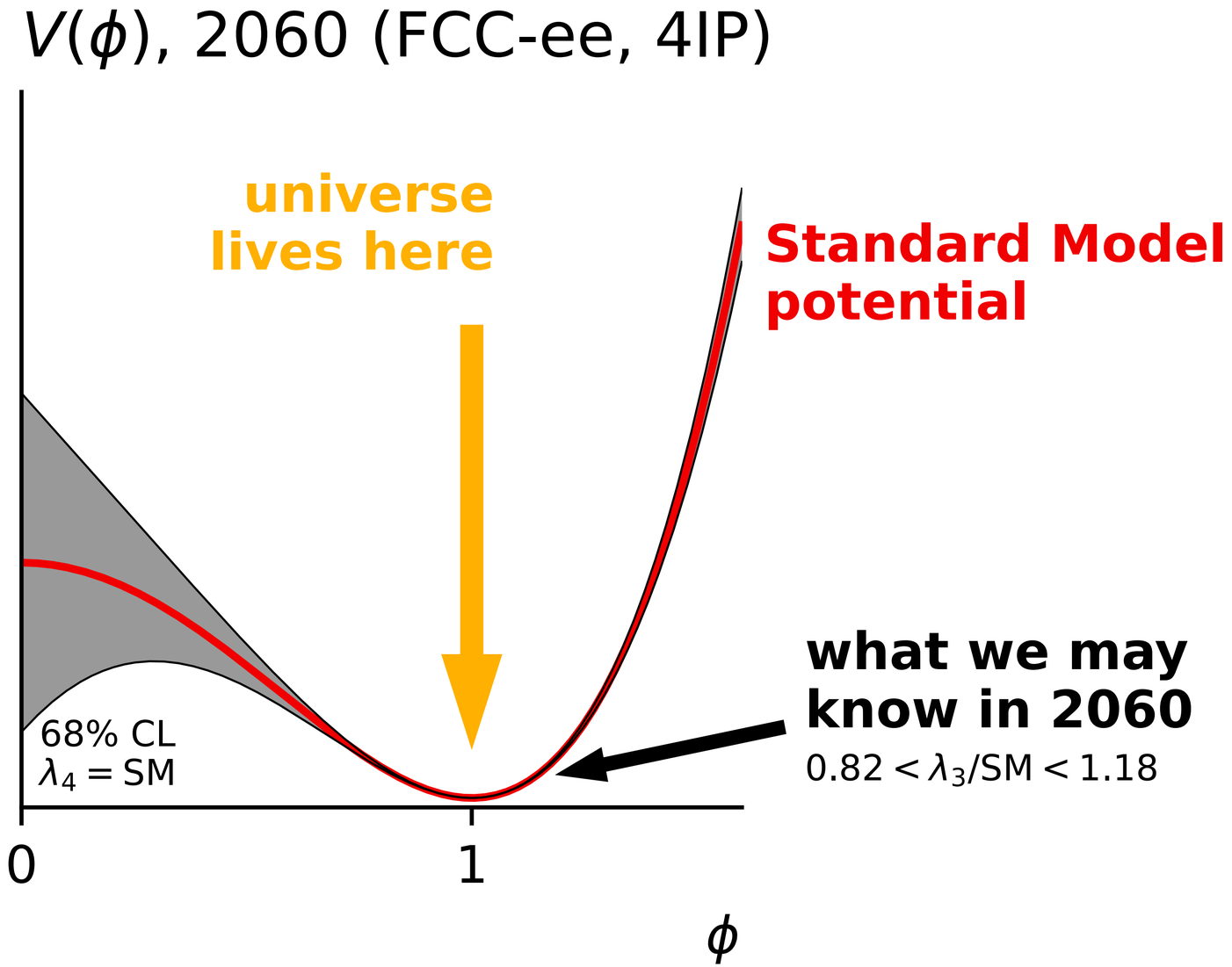


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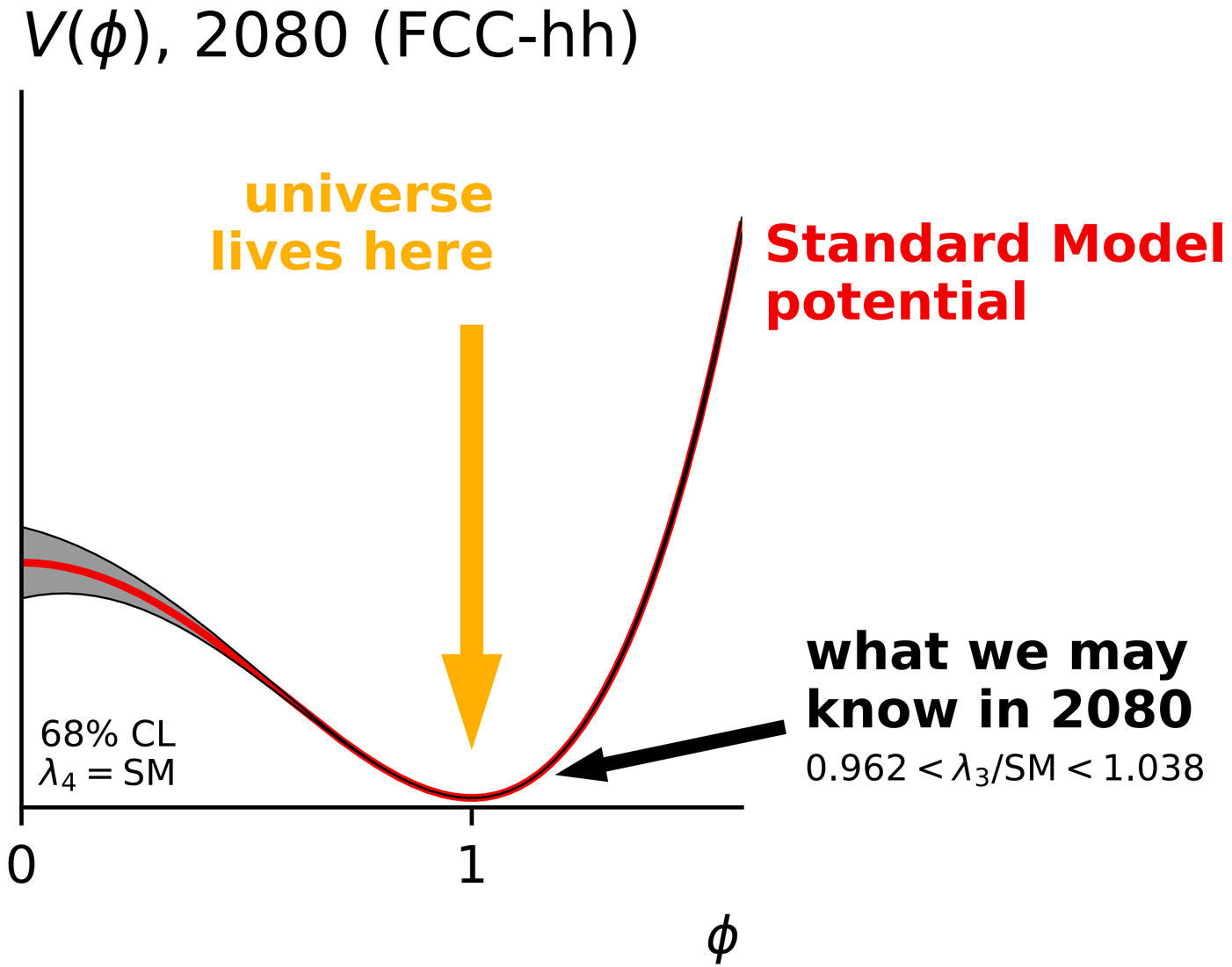


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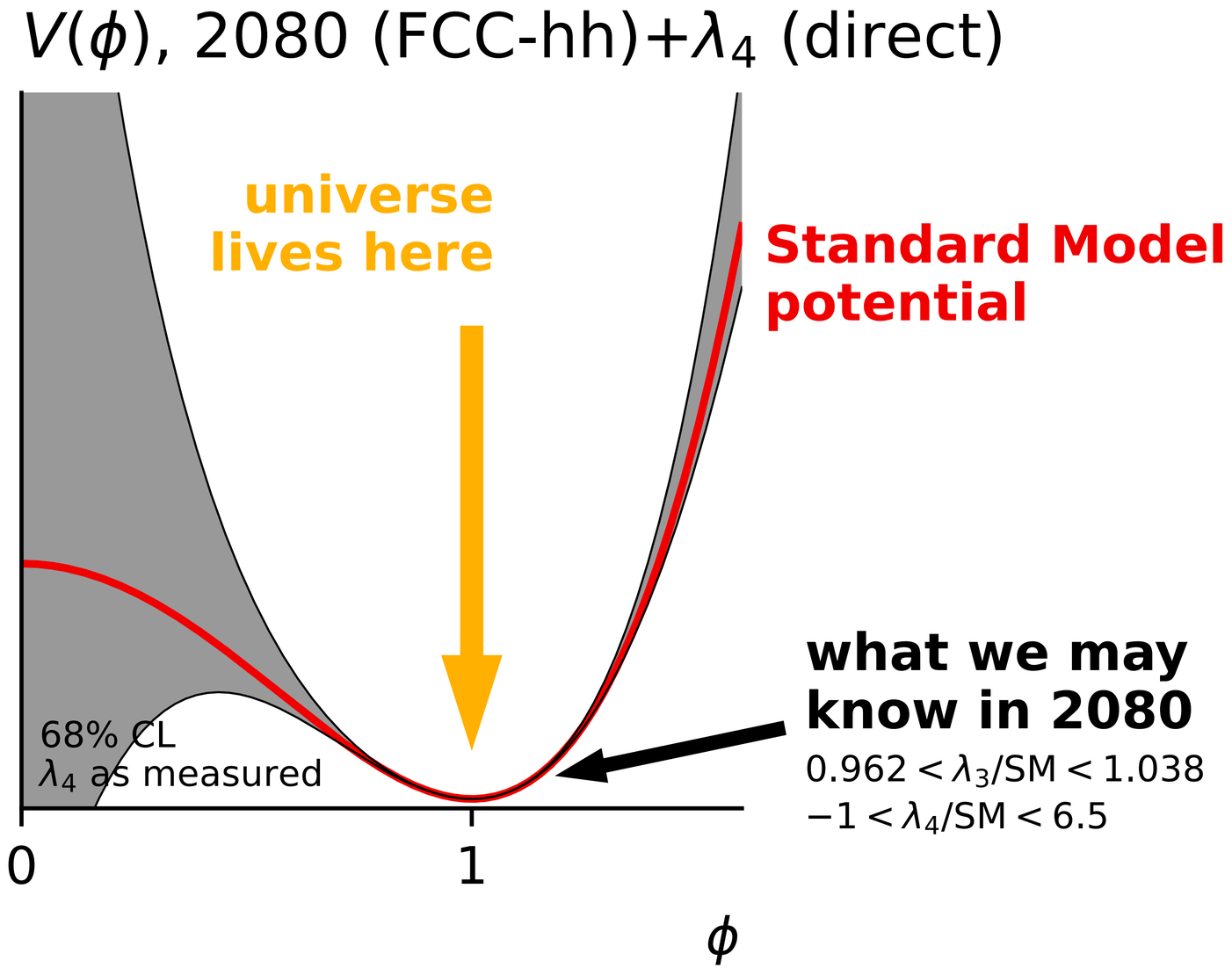


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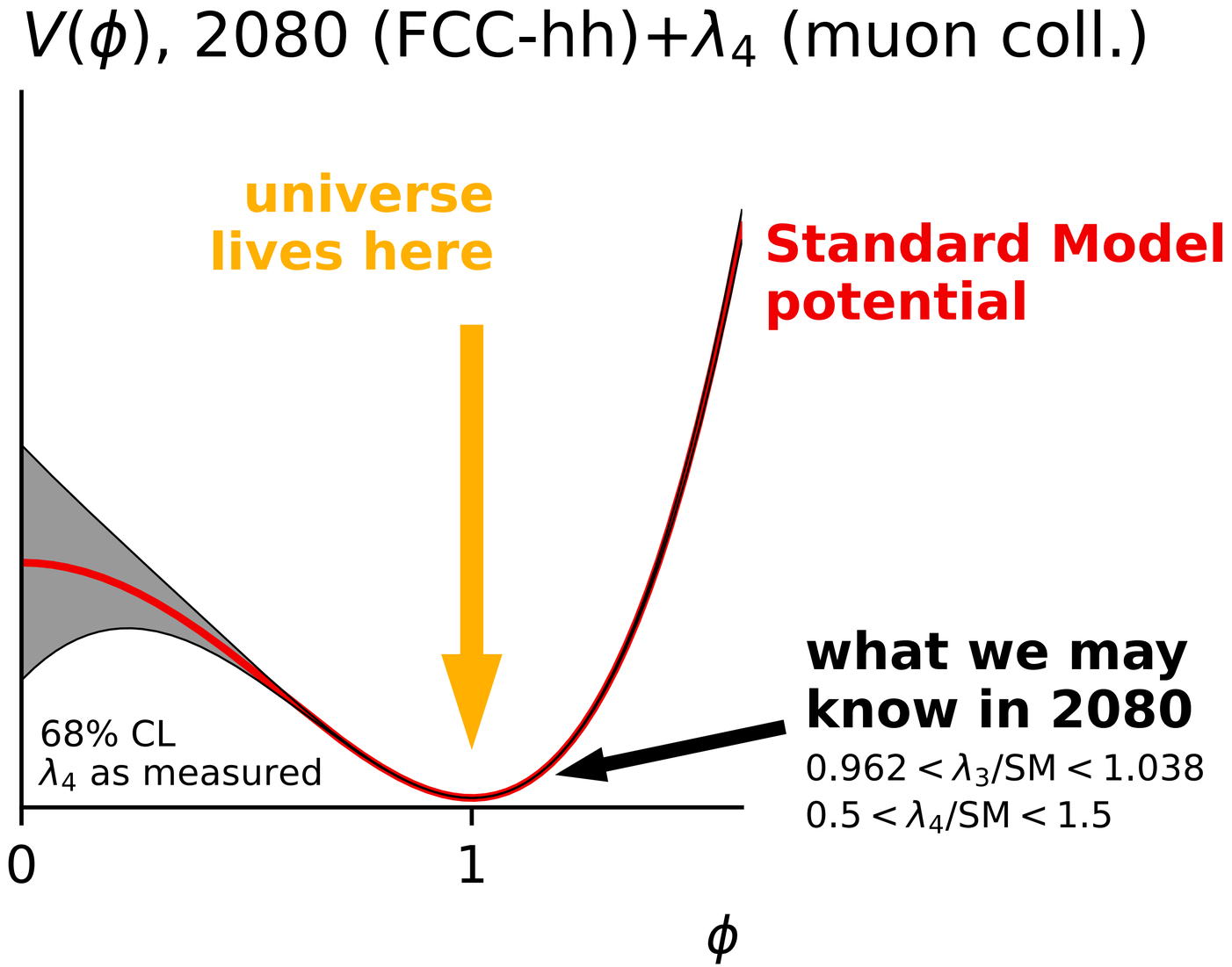


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Higgs potential



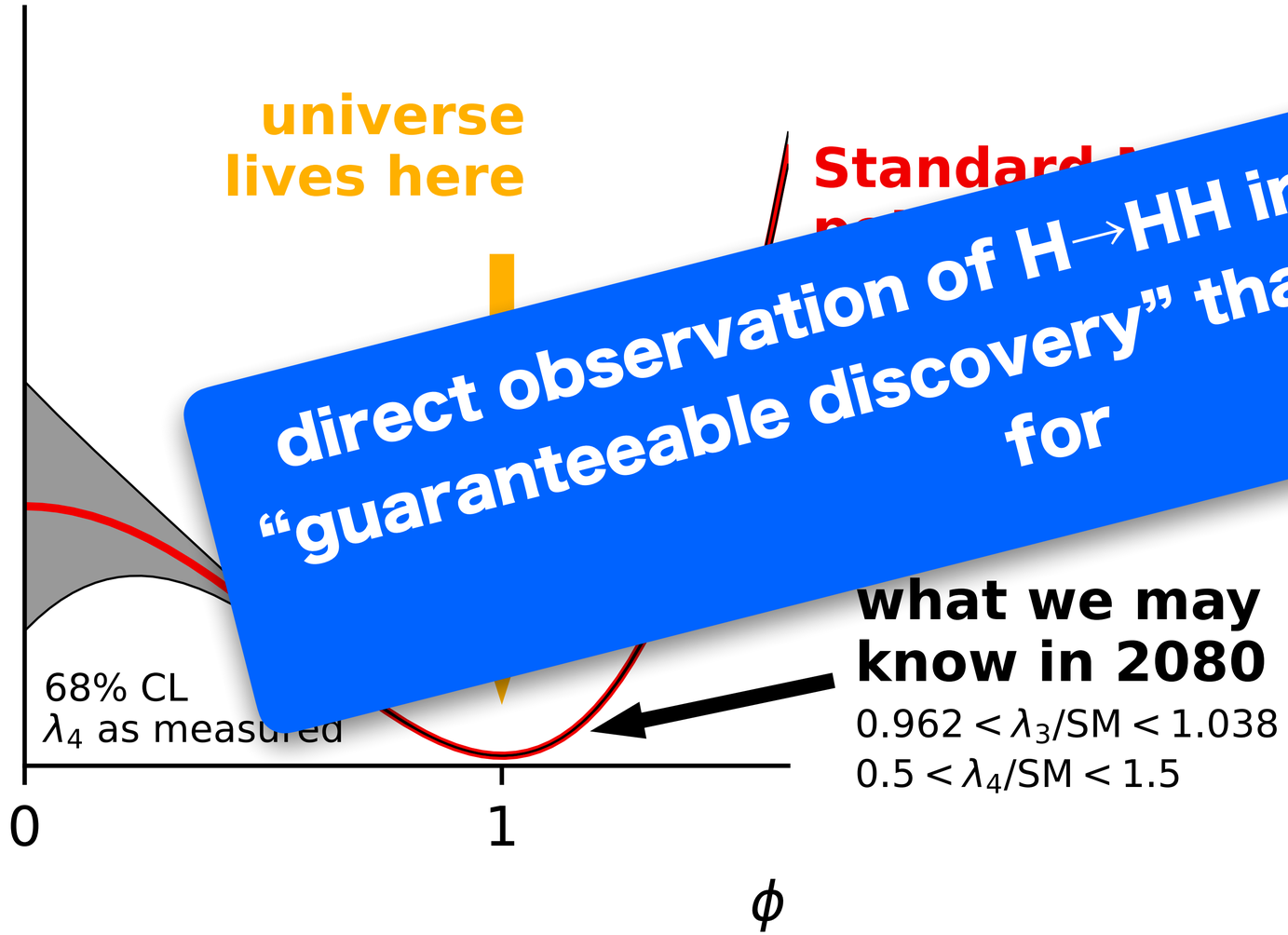
Studying H→HH probes specific mathematical property of the potential's shape:

its third derivative (λ_3), i.e. how asymmetric it is at the minimum

[reconstruction in plot assumes higher derivatives as in SM]

Higgs potential

$V(\phi)$, 2080 (FCC-hh)+λ_4 (muon coll.)



what we may know in 2080

$0.962 < \lambda_3/\mathrm{SM} < 1.038$

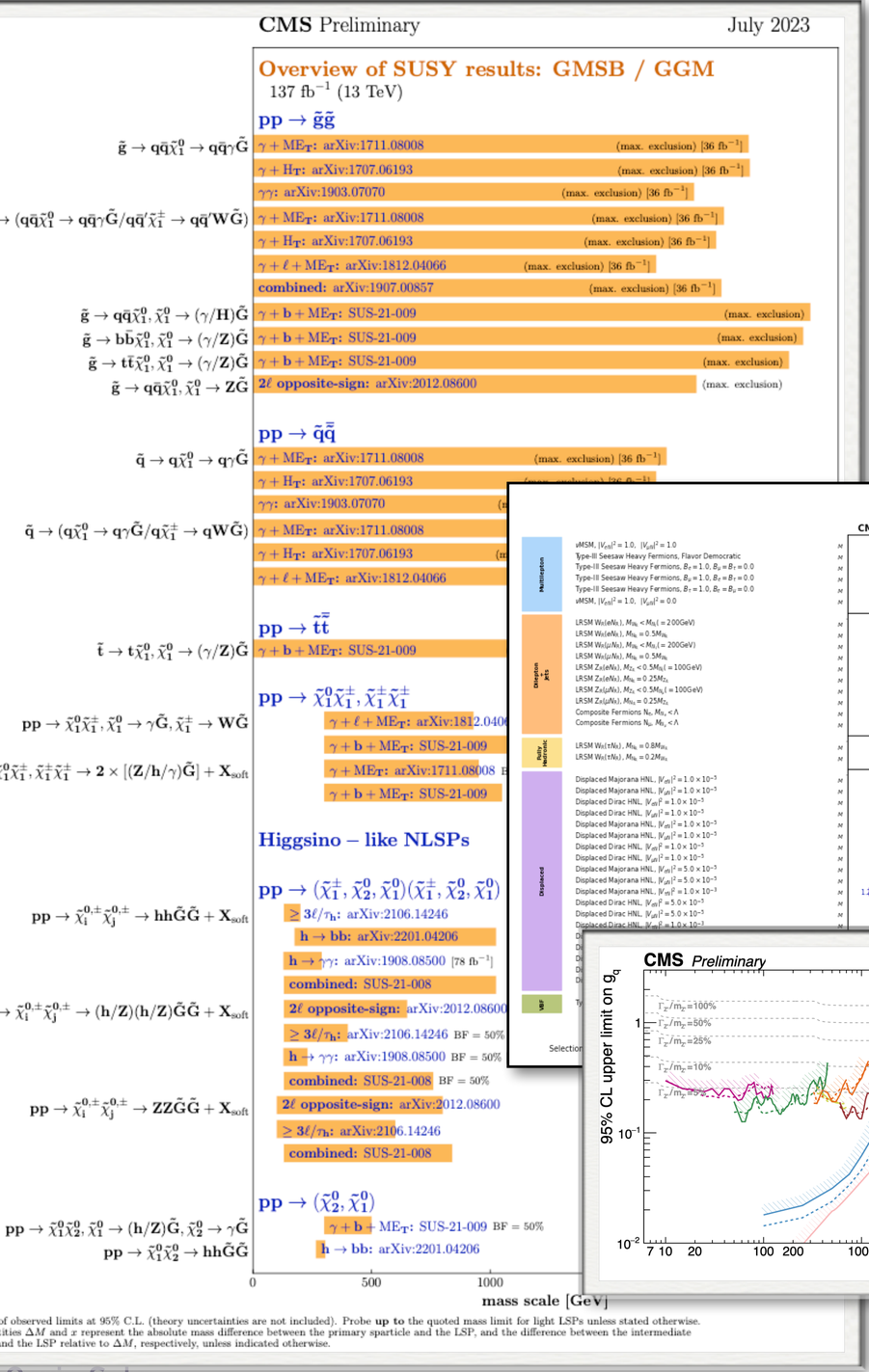
$0.5 < \lambda_4/\mathrm{SM} < 1.5$

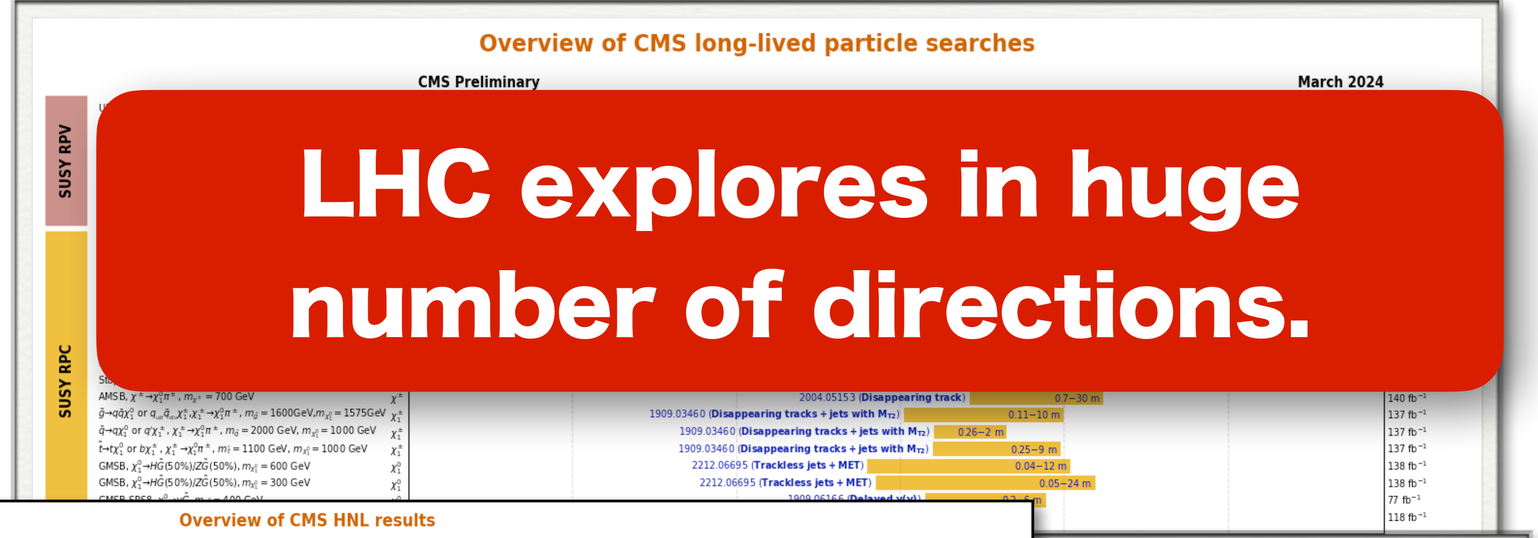
direct observation of H→HH interaction is a "guaranteeable discovery" that HEP is aiming for

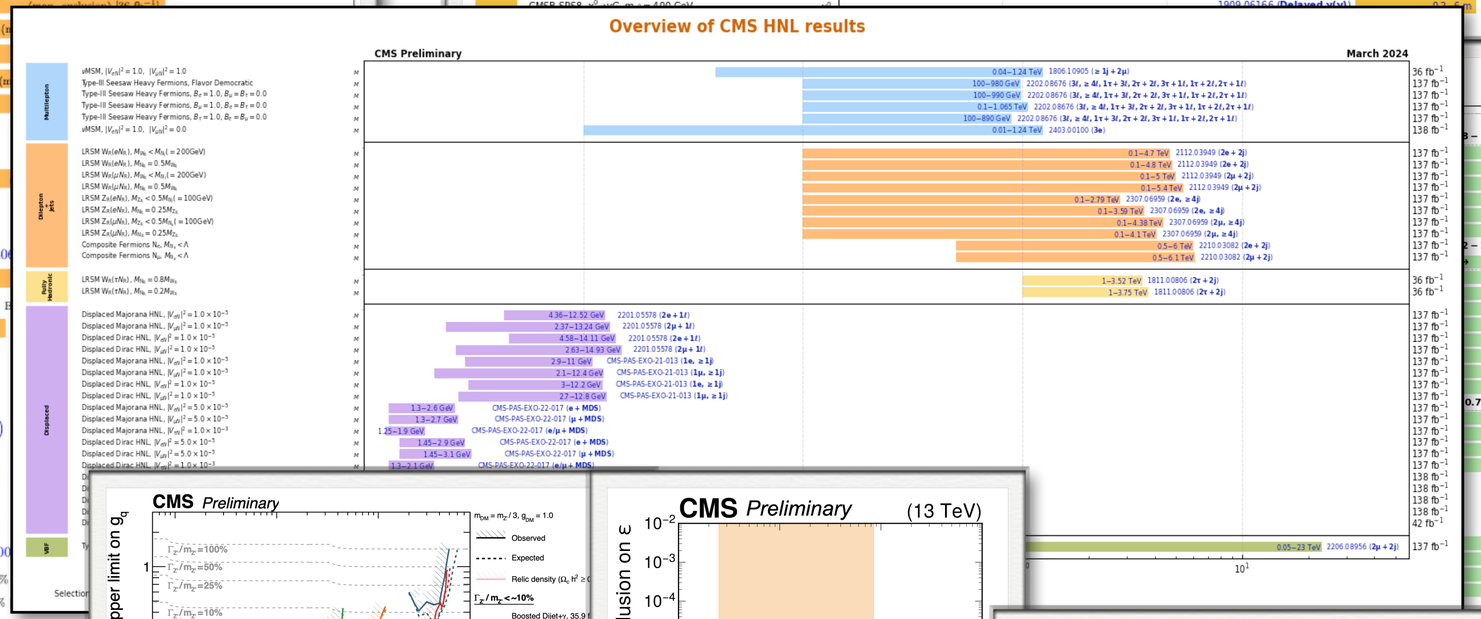
Studying H→HH probes
spe… …tical property
…l's shape:
…ive (λ_3),
…metric it is
at the minimum

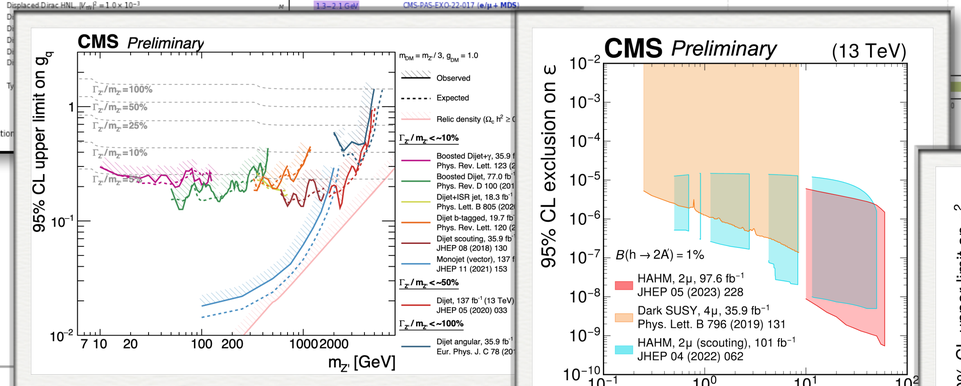
[reconstruction in plot assumes higher derivatives as in SM]

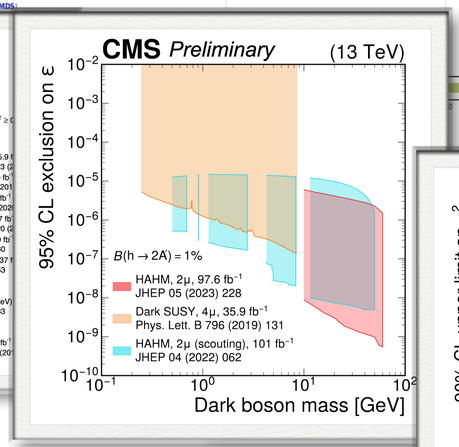
LHC explores in huge number of directions.

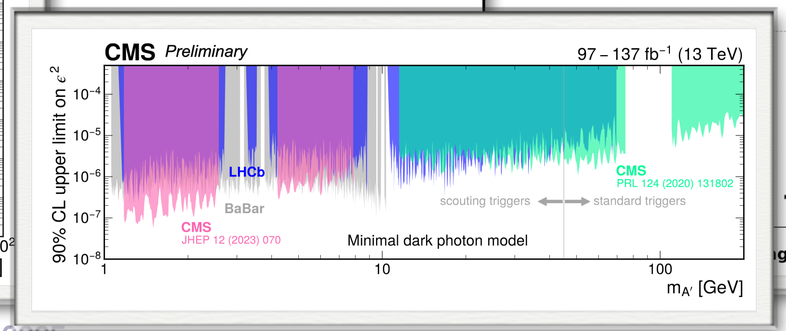


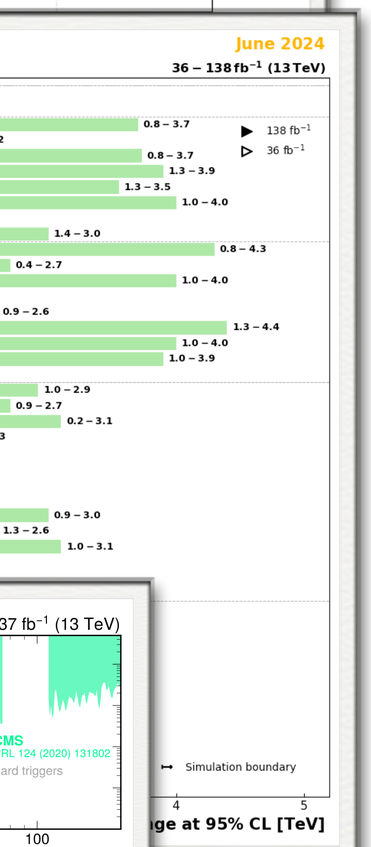




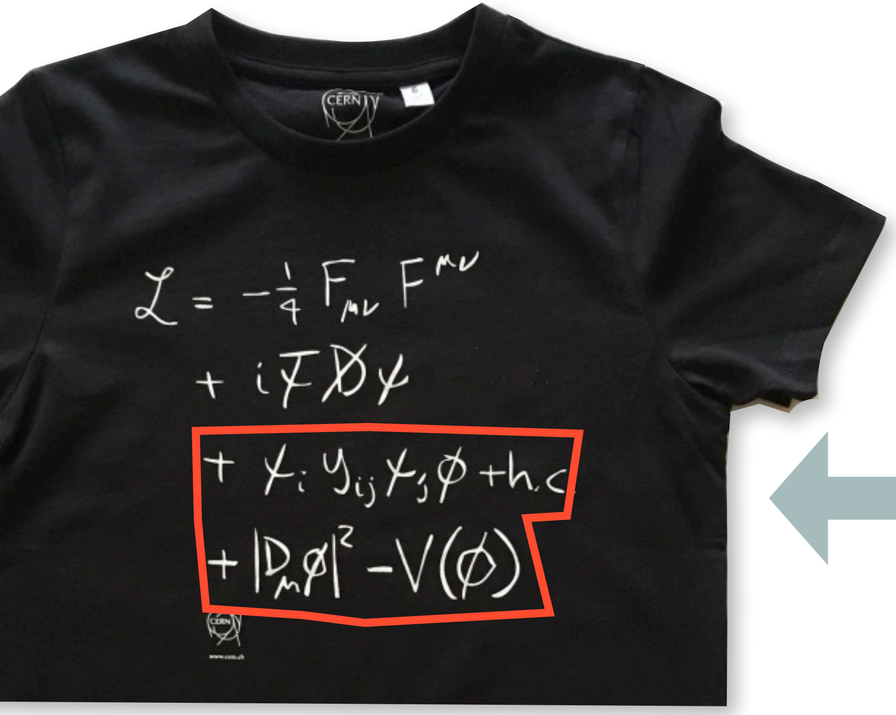






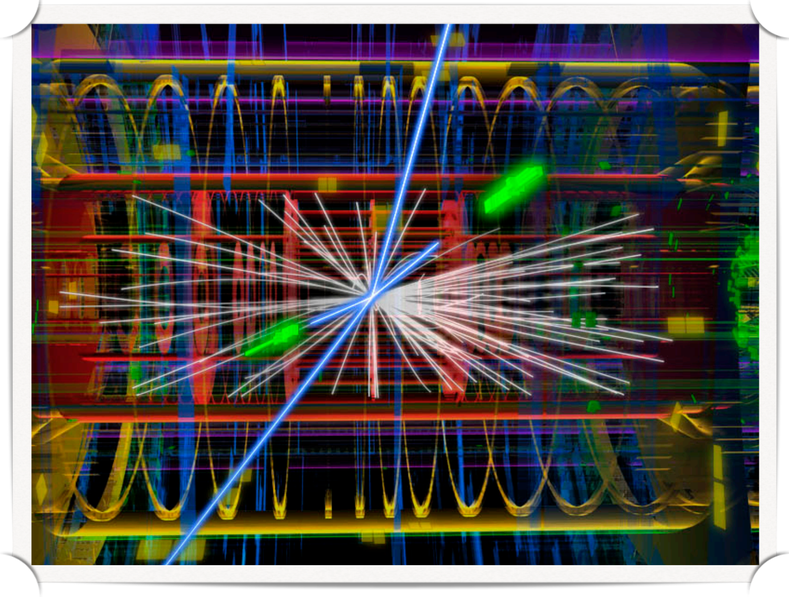


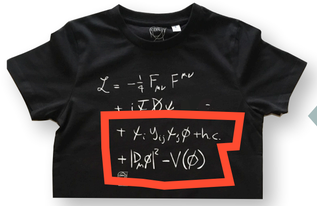
UNDERLYING THEORY

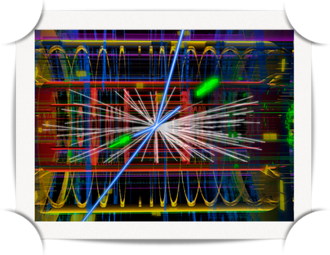


how do you make quantitative connection?

EXPERIMENTAL DATA



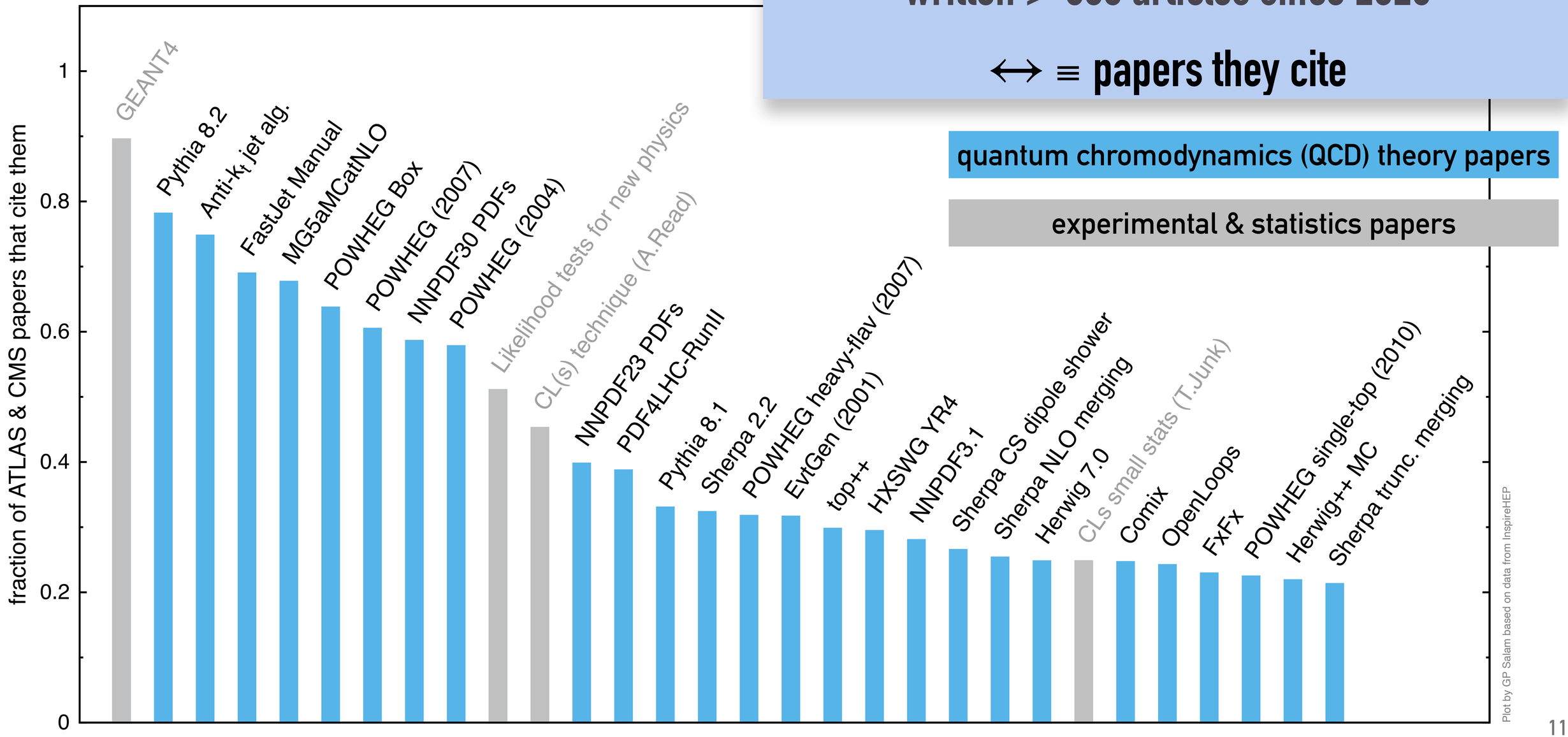


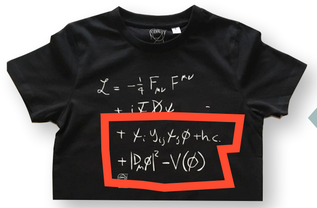


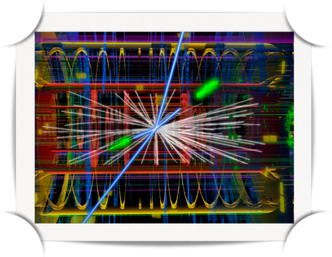
Lagrangian ↔ data

ATLAS and CMS (big LHC expts.) have written > 800 articles since 2020

↔ ≡ papers they cite







predicting full particle structure that comes out of a collision

fraction of ATLAS & CMS papers that cite them

0
0.2
0.4
0.6
0.8
1

GEANT4
Pythia 8.2
Anti-k_t jet alg.
FastJet Manual
MG5aMCatNLO
POWHEG Box
POWHEG (2007)
NNPDF30 PDFs
POWHEG (2004)
Likelihood tests for new physics
CL(s) technique (A.Read)
NNPDF23 PDFs
PDF4LHC-RunII
Pythia 8.1
Sherpa 2.2
POWHEG heavy-flav (2007)
EvtGen (2001)
top++
HXSWG YR4
NNPDF3.1
Sherpa CS dipole shower
Sherpa NLO merging
Herwig 7.0
CLs small stats (T.Junk)
Comix
OpenLoops
FxFx
POWHEG single-top (2010)
Herwig++ MC
Sherpa trunc. merging

Plot by GP Salam based on data from InspireHEP

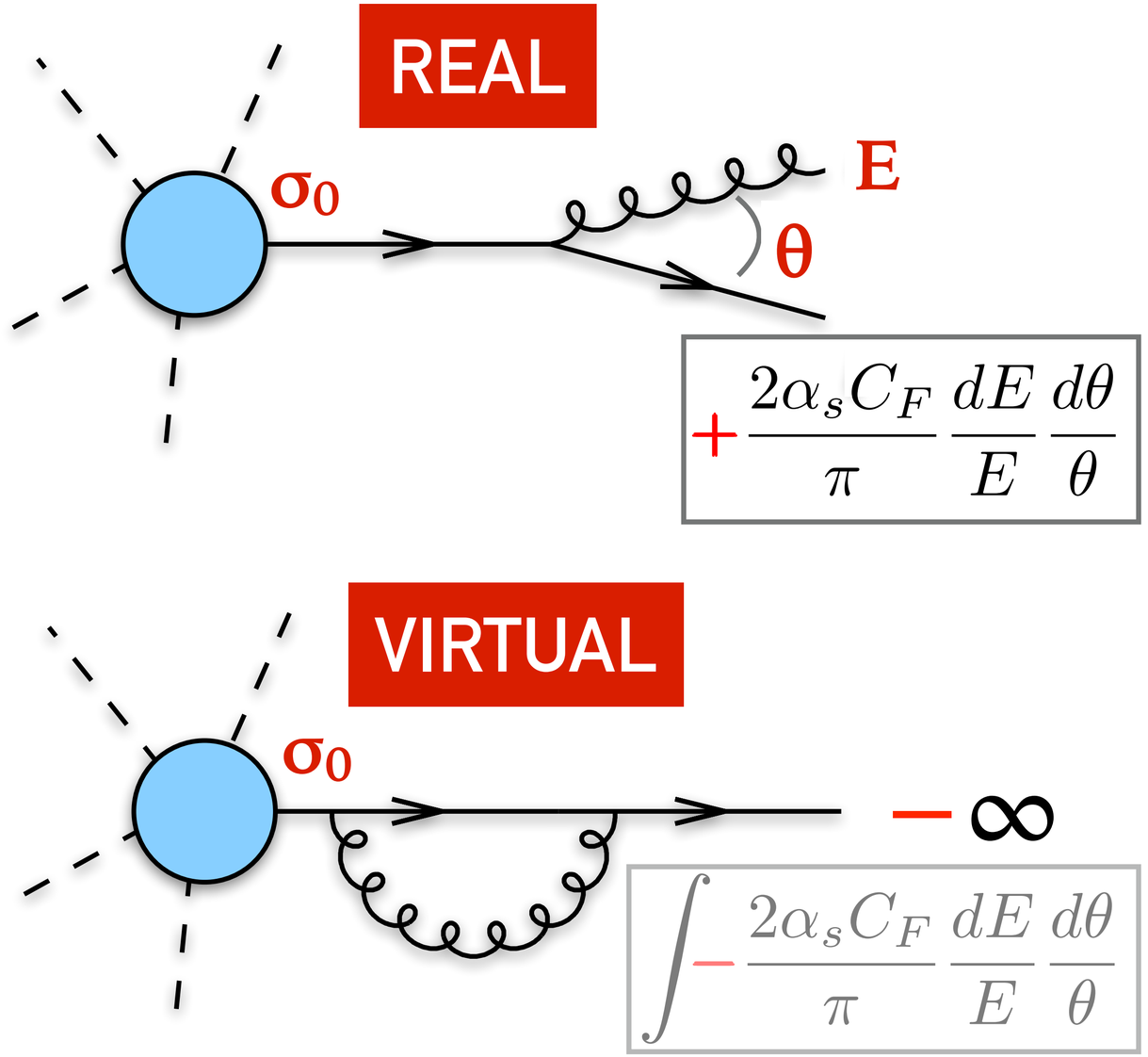
Why not just plain (N)NLO?

i.e. get scattering cross-sections from first few orders of perturbative expansion in the strong coupling α_s

$$\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \cdots$$

LO NLO NNLO

What kind of contributions do we get at NLO?



Divergences are present in both real and virtual diagrams.

They arise when an emission has a small energy ($E \ll 1$) or a small angle ($\theta \ll 1$).

In dim-reg, this brings $1/\varepsilon^2$ for each order in α_s.

What a NLO calculation gives you (here, Event2, $e^+e^- \to q\bar{q}$)

LO (2-particle) tree-level event
with weight 1.00000
px, py, pz, E = -1.32 -1.38 -49.96 50.00
px, py, pz, E = 1.32 1.38 49.96 50.00

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LO event ($q\bar{q}$)

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px, py, pz, E = 1.32 1.38 49.96 50.00

NLO (3-particle) tree-level event
with **weight 893.22103**, multiplying (alphas/2pi)
px, py, pz, E = -1.60 -1.75 -49.87 49.93
px, py, pz, E = 1.31 1.36 49.25 49.29
px, py, pz, E = 0.30 0.39 0.62 0.79

LO event ($q\bar{q}$)

NLO event, with real emission
~ LO event + extra soft gluon
and **large positive weight**

What a NLO calculation gives you (here, Event2, $e^+e^- \to q\bar{q}$)

LO (2-particle) tree-level event
with weight 1.00000
px, py, pz, E = -1.32 -1.38 -49.96 50.00
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px, py, pz, E = 1.31 1.36 49.25 49.29
px, py, pz, E = 0.30 0.39 0.62 0.79

NLO (2-particle) virtual subtraction event
with **weight -84.49299**, multiplying (alphas/2pi)
px, py, pz, E = -1.32 -1.38 -49.96 50.00
px, py, pz, E = 1.32 1.38 49.96 50.00

NLO (2-particle) virtual subtraction event
with **weight -808.58646**, multiplying (alphas/2pi)
px, py, pz, E = -1.61 -1.75 -49.94 50.00
px, py, pz, E = 1.61 1.75 49.94 50.00

NLO (2-particle) virtual finite event
with weight 2.66667, multiplying (alphas/2pi)
px, py, pz, E = -1.32 -1.38 -49.96 50.00
px, py, pz, E = 1.32 1.38 49.96 50.00

LO event ($q\bar{q}$)

NLO event, with real emission
~ LO event + extra soft gluon
and **large positive weight**

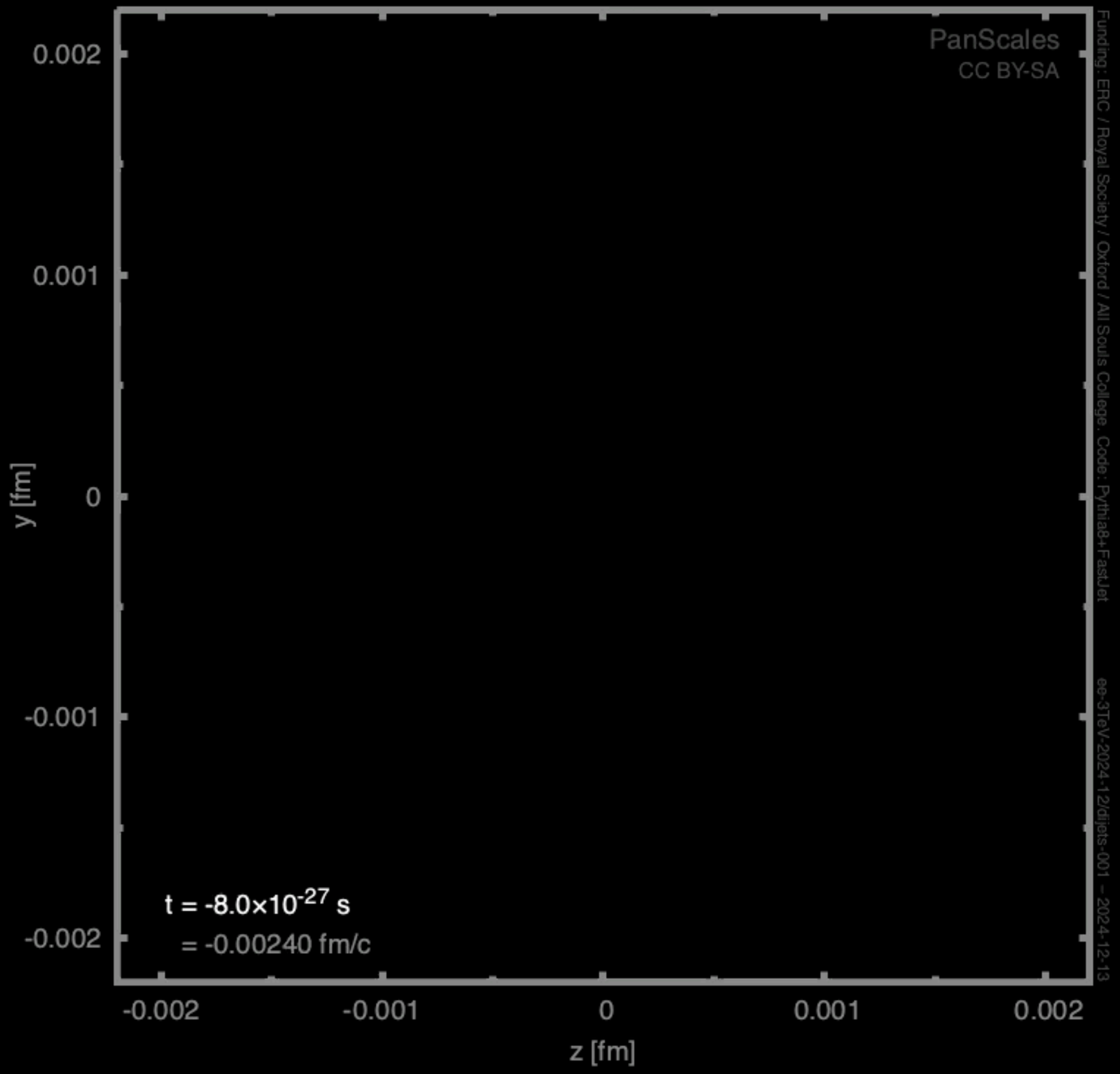
NLO event, “virtual” correction
~ LO event
and **large negative weight**

event weights are ~ probabilities

- real life doesn't have negative probabilities
- real life doesn't have (near-)divergent probabilities
- you can avoid these problems in perturbation theory if you ask very limited kinds of questions, i.e. nearly always summing real & virtual divergences (infrared safe observable, single momentum scale)*
- but experiments don't limit themselves to those kinds of questions

* though there can still be nasty surprises, cf. Chen et al 2102.07607, GPS & Slade 2106.08329

What actually happens in a collision?

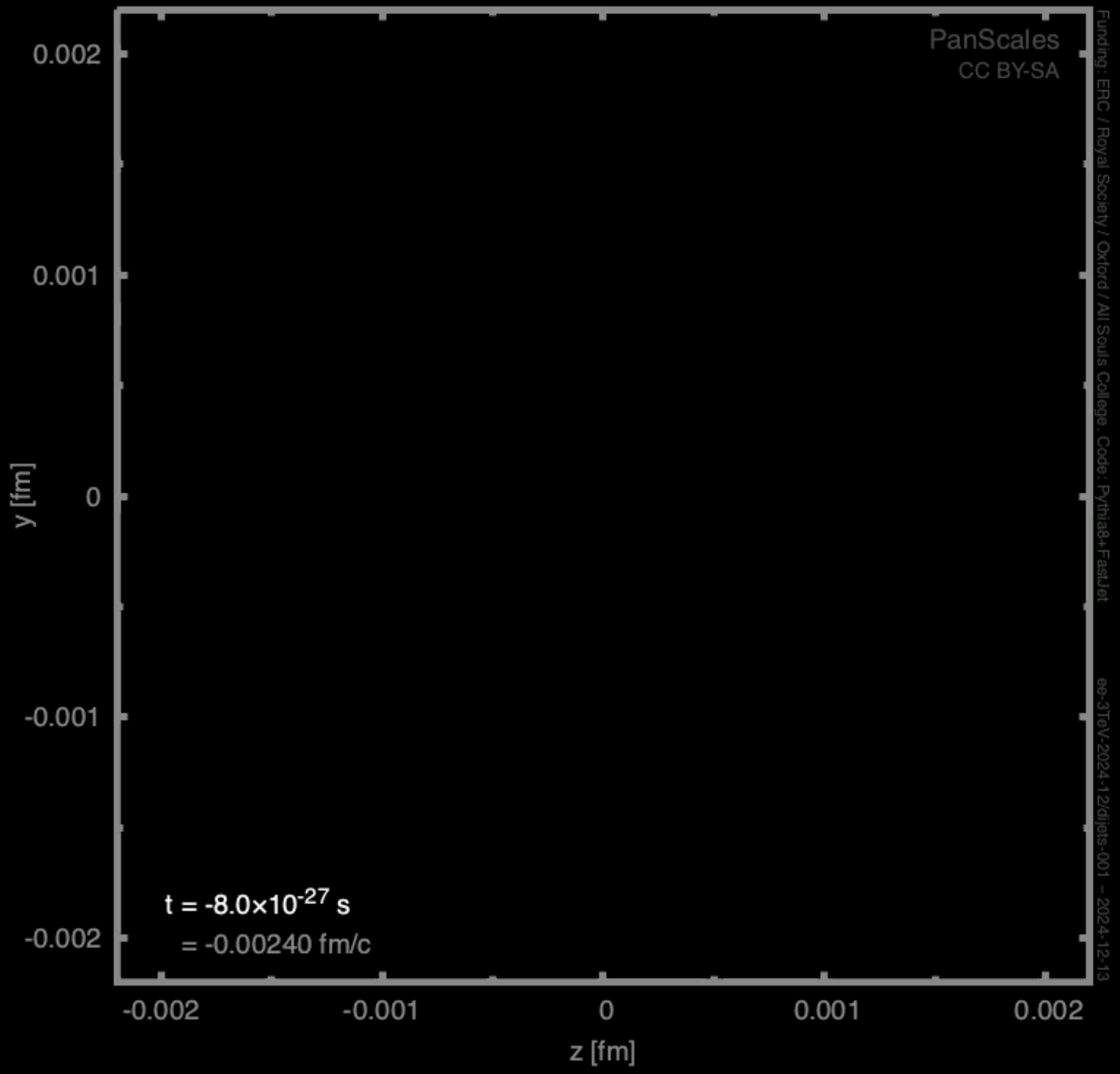


incoming beam particle

intermediate particle (quark or gluon)

final particle (hadron)

Event evolution spans 7 orders of magnitude in space-time



incoming beam particle

intermediate particle (quark or gluon)

final particle (hadron)

Event evolution spans 7 orders of magnitude in space-time

simulations use General Purpose Monte Carlo event generators and LO/NLO/NNLO matching codes



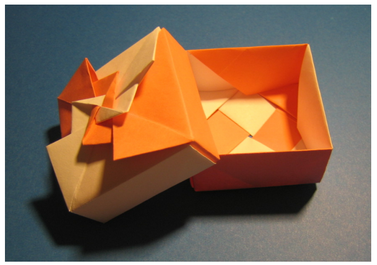
Herwig 7



Pythia 8



Sherpa 2

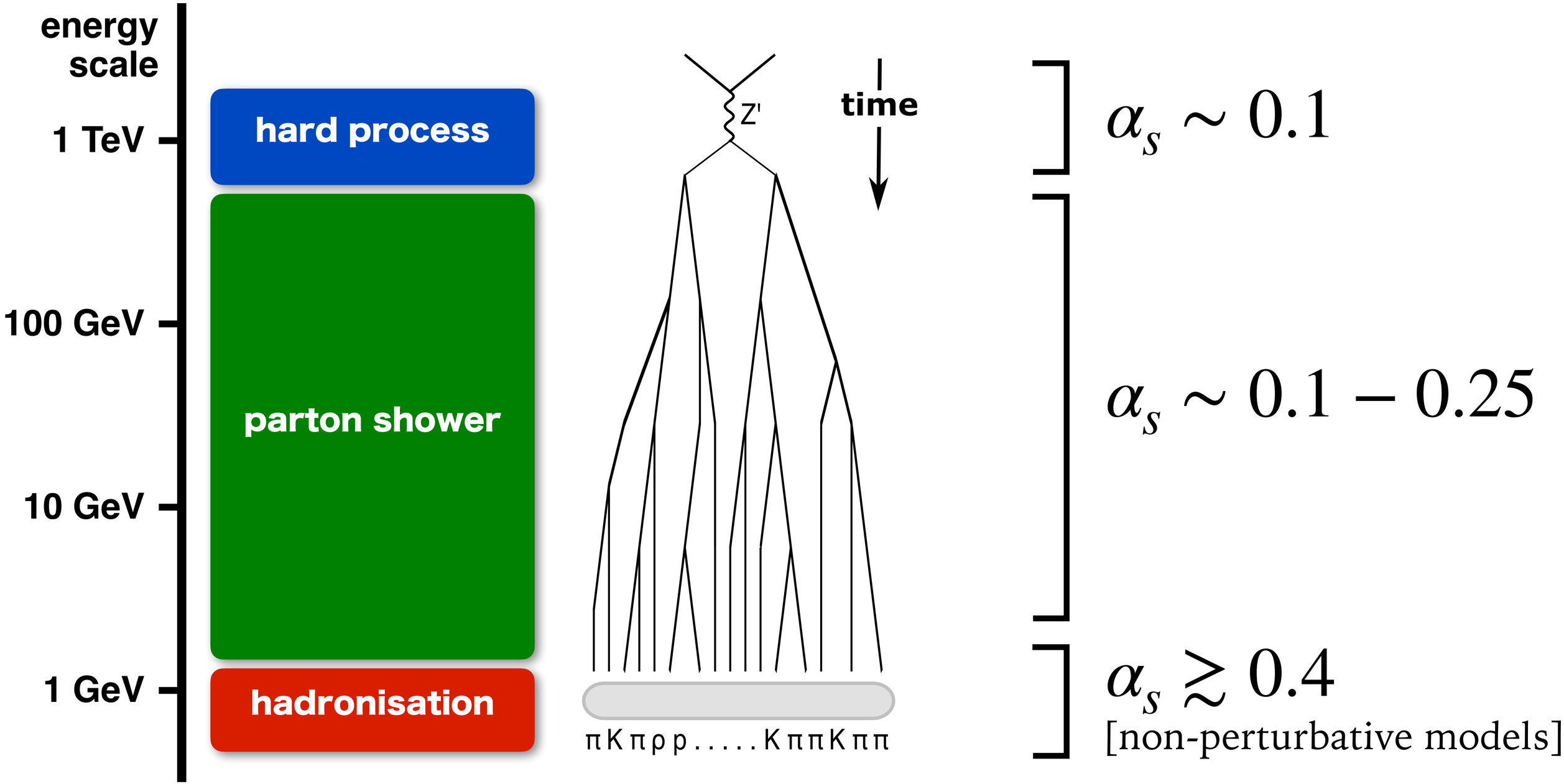


POWHEG BOX
+ MiNNLO

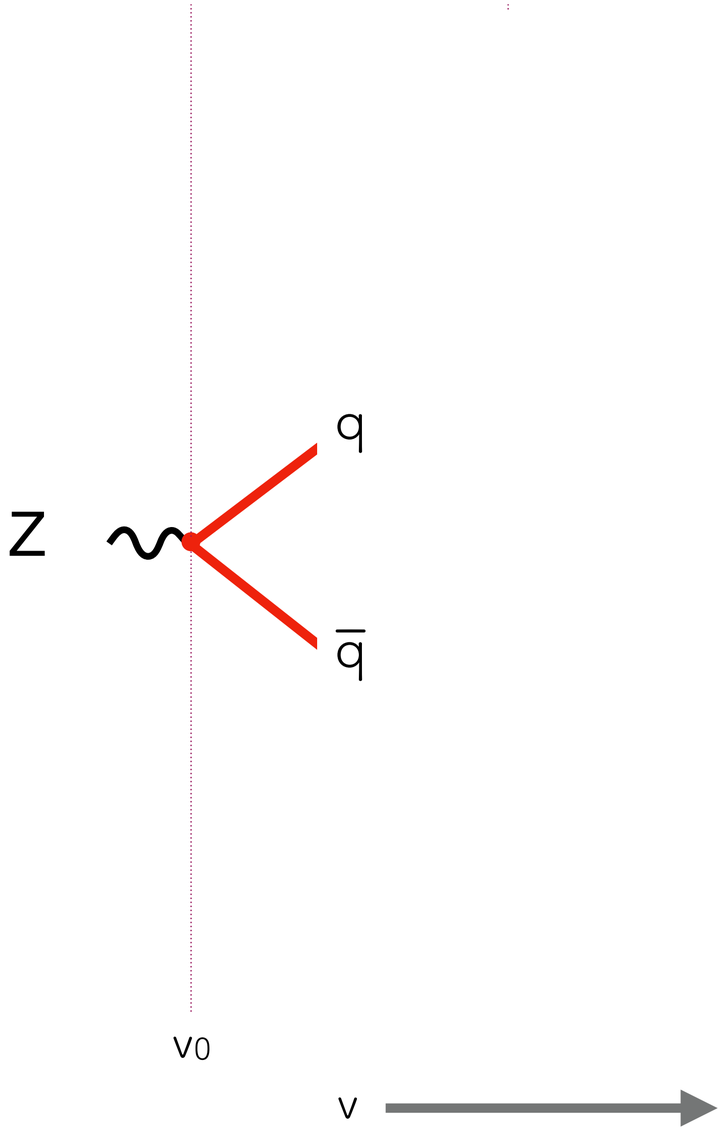


Madgraph5
_aMCatNLO

used in ~95% of ATLAS/CMS publications
they do an amazing job of simulation vast swathes of data;
collider physics would be unrecognisable without them


energy scale
1 TeV
100 GeV
10 GeV
1 GeV
hard process
parton shower
hadronisation
Z'
time
π K π ρ p . . . . . K π π K π π
$\alpha_s \sim 0.1$
$\alpha_s \sim 0.1 - 0.25$
$\alpha_s \gtrsim 0.4$
[non-perturbative models]

QCD parton shower: an evolution equation (in evolution scale v, e.g. trans.mom.)

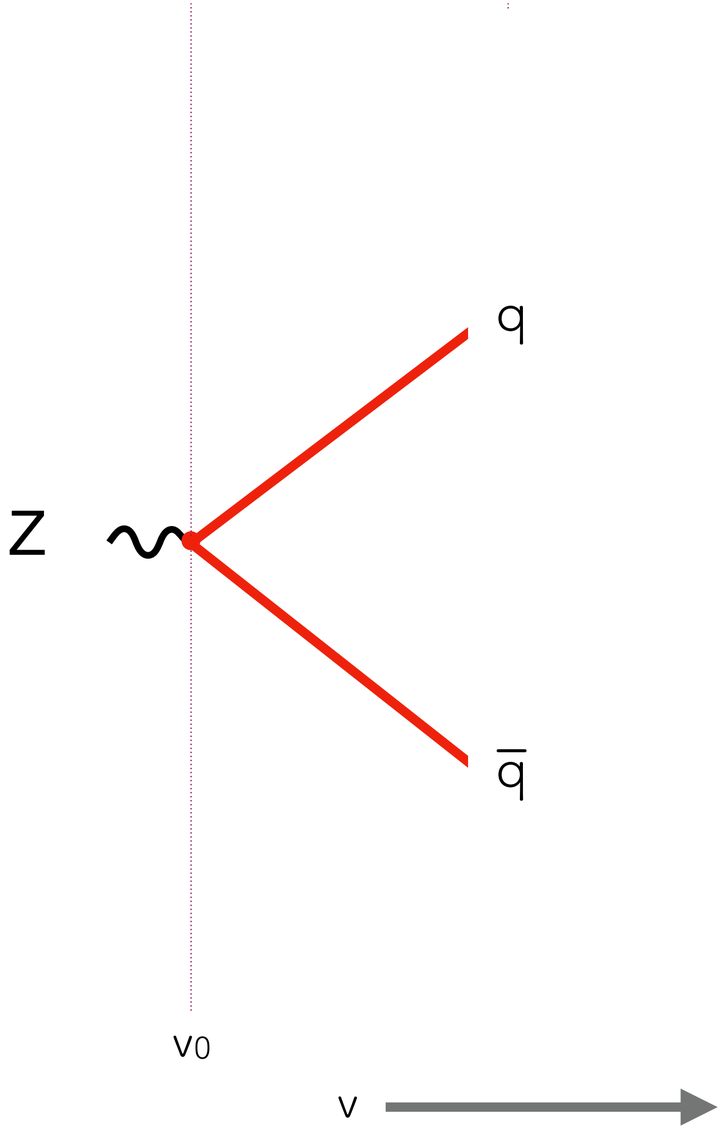


Start with $q\bar{q}$ state.

Throw a random number to determine down to what scale state persists unchanged

$$\frac{dP_2(v)}{dv} = -f^{q\bar{q}}_{2\to 3}(v)\, P_2(v)$$

QCD parton shower: an evolution equation (in evolution scale v, e.g. trans.mom.)

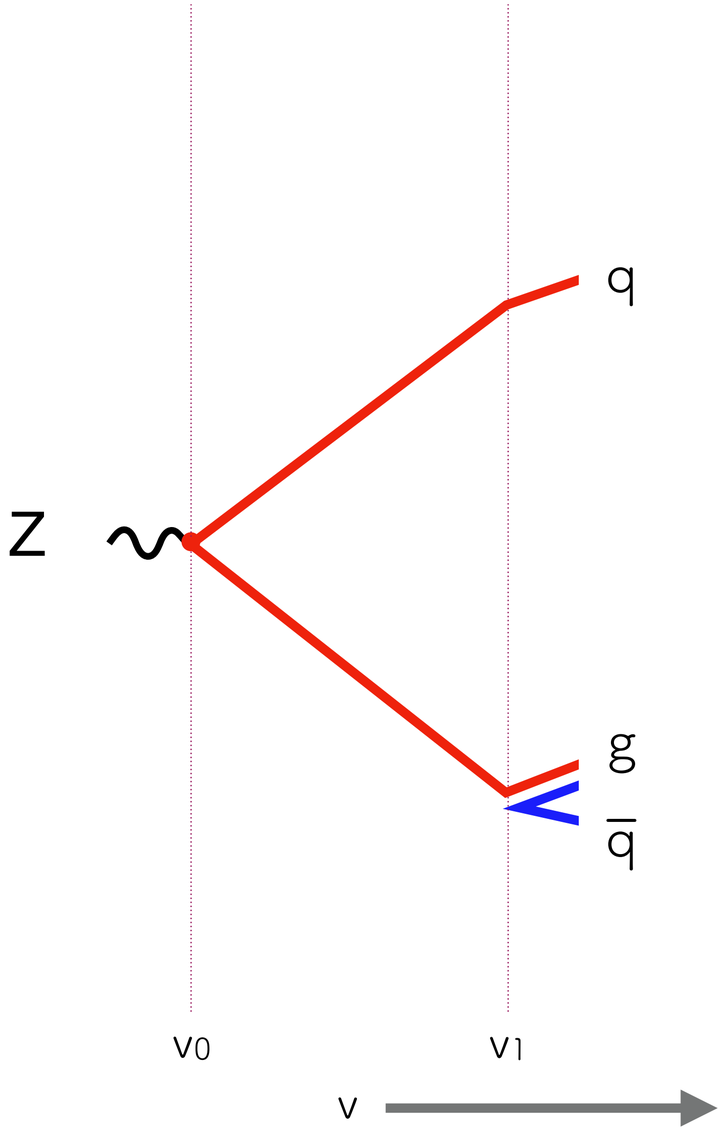


Start with q-qbar state.

Throw a random number to determine down to what scale state persists unchanged

$$\frac{dP_2(v)}{dv} = -f^{q\bar{q}}_{2\to 3}(v)\, P_2(v)$$

QCD **parton shower**: an evolution equation (in **evolution scale v**, e.g. trans.mom.)



Start with q-qbar state.

Throw a random number to determine down to what scale state persists unchanged

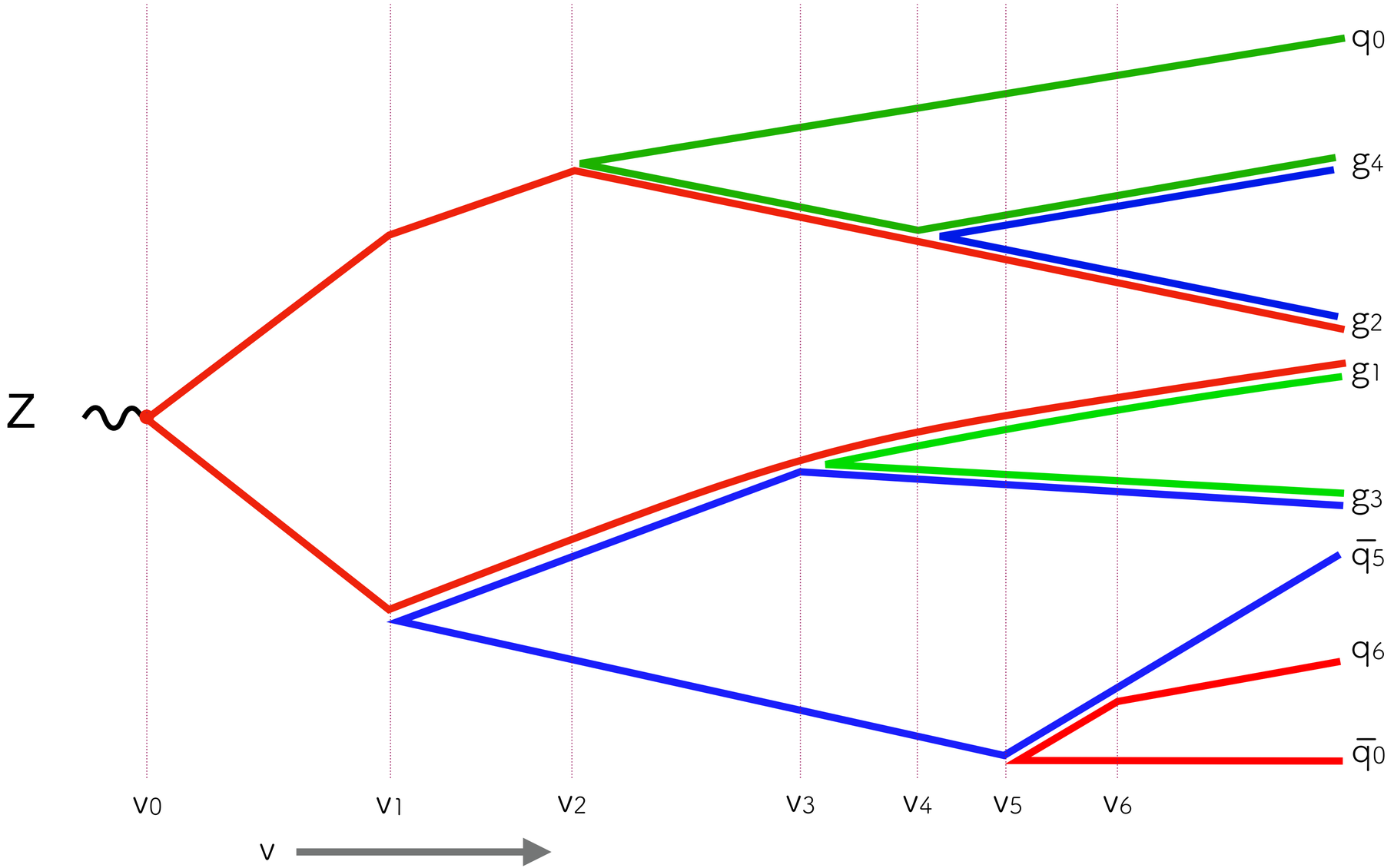
At some point, **state splits** (2→3, i.e. emits gluon). Evolution equation changes

$$\frac{dP_3(v)}{dv} = -\left[f^{qg}_{2\to3}(v) + f^{g\bar{q}}_{2\to3}(v)\right] P_3(v)$$

gluon is part of two dipoles (qg), $(g\bar{q})$, each treated as independent

(many showers use a large N_C limit)

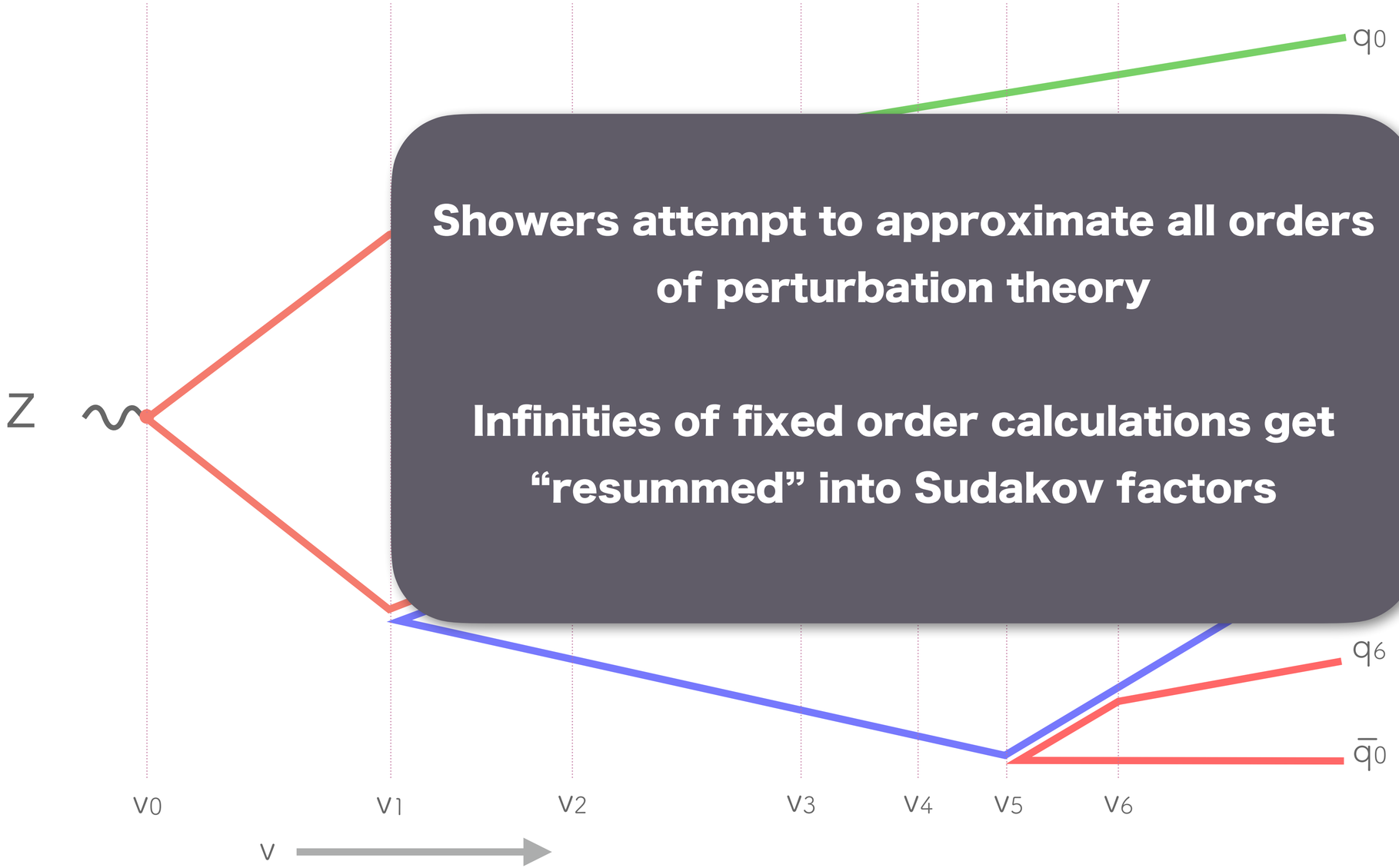
QCD parton shower: an evolution equation (in evolution scale v, e.g. trans.mom.)



self-similar evolution continues until it reaches a non-perturbative scale

(made tractable in part by large-N_C limit, with non-interfering colour dipoles)

QCD parton shower: an evolution equation (in evolution scale v, e.g. trans.mom.)



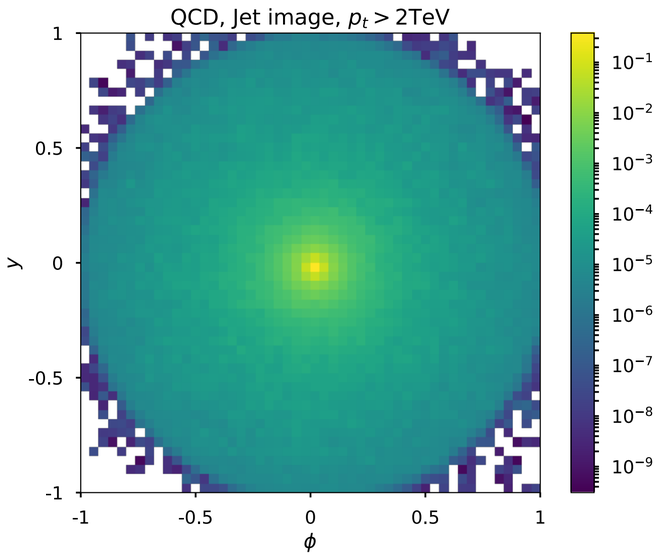
Showers attempt to approximate all orders of perturbation theory

Infinities of fixed order calculations get "resummed" into Sudakov factors

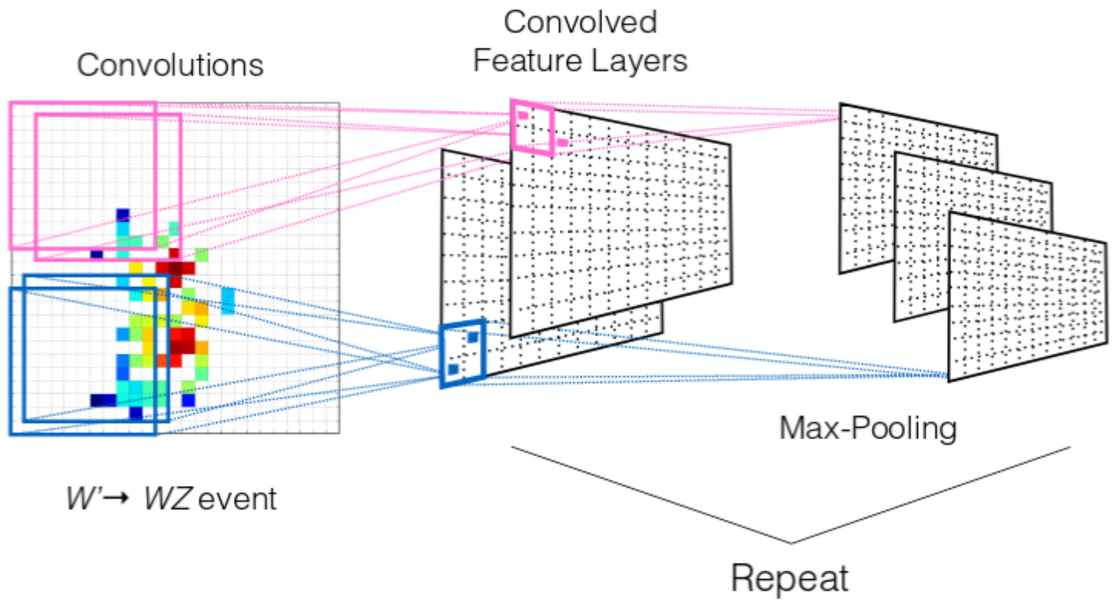
self-similar evolution continues until it reaches a non-perturbative scale

(made tractable in part by large-N_C limit, with non-interfering colour dipoles)

Machine learning and jet/event structure

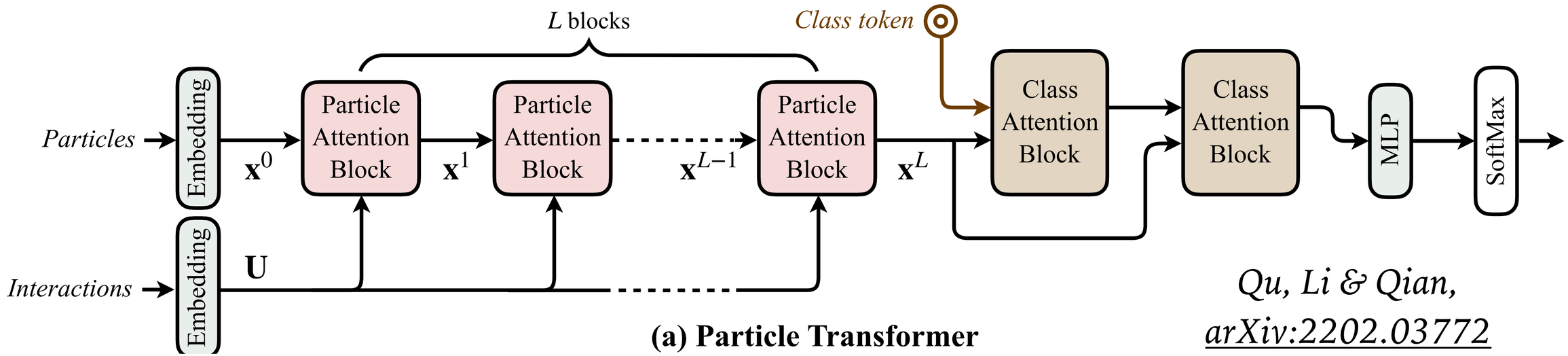


2021 Young Experimental Physicist
EPS HEPP prize



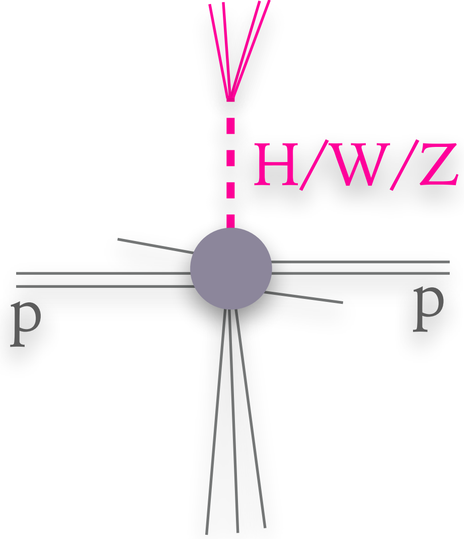
[Cogan, Kagan, Strauss, Schwartzman JHEP 1502 (2015) 118]

[de Oliveira, Kagan, Mackey, Nachman, Schwartzman JHEP 1607 (2016) 069]

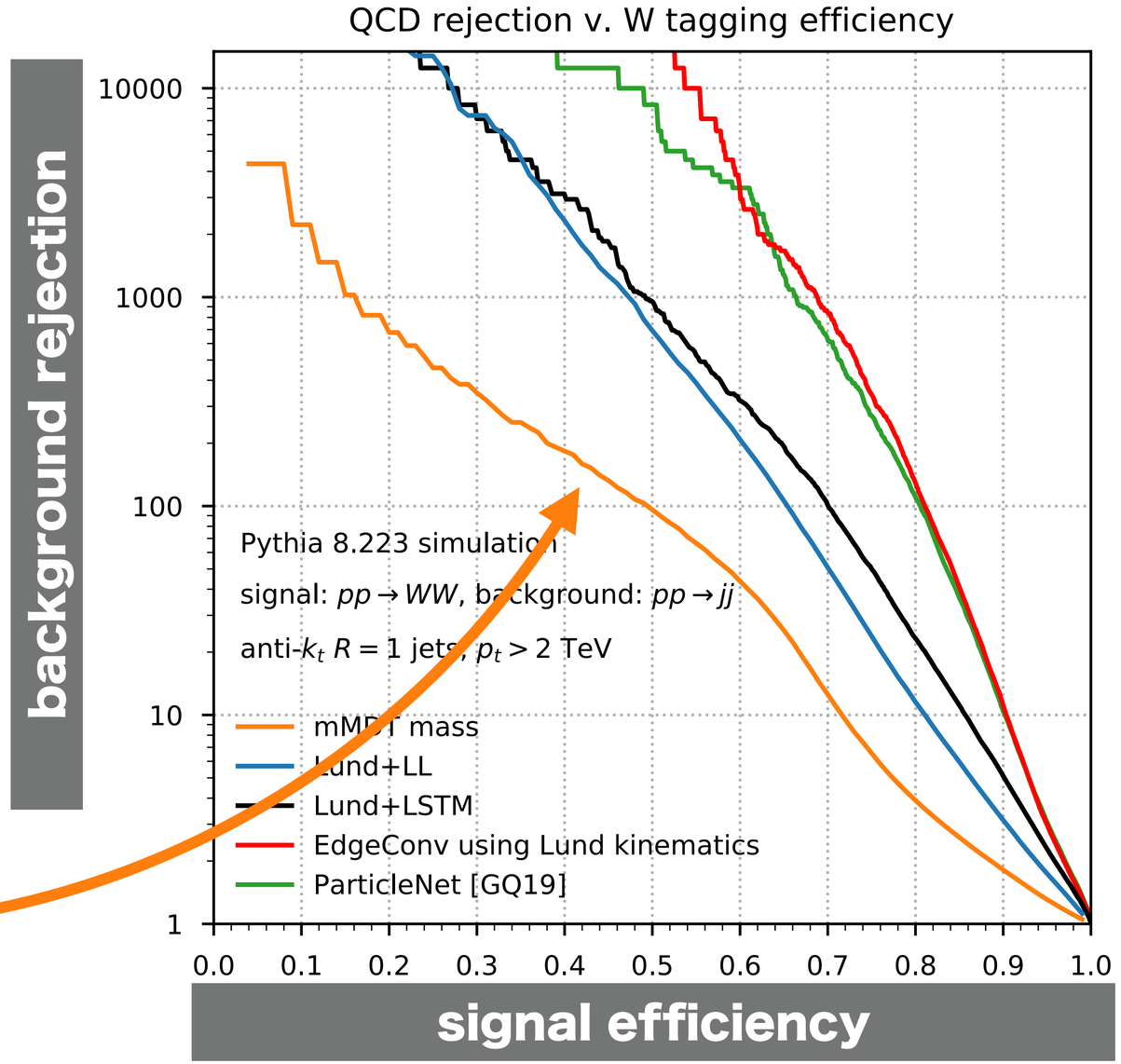


Qu, Li & Qian,
arXiv:2202.03772

using full jet/event information for H/W/Z-boson tagging

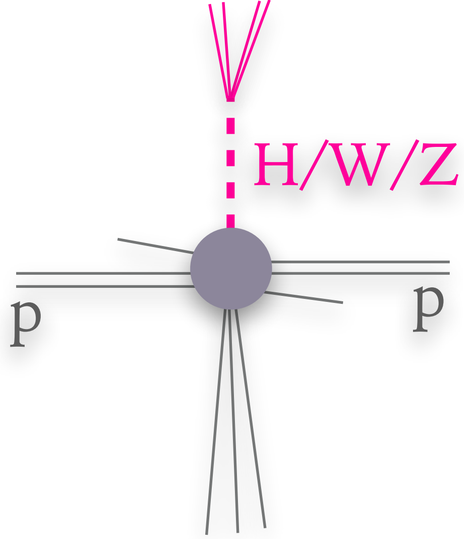


adapted from Dreyer & Qu 2012.08526

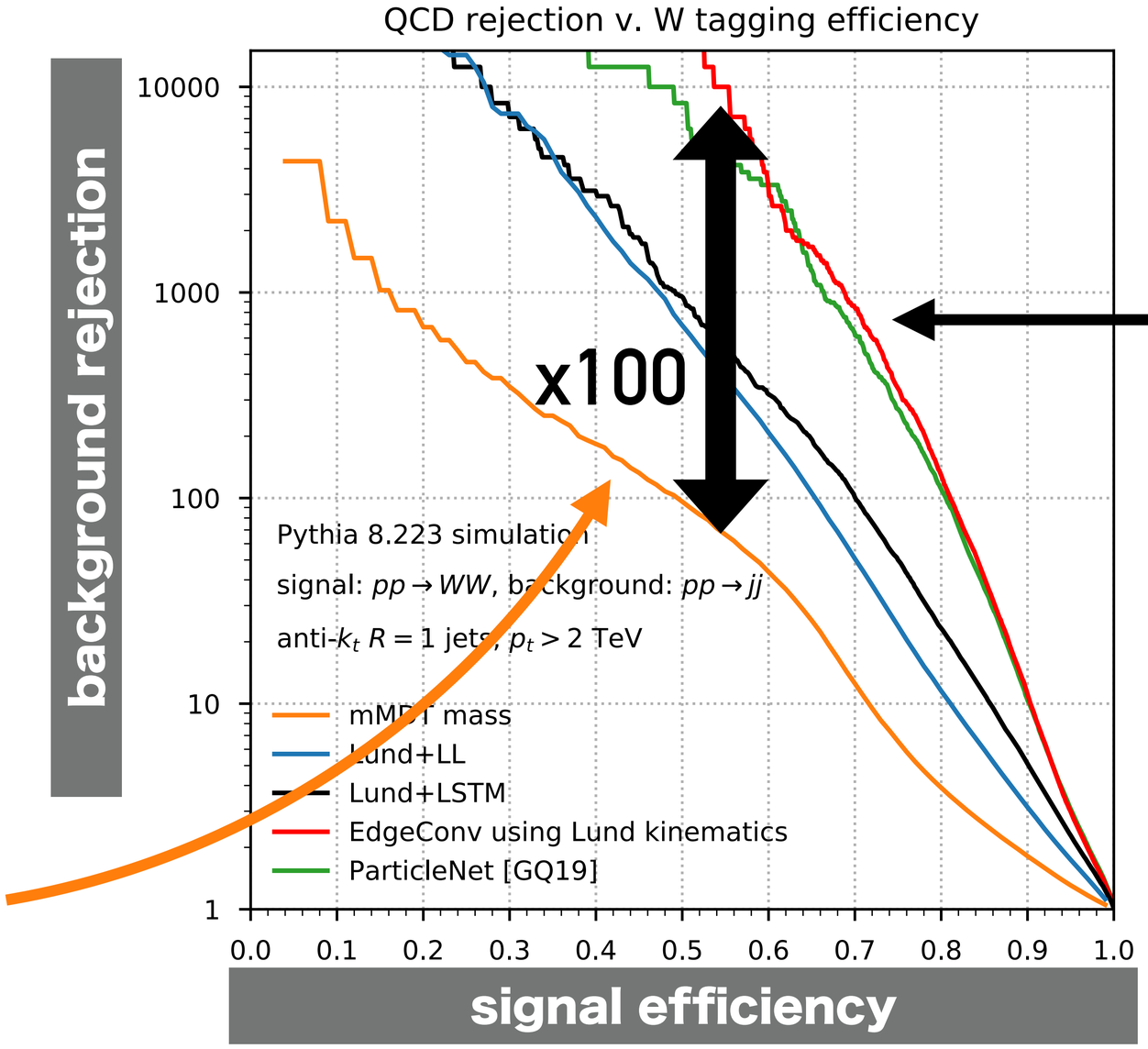


QCD rejection with just jet mass (SD/mMDT) i.e. 2008 tools & their 2013/14 descendants

using full jet/event information for H/W/Z-boson tagging



adapted from Dreyer & Qu 2012.08526



QCD rejection with just jet mass (SD/mMDT) i.e. 2008 tools & their 2013/14 descendants

QCD rejection with use of full jet substructure (2021 tools)

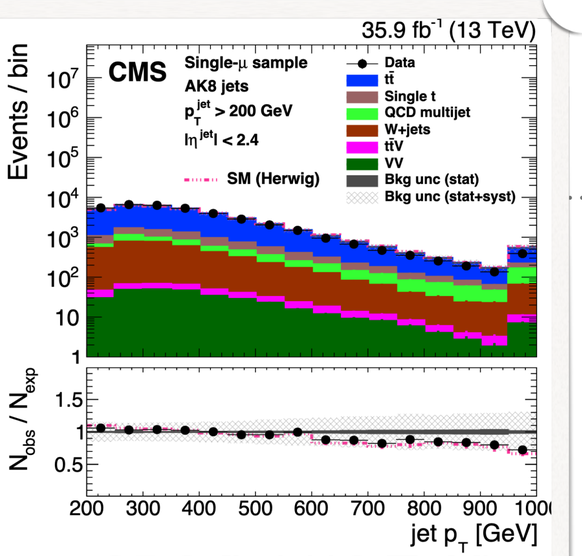
100x better

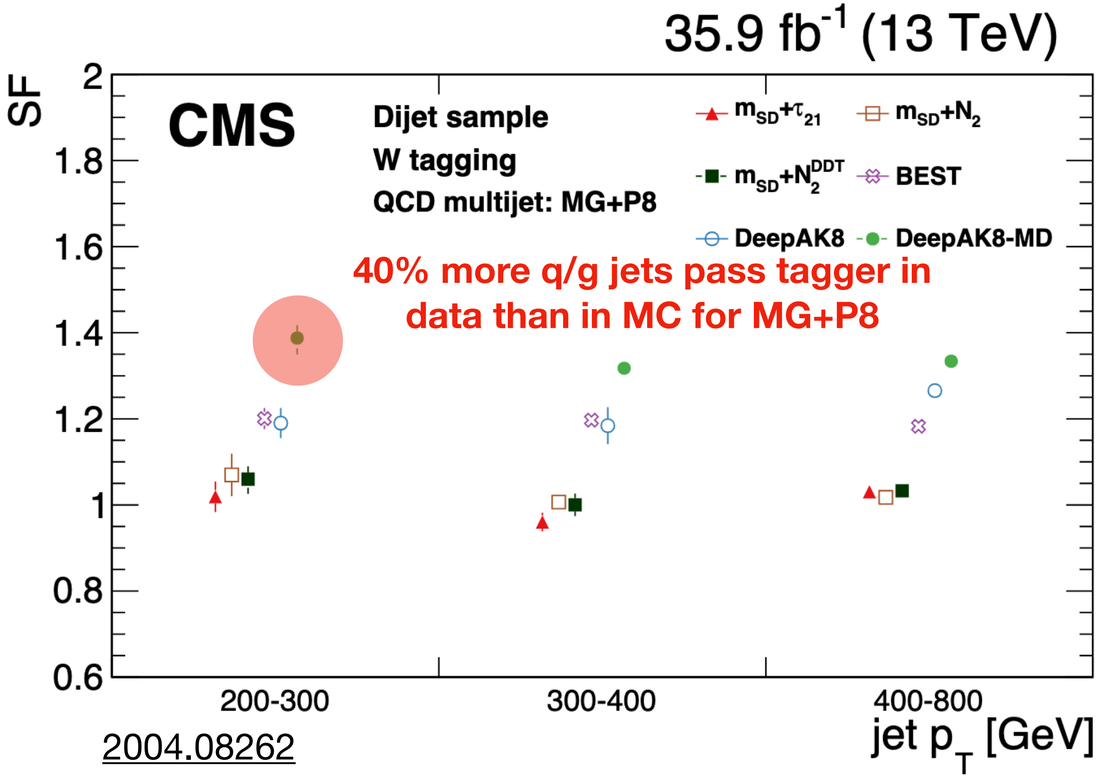
First started to be exploited by Thaler & Van Tilburg with "N-subjettiness" (2010/11)

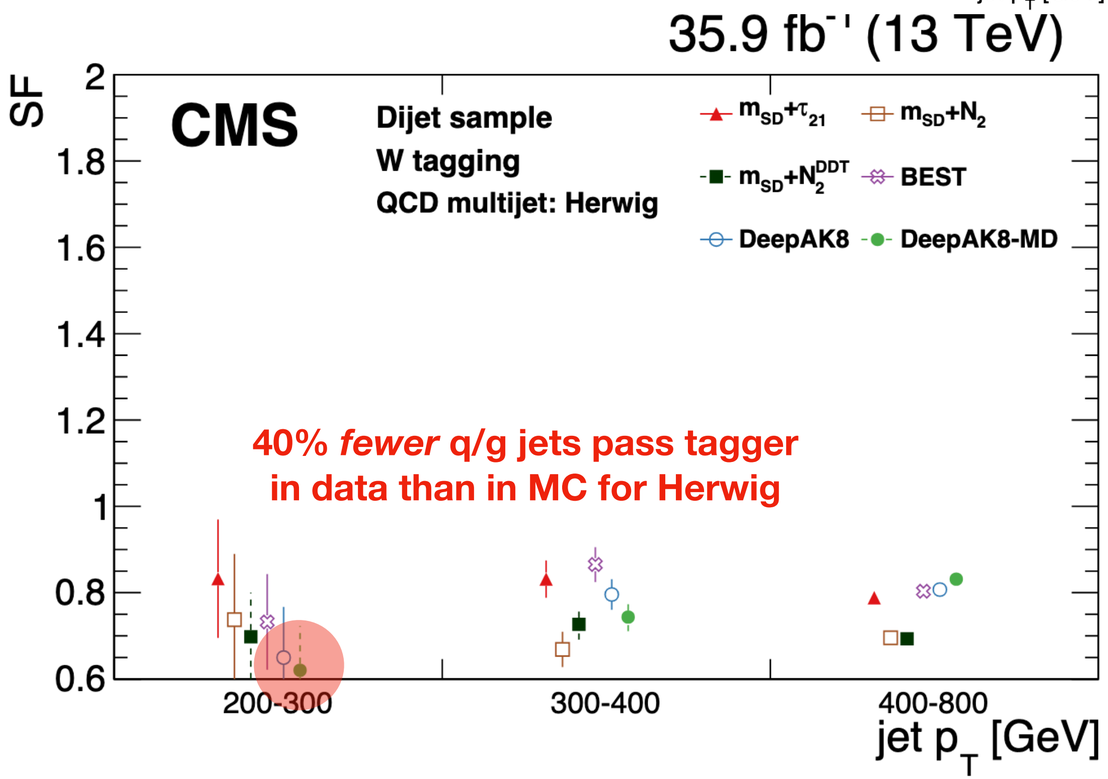
Jennifer Roloff, PSR25

tagging: a diversion

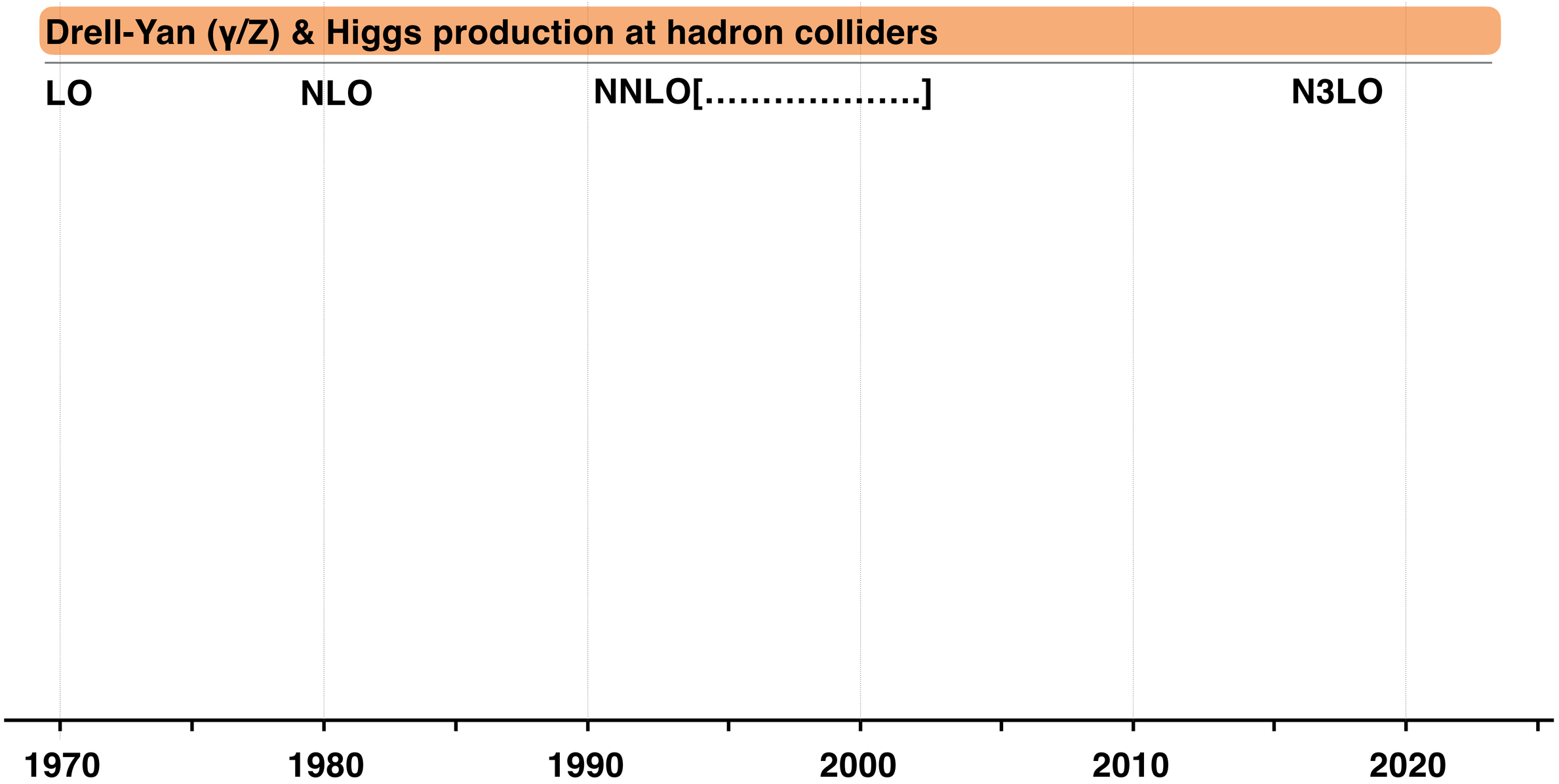
- Taggers typically trained on MC — also need to determine the efficiency in data (***scale factors***)
- Scale factors (SF) can depend on the specific MC generator that is used
  - Can be large scale factors *and* large differences across MC predictions
- ML is learning features that don't exist on data — better MC would *reduce modeling uncertainties* and potentially *improve tagger performance*



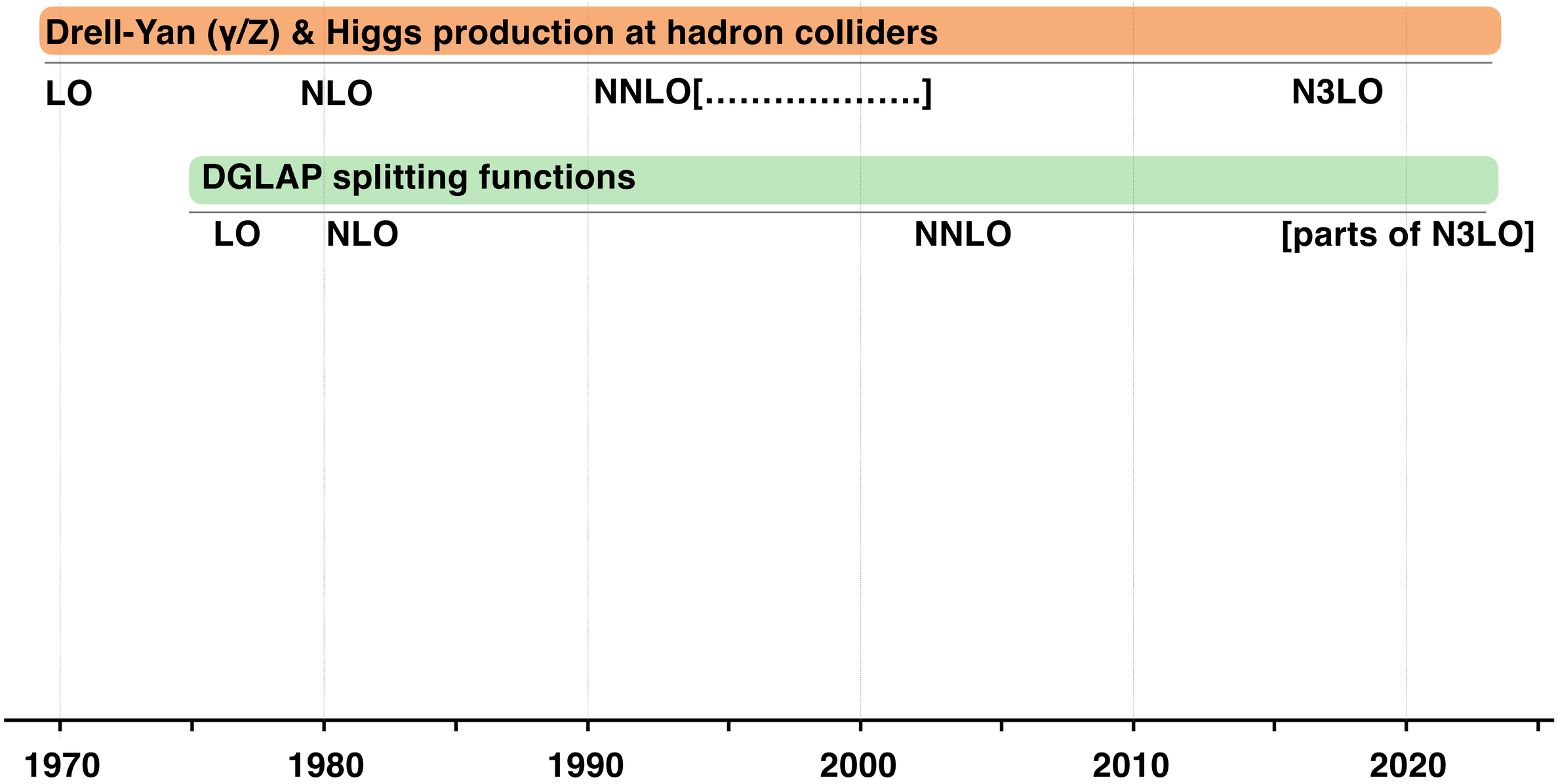




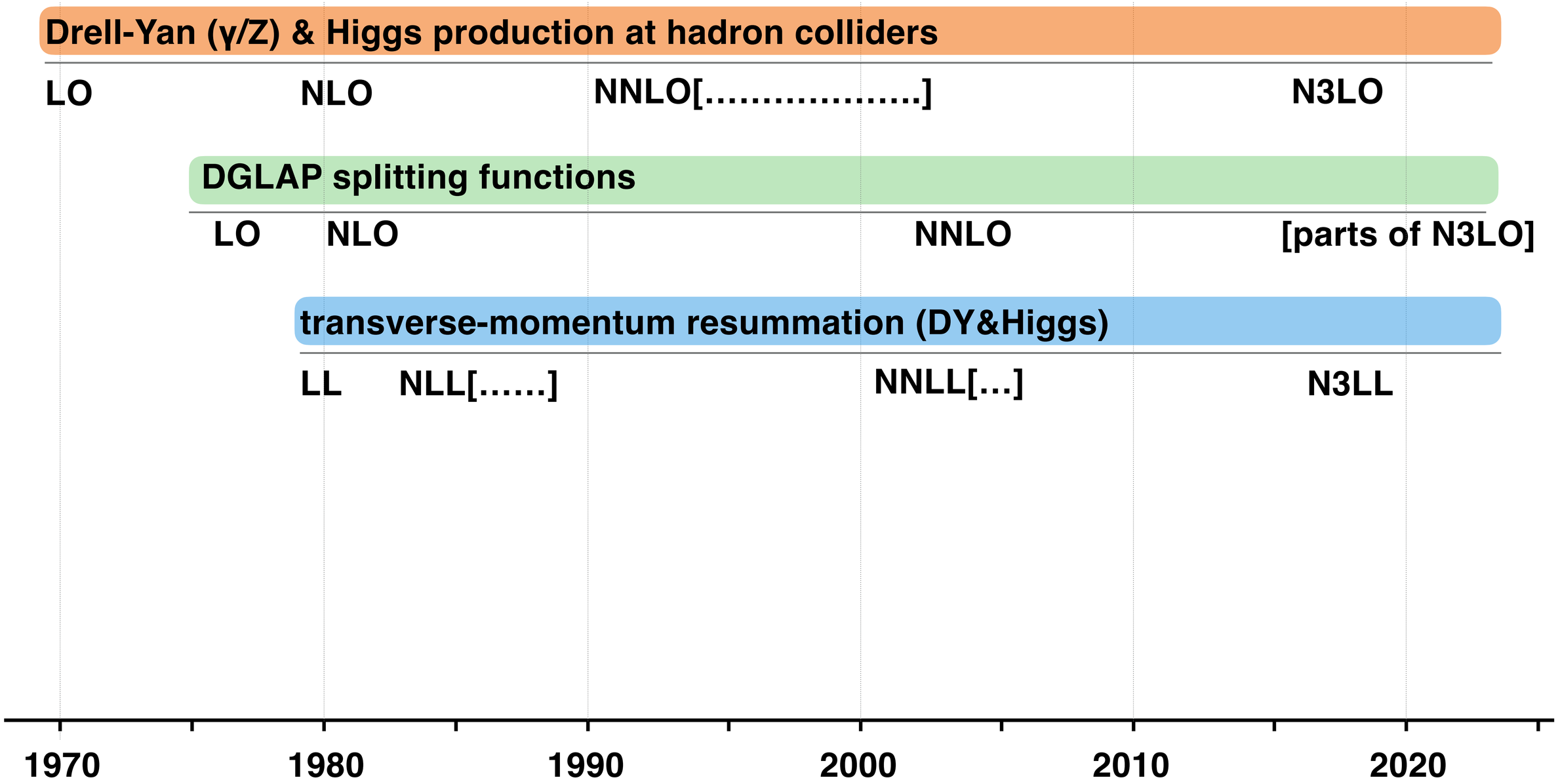
selected collider-QCD accuracy milestones



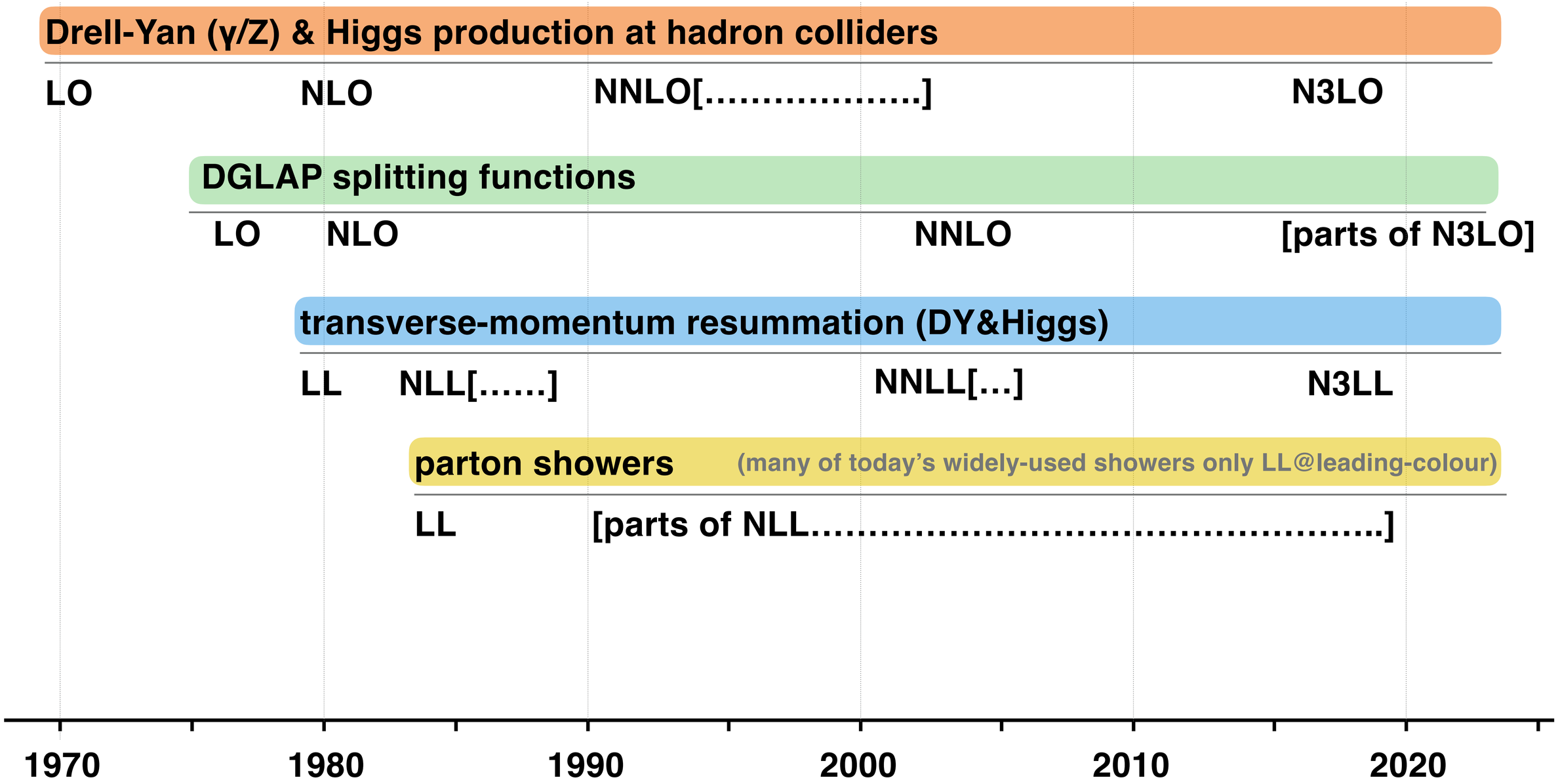
selected collider-QCD accuracy milestones



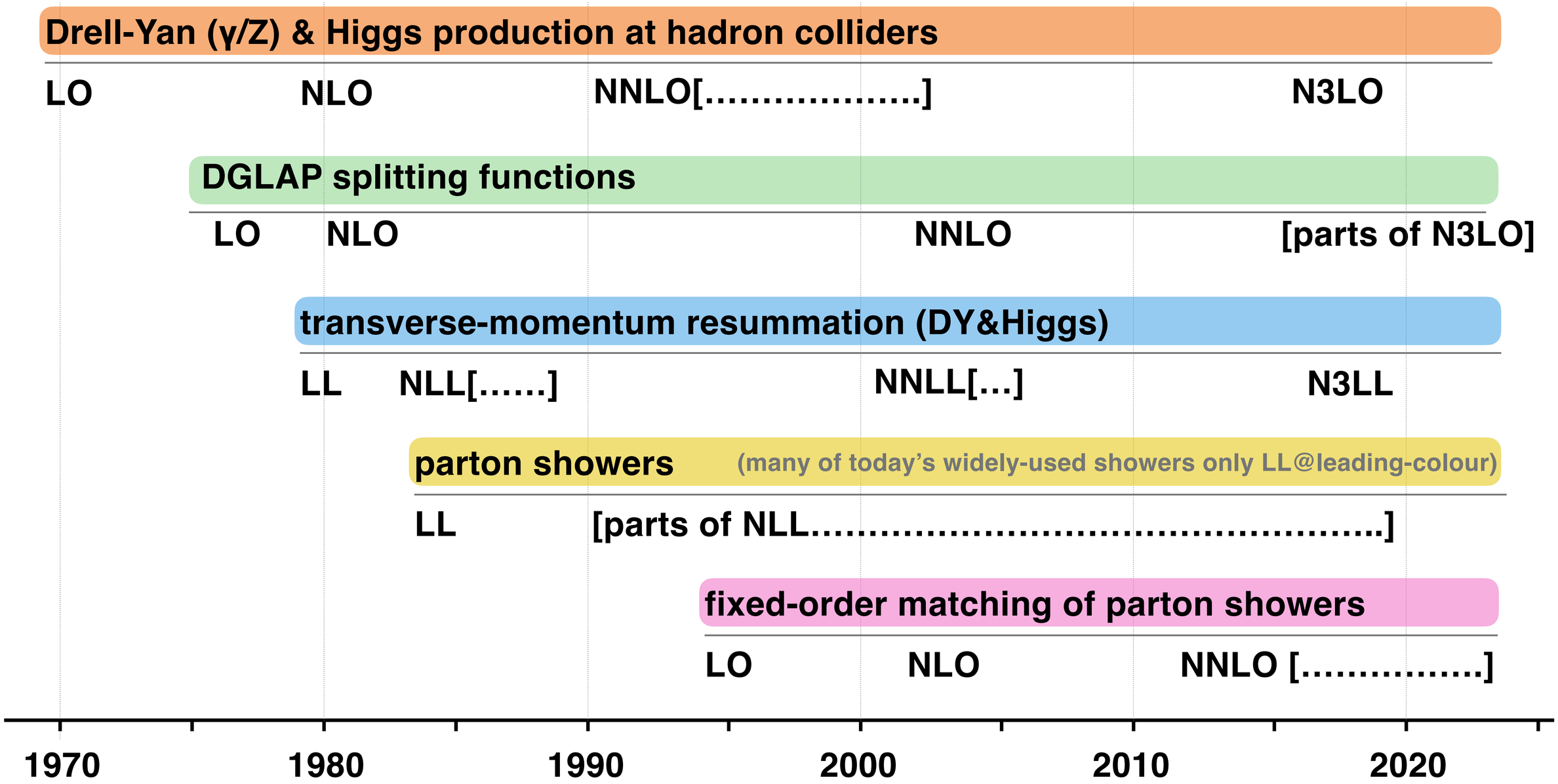
selected collider-QCD accuracy milestones



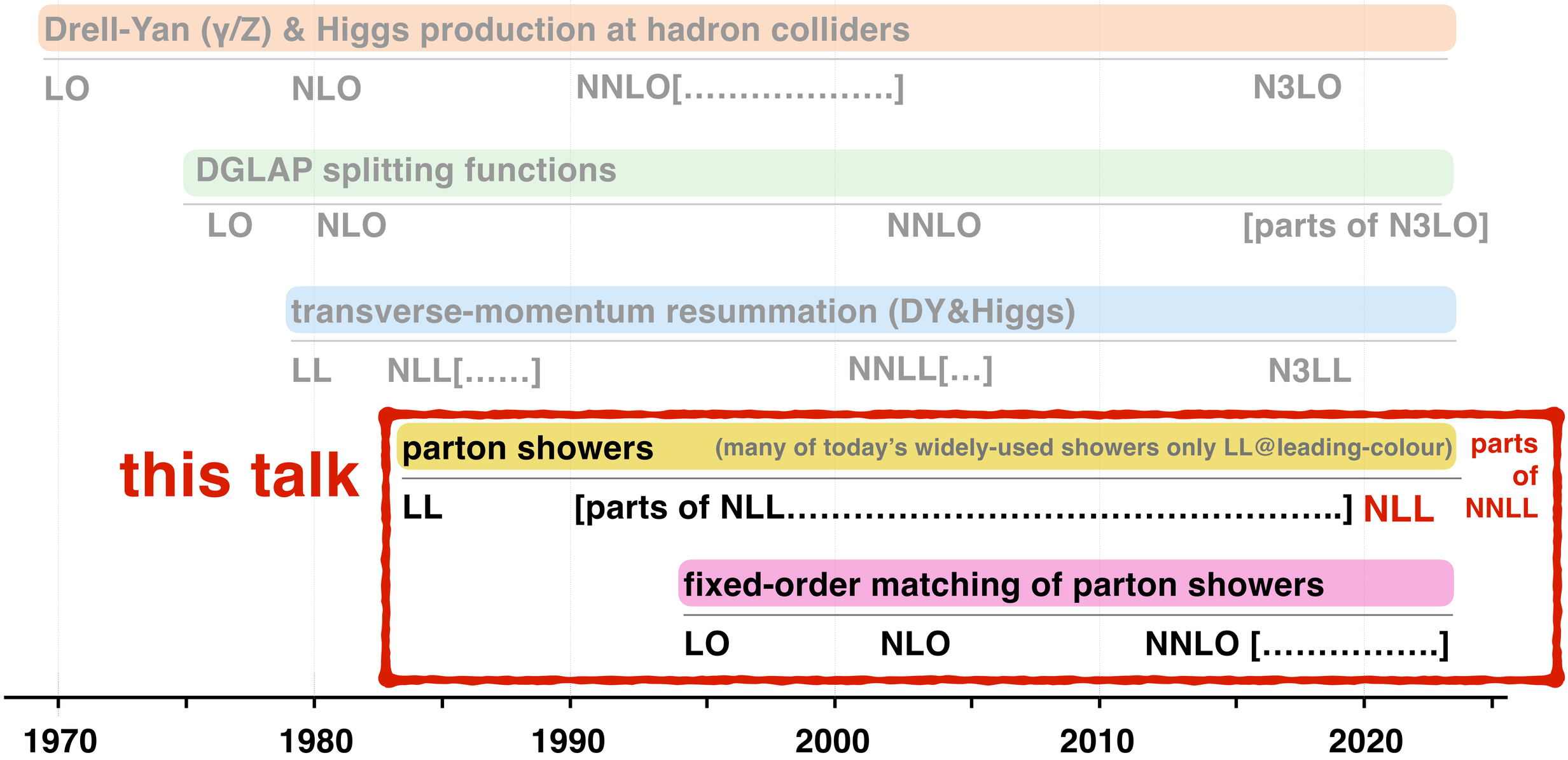
selected collider-QCD accuracy milestones



selected collider-QCD accuracy milestones



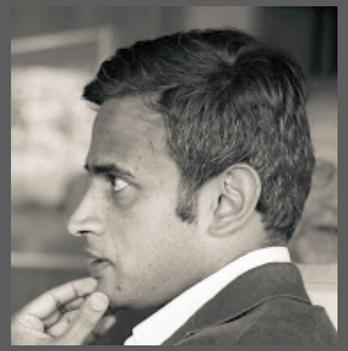
selected collider-QCD accuracy milestones



can we make perturbative QCD simultaneously physical and systematically improvable?



Melissa van Beekveld
NIKEHF



Mrinal Dasgupta
Manchester



Basem El-Menoufi
Monash



Silvia Ferrario Ravasio
CERN



Keith Hamilton
Univ. Coll. London



Jack Helliwell
Monash



Alexander Karlberg
CERN



Pier Monni
CERN



GPS
Oxford



Nicolas Schalch
Oxford



Ludovic Scyboz
Monash



Alba Soto-Ontoso
Granada



Grégory Soyez
IPhT, Saclay



Silvia Zanoli
Oxford

PanScales

A project to bring logarithmic understanding and accuracy to parton showers



ERC funded
2018–2024

former members



Frédéric Dreyer



Rob Verheyen



Rok Medves



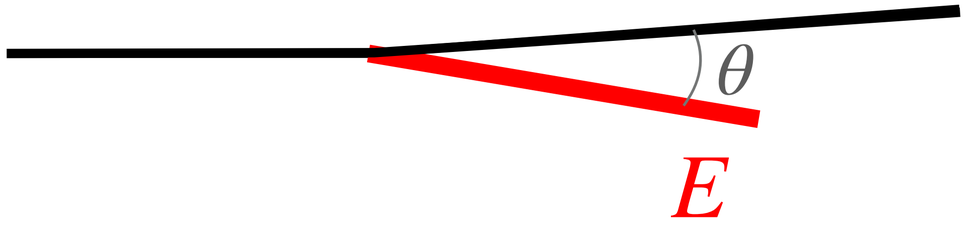
Emma Slade

the Lund plane

organisation of phase space that highlights QCD divergences and logarithms

Original concept: B. Andersson, G. Gustafson L. Lonnblad and Pettersson, 1989
Event-by-event definition: Dreyer, GPS & Soyez, 1807.04758

Phase space: two key variables (+ azimuth)



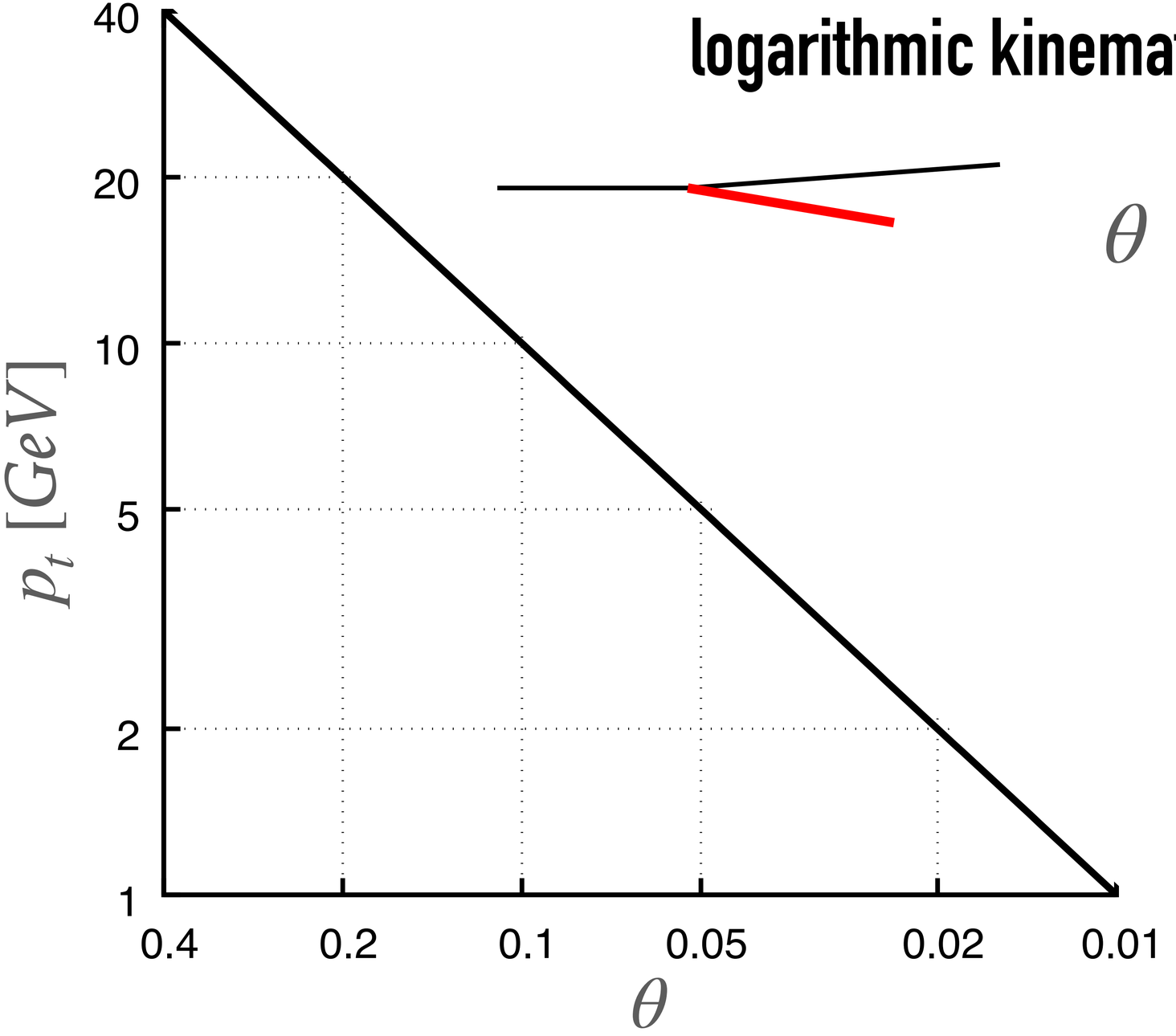
θ $(or\ \eta = -\ln\tan\frac{\theta}{2})$ — *η is called (pseudo)rapidity*

$p_t = E\theta$ — *p_t $(or\ p_\perp)$ is a transverse momentum*

$$d\Phi\,|M^2| = \frac{2\alpha_s(p_t)C}{\pi}\,\frac{d\theta}{\theta}\,\frac{dp_t}{p_t}\,\frac{d\phi}{2\pi}$$

emission probability in low-energy, small-angle limit

jet with $R = 0.4$, $p_t = 200\ GeV$



logarithmic kinematic plane whose two variables are

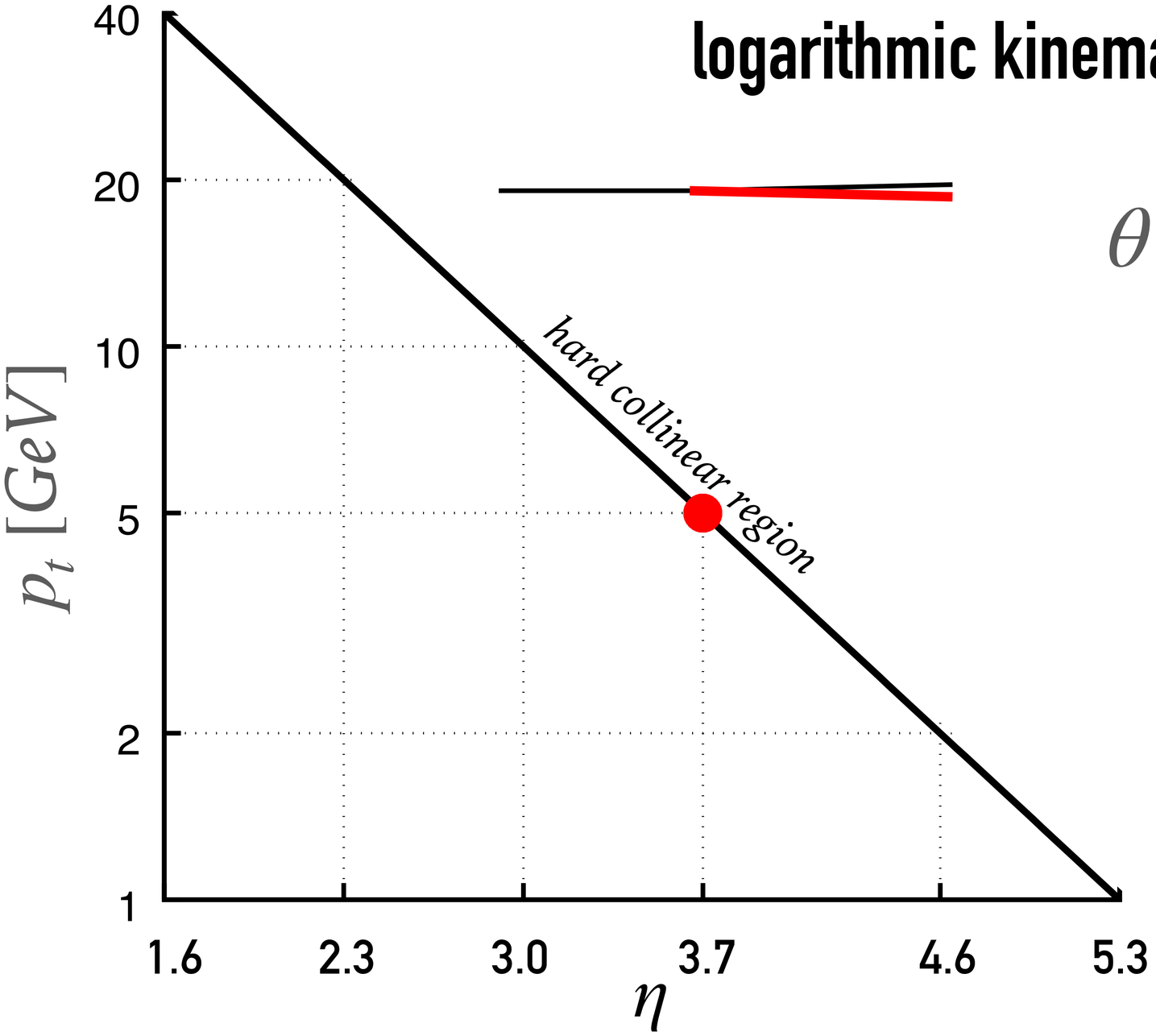
$$\theta \ (or\ \eta = -\ln\tan\frac{\theta}{2})$$

$$p_t = E\theta$$

Introduced for understanding Parton Shower Monte Carlos by
B. Andersson, G. Gustafson L. Lonnblad and Pettersson 1989

The Lund Plane

jet with $R = 0.4$, $p_t = 200\ GeV$



logarithmic kinematic plane whose two variables are

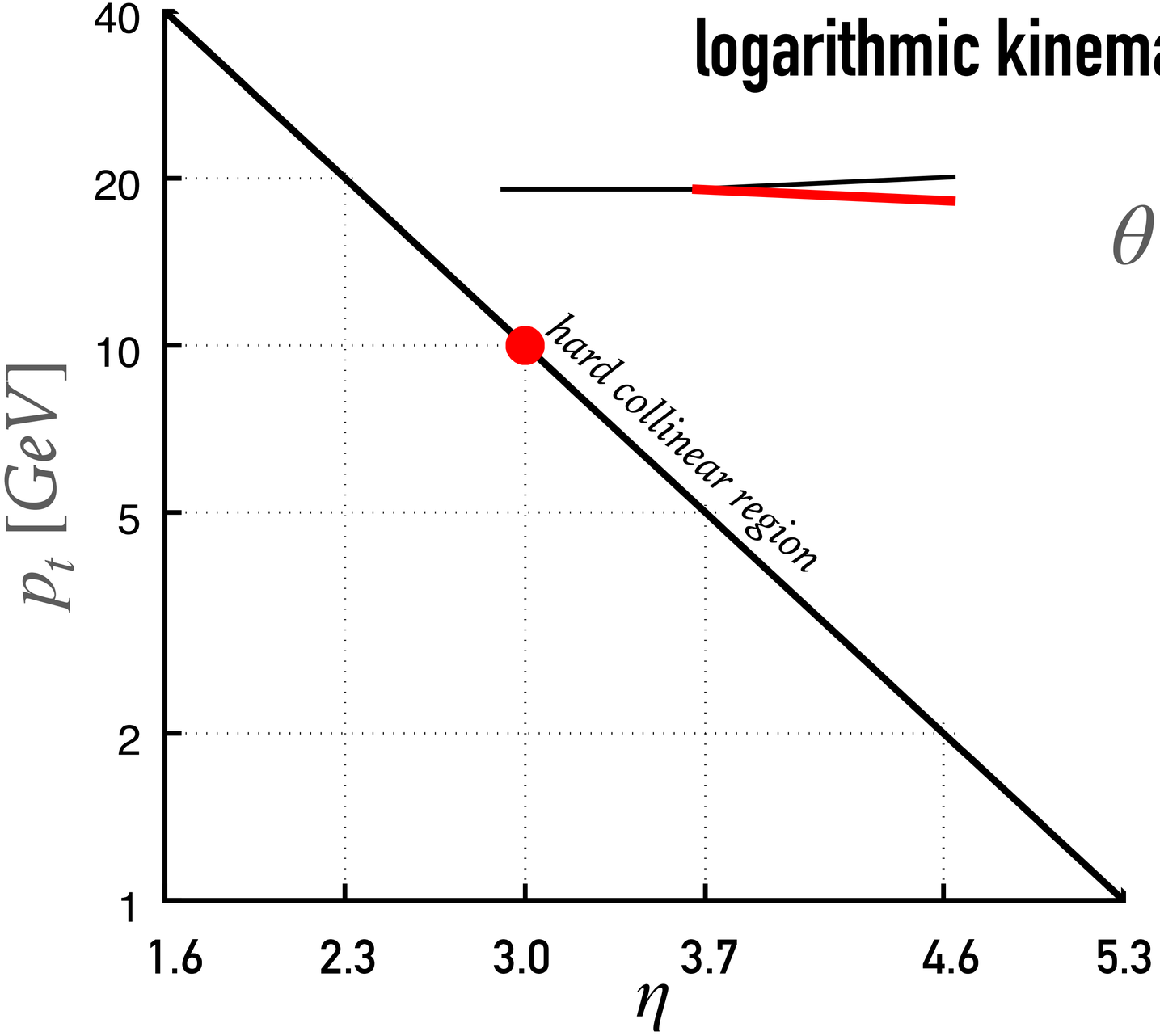
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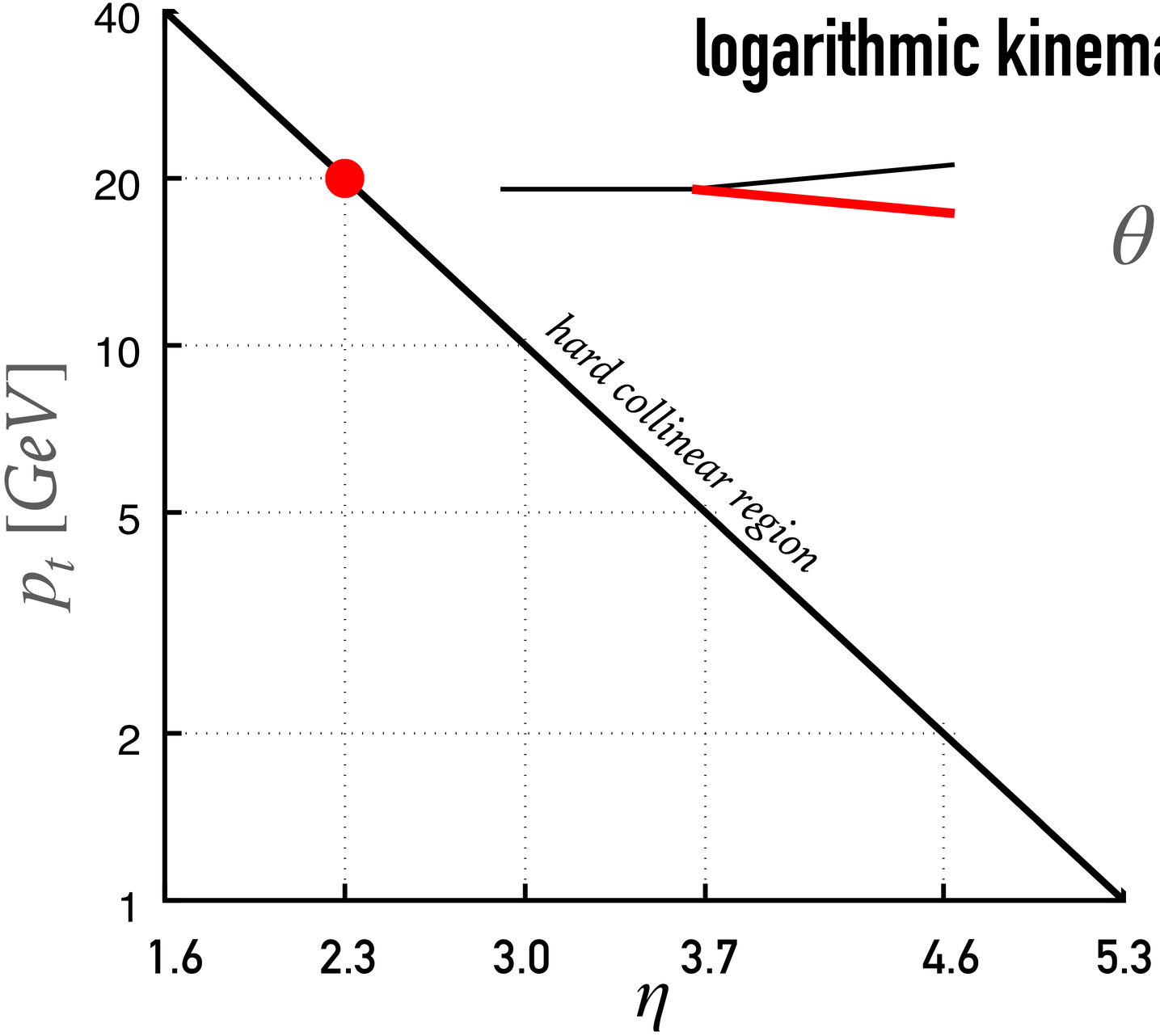
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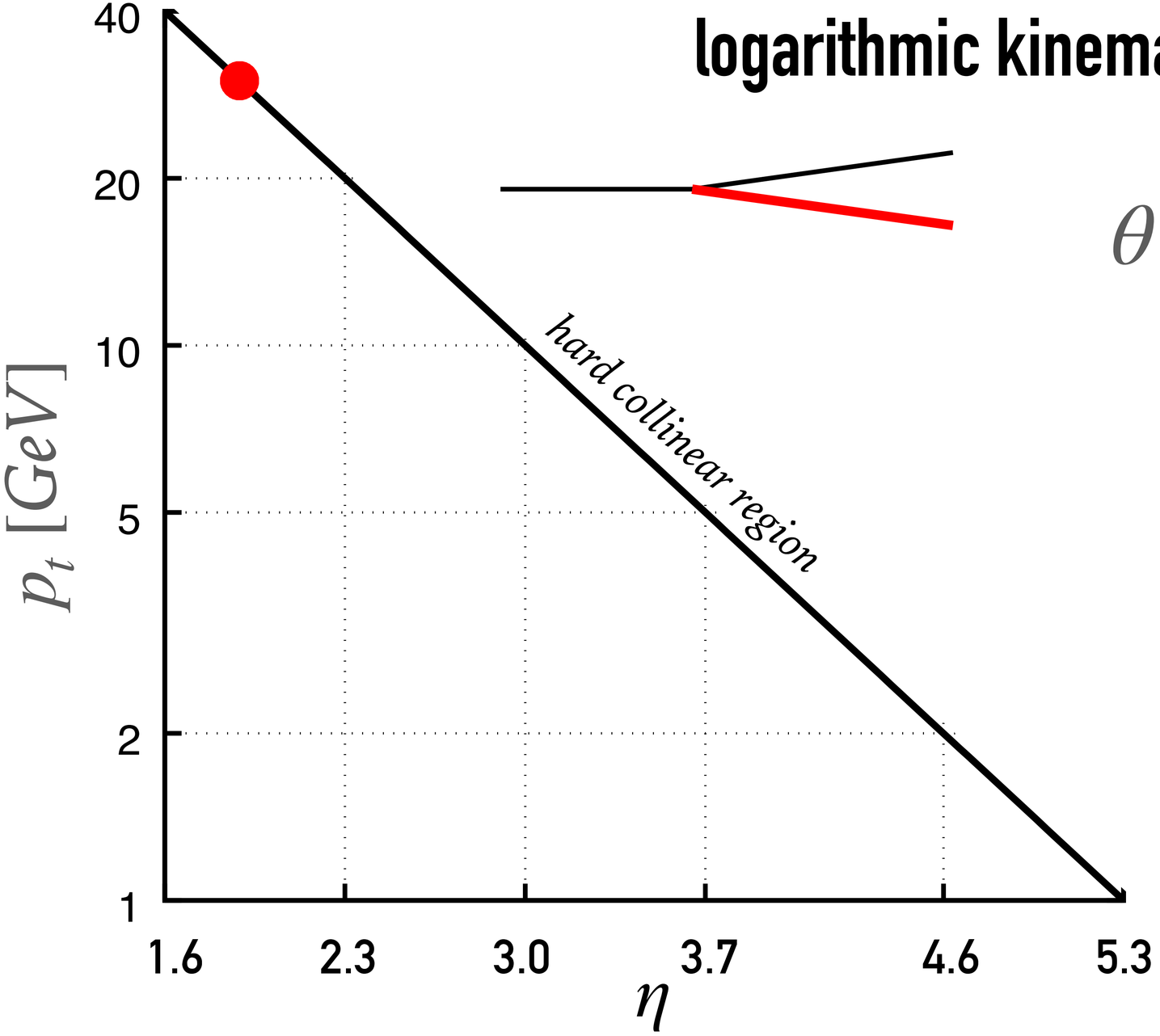
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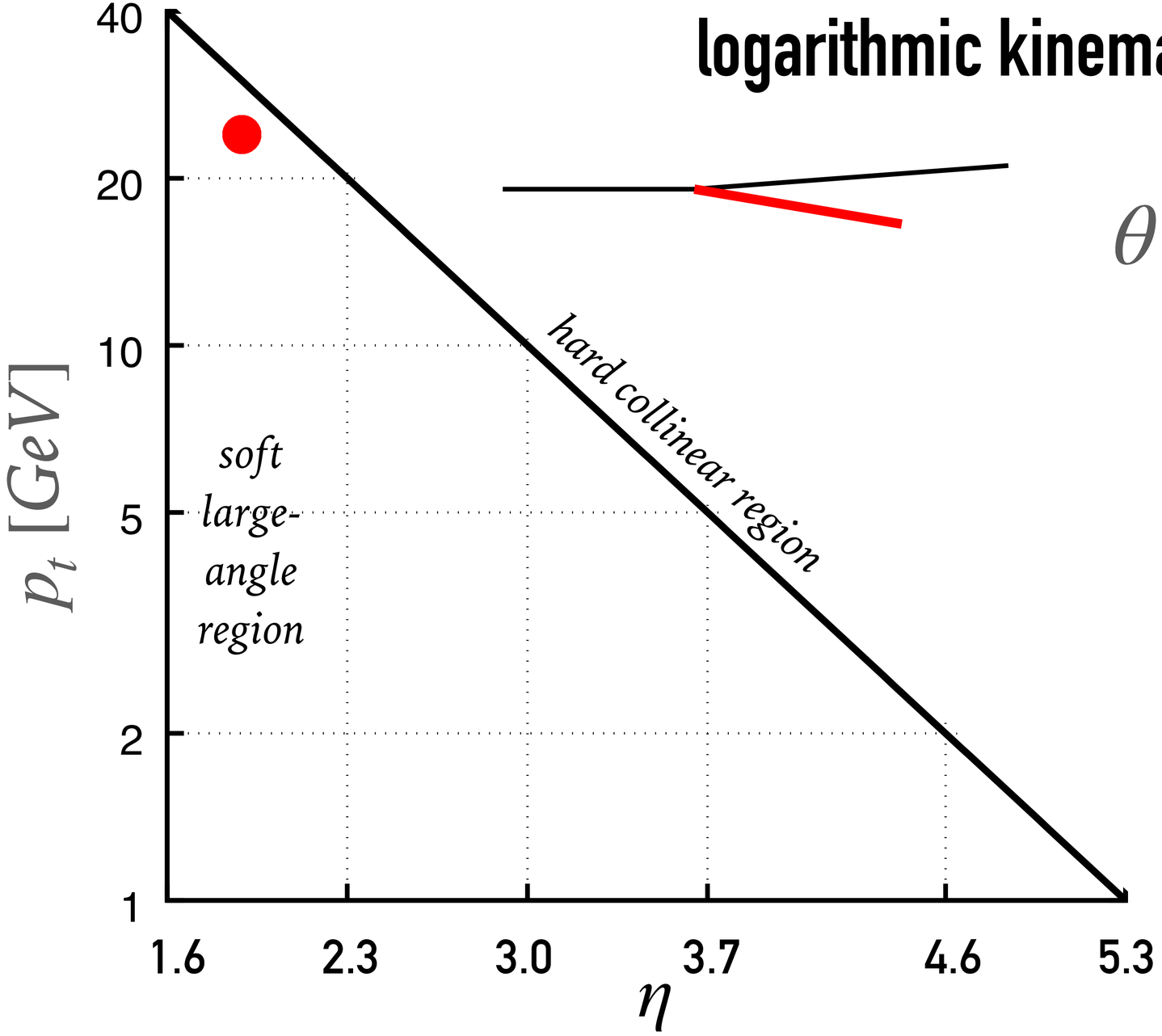
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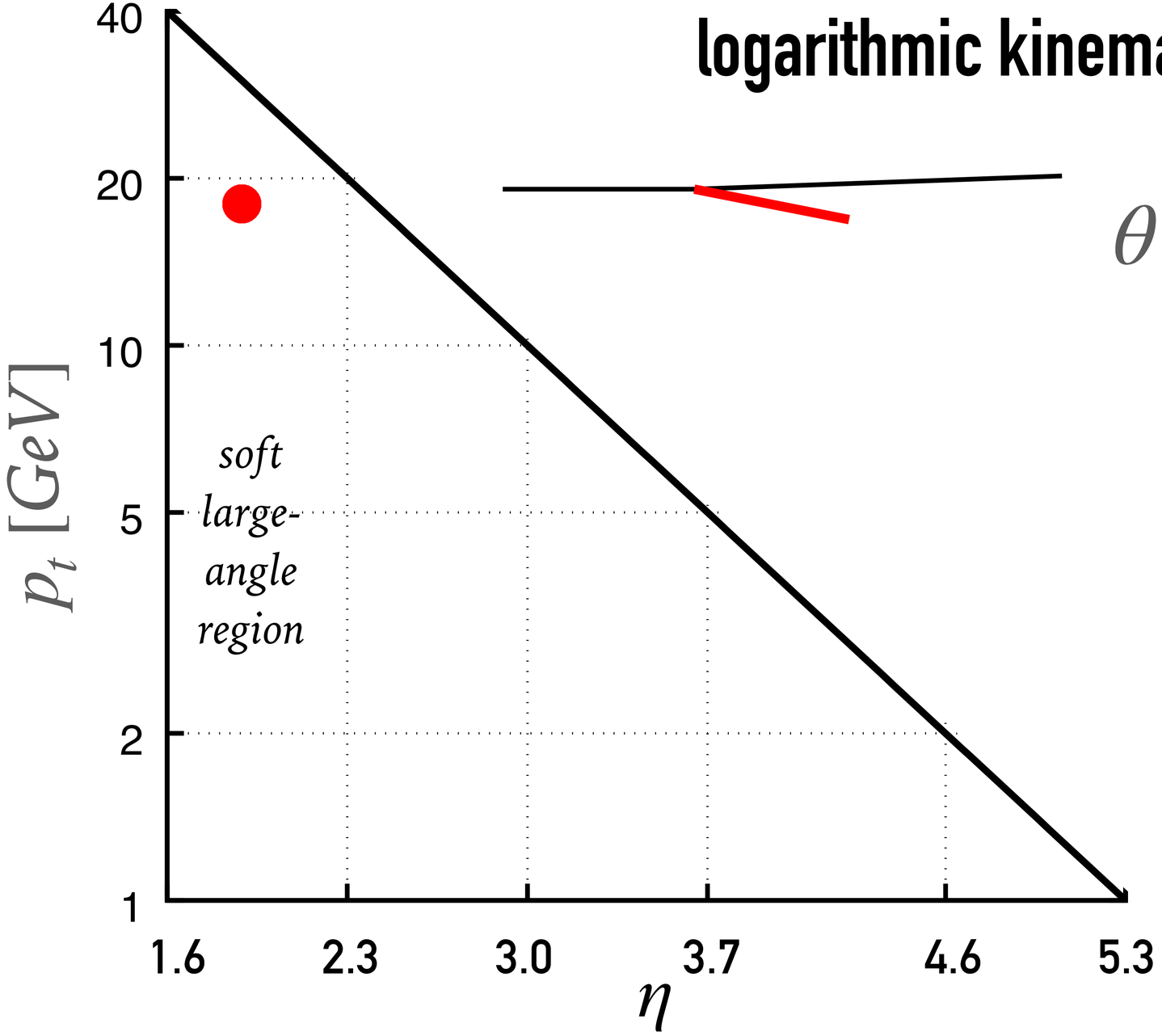
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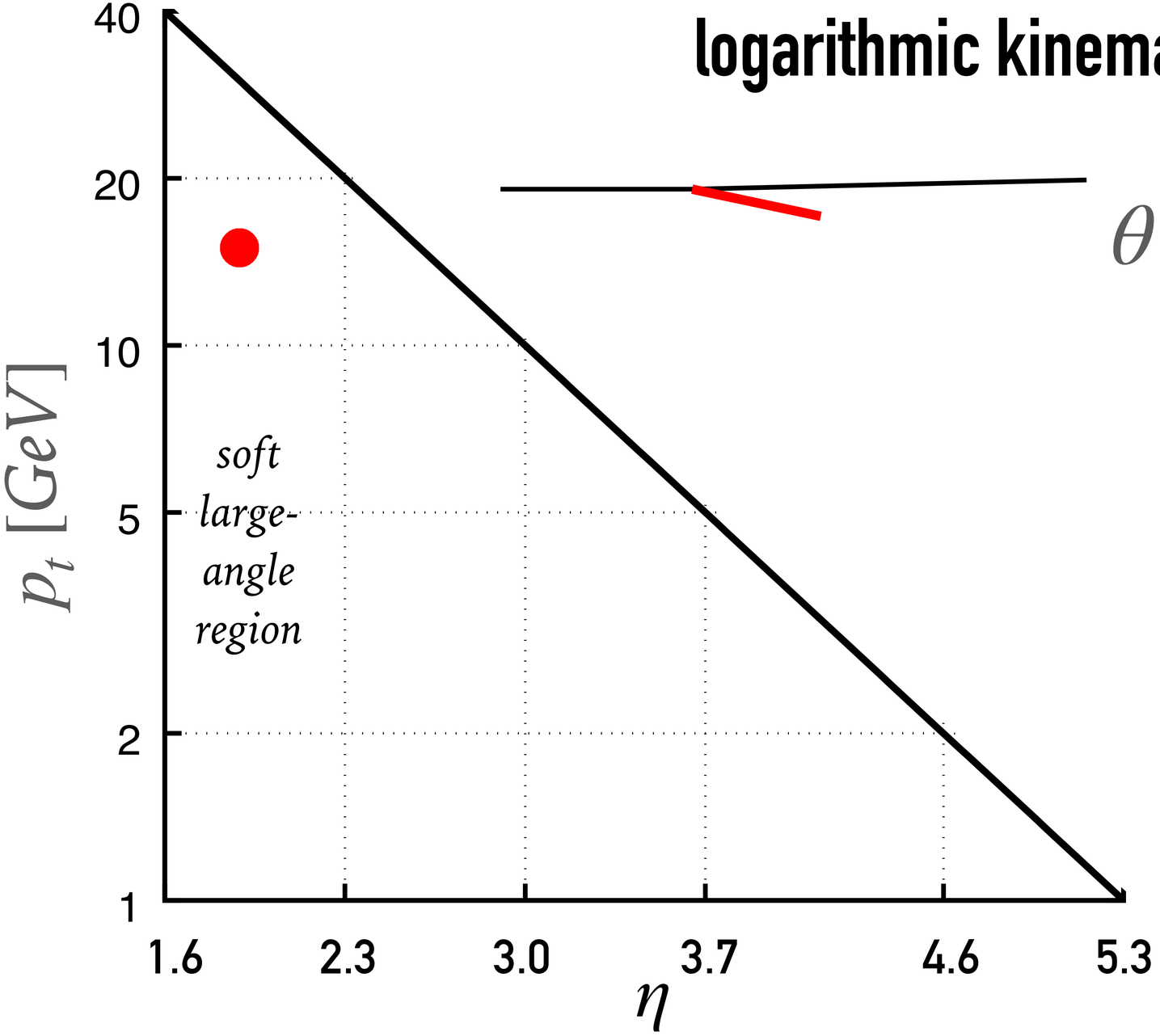
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logarithmic kinematic plane whose two variables are

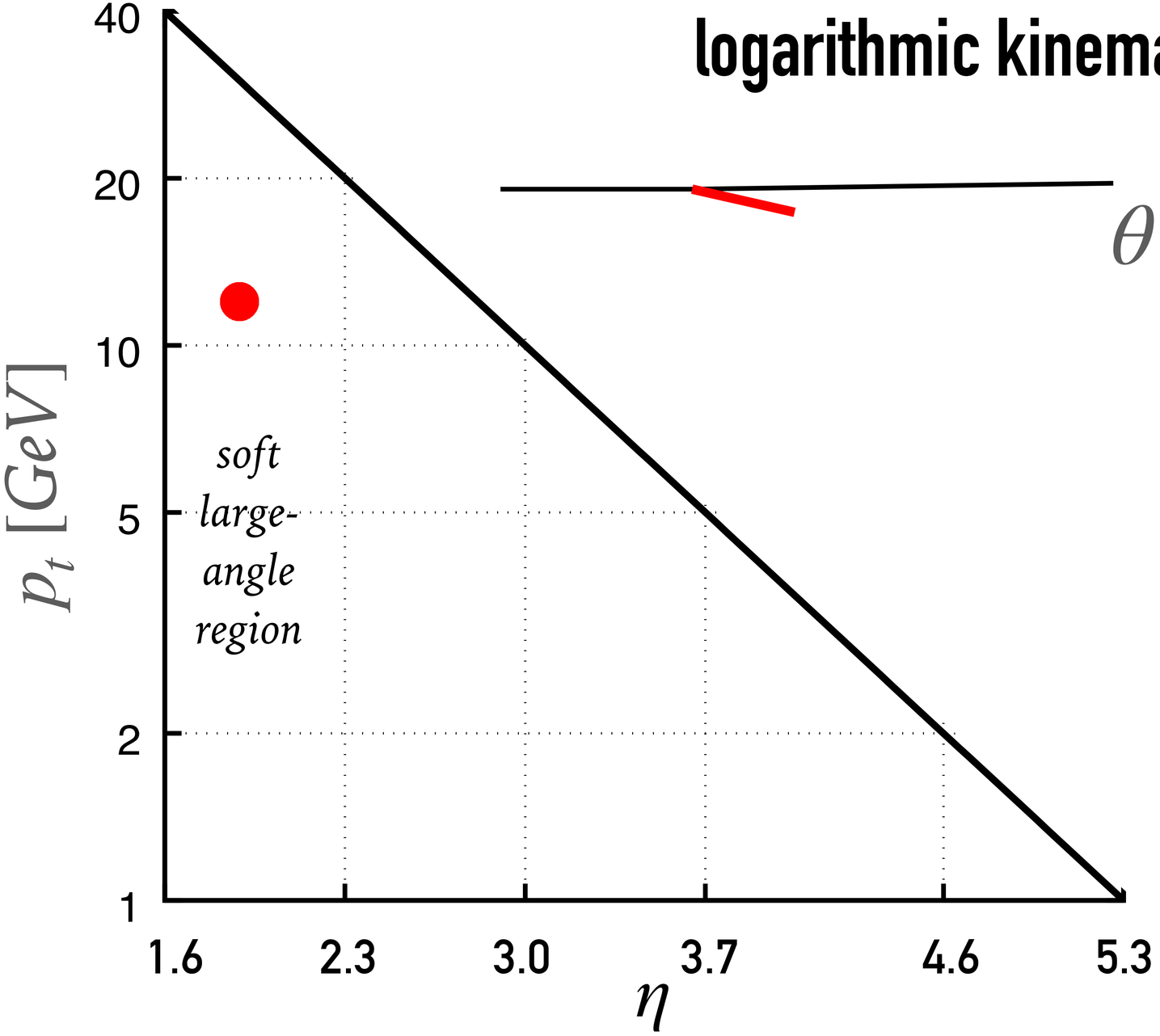
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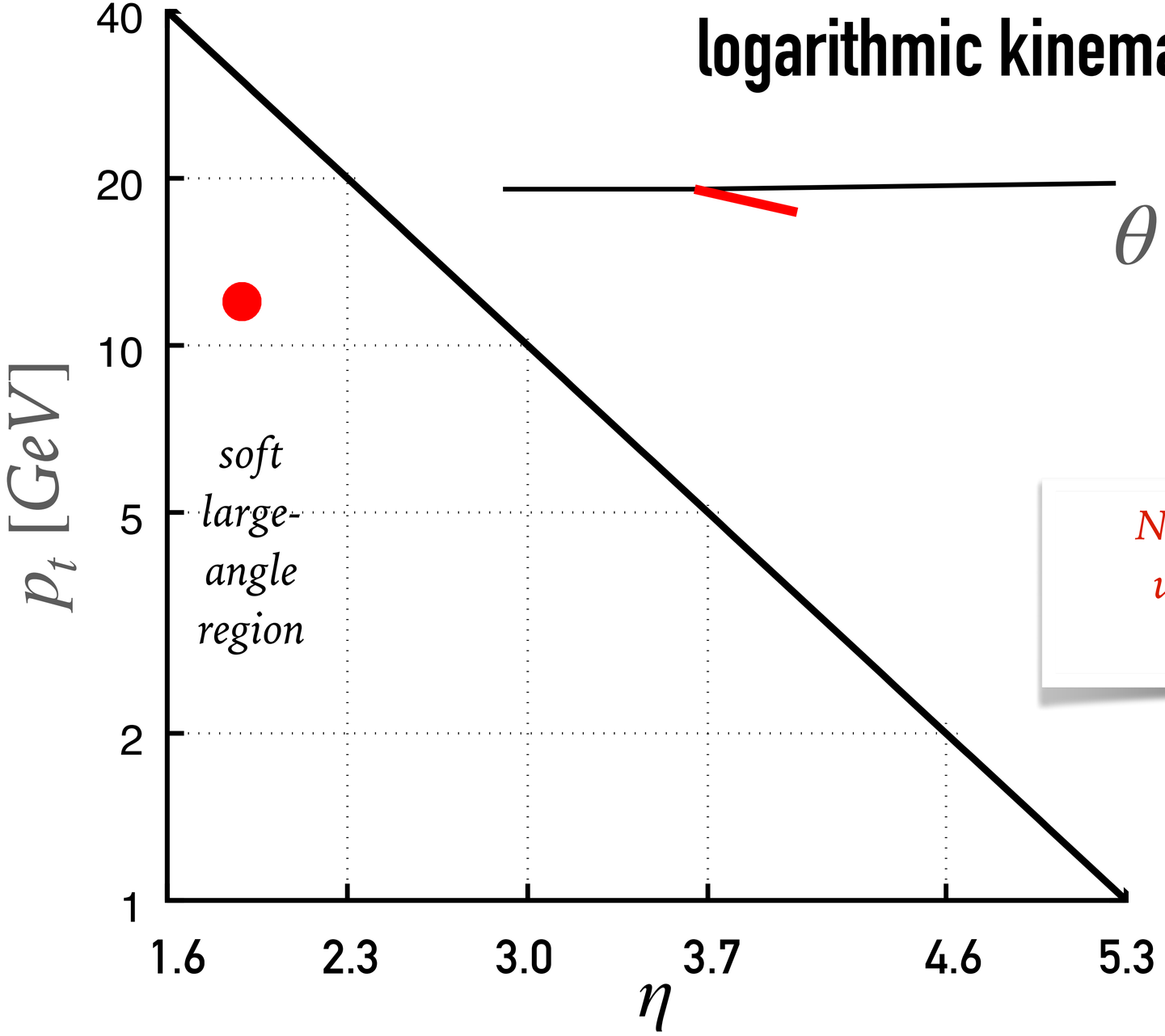
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The Lund Plane

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logarithmic kinematic plane whose two variables are

$$\theta \ (or \ \eta = -\ln\tan\frac{\theta}{2})$$

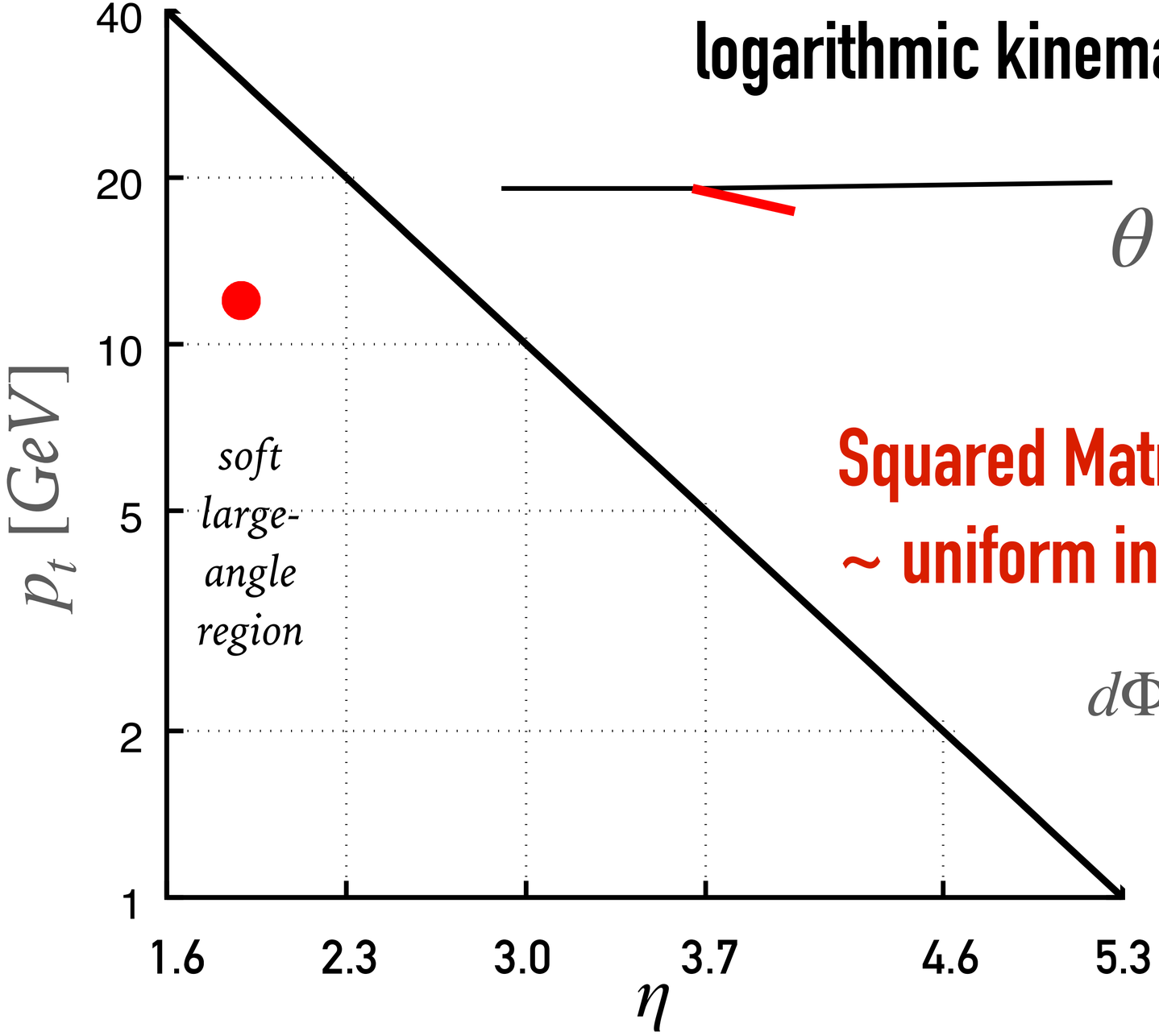
$$p_t = E\theta$$

NB: Lund plane can be constructed event-by-event using Cambridge/Aachen jet clustering sequence,
cf. Dreyer, GPS & Soyez '18

Introduced for understanding Parton Shower Monte Carlos by
B. Andersson, G. Gustafson L. Lonnblad and Pettersson 1989

The Lund Plane

jet with $R = 0.4$, $p_t = 200$ GeV



logarithmic kinematic plane whose two variables are

$$\theta \ (or \ \eta = -\ln\tan\frac{\theta}{2})$$

$$p_t = E\theta$$

Squared Matrix Element $\times$ phasespace

$\sim$ uniform in ln p_t and η

$$d\Phi\,|M^2| = \frac{2\alpha_s(p_t)C}{\pi}\frac{dp_t}{p_t}\frac{d\theta}{\theta}\frac{d\phi}{2\pi}$$

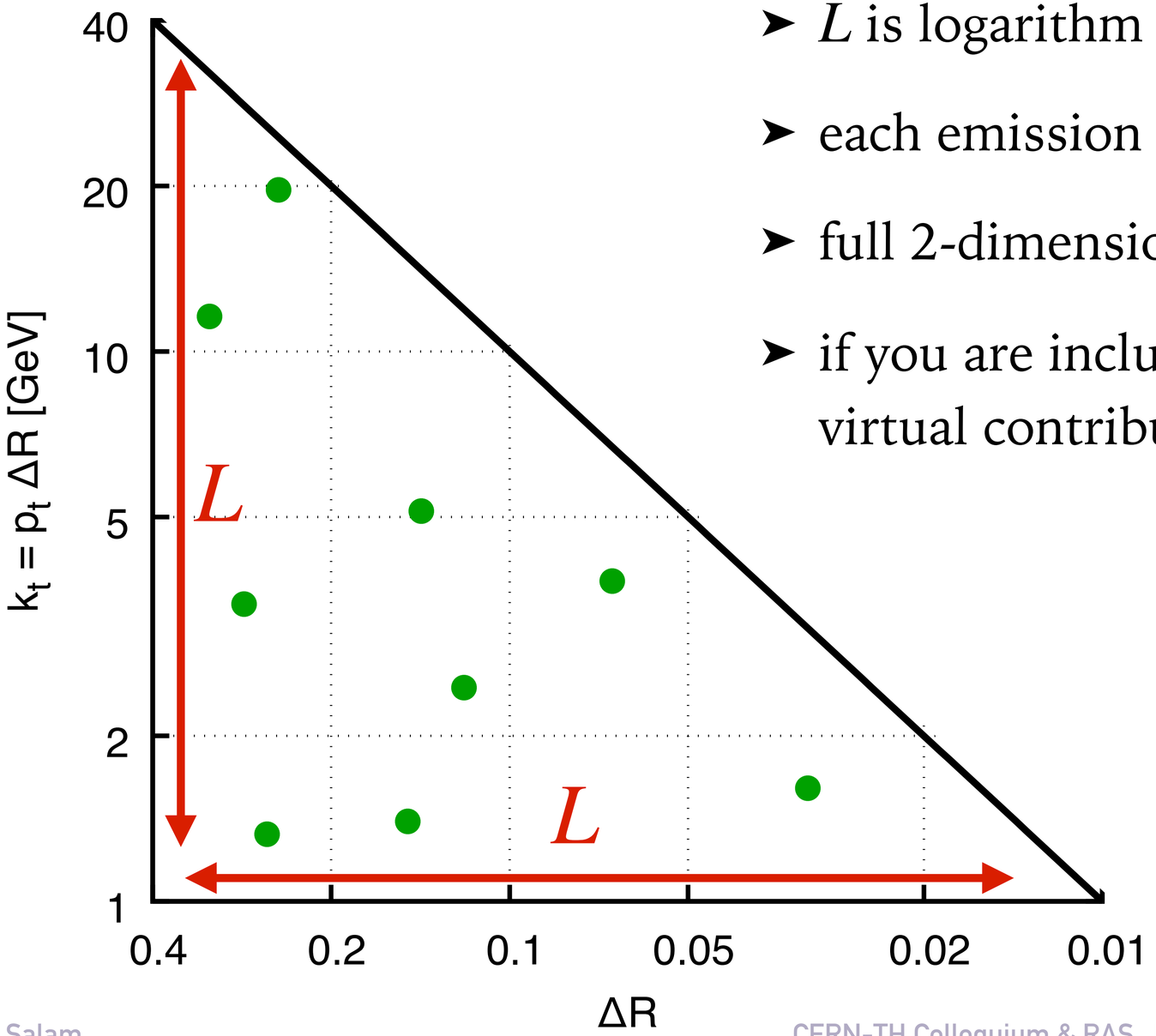
Introduced for understanding Parton Shower Monte Carlos by
B. Andersson, G. Gustafson L. Lonnblad and Pettersson 1989

The Lund Plane

Element #1: criteria for accuracy

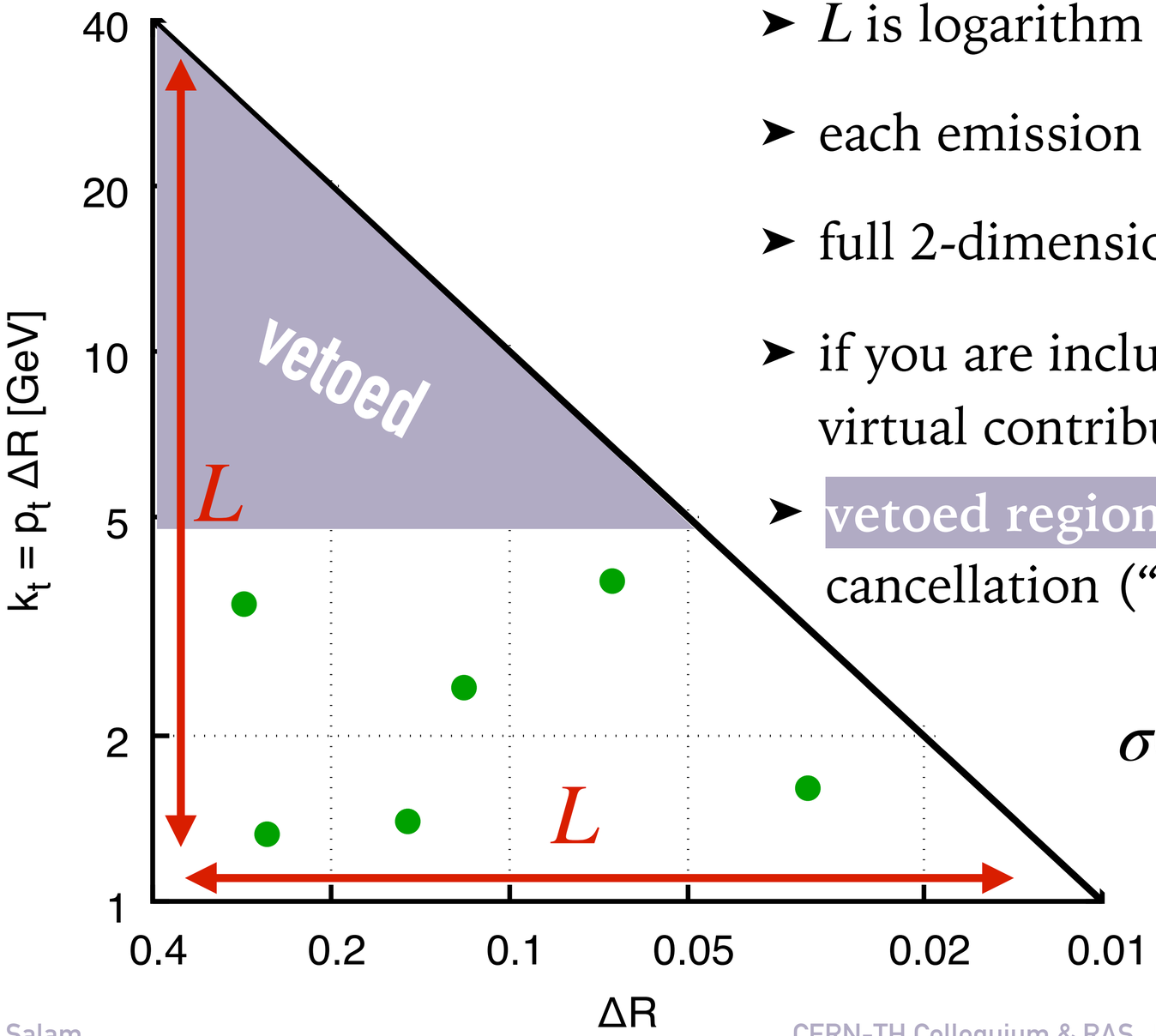
parton showers span disparate scales
natural language is “logarithmic” accuracy

Double (or leading) logarithms: $\alpha_s^n L^{2n}$



- L is logarithm of some ratio of momentum scales
- each emission "costs" a power of α_s
- full 2-dimensions of phase space → factor of L^2
- if you are inclusive, real $\alpha_s L^2$ terms cancel against virtual contributions (unitarity)

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- if you are inclusive, real $\alpha_s L^2$ terms cancel against virtual contributions (unitarity)
- vetoed regions of phase space break the cancellation ("Sudakov" form factor)

$$\sigma(p_{t,Z} < e^L) \sim \sigma_{tot} \exp\left[-c \cdot \alpha_s L^2\right]$$

Logarithmic accuracy hierarchy, with $\alpha_s L \sim 1$ (as used in this talk)

[depending on observable, take log of cross section, possibly also Fourier/etc. transform]

$$\alpha_s L^2 + \alpha_s^2 L^3 + \alpha_s^3 L^4 + \cdots \equiv \alpha_s^n L^{n+1} \sim \frac{1}{\alpha_s}$$ leading logarithms (LL)

$$\alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \cdots \equiv \alpha_s^n L^n \sim 1$$ next-to-leading logarithms (NLL)

[also called *single logarithms*, SL]

$$\alpha_s + \alpha_s^2 L + \alpha_s^3 L^2 + \cdots \equiv \alpha_s^n L^{n-1} \sim \alpha_s$$ next-to-next-to-leading logarithms (NNLL)

etc.

Defining what we mean by NLL

Dasgupta, Dreyer, Hamilton, Monni, GPS '18
ibid + Soyez '20

A Matrix Element condition

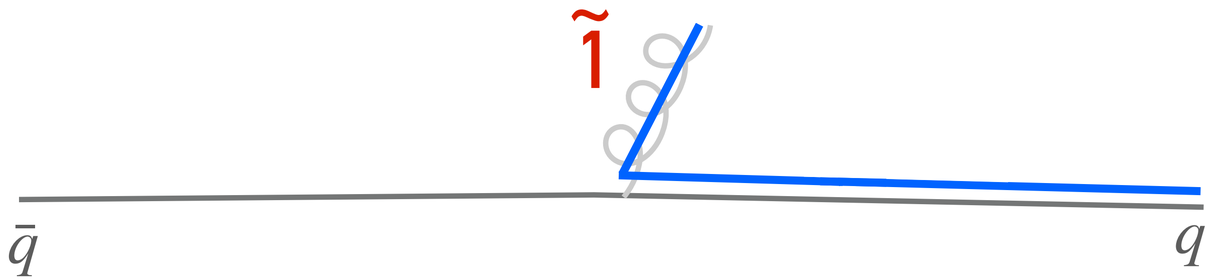
- correctly reproduce n-parton tree-level matrix element for arbitrary configurations, so long as all emissions well separated in the Lund diagram
  (because those configurations are the most likely ones)
- supplement with unitarity, 2-loop running coupling & cusp anomalous dimension

Resummation condition: reproduce NLL results for all standard resummations

- global event shapes
- non-global observables
- fragmentation functions
- multiplicities
- …

1. Recoil: the core of any shower

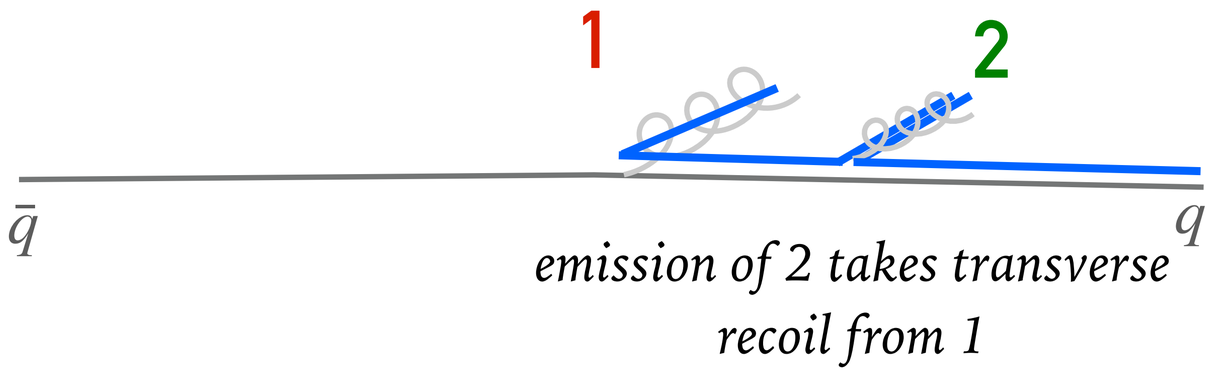
Dipole showers conserve momentum at each step. Traditional dipole-local recoil:



$$\mathrm{d}\mathcal{P}^{\mathrm{FS}}_{\tilde{\imath}\to ik} = \frac{\alpha_s(k_\perp^2)}{2\pi}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{\mathrm{d}z}{z}\frac{\mathrm{d}\varphi}{2\pi}N_{ik}^{\mathrm{sym}}\left[zP_{\tilde{\imath}\to ik}(z)\right]$$

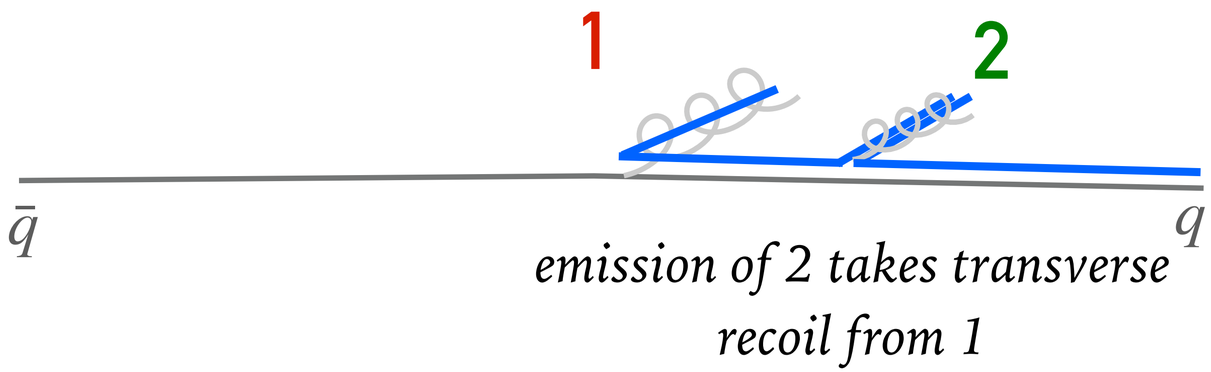
1. Recoil: the core of any shower

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Dipole showers conserve momentum at each step. Traditional dipole-local recoil:

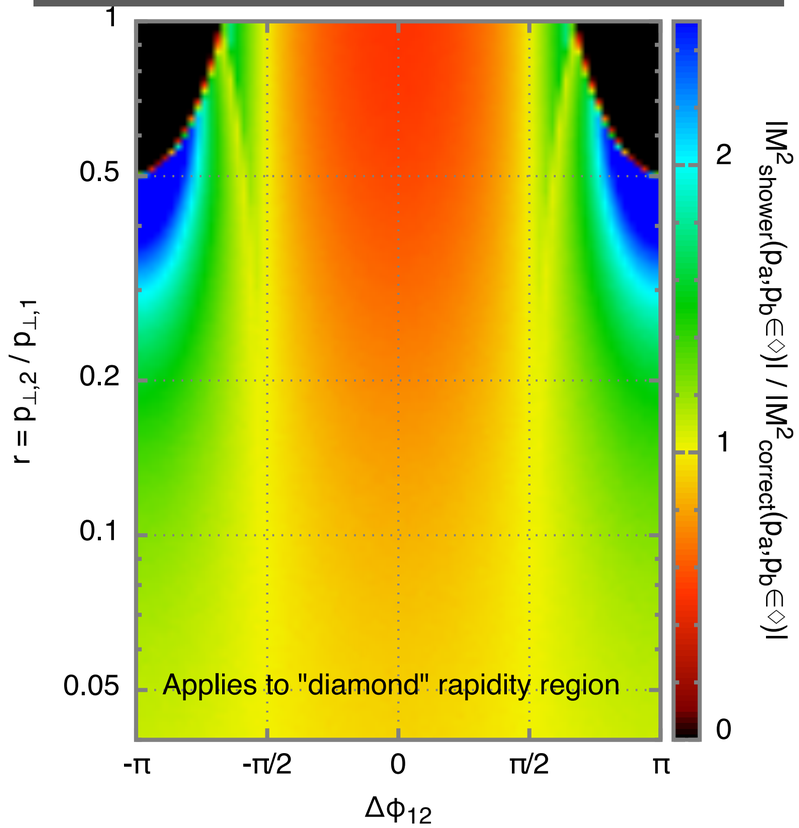


Shower initially generated matrix element for particle $\tilde{1}$, whose momentum differs (by ~ 50%) from final particle 1.

Matrix element is incorrect wrt final momentum 1.

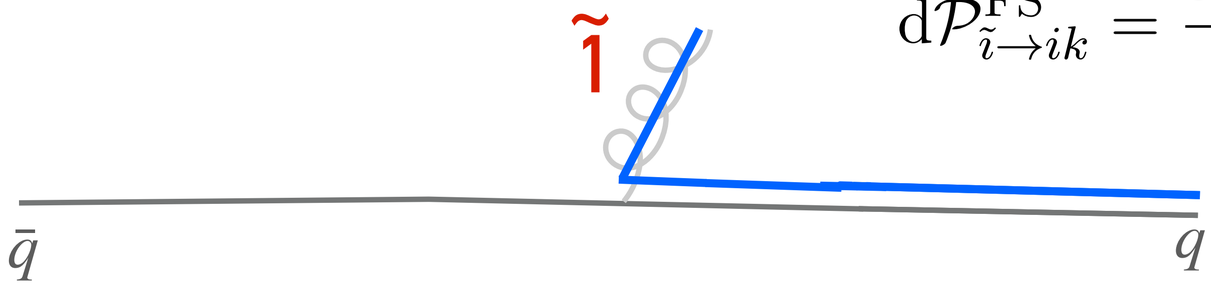
First observed: Andersson, Gustafson, Sjogren '92
Closely related effect present for Z p_t: Nagy & Soper 0912.4534
Impact on log accuracy across many observables: Dasgupta, Dreyer, Hamilton, Monni, GPS, 1805.09327

ratio of effective shower matrix element to exact one



1. Correct recoil rule: no side effects on other distant emissions

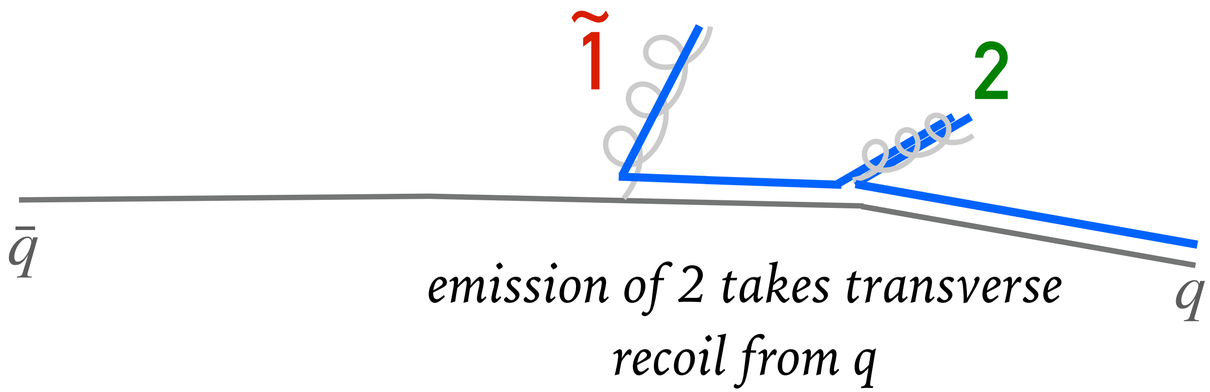
One approach



$$\mathrm{d}\mathcal{P}^{\mathrm{FS}}_{\tilde{i}\to ik} = \frac{\alpha_s(k_\perp^2)}{2\pi}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{\mathrm{d}z}{z}\frac{\mathrm{d}\varphi}{2\pi}N_{ik}^{\mathrm{sym}}\left[zP_{\tilde{i}\to ik}(z)\right]$$

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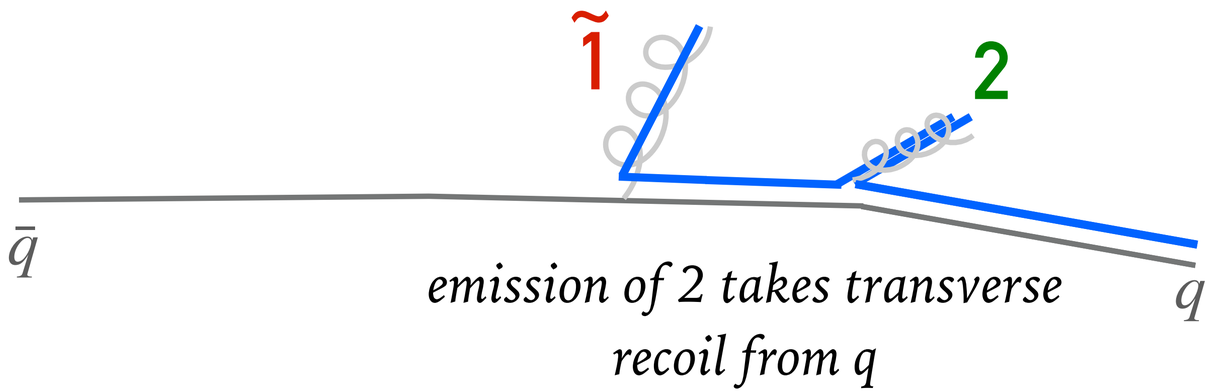
One approach

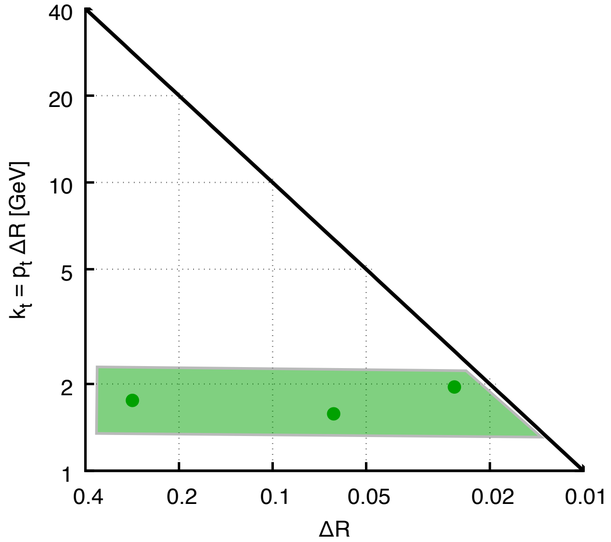


θ_{1q} left almost unchanged if ⊥ recoil from emission of 2 taken by (much harder) q

1. Correct recoil rule: no side effects on other distant emissions

One approach





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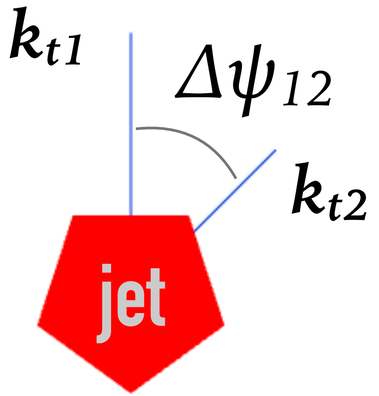
Can be achieved in multiple ways:

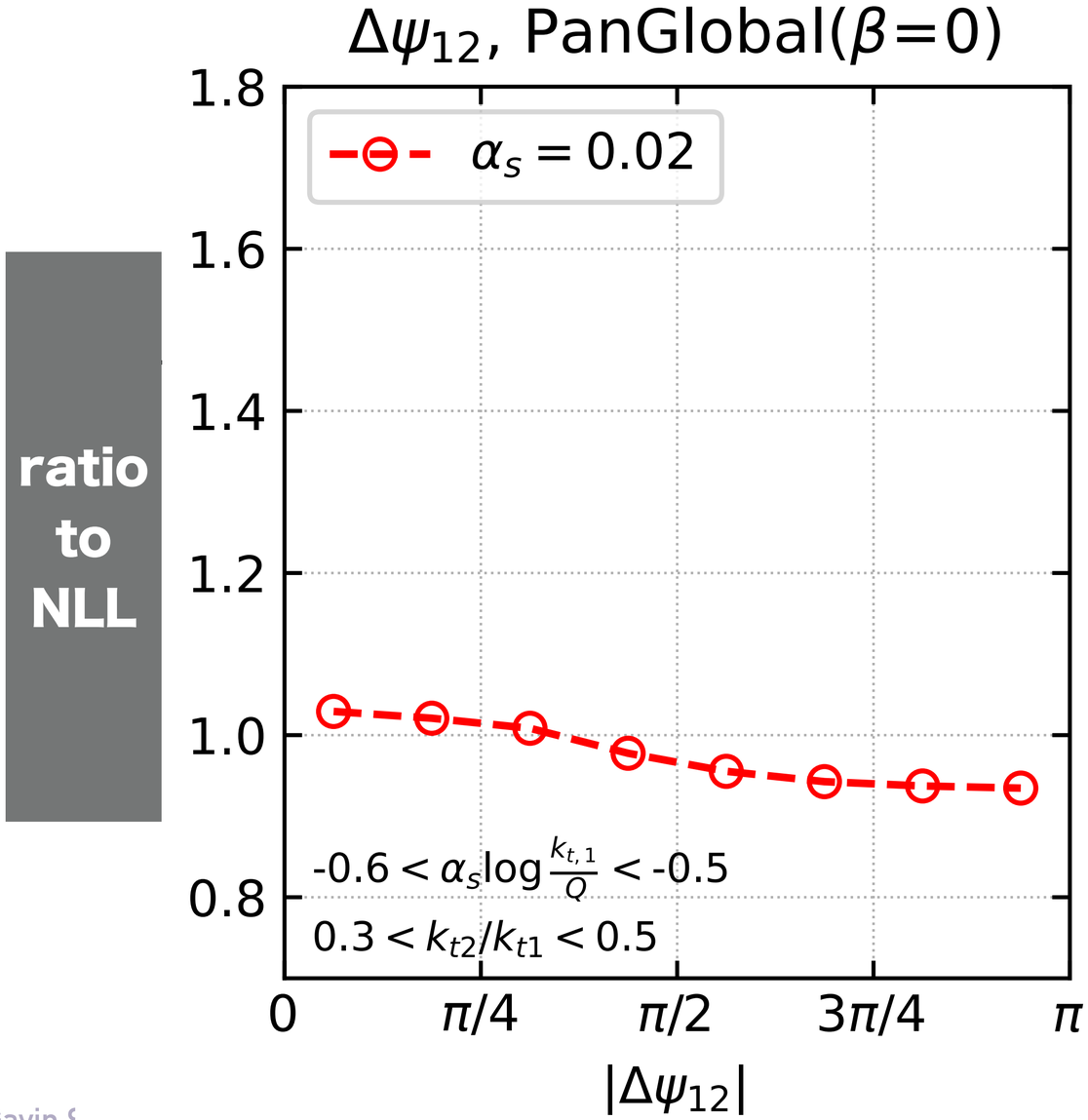
- global transverse recoil
  (Dasgupta et al 2002.11114, "**PanGlobal**"; Holguin Seymour & Forshaw 2003.06400; Alaric 2208.06057 + ⋯, Apollo, 2403.19452)
- local transverse recoil, with non-standard shower ordering & dipole partition
  (2002.11114 "**PanLocal**"; Nagy & Soper 0912.4534 + ⋯, "Deductor")

Element #2: testing correctness

Parton showers operate at all orders and mix many effects. How can you separate out just the orders you aim to control to test they're correct?

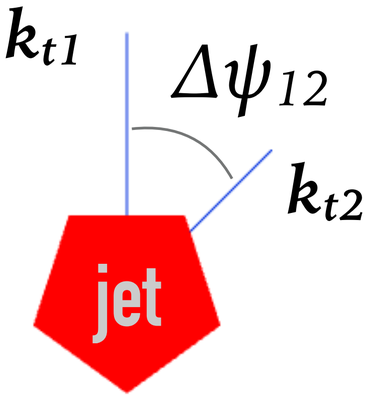
Tests (2): full shower v. all-order NLL

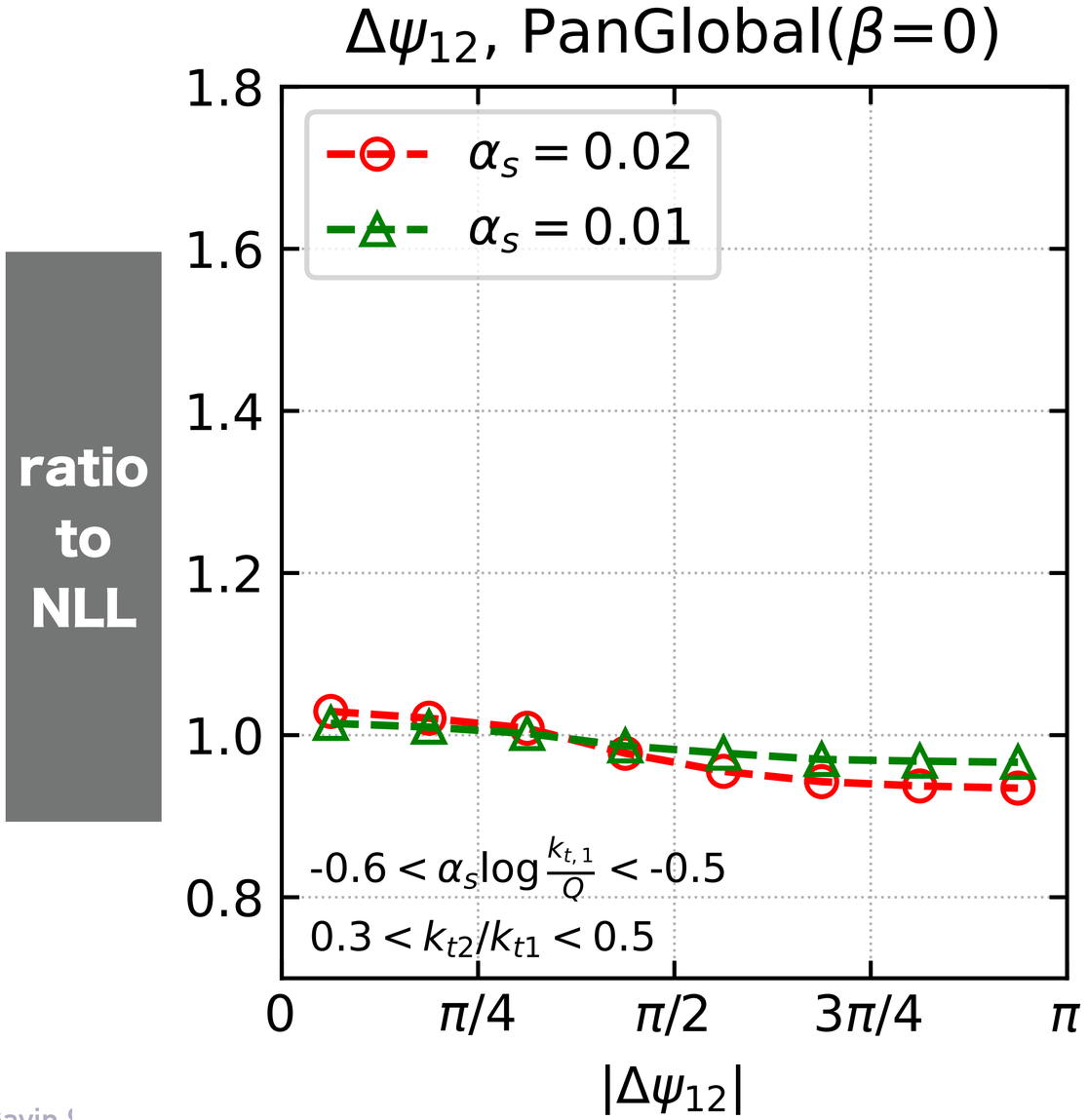




- run full shower with specific value of $\alpha_s(Q)$ & measure an observable: azimuth between two highest-k_t emissions (soft-collinear)
- ratio to NLL should be flat ≡ 1
- it isn't: have we got an NLL mistake? Or a residual subleading (NNLL) term?

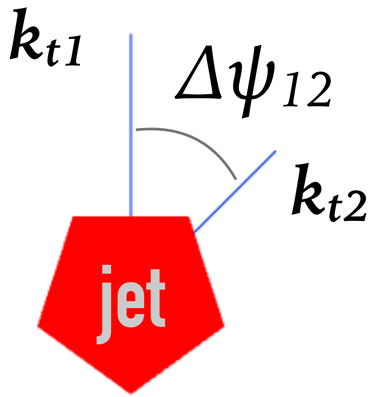
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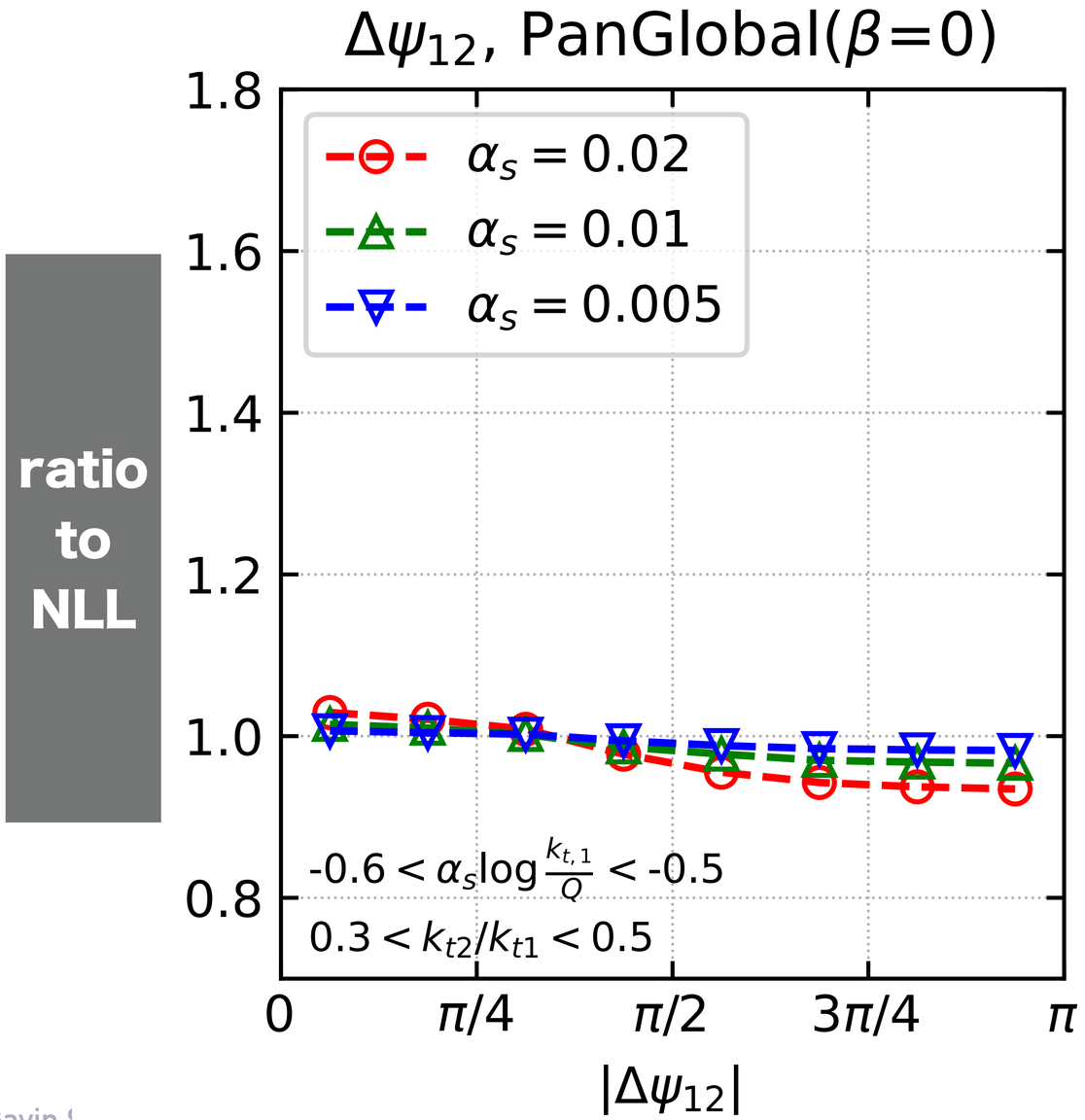




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- try reducing $\alpha_s(Q)$, while keeping constant $\alpha_s L$ $[L \equiv \ln k_{t1}/Q]$
- NLL effects, $(\alpha_s L)^n$, should be unchanged, subleading ones, $\alpha_s(\alpha_s L)^n$, $\to 0$

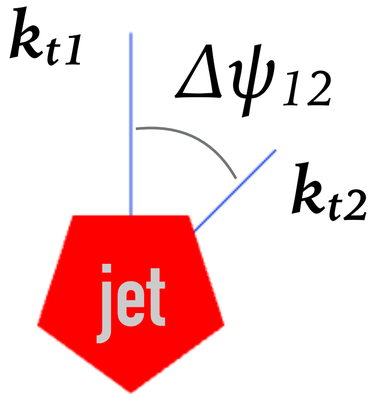
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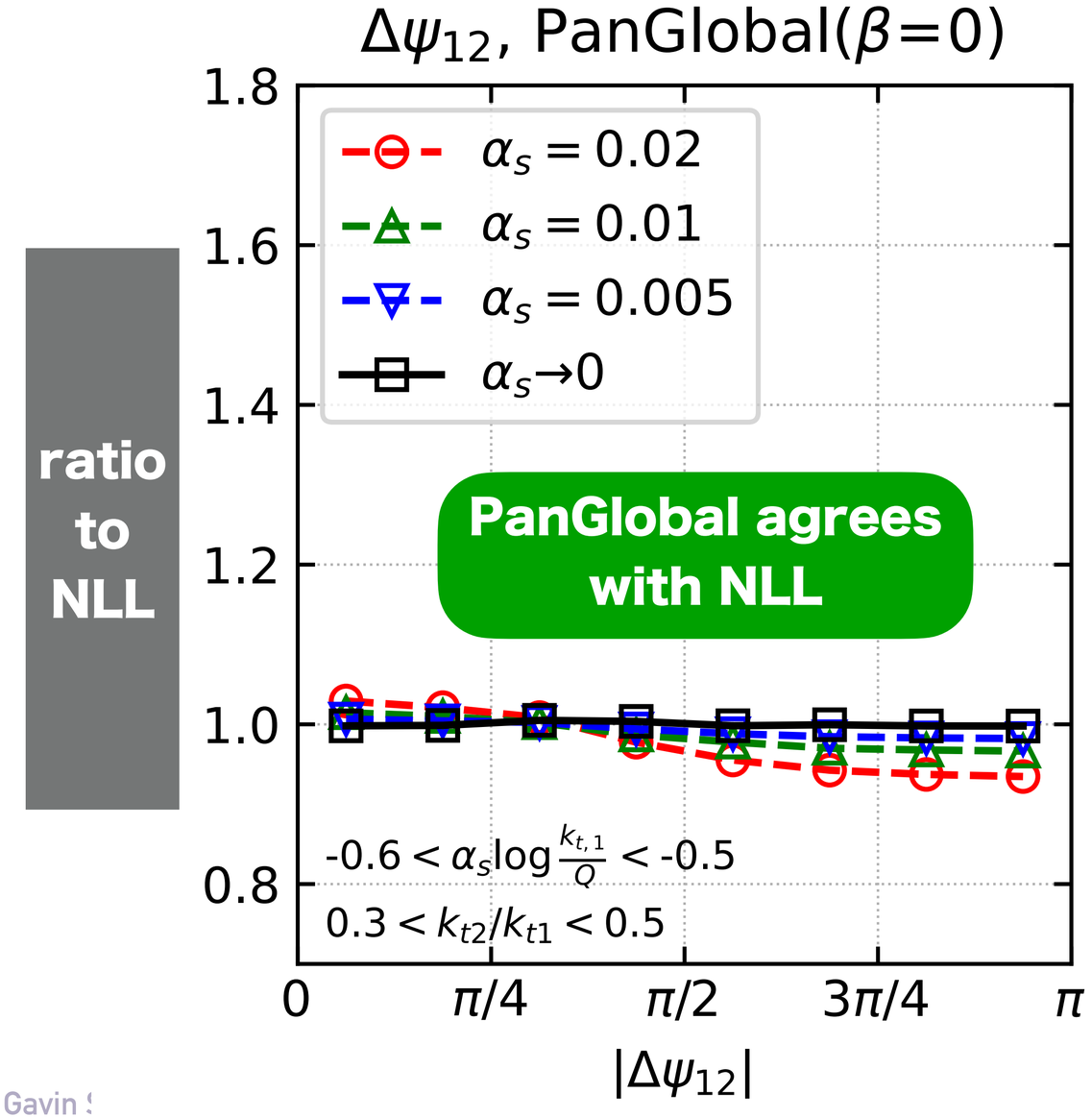




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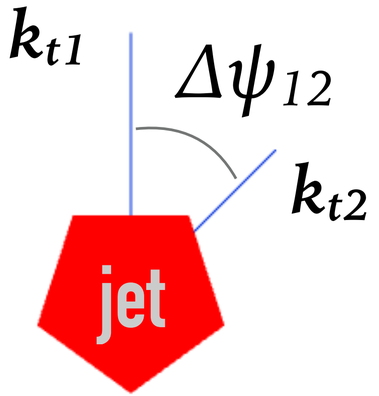




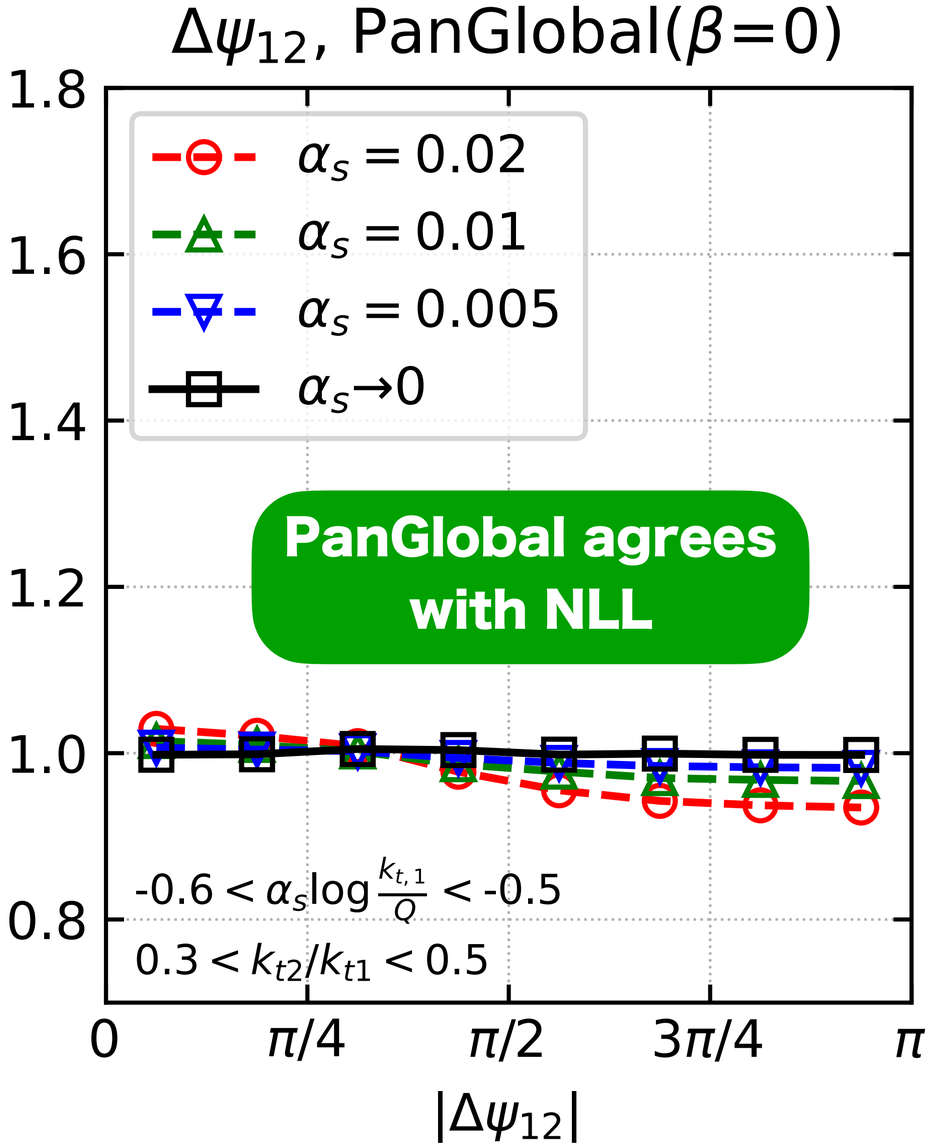
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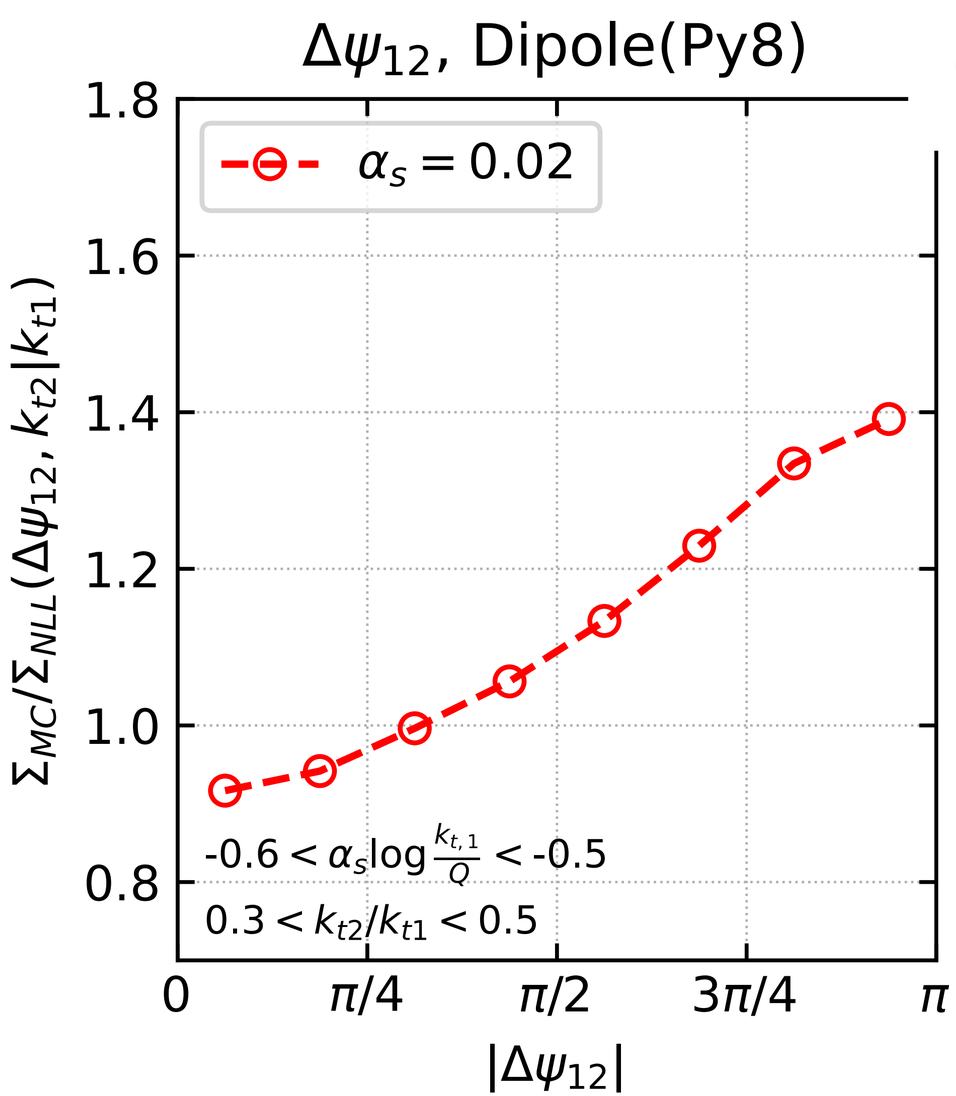
✓ **extrapolation $\alpha_s \to 0$ agrees with NLL**

Tests (2): full shower v. all-order NLL

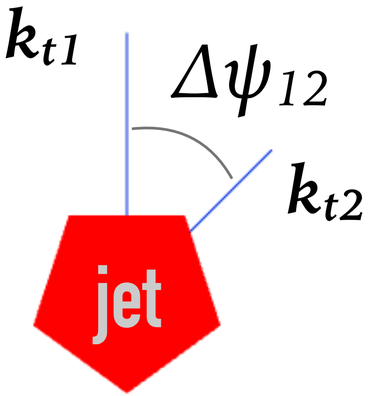


ratio to NLL

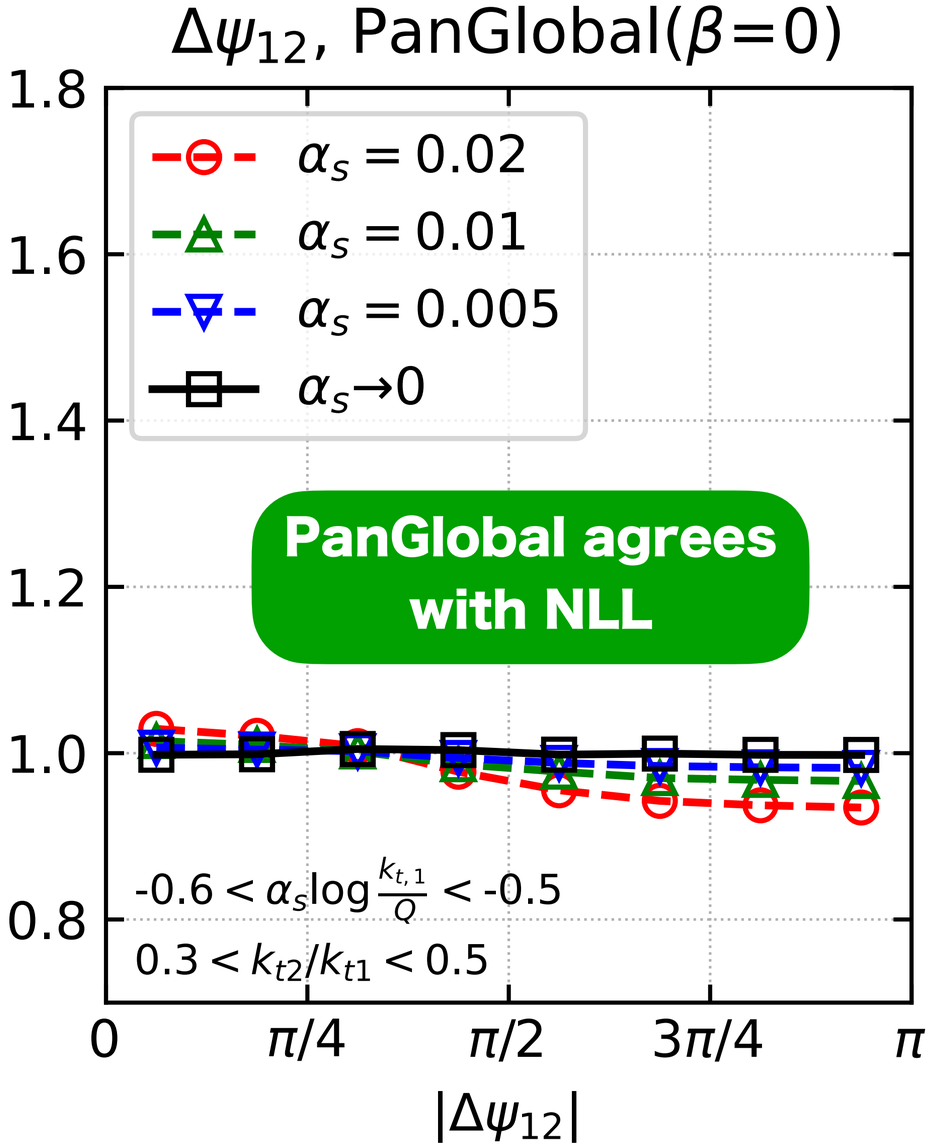


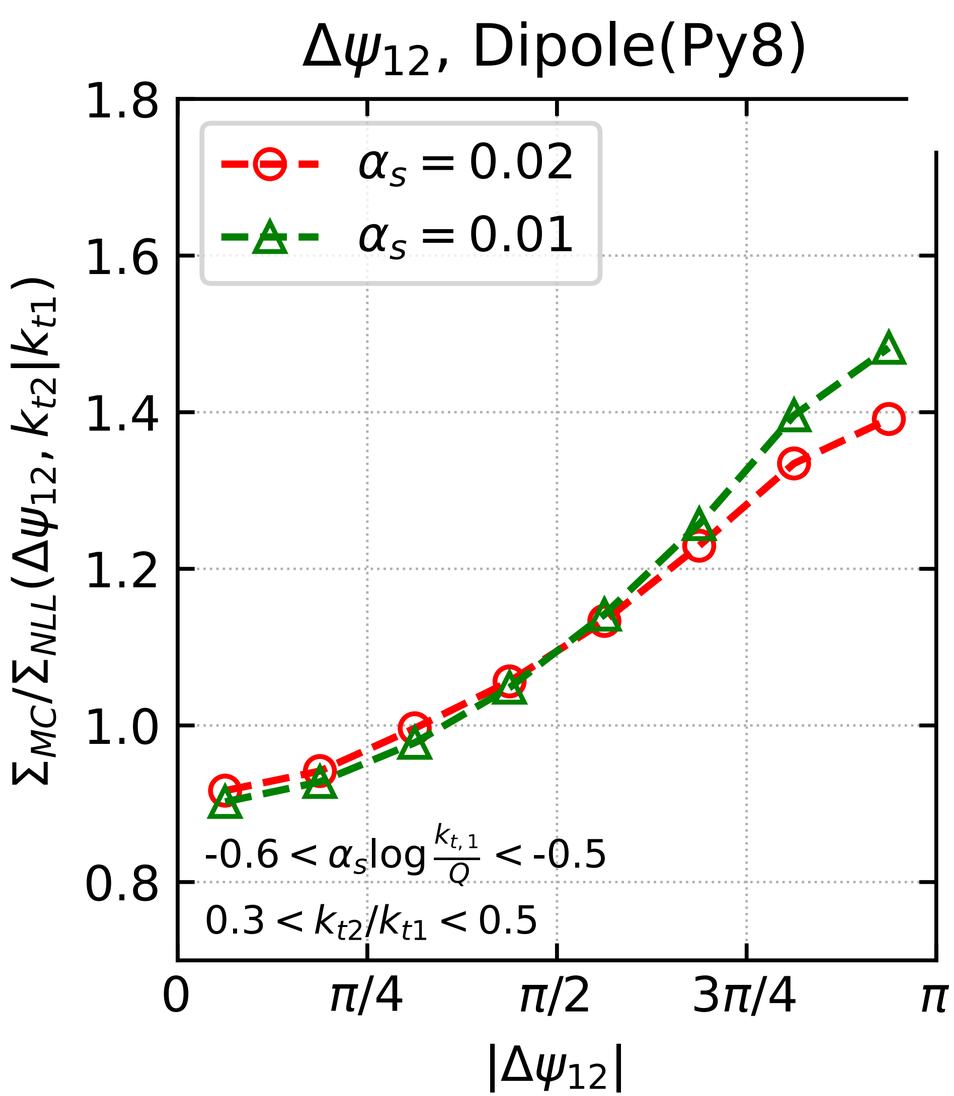


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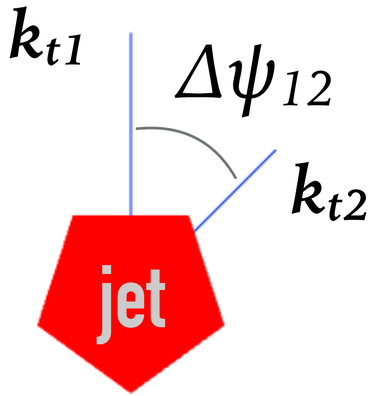


ratio to NLL

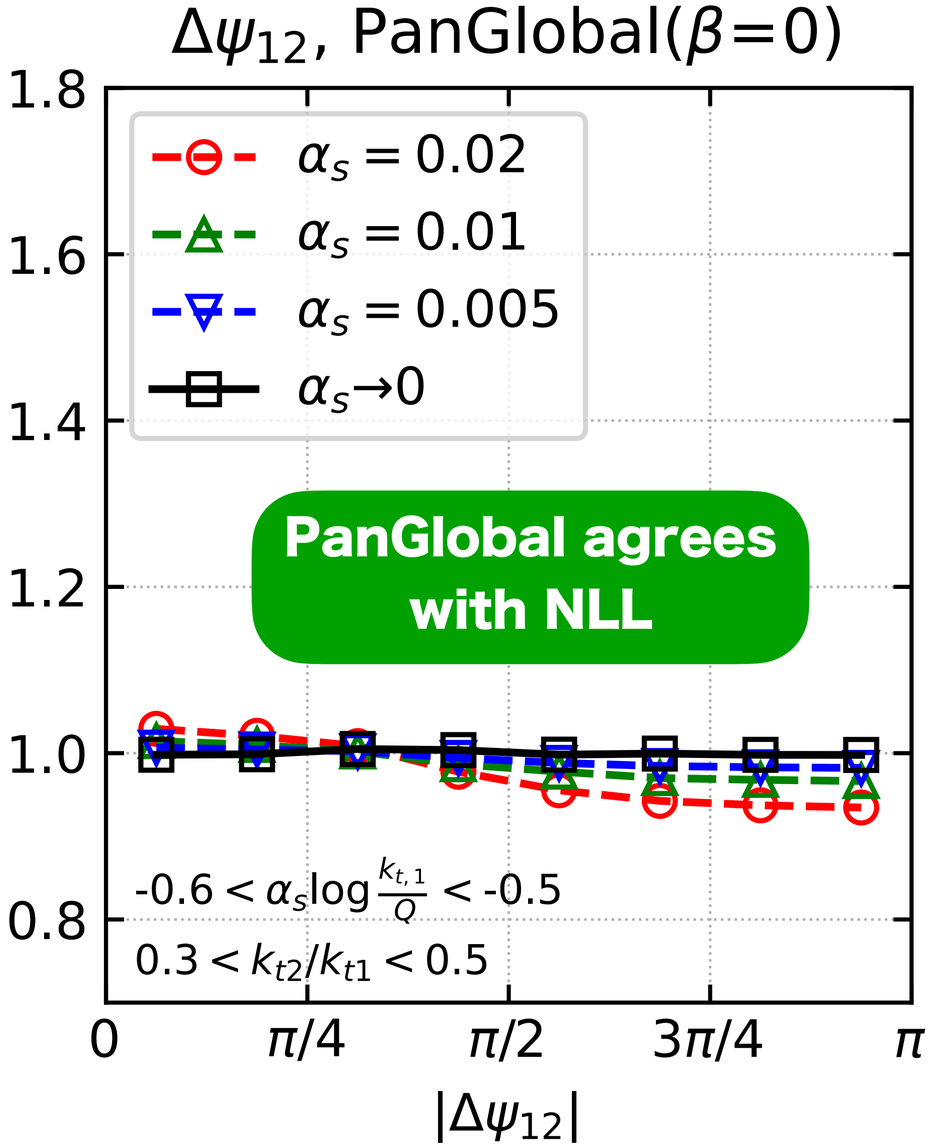


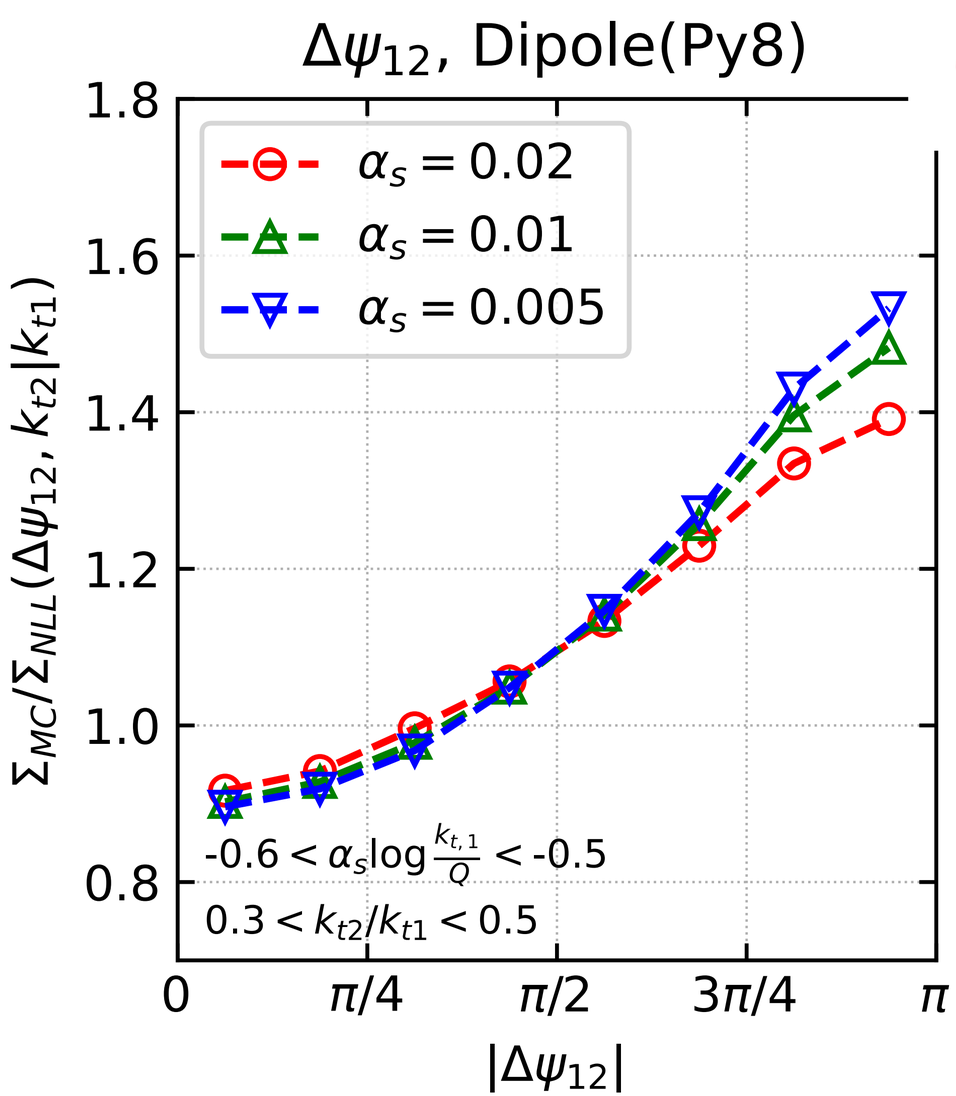


Tests (2): full shower v. all-order NLL

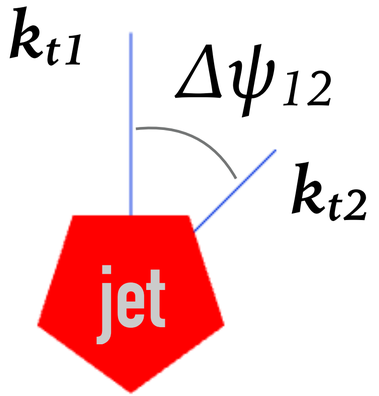


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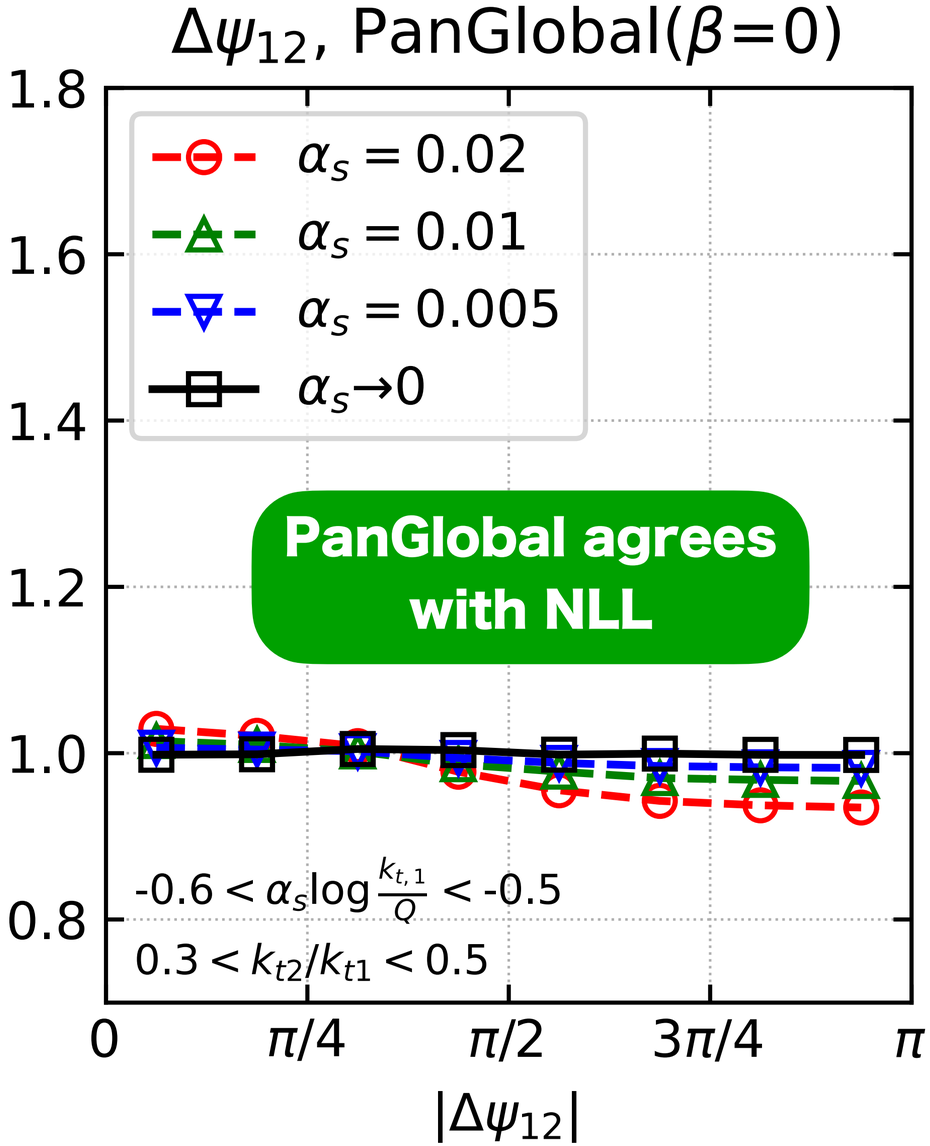


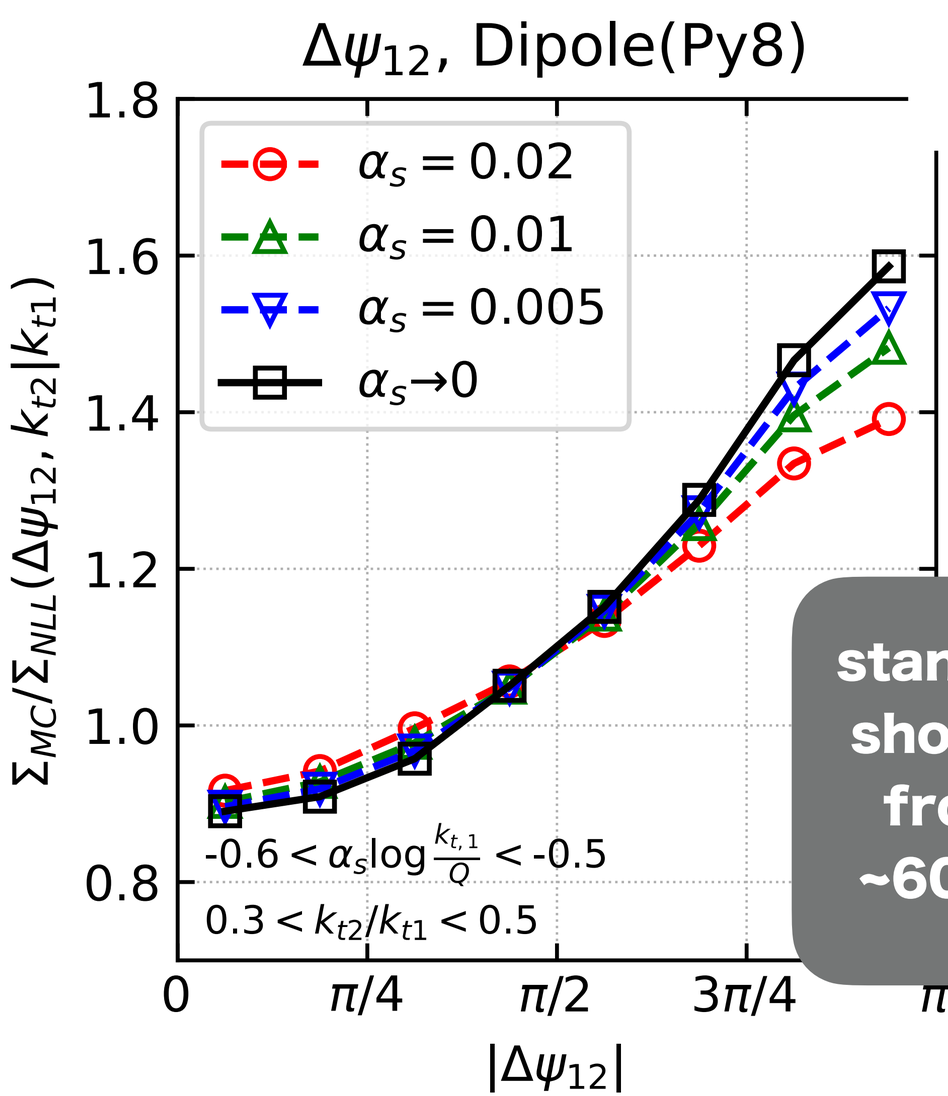


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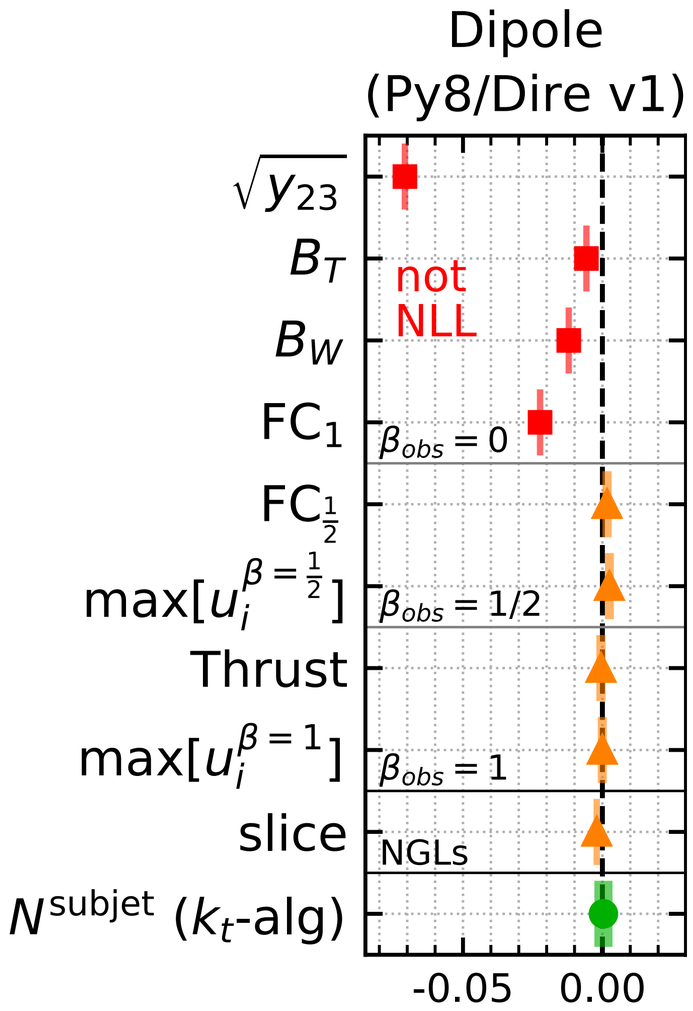
ratio to NLL





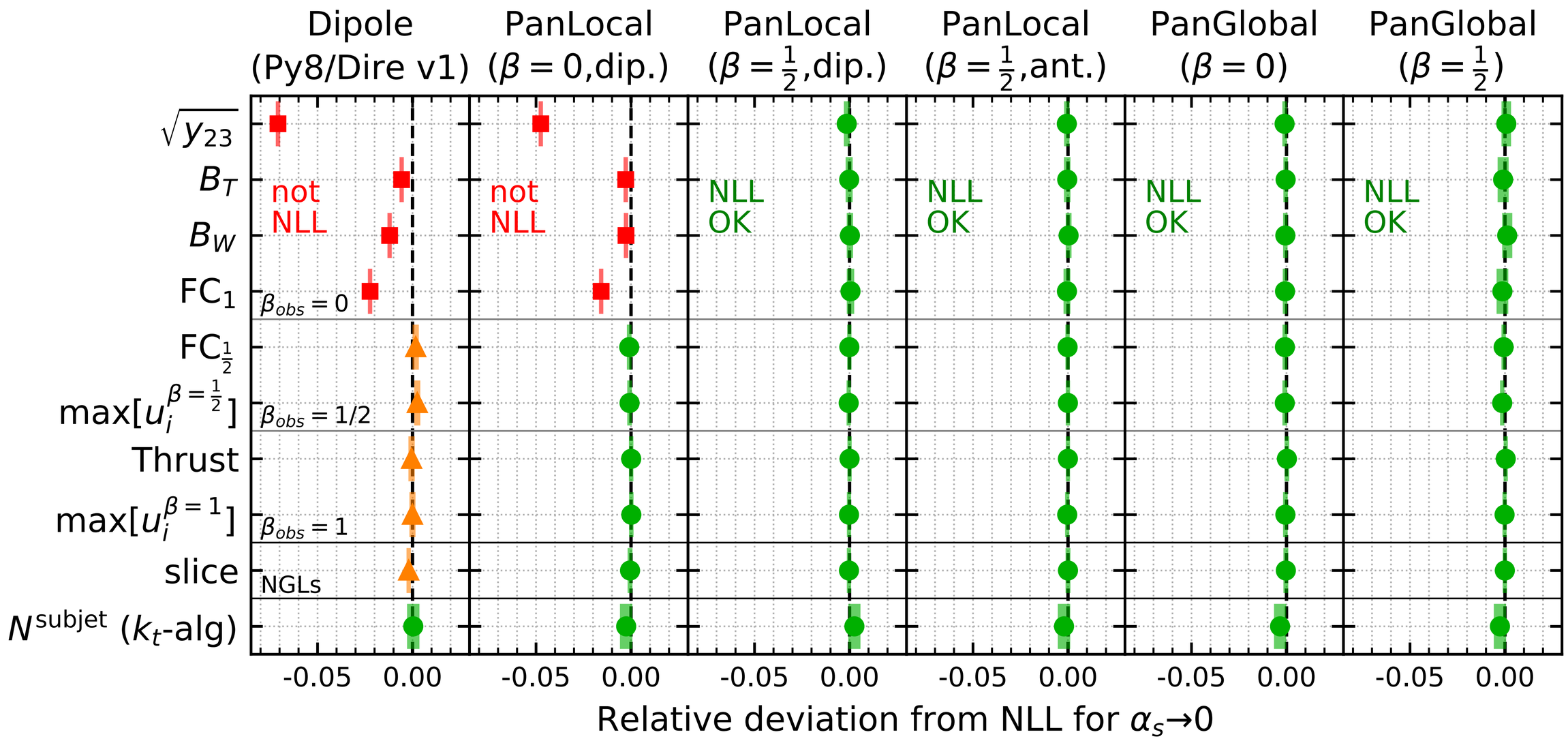
standard dipole shower differs from NLL by ~60%, i.e. O(1)

Test class 2: full shower v. all-order NLL — many observables



Relative deviation from NLL for $\alpha_s \to 0$

Test class 2: full shower v. all-order NLL — many observables



NLL accuracy is becoming the new standard

Logarithmic accuracy of parton showers: a fixed-order study
Dasgupta, Dreyer, Hamilton, Monni, Salam [1805.09327]

Parton showers beyond leading logarithmic accuracy
Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

A new approach to color-coherent parton evolution
Herren, Höche, Krauss, Reichelt, Schönherr [2208.06057]

Colour and logarithmic accuracy in final-state parton showers
Hamilton, Medves, Salam, Scyboz, Soyez [2011.10054]

Spin correlations in final-state parton showers and jet observables
Karlberg, Salam, Scyboz, Verheyen [2103.16526]

Alaric

New approach to QCD final-state evolution in processes with massive partons
Assi, Höche [2307.00728]

The Alaric parton shower for hadron colliders
Höche, Krauss, Reichelt [2404.14360]

Soft spin correlations in final-sta parton showers
Hamilton, Karlberg, Salam, Scyboz, Verhe [2111.01161]

PanScales parton showers for hadron collisions: formulation and fixed-order studies
van Beekveld, Ferrario Ravasio, Salam, Soto Ontoso, Soyez, Verheyen [2205.02237]

PanScales

A partitioned dipole-antenna shower with improved transverse recoil
Preuss [2403.19452]

Apollo

PanScales parton showers for hadron collisions: all-order validation
van Beekveld, Ferrario Ravasio, Hamilton, Salam, Soto Ontoso, Soyez, Verheyen [2207.09467]

Next-to-leading-logarithmic PanScales showers for deep inelastic scattering and vector boson fusion
van Beekveld, Ferrario Ravasio [2305.08645]

Summation of large logarithms by parton showers
Nagy, Soper [2011.04773]

Deductor

Summation by parton showers of large logarithms in electron-positron annihilation
Nagy, Soper [2011.04777]

Introduction to the PanScales framework, version 0.1
van Beekveld, Dasgupta, El-Menoufi, Ferrario Ravasio, Hamilton, Helliwell, Karlberg, Medves, Monnim Salam, Scyboz, Soto Ontoso, Soyez, Verheyen [2312.13275]

Logarithmic accuracy of angular-ordered parton showers
Bewick, Ferrario Ravasio, Richar Seymour [1904.11866]

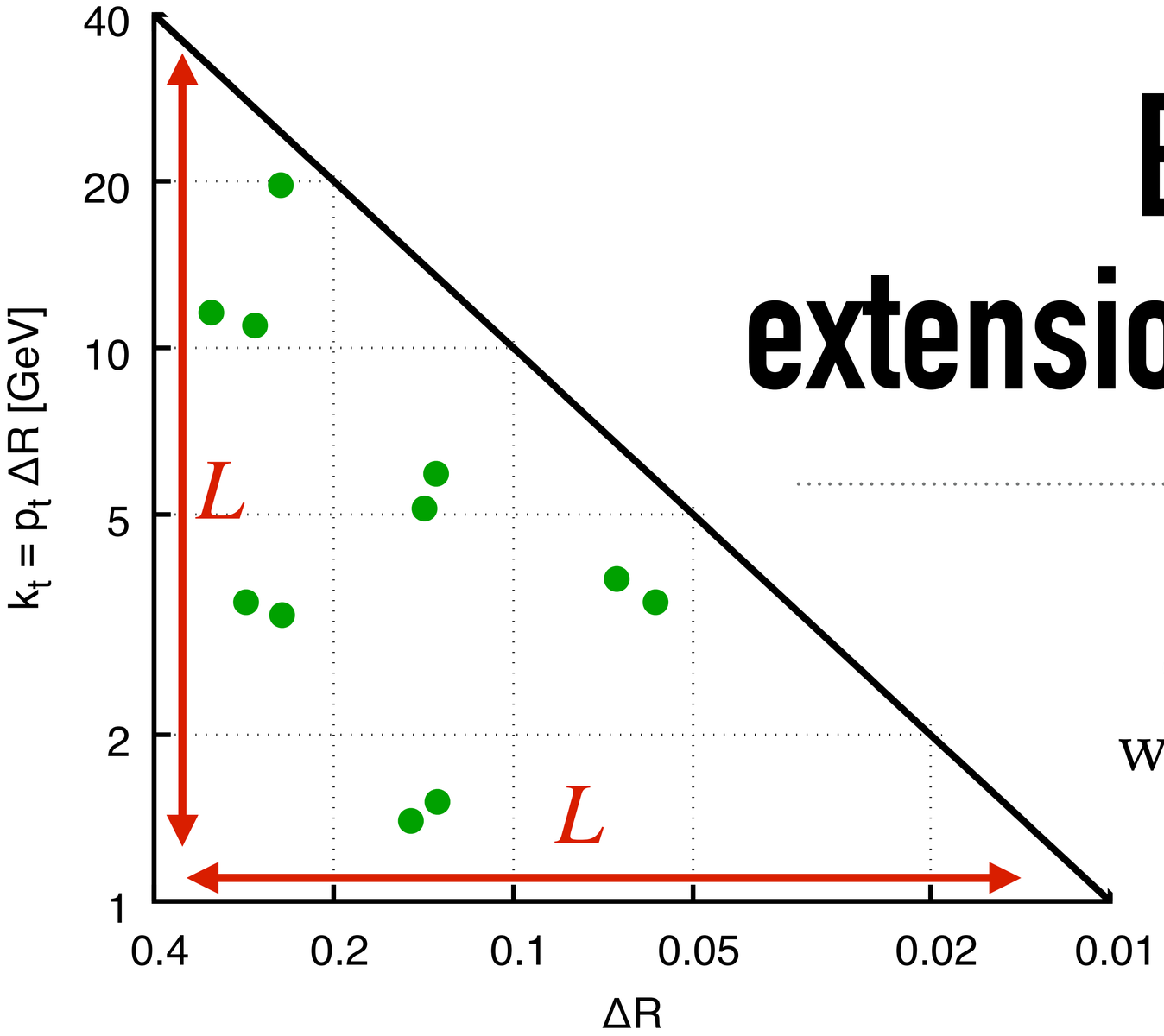
Herwig

Building a consistent parton shower
Forshaw, Holguin, Plätzer [2003.06400]

Initial state radiation in the Herwig 7 angular-ordered parton shower
Bewick, Ferrario Ravasio, Richardson, Seymour [2107.04051]

Improvements on dipole shower colour
Forshaw, Holguin, Plätzer [2011.15087]

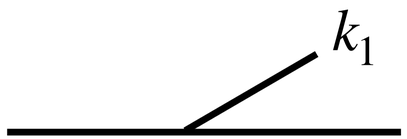
slide from M. van Beekveld



Element #3: extension to higher orders

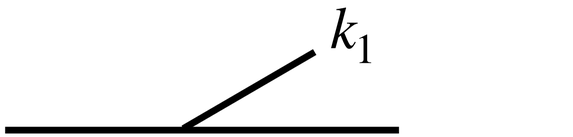
E.g. at NNLL, effective matrix element should be correct even where there are pairs of emissions close by in the Lund plane

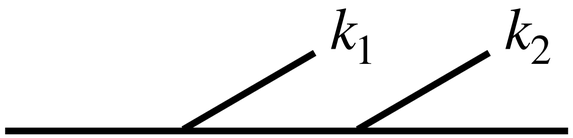
Make each new emission's distribution conditional on previous emission



Distribute k_1 according to $M^2(k_1)$

Make each new emission's distribution conditional on previous emission

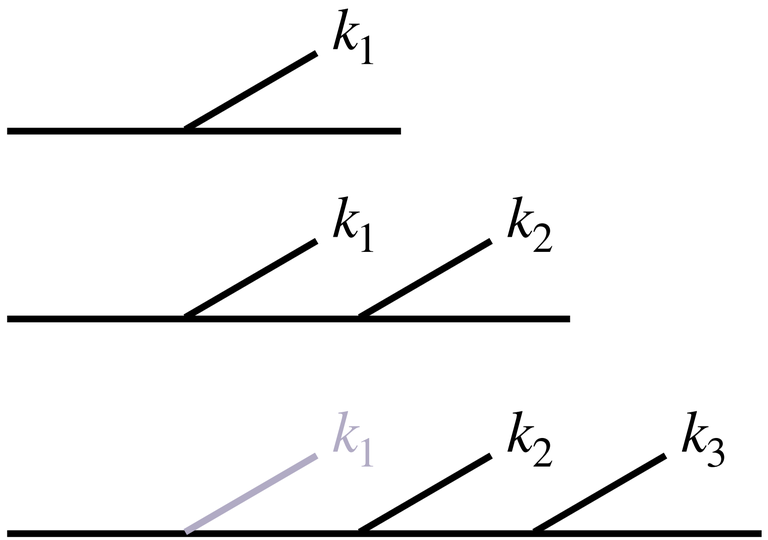




Distribute k_1 according to $M^2(k_1)$

Distribute k_2 according to $M^2(k_1, k_2)/M^2(k_1)$

Make each new emission's distribution conditional on previous emission

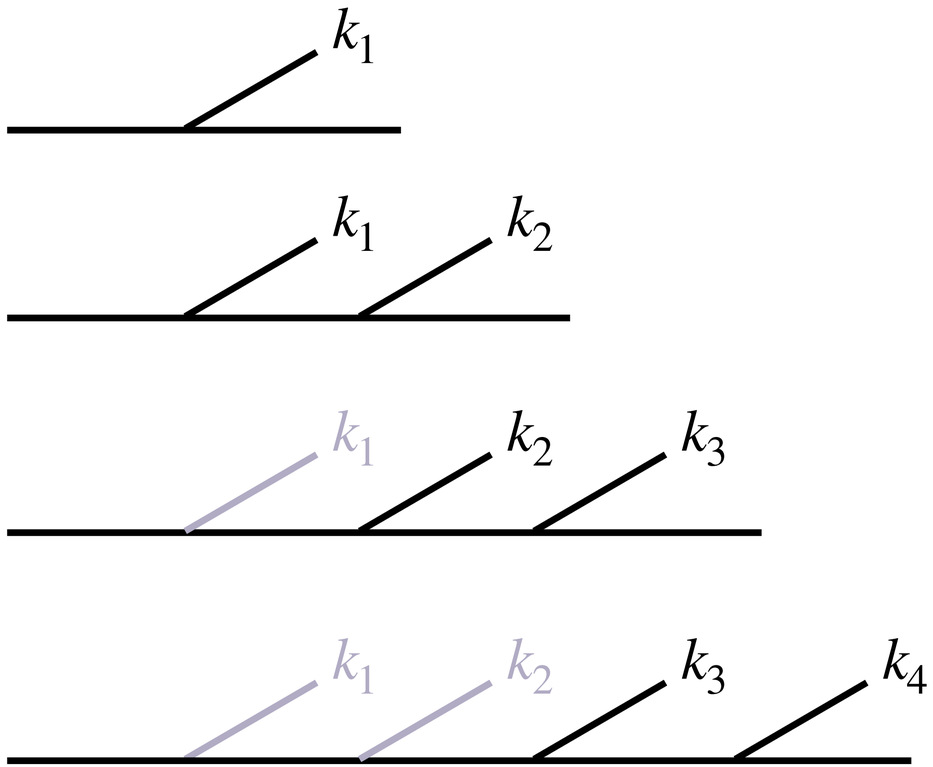


Distribute k_1 according to $M^2(k_1)$

Distribute k_2 according to $M^2(k_1, k_2)/M^2(k_1)$

Distribute k_3 according to $M^2(k_2, k_3)/M^2(k_2)$

Make each new emission's distribution conditional on previous emission



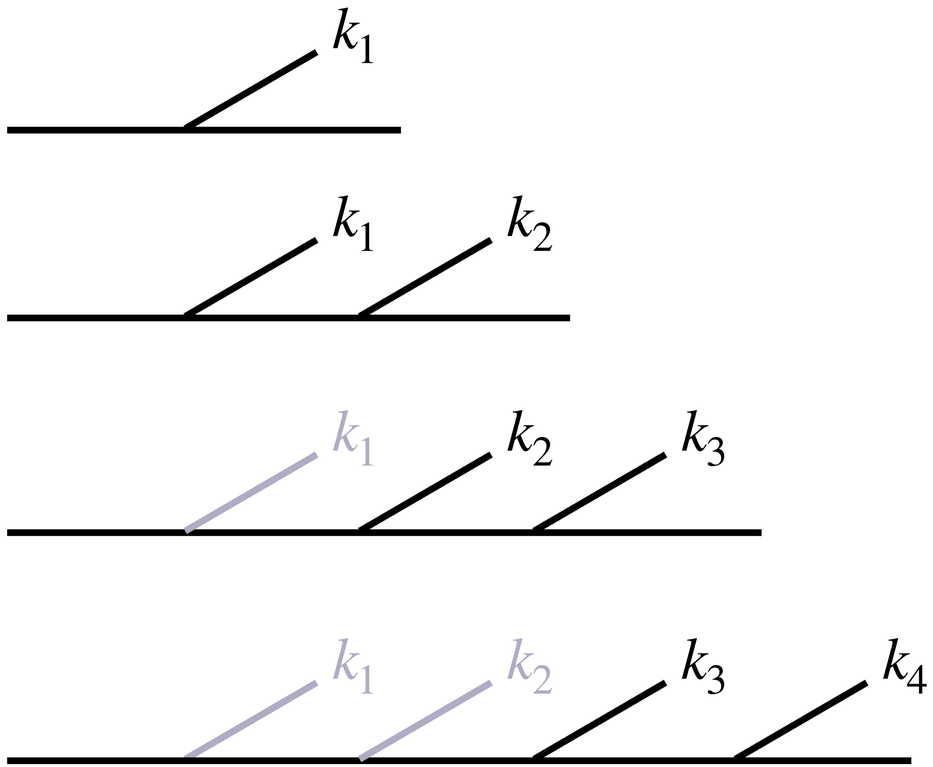
Distribute k_1 according to $M^2(k_1)$

Distribute k_2 according to $M^2(k_1, k_2)/M^2(k_1)$

Distribute k_3 according to $M^2(k_2, k_3)/M^2(k_2)$

Distribute k_4 according to $M^2(k_3, k_4)/M^2(k_3)$

Make each new emission's distribution conditional on previous emission



Distribute k_1 according to $M^2(k_1)$

Distribute k_2 according to $M^2(k_1, k_2)/M^2(k_1)$

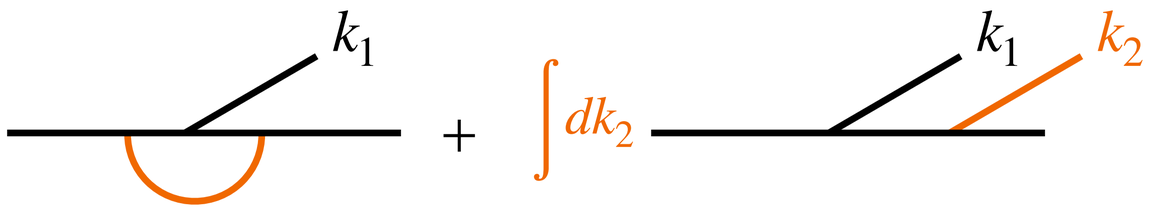
Distribute k_3 according to $M^2(k_2, k_3)/M^2(k_2)$

Distribute k_4 according to $M^2(k_3, k_4)/M^2(k_3)$

Relies on factorisation: e.g. $M^2(k_1, k_2, k_3, k_4)/M^2(k_1, k_2, k_3) \to M^2(k_3, k_4)/M^2(k_3)$
if 3 and 4 well separated in Lund plane from 1 and 2

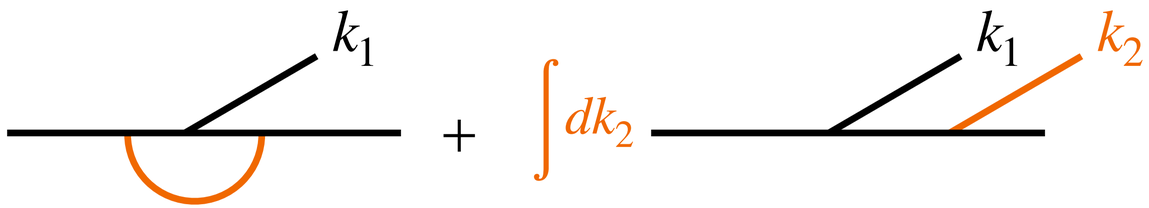
[factorised matrix elements given in Dokshitzer, Marchesini & Oriani '92, Campbell & Glover, hep-ph/9710255, Catani & Grazzini hep-ph/9810389, etc.]

Account for virtual corrections associated with each emission

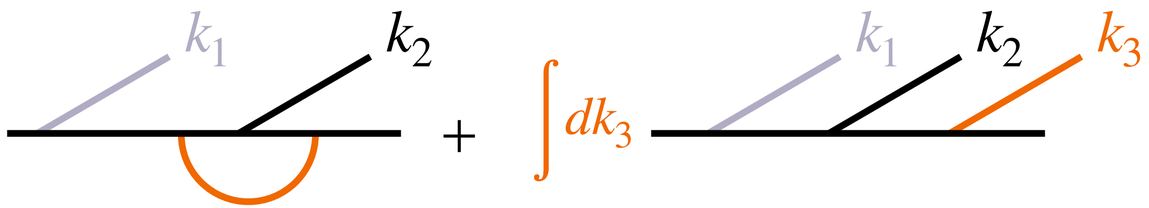


NLO correction to k_1 emission intensity sums loop correction and all possible scenarios for the next emission

Account for virtual corrections associated with each emission



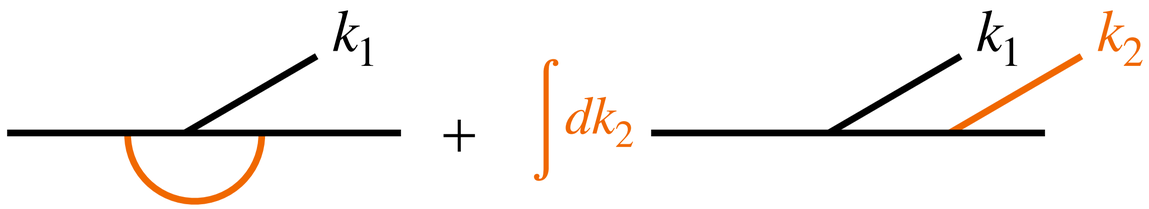
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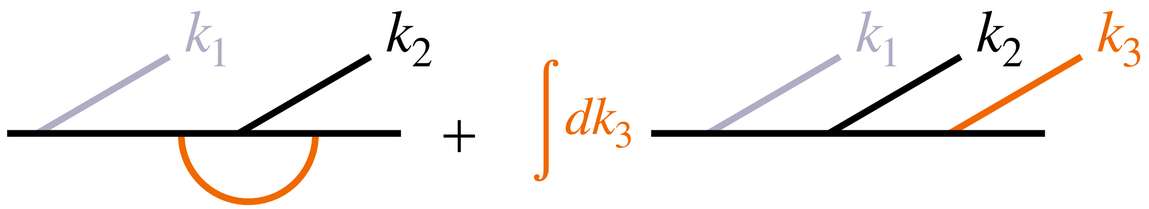
NLO correction to k_2 emission intensity sums loop correction and all possible scenarios for the next emission

etc.

Account for virtual corrections associated with each emission



NLO correction to k_1 emission intensity sums loop correction and all possible scenarios for the next emission

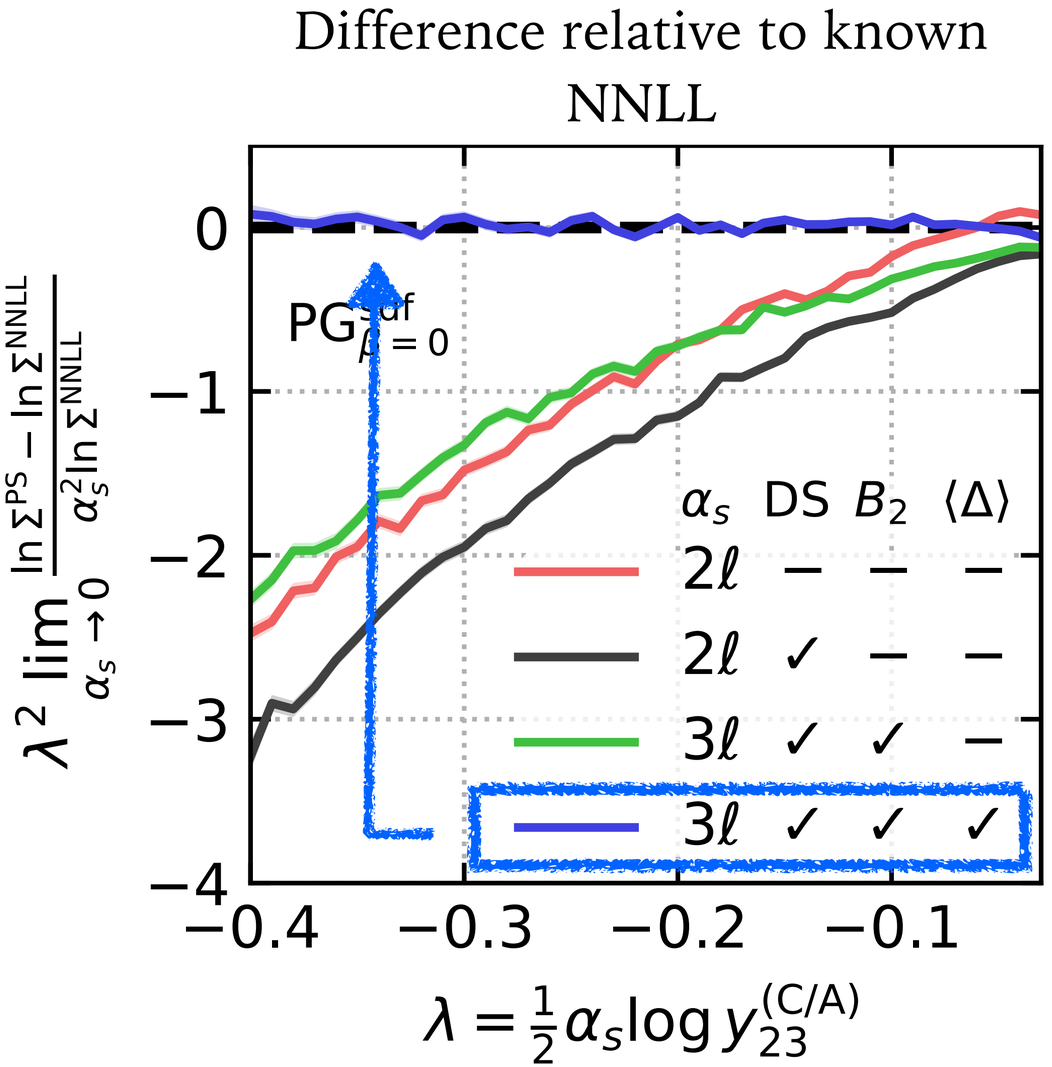


NLO correction to k_2 emission intensity sums loop correction and all possible scenarios for the next emission

etc.

Again relies on factorisation, e.g. when 1 and 2 are well separated in the Lund plane + careful nesting, cf. van Beekveld, Dasgupta, El-Menoufi, Helliwell, Monni, GPS 2409.08316 (see also Hartgring, Laenen & Skands, 1303.4974, Campbell et al 2108.07133 at fixed order)

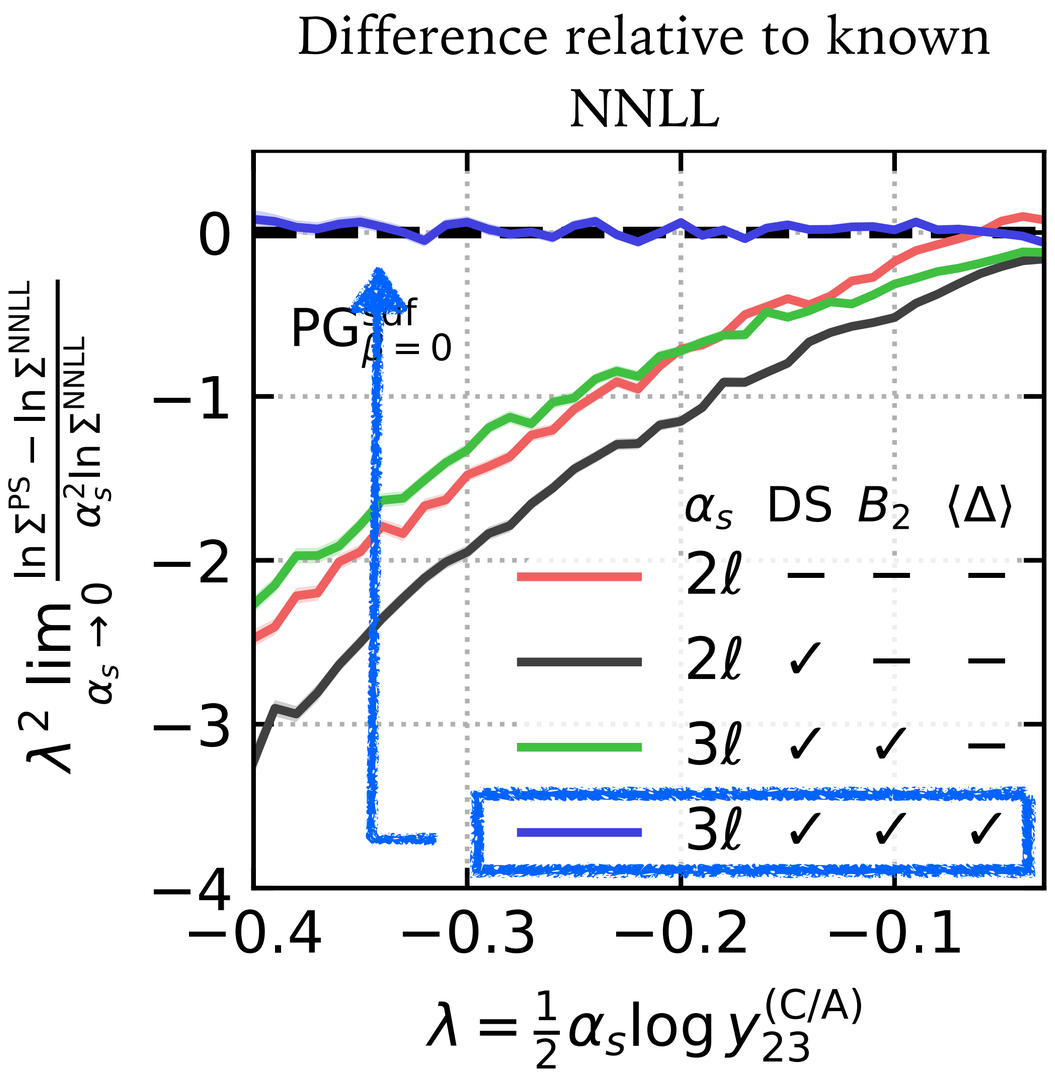
Testing NNLL for event shapes (so far only for e⁺e⁻ collisions)

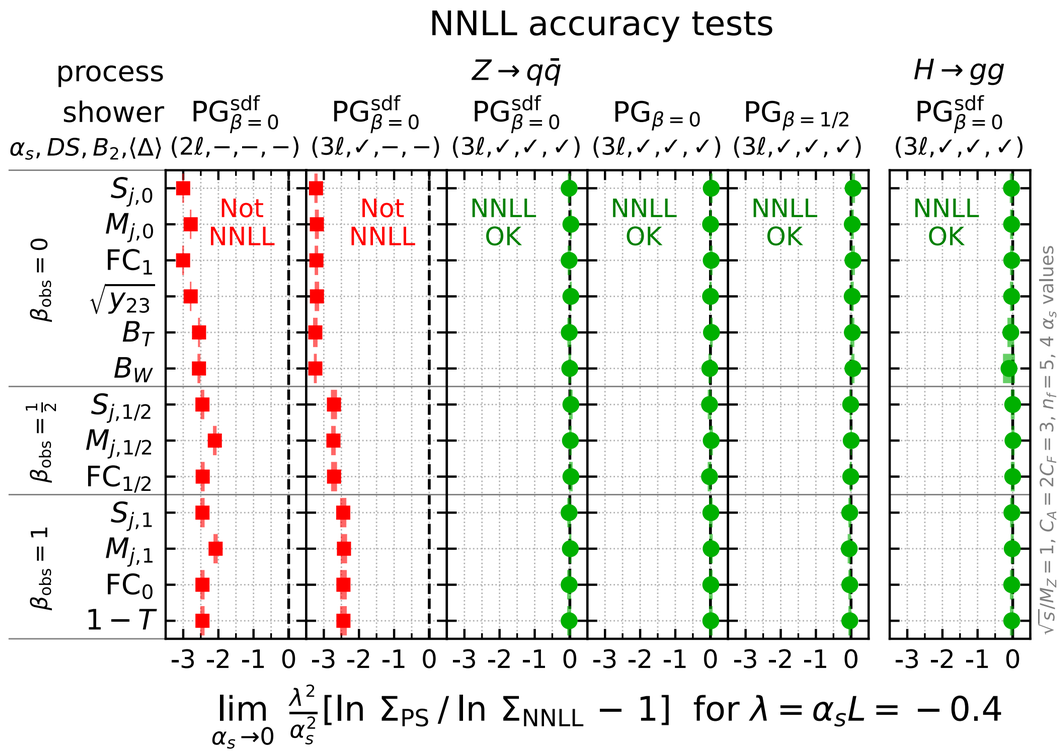


need to analyse and account for all possible sources of NNLL contribution

(some, which don't affect event shapes, are still work in progress)

Testing NNLL for event shapes (so far only for e⁺e⁻ collisions)

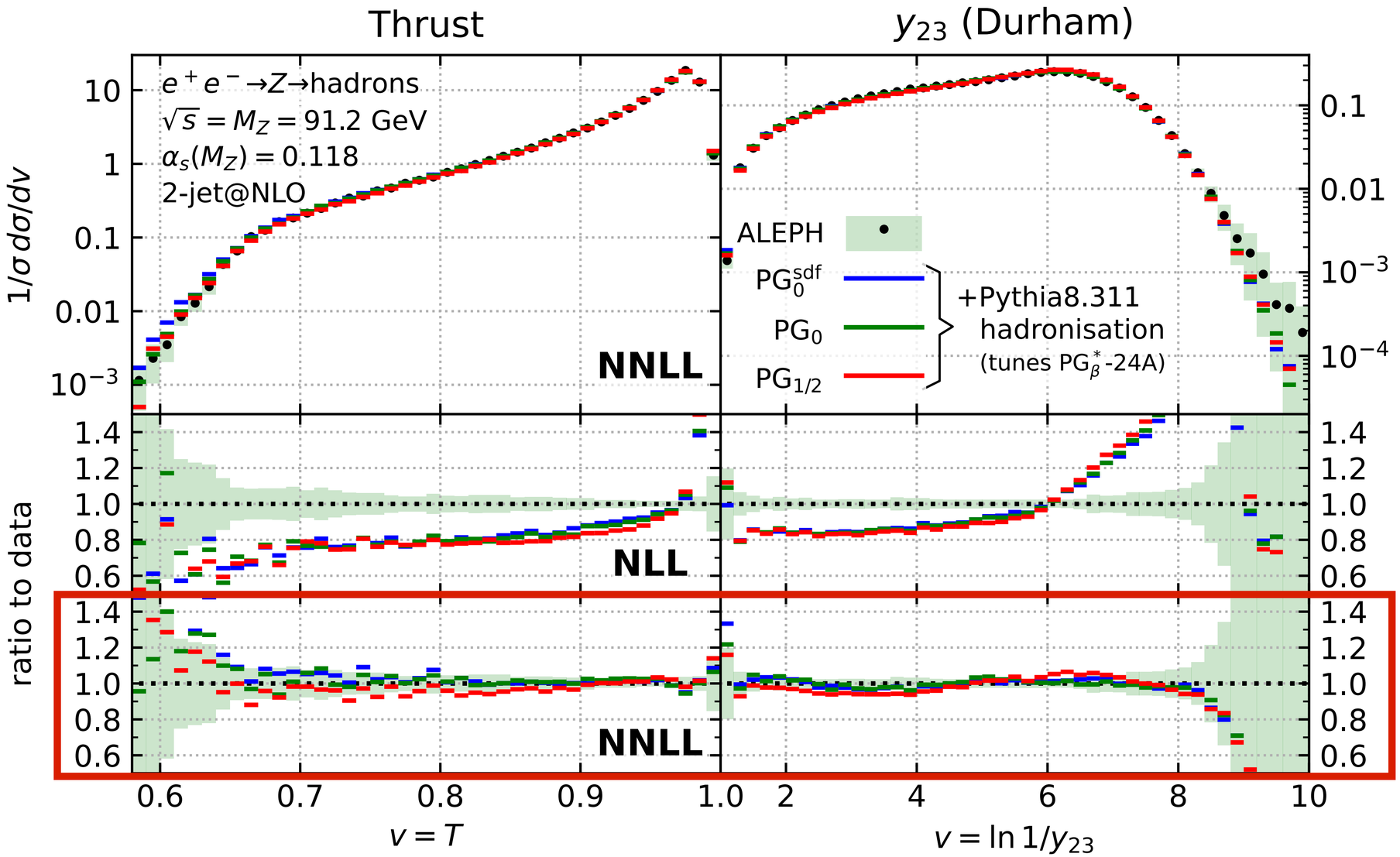




need to analyse and account for all possible sources of NNLL contribution

(some, which don't affect event shapes, are still work in progress)

Comparing to LEP event-shape data



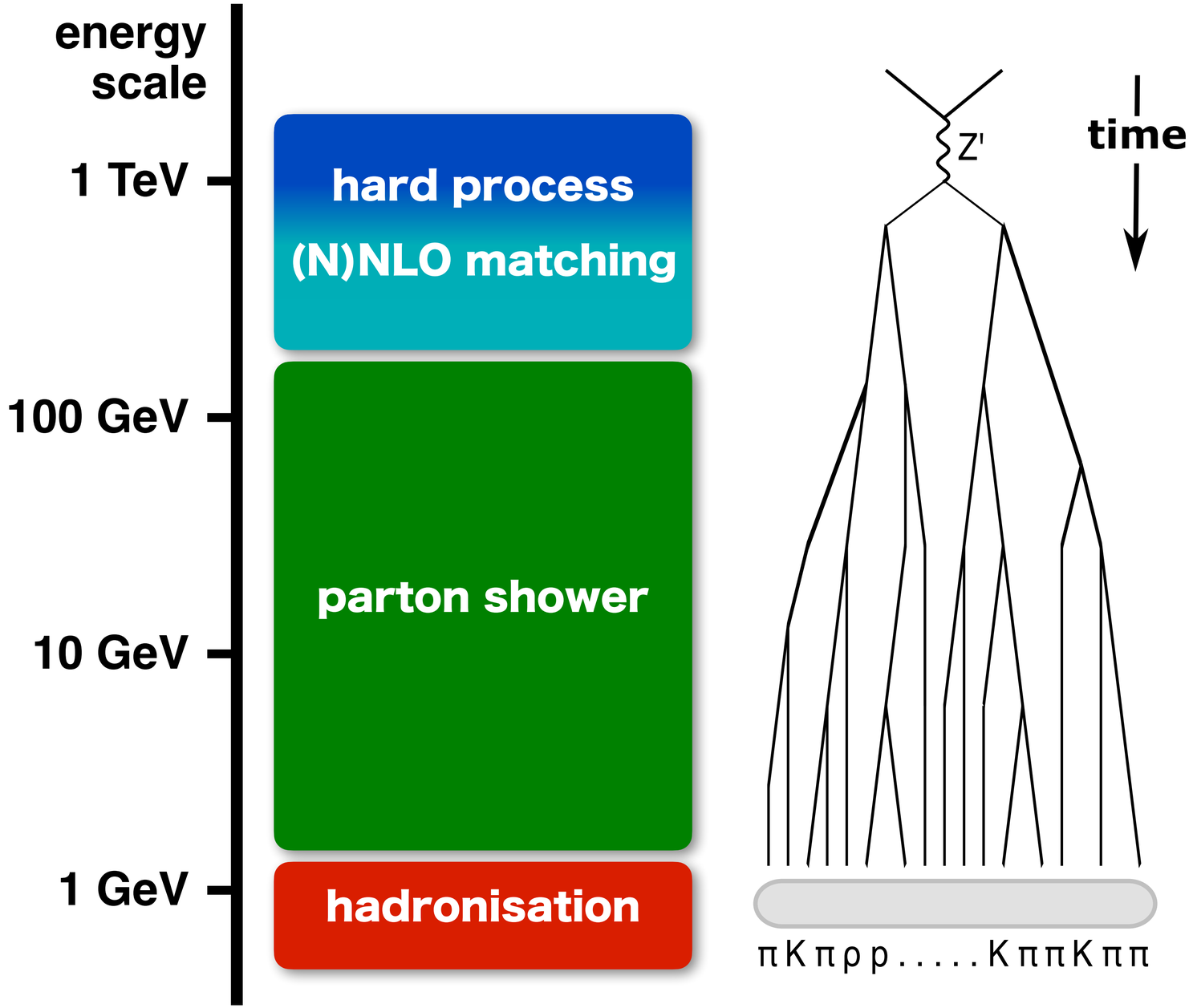
NNLL brings 20% effects ($\sim \alpha_s$)

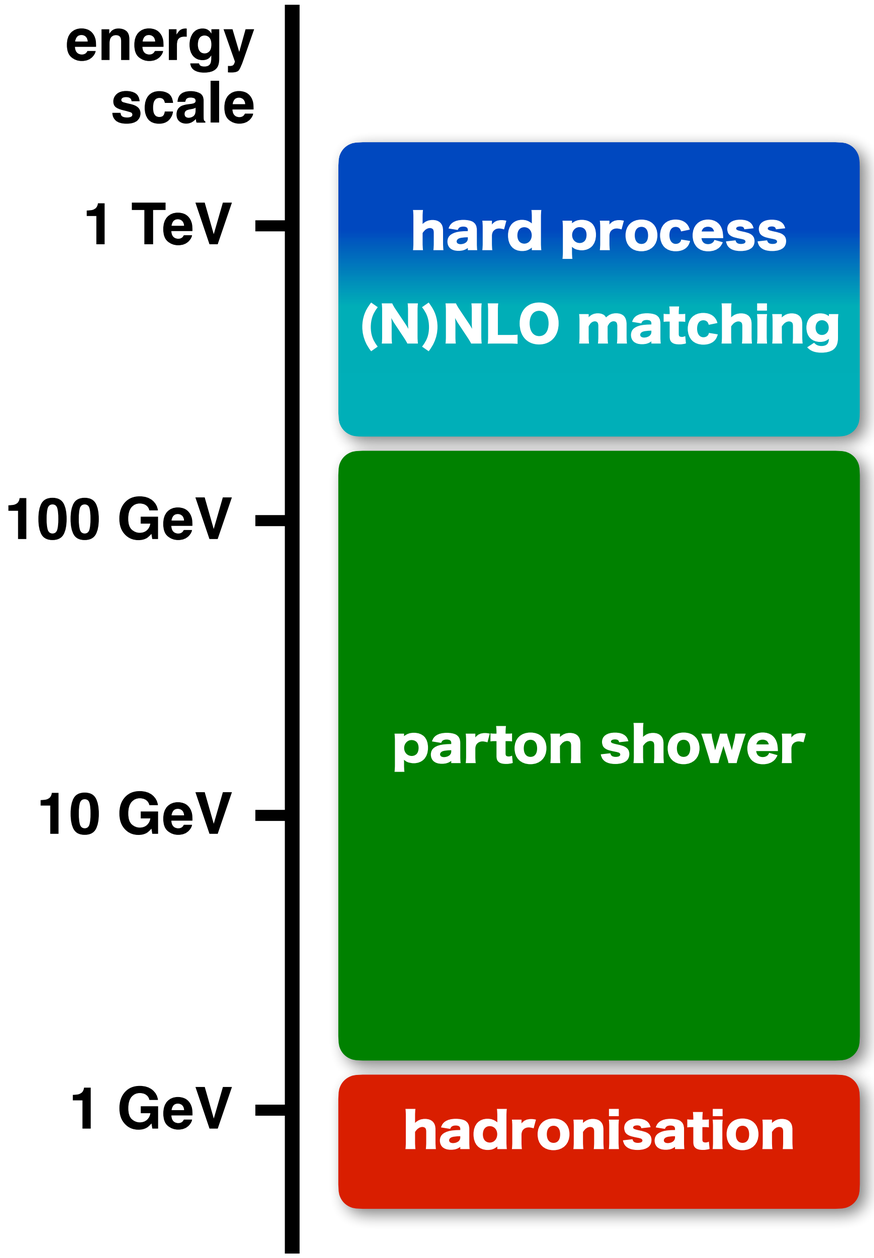
Dramatically improves agreement with data, using a "normal" $\alpha_s = 0.118$

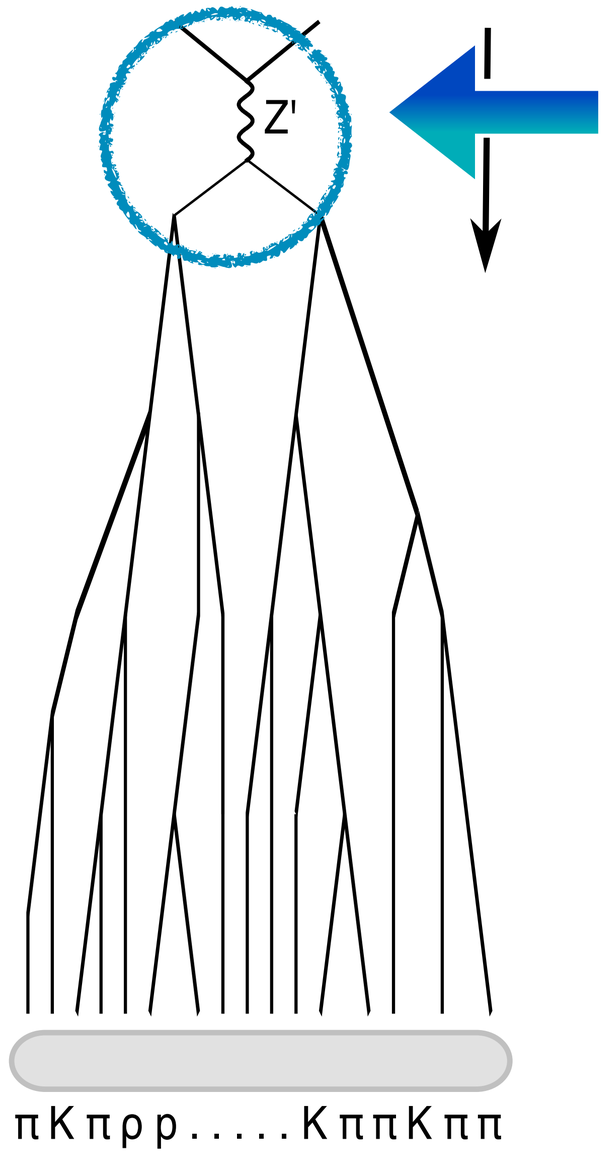
NB: 3-jet @ NLO still missing for robust pheno conclusions

← NNLL

Element #4: positive-definite NLO event normalisation


energy scale
1 TeV
100 GeV
10 GeV
1 GeV
hard process
(N)NLO matching
parton shower
hadronisation
Z'
time
π K π ρ p . . . . . K π π K π π





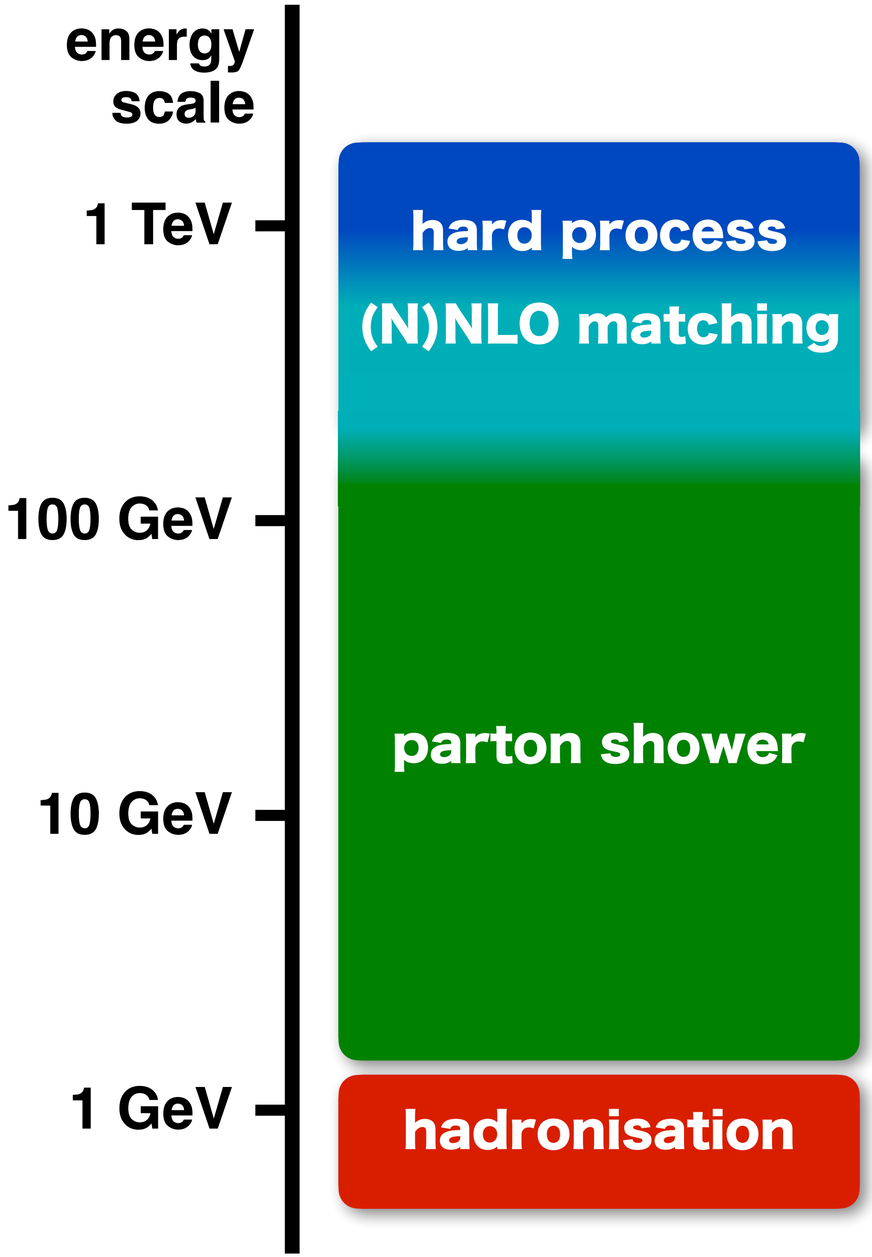
Key innovation of 2002–'04:
correct or replace
first step so that perturbative expansion of hard process + parton-shower is equivalent to the true NLO.

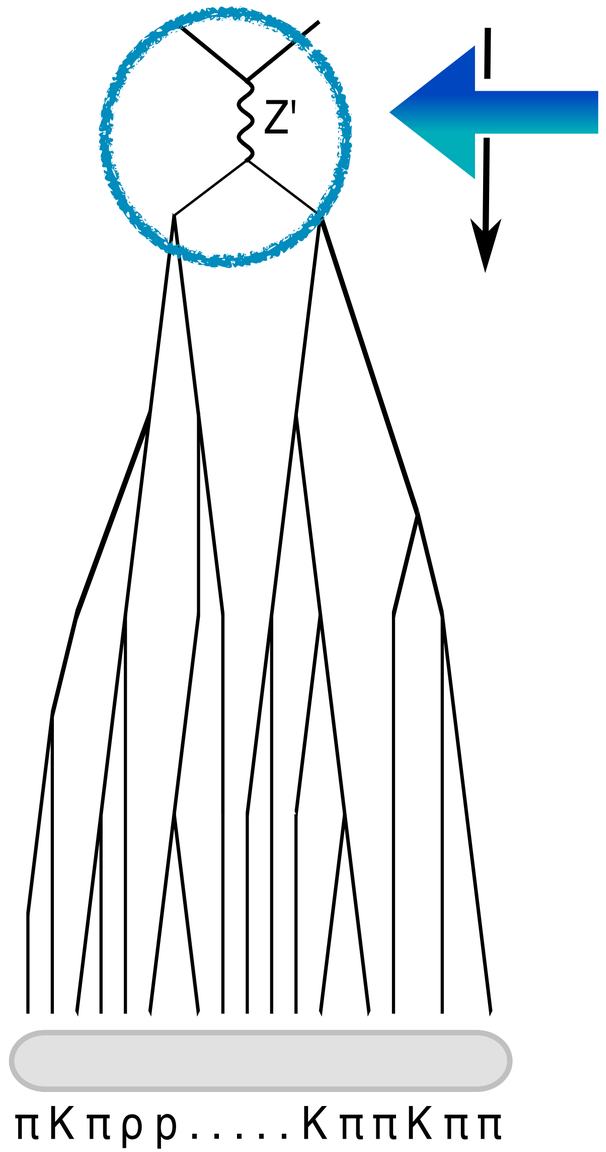
Frixione & Webber: MC@NLO
hep-ph/0204244

Nason: POWHEG
hep-ph/0409146

[>7500 citations; these methods used also in Sherpa, Herwig]

log-accuracy impact of matching/shower interface:
Hamilton, Karlberg, GPS, Scyboz, Verheyen, 2301.09645





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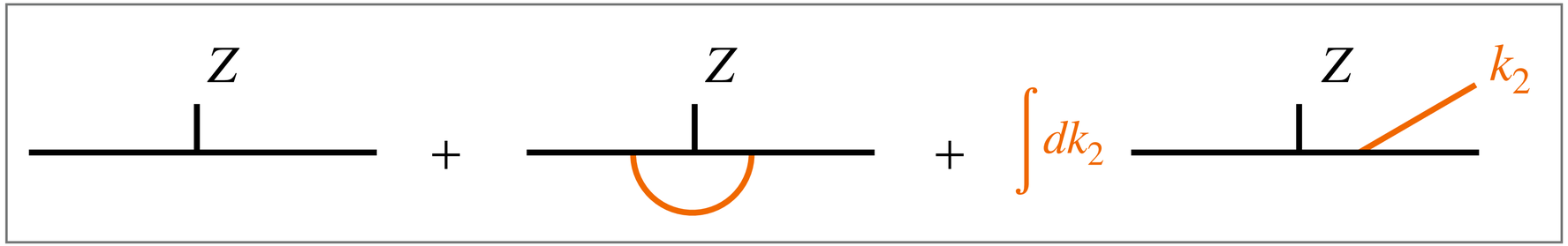
Nason: POWHEG
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[>7500 citations; these methods used also in Sherpa, Herwig]

Key features of MC@NLO and POWHEG events

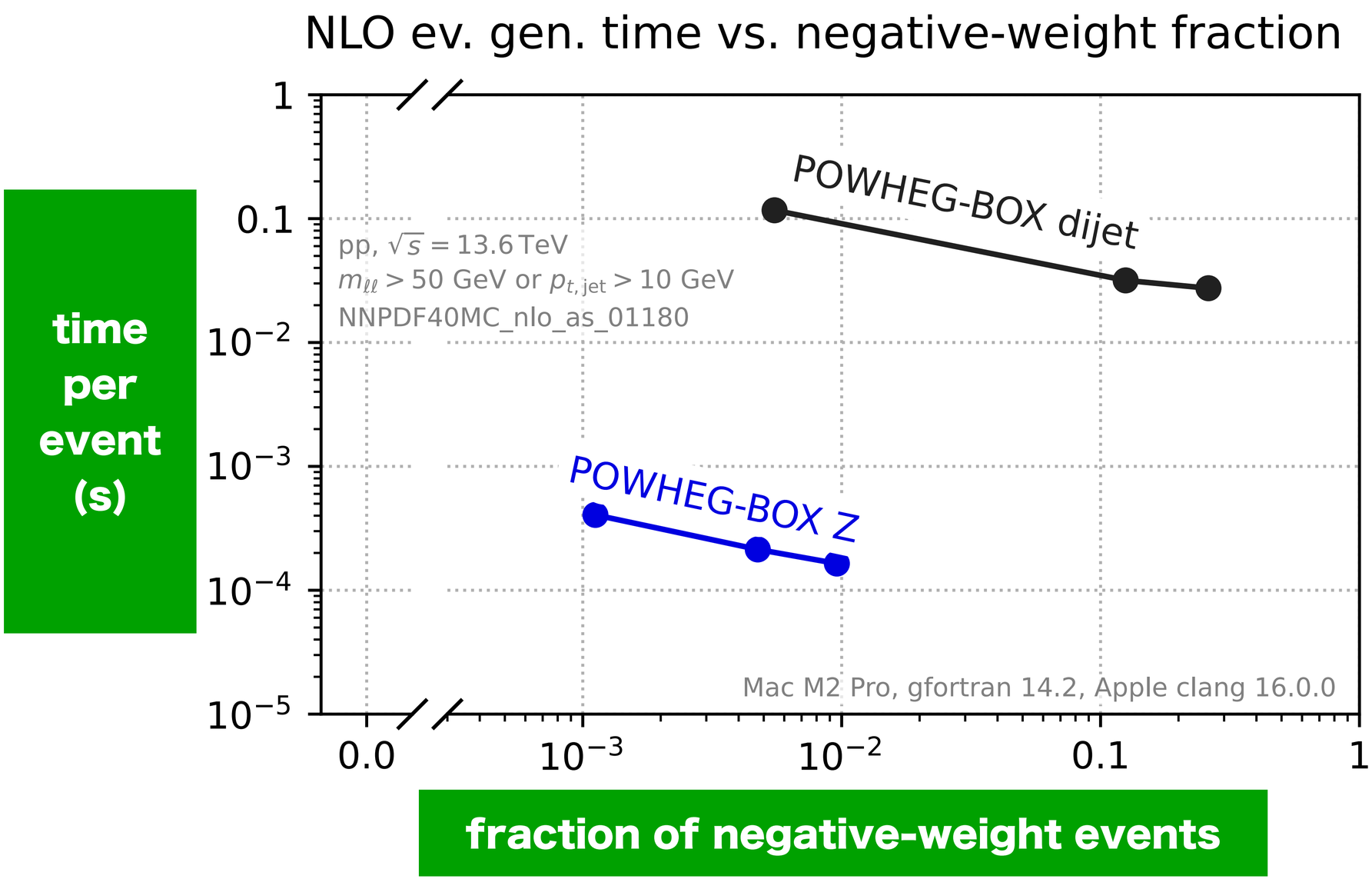
- MC@NLO and POWHEG methods, supplemented with parton showers + hadronisation models, provide NLO-accurate realistic hadron-level events
- they avoid the problem of (near) divergent event weights
- instead the event weights are just ± 1: big advantage over "pure" NLO

But the event sample isn't quite like a true physical event sample, because there are still some negative weights



Born event ($q\bar{q} \to Z$) generated with NLO cross section, estimated by 1-point Monte Carlo evaluation of real integral

NLO state of the art [POWHEG]



More complex processes tend to have higher fractions of negative weights.

Mitigation options (e.g. folding, Nason 0709.2085) often trade off negative weight fraction v. generation time.

Problem usually worse for NNLO event generation

Some LHC experiments' statements on negative weights and machine learning

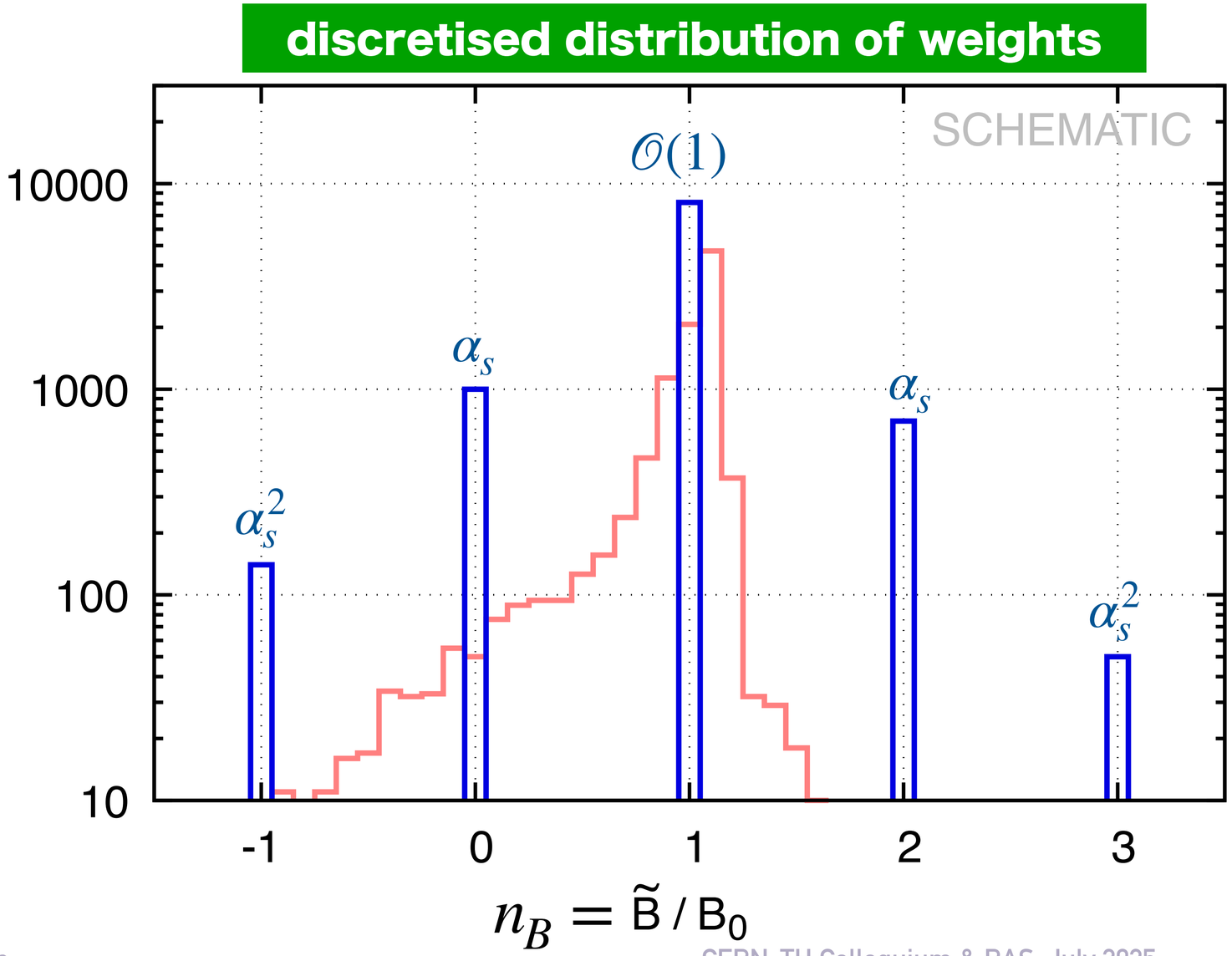
- ATLAS 2211.01136: "To avoid the use of negative weights present in the nominal NLO sample in the training of the multivariate discriminant used to separate SM $t\bar{t}t\bar{t}$ events from background […], **a sample was produced with similar generator settings, but at LO.**"

- CMS 2411.03023: "However, the binary cross-entropy given by Eq. (2), can become negatively unbounded for negative event weights, **making the classification task potentially impossible**"

- ATLAS 2412.15123: "Since XGBoost [ML framework] cannot handle negative-weight events, **the absolute value of each event weight is used.**"

Robust positivity with NLO accuracy

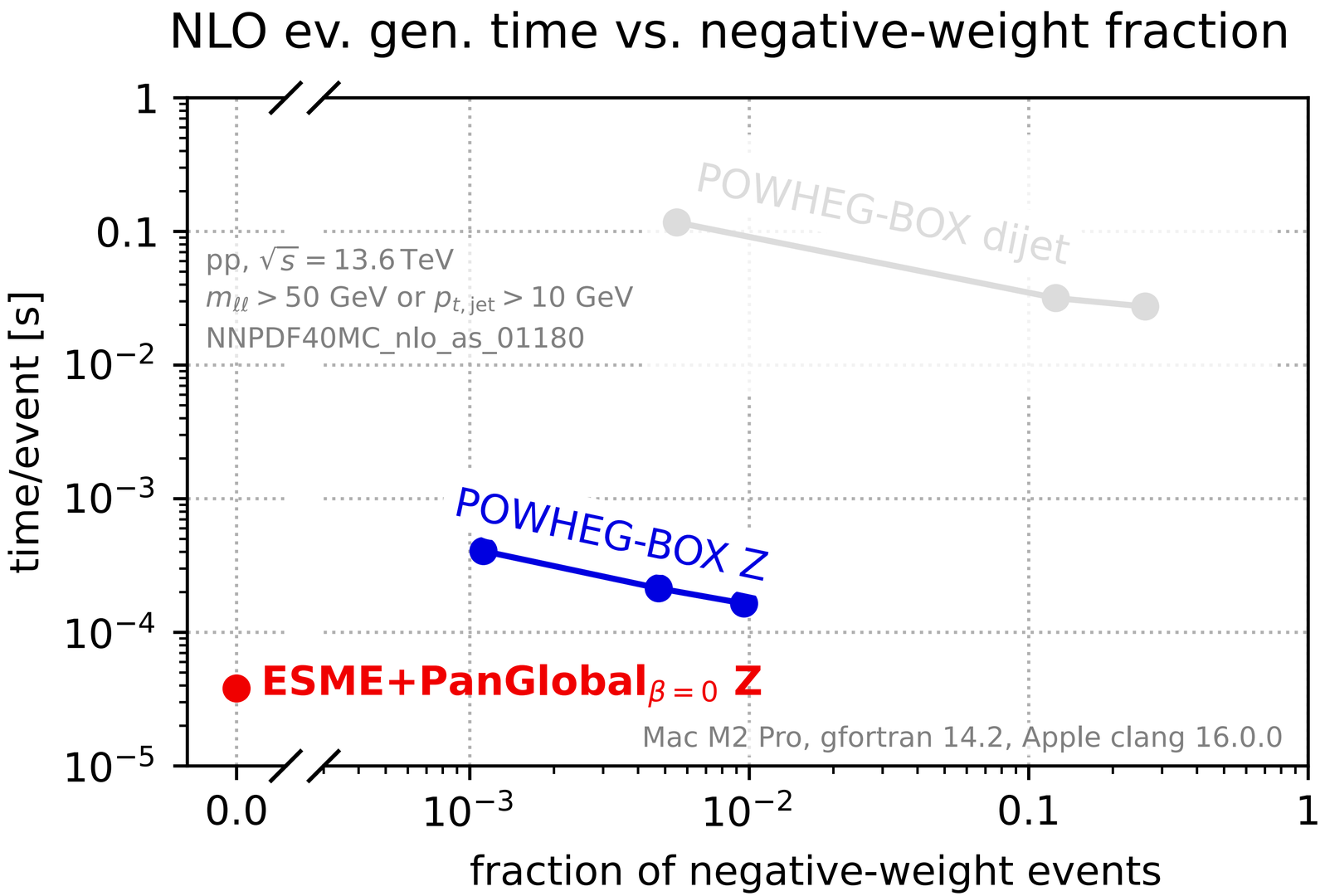
van Beekveld, Ferrario Ravasio, Helliwell, Karlberg, GPS, Scyboz, Soto-Ontoso, Soyez, Zanoli, 2504.05377



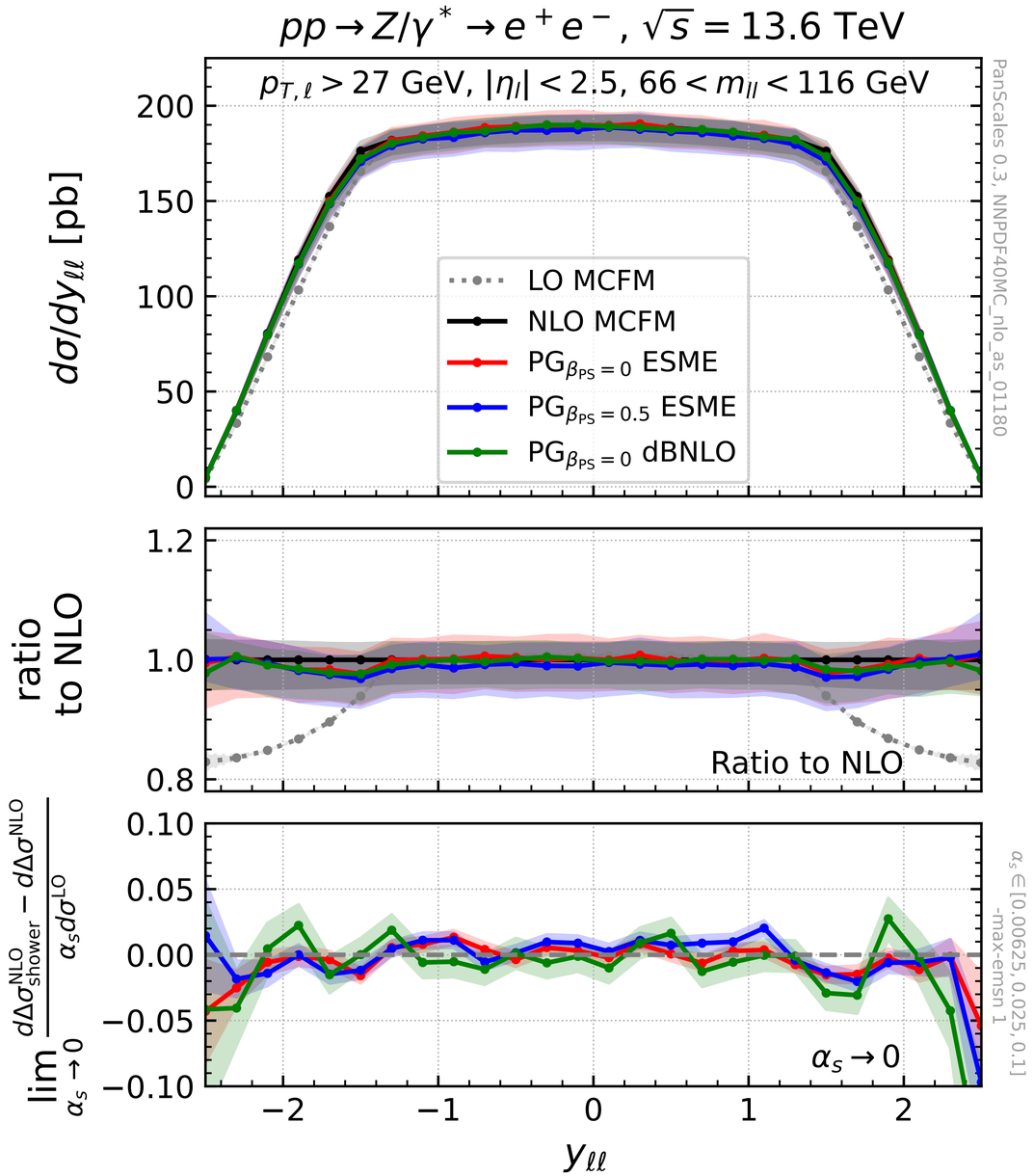
Now if you discard negative-weight events you have a guarantee that you only change the result at NNLO or beyond

Because each decrement of n_B costs a power of α_s

[discretisation explored also in Lavesson, Lönnblad, 0811.2912]


NLO ev. gen. time vs. negative-weight fraction
POWHEG-BOX dijet
pp, $\sqrt{s} = 13.6\,\text{TeV}$
$m_{\ell\ell} > 50$ GeV or $p_{t,\text{jet}} > 10$ GeV
NNPDF40MC_nlo_as_01180
POWHEG-BOX Z
ESME+PanGlobal$_{\beta=0}$ Z
Mac M2 Pro, gfortran 14.2, Apple clang 16.0.0
time/event [s]
fraction of negative-weight events
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.0
10^{-3}
10^{-2}
0.1
1

Does ESME give the correct answer? → Yes

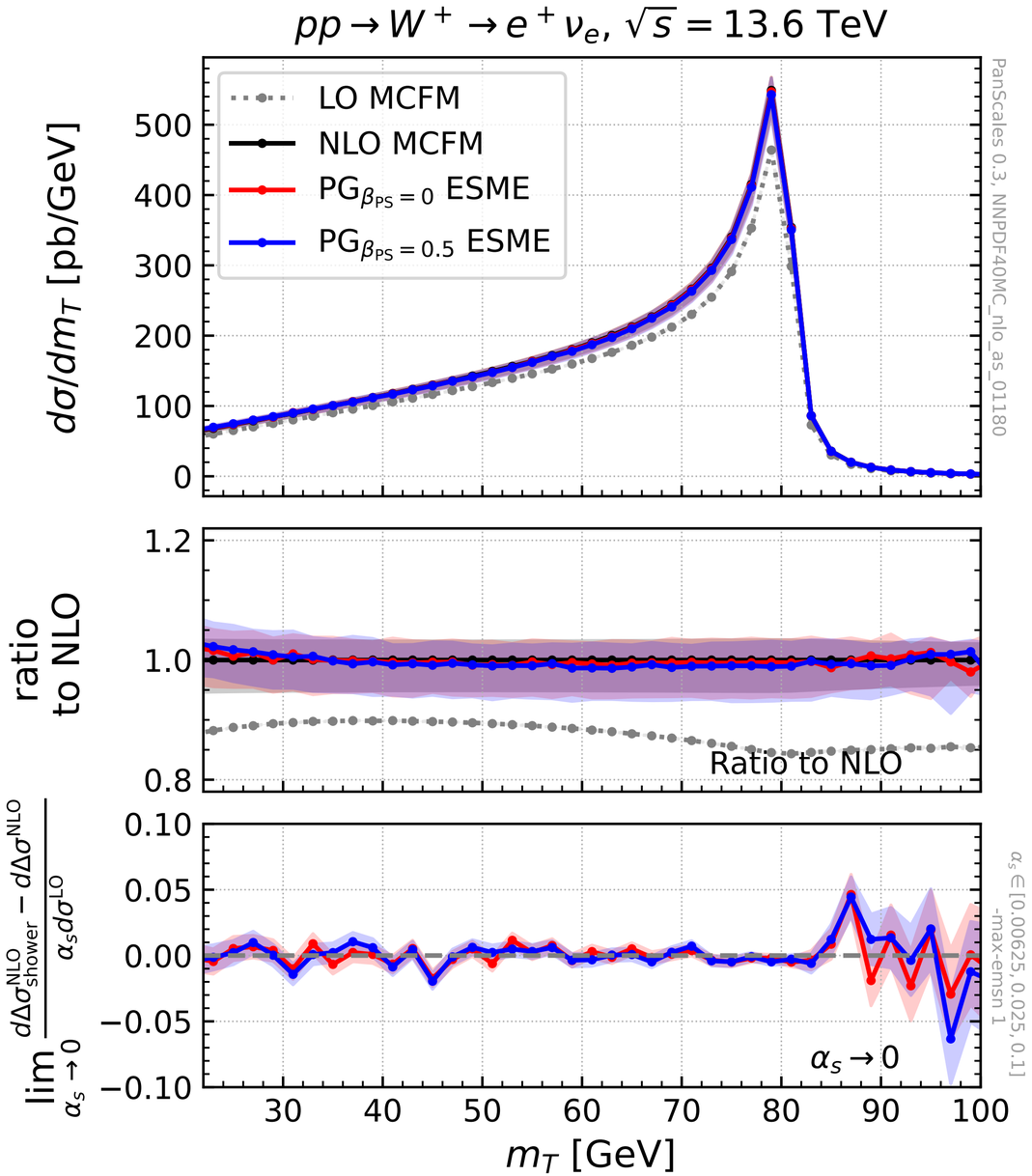


Comparison to exact NLO
(and to more standard method, here labelled dBNLO)

Ratio to exact NLO

Check of NLO coefficient

Does ESME give the correct answer? → Yes



Comparison to exact NLO
(and to more standard method, here labelled dBNLO)

Ratio to exact NLO

Check of NLO coefficient

Conclusions

Outlook

Physical predictions for general observables require all-order QCD – parton showers provide that with full flexibility.

NLL-accurate showers provide a baseline for a systematic expansion, so that whatever part of the event you look at, higher terms in the expansion will improve the prediction.

First indications are that full NNLL is essential for precision collider phenomenology.

Open practical questions:

- NNLL for e^+e^-: including fully differential $1\to 3$ & 1-loop $1\to 2$ collinear splitting
- NNLL with initial-state hadrons
- log-accurate treatment of quark masses
- generalising positive-definite NLO to wider range of processes and to NNLO and beyond
- some aspects of sub-leading colour

Open conceptual question: formalising this for any logarithmic / fixed order.

PanScales code is available publicly: https://gitlab.com/panscales/panscales-0.X

Backup

NLO Born normalisation, inclusive over subsequent branching

Born +NLO norm. = *Born* + *1-loop virtual* + *real*

(LO + NLO) *(LO)* *(NLO)* *(NLO)*

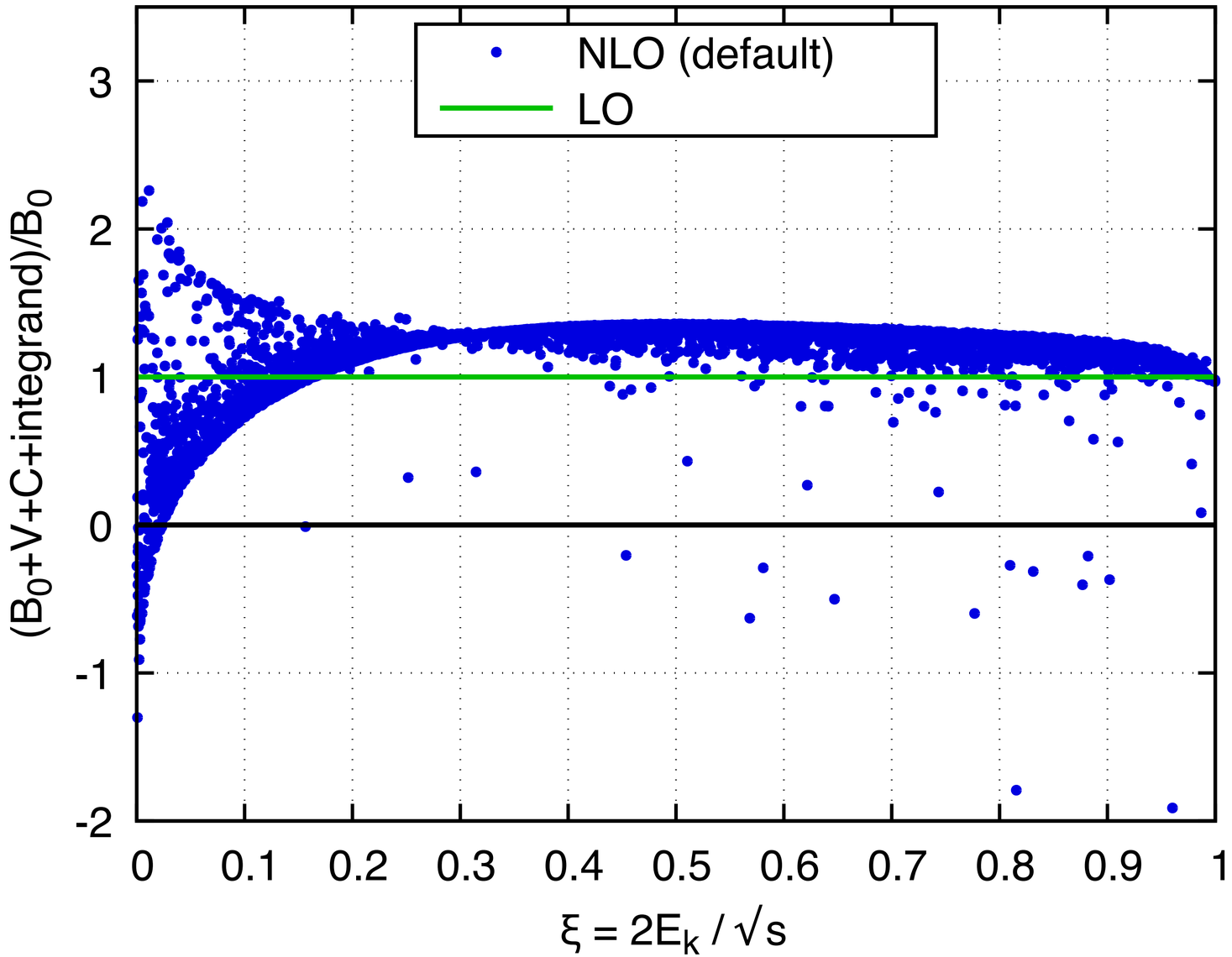
$$\bar{B}(\Phi_{\mathrm{B}}) = B_0(\Phi_{\mathrm{B}}) + \underbrace{V(\Phi_{\mathrm{B}}) + \int R(\Phi_{\mathrm{B}}, \Phi_{\mathrm{rad}})\, d\Phi_{\mathrm{rad}}}_{\text{relative order } \alpha_s},$$

Made explicitly finite with counterterms

counterterm (integrated) *counterterm (differential)*

$$\bar{B}(\Phi_{\mathrm{B}}) = B_0(\Phi_{\mathrm{B}}) + \underbrace{V(\Phi_{\mathrm{B}}) + C_{\mathrm{int}}(\Phi_{\mathrm{B}}) + \int \left[R(\Phi) - C(\Phi)\right] d\Phi_{\mathrm{rad}}}_{\text{relative order } \alpha_s},$$

$$\bar{B}(\Phi_{\rm B}) = B_0(\Phi_{\rm B}) + V(\Phi_{\rm B}) + C_{\rm int}(\Phi_{\rm B}) + \int [R(\Phi) - C(\Phi)]\, d\Phi_{\rm rad},$$



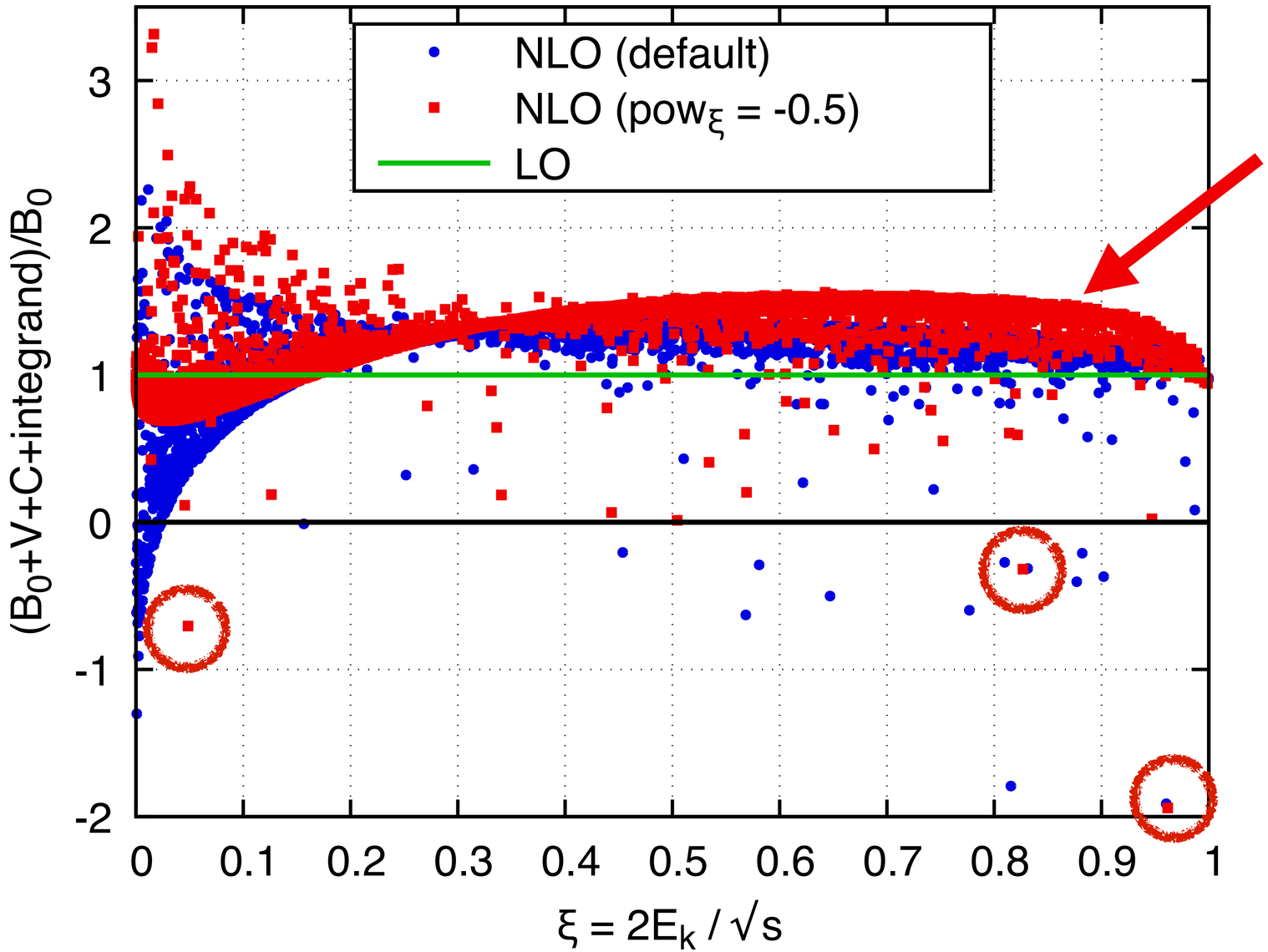
each point ≡ one Born phase space choice and one radiation phase space choice

Plotted as a function of one of the 3 real radiation phase space variables

Result should be $1 + \mathcal{O}(\alpha_s)$

But sometimes coefficient in front of α_s is large and result is negative

$$\bar{B}(\Phi_{\mathrm{B}}) = B_0(\Phi_{\mathrm{B}}) + V(\Phi_{\mathrm{B}}) + C_{\mathrm{int}}(\Phi_{\mathrm{B}}) + \int [R(\Phi) - C(\Phi)]\, d\Phi_{\mathrm{rad}},$$

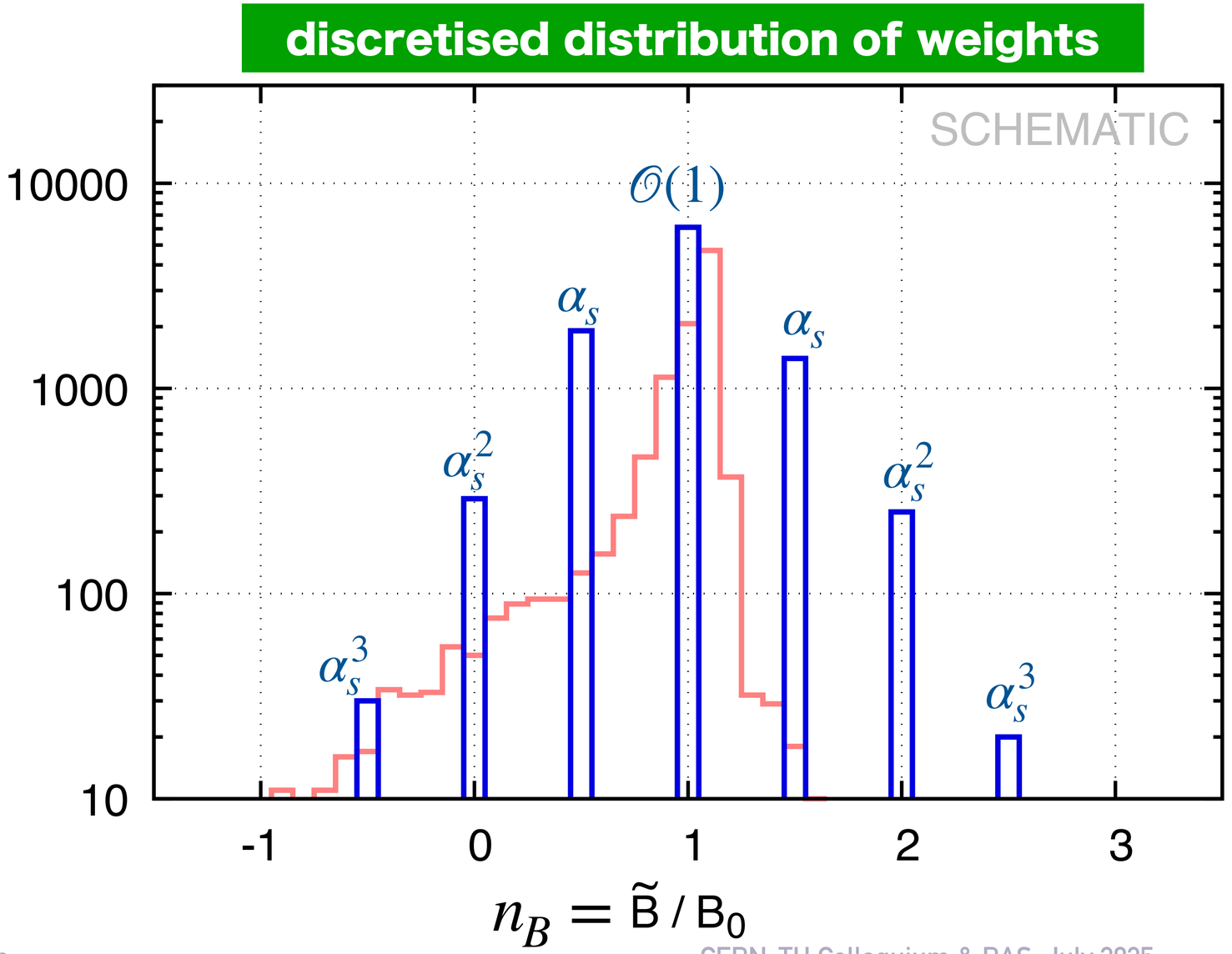


reweighting of integration variables can help*

But still some negative-weight events and reweighting not always easy or successful

* recent proposals can be viewed as doing something similar: "Born spreading" [Frederix & Torrielli, 2310.04160] and "ARCANE" [Shyamsunda, 2502.08052, 2502.08053]

Robust positivity with NLO accuracy (spurious terms from N3LO)



Multiply M, R, C in the algorithm by factor p (here $p = 2$)

Increment n_B by $\pm\frac{1}{p}$

Algorithm gives exactly the same $\langle n_B \rangle$

Keeping only positive-weight events changes integral by just α_s^{p+1}

Algorithm 1 General algorithm to convert NLO subtraction integral to integer

Set $n_{\mathrm{B}} = 1$ and $v = v_{\max}$
while $v > v_{\min}$ **do**
    generate next v and Φ_2 according to Sudakov with density $\rho(v) d\ln v$, Eq. (3.6)
    generate random number $0 < r < 1$
    if $r < |R(\Phi) - C(\Phi)|/M(\Phi)$ **then** [$M(\Phi)$ is an overestimate for $\max(R, C)$]
        if $R(\Phi) > C(\Phi)$: $n_{\mathrm{B}} \to n_{\mathrm{B}} + 1$
        else: $n_{\mathrm{B}} \to n_{\mathrm{B}} - 1$
return n_{B}

n_B then multiplies Born matrix element $B_0(\Phi_B)$ — this is exact

Using instead $\max(n_B,0)$ removes negative weights, but remains NLO accurate

Can easily be generalised to give integral up to any N^{p}LO accuracy